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## UMI

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# Improved geodetic control by GPS for establishing a Geographic Information System for Sri Lanka 

by
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A dissertation submitted to the graduate faculty in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY

Major: Civil Engineering (Geometronics)
Major Professor: K. Jeyapalan

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## TABLE OF CONTENTS

1 INTRODUCTION ..... 1
1.1 Research objectives ..... 3
1.2 Present available accuracy of the network ..... 4
1.3 GPS data collection procedure for a new adjustment ..... 5
1.4 Transformation parameters ..... 5
1.5 Spatial data for the cadastral map layer ..... 6
1.6 Improving digitized coordinates by a sequential adjustment ..... 7
2 CONCEPTS OF A GIS AND THE SRI LANKAN GIS ..... 9
2.1 Components of a GIS data base ..... 9
2.2 The Sri Lankan GIS ..... 10
3 EXISTING GEODETIC CONTROL OF SRI LANKA ..... 12
3.1 History ..... 12
3.2 Need for a re-adjustment ..... 13
3.3 Means of evaluating weaknesses of the network ..... 14
3.3.1 Old observations ..... 14
3.3.2 Comparison of distances ..... 16
3.3.3 Evaluation of the old adjustment methodology ..... 18
3.3.4 Effect of the Geoid and the Ellipsoid ..... 20
3.4 The present accuracy of the network ..... 21
4 PRINCIPLES AND APPLICATIONS OF GPS ..... 24
4.1 Components of GPS ..... 25
4.2 Orbital motion of a satellite ..... 27
4.3 GPS satellite transmissions ..... 29
4.4 Positioning methods ..... 29
4.4.1 Stand along mode ..... 30
4.4.2 Relative mode (differential GPS) ..... 31
4.4.3 Kinematic and static modes ..... 31
4.5 Computations in differential GPS ..... 33
4.6 Errors effecting GPS positioning ..... 36
4.7 Specifications of differential GPS ..... 38
4.8 Getting GPS data in to a GIS ..... 39
5 A NEW GEODETIC CONTROL SYSTEM ..... 41
5.1 Reconnaissance ..... 41
5.1.1 Field measurements used for old networks ..... 43
5.2 Geodetic control and its users ..... 44
5.3 GPS observations for new adjustment ..... 44
5.3.1 Expected and predicted accuracy of the network ..... 45
5.3.2 Simulated data ..... 47
5.3.3 Fixed stations for the adjustment ..... 49
5.3.4 Procedure for GPS observation to achieve the required positional accuracy of the network ..... 50
5.3.5 Pre-adjustment using simulated data ..... 52
5.3.6 Observation and processing of actual GPS data ..... 52
5.4 Gravity measurements ..... 57
5.5 Azimuth observations ..... 58
5.6 Mathematical surface for the geodetic network ..... 59
5.7 Everest vs. GRS80 for Sri Lanka ..... 61
5.7.1 Everest ellipsoid ..... 61
5.7.2 GRS80 ellipsoid ..... 62
5.8 Reduction of angle and distance observations ..... 63
5.9 The least squares adjustment ..... 64
5.10 Approximate values of parameters for the new adjustment ..... 65
5.11 Weights and priori variance ..... 66
5.12 Observation equations ..... 67
5.12.1 Directions ..... 67
5.12.2 Distances ..... 68
5.13 Solution for parameters ..... 68
5.14 Evaluation of the adjustment ..... 71
5.14.1 Üsing GPS observations for evaluation ..... 71
5.14.2 Variance-covariance matrices for parameters and adjusted obser- vations ..... 72
6 TRANSFORMATION PARAMETERS FOR THE NEW ADJUST- MENT ..... 74
6.1 Mathematical models for coordinate transformations ..... 76
6.2 Methods used for the coordinate transformation from NAD27 to NAD83 ..... 78
6.3 Accuracy of coordinate transformation of Sri Lanka ..... 80
6.4 Two dimensional coordinates ..... 81
6.4.1 Projection Surfaces and their classifications ..... 84
6.4.2 Map projection for Sri Lanka ..... 86
6.4.3 Normal Cylindrical Projections ..... 87
6.4.4 Transverse Mercator Projection ..... 89
7 GETTING SPATIAL DATA INTO THE GIS ..... 92
7.1 Data layers for cadastral and engineering applications ..... 93
7.1.1 Scanning large scale cadastral maps ..... 94
7.1.2 Major problems faced in scanning and vectorization of cadastral maps ..... 95
7.1.3 Improving digitized coordinates using a least squares sequential adjustment ..... 96
7.2 Spatial data sources for data analysis in the Sri Lankan GIS ..... 98
7.2.1 1: 500,000 scale road map ..... 100
7.2.2 1: 100,000 scale Land use map ..... 101
7.2.3 1:50,000 Topographic map ..... 102
7.2.4 $1: 10,000$ Topographic map ..... 103
8 CONCLUSION ..... 104
8.1 New geodetic network ..... 105
8.1.1 GPS data for the geodetic network ..... 105
8.1.2 Reference ellipsoid for Sri Lanka ..... 106
8.2 Coordinate transformation ..... 107
8.3 Spatial data for cadastral and engineering applications ..... 107
8.3.1 Updating coordinates by a sequential adjustment ..... 108
8.4 Spatial data from small scale maps ..... 108
8.5 Summary of recommendations ..... 109
APPENDIX A EXISTING GEODETIC CONTROL ..... 111
APPENDIX B PROPOSED LAND INFORMATION NETWORK FOR SRI LANKA ..... 119
APPENDIX C RESULTS OBTAINED FOR A NEW ADJUSTMENT USING OLD OBSERVATIONS ..... 120
APPENDIX D "Con_cord" MAIN MENU ..... 128
APPENDIX E INPUT FILE FOR A GEOLAB ADJUSTMENT WITH NEW GPS OBSERVATIONS ..... 129
APPENDIX F A PART OF THE OUTPUT FILE FOR AN ADJUST- MENT WITH NEW GPS OBSERVATIONS ..... 142
APPENDIX G RESULIS OF SCANNING AND VECTORIZATION OF CADASTRAL MAPS ..... 158
APPENDIX H PART OF A SRI LANKAN CADASTRAL MAP ..... 166
APPENDIX I GEO-REFERENCING ERRORS DUE TO MOUN- TAIN PEAKS IN 1 : 500,000 MAP ..... 167
APPENDIX J SUBROUTINES USED FOR "Concord" AND THE SEQUENTIAL LEAST SQUARES ADJUSTMENT ..... 168
REFERENCES ..... 182

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## 1 INTRODUCTION

Sri Lanka is in the process of establishing a Geographic Information System (GIS). A GIS is a combination of spatial data which provides the relative location of points, lines or areas on the earth surface and attribute data related to these individual points, lines and polygons (Burrough, 1986).

Many base map layers, such as land use, elevation, soil type, transportation and cadastral layer showing property boundaries should be included in the GIS. Of these many layers, the cadastral map layer requires the highest level of spatial accuracy in the GIS because it is expected to resolve land registration, land transaction and boundary disputes in the country (Berugoda, 1987).

Currently, data for the base map layers are obtained from hard copy maps which are drawn to different scales. In Sri Lanka, a number of important data layers can be obtained from the $1: 10,000$ or $1: 50,000$ map series. Unfortunately, these series either do not cover the entire country or are not current. Therefore, many data layers have to be created using other map series; the $1: 100,000$ series can provide data for the land use layer, the $1: 500,000$ series for the transportation layer and the large scale series such as 1:1000 or 1:2000 for the cadastral layer.

When many different types of spatial data in the GIS are obtained from hard copy maps of different scales, a reliable spatial linkage mechanism is required for overlapping these digital layers. If the linkage mechanism is not reliable, the overlapping is not perfect and, as a result, the GIS will give inaccurate information at the data analysis stage.

Grid or projection coordinates obtained from hard copy maps are used in GIS as the linkage mechanism. In Sri Lanka a computerized GIS, even without property boundary information, is not available today. All the cadastral information requirements are obtained from hard copy maps (Manual GIS as opposed to computerized GIS). For cadastral layers in a GIS, this type of spatial linkage does not provide adequate coordinate control because the property boundaries, right of ways, utility lines etc, have to be determined within a tolerance of few centimeters. The solution to this problem is to develop an accurate geodetic control network covering the entire country, which can be used as the linkage mechanism of the GIS. The linkage mechanism of a GIS in a country is the geodetic control network (Committee on Geodesy, 1980). In "Arc/Info", a GIS software used in this study, this linkage mechanism is referred to as "tic file" in the master control coverage.

To develop an accurate linkage mechanism for the GIS, the geodetic control of Sri Lanka must be accurate for all data layers. The spatial accuracy needed in the GIS is dependent on the type of data layers that are required. Since the relative coordinates of property boundaries should be determined within an accuracy of few millimeters, the geodetic control must provide these with relative accuracy together with 5 cm absolute accuracy for cadastral surveying requirements of the GIS. Figure 1.1 shows how a geodetic control network functions in a GIS.

Today, using modern surveying instruments and Global Positioning System (GPS) of satellites, a cadastral surveying can be done with centimeter level relative accuracy and 5 cm level absolute accuracy.

In order to provide the centimeter level accuracy in parcel boundary surveys for GIS applications in Sri Lanka, we need to establish control stations at 10 km apart, having an absolute accuracy of 5 cm and relative accuracy of $1: 1,000,000$. These control stations are the " C " order points in a modern GPS geodetic network.

The control network with " $C$ " order points can be established using a primary control


Figure 1.1 Geodetic control as the linkage mechanism of the GIS
network that has better relative and absolute accuracies. Such a network will have "A" order points within 1 cm absolute accuracy ( 100 km apart) and " B " order points with 2 cm absolute accuracy ( 25 km apart). Since this control network provides 1 part per million (ppm) relative accuracy in the " $B$ " and " $C$ " order points, it meets the specifications of the GPS network given by the Federal Geodetic Control Committee of the United States (FGCC, 1988).

### 1.1 Research objectives

The objectives of this research are to identify and describe the procedures of establishing a GIS for Sri Lanka and to find solutions for the ensuing problems. Specifically,

- it investigates the accuracy of the existing geodetic control network which was established in 1932 (Jackson and Price, 1933). In this stage of research we shall determine the suitability of the existing geodetic control as the linkage mechanism of the GIS.
- it develops the best procedure for establishing a new GPS geodetic network to
satisfy the needs of the comprehensive GIS, including an appropriate procedure for collection of GPS data which will provide the expected spatial accuracy after the adjustment of the network.
- it develops a suitable procedure for transformation between old and new coordinate system. It will transform latitudes and longitudes calculated on two different reference ellipsoids enabling us to combine the future with the past. This transformation methodology will also facilitate the incorporation of maps of the old plane coordinate system in the newly calculated linkage mechanism.
- it recommends a suitable cost effective, accurate and fast methodology to get cadastral level spatial information into the GIS from large scale cadastral maps and spatial information from small scale maps. A procedure for using scanners to produce raster data from hard copy maps and converting these raster data to vector is described here. These vector data are improved by a sequential least squares adjustment.


### 1.2 Present available accuracy of the network

The present geodetic control of the country was analyzed. It was found to provide only about $1: 30,000$ accuracy in the central regions of the country and the accuracy is even much weaker in the northern and north-eastern regions. Therefore, the existing geodetic network is not acceptable as the linkage mechanism of the GIS. It is also not suitable basis for the data collection for the cadastral map layer of the GIS because the desired accuracy is $1: 1,000,000$ for points which are 25 km apart (FGCC,1988). A detailed analysis of the present geodetic network is given in Chapter 3.

Due to the poor accuracy of the existing network, it is necessary to establish a new geodetic network before creating the GIS. The old geodetic network was established by a triangulation method. Today, the Global Positioning Systems (GPS) provide the
best technology for establishing highly accurate geodetic networks (Leick, 1995). A suitable GPS data collection procedure, which will provide the expected accuracy of the new geodetic network, has to be developed. The GPS data collection procedure and recommendations are discussed in section 4.3 and 5.1.

### 1.3 GPS data collection procedure for a new adjustment

Working with simulated data, and adjusting the network with the least squares adjustment software "Geolab" (Bitwise Ideas, Inc.), it was found that a relative accuracy better than 1 cm can be achieved for "A" order points, 100 km apart, if random errors can be limited to 1 mm and the GPS observation procedure designed to limit ppm errors to $10^{-8}$. These " $A$ " order points can be taken as fixed points for adjusting " $B$ " order points in the network. Details of this analysis are given in section 5.3.2.

For "B" order points, 25 km apart, the Survey Department of Sri Lanka proposes to take GPS observations, according to triangulation lines which were used in the old geodetic control network. Again, working with simulated data, it was found that only a 25 cm average absolute accuracy can be obtained with 5 mm random errors and $10^{-8}$ ppm errors with proposed procedure. This accuracy can be improved to 2 cm average relative accuracy by using the procedure explained in section 5.3.4.

### 1.4 Transformation parameters

Transformation parameters are necessary to transform coordinates between old and new systems in order to incorporate all the old maps into the new system. For this purpose, it is required to calculate a suitable set of transformation parameters for the country. Using "Geolab" and all currently available data, a new set of coordinates were obtained for the purpose of calculating transformation parameters. These parameters are expected to be comparable with parameters to be calculated after a complete adjustment
proposed in this research because the differences between new and old coordinates will be mainly due to the adjustment procedure used in the old adjustment, discussed in Chapter 3.

The calculated parameters show that a single set of transformation parameters for the entire country provides 1 m accuracy for coordinate transformations. When dealing with small scale maps, this transformation accuracy is considered to be satisfactory according to the spatial accuracy of small scale maps given in Table 7.2. Nine sets of parameters calculated at provincial levels provided an accuracy of 30 cm ; and 25 sets of parameters calculated at district levels provided 14 cm accuracy for coordinate transformations. Therefore, the district level parameters are suitable for coordinate transformation of large scale maps. Details of the calculation of transformation parameters are given in chapter 6. A software "Con_cord" was developed for the purpose of transforming many different types of coordinates to and from each other. The main menu of "Con_cord" is given in Appendix D, and subroutines developed for this purpose are given in Appendix J.

### 1.5 Spatial data for the cadastral map layer

Highly accurate spatial data are required for the cadastral map layer of the GIS, because it provides coordinates for most of the cadastral and engineering needs of the country. The best possible way of getting highly accurate data into the GIS is by direct entry of coordinates and measurements obtained by GPS or surveying (Byrene, 1991). As cost and time factors play a major role in establishing a GIS, it is not advisable to neglect already available cadastral and town survey maps in the country. The common method of converting these large scale maps to digital form is by manual digitizing, but this procedure is very costly, tedious and time consuming. Also, it can introduce more human errors to digitized coordinates. As the large scale cadastral and town survey
maps are prepared using only one color and one line-thickness, it may be possible to use scanning and vectorization techniques to convert these large scale maps into the digital form. Working with a simulated cadastral map in the scale of $1: 2000$ it was found that the manual digitizing provided an average accuracy of 26 cm each in X and $Y$ directions for coordinates of property boundaries. A scanning procedure which used 600 dpi and vectorization capability in "ArcInfo" provided 34 and 23 cm accuracy respectively in X and Y for coordinates of same points. These results show that the scanning technique may be used for converting cadastral maps into the digital form if other related difficulties in this can be overcome.

Two major difficulties were observed in scanning and vectorizing large scale maps. They are the creation of unnecessary nodes and the non-creation of some of the required nodes. Creation of unnecessary nodes (property corners) was about $65 \%$ for the used map. This additional $65 \%$ represented about $15 \%$ of nodes which are not created at the correct locations (see Appendix G). The "Cleaning" or "Building" procedure in "Arc/Info" could not be satisfactorily used to overcome these two problems. Instead, manual editing, which is not satisfactory for large projects, had to be used. However, an automated procedure must be developed in future research in order to remove unnecessary nodes and create any new nodes which are required but not created by scanning and vectorization. Details and statistics for the manual digitizing, scanning and vectorization done in this study are given in Chapter 7.

### 1.6 Improving digitized coordinates by a sequential adjustment

Coordinates obtained by manual digitizing can be improved by a least squares sequential adjustment (Tameem, 1994). This procedure can also be used to improve coordinates obtained from scanning and vectorization. A software was prepared to improve the coordinates obtained by scanning and vectorization, using linear, angular and area
conditions. The procedure is discussed in detail in Chapter 7.
Although GIS software moduies such as "CoGo" in ArcInfo can be used to input these linear. angular and area measurements in to a GIS system (ESRI. 1995). a sequential least square adjustment can be considered as a better procedure to upgrade property corners digitized from maps. This methodology will upgrade coordinates of property corners whenever and wherever additional measurements are available. The development of an automated procedure integrating GIS software with leasr squares sequential adjustment software facilitating the use of this methodology for large cadastral maps or large projects is recommended for future research study:

## 2 CONCEPTS OF A GIS AND THE SRI LANKAN GIS

The concept of GIS is not new to the world. Manual system of maps, fiscal data and census records have been in use for several centuries. The "Doomsday Book", a record of cadastral survey made under William the Conqueror in 1805 is a manual GIS (Committee on Geodesy, 1980). However, its capabilities are limited when compared to a modern computerized GIS. Only in the last two decades, with the advent of modern computer technology, has the GIS expanded its capabilities and possible applications.

A GIS is an important decision making tool for a developing country like Sri Lanka. Available resources, mainly earth-related resources, are limited; a small country must manage its resources efficiently. A GIS provides attractive long and short term benefits which more than offset the initial cost of establishing a GIS (Dale and McLaughlin, 1987).

The strength of the GIS depends on the data base. In order to provide accurate information for decision making, the data base must be well interpreted and spatially accurate.

### 2.1 Components of a GIS data base

The GIS data base is a combination of a number of digital maps known as data layers, which have the spatial and attribute component. A GIS is more effective and efficient if all related data layers are included in it. Therefore, it is necessary to include a large quantity of attribute data in the data base. However, the more attribute data are
included, the more costly and time consuming it is. The lack of availability of trained personal also makes it necessary to limit the amount of attribute data included in a data base, at least in the initial stage.

### 2.2 The Sri Lankan GIS

Sri Lanka is still in its initial stage of establishing a computerized and integrated GIS. Several government and affiliated organizations such as the Ministry of Planning, Department of forests, International Irrigation Management Institute of the U.N. (IIMI) and Integrated Rural Development Projects (IRDP) have established GIS. These systems are limited to small geographical areas and none of them have the capability of handling cadastral and boundary problems. They provide solutions to problems of resource management within each agency. They are, however, incapable of solving problems at the spatial and attribute levels of the country as a whole.

A proposal for establishing an integrated GIS for the country was made and accepted by the government in 1987. The system will be coordinated by the Land Use Policy Planning Division (LUPPD) of the Ministry of Lands and Land development. Although a final decision has not been made about the data layers and the time frame, a proposal to establish the system in 3 phases (Berugoda, 1987) has to be decided during phase 1. The proposal is describe in Appendix B.

Establishing a cadastral map layer to show the property boundaries of the country has been given priority and is to be done in phase 1. This work will be done by the Survey Department of Sri Lanka. In order to expedite the work, the Department has acquired modern surveying instruments such as "total stations" and GPS receivers. Arrangements were in progress at the beginning of 1996 to purchase hardware and software for the GIS. Assessment of the geodetic network and the work required to improve it must be done before the data collection for the cadastral map layer of the GIS.

When a GIS is established, it will be the major spatial information supplier for many decision-making processes. The cadastral map layer will have to provide accurate spatial information for almost all cadastral and engineering applications. Therefore. it is important to know the spatial accuracy of the system before it is established. The spatial accuracy of the GIS is dependent on the linkage mechanism of the GIS or the geodetic reference framework. Hence. the strengths and weaknesses of the reference network must be evaluated before the data acquisition for the GIS. If the present accuracy of the geodetic network does not provide the required spatial accuracy: the problem must be solved before establishing the GIS.

## 3 EXISTING GEODETIC CONTROL OF SRI LANKA

### 3.1 History

Systematic triangulation in Sri Lanka (then Ceylon) commenced in 1857 with the measurement of the Negombo base (Jackson and Price, 1933). During that time, triangulation was the only means of establishing a geodetic control network (Kahmen and Faig, 1988). Prior to 1857 , all the surveys in Sri Lanka were done in a sporadic basis, using magnetic azimuths and sometimes even without connecting them with each other. After the base measurement, trignometrical observations were made until 1887 throughout the country, except for most northern parts. The work was connected to the Indian network. This 1857 coordinate computation was found to be inconsistent and as a result, a new adjustment called a "new fixing" was made in 1890 (Jackson and Price, 1933).

The angular measurements used for both of the above adjustments were obtained using 13" vernear theodolites. The perspective of computation of coordinates was topographic mapping of the country. During the early $19^{\text {th }}$ century, a considerable amount of first order angular observations were also made using $8^{\prime \prime}$ vernear or $5^{\prime \prime}$ micrometer theodolites in order to fill open spaces of the principal triangulation network (Jackson and Price, 1933). Further, Jackson and Price say (1933, p2), "with the commencement of systematic large scale cadastral surveys in the country in early 1920s, the geodetic control available during that time was found to be not satisfactory and many inconsistencies and errors were observed."

In 1929, the surveyor general of Sri Lanka, Dowson, decided to investigate the condition of the network and make a possible re-computation. As a result, Price and Jackson were appointed to investigate the possibility of a re-adjustment of the network. They concluded that the observations made during 1860 and 1890 were satisfactory and recommended a re-computation (Price and Jackson, 1930).

Two base lines were re-measured and two astronomical azimuths of base lines were observed for the readjustment completed in 1932. In addition, angles of figure IX (see Appendix A), consisting of 6 stations, were re-observed. All the observations used for the 1932 adjustment were the angular observations made during 1858 to 1906 except for the two base measurements, two astronomical observations and re-observation of figure IX. The adjusted network in 1932 and its station names are also given in Appendix A.

### 3.2 Need for a re-adjustment

As mentioned in Chapter 1, geodetic control should provide sufficient accuracy for all the data layers of the GIS. The purpose of the 1932 coordinate adjustment of Sri Lanka was to provide a consistent set of results upon which secondary and minor triangulation could be based without serious distortions, and the triangulation was not observed for cadastral surveys (Jackson and Price, 1933).

The reason for re-adjusting the north American datum of 1927 (NAD27) is given as, "The weaknesses of NAD27 became apparent in several ways. Surveyors were buying accurate electronic distance measurement instruments and finding unexplainable discrepancies between the existing control network and distances measured by their own independent surveys (Schwartz, 1985, p8)."

This idea is valid for all the countries that have geodetic control networks established during the early part of the century. As mentioned in the previous section, Sri Lankan primary network was established in 1932 only to facilitate lower order triangulation
projects, but not for cadastral surveying needs. Therefore, the accuracy of the network may be lower than cadastral surveying needs. Hence, the densified control such as traverses, established using the primary control have a very low accuracy. Therefore, the old network needs to be completely analyzed and the existing accuracy has to be determined before establishing the Sri Lankan GIS.

During the last 15 years, surveyors in Sri Lanka started using modern surveying instruments such as Electro Distance Measurements (E.D.M.) and found similar diffculties observed by the surveyors in the U.S., as described by Schwartz. These surveyors were usually forced to introduce distortions into their more accurate measurements to connect the work to the less accurate national geodetic control network.

In addition to the problems related to surveying, the low accuracy of the geodetic network was a major problem for other engineering projects such as dam movement monitoring, geological projects and seismic studies.

### 3.3 Means of evaluating weaknesses of the network

There may be a number of reasons for the weaknesses of the network. A research was conducted in the following four areas to see the reasons for the weaknesses.

1. Evaluation of observations used for 1932 adjustment
2. Comparison of new distance measurements with the distances obtained from the network
3. Evaluation of the methodology used for 1932 adjustment
4. Evaluation of the effect of the reference ellipsoid and geoid

### 3.3.1 Old observations

Observations used for 1932 adjustment can be summarized as follows (Jackson, 1932):

1. Two base line measurements which are in triangles 1 and 43.
2. 500 angle observations.
3. Two astronomical azimuth observations on two base lines.

Both base lines were about $5 \frac{1}{2}$ miles long and about 127 miles apart. Both were measured in 1930, using "invar" tapes and the procedure was quite conventional for that period of time. Base lines were measured both forward and backward but the backward measurement for the base 1 has not been used for the adjustment due to the unfavorable conditions during measurements, suspecting errors in the measurement.

The average length of a side of a triangle was about 16 miles and the minimum and maximum being $5 \frac{1}{2}$ and 50 miles, respectively. Angles were measured by 10 observers using 3 different types of vernier theodolites ( $8^{\prime \prime}, 12^{\prime \prime}$, and $13^{\prime \prime}$ ) and a $5^{\prime \prime}$ micrometer theodolite. It was observed that the number of repetitions or the standard deviation of angle observations has not been used as weights in the adjustment (Jackson, 1932). According to the condition of angle observations, the present geodetic control network of Sri Lanka can be classified as third order triangulation. (For classification of networks, see Moffit and Bouchard, g $^{\text {th }}$ ed. pp360-362)

A least squares adjustment which is described in detail in Chapter ${ }^{5}$, provides most probable values for unknowns (coordinates of points in a geodetic adjustment). Also, a least squares adjustment can be used to get an analysis about the quality of observations. In the 1932 adjustment, observations were measured angles, base line distances and azimuths. A new adjustment was performed using "Geolab" and old observations. Misclosures obtained for the old observations in this new adjustment are given in Table 3.1.

These values indicate that the old observations do not provide a high accuracy in a new adjustment. The absolute accuracy obtained using these observations ranges from 11.4 meters in the northern regions of the country to 0.3 meters in the central regions. The average absolute accuracy obtained is only 5.3 meters. Absolute accuracy obtained

Table 3.1 Misclosures in the observations used in 1932 adjustment

| Type of Observation | Maximum | Mean | RMS |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| Angles | 18.4 sec | -0.087 sec | 1.95 sec |
| Baselines | 0.17 m | 0.0004 m | 0.024 m |
| Azimuths | 2.66 sec | 0.0001 sec | 0.075 sec |

for all the points (using old observations) are given in Appendix $C$ in the form of error ellipses.

### 3.3.2 Comparison of distances

The 1932 geodetic control adjustment was entirely a triangulation project adjusted using only 2 base lines and the method of condition equations. During the past few years, distances among triangulation stations in triangles $21,22,48,49,50,54,55,56,65,66$, $67,108,109,117$ and 118 were measured by the Institute of Surveying and Mapping in Sri Lanka, using leng range EDM instruments. These distances were reduced to the ellipsoid (Everest ellipsoid) and compared with the distances obtained by inverse calculation of geodetic coordinates (latitudes and longitudes) of 1932 adjustment. Results are given in Table 3.2. Station numbers and names for all stations are given in Appendix A.

Table 3.2 shows that the measured distances are not compatible with calculated distances. The differences are extremely large in many cases. In the 29 cases used for this study, differences between measured and calculated distances varied from 17 cm to 3.32 m . The average difference was 93.7 cm for 29 distances. We have to expect many more discrepancies in lower order control points which will be used for surveying property boundaries at the village level because they are established using the primary control network. Therefore, the coordinates of the 1932 adjustment are extremely weak and not suitable to use either as the linkage mechanism of the GIS or as a control for surveying with modern equipments. Existing accuracy of the network is discussed in

Table 3.2 Comparison of measured distances with calculated distances from 1932 coordinates.

| Station from | Station to | $\begin{array}{r} \text { Measured } \\ \text { (meters) } \\ \hline \end{array}$ | Calculated (meters) | $\begin{gathered} \hline \text { Differences } \\ \text { (meters) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 103 | 105 | 14625.318 | 14626.080 | -0.76? |
| 103 | 99 | 22014.050 | 22014.560 | -0.510 |
| 100 | 10.3 | 30386.142 | 30386.530 | -0.388 |
| 100 | 99 | 25516.174 | 28.516.9.50 | -0.176 |
| 83 | 91 | 27818.916 | 27818.540 | 0.370 |
| 90 | 91 | 18855.027 | 18854.610 | 0.417 |
| S3 | 90 | 32869.964 | 32869.470 | 0.494 |
| 38 | 39 | 41238.347 | 41239.830 | -1.483 |
| 39 | 47 | 12296.780 | 12296.970 | -0.190 |
| 33 | 34 | 27060.375 | 27061.750 | -1.375 |
| 37 | 47 | 28539.717 | 28.540 .260 | -0.54.3 |
| 37 | 39 | 27121.144 | 27121.510 | -0.366 |
| 38 | 40 | 28647.280 | 28647.930 | -0.6.50 |
| 61 | 40 | 35994.657 | 3.5995 .590 | -0.933 |
| 61 | 39 | 21888.434 | 21889.710 | -1.276 |
| 38 | 37 | 31.312 .177 | 31313.790 | -1.613 |
| 33 | 38 | 42922.806 | 42925.250 | -2.444 |
| 39 | 49 | 32466.006 | 32467.250 | -1.244 |
| 40 | 39 | 45741.688 | 4.5743 .250 | -1.562 |
| . 33 | 40 | 61106.702 | 61110.030 | $-3.328$ |
| 61 | 49 | 41844.659 | 41846.130 | -1.471 |
| 79 | 82 | 33952.457 | 33951.770 | 0.685 |
| 79 | 78 | 1.5364 .904 | 1.5365 .670 | -0.166 |
| 80 | 79 | 25832.662 | 25832.280 | 0.382 |
| 80 | 82 | 19029.582 | 19029.090 | 0.492 |
| 53 | 56 | 16328.590 | 16328.850 | -0.260 |
| . 53 | . 57 | 29502.465 | 29.503 .210 | -0.74.5 |
| 56 | 57 | 14360.815 | 14361.440 | -0.625 |
| 49 | 60 | 50179.715 | 50181.350 | -1.63.5 |
| RMS of the differences Maximum Difference |  |  |  | 0.937 m |
|  |  |  |  | 3.328 m |

section 3.4 in detail.

### 3.3.3 Evaluation of the old adjustment methodology

In order to evaluate the methodology used for the 1932 adjustment, a new adjustment was done using the same observations used for the old adjustment. This new adjustment was performed using a least square adjustment software "Geolab V 1.9" (Bitwise Ideas, Inc.). Resulting coordinates of this new adjustment were used to do a comparison similar to the one described in section 3.3.2. "Geolab" adjusts the network, forming an observation equation for each observation, and then performs a least squares adjustment.

The 1932 adjustment was done by dividing the total figure into 17 smaller figures (see Appendix A). This was required for the formation of condition equations. 151 polygon conditions, 55 center conditions and 78 side conditions were used, amounting to 284 conditions for the entire network (Jackson and Price, 1932). After the angles were adjusted by a least square solution, Clerk's formulas (Bomford, 1980) and seven figure log tables were used for the calculations of geodetic coordinates (latitudes and longitudes).

The comparison of distances was done by first converting the geodetic coordinates of the new adjustment to three dimensional Cartesian coordinates, which were then used for the calculation of ellipsoidal distances between stations (Fortran subroutines used for these calculations are given in Appendix J). Calculated distances were again compared with the measured and reduced distances. Results are given in Table 3.3.

Table 3.3 shows that the distances calculated using the coordinates of the new adjustment (with the same observations as the old adjustment) consistent with the measured values much better than the distances obtained using 1932 coordinates. The average deference between distances were reduced to 46.9 cm from 93.7 cm . This is an approximate improvement from the accuracy of $1: 30,000$ to $1: 60,000$. Hence, the methodology used for the calculation of 1932 adjustment can be significantly improved using a least

Table 3.3 Comparison of measured distances with calculated distances from a new adjustment using same old observations.

| Station from | Station to | Measured (meters) | Calculated (meters) | Differences (meters) |
| :---: | :---: | :---: | :---: | :---: |
| 103 | 10.5 | 14625.318 | 1462.5.52 | -0.202 |
| 103 | 99 | 22014.050 | 22014.01 | 0.040 |
| 100 | 103 | 30386.142 | 3038.5.52 | 0.392 |
| 100 | 99 | 28516.714 | 28.516 .01 | 0.764 |
| 8.3 | 91 | 27818.916 | 27818.06 | 0.856 |
| 90 | 91 | 18855.027 | 188.54 .30 | 0.72 |
| 8.3 | 90 | 32869.964 | 32869.01 | 0.9.54 |
| 38 | 39 | 41238.347 | 41238.40 | -0.0.5.3 |
| 39 | 47 | 12296.780 | 12296.4.5 | 0.3:30 |
| 33 | 34 | 27060.375 | 27060.12 | -0.34.5 |
| 37 | 47 | 28539.717 | 28539.47 | 0.247 |
| 37 | 39 | 27121.144 | 27120.88 | $0.26 \div$ |
| 38 | 40 | 28647.280 | 28646.97 | 0.310 |
| 61 | 40 | 35994.657 | 35994.29 | 0.367 |
| 61 | 39 | 21885.434 | 21888.76 | -0.326 |
| 38 | 37 | 31312.17 | 31.312 .63 | -0.4.53 |
| 33 | 38 | 42922.806 | 4.29 .3 .57 | -0.764 |
| 39 | 49 | 32466.006 | 3246.5.56 | 0.446 |
| 40 | 39 | 45741.688 | 4.5741 .54 | 0.148 |
| 33 | 40 | 61106.702 | 611107.56 | -0.8.58 |
| 61 | 49 | 41844.659 | 41844.11 | 0.549 |
| 79 | S2 | $3.3952 .45{ }^{\circ}$ | 339.51 .44 | 1.017 |
| 79 | 78 | 1.5364 .904 | 15365.75 | -0.846 |
| 80 | 79 | 25832.662 | 25831.91 | 0.752 |
| S0 | 82 | 19029.582 | 19028.93 | 0.652 |
| 53 | 56 | 16328.590 | 16328.46 | 0.130 |
| 53 | 57 | 29502.46 .5 | 29502.45 | 0.01 .5 |
| . 56 | 57 | 14360.81 .5 | 14361.12 | -0.305 |
| 49 | 60 | 50179.115 | 50179.07 | 0.64.5 |
| RMSS of differences Maximum difference |  |  |  | 0.469 m |
|  |  |  |  | 1.846 m |

squares adjustment which considers the entire network as one figure.

### 3.3.4 Effect of the Geoid and the Ellipsoid

When measurements or calculations are made on the earth surface, we have to consider 3 surfaces (see Figure 3.1). They are:

1. The physical surface of the earth which cannot be mathematically defined.
2. The "Geoid", which is the equipotential surface of the earth.
3. The "Ellipsoid", which is the best mathematical approximation used for the calculations on the earth surface.


Figure 3.1 Physical surface of the earth, the geoid and the ellipsoid

When distances are measured on the earth surface, they should be reduced to the ellipsoid before the adjustment because all the calculations are done on the ellipsoid. When horizontal angles or azimuths are measured at a station, they are measured using the horizontal plane at the point of measurement. This is the plane perpendicular to the direction of gravity at the point. The direction of the gravity is perpendicular to the geoid but not to the ellipsoid. Hence, corrections have to be made to the measured angles and azimuths. This correction is known as the correction for the deviation of the vertical. Corrections required for observations in a geodetic networks are given in section 5.8.

According to Jackson (1933), two base lines measured for Sri Lankan triangulation were reduced to the mean sea level. This shows that the geoid was taken as the mean sea level and the ellipsoid was considered as coinciding with the geoid (at the mean sea level). Therefore, geoid undulations were neglected for the calculations. Geoid undulations have to be determined by sufficient gravity measurements in the country. The effect of geoid undulations is about 1 part per million for a geoid undulation of 6 meters (Bomford, 1980). Although the effect of neglecting geoid undulation is fairly small when a local ellipsoid is used, (see section 5.6 for different types of ellipsoids) this negligence is one of the factors for the low accuracy of the present geodetic network.

### 3.4 The present accuracy of the network

Expected accuracy of the a geodetic network can be evaluated by studying the classification of networks. Generally, classification of geodetic networks is done as first, second and third order. Sometimes these three orders are subdivided into classes, such as "second order first class" or "second order second class" (Moffit and Bouchard, 1991). According to the specifications of geodetic networks, closure in lengths for first order networks is 1 part in 100,000 . For second order it is 1 part in 50,000 to 1 part in 20,000 in (class1 and class 2) and for the third order 1 part in 10,000 to 1 part in 5,000 (Moffit and Bouchard, 1991). For modern GPS geodetic networks, allowable errors in lengths given by FGCC (1988) are as follows:

1. Primary networks (Order A) I part in 10 million
2. Secondary networks (Order B) 1 part in 1 million

A comparison of measured distances and calculated distances from available coordinates was used to evaluate the classification of the Sri Lankan geodetic network. According to the Table 3.2, the average difference of lengths is 93 cm . As the average distance of a line of the network is 29.5 km , the available accuracy is approximately
$1: 30,000$. When shorter lines such as the line of station 33 and 40 is considered, the accuracy is only in $1: 17,000$. Therefore, the accuracy of the network is between second order class 2 and third order class 1 , according to the measurements taken in central and south-central regions of the country.

Since the length measurements were not observed in the northern and eastern regions of the country, the strengths of the network of those areas were found by comparing coordinates with the other parts of the network (same observations used for both adjustments). This comparison was made assuming that the reliability of angle observations is the same for the entire country and the weakness of the old adjustment is basically due to the methodology used for the calculations.

In order to obtain linear values of differences, both sets of coordinates were converted to plane coordinates using the Transverse Mercator projection. Software used for this transformation is given in Appendix J. Differences obtained in eastings and northings between old and new coordinates are shown in the map in Figure 3.2 and are also given in Appendix C in a tabular form.

These differences show that the northern and eastern parts of the present network are comparatively weaker than other areas. Therefore, when the entire network is considered, it is not possible to classify the existing Sri Lankan geodetic network even up to the third order.


Coordinare Differences in Nortinings:


Differences in Eastings

Figure 3.2 Differences in Northings and Eastings between 1932 adjustment and a new adjustment using same observations. The largest circle indicates 14.2 meters

## 4 PRINCIPLES AND APPLICATIONS OF GPS

Principles and possible error sources of Global Positioning Systems (GPS) have to be identified before establishing a GPS geodetic control network. Therefore, a short description of the principles and applications of GPS is given in this chapter. It will provide basic knowledge about GPS essential to all who will be involved in establishing the new GPS geodetic control network in Sri Lanka.

In the past few decades, a number of positioning systems have been established and tested, but their success were very limited for accurate and widely-used positioning. These techniques include satellite photography, Very Long Base line Interferometry (VLBI), Lunar Laser Ranging (LLR), Satellite Laser Ranging (SLR) and Doppler Positioning Systems. All these methods were not satisfactory as a suitable method for establishing geodetic control networks (Leick, 1995).

Today, NAVISTAR (Navigation System with Time and Ranging) GPS provides accurate positioning for high order geodetic networks with convenience and a lower cost than older surveying and other positioning methods.

Older positioning methods require a considerably long field time and much more trained labour. Other than these obstacles, a real time accurate positioning with these conventional methods was nearly impossible (Leick, 1995). Thus, GPS has already proved that it is a far better positioning technology than any other previously available technique.

Accurate and current data collection for GIS can be done by using GPS, especially when the spatial data is required in relatively high accuracy such as creating the cadastral
map layer. GPS has already acquired a wide popularity in applications such as land, marine and air transportation; recreational and military applications; surveying and geodesy; and space applications. In the field of geodetic surveying, GPS has already become the primary source of data collection for geodetic network adjustments. The differential GPS positioning technique, which will be discussed in detail in section 4.4.2, easily provides the required level of accuracy.

Another important area of GPS application is photogrammetry. Photogrammetry provides a reasonably accurate, current and detailed three-dimensional spatial data for GIS. The biggest concern for photogrammetry has been the need for ground control points (GCP) which are used to transform the coordinate system of aerial photographs to the ground coordinate system. Establishing suitable GCP using conventional methods is a fairly tedious and costly process. GPS has been used to get up to 30 cm accuracy for ground control points required for photogrammetric applications (Wells, 1987). Hence, it can be very effectively used in photogrammetry as a substitute for GCP needs. Main features of GPS can be described as follows:

1. High accuracy
2. Low cost to users
3. A unified world wide coordinate system
4. Suitability for many different types of applications

### 4.1 Components of GPS

Positioning with GPS is a result of three major components (Puterski, 1992).

1. Space segment
2. User segment
3. Control segment

The space segment consists of GPS satellites which transmit signals. These signals
contain a number of information related to the satellite as well as the GPS time which is accurately determined by an atomic clock. Satellites were launched by NASA as 3 separate blocks called block 1, block 2 and block 2A. By the end of 1995, another 20 GPS satellites named block 2 R were in development (Leick, 1995). Satellites were placed on 6 orbital planes, which are approximately 55 degrees inclination to the equator. Semi major axes of orbital paths of satellites are approximately $26,000 \mathrm{~km}$ from the center of the earth and the orbital period is slightly less than 12 hours. Because of the high altitude of satellites and due to the optimization of satellite visibility by keeping satellites unevenly in the orbital plane, more than 4 GPS satellites can be tracked from anywhere in the world, even after the mask angle of $15^{\circ}$ (Leick, 1995).

User segment or the data collection segment consists of a GPS receiver and an antenna. The GPS data received by the antenna is transferred to the receiver through a cable. The GPS receiver is a combination of an amplifier, radio signal micro processor, control and display device, data recording unit and a power supply (Wells, 1987). The software and electronics in the GPS receiver decode the timing signals from 4 or more GPS satellites and first calculate the "pseudo range" (uncorrected distance) to the satellites and then compute the latitude, longitude and the elevation of the occupied station with respect to the GRS80 ellipsoid. All the data collected during the session are stored in the memory of the GPS receiver, so that they can be processed with respect to a control (known) station in order to get the geodetic level accuracy. Real time data processing can also be done with transmitted data from the control station. Details of these procedure are given in section 4.5.

Continuous monitoring and correcting the satellite orbit are required for efficient use of the GPS system. For this purpose, five monitoring stations continuously track all GPS signals for the purpose of controlling satellites and predicting their orbits (Wells, 1987). Locations of these stations are given in Table 4.1.

Table 4.1 Locations of GPS control stations

| Station | Location |
| :--- | :--- |
|  |  |
| Colorado Springs | U.S. |
| Hawaii (U.S.) | Pacific Ocean |
| Ascencion (U.K.) | Atlantic Ocean |
| Diego Garcia (U.K.) | Indian Ocean |
| Kwajalein (U.S.) | Pacific Ocean |

Positions of these monitoring stations are known to a very high accuracy. Colorado Springs station works as the master control station. Tracking data from the monitoring stations are transformed to the master control station for processing. This processing enables the accurate determination of satellite ephemerides, satellite clock corrections and other broadcasting message data which are transmitted back to satellites (Leick, 1995).

### 4.2 Orbital motion of a satellite

A short description of the orbital motion of a satellite is given in this section in order to understand the positioning of satellites in space. The motion of a satellite in an orbit is a result of the earth's gravitational attraction, attractions of the sun, moon and many other forces such as the pressure by solar radiation particles and the atmospheric drag (Wells, 1987). Kepler's laws describe the motion of a satellite in an ideal situation. A set of 6 parameters can be used to define the location of a satellite in the space at any given time.

These 6 parameters given according to figure 4.1 are:

1. Greenwich hour angle of the ascending node ( $\Omega$ )
2. Inclination (i). The angle between the equatorial and orbital planes
3. Argument of perigee $(\omega)$. The angle between the nodal and perigee directions measured in the orbital planes


Figure - 1.1 Parametors defining the position of a satellite in the space at a given lime.
4. Semi major axis of the elliptical orbit (a)
5. Eccentricity of the orbit (e)
6. Time at perigee $(t)$

Apogee and perigee are the two points on the orbit where the distance from the earth to the satellite is maximum and minimum respectively.

The parameters $\Omega$ and $i$ define the orientation of the orbital plane in space; $\omega$ defines the location of the perigee on the orbit and $a$ and $e$ define the size and shape of the orbit (Wells, 1987).

### 4.3 GPS satellite transmissions

GPS satellite transmissions are based on two high frequency wave bands. High frequencies are required to minimize ionospheric effects. These two wave bands are usually called $L 1$ (Link 1) and $L 2$ (Link2). Frequencies of $L 1$ and $L 2$ are 1575.42 $M H z$ and 1227.60 MHz (Leick, 1995). These two carrier waves carry a number of modulated signals known as PRN (Pseudo Random Noise) codes. Out of these PRN codes, $C / A$ code (Coarse/Acquisition code), and Pcode (Precise or protected code) are used for the determination of positions by GPS receivers. Although Pcode was considered as protected for military use, with new receiver designs, now there are no significant differences in position determination by Pcode or C/A code (Hunn, 1989). According to Puterski (1992), "Another PRN code, the $Y$ code is now being transmitted on block 2 satellites. It is also for military use and its signal structure is more secure then $P$ code."

### 4.4 Positioning methods

Positioning with GPS can basically be divided into two modes:

1. Stand along mode (Absolute positioning)

## 2. Relative mode (Differential positioning)

In both of these modes, the distance traveled by a wave between the satellites and the receiver is determined by measuring time delays or by using the technique of phase measurement. Measuring time delays to calculate the pseudo-range is usually called "pseudo-ranging" (Moffit and Bouchard, 1991).

By knowing the time of transmission and the receiving time. the pseudo-range can be writien as:

$$
r=V_{r} . \Delta t
$$

where $r$ is the pseudo range. $V_{r}$ is the velocity of light and $\Delta t$ is the travel time between broadcast and reception. Since the synchronization of the satellite clock and the receiver clock is not exact, the calculated range is not the true range. This is the reason for referring the calculated distance as pseudo-range (Mofft and Bouchard. 1989).

The technique of phase measurement is the way of distance measurement using the number of phase cycles over the transmission path and the phase difference at the receiver. This is done by mixing the incoming signal with a known signal generated by the receiver (Wells, 198i).

### 4.4.1 Stand along mode

In the stand-along mode, the process is carried out between satellites and a single GPS receiver. As the data is collected and processed by only one receiver, positioning with stand-along mode is subject to all the errors associated with GPS positioning. including clock errors in the receiver and satellites and errors due to the atmosphere and troposphere. The most common method used for stand-along mode is pseudo-ranging. This is generally known as $C / A$ code pseudo-ranging or $P$ code pseudo-ranging.

The accuracy of pseudo-ranging depends on the frequency (or the wave length) of the wave tracked by the receiver. The effective wave length of $C / A$ code is 29.3 .3 meters and
the wave length of $P$ code is one tenth of the $C / A$ code (Moffit and Bouchard, 1991). The accuracy of pseudo-ranging in stand along mode by $C / A$ code is known as around 20 meters and around 5 meters for the $P$ code receivers (Leick. 1995).

### 4.4.2 Relative mode (differential GPS)

Relative GPS positioning must be used when accurate positioning is needed for projects such as establishing geodetic networks. Relative positioning is commonly known as differential GPS. In this technique. one GPS receiver is placed at a station that the coordinates are known to a high degree while the $2^{\text {ne }}$ receiver is placed at the point where the position determination is required.

Usually, data from satellites are collected at both stations and down-loaded in to a computer for processing. Software for this processing is usually provided with the GPS receivers. For "Ashtech " GPS receivers. "GPPS". "PNAV" and "Prism" are used for differential post processing. When accurate positioning is required in real time (real time differential GPS). data from the known station are transmitted to the receiver at the unknown station and processed in real time. As the data are collected at two stations and processed in differential GPS. atmospheric and ionospheric errors effecting both receivers are assumed as same, when two stations are not very far apart. So. these errors can be eliminated during the computation stage (Leick. 1995).

### 4.4.3 Kinematic and static modes

Differential GPS observations can be basically made using 4 different modes. They are:

1. Static mode
2. Pseudo static mode
3. Kinematic mode
4. Pseudo kinematic mode

In static mode, receivers are placed at two stations and data collection is done for a long period of time. The length of data collection depends on the type of receivers and the required accuracy in positioning. Generally the length of data collection is 45 minutes to 2 hours with 20 second epochs. For the projects which need very high positional accuracy, this length of time can be 24 hours or more (Soler and Hall. 199.5).

The pseudo static mode is somewhat similar to the static mode. but the length of data collection is much shorter than the static mode. The usual period of data collection is about 15 minutes. Pseudo static mode can be used when the positioning is not required to a high precision or the data collection has to be done in a short period of time.

In the kinematic mode. the receiver at the known station (base station) is kept fixed and the other receiver or a number of receivers (rover units) move from one station to another station after the data collection for a short period of time. U"sually rover units collect data from 2 to 5 minutes at one station. This technique is useful when data to be collected at a large number of points but with a high positional accuracy is not required. Generally, positional data for small scale mapping can be collected using the kinematic mode.

Pseudo kinematic mode is a modification of kinematic mode but provides much more accurate results than the kinematic mode. For this mode. a second control point is required. The rover unit is first placed at the second control point for a short period of time for initialization while one receiver is at the base station. Then the rover is moved to other points where position determination is required and finally brought back to the second control point. All this time, one receiver collects data at the base station. This mode is effectively used for photogrammetry without ground control points (Jeyapalan. 1995).

### 4.5 Computations in differential GPS

There are at least 4 types of GPS signal measurements that have been used for relative positioning techniques. They are "pseudo-range, carrier phase, code phase, and integrated Doppler" (FGCC, 1988). These measurements involve advanced techniques in electronics in GPS receivers. The most common way of signal measurement is pseudo ranging while the phase measurement gives the most precise measurements (FGCC, 1988). All four techniques of differential GPS involve the solution of observation equations for each measurement. By solving these equations and also using the techniques of double difference and triple difference, which are discussed later in this section, we can eliminate clock errors in receivers and satellites, ionospheric and tropospheric errors, and orbital errors of satellites. Making use of different types of measurements depend on the design of receivers for different applications, but a combination solution of pseudo-ranging and carrier phases are becoming more common (Leick, 1995).

Assume that satellites A, B, C and D as shown in Figure 4.2 are observed by a receiver at station 1 . Then the pseudo-ranges from the station to each satellite can be written as (Moffit and Bouchard, 1991):

$$
\begin{aligned}
& {\left[\left(X_{1}-X_{A}\right)^{2}+\left(Y_{1}-Y_{A}\right)^{2}+\left(Z_{1}-Z_{A}\right)^{2}\right]^{1 / 2}=r_{1 A}-V_{r}\left(\Delta t_{1}-\Delta t_{A}\right)} \\
& {\left[\left(X_{1}-X_{B}\right)^{2}+\left(Y_{1}-Y_{B}\right)^{2}+\left(Z_{1}-Z_{B}\right)^{2}\right]^{1 / 2}=r_{1 B}-V_{r}\left(\Delta t_{1}-\Delta t_{B}\right)} \\
& {\left[\left(X_{1}-X_{C}\right)^{2}+\left(Y_{1}-Y_{C}\right)^{2}+\left(Z_{1}-Z_{C}\right)^{2}\right]^{1 / 2}=r_{1 C}-V_{r}\left(\Delta t_{1}-\Delta t_{C}\right)} \\
& {\left[\left(X_{1}-X_{D}\right)^{2}+\left(Y_{1}-Y_{D}\right)^{2}+\left(Z_{1}-Z_{D}\right)^{2}\right]^{1 / 2}=r_{1 D}-V_{r}\left(\Delta t_{1}-\Delta t_{D}\right)}
\end{aligned}
$$

Where $X, Y, Z$ are the earth centered cartesian coordinates of point 1 and satellites $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and $\mathrm{D} ; \Delta t$ is the receiver clock error at station $1 ; \Delta t_{A}$ is the satellite clock error of satellite $A$. Satellite clock errors are considered to be known and also can be measured by calibration (Moffit and Bouchard, 1991). When the pseudo-ranges $r_{1 A}, r_{1 B}$,

known station

Figure 4.2 Data collection procedure for differential GPS
$r_{1 C}$ and $r_{1 D}$ are measured. only unknowns $X, Y, Z$ and $\Delta t_{1}$ can be calculaied by solving 4 equations to obtain a solution in the stand along mode.

These 4 equations are only for one instance of range measurement. In practice. measurements are made for a long period of time with different epochs (time intervals) depending on the receiver and required accuracy of positioning. For geodetic work, epochs ranging from a fraction of a second up to 20 seconds are used for observations. These redundant observations are then used in a least squares solution to obtain a solution for 4 parameters.

When differential GPS observations are made at a known station and at an unknown station, satellite clock errors are considered as unknowns. the unknowns $\Delta t_{1}, \Delta t_{2}, \Delta t_{A}$, $\Delta t_{B}, \Delta t_{C}, \Delta t_{D}$ and also other errors due to the ionospheric and tropospheric delays can be eliminated by using the methodology of double and triple differencing, which is described later in this section.

When measurements are done using the phase difference, the difference between the
the satellite carrier phase received by the receiver and the phase of a similar wave generated in the receiver are compared and measured (Wells,1987). This measurement process cannot account for the number of whole carrier waves between the receiver and the satellites. The carrier phase observable at the receiver is a function of integer ambiguity, time taken for traveling, ionospheric and tropospheric effect, hardware delays at the satellite, and the receiver delays due to multi-path and random carrier phase measurement noises (Leick, 1995).

Taking the wave length as $\lambda$ and $\Delta e$ for the combined delays due to hardware at receiver and satellites, multi-path and noises (explained at the end of this section), we can write a simplified equation for a pseudo-range, using the phase measurement (Jeyapalan, 1995):

$$
\phi=n \lambda+L+\Delta t_{s}+\Delta t_{r}+\Delta e
$$

where $n$ is the total number of cycles between the satellite and receiver (integer ambiguity), $L$ is the measured phase difference, and $\Delta t_{s}$ and $\Delta t_{r}$ are the effects of satellite and receiver clock errors respectively. For a differential solution, if the data is collected at station 1 and 2 from satellite $p$, pseudo-ranges from both stations to the same satellite can be written as follows:

$$
\begin{align*}
& \phi_{1}=n_{1} \lambda+L_{1}+\Delta t_{s}+\Delta t_{1}+\delta e_{1}  \tag{4.1}\\
& \phi_{2}=n_{2} \lambda+L_{2}+\Delta t_{s}+\Delta t_{2}+\delta e_{2} \tag{4.2}
\end{align*}
$$

subtracting the above two equations assuming delays other than receiver clock errors are the same ( $\delta e_{1}=\delta e_{2}$ ) for both stations, we can write the first difference as:

$$
\begin{equation*}
\phi_{1}-\phi_{2}=\left(n_{2}-n_{1}\right) \lambda+\left(L_{2}-L_{1}\right)+\left(\Delta t_{2}-\Delta t_{1}\right) \tag{4.3}
\end{equation*}
$$

we can write this equation (for satellite $p$ ) in a simplified form as:

$$
\begin{equation*}
\Delta \phi_{p}=n_{p} \lambda+L_{p}+\left(\Delta t_{2}-\Delta t_{1}\right) \tag{4.4}
\end{equation*}
$$

If observations are taken from another satellite (say $q$ ):

$$
\begin{equation*}
\Delta \phi_{q}=n_{q} \lambda+L_{q}+\left(\Delta t_{2}-\Delta t_{1}\right) \tag{4.5}
\end{equation*}
$$

By subtracting equation 4.4 and 4.5 , we can obtain an equation eliminating receiver clock ambiguities. This equation is known as the double difference.

$$
\begin{equation*}
\Delta \phi_{p q}=\left(n_{p}-n_{q}\right) \lambda+L_{p}-L_{q} \tag{4.6}
\end{equation*}
$$

By measuring phase differences and writing observation equations, using parameters as coordinates of unknown station and the integer ambiguity, we can do a least square adjustment to obtain a solution for parameters.

By adding any errors due to integer ambiguities $\Delta N_{p q}$ to the equation obtained from the double difference at time $t_{1}$ and $t_{2}$,

$$
\begin{gather*}
\Delta \phi_{p q}^{t 1}=\left(n_{p}-n_{q}\right) \lambda+\left(L_{p}-L_{q}\right)+\Delta N_{p q}  \tag{4.7}\\
\Delta \phi_{p q}^{t 2}=\left(n_{p}-n_{q}\right)^{\prime} \lambda+\left(L_{p}-L_{q}\right)^{\prime}+\Delta N_{p q} \tag{4.8}
\end{gather*}
$$

The difference between equations 4.7 and 4.8 at 2 different epochs is known as the triple difference. Writing observation equations for the triple difference solution, again we can do a least square solution for unknowns. Both double difference and triple difference solutions give satisfactory results in geodetic accuracy, but because of the additional differencing over time, the triple differences lose some geometric strength (Leick, 1995).

### 4.6 Errors effecting GPS positioning

Positional accuracy of GPS can be affected by a number of factors. They are (Puterski, 1992):

1. Geometry of observing satellites
2. Variations in satellite and receiver clocks
3. Errors in satellite ephemerides
4. Errors due to the electronics of receivers
5. Effects by the ionosphere and troposphere
6. Cycle slips
7. Multi-path effect
8. Selective availability

When differential GPS is used, many of the effects of these errors can be mathematically eliminated during data processing.

Geometry of satellites is the position of satellites in space at the time of data collection, relative to each other and relative to receivers. Widely spread positions of satellites give better results. This effect is expressed in GPS as GDOP (Geometric dilution of Precision). The effect of GDOP in three dimension is expressed as PDOP (Position dilution of precision) (Wells, 1987). PDOP is measured in a scale from 0 to 10 , where 0 gives the best positional accuracy.

Measurement of pseudo-ranges depends on the satellite clocks and the clock in the receiver. Satellite clocks are highly accurate atomic clocks but the receiver clocks are built less accurately than satellite clocks due to the cost factor. Cesium atomic clock loses only one second in 300,000 years( Leick, 1992). GPS time is realized by an atomic clock maintained at the master control station at Colorado Springs, Colorado. There is 1 micro second difference between the GPS time and the Universal Coordinated time (UTC) (Leick, 1995).

A temporary interruption in GPS data collection is referred to as a cycle slip. When the data is collected for a significantly long period of time, a short cycle slip can be neglected but longer cycle slips have to be compensated with additional time periods of data collection. Usually receivers are capable of informing the observer about cycle slips, using a sound signal. For differential GPS, data can be edited before post processing in order to eliminate the effect of cycle slips.

Multi-path is the effect of large reflecting surfaces close to a GPS receiver. This error is caused by a reflected GPS signal received by the receiver in addition to the direct signal. This error has to be minimized during the reconnaissance stage. So, the locations of GPS stations have to be selected away from large metal buildings and considerably large bodies of water.

The operation of GPS satellites is performed by the United States Department of Defense (DOD). DOD can intentionally degrade the clock and ephemeris signals from satellites. This is known as selective availability. The effect of selective availability of positioning with $C / A$ code in stand along mode can be somewhere between 50 to 200 meters (Puterski, 1992). Selective availability has only a very little effect on differential GPS (FGCC, 1988).

Ionosphere is generally considered to be the region of the atmosphere from approximately 50 to 1000 km in altitude and the troposphere is known as the atmosphere close to the earth surface, generally up to 80 km . A GPS signal travels through ionosphere and the troposphere can be affected by the ionized medium of the ionosphere (Wells, 1987) and the refraction by the troposphere. The ionospheric and tropospheric effects can be eliminated in differential GPS, but it effects the position determination by stand along mode. This error can be in the range from 1 to 10 ppm (FGCC, 1988).

### 4.7 Specifications of differential GPS

Specifications for GPS surveying in the United States are determined and published by the Federal Geodetic Control Committee (FGCC). According to the preliminary document published by FGCC in 1988, which is the latest version of specifications available today, GPS surveys have been categorized into 4 major orders. They are called $A A, A, B$ and $D$. Out of these 4 categories only $A A, A$ and $B$ are considered as geodetic level networks. Expected absolute accuracy of each of these networks is given in Table 4.2

Table 4.2 Expected absolute accuracy of GPS networks

| Order | Expected accuracy |  |
| :--- | :--- | :--- |
|  |  |  |
| AA | $1: 100,000,000$ | 0.01 ppm |
| A | $1: 10,000,000$ | 0.1 ppm |
| B | $1: 1,000,000$ | 1 ppm |
| D-1 | $1: 1,00,000$ | 10 ppm |
| D-2-I | $1: 50,000$ | 20 ppm |
| D-2-II | $1: 20,000$ | 50 ppm |
| D-3 | $1: 10,000$ | 100 ppm |

(FGCC,1988).

### 4.8 Getting GPS data in to a GIS

GPS gives highly accurate positional data for a GIS, but a few more steps are needed before using them with other spatial and attribute data in the GIS. Two major concerns are the reference ellipsoid and the map projection. As explained in chapter 3, reference ellipsoid is the mathematical representation of the earth, which is used for the calculation of latitudes and longitudes. Plane coordinates are calculated using these latitudes and longitudes, which are used for large scale mapping.

Many different reference ellipsoids are being used for different parts of the world. Before 1983, the U.S. used the Clerk's (1866) ellipsoid. So, NAD27 coordinates are based on the clerk's ellipsoid. NAD83 coordinates were calculated using GRS80 ellipsoid. This ellipsoid is the same ellipsoid used by GPS. Currently, Sri Lanka uses the Everest ellipsoid and hence all maps prepared by Sri Lanka are based on the Everest ellipsoid. The effect of different ellipsoids at different locations varies and has to be corrected using datum transformation techniques explained in chapter 6 . With the wide use of GPS technology in many civilian applications, today's trend is to use a single ellipsoid for the entire world. Achieving this objective is far from over and especially developing
countries have to do much more work in this area of the subject. It will take many more years to establish a common ellipsoid for the whole world and prepare maps using this common ellipsoid. Until then, GPS data will have to be adjusted for the relevant ellipsoid which other maps of the GIS data are based on.

Maps prepared in the U.S. using NAD83 coordinates have the same reference ellipsoid as GPS. Therefore, the GPS data are compatible with the coordinates given in new maps. Older maps of the ['.S. as well as maps of Sri Lanka do not have the same reference ellipsoid as GPS and have to be brought to the same ellipsoid before using them for data analysis in the GIS.

A satisfactory GIS must have a variety of spatial and attribute information. Today: the primary way of collecting spatial data for a GIS is from hard copy maps, which are prepared using a map projection. Positional data collected by GPS are directly on the reference ellipsoid and are not related to any map projection. They are usually given as latitudes and longitudes (geodetic coordinates) or as earth centered 3 dimensional cartesian coordinates. Therefore, these geodetic or cartesian coordinates have to be transformed to the same map projection used in the GIS.

Many popular GIS packages have the capability of transforming ellipsoidal or cartesian coordinates to a given map projection. For example, Arc Info Rev 0.0 is equipped with the NADCON ellipsoidal transformation module which is developed by SiGS (ESRI. 1995). Thus, maps prepared on NAD27 datum can be transformed to the GRS80 datum. In addition, almost all popular GIS software has the capability of transforming coordinates from one map projection to another map projection.

## 5 A NEW GEODETIC CONTROL SYSTEM

According to the analysis done in Chapter 3, it appears that the existing geodetic control of Sri Lanka does not satisfy the needs of GIS. The network is not suitable as the linkage mechanism. Also, it is not suitable for GPS observations and long range EDM observations, which will be major techniques for spatial data collection for the establishment and maintenance of future GIS. Therefore, the geodetic network of Sri Lanka has to be strengthened and a new set of coordinates adopted before any significant work can be started for data collection for GIS. This chapter discusses the procedure for the establishment of a new geodetic control network for Sri Lanka.

### 5.1 Reconnaissance

Establishment of a Geodetic control network starts with reconnaissance. In this stage, points with coordinates to be determined are physically selected on the ground. The success of the geodetic control network is greatly dependent on reconnaissance because the collection of data, cost of data collection, accuracy of the network and future use of the network depend on the locations of points of the control network. In early days, when triangulation was the only means of establishing geodetic networks, reconnaissance was basically concentrated on 2 factors. They were:

1. The strength of the figure
2. Inter visibility among stations

A considerable attention was paid to the geometric strength of figure because a net-
work with equilateral triangles was considered the strongest. Generally triangles with small angles such as $10^{\circ}$ or $15^{\circ}$ degrees were considered weak. Consideration of the strength of the figure was important because the influence of small angles on trigonometric formulas used for the calculation of distances was proportionately higher than larger angles. The errors of measurements of small angles made a disproportionately large contribution to the errors of calculation of lengths. Today, with the use of EDM instruments and GPS, paying attention to the strength of a figure has become unnecessary.

Inter-visibility between stations is important for angle measurements as well as distance measurements using EDM instruments, but is not required for GPS. Inter visibility depends on 3 factors:

1. Earth curvature
2. Obstructions due to terrain conditions
3. Secondary obstructions such as trees

Maximum possible sighting distance between two stations due to the earth curvature can be mathematically calculated. But the geographical area of Sri Lanka is not very large and satisfactory control points have already been selected for previous adjustments. Thus, the effect of the earth curvature is not a major concern for the new geodetic control adjustment in the country.

Obstructions due to local conditions such as trees are important even for GPS observations, because the mask angle for GPS observations has to be used as $10^{\circ}$ to $15^{\circ}$ degrees for geodetic measurements (FGCC, 1988). Hence, all the stations of the old network has to be checked for local obstructions, as a part of the reconnaissance of the new network.

Stations used in geodetic networks established by triangulation are generally located on hill tops or remote areas. This was due to the easy sighting between stations. Also, this way of selecting points provided a better protection for monuments used in early
geodetic networks. As the long term protection of control points (monuments) is one of the major concerns in the reconnaissance stage, it is important to establish them in protected areas. Another important consideration is the easy access of monuments for day to day work. These two factors usually contradict each other. For the long term safety of monuments, it is better to establish them away from populated areas but for easier access, they have to be close to population centers. With the usage of GPS for geodetic observations we can select locations for control points in any convenient place.

Although the strength of the figure is not important for GPS networks, the number of vectors observed for one station play a major role when the network is adjusted. For example, if the location of a station is determined using only two vectors of GPS observations, the positional accuracy of that station can be weak. GPS observational requirements for Sri Lanka, which are discussed in section 5.3.4 are important when dealing with this problem. Therefore, the planning of the observational procedure has to be done with great care, in order to achieve good positional accuracy for the network.

### 5.1.1 Field measurements used for old networks

Older geodetic networks established in the 1960s or before were established using triangulation (Kahmen and Faig, 1988). Triangulation is based on the measurement of two or more base lines, a few astronomical azimuth observations and the angular observations in all the triangles. The measurement procedure for base lines, which usually used "Invar" tapes, were very tedious, time consuming and costly. Angle observations were done in the night to avoid the effect of refraction, using suitable light sources as targets. For all these reasons, the workload for observations of geodetic surveys was very large. The average time for angle observations at one station was 6 weeks for the observation of the German first order network (Kahmen and Faig, 1988).

The existing Sri Lankan primary geodetic network has 110 stations. Although some of them cannot be used for angle observations due to obstructions such as buildings,
only a very few stations have been destroyed during the last 64 years (since the last adjustment). Further, almost all stations of the Sri Lankan network are established on permanent natural features such as huge rocks. Hence, the possibility of disturbances to monuments is low. Therefore, it is economical and fast to use all available old stations for the new network. Also, it will facilitate the calculation of accurate transformation parameters between two systems.

### 5.2 Geodetic control and its users

Good geodetic control is an essential part of a GIS. Hence. All the GIS users are direct beneficiaries of a good geodetic control system. Even without a GIS, geodetic control plays an important role in many disciplines. Users of a geodetic network can be basically grouped in to three categories (Dewhurst, 1990):

1. Primary users: those who employ the coordinate information directly. Geodesists, Geographers and Land Surveyors are in this category.
2. Secondary users: those who employ the work of the primary users. They usually add information and convert the work of primary users into a more general and usable form. Cartographers and digital map makers fall into this category.
3. Tertiary users: those who use the work of secondary users for learning, planing and many engineering works. Engineers, planning specialists, attorneys, students, social scientists, economists and legislators can be put in to this category.

### 5.3 GPS observations for new adjustment

Frequent changes of coordinates of a geodetic network are not possible due to many different levels of users in the country. Hence, when a coordinate system is established,
it must be good for present as well as future applications. Therefore, coordinates of a geodetic network have to be calculated with the highest possible accuracy. GPS has been proven the best and most accurate data collection method available today for establishing geodetic networks (Leick 1995, Puteski 1992, Wells 1987), if the correct procedure is adopted for GPS observations.

In the U.S., the new geodetic control network adjusted in 1983 (NAD83) is now considered obsolete, mainly due to the obtainable accuracy of GPS. NAD83 was established using a very little GPS observation (Snay, 1989), due to the availability of this new technology during that time. Therefore, Sri Lanka must use GPS for establishing the new geodetic network if the country is to avoid another geodetic project within a short period of time.

### 5.3.1 Expected and predicted accuracy of the network

Regarding the new geodetic control in the U.S., NAD83, Schwartz (1983, p9) says,"as the number of types of coordinate users increased with the rapid population growth, so did their accuracy needs. The number of coordinate users have rapidly increased due to the need of growing population centers. These population centers need accurate maps for tax assessment and land use planning and the construction and maintenance of sewer and water supply lines, highways, bridges, tunnels, telephone lines, pipe lines, power transmission lines and many other related services."

Also, accuracy needs were increased due to the availability of modern technology for surveying, mapping and other engineering areas. The increase of land values due to the population growth is another contributing factor to the demand of more accurate geodetic control.

In the early part of this century, even up to the 1960s, geodetic control was established using purely "Triangulation" techniques. Those networks satisfied the needs of mapping, boundary determination and many other construction work such as highways, dams and
irrigation projects. Accuracy expectations of those networks were not as high as today, due to the non availability of convenient and accurate long range distance measurement techniques. Accuracy expected by those geodetic networks was around 1 part in 50,000 but by 1983 this demand increased up to the accuracy of 1 part in 100,000 (Schwarz, 1983).

Today, with the wide usage of GPS technology, this demand has increased to at least 1 part in 1 million. The HARN (High Accuracy Reference Network) project, which is being established in the state of Iowa and throughout the U.S., is aiming at accuracy of 1 part in 10 million, hoping that it will at least satisfy the needs of geodetic control in the early part of $21^{s t}$ century (A order accuracy by FGCC, 1988).

Geodetic control of a country is determined by calculating latitudes and longitudes of a number of permanently marked points. As these points were established using triangulation, they are usually known as triangulation points or some times as primary control points. In the triangulation, a few lengths (at least two) between network points, which are known as "base lines", were measured to high accuracy. The method used for these length measurements was very tedious and time consuming. Then almost all the angles in the network were observed. By using these measurements and fixing one point usually called the "datum point", geodetic coordinates of other points were calculated using the principal of "least square". Geodetic coordinates of the datum point were usually assumed as equal to the values obtained using astronomical methods.

Once the geodetic coordinates for a country are finalized, they are usually published by the government of the country. In the U.S., the National Geodetic Survey (NGS) is the government agency which performs this work and in Sri Lanka, it is the Survey Department. Once the coordinates are published, they become permanent coordinates of the triangulation points until a re-calculation is done and a new set of coordinates are published. The general understanding in the past was that the coordinates published can be used without a change for about 50 years, but the rapid development of technology
has made it impossible to keep up this time frame. In the U.S., NAD27 (adjusted in 1927) was replaced by NAD83 after 56 years but now the HARN coordinates are going to replace them again after only 16 years.

Sri Lanka uses the coordinates calculated and published in 1932. Thus, for the last 64 years, geodetic needs of the country were fulfilled by these coordinates, although they could not cope with the present needs of surveying and engineering technologies. The Survey Department of Sri Lanka has proposed to established a new geodetic network consisting of 240 points, including about 50 points used for the 1932 adjustment. Observations for the new adjustment using long range EDM (Electro Distance Measurement) instruments and GPS were expected to be commenced in the early part of 1996.

As mentioned in Chapter 1, an absolute accuracy of $1: 1,000,000$ in primary control points will provide sufficient geodetic control for GIS applications. As the average distance between the primary control points is about $30 \mathrm{~km}, 2 \mathrm{~cm}$ absolute accuracy of points will provide the accuracy of 1 ppm for the Sri Lankan network.

It is extremely useful if the final accuracy of the network can be predicted before the adjustment, according to the quality of GPS observations. Then, the observation procedure for data collection can be adjusted to get the desired accuracy. In order to do this prediction, simulated sets of GPS observations were calculated and a least squares adjustment was performed (using "Geolab"). Using this method the final accuracy of the Sri Lankan network could be predicted for a number of GPS observation methods. The complete procedure for obtaining simulated data and the predicted accuracies of Sri Lankan network are discussed in next two subsections.

### 5.3.2 Simulated data

Positional data obtained by differential GPS (see Chapter 4) are as follows:

1. Earth centered $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ coordinate differences for vectors between stations ( $\Delta X, \Delta Y, \Delta Z$ ). These coordinates are based on the GPS ellipsoid (GRS80).
2. Mark to mark distances between stations.
3. Azimuth between two stations according to the GPS ellipsoid
4. Ellipsoidal height difference according to the GPS ellipsoid

Out of all these data, $\Delta X, \Delta Y$ and $\Delta Z$ between stations are calculated by GPS first, and then other data are derived using geodetic formulas and the GPS ellipsoid (Jeyapalan, 1995).

Simulated mark to mark distances and $\Delta X, \Delta Y, \Delta Z$ were obtained as follows:

1. A least squares adjustment was performed using all currently available data to obtain latitudes and longitudes of stations
2. Mark to mark distance and $\Delta X, \Delta Y, \Delta Z$ were calculate using latitudes and longitudes.

The software "Con_cord" mentioned in Chapter 1 was used for the calculations. Also, "Geolab", the least squares adjustment software, provides $\Delta X, \Delta Y, \Delta Z$ and mark to mark distances for adjoining stations.

These distances and $\Delta X, \Delta Y, \Delta Z$ were simulated (introduced random errors according to a normal curve) using the following mathematical model:

$$
S_{s}=S_{A} \pm \sigma \pm n
$$

where,
$S_{s}=$ Simulated values
$S_{A}=$ Values before the simulation
$\sigma=$ Random error introduced according to a selected std. deviation
$n=$ Error introduced as a ppm correction
Values of $\sigma$ were obtained using the random values generation command "rnorm" in the statistical package "Splus"(Statistical Science Inc. 1993), using the mean as zero and the standard deviation as 1 mm for "A" order control points and 5 mm for " B " order points. $n$, the ppm error were used as 1 ppm or .01 ppm in different cases as shown in Table 5.1.

Table 5.1 Positional accuracy obtained for "A" order points, using different simulations

| Number <br> of <br> points | Point <br> numbers | Random <br> error <br> introduced <br> mm | ppm <br> error <br> introduced <br> mm | Average <br> positional <br> accuracy <br> cm | Range of <br> positional <br> accuracy <br> cm |
| :--- | :--- | :---: | :---: | ---: | :---: |
|  |  |  |  |  |  |
| 4 | $34,77,88,104$ | 1 | $10^{-8}$ | .50 | $.42-0.59$ |
| 4 | $1,77,88,104$ | 1 | $10^{-8}$ | 1.48 | $1.41-1.58$ |
| 5 | $34,77,80,88,104$ | 1 | $10^{-8}$ | .38 | $.28-0.53$ |
| 5 | $34,77,80,88,104$ | 1 | $10^{-6}$ | 6.05 | $5.38-7.56$ |
| 6 | $1,34,77,80,88,104$ | 1 | $10^{-8}$ | 1.25 | $0.82-1.92$ |

### 5.3.3 Fixed stations for the adjustment

When a geodetic adjustment is performed, one or more points have to be fixed and the coordinates have to be taken as correct. These points are called fixed points or "datum" points. Only one datum point was used for the adjustment performed in earlier days. For the 1932 adjustment in Sri Lanka, "Kandawala" was used as the datum point and for the NAD27 in the U.S. the station "Meades Ranch" in Kansas was the datum point.

When only one station is fixed, coordinates of all other stations are calculated relative to the fixed station. Theoretically, fixing more stations should give more accurate results, but before the technology of GPS, this theory was not practical. These fixed points are the 100 km apart "A" order points in the recommended Sri Lankan geodetic network.

We can fix more than one point, if only the coordinates of those points can be calculated to a very high accuracy. At least 1 cm absolute accuracy is required for fixing points in the primary network adjustment. Working with simulated data with 1 mm random error and different ppm errors, absolute accuracies were obtained for "A" order points as shown in Table 5.1.

Figure 5.1 shows the location of these points. It can be seen from the Table 5.1 that a $1 / 2 \mathrm{~cm}$ positional accuracy can be obtained for a few control points, if random
errors can be limited to 1 mm and GPS observations can be taken to the accuracy of .01 ppm. This indicates that the Sri Lankan geodetic network can be adjusted using 4 or 5 fixed points taking GPS observations at those locations to the accuracy of 0.01 ppm and limiting random errors to 1 mm .


Figure 5.1 Locations of points chosen for fixing
5.3.4 Procedure for GPS observation to achieve the required positional accuracy of the network

The goal of the proposed geodetic control project is to achieve 2 cm absolute accuracy for all primary points. Hence, the GPS observation procedure has to be designed to achieve this objective.

A number of distance measurement methodologies were tested using different types of simulated data to see the final positional accuracy of the network. Standard deviation
of 5 mm (for random errors) was used for data simulation. Different methodologies used, ppm errors and the positional accuracies obtained in each case are given in Table 5.2.

Table 5.2 shows that the GPS observation taken using triangulation lines (see Appendix A), with 5 mm random errors and $10^{-10} \mathrm{ppm}$ error will provide approximately 6 cm absolute accuracy for primary control points. The same procedure with $10^{-8} \mathrm{ppm}$ provides only 24.9 cm accuracy for primary control points. Also, simulated data shows that a better positional accuracy can be obtained with only $10^{-8} \mathrm{ppm}$ errors, if observations are not limited to triangulation lines. It was possible to obtain an average of 2.68 cm positional accuracy for the network, when all adjoining lines shorter than 75 km were included in the adjustment. In this adjustment, the lowest positional accuracy obtained was only 5.60 cm .

Table 5.2 Positional accuracy obtained for " B " order points using different GPS observation methods.

| Methodology used | Number of lines | Number of fixed stations | Random error introduced ( mm ) | ppm error introduced (mm) | Average positional accuracy (cm) | Range of positional accuracy (cm) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Triangulation | 110 | 4 | 5 | $10^{-8}$ | 24.94 | 14.39-40.04 |
| lines |  | 4 | 5 | $10^{-10}$ | 6.40 | 1.25-21.74 |
| Including | 127 | 4 | 5 | $10^{-6}$ | 14.75 | 5.8-27.49 |
| additional |  | 4 | 5 | $10^{-8}$ | 2.64 | 1.03-5.69 |
| lines( F 7 km ) |  | 4 | 5 | $10^{-10}$ | 2.68 | $1.00-5.60$ |
| All Possible lines | 2500 | 4 | 5 | $10^{-8}$ | 8.92 | $3.28-23.68$ |

The accuracy of 2.68 cm at stations easily provides the accuracy of $1: 1,000,000$ for the network, when compared with the average length of lines in the network. Therefore, the following procedure can be recommended in order to achieve a $1: 1,000,000$ absolute accuracy for the primary network.

1. Fix 4 points in 4 corners, which were calculated to the positional accuracy of $1 / 2 \mathrm{~cm}$ as shown in Figure 5.1
2. Limit random errors to 5 mm
3. Observe all possible adjoining distances which are shorter than 75 km using a GPS procedure which gives 0.01 ppm accuracy

### 5.3.5 Pre-adjustment using simulated data

For the prediction of co-ordinates for the Sri Lankan primary network, a least square adjustment was performed using simulated values. These simulated values were obtained using the procedure explained in section 5.3.2.

Using the Reference ellipsoid as "Everest", these simulated data for 50 stations were used to calculate the co-ordinates of fifty stations in the Sri Lankan primary network. Station numbers $34,77,80,88$ and 104 which were shown in figure 5.1 were treated as fixed. The data were processed using "Geolab". At the stage of simulating the GPS data, the elevations of the stations were not known to any reasonable degree of correctness. So, they were excluded from the purview of this evaluation and only the planimetric location have been considered. The results using these simulated values were compared with adjusted values obtained using actual data and the results are discussed in section 5.3.6.

### 5.3.6 Observation and processing of actual GPS data

GPS observations were performed at 445 stations mentioned in the previous section. These data were collected as a readjustment project to the primary control network of Sri Lanka launched by the Survey Department. Five Wild GPS receivers (System 300 Model SR399E) instruments were used for observations (see figure 5.2).

Except at the station 111, Samadi ISMD, the data collection time at each station was 3 hours. Seven days continuous data collection were done at the Samadi ISMD and


Figure 5.2 Stations used for new GPS observations. TO are old trignometrical stations and TN are newly established points.
data were processed in the single point mode. It is expected that the absolute accuracy obtained for this point by 7 days data collection is one meter. GPS data collected at the above mentioned trigonometrical stations were processed using two different methods.

In method I, the station Samadi ISMD, coded 111, was considered as fixed. Then the data of 45 stations were processed using "Geolab" to solve GPS vectors and to determine the co-ordinates in terms of latitudes and longitudes of each station.

Two steps were used to perform method II. In the first step, the vectors that were observed more than once, from the base station were isolated and and corresponding stations were identified using GPS observed vectors shown in figure 5.2. There were a total of 18 such stations. The observation data for these 18 stations were processed treating station 111 , Samadi ISMD as fixed and all other seventeen stations as free and co-ordinates of seventeen stations were determined in terms of latitude, longitude and elevation of each station. The input file and the output file for this adjustment are given in Appendix E and Appendix F respectively. The results obtained by the above step for 18 stations are free from the effects of possible errors in measurement vectors that were measured only once. Subsequently they are treated as fixed to adjust all the observed stations.

A comparison of results obtained through the above mentioned two methods is given in table 5.3. The mean difference between the two methods shows only .0005579 seconds in latitude and 0.000513 seconds in longitude. The standard error is 0.00339 seconds in latitude and 0.00462 in longitude. The values are considerably small. So the results obtained by the two methods do not show a significant difference.

A comparison of results obtained by adjusting simulated data with the results obtained by processing observed GPS data (fixing only the base station), given in table 5.4 , indicates a mean difference of 1.19845 seconds in latitude and 7.744308 seconds in longitude. The standard error is 0.13117 seconds in latitude and 0.21002 seconds in longitude. The major reason for these differences can be due to the datum shift between



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 Latitude





Table 5.4 Adjustment of simulated and actual GPS observations.

|  |  | Old Adjusted Values |  |  |  |  |  | Now Adjusted Values |  |  |  |  |  |  |  | Lat Diff sec | Long Dill sec | Elev Dill |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stn ID | Station Name | Latilude |  |  | Longilude |  |  | Allitude | Latilude |  |  | Longilude |  |  | Adj Elev |  |  |  |
| 111 | SAMADI_ISMD |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 34 | IRATTAPEKULA | 8 | 42 | 23.859900 | 80 | 28 | 57.164788 | 134 | 8 | 42 | 24.757215 | 80 | 29 | 4.604413 | 34 | -0.897315 | -7.439625 | 100 |
| 36 | ISSENBESSAWAGAL | 8 | 34 | 5.203913 | 80 | 28 | 45.782570 | 100 | 8 | 34 | 6.635614 | 80 | 28 | 52.714121 | 63 | -1.431701 | -6.931551 | 37 |
| 37 | BOGAHAWEWA | 8 | 19 | 16.580340 | 80 | 13 | 55.872460 | 105 | 8 | 19 | 17.532417 | 80 | 14 | 3.425888 | 2 | -0.952077 | -7.553428 | 103 |
| 38 | KATUPOTAKANDA | 8 | 19 | 42.937000 | 80 | 30 | 58.909000 | 404 | 8 | 19 | 43.897771 | 80 | 31 | 6.353465 | 302 | -0.960771 | -7.444465 | 101 |
| 39 | TAMBUTTKANDA | 8 | 4 | 34.294178 | 80 | 14 | 27.110663 | 281 | 8 | 1 | 35.293906 | 80 | 14 | 34.658723 | 180 | -0.999728 | -7.548060 | 101 |
| 40 | RITIGALA | 8 | 6 | 33.153685 | 80 | 39 | 16.374435 | 768 | 8 | 6 | 34.135726 | 80 | 39 | 23.753609 | 666 | -0.982041 | -7.379174 | 101 |
| 47 | MADAMOLA | 8 | 5 | 3.539557 | 80 | 7 | 46.517521 | 264 | 8 | 5 | 4.537904 | 80 | 7 | 54.107387 | 160 | -0.998347 | -7.589866 | 104 |
| 48 | CROWS_NEST | 8 | 2 | 33.513049 | 79 | 53 | 0.912860 | 100 | 8 | 2 | 34.513212 | 79 | 53 | 8.594639 | -29 | -1.000163 | -7.681779 | 129 |
| 49 | PARAMAKANDA | 7 | 54 | 22.258701 | 80 | 0 | 2.746281 | 164 | 7 | 54 | 23.286765 | 80 | 0 | 10.384913 | 60 | -1.028064 | -7.638632 | 104 |
| 51 | MEDAGAMA | 7 | 34 | 26.250752 | 80 | 7 | 30.870805 | 100 | 7 | 34 | 27.297564 | 80 | 7 | 38.386485 | 216 | -1.046812 | -7.515680 | -116 |
| 56 | NARANGAMA | 7 | 19 | 50.737116 | 80 | 6 | 46,260501 | 201 | 7 | 19 | 51.902324 | 80 | 6 | 53.870618 | 97 | -1.165208 | -7.610117 | 103 |
| 57 | ENGODA | 7 | 14 | 6.338846 | 80 | 12 | 2.862389 | 309 | 7 | 14 | 7.508936 | 80 | 12 | 10.427022 | 209 | -1.170090 | -7.564633 | 101 |
| 59 | AMBOKKA | 7 | 36 | 16.733058 | 80 | 34 | 32.201407 | 100 | 7 | 36 | 17.796710 | 80 | 34 | 39.632167 | 1132 | -1.063652 | -7.430760 | -1032 |
| 60 | YAKDESSAGALA | 7 | 34 | 58.048355 | 80 | 19 | 11.477374 | 526 | 7 | 34 | 59,112490 | 80 | 19 | 18.974153 | 422 | -1.064135 | -7.496779 | 104 |
| 61 | GALGIRIYA | 7 | 56 | 3.320560 | 80 | 22 | 45.190946 | 572 | 7 | 56 | 4.343467 | 80 | 22 | 52.682986 | 471 | -1.022907 | -7.492040 | 101 |
| 73 | dorapatacala | 7 | 5 | 58.365809 | 81 | 9 | 34.930412 | 1517 | 7 | 5 | 59.503455 | 81 | 9 | 42.098141 | 1415 | -1.137646 | -7.167729 | 102 |
| 74 | KNUCKLES | 7 | 23 | 45.847294 | 80 | 48 | 27.537084 | 100 | 7 | 23 | 46.937584 | 80 | 48 | 34.850771 | 1762 | -1.090290 | -7.313687 | -1662 |
| 75 | GOMMOLIYA | 6 | 59 | 15,472410 | 80 | 55 | 7.529257 | 100 | 6 | 59 | 16.637164 | 80 | 55 | 14.777842 | 1581 | -1.164754 | -7.248585 | -1481 |
| 76 | NAMUNUKULA | 6 | 55 | 57.711201 | 81 | 6 | 45.567178 | 100 | 6 | 55 | 58.843032 | 81 | 6 | 52.715544 | 1934 | -1.131831 | -7.148366 | -1834 |
| 78 | UGALA | 6 | 42 | 25.009359 | 81 | 16 | 10.810282 | 576 | 6 | 42 | 26.214325 | 81 | 16 | 17.911791 | 475 | -1.204966 | -7.101509 | 101 |
| 80 | BERAGALA_NORTH | 6 | 16 | 25.176000 | 80 | 54 | 44.130000 | 1778 | 6 | 46 | 26.387026 | 80 | 54 | 51.378719 | 1678 | -1.211026 | -7.248719 | 100 |
| 82 | KIRIOLUHENA | 6 | 37 | 16,692197 | 80 | 49 | 58.281216 | 734 | 6 | 37 | 17.947623 | 80 | 50 | 5.540478 | 632 | -1.255426 | -7.259262 | 102 |
| 83 | GONGALA | 6 | 23 | 7.720707 | 30 | 39 | 7.170754 | 1361 | 6 | 23 | 9.044697 | 80 | 39 | 14.478505 | 1258 | -1.315910 | -7.307751 | 102 |
| 84 | HABURUGALA | 6 | 19 | 47.791246 | 80 | 57 | 27.252158 | 100 | 6 | 19 | 49.093556 | 80 | 57 | 34.439380 | 3 | -1.302310 | -7.187222 | 97 |
| 86 | KARAMBAGALA | 6 | 14 | 15.474057 | 81 | 0 | 9.614992 | 100 | 6 | 14 | 17.281771 | 81 | 0 | 16.604945 | 19 | -1.807714 | -6.989953 | 81 |
| 88 | KADURUPOKUNA | 6 | 0 | 53.596000 | 80 | 46 | 32.876000 | 100 | 6 | 0 | 54.571854 | 80 | 46 | 40.749550 | -39 | -0.975854 | -7.873550 | 139 |
| 89 | HAMBANTOTA_T | 6 | 7 | 18.088765 | 81 | 7 | 29.862648 | 32 | 6 | 7 | 19.402884 | 81 | 7 | 36.966446 | -70 | -1.314119 | -7.103798 | 102 |
| 90 | AMANGALAKANDA | 6 | 5 | 22.266423 | 80 | 40 | 38.149575 | 227 | 6 | 5 | 23.631969 | 80 | 40 | 45.406161 | 124 | -1.365546 | -7.256586 | 103 |
| 91 | URUMUTTA | 6 | 10 | 4.664991 | 80 | 31 | 33.659379 | 434 | 6 | 10 | 6.039856 | 80 | 31 | 40.985065 | 329 | -1.374865 | -7.325686 | 105 |
| 92 | MORAWAKA | 6 | 17 | 3.070246 | 80 | 30 | 43.535120 | 100 | 6 | 17 | 4.413762 | 80 | 30 | 50.876483 | 623 | -1.343516 | -7.341363 | - 523 |
| 93 | HINDEKNATU | 6 | 7 | 30.454696 | 80 | 23 | 57.748909 | 100 | 6 | 7 | 31.850067 | 80 | 24 | 5.125776 | 307 | -1,395371 | -7.376867 | -207 |
| 94 | GALLE_TOWER | 6 | 1 | 4.411401 | 80 | 14 | 34.641509 | 100 | 6 | , | 5.791578 | 80 | 14 | 43.427665 | -31 | -1.380177 | -8.786156 | 131 |
| 95 | KURUNDAKANDA | 6 | 4 | 44.899620 | 80 | 13 | 56.780688 | 100 | 6 | 1 | 46.323352 | B0 | 14 | 4.250551 | . 13 | -1.423724 | -7.469863 | 113 |
| 96 | AMUNERIYAGODA | 6 | 7 | 1.292483 | 80 | 8 | 6.340285 | 100 | 6 | 7 | 2.702443 | B0 | 8 | 13.839409 | -67 | -1.409960 | -7.499124 | 167 |
| 97 | PANILKANDA | 6 | 16 | 11.145494 | 80 | 6 | 12.295493 | 100 | 6 | 16 | 12.515713 | 80 | 6 | 19.824587 | . 49 | -1.370219 | -7.529094 | 149 |
| 98 | HAYCOCK | 6 | 19 | 54.239095 | 80 | 17 | 51.323227 | 100 | 6 | 19 | 55.590030 | 80 | 17 | 58.768263 | 558 | -1.350935 | -7.445036 | -458 |
| 99 | BOMBUWALA | 6 | 34 | 48.161660 | 80 | 1 | 5,478606 | 100 | 6 | 34 | 49.429162 | 80 | 1 | 13.067383 | 55 | -1.267502 | -7.588777 | 45 |
| 103 | OLEBODA | 6 | 46 | 44.564128 | 80 | 0 | 45,992191 | 100 | 6 | 46 | 45.808193 | 80 | 0 | 53.589338 | 11 | -1.244065 | -7.597147 | 89 |
| 107 | ASGIRIYA | 7 | 6 | 31.575331 | 79 | 59 | 16.785001 | 100 | 7 | 6 | 32.738309 | 79 | 59 | 24.399076 | -34 | -1.163058 | -7.614075 | 134 |
| 108 | ALUTAIPOLA | 7 | 12 | 17.809067 | 79 | 57 | 40.574854 | 100 | 7 | 12 | 18.963643 | 79 | 57 | 48.201704 | . 47 | -1.154576 | -7.626850 | 147 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | Mean |  | -1.198459 | -7.443084 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | rins of dillere | ence= | 0.1855146 | 0.2970142 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | rins of single | e value= | 0.1311786 |  |  |

the co-ordinates used for obtaining simulated vectors.
So it can be concluded that an adjustment of a network using simulated GPS vectors give a reasonable estimate for the co-ordinates of a geodetic network.

### 5.4 Gravity measurements

Gravity measurements are used to calculate gravity anomalies at points. Then these gravity anomalies are used to obtain the Geoid Undulation at station. Geoid undulation is used to calculate the correction of the Deflection of vertical. Figure 3.1 shows the geoid, ellipsoid and geoid undulation. Gravity anomaly is defined as the difference between the measured gravity and normal gravity, which is defined by the following equation (Ewing and Mitchell, 1979).

$$
g_{\phi}=g_{e}\left(1+\beta \operatorname{Sin}^{2} \phi-\beta_{1} \operatorname{Sin}^{2} 2 \phi\right)
$$

where $\beta$ and $\beta_{1}$ are constants, $g_{e}$ is the gravity at the equator and $g_{\phi}$ is the gravity at a location of latitude $\phi$. For GRS80, the normal gravity is defined by the International Union of Geodesy and Geophysics (I.U.G \& G.) as follows (Ewing and Mitchell, 1979). Unit of gravity given in gals.

$$
g_{\phi}=978.0490\left(1+0.0052884 \operatorname{Sin}^{2} \phi-0.0000059 \operatorname{Sin}^{2} 2 \phi\right)
$$

Applications of the deflection of vertical are discussed in section 3.3.4. Determination of accurate geoid undulations, or (in other words) the determination of a good geoid model for a country is important not only for the adjustment of the new network, but also for many other applications such as resource exploration, seismic studies and other scientific studies in geology. This task can be performed by taking gravity measurements at about $1 / 2$ degree intervals throughout the country. Selecting locations at all the old primary control points and new proposed points, gravity measurements can be obtained at approximately 15 mile intervals. This is a satisfactory interval for gravity measure-
ments of the country. When the NAD83 was adjusted in the U.S., gravity measurements at approximately $5 \operatorname{arc}$ minutes (less than 10 miles) were used for the calculation of the geoid model and hence the calculation of the correction for deflection of verticals (Makey, 1989).

### 5.5 Azimuth observations

Azimuth observations provide the orientation to the geodetic network. The traditional method was to do astronomical azimuth observations and consider them as geodetic azimuths for the calculations. When the Sri Lankan adjustment was done in 1932, only two astronomical observations were used for the adjustment (Jackson, 1933). These two observations were at "Vavunative" and "Kandawala" which are given as station numbers 67 and 54. Four sets of astronomical observations at Vavunative (to Tavelamunai) and six sets at Kandawala (to Halgastota) were observed. Computed azimuths of each set for these observations are given below:

Vavunative : $160^{\circ} 54^{\prime}$ and seconds $24.94,33.70,27.42,30.24$,
Kandawala: $176^{\circ} 41^{\prime}$ and seconds $32.83,34.87,36.43,35.80,27.65,33.07$
It shows that the standard errors for these two sets are 3.76 and 3.18 seconds respectively for both stations.

5000 astronomical azimuth observations were used for the NAD83 adjustment of the U. S. These astronomical azimuths, which were observed by the National Geodetic Survey (NGS) prior to 1970, are accurate up to 1.1 arc seconds (Gergen, 1989).

A considerable amount of azimuth observations are required to do a good orientation of the ellipsoid, which is the mathematical surface of the geodetic network. Astronomical azimuth observations used for the 1932 adjustment in Sri Lanka and also the observations used for NAD83 show that the accuracy of astronomical azimuth determination is usually low.

GPS provides accurate azimuths based on the GRS80 ellipsoid (Wells, 1987). Usage of GPS azimuths for the new adjustment depends on the selection of the mathematical surface for the calculations. This is one of the criteria for the selection of a suitable mathematical surface (ellipsoid) for Sri Lanka, which is discussed in detail in the next section.

### 5.6 Mathematical surface for the geodetic network

When we work with coordinate systems of the earth, or when taking measurements on the earth, we have to consider three surfaces. They are the actual physical surface of the earth, the geoid and the reference ellipsoid (see Figure 3.1).

The actual physical surface of the earth is so rugged that it is not suitable for mathematical calculations. An equipotential surface, usually taken as the mean sea level is defined as the "geoid". The reference ellipsoid is the mathematical surface used for calculations on the earth surface. This is the best approximation to the earth (Bomford, 1980). Therefore, the ellipsoid (or some times known as spheroid) was used as the mathematical approximation to the earth surface and for all geodetic calculations for centuries. The ellipsoid is defined as a three dimensional surface which can be obtained by rotating an ellipse around its minor axis.

When defining an ellipsoid for geodetic calculations, the following independent constants have to be defined (Bomford, 1980):

1. Length of the semi major axis.
2. Length of the semi minor axis (or flatness).
3. Direction of the minor axis (usually taken as parallel to the earth's mean polar axis).
4. The definition of the center of the ellipsoid (the earth's center of Gravity is used for GRS 80 ellipsoid, which is the accepted ellipsoid by the

International Association of Geodesy).
5. An initial point of zero longitude. Usually Greenwich is used by all nations.
6. The definition of one to one correspondence between ground points and the points on the ellipsoid.

The definition of the earth's center of gravity was possible only recently, when satellite geodesy became a possibility. As a result, three constants had to be defined in place of the center of the ellipsoid before the era of artificial satellites.

1. A locally defined origin (datum point). This is a station defining the connection between the earth surface and the ellipsoid. Also, this point defines the height of the earth surface above the ellipsoid.
2. Astronomic azimuth at the origin is defined as equal to the geodetic azimuth
3. Astronomic latitude at the origin is defined as equal to the geodetic latitude.

Table 5.5 Sizes of different ellipsoids and the year they were introduced. (Source: Bomford, 1980 and Moffit and Bouchard, 1992)

| Ellipsoid | year | Semi major axis | semi minor axis |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| Everest | 1830 | 6377310 | 6356109 |
| Bessel | 1841 | 6377397 | 6356079 |
| Clarke | 1866 | 6378206 | 6356584 |
| Clarke | 1880 | 6378301 | 6356566 |
| Hayford | 1909 | 6378388 | 6356912 |
| Krassovsky | 1948 | 6378245 | 6356863 |
| Fischer | 1960 | 6378155 | 6356773 |
| WGS72 | 1974 | 7378135 | 6356750 |
| GRS80 | 1977 | 6378137.0 | 6356752.314 |

Different reference ellipsoids were used by different countries. Table 5.3 shows their semi major and semi minor axis. Reference ellipsoids defined after the reliable determination of gravitational center of the earth use the center of the earth as the center of the ellipsoid. Older ellipsoids use the datum point to relate the ellipsoid to the earth surface. Ellipsoids which use datum points are called local reference ellipsoids. For the
geodetic control adjustment of Sri Lanka in 1932, the "Everest ellipsoid was used as the mathematical surface. For the NAD27 in the U.S., Clark's 1866 ellipsoid was used. When the NAD83 was computed, the earth centered ellipsoid GRS80 was adopted as the mathematical figure of the earth by all North American Countries (Schwartz, 1985).

Although a well-defined earth-centered ellipsoid is available today, and accepted by the International Association of Geodesy (IAG), many countries still continue to use local ellipsoids. According to Bomford, "International values have been agreed on, but in most countries, past history continues to dictate the adoption of others." The reluctance for a change is due to the lack of understanding about the importance of a common, earth-centered ellipsoid for the world (Bomford, 1980). With the wide use of GPS in civilian applications, the concept of a common ellipsoid will be more popular in future.

### 5.7 Everest vs. GRS80 for Sri Lanka

It is very controversial when a suitable reference ellipsoid is recommended for the adjustment of a geodetic network. Sri Lanka has two choices when adopting a reference ellipsoid. The first is to use the Everest ellipsoid, which was used for the 1932 adjustment. The second is to use the earth centered reference ellipsoid adopted by the International Association of Geodesy, the GRS80. Characteristics of both reference ellipsoids are discussed below in detail in order to evaluate the advantages and disadvantages of both ellipsoids before using them in a new adjustment.

### 5.7.1 Everest ellipsoid

(1) Locally fit to the earth surface. Therefore, the geoid undulation is much smaller than an earth-centered ellipsoid. Geoid undulation is used to apply the correction for deviation of vertical (see Stoke's and Vening Meinesz formulas in Heiskanen and Moritz, 1967). When gravity observations are not available for the calculation of the correction
for the deflection of vertical, smaller geoid undulations are important because this correction can then be neglected. If sufficient gravity measurements are available for the calculation of the deflection of vertical there is no significant importance of having a smaller geoid undulation.
(2) Today, the size of the earth is known to the nearest meter (Bomford, 1980). New values (GRS80) for the size of the earth to the nearest meter are:

Semi major axis: 6378137
Semi minor axis: 6356752
The Everest ellipsoid used in 1932 for the adjustment of the Sri Lankan network uses the following values as semi major and semi minor axis of the earth (Jackson and Price, 1933):

Semi major axis: 6377310
Semi minor axis: 6356109
This shows that the Everest ellipsoid is approximately 827 meters shorter in its major axis and 643 meters shorter in the minor axis than the current values. Therefore, the Everest ellipsoid is not a good approximation of the earth. There is no reason to use a distorted ellipsoid as the mathematical figure of the earth surface.

### 5.7.2 GRS80 ellipsoid

(1) This is an earth-centered ellipsoid. The Geoid undulation for Sri Lanka for GRS80 is about 100 meters, according to the world geoid model (Rapp, 1988). If GRS80 is used as the mathematical surface for Sri Lanka, this large geoid undulation has to be calculated and used for the correction of deflection of the verticals. For this, sufficient gravity measurements have to be taken.
(2) GRS80 is the currently available best defined approximation to the earth and accepted by the international Association of Geodesy.
(3) This ellipsoid is used as the reference ellipsoid for GPS. Therefore, the azimuths and ellipsoidal heights given by GPS can be directly used for the adjustment.
(4) Normal gravity of GRS80 is defined by the International Union of Geodesy and Geophysics. Hence, the gravity anomalies can be calculated after gravity measurements, which are used for the calculation of deviation of vertical.
(5) There are some concerns about the national security of a country if a common earth centered ellipsoid is used for the calculation of coordinates because then the precise locations become wide open information. This concern has not prevented the use of GRS80 reference ellipsoid for NAD83 by the U.S. and all other North American nations. This concern cannot be taken as a major issue when selecting a mathematical figure for the coordinate calculation of Sri Lanka.

When the above characteristics of Everest and GRS80 ellipsoids are considered, it can be recommended that the GRS80 is a better choice for the new geodetic control network for Sri Lanka. Also, it should be recommended that enough gravity measurements be taken and a good geoid model be defined for the country in order to calculate the correction for deviation of vertical. GRS80 ellipsoid will provide all the conveniences needed for the wide use of GPS in the country.

### 5.8 Reduction of angle and distance observations

Measured distances and directions have to be ccrrected for the ellipsoid before they are used in the adjustment. There are corrections due to the shape of the earth and the variations of gravity. These corrections are explained in detail in almost all Geodesy books (eg. Bomford or Heiskanen and Moritz). A list of corrections required for the geodetic calculations are given below(Jeyapalan, 1994):

1. Reduction of measured mark to mark distances to the ellipsoidal arc.
2. Correction for directions due to deviation of vertical
3. Correction for directions due to skew normals
4. Correction for directions due to convergence of meridians

### 5.9 The least squares adjustment

Since its introduction by Gauss in 1801, (published by Legendre in 1820) least squares adjustment plays a major role in many calculations (Uotila, 1986). When more measurements are made than the minimum requirement for the calculation of unknowns (redundant observations), the least squares principle provides the best accepted procedure for getting the most probable value for unknowns.

The basic idea of the least squares principle is very simple but it provides excellent results for calculations with redundant observations. The principle assumes that the sum of squares of residuals of observations be minimum for the best solution for unknowns and gives the most probable values for unknown parameters. Residuals are defined as the difference between the most probable values and the measured values of observations. The objectives of a least squares solution can be given as follows (Bomford, 1980):

1. To produce a unique value for unknowns
2. To obtain a solution which has the maximum probability
3. To get an indication about the precision with which the unknowns have been determined.

When adjusting a geodetic control network, the least squares solution can be performed in two ways. They are:

1. The method of observation equations
2. The method of condition equations

Observed directions, lengths, coordinates etc, are written as a function of unknowns in the method of observation equations. In the method of condition equations, the errors of all observations are taken as unknowns. Then, these errors are determined and the
observations are corrected. Once the observations are corrected, they can be used to determine unknowns.

A number of least squares solutions were performed for the Sri Lankan network, using different combinations of available observations. First, using only the observations used for the 1932 adjustment and then using all the available data including 29 new distance observations and 21 height observations. The first adjustment provided the extent of errors in the existing system due to improper adjustment procedures. The second adjustment provided better values for coordinates, but the absolute accuracies obtained are only within 10 meters. Therefore, Sri Lanka has to perform a new geodetic control adjustment using new reliable observations in order to provide the required accuracy for GIS. Procedure and recommendations for taking GPS observations for this purpose is discussed in sections 5.3.4.

### 5.10 Approximate values of parameters for the new adjustment

In the method of observation equations, observations are written as a function of parameters (Uotila, 1986). This function is the mathematical model for the adjustment. If there are $s$ number of observations with $t$ unknowns, observation equations can be written as follows:

$$
\begin{aligned}
a_{1} x_{1}+b_{1} x_{2}+\cdots+t_{1} x_{t} & =k_{1}+v_{1} \\
a_{2} x_{2}+b_{2} x_{2}+\cdots+t_{2} x_{t} & =k_{2}+v_{2} \\
\ldots \cdots \cdots \cdots \cdots \cdots \cdots & =\cdots \cdots \\
\ldots \cdots \cdots \cdots \cdots \cdots \cdots & =\cdots \cdots \\
a_{s} x_{1}+b_{s} x_{2}+\cdots+t_{s} x_{t} & =k_{s}+v_{s}
\end{aligned}
$$

As there are redundant observations, $(s-t)$ is defined as the degree of freedom. Parameters, ( $x$ values) are $\delta \dot{\phi} \mathrm{s}$ and $\delta \lambda \mathrm{s}$, where

$$
\begin{aligned}
\delta \phi & =\phi_{0}-\phi \\
\delta \lambda & =\lambda_{0}-\lambda
\end{aligned}
$$

$\phi_{0}$ and $\lambda_{0}$ are approximate values for parameters and $\phi$ and $\lambda$ are the most probable values of parameters. $v$ 's are residuals of observations and $k$ 's are the differences between actual observations and the values calculated for observations using approximate values of parameters. In a real world situation, it can be seen that many of $a$ 's and $b$ 's will be zero because observations do not depend on all the parameters.

Hence, it is required to calculate approximate values of parameters before the adjustment. As many of the network points for the new adjustment for Sri Lanka will be from the old adjustment done in 1932, coordinates obtained by the 1932 adjustment can be used as approximate values for the new adjustment. For other new GPS points, approximate values can be obtained by processing GPS vectors with respect to any other old station. Old coordinates, which can be used as approximate values, are given in Appendix A.

### 5.11 Weights and priori variance

When the reliability of observations are not the same, it is appropriate to use different weights for different observations in the adjustment. Assume that there are $l_{1}, l_{2}, \cdots, l_{n}$ observations and the variance-covariance matrix for these observations is $\sum_{L^{b}}$. The variance-covariance matrix is formed by the variances of each observation in its diagonal elements and co-variances in non-diagonal elements. When the observations are uncorrelated, the variance-covariance matrix becomes a diagonal matrix with only variances in the diagonal elements. Weights for observations can be written in a matrix form as
follows:

$$
P_{x}=\sigma_{0}^{2} \sum_{L^{b}}^{-1}
$$

where, $P_{x}$ is called the weight matrix and $\sigma_{0}^{2}$ is called the priori variance of observations. $\sigma_{0}^{2}$ can be any pre selected number and it is unit less (Uotila, 1988). When observations are un-corelated, we can select $\sigma_{0}^{2}$ as one and define the weight matrix as $\Sigma_{x}^{-1}$, which will be formed only by the variances.

For the adjustment of Sri Lankan geodetic network, variances for each observations have to be calculated in order to form the variance-covariance matrix, assuming that there are no co-relations among different types observations.

### 5.12 Observation equations

Directions and distances are the usual observations for many least squares adjustments in surveying and geodesy. Observation equations for these two types of observations can be written as follows (Oliver, 1977):

### 5.12.1 Directions

Assume that:
$\alpha=$ Observed direction (according to an approximate azimuth)
$z=$ Orientation correction
$v=$ Residual
$\alpha_{0}=$ Approximate value of observation calculated using approximate values of parameters
$d \alpha=$ Correction for the approximate value
Then,

$$
\alpha+z+v=\alpha_{0}+d \alpha
$$

This observation equation for the azimuth from station 1 to station 2 can be written in terms of parameters, which are the corrections for approximate values of latitudes and longitudes.

$$
d \alpha=\frac{\rho_{1} \operatorname{Sin} \alpha_{12}^{0}}{s} d \phi_{1}+\frac{\rho_{2} \operatorname{Sin} \alpha_{21}^{0}}{s} d \phi_{2}+\frac{\nu_{2} \operatorname{Cos} \phi_{2} \operatorname{Cos} \alpha_{21}}{s}\left(d \lambda_{1}-d \lambda_{2}\right)
$$

where $\rho$ and $\nu$ are the radii of curvatures of prime vertical and meridian. $s$ is the distance between two points, which can be calculated using approximate values of parameters. $\alpha_{12}$ is the azimuth from station 1 to station 2 and $\phi_{1}, \phi_{2}, \lambda_{1}, \lambda_{2}$ are latitudes and longitudes of stations 1 and 2 .

### 5.12.2 Distances

If $s$ is the observed distance and $s_{0}$ is the approximate value for the distance obtained using the approximate values of parameters,

$$
s+v=s_{0}+d s
$$

This observation equation can be written in terms of parameters as (Olliver, 1977):

$$
d s=-\rho_{1} \operatorname{Cos} \alpha_{12}^{0} \operatorname{Sin} 1 " d \phi_{1}-\rho_{2} \operatorname{Cos} \alpha_{21}^{0} \operatorname{Sin} 1 " d \phi_{2}+\nu_{2} \operatorname{Cos} \phi_{2} \operatorname{Sin} \alpha_{21}^{0} \operatorname{Sin} 1 "\left(d \lambda_{1}-d \lambda_{2}\right)
$$

Those observation equations provide the accuracy better than $\pm 0.05, \pm 0.15$ and $\pm 0.5$ seconds for lines of $20 \mathrm{~km}, 50 \mathrm{~km}$ and 200 km respectively (Olliver,1977). Since the iteration procedure is used to calculate the most probable values of parameters, the accuracy of those observation equations are satisfactory for a least squares adjustment with an iterative procedure.

### 5.13 Solution for parameters

When a least squares solution is obtained using observation equations, the procedure is to form normal equations first and then perform a solution for parameters. A short
description of the procedure for obtaining this solution and the method of evaluating results are given below (Uotila, 1986):

Assume that $l_{1}^{b}, l_{2}^{b}, \cdots, \cdots, l_{n}^{b}$ are observations and $v_{1}, v_{2}, \cdots, \cdots, v_{n}$ are residuals associated with observations. Parameters are $x_{1}, x_{2}, \cdots, \cdots, x_{m}$. If the mathematical model is linear,

$$
\begin{aligned}
& l_{1}^{b}-v_{1}=a_{11} x_{1}+a_{12} x_{2}+\cdots+\cdots+a_{1 m} x_{m} \\
& l_{2}^{b}-v_{2}=a_{21} x_{1}+a_{22} x_{2}+\cdots+\cdots+a_{2 m} x_{m} \\
& \cdots \cdots=\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \\
& l_{n}^{b}-v_{n}=a_{n 1} x_{1}+a_{n 2} x_{2}+\cdots+\cdots+a_{n m} x_{m}
\end{aligned}
$$

These equations can be written in matrix form as follows:

$$
L_{b}-\epsilon=A . X
$$

When the expected values are considered, expected values of residuals become zero. So, the equation can be written as:

$$
\hat{L}_{a}=A \cdot \hat{X}_{a}
$$

where $\hat{L_{a}}$ is the estimate for true values of observations and $\hat{X}$ is the expected values of parameters.

Residuals $V$, is given by:

$$
V=\hat{L_{a}}-L_{b}
$$

Then the observation equation can be written as:

$$
V=A \cdot \hat{X}_{a}-L_{b}
$$

For a non linear mathematical model, observation equation can be obtained using the Taylor's series (Uotila, 1986):

$$
L_{b}-\epsilon=F\left(X^{a}\right)=F\left(X^{0}\right)+\left.\left(\frac{\partial F}{\partial X_{a}}\right)\right|_{X_{a}=X_{0}} .\left(X_{a}-X_{0}\right)+\cdots+\cdots
$$

where $X_{0}$ are approximate values of parameters.
Using approximate values of parameters, we can calculate a theoretical value for observations. These are known as approximate values for observations and denoted as $L_{0}$. Assuming,

$$
X=\hat{X}_{a}-X_{0} \quad \text { and } \quad L=L_{0}-L_{b}
$$

the observation equation for a non-linear mathematical model can be written as:

$$
V=A \hat{X}_{a}+L
$$

where,

$$
A=\left.\frac{\partial F}{\partial X_{a}}\right|_{X_{a}=X_{0}}
$$

In other words, $A$ is the matrix of partial derivatives of observations with respect to parameters. When there are $n$ observation equations and $m$ parameters, order of the $A$ matrix become $n \times m$.

For a least squares solution, we have to minimize $\sum v^{2}$ or in matrix form $V^{T} V$, assuming the weight matrix as the identity matrix. Including weights, we can minimize $V^{T} \Sigma^{-1} V$. where $\sum$ is the variance-covariance matrix for observations. If observations are uncorrelated, $\Sigma$ becomes a diagonal matrix.

Considering a constant, $\sigma_{0}^{2}$, the weight matrix $P$ can be defined as:

$$
P=\sigma_{0}^{2} * \Sigma^{-1}
$$

The constant $\sigma_{0}^{2}$ is known as the priori variance of unit weight and usually taken as one.
$V^{T} P V$ can be minimized by taking partial derivatives of functions with respect to each variable and making them equal to zero (Uotila, 1986). The solution for parameters can be written in matrix form as:

$$
\hat{X}_{a}=-\left(A^{T} P A\right)^{-1} \cdot\left(A^{T} P L\right)
$$

The results of a new least squares adjustment for the primary geodetic control of Sri Lanka obtained using observation equations (Using "Geolab") are given in Appendix C. The Everest ellipsoid was used as the mathematical figure for this adjustment. Observations used are the same observations used for the 1932 adjustment.

### 5.14 Evaluation of the adjustment

The evaluation of the adjustment of a geodetic network is necessary to see acceptability of the results of the adjustment. In early days this evaluation was done using statistical methods. That is by calculating two variance-covariance matrices for adjusted observations and parameters. Also, a constant called a posteriori variance of unit weight, $\left(\hat{\sigma_{0}^{2}}\right)$ is calculated after the adjustment, and compared with the priori variance of unit weight. Calculation of the posteriori variance of unit weight is discussed in section

### 5.14.2.

In addition to these error statistics, today, additional GPS observations can be used for the evaluation of the geodetic network, as GPS provides centimeter level positional accuracy for relative positioning. In order to get this centimeter level accuracy, a correct procedure has to be used as described in the next section.

### 5.14.1 Using GPS observations for evaluation

GPS provides positional accuracy higher than 1 ppm and hence can be used to do an accuracy analysis of a geodetic network (Snay, 1989). Also, GPS can be used to evaluate the accuracy of the densified control (with " $C$ " order points), which is the next step after establishing the primary control.

For the evaluation of the primary network ("A" and "B" order points), new GPS observations can be taken at primary stations. Then the vectors obtained by new observations can be compared with the vectors obtained by reverse calculations of coordinates of the new adjustment.

When the NAD83 coordinate system was established for North America, an accuracy analysis was made using additional GPS observations. For this analysis, GPS data between stations were taken and processed. Components parallel to two stations (collinear component) and the component perpendicular to two stations (transverse component)
were compared. This comparison and evaluation was based on the following mathematical model (Snay, 1989).

$$
e=a . K^{b}
$$

where $e$ is the RMS (Root Mean Square) error of the vector component in meters and $K$ is the inter-station distance in kilometers. Values for $a$ and $b$ for different levels of network were used as given in Table 5.6.

Table 5.6 Parameters used for equation 5.1 for the evaluation of different levels of networks (Snay, 1989)

| Order | Collinear component | Transverse component |
| :--- | :--- | :--- |
|  |  |  |
| First | $a=0.008, b=0.7$ | $a=0.02, b=0.5$ |
| Second | $a=0.01, b=0.7$ | $a=0.025, b=0.5$ |
| Third | $a=0.01, b=0.7$ | $a=0.03, b=0.5$ |

This methodology can also be used for the evaluation of the new geodetic control network of Sri Lanka, in addition to the statistical evaluations discussed in the next section.

### 5.14.2 Variance-covariance matrices for parameters and adjusted observations

Results of the new adjustment can be evaluated using the variance covariance matrices of adjusted parameters, adjusted observations and the posteriori variance of unit weight (Uotila, 1986).

Variance-covariance matrix of adjusted parameters gives a measure about the absolute accuracy of adjusted parameters and can be calculated as follows (Uotila, 1986): As given in section 5.13

$$
\begin{aligned}
\hat{X}_{a} & =X_{0}+X \\
L & =L_{0}-L_{b}
\end{aligned}
$$

Then

$$
\begin{array}{r}
\hat{X}_{a}=X_{0}-\left(A^{T} P A\right)^{-1}\left(A^{T} P L\right) \\
=X_{0}-\left(A^{T} P A\right)^{-1}\left[A^{T} P\left(L_{0}-L_{b}\right)\right] \\
=X_{0}-\left(A^{T} P A\right)^{-1} A^{T} P L_{0}+\left(A^{T} P A\right)^{-1} A^{T} P L_{b}
\end{array}
$$

But. for a mathematical model

$$
Y=G \cdot X+C
$$

The variance-covariance matrix of $Y$ is given as a function of the tariance-covariance matrix of X (Uotila. 1986):

$$
\Sigma_{Y}=G \Sigma_{x} G^{T}
$$

Ūsing this relationship, we can write the variance covariance matrix for adjusted parameters as:

$$
\sum_{\lambda_{a}}=\left(A^{T} P A\right)^{-1} A^{T} P \sum_{L_{B}} P A\left(A^{T} P A\right)^{-1}
$$

Using the relationship of

$$
P_{z}=\sigma_{0}^{2} \sum_{L^{b}}^{-1}
$$

this relationship can be simplified as:

$$
\sum_{\hat{k}_{a}}=\sigma_{0}^{2} \cdot\left(A^{T} P . A\right)^{-1}
$$

Variance covariance matrix of adjusted observations can be obtained using the following equation (Cotila. 1986), and be used to evaluate the quality of each observation.

$$
\sum_{\dot{L_{a}}}=\dot{\sigma_{0}^{2}} \cdot\left(. A^{T} P A\right)^{-1} . A^{T}
$$

where $\hat{\sigma}_{0}^{2}$ is the posteriori variance of unit weight.
The posteriori variance of unit weight is given by:

$$
\hat{\sigma_{0}^{2}}=\frac{V^{T} P V}{n-u}
$$

where $n$ is the number of observations, $u$ is the number of parameters and $(n-u)$ is the degree of freedom. The posteriori variance of unit weight has to be compared to the priori variance of unit weight, which is chosen before the adjustment. A good adjustment gives equal or closer vaiues for both of these constants.

## 6 TRANSFORMATION PARAMETERS FOR THE NEW ADJUSTMENT

When a new geodetic network adjustment is performed. we get a new set of latitudes and longitudes of stations, which are different from the old set of latitudes and longitudes. Maps are prepared using a two dimensional coordinate system designed using a suitable map projection for the country. Therefore, after the new adjustment there are three sets of different coordinates in the country. They are:

1. Old geodetic coordinates (Latitudes and Longitudes)
2. New geodetic coordinates
3. Old two dimensional coordinates

As mentioned in Chapter 5, there are three categories of coordinate users in a country. Primary users (geodesists) establish the coordinate system. All the other users who usually have only a little knowledge about coordinate adjustments, make use of those coordinates. When one set of coordinates are replaced by another, secondary and tertiary users have to be well-informed about the change and sufficiently supported to overcome any difficulty due to the new introduction of coordinates. Therefore, primary users (Geodesists) have the responsibility of providing an easy and accurate method to transform coordinates between old and new systems. Also, latitudes and longitudes have to be converted to a plane coordinate system using a suitable map projection for the country.

Transformation of coordinates must be unique and uniform. Also, the extent of
possible errors in transformation of coordinates must be known. Availability of these information to secondary and tertiary users will prevent any possible confusion. Therefore, the responsibility of primary users are not only readjusting and publishing new coordinates, but also devising a good system of coordinate transformation of the country.

There can be two major differences between different rectangular coordinate systems. They are:

1. Shift of the origin
2. Rotation of axes

Transformation of coordinates of a country from one system to another can be done in several ways. Some popular options are:

1. Direct transformation of latitudes and longitudes
2. Convert latitudes and longitudes to a rectangular earth centered coordinate system before the transformation
3. Convert to a two dimensional coordinate system using a map projection and then do the transformation

In addition to these methods of coordinate transformations, we can do the transformation and calculate transformation parameters using different mathematical models. Examples are:

1. Affine transformations
2. Transformations using a rotation and a transformation matrix

Affine transformations can be done as linear or a higher order transformations. Some conditions such as perpendicularity of axis, a common scale factor etc, can also be applied to these mathematical models. Application of conditions eliminates some of the parameters and hence reduces the need of points common to both new and old systems. Usually a satisfactorily accurate system of coordinate transformation and a mathematical model for a country has to be accepted after studies of achievable accuracy in
coordinate transformation. For the coordinate transformation in the U.S. from NAD27 to NAD83, many transformation methodologies have been introduced by the DMA. (Defense Mapping Agency) and NGS (:National Geodetic Surveys). "NADCON" is one of the softwares introduced by the NGS for this purpose. NADCON provides $0.15-0.50 \mathrm{ac}$ curacy of coordinate transformations for the U.S. (Dewhurst, 1990). For the generation of a data base for GIS applications. which include transformation parameters to convert $\mathcal{N} \mathrm{AD} 27$ to NAD83, it has been proposed to calculate transformation parameters for each 7.5 by 7.5 minute rectangle blocks in the continental United States. It is expected that these transformation parameters provide the accuracy of coordinate transformation within centimeter level (Shrestha and Dicks, 1990). Coordinate transformation accuracies obtained for Sri Lanka is given in section 6.3.

### 6.1 Mathematical models for coordinate transformations

For coordinate transformations, assuming a shift of the origin of the coordinate system (translation) and a rotation of axes, parameters for transformation of geodetic coordinates can be calculated using number of methods:

1. Convert latitudes and longitudes to global $X, Y, Z$ coordinates and then do a rotation and a translation
2. Convert latitudes and longitudes to global $X, Y, Z$ coordinates and then use a linear affine transformation.
3. Convert old latitudes and longitudes directly to new latitudes and longitudes using a second or higher order polynomial

In the first method, the mathematical model can be written as:

$$
\left(\begin{array}{l}
X^{\prime} \\
Y^{\prime} \\
Z^{\prime}
\end{array}\right)=R \cdot\left(\begin{array}{l}
X \\
Y \\
Z
\end{array}\right)+\left(\begin{array}{c}
\Delta X \\
\Delta Y \\
\Delta Z
\end{array}\right)
$$

where, $X^{\prime}, Y^{\prime}, Z^{\prime}$ are new coordinates, $X, Y, Z$ are old coordinates, $\Delta X, \Delta Y, \Delta Z$ are translations and $R$ is the rotation matrix. Assuming that the rotation around $Z$ axis is $\kappa$, rotation around $Y$ axis is $\phi$ and the rotation around $X$ axis is $\omega$, the rotation matrix $R$ can be written as:

$$
R=\left(\begin{array}{ccc}
\cos \kappa & -\sin \kappa & 0 \\
\sin \kappa & \cos \kappa & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
\cos \phi & 0 & \sin \phi \\
0 & 1 & 0 \\
-\sin \phi & 0 & \cos \phi
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \omega & -\sin \omega \\
0 & \sin \omega & \cos \omega
\end{array}\right)
$$

In this mathematical model, unknowns are $\kappa, \phi, \omega$ and $\Delta X, \Delta Y, \Delta Z$. A least squares solution for parameters in this mathematical model can be obtained using at least 3 points common to old and new systems.

In the second case, the mathematical model for the affine transformation is:

$$
\begin{aligned}
X^{\prime} & =a_{1} X+b_{1} Y+c_{1} Z+\Delta X_{0} \\
Y^{\prime} & =a_{2} X+b_{2} Y+c_{2} Z+\Delta Y_{0} \\
Z^{\prime} & =a_{3} X+b_{3} Y+c_{3} Z+\Delta Z_{0}
\end{aligned}
$$

There are 12 unknowns in this mathematical model. So, we need at least 4 common points, in order to calculate transformation parameters. More than 4 points will provide a least squares solution.

In the third case, old latitudes and longitudes are directly transformed to new latitudes and longitudes. For this case, a second order mathematical model can be written as:

$$
\begin{aligned}
& \phi^{\prime}=\phi+a_{1} \phi+a_{2} \lambda+a_{3} \phi^{2}+a_{4} \lambda^{2}+a_{5} \phi \lambda+\Delta \phi \\
& \lambda^{\prime}=\lambda+b_{1} \phi+b_{2} \lambda+b_{3} \phi^{2}+b_{4} \lambda^{2}+b_{5} \phi \lambda+\Delta \lambda
\end{aligned}
$$

where, $\phi^{\prime}, \lambda^{\prime}$ are new latitudes and longitudes, and $\phi, \lambda$ are old values. In this mathematical model, there are 12 unknowns, and geodetic coordinates are not transformed into global $X, Y, Z$ coordinates. Hence, at least 6 common points are required for the transformation by this mathematical model.

Out of these three mathematical models, the second case, the direct transformation of global $X, Y$ and $Z$ coordinates has an advantage because GPS provides global $X, Y$ and $Z$ coordinates with respect to GRS80 ellipsoid (Jeyapalan: 1995). The software (Coneord) written for coordinate transformations in Sri Lanka uses this mathematical model.

### 6.2 Methods used for the coordinate transformation from NAD27 to NAD83

Several procedures of coordinate transformation were used in the U.S., when coordinates from NAD27 were transformed to NAD83. These different procedures used different mathematical models and are capable of producing different results. A summary of their characteristics and accuracies are given in Table 6.1. As the common control points for NAD27 and NAD83 are not densely spreaded, it has become necessary to use a number of different interpolation and extrapolation techniques for many of these methods in order to get a reasonable accuracy in coordinate transformations. As the average distance between primary control points in Sri Lanka is 30 km , and also, all possible old primary points are to be included into the new adjustment, we can expect that the interpolation and extrapolation techniques need not to be used to achieve a sufficient accuracy in coordinate transformation for Sri Lanka.

In the regression analysis method, coordinate difference between NAD27 and NAD83 (datum shift) was defined using a two dimensional polynomial. This procedure was assumed as a solution for thinly spreaded common points for both systems. "LEFT1" is

Table 6.1 Comparison of various transformation methodologies used in the U.S. for the transformation of coordinates from N.AD27 to NAD83. Source: (Dewhurst, 1990)

| Methods Originator | Advantages | Disadvantages | Apcroximate zemary <br> (m) | Field use (Yes/No) |
| :---: | :---: | :---: | :---: | :---: |
| Molodersiy DMA Abridged Molodensky | Defined woridwide | General <br> Doppler-derived | - 5-10 | Yes |
| $\begin{array}{ll} \text { Regression } \\ \text { anaiysis } \end{array} \quad \text { DMA }$ | Defined woridwide | Incerurate <br> Cumbersome Local Dependency | -3.5 | Yes |
| LEFH NGS | Documented | Eremal data required <br> Geomery dependent Awhoward Expert sequised | 1.9 | No |
| NADCON NGS | Far <br> Acrurate <br> Coatinuous <br> Sandardized <br> Singie source <br> Consistent | Interpolation and errapolation | $0.15 \cdot 0.5$ | Yes |
| independently varies derived | Tailored for user | Not sandardized Expers may be required Discontinuous | Varies | Pertaps |

a computer program which uses actual NAD27 and NAD83 coordinates with an affine transformation to calculate parameters and then do the conversion. "Nadcon" uses a similar technique but uses interpolated gridded points as common points for the calculation of parameters (Dewhurst, 1990).

### 6.3 Accuracy of coordinate transformation of Sri Lanka

Accuracy of a coordinate transformation can be correctly evaluated only after the finalization of the new set of coordinates. A final set of transformation parameters can also be calculated only when the new coordinates are finalized. When an adjustment is performed using a methodology similar to the proposed methodology for the new adjustment but using only a limited number of new observations, a fairly accurate study can be done about the deviation pattern between the old and new coordinates. This pattern of coordinate differences can be expected to be consistent for Sri Lanka, because all the observations used for the old adjustment have approximately the same precision and also it seems that the differences are mainly due to the poor methodology used in the adjustment procedure of the old adjustment.

Using a new least square adjustment performed using old observations and all currently available new observations, a new set of coordinates were calculated. These new coordinates were used to calculate transformation parameters. Transformation parameters were calculated for number of different sizes of geographical areas in the country. These transformation parameters were used to transform old coordinates to new coordinates and compared with the actual values of new coordinates. Results obtained for each method of transformations are given in Tables 6.1 and 6.2.

Results in Table 6.3 show that a set of transformation parameters with an average accuracy of 14 cm can be published for the district level. This accuracy will satisfy the needs of almost all secondary and tertiary level coordinate users in the country. If

Table 6.2 Accuracies of different coordinate transformations for Sri Lanka, up to provincial level

| Methodology <br> used | Number of <br> points <br> used | Zone or <br> province | Max \& min <br> errors (m) in <br> X, Y or Z | Average of <br> RMS |
| :--- | :--- | :--- | :---: | ---: |
|  |  |  |  |  |
| One set of <br> (meters) |  |  |  |  |
| parameters for <br> entire country | 110 |  | $0.002-3.525$ | 1.059 |
|  |  |  |  |  |
| Two equal | 5.5 | Northern | $0.001-1.344$ | 0.801 |
| zones | 5.5 | Southern | $0.006-2.924$ | 0.553 |
|  |  |  |  |  |
| According to | 17 | Western | $0.000-0.35 .5$ | 0.166 |
| provinces | 13 | Central | $0.020-0.360$ | 0.253 |
|  | 35 | Northern | $0.006-0.52 .5$ | 0.250 |
|  | 16 | Sabara | $0.002-0.816$ | 0.408 |
|  | 18 | Uva | $0.003-0.556$ | 0.359 |
|  | 16 | Southern | $0.001-0.541$ | 0.24 .5 |
|  | 18 | NCP | $0.009-0.588$ | 0.366 |
|  | 13 | NWP | $0.008-0.6 .24$ | 0.345 |
|  | 18 | Eastern | $0.004-1.079$ | 0.562 |

more accuracy in coordinate transformation is required, it has to be done using actual values of old and new coordinates of common points. The software "Con_cord" can be used with 4 common points for this purpose. "Con_cord" was written to transform coordinates using a 12 parameter affine transformation described in section 7.1. using the exact values of common points. No interpolation or extrapolation technique was used for transformations. The main menu of "Con_cord" is given in Appendix D.

### 6.4 Two dimensional coordinates

When latitudes and longitudes of geodetic control stations are calculated, next step is to convert them to a plane coordinates using a suitable map projection for the country. Plane coordinates are used for small and large scale mapping. Although GPS and other

Table 6.3 Accuracies of district level parameters for coordinate transformations

| District | Number of <br> points <br> used | Max \& min <br> errors (m) in <br> X, Y or Z | Average of <br> R.ISS <br> (meters) |
| :--- | :--- | :---: | :--- |
| Colombo | 5 | $.002-.144$ | 0.107 |
| Gampaha | 9 | $.003-.141$ | 0.068 |
| Kalutara | 5 | $.005-.099$ | 0.083 |
| Galle | 7 | $.000-.081$ | 0.064 |
| Matara | 5 | $.001-.088$ | 0.065 |
| Hambantota | 9 | $.004-.150$ | 0.126 |
| Ampara | 5 | $.000-.030$ | 0.020 |
| Batticalo | 8 | $.003-.085$ | 0.078 |
| Trincomalee | 6 | $.002-.264$ | 0.167 |
| Mullatiw | 12 | $.001-.284$ | 0.129 |
| Killinochchi | 10 | $.002-.2 .56$ | 0.135 |
| Jaffna | 5 | $.000-.010$ | 0.007 |
| Mannar | 11 | $.004-.191$ | 0.152 |
| Vaunia | 8 | $.002-.162$ | 0.093 |
| Putlam | 10 | $.007-.391$ | 0.311 |
| Kurunegala | 12 | $.005-.325$ | 0.288 |
| Anuradhapura | 15 | $.003-.593$ | 0.308 |
| Polonnaruwa | 10 | $.000-.212$ | 0.152 |
| Matale | 5 | $.009-.151$ | 0.118 |
| Kandy | 6 | $.005-.019$ | 0.040 |
| N' eliya | 6 | $.003-.129$ | 0.074 |
| Badulla | 7 | $.010-.360$ | 0.202 |
| Monaragala | 11 | $.011-.608$ | 0.383 |
| Ratnapura | 9 | $.002-.516$ | 0.281 |
| Kegalle | 5 | $.007-.096$ | 0.084 |

spatial data collection methods such as surveying provide accurate spatial data, hard copy maps are still the major source of spatial data for a GIS. In this process, hard copy maps have to be transformed into the digital form using a digitizer or a scanner. Digitizers provide the spatial data in "vector" format and scanners provide data in "raster" format.

When hard copy maps are used as the primary source of spatial data in a GIS, it is important to understand basic concepts of producing maps and the errors which can be associated in the process of map making. In order to achieve this objective, a short description about map projections are given in the following sections.

Maps are produced using the measurements taken on the earth surface, which is a curved surface. As the actual earth surface is very irregular and cannot be easily defined by mathematical relationships, an ellipsoid is accepted as the curved surface of the earth (see Figure 3.1). Therefore, the spatial data collected on the earth surface have to be reduced to the ellipsoid and then transformed in to the two dimensional surface of the map. This transformation procedure from the three dimensional earth surface to the two dimensional map surface is known as a map projection.

There are many map projections, but only a few have a significant importance for wide use of mapping. The "Transverse Mercator" and "Lambert Conformal" projections, both of which were developed by Lambert in 1772 , can be described as the most useful and widely used map projections for large scale mapping in the world (Snyder, 1987). These two map projections are used for State Plane Coordinate system (STPL) of the United States. The Transverse Mercator projection facilitates the Universal Transverse Mercator (UTM) projection which is used as a common map projection for mapping the entire world. Sri Lanka uses the Transverse Mercator projection (National Atlas, 1988). As the country has a more North-South expansion, Transverse Mercator projection provides satisfactory results as a conformal projection for mapping (Snyder, 1987).

The transformation of data from a 3 dimensional earth surface to a 2 dimensional map
cannot be done without introducing errors, which are generally known as distortions. When a relatively small area of the earth surface is to be mapped, these distortions can be assumed as small and neglected but for large areas they have to be considered and corrected. Distortions in map projections can be categorized into 3 groups:

1. Distortions in distances (Linear Distortions)
2. Distortions in angles (Angular Distortions)
3. Distortions in areas of the map (Area Distortions)

### 6.4.1 Projection Surfaces and their classifications

In map projections, three dimensional earth surface is transformed into the two dimensional map using an intermediate surface. These intermediate surfaces are called projection surfaces. Today, all the projections used for map projections are based on 3 projection surfaces (Pearson, 1990). They are a plane. a cone or a cylinder. Projections based on these three surfaces are known as Azimuthal, Conical or Cylindrical projections. respectively. Figure 6.1 shows these three basic map projections.


Figure 6.1 Projection surfaces used for azimuthal, cylindrical and conical projections.

The position of the projection surface to the earth gives another way of classifying map projections. They are known as "Regular", "Transverse" or "Oblique". In the cases of conical and cylindrical map projections, when the polar axis of the earth is parallel to the axis of the cone or the cylinder, they are known as regular map projections. Transverse projections are generated when these two axes are perpendicular to each other. All other cases which do not fall into categories of regular or transverse fall into the category of "Oblique".


Figure 6.2 Polar equatorial and oblique map projections.

Azimuthal projections are classified as "Polar", "Equatorial" or "Oblique", depending on the common point of both surfaces to the earth and the projection surface. If the common point is the pole or a point on the equator, they are known as "Polar" and "Equatorial" respectively. All other cases are categorized as "Oblique". Figure 6.2 shows this categorization.

In some cases, the projection surface can be tangent or secant to the earth surface, as shown in Figure 6.3. This can be described as the third way of classifying map projections. When a cone or a cylinder is used as a tangent, there is only one line common to the earth surface and this line is known as the standard parallel. In secant position there are two standard parallels.


Figure 6.3 Tangent and secant map projections.

All the map projections can be classified using another characteristic of map projections. considering whether they can be obtained graphically or mathematically. In graphical projections, the surface of the earth is projected to the mapping plane through a point using simple geometric techniques. Mathematical projections can be derived only by mathematical calculations. The Transverse Mercator projection. which is used for Sri Lanka. can be described as a mathematical projection.

### 6.4.2 Map projection for Sri Lanka

As mentioned in the previous section, out of hundreds of available map projections. only a few can be used for large scale mapping, due to their nature of distortions. Lambert Conformal and Transverse Mercator projections, which are conical and cylindrical. respectively; are the two most popular and widely-used map projections. Transverse Nercator projection has been used for all mapping in the country (National Atlas, 1988). This projection is appropriate as a conformal projection for mapping in the country. Transverse Mercator projection is a conformal and transverse map projection. The transformation equations for this projection are obtained by imposing conditions on normal cylindrical projections.

### 6.4.3 Normal Cylindrical Projections

Normal cylindrical projections are tangent and regular, because the projection surface (the cylinder) touches the earth surface at the equator as shown in Figure 6.4.

In normal cylindrical projections, which are sometimes known as "Casini" projections, meridians of the datum surface are projected as equally spaced straight lines on the projection surface. Projection equations for Casini projection can be obtained as follows (Muller, 1988):


Figure 6.4 Datum and projection surface for cylindrical projections. (Source: Muller, 1988)

In Figure 6.5, NAB is a rectangular spherical triangle. Using the "Naphire's" rule for spherical triangles, we can get the transformation equation for $X$.

Napier's rules are:
(1) Sine of any section of the circle (Figure 6.6) is equal to the product of cosines of two opposite sections.
(2) Sine of any section of the circle is equal to the product of tangents of two adjacent


Figure 6.5 Spherical triangle in the datum surface used for the calculation of "Casini" projection
sections.
According to Figure 6.6, using the relationship for opposite sections,

$$
\begin{equation*}
\operatorname{Sin} \theta=\operatorname{Cos}(90-\Delta \lambda) \cdot \operatorname{Cos} \phi \operatorname{Sin} \frac{x}{r}=\operatorname{Sin} \Delta \lambda \cdot \operatorname{Cos} \phi \tag{6.1}
\end{equation*}
$$

Using the relationship for adjacent sections,

$$
\begin{array}{r}
\operatorname{Sin}(90-\Delta \lambda)=\tan \left(90-\phi_{0}-\beta\right) \cdot \tan \phi \\
\operatorname{Cos} \Delta \lambda=\operatorname{Cot}\left(\phi_{0}+\frac{Y}{r}\right) \tan \phi \quad \operatorname{Cot}\left(\phi_{0}+\frac{Y}{r}\right)=\operatorname{Cos} \Delta \lambda \cdot \operatorname{Cot} \phi \tag{6.3}
\end{array}
$$

Equations 6.11 and 6.14 can be used to calculate the $X$ and $Y$ coordinates of point $A$.
In order to calculate the convergence, we can again use the naphier's rule,

$$
\begin{array}{r}
\operatorname{Sin}[90-(90-\phi)]=\tan (90-\Delta \lambda) \cdot \tan [90-(90-c 0] \\
\operatorname{Sin} \phi=\operatorname{Cot} \Delta \lambda \cdot \tan c \\
\tan c=\operatorname{Sin} \phi \cdot \tan \Delta \lambda \tag{6.6}
\end{array}
$$

Since the angles $c$ and $\Delta \lambda$ are small, the convergence can be written as:

$$
c=\Delta \lambda \cdot \operatorname{Sin} \phi
$$



Figure 6.6 Circle used for Naphire's rule for rectangular spherical triangles

### 6.4.4 Transverse Mercator Projection

Transverse Wercator projection is obtained by imposing the condition of conformality and placing the cylinder in the transverse position in "Casini" projection as shown in Figure 6.7. This projection gives good results for geographical areas, which have a greater expansion in north-south direction. Characteristics of the Transverse Mercator Projection can be summarized as follows (Snyder (1987):

1. Cylindrical (Transverse)
2. Conformal
3. Central meridian and parallels are straight lines
4. Other meridians and parallels are complex curves
5. Scale is true along the central meridian

Transformation equations of Transverse Mercator projection can be obtained by first, imposing the condition of conformality to the Casini projection and then using the transverse position of the cylinder.

Projection equations for Transverse Mercator projection for the case of a globe can


Figure 6.7 Position of the cylinder in the transverse position in "Transverse Mercator" projection (source: Muller, 1988)
be taken as follows (Snyder, 1987):

$$
\begin{array}{r}
x=R \cdot K_{0}^{\prime} \cdot \operatorname{arctanh}(B) \quad \text { or } \quad x=\frac{1}{2} \cdot R \cdot k_{0} \cdot \ln [(1+B)(1-B)] \\
y=R \cdot k_{0} \cdot\left(\arctan \left[\tan \phi / \operatorname{Cos}\left(\lambda-\lambda_{0}\right)\right]-\dot{\phi}_{0}\right) \\
k=\frac{k_{0}}{\left(1-B^{2}\right)^{1 / 2}} \tag{6.9}
\end{array}
$$

where

$$
B=\operatorname{Cos} \dot{\phi} \cdot \operatorname{Sin}\left(\lambda-\lambda_{0}\right)
$$

$k_{0}=$ Scale factor along the central meridian $\lambda_{0}$
$\dot{\varphi}_{0} . \lambda_{0}$ coordinates of the origin
$y$ axis along the central meridian

Reveres formulas and the formulas in a case of an ellipsoid for Transverse Mercator projection are given in Snyder, 1987, along with transformation equations for many other map projections. For the conversion of latitudes and longitudes to Transverse Mercator projection in Sri Lanka, the central meridian and the standard parallel are chosen through the station number 65 (Pidurutalagala) of the geodetic network. Plane coordinates used in the country today use an origin located 200 km west and 200 km
south of "Pidurutalagala" in order to avoid positive and negative values in coordinates (National Atlas, 1988). The software "Con_cord", mentioned earlier in this chapter can be used to convert latitudes and longitudes of the country to Transverse Mercator projection coordinates.

## 7 GETTING SPATIAL DATA INTO THE GIS

Creating a GIS with a cadastral map layer involves activities such as:

1. Establishing suitable geodetic control
2. Data capture
3. Getting data into the GIS

When the geodetic control is established as discussed in Chapter 5. the next step is the capturing of spatial data for the GIS. This has to be done considering two different utilizations of the GIS data base:

1. Data required for cadastral and engineering applications
2. Data required only for analysis purposes. and can be considered as information for decision making.

The cost for capturing spatial data becomes extremely high when the required spatial accuracy is increased (Henson, 1984). Due to this, it is important to consider the accuracy requirements of each data layer of the GIS. Spatial accuracy required for analysis and decision making depends on many factors such as type of the project. location, time etc, but not up to the level of accuracy required for cadastral and engineering needs. Hence, the possibilities of data capturing techniques for analysis and also for cadastral and engineering needs are discussed in the next few sections of this Chapter.

### 7.1 Data layers for cadastral and engineering applications

High spatial accuracy is the important part of the cadastral map layer which provides accurate coordinates for cadastral and engineering applications. The most accurate way of getting these high accuracy data into the cadastral map layer of the GIS is the direct input of digital values into the system (Byrene, 1991).

It is obvious that the capturing of highly accurate spatial data is very costly. According to Hensen (1984), the cost of data acquisition is extremely high when the requirement of the spatial accuracy becomes higher than 1 foot. Due to this reason, all the available and suitable resources of spatial data have to be used in the process of creating the cadastral map layer of the country.

There is a considerable amount of cadastral and town survey maps available in Sri Lanka. It can be estimated that these maps cover about half of the entire geographical area of the country. Almost all of these large scale maps have been prepared connecting the coordinates to the national geodetic network. They are excellent sources of getting spatial data into the cadastral map layer of the GIS. There are two ways of converting these hard copy maps into the digital form:

1. Manual digitizing.
2. Scanning and then converting raster data into the vector form.

Manual digitizing has been the popular and most widely used way of converting hard copy maps into the digital form. This technique provides highly accurate digital data, but the technique has the following disadvantages which are not found in scanning;

1. Very time consuming and tedious
2. Costlier than scanning
3. Open for human errors

Due to these reasons, it is extremely useful if a suitable scanning and vectorization procedure for large scale maps can be developed.

### 7.1.1 Scanning large scale cadastral maps

Scanning small scale maps and converting them into a usable form of vector data is not easy due to the availability of many different types of information in many different forms. If scanned, much more effort is required to edit them for the purpose of preparing them in a usable form.

Large scale cadastral and town survey maps are usually prepared using black lines which have a single line-thickness (see Appendix H). Details of these maps are restricted only to boundary lines and reference numbers. Due to these reasons, large scale maps are much more suitable for scanning and vectorization than small scale topographic or thematic maps.

A simulated cadastral map, which is given in Appendix $G$ was scanned and converted into the vector form using "Arc/Info", in order to find the obtainable accuracy of coordinates. The same map was manually digitized. Coordinates obtained from both cases were compared with actual coordinates. Results of this comparison are given in Table 7.1.

Table 7.1 Comparison of coordinates by manual digitizing and scanning and vectorization

|  | Number of <br> points | Mean difference <br> in X (m) | Mean difference <br> in $Y(\mathrm{~m})$ | Std. dev. <br> in X $(\mathrm{m})$ | Std. dev. <br> in Y (m) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Manual |  |  |  |  |  |
| Digitizing <br> Scanning and <br> Vectorization | 38 | 026 | 0.26 | 0.34 | 0.23 |

Although the accuracy of manual digitizing can be varied according to the operator, results in Table 7.1 show that the coordinates obtained by scanning and vectorization process using "Arc/Info" are comparable with the coordinates obtained by manual digitizing if the correct nodes can be identified. Coordinates obtained by these two cases
are given in Appendix G.

### 7.1.2 Major problems faced in scanning and vectorization of cadastral maps

The procedures adopted for scanning and vectorizing the cadastral map were as follows:

1. Scanning the map using a "HP Scanjet IIc" scanner. Different dpi (dots per inch) were tested but the comparison is based on 600 dpi .
2. Getting the scanned image to "Grid" in ArcInfo.
3. Convert the grid into a line coverage in ArcInfo.
4. Transforming the line coverage to the ground coordinate system.

The original map and the map obtained by scanning and vectorization are given in Appendix G. It can be seen that many unnecessary nodes have been created by scanning and vectorization process. Although only 87 nodes were expected, scanning and vectorization produced 130 nodes. In this case, about $65 \%$ extra nodes have been created. But a map which has mostly rectangular property boundaries will produce much smaller amount of extra nodes. Maps without traverse lines will produce even lesser extra nodes.

Also, it was observed that a few number of necessary nodes have not been created in the process. Instead of the correct nodes, a number of wrong nodes were observed around the correct location. The percentage of these missing nodes was about $15 \%$. Both of these problems were solved using manual editing on the screen. An automated procedure has to be developed to deal with these two problems, in order to use scanning for the conversion of large cadastral maps into the digital form.

### 7.1.3 Improving digitized coordinates using a least squares sequential adjustment

When coordinates of points are obtained by converting a hard copy map into digital form using manual digitizing or scanning and vectorization, some of the errors which can be contained in those coordinates are:

1. Plotting errors, which are introduced in the mapping stage. These errors can be due to human errors or due to the scale of the map.
2. Errors due to shrinkage and expansions of the map sheet.
3. Errors due to the digitization process.

These 3 types of errors can be partly eliminated and the coordinates can be improved by using a sequential least squares adjustment. This methodology has been proposed to update digital cadastral data using area, linear, angular and tangency conditions (Tamim and Schaffrin, 1985). According to Tamim and Schaffrin, this methodology has reduced errors of manually digitized coordinates by $75 \%$. The same methodology can be used to improve coordinates obtained by scanning and vectorization.

Mathematics behind the sequential least squares adjustment can be explained as follows (Uotila, 1986):

As shown in Chapter 5, the solution for parameters can be written according to equation in section 5.11 .

$$
\hat{X}_{a}=-\left(A^{T} P A\right)^{-1} \cdot\left(A^{T} P L\right)
$$

In this case, parameters are the corrected coordinates of nodes. Digitized coordinates can be considered as the first set of observations and the conditions such as linear, angular and area can be considered as the second set of observations. When there are two sets of observations for the same set of parameters, they can be written in a mathematical model as follows:

$$
L_{1}^{a}=F_{1}\left(X_{a}\right)
$$

$$
L_{2}^{a}=F_{2}\left(X_{a}\right)
$$

Using the equation in section 4.11, the solution for the common set of parameters can be written as (Uotila, 1986):

$$
\hat{X}_{c}=-\left(A_{1}^{T} P_{1} A_{1}+A_{2}^{T} P_{2} A_{2}\right)^{-1} \cdot\left(A_{1}^{T} P_{1} L_{1}+A_{2}^{T} P_{2} L_{2}\right)
$$

and the variance-covariance matrix for adjusted parameters, as shown in equation 5.13 becomes:

$$
\sum_{\dot{X}_{a}}=\sigma_{0}^{2} \cdot\left(A_{1}^{T} P_{1} A_{1}+A_{2}^{T} P_{2} A_{2}\right)^{-1}
$$

The equation for the solution of parameters can be written in another form to show the effect of the second set of observations (Uotila, 1986). If the solution by only the first set of parameters is $\hat{X}_{a}{ }^{*}$ then,

$$
\hat{X}_{a}=\hat{X}_{a}{ }^{\pi}+\Delta X
$$

where, $\Delta X$ is the effect of the second set of observations which are linear, angular and area conditions in this case.

It can be shown that $\Delta X$ is given by:

$$
\Delta X=-N_{1}^{-1} \cdot A_{2}^{T}\left(A_{2} N_{1}^{-1} A_{2}^{T}+P_{2}^{-1}\right)^{-1} \cdot\left(A_{2} \hat{X^{2}}+L_{2}\right)
$$

where, $\quad N_{1}=\left(A_{1}^{T} P_{1} A_{1}\right)^{-1}$

Using this mathematical approach, a software was written to do the sequential least squares adjustment. This software allows manual entry of linear, angular and area conditions. It is required to extract the node coordinates of property boundaries before this adjustment. A manual approach was used to perform this step. It is required to integrate GIS software with the sequential least squares adjustment (SLS) software in order to use this methodology for larger projects. The new procedure should be capable of first getting node coordinates from the map and secondly, to perform the SLS
adjustment. Thirdly, it should be able to upgrade the map using adjusted coordinates. The subroutine written for SLS is given in Appendix J.

### 7.2 Spatial data sources for data analysis in the Sri Lankan GIS

For analysis and decision making purposes the most widely used method is acquiring data from available hard copy maps. Usually, these small scale hard copy maps contain many errors due to two reasons. First is the inaccurate data used for mapping and the second is the errors introduced in the process of map making. Errors in mapping process can be due to following characteristics of maps (Robinson and Howard, 1984).

1. Scale of the map.
2. The map projection.
3. Generalizations involved in the map.
4. Symbolizations used in the map.

Generalizations and symbolizations are required in mapping in order to store different types of data according to the scale of the map and for convenient reading of the map. Amount of spatial errors introduced in generalization and symbolization is directly proportional to the scale of the map (Robinson and Howard, 1984). Errors introduced by the map projection depend on the type of map projection used for mapping and the location of the mapping area. Details of map projections are discussed in chapter 6.

In Sri Lanka, maps which can be used for the creation of the GIS are as follows:

1. A road map in the scale of $1: 500,000$.
2. A map covering the entire country in the scale of $1: 250,000$.
3. One inch to one mile series map ( $1: 63360$ ).
4. One hundred thousand scale land use map series ( $1: 100,000$ ).
5. Fifty thousand topographic map series ( $1: 50,000$ ).
6. Ten thousand topographic map series ( $1: 10,000$ ).
7. Block survey maps which separate state and private properties, usually in 4 chain scale ( $1: 3168$ ).
8. Large scale cadastral survey maps ( $1: 4000,1: 2000$ or larger).
9. Large scale town survey maps (1 chain, $1: 1000$ or $1: 500$ ).

Block survey, town survey and cadastral survey maps are prepared in large scales, without generalizations and symbolizations. They are good sources of obtaining spatial data with higher accuracy. Procedures of getting data to the GIS from these large scale maps were discussed in the section 7.1.

The road map and the map in $1: 250,000$ have been derived using the one inch map series of the country (National Atlas, 1988), and 1 inch to one mile maps have been replaced by $1: 50,000$ map series. All these small scale map series are prepared by the Survey Department, which is the government agency responsible for surveying and mapping activities in the country.

According to the map accuracy standard of the U.S., which are usually met by the department, the available spatial accuracy of small scale map series are given in Table 7.2. (Manual of Photogrammetry, 1987)

Publishing of 1:100,000 land use map series and 1:50,000 topographic map series was started in 1982 and 1977, respectively (National Atlas, 1988). Both of these map series were completed in 1990 (informal departmental sources). It is required to update

Table 7.2 Spatial accuracy of small scale maps

| Map series | liner accuracy |
| :---: | :---: |
|  |  |
| $1: 500,000$ | 250 meters |
| $1: 250,000$ | 125 meters |
| $1: 100,000$ | 50 meters |
| $1: 50,000$ | 25 meters |
| $1: 10,000$ | 8 meters |

these maps frequently, but much of the information available in those maps is older than five years. Production of $1: 10,000$ map series also started in early 1980 s and it is not yet completed.

When completed, $1: 10,000$ map series will provide information for many data layers of the national comprehensive GIS. But smaller scale maps have to be used for the areas for which suitable larger scale maps are not available.

Converting small scale maps into digital form has to be done using manual digitizing. Because of many different types of information, different colors, line types and symbols, it is extremely difficult to use scanning for the conversion of these maps into the digital form. When the digitization is done, it is required to convert the data in the digitizer coordinates to the real world coordinates (geo-referencing). Details of converting each map series into digital form and procedures for geo-referencing is discussed below:

### 7.2.1 1 : 500,000 scale road map

The road map is the most widely used map among the general public. According to Table 7.2, the positional accuracy which can be obtained from this map is only about 250 meters. When digitizing errors are added, this positional accuracy will even be lower than 250 meters. Hence, this map cannot be recommended for creating the national GIS. Instead, it can be used as a quick data capturing source for other purposes such as training.

Data available in $1: 500,000$ map are mainly the road network, water features and the shore line of the country. When digitized, geo-referencing of this map can be done using coordinates of grid intersections. Geodetic network points are not shown in this map. Instead, some of the mountain peaks where those geodetic control points are located are shown in this map. In order to determine the possibility of using peaks as geodetic control points, a geo-referencing transformation was performed for a digitized coverage of this map. Thirteen mountain peaks were used for the transformation.

Errors obtained in transforming digitizer coordinates to the real world coordinates are given in Appendix I and the map created is given in Appendix A. Although the RMS (Root Mean Squares) error expected according to map accuracy standards was 250 meters, an error of 508 meters were observed for 11 tic points (assuming there are blunders in point numbers 81 and 101). This indicates that the locations of geodetic control network points cannot be accurately identified in 1:500,000 road map of the country. Although the large RMS error can be partly due to digitization and mapping errors, the main two reasons are the wrong location identification and the unsuitability of the symbol used for peaks, as control points. It can be recommended that the geodetic control points should be shown on new versions of this map, so that they can be used mainly for geo-referencing. Also, it will create an awareness among general public about the geodetic control network, which will ultimately help the protection of geodetic network monuments.

### 7.2.2 1 : 100,000 scale Land use map

This map series has been prepared on a district basis. Information shown in the map is basically restricted to vegetation types, but major roads and some of the administrative boundaries are also shown in the map. When digitized, like in the $1: 500,000 \mathrm{map}$, digitizer coordinates can be transformed to the real world coordinates using coordinates of grid intersections. Points of the geodetic control network are shown in this map as mountains. The solid triangular shape symbol used is not suitable for geodetic control points. It can be recommended that the mountains and geodetic control points are shown using two different symbols in future versions of this map.

The 1: 100,000 map for "Matale" district was digitized and transformed to real world coordinates using geodetic control points identified on the map. This map contained only one primary control point (station 59 - "Ambokka") in the old network. However, eleven secondary control points were identified (total 14 in the district) on the map and used
as tic points for geo-referencing. Names and coordinates of those 11 secondary control points are shown in Table 7.3. Similar to the $1: 500,000$ map, the RMS error observed was larger than expected. This can also be attributed to the same reasons given in the previous section.

Table 7.3 Secondary geodetic control points in Matale district: coordinates are shown as Pidurutalagala as the origin

| Name of the station | X-coordinate (m) | Y-coordinate (m) |
| :--- | ---: | ---: |
|  |  |  |
| Etapola | +54095.96 | -21221.72 |
| Karanampota | +81527.74 | -26167.65 |
| Beliyakanda | +89573.53 | -20527.84 |
| Nalanda Rock | +77057.48 | -14885.35 |
| Kirigalpotta | +52631.44 | -4816.28 |
| Patanagala | +54592.85 | +24394.29 |
| Wahigala | +85539.59 | -6978.84 |
| Erawalagala | +95598.03 | -4362.63 |
| Sigiriya Rock | +112310.50 | +15967.85 |
| Kudapatana | +67975.93 | +2077.10 |
| Mariyakanda | +60601.62 | +11661.35 |

### 7.2.3 $1: 50,000$ Topographic map

This is the most recent and complete map series available in the country. These maps will provide following data for the national GIS:

1. Road network
2. Water features
3. Vegetation
4. Elevation
5. Administrative boundaries

In this map series, geodetic control points are shown using a well-defined point in a triangular shape symbol. Hence, they can be effectively used as tic points for digitizing and geo-referencing.

Some of the information available in sheet number 48, "Matale", were digitized and transformed to the real world coordinates using geodetic control points. Due to the large amount of information available in the map, it was observed that this map series is not appropriate for digitizing contours. However, other information can be effectively picked up using this map series.

### 7.2.4 $1: 10,000$ Topographic map

Once completed, this map series will provide the best spatial information for many data layers of the national GIS. As there are 1834 map sheets in this series (National Atlas, 1988), it takes a considerable time to complete. Publishing of this series was started by the Survey Department of Sri Lanka in early 1980s. The series can be estimated to be completed in the year 2000 (informal departmental sources). By its completion, published map sheets in early stage will be about 20 years old. Therefore, the department has to accelerate revision of this map series in order to use them effectively in creating the national GIS.

Due to the large scale, one sheet of this map series cover only 40 square km ( 15.6 sq. miles). Hence, it cannot be expected that at least 4 primary or secondary geodetic control points are available in one map sheet. Thus, additional points are required with known ground coordinates for the purpose of geo-referencing. Although surveying and photogrammetry can be used, GPS can be recommended as the best procedure for obtaining ground coordinates of points identified on the map.

## 8 CONCLUSION

An appropriate GIS is a good tool for decision making and hence facilitating the proper use of resources in a country. Geodetic control network, which is the linkage mechanism for data layers, is an important component of a GIS (Dale and Mclaughlin, 1988). When property boundary level information is to be handled by the GIS of a country, it is critical to have an accurate geodetic control network because property boundaries have to be determined within a tolerance of few centimeters.

The present Sri Lankan geodetic control network does not provide adequate spatial accuracy for the GIS. The present absolute accuracy of the network was found to be less than $1: 30,000$ (see chapter 3 ). A preferable absolute spatial accuracy for a geodetic network for the use of GIS and GPS is $1: 1,000,000$ or higher (FGCC, 1988) in order to provide centimeter-level relative accuracy for property boundaries.

It was also found (using "Geolab" software), that the angle and base line measurements used for the old adjustment can provide only up to an accuracy of $1: 60,000$, if the network is newly adjusted using old observations. The absolute accuracy of individual points obtained in this adjustment varied from 5 mm to 11.43 meters in the $9.5 \%$ confidence interval. This indicated that a set of new observations are necessary for a new geodetic adjustment.

### 8.1 New geodetic network

In addition to performing as the linkage mechanism of the GIS, a geodetic control network provides coordinate control for surveying and mapping. For cadastral mapping, we need to obtain centimeter-level accuracy. Thus, it can be recommended that a geodetic network that has "C" order points (see chapter 5) with 5 cm absolute accuracy ( 10 km apart) should be established for Sri Lanka. The control network with "C" order points can be established using a primary control network. This primary control network should have " $A$ " and " $B$ " order points (see chapter 5 ) with better relative and absolute accuracy.

### 8.1.1 GPS data for the geodetic network

GPS has been realized as the best available technology for data collection for a new adjustment of a geodetic network (Leick, 1995. Wells, 1986. Puterski, 199.2). When GPS is used as observations for the adjustment, the accuracy of the adjustment is mainly dependent upon the procedure used for GPS data collection. Federal geodetic control committee (FGCC) of the U.S. has published a common specifications for GPS data collection. However, these specifications can provide varying results for different networks, according to the densification of points and their locations. Hence. it is extremely useful to develop a method to predict the accuracy of the adjustment according to the quality of GPS observations. Then the procedure for GPS observations can be designed to achieve the desired accuracy of the network.

A set of simulated data was used for the prediction of the accuracy of the geodetic network of Sri Lanka. Simulated data were obtained by introducing a random error and a systematic error (as a part per million correction) to the distances obtained using the coordinates of a new adjustment. This new adjustment was performed using old available data and the "Geolab", as explained in section 5.1.2.

Before the 1960 's, geodetic network adjustments were usually done by the method of "Triangulation" (Kahmen and Faig, 1988). These adjustments used only one point, called the "datum point" as a fixed point. Coordinates of these fixed points obtained by astronomical methods were assumed as correct. Today, GPS allows us to fix more than one point, if the positional accuracy of those points can be determined to a very high accuracy. Fixing more points provides a greater control for the coordinate adjustment. Simulated data has shown (see Chapter 5) that an average accuracy better than 1 cm can be obtained for a few control points which are approximately 100 km apart. This has been achieved by limiting random errors to 1 mm and using a GPS procedure which provides a $10^{-8} \mathrm{ppm}$ measuring accuracy, as explained in section 5.3.2. This procedure can be used to find the coordinates of points which can be used as fixed stations in the adjustment.

Survey Department of Sri Lanka has proposed to take GPS observations in Sri Lanka according to trignometrical lines, as shown in Appendix A. Simulated data has shown that this procedure can provide only an average positional accuracy of 24.9 cm , if GPS data is good up to $10^{-8}$ with 5 mm random errors. Also, it was found that a better positional accuracy (average of 2.64 cm ) can be achieved by increasing the number of observed lines as explained in chapter 5. This accuracy was obtained by introducing the same $10^{-8} \mathrm{ppm}$ error to simulated values. Therefore, it can be recommended that the GPS observations should not be restricted to triangulation lines. As there are no inter-visibility requirements for GPS, any line can be used for GPS observations.

### 8.1.2 Reference ellipsoid for Sri Lanka

When the data is collected for the adjustment, the next important decision to make is the selection of a reference ellipsoid for calculations. The Everest ellipsoid has been used for the old Sri Lankan adjustment (Jackson and Price, 1933). As the Everest ellipsoid is locally fit to the country, it has the advantage of providing smaller geoid undulations
(Bomford, 1980). But the international ellipsoid of GRS80 is recommended for the new adjustment of Sri Lanka due to the reasons given in section 5.7.2. Use of GRS80 ellipsoid will greatly facilitate the wide use of GPS technology, which is becoming inevitable in many civilian applications.

### 8.2 Coordinate transformation

When a new set of coordinates are calculated for a country, it is required to calculate transformation parameters in order to integrate future and present work. Also, transformation parameters facilitate the use of maps prepared using the old system of coordinates with the newly adjusted linkage mechanism of the GIS. These parameters should provide sufficient accuracy for conversion of maps. It was found that a single set of parameters calculated for the entire country provides only 1 meter accuracy whereas a set of transformation parameters calculated for district level provides an average accuracy of 14 cm . Also, it was found that provincial level parameters provide about 30 cm accuracy for coordinate transformations in the country. Accuracy obtained for each district is given in Table 6.3. Parameters were calculated without using any interpolation or extrapolation technique, but using control points in and around the district boundaries. The software "Con_cord" was developed for all types of coordinate transformation of the country. The main menu of "Con_cord" is given in Appendix D.

### 8.3 Spatial data for cadastral and engineering applications

The most accurate way of creating the cadastral map layer is the direct entry of GPS and surveying data into the GIS (Byrne, 1991). This will provide highly accurate spatial data for all cadastral and engineering applications. It is extremely difficult to survey all land parcels in the country as a new project due to time and cost factors. Hence, the conversion of available cadastral maps into digital form is required. Scanning
and vectorization capabilities available today provide comparable results as in manual digitization as shown in section 7.1.1. It was found that the scanning and vectorization provide about $85 \%$ of correct nodes. $15 \%$ of missing nodes were created using manual methods. An automated method for this purpose is required for large cadastral maps or large projects.

### 8.3.1 Updating coordinates by a sequential adjustment

A sequential least squares adjustment (Tamim and Schffrin, 1995) is a useful mathematical procedure for the elimination of digitization and mapping errors from digitized coordinates. As shown in section 7.1.1, digitized coordinates provide approximately 36 cm spatial accuracy ( 26 cm each in X and Y ) for a cadastral map in the scale of $1: 2000$. In order to improve this accuracy of digitized coordinates, measurements in better accuracy are required for the adjustment as the second set of observations in a sequential least squares adjustment. Surveying and GPS provide this type of spatially accurate measurements.

A software was written to upgrade digitized coordinates using linear, angular and area observations as a second set of measurements and the mathematical concept explained in section 7.1.3. This software can upgrade the coordinates of a particular point where linear, angular or area measurements are available. This methodology can be mentioned as important when coordinates of a few points have to be upgraded with respect to other points. The subroutine written for this purpose is given in Appendix H.

### 8.4 Spatial data from small scale maps

A GIS data layer can be created using small scale maps, only if accurate measurements are not required (see Table 7.2 for spatial accuracy of small scale maps). In Sri Lanka $1: 500,000,1: 100,000$ and $1: 50,000$ maps are available for this purpose.

Production of a $1: 10,000$ map series is in progress.
According to the map accuracy standards, the spatial accuracy of the above scaled maps varies from 250 meters to 8 meters, respectively. Thus, when digitized, conversion of digitizer coordinates to real world coordinates (geo-referencing) can be done using the coordinates of intersections of grid lines. Using points of the geodetic control network is a better method of geo-referencing because those points are actual points available on ground that can be checked. Geodetic control points are not shown in $1: 500,000$ and 1: 100,000 maps in the country. Instead, mountain peaks where geodetic control points are located are shown. In some cases, locations of these two types may not be the same. Although these mountain peaks can be used as geodetic control points for geo-referencing: they provided lesser accurate results. Possible reasons for this low accuracy are discussed in sections 7.2.1 and 7.2.2. It can be recommended that geodetic control points are shown in revised versions of $1: 500.000$ and 1:100.000 maps in Sri Lanka. Geodetic control points mapped in $1: 50,000$ maps can be effectively used for geo-referencing. Due to the small area, sufficient " $A$ ", " $B$ " or " $C$ " order geodetic control points can not be shown in $1: 10,000$ maps, which can be used for geo-referencing. Surveying, GPS or photogrammetric methods have to be used to obtain ground coordinates when more control points (tic points) are required. The software developed ("Con_cord"') can be used to convert these coordinates to any appropriate form in order to use them in the GIS.

### 8.5 Summary of recommendations

In a brief summary, it can be recommended that a new geodetic network for Sri Lanka, which provides $1: 1,000,000$ absolute accuracy for " C " order points ( 10 km apart) be established before establishing the GIS. This accuracy will also satisfy the needs of GPS applications. A few points that are about 100 km apart can be used as fixed points for
the adjustment. Coordinates of these points have to be determined up to 1 cm accuracy. A GPS observation procedure which provides $10^{-8} \mathrm{ppm}$ measuring accuracy should be used to obtain 1 cm absolute accuracy for these "A" order points. " B " order points which have 2 cm absolute accuracy ( 25 km apart) should be established in the primary network. The GRS80 reference ellipsoid should be used for the new adjustment.

All large scale cadastral maps have to be digitized and incorporated into the cadastral map layer. New surveving and GPS measurements should be entered to the GIS as digital values. Digitized data can be maintained as a separate layer in order to separate digitized data from more accurate data acquired by surveying and GPS and directly entered to the system. An automated system has to be developed to obtain all the required nodes in a scanned and vectorized cadastral map. Cintil then. manual digitizers have to be used for converting hard copy maps to digital form. When the $1: 10,000$ map series is completed, it can be used as the base map for creating transportation, vegetation, water features and elevation layers of the national GIS. In the areas where $1: 10,000$ maps are not available, the $1: 50,000$ map series has to be used. These data can later be upgraded when 1: 10,000 maps are produced.

## APPENDIX A EXISTING GEODETIC CONTROL

Locations of geodetic control points and major roads


## Observed lines of the network

(Source: Jackson, 1933)


## Station numbers and names of the primary network

| NUMJ3ER | WNMT. |
| :---: | :---: |
| 001 | F'unkudutivu |
| 002 | Kalmumai |
| 00 | Karaciwn |
| 004 |  |
| 005 | Chundikulam |
| 900 | Kilinsonchi |
| 007 | F'alasilkulam |
| !10 | Levils Foint |
| jus | F'aletivu |
| 019 | Velantulan |
| Oi | ILupFaikadavai |
| リン | Yianchuravilla |
| 013 | Narimbatabrhan |
| 01 | Hutamalai |
| 015 | Kumpudumatai |
| 018 | Chenmana |
| 017 | Brante Tant |
| 12 | Mantei |
| $11 \%$ | Freslei |
| 00 | 1ranativu |
| 2 | Laiserativi |
| 02 | Futumulisumeu |
| (1)3 | Watersinerl foomt |
| 924 | Gth lille Et-g |
| 025 | Vedubianari |
| CR | Ruriohudati |
| T | Katurala |
| S | Hablutamda |
| Ser | Remaisamay |
| 009 | kiolavil |
| U3 | Kantabhanimalat |
| 08 | Andambammatij |
| 48 |  |
| \% 4 | Irataperivabumat |
| 0.35 | HEnLingo |
| 0.6 | 1ssenbessabagata |
| 0.37 | Bogahawrwa |
| 038 | Katupothatamo a |
| 03.1 | l'ambutalianda |
| (14) | Fitigala |


| 041 | Omali らuoin |
| :---: | :---: |
| 040 | Gnntalawe |
| ! | Chatpr] [ij]! |
| リ.4 | Kuclremialai |
| 11.15 | Kulettandi |
| 946 | Ar-uakaiu |
| \% 7 | Maclamola |
| $9+3$ | Crow's Nes: |
| 149 |  |
| !eu | ©hill |
| 15 |  |
| 15: |  |
| $45: 1$ |  |
| 65 |  |
| 165 |  |
| ! ! 1 |  |
| 057 | Engoria |
| 058 | AllagaliE |
| 059 | Ambolitra |
| 060 | Yahdeseagala |
| 061 | Galgirlya |
| 062 | Gumner'a Quoin |
| 063 | Baron'e Cap |
| 064 | Kohagala |
| 1065 | Fidurutalagala |
| 060 | Kuchalanaimelai |
| O67 | Vavunativu |
| 068 | Tavelamunal |
| 069 | Kadragala |
| 070 | Friare Hood |
| 071 | Pambuthimalai |
| 072 | Wadinagala |
| 073 | Jorepotagala |
| 074 | Knuchles |
| 075 | Gommaliya |
| 076 | Namunakuli |
| 077 | Maragalakancia |
| 078 | Ulgala |
| 079 | Wadinahela |
| 080 | Berragala North |
| 081 | Kirigalpota |
| 082 | Kirioluahena |
| 083 | Gongala |
| 084 | Haburugala |
| 085 | Kataragama Peak |
| 086 | Karambagala |
| 087 | Gonadeniya |
| 088 | Kadurupokuna |
| 089 | Hambantota Tower |
| $\bigcirc 90$ | Amangalahanda |
| 091 | Uramutta |

092
093
094
095
096
097
098 099 100 101 102 103 104 105 106 107 108 109 110

Morawaka
Hidelnattu
Galle Tower
Kurundahanda
Amunerijragoda
Fanillanda
fiaycock
Bombuwela
Kukulugala
Adam's Fealz
Yainanatuwa
Oleloocia
Colombo Tower
Nawalgama
Randena
Asgiriya
Alutaipola
Halgastota
Eupala

Breakdown of 17 smaller figures for the 1932 adjustment
(Source: Jackson, 1933)


Latitudes，longitudes and plane coordinates of 1932 adjustment

| Str． | Latitude |  |  | Longitude |  |  | Nortining | Easこing |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \＃ | D | M | Sec． | D | M | Sec | （meters） | （meters） |
| 1 | 9 | 34 | 03.530 | 79 | 51 | 06.800 | 14118.259 | －5015．813 |
| 2 | 9 | 35 | 55.455 | 80 | 02 | 59.364 | 14286.612 | －3930．405 |
| 3 | $\bigcirc$ | 35 | 04.435 | 80 | 11 | 43.087 | 14207.200 | －314i．319 |
| ； | ¢ | 30 | 37.810 | 80 | 23 | 28.900 | 13798.561 | －207E．573 |
| E | 9 | 28 | 54.107 | 80 | 35 | 05.755 | 13639．339 | －：0こ5．335 |
| 6 | 9 | 23 | 36.844 | 80 | 24 | 18.972 | 13155.643 | －2000．347 |
| $\div$ | $\bigcirc$ | 24 | 34.453 | 80 | 12 | 32.565 | 13245.033 | －307： 389 |
| E | c | 23 | 46.521 | 80 | 03 | 31.570 | －13173．329 | －38 |
| E | ¢ | 28 | 49.599 | 80 | 00 | 54.535 | 13636.571 | －4i2c．013 |
| 10 | g | 11 | 46.274 | 8 C | 06 | 15.742 | －2072．947 | －364\％．995 |
| $\because$ | － | 05 | 35.259 | 80 | 04 | 37.714 | 11506．651 | －玉7Ei．e24 |
| 12 | 8 | こう | 20.736 | 80 | 08 | 42.502 | 10567．355 | －3424．91： |
| $\pm 3$ | ê | 43 | 03.156 | 80 | 08 | 26.935 | 9441.269 | －3450．451 |
| 14 | 2 | 35 | 16.837 | 80 | 15 | 16.082 | 8728．130 | －273ミ．637 |
| 三ミ | $\hat{*}$ | 34 | $\bigcirc 2.969$ | 80 | 03 | 03.144 | 3632．5：7 | －394こ．902 |
| 三 | 8 | 42 | 52.064 | 79 | 58 | 34.280 | 0425.030 | －435． 898 |
| － | $\xi$ | 52 | 04.960 | 30 | 00 | 48.591 | 10259.934 | －4143．130 |
| iE | 5 | 57 | 14.788 | $7 \Xi$ | 57 | 36.908 | 10743.594 | －443ミ． 99 |
| － | g |  | 08.555 | 79 | 51 | 26.378 | ：$: 376.905$ | －49EE．22： |
| ここ | $\cong$ | 18 | 10.284 | 79 | 59 | 05.783 | $\div 2660.703$ | －429－ 5 52 |
| 2－ | $\underline{s}$ | 20 | 19.016 | 79 | 53 | 04.104 | $\bigcirc 3408.367$ | －4342． 222 |
| 22 | $\underline{C}$ | 13 | 06.045 | 80 | 41 | 22.101 | 12649.517 | －445．043 |
| 23 | c | 13 | 25.239 | 80 | 34 | 44.455 | $\underline{12220.957}$ | －1052．400 |
| 29 | E | i | 51.575 | 80 | 43 | 52.532 | 12077.662 | －22こ．944 |
| 25 | ¢ | 01 | 06.341 | 80 | 48 | 4：．384 | 11092．394 | 2：－．405 |
| 2 2 | $\underline{5}$ | 02 | 20.355 | 80 | 37 | 00.032 | 11205． 578 | －84？．156 |
| 27 | 8 | 50 | 03.567 | 80 | 39 | 33.817 | $\bigcirc 0080.435$ | －61\％．075 |
| $2 \%$ | 2 | 45 | 35.457 | 80 | 32 | 39.183 | 9671．323 | －124：．030 |
| 2 O | $\bigcirc$ | 02 | 13．387 | 80 | 28 | 46.268 | 11192.345 | －159 ． 633 |
| 3 C | 9 | ： 6 | 13.515 | 80 | 24 | 20．735 | 12478．660 | －199ミ． 374 |
| 三 | 3 | 57 | 52.442 | 80 | 55 | 26.931 | 10796.475 | 8ここ． 123 |
| 32 | $E$ | 45 | 34.719 | 81 | 05 | 22.390 | 9670.555 | 273E．109 |
| －3 | 2 | 19 | 26.933 | 80 | 43 | 24.454 | 9108.245 | －25ड．915 |
| 24 | E | 42 | 24.064 | 80 | 28 | 57.227 | 5379.302 | － 55 － 409 |
| 三こ | 3 | 43 | 07.737 | 80 | 22 | 07.227 | 9446.550 | －22Ci． 229 |
| ミう | $\varepsilon$ | 34 | 05.385 | 80 | 28 | 45.831 | 8617.847 | －15£ 3.305 |
| 37 | 5 | 19 | こう．710 | 80 | 13 | 53.876 | 7262.301 | －295こ．745 |
| 3 c | 8 | 19 | 43.072 | 80 | 30 | 58.961 | 7300.992 | －1357．839 |
| 39 | 3 | 04 | 34.414 | 80 | 14 | 27.112 | 5914.986 | －2903．022 |
| 40 | $\varepsilon$ | 06 | 33.253 | 80 | 39 | 15.424 | 6094.643 | －64：．691 |
| 41 | 8 | 01 | 42.262 | 81 | 18 | 12.180 | 5652.121 | 2912.885 |
| 42 | 8 | 26 | 27.851 | 81 | 02 | 18.904 | 7919.110 | 1460.597 |
| 43 | 8 | 32 | 51.523 | 81 | 14 | 32.287 | 8506.028 | 2574．851 |
| 44 | 8 | 30 | 28.521 | 79 | 52 | 12.747 | 8291.842 | －4933．254 |
| 45 | 8 | 22 | 22.213 | 80 | 02 | 36.142 | 7547.228 | －3985．980 |
| 45 | 8 | 16 | 29.531 | 79 | 49 | 40.456 | 7011.145 | －5167．846 |
| 47 | 8 | 05 | 03.660 | 80 | 07 | 46.512 | 5960.522 | －3517．562 |
| 48 | 8 | 02 | 33.625 | 79 | 53 | 00.871 | 5733.939 | －4865．803 |


| 49 | 7 | 54 | 22.353 | 80 | 00 | 02.720 | 4982.445 | －4225．171 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 7 | 34 | 35.817 | 79 | 47 | 08.163 | 3172.969 | －5408．614 |
| E1 | 7 | 34 | 26.315 | 80 | 07 | 30.836 | 3154.959 | －3545．730 |
| 52 | 7 | 21 | 55.982 | 79 | 53 | 45.559 | 2011.373 | －4805．417 |
| 53 | 7 | 28 | 33.398 | 80 | 05 | 09.066 | 2616.394 | －3762．574 |
| 54 | 7 | 14 | 06.838 | 79 | 52 | 36.670 | 1295.180 | －4911．853 |
| 55 | 7 | 18 | 13.607 | 79 | 57 | 10.778 | 1671.211 | －4493．206 |
| 56 | 7 | 19 | 50.761 | 80 | 06 | 46.224 | 1818.121 | －3615．695 |
| 57 | 7 | 14 | 06.356 | 80 | 12 | 02.843 | 1291.580 | －3133．699 |
| 58 | 7 | 17 | 46.976 | 80 | 28 | 05.304 | 1627.027 | －1664．485 |
| 59 | 7 | 36 | 16.779 | 80 | 34 | 32.227 | 3321.231 | －1075．408 |
| 60 | 7 | 34 | 58.108 | 80 | 19 | 11.479 | 3202.157 | －2478．207 |
| \％： | 7 | 56 | 03.408 | 80 | 22 | 45.215 | 5133.850 | －2150．795 |
| 62 | 7 | 51 | 37.027 | 81 | 08 | 33.946 | 4727.001 | 2033.703 |
| ¢ 3 | 7 | 50 | 04.728 | 81 | 23 | 27.247 | 4587.669 | 3393.951 |
| $5:$ | 7 | 24 | 58.213 | 81 | $: 2$ | 24.320 | 2286.076 | 2355.012 |
| 65 | 7 | 00 | 01.729 | 80 | 46 | 18.160 | 0.0 | 0.0 |
| 5 | 7 | 42 | 37.320 | 81 | 30 | 50.742 | 3905.565 | 4070.471 |
| 67 | 7 | 41 | 42.684 | 81 | 39 | 12.635 | 3823.587 | $48: 5.205$ |
| E | 7 | 37 | 02.735 | 81 | 40 | 49.752 | 3395.396 | 4933.942 |
| ¢ | 7 | 32 | 30.361 | 81 | 32 | 55.524 | 2979.073 | 4263.724 |
| $\bigcirc$ | 7 | 25 | 31.577 | 81 | 30 | 20.888 | 2430.801 | 4027.492 |
| －： | 7 | 19 | 42.330 | 81 | 32 | 51.563 | 1806.275 | 4258.221 |
| 72 | 7 | 05 | 56.355 | 81 | 30 | 59.941 | 544.744 | $4090 .: 15$ |
| 73 | 7 | 05 | 58.339 | 81 | 09 | 34.959 | 545.378 | 2130.281 |
| 7 | 7 | 23 | 45.860 | 80 | 48 | 27.553 | 2174.441 | 397.209 |
| 7 | 5 | 59 | 15.456 | 80 | 55 | 07.540 | －70．509 | 807.535 |
| 75 | 6 | 55 | 57.677 | 81 | 05 | 45.581 | －371．953 | 1872.524 |
| 7 | 6 | 52 | 28．451 | 81 | 23 | 03.712 | －705．193 | 3365.390 |
| － 5 | 5 | 42 | 24.956 | 81 | 16 | 10.805 | －1512．119 | 2736.250 |
| $\bigcirc$ | 5 | 4 | 35.517 | 8. | 07 | 32.937 | －1588．259 | －97c．： 58 |
| g | 6 | 40 | 25.149 | 80 | 54 | 4i．：31 | －1246．565 | 772．：91 |
| 3 S | 6 | 47 | 55.642 | 80 | 45 | 52.054 | －1108．609 | －39．840 |
| E | 5 | 37 | $\underline{15.594}$ | 80 | 49 | 58.272 | －2084．146 | 335.229 |
| E | 5 | 23 | 07.719 | 80 | 39 | 07.179 | －3380．310 | － 658.256 |
| 6 | 6 | 19 | 47.796 | 80 | 57 | 27.250 | －3685．445 | 1022.039 |
| ミミ | ¢ | 23 | 11.560 | 81 | 20 | 02.605 | －3372．833 | 3092.057 |
| ¢5 | б | 14 | 15.486 | 31 | 00 | 09.610 | －4192．717 | 1270.270 |
| ¢ 7 | $\leqslant$ | 12 | 44.277 | 80 | 48 | 46.658 | －4332．244 | 225.383 |
| ミ |  | 00 | 53.538 | 80 | 46 | 32.895 | －5417．243 | 22.521 |
| E9 | 5 | 07 | 18.113 | 81 | 07 | 29.852 | －4829．596 | 1943.291 |
| go | 5 | 05 | 22.278 | 80 | 40 | 38.175 | －5007．045 | －519．564 |
| O： | 5 | 10 | 04.555 | 80 | 31 | 33.592 | －4575．649 | －1351．448 |
| ¢ 2 | 6 | 17 | 03.060 | 80 | 30 | 43.554 | －3936．791 | －1427．744 |
| E | 6 | 07 | 30.427 | 80 | 23 | 57.782 | －4810．724 | －2048． 239 |
| 94 | 5 | 02 | 04.356 | 80 | 14 | 34.674 | －5399．472 | －2909． 324 |
| ¢ | 5 | 04 | 44.848 | 80 | 13 | 56.805 | －5062．758 | －2950． 371 |
| ES | б | 07 | 01.235 | 80 | 08 | 06.356 | －4853．931 | －3502．219 |
| 97 | 6 | 13 | 11.098 | 80 | 06 | 12.295 | －4014．139 | －3675．472 |
| 98 | 6 | 19 | 54.209 | 80 | 17 | 51.327 | －3674．647 | －2607．213 |
| 99 | $\sigma$ | 34 | 48.126 | 80 | 01 | 05.461 | －2307．881 | －4141．741 |
| 100 | 6 | 39 | 48.826 | 80 | 15 | 43.886 | －1850．445 | －2800．051 |
| 101 | 6 | 48 | 32.544 | 80 | 29 | 50.403 | －1051．839 | －1507．365 |
| 102 | 6 | 50 | 42.472 | 80 | 14 | 35.381 | －302．610 | －2902．936 |
| 103 | 6 | 46 | 44.543 | 80 | 00 | 45.955 | －1213．905 | －4169．839 |
| 104 | 5 | 56 | 03.800 | 79 | 50 | 26.480 | －358．261 | －5113．587 |
| 105 | 6 | 54 | 39.659 | 80 | 00 | 14.664 | －488．347 | －4216．438 |
| 106 | 7 | 03 | 01.670 | 79 | 53 | 41.461 | 279.264 | －4314：990 |
| 107 | 7 | 06 | 31.582 | 79 | 59 | 16.745 | 598.885 | －4303．000 |
| 108 | 7 | 12 | 17.820 | 79 | 57 | 40.537 | 1127.841 | －4448．807 |
| 109 | 7 | 09 | 20.658 | 79 | 52 | 53.229 | 858.126 | －4887．453 |
| 0 | 7 | 15 | 04.529 | 80 | 03 | 11.824 | 1381.555 | －3943．230 |

## APPENDIX B PROPOSED LAND INFORMATION NETWORK FOR SRI LANKA

(Source: Berugoda. 198i)
STAGE - 1
sincie-11
SIAGE-111


# APPENDIX C RESULTS OBTAINED FOR A NEW ADJUSTMENT USING OLD OBSERVATIONS 

Absolute accuracy which can be obtained by old observations

| IDENT. | MAJOR SEMI-AXIS | MINOR SEMI-AXIS | AZ (MAJ) |
| :---: | :---: | :---: | :---: |
| 01 | 11.4343 | 9.7183 | 179.36 |
| 02 | 11.3311 | 9.4292 | 4.69 |
| 03 | 11.1273 | 9.1500 | 8.50 |
| 04 | 10.6479 | 8.6689 | 13.15 |
| 05 | 10.6014 | 8.5026 | 16.94 |
| 06 | 9.9966 | 8.0235 | 14.14 |
| 07 | 10.0918 | 8.1935 | 9.78 |
| 08 | 10.1314 | 8.3261 | 6.43 |
| 09 | 10.6572 | 8.8509 | 4.83 |
| 10 | 8.9914 | 7.2176 | 9.11 |
| 11 | 8.4501 | 6.7107 | 8.73 |
| 12 | 7.5309 | 5.7912 | 11.16 |
| 13 | 6.5351 | 4.8174 | 11.77 |
| 14 | 5.9166 | 4.1960 | 15.84 |
| 15 | 5.9446 | 4.2675 | 9.30 |
| 16 | 6.6899 | 5.0271 | 7.80 |
| 17 | 7.3701 | 5.6853 | 8.47 |
| 18 | 7.8642 | 6.2233 | 6.69 |
| 19 | 8.6290 | 7.0304 | 3.66 |
| 20 | 9.6894 | 7.9724 | 5.69 |
| 21 | 10.6071 | 8.9471 | 1.80 |
| 22 | 9.7166 | 7.5965 | 20.74 |
| 23 | 9.1768 | 7.1245 | 19.46 |
| 24 | 9.2262 | 7.0950 | 22.74 |
| 25 | 8.4672 | 6.3043 | 26.77 |
| 26 | 8.2693 | 6.2245 | 22.33 |
| 27 | 7.3413 | 5.3079 | 25.97 |
| 28 | 6.8722 | 4.9220 | 23.91 |
| 29 | 8.1501 | 6.1864 | 19.31 |
| 30 | 9.3291 | 7.3815 | 15.34 |
| 31 | 8.4497 | 6.2333 | 30.05 |
| 32 | 8.0326 | 5.7115 | 37.34 |
| 33 | 6.6556 | 4.6727 | 30.00 |
| 34 | 6.5704 | 4.6769 | 22.40 |


| IDENT. | MAJOR SEMI-AXIS | MINOR SEMI-AXIS | AZ (MAJ) |
| :---: | :---: | :---: | :---: |
| 35 | 6.5491 | 4.6943 | 18.60 |
| 36 | 5.9500 | 4.1578 | 23.62 |
| 37 | 4.7334 | 3.2652 | 16.29 |
| 38 | 5.0001 | 3.3792 | 28.23 |
| 39 | 3.7038 | 2.5198 | 19.70 |
| 40 | 4.4473 | 2.8656 | 38.81 |
| 41 | 5.9689 | 3.8453 | 58.81 |
| 42 | 6.3957 | 4.5559 | 43.06 |
| 43 | 7.4268 | 5.4378 | 48.57 |
| 44 | 5.8672 | 4.2166 | 0.63 |
| 45 | 5.0092 | 3.5350 | 7.12 |
| 46 | 4.8802 | 3.4996 | 174.98 |
| 47 | 3.7103 | 2.5668 | 12.58 |
| 48 | 3.8072 | 2.7298 | 173.36 |
| 49 | 3.0168 | 2.2105 | 1.77 |
| 50 | 2.6766 | 1.1876 | 139.21 |
| 51 | 1.7023 | 1.0511 | 24.31 |
| 52 | 0.5073 | 0.3762 | 168.09 |
| 53 | 1.0733 | 0.7558 | 37.57 |
| 55 | 0.3304 | 0.2075 | 47.05 |
| 56 | 0.8113 | 0.5013 | 64.90 |
| 57 | 1.0183 | 0.5754 | 88.84 |
| 58 | 2.0129 | 1.1858 | 83.30 |
| 59 | 2.8253 | 1.6989 | 59.57 |
| 60 | 2.0628 | 1.2821 | 47.25 |
| 61 | 3.3098 | 2.1718 | 32.16 |
| 62 | 5.0697 | 3.1836 | 53.21 |
| 63 | 5.8465 | 3.5920 | 67.51 |
| 64 | 4.7213 | 2.8156 | 80.80 |
| 65 | 3.1338 | 1.8499 | 103.75 |
| 66 | 6.1017 | 3.4691 | 75.19 |
| 67 | 6.3799 | 3.6041 | 76.42 |
| 68 | 6.3836 | 3.6053 | 76.98 |
| 69 | 6.0907 | 3.4315 | 78.15 |
| 70 | 5.8632 | 3.4815 | 82.13 |
| 71 | 5.9813 | 3.6786 | 87.11 |
| 72 | 5.8771 | 3.6476 | 95.47 |
| 73 | 4.4930 | 2.7265 | 94.99 |
| 74 | 3.3104 | 1.9432 | 77.57 |
| 75 | 3.6700 | 2.1770 | 102.45 |
| 76 | 4.4254 | 2.6730 | 102.93 |
| 77 | 5.5250 | 3.4692 | 103.05 |
| 78 | 5.3030 | 3.3504 | 109.88 |
| 79 | 4.8199 | 2.9878 | 112.53 |
| 80 | 3.9036 | 2.3620 | 113.32 |
| 81 | 3.3780 | 2.0204 | 115.47 |
| 82 | 3.9406 | 2.4355 | 122.77 |

2-D AND 1-D STATION CONFIDENCE REGIONS ( 95.000 \%):

| IDENT. | MAJOR SEMI-AXIS | MINOR SEMI-AXIS | AZ (MAJ) |
| :--- | ---: | ---: | ---: |
| 83 | 4.0931 | 2.5929 | 139.21 |
| 84 | 5.0811 | 3.2446 | 129.87 |
| 85 | 6.2425 | 4.0033 | 119.42 |
| 86 | 5.5012 | 3.5525 | 131.47 |
| 87 | 5.0685 | 3.2603 | 138.04 |
| 88 | 5.7551 | 3.7908 | 144.65 |
| 89 | 6.2652 | 4.1276 | 131.43 |
| 90 | 5.2342 | 3.4484 | 145.04 |
| 91 | 4.6180 | 3.0153 | 149.23 |
| 92 | 4.1379 | 2.6690 | 147.32 |
| 93 | 4.6210 | 3.0461 | 156.60 |
| 94 | 4.9764 | 3.4190 | 166.55 |
| 95 | 4.6586 | 3.1917 | 165.99 |
| 96 | 4.4364 | 3.1345 | 170.64 |
| 97 | 3.7373 | 2.5893 | 167.34 |
| 98 | 3.6466 | 2.3397 | 157.38 |
| 99 | 2.3923 | 1.5161 | 168.92 |
| 100 | 2.3204 | 1.4338 | 144.50 |
| 101 | 2.5003 | 1.4725 | 123.68 |
| 102 | 1.4604 | 0.8455 | 128.44 |
| 103 | 1.5687 | 0.9466 | 166.22 |
| 104 | 1.0059 | 0.6129 | 12.22 |
| 105 | 0.5436 | 0.6193 | 159.91 |
| 106 | 0.4867 | 0.3020 | 174.47 |
| 107 | 0.2650 | 0.2624 | 139.54 |
| 108 | 0.2162 | 0.0955 | 176.77 |
| 109 | 0.5478 | 0.3118 | 85.23 |

## Differences between old and new adjustments

(same observations)

Station Difference in Difference in Station Differnce in Diference in Number eastings ( $m$ ) northings ( $m$ ) .Number eastings ( $m$ ) northings ( $m$ )

| 1 | -0.619 | 14.114 | $\underline{2}$ | 1.208 | 14.208 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 2.435 | 1.3.902 | 4 | 3.842 | 12.378 |
| 5 | 5.0 .59 | 11.515 | 6 | 3.498 | 11.564 |
| \% | 2.069 | 112.168 | 8 | 0.910 | 12.150 |
| 9 | 0.698 | 13.147 | 10 | 1.196 | 10.611 |
| 11 | 0.831 | 9.631 | 12 | 1.022 | 8.010 |
| 13 | 0.522 | 6.601 | 14 | 0.826 | 5.803 |
| 1.5 | -0.108 | 5.945 | 16 | -0.418 | 6.78 .5 |
| 17 | -0.009 | 7.783 | 18 | -0.42.4 | 8.612 |
| 19 | -1.051 | 9.750 | 20 | 0.216 | 11.71 .58 |
| 21 | -0.472 | 12.914 | $\underline{2} 2$ | 5.106 | 9.833 |
| 23 | 4.067 | 9.774 | 24 | 4.727 | 9.127 |
| 25 | 5.323 | 8.037 | 26 | 3.8:36 | 8.437 |
| 27 | 3.771 | 6.998 | 28 | 2.957 | 6.616 |
| 29 | 2.809 | 8.705 | 30 | 3.095 | 10.65:3 |
| 31 | 5.920 | 7.430 | 32 | 6.594 | 5.583 |
| 33 | 3.850 | 5.936 | 34 | 2.464 | $6.28: 3$ |
| 35 | 1.725 | 6.524 | 36 | 2.184 | 5. 411 |
| 37 | 0.277 | 4.166 | 38 | 1.806 | 3.909 |
| 39 | 0.118 | 3.776 | 40 | 1.781 | 3.035 |
| 41 | 3.852 | 1.445 | 42 | 4.690 | 3.55.3 |
| 43 | 6.834 | 3.499 | 44 | -1.344 | 5.580 |
| 45 | -0.284 | 4.586 | 46 | $-1.473$ | 4.272 |
| 47 | -0.007 | 3.830 | 48 | -1.215 | 3.610 |
| 49 | -0.779 | 2.986 | 50 | -3.061 | 2.080 |
| 51 | -0.958 | 2.009 | 52 | -1.288 | 0.475 |
| 53 | -1.201 | 0.951 | 54 | -1.174 | 0.329 |
| 55 | -1.143 | 0.459 | 56 | -0.975 | 0.716 |
| 57 | -0.421 | 0.516 | 58 | 0.115 | 0.573 |
| 59 | 0.977 | 1.431 | 60 | 0.201 | 1.889 |
| 61 | 20.814 | 2.698 | 62 | 2.840 | 1.217 |

Station Difference in Difference in Station Differnce in Difference in Number eastings ( $m$ ) northings ( m ) Number eastings ( m ) northings (m)

| 71 | 2.718 | -1.410 | $i 2$ | 1.923 | -1.907 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 73 | 1.065 | -0.907 | 74 | 0.71 .5 | 0.392 |
| 75 | 0.433 | -0.195 | 76 | 0.496 | -1.130 |
| 77 | 0.659 | -1.976 | 78 | 0.006 | -1.762 |
| 79 | -0.521 | -1.318 | 80 | 0.001 | -0.721 |
| 81 | 0.147 | -0.502 | 82 | -0.107 | -0.1 .58 |
| 83 | 0.538 | -0.700 | 84 | 0.232 | -0.173 |
| 85 | -1.276 | -0.238 | 86 | 0.192 | 0.005 |
| 87 | 0.557 | -0.119 | 88 | 0.951 | 0.446 |
| 89 | 0.103 | -0.476 | 90 | 1.238 | -0.192 |
| 91 | 1.355 | -0.787 | 92 | 0.925 | -0.81 .5 |
| 93 | 1.324 | -1.279 | 94 | 1.181 | -2.072 |
| 95 | 0.971 | -1.943 | 96 | 0.669 | -2.117 |
| 97 | 0.187 | -1.574 | 98 | 0.394 | -1.285 |
| 99 | -0.507 | -1.302 | 100 | -0.561 | -0.317 |
| 101 | -0.158 | -0.001 | 102 | -0.370 | -0.114 |
| 103 | -1.108 | -0.799 | 104 | -1.257 | 0.006 |
| 105 | -0.874 | -0.243 | 106 | -1.109 | 0.130 |
| 107 | -0.972 | 0.229 | 108 | -1.046 | 0.343 |
| 109 | -1.132 | 0.258 | 110 | -0.942 | 0.486 |

Latitudes and longitudes of the new adjustment
(Same observations)

| Str. | Latitude |  |  | Longitude |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | D | M | Sec. | D |  | Sec. |
| 01 | 9 | 34 | 3.142831 | 79 | 51 | 6.752304 |
| 02 | 9 | 35 | 55.069647 | 80 | 2 | 59.272577 |
| 03 | 9 | 35 | 4.064235 | 80 | 11 | 42.968155 |
| 04 | 9 | 30 | 37.481156 | 80 | 23 | 28.754690 |
| 05 | 9 | 28 | 53.806294 | 80 | 35 | 5.585413 |
| 05 | 9 | 23 | 36.531446 | 80 | 24 | 18.840453 |
| 07 | 9 | 24 | 34.118687 | 80 | 12 | 32.452759 |
| 08 | 9 | 23 | 46.175456 | 80 | 3 | 31.491E51 |
| 09 | 9 | 28 | 49.237016 | 80 | 0 | 54.459082 |
| 10 | 9 | 11 | 45.972997 | 80 | 6 | 15.659998 |
| 11 | 9 | 5 | 34.982887 | 80 | 4 | 37.643E70 |
| 12 | 8 | 55 | 20.503318 | 80 | 8 | 42.433615 |
| 13 | 8 | 43 | 2.956404 | 80 | 8 | $26.888=10$ |
| 14 | 8 | 35 | 16.663699 | 80 | 16 | 16.041228 |
| 15 | 8 | 34 | 12.777129 | 80 | 3 | 3.120603 |
| 16 | 8 | 42 | 51.851859 | 79 | 58 | 34.256076 |
| 17 | 8 | 52 | 4.727227 | 80 | 0 | 48.551711 |
| 18 | 8 | 57 | 14.533340 | 79 | 57 | 36.874039 |
| 19 | 9 | 4 | 8.269456 | 79 | 51 | 26.354E97 |
| 20 | 9 | 18 | 9.953590 | 79 | 59 | 6.722702 |
| 21 | 9 | 26 | 18.656398 | 79 | 53 | 4.055783 |
| 22 | 9 | 18 | 5.785885 | 80 | 41 | 21.941701 |
| 23 | 9 | 13 | 24.974096 | 80 | 34 | 44.322757 |
| 24 | 9 | 11 | 51.332341 | 80 | 43 | 52.389772 |
| 25 | 9 | 1 | 6.121090 | 80 | 48 | 41.230744 |
| 25 | 9 | 2 | 20.118880 | 80 | 36 | 59.913225 |
| 27 | 8 | 50 | 3.366647 | 80 | 39 | 33.707205 |
| 28 | 8 | 45 | 35.260395 | 80 | 32 | 39.094525 |
| 29 | 9 | 2 | 11.140321 | 80 | 28 | 46.172836 |
| 30 | 9 | 16 | 13.221992 | 80 | 24 | 20.619¢73 |
| 31 | 8 | 57 | 52.239311 | 80 | 55 | 26.766107 |
| 32 | 8 | 45 | 34.564537 | 81 | 5 | 22.223902 |
| 33 | 8 | 39 | 26.759617 | 80 | 43 | 24.358131 |
| 34 | 8 | 42 | 23.876598 | 80 | 28 | 57.152504 |
| 35 | 8 | 43 | 7.545595 | 80 | 22 | 7.168877 |
| 36 | 8 | 34 | 5.217740 | 80 | 28 | 45.760723 |
| 37 | 8 | 19 | 16.573156 | 80 | 13 | 55.842415 |
| 38 | 8 | 19 | 42.937560 | 80 | 30 | 58.890890 |
| 39 | 8 | 4 | 34.297189 | 80 | 14 | 27.085516 |
| 40 | 8 | 6 | 33.151541 | 80 | 39 | 16.342424 |
| 41 | 8 | 1 | 42.191119 | 81 | 18 | 12.034090 |
| 42 | 8 | 26 | 27.724908 | 81 | 2 | 18.760454 |
| 43 | 8 | 32 | 51.392653 | 81 | 14 | 32.104360 |


| 44 | 8 | 30 | 28.330550 | 79 | 52 | 12.764103 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 45 | 8 | 22 | 22.059235 | 80 | 2 | 36.132775 |
| 46 | 8 | 16 | 29.380920 | 79 | 49 | 40.489539 |
| 47 | 8 | 5 | 3.539194 | 80 | 7 | 46.503079 |
| 48 | 8 | 2 | 33.504023 | 79 | 53 | 0.905703 |
| 49 | 7 | 54 | 22.259194 | 80 | 0 | 2.740921 |
| 50 | 7 | 34 | 35.781774 | 79 | 47 | 8.211219 |
| 51 | 7 | 34 | 26.272243 | 80 | 7 | 30.837909 |
| 52 | 7 | 21 | 55.978475 | 79 | 53 | 45.560412 |
| 53 | 7 | 28 | 33.377433 | 80 | 5 | 9.067767 |
| 54 | 7 | 14 | 6.838000 | 79 | 52 | 36.670000 |
| 55 | 7 | 18 | 13.603733 | 79 | 57 | 10.776079 |
| 56 | 7 | 19 | 50.752319 | 80 | 6 | 46.214924 |
| 57 | 7 | 14 | 6.353127 | 80 | 12 | 2.827299 |
| 58 | 7 | 17 | 46.965308 | 80 | 28 | 6.266452 |
| 59 | 7 | 36 | 16.740877 | 80 | 34 | 32.169687 |
| 60 | 7 | 34 | 58.064950 | 80 | 19 | 11.452583 |
| 61 | 7 | 50 | 3.317675 | 80 | 22 | 45.171724 |
| 62 | 7 | 51 | 36.969870 | 81 | 8 | 33.825714 |
| 63 | 7 | 50 | 4.682280 | 81 | 23 | 27.098280 |
| 64 | 7 | 24 | 58.205514 | 81 | 12 | 24.204869 |
| 65 | 7 | 0 | 1.736267 | 80 | 46 | 18.109007 |
| 66 | 7 | 42 | 37.290945 | 81 | 30 | 50.588729 |
| 67 | 7 | 41 | 42.663768 | 81 | 39 | 12.467060 |
| 58 | 7 | 37 | 2.722145 | 81 | 40 | 49.585433 |
| 69 | 7 | 32 | 30.352491 | 81 | 32 | 56.373256 |
| 70 | 7 | 26 | 31.579367 | 81 | 30 | 20.747231 |
| 71 | 7 | 19 | 42.343085 | 81 | 32 | 51.424736 |
| 72 | 7 | 5 | 56.385915 | 81 | 30 | 59.816133 |
| 73 | 7 | 5 | 58.354492 | 81 | 9 | 34.863741 |
| 74 | 7 | 23 | 45.843589 | 80 | 48 | 27.490447 |
| 75 | 6 | 59 | 15.473374 | 80 | 55 | 7.469654 |
| 76 | 6 | 55 | 57.706058 | 81 | 6 | 45.496038 |
| 77 | 6 | 52 | 18.495971 | 31 | 23 | 3.616348 |
| 78 | 6 | 42 | 25.003801 | 81 | 16 | 10.731433 |
| 79 | 6 | 41 | 35.555898 | 81 | 7 | 52.860862 |
| 80 | 6 | 46 | 25.178413 | 80 | 54 | 44.063118 |
| 81 | 6 | 47 | 55.670348 | 80 | 45 | 51.991990 |
| 82 | 6 | 37 | 16.728810 | 80 | 49 | 58.207231 |
| 83 | 6 | 23 | 7.773627 | 80 | 39 | 7.090805 |
| 84 | 6 | 19 | 47.833890 | 80 | 57 | 27.168162 |
| 85 | 6 | 23 | 11.589482 | 81 | 20 | 2.547581 |
| 86 | 6 | 14 | 15.519716 | 81 | 0 | 9.527095 |
| 87 | 6 | 12 | 44.317037 | 80 | 48 | 46.568408 |
| 88 | 6 | 0 | 53.670106 | 80 | 46 | 32.794068 |
| 89 | 6 | 7 | 18.135258 | 81 | 7 | 29.771202 |
| 90 | 6 | 5 | 22.344014 | 80 | 40 | 38.052171 |
| 91 | 6 | 10 | 4.736934 | 80 | 31 | 33.588603 |
| 92 | 6 | 17 | 3.132202 | 80 | 30 | 43.467792 |
| 93 | 6 | 7 | 30.531891 | 80 | 23 | 57.692734 |
| 94 | 6 | 1 | 4.496680 | 80 | 14 | 34.607476 |
| 95 | 6 | 4 | 44.977753 | 80 | 13 | 56.745888 |
| 96 | 6 | 7 | 1.363530 | 80 | 8 | 6.317087 |
| 97 | 6 | 16 | 11.196710 | 80 | 6 | 12.270286 |


| 98 | 6 | 19 | 54.294553 | 80 | 17 | 51.276721 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 99 | 6 | 34 | 48.174668 | 80 | 1 | 5.451987 |
| 100 | 6 | 39 | 48.859575 | 80 | 15 | 43.851652 |
| 101 | 6 | 48 | 32.561301 | 80 | 29 | 50.354755 |
| 102 | 6 | 56 | 42.484510 | 80 | 14 | 35.359050 |
| 103 | 6 | 46 | 44.572013 | 80 | 0 | 45.944699 |
| 104 | 6 | 56 | 3.813689 | 79 | 50 | 26.483182 |
| 105 | 6 | 54 | 39.672604 | 80 | 0 | 14.657271 |
| 106 | 7 | 3 | 1.676059 | 79 | 53 | 41.459824 |
| 107 | 7 | 6 | 31.584779 | 79 | 59 | 16.740164 |
| 108 | 7 | 12 | 17.820254 | 79 | 57 | 40.533207 |
| 109 | 7 | 9 | 20.660517 | 79 | 52 | 53.228472 |
| 110 | 7 | 15 | 4.525925 | 80 | 3 | 11.815395 |

## APPENDIX E INPUT FILE FOR A GEOLAB

## ADJUSTMENT WITH NEW GPS OBSERVATIONS

* Welcome to GeoLab, the survey laboratory of software tools you've
* been waiting for. This file is an example of the text input file
* which GeoLab reads, interprets, and processes.
* 
* GeoLab input files may have initial comments like this - the
* records beginning with '*' are completely ignored.
* 
* The following title record must be the first (non *) record:

GeoLab Adjustment of Sri Lanka full figure with Fig $1 \& 2$ and HDC $10 / 11$
*

* The second record must be the options record:
$005 \quad 01131112 \quad 0005500.001 \quad 95.000$
* Reference ellipsoid is Everest ellipsoid used in 1932 adjustment

| 30 | 6377299.151 | 6356098.145 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- |


|  | Station | Latitude | ongitude Elevation |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| * + | 1001 |  | 3000.00000 w | 9000.00000 | 1955.800 |
| 14 | 111 | SAMADI_ISMD | 6492.70019 e | 805740.91115 | 1160.5110 |
| 4 | 34 | IRATTAPEKULA | 84224.77066 e | 8029 4.58251 | 33.5720 |
| $\rightarrow$ | 36 | ISSENBESSAWAGAL | 834 6.65002e | 802852.70220 | 62.7633 |
| 4 | 37 | BOGAHAWEWA | $81917.53231 e$ | 80143.42980 | 2.3157 |
| $1+$ | 38 | KATUPOTAKANDA | 81943.89753 e | 9031 6.36227 | 302.1105 |
| 14 | 39 | TAMBUTTKANDA | 8435.29376 e | 801434.66119 | 180.1575 |
| 14 | +0 | RITIGALA | 8634.14244 e | 803923.77113 | 666.6753 |
| 4 | 47 | MADAMOLA | $854.53802 e$ | 80754.10847 | 160.0959 |
| $+$ | 48 | CROWS_NEST | ¢ 234.51353 e | 7953 8.59602 | 29.0952 |
| 4 | $\pm 9$ | PARAMAKANDA | $75+23.28671 \mathrm{e}$ | $80 \quad 010.38595$ | 59.5401 |
| $+$ | 51 | MEDAGAMA | $73427.34187 e$ | 800738.44112 | 217.0589 |
| 4 | 56 | NARANGAMA. | 71951.86535 e | 80653.82980 | 95.9161 |
| 14 | 57 | ENGODA | 714.7 .4847 je | 801210.40046 | 207.8175 |
| 4 | 59 | AMBOKKA | 73617.79764 e | 803439.63208 | 1131.6784 |
| 14 | 60 | YAKDESSAGALA | $73459.12325 e$ | 801918.98921 | 422.2009 |
| 4 | 61 | GALGIRIYA | 7564.34335 e | 802252.68669 | 470.9143 |
| 14 | 73 | DORAPATAGALA | 7559.50407 e | 81942.09889 | 1414.7603 |
| 14 | 74 | KNUCKLES | 72346.93982 e | 804834.85377 | 1762.2261 |
| 14 | 75 | GOMMOLIYA | 65916.63785 e | 805514.77851 | 1580.9060 |
| 14 | 76 | NAMUNUKULA | 65558.84319 e | 81652.71581 | 1933.7946 |
| 4 | 77 | MARAGALAKANDA | 65253.88094 e | 812322.87867 | 1017.6951 |
| 4 | 78 | UGALA | 64226.21453 e | 811617.91375 | 475.1076 |
| 4 | 80 | BERAGALA_NORTH | 64626.38687 e | 805451.37911 | 1677.9687 |
|  | 82 | KIRIOLUHENA | 63717.94763 e | 80505.54051 | 632.4 |


| 4 | 83 | GONGALA | 6239.04441 e | 803914.47894 | 1258.4147 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14 | 84 | HABURUGALA | 61949.09349 e | 805734.43936 | 2.5440 |
| 4 | 85 | KATARAGAMA_PEA. | $6234.37495 e$ | 811929.05240 | 263.4605 |
| 4 | 86 | KARAMBAGALA | 61417.28161 e | 81016.60445 | 18.6120 |
| 14 | 88 | KADURUPOKUNA | 6054.57172 e | 804640.74918 | 39.4073 |
| 4 | 89 | HAMBANTOTA_T | 6719.40276 e | 81736.96665 | 69.6527 |
| 14 | 90 | AMANGALAKANDA | 60523.63198 e | 804045.40534 | 123.8126 |
| 4 | 91 | URUMUTTA | 6106.03930 e | 803140.98585 | 329.5039 |
| 4 | 92 | MORAWAKA | 6174.41334 e | 803050.87691 | 623.2785 |
| 4 | 93 | HINDEKNATU | $6731.85032 e$ | $8024 \quad 5.12765$ | 307.0154 |
| 14 | 94 | GALLE TOWER | 615.79160 e | 801443.42696 | 30.8270 |
| $t$ | 95 | KURUNDAKANDA | 6446.31613 e | 80144.25130 | 12.8704 |
| $1+$ | 96 | AMUNERIYAGODA | 672.70254 e | 80813.33751 | 66.5288 |
| $+$ | 97 | PANILKANDA | 61612.51503 e | $80619.924+1$ | +8.6191 |
| 4 | 98 | HAYCOCK | 61955.58962 e | 801758.76868 | 538.2764 |
| 4 | 99 | BOMBUWALA | 63449.42735 e | 80113.06479 | 55.3614 |
| 14 | 103 | OLEBODA | 64645.80610 e | $80 \quad 053.58709$ | 11.1961 |
| $t$ | 107 | ASGIRIYA | 7632.73594 e | 795924.39626 | 34.2910 |
| 14 | 108 | ALUTAIPOLA | 71218.95729 e | 795748.19502 | +7.2406 |
| 4 | 112 | GONADENIYA | $61245.72071 e$ | $804853.8+195$ | 163.2035 |
| 4 | 113 | PUGALA | $7155.65395 e$ | $80 \quad 319 .+5020$ | 66.5118 |
| 913DD |  |  |  |  |  |
| *96 | 1007 | $3001.94870 w$ | 903052.04613 | 1937.020 |  |
| 96 | 111 | 6492.70019 e | 805740.91115 | 1160.811 |  |
| 96 | 34 | 84224.77066 e | 80294.58251 | 33.872 |  |
|  | 36 | 834 6.65002e | 802852.70220 | 62.763 |  |
| 96 | 38 | $81943.89686 e$ | 80316.36009 | 302.154 |  |
| 96 | 39 | $8435.293+2 e$ | $801+34.66110$ | 180.187 |  |
| 96 | 39 | 8 + 35.29436 e | $801+34.66166$ | 180.100 |  |
| 96 | 40 | 8634.14219 e | 803923.76941 | 666.721 |  |
| 96 | 40 | 8634.14255 e | 803923.77384 | 666.606 |  |
| 96 | 48 | 8234.51315 e | 79538.59569 | -29.100 |  |
| 96 | 57 | 7147.48444 e | 801210.39949 | 207.717 |  |
| 96 | 57 | 7147.48503 e | 801210.40143 | 207.918 |  |
| 96 | 59 | 73617.79760 e | 803439.63353 | 1131.621 |  |
| 96 | 60 | 73459.12345 e | 801918.98816 | 422.245 |  |
| 96 | 60 | $73459.12276 e$ | 801918.98898 | 422.208 |  |
| 96 | 61 | 7564.34335 e | 802252.68669 | 470.914 |  |
| 96 | 73 | 7559.50428 e | 81942.10051 | 1414.766 |  |
| 96 | 74 | 723 46.93920e | 804834.85430 | 1762.168 |  |
| 96 | 74 | 72346.93942 e | 804834.85203 | 1762.287 |  |
| 96 | 75 | 65916.63839 e | 805514.77946 | 1580.898 |  |
| 96 | 75 | 65916.63924 e | 805514.77632 | 1580.928 |  |
| 96 | 76 | 65558.84337 e | 81652.71589 | 1933.775 |  |
| 96 | 76 | 65558.84316 e | 81652.71723 | 1933.782 |  |
| 96 | 77 | 65253.88094 e | 812322.87767 | 1017.692 |  |
| 96 | 78 | 64226.21453 e | 811617.91275 | 475.112 |  |
| 96 | 80 | 64626.38687 e | 805451.37911 | 1677.968 |  |
| 96 | 82 | 63717.94760 e | 80505.54245 | 632.402 |  |
| 96 | 82 | 63717.94783 e | 30505.53927 | 632.404 |  |
| 96 | 82 | 63717.94767 e | $8050 \quad 5.53940$ | 632.484 |  |

$\left.\begin{array}{lllllll}96 & 83 & & 623 & 9.04441 e & 80 & 39 \\ 96\end{array}\right)$


| 98 | 0.00009 | 0.00009 | 0.00009 |
| :--- | :--- | :--- | :--- |
| 98 | 0.00009 | 0.00009 | 0.00009 |

913DD
*96 1007
9639
$\left.\begin{array}{rrrrrr}30 & 0 & 1.94870 w & 903052.04613 & 1937.020 \\ 8 & 4 & 35.29436 e & 80 & 1434.66166 & 180.100 \\ 8 & 5 & 4.53802 e & 80 & 754.10847 & 160.095 \\ 8 & 2 & 34.51347 e & 79 & 53 & 8.59622\end{array}\right)-29.114$

9648
97pdvdiagonal
980.00009
$0.00009 \quad 0.00009$
$98 \quad 0.00009$
$0.00009 \quad 0.00009$
75

913DD
*96 1007
9640

| 30 | 0 | $1.94870 w$ | 903052.04613 | 1937.020 |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 8 | 634.14255 e | 803923.77384 | 666.606 |  |  |
| 842 | 24.75366 e | 8029 | 4.63066 | 33.500 |  |
| 8 | 19 | 43.89796 e | 80 | 31 | 6.36493 | 302.063

9638
97pdvdiagonal
$98 \quad 0.00009$
$0.00009 \quad 0.00009$
980.00009
$0.00009 \quad 0.00009$
75

913DD
*96 1007
$96+7$
9648
9649
96 111
97pdvdiagonal

| 98 | 0.00009 | 0.00009 | 0.00009 |
| :--- | :--- | :--- | :--- |
| 98 | 0.00009 | 0.00009 | 0.00009 |
| 98 | 0.00009 | 0.00009 | 0.00009 |

75
$\left.\begin{array}{rrrlrr}30 & 0 & 1.94870 w & 90 & 30 & 52.04613 \\ 8 & 5 & 4.53802 \mathrm{e} & 80 & 7535.10847 & 160.095 \\ 8 & 2 & 34.51359 \mathrm{e} & 79 & 53 & 8.59602\end{array}\right)-29.096$

913DD
*96 1007
9649
$30 \quad 0 \quad 1.94870 w \quad 903052.04613 \quad 1937.020$
$75423.28671 \mathrm{e} 80 \quad 010.38595 \quad 59.540$
$8435.29400 e \quad 801434.66123 \quad 180.131$
649 2.69983e $805740.91066 \quad 1160.837$
96 111
97pdvdiagonal
980.00009
980.00009
$0.00009 \quad 0.00009$

75

913DD
*96 1007
9651
9640
96111
$0.00009 \quad 0.00009$

97pdvdiagonal
$30 \quad 0 \quad 1.94870 w \quad 903052.046131937 .020$
$\begin{array}{llll}73427.34187 e & 80 & 738.44112 & 217.058\end{array}$ $8634.14240 \mathrm{e} \quad 803923.76992 \quad 666.727$ $649 \quad 2.70040 \mathrm{e} \quad 805740.91194 \quad 1160.809$

| 98 | 0.00009 | $0.00009 \quad 0.00009$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 98 | 0.00009 | 0.00009 | 0.000 |  |  |
| 75 |  |  |  |  |  |
| $913 D D$ |  |  |  |  |  |
| *96 | 1007 | 300 | 1.94870w | 903052.04613 | 1937.020 |
|  | 56 | 719 | 51.86535e | 80653.82980 | 95.916 |
| 96 | 51 | 734 | 27.21157e | 80738.28770 | 212.564 |
|  | 111 | 649 | 2.70010 e | 805740.91506 | 1160.728 |
| 97pdvdiagonal |  |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.0000 |  |  |
| 98 | 0.00009 | 0.00009 | 0.000 |  |  |
| 75 |  |  |  |  |  |
| $913 D D$ |  |  |  |  |  |
| *96 | 1007 | 300 | 1.94870w | 903052.04613 | 1937.020 |
|  | 57 | 714 | 7.48444e | 801210.39949 | 207.717 |
|  | 51 | 734 | 27.20682e | 80738.28499 | 212.492 |
|  | 56 | 719 | 51.86535e | 80653.82980 | 95.916 |
| 97pdvdiagonal |  |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.0000 |  |  |
| 98 | 0.00009 | 0.00009 | 0.000 |  |  |
| 75 |  |  |  |  |  |
| 913DD |  |  |  |  |  |
| *96 | 1007 | 300 | $1.94870 w$ | 903052.04613 | 1937.020 |
|  | 57 | 714 | 7.48503e | 801210.40143 | 207.918 |
| 96 | 108 | 712 | 18.95725e | 795748.19521 | -47.215 |
| 96 | 113 | 715 | 5.65395e | 80319.45020 | 66.511 |
| 97pdvdiagona! |  |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.0000 |  |  |
| 98 | 0.00009 | 0.00009 | 0.000 |  |  |
| 75 |  |  |  |  |  |
| 913DD |  |  |  |  |  |
| *96 | 1007 | 300 | 1.94870w | 903052.04613 | 1937.020 |
| 96 | 108 | 712 | 18.95725e | 795748.19521 | -47.215 |
| 96 | 113 | 715 | 5.65509e | 80319.45028 | 66.519 |
| 97pdvdiagonal |  |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.000 |  |  |
| 75 |  |  |  |  |  |
| 913DD |  |  |  |  |  |
| *96 | 1007 | 300 | 1.94870w | 903052.04613 | 1937.020 |
| 96 | 60 | 734 | 59.12276 e | 801918.98898 | 422.208 |
| 96 | 40 | 86 | 34.14240e | 803923.76986 | 666.728 |
| 96 | 51 | 734 | 27.34187e | 80738.44112 | 217.058 |
| 97pdvdiagonal |  |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.000 |  |  |
| 98 | 0.00009 | 0.00009 | 0.000 |  |  |
| 75 |  |  |  |  |  |


| 913DD |  |  |
| :---: | :---: | :---: |
| *96 1007 | 3001.94870 w 903052.04613 | 1937.020 |
| 9661 | 7564.34335 e 802252.68669 | 470.914 |
| 9639 | 8435.29355 e 801434.66090 | 180.185 |
| 9638 | 81943.89776 e 80316.36240 | 302.110 |
| 97pdvdiagonal |  |  |
| 980.00009 | 0.000090 .00009 |  |
| 980.00009 | 0.000090 .00009 |  |
| 75 |  |  |
| $913 D D$ |  |  |
| *96 1007 | $3001.94870 w 903052.04613$ | 1937.020 |
| 9673 | 7 559.5056le 81 942.09897 | 1414.757 |
| 9675 | $65916.63887 \mathrm{e} 8055 \mathrm{l}+.77921$ | 1580.892 |
| 96111 | 649 2.70226e 805740.91087 | 1160.831 |
| 97pdvdiagona! |  |  |
| 980.00009 | 0.000090 .00009 |  |
| 980.00009 | 0.000090 .00009 |  |
| 75 |  |  |
| $913 D D$ |  |  |
| *961007 | $3001.94870 w 903052.04613$ | 1937.020 |
| 9673 | 7 5 59.50586e 81942.09808 | 1414.765 |
| 9674 | 72346.94355 e 804834.85568 | 1762.223 |
| 97pdvdiagona! |  |  |
| 980.00009 | 0.000090 .00009 |  |
| 75 |  |  |
| 913DD |  |  |
| *961007 | 30 0 1.94870w 903052.04613 | 1937.020 |
| 9674 | $72346.93942 e 804834.85203$ | 1762.287 |
| 9659 | 73617.79764 e 803439.63208 | 1131.678 |
| 9660 | 73459.12361 e 801918.98774 | 422.257 |
| 97pdvdiagonal |  |  |
| 980.00009 | $0.00009 \quad 0.00009$ |  |
| 980.00009 | 0.000090 .00009 |  |
| 75 |  |  |
| 913DD |  |  |
| *961007 | $3001.94870 w 903052.04613$ | 1937.020 |
| 9675 | - 5916.63924 e 805514.77632 | 1580.928 |
| 9673 | 7559.50586 e 81942.09808 | 1414.765 |
| 9674 | 72346.94105 e 804834.85035 | 1762.249 |
| 97pdvdiagonal |  |  |
| 980.00009 | 0.000090 .00009 |  |
| 980.00009 | 0.000090 .00009 |  |
| 75 |  |  |
| 913DD |  |  |
| *96 1007 | $30 \quad 0 \quad 1.94870 w 903052.04613$ | 1937.020 |
| 9676 | $65558.84337 e 81652.71589$ | 1933.775 |
| 9673 | 7559.50561 e 81942.09897 | 1414.757 |

9675
97pdvdiagonal

| 98 | 0.00009 | 0.00009 | 0.00009 |
| :--- | :--- | :--- | :--- |
| 98 | 0.00009 | 0.00009 | 0.00009 |

$0.00009 \quad 0.00009$ $0.00009 \quad 0.00009$
$65916.63686 \mathrm{e} 805514.77936 \quad 1580.866$
$30 \quad 0 \quad 1.94870 w 903052.046131937 .020$
$65558.84316 \mathrm{e} 81 \quad 652.71723 \quad 1933.782$
65253.88050 e 81 2322.879011017 .728
64226.21442 e 81 $1617.91310 \quad 475.107$
$63717.94755 \mathrm{e} \quad 8050 \quad 5.54262 \quad 632.372$
$0.00009 \quad 0.00009$
$0.00009 \quad 0.00009$
$0.00009 \quad 0.00009$

913DD
*96 1007
9676
9677
9678
9682
97pdvdiagonal
$98 \quad 0.00009$
980.00009
980.00009

75
913DD
*96 1007
9678
$96 \quad 77$
$96 \quad 32$
97pdvdiagonal 980.00009
$0.00009 \quad 0.00009$
980.00009
$0.00009 \quad 0.00009$
75

913DD
*96 1007
9680
$96 \quad 73$
9675
9676
97pdvdiagonal
$98 \quad 0.00009$
980.00009
$98 \quad 0.00009$
75
913DD
*96 1007
9682
$96 \quad 77$
9677
97pdvdiagonal
$98 \quad 0.00009$
$98 \quad 0.00009$
75

913DD
*96 1007
$0.00009 \quad 0.00009$
$30 \quad 1.94870 \mathrm{w} 903052.04613 \quad 1937.020$
$63717.94760 \mathrm{e} 8050 \quad 5.54245 \quad 632.402$
$65253.88027 e 812322.87779 \quad 1017.763$
$65253.88091 \mathrm{e} 812322.88085 \quad 1017.623$
$0.00009 \quad 0.00009$

| 96 | 82 | 63717.94783 e | 80505.53927 | 632.404 |
| :---: | :---: | :---: | :---: | :---: |
| 96 | 84 | 61949.09384 e | 805734.43842 | 2.556 |
|  | 85 | 6234.37495 e | 811929.05254 | 263.444 |
| 97pdvdiagonal |  |  |  |  |
| 98 | 0.00009 | $0.00009 \quad 0.00009$ |  |  |
| 98 | 0.00009 | 0.000090 .00009 |  |  |
| 75 |  |  |  |  |
| $913 D D$ |  |  |  |  |
|  | 1007 | 300 l.94870w | 903052.04613 | 1937.020 |
| 96 | 83 | 623 9.0444le | 803914.47894 | 1258.414 |
| 96 | 91 | 610 6.03930e | 803140.98585 | 329.503 |
| 96 | 88 | 6054.57160 e | 804640.74985 | -39.478 |
| 97 pdivdiagonal |  |  |  |  |
| 98 | 0.00009 | $0.00009 \quad 0.00009$ |  |  |
| 98 | 0.00009 | 0.000090 .00009 |  |  |
| 75 |  |  |  |  |
| $913 D D$ |  |  |  |  |
| *96 | 1007 | $3001.94870 w$ | 903052.04613 | 1937.020 |
| 96 | 84 | 61949.09384 e | 805734.43842 | 2.556 |
| 96 | 36 | 61417.28161 e | 81016.60445 | 18.612 |
| 96 | 85 | 623 4.37507e | 811929.05247 | 263.451 |
| 96 | 111 | 649 2.70043e | 805740.91074 | 1160.833 |
| 97 pdivdiagonal |  |  |  |  |
| 98 | 0.00009 | $0.00009 \quad 0.00009$ |  |  |
| 98 | 0.00009 | $0.00009 \quad 0.00009$ |  |  |
| 98 | 0.00009 | $0.00009 \quad 0.00009$ |  |  |
| 75 |  |  |  |  |

$913 D D$
*96 1007
9634
9688
9689
$96 \quad 112$
97pdivdiagonal
$98 \quad 0.00009$
$3001.94870 w 903052.04613$
1937.020
$61949.09342 \mathrm{e} 805734.43936 \quad 2.504$
$6054.57181 \mathrm{e} 804640.74891-39.476$
6719.40276 e 81736.96665 -69.652
$61245.72071 \mathrm{e} 804853.84195 \quad 163.203$
$98 \quad 0.00009$
980.00009

75
913DD
*96 1007
9686
9682
9685
96111
97pdvdiagonal

| 98 | 0.00009 | 0.00009 | 0.00009 |
| :--- | :--- | :--- | :--- |
| 98 | 0.00009 | 0.00009 | 0.00009 |
| 98 | 0.00009 | 0.00009 | 0.00009 |

913DD
*96 1007
9689
$3001.94870 w 903052.046131937 .020$
$\begin{array}{lllll}6 & 719.40276 e & 81 & 736.96665 & -69.652\end{array}$
$6054.57167 e \quad 804640.74921 \quad-39.477$
$649 \quad 2.70014 \mathrm{e} \quad 805740.91126 \quad 1160.806$

96111
97pdvdiagonal

| 98 | 0.00009 | 0.00009 | 0.00009 |
| :--- | :--- | :--- | :--- |
| 98 | 0.00009 | 0.00009 | 0.00009 |

75

913DD
*96 1007
$96 \quad 112$
9688
9689
96 111
97pdydiagonal

| 98 | 0.00009 | 0.00009 | 0.00009 |
| :--- | :--- | :--- | :--- |
| 98 | 0.00009 | 0.00009 | 0.00009 |
| 98 | 0.00009 | 0.00009 | 0.00009 |

75
913DD
*96 1007
9690
9688
96 83
96 111
97pdvdiagonal
980.00009
980.00009
$98 \quad 0.00009$
75
913DD
*96 1007
9690
9693
9694
9694
9695
9696
9696
96111
96 111
97pdivdiagonal
9800009

| 98 | 0.00009 | 0.00009 | 0.00009 |
| :--- | :--- | :--- | :--- |
| 98 | 0.00009 | 0.00009 | 0.00009 |

$\begin{array}{llll}98 & 0.00009 & 0.00009 & 0.00009\end{array}$

| 98 | 0.00009 | 0.00009 | 0.00009 |
| :--- | :--- | :--- | :--- |
| 98 | 0.00009 | 0.00009 | 0.00009 |
| 98 | 0.00009 | 0.00009 | 0.00009 |
| 98 | 0.00009 | 0.00009 | 0.00009 |
| 75 |  |  |  |

913DD
*96 1007
9691
9688
$3001.94870 w 903052.04613 \quad 1937.020$
$6106.03930 \mathrm{e} 803140.98585 \quad 329.503$
$6054.57147 e 804640.75081-39.378$
$60523.63046 e 804045.40727 \quad 123.803$
97pdvdiagonal
$\begin{array}{llll}98 & 0.00009 & 0.00009 & 0.00009\end{array}$
$\begin{array}{llll}98 & 0.00009 & 0.00009 & 0.00009\end{array}$
75
913DD
*96 1007
9692
9696
9697
9698
97pdvdiagonal
0.00009
$\begin{array}{llll}98 & 0.00009 & 0.00009 & 0.00009 \\ 98 & 0.00009 & 0.00009 & 0.00009\end{array}$
75

| 30 | 0 | $1.94870 w$ | 9 |
| ---: | ---: | ---: | ---: |
| 617 | $4.41334 e$ | 8 |  |
| 6 | 7 | 2.70187 e | 8 |
| 616 | 12.51500 e | 8 |  |
| 619 | 55.58962 e | 8 |  |
|  |  |  |  |
| 0009 | 0.00009 |  |  |
| 0009 | 0.00009 |  |  |
| 0009 | 0.00009 |  |  |

$913 D D$
*96 1007
9693
96111
97pdvdiagonal
980.00009
$0.00009 \quad 0.00009$
75

| 30 | 0 | $1.94870 w$ | 90 | 30 |
| ---: | ---: | ---: | ---: | ---: |
| 6 | 732.04613 | 1937.020 |  |  |
| 649 | $2.70032 e$ | 80 | 57 | 50.12765 |

913DD
*96 1007
9695
9696
96111
97pdvdiagonal
$98 \quad 0.00009$
$98 \quad 0.00009$
75
$30 \quad 0 \quad 1.94870 \mathrm{w} 903052.04613 \quad 1937.020$
$6446.31613 \mathrm{e} 80144.25130-12.870$
67 2.69588e $80813.84720-66.522$
649 2.69147e $805740.90359 \quad 1160.494$
$0.00009 \quad 0.00009$

913DD
*96 1007
$30 \quad 0 \quad 1.94870 \mathrm{w} \quad 903052.04613 \quad 1937.020$
9696
$672.69588 \mathrm{e} 80813.84720 \quad-66.522$
9693
$6731.84344 \mathrm{e} 80245.13196 \quad 306.978$
9694
96111

| 6 | 1 | $5.78475 e$ | 801443.43927 | -30.807 |
| :--- | :--- | :--- | :--- | :--- |

649 2.69350e $805740.91590 \quad 1160.784$

|  | 111 |  | 2.69281 l 805740.92154 | 1160.798 |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 649 | 2.69360 e 805740.92872 | 1160.805 |
| 97 pdvdiagonal |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 75 |  |  |  |  |
| 913 DD |  |  |  |  |
|  | 1007 | 300 | 1.94870w 903052.04613 | 1937.020 |
|  |  | 616 | 12.51503e 80619.82441 | -48.619 |
|  | 96 | 67 | 2.70066 e 80813.84052 | -66.651 |
|  | 111 | 649 | 2.70004 e 805740.90882 | 1160.805 |
| 97pdvdiagonal |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 75 |  |  |  |  |
| $913 D D$ |  |  |  |  |
|  | 1007 | 300 | 1.94870w 903052.04613 | 1937.020 |
|  | 98 | 619 | 55.58962e 801758.76868 | 558.276 |
|  | 96 | 67 | 2.70188 e 80813.84009 | -66.669 |
|  | 97 | 616 | 12.51503 e 80619.82441 | -48.619 |
|  | 111 | 649 | 2.70020e 805740.91188 | 1160.801 |
| 97pdvdiagonal |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
|  |  |  |  |  |
| $913 D D$ |  |  |  |  |
|  | 1007 | 300 | $1.94870 w 903052.04613$ | 1937.020 |
|  | 103 | 646 | 45.80646e $80 \quad 053.58791$ | 11.200 |
|  | 107 | 76 | 32.73594e 7959 24.39626 | -34.291 |
|  | 108 | 712 | 18.95740e 795748.19498 | -47.261 |
| 97pdvdiagonal |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 7 |  |  |  |  |
| 913DD |  |  |  |  |
|  | 1007 | 300 | $1.94870 w 903052.04613$ | 1937.020 |
|  | 103 | 646 | 45.8055le 80053.58542 | 11.168 |
|  |  | 634 | 49.42735e $80 \quad 113.06479$ | 55.361 |
| 97pdvdiagonal |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |
| 75 |  |  |  |  |
| $913 D \mathrm{D}$ |  |  |  |  |
|  | 1007 | 300 | $1.94870 w 903052.04613$ | 1937.020 |

## 141

| 96 | 107 | 7 | $632.73594 e$ | 795924.39626 | -34.291 |
| :--- | :--- | ---: | ---: | ---: | ---: |
| 96 | 108 | 712 | 18.95770 e | 795748.19319 | -47.182 |
| 96 | 111 | 649 | $2.7005 l \mathrm{e}$ | 805740.91265 | 1160.794 |
| 97 pdvdiagonal |  |  |  |  |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |  |
| 98 | 0.00009 | 0.00009 | 0.00009 |  |  |
| 75 |  |  |  |  |  |

* The next record causes all data to the end of this file to be ignored.


## APPENDIX F A PART OF THE OUTPUT FILE FOR AN ADJUSTMENT WITH NEW GPS OBSERVATIONS



Iowa State University, Dept. of Civil Eng.
GeoLab Adjustment of Sri Lanka full figure with Fig $1 \& 2$ and HDC $10 / 11$
$\mathrm{A}=6377299.151 \mathrm{~B}=6356098.145 \mathrm{X} 0=0.000 \mathrm{Y} 0=0.000 \mathrm{Z}=0.000$
GETUP:

| SUMMARY OF SELECTED OPTIONS |  |
| :---: | :---: |
| OPTION | SELECTION |
| Computation Mode | Adjustment |
| Linear Unit | Metre |
| Maximum Iterations | 5 |
| Confidence Regions Selected | Point and Connecred Relative |
| Confidence Region Dimensions | 1-D, 2-D, and 3-D |
| Print Input Station Data | Off |
| Variance Factor Knowledge | Known |
| Confidence Level for Statistics | 95.000 |
| Dual-Height Mode | Off |
| Print Solution Vector | On All Iterations |
| Printed Ellipsoidal Coordinates | 6 Decimal Places |
| Print Adjusted $X, Y, Z$ | On |
| Print Histograms | On |
| Print Misclosures | All on First Pass Only |
| Print Residuals | All |
| Variance Factor Usage | Scale Confidence Regions |
| Residual Rejection Criterion | Tau Max |
| Angular Misclosure Limit Factor | 5 |
| Linear Misclosure Limit Factor | 5 |
| Convergence Criterion | 0.001000 |

GETUP successfully completed.

GeoLab - VI.91S, (C) 1985/86/87/88/89 BitWise Ideas Inc. [103209264]

Iowa State University, Dept. of Civil Eng.
GeoLab Adjustment of Sri Lanka full figure with Fig 1\&2 and HDC $10 / 11$ $A=6377299.151 \mathrm{~B}=6356098.145 \mathrm{X} 0=0.000 \mathrm{Y} 0=0.000 \mathrm{ZO}=0.000$

FORMEQ:
NOTE 6: Reordering was done.

| AT | TO | OBS TYPE | OBSERVATION APPROX.SIG. MISCLOSURE |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 111 | 34 | 3-D X-Coord Diff | 47162.8039 | 0.0095 | 0.0000 |
| 111 | 34 | 3-D Y-Coord Diff | -37431.0617 | 0.0095 | 0.0000 |
| 111 | 34 | 3-D Z-Coord Diff | 206892.0978 | 0.0095 | 0.0000 |
| 111 | 36 | 3-D X-Coord Diff | 47905.6901 | 0.0095 | 0.0000 |
| 111 | 36 | 3-D Y-Coord Diff | -35196.3934 | 0.0095 | 0.0000 |
| 111 | 36 | 3-D Z-Coord Diff | 191767.7740 | 0.0095 | 0.0000 |
| 111 | 38 | 3-D X-Coord Diff | 44555.5608 | 0.0095 | -0.0734 |
| 111 | 38 | 3-D Y-Coord Diff | -30447.4693 | 0.0095 | -0.0349 |
| 111 | 38 | 3-D Z-Coord Diff | 165586.5749 | 0.0095 | 0.0140 |
| 111 | 39 | 3-D X-Coord Diff | 75124.8979 | 0.0095 | -0.0080 |
| 111 | 39 | 3-D Y-Coord Diff | -31711.7320 | 0.0095 | . 0.0303 |
| 111 | 39 | 3-D Z-Coord Diff | 137942.2853 | 0.0095 | 0.0061 |
| 111 | 39 | 3-D X-Coord Diff | 75124.3657 | 0.0095 | 0.0242 |
| 111 | 39 | 3-D Y-Coord Diff | -31711.8180 | 0.0095 | 0.0557 |
| 111 | 39 | 3-D 2-Coord Diff | 137942.3017 | 0.0095 | -0.0102 |
| 111 | 40 | 3-D X-Coord Diff | 30163.6713 | 0.0095 | -0.0595 |
| 111 | 40 | 3-D Y-Coord Diff | -24180.0591 | 0.0095 | -0.0375 |
| 111 | 40 | 3-D Z-Coord Diff | 141625.5592 | 0.0095 | 0.0011 |
| 111 | 40 | 3-D X-Coord Diff | 30163.5188 | 0.0095 | 0.0930 |
| 111 | 40 | 3-D Y-Coord Diff | -24180.1510 | 0.0095 | 0.0544 |
| 111 | 40 | 3-D Z-Coord Diff | 141625.5539 | 0.0095 | 0.0064 |
| 111 | 48 | 3-D X-Coord Diff | 113961.2016 | 0.0095 | -0.0095 |
| 111 | 48 | 3-D Y-Coord Diff | -38196.9196 | 0.0095 | 0.0041 |
| 111 | 48 | 3-D Z-Coord Diff | 134239.3210 | 0.0095 | 0.0121 |
| 111 | 57 | 3-D X-Coord Diff | 81590.3548 | 0.0095 | -0.0126 |
| 111 | 57 | 3-D Y-Coord Diff | -20224.2344 | 0.0095 | 0.1017 |
| 111 | 57 | 3-D Z-Coord Diff | 45764.1341 | 0.0095 | 0.0214 |
| 111 | 57 | 3-D X-Coord Diff | 81590.3297 | 0.0095 | 0.0125 |
| 111 | 57 | 3-D Y-Coord Diff | -20224.0300 | 0.0095 | -0.1027 |
| 111 | 57 | 3-D Z-Coord Diff | 45764.1774 | 0.0095 | -0.0219 |
| 111 | 59 | 3-D X-Coord Diff | 40070.1160 | 0.0095 | 0.0531 |
| 111 | 59 | 3-D Y-Coord Diff | -17618.2362 | 0.0095 | 0.0483 |
| 111 | 59 | 3-D Z-Coord Diff | 86409.8700 | 0.0095 | 0.0088 |
| 111 | 60 | 3-D X-Coord Diff | 67835.2666 | 0.0095 | -0.0391 |
| 111 | 60 | 3-D Y-Coord Diff | -22678.6666 | 0.0095 | -0.0378 |
| 111 | 60 | 3-D Z-Coord Diff | 83920.3035 | 0.0095 | -0.0120 |
| 111 | 60 | 3-D X-Coord Diff | 67835.2361 | 0.0095 | -0.0086 |
| 111 | 60 | 3-D Y-Coord Diff | -22678.6957 | 0.0095 | -0.0086 |








| 111 | 88 | 3-D X-Coord D |
| :---: | :---: | :---: |
| 111 | 88 | 3-D Y-Coord Diff |
| 111 | 88 | 3-D Z-Coord Diff |
| 111 | 88 | 3-D X-Coord Diff |
| 11 | 88 | 3-D Y-Coord Diff |
| ! | 88 | 3-D Z-Coord Diff |
| 111 | 90 | 3-D X-Coord Diff |
| 11 | 90 | 3-D Y-Coord Diff |
| 111 | 90 | 3-D Z-Coord Diff |
| 111 | 91 | 3-D X-Coord Diff |
| 111 | 91 | 3-D Y-Coord Diff |
| 111 | 91 | 3-D Z-Coord Diff |
| 111 | 92 | 3-D X-Coord Diff |
| 111 | 92 | 3-D Y-Coord Diff |
| 111 | 92 | 3-D Z-Coord Diff |
| 111 | 94 | 3-D X-Coord Diff |
| 111 | 94 | 3-D Y-Coord Diff |
| 111 | 94 | 3-D Z-Coord Diff |
| 111 | 94 | 3-D X-Coord Diff |
| 111 | 94 | 3-D Y-Coord Diff |
| 111 | 94 | 3-D Z-Coord Diff |
| 111 | 96 | 3-D X-Coord Diff |
| 111 | 96 | 3-D Y-Coord Diff |
| 111 | 96 | 3-D Z-Coord Diff |
| 111 | 99 | 3-D X-Coord Diff |
| 111 | 99 | 3-D Y-Coord Diff |
| 111 | 99 | 3-D Z-Coord Diff |
| 111 | 103 | 3-D X-Coord Diff |
| 111 | 103 | 3-D Y-Coord Diff |
| 111 | 103 | 3-D Z-Coord Diff |
| 111 | 103 | 3-D X-Coord Diff |
| 111 | 103 | 3-D Y-Coord Diff |
| 111 | 103 | 3-D Z-Coord Diff |
| 111 | 108 | 3-D X-Coord Diff |
| 111 | 108 | 3-D Y-Coord Diff |
| 111 | 108 | 3-D Z-Coord Diff |
| 111 | 108 | 3-D X-Coord Diff |
| 111 | 108 | 3-D Y-Coord Diff |
| 111 | 108 | 3-D Z-Coord Diff |
| 111 | 113 | 3-D X-Coord Diff |
| 111 | 113 | 3-D Y-Coord Diff |
| 111 | 113 | 3-D Z-Coord Diff |
| 36 | 34 | 3-D X-Coord Diff |
| 36 | 34 | 3-D Y-Coord Diff |
| 36 | 34 | 3-D Z-Coord Diff |
| 36 | 40 | 3-D X-Coord Diff |
| 36 | 40 | 3-D Y-Coord Diff |
| 36 | 40 | 3-D Z-Coord Diff |
| 37 | 38 | 3-D X-Coord Diff |
| 37 | 38 | 3-D Y-Coord Diff |
| 37 | 38 | 3-D Z-Coord Diff |
| 37 | 61 | 3-D X-Coord Diff |


| 21412.1570 | 0.0095 | 0.0088 |
| :---: | :---: | :---: |
| 5392.5472 | 0.0095 | 0.0621 |
| -88299.0526 | 0.0095 | 0.0060 |
| 21412.1539 | 0.0095 | 0.0119 |
| 5392.6451 | 0.0095 | -0.0358 |
| -88299.0394 | 0.0095 | -0.0072 |
| 32080.8756 | 0.0095 | 0.0717 |
| 2932.5397 | 0.0095 | -0.0299 |
| -80063.3347 | 0.0095 | -0.0112 |
| 48475.9345 | 0.0095 | -0.0098 |
| -512.0900 | 0.0095 | 0.0028 |
| -71416.1593 | 0.0095 | -0.0011 |
| 49813.7399 | 0.0095 | 0.0000 |
| -1852.5610 | 0.0095 | 0.0000 |
| -58608.2083 | 0.0095 | 0.0000 |
| 79555.7396 | 0.0095 | 0.1977 |
| -4353.0015 | 0.0095 | -0.0970 |
| -87955.3753 | 0.0095 | -0.0408 |
| 79556.1688 | 0.0095 | -0.2314 |
| -4353.1895 | 0.0095 | 0.0910 |
| -87955.4520 | 0.0095 | 0.0359 |
| 91155.4792 | 0.0095 | 0.0891 |
| .7570.6751 | 0.0095 | 0.0964 |
| -77057.7777 | 0.0095 | 0.0322 |
| 102937.6885 | 0.0095 | 0.0416 |
| -15255.7512 | 0.0095 | 0.0111 |
| -26162.4078 | 0.0095 | -0.0267 |
| 103075.4900 | 0.0095 | 0.0243 |
| -17923.5475 | 0.0095 | -0.0070 |
| -4311.8818 | 0.0095 | -0.0115 |
| 103075.5604 | 0.0095 | -0.0460 |
| -17923.5887 | 0.0095 | 0.0342 |
| -4311.9146 | 0.0095 | 0.0213 |
| 107671.7354 | 0.0095 | 0.0139 |
| -24615.4441 | 0.0095 | -0.0465 |
| 42424.7954 | 0.0095 | -0.0077 |
| 107671.7407 | 0.0095 | 0.0086 |
| -24615.5110 | 0.0095 | 0.0203 |
| 42424.7903 | 0.0095 | -0.0025 |
| 97572.7128 | 0.0095 | 0.0114 |
| -23376.0918 | 0.0095 | -0.0041 |
| 47518.9382 | 0.0095 | 0.0007 |
| -743.2797 | 0.0095 | 0.3935 |
| -2234.9679 | 0.0095 | 0.2996 |
| 15124.3148 | 0.0095 | 0.0090 |
| -17741.1386 | 0.0095 | -0.9397 |
| 11016.0226 | 0.0095 | 0.2742 |
| -50141.6731 | 0.0095 | -0.5406 |
| -30826.2458 | 0.0095 | 0.0058 |
| 5408.8775 | 0.0095 | -0.0023 |
| 844.8153 | 0.0095 | -0.0045 |
| -14872.7003 | 0.0095 | 0.0059 |












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| 75 | 74 | 3-D X-Coord Diff |
| :---: | :---: | :---: |
| 75 | 74 | 3-D Y-Coord Diff |
| 75 | 74 | 3-D Z-Coord Diff |
| 76 | 73 | 3-D X-Coord Diff |
| 76 | 73 | 3-D Y-Coord Diff |
| 76 | 73 | 3-D Z-Coord Diff |
| 76 | 75 | 3-D X-Coord Diff |
| 76 | 75 | 3-D Y-Coord Diff |
| 76 | 75 | 3-D Z-Coord Diff |
| 76 | 77 | 3-D X-Coord Diff |
| 76 | 77 | 3-D Y-Coord Diff |
| 76 | 77 | 3-D Z-Coord Diff |
| 76 | 78 | 3-D X-Coord Diff |
| 76 | 78 | 3-D Y-Coord Diff |
| 76 | 78 | 3-D Z-Coord Diff |
| 76 | 82 | 3-D X-Coord Diff |
| 76 | 82 | 3-D Y-Coord Diff |
| 76 | 82 | 3-D Z-Coord Diff |
| 78 | 77 | 3-D X-Coord Diff |
| 78 | 77 | 3-D Y-Coord Diff |
| 78 | 77 | 3-D Z-Coord Diff |
| 78 | 82 | 3-D X-Coord Diff |
| 78 | 82 | 3-D Y-Coord Diff |
| 78 | 82 | 3-D Z-Coord Diff |
| 80 | 73 | 3-D X-Coord Diff |
| 80 | 73 | 3-D Y-Coord Diff |
| SO | 73 | 3-D Z-Coord Diff |
| 80 | 75 | 3-D X-Coord Diff |
| SO | 75 | 3-D Y-Coord Diff |
| 80 | 75 | 3-D Z-Coord Diff |
| 80 | 76 | 3-D X-Coord Diff |
| 80 | 76 | 3-D Y-Coord Diff |
| 80 | 76 | 3-D Z-Coord Diff |
| 82 | 77 | 3-D X-Coord Diff |
| 82 | 77 | 3-D Y-Coord Diff |
| 82 | 77 | 3-D Z-Coord Diff |
| 82 | 77 | 3-D X-Coord Diff |
| 82 | 77 | 3-D Y-Coord Diff |
| 82 | 77 | 3-D Z-Coord Diff |
| 82 | 84 | 3-D X-Coord Diff |
| 82 | 84 | 3-D Y-Coord Diff |
| 82 | 84 | 3-D Z-Coord Diff |
| 82 | 85 | 3-D X-Coord Diff |
| 82 | 85 | 3-D Y-Coord Diff |
| 82 | 85 | 3-D Z-Coord Diff |
| 83 | 91 | 3-D X-Coord Diff |
| 83 | 91 | 3-D Y-Coord Diff |
| 83 | 91 | 3-D Z-Coord Diff |
| 83 | 88 | 3-D X-Coord Diff |
| 83 | 88 | 3-D Y-Coord Diff |
| 83 | 88 | 3-D Z-Coord Diff |
| 84 | 86 | 3-D X-Coord Diff |


| 11246.4148 | 0.0095 | -0.0374 |
| :---: | :---: | :---: |
| -7354.7478 | 0.0095 | 0.0049 |
| 44843.1893 | 0.0095 | 0.0046 |
| -5563.9769 | 0.0095 | -0.0017 |
| -1935.5222 | 0.0095 | -0.0106 |
| 18254.0119 | 0.0095 | -0.0434 |
| 20994.2689 | 0.0095 | 0.0260 |
| -4418.7981 | 0.0095 | 0.0125 |
| 5989.8056 | 0.0095 | 0.0383 |
| -30080.8334 | 0.0095 | -0.0397 |
| 4400.0968 | 0.0095 | -0.0405 |
| -5751.7823 | 0.0095 | 0.0071 |
| -16918.3370 | 0.0095 | -0.0647 |
| 4155.7405 | 0.0095 | -0.0023 |
| -24962.8853 | 0.0095 | 0.0010 |
| 30981.6155 | 0.0095 | 0.0255 |
| -2114.5600 | 0.0095 | 0.0247 |
| -34350.4766 | 0.0095 | 0.0048 |
| -13162.5008 | 0.0095 | 0.0293 |
| 244.3234 | 0.0095 | -0.0053 |
| 19211.1023 | 0.0095 | 0.0067 |
| 47899.9729 | 0.0095 | 0.0697 |
| -6270.2621 | 0.0095 | -0.0114 |
| -9387.5729 | 0.0095 | -0.0146 |
| -27730.5104 | 0.0095 | -0.0136 |
| -298.2663 | 0.0095 | -0.0298 |
| 35748.0245 | 0.0095 | $0.018+$ |
| -1172.2633 | 0.0095 | 0.0127 |
| -2781.5368 | 0.0095 | -0.0120 |
| 23483.8789 | 0.0095 | 0.0393 |
| -22166.5313 | 0.0095 | -0.0142 |
| 1637.2659 | 0.0095 | -0.0292 |
| 17494.0760 | 0.0095 | -0.0016 |
| -61062.4164 | 0.0095 | -0.0977 |
| 6514.6580 | 0.0095 | -0.0663 |
| 28598.6864 | 0.0095 | 0.0101 |
| . 61062.5304 | 0.0095 | 0.0163 |
| 6514.5323 | 0.0095 | 0.0594 |
| 28598.6892 | 0.0095 | 0.0074 |
| -13142.1548 | 0.0095 | 0.8579 |
| 5152.0842 | 0.0095 | 5.3337 |
| -32084.3497 | 0.0095 | 0.5946 |
| 53120.4287 | 0.0095 | 0.0427 |
| 10972.5685 | 0.0095 | -0.0005 |
| 26093.8637 | 0.0095 | 0.0068 |
| 14030.7828 | 0.0095 | 0.0000 |
| -595.0983 | 0.0095 | 0.0000 |
| 24011.4872 | 0.0095 | 0.0000 |
| 13033.0076 | 0.0095 | 0.0316 |
| 5309.5322 | 0.0095 | 0.0660 |
| 40894.3846 | 0.0095 | 0.0111 |
| -4744.9556 | 0.0095 | -0.8219 |


| 84 | 86 | 3-D Y-Coord Diff | 1898.8033 | 0.0095 | -5.3493 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 84 | 86 | 3-D Z-Coord Diff | -10129.1106 | 0.0095 | -0.5896 |
| 84 | 85 | 3-D X-Coord Diff | - -39979.2708 | 0.0095 | -0.8183 |
| 84 | 85 | 3-D Y-Coord Diff | 5820.4905 | 0.0095 | -5.3404 |
| 84 | 85 | 3-D Z-Coord Diff | 5990.4905 | 0.0095 | -0.5922 |
| 84 | 111 | 3-D X-Coord Diff | -984.5384 | 0.0095 | -0.8376 |
| 84 | 111 | 3-D Y-Coord Diff | -4923.0524 | 0.0095 | -5.3680 |
| 84 | 111 | 3-D Z-Coord Diff | 53648.3791 | 0.0095 | -0.5995 |
| 84 | 88 | 3-D X-Coord Diff | 20427.6453 | 0.0095 | -0.8554 |
| 84 | 88 | 3-D Y-Coord Diff | 469.5139 | 0.0095 | -5.3250 |
| 84 | 88 | 3-D Z-Coord Diff | -34650.6634 | 0.0095 | -0.6036 |
| 84 | 89 | 3-D X-Coord Diff | -17916.1757 | 0.0095 | -0.8583 |
| 84 | 89 | 3-D Y-Coord Diff | 5279.4244 | 0.0095 | -5.3944 |
| 84 | 89 | 3-D Z-Coord Diff | -22899.6512 | 0.0095 | -0.6081 |
| 84 | 112 | 3-D X-Coord Diff | 16048.6905 | 0.0095 | -0.8583 |
| 84 | 112 | 3-D Y-Coord Diff | -973.6772 | 0.0095 | -5.5944 |
| 84 | 112 | 3-D Z-Coord Diff | -12909.2877 | 0.0095 | -0.6081 |
| 86 | 82 | 3-D X-Coord Diff | 17887.1058 | 0.0095 | -0.0315 |
| 86 | 82 | 3-D Y-Coord Diff | -7050.7981 | 0.0095 | -0.0738 |
| 86 | 82 | 3-D Z-Coord Diff | 42213.4520 | 0.0095 | 0.0033 |
| 86 | 85 | 3-D X-Coord Diff | -35233.3275 | 0.0095 | 0.0159 |
| 36 | 85 | 3-D Y-Coord Diff | 3921.7000 | 0.0095 | -0.0040 |
| 86 | 85 | 3-D Z-Coord Diff | 16119.5847 | 0.0095 | 0.0157 |
| 86 | 111 | 3-D X-Coord Diff | 3760.4066 | 0.0095 | -0.005? |
| 86 | 111 | 3-D Y-Coord Diff | -6821.8459 | 0.0095 | -0.0285 |
| 86 | 111 | 3-D Z-Coord Diff | 63777.4734 | 0.0095 | 0.0064 |
| 39 | 88 | 3-D X-Coord Diff | 38343.8118 | 0.0095 | 0.0120 |
| 89 | 83 | 3-D Y-Coord Diff | -4809.9095 | 0.0095 | 0.0684 |
| 89 | 88 | 3-D Z-Coord Diff | -11751.0166 | 0.0095 | 0.0089 |
| 89 | 111 | 3-D X-Coord Diff | 16931.6538 | 0.0095 | 0.0041 |
| 89 | 111 | 3-D Y-Coord Diff | -10202.4546 | 0.0095 | 0.0042 |
| 39 | 111 | 3-D Z-Coord Diff | 76548.0368 | 0.0095 | 0.0021 |
| 112 | 88 | 3-D X-Coord Diff | 4378.9253 | 0.0095 | 0.0324 |
| 112 | 88 | 3-D Y-Coord Diff | 1443.1953 | 0.0095 | 0.0651 |
| 112 | 88 | 3-D Z-Coord Diff | -21741.3607 | 0.0095 | -0.0106 |
| 112 | 89 | 3-D X-Coord Diff | -33964.8178 | 0.0095 | -0.0483 |
| 112 | 89 | 3-D Y-Coord Diff | 6253.0982 | 0.0095 | 0.0033 |
| 112 | 89 | 3-D Z-Coord Diff | -9990.3554 | 0.0095 | -0.0082 |
| 112 | 111 | 3-D X-Coord Diff | -17033.1786 | 0.0095 | -0.0296 |
| 112 | 111 | 3-D Y-Coord Diff | -3949.3639 | 0.0095 | 0.0150 |
| 112 | 111 | 3-D Z-Cuord Diff | 66557.6763 | 0.0095 | -0.0009 |
| 90 | 88 | 3-D X-Coord Diff | -10668.7640 | 0.0095 | -0.0176 |
| 90 | 88 | 3-D Y-Coord Diff | 2460.1539 | 0.0095 | -0.0544 |
| 90 | 88 | 3-D Z-Coord Diff | -8235.7040 | 0.0095 | 0.0033 |
| 90 | 83 | 3-D X-Coord Diff | 2364.2410 | 0.0095 | -0.0465 |
| 90 | 83 | 3-D Y-Coord Diff | -2849.4963 | 0.0095 | -0.0025 |
| 90 | 83 | 3-D Z-Coord Diff | 32658.6849 | 0.0095 | -0.0122 |
| 90 | 111 | 3-D X-Coord Diff | -32080.9011 | 0.0095 | -0.0462 |
| 90 | 111 | 3-D Y-Coord Diff | -2932.5071 | 0.0095 | -0.0027 |
| 90 | 111 | 3-D Z-Coord Diff | 80063.3578 | 0.0095 | -0.0119 |
| 90 | 93 | 3-D X-Coord Diff | 30296.1621 | 0.0095 | -0.0733 |
| 90 | 93 | 3-D Y-Coord Diff | -5288.0770 | 0.0095 | 0.0302 |


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| 96 | 111 | 3-D X-Coord Diff | -91155.4241 | 0.0095 | -0.1443 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 96 | 111 | 3-D Y-Coord Diff | 7570.5209 | 0.0095 | 0.0578 |
| 96 | 111 | 3-D Z-Coord Diff | 77057.7410 | 0.0095 | 0.0045 |
| 96 | 111 | 3-D X-Coord Diff | -91155.5925 | 0.0095 | 0.0241 |
| 96 | 111 | 3-D Y-Coord Diff | 7570.5643 | 0.0095 | 0.0144 |
| 96 | 111 | 3-D Z-Coord Diff | 77057.7216 | 0.0095 | 0.0239 |
| 96 | 111 | 3-D X-Coord Diff | -91155.8096 | 0.0095 | 0.2412 |
| 96 | 111 | 3-D Y-Coord Diff | 7570.6029 | 0.0095 | -0.0242 |
| 96 | 111 | 3-D Z-Coord Diff | 77057.7465 | 0.0095 | -0.0010 |
| 97 | 96 | 3-D X-Coord Diff | -3143.1874 | 0.0095 | 0.1111 |
| 97 | 96 | 3-D Y-Coord Diff | 2378.6778 | 0.0095 | 0.0986 |
| 97 | 96 | 3-D Z-Coord Diff | -16791.4961 | 0.0095 | 0.0705 |
| 97 | 111 | 3-D X-Coord Diff | -94298.5749 | 0.0095 | -0.0698 |
| 97 | 111 | 3-D Y-Coord Diff | 9949.3385 | 0.0095 | 0.0166 |
| 97 | 111 | 3-D Z-Coord Diff | 60266.3147 | 0.0095 | 0.0053 |
| 98 | 96 | 3-D X-Coord Diff | 18049.9850 | 0.0095 | 0.1018 |
| 98 | 96 | 3-D Y-Coord Diff | -1130.0125 | 0.0095 | 0.1224 |
| 98 | 96 | 3-D Z-Coord Diff | -23669.0135 | 0.0095 | 0.0352 |
| 98 | 97 | 3-D X-Coord Diff | 21193.1632 | 0.0095 | 0.0000 |
| 98 | 97 | 3-D Y-Coord Diff | -3508.6665 | 0.0095 | 0.0000 |
| 98 | 97 | 3-D Z-Coord Diff | -6877.5527 | 0.0095 | 0.0000 |
| 98 | 111 | 3-D X-Coord Diff | -73105.5052 | 0.0095 | 0.0237 |
| 98 | 111 | 3-D Y-Coord Diff | 6440.6823 | 0.0095 | 0.0063 |
| 98 | 111 | 3-D Z-Coord Diff | 53388.7664 | 0.0095 | 0.0009 |
| 103 | 107 | 3-D X-Coord Diff | 1922.8517 | 0.0095 | -0.0243 |
| 103 | 107 | 3-D Y-Coord Diff | -4860.8134 | 0.0095 | 0.0070 |
| 103 | 107 | 3-D Z-Coord Diff | 36186.1523 | 0.0095 | 0.0115 |
| 103 | 108 | 3-D X-Coord Diff | 4596.2567 | 0.0095 | -0.0218 |
| 103 | 108 | 3-D Y-Coord Diff | -6691.9643 | 0.0095 | 0.0281 |
| 103 | 108 | 3-D Z-Coord Diff | 46736.6703 | 0.0095 | 0.0107 |
| 103 | 99 | 3-D X-Coord Diff | -137.8304 | 0.0095 | 0.0460 |
| 103 | 99 | 3-D Y-Coord Diff | 2667.8486 | 0.0095 | -0.0342 |
| 103 | 99 | 3-D Z-Coord Diff | -21850.5198 | 0.0095 | -0.0213 |
| 107 | 108 | 3-D X-Coord Diff | 2673.4726 | 0.0095 | -0.0650 |
| 107 | 108 | 3-D Y-Coord Diff | -1831.0844 | 0.0095 | -0.0453 |
| 107 | 108 | 3-D Z-Coord Diff | 10550.5371 | 0.0095 | -0.0198 |
| 107 | 111 | 3-D X-Coord Diff | -104998.3900 | 0.0095 | 0.0483 |
| 107 | 111 | 3-D Y-Coord Diff | 22784.3503 | 0.0095 | 0.0106 |
| 107 | 111 | 3-D Z-Coord Diff | -31874.2627 | 0.0095 | -0.0077 |

FORMEQ successfully completed.

Iowa State University, Dept. of Civil Eng.
GeoLab Adjustment of Sri Lanka full figure with Fig $1 \& 2$ and HDC 10/11 $\mathrm{A}=6377299.151 \mathrm{~B}=6356098.145 \mathrm{XO}=0.000 \mathrm{Y} 0=0.000 \mathrm{ZO}=0.000$

SOLVE:
Adjusted Values (Iteration Count $=2$ ):
CODE IDENT. TYPE INITIAL DX ADJUSTED

| 14 | 84 | LATITUDE | 61949.093490 | FIXED |
| :---: | :---: | :---: | :---: | :---: |
| 14 | 84 | LONGITUDE | 805734.439360 | FIXED |
| 14 | 84 | HEIGHT | 8.00000 | FIXED |
| 14 | 38 | LATITUDE | 81943.897530 | FIXED |
| 14 | 38 | LONGITUDE | 80316.362270 | FIXED |
| 14 | 38 | HEIGHT | 302.11000 | FIXED |
| 14 | 39 | LATITUDE | 8435.293760 | FIXED |
| 14 | 39 | LONGITUDE | 801434.661190 | FIXED |
| 14 | 39 | HEIGHT | 180.15700 | FIXED |
| 14 | 111 | LATITUDE | 6492.700190 | FIXED |
| 14 | 111 | LONGITUDE | 805740.911150 | FIXED |
| 14 | 111 | HEIGHT | 1160.81100 | FIXED |
| 14 | 103 | LATITUDE | 64645.806100 | FIXED |
| 14 | 103 | LONGITUDE | $80 \quad 53.587090$ | FIXED |
| 14 | 103 | HEIGHT | 11.19600 | FIXED |
| 14 | 108 | LATITUDE | 71218.957290 | FIXED |
| 14 | 108 | LONGITUDE | 795748.195020 | FIXED |
| 14 | 108 | HEIGHT | -47.24000 | FIXED |
| 14 | 40 | LATITUDE | $8 \quad 634.142440$ | FIXED |
| 14 | 40 | LONGITUDE | 803923.771130 | FIXED |
| 14 | 40 | HEIGHT | 666.67500 | FIXED |
| 14 | 60 | LATITUDE | 73459.123250 | FIXED |
| 14 | 60 | LONGITUDE | 801918.989210 | FIXED |
| 14 | 60 | HEIGHT | 422.20000 | FIXED |
| 14 | 90 | LATITUDE | $6 \quad 523.631980$ | FIXED |
| 14 | 90 | LONGITUDE | 804045.405340 | FIXED |
| 14 | 90 | HEIGHT | 123.81200 | FIXED |
| 14 | 82 | LATITUDE | 63717.947630 | FIXED |
| 14 | 82 | LONGITUDE | 80505.540510 | FIXED |
| 14 | 82 | HEIGHT | 632.41300 | FIXED |
| 14 | 73 | LATITUDE | 7559.504070 | FIXED |
| 14 | 73 | LONGITUDE | 81942.098890 | FIXED |
| 14 | 73 | HEIGHT | 1414.76000 | FIXED |
| 14 | 57 | LATITUDE | $714 \quad 7.484730$ | FIXED |
| 14 | 57 | LONGITUDE | 801210.400460 | FIXED |
| 14 | 57 | HEIGHT | 207.81700 | FIXED |
| 14 | 74 | LATITUDE | 72346.939820 | FIXED |
| 14 | 74 | LONGITUDE | 804834.853770 | FIXED |
| 14 | 74 | HEIGHT | 1762.22600 | FIXED |


| 14 | 75 | LATITUDE | 65916.637850 | FIXED |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14 | 75 | LONGITUDE | 805514.778510 | FIXED |  |
| 14 | 75 | HEIGHT | 1580.90600 | FIXED |  |
| 14 | 76 | LATITUDE | 65558.843190 | FIXED |  |
| 14 | 76 | LONGITUDE | 81652.715810 | FIXED |  |
| 14 | 76 | HEIGHT | 1933.79400 | FIXED |  |
| 14 | 88 | LATITUDE | 6054.571720 | FIXED |  |
| 14 | 88 | LONGITUDE | 804640.749180 | FIXED |  |
| 14 | 88 | HEIGHT | -39.40700 | FIXED |  |
| 14 | 94 | Latitude | 615.791600 | FIXED |  |
| 14 | 94 | LONGITUDE | 801443.426960 | FIXED |  |
| 14 | 94 | HEIGHT | -30.82700 | FIXED |  |
| 14 | 96 | LATITUDE | 672.702540 | FIXED |  |
| 14 | 96 | LONGITUDE | 80813.837510 | FIXED |  |
| 14 | 96 | HEIGHT | -66.52800 | FIXED |  |
| 4 | 34 | LATITUDE | 84224.759336 | -0.000000 | $8+224.759336$ |
| 4 | 34 | LONGITUDE | 80294.613210 | 0.000000 | 80294.613210 |
| 4 | 34 | HEIGHT | 33.63347 | 0.00000 | 33.63347 |
| 4 | 36 | LATITUDE | 8346.637746 | -0.000000 | 8 It 6.637746 |
| 4 | 36 | LONGITUDE | 802852.722915 | -0.000000 | 302852.722915 |
| 4 | 36 | HEIGHT | 62.80333 | 0.00000 | 62.80333 |
| 4 | 37 | LATITUDE | 81917.532289 | 0.000000 | 81917.532289 |
| 4 | 37 | LONGITUDE | 80143.429650 | -0.000000 | $8014 \quad 3.429650$ |
| 4 | 37 | HEIGHT | 2.30618 | -0.00000 | 2.30618 |
| 4 | 47 | LATITUDE | $8 \quad 54.537831$ | 0.000000 | 8 5 4.537831 |
| 4 | 47 | LONGITUDE | 80754.108621 | 0.000000 | 80754.108621 |
| 4 | 47 | HEIGHT | 160.10834 | -0.00000 | 160.10834 |
| 4 | 48 | LATITUDE | 8234.513141 | -0.000000 | 8234.513141 |
| 4 | 48 | LONGITUDE | 79538.595875 | 0.000000 | 79538.595875 |
| 4 | 48 | HEIGHT | -29.07986 | 0.00000 | -29.07986 |
| 4 | 49 | LATITUDE | 75423.286694 | 0.000000 | 75423.286694 |
| 4 | 49 | LONGITUDE | $80 \quad 10.386147$ | 0.000000 | $30 \quad 10.386147$ |
| 4 | 49 | HEIGHT | 59.54445 | 0.00000 | 59.54445 |
| 4 | 51 | LATITUDE | 73427.294462 | -0.000000 | 73427.294462 |
| 4 | 51 | LONGITUDE | 80738.385883 | -0.000000 | $80 \quad 738.385883$ |
| 4 | 51 | HEIGHT | 215.45975 | 0.00000 | 215.45975 |
| 4 | 56 | LATITUDE | 71951.893230 | -0.000000 | 71951.893230 |
| 4 | 56 | LONGITUDE | $80 \quad 653.861550$ | -0.000000 | 80653.861550 |
| 4 | 56 | HEIGHT | 96.93900 | 0.00000 | 96.93900 |


| 4 | 59 | LATITUDE | 73617.797824 | -0.000000 |  | 3617.797824 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 59 | LO:VGITUDE | 803439.633671 | 0.000000 |  | 3439.633671 |
| 4 | 59 | HEIGHT | 1131.61892 | 0.00000 |  | 1131.61892 |
| 4 | 61 | LATITUDE | 7564.343344 | -0.000000 |  | 564.343344 |
| 4 | 61 | LONGITUDE | 802252.686741 | -0.000000 | 80 | 2252.686741 |
| 4 | 61 | HEIGHT | 470.90456 | 0.00000 |  | 470.90456 |
| 4 | 77 | LATITUDE | 65253.880656 | -0.000000 | 6 | 5253.880656 |
| 4 | 77 | LONGITUDE | 812322.877557 | 0.000000 |  | 2322.877557 |
| 4 | 77 | HEIGHT | 1017.70810 | 0.00000 |  | 1017.70810 |
| 4 | 78 | LATITUDE | 64226.214378 | -0.000000 | 6 | 4226.214378 |
| 4 | 78 | LONGITUDE | 811617.911887 | 0.000000 |  | 1617.911887 |
| 4 | 78 | HEIGHT | 475.11392 | 0.00000 |  | 475.11392 |
| 4 | 80 | LATITUDE | 64626.387393 | 0.000000 | 6 | 4626.387393 |
| 4 | 80 | LONGITUDE | 805451.379140 | -0.000000 |  | 5451.379140 |
| 4 | 80 | HEIGHT | 1677.95166 | 0.00000 |  | 1677.95166 |
| 4 | 83 | LATITUDE | 6239.044654 | 0.000000 | 6 | $23 \quad 9.044654$ |
| 4 | 83 | LONGITUDE | 803914.478108 | 0.000000 | 80 | 3914.478108 |
| 4 | 83 | HEIGHT | 1258.43458 | -0.00000 |  | 1258.43458 |
| 4 | 85 | LATITUDE | 6234.374667 | 0.000000 | 6 | $23 \quad 4.374667$ |
| 4 | 85 | LONGITLDE | 811929.052956 | -0.000000 | 81 | 1929.052956 |
| 4 | 85 | HEIGHT | 265.25578 | -0.00000 |  | 265.25578 |
| 4 | 86 | LATITUDE | 61417.281846 | -0.000000 | 6 | 1417.281846 |
| 4 | 86 | LONGITUDE | 81016.604902 | -0.000000 | 81 | 016.604902 |
| 4 | 86 | HEIGHT | 20.39566 | -0.00000 |  | 20.39566 |
| 4 | 89 | LATITUDE | $6 \quad 719.403033$ | -0.000000 | 6 | 719.403033 |
| 4 | 89 | LONGITUDE | 81736.966143 | 0.000000 | 81 | 736.966143 |
| 4 | 89 | HEIGHT | -67.79383 | -0.00000 |  | -67.79383 |
| 4 | 91 | LATITUDE | $610 \quad 6.039814$ | 0.000000 | 6 | $10 \quad 6.039814$ |
| 4 | 91 | LONGITUDE | 803140.984669 | -0.000000 | 80 | 3140.984669 |
| 4 | 91 | HEIGHT | 329.50302 | -0.00000 |  | 329.50302 |
| 4 | 92 | LATITUDE | 6174.413811 | -0.000000 | 6 | $17 \quad 4.413811$ |
| 4 | 92 | LONGITUDE | 803050.875534 | -0.000000 | 80 | 3050.875534 |
| 4 | 92 | HEIGHT | 623.33939 | 0.00000 |  | 623.33939 |
| 4 | 93 | LATITUDE | 6731.850103 | -0.000000 | 6 | 731.850103 |
| 4 | 93 | LONGITUDE | 80245.124870 | -0.000000 | 80 | $24 \quad 5.124870$ |
| 4 | 93 | HEIGHT | 306.99007 | 0.00000 |  | 306.99007 |
| 4 | 95 | LATITUDE | 6446.323388 | -0.000000 | 6 | 446.323388 |
| 4 | 95 | LONGITUDE | 80144.249645 | 0.000000 | 80 | 144.249645 |
| 4 | 95 | HEIGHT | -12.70449 | 0.00000 |  | -12.70449 |


| 4 | 97 | LATITUDE | 6 | 16 | 12.515761 | -0.000000 | 6 | 16 | 12.515761 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $+$ | 97 | LONGITUDE | 80 | 6 | 19.823637 | 0.000000 | 80 | 6 | 19.823637 |
| 4 | 97 | HEIGHT |  |  | -48.54992 | 0.00000 |  |  | -48.54992 |
| 4 | 98 | LATITUDE | 6 | 19 | 55.590078 | -0.000000 | 6 | 19 | 55.590078 |
| 4 | 98 | LONGITUDE | 80 | 17 | 58.767313 | 0.000000 | 80 | 17 | 58.767313 |
| 4 | 98 | HEIGHT |  |  | 558.34642 | -0.00000 |  |  | 558.34642 |
| 4 | 99 | LATITUDE | 6 | 34 | 49.428112 | -0.000000 | 6 | 34 | 49.428112 |
| 4 | 99 | LONGITUDE | 80 | 1 | 13.066260 | -0.000000 | 80 | 1 | 13.066260 |
| 4 | 99 | HEIGHT |  |  | 55.36747 | 0.00000 |  |  | 55.36747 |
| 4 | 107 | LATITUDE | 7 | 6 | 32.735574 | -0.000000 | 7 | 6 | 32.735574 |
| 4 | 107 | LONGITUDE | 79 | 59 | 24.396100 | -0.000000 | 79 | 59 | 24.396100 |
| 4 | 107 | HEIGHT |  |  | -34.30576 | 0.00000 |  |  | -34.30576 |
| 4 | 112 | LATITUDE | 6 | 12 | 45.720638 | 0.000000 | $\sigma$ | 12 | 45.720638 |
| 4 | 112 | LONGITUDE | 80 | 48 | 53.842491 | -0.000000 | 80 | 48 | 53.842491 |
| 4 | 112 | HEIGHT |  |  | 165.05970 | 0.00000 |  |  | 165.05970 |
| 4 | 113 | LATITUDE | 7 | 15 | 5.654234 | 0.000000 | 7 | 15 | 5.654234 |
| 4 | 113 | LONGITUDE | 80 | 3 | 19.449968 | 0.000000 | 80 | 3 | 19.449968 |
| 4 | 113 | HEIGHT |  |  | 66.47236 | -0.00000 |  |  | 66.47236 |

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GeoLab Adjustment of Sri Lanka full figure with Fig $1 \& 2$ and HDC 10/11
$A=6377299.151 \mathrm{~B}=6356098.145 \mathrm{XO}=0.000 \mathrm{Y} 0=0.000 \mathrm{ZO}=0.000$

Adjusted Cartesian Coordinates:

| CODE | IDENT. | X-COORDINATE | Y-COORDINATE | Z-COORDINATE |
| :---: | :---: | :---: | :---: | :---: |
| 14 | 111 | 995020.3642 | 6255009.6592 | 752170.3884 |
| 4 | 34 | 1042182.2124 | 6217578.5720 | 959062.1062 |
| 4 | 36 | 1042925.4455 | 6219813.4653 | 943937.7955 |
| 4 | 37 | 1070402.0946 | 6219153.2706 | 916912.1646 |
| 14 | 38 | 1039575.8516 | 6224562.1550 | 917756.9773 |
| 14 | 39 | 1070145.2541 | 6223297.8970 | 890112.6799 |
| 14 | 40 | 1025183.9760 | 6230829.5626 | 893795.9487 |
| 4 | 47 | 1082203.4162 | 6221064.0464 | 890999.3101 |
| 4 | 48 | 1108981.5637 | 6216812.7603 | 886409.7119 |
| 4 | 49 | 1096645.1778 | 6221215.6639 | 871478.0756 |
| 4 | 51 | 1084005.2575 | 6228610.0394 | 835094.1852 |
| 4 | 56 | 1085928.3525 | 6231696.3182 | 808414.8177 |
| 14 | 57 | 1076610.7064 | 6234785.5266 | 797934.5440 |
| 4 | 59 | 1035090.4754 | 6237391.4208 | 838580.2649 |
| 14 | 60 | 1062855.5917 | 6232330.9549 | 836090.6799 |
| 4 | 61 | 1055529.3941 | 6228302.7803 | 874609.5932 |
| 14 | 73 | 972601.4175 | 6254959.6643 | 783210.8042 |
| 14 | 74 | 1010406.0683 | \$245121.6685 | 815789.8735 |


| 14 | 75 | 999159.6910 | 6252476.4115 | 770946.6795 |
| ---: | ---: | ---: | ---: | ---: |
| 14 | 76 | 978165.3961 | 6256895.1971 | 764956.8356 |
| 4 | 77 | 948084.5588 | 6261295.2622 | 759205.0533 |
| 4 | 78 | 961247.0521 | 6261050.9339 | 739993.9475 |
| 4 | 80 | 1000331.9377 | 6255257.9426 | 747462.7753 |
| 14 | 82 | 1009147.0371 | 6254780.6617 | 730606.3639 |
| 4 | 83 | 1029465.5345 | 6255092.6855 | 704765.7250 |
| 14 | 84 | 996005.7402 | 6259938.0796 | 698522.6088 |
| 4 | 85 | 956026.9035 | 6265754.9974 | 704512.6981 |
| 4 | 86 | 991260.2261 | 6261833.2863 | 688393.1096 |
| 14 | 88 | 1016432.5301 | 6260402.2685 | 663871.3418 |
| 4 | 89 | 978089.0065 | 6265213.9318 | 675622.5360 |
| 14 | 90 | 1027101.3116 | 6257942.1690 | 672107.0425 |
| 4 | 91 | 1043496.3244 | 6254497.5643 | 680754.2437 |
| 4 | 92 | 1044834.1557 | 6253157.1498 | 693562.2012 |
| 4 | 93 | 1057397.4807 | 6252654.0842 | 676042.6898 |
| 14 | 94 | 1074576.3015 | 6250656.5608 | 664214.9723 |
| 4 | 95 | 1075645.4958 | 6249766.6374 | 6709533.0389 |
| 14 | 96 | 1086175.9326 | 6247439.0805 | 675112.6429 |
| 4 | 97 | 1089319.0437 | 6245060.3652 | 691904.0983 |
| 4 | 98 | 1068125.8987 | 6248569.0309 | 698781.6429 |
| 4 | 99 | 1097958.0504 | 6239753.9306 | 726007.9780 |
| 14 | 103 | 1098095.8785 | 6237086.1047 | 747858.4951 |
| 4 | 107 | 1100018.7084 | 6232225.2844 | 784044.6458 |
| 14 | 108 | 1102692.1135 | 6230394.1685 | 794595.1762 |
| 4 | 112 | 1012053.8507 | 6258960.8331 | 635612.9117 |
| 4 | 113 | 1092593.0887 | 6231633.5232 | 799689.3311 |

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## APPENDIX G RESULTS OF SCANNING AND VECTORIZATION OF CADASTRAL MAPS

Simulated cadastral map used for scanning and vectorization
Scale 1: 2000


## Coordinates of property corners of the simulated map

Station Number $X$ coordinate (m) Y coordinate (m)

| 1 | 1.01 | 0.97 |
| :--- | :---: | :---: |
| 2 | 48.23 | 1.22 |
| 3 | 101.12 | 0.64 |
| 4 | 150.71 | 2.46 |
| 5 | 198.37 | 0.41 |
| 6 | 199.01 | 50.22 |
| 7 | 152.87 | 49.64 |
| 8 | 100.43 | 51.26 |
| 9 | 50.96 | 50.55 |
| 10 | 0.73 | 49.21 |
| 11 | 91.32 | 96.43 |
| 12 | 146.65 | 103.21 |
| 13 | 262.16 | 101.32 |
| 14 | 190.32 | 94.31 |
| 15 | 146.27 | 110.21 |
| 16 | 90.46 | 103.26 |
| 17 | 55.72 | 106.56 |
| 18 | 13.24 | 109.56 |
| 19 | 3.44 | 148.26 |
| 20 | 54.44 | 150.35 |
| 21 | 94.17 | 147.32 |
| 22 | 152.11 | 15.77 |
| 23 | 206.14 | 147.14 |
| 24 | 250.32 | 154.31 |
| 25 | 154.23 | 190.19 |
| 26 | 48.22 | 194.23 |
| 27 | 0.16 | 208.22 |
| 28 | 48.36 | 201.43 |
| 29 | 100.41 | 200.32 |
| 30 | 154.21 | 198.46 |
| 31 | 195.34 | 202.21 |
| 32 | 250.94 | 206.92 |
| 33 | 247.36 | 251.26 |
| 34 | 196.77 | 246.31 |
| 35 | 149.44 | 249.32 |
| 36 | 96.52 | 251.33 |
| 37 | 50.02 | 249.47 |
| 38 | 6.73 | 251.22 |
|  |  |  |

Map obtained after scanning and vectorization


Node coordinates obtained by manual digitizing



| 65 | 44 |  | 259.261 | 112.235 |
| :---: | :---: | :---: | :---: | :---: |
| 67 | 45 | 15 | 146.517 | $110.44 \%$ |
| 73 | 46 |  | 8.953 | 109.646 |
| 74 | 47 | 18 | 12.596 | 109.419 |
| 77 | 48 | 17 | 55.598 | 205.428 |
| 81 | 49 | 16 | 90.551 | 103.444 |
| 76 | 50 | 12 | 147.254 | 102.848 |
| 85 | 51 |  | 10.179 | 102.550 |
| 54 | 52 |  | 11.303 | 102.576 |
| 68 | 53 | 13 | 262.181 | 101.804 |
| 68 | 54 |  | 261.517 | 101.736 |
| 70 | 55 |  | 261.563 | 101.523 |
| 71 | 56 |  | 246.549 | 100.182 |
| 79 | 57 |  | 174.810 | 100.018 |
| 91 | 58 |  | 55.003 | 99.302 |
| 96 | 59 |  | 2.493 | 98.596 |
| 93 | 60 | 11 | 92.903 | 96.54 C |
| 89 | 62 |  | 174.623 | 94.680 |
| 86 | 62 | 14 | 190.658 | 94.379 |
| 98 | 63 |  | 191.560 | 89.449 |
| 100 | 64 |  | 197.833 | 87.376 |
| 101 | 65 |  | 198.019 | 5i.17c |
| 110 | 65 | 8 | 200.747 | 51.392 |
| 112 | 67 | 9 | 50.945 | $50.7 \mathrm{J2}$ |
| 103 | 68 | 6 | 198.665 | 50.636 |
| 104 | 59 |  | 194.154 | 50.506 |
| 106 | 70 |  | 273.057 | 49.899 |
| 108 | 71 | 7 | 152.435 | 49.470 |
| 114 | 72 | 10 | 0.847 | 49.397 |
| 116 | 73 |  | 0.567 | 43.609 |
| 118 | 74 |  | 172.302 | 29.825 |
| 120 | 75 |  | 15:.537 | 24.673 |
| 122 | 76 |  | 100.896 | 12.107 |
| 125 | 77 |  | 150.750 | 2.916 |
| 127 | 78 | 4 | 150.743 | 2.712 |
| 133 | 79 | 1 | 0.927 | 1.511 |
| 124 | 80 | 5 | 198.278 | 1.165 |
| 132 | 81 | 2 | 47.797 | 2.015 |
| 134 | 82 |  | -4.251 | 0.906 |
| 130 | 83 |  | 55.076 | 0.732 |
| 129 | 84 | 3 | 100.940 | 0.670 |
| 128 | 85 |  | 204.807 | 0.181 |
| 136 | 86 |  | -9.392 | -6.24i |
| 136 | 87 |  | 211.668 | -7.565 |

Node coordinates obtained by scanning and vectorization


| 136 | 1 |  | 126.600 | 252.429 |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 2 |  | 121.492 | 251.656 |
| 1 | 3 |  | 122.673 | 251.493 |
| 139 | 4 |  | 96.426 | 25:.152 |
| 2 | 5 |  | 226.395 | 250.835 |
| 3 | 5 |  | 123.830 | 25c.765 |
| 6 | 7 |  | 122.676 | 250.470 |
| 1 | 8 |  | 121.664 | 250.463 |
| 5 | 9 |  | 215.605 | 250.435 |
| 141 | 10 | 37 | 50.536 | 245.579 |
| 10 | :: | 35 | 149.5771 | 249.183 |
| 144 | 12 |  | 170.8301 | 247.595 |
| 147 | 13 |  | 199.418 | 246.129 |
| 147 | 14 | 34 | 196.989 | 246.067 |
| 150 | 15 |  | 248.285 | 235.125 |
| 72 | i 5 |  | 249.459 | 231.752 |
| 12 | 17 | 27 | -0.310 | 209.204 |
| 69 | 18 |  | -3.639 | 207.623 |
| 70 | 13 |  | -5.8731 | 20i.3: 3 |
| 72 | 20 |  | 256.359 | 206.75\% |
| 11 | 21 |  | 10.824 | 205.740 |
| 78 | 22 |  | 11.162 | 206.576 |
| 72 | 23 | 32 | 250.680 | 206.3: |
| 74 | 24 |  | -3.724 | 202.385 |
| 75 | 25 |  | -6.365 | 202.353 |
| 12 | 26 |  | -0.123 | 201.600 |
| 30 | 27 | 31 | 195.58: | 201.539 |
| 78 | 28 | 28 | 48.367 | 201.465 |
| 13 | 29 |  | 100.052 | 199.931 |
| 153 | 30 |  | $257.8: 2$ | 199.895 |
| 13 | 31 | 29 | 99.884 | 199.80 S |
| 14 | 32 |  | 99.715 | 199.808 |
| 73 | 33 |  | 250.983 | 199.765 |
| 16 | 34 |  | 99.884 | 299.639 |
| 15 | 35 |  | 99.547 | 199.636 |
| 18 | 36 |  | 99.885 | 199.297 |
| 153 | 37 |  | 250.985 | 299.116 |
| 20 | 38 | 30 | 154.021 | 198.283 |
| 255 | 39 |  | -3.439 | 195.824 |
| 77 | 40 |  | 0.200 | 195.251 |
| 83 | 41 | 26 | 49.044 | 194.401 |
| 85 | 42 |  | 196.806 | 194.128 |
| 87 | 43 |  | 99.062 | 192.125 |


| 88 | 44 |  | 144.430 | 190.712 |
| :---: | :---: | :---: | :---: | :---: |
| 23 | 45 |  | 145.948 | 190.381 |
| 22 | 46 | 25 | 153.707 | 190.090 |
| 25 | 47 |  | 50.684 | 180.697 |
| 92 | 48 |  | 95.385 | 167.970 |
| 82 | 49 |  | 1.371 | 165.168 |
| 95 | 50 |  | 1.762 | 161.033 |
| 158 | 51 | 24 | 250.256 | 254.547 |
| 24 | 52 |  | 151.959 | 152.024 |
| 162 | 53 |  | 148.546 | 151.706 |
| 25 | 54 | 22 | 151.623 | 151.339 |
| 26 | 55 | 20 | 54.153 | 150.536 |
| 27 | 56 | 19 | 151.739 | 150.457 |
| 254 | 57 |  | 3.236 | 148.490 |
| 25 | 58 | 21 | 93.960 | 147.552 |
| 97 | 591 | 23 | 205.767 | 147.25: |
| 105 | 60 |  | 4.850 | 138.698 |
| 99 | 51 |  | 259.564 | 112.427 |
| 102 | 62 | 15 | 146.343) | :10.178 |
| 167 | $5]$ |  | 8.179 | 109.5:01 |
| 105 | 64 | 18 | 22.353 | 109.329 |
| 29 | 65 |  | 55.458 | 105.5:7 |
| 105 | 65 | 17 | 55.221 | 206.3:4 |
| 30 | 67 | 16 | 90.2051 | 203.54, |
| 21 | 68 |  | 89.522 | 203.154 |
| 35 | 59 |  | 246.700 | 203.013 |
| 103 | 70 |  | 90.54:1 | 102.990 |
| 35 | 71 | 12 | 147.2521 | 202.590 |
| $\pm 13$ | 72 |  | E1.052 | 102.452 |
| 33 | 73 |  | 11.286 | 102.476 |
| 34 | 74 |  | 9.685 | 102.210 |
| 33 | 75 |  | 10.781 | 102.:52 |
| 16 | 76 |  | 10.112 | 100.591 |
| . 30 | 77 |  | 247.890 | 100.427 |
| 108 | 78 |  | :74.703 | 99.807 |
| $1: \pm$ | 79 |  | 54.640 | 99.332 |
| 38 | 80 |  | 245.280 | 99.130 |
| 170 | 61 |  | 10.291 | 97.532 |
| 113 | 82 | 11 | 91.490 | 96.512 |
| 210 | 83 |  | 174.556 | 95.080 |
| 119 | 84 | 14 | 190.528 | 94.466 |
| 213 | 85 |  | 177.080 | 93.995 |
| 123 | 86 |  | 177.418 | 03.827 |
| 122 | 87 |  | 191.132 | 89.521 |
| 124 | 88 |  | 198.009 | 87.646 |
| 125 | 89 |  | 2.131 | 59.840 |
| 125 | 90 |  | 2.132 | 59.754 |
| 127 | 91 |  | 198.062 | 53.815 |
| 39 | 92 |  | 99.964 | 51.515 |
| 42 | 93 |  | 100.362 | 51.347 |
| 40 | 94 | 8 | 100.133 | 51.345 |
| 39 | 95 |  | 99.796 | 51.343 |
| 44 | 96 |  | 195.162 | 50.682 |


| 131 | 97 | 9 | 50.406 | 50.426 |
| :---: | :---: | :---: | :---: | :---: |
| 44 | 98 | 6 | 198.620 | 50.278 |
| 45 | 99 |  | 193.140 | 49.985 |
| 48 | 100 |  | 172.820 | 49.940 |
| 47 | 101 |  | 246.175 | 49.938 |
| 43 | 102 |  | 144.826 | 49.929 |
| 46 | 103 | 7 | 152.416 | 49.637 |
| 49 | 104 | 10 | 0.389 | 49.589 |
| 49 | 105 |  | -0.362 | 46.76 |
| 67 | 106 |  | -0.698 | 45.536 |
| 56 | 107 |  | -0.361 | 4¢.427 |
| 173 | 108 |  | 171.746 | $29.14 \%$ |
| 50 | 209 |  | 151.475 | 2i.37i |
| 51 | 110 |  | 151.224 | 23.770 |
| 52 | 121 |  | 100.751 | 2i.75 |
| 54 | 112 | 4 | 150.356 | 2.353 |
| 55 | 113 |  | 60.306 | 1.769 |
| 57 | 114 |  | 59.548 | 2.336 |
| 56 | 115 | 1 | 0.273 | 4.200 |
| 177 | 116 |  | 20.510 | 0.999 |
| 176 | 117 |  | -6.135 | C.997 |
| 51 | 118 |  | 20.8:7 | $0.83: 1$ |
| 58 | 119 |  | 60.815 | 0.749 |
| 63 | 120 |  | 198.086 | 0.615 |
| 178 | 122 |  | 47.409 | 0.580 |
| 60 | :22 |  | 32.569 | 0.480 |
| 60 | 123 |  | 30.798 | $0.36:$ |
| 62 | 124 |  | 100.783 | 0.325 |
| 186 | 125 |  | 205.338 | 0.321 |
| 180 | 126 |  | 47.158 | -0.36i |
| 65 | 127 |  | 32.487 | -0.45s |
| 64 | 228 |  | 30.801 | -0.470 |
| 68 | 129 |  | -i1.005 | -6.3i2 |
| 59 | 130 |  | 212.105 | -7.314 |

APPENDIX H PART OF A SRI LANKAN CADASTRAL MAP


## APPENDIX I GEO-REFERENCING ERRORS DUE TO MOUNTAIN PEAKS IN 1 : 500,000 MAP

| Transforming coordinates for coverage boun-orig |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Scale $(X, Y)=$ (12006. 772, 12861. 982) $\quad$ Translation $=(-179101.258,-136746.029)$ Rotation (degrees) $=(0.358)$ Fils Error (input, output $)=(0.078,997.219)$ |  |  |  |  |
| tic id | input output $\mathbb{Z}$ | input y <br> output y | $\pm$ error | y error |
| 98 | 99.854 -5249.999 | $\begin{array}{r} 4.808 \\ -72029 \end{array}$ | -841.803 |  |
| 83 | 13.004 | -7922. 543 | -841.803 | -268.586 |
|  | -13841.900 | -68000. 898 | 257.130 | -365.927 |
| 85 | 18.8199 62202.199 | - 57.2620 | -846. 626 | 149.857 |
| 79 | 17.143 | 7.849 |  |  |
|  | 39758. 102 | -3396\%. 500 | 54.802 | -585. 785 |
| 81 | -801.400 | -22201.600 | 972.468 | -1263. 603 |
| 100 | 9.638 | 7.688 |  |  |
| 100 | $-56327.398$ | -37224. 8988 | -36. 097 | 60.099 |
| 101 | -30322.301 | $\begin{array}{r} 9.065 \\ -21159.600 \end{array}$ | 1101.406 | 1860.085 |
| 65 | 14.071 | 10.549 |  |  |
|  | 0. 000 | 0.000 | 255.233 | -44. 712 |
| 76 | 16.920 | $\begin{array}{r} 10.026 \\ -748 \% 400 \end{array}$ | -888.320 | 930.866 |
| 70 | 210.459 | 14.337 |  |  |
|  | 21020. 102 | 48899. 298 | 740.090 | 239.971 |
| 64 |  |  |  |  |
| 61 | 42016. 10.686 | $4598 \varepsilon^{2} .500$ 18.573 | 465.975 | -355.885 |
|  | -43367. 100 | 103276.500 | -444. 277 | -364. 134 |
| 62 | $40911.500$ | $\begin{aligned} & 17.929 \\ & 95092.000 \end{aligned}$ | -790.983 | 12. 755 |
| Prrc: 1 |  |  |  |  |

## APPENDIX J SUBROUTINES USED FOR "Concord" AND <br> THE SEQUENTIAL LEAST SQUARES ADJUSTMENT

Subroutine TOXYZ(N)

|  | Calculation of Global X, Y, Z coordinates <br> Given geodetic latitude and longotude and height of a stations <br> Lat, Lon are in "input.d" file (stn, LatD, LatM, LatS, Lond, LonM, Lons) <br> Outpur X,Y,Z coordinates are in "output.d"Eile ( stn, X, Y, Z) <br> N is the number of stations. |
| :---: | :---: |
|  | Implicit real*8( $a-h, p-z$ ) <br> Dimension istn(N), latd(N), latm(N), alats(N), $\operatorname{ceno}(N), a n(N), x(N), Y(N), z(N)$ |
|  | open (unit=i, file='input_f. $d^{\prime}$ ) <br> open (unit=2, file='outpū_f.d') |
|  | pi $=3.1415926535897932 \mathrm{do}$ |
| c | Coordinates are calculated as on the ellipsoid. So, height is taken as 0.0 |
| $c$ | otherwise heights of stations have to be read and |
| c | assigned to h . $h=0.0 \mathrm{do}$ |

2

$$
\begin{aligned}
& \text { write (*, 2) n } \\
& \text { format(//,12x,'Number of Stations are',i3) } \\
& \text { co } 100 i=1, n \\
& \text { read ( } 1, * \text { ) istn (i), latd(i), latm(i), alats (i), } \\
& +\quad \text { read(1, iond(i), lonm(i), alons (i) } \\
& \begin{aligned}
\text { alat }(i)= & (l a t d(i)+l a t m(i) / 60.0 d 0+a l a t s(i) / 3600.0 d 0) \\
+ & \text { *pi/180.0d0 } \\
+\quad \text { alon }(i)= & (\operatorname{lond}(i)+10 n m(i) / 60.0 d 0+a l o n s(i) / 3600.0 d 0) \\
& \text { *pi/180.0d0 }
\end{aligned} \\
& \text { smajor }=20922931.80 \mathrm{dO} * 0.3047995 \mathrm{dO} \\
& \text { sminor }=20853374.58 \mathrm{~d} 0 \text { * } 0.3047995 \mathrm{~d} 0 \\
& \text { esq }=\text { (smajor**2 - sminor**2)/smajor**2 }
\end{aligned}
$$

c calculating the Radius of curvature in the prime vertical deno(i) $=(1.0 \mathrm{~d} 0$-esq*dsin(alat(i))**2)**0.5 an(i) = smajor/deno(i)

$$
\begin{aligned}
& x(i)=(\operatorname{an}(i)+h) * \operatorname{dcos}(\operatorname{alat}(i)) * \operatorname{dcos}(a \operatorname{lon}(i)) \\
& y(i)=(a n(i)+h) * \operatorname{dcos}(\operatorname{alat}(i)) \star \operatorname{dsin}(a l o n(i)) \\
& z(i)=(\operatorname{an}(i) *(1.0 d 0-\operatorname{sc} q)+h) * d s i n(a 1 a t(i))
\end{aligned}
$$

c writing data
15 format (/,i3,i5,i5, $£ 8.5, i 3, i 5, f 8.5)$ write(*,15)istn(i), latd(i), latm(i), alats(i), lond(i), $+\quad$ lonm(i), alons(i)
$c \quad$ Writing the calculated values
write ( 2,16 ) istn (i), $x(i), y(i), z(i)$ write (*, 16) istn(i), x(i), $\underline{y}(i), z(i)$
16 format(i3, 3f20.8)
100 continue
=eturn end

SUBROUTINE PARAM(N,M)
c Program for the calculation of Transformation parameters c to transform old global XYZ coordinates to new XYZ c global coordinates
c 12 parameter linear affine transformation
c at least 4 points are required for the transformation
$c \mathrm{~N}$ is the number of common points and $\mathrm{M}=3 * \mathrm{~N}$
c Old XYZ coordinates are in "from.d" file and
c New XYZ coordinates are in "to.d" file.
c Parameters will be written to "para.d" file

```
    implicit real *8(a-h,0-z)
    dimension istnf(N),xf(N),yf(N),zf(N),istnt(N),
        +
        xt (N),yt(N),zt(N),a(M,12), al(M,1), at (12,M),
        ata(12,12),atl(12,1),b(12),x(12,1)
            OPEN (UNIT=1, EILE = 'from.d')
            OREN (UNIT=2, EILE = 'tO.d')
            OREN (UNIT=3, EILE = 'para.d')
```

                DO 10 i \(=1, n\)
                read(1,*)istnf(i), xf(i),yf(i),zf(i)
                read( \(2, *\) )istnt (i), xt(i),yt(i), zt(i)
                \(x f(i)=x f(i)-1000000.000\)
                \(y f(i)=y f(i)-6200000.0 \mathrm{dO}\)
                \(z f(i)=z f(i)-800000.0 \mathrm{dO}\)
                \(x t\) (i) \(=x t\) (i) -1000000.0 dO
                \(y t(i)=y t(i)-6200000.0 \mathrm{do}\)
                \(z t(i)=z t(i)-800000.0 \mathrm{do}\)
    10 CONTINUE
    c Writing old and new XYX values on the screen do $111 i=1,10$
write (*, *) istnf(i), xf(i), yf(i), $2 f(i)$
111 write(*,*)istnt(i),xt(i),yt(i),zt(i)
c forming A matrix for 12 parameters

$$
j=1
$$

do $200 \quad i=1, m$
$a(i, 1)=x f(j)$
$a(i, 2)=y f(j)$
$a(i, 3)=z f(j)$
$a(i, 10)=1.0 \mathrm{do}$
$a(i+1,4)=x f(j)$
$a(i+1,5)=y f(j)$
$a(i+1,6)=2 f(j)$
$a(i+1,11)=1.0 \mathrm{dO}$

```
    a(i+2,7)=xf(j)
    a(i+2,8)=Yf(j)
    a(i+2,9)=zf(j)
    a(i+2,12)=1.0d0
    j=j+1
    continue
    c forming I matrix for }12\mathrm{ parameter transformation
        do 300 i=1,m
        al(j,1)=xt(i)
        aI(j+1,I)=yt (i)
        al(j+2, ( ) =zt(i)
        3 0 0
        continue
    c Subroutine TRANS is to transform a matrix
        CAII TRANS (A,AT,m,12)
    c Subroutine AB is to multiply A and B matrices
                            CAII AB (AT,A,ATA,12,m,12)
    CAII AB (AT,AL,ATL, 12,m,1)
c Subroutine INVERT calculate the inversion of a matrix
\(c\) and replaces the original matrix by the inversion.
CAII INVERT (ATA, B, 12)
c
After inversioin ATA is replaced by ATAinverse
CAII AB (ATA, ATL, X, 12,12,1)
C Writing calculated parameters.
print*' \({ }^{\prime}\) parameters'
do \(17 \quad i=1,12\)
write ( \(3, *\) ) \(\times(i, 1)\)
write (*, *) \(\mathrm{x}(\mathrm{i}, 1)\)
17 CONTINUE
CIOSE (UNIT=1)
CLOSE (UNIT=2)
CLOSE (UNIT=3)
RETURN
END
```


## SUBROUTINE convert (N,M)

c Subroutine to transform global XYZ XYZ coordinates c from one system to another system, using available $c$ parameters at least 4 common points are required for c the transformation
$c \mathrm{~N}$ is number of points and $\mathrm{M}=3 * \mathrm{~N}$
IMPIICIT REAL *8 (A-H, O-Z)
dimension istn(N), $X f(N), y f(N), z f(N)$,
$+\quad a(M, 12)$, al (M), at (12, M), istnt (N), xt (N), $+\quad a t a(12,12), a t l(12,1), B(12), x(12), y t(N), z t(N)$, $\mathrm{xn}(\mathrm{N}), \mathrm{yn}(\mathrm{N}), \mathrm{zn}(\mathrm{N}), \mathrm{xd}(\mathrm{N}), \mathrm{yd}(\mathrm{N}), \mathrm{zd}(\mathrm{N})$

OPEN (UNIT=1, EILE $=$ 'from. $\mathrm{d}^{\prime}$ )
OPEN (UNIT=3, EILE = 'Dara.d')
open (unit=4, file='to.dr)

10 CONTINUE
do $20 i=1,12$ read (3, *) x (i) continue
20
c forming A matrix for 12 parameters

$$
j=1
$$

$$
\mathrm{do} 200 \mathrm{i}=1 \mathrm{~m}
$$

$a(i, 1)=x f(j)$
$a(i, 2)=y f(j)$
a $(i, 3)=z f(j)$
$a(i, 10)=1.0 \mathrm{~d} 0$
$a(i+1,4)=x f(j)$
$a(i+1,5)=y f(j)$
$a(i+1,6)=\mathbf{z f}(j)$
$a(i+1,11)=1.0 \mathrm{~d} 0$
$a(i+2,7)=x f(j)$
$a(i+2,8)=y f(j)$
$a(i+2,9)=z f(j)$
$a(i+2,12)=1.0 d 0$
$j=j+1$
$i=i+2$
continue

$$
\begin{aligned}
& \text { DO } 10 i=1, n \\
& x f(i)=x f(i)-1000000.0 \mathrm{dO} \\
& y f(i)=y f(i)-6200000.0 d 0 \\
& z f(i)=z f(i)-800000.0 \mathrm{dO} \\
& x t(i)=x t(i)-1000000.0 d 0 \\
& y t(i)=y t(i)-6200000.0 \mathrm{do} \\
& z t(i)=z t(i)-800000.0 \mathrm{do}
\end{aligned}
$$

c Subroutine $A B$ is for matrix multiplication $C A I I A B(A, x, A I, m, 12, I)$ $j=1$ do $300 i=1, m$ $x \Omega(j)=a l(i)$ yn $(j)=a 1(i+1)$

```
zr(j)=al(i+2)
```

$i=i+2$

$$
j=j+1
$$

$$
\text { do } 400 \quad i=1, n
$$

$$
x d(i)=x t(i)-x n(i)
$$

$$
y d(i)=y t(i)-y n(i)
$$

$$
\operatorname{zd}(i)=z t(i)-z n(i)
$$

$$
\begin{aligned}
& x n(i)=x n(i)+1000000.0 \mathrm{do} \\
& \mathrm{yn}(i)=y n(i)+6200000.0 \mathrm{do} \\
& 2 n(i)=2 n(i)+800000.0 \mathrm{~d} 0
\end{aligned}
$$

400
continue
C Writing calculated new coordinates
$\underset{j=1}{j=1}$
do $17 \quad i=1, n$
write ( $2, *$ ) istn(i), $x n(i), y n(i), z n(i), x d(i), y d(i), z d(i)$ write (*, *)istn(i), xn(i),yn(i), zn(i),xd(i),yd(i),zd(i)
format (i5, 3f15.4, 3f8.3)
16
continue
CLOSE (UNIT $=1$ )
CIOSE (UNIT=2) CEOSE (UNIT=3)
return
end

## SUBROUTINE FROMXYZ (N)

c Calculation of Latitudes and Longitudes from Global X, Y, c $Z$ coordinates
c Input data is in "input_r.d" file (format : station, X, Y, 2) c Output values are in "output_r.d" file
$c \mathrm{~N}$ is the number of stations

$$
\text { implicit real*8 }(a-h, p-z)
$$

dimension istn(N), alat(N), alon(N), $x(N), y(N), z(N)$, $+\quad \operatorname{an}(N)$, deno $(N), \operatorname{latd}(N), \operatorname{latm}(N)$, alats $(N), 10 n d(N)$, lonm(N), alons(N)
open(unit=1, file='input r. $\mathrm{d}^{\prime}$ ) open(unit=2, file='output_r.d')
$p i=3.1415926535897932 \mathrm{do}$
$h=0.0 \mathrm{dO}$
co $100 i=1, n$
read(1,*)istn(i), x(i),y(i),z(i)
smajor $=20922931.80 \mathrm{dO} * 0.3047995 \mathrm{dO}$
sminor $=20853374.58 \mathrm{do} * 0.3047995 \mathrm{do}$
esq $=$ (smajor**2 - sminoこ**2)/smajor**2
c calculating longitude
alon(i) $=\operatorname{datan}(y(i) / x(i))$
c calculating an approximate value for latitude
$\operatorname{alat}(i)=\operatorname{atan}(z(i) /(\operatorname{sqrt}(x(i) * * 2+y(i) * * 2)))$
do $52 \mathrm{j}=1,10$
c calculating the Radius of curvature in the prime vertical
deno $(j)=(1.0 d 0-e s q * d s i n(a l a t(i)) * * 2) * * 0.5$ an $(j)=$ smajor/deno(j)
c calculating latitude
alat $(i)=\operatorname{datan}(z(i) /(s q r t(x(i) * * 2+y(i) * * 2)) *$
$+$
52 continue
c Subroutine "RATODMS" converts radians to Degrees, minutes $c$ and seconds.

```
        call ratodms(alat(i),latd(i),latm(i),alats(i))
```

        call ratodms (alon(i), iond(i), lonm(i), alons(i))
    11
1 0 0

```
```

```
    WRITE (2,1I)ISTN(I), IAATD(I), IATM(I), A工ATS(I),
```

```
    WRITE (2,1I)ISTN(I), IAATD(I), IATM(I), A工ATS(I),
    +
    +
    WRITE (*, 1I) ISTN(I), I_ATD(I), IAATM(I), ALATS (I),
    WRITE (*, 1I) ISTN(I), I_ATD(I), IAATM(I), ALATS (I),
    +
    +
```

                EORMAT(I4,I5, I5,E20.8,5X,I5,I5,F2O.8)
    ```
                EORMAT(I4,I5, I5,E20.8,5X,I5,I5,F2O.8)
            continue
            continue
        return
        return
        end
```

        end
    ```
    SUBROUTINE RATODMS (RAD, IDD, MM, SS)
    IMPIICIT REAL *8 (A-H, P-Z)
    \(P I=4 . O D O * D A T A N(1 . O D O)\)
    \(D E G=R A D / R I * 180.0 D 0\)
        LDD \(=\) IDINT (DEG)
        \(A M M=\) (DEG - IDD)*60.ODO
        \(M M=\) IDINT (AMM)
        \(S S=(A M M-M M) * 60.0 D 0\)
    RETURN
    END

SUBROUTINE MERCA（ALAT，ALON，X，Y）
C Subroutine to calculate plane coordinates from geodetic C coordinates for Sri lanka
C Transverse mercator projection
C Central meridian and standard parallel through C PIDURUTAIAGAIA．
C Equations Bomford，Geodesy，4th edition， 2.114 and
\(C\) and A． 68 （Latitudes and Longitudes must be in radians）
IMPIICIT REAL＊ 8 （ \(\mathrm{A}-\mathrm{F}, \mathrm{P}-\mathrm{Z}\) ）

> SMAJOR \(=20922931.80 D 0 * 0.3047995 D 0\)
> SMINOR \(=20853374.58 D 0 * 0.3047995 D 0\)
c Calculating e－squared（E）
\(E=(S M A J O R * * 2-S M I N O R * * 2) /\) SMAJOR＊＊2
c Calculating radius of curvature in prime vertical AN \(=\) SMAJOR \(/(1.0 D 0-E * D S I N(A I A T) * * 2) \pi * 0.5\) \(A M=S M A J O R *(1.0 D 0-E) /((1.0 D O-E * D S I N(A I A T) * * 2) * * 1.5)\)
c Calculating coefficents for Bomford A． 68
\[
\begin{aligned}
& A O=1.0 D 0-1.0 D 0 / 4.0 D 0 * E-3.0 D 0 / 64.0 D 0 * E * * 2+5.0 D 0 / \\
& +\quad 256.000 * E * * 3 \\
& \text { A2 }=3.0 D 0 / 8.0 D 0 *(E+1.0 D 0 / 4.0 D 0 * E * * 2+15.0 D 0 / \\
& \text { + 128.0D0*E**3) } \\
& \mathrm{A} 4=15.0 \mathrm{DO} / 256.0 \mathrm{D} *(E * * 2+3.000 / 4.000 * \Xi * * 3) \\
& \mathrm{A} 6=35.0 \mathrm{DO} / 3072.0 \mathrm{DO} \mathrm{E} \text { ** } 3
\end{aligned}
\]
c Calculating the meridian distance for a station \(S=S M A J O R *(A 0 * A 工 A T-A 2 * D S I N(2 . O D O * A 工 A T)+A 4 *\)
 \(P I=3.1415926535897932 D 0\) CIAT \(=(7.000+00 / 60 .+1.729 D 0 / 3600.0 D 0) *\) PI／180．0D0 \(C L O N=(80.0 D 0+46.0 D 0 / 60.0 D 0+18.160 D 0 / 3600.0 D 0) *\) \(+\quad\) PI／180．0D
c Calculating the meridian distance to Pidurutalagala \(S C=\) SMAJOR＊（AO＊CLAT－A2＊DSIN（2．ODO＊CLAT）＋AA＊ \(+\quad \operatorname{DSIN}(4.0 D 0 * C L A T)-A 6 * D S I N(6.0 D 0 * C L A T))\)
c Coefficents in Bomford 2.114
```

    ERS = E/(I-E)
        G = (DCOS (ALAT)**2)* (1.ODO-DTAN (A工AT)**2 +
    + EPS*DCOS (ALAT)**2)
H=DSIN (ALAT)* (DCOS (AIAAT)**2)* (5.0DO-
+ DTFN (ALAT)**2 +9.ODO*EPS*DCOS (ALAT)**2 +
+ 4.ODO*EPS**2*DCOS (ALAT)**4)
AJ = (DCOS (AT,AT)**4)*(5.0D0-18.0D0*(DTAN (ALAT)**2) +
+ (DTAN (ALAT)**4) +14.0D0*EPS*(DCOS (ALAT)**2) +
+ 58.0DO*EPS*(DSIN(AIAT)**2))

```
c Scale factor from the Jackson's report \(Z=0.9999238418 \mathrm{DO}\)
c Calculating northings (from the equator first)
\[
\begin{aligned}
Y= & S+0.5 D O *((A L O N-C L O N) * * 2) \star A N *(D C O S(A L A T)) * \\
+ & (D S I N(A L A T))+E / 24 . O D O *((A L O N-C L O N) * * 4) * \\
+\quad & A N * D C O S(A L A T) \\
Y C= & S C \\
Y= & (Y-Y C) * Z
\end{aligned}
\]
c Calculating eastings
\[
\begin{aligned}
X= & 2 \star A N * D C O S(A I A T) *((A I O N-C I O N)+(G *(A I O N-C I O N) \\
+ & \star * 3) / 6.0 D 0+(A J *(A I O N-C I O N) * * 5) / 120.0 D 0)
\end{aligned}
\]

RETURN END

\section*{SUBROUTINE INVERT (A, B, I)}

C Subroutine to invert the matrix A and replace A by its inverse
C Subroutine provided by the Ohio State University.
IMPIICIT REAL* 8 ( \(A-H, O-Z\) )
DIMENSION \(A(I, I), B(I)\)
IE ( I .EQ. 1) GO TO 10
\(I M=I-1\)
DO \(5 \mathrm{~K}=1, I\)
DO \(2 \mathrm{~J}=1, \mathrm{IM}\)
2
\(B(J)=A(1, J+1) / A(1,1)\)
\(B(I)=1 . / A(1, I)\)
DO \(4 \mathrm{~L}=1, I M\)
DO \(3 \mathrm{~J}=1, \mathrm{IM}\)
\(A(L, J)=A(I+1, J+1)-A(I+1,1) * B(J)\)
\(A(L, I)=-A(I+1, I) * B(i)\)
DO \(5 \mathrm{~J}=1, I\)
\(A(i, j)=B(J)\) RETURN
\(10 \quad \mathrm{~A}(1,1)=1 . / \mathrm{A}(1,1)\) RETURN END

S' Subroutine used for sequential least squares adjustment

SUBROUTINE SEQUENTIAL (N, M)
c Subroutine to upgrade digitized and georeferenced c coordinates of property corners using linear, angular c and area measurements.
\(c \quad N\) is the \(n u m b e r\) of property corners, and \(M=2 * N\)
c Coordinates of property corners are in "node. d" file
c Upgraded values will be written to "new.d" file
```

        implicit real*8 (a-h,o-z)
    dimension inode(N), x(N), }Y(N), al(M,M), al1(M,1)
        + a2(5,M),al2(5,1),alt (M,M),a2t(M,5),altal(M,M),
        + 22ta2(M,M),ata(M,M),altll(M,I), a2tl2(M,I),
        + atl(M,I), xhat (M,I),b(M)
    ```
            open (unit=1,file='node. \(\mathrm{d}^{\prime}\) )
            open (unit=2, £ile='new.d')
            print*,' Number of nodes are:'
            Write (*, *) N
c reading \(\frac{n}{\pi}\) of available liner, angular and area measurements
print*,'No of distance conditions ?'
read (*, *) nd
print*,' No of angle conditions ?'
read ( \(*, *\) ) ла
print*,'No of area conditions ?'
read (*,*) nr
\(C\) reading nodes and their \(X, Y\)
do \(10 \quad i=1, n\)
            read ( \(1, *\) ) inode (i), \(x(i), Y(i)\)
                        continue
กn2 2 2 \({ }^{\text {* }}\)
nt=nd+na+nr
c A and Imatrices for first set of observations (I)
c Eirst set of observations are the digitized coordinates.
                        do \(20 i=1, \operatorname{mn} 2\)
        a1 \((i, i)=1.0\)
    continue
c Formation of Il matrix
do \(30 i=1, n n 2,2\)
\(\operatorname{all}(i, I)=0.0\)
\(\operatorname{all}(i+1, I)=0.0\)
30
continue
c A and I matrices for linear, angular and area measurements c first for distances (Uotila page 67)
if (nd.eq. 0) go to 41
do \(40 i=1, n d\)
print*,'Enter both nodes for distance condition ?'
read(*, *)k1,k2
L1 \(=2 * k 1-1\)
L2 \(2=2 * k 2-1\)
print*,'enter the distance (same units as coordinates) ?' read(*,*)d
c Calculating approximate distances
\(\operatorname{ddo}=\operatorname{dsqrt}((x(k 1)-x(k 2)) * * 2+(y(k 1)-y(k 2)) * * 2)\)
\(a 2(i, L I)=(x(k 1)-x(k 2)) / d\)
\(\mathrm{a} 2(i, L I+1)=(\underline{y}(k 1)-y(k 2)) / d\)
\(\mathrm{a} 2(\mathrm{i}, \mathrm{L} 2)=(\mathrm{x}(\overline{\mathrm{k}} 2)-\mathrm{x}(\mathrm{k} 1)) / \mathrm{d}\)
\(a 2(i, L 2+1)=(\mathrm{y}(\mathrm{k} 2)-\mathrm{y}(\mathrm{k} 1)) / \mathrm{d}\)
a12 (i, 1) =ddo-d
40 continue
41 continue
c Eormation of A2 and 12 matrices for angles
c Uotila page 68
ro \(=180 * 60 * 60 * 7 / 22.0\)
\(j=n d+1\)
if (na.eq. 0) go to 51
do \(50 \quad i=1\), na
print*,'Select end-middle-end nodes for angles' read (*, *) k1,k2,k3 LI \(=2 * k 1-1\) \(\mathrm{L} 2=2 * \mathrm{k} 2-1\) L3 \(=2\) * \(k 3-1\)
print*,'Enter the angle in degrees'
read(*,*)ang
c Calculating distances among 3 points
\[
\begin{aligned}
& \text { disl }=\operatorname{dsqrt}((x(k 1)-x(k 2)) * * 2+(y(k 1)-y(k 2)) * * 2) \\
& \text { dis2 }=\operatorname{dsqrt}((x(k 3)-x(k 2)) * * 2+(y(k 3)-y(k 2)) * * 2) \\
& a z 1=\operatorname{datan}((x(k 3)-x(k 2)) /(y(k 3)-y(k 2))) \\
& a z 2=\operatorname{datan}((x(k I)-x(k 2)) /(y(k 1)-y(k 2))) \\
& \text { ango }=\text { abs (az1-az2)*180*7/22.0 } \\
& \mathrm{a} 2(\mathrm{j}, \mathrm{II})=-\mathrm{ra}^{*}(\mathrm{y}(\mathrm{k} 1)-\mathrm{y}(\mathrm{k} 2)) / \mathrm{di} s 1 * * 2 \\
& \mathrm{a} 2(\mathrm{j}, \mathrm{II} \mathrm{l} 1)=-\mathrm{ro*}(\mathrm{x}(\mathrm{k} 1)-\mathrm{x}(\mathrm{k} 2)) / \mathrm{dis} 1 * * 2 \\
& a 2(j, I 2)=-r 0^{*}(\underline{y}(k 2)-y(k 3)) / d i s 2 * * 2+
\end{aligned}
\]
```

+ (y(k1)-y(k2))/dis1**2
a2(j,L2+1) = -Io*(x(k3)-x(k2))/dis2**2 +
+ x(k1)-x(k2))/disi**2
a2(j, L3) = -ro* (y(k3)-y(k2))/dis2**2
a2(j,L3+1) = -IO*(x(k3)-x(k2))/dis2**2
al2(j,1 )= (ango-ang)*3600.0
j=j+1
continue
continue

```
51
c Eormation of A2 and L2 for area conditions if (nr eq. 0) go to 61
\(j=n d+n a+1\)
do \(60 i=1\), \(n r\)
print*,'Select 4 points for area condition' print*,'In clockwise or anti-clockwise orcier' read (*, *) kl,k2,k3,k4 I \(1=2 * k 1-1\)
L2 \(2=2 * k 2-1\)
L3 \(=2 * k 3-1\)
I \(4=2 * k 4-1\)
print*,'enter the area in square meters' read (*, *) area
c Calculating approximate value for the area areao \(=\operatorname{abs}((x(k 1) * y(k 2)+x(k 2) * y(k 3)+x(k 3) * y(k 4)+\)
    \(+\)
                        \(x(k 4) \star y(k 1)-x(k 1) * y(k 4)-x(k 4) * y(k 3)-x(k 3) *\)
                            \(y(k 2)-x(k 2) * y(k 1)) / 2.0)\)
                \(a 2(j, 工 1)=y(k 2)-y(k 4)\)
                a2 \((j, L 1+1)=x(k 4)-x(k 2)\)
                \(a 2(j, L 2)=y(k 3)-y(k I)\)
                a2 \((j, I 2+1)=x(k 1)-x(k 3)\)
                a2 \((j, \pm 3)=y(k 4)-y(k 2)\)
                a2 \((j, \pm 3+1)=x(k 2)-x(k 4)\)
                a2 \(\langle j, I 4\) ) \(=y(k 1)-y(k 3)\)
                a2 \(\langle j, I 4+1)^{\prime}=x(k 3)-x(k 1)\)
                al2 (j, 1) =areao-area
                        \(j=j+1\)
    60 continue
    61 continue
c Subroutine "TRANS" transforms a matrix
c Subroutine "AB" is to multiply two matrices
c Subroutine "ADD" is to add two matrices
c Subroutine "INVERT" is to invert a matrix and replace
\(c\) thc the original martix by the inverse.
```

call trans (al,ait,nn2,n\Omega2)
call trans (a2, a2t,nt,nn2)
cal1 ab(alt,al,a1tal,nn2,nn2,nn2)
call ab (a2t,a2,a2ta2,nn2,nt,nn2)
call add(a1tal,a2ta2,ata,nn2,nn2)

```
call \(a b(a 1 t, a 11, a 1 t 11, n n 2, n n 2,1)\)
call \(a b(a 2 t, a 12, a 2 t 12, n n 2, n t, 1)\)
call add(a1t11,a2t12,at1,nn2,1)
c ata is replaced by ata-inverse
call invert (ata, \(b, n n 2\) )
call \(a b(a t a, ~ a t 1, x h a t, n n 2, n n 2, I)\)
do \(70 \quad i=1, \mathrm{nn} 2\)
xhat (i, 1) \(=-1\) *xhat ( \(i, 1\) )
continue
co \(80 \mathrm{i}=1, \mathrm{nn} 2,2\)
\(c\) Writing \(d x\) and dy on the screen
write (*, *)istn(i), xhat (i, 1), xhat (i+1,1) continue
cio \(90 \quad i=1, n n\)
\(x(i)=x(i)+x h a t(i, 1)\)
\(\underline{v}(i)=y(i)+x h a t(i+1, i)\)
c Writing updated property corners.

90

> write \((13, *) i s t n(i), x(i), Y(i)\)
> write \((2, *) i \operatorname{stn}(i), x(i), Y(i)\)
> continue
return
end

\section*{REFERENCES}

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\section*{IMAGE EVALUATION}

TEST TARGET (Q A-3)
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