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### Performance Analysis of WDM Optical Networks with Grooming Capabilities

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#### ABSTRACT

In this paper, we analyze the performance of WDM networks with traffic grooming capabilities supporting lowrate circuit-switched traffic streams. Traffic grooming in WDM networks collectively refers to the multiplexing, demultiplexing and switching of lower-rate traffic streams onto high capacity lightpaths. Networks which perform grooming only at the OADMs present in the nodes are referred to as Constrained Grooming Networks. Networks whose nodes switch traffic streams between wavelengths and perform grooming at the OADMs are referred to as Sparse Grooming Networks. Given the network topology, the traffic matrix and the node locations of grooming and traffic stream switching, we present an analytical model, using link-independence and wavelength-independence assumptions, to calculate the blocking performance. We illustrate the benefits of sparse grooming over constrained grooming in the mesh-torus and ring network topologies, using both simulation and analytical results.

Keywords: Wavelength-Routing, Traffic Grooming, Blocking Analysis

#### 1. INTRODUCTION

Wavelength Division Multiplexing(WDM) divides the large bandwidth available in optic fiber into multiple channels with each of the channels operating at different wavelengths and at moderate data rates of around 2.5 Gbps (OC-48) to 10 Gbps (OC-192). While there is no doubt that WDM has gained commercial acceptance as a dominant communication technology, the exact role of WDM as a switching technology in next-generation broadband networks is still a topic of debate. While proponents of all-optical networks predict that all-optical switching, in conjunction with wavelength-routing and wavelength conversion, will fully replace electronic switching, there has also been considerable support for networks with terabit-per-second electronic switching using WDM solely as point-to-point transport. However, it is likely the case that networks of the future will employ a hybrid, layered architecture, using both wavelength-routing and electronic switching technologies. In such WDM networks, while the capacity of a wavelength has steadily increased from OC-48 to OC-192 (and on to OC-768 in the future), the networks are required to provide dynamic services to the user at a rate that is much lower than the full wavelength capacity. These sub-rate traffic connections can vary in range from say, STS-1 (51.84 Mbps) capacity upto the full wavelength capacity. In addition, in networks of practical size, the number of source-destination traffic connections is still an order of magnitude higher than the number of available wavelengths. Designing such a network, to provide a varied range of sub-rate traffic services in a cost-effective manner, is a difficult task.

In this respect, WDM technology based on wavelength-routing helps us in bringing down the cost of the network by reducing the amount of electronic TDM equipment. A key factor that enables this cost-reduction is the amount of *optical passthrough* that is possible in the network. Optical Add/Drop Multiplexers (OADMs) and Optical Wavelength Cross-Connects (OXCs) accomplish this by allowing the wavelengths to be selectively dropped at the TDM equipment or to pass optically through the node unaffected. The amount of passthrough is affected by the physical topology of the network, the traffic between the nodes and more importantly, the ability to efficiently provision and groom the traffic. Traffic grooming in WDM networks can be defined as the act of multiplexing, demultiplexing and switching lower rate traffic streams onto higher capacity lightpaths. Efficient traffic grooming improves the amount of optical passthrough and wavelength utilization in the network. Traffic grooming is performed at two points in a WDM network. The multiplexing and demultiplexing of traffic streams, in the time domain, onto lightpaths is

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performed by the OADMs at the nodes. The switching of traffic streams from one wavelength to another is performed by the crossconnects present at the nodes. This switching can be carried out in space, time and wavelength. A traffic stream on a set of time slots on a wavelength on an incoming fiber can be switched to a different set of time slots on a different wavelength on an outgoing fiber. However, this *traffic stream switching* capability comes at the cost of increased crossconnect complexity. In addition to space-switching of wavelengths, the crossconnect may have to be provided with wavelength conversion and time-slot interchange equipment. Since, all-optical wavelength conversion and photonic time-slot interchange devices are still being prototyped, currently it is more attractive to use electronic implementation of these features in the network. In the future, all-optical traffic grooming might prove to be more cost-effective and manageable than its electronic equivalent.

In this paper, we present simulation and analytical results of the performance of WDM networks with traffic grooming capabilities. Specifically, we compare the performance in networks where grooming is carried out only at the OADMs in the nodes and in networks with additional traffic switching capability at the crossconnects. We will also specify a simple analytical model using link and wavelength independence assumptions. The model takes into account the capacity distributions of the wavelengths for calculating the blocking probabilities and the interdependence between call blocking and the offered network load. It is important to quantify this effect of grooming on network performance, to assess its usefulness in future WDM networks.

#### 1.1. BACKGROUND

Most of the work in the literature on traffic grooming has been concentrated on providing efficient network designs in WDM rings for improving the overall network cost. Specifically, these studies have dealt with reducing the higherlayer electronic processing equipment whose cost dominate over the cost of the number of wavelengths in the ring network. Gerstel, Ramaswami and Sasaki<sup>1</sup> discussed traffic grooming for a number of ring architectures. Gerstel et al.<sup>2</sup> further analyzed traffic grooming issues in UPSR and BLSR WDM rings and considered the traffic grooming and lightpath assignment as a single problem. Traffic grooming algorithms for assigning low rate circuits to wavelengths in unidirectional SONET/WDM rings have also been proposed.<sup>3,4</sup> Uniform traffic was used in all of these papers. Zhang and Qiao<sup>5</sup> proposed algorithms for traffic grooming and wavelength assignment which can be applied to both unidirectional and bidirectional rings with an arbitrary number of nodes under both uniform and non-uniform traffic. They also derived lower bounds on the number of SONET ADMs and wavelengths required for a given traffic pattern. Wan et al.<sup>6</sup> specified approximation algorithms for traffic grooming of arbitrary traffic in SONET/WDM rings. Cinkler et al.<sup>7</sup> considered the optimization problem of reducing resource usage at electronic layers in arbitrary mesh networks.

It is well-known that the wavelength continuity constraint in WDM networks contributes to the increase in probability of a call request being blocked. In this respect, a lot of work has been done in studying the call blocking performance of wavelength-routed networks using different routing and wavelength assignment schemes. It has been established that wavelength conversion, that is, the ability of a routing node to convert one wavelength to another, plays an important role in improving the blocking performance. Wavelength conversion for WDM networks was proposed by Lee and Li<sup>8</sup> and it was shown that wavelength converters reduce wavelength conflicts and improve the performance by reducing the blocking probability. Lower bounds on the blocking probability for an arbitrary network for any routing and wavelength assignment algorithm were derived by Ramaswami and Sivarajan.<sup>9</sup> It was further shown that use of wavelength converters resulted in a 10 to 40 percent increase in wavelength reuse. Birman<sup>10</sup> proposed a reduced load approximation scheme to calculate the blocking probabilities for their optical network model for two routing schemes: fixed routing and least-loaded routing. However, their model did not consider the load correlation between links. Barry and Humblet<sup>11</sup> proposed analytical models of networks, using fixed routing and random wavelength assignment, taking wavelength correlation into account. They also studied the effects of path length, switch size and hop number on the blocking probability in networks with and without wavelength conversion.

Subramaniam et al.<sup>12</sup> studied sparse wavelength conversion and its effects on blocking performance. Their analytical model takes into account both wavelength correlation and the dynamic nature of the traffic. Sridharan and Sivarajan<sup>13</sup> presented a new analytical technique based on the inclusion-exclusion principle for networks with no wavelength conversion and random wavelength assignment. Their model improves on Birman's model in that it is independent of hop-length and scales only with the capacity of the link. While the above models concentrated on fixed routing, models based on Least Loaded Routing<sup>14</sup> and Alternate Routing<sup>15</sup> have also been studied. Analytical models<sup>15-17</sup> for first-fit wavelength assignment scheme, based on overflow traffic model, have also been proposed. Karasan and Ayanoglu<sup>18</sup> provide a review of the various schemes and their associated analytical models.

There has also been considerable interest in studying the performance of networks which utilize both TDM and WDM. Sabry and Midwinter<sup>19</sup> proposed preliminary models for their multiwavelength TDM network in which time-slots on a wavelength could be dedicated to each source-destination pair. Yates<sup>20</sup> studied the performance improvements offered by wavelength converters and time-slot interchangers in shared-wavelength TDM networks and dedicated-wavelength TDM networks. However, the analysis was restricted to the case where all calls have uniform bandwidth requirements and occupied one time slot on a wavelength. The rest of the paper is organized as follows. In Section 2, we present the network model and connection setup and release procedure for the traffic grooming networks. In Section 3, we will present the analytical model using link and wavelength independence assumptions. In Section 4, we illustrate our simulation and analytical results on mesh-torus and ring topologies and study the effect of grooming on these topologies. Conclusions will be presented in Section 5.

#### 2. NETWORK MODEL

We consider a WDM network with network nodes of two types: Wavelength-Selective Crossconnect (WSXC) nodes and Wavelength-Grooming Crossconnect (WGXC) nodes. These networks nodes are interconnected by fiber-optic links which can be either bi-directional or uni-directional. WSXC nodes have OXCs, which space-switch full wavelengths from an input port to an output port, and OADMs, which groom the traffic streams onto the added/dropped wavelengths. However, WSXC nodes cannot switch traffic streams between wavelengths. WGXC nodes, in addition to having the functionality of a WSXC, are capable of time-slot interchange and can switch lower-rate traffic streams from a set of time slots on one wavelength on an input port to a different set of time slots on another wavelength on an output port. We assume that this switching is fully non-blocking and can be performed for all wavelengths from any input port to any output port. Hence, full wavelength conversion capability is implicitly available at the WGXC node. Such a node is said to have full grooming capability. If switching of lower-rate traffic streams is performed only on a restricted number of wavelengths, then the node is said to have *limited grooming capability*. We assume that all the WGXC nodes in our network are provided with full grooming capability. Since the hardware complexity of WGXC nodes is more than that of WSXC nodes, WGXC nodes also cost significantly more than WSXC nodes. Therefore, we assume the practical situation in which only some of the nodes of the network are WGXC nodes and the rest of the nodes are WSXC nodes. Such a network is referred to as a sparse grooming network. On the other hand, a network with only WSXC nodes and no WGXC nodes is referred to as a constrained grooming network, since grooming is constrained to the OADMs at the nodes.

We assume a dynamic traffic model in which low-rate traffic sessions arrive and depart from the network in a random manner. Such a traffic session is routed along a path traversing through intermediate WSXC and WGXC nodes between the source and destination. If the path traverses through one or more intermediate WGXC nodes then the traffic session involves more than one lightpath. Lightpaths between the source, destination or intermediate WGXC nodes wavelength continuity constraint, that is, the traffic stream occupies the same wavelength on all the links of the path between the source, destination or intermediate WGXC nodes. However lightpaths between WGXC nodes can be routed on different wavelengths. In this manner, each lightpath typically carries many multiplexed lower-speed traffic streams. During connection setup, it should be confirmed whether the lightpaths, that have been established earlier, have the required amount of capacity before they can be used to accommodate the new traffic session.

#### 2.1. Connection Setup and Release

The connection setup and release procedure in traffic grooming networks is different from the lightpath establishment process of conventional wavelength routing networks. Consider an example of a sub-network, shown in Figure. 1, which can be a part of a bigger mesh network. Assume a single wavelength is currently available on the path from A to F. Let the capacity of the wavelength be C. Further assume that all the nodes on the path are WSXC nodes. Suppose a request arrives for a connection from node B to E for a line capacity of C/4. This is established immediately on the available wavelength by configuring the OADMs at nodes B and E, and by configuring the OXCs at nodes C and D. Note that we add/drop the wavelength only at nodes B and E, and not at nodes C and D. Let a second request arrive for a connection from node A to node F for a line capacity of C/4. This is also established on the same wavelength by setting up lightpaths from A to B and from E to F, and by using the same lightpath on the wavelength between B to E that was established for the first connection. In this process, the first traffic stream is not disturbed. The wavelength is now add/dropped at four nodes, namely, A, B, E and F. Let a third request arrive for a connection from A to G for line capacity C/4. However, this connection cannot be established on the



Figure 1. Network Example

wavelength and will be blocked. The reason is, the path for the third connection request from A to G deviates away from the path of the lightpath on the wavelength and node D is only a WSXC node.

Let a fourth request arrive for a connection from node C to D for a line capacity of C/4. At this point, we have two options. (a) We can assume any lightpath that has been established should not be disturbed. Therefore, the traffic stream cannot be established on the same wavelength and is blocked. (b) However, if a temporary disturbance to the lightpath is acceptable. We can establish the third call on the same wavelength by add/dropping the wavelength at node C and D. The lightpath is now split into three parts. This temporary disturbance can be made possible by the presence of fast OADMs and fast reconfigurable OXCs at the nodes. For our network model, we assume the latter case (option b) and assume the nodes have fast OADMs and OXCs. On the other hand, if nodes C and D happen to be WGXC nodes, then it is possible to satisfy all the call requests.

When a call leaves the network, the lightpaths that are used to hold the traffic connection release the capacity used by the traffic stream. However, the lightpaths themselves might continue to operate over the wavelengths since they might have other traffic streams multiplexed over them. If the traffic stream was the sole one to have used the lightpath, then the lightpath itself can be released and the wavelengths on the links can be freed.

#### 3. ANALYTICAL MODEL

In this section, we will present the approximate analytical model based on link and wavelength independence assumptions. Our model is different from previous proposed models in that it considers the wavelength capacity distribution of the lightpaths and helps us in computing the blocking probabilities of traffic streams at various line-speeds. We consider a network with V nodes and L links. We assume that each node has already been configured either as a WSXC node or as a WGXC node. Each link is bidirectional and consists of a pair of fibers with W wavelengths each in each direction. Each wavelength, with capacity C, has a line-speed indicated by a parameter g (C is assumed to be divisible by g) referred to as the granularity. A lightpath that traverses a wavelength can support a maximum of g traffic streams.

Calls arrive at a node according to a Poisson process with rate  $\lambda$ . Each call is equally likely to be destined to any of the remaining V - 1 nodes. The arrival rate of calls  $\lambda_{sd}$  for a node pair (s, d) is then  $\lambda/(V - 1)$ . Each call can request a line-speed j (of capacity jC/g), where  $1 \leq j \leq g$ . The arrival rate of calls at a source-destination pair and requesting a line-speed j is  $\lambda_{sd}(j)$ . Each set of calls of line-speed j from a source to a destination requests equal total capacity of calls in its line speed class. In other words, if the combined capacity of calls to a node pair is say, Kg, then each line-speed class contributes a capacity of K through its call arrivals. For example, line-speed 1 traffic will have K call arrivals, line-speed 2 will have K/2 call arrivals and similarly line-speed j traffic will have K/j call arrivals. Therefore, the probability,  $r_j$  that a call is of line-speed j is

$$r_j = \frac{1/j}{\sum_{i=1}^g 1/i}.$$
 (1)

The expected value of  $j, E\{j\}$  is then given by

$$\sum_{j=1}^{g} jr_j = \frac{g}{\sum_{i=1}^{g} 1/i}.$$
(2)

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The arrival rate per unit line-speed per s-d pair is now defined as

$$\hat{\lambda_{sd}} = \lambda_{sd} E\{j\}. \tag{3}$$

Here the term "unit line-speed" refers to the capacity of the lowest granularity traffic stream that can be groomed on to the lightpath.  $\hat{\lambda_{sd}}$  is essentially the arrival rate of calls at s-d pairs in the network if all call requests are of the lowest granularity, i.e. of line-speed 1.

The call holding time is assumed to be exponentially distributed with unit mean. We use fixed-path routing to route calls. Each call uses a prespecified path. If the path cannot accommodate the call, then the call is assumed to be blocked and is lost. Traffic requests cannot be split up among wavelengths on a link. We further assume that fast OADMs and fast reconfigurable OXCs are available in the nodes. Hence a lightpath can be disturbed to multiplex a traffic stream into it. We assume random wavelength assignment that is, the wavelength which has to be assigned to the call is chosen randomly from the set of available wavelengths on the path. Other wavelength assignment strategies like first-fit and most-used can provide better performance but are more difficult to analyze. We further assume link and wavelength independence. Link independence can be defined as the independence of events on different links of the network. Wavelength independence assumes that wavelength usage is uniformly distributed over the entire set of wavelengths and is independent of the utilization of other wavelengths on the same link and on other links. Each node is assumed to have enough WADMs (receivers/transmitters) to handle all the traffic that originate from and sink into the node. The traffic from a node is hence limited by its degree, the number of wavelengths on the fibers and the capacity of the wavelengths.

#### 3.1. Analysis for a Single Wavelength Link

Consider a single link, single wavelength system with wavelength capacity, C and granularity g. Let  $\lambda_l(j)$  be the link arrival rates of traffic streams of line-speed j (capacity jC/g). Let  $n_j$  be the number of traffic streams of line-speed j multiplexed into the wavelength. The wavelength can contain  $(n_1, n_2, ..., n_j, ..., n_g)$  traffic streams of line-speeds from 1 to g provided:

$$0 \le n_j \le \lfloor \frac{g}{j} \rfloor; \tag{4}$$

$$\sum_{j=1}^{g} jn_j \le g. \tag{5}$$

We can model the above system as a Markov chain with a state space given by  $(n_1, n_2, ..., n_j, ..., n_g)$  where  $n = \sum_{j=1}^{g} n_j$  is the total number of calls in the system and values of  $n_j$  satisfy constraints (4) and (5). The generator matrix  $Q^*$  of the Markov process governing the system can be formed according to the following rules. For a state transition:

- 1. From state  $(n_1, n_2, ..., n_j, ..., n_g)$  to state  $(n_1, n_2, ..., n_j + 1, ..., n_g)$ , provided constraints (4) and (5) are satisfied. We denote the arrival of a call of line-speed j and the transition rate is given by  $\lambda_l(j)$ .
- 2. From state  $(n_1, n_2, ..., n_j, ..., n_g)$  to state  $(n_1, n_2, ..., n_j 1, ..., n_g)$  provided  $n_j > 0$ . We denote the departure of a call of line-speed j. The transition rate is given by  $n_j$  since  $\mu = 1$ .
- 3. The diagonal elements of the matrix are negative such that the sum of all the elements of any row in the matrix equals zero. Specifically their value is given by  $-\lambda_s n$  where  $\lambda_s$  is the sum of only those  $\lambda_l(j)$  rates, with valid single-call arrivals so that constraints (4) and (5) would not be violated.
- 4. For all other state transitions, the transition rate is zero.

The stationary probability vector X at an arbitrary time for the generator  $Q^*$  is the unique solution to the equations:

$$XQ^* = 0, Xe = 1. (6)$$

where e is a column vector of 1s. Hence the elements of vector X are  $x(n_1, n_2, ..., n_j, ..., n_g)$  which correspondingly provide the probability that the system is in state  $(n_1, n_2, ..., n_j, ..., n_g)$  Once we calculate the steady-state probability vector X, we obtain the link blocking probability  $P_l(j)$  of class-j traffic stream by

$$P_l(j) = 1 - \sum_{i=0}^{g-j} X_c(i), \tag{7}$$

where  $X_c(i)$  is the sum of only those probability values  $x(n_1, n_2, ..., n_j, ..., n_g)$  of X such that the corresponding state  $(n_1, n_2, ..., n_j, ..., n_g)$  yields a capacity of i, i.e.,  $\sum_{k=1}^g kn_k = i$ .

#### 3.2. Blocking Probability Analysis of the Network

Having obtained the individual blocking probabilites of traffic streams of various line-speeds for a single-wavelength link, we now consider the computation of blocking probability for a single path with multiple wavelengths. Consider a path p of an end-to-end call requesting line-speed of j from a source node s to a destination node d in the network. We define a segment to be the set of links on the path between two consecutive WGXC nodes or between the source(or destination) and a WGXC node. If the path contains no intermediate WGXC nodes, then it consists of a single segment from source to destination. Then

$$f_{mk}(j) = 1 - (1 - \{\overline{P}_{mm_1}(j)\overline{P}_{m_1m_2}(j)...\overline{P}_{m_nk}\})^W$$
(8)

is the success probability in the segment from node m to node k on the path, where  $\overline{P}_{xy}(j) = 1 - P_{xy}(j)$  for link  $l_{xy}$  and  $m_1, m_2, ..., m_n$  are the WSXC nodes in the segment between nodes m and k. Let the number of WGXC nodes on the path (not including the source and destination nodes) of the path p be k. This divides the path p into k+1 segments. Hence the success probability of the end-to-end call of line-speed j is given by

$$S_{sd}(j) = \prod_{i=0}^{k} f(s_i),$$
(9)

where  $f(s_i)$  corresponds to the success probability of the segment  $s_i$  between the WGXC nodes.  $f(s_0)$  corresponds to the success probability of the segment between the source node and the first WGXC node on the path and  $f(s_k)$ corresponds to the success probability of the segment between the last WGXC node and the destination node. The blocking probability for the path is then obtained as

$$P_{sd}(j) = 1 - S_{sd}(j). (10)$$

Using the above formula, we can calculate the blocking probabilities for all the paths for all line speeds in the network. From this, we obtain the blocking performance  $\Gamma(j)$  for a given linespeed j for the network by

$$\Gamma(j) = \frac{\sum_{\forall s,d} \lambda_{sd}(j) P_{sd}(j)}{\sum_{\forall s,d} \lambda_{sd}(j)}.$$
(11)

Typically, the traffic in a network is specified in terms of a traffic matrix (say A) which specifies the offered load between node pairs. In our case, we have j traffic matrices, with each traffic matrix  $A_j$ . Using this information, we need to estimate the load at the links. However, the offered load at the node pairs is not entirely carried by the links of the network as some of the calls are blocked. On the other hand, the extent of call blocking is in turn dependent on the offered load. This interdependence leads us to a set of coupled non-linear equations called the Erlang Map<sup>21</sup> and its solution is called the Erlang fixed point. In our case, the holding time of all of our calls between the node pairs is exponentially distributed with unit mean, hence the offered loads at the links and nodes are equal to their respective arrival rates. Hence if the probability of blocking of a line-speed j call on a link l on the path is  $P_l(j)$ , the probability of blocking of a line-speed j call on path p from s to d is  $P_{sd}(j)$ , and the arrival rate of of line-speed jcall on the path p is  $\lambda_{sd}(j)$ , then a good approximation<sup>21</sup> for the arrival rate of a class-j call at the link l at a single wavelength is given by

$$\lambda_l(j) = \sum_{\forall p \mid l \in p} \frac{\lambda_{sd}(j)}{W} \frac{(1 - P_{sd}(j))}{(1 - P_l(j))}.$$
(12)

We use the following iterative procedure to solve for  $\Gamma(j)$  for all paths. We define  $\lambda_l^{(i)}(j)$ ,  $P_{sd}^{(i)}(j)$ ,  $\Gamma^{(i)}(j)$  and  $P_l^{(i)}(j)$  as the values obtained at the end of the *i*th iteration for the respective variables without the superscript (i). First we set  $P_{sd}^{(0)}(j)$ ,  $\Gamma^{(0)}(j)$ , and  $P_l^{(0)}(j)$  to zero and *i* to 1 as part of the initial conditions. Then we follow the iterative method specified below:

- 1. Determine the link arrivals rates,  $\lambda_l^{(i)}(j)$ , from (12) for all links.
- 2. Use the methods specified in Sub-section 3.1 to calculate  $P_{I}^{(0)}(j)$  for all links.
- 3. Calculate  $P_{sd}^{(i)}(j)$  using (8), (9) and (10).
- 4. Calculate  $\Gamma^{(i)}(j)$  using (11).
- 5. If the absolute, percentage difference between  $\Gamma^{(i)}(j)$  and  $\Gamma^{(i-1)}(j)$  is smaller than a preselected threshold value,  $\epsilon$ , (in our case,  $\epsilon = 1e 3$ ), then terminate. Otherwise, increment *i* and go to step 1.

It should be mentioned that the above iterative procedure does not always converge to a solution. Although there exists methods such as Newton's method that are guaranteed to converge, we use this method as it is computationally efficient and simpler, and converges within a few iterations for most cases.

#### 4. NUMERICAL RESULTS

In this section, we present the effect of traffic grooming through simulation and analytical results on two network topologies, a 16-node mesh torus and an 8-node ring, under both constrained grooming (with all WSXC nodes) and sparse grooming (with all WGXC nodes) conditions. For both topologies, as shown in Figures 2-5, we observe that the blocking probability increases as the line-speed increases. We also observe that the analytical model provides fair estimates of blocking probability at different line-speeds. For the case of the mesh-torus, we find that sparse grooming offers an order of magnitude decrease in blocking probability for low line-speed connections. The reason is partly due to the fact that low line-speed connections groom better and fit easier into wavelengths than high line-speed connections. However, in the case of the ring, introduction of sparse grooming offers only a small improvement when compared to the mesh-torus. This is because the mesh-torus has better mixing of traffic connections and lower link-load correlation than the ring.

#### 5. CONCLUSION

In this paper, we studied the performance of WDM networks with constrained and sparse grooming capabilities. Given a topology, the traffic demand and the types of nodes in the network, we specified an analytical model, based on link independence and wavelength independence assumptions, to calculate the blocking performance. The analytical model incorporates the interdependence between call blocking and the offered load. The model also takes into account the capacity distribution of the wavelengths for calculating the blocking probabilities of traffic streams with different line-speeds. Simulation and analytical results were obtained for mesh-torus and ring topologies. The analytical model provides fair estimates of the blocking performance for constrained and sparse grooming networks. In our study, we found that sparse grooming networks performance between high and low line-speed traffic streams increased, as the traffic-stream switching capability of the network was increased. The difference in blocking is referred to as capacity fairness and has been addressed in.<sup>22</sup> This work can be extended in many directions which include: (i) Improving the analytical model to capture the load correlation between links and the wavelength dependence on the links, (ii) Analysing the network performance of such traffic grooming networks using different wavelength assignment algorithms.



Figure 2. Blocking probability vs. load per station in Erlangs for a Constrained Grooming  $4 \times 4$  mesh-torus network with W = 5 and g = 4. (LS: Line-Speed, Sim: Simulation, Inde: Independence Model)



Figure 3. Blocking probability vs. load per station in Erlangs for a Sparse Grooming  $4 \times 4$  mesh-torus network with W = 5 and g = 4. (LS: Line-Speed, Sim: Simulation, Inde: Independence Model)



Figure 4. Blocking Probability vs load per station in Erlangs for a Constrained Grooming 8-node ring network with W = 5 and g = 2. (LS: Line-Speed, Sim: Simulation, Inde: Independence Model)



Figure 5. Blocking Probability vs load per station in Erlangs for a Sparse Grooming 8-node ring network with W = 5 and g = 2. (LS: Line-Speed, Sim: Simulation, Inde: Independence Model)

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