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INTERREGIONAL COMPETITIVE POSITION OF THE HOG-PORK INDUSTRY IN SOUTHEAST UNITED STATES

by

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INTRODUCTION

All sectors of the hog-pork industry need reliable information about effects of prospective changes in livestock and meat marketing for purposes of decision-making in resource allocation and capital investment planning. However, much more than the marketing system is involved in supplying this information and analysis. To make sound investment decisions in the hog-pork industry, the entire industry must be analyzed within a framework of comparative advantage and interregional competition. In this study the Southeast hog-pork industry provides a setting for the factual problem.

As pointed out by Maki et al. (30, p. 699), "Historically, our livestock and meat markets have adjusted to changing patterns of production and consumption by gradually modifying, expanding or relocating existing facilities and by adopting new methods of livestock procurement and distribution. Sharp changes in these historical patterns of marketing and distribution may occur in future years. These changes involve the entire marketing process." The Southeast* hog-pork industry, particularly, has undergone important changes since World War II.

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*As defined here, Southeast United States includes the states of Virginia, North Carolina, South Carolina, Georgia, Alabama, Florida, Maryland, Delaware and the District of Columbia.

The Southeast is currently a deficit region with respect to pork production (39). In 1960, commercial hog slaughter in the Southeast was about one-half of Southeast pork consumption (11). However, a decline in farm population and increasing urbanization is currently taking place in the Southeast. Associated with this urbanization is increasing consumer income. These changes, along with changes in tastes and preferences, are affecting the pork consumption patterns of consumers in the Southeast.

The Problem

To comprehensively analyze the competitive position of the Southeast hog-pork industry, all sectors of the industry must be examined. These sectors would include (a) production of feeder pigs, (b) feeding of slaughter hogs, (c) hog slaughter and (d) pork consumption. In addition, the more important costs involved with each of these sectors, or dimensions, should be included in the analysis. These costs would pertain to each of the four major sectors. For example, the production of feeder pigs is affected by (a) regional differences in input-output ratios for feeder pigs as well as for enterprises that compete for similar resources, (b) regional differences in costs of factors of productions and (c) transportation and procurement costs for feeder pigs between regions.

The feeding of slaughter hogs is influenced by a similar set of variables, namely, (a) regional differences in input-output ratios for feeding slaughter hogs as well as for enterprises that compete for similar resources, (b) regional differences in costs of factors of production and (c) transportation and procurement costs for slaughter hogs between regions. Hog slaughter, on the other hand, is influenced by (a) regional differences in hog slaughter costs as well as costs for other slaughtering facilities that compete for similar resources and (b) transportation and distribution costs for pork between regions. Finally, pork consumption is affected by regional differences in demand for pork as well as cross relationships with competing meats.

Because of the nature of the problem to which this study is addressed, its objective is twofold. The first task is to develop an operational model capable of analyzing a spatially separated hog-pork industry. The second part of the objective is to apply this model to the United States hog-pork industry with special emphasis on the Southeast in an analysis of the feeding, slaughter and consumption sectors presented in the preceding discussion. Estimates of transfer costs between these three sectors are used in this partial equilibrium analysis. Under specific assumptions regarding the production of feeder pigs, the model used in this study <u>simultaneously</u> derives patterns of interregional hog and pork shipments that minimize total transportation and slaughtering costs.

Procedure

The second section of this study describes the economic setting of the hog-pork industry. The third section develops the theoretical framework for the study. The fourth section presents the economic model used in the study plus a small illustration of the model which shows the exact procedure for solution. The following section includes the empirical analyses of hog-pork distribution for 1960 and 1970. The sixth and final sections present the conclusions from the analyses and discuss the limitations of the study as well as suggestions for future research.

SOUTHEAST HOG-PORK INDUSTRY

To illustrate the nature of the factual problem, this section is devoted to a descriptive analysis of both the past and present Southeast hog-pork industry. The information presented in this section is used in the next section to develop relevant hypotheses for analyzing the competitive position of the Southeast hog-pork industry.

Delineation of Substate Areas

The areas used throughout this study are delineated in Figure 1. The Southeast is defined as areas 1 through 12 and the non-Southeast as areas 13 through 22. Areas 2 through 12 are the same as those used by the Southern Regional Livestock Marketing Research Committee for the presentation of 1960 hog and pork shipment estimates developed by the Committee (11, 12, 37). Areas 2 through 12 are not restricted by state boundaries whereas the other areas in Figure 1 are either states or combinations of states.

The data and model used in this study are specified for the areas defined in Figure 1 for two reasons: (a) to conduct the analyses in this study for Southeast substate areas that are relatively homogeneous with respect to hog production, hog slaughter and pork consumption and (b) to allow

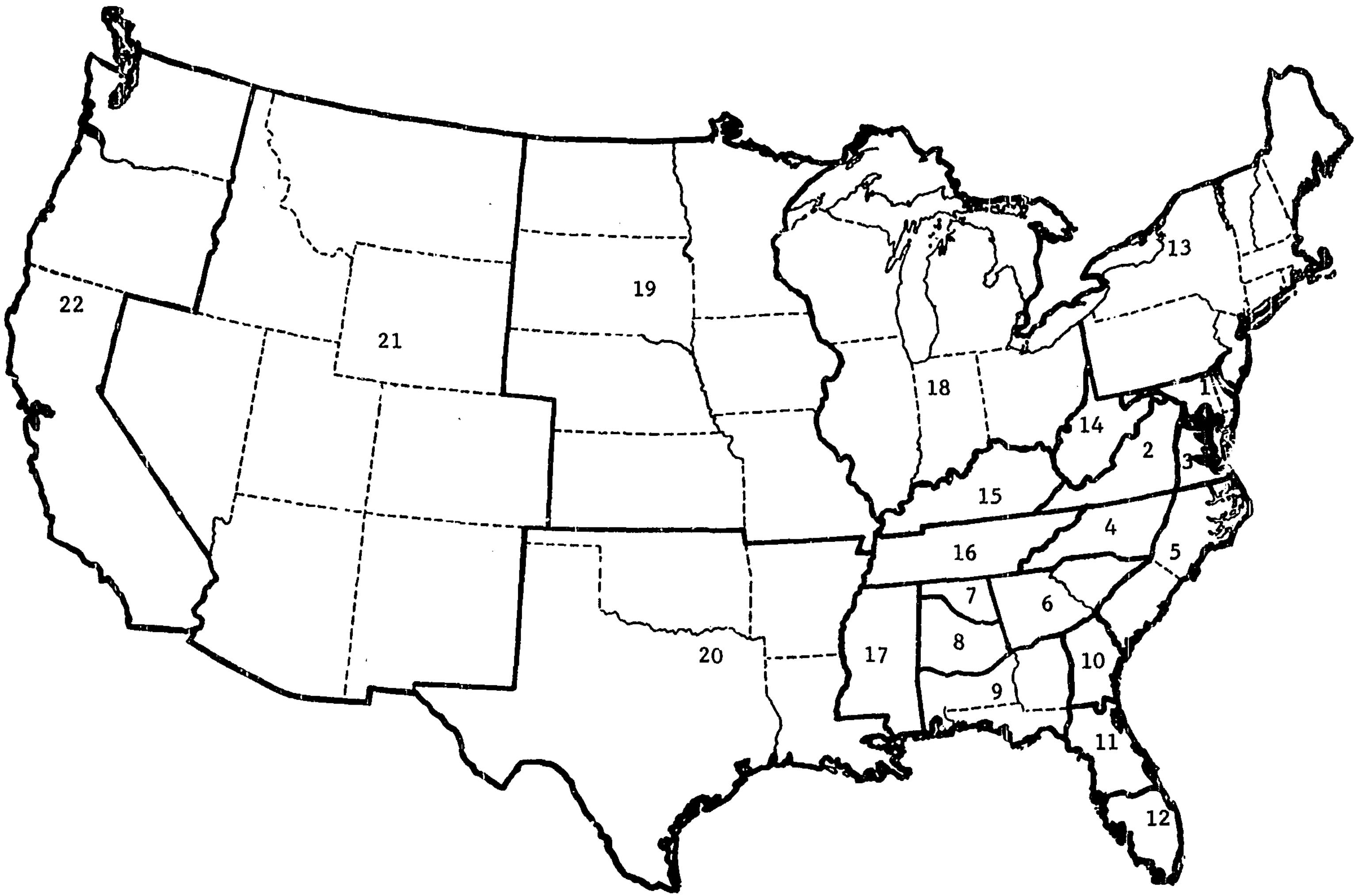

Figure 1. Delineation of Southeast areas (1-12) and non-Southeast areas (13-22).

for comparative analysis between the actual 1960 Southeast hog and pork shipments and the solution to the model for 1960.

Hog Production, Marketings and Slaughter

Figure 2 shows that hog marketings in the Southeast have increased substantially from 1947 through 1961. Figures 3 and 4, however, indicate that hog production has remained relatively constant during this same period.* Marketings plus farm slaughter and January 1 inventory are used here as measures of production.**

As illustrated in Figure 5, farm slaughter has decreased in the Southeast at almost the same rate since 1947 as in the non-Southeast. However, farm slaughter constitutes a much larger share of production in the Southeast. In 1947, farm slaughter in the Southeast represented 50.3 percent of marketings plus farm slaughter, but in 1961 farm slaughter was only 18.7 percent of marketings plus farm

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*Production has increased in some of the Southeast areas and decreased in others during this period, but for the entire Southeast it has remained relatively constant.

**Appendix A presents a procedure for estimating the number of hogs available for commercial slaughter by the areas outlined in Figure 1.

Figure 2. Index numbers of hog marketings in the Southeast and non-Southeast, 1947-61 (40, 43, 47, 48)

Figure 3. Index numbers of hog marketings plus farm hog slaughter in the Southeast and non-Southeast, 1947-61 (40, 43, 47, 48)

Figure 4. Index numbers of January 1 inventory of all hogs and pigs on farms in the Southeast and non-Southeast, 1947-61 (40, 43, 47, 48)


Figure 2
Index Numbers
(1947-49 Avg.=100)
170
160
150
140
130
120
110
100
90
0
Southeast
Non-Southeast

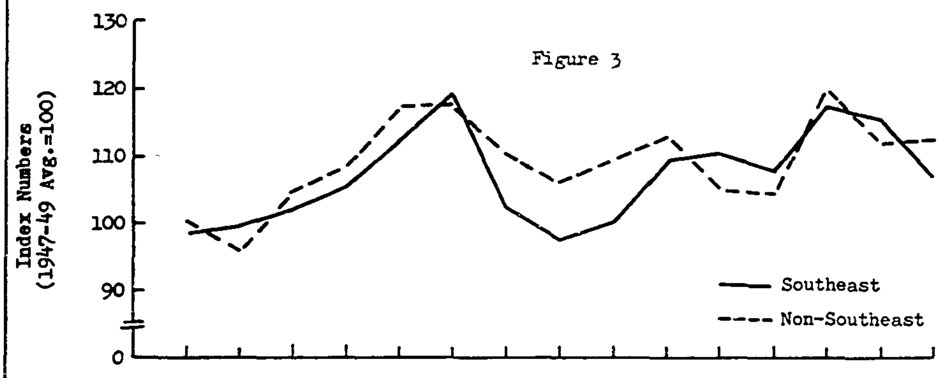
Figure 3
Index Numbers
(1947-49 Avg.=100)
130
120
110
100
90
0
Southeast
Non-Southeast

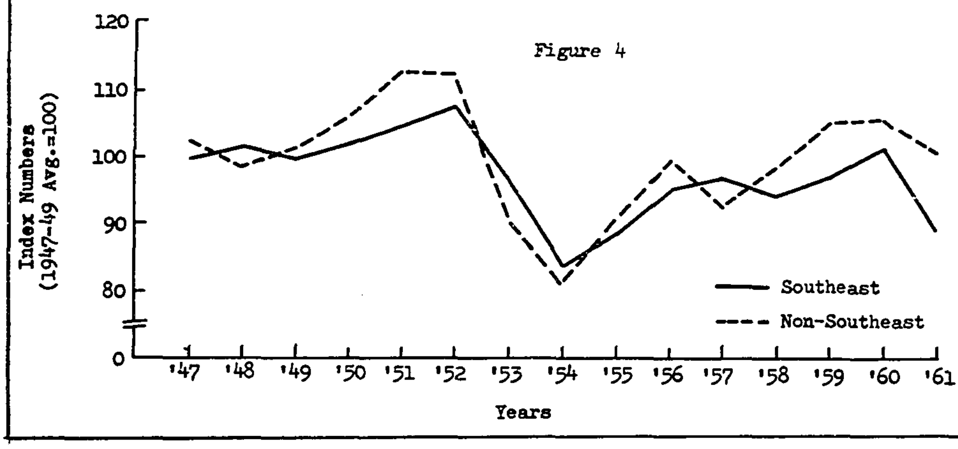
Figure 4
Index Numbers
(1947-49 Avg.=100)
120
110
100
90
80
0
Southeast
Non-Southeast
'47 '48 '49 '50 '51 '52 '53 '54 '55 '56 '57 '58 '59 '60 '61
Years

slaughter (see Figure 6). It follows that the substantial increase in marketings in the Southeast since 1947 has resulted mainly from a decrease in farm slaughter.

Figure 7 indicates that commercial hog slaughter has increased in the Southeast at a faster rate since 1947 than in the non-Southeast. In 1947 the Southeast accounted for about 8 percent of the commercial slaughter in the United States. This increased to almost 10 percent by 1961. Commercial slaughter in the Southeast increased from 4,946,600 head in 1947 to 7,568,300 head in 1961, while at the same time farm slaughter decreased from 3,428,000 head to 1,375,000 head.

The reported data show that approximately three-fourths of the increase in commercial hog slaughter in the Southeast from 1947 through 1961 has resulted from a substantial decrease in farm slaughter and not from an increase in production. Since farm slaughter in 1961 was less than one-fifth of production, then it follows that if commercial slaughter in the Southeast is to increase during the 1960's at a rate similar to that from 1947 through 1961, new sources of slaughter hogs must be found. Some logical possibilities might be (a) a continuing decline in farm slaughter relative to production in the Southeast, (b) an increase in production in the Southeast and (c) an increase in inshipments of live hogs from areas outside the Southeast.

Figure 5. Index numbers of farm hog slaughter in the Southeast and non-Southeast, 1947-61 (40, 43, 47, 48)

Figure 6. Farm hog slaughter as a percent of hog marketings plus farm hog slaughter in the Southeast and non-Southeast, 1947-61 (40, 43, 47, 48)

Figure 7. Index numbers of commercial hog slaughter in the Southeast and non-Southeast, 1947-61 (46)

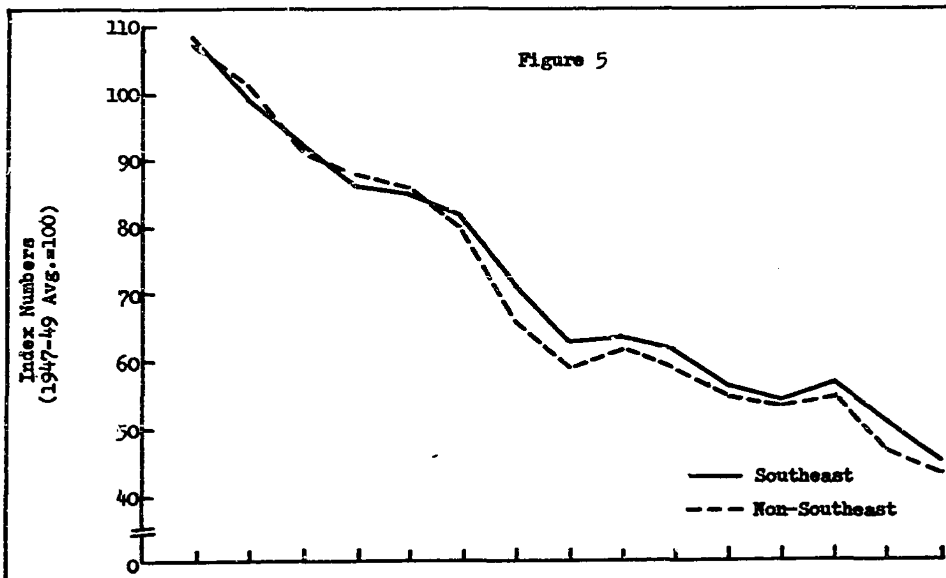
Figure 5
Index Numbers
(1947-49 Avg.=100)
Southeast
Non-Southeast

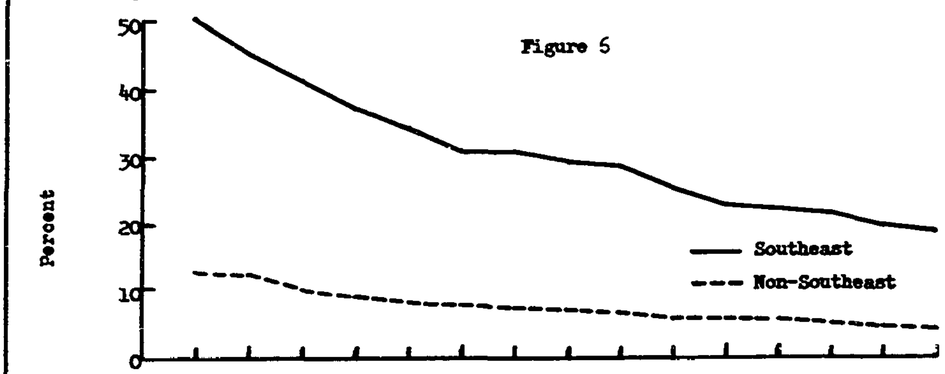
Figure 6
Percent
Southeast
Non-Southeast


Figure 7
Index Numbers
(1947-49 Avg.=100)
Southeast
Non-Southeast
'47 '48 '49 '50 '51 '52 '53 '54 '55 '56 '57 '58 '59 '60 '61
Years

However, the possibility that it will be economically unsound for Southeast commercial hog slaughter to increase between 1960 and 1970 at the same rate as occurred from 1947 through 1961 must not be overlooked.

Pork Consumption

Pork consumption is estimated for 1960 and 1970 by the areas in Figure 1. These estimates and the procedure used for developing them are presented in Appendix B.

In 1960 the total commercial slaughter of hogs in the Southeast was 7,478,500 head (46). Assuming an average weight per head of 210 pounds and a dressing percentage for pork (excluding lard and inedibles) of 58.5 percent (45, 46), this would amount to 918,733,725 pounds of pork. Southeast consumption of commercially slaughtered pork in 1960 is estimated to be 1,701,406,366 pounds. In other words, only about 54 percent of the commercially slaughtered pork consumed in the Southeast in 1960 could have been supplied by Southeast commercial hog slaughter. Southeast consumption of commercially slaughtered pork in 1970 is estimated to be 2,023,063,300 pounds. Therefore, if 1970 commercial hog slaughter in the Southeast is equal to 1960 slaughter this could only supply about 45 percent of the estimated 1970 Southeast consumption of commercially slaughtered pork.

This would mean that the Southeast would be even more deficit in pork in 1970 than it was in 1960.

Slaughter Facilities and Capacities

On March 1, 1960, there were 3,144 Federally inspected, large non-Federally inspected and medium non-Federally inspected livestock slaughter plants in the United States. There were also about 6,500 small non-Federally inspected plants. Commercial slaughter estimates released by the U. S. Department of Agriculture include slaughter from each of these four classes of livestock slaughter plants. In 1959, slaughter in the 3,144 plants accounted for 99 percent of the total commercial livestock slaughter in the United States (41).

The number and location of Federally inspected livestock slaughter plants in 1960 for the 12 Southeast areas are shown in Figure 8. The number of this class of plants has increased from 38 in 1955 to 47 in 1960. In 1955 the percentage of these plants that were slaughtering hogs was 81.6; however, this had dropped to 68.1 percent by 1960. This indicates a trend toward specialization at a level similar to the non-Southeast where 55.4 percent of the Federally inspected plants were slaughtering hogs in 1955 and 47.6 percent in 1960 (refer to Table 1).

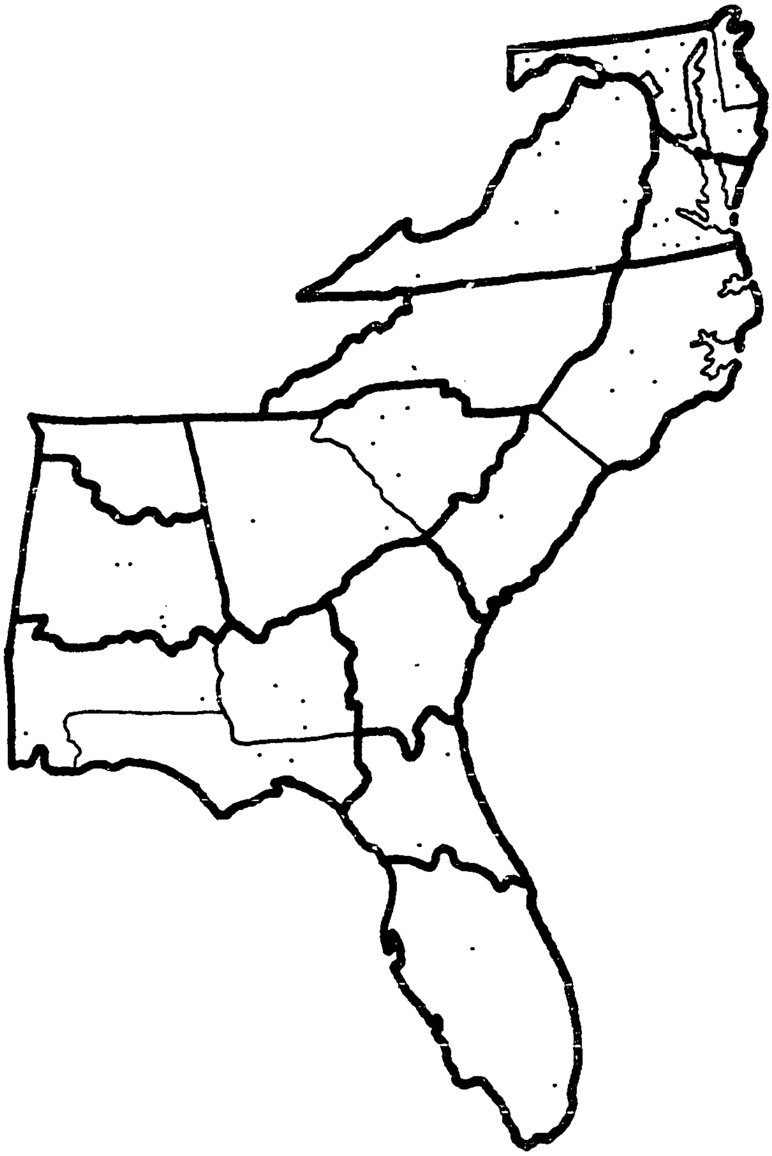

Figure 8. Federally inspected livestock slaughter plants, March 1, 1960 (49).

Table 1. Number of Federally inspected, large non-Federally inspected and medium non-Federally inspected slaughter plants and percent slaughtering hogs, by states in the Southeast, March 1, 1955 and 1960 (41)

State	Number of plants						Percent slaughtering hogs			
	Fed. inspected		Large non-FIS		Medium non-FIS		Fed. inspected		Large & med. non-FIS	
	1955	1960	1955	1960	1955	1960	1955	1960	1955	1960
Del.-Md.[a]	11	10	17	16	39	22	81.8	70.0	71.4	68.4
Va.	9	12	12	11	25	19	88.9	75.0	86.5	90.0
N. Car.	2	3	33	29	65	55	100.0	100.0	91.8	94.0
S. Car.	1	5	11	13	34	32	100.0	60.0	100.0	100.0
Ga.	7	5	33	38	48	56	85.7	80.0	98.8	100.0
Ala.	4	6	11	16	47	34	100.0	66.7	100.0	100.0
Fla.	4	6	26	19	36	29	25.0	33.3	83.9	72.9
Southeast	38	47	143	142	294	247	81.6	68.1	90.8	91.5
Non-Southeast	417	483	809	760	1516	1465	55.4	47.6	76.1	79.2

[a]The District of Columbia is included with Delaware and Maryland.

Figure 9 shows the number and location of large non-Federally inspected livestock slaughter plants in 1960 for the Southeast. The number of these plants remained almost constant from 1955 to 1960, with a loss of one from 143 to 142. Figure 10 indicates the number and location of medium non-Federally inspected livestock slaughter plants in the Southeast on March 1, 1960. The number of these plants dropped from 294 in 1955 to 247 in 1960 or about 16 percent. The percent of large and medium plants combined, that slaughtered hogs in 1960, was about the same as in 1955--a little over 90 percent (refer to Table 1).

Estimates of 1960 commercial hog slaughter capacity for the 22 hog slaughter areas are developed from unpublished data received from the Meat Inspection Division of the U. S. Department of Agriculture and from a general information schedule taken from packing plants in the Southeast by the Southern Regional Livestock Marketing Research Committee (37, 42).* These estimates are presented in Table 2 for both Federally and non-Federally inspected slaughter facilities.

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*Rizek, Robert L., Marketing Econ. Div., Econ. Res. Ser., U. S. Dept. of Agr., Ames, Iowa. Estimates of 1960 commercial hog slaughter capacity. Private communication. 1963.

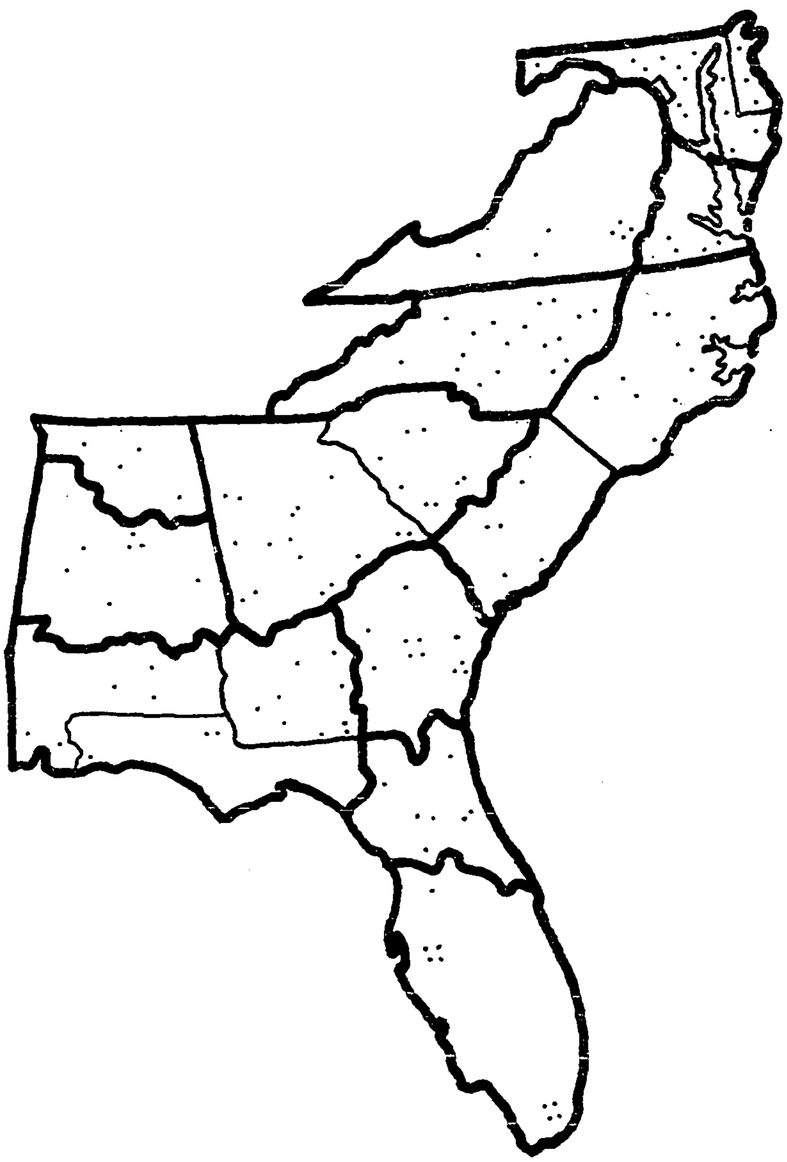

Figure 9. Large non-Federally inspected livestock slaughter plants, March 1, 1960 (49).

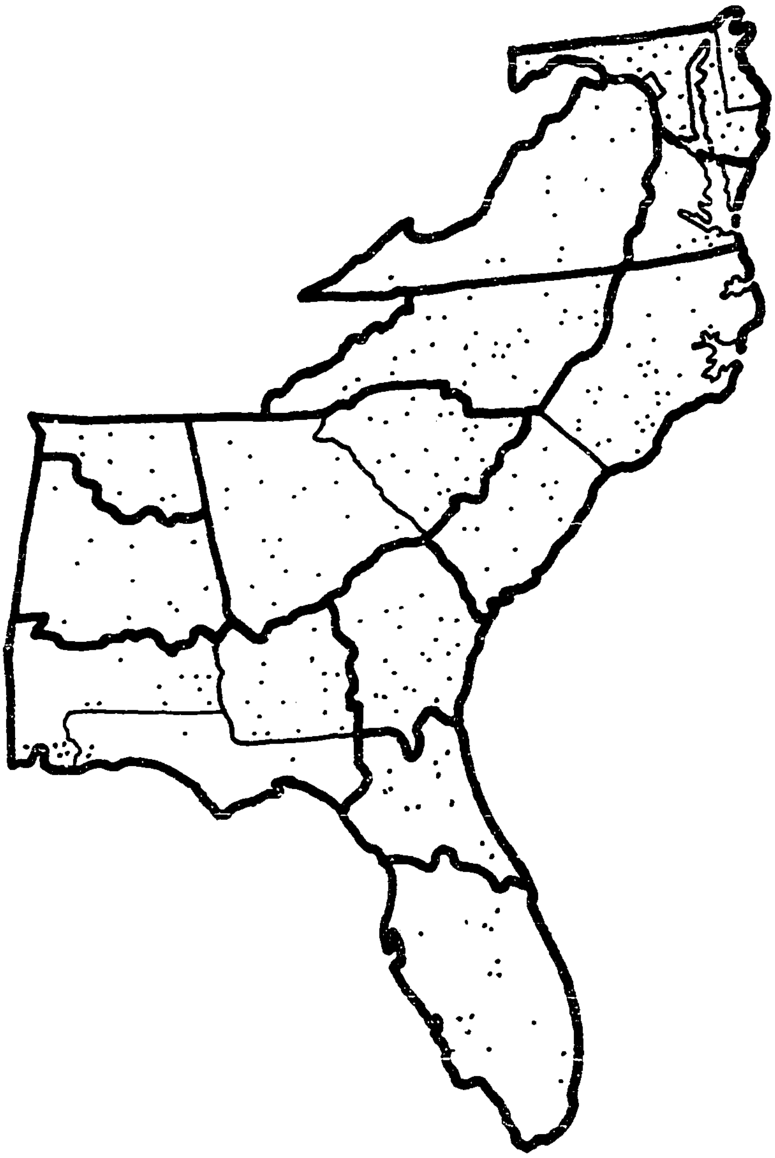

Figure 10. Medium non-Federally inspected slaughter plants, March 1, 1960 (49).

Table 2. Estimated commercial hog slaughter capacity by areas for 1960 (37, 42)[a]

Hog slght. area	FIS plants	Non-FIS plants	Total capacity Head	Live-weight[b]	Dressed weight[c]
	(thousand head)		(thousand)	(million pounds)	
1	1,472	183	1,655	414	242
2	-[d]	-[d]	1,693	423	247
3	-[d]	-[d]	1,272	318	186
4	-[d]	-[d]	429	107	63
5	-[d]	-[d]	1,446	362	212
6	-[d]	-[d]	1,470	368	215
7	-[d]	-[d]	116	29	17
8	-[d]	-[d]	666	167	98
9	-[d]	-[d]	1,707	427	250
10	-[d]	-[d]	422	106	62
11	-[d]	-[d]	589	147	86
12	-[d]	-[d]	170	43	25
13	10,118	2,039	12,157	3,039	1,778
14	none	224	224	56	33
15	1,556	343	1,899	475	278
16	2,955	537	3,492	873	511
17	613	344	957	239	140
18	24,708	5,199	29,907	7,477	4,374
19	39,830	696	39,527	9,882	5,781
20	3,960	1,482	5,442	1,361	796
21	3,772	569	4,341	1,085	635
22	6,578	630	7,208	1,802	1,054
Total			116,789	29,200	17,083

[a]Estimated from rated hourly capacities using a 7.2 hour work day, 5 day work week and 252 day work year.

[b]Computed with a per head weight of 250 pounds.

[c]Computed with a pork (excluding lard and inedibles) dressing percentage of 58.5 percent (45, 46).

[d]Insufficient information available to estimate Federally inspected and non-Federally inspected slaughter capacity separately by areas smaller than a state.

THEORETICAL FRAMEWORK

Northrup (34, p. 128) states, "that a scientifically significant operation is one in which the theoretical concept defines the operation, rather than one in which the denotatively given operation defines the scientific concept." Northrup (34, p. 29) states further that "in the handling of any specific problem certain stages are to be noted. The first stage is concerned with the analysis of this problem. This analysis leads one to the relevant facts to be observed, and these relevant facts in turn suggest the relevant hypothesis. This inquiry, as it proceeds, exhibits at least three major stages: (1) the analysis of the problem which initiates the inquiry, (2) the Baconian inductive observation of the relevant facts to which the analysis of the problem leads one, and (3) the designation of relevant hypotheses suggested by the relevant facts." Northrup also implies a fourth stage to test the hypotheses generated in stage three.

This study has proceeded through the first two stages of scientific inquiry. For the most part, the inductive stage has been accomplished in earlier studies of the livestock-meat economy in the Southeast (11, 12, 37, 39). The formulation of the empirical problem also has been assisted by the earlier studies. The results of these studies are now brought to focus on the problems of a particular segment of the livestock-meat economy in the Southeast that are related to its competitive relationship with other livestock areas in the United States. The first two stages of inquiry point, therefore, to a particular theoretical framework that would serve as a basis for prediction as well as analysis. This theoretical framework is encompassed within the concept of interregional competition.

Interregional Competition

Mighell and Black (31, p. 13) state that, "the economic principle underlying interregional competition is the familiar Principle of Comparative Advantage. It is, in fact, in interregional competition within one national boundary that this principle is exemplified in its pure form." Johnson (22, p. 224) noted further that, "the term 'comparative advantage' as used in agriculture is thus an outgrowth of the analysis by the English classical economists of the reason for and the effects of international trade." The fundamental principles of trade were, "therefore first stated in terms of international trade, which, after all, is only a special case of interregional trade." (31, p. 14)

The first of these principles was that of specialization which is credited to Adam Smith and his generation.

It states that, "Each area tends to produce only a few things -- frequently only one -- and to sell its surplus of these and with the proceeds buy the other things it needs." (31, p. 14) When this principle is applied to individuals, we have Durkheim's (5) concept of division of labor. The gains of specialization result because different areas are, first, endowed with different kinds and amounts of natural resources, and second, certain areas have developed over time specific kinds of equipment and technological advances which further adapt them to the production of a certain commodity or commodities.

The concept of specialization fails, however, to answer the basic question: What commodities will a region produce and what commodities will it purchase? Another generation of economists, notably David Ricardo, first attempted to answer this question with the principle of comparative advantage. As Viner (53, p. 50) states it, "according to classical theory, the gain to a country or region from foreign trade consists in getting indirectly in exchange for those products in which a country has comparative advantage in production (or less comparative disadvantage) more goods, or better goods, than could be produced at home with the same quantity of productive resources...."

One of the first applications of the principle of comparative advantage was presented by Von Thünen (2, pp. 192-

194) and Weber (54), who used this problem in their theory of location. Weber, especially, was concerned with transportation costs and his analysis was conducted for the purpose of finding the geographic point at which sums of costs of transporting raw materials to the factory, and of the finished product to the consuming center, are at a minimum. The transportation model in use today is a mathematical expression of this type of analysis. Because Weber assumed that demand, location of raw materials and the location of markets were given, his partial equilibrium approach is considered inadequate by modern location theorists.

Hoover (20) also emphasized the cost approach in his analyses of location problems. He notes that the more important costs involved in the location of a productive enterprise are "(a) procurement: purchasing and bringing the necessary materials to the site of processing, (b) processing: transforming the materials into more valuable forms (products) and (c) distribution: selling and delivering the products." (20, p. 7) Hoover also brings in the economies of size achieved by larger plants, land values, taxes, wages, organized labor and consumer and producer services.

A further stage in the development of the principle of comparative advantage started with the equilibrium type of analysis by Marshall. Two of the first attempts to tie demand and price in with comparative advantage were by Ohlin (35) and Lösch (29). Ohlin's mutual interdependence theory of interregional trade begins with the general equilibrium type of analysis. Lösch's major contribution was to press forward from Weber's partial equilibrium approach and develop a static general equilibrium approach to location theory. The fact that he only described the concept of the interrelationships of the full general equilibrium need not distract from his contribution. Isard (21) later attempted to combine the cost concepts of Weber and Hoover with the market concepts of Lösch into an even more general theory.

With price taken as given, Black (2, p. 137) then states that, "Each area tends to produce those products for which its ratio of advantage is greatest as compared with other areas, or its ratio of disadvantage is least, up to the point where the land may be needed by some products less advantaged in the area in order to meet the demand for them at such prices as will come to prevail under such circumstances." Black's statement allows for a general supply-demand equilibrium between regions but it does not include the mechanism for it; i.e., price.

Changes occur on the supply side as equipment wears out, resources are depleted, weather conditions vary, new technology is introduced, etc. Likewise, demand changes as consuming centers grow, incomes increase and tastes change. The results of these changes are shifts in the region's composite demand and supply curves for a commodity. These shifts result in a new equilibrium price. The spatial equilibrium model is an algorithm which has been developed for the analytical solution of these kinds of equilibrium problems.

The conceptual framework of the hog-pork industry presented in the introduction is a step toward the general equilibrium approach. However, there currently is no analytical procedure that will economically furnish an equilibrium solution for this conceptual model when supply and demand are functions of price and processing costs are a function of slaughter volume. Considerable work on these types of analytical procedures is being conducted but the development of a completely operational and economical methodology is still forthcoming.

Economic Model

The economic model follows the conceptual framework of Weber and Hoover in that supply and demand are fixed and the critical variables become the transfer costs. The model assumes the supply of hogs available for commercial slaughter, commercial hog slaughter capacity and the demand for commercially slaughtered pork in each geographic area are given. The economic model is based, therefore, on an efficiency criterion, namely, that the location of hog slaughter will occur at those points that minimize the total transportation and marketing bill.

The economic model is not restricted to two sectors or dimensions of the hog-pork industry, nor is it restricted to the three dimensions (slaughter hog production, hog slaughter and pork consumption) included in this study. The major restrictions on the model, other than those mentioned above, are the necessary input-output coefficients and transfer costs for each additional dimension of the industry that is specified. For this reason the feeder pig dimension in the conceptual framework presented earlier is not included in the economic model.

Total pork demand in the Southeast can logically be supplied by three sources: (a) hogs produced and slaughtered in the Southeast, (b) hogs shipped in from outside the Southeast and then slaughtered in the Southeast and (c) pork shipped in from outside the Southeast. The analyses presented in a later section indicate which of these sources should supply the pork consumed in the Southeast. These analyses are conducted under various hypotheses regarding transfer cost. The different models furnish equilibrium solutions for the spatial distribution of hogs and pork simultaneously. The solutions also indicate the areas in which excess slaughter capacity would occur when total transfer costs are minimized.

The theory of comparative advantage helps explain why the Southeast produces some feed grains and hogs at the same time that the bulk of production of both occurs in the Midwest. Facilities for slaughtering hogs thus have developed in the Southeast. However, due to the limited supply of local slaughter hogs, these plants find it necessary occasionally to supplement their supply of local hogs with inshipments from areas outside the Southeast. It may at first appear economically unsound to procure hogs, at various times during the year, from areas outside the Southeast for slaughter in the Southeast. But if Southeast hog supplies were adequate to meet the requirements of Southeast slaughter facilities, say 75 percent of the time, then it is logical that the total cost to the Southeast pork consumer would be less when 25 percent of the hogs slaughtered in the Southeast are shipped in from outside the Southeast than if all hogs produced in the Southeast were shipped outside the Southeast for slaughter and then returned as pork for consumption in the Southeast. This assumes, of course, that the cost of slaughtering hogs is not substantially less outside the Southeast than it is within the Southeast.

The next section presents the procedure used to achieve solutions to the various economic models. A sample problem using the generalized distribution problem is presented also.

ANALYTICAL PROCEDURES

Hitchcock (19) first presented the transportation problem as a tool for solving the optimum distribution of a homogeneous product from several sources to numerous localities. Dantzig and Koopmans (4, 28) have reported on the application of the simplex method to this same transportation problem.

From the transportation problem has evolved a more sophisticated algorithm, the spatial equilibrium model, which allows the quantities available for shipment at the surplus points, as well as the quantities demanded at the deficit points, to be a function of price. Samuelson (38), Baumol (1) and Enke (10) have reported on the spatial equilibrium model.

During the past ten years, numerous agricultural economists have employed the transportation problem and its partner, the spatial equilibrium model, in their research. Fox (13) used the spatial equilibrium model in analyzing the United States livestock-feed economy. Judge (23) applied the same model to eggs, and Henry and Bishop (18) used the transportation problem in an analysis of interregional competition in the broiler industry.

Judge and Wallace (24, 25, 26) have presented a series of three spatial price equilibrium analyses of the United

States livestock economy. The third in this series (26) exhibits spatial price equilibrium models of the pork marketing system. These models consist of both 21- and 29-region delineations for the United States. Also, the Northcentral Regional Livestock Marketing Research Committee is currently conducting a study of the United States livestock-meat economy under project NCM-25. The Northcentral study uses a 26-region delineation of the United States.

The regions employed by Judge and Wallace, as well as the Northcentral Committee, are either states or combinations of states. In this study the same can be said for the non-Southeast areas. However, the substate areas chosen for the Southeast are not restricted by state boundaries, but instead are delineated across state lines so as to more accurately define homogeneous production, slaughter and consumption areas (see Figure 1). Also, the actual Southeast shipment data for 1960 that are used for comparative analysis are presented by these same substate areas (refer to Appendix C).

The transportation problem, in its usual form, is defined by two dimensions--origins and destinations (27). However, the distribution problem in this study has three dimensions--hog supply areas, hog slaughter areas and pork demand areas. The more common algorithms used to solve the transportation problem, e.g., stepping stone method, MODI method, and inspection method (17), are inadequate to find the desired solution to the multi-dimensional or generalized distribution problem.

Dwyer and Galler (6, 7, 8, 9, 16) have developed a procedure for solving the generalized distribution problem. This procedure is based on the method of reduced matrices. Using this procedure, Galler (14, 15) has written an IBM 704 program for the solution of the generalized distribution problem. This procedure and IBM 704 program are used to find a solution to the three-dimensional generalized distribution model employed in this study.

The General Problem

The formal statement of the generalized distribution problem is similar to that of the transportation problem (19). The major difference is the "dimensional" feature, i.e., the generalized distribution problem considers the two transportation problem dimensions--origins and destinations--plus an infinite number of dimensions (processing, warehousing, etc.) between the origins and destinations.

The statement of the generalized distribution problem and the illustration of the method of reduced matrices for solving it will be limited to three dimensions. Generalization to a n-dimensional problem is obvious.

Let c_{ijk} be the known cost of transferring a unit item at origin i to destination k via intermediate point j. The units available at i for shipment are denoted by e_i, the units demanded at k by g_k and the capacity of the intermediate point by f_j. The sum of the frequencies for the areas or plants in each dimension must meet the restriction

$$\sum_{i=1}^{m_1} e_i = \sum_{j=1}^{m_2} f_j = \sum_{k=1}^{m_3} g_k = N \qquad \text{(Eq. 1)}$$

where m_1, m_2 and m_3 are the respective numbers of origins, intermediate points and destinations and N is the sum of the frequencies for any dimension.*

Now let x_{ijk} be the units which are shipped via route ijk, under the restrictions that

$$x_{ijk} \geq 0, \qquad \text{(Eq. 2)}$$

$$\sum_{j=1}^{m_2} \sum_{k=1}^{m_3} x_{ijk} = e_i, \qquad \text{(Eq. 3)}$$

*In practical cases this will rarely occur; however, the formal restriction can easily be satisfied by including "dummy" areas and frequencies, and by using a common high transfer cost for all transfers to or from the "dummy" areas.

$$\sum_{i=1}^{m_1} \sum_{k=1}^{m_3} x_{ijk} = f_j \qquad \text{(Eq. 4)}$$

and

$$\sum_{i=1}^{m_1} \sum_{j=1}^{m_2} x_{ijk} = g_k \qquad \text{(Eq. 5)}$$

so that we minimize the total transfer cost

$$T = \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} \sum_{k=1}^{m_3} x_{ijk} c_{ijk}. \qquad \text{(Eq. 6)}$$

The Method of Reduced Matrices

The method most frequently used to solve the two-dimensional transportation problem is the simplex method (4). This method first finds a feasible solution which satisfies restrictions similar to Equations 1 through 5 and then improves this solution until the total transportation cost is minimized.

Two major difficulties arise when the simplex method is used to solve a n-dimensional distribution problem: (a) finding a good feasible solution is much more complicated and (b) the number of restriction equations increases rapidly. The method of reduced matrices eliminates the first difficulty and lessens the second.

The method of reduced matrices is based on the property of the cost matrix (an $m_1 \times m_2 \times m_3$ matrix of c_{ijk} elements) which allows the subtraction of a constant from all elements of some row i, column j or layer k without affecting the final allocation of shipments (x_{ijk}). The optimal value of Equation 6 is affected but not the magnitude of the shipments. A necessary condition for the existence of a solution is the existence of constants u_i, v_j and w_k (sometimes referred to as shadow prices or indirect costs of not using a route) such that

$$c_{ijk} - u_i - v_j - w_k = 0 \text{ whenever } x_{ijk} \neq 0 \qquad \text{(Eq. 7)}$$

and

$$c_{ijk} - u_i - v_j - w_k \geq 0 \text{ in any case.} \qquad \text{(Eq. 8)}$$

If the minimal cost element of each row (column, layer) is subtracted from each element of that row (column, layer), Equation 8 is satisfied. The resulting cost matrix is said to be reduced and Equation 7 is satisfied at the zero matrix locations produced by these subtractions. These zero cost elements provide a set of matrix locations for a possible allocation of shipments. Also, each subtraction from the cost matrix increases the value of the bounding sum

$$S = \sum_{i=1}^{m_1} u_i e_i + \sum_{j=1}^{m_2} v_j f_j + \sum_{k=1}^{m_3} w_k g_k. \qquad \text{(Eq. 9)}$$

From Equations 3, 4, 5 and 8 it can be shown that

$$S = \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} \sum_{k=1}^{m_3} (u_i + v_j + w_k) x_{ijk} \leq \sum_{i=1}^{m_1} \sum_{j=1}^{m_2} \sum_{k=1}^{m_3} c_{ijk} x_{ijk} = T \qquad \text{(Eq. 10)}$$

and the inequality becomes an equality when the values of u_i, v_j and w_k are such that Equations 3, 4, 5, 7 and 8 are true. Then we have

$$S = T. \qquad \text{(Eq. 11)}$$

After the marginal transformations (subtraction of the minimal cost element in each row, column and layer), one may attempt to find a solution by putting $x_{ijk} = 0$ wherever $c_{ijk} > 0$, but this usually leads to a set of equations for Equations 3, 4 and 5 with $x_{ijk} \neq 0$ which is inconsistent and thus a nonacceptable solution. It is at this point that the method of reduced matrices performs its most important function. The method uses the information gained by the marginal transformation to perform an additional general transformation(s) on the cost matrix which produces another zero(s) in the matrix without losing those zeros produced by the marginal transformations. This general transformation increases the magnitude of S and after a small number of iterations S = T. At this point a solution is available for Equations 3, 4 and 5 that also satisfies Equations 7 and 8 but not necessarily Equation 2, since one or more of the x_{ijk} values may be negative. These negative values must be removed in order to derive an acceptable solution. Another series of transformations eliminates these negative x_{ijk} values, increases the value of S and the zero cost elements associated with these negative x_{ijk} values become non-zero. The resulting cost matrix is said to be <u>finally reduced</u>. At this point Equations 2 through 5, 7, 8 and 11 are all satisfied.

An important fact which is significant, not only for the method of reduced matrices but also for any other method such as the simplex method, is that the relation

$$\max S = \min T \qquad \text{(Eq. 12)}$$

is obtained with positive rational x_{ijk}. After the negative x_{ijk} have been eliminated, there is no way by which the cost matrix may be reduced further (thus increasing S) without violating Equations 7 or 8 and an optimal (min T) solution has been achieved.

An Illustration

The sample problem presented here to illustrate the method of reduced matrices is three-dimensional. It is identical to the model used in this study except for the small number of areas in each dimension. The three dimensions in the sample problem consist of (a) four hog supply areas with supplies, e_i, (b) two hog slaughter areas with capacities, f_j, and (c) three pork demand areas with demands, g_k. The frequencies and shipments are expressed in hundredweight pork equivalent units.

In order to satisfy the formal requirements of the procedure, the sum of the frequencies for the areas or plants in each dimension is

$$\sum_{i=1}^{4} e_i = \sum_{j=1}^{2} f_j = \sum_{k=1}^{3} g_k = 100. \qquad \text{(Eq. 13)}$$

The transfer cost matrix is shown in Table 3. The frequencies for the areas or plants are listed on the borders of the cost matrix, e.g., the transfer cost element c_{223} = 28 and the frequencies associated with it are $e_2 = 32$, $f_2 = 60$ and $g_3 = 33$. The transfer costs are expressed in dollars per hundredweight pork equivalent units.

Table 3. Matrix of transfer cost elements, c_{ijk}, and related frequencies

e_i \ f_j / g_k	40			60		
	25	42	33	25	42	33
18	8	12	19	6	11	15
32	22	17	14	27	23	28
21	26	21	18	9	7	24
29	20	16	25	10	13	5

Marginal transformations

The first step in the procedure is called the marginal transformations. It is accomplished by subtracting the minimum cost element in each row (hog supply areas), each column (hog slaughter areas) and each layer (pork demand areas) from the other cost elements in their respective rows, columns or layers. The resulting cost matrix is said to be reduced. There is nothing unique about this step, and thus this marginal transformation process for an n-dimensional problem can be formed in n! different orders. For machine problems Galler and Dwyer have found that the subtractions in the order layers, columns, rows simplify the procedure (16, p. 58). A maximum of n steps is required for these marginal transformations.

The minimum cost elements in the three layers in Table 3 are 6, 7 and 5. The subtraction of these elements from their respective layers results in the cost matrix shown in Table 4.

Table 4. Matrix of transfer cost elements after layer subtractions

e_i \ f_j / g_k	40			60		
	25	42	33	25	42	33
18	2	5	14	0	4	10
32	16	10	9	21	16	23
21	20	14	13	3	0	19
29	14	9	20	4	6	0

The minimum cost elements in the two columns, after the layer subtractions, are 2 and 0. The result of these subtractions is shown in Table 5.

The minimum cost elements in the four rows, after the layer and column subtractions, are 0, 7, 0 and 0. The result of these subtractions is called a <u>reduced</u> matrix and is shown in Table 6.

Table 5. Matrix of transfer cost elements after column subtractions

e_i \ f_j	40			60		
e_i \ g_k	25	42	33	25	42	33
18	0	3	12	0	4	10
32	14	8	7	21	16	23
21	18	12	11	3	0	19
29	12	7	18	4	6	0

Table 6. Matrix of transfer cost elements after row subtractions

e_i \ f_j	40			60		
e_i \ g_k	25	42	33	25	42	33
18	0	3	12	0	4	10
32	7	1	0	14	9	16
21	18	12	11	3	0	19
29	12	7	18	4	6	0

General transformations

The general transformation is the most important feature of the method of reduced matrices. Through a succession of transformations, this step generates a sufficient number of zero cost elements at specific locations in the cost matrix so as to allow assignment of values, x_{ijk}, at these locations which will minimize the total cost of distribution.

The sample problem at hand can be expressed in two dimensions by the following system of equations

$$Ax = y \qquad \text{(Eq. 14)}$$

where $A = (a_{uv})$ is a p x q matrix with $p \leq q$ (here $p = m_1 + m_2 + m_3 = 9$ and $q = m_1 m_2 m_3 = 24$)

$x = (x_1, \ldots, x_q)$ is a q x 1 vector with each x_v being a non-negative shipment (x_{ijk}) and

$y = (y_1, \ldots, y_p)$ is a p x 1 vector with each y_u being a non-negative frequency (e_i, f_j or g_k) for a row, column or layer.

The matrix A for the sample problem is shown in Table 7. In the left-most column labeled "equation," the first digit is the dimension and the second digit is the area, e.g., the equation for layer 2 (pork demand area 2) is listed as 32. Each column of matrix A is headed by one of the feasible routes. The elements a_{uv} are either "0" or "1" and the zeros are suppressed. For a specific row in matrix A, the dimension number indicates that a "1" should appear under those routes which have a subscript equal to the area

Table 7. Matrix A and computations for the general transformations

Matrix A

Dimen. (Equation)	Area (Route →)	111	112	113	121	122	123	211	212	213	221	222	223	311	312	313	321	322	323	411	412	413	421	422	423
1	i=1	1	1	1	1	1	1																		
1	i=2							1	1	1	1	1	1												
1	i=3													1	1	1	1	1	1						
1	i=4																			1	1	1	1	1	1
2	j=1	1	1	1				1	1	1				1	1	1				1	1	1			
2	j=2				1	1	1				1	1	1				1	1	1				1	1	1
3	k=1	1			1			1			1			1			1			1			1		
3	k=2		1			1			1			1			1			1			1			1	
3	k=3			1			1			1			1			1			1			1			1

Computations for the General Transformations

	111	112	113	121	122	123	211	212	213	221	222	223	311	312	313	321	322	323	411	412	413	421	422	423
c_{ijk}	0	3	12	0	4	10	7	1	0	14	9	16	18	12	11	3	0	19	12	7	18	4	6	0
r_{4v}	0	0	-1	0	0	-1	1	1	0	1	1	0	0	0	-1	0	0	-1	1	1	0	1	1	0
θr_{4v}	0	0	-1	0	0	-1	1	1	0	1	1	0	0	0	-1	0	0	-1	1	1	0	1	1	0
c'_{ijk}	0	3	13	0	4	11	6	0	0	13	8	16	18	12	12	3	0	20	11	6	18	3	5	0
r_{7v}	0	-1	-1	0	-1	-1	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0	1	0	0
θr_{7v}	0	-3	-3	0	-3	-3	3	0	0	3	0	0	3	0	0	3	0	0	3	0	0	3	0	0
c''_{ijk}	0	6	16	0	7	14	3	0	0	10	8	16	15	12	12	0	0	20	8	6	18	0	5	0

designation for that row. For example, in the eighth row (equation 32) the dimension number 3 indicates that a "1" should appear wherever the third subscript of the routes is equal to 2.

All the routes in matrix A with $c_{ijk} = 0$ after the marginal transformations are defined as a set Z. These routes are 111, 121, 213, 322 and 423 (refer to Table 6).

Now consider the system of equations

$$Bx = y \qquad \text{(Eq. 15)}$$

where $B = (b_{ut})$ is a p x z matrix of coefficients from matrix A for those routes in set Z. Therefore, matrix B contains the qth column of matrix A if and only if $c_{ijk} = 0$ for that column. The system Bx = y is shown in Table 8.

Table 8. System Bx = y

Equation: Dimen.	Area	Route: 111	121	213	322	423	y
1	i=1	1	1				18
1	i=2			1			32
1	i=3				1		21
1	i=4					1	29
2	j=1	1		1			40
2	j=2		1		1	1	60
3	k=1	1	1				25
3	k=2				1		42
3	k=3			1		1	33

By a series of row transformations, matrix B may be reduced to a matrix $D = (d_{ut})$ in echelon form* which is not necessarily unique.**

Then

$$D = \pi B \qquad \text{(Eq. 16)}$$

where $\pi = (\pi_{uu})$ is the result of applying the same row transformation used on the system $Bx = y$ (in the same order) to a p x p identity matrix I.

By performing the row transformations 32 - 13, 31 - 11, 33 - 12, 22 - 14, 14 - 33, 22 - 13, 21 - 12, 11 - 21 and 11 - 22 to matrix B in Table 8, matrix D in Table 9 is generated in echelon form.

The same row transformations are then performed on a 9 x 9 identity matrix and the result is the matrix π in Table 10.

---

*Any matrix M (with m rows) can be reduced to the echelon form N by elementary row operations where N is a matrix with the following characteristics: (a) r (rank) non-zero rows, (b) m-r zero rows, (c) in each non-zero row the first non-zero element is a "1," (d) these "1's" will be in columns $c_1, c_2, \ldots, c_r$ where c_1 is not necessarily the first column in matrix N, and (e) in each column c_i the only non-zero element will be the element "1" in the ith row.

**The same transformations are performed on the y vector of the system $Bx = y$. Hereafter any reference to the matrices B and D will include the y vector.

Table 9. Matrix D

Equation: Dimen.	Area	Route 111	121	213	322	423	y'
1	i=1						0
1	i=2			1			32
1	i=3				1		21
→ 1	i=4						28
2	j=1	1					8
2	j=2		1				10
→ 3	k=1						7
→ 3	k=2						21
3	k=3					1	1

Table 10. Transformation matrix π

Equation: Dimen.	Area	1	2	3	4	5	6	7	8	9
1	i=1	1	1	1	1	-1	-1			
1	i=2		1							
1	i=3			1						
1	i=4		1		1					-1
2	j=1		-1			1				
2	j=2			-1	-1					
3	k=1	-1						1		
3	k=2			-1					1	
3	k=3		-1							1

By definition an <u>inconsistent row</u> occurs when a specific row, $d_{u_o t}$, in matrix D contains only zeros while $y' \neq 0$. The amount of inconsistency is indicated by the magnitude of y'.

The existence of inconsistent rows shows that a solution of the desired form is impossible until set Z is enlarged. In order to decide which route to add to set Z, we define θ by the equation

$$\theta = \min \frac{c_{ijk}}{r_{u_o v}} \qquad \text{(Eq. 17)}$$

under the restrictions that the c_{ijk} elements for the routes contained in set Z and all $r_{u_o v} \leq 0$ be omitted, and where

$$r_{u_o v} = \sum_{u=1}^{p} \pi_{u_o u} a_{uv}. \qquad \text{(Eq. 18)}$$

In Table 9 three inconsistent rows are evident. The most inconsistent row is equation 14 with $y' = 28$. Equation 14 (the fourth row) of the π matrix, π_{4u}, is multiplied times matrix A in Table 7 according to Equation 18. This generates 24 r_{4v} terms and these are recorded below matrix A in Table 7. The c_{ijk} elements in Table 6 are also recorded in Table 7 just above the r_{4v} terms, thus forming the ratio θ. All c_{ijk} elements corresponding to routes contained in set Z and all $r_{4v} \leq 0$ are eliminated. The minimum θ =

$\frac{c_{212}}{1} = 1$ occurs for column (route) 212. The transformation is now defined as

$$c'_{ijk} = c_{ijk} - \theta r_{u_o v}. \qquad \text{(Eq. 19)}$$

The 24 θr_{4v} terms are recorded in Table 7 just below the r_{4v} terms and the c'_{ijk} elements are recorded below the θr_{4v} terms. The first general transformation is now complete and the cost matrix containing the c'_{ijk} elements is shown in Table 11.

Table 11. Matrix of cost elements, c'_{ijk}, after first general transformation

e_i \ f_j / g_k	40			60		
	25	42	33	25	42	33
18	0	3	13	0	4	11
32	6	0	0	13	11	16
21	18	12	12	3	0	20
29	11	6	18	3	5	0

Each general transformation process transforms <u>at least</u> one inconsistent row (equation) of matrix D to a consistent row. A matrix D with h inconsistent equations is reduced to a consistent set of equations in most h general transformations.

The second general transformation is initiated by adding 212 to set Z to form set Z'. The route 212 is also added to the matrix B to form a new system of equations

$$B'x = y \qquad \text{(Eq. 20)}$$

where matrix B' contains the columns of matrix B with any new columns (in this case 212) added to the right (refer to Table 12).

The matrix B' is reduced to an echelon matrix D' (thus generating a transformation matrix π') by first applying the same row transformations (in the same order) used to reduce matrix B to matrix D, followed by three additional row transformations: 33 + 14, 12 - 32 and 32 - 14. The resulting matrices D' and π' are shown in Tables 13 and 14.

Table 12. System B'x = y

Equation: Dimen.	Area	111	121	213	322	423	212	y
1	i=1	1	1					18
1	i=2			1			1	32
1	i=3				1			21
1	i=4					1		29
2	j=1	1		1			1	40
2	j=2		1		1	1		60
3	k=1	1	1					25
3	k=2				1		1	42
3	k=3			1		1		33

Table 13. Matrix D'

Equation: Dimen.	Equation: Area	Route: 111	121	213	322	423	212	y'
1	i=1							0
1	i=2			1				11
1	i=3				1			21
1	i=4						1	28
2	j=1	1						8
2	j=2		1					10
→ 3	k=1							7
→ 3	k=2							-7
3	k=3					1		29

Table 14. Transformation matrix π'

Equations: Dimen.	Equations: Area									
1	i=1	1	1	1	1	-1	-1			
1	i=2		1	1					-1	
1	i=3			1						
1	i=4		1		1					
2	j=1		-1			1				
2	j=2			-1	-1					
3	k=1	-1						1		
3	k=2		-1	-1	-1				1	
3	k=3				1					1

Two inconsistent equations, 31 and 32, are evident in matrix D'. The most inconsistent is equation 31 with y' = 7. The corresponding seventh row of the matrix π' is used to perform the computation defined by Equation 18. The resulting 24 r_{7v} terms are recorded below the c'_{ijk} elements in Table 7. The θr_{7v} terms and the c''_{ijk} elements are computed according to Equation 19 and recorded in Table 7. The c'_{ijk} elements corresponding to the routes contained in set Z' and all $r_{7v} \leq 0$ are eliminated. The minimum $\theta = \frac{c_{421}}{1} = 3$ occurs for column (route) 421. (In this example column 321 also generates a $\theta = 3$, indicating an equally acceptable route.) The second general transformation is now complete and the cost matrix containing the c''_{ijk} elements is shown in Table 15.

Table 15. Matrix of cost elements, c''_{ijk}, after the second general transformation

e_i \ f_j / g_k	40			60		
	25	42	33	25	42	33
18	0	6	16	0	7	14
32	3	0	0	10	8	16
21	15	12	12	0	0	20
29	8	6	18	0	5	0

The third general transformation is initiated by adding route 421 to set Z' to form set Z". The column 421 is also added to matrix B' to form a new system of equations

$$B''x = y \qquad \text{(Eq. 21)}$$

where matrix B" contains the columns of matrix B' with column 421 added to the right (refer to Table 16).

The matrix B" is reduced to an echelon matrix D" (thus generating a transformation matrix π'') by first applying the same row transformations (in the same order) used to reduce matrix B' to matrix D', followed by three additional row transformations: 14 - 31, 33 - 31 and 32 + 31. The resulting matrices D" and π'' are shown in Tables 17 and 18.

Table 16. System $B''x = y$

Equation: Dimen.	Equation: Area	Route: 111	121	213	322	423	212	421	y
1	i=1	1	1						18
1	i=2			1			1		32
1	i=3				1				21
1	i=4					1		1	29
2	j=1	1		1			1		40
2	j=2		1		1	1		1	60
3	k=1	1	1					1	25
3	k=2				1		1		42
3	k=3			1		1			33

Table 17. Matrix D″

Equation: Dimen.	Area	111	121	213	322	423	212	421	y″
1	i=1								0
1	i=2			1					11
1	i=3				1				21
1	i=4						1		21
2	j=1	1							8
2	j-2		1						10
3	k=1							1	7
3	k=2								0
3	k=3					1			22

Table 18. Transformation matrix π''

Equation: Dimen.	Area									
1	i=1	1	1	1	1	-1	-1			
1	i=2		1	1					-1	
1	i=3			1						
1	i=4	1	1		1			-1		
2	j=1		-1			1				
2	j=2			-1	-1					
3	k=1	-1						1		
3	k=2	-1	-1	-1	-1			1	1	
3	k=3	1			1			-1		1

No inconsistent equations are evident in matrix D" and all y" values are zero or positive. Therefore, the cost matrix containing the c''_{ijk} elements (refer to Table 15) is said to be <u>finally reduced</u>.

The shipments, x_{ijk}, and route assignments for the final solution shown in Table 19 are taken from matrix D" in Table 17. The shipments are listed under the y" column and the routes are indicated by the "l's."

Table 19. Final solution; elements in the matrix are x_{ijk} shipments

e_i \ f_j	40			60		
e_i \ g_k	25	42	33	25	42	33
18	8			10		
32		21	11			
21				Par[a]	21	
29				7		22

[a]Par for route 321 in the final solution is an abbreviation for the word parameter, which in this case means that this route is an equally acceptable route but is not used. It also indicates that the final shipment pattern in Table 19 is not unique even though T is minimized. If route 321 is included in matrix B" instead of route 421, an equally acceptable final solution is produced with T still minimized at 962 dollars.

The minimum total transfer cost is

$$\min T = (8)(8) + (10)(6) + (21)(17) + (11)(14) + (21)(7) + (7)(10) + (22)(5) = 962 \text{ dollars.} \quad \text{(Eq. 22)}$$

This is the same solution achieved on the IBM 704 in approximately 90 seconds of program reading time (on-line) and 50 seconds of computing time.

<u>Elimination of negative solutions</u>

In matrix D' the y' value for equation 32 is -7. This negative y' value was eliminated in the matrix D". It is entirely possible, however, for negative values to occur in the transformed y vector of a D matrix that has no inconsistent equations. Negative values in the transformed y vector provide an algebraic solution but they do not meet the restriction that all $x_{ijk} \geq 0$.

These negative values in the transformed y vector can be eliminated by a succession of row transformations similar to those used for the general transformation (16, pp. 61-62).

<u>Adjustments of fractional solutions</u>

The attainment of a positive solution (all positive values in the transformed y vector of a D matrix with no inconsistent equations), either fractional or integral, indicates the completion of a method of reduced matrices. It is entirely possible that an acceptable solution must be integral, e.g., units being shipped are tractors. In order to handle such situations, Galler and Dwyer have developed a combinatorial procedure which uses the transformation matrix π (16, pp. 62-63). This procedure has been incorporated into the IBM programs now available.

Computer Programs

The method of reduced matrices has been programmed for high-speed electronic computers. One IBM 704 program written by Galler (15) features the exact solution just illustrated. Another IBM 704 program, also written by Galler (14), is based on the calculation of weighted deviates of the elements of the cost matrix which results in an approximate solution to the generalized distribution problem. The approximate solution can be used in place of the exact solution when the latter would require an excessive amount of computer time or when the available data are also approximate. This program has been shown to be about 98 percent efficient (9, p. 313). Galler has recently revised both of these IBM 704 programs for use on the IBM 7090.*

The size of the problems that can be solved by these programs is limited by

(a) $$P = \sum_{i=1}^{n} m_i \leq 900$$

where m_i denotes the number of areas in a dimension,

(b) $Q = m_1 m_2, \ldots, m_n \leq 900{,}000$ for the exact solution and $\leq 60{,}000$ for the approximate solution,

(c) $n \leq 20$, and

(d) the cost elements c_{ijk} must be integers less than 10,000.

Computing time for the exact solution is approximately $\frac{P^2}{100}$ minutes.**

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*Galler, Bernard A., Computing Center, University of Michigan, Ann Arbor, Michigan. Information on available computer programs. Private communication. 1962.

**This is Galler's method for estimating computing time. Several factors affect computing time and in many cases this method is not satisfactory, e.g., the 23x22x23 problems solved for the 1960 models in this study each required 2 hours and 20 minutes computing time for the exact solution on an IBM 704, while the 23x22x23 problem solved for the first 1970 model in this study required only 1 hour and 23 minutes computing time.

ANALYSIS OF HOG-PORK DISTRIBUTION

The economic model used for the empirical analysis is basically the same as the sample problem solved in the preceding section. However, additional areas have been added to each dimension to correspond with the areas in Figure 1. The exact solution procedure of the reduced matrices method is used to secure solutions to the 1960 and 1970 models in this section.

General Assumptions of the Model

The generalized distribution model of the hog-pork industry used in this study is based on several assumptions. They are:

(a) The number of slaughter hogs available in each hog supply area for shipment to hog slaughter areas for commercial slaughter is known.

(b) The commercial hog slaughter capacity of each hog slaughter area is known.

(c) The amount of commercially slaughtered pork required by each pork demand area is known.

(d) The cost of transferring product flows from origins to destinations via intermediate slaughtering points is known.

(e) All shipments of hogs (in pork units) and pork are assumed to be homogeneous.

(f) The hog-pork industry is assumed to operate within a framework of perfect competition.

(g) There are no interarea constraints, other than those specified above, on product flows.

The data on actual Southeast pork shipments in 1960 include the restriction that only meat from Federally inspected plants can be shipped across state lines (refer to Appendix C). Five of the areas delineated in Figure 1 had no Federally inspected hog slaughter in 1960; i.e., areas 4, 7, 10, 11 and 12. Therefore, the actual movements show no pork shipments out of these areas.

For all solutions of the model, the assumption is made that all pork shipped by packers in the hog slaughter areas is Federally inspected and thus may be shipped to any pork demand area. This falls under assumption (g) above and allows for insights into the advantages that Federal inspection may provide to those areas that currently have no Federally inspected hog slaughter.

Least-Cost Hog-Pork Distribution, 1960

The model is first solved using 1960 annual estimates of hogs sold for commercial slaughter, e_i, commercial hog slaughter capacities, f_j, and commercially slaughtered pork consumption, g_k.* Next the model is solved using 1960 quarterly estimates of the same three inputs for the 22 areas delineated in Figure 1.

A "dummy" hog supply and pork demand area, 23, is included in order to compensate for the excess hog slaughter capacity. This permits the sum of the input frequencies, N, to be equal in each dimension, i.e.,

$$N = \sum_{i=1}^{23} e_i = \sum_{j=1}^{22} f_j = \sum_{k=1}^{23} g_k.$$

Input estimates

The 1960 production of hogs sold for commercial slaughter in the 22 areas combined is estimated to be 10,917 million pounds (in pork units). The 1960 consumption of commercially slaughtered pork for the 22 areas combined is estimated to be 10,830 million pounds. In order to meet the formal restrictions of the model it is assumed that all hogs produced in 1960 are consumed in 1960. Therefore, the estimate of pork consumption for each area is expanded by .8 of 1 percent (the amount by which estimated 1960 hog production exceeds estimated 1960 pork consumption for the 22 areas

*Refer to Appendixes A and B for the procedures used to estimate e_i and g_k for each area. Hog slaughter capacities, f_j, are presented in an earlier section.

combined). The original estimates of pork consumption and the adjusted estimates are given for each area in Table 20.

For the quarterly solutions, the 1960 annual estimates of hog production and pork consumption are disaggregated by areas to quarterly estimates according to the procedures outlined in Appendixes A and B. After the annual estimates are disaggregated to quarterly estimates, production and consumption for each quarter for the 22 areas combined are not equal. This results because hog production and pork consumption do not vary seasonally in the same manner.

Again to meet the formal requirements of the model, consumption is adjusted upward during the first two quarters and down during the last two quarters for each area separately. This in effect forces the pork consumption estimates into the seasonal pattern for hog production. It means that consumption is adjusted from an estimate of retail consumption to an estimate of wholesale consumption. Since all hogs sold for commercial slaughter in a given quarter must be slaughtered during that quarter, the model now allows all pork emanating from commercial slaughter to be distributed to the pork demand areas in the same quarter. As a result, during the first two quarters of 1960 approximately 6 percent more pork will be shipped to the demand areas than is actually consumed. During the second quarter about 4 1/2 percent extra will be shipped. However, during

Table 20. Area retail estimates of commercially slaughtered pork consumption, g_k, both annual and quarterly for 1960

Area	Original annual estimate	Adjusted annual estimate[b]	Quarterly retail estimates[a] Jan.-Mar.	Apr.-June	July-Sept.	Oct.-Dec.
	(million pounds)					
1	233	235	61	56	55	63
2	180	181	45	41	42	53
3	67	68	17	15	16	20
4	199	201	50	45	46	59
5	126	127	31	29	29	37
6	280	282	70	64	65	83
7	40	40	10	9	9	12
8	130	131	32	30	30	39
9	103	104	26	24	24	31
10	29	29	7	7	7	9
11	61	61	15	14	14	18
12	255	257	64	58	59	76
13	2,426	2,446	636	582	567	660
14	120	121	31	29	28	33
15	180	181	47	43	42	49
16	213	215	56	51	50	58
17	130	131	34	31	30	35
18	2,426	2,446	636	582	567	660
19	933	941	245	224	218	254
20	1,170	1,179	307	281	274	318
21	373	376	98	89	87	102
22	1,156	1,165	303	277	270	315
Total	10,830	10,917	2821	2581	2529	2984

[a]Quarterly estimates computed from adjusted annual estimates according to procedures outlined in Appendixes A and B. Quarterly estimates will not sum to annual estimates in all areas due to rounding errors.

[b]Original estimate increased by .8 of 1 percent in order to make consumption equal to production for the 22 areas combined.

the third quarter the amount of pork shipped will be about 4 1/2 percent less than the amount consumed and the deficit for the fourth quarter will be approximately 6 percent. The excess of wholesale consumption over retail consumption during the first two quarters of 1960 compensates for the deficit during the last two quarters. The quarterly estimates of retail consumption are given in Table 20.

By using estimates of wholesale pork consumption instead of retail pork consumption, the inventory problem is avoided. In order to adequately handle the inventory problem by quarters, another dimension would need to be included in the model. This would require quarterly estimates of fresh pork storage as well as estimates of the amount of excess pork (over and above quarterly consumption) used in processed meat products. Table 21 gives the 1960 quarterly distribution of hog supplies and retail pork consumption for the Southeast and non-Southeast. The inventory problem results from the excess of production over consumption the early part of the year and an opposite situation during the latter part of the year.

The final 1960 annual and quarterly estimates of hogs sold for commercial slaughter, e_i, commercial hog slaughter capacities, f_j, and commercially slaughtered pork consumption, g_k, used in the models are given in Table 22. The quarterly estimates of hog slaughter capacity are 25 percent

Table 21. Quarterly distribution of hogs available for commercial slaughter and commercially slaughtered pork consumption, 1960[a]

Area	Hog supplies as a percent of annual total					Retail pork consumption as a percent of annual total[b]				
	Jan.-Mar.	Apr.-June	July-Sept.	Oct.-Dec.	Total	Jan.-Mar.	Apr.-June	July-Sept.	Oct.-Dec.	Total
			(percent)					(percent)		
Southeast	28.0	24.6	22.2	25.2	100.0	24.8	22.6	23.1	29.5	100.0
Non-Southeast	27.4	24.7	22.4	25.5	100.0	26.0	23.8	23.2	27.0	100.0
United States	27.4	24.7	22.4	25.5	100.0	25.9 (26.7)	23.6 (24.0)	23.2 (23.3)	27.3 (26.0)	100.0 (100.0)

[a]See Appendixes A and B for procedures used to estimate quarterly hog supplies and pork consumption.

[b]Quarterly distribution of United States pork consumption, listed in parentheses, is computed from (46) and includes consumption of farm slaughtered pork.

ble 22. Area estimates of hogs sold for commercial slaughter, e_i, commercial hog slaughter capacities, f_j, and commercially slaughtered pork consumption, g_k, all in pork (excluding lard and inedibles) equivalent units for 1960[a]

ea	Hog supplies, e_i					Slaughter capacities, f_j					Pork consumption, g_k				
	Jan.-Mar.	Apr.-June	July-Sept.	Oct.-Dec.	Total	Jan.-Mar.	Apr.-June	July-Sept.	Oct.-Dec.	Total	Jan.-Mar.	Apr.-June	July-Sept.	Oct.-Dec.	Total
							(million pounds)								
1	9	8	7	8	31	61	61	61	61	242	65	59	53	59	235
2	12	11	10	11	43	62	62	62	62	247	48	43	41	49	181
3	8	7	7	8	30	47	47	47	47	186	18	16	15	19	68
4	17	15	13	15	59	16	16	16	16	63	53	47	44	55	201
5	53	46	41	47	188	53	53	53	53	212	33	30	28	35	127
6	16	14	13	14	58	54	54	54	54	215	74	67	63	78	282
7	12	11	10	11	44	4	4	4	4	17	11	9	9	11	40
8	10	9	8	9	37	25	25	25	25	98	34	31	29	36	131
9	49	43	38	44	174	63	63	63	63	250	28	25	23	29	104
10	25	22	20	23	90	16	16	16	16	62	7	7	7	8	29
11	5	4	4	4	18	22	22	22	22	86	16	15	14	17	61
12	1	1	1	1	4	6	6	6	6	25	68	60	57	71	257
13	37	32	31	37	138	445	445	445	445	1,778	675	607	548	616	2,446
14	3	2	2	2	9	8	8	8	8	33	33	30	27	31	121
15	59	56	54	59	228	70	70	70	70	278	50	45	41	46	181
16	48	48	42	44	183	128	128	128	128	511	59	53	48	54	215
17	21	19	17	19	76	35	35	35	35	140	36	32	29	33	131
18	1,017	914	837	889	3,657	1,094	1,094	1,094	1,094	4,374	675	607	548	616	2,446
19	1,442	1,299	1,171	1,411	5,323	1,445	1,445	1,445	1,445	5,781	260	234	210	237	941
20	90	78	67	79	314	199	199	199	199	796	325	293	264	297	1,179
21	30	27	25	25	107	159	159	159	159	635	104	93	84	95	376
22	29	26	25	26	106	264	264	264	264	1,054	321	289	261	294	1,165
23	1,283	1,584	1,833	1,490	6,166						1,283	1,584	1,833	1,490	6,166
otal	4,276	4,276	4,276	4,276	17,083	4,276	4,276	4,276	4,276	17,083	4,276	4,276	4,276	4,276	17,083

[a]Quarterly estimates will not sum to annual estimates in all areas due to rounding errors.

of annual capacity.

Transfer costs

The transfer costs, c_{ijk}, used in the model are a combination of three separate costs: (a) cost of transporting hogs (in pork units) from hog supply areas to hog slaughter areas, (b) cost of slaughtering hogs to fresh pork at the point of slaughter and (c) cost of transporting pork from hog slaughter areas to pork demand areas. For example, transfer cost $c_{16,9,12}$ includes the cost of transporting hogs from hog supply area 16 to hog slaughter area 9, the slaughtering cost in hog slaughter area 9 and the cost of transporting pork from hog slaughter area 9 to pork demand area 12, all in dollars per hundredweight pork equivalent (excluding inedibles and lard).

Institutional and directional value-of-service rates are used for (a) and (c). Both rail and truck transport cost functions are used to compute transport costs from hog supply areas to hog slaughter areas (11, p. 86).* Only rail transport cost functions are used to compute transport costs from hog slaughter areas to pork demand areas.*

*Maki, Wilbur R., Dept. of Econ. and Soc., Iowa State University, Ames, Iowa. Rail transport cost functions for hogs and fresh meat. Private communication. 1963.

Rand McNally highway mileages are used for both sets of transportation costs (32). The transportation centers used for the 22 areas are given in Table 23. Even though the 22 areas are the same geographically in each dimension, different transport centers are selected for some areas depending on the dimension involved. The centers were selected on the basis of the concentration of hog production for the ith dimension, slaughter facilities for the jth dimension and human population for the kth dimension.

Only a very limited amount of data are available on (b), the cost of slaughtering hogs to fresh pork by regions in the United States. The most current information is being assembled under a U. S. Department of Agriculture project (44, p. 40).* On the basis of cost data from a sample of 22 packers in the Northcentral, Northeast and South during 1959-60 and from a sample of 24 packers in the same regions during 1960-61, a cost for slaughtering hogs to fresh pork of $1.75 per hundred pounds of fresh pork is used for the Southeast areas (1-12) and a cost of $2.50 is used for the non-Southeast areas (13-22). These costs include kill and cut labor, packaging, order filling and shipping room.

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*Agnew, Donald B., Marketing Econ. Div., Econ. Res. Ser., U. S. Dept. of Agr., Washington, D. C. Information on slaughtering costs for fresh pork operations. Private communication. 1963.

Table 23. Transportation centers used for the 22 areas in each of the three dimensions: hog supply, hog slaughter and pork demand

Area	Transportation centers		
	Hog supply areas	Hog slaughter areas	Pork demand areas
1	Baltimore, Md.	Baltimore, Md.	Baltimore, Md.
2	Roanoke, Va.	Roanoke, Va.	Richmond, Va.
3	Franklin, Va.	Franklin, Va.	Norfolk, Va.
4	Greensboro, N.C.	Greensboro, N.C.	Greensboro, N.C.
5	Fayetteville, N.C.	Wilmington, N.C.	Charleston, S.C.
6	Greenville, S.C.	Greenville, S.C.	Atlanta, Ga.
7	Huntsville, Ala.	Huntsville, Ala.	Gadsden, Ala.
8	Birmingham, Ala.	Birmingham, Ala.	Birmingham, Ala.
9	Dothan, Ala.	Albany, Ga.	Mobile, Ala.
10	Swainsboro, Ga.	Swainsboro, Ga.	Savannah, Ga.
11	Gainesville, Fla.	Jacksonville, Fla.	Jacksonville, Fla.
12	Tampa, Fla.	Tampa, Fla.	Tampa, Fla.
13	Harrisburg, Pa.	New York, N.Y.	New York, N.Y.
14	Charleston, W.Va.	Charleston, W.Va.	Charleston, W.Va.
15	Lexington, Ky.	Louisville, Ky.	Louisville, Ky.
16	Nashville, Tenn.	Nashville, Tenn.	Nashville, Tenn.
17	Jackson, Miss.	Jackson, Miss.	Jackson, Miss.
18	Indianapolis, Ind.	Chicago, Ill.	Chicago, Ill.
19	Des Moines, Iowa	Omaha, Nebr.	Omaha, Nebr.
20	Dallas, Texas	Dallas, Texas	Dallas, Texas
21	Denver, Colo.	Denver, Colo.	Denver, Colo.
22	Fresno, Cal.	Los Angeles, Cal.	Los Angeles, Cal.

These cost estimates represent a weighted average of the plants sampled in the Southeast and non-Southeast. It should be pointed out that the cost variation within the two regions is nearly as great as between the regions. The major contributors to the lower average cost in the Southeast are lower labor costs and newer, more efficient plants.

Because of the lack of sufficient data to accurately estimate area differences in hog slaughter costs, the \$.75 differential between the Southeast and non-Southeast areas is included in the 1960 models (and left out of the 1970 models) as a hypothesis to be tested.

The three separate costs, (a), (b) and (c), are combined into a set of transfer costs, c_{ijk}, by an IBM 1620 program. A 23 x 22 matrix of hog transport cost for (a), a 22 x 1 vector of slaughter costs for (b) and a 22 x 23 matrix of pork transport costs for (c) are used as input for the IBM 1620 program. From these costs the program computes all possible c_{ijk} transfer cost elements (23 x 22 x 23 = 11,638). The program punches the output (c_{ijk} cost elements) onto cards in the exact format required for input into the Galler IBM 704 generalized distribution program (15).

Solutions of the models

The least-cost routes and shipments for the 1960 annual solution are given in Table 24. For example, the fourth route listed in Table 24, 4-4-13, indicates that 59 million pounds of hogs (in pork units) were produced in hog supply area 4, slaughtered in hog slaughter area 4 and shipped to pork demand area 13 for consumption. These routes and shipments are depicted in Figures 11 and 12. The 1960 first

Table 24. Least-cost solution for 1960 annual model, shipments in pork (excluding lard and inedibles) equivalent units

Route: ith hog supply area	Route: jth hog slgt. area	Route: kth pork demand area	Million pounds shipped
1	1	1	31
2	2	4	43
3	3	2	30
4	4	13	59
5	5	13	188
6	6	13	58
7	7	13	17
7	8	13	27
8	8	13	37
9	9	12	174
10	10	5	33
10	10	10	29
10	11	11	28
11	11	11	18
12	12	12	4
13	1	1	131
13	1	13	7
14	14	14	9
15	15	13	228
16	16	13	95
16	16	14	88
17	17	20	76
18	1	1	73
18	2	2	63
18	2	4	141
18	3	2	88
18	3	3	68
18	13	13	1511
18	14	14	24
18	18	18	1689
19	4	13	4
19	5	13	24
19	6	13	157
19	8	13	34
19	9	12	76
19	11	11	15
19	12	12	3
19	18	18	757
19	19	4	17
19	19	5	94
19	19	6	282
19	19	7	40
19	19	8	131
19	19	9	104
19	19	15	181
19	19	16	215
19	19	17	131
19	19	19	941
19	19	20	1103
19	21	21	269
19	22	22	745
20	20	22	314
21	21	21	107
22	22	22	106
23	11	23	25[a]
23	12	23	18[a]
23	13	23	267[a]
23	15	23	50[a]
23	16	23	328[a]
23	17	23	64[a]
23	18	23	1928[a]
23	19	23	2542[a]
23	20	23	482[a]
23	21	23	259[a]
23	22	23	203[a]

Parameter Routes[b]

1	1	13
2	2	2
3	3	3
10	10	13
18	1	13

[a]Amount of excess slaughter capacity in specified hog slaughter area.

[b]Parameter routes are equally acceptable routes and could have been used in lieu of routes that were used without changing the total cost of all shipments.

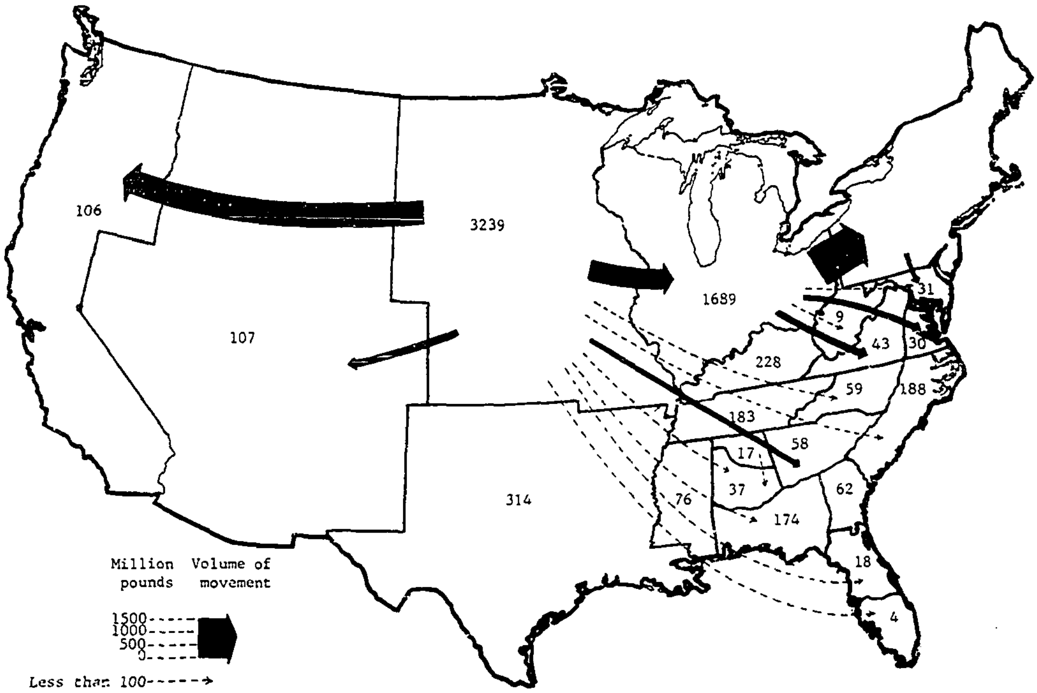

Figure 11. Shipments of hogs (in pork units) from hog supply areas to hog slaughter areas, 1960 annual solution.

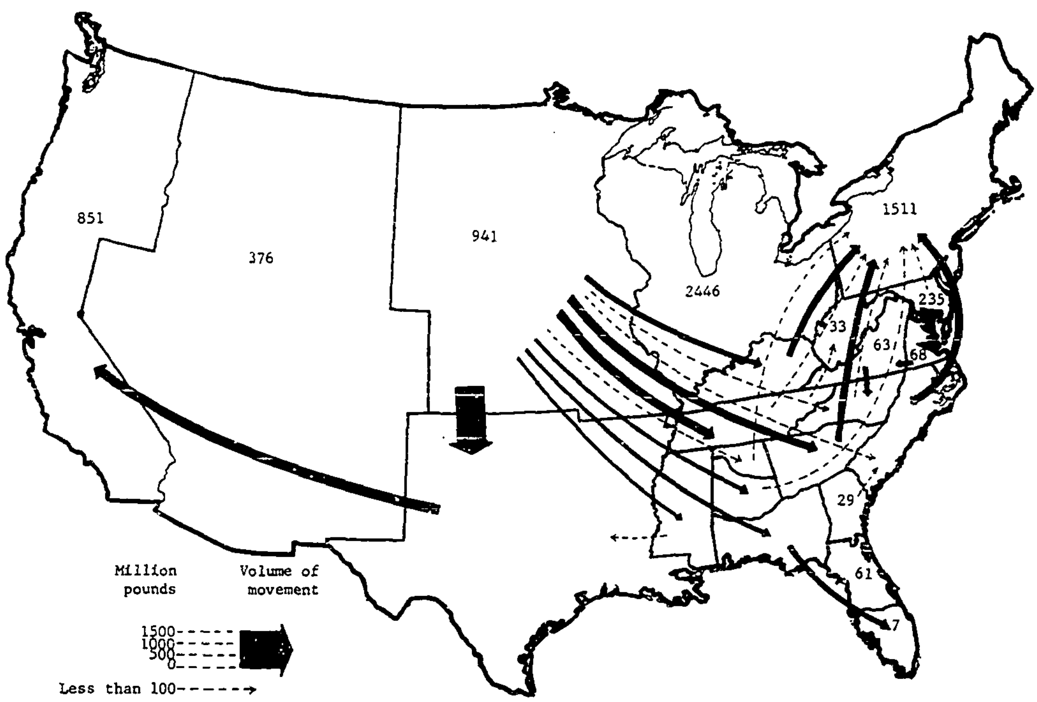

Figure 12. Shipments of pork from hog slaughter areas to pork demand areas, 1960 annual solution.

quarter solution is presented in Table 25 and Figures 13 and 14, the second quarter solution in Table 26 and Figures 15 and 16, the third quarter solution in Table 27 and Figures 17 and 18 and the fourth quarter solution in Table 28 and Figures 19 and 20.

The routes from hog supply area 23 to pork demand area 23 indicated the amount of excess slaughter capacity in the specified hog slaughter areas (refer to Tables 24 through 28). For example, in Table 24 route 23-13-23 indicates that the amount of excess slaughter capacity in hog slaughter area 13 is 267 million pounds for the 1960 annual solution. Only the "dummy" hog supply area, 23, can ship to the "dummy" demand area, 23, and these shipments must go through the excess slaughter capacity in the 22 hog slaughter areas. In order to accomplish this, the 22 $c_{23,j,23}$ cost elements must be (a) equal to each other and (b) smaller than any other c_{ijk} cost element in the problem. For this study the $c_{23,j,23}$ cost elements were set at $1.00.

The 23-j-k (where $k \neq 23$) routes represent routes from the "dummy" hog supply area to any "real" pork demand area. Likewise, the i-j-23 (where $i \neq 23$) routes represent routes from any "real" hog supply area to the "dummy" pork demand area. These are not logical routes and to prevent their use a transfer cost of $99.99 is used.

Table 25. Least-cost solution for 1960 first quarter model, shipments in pork (excluding lard and inedibles) equivalent units

Route			Million pounds shipped
ith hog supply area	jth hog slgt. area	kth pork demand area	
1	1	1	9
2	2	4	12
3	3	2	8
4	3	2	1
4	4	13	16
5	5	13	53
6	6	13	16
7	7	13	4
7	8	13	8
8	8	13	10
9	9	12	49
10	10	5	9
10	10	10	7
10	11	11	9
11	11	11	5
12	12	12	1
13	1	1	37
14	14	14	3
15	15	13	59
16	16	13	23
16	16	14	25
17	17	20	21
18	1	1	15
18	2	1	4
18	2	2	19
18	2	4	27
18	3	2	20
18	3	3	18
18	13	13	441
18	14	14	5
18	18	18	468
19	6	13	38
19	8	13	7
19	9	12	14
19	11	11	2
19	12	12	4
19	18	18	207
19	19	4	14
19	19	5	24
19	19	6	74
19	19	7	11
19	19	8	34
19	19	9	28
19	19	15	50
19	19	16	59
19	19	17	36
19	19	19	260
19	19	20	304
19	21	21	74
19	22	22	202
20	20	22	90
21	21	21	30
22	22	22	29
23	11	23	6[a]
23	12	23	1[a]
23	13	23	4[a]
23	15	23	11[a]
23	16	23	80[a]
23	17	23	14[a]
23	18	23	419[a]
23	19	23	551[a]
23	20	23	109[a]
23	21	23	55[a]
23	22	23	33[a]

Parameter Routes[b]

2	2	1
2	2	2
3	3	3
4	3	3
5	3	2
5	3	3
10	10	13

[a]Amount of excess slaughter capacity in specified hog slaughter area.

[b]Parameter routes are equally acceptable routes and could have been used in lieu of routes that were used without changing the total cost of all shipments.

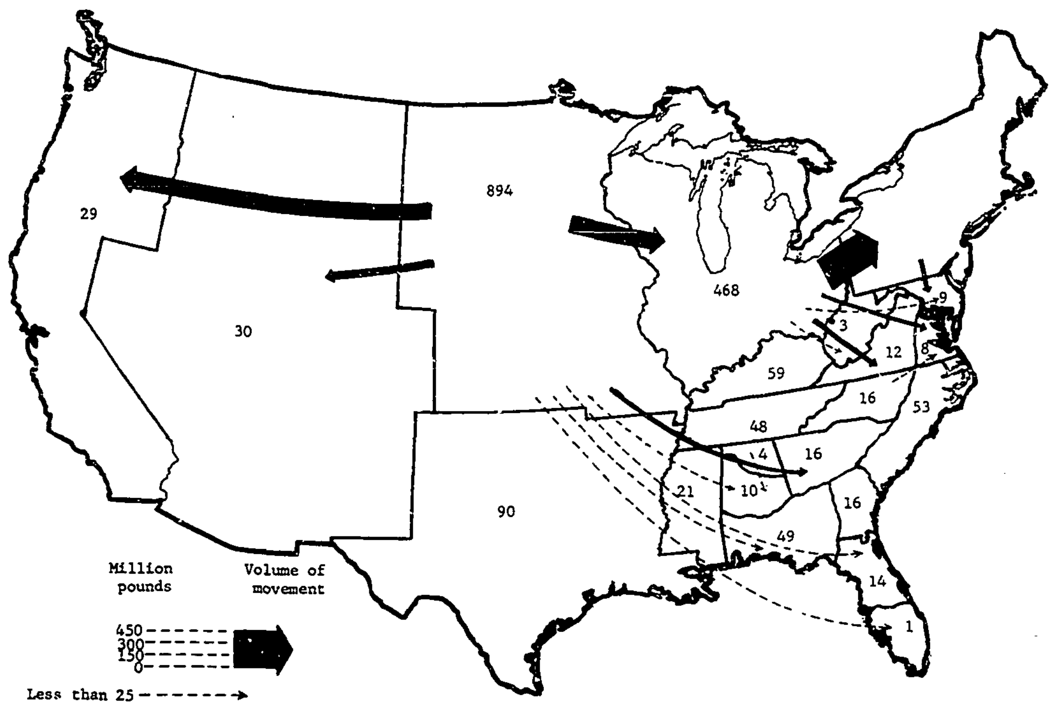

Figure 13. Shipments of hogs (in pork units) from hog supply areas to hog slaughter areas, 1960 first quarter solution.

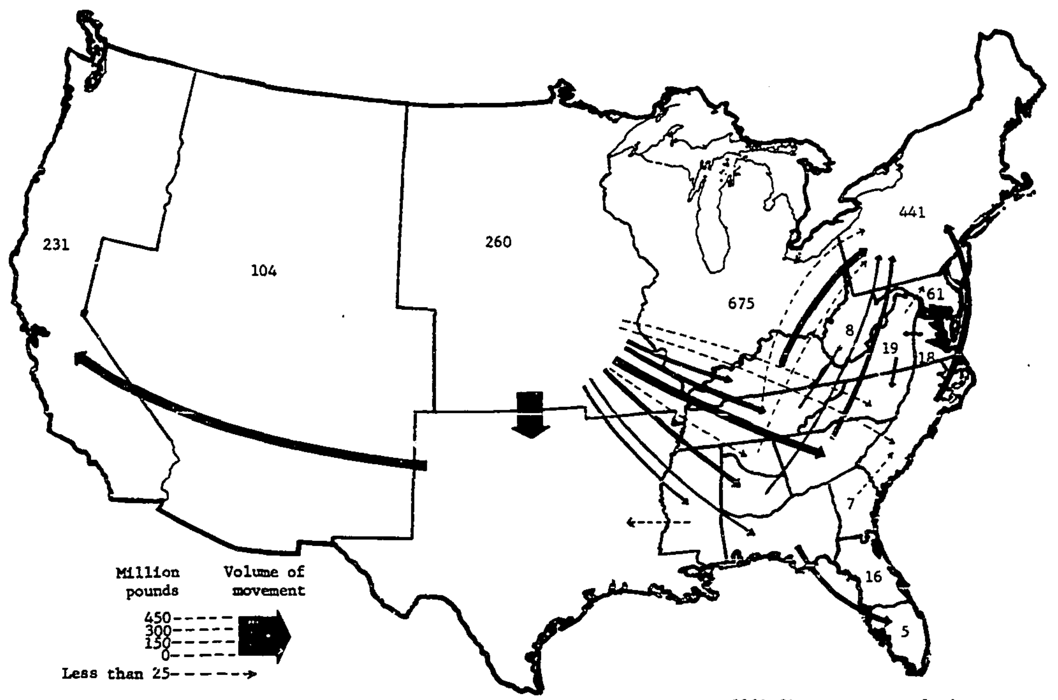

Figure 14. Shipments of pork from hog slaughter areas to pork demand areas, 1960 first quarter solution.

Table 26. Least-cost solution for 1960 second quarter model, shipments in pork (excluding lard and inedibles) equivalent units

Route: ith hog supply area	Route: jth hog slgt. area	Route: kth pork demand area	Million pounds shipped
1	1	1	8
2	2	4	11
3	3	2	7
4	4	13	15
5	5	13	46
6	6	13	14
7	7	13	4
7	8	13	7
8	8	13	9
9	9	12	43
10	10	5	9
10	10	10	7
10	11	11	6
11	11	11	4
12	12	12	1
13	1	1	30
13	1	13	2
14	14	14	2
15	15	13	56
16	16	13	26
16	16	14	22
17	17	20	19
18	1	1	21
18	2	2	12
18	2	4	36
18	2	5	3
18	3	2	24
18	3	3	16
18	13	13	371
18	14	14	6
18	18	18	425
19	4	13	1
19	5	13	7
19	6	13	40
19	8	13	9
19	9	11	4
19	9	12	16
19	11	11	1
19	18	18	182
19	19	5	18
19	19	6	67
19	19	7	9
19	19	8	31
19	19	9	25
19	19	15	45
19	19	16	53
19	19	17	32
19	19	19	234
19	19	20	274
19	21	21	66
19	22	22	185
20	20	22	78
21	21	21	27
22	22	22	26
23	11	23	11[a]
23	12	23	5[a]
23	13	23	74[a]
23	15	23	14[a]
23	16	23	80[a]
23	17	23	16[a]
23	18	23	487[a]
23	19	23	657[a]
23	20	23	121[a]
23	21	23	66[a]
23	22	23	53[a]

Parameter Routes[b]

1	1	13
2	2	2
2	2	5
2	2	14
3	3	3
9	9	11
10	10	13
18	1	13
18	2	14

[a]Amount of excess slaughter capacity in specified hog slaughter area.

[b]Parameter routes are equally acceptable routes and could have been used in lieu of routes that were used without changing the total cost of all shipments.

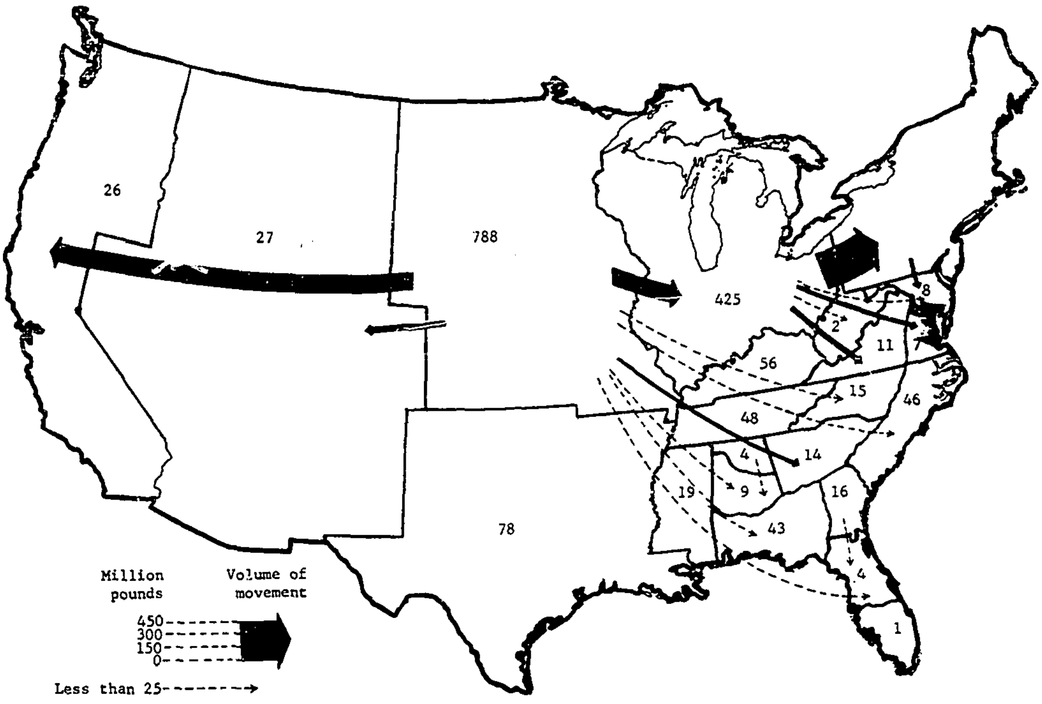

Figure 15. Shipments of hogs (in pork units) from hog supply areas to hog slaughter areas, 1960 second quarter solution.

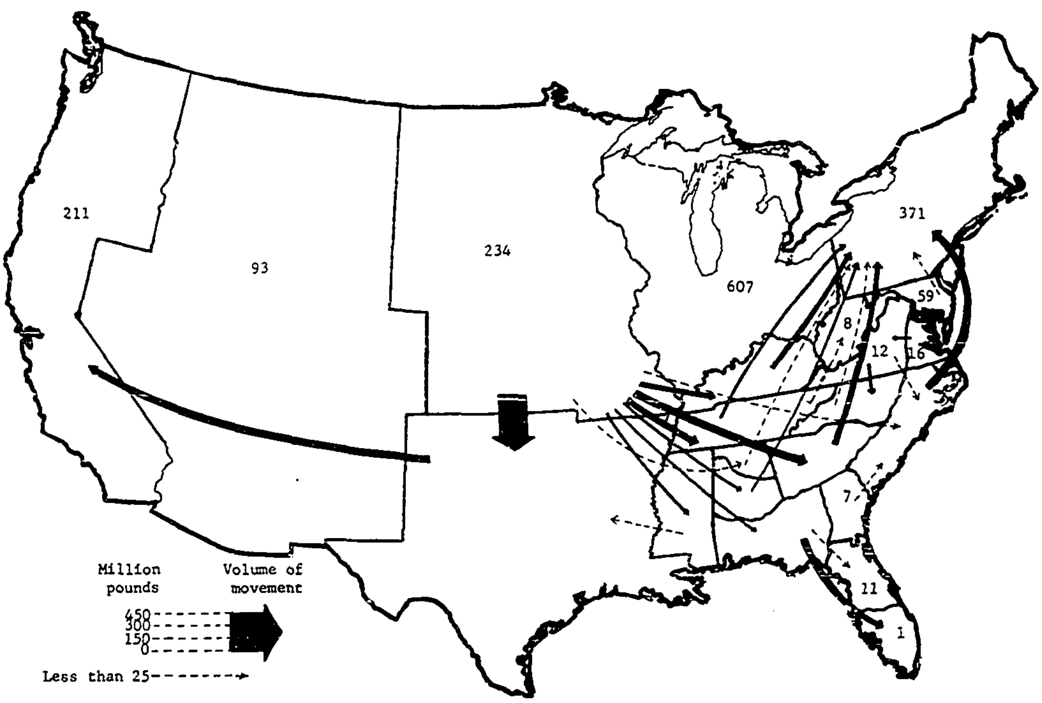

Figure 16. Shipments of pork from hog slaughter areas to pork demand areas, 1960 second quarter solution.

Table 27. Least-cost solution for 1960 third quarter model, shipments in pork (excluding lard and inedibles) equivalent units

Route			Million pounds shipped
ith hog supply area	jth hog slgt. area	kth pork demand area	
1	1	1	7
2	2	4	10
3	3	2	7
4	4	13	13
5	5	13	41
6	6	13	13
7	7	13	4
7	8	13	6
8	8	13	8
9	9	12	38
10	10	5	9
10	10	10	7
10	11	11	4
11	11	11	4
12	12	12	1
13	1	1	23
13	1	13	8
14	14	14	2
15	15	13	54
16	16	13	23
16	16	14	19
17	17	20	17
18	1	1	23
18	2	2	9
18	2	4	34
18	2	5	9
18	3	2	25
18	3	3	15
18	13	13	311
18	14	14	6
18	18	18	405
19	4	13	3
19	5	13	12
19	6	13	41
19	8	13	11
19	9	11	6
19	9	12	18
19	18	18	143
19	19	5	10
19	19	6	63
19	19	7	9
19	19	8	29
19	19	9	23
19	19	15	41
19	19	16	48
19	19	17	29
19	19	19	210
19	19	20	247
19	21	21	59
19	22	22	169
20	20	22	67
21	21	21	25
22	22	22	25
23	9	23	1[a]
23	11	23	14[a]
23	12	23	5[a]
23	13	23	134[a]
23	15	23	16[a]
23	16	23	86[a]
23	17	23	18[a]
23	18	23	546[a]
23	19	23	736[a]
23	20	23	132[a]
23	21	23	75[a]
23	22	23	70[a]
	Parameter Routes[b]		
1	1	13	
2	2	2	
2	2	5	
2	2	14	
3	3	3	
9	9	11	
10	10	13	
18	1	13	
18	2	14	

[a]Amount of excess slaughter capacity in specified hog slaughter area.

[b]Parameter routes are equally acceptable routes and could have been used in lieu of routes that were used without changing the total cost of all shipments.

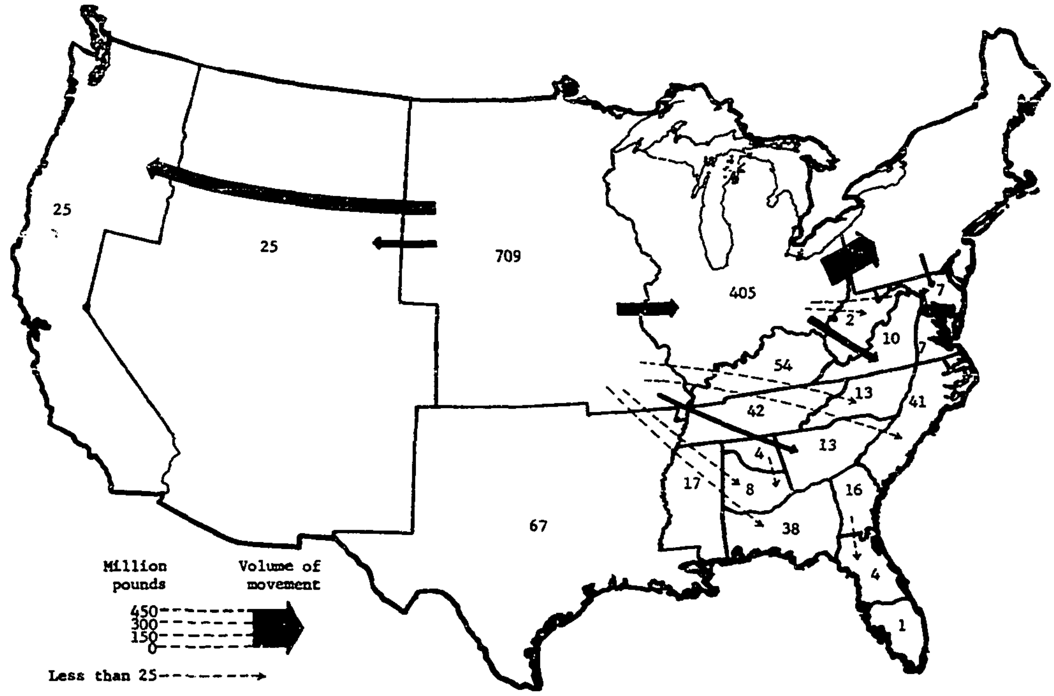

Figure 17. Shipments of hogs (in pork units) from hog supply areas to hog slaughter areas, 1960 third quarter solution.

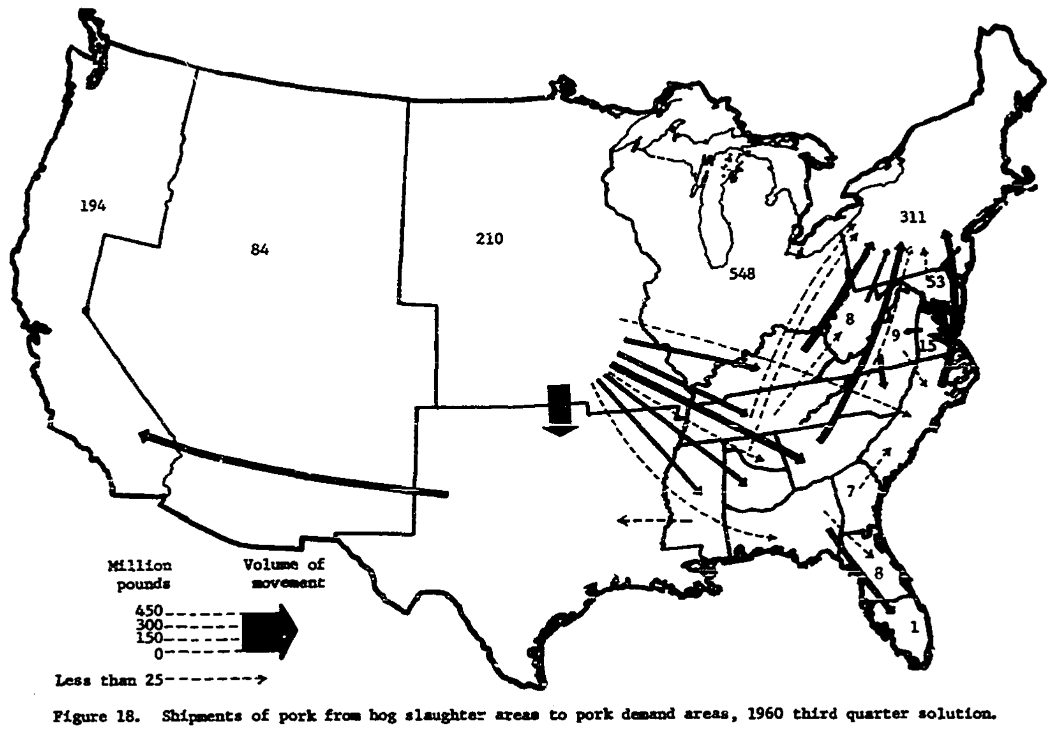

Figure 18. Shipments of pork from hog slaughter areas to pork demand areas, 1960 third quarter solution.

Table 28. Least-cost solution for 1960 fourth quarter model, shipments in pork (excluding lard and inedibles) equivalent units

Route			Million pounds shipped
ith hog supply area	jth hog slgt. area	kth pork demand area	
1	1	1	8
2	2	4	11
3	3	2	8
4	4	13	15
5	5	13	47
6	6	13	14
7	7	13	4
7	8	13	7
8	8	13	9
9	9	12	44
10	10	5	8
10	10	10	8
10	11	11	7
11	11	11	4
12	12	12	1
13	1	1	35
13	1	13	2
14	14	14	2
15	15	13	59
16	16	13	21
16	16	14	23
17	17	20	19
18	1	1	16
18	2	2	21
18	2	4	30
18	3	2	20
18	3	3	19
18	13	13	382
18	14	14	6
18	18	18	395
19	4	13	1
19	5	13	6
19	6	13	40
19	8	13	9
19	9	12	19
19	11	11	6
19	11	12	2
19	12	12	5
19	18	18	221
19	19	4	14
19	19	5	27
19	19	6	78
19	19	7	11
19	19	8	36
19	19	9	29
19	19	15	46
19	19	16	54
19	19	17	33
19	19	19	237
19	19	20	278
19	21	21	70
19	22	22	189
20	20	22	79
21	21	21	25
22	22	22	26
23	11	23	3[a]
23	13	23	63[a]
23	15	23	11[a]
23	16	23	84[a]
23	17	23	16[a]
23	18	23	478[a]
23	19	23	602[a]
23	20	23	120[a]
23	21	23	64[a]
23	22	23	49[a]

Parameter Routes[b]

1	1	13
2	2	2
3	3	3
10	10	13
10	11	12
11	11	12
18	1	13

[a]Amount of excess slaughter capacity in specified hog slaughter area.

[b]Parameter routes are equally acceptable routes and could have been used in lieu of routes that were used without changing the total cost of all shipments.

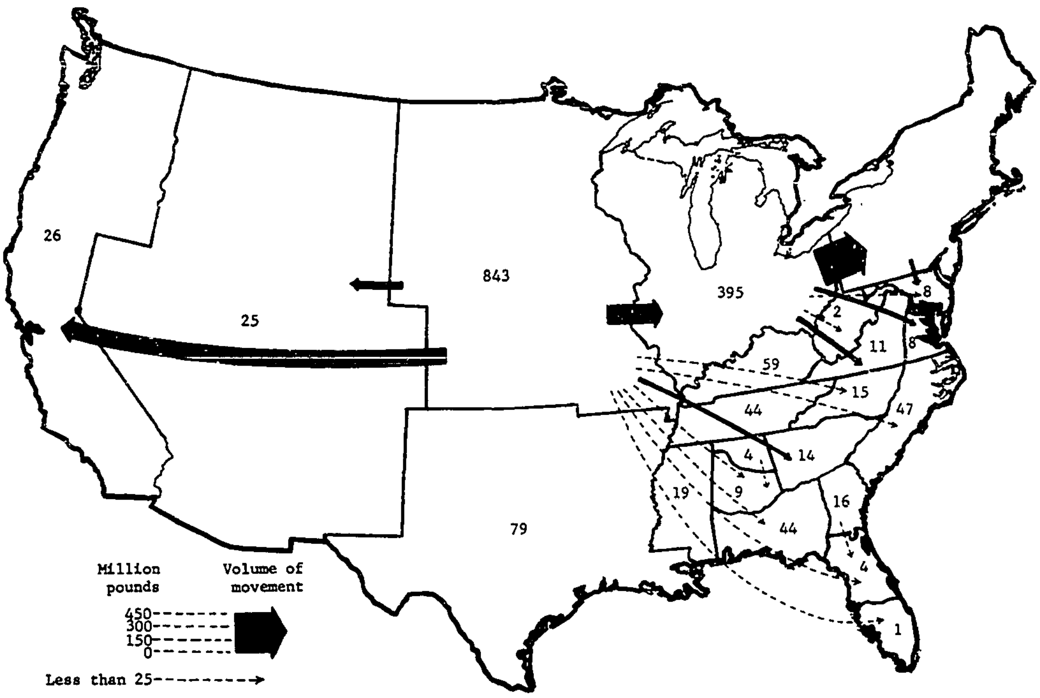

Figure 19. Shipments of hogs (in pork units) from hog supply areas to hog slaughter areas, 1960 fourth quarter solution.

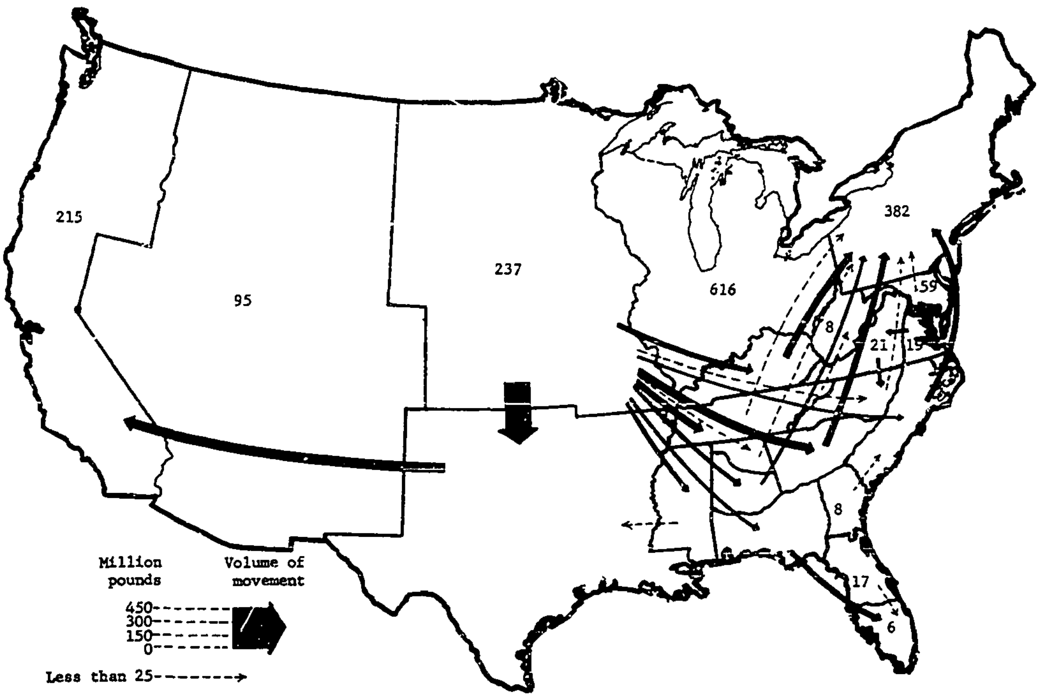

Figure 20. Shipments of pork from hog slaughter areas to pork demand areas, 1960 fourth quarter solution.

Least-Cost Hog-Pork Distribution, 1970

Two separate annual models are formulated for 1970 and one solution is computed for each model. The first model is a full three-dimensional model using 1970 estimates of hogs available for commercial slaughter, commercial hog slaughter capacities and commercially slaughtered pork consumption. The second model eliminates the fixed slaughter capacity restriction of the 1960 models and the first 1970 model. By relaxing this restriction the model takes on a long-run normative character and the amount of hog slaughter in each area is determined within the model. The assumption is made that by 1970 sufficient slaughter facilities could be built in each area to satisfy the requirements of the solution to the normative model.

Input estimates

The 1970 annual estimates of hogs available for commercial slaughter, e_i, commercial hog slaughter capacities, f_j, and commercially slaughtered pork consumption, g_k, for the 22 areas are given in Table 29.

The procedure used to estimate 1970 pork consumption is outlined in Appendix B. The assumption is made that the United States production of hogs for commercial slaughter

Table 29. Area estimates of hogs available for commercial slaughter, e_i, commercial hog slaughter capacities, f_j, and commercially slaughtered pork consumption, g_k, all in pork (excluding lard and inedibles) equivalent units for 1970

Area	Hog supplies, e_i	Slaughter capacities, f_j	Pork consumption, g_k
	(million pounds)		
1	35	154	275
2	48	313	206
3	34	235	82
4	66	79	210
5	210	263	150
6	65	254	306
7	49	27	45
8	42	153	125
9	195	304	127
10	101	73	37
11	20	65	67
12	4	19	393
13	155	1,131	2,588
14	10	41	101
15	256	349	181
16	205	641	226
17	86	220	134
18	4,102	4,061	2,750
19	5,971	6,374	1,006
20	352	617	1,323
21	120	485	468
22	119	867	1,445
23	4,480		4,480
Total	16,725	16,725	16,725

in 1970 will equal commercially slaughtered pork consumption for the United States in 1970. This 1970 estimate of hogs available for commercial slaughter in the United States is allocated among the 22 hog supply areas on the basis of each area's share of the 1960 supply of hogs sold for commercial slaughter in the United States. The allocating procedure is based on the assumption that each area will maintain the same relative share of the United States supply in 1970 that it held in 1960.

The 1970 estimates of commercial hog slaughter capacities are based on a projection of the increase or decrease, by areas, from 1955 through 1962. These projections are developed from unpublished data supplied by the U. S. Department of Agriculture and the Southern Regional Livestock Marketing Research Committee (37, 42).*

Transfer costs

It is extremely difficult to estimate the institutional or regulatory changes that may occur in the establishment of transportation rates by 1970. Because of this, it is assumed that the forces of competition within the rail-truck transportation system will tend to eliminate institutional and directional differences in the 1960 transport costs for hogs and pork. On the basis of this assumption the 484 (22 x 22) mileage-cost observations computed for both hogs

*Rizek, Robert L., Marketing Econ. Div., Econ. Res. Ser., U. S. Dept. of Agr., Ames, Iowa. Estimates of 1970 commercial hog slaughter capacity. Private communication. 1963.

and pork for the 1960 models are used to fit a single normative hog transport cost function and a similar pork transport cost function for the entire United States. Several regression models are used with the following functions providing the best fit:

$$\text{Hogs:} \quad Y = .47232 + \underset{(.000066969)^{**}}{.00233\, X} - \underset{(.000025875)^{**}}{.00055\, \frac{X^2}{1000}} \; ; \; R^2 = .847$$

$$\text{Pork:} \quad Y = 1.05994 + \underset{(.000060837)^{**}}{.00057\, X} - \underset{(.000024157)^{**}}{.000072996\, \frac{X^2}{1000}} \; ; \; R^2 = .764$$

where

Y = transportation cost in dollars per 100 pounds fresh pork (hog function in pork equivalent units also) and

X = miles hauled.

These two functions are used to estimate 1970 hog and pork transportation costs. The relationship between these two functions is given in Table 30. Up to approximately 350 miles the cost (in pork equivalent units) is less for hogs than for pork. From 350 miles to 3000 miles the cost is less for pork. The mileages used are 1960 Rand McNally highway mileages (32). However, these mileages are compared with estimated mileages between the transport centers via

Table 30. Comparison of hog and pork transportation costs generated by estimated 1970 transport cost functions, in pork (excluding lard and inedibles) equivalent units[a]

Distance	Hog costs	Pork costs
(miles)	(dollars per 100 pounds pork equivalent units)	
200	.92	1.17
400	1.32	1.28
600	1.67	1.38
800	1.98	1.47
1000	2.25	1.56
1200	2.48	1.64
1400	2.66	1.71
1600	2.79	1.79
1800	2.88	1.85
2000	2.93	1.91
2200	2.94	1.96
2400	2.90	2.01
2600	2.81	2.05
2800	2.68	2.08
3000	2.51	2.11

[a]Conversion of hog transportation costs to pork equivalent units is based on a yield of 58.5 percent.

interstate highways (see Figure 21).* In most cases the distances via interstate routes (assuming the system is completed by 1970) are about 2-5 percent greater than via the state and Federal highways. But the variability in the

*Portland Cement Association, 33 West Grand Avenue, Chicago, Ill. Approximate mileages between points on the interstate highway system. Private communication. 1962.

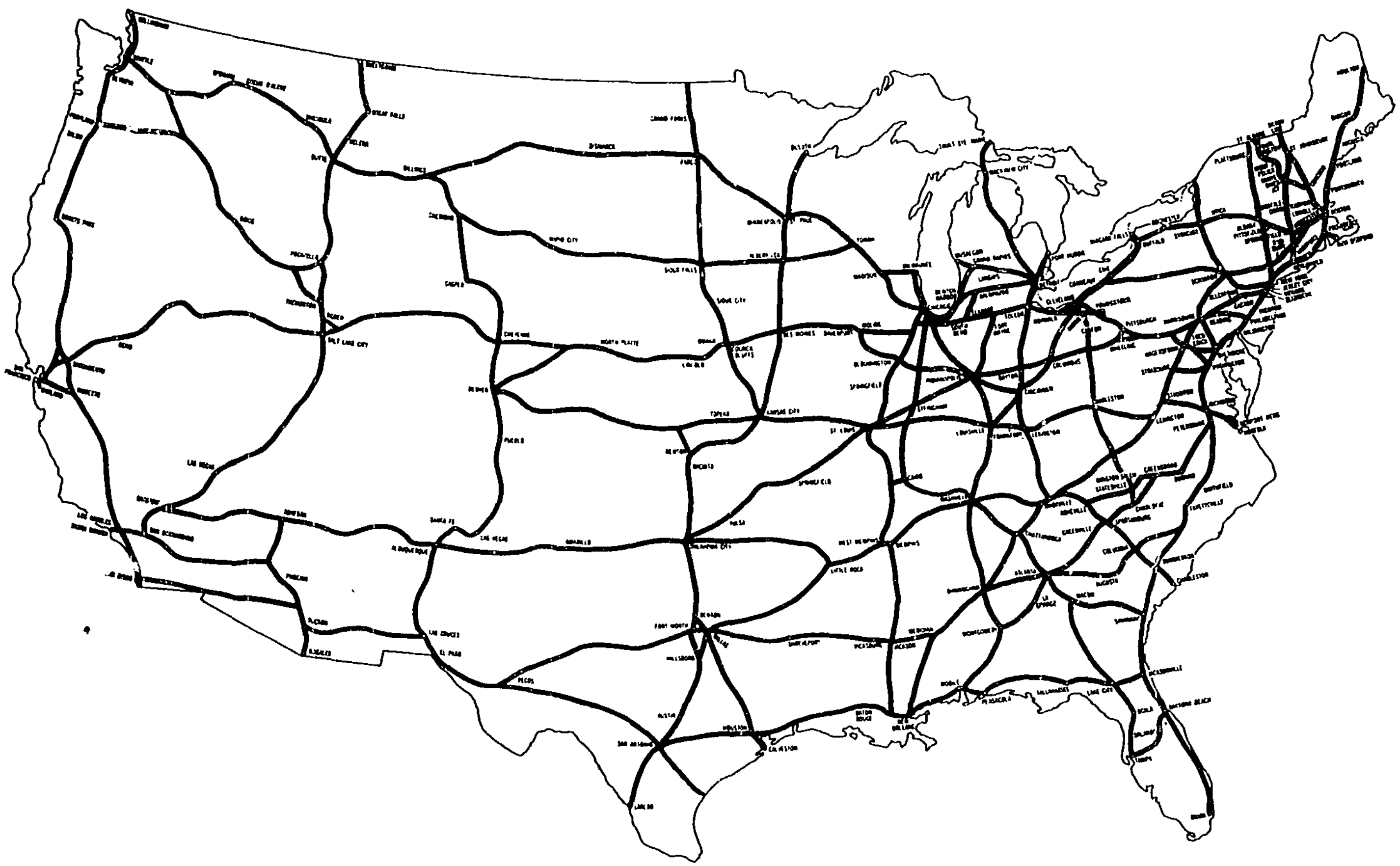

Figure 21. National system of interstate highways (52).

quality of the highways will be nearly eliminated when the interstate system is complete. By using the interstate system between the transport centers the distances are more comparable, i.e., the time-in-transit is directly proportional to the distance hauled.

Due to this feature, time-in-transit during 1970 is assumed to be one hour for each 50 miles hauled. In an attempt to more realistically reflect the cost of transporting hogs for slaughter, each hog transportation cost is adjusted for tissue shrinkage (refer to Table 31). Since the supplies of slaughter hogs are expressed in pork equivalent units, it is not necessary to adjust the hog costs for excretory shrinkage. The tissue shrinkage rates used are developed from the estimates discussed by Fishel _et al_. (11, pp. 92-94).

Solutions of the models

The solution for the 1970 model with slaughter capacities fixed is presented in Table 32 and Figures 22 and 23. The solution for the 1970 model with no limitations on hog slaughter capacity in any area is presented in Table 33 and Figures 24 and 25. A comparison of 1970 estimated hog slaughter capacities and the capacities specified by the second 1970 model is given in Table 34.

Table 31. Estimates of tissue shrinkage used to adjust 1970 hog transportation costs (11, pp. 92-94)

Distance hauled	Time-in-transit	Tissue shrink as a percent of live-weight	Reduction in dressing percentage from 70%	Cost/cwt. carcass at $15.00/cwt. liveweight	Loss from haul per cwt. carcass
(miles)	(hours)	(percent)	(percent)	(dollars)	(dollars)
0	0	.0	70.0	21.43	.00
0-250	0-5	.3	69.7	21.53	.09
250-500	5-10	.5	69.5	21.58	.15
500-750	10-15	.8	69.2	21.68	.25
750-1000	15-20	1.1	68.9	21.77	.34
1000-1250	20-25	1.3	68.7	21.83	.40
1250-1500	25-30	1.5	68.5	21.90	.47
1500-1750	30-35	1.7	68.3	21.96	.53
1750-2000	35-40	1.9	68.1	22.03	.60
2000-2250	40-45	2.1	67.9	22.09	.66
2250-2500	45-50	2.3	67.7	22.16	.73
2500-2750	50-55	2.5	67.5	22.22	.79
2750-3000	55-60	2.7	67.3	22.29	.86

The solution to the second 1970 model is achieved by using an IBM 1620 program for the standard two-dimensional transportation problem. This is accomplished by a procedure developed by Pherson and Firch (36). This procedure requires that the three-dimensional c_{ijk} transfer cost matrix used in the first 1970 model be scanned for the minimum cost for shipments from each ith area to each kth area. For example, in this model there are 22 ways (j=22) of going from area i=1 to area k=1, and so on. An IBM 1620 program is used to select the minimum i-j-k route for every i-k

Table 32. Least-cost solution for 1970 annual model with hog slaughter capacities fixed, shipments in pork (excluding lard and inedibles) equivalent units

Route: ith hog supply area	Route: jth hog slgt. area	Route: kth pork demand area	Million pounds shipped
1	1	1	35
2	2	2	48
3	3	3	34
4	4	4	66
5	4	4	13
5	5	3	48
5	5	5	149
6	6	5	1
6	6	10	37
6	6	11	27
7	7	12	27
7	8	12	22
8	8	12	42
9	9	12	195
10	9	12	8
10	10	12	73
10	11	11	20
11	11	11	20
12	12	12	4
13	1	1	119
13	13	13	36
14	14	2	10
15	14	2	31
15	16	6	101
15	16	12	22
15	16	16	102
16	16	6	205
17	17	9	86
18	15	2	37
18	15	4	131
18	15	15	181
18	18	1	121
18	18	2	80
18	18	13	2552
18	18	14	101
18	18	18	899
19	19	7	45
19	19	8	125
19	19	9	41
19	19	16	124
19	19	17	134
19	19	18	1851
19	19	19	1006
19	19	20	971
19	19	21	468
19	19	22	1206
20	20	20	352
21	21	22	120
22	22	22	119
23	2	23	265[a]
23	3	23	201[a]
23	5	23	66[a]
23	6	23	189[a]
23	8	23	89[a]
23	9	23	101[a]
23	11	23	25[a]
23	12	23	15[a]
23	13	23	1095[a]
23	16	23	211[a]
23	17	23	134[a]
23	18	23	308[a]
23	19	23	403[a]
23	20	23	265[a]
23	21	23	365[a]
23	22	23	748[a]

Parameter Routes[b]

16	16	12
16	16	16
18	15	14
19	19	14

[a]Amount of excess slaughter capacity in specified hog slaughter area.

[b]Parameter routes are equally acceptable routes and could have been used in lieu of routes that were used without changing the total cost of all shipments.

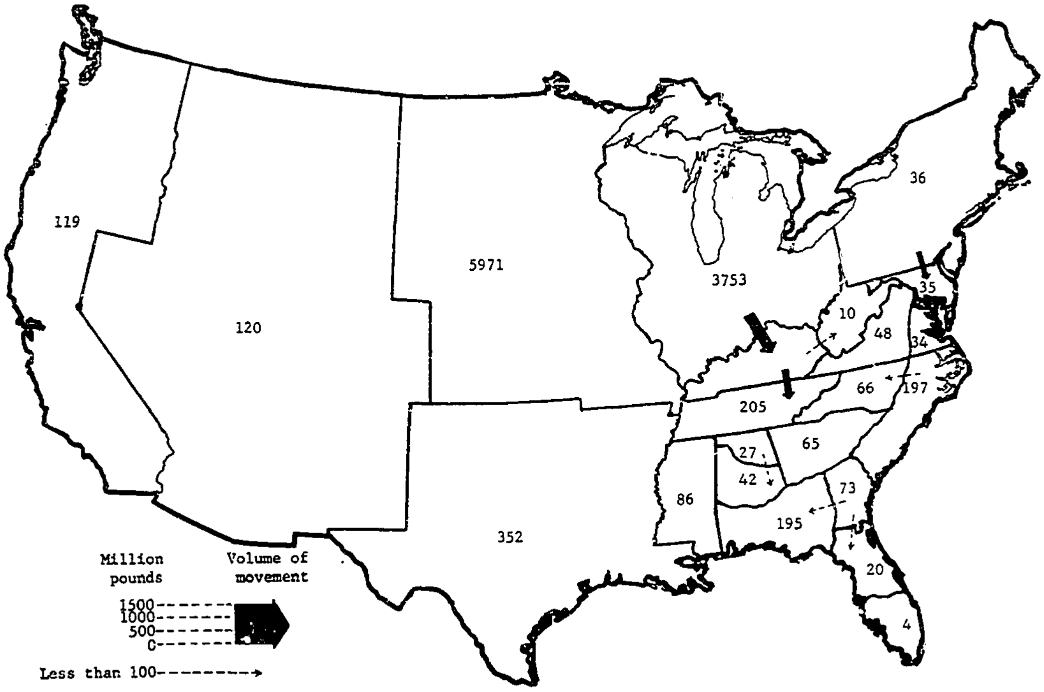

Figure 22. Shipments of hogs (in pork units) from hog supply areas to hog slaughter areas, 1970 annual solution.

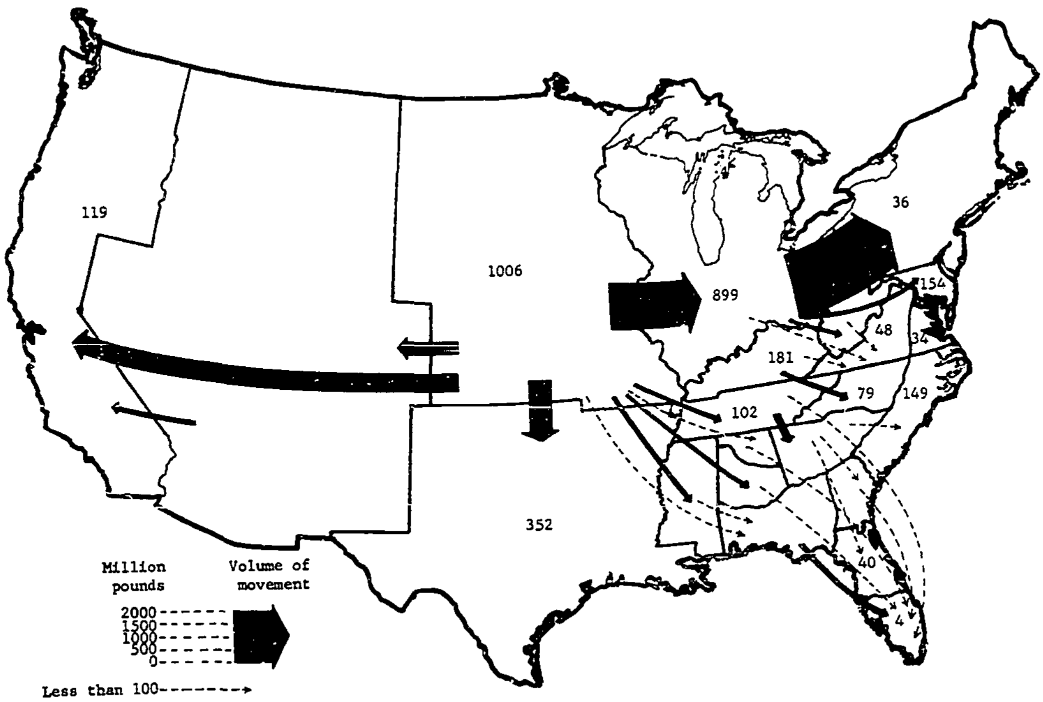

Figure 23. Shipments of pork from hog slaughter areas to pork demand areas, 1970 annual solution.

Table 33. Least-cost solution for 1970 annual model with hog slaughter capacity in each area unlimited, shipments in pork (excluding lard and inedibles) equivalent units

Route			
ith hog supply area	jth hog slgt. area	kth pork demand area	Million pounds shipped
1	1	1	35
2	2	3	48
3	3	3	34
4	4	4	66
5	4	4	144
5	5	5	66
6	6	5	65
7	7	9	49
8	8	9	42
9	9	12	195
10	10	10	37
10	10	12	64
11	11	11	20
12	12	12	4
13	1	1	155
14	14	2	10
15	15	1	6
15	15	15	181
15	15	16	21
15	15	17	48
16	16	16	205
17	17	17	86
18	15	1	79
18	15	2	196
18	15	5	19
18	15	6	306
18	15	7	45
18	15	8	125
18	15	9	36
18	15	11	47
18	15	12	130
18	15	13	2588
18	15	14	101
18	18	18	430
19	19	18	2320
19	19	19	1006
19	19	20	971
19	19	21	468
19	19	22	1206
20	20	20	352
21	21	22	120
22	22	22	119

combination. The cost elements for these routes are then used as cost inputs for the two-dimensional (i=22, k=22 in this case) problem with no limitation on hog slaughter capacities.

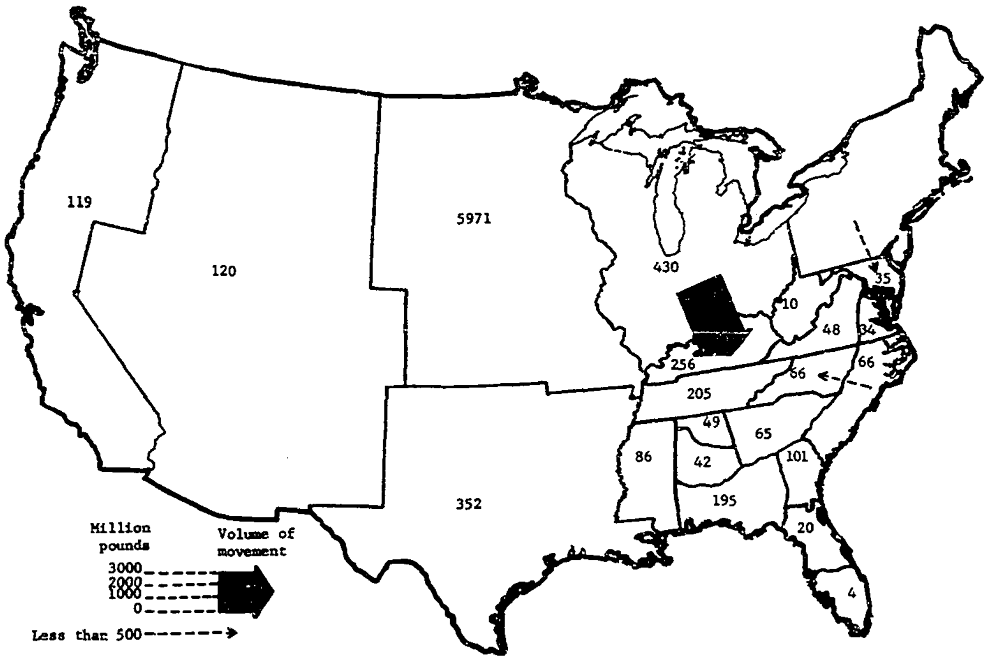

Figure 24. Shipments of hogs (in pork units) from hog supply area to hog slaughter area, 1970 annual solution with hog slaughter capacity in each area unlimited.

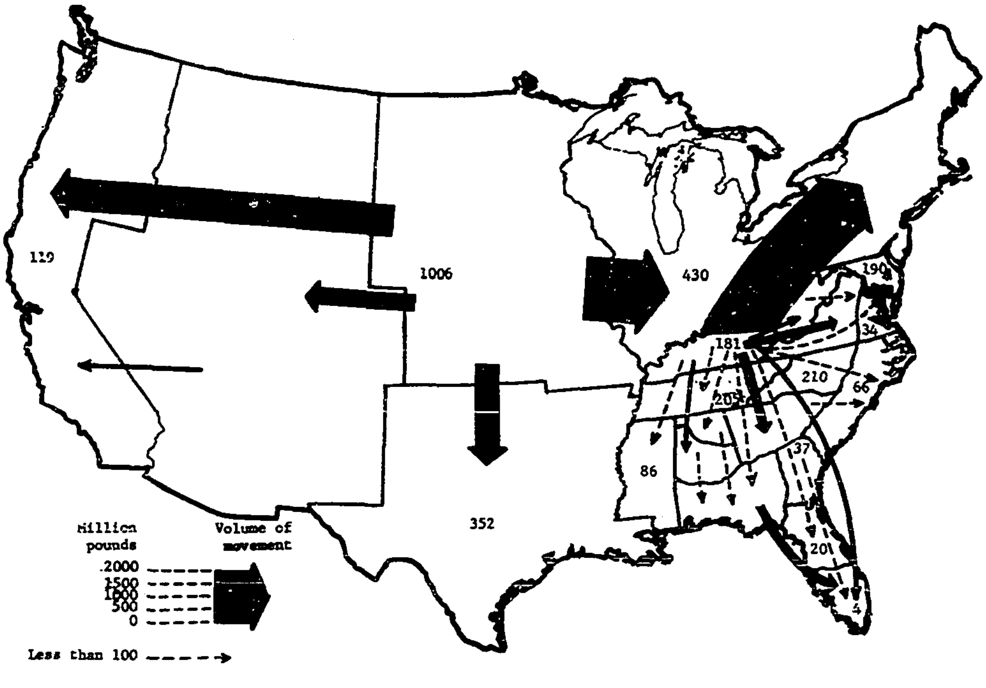

Figure 25. Shipments of pork from hog slaughter areas to pork demand areas, 1970 annual solution with hog slaughter capacity in each area unlimited.

Table 34. Comparison of 1960 and 1970 estimated hog slaughter capacities and the capacities specified for the areas by the 1970 model with hog slaughter capacity in each area unlimited

Area	Estimated 1960 capacities	Estimated 1970 capacities	Specified 1970 capacities	Col. 3 as a percent of Col. 1	Col. 3 as a percent of Col. 2
	(million pounds pork equivalent)			(percent)	
1	242	154	190	78.5	123.4
2	247	313	48	19.4	15.3
3	186	235	34	18.3	14.5
4	63	79	210	333.3	265.8
5	212	263	66	31.1	25.1
6	215	254	65	30.2	25.6
7	17	27	49	288.2	181.5
8	98	153	42	42.9	27.5
9	250	304	195	78.0	64.1
10	62	73	101	162.9	138.4
11	86	65	20	23.3	30.8
12	25	19	4	16.0	21.1
13	1,778	1,131	0	.0	.0
14	33	41	10	30.3	24.4
15	278	349	3,928	1412.9	1125.5
16	511	641	205	40.1	32.0
17	140	220	86	61.4	39.1
18	4,374	4,061	430	9.8	10.6
19	5,781	6,374	5,971	103.3	93.7
20	796	617	352	44.2	57.1
21	635	485	120	18.9	24.7
22	1,054	867	119	11.3	13.7
Total	17,083	16,725	12,245		

CONCLUSIONS AND IMPLICATIONS

The most significant parts of the solutions to the 1960 annual and quarterly models are the shipments of hogs from the Northcentral (areas 18 and 19) to the Southeast and shipments of pork from the Southeast to the Northeast (area 13). At the same time pork is shipped in from the Northcentral to supply a large share of the demand in the Southeast. This results from the $.75 per hundredweight pork equivalent differential between slaughtering costs in the Southeast and non-Southeast that is hypothesized in the 1960 models. This differential makes it economical to ship hogs into the Southeast for slaughter and then ship out enough pork to the Northeast to supply over one-third of the pork consumed in area 13 during 1960.

This differential in slaughtering costs also is chiefly responsible for the result that all of the Southeast areas, except 11 and 12, use 100 percent of their hog slaughter capacity in the 1960 solutions. (The only minor exceptions to this are in the third quarter solution where area 9 has 1 million pounds excess capacity out of a total capacity of 63 million pounds and in the fourth quarter solution where area 12 operates at full capacity.) This differential in slaughtering costs, along with the result that hogs can be shipped from area 18 to area 13 more cheaply than pork, explains why no pork is shipped from the two Northcentral areas to the Northeast in any of the 1960 solutions.

It should also be pointed out that all hog and pork supplies in each area are allowed to move, i.e., the models are not "netted-out" to establish surplus and deficit areas before the solutions are run. Because of this procedure, and because of the selection of different points within an area to represent the hog supply, hog slaughter and pork demand centers, some areas in the solutions ship their entire supply of hogs or pork out to other areas and then receive shipments of hogs or pork from other areas to satisfy their own requirements. Some of this arbitrage, however, is accounted for by the parameter routes in the solutions.

Two major conclusions result from the solutions to the 1960 models: (a) if the Southeast hog slaughtering costs are at least $.75 per hundredweight pork equivalent less than the non-Southeast slaughtering costs, then it is economically feasible from the standpoint of the entire United States hog-pork industry (under the assumptions stated earlier) to ship hogs into the Southeast from the Northcentral and slaughter at full capacity; and (b) some Southeast areas may slaughter at a very low percent of capacity during certain seasons and at full capacity during other seasons. The actual hog and pork movement estimates for 1959 and 1960 presented in Appendix C indicate substantial shipments of hogs into the Southeast from the North-central areas as well as shipments of pork into the North-east from the Southeast.

As noted in the analysis section, five Southeast areas had no Federally inspected slaughter in 1960. However, both the 1960 and 1970 models placed no restrictions on the outshipment of pork from these five areas across state lines. As a result, the solutions to the 1960 models show shipments of pork from areas 4, 7 and 10 across state lines to other areas. The solutions to the 1970 models show pork shipments from areas 7 and 10 across state lines but all the hogs slaughtered in area 4 are consumed in that area. Neither the 1960 or 1970 model solutions show outshipments of pork across state lines from areas 11 and 12, the other two Southeast areas with no Federally inspected hog slaughter in 1960.

Three important changes are made in the transfer costs before the 1970 models are solved: (a) normative cost functions are used to estimate hog and pork transportation costs during 1970, (b) the hog transportation costs are adjusted for tissue shrinkage and (c) a zero slaughtering cost differential between the Southeast and non-Southeast is used on the assumption that by 1970 labor mobility and unionization will eliminate a large portion of the major contributor to the 1960 cost differential.

The most important deviations of the solution to the first 1970 model from the solutions to the 1960 models are the absence of shipments of hogs into the Southeast from the Northcentral and the absence of any shipments of pork from the Southeast to the Northeast. This results from (a) elimination of the slaughtering cost differential, (b) use of normative hog and pork transport cost functions which produce a lower transportation cost for pork than for hogs when the haul is 350 miles or more and (c) adjustment of the hog transportation costs for tissue shrinkage. Also evident is the inshipment from area 18 of almost all of the pork consumed in area 13, with area 13 slaughtering only those hogs produced in area 13. As a result of this large out-shipment of pork from area 18, area 19 is forced to supply over two-thirds of the pork consumed in area 18.

Also noticeable in the first 1970 solution is the small amount of hog movement between areas with most of the hogs produced in each area slaughtered in the same area. This results from the normative transportation cost functions and the adjustment for tissue shrinkage.

In the second 1970 model, with hog slaughter capacity in each area unlimited, with one large exception there is again almost no movement of hogs between areas. The exception is the shipment of 3672 million pounds of hogs (in pork units) from area 18 to area 15. This is caused by the location of the three transportation centers chosen for the two areas; Indianapolis-Chicago-Chicago and Lexington-Louisville-Louisville for the i-j-k dimensions (refer to Table 23). Because Louisville is chosen as the hog slaughter center for area 15, it is more economical to route hogs from the area 18 hog transportation center, Indianapolis, to Louisville for slaughter and on to the Southeast and Northeast for consumption than it is to route them to Chicago for slaughter and then back east again.

The larger the geographic area the more difficult it is to select a center that adequately represents the entire area. Also, the selection of a hog supply center different from the hog slaughter center in the same area can also cause this kind of movement.

For this particular model the situation can be overcome by aggregating the results from areas 15 and 18. By doing this, the specified 1970 slaughter capacity for the two areas combined, as a percent of estimated 1960 capacity, is 93.7 percent (refer to Table 34). Likewise, the combined specified 1970 capacity, as a percent of estimated 1970 capacity, is 98.8 percent. The same problem exists, only in minor proportions, with areas 2, 3 and 4. By combining these areas, the results in Table 34 would be 58.9 percent for the combined 1970 specified capacity as a percent of combined 1960 estimated capacity, and 46.6 percent for the specified capacity as a percent of combined 1970 estimated capacity.

However, it is not necessary to explain all of the large shipment of hogs from area 18 to area 15 by shortcomings in the model. If Indianapolis does, at least in part, represent the center of hog production in area 18, then it certainly is economically sound for at least part of these hogs to move for slaughter in the same general direction as their final destination for consumption, namely east and south. The real fallacy in the model is that even though Louisville is in area 15, it really represents the Ohio River Valley area and Evansville, Cincinnati or several other points could do the same job.

The major conclusion from the 1970 models, keeping in mind all assumptions, is that given a zero slaughtering cost differential between the Southeast and non-Southeast, the deficit pork condition in the Southeast is most economically satisfied (from the standpoint of the entire United States hog-pork industry) by slaughtering hogs produced in the Northcentral region in the Ohio River Valley area and then shipping the pork on to the Southeast. If, however, substantial savings are evident in slaughtering costs in the Southeast, this distribution pattern for hogs and pork need not prevail.

As mentioned in the introduction, the objective of this study is to develop an operational model capable of analyzing a spatially separated hog-pork industry and use this model for a preliminary analysis of the interregional competitive position of the Southeast hog-pork industry. The results of this study show that under certain conditions the Southeast can compete favorably with the Northcentral region in the slaughter of hogs even to the extent of shipping hogs in from the Northcentral for slaughter in the Southeast. However, much more information is needed to accurately establish the interregional competitive position of the Southeast hog-pork industry. These research and data needs are discussed in the next section.

LIMITATIONS OF THE STUDY AND POSSIBLE FUTURE RESEARCH

Before a complete analysis can be made of the interregional competitive position of the Southeast hog-pork industry, the models used in this study must be expanded to include all sectors of the industry discussed in the introduction. This, in effect, means tying both the feeder pig and the hog feeding sectors of the industry in with the processing and distribution sectors.

In order to make such a model operational, several additional pieces of information not included in this study are needed. The more important ones are: (a) regional differences in input-output ratios for feeder pigs as well as for enterprises that compete for similar resources, (b) regional differences in input-output ratios for feeding slaughter hogs as well as for enterprises that compete for similar resources, (c) regional differences in costs of factors required for producing feeder pigs and slaughter hogs, (d) costs for transporting feeder pigs between regions, (e) regional differences in procurement costs for both feeder pigs and slaughter hogs, (f) more refined regional differences in hog slaughter and processing costs than were used in this study and (g) additional information on storage and processing in order to include these sectors in the economic model.

Several regional livestock production and marketing research committees are directing their work toward these needed pieces of information. Also, several state projects have been conducted or are being formulated in an attempt to answer many of the production questions.

Finally, additional methodological research is needed to allow the solution of this more general model of the hog-pork industry with supply expressed as a function of price, slaughter costs as a function of volume, demand as a function of price and so on.

The above are only some of the more critical pieces of information that are needed. It should be obvious that much is left to be done.

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APPENDIX A

In an earlier section two series of estimates are used as measures of hog production: (a) hog marketings plus farm hog slaughter and (b) January 1 inventory of all hogs and pigs on farms. Neither of these is a satisfactory measure of hogs sold for commercial slaughter. For this reason, the following procedure is used to estimate hogs sold for commercial slaughter in the 22 hog supply areas delineated in Figure 1.

The following equations indicate that the difference between (a) liveweight of hogs produced in year t and available for commercial slaughter and (b) liveweight of hogs sold for commercial slaughter in year t, by states, is the liveweight of inventory change in year t:

$$X_{kt} = M_{kt} - S_{kt} \pm I_{kt} \quad \text{(Eq. 23)}$$

and

$$Y_{kt} = M_{kt} - S_{kt} \quad \text{(Eq. 24)}$$

where

X_{kt} = liveweight of hogs produced in kth state during year t and available for commercial slaughter,

Y_{kt} = liveweight of hogs sold for commercial slaughter in kth state during year t,

M_{kt} = liveweight of hog marketings in kth state during year t (48),*

S_{kt} = liveweight of inshipments in kth state during year t (48),** and

I_{kt} = liveweight of inventory change in kth state during year t (48).

Equation 24 is used to derive liveweight estimates of e_i, pork equivalent weight of hogs sold for commercial slaughter, for each of the 22 hog supply areas during 1960. Y_{kt} is first computed for each of the 48 continental states. Each Y_{kt} is then adjusted as follows:

$$b_t = \frac{CS_t}{\sum_k Y_{kt}} \qquad \text{(Eq. 25)}$$

and

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*Marketings for each state include all shipments of feeders out of the state. By subtracting inshipments from marketings, each state is credited with producing the weight of its outshipments of feeders but is not credited with producing the weight of its inshipments of feeders.

**Hannawald, E. B., Agr. Est. Div., Stat. Rep. Ser., U. S. Dept. of Agr., Washington, D. C. Information on average weights of inshipments by states during 1960. Private communication. 1963.

$$Y'_{kt} = b_t Y_{kt} \qquad \text{(Eq. 26)}$$

where

b_t = adjustment factor for year t,

CS_t = liveweight of total U. S. commercial hog slaughter during year t, and

Y'_{kt} = liveweight of hogs sold for commercial slaughter in kth state during year t, adjusted for total U. S. commercial hog slaughter.

The results from Equation 26 are aggregated according to Figure 1 to provide liveweight estimates of e_i for hog supply areas 1 and 13 through 22. For hog supply areas 2 through 12 an additional procedure is required. The results from Equation 26 are disaggregated to the substate areas in Figure 26 according to the following:

$$p_{jkt} = \frac{J_{jkt}}{\sum_j J_{jkt}} \qquad \text{(Eq. 27)}$$

and

$$Y'_{jkt} = p_{jkt} Y'_{kt} \qquad \text{(Eq. 28)}$$

where

p_{jkt} = proportional factor for year t,

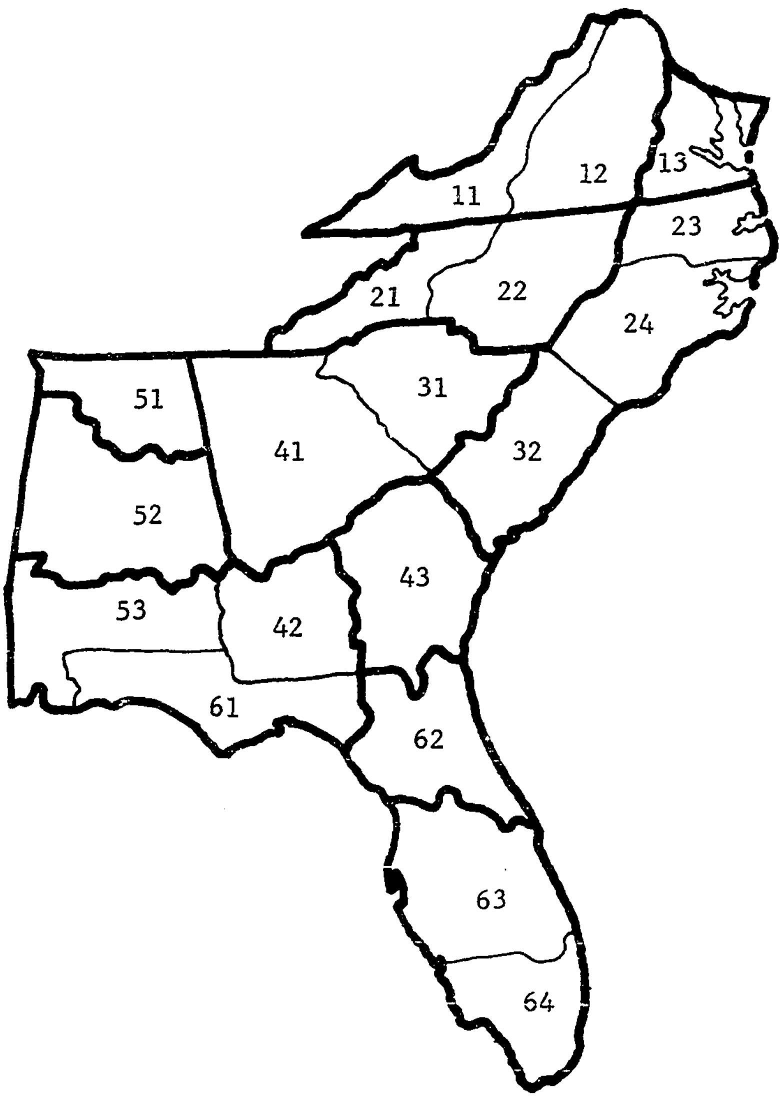

Figure 26. Southeast substate areas (indicated above by numbers) delineated by the Southern Regional Livestock Marketing Research Committee and used for developing hog supply, hog slaughter capacity and pork consumption estimates for areas 2 through 12 in Figure 1 (indicated above by heavy black lines).

J_{jkt} = January 1 inventory of all hogs and pigs on farms in jth substate area of kth state during year t, and

Y'_{jkt} = liveweight of hogs sold for commercial slaughter in jth substate area of kth state during year t, adjusted for total U. S. commercial hog slaughter.

The results from Equation 28 are aggregated according to Figure 26 to furnish liveweight estimates of e_i for hog supply areas 2 through 12. The liveweight estimates of e_i for all 22 hog supply areas are converted to pork (excluding lard and inedibles) equivalent units by multiplying times .585 (45, 46).

The 1960 estimates of e_i resulting from the above procedure are disaggregated to quarterly estimates as follows:

$$a_{iq} = \frac{CS_{iq}}{CS_i} \qquad \begin{array}{l} i = 1,\ldots,22 \\ q = 1 \text{ for Jan.-Mar.,} \\ \quad\;\; 2 \text{ for Apr.-June,} \\ \quad\;\; 3 \text{ for July-Sept., and} \\ \quad\;\; 4 \text{ for Oct.-Dec.} \end{array} \qquad \text{(Eq. 29)}$$

and

$$e_{iq} = a_{iq} e_i \qquad \text{(Eq. 30)}$$

where

a_{iq} = quarterly slaughter as a percent of annual slaughter in ith hog supply area during qth quarter in 1960,

CS_i = liveweight of commercial hog slaughter in ith hog supply area during 1960 (46),

CS_{iq} = liveweight of commercial hog slaughter in ith hog supply area during qth quarter in 1960 (46),

e_{iq} = pork (excluding lard and inedibles) equivalent estimate of hogs sold for commercial slaughter in ith hog supply area during qth quarter in 1960, and

e_i = pork (excluding lard and inedibles) equivalent estimate of hogs sold for commercial slaughter in ith hog supply area during 1960.

A separate set of a_{iq}'s is computed for hog supply areas 1 and 13 through 22. However, only one set of a_{iq}'s is computed for hog supply areas 2 through 12 based on the combined commercial hog slaughter in the six states involved.

APPENDIX B

The derivation of pork consumption estimates involves basically two procedures: (a) estimation of population and (b) estimation of per capita consumption. The procedures outlined below are essentially those used by Stout et al. (39) for estimating pork consumption in the South during 1956, 1957 and 1958. The substate areas used in estimating pork consumption, g_k, for pork demand areas 2 through 12 in Figure 1 are delineated in Figure 26. For pork demand areas 1 and 13 through 22, all references to substate areas in the following procedures can be ignored because pork consumption for these areas is estimated by states and then aggregated where necessary.

Procedure for Estimating Population

Census population data are used in deriving 1960 pork consumption estimates; however, an estimation procedure is required for developing similar population figures for 1970.

Ratio estimates of population for each type of residence by substate areas for 1970 are derived by the following equation:

$$\overline{P}_{ijk} = \frac{P'_{ijk}}{P''_{ijk}} (P'_{ijk})$$

i = 1 for urban, 2 for rural nonfarm and 3 for farm (Eq. 31)

j = 1,...,4 for substate areas in Va., N. C., S. C., Ga., Ala. and Fla. (see Figure 26)--the j subscript is ignored for all other states

k = 1,...,48 for the continental states

where

$\overline{P}_{ijk}$ = ratio estimate of 1970 population for ith type of residence in jth substate area of kth state,

P'_{ijk} = 1960 population for ith type of residence in jth substate area of kth state (51), and

P''_{ijk} = 1950 population for ith type of residence in jth substate area of kth state (50).

The 1970 ratio estimates for each state are adjusted by

$$\overline{P}_k = \sum_i \sum_j \overline{P}_{ijk} \qquad \text{(Eq. 32)}$$

and

$$\frac{P_k}{\overline{P}_k} = b_k \qquad \text{(Eq. 33)}$$

where

$\overline{P}_k$ = ratio estimate of 1970 population in kth state,

P_k = National Education Association estimate of 1970 population in kth state (33), and

b_k = adjustment factor for 1970 population estimates in kth state.

With the use of the appropriate adjustment factor, b_k, for those substate areas within the kth state, the final 1970 population estimates are derived from

$$P_{ijk} = b_k \overline{P}_{ijk} \quad \text{(Eq. 34)}$$

and

$$P_{jk} = \sum_i P_{ijk} \quad \text{(Eq. 35)}$$

where

P_{ijk} = final estimate of 1970 population for ith type of residence in jth substate area of kth state and

P_{jk} = final estimate of 1970 population in jth substate area of kth state.

The above procedure is valid under two assumptions:

(a) The relative change in population by type of residence within each substate area will continue at the same rate between 1960 and 1970 as occurred between 1950 and 1960.

(b) The relative change in population for each sub-state area within a state will continue at the same rate between 1960 and 1970 as occurred between 1950 and 1960.

(c) The National Education Association's 1970 population estimates by states are valid.

Procedure for Estimating Per Capita Consumption

The procedure used to estimate per capita consumption for the South (areas 2-12, 14, 15, 16, 17 and 20) in Figure 1 involves the adjustment of U. S. per capita consumption for (a) regional differences in consumption, (b) racial differences in consumption and (c) differences in type of residence. Per capita consumption estimates for the Northeast (area 13), the Northcentral (areas 18 and 19) and the West (areas 21 and 22) in Figure 1 are only adjusted for (a) and (c) above.

Racial differences in per capita consumption for each type of residence in the South are derived by the following set of equations:

$$T_1 = w_1 W_1 + n_1 N_1 \qquad \text{(Eq. 36)}$$

and

$$w_i = b_i n_i \qquad \text{(Eq. 37)}$$

where

T_i = weekly per capita consumption of pork for ith type of residence in the South during 1955 (3, p. 11),

W_i = white proportion of population for ith type of residence in the South during 1960 (51),

N_i = nonwhite proportion of population for ith type of residence in the South during 1960 (51),

w_i = weekly white per capita consumption of pork for ith type of residence in the South,

n_i = weekly nonwhite per capita consumption of pork for ith type of residence in the South, and

b_i = ratio of white per capita consumption to nonwhite per capita consumption for ith type of residence in the South (39, p. 20).

The parameters w_i and n_i are obtained for all three types of residences in the South by simultaneously solving the above set of equations. These parameters are then used to obtain estimates of per capita consumption for each type of residence in each substate area or state in the South during 1960 and 1970 by the equation

$$T_{ijkn} = w_i W_{ijkn} + n_i N_{ijkn} \qquad n = \text{year (1960 and 1970)} \qquad \text{(Eq. 38)}$$

where

T_{ijkn} = weekly per capita consumption of pork for ith type of residence in jth substate area of kth state during nth year, adjusted for racial composition and regional differences in consumption,

W_{ijkn} = white proportion of population for ith type of residence in jth substate area of kth state during nth year (50, 51),* and

N_{ijkn} = nonwhite proportion of population for ith type of residence in jth substate area of kth state during nth year (50, 51).*

The final estimate of annual per capita pork consumption in jth substate area of kth state for nth year is obtained from

$$Z_{jkn} = \frac{X_n}{Y} \cdot \frac{\sum_i (T_{ijkn} \cdot P_{ijkn})}{P_{jkn}} \qquad \text{(Eq. 39)}$$

where

Z_{jkn} = annual per capita consumption of pork in jth substate area of kth state during nth year,

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*The 1970 white and nonwhite proportion for each type of residence in each substate area or state in the South is estimated by the procedure defined by Equations 31 through 35.

adjusted for region, race and type of residence,

X_n = annual U. S. per capita consumption of pork for nth year (46),*

Y = weekly U. S. per capita consumption of pork (3, p. 11),

P_{ijkn} = population for ith type of residence in jth substate area of kth state during nth year, and

P_{jkn} = population for all residences in jth substate area of kth state during nth year.

This estimating procedure is valid under the following assumptions:

(a) The change in population by race for each type of residence in each substate area will continue at the same rate between 1960 and 1970 as occurred between 1950 and 1960.

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*An estimate of 1970 U. S. per capita pork consumption was derived by least squares regression of U. S. per capita pork consumption (Y) on time (X). The regression equation was $Y = 70.158 - .4958X$ with an R value of .519. Another regression was run with an additional term (X^2) added. The result was $Y = 70.9300 - .7523X + .0153X^2$ with an R value of .571 and an R^2 value of .326. The first equation gives an estimate of 58.2 pounds (Y) for 1970 and the second equation gives an estimate of 61.7 pounds for 1970. Due to the nature of the data and the poor fit of the estimating equations, the author has elected to use 60 pounds as an estimate of 1970 U. S. per capita pork consumption in this study.

(b) The annual per capita consumption of pork for each type of residence in the South is proportionally related to annual U. S. per capita consumption of pork in 1960 and 1970 as it was in 1955 and that the relationship is stable over all seasons and over the hog production cycle.

(c) The racial patterns of pork consumption are uniform for each type of residence and these patterns for 1960 and 1970 are accurately characterized by the Atlanta Consumer Panel for urban and rural nonfarm residents and by the Moser study for farm residents.

(d) The annual U. S. per capita pork consumption in 1970 will be 60 pounds.

Pork Consumption, 1960 and 1970

The final estimate of total pork consumption in jth substate area of kth state during nth year, C_{jkn}, is obtained by the equation

$$C_{jkn} = Z_{jkn} \cdot P_{jkn}. \qquad \text{(Eq. 40)}$$

Since the analytical model only considers hogs sold for commercial slaughter and commercial hog slaughter capacity, the estimates of total pork consumption, C_{jkn}, are adjusted for farm slaughter by the following:

$$C'_{jkn} = C_{jkn} - (FS_{jkn})(AW_n)(DP_n) \qquad \text{(Eq. 41)}$$

where

C'_{jkn} = estimate of pork (excluding lard and inedibles) consumption (emanating from commercial slaughter) in jth substate area of kth state during nth year,

FS_{jkn} = number of hogs slaughtered on farms in jth substate area of kth state during nth year (40, 48),*

AW_n = average liveweight of hogs slaughtered on farms in the United States during 1960 (46), and

DP_n = dressing percentage of hogs slaughtered on farms in the United States during 1960 (46).

For n = 1970, the 1960 estimates of AW_n and DP_n are used to adjust 1970 estimates of C_{jkn}. However, $FS_{kn} = \sum_j FS_{jkn}$ is assumed to decrease between 1960 and 1970 at the same rate as between 1950 and 1960. A 1970 ratio estimate of FS_{kn} is computed by the procedure defined by Equation 31.

The estimates of pork consumption, C'_{jkn}, for the substate areas defined in Figure 26, are aggregated to furnish estimates of g_k for pork demand areas 2 through 12 in

*Substate area estimates are computed by the procedure defined by Equations 27 and 28 in Appendix A.

Figure 1.

The procedures presented by Equations 31 through 41 are used to estimate pork consumption, C'_{kn}, for all states not included in pork demand areas 2 through 12. These state estimates are then aggregated, where necessary, to furnish estimates of g_k for pork demand areas 1 and 13 through 22 in Figure 1.

The 1960 estimates of g_k resulting from the above procedures are disaggregated to quarterly estimates by the equation

$$g_{kq} = a_{kq} g_k \qquad \begin{array}{l} k = 1, \ldots, 22 \\ g = 1 \text{ for Jan.-Mar.,} \\ \quad\; 2 \text{ for Apr.-June,} \\ \quad\; 3 \text{ for July-Sept. and} \\ \quad\; 4 \text{ for Oct.-Dec.} \end{array} \qquad \text{(Eq. 42)}$$

where

g_{kq} = estimate of pork (excluding lard and inedibles) consumption (emanating from commercial slaughter) in kth pork demand area during qth quarter in 1960,

a_{kq} = quarterly consumption as a percent of annual consumption in kth pork demand area during qth quarter in 1960, and

g_k = estimate of pork (excluding lard and inedibles) consumption (emanating from commercial slaughter) in kth pork demand area during 1960.

The 1960 estimates of g_{k} for pork demand areas 2 through 12 are disaggregated to quarterly estimates on the basis of consumption data from the Atlanta Consumer Panel (39, p. 27). For pork demand areas 1 and 13 through 22, similar data on seasonal pork consumption from the Michigan State University Consumer Panel are used.* These data are presented in Table 35.

Table 35. Seasonal distribution of pork consumption

Quarter	Atlanta Consumer Panel 1959	Michigan State University Consumer Panel 1956-58
	(percent of annual total)	
January-March	24.8	26.0
April-June	22.6	23.8
July-September	23.1	23.2
October-December	29.5	27.0
Total	100.0	100.0

*Riley, Harold M., Dept. of Agr. Econ., Michigan State University, East Lansing, Michigan. Information on seasonal distribution of pork consumption from Michigan State University Consumer Panel. Private communication. 1963.

APPENDIX C

The Southern Regional Livestock Marketing Research Committee, under research project SM-23, has investigated the movement of hogs and pork in the Southeast (11, 12, 37).

The actual hog and pork movement estimates developed by the Committee for one-week periods during November 1959 and February, May and August 1960 are presented in Tables 36 through 43 (12, 37).

Tables 36 through 39 give the movements of slaughter hogs into, within and out of areas 2 through 12 delineated in Figure 1. Tables 40 through 43 give the shipments of pork by Southeast packers within and out of areas 2 through 12.

Table 36. Shipments of slaughter hogs within, into and out of the Southeast areas delineated in Figure 1, November 15-21, 1959 (12)[a]

Origin	Destination																		
	1	2	3	4	5	6	7	8	9	10	11	13	14	15	16	17	18	Unk.[b]	Total
	(head)																		
1		323																	323
2	269	2451		7								1176	206		23			1	4133
3	737	281	2507									89						21	3635
4	8	651		2182	214												12		3067
5		3130	11711	88	11443	232								328			11		26943
6				141		3876			392	13							6	6	4434
7					209	113	949	2014							1482	594	1		5362
8						1213	148	2392							14	32			3799
9					857	2117		309	29748	655	4442						67	250	38445
10		469	1154		2625	3081			1832	5438	582						110	6099	21390
11											2053								2053
15			363		637	3555		1063			231								5849
16				239		2220	394	3063			86								6002
17							4	33	30										67
18		7814	3575	1816	3886	2339	366	162	341										20299
19		2397	1111		1225	1753	124	1407	1029		618								9664
Total	1014	17516	20421	4473	21096	20499	1985	10443	33372	6106	6012	1265	206	328	1519	626	207	6377	155465

[a]No data available for area 12. No shipments out of the Southeast into areas 19, 20, 21 and 22. No shipments into the Southeast from areas 13, 14, 20, 21 and 22.

[b]Destination unknown.

Table 37. Shipments of slaughter hogs within, into and out of the Southeast areas delineated in Figure 1, February 14-20, 1960 (12)[a]

Origin	Destination																
	1	2	3	4	5	6	7	8	9	10	11	13	16	17	18	20	Total
								(head)									
2	625	3509	91									2908	235				7368
3	1255	794	5360									17					7426
4		787	13836	3014	268												17905
5	266	8087		147	16553	428									60		25541
6					294	5996									18		6308
7						142	1168	1857	128				1434	70			4799
8						4385		2224						20			6629
9					236	3299		413	24392	261	2338				76		31015
10		587	403		1522	3368			692	4169					74	21	10836
11											4102						4102
13		20															20
15			331	299	913	4972		810									7325
16						4630	257	2989	292								8168
17							12	54									66
18		4912		841	173	1033		291	557								7807
Total	2146	18696	20021	4301	19959	28253	1437	8638	26061	4430	6440	2925	1669	90	228	21	145315

[a]No data available for area 12. No shipments out of the Southeast into areas 14, 15, 19, 21 and 22. No shipments into the Southeast from areas 1, 14, 19, 20, 21 and 22.

Table 38. Shipments of slaughter hogs within, into and out of the Southeast areas delineated in Figure 1, May 15-21, 1960 (12)[a]

Origin	Destination 1	2	3	4	5	6	7	8	9	10	11	13	14	15	16	17	18	Unk.[b]	Total
									(head)										
2	441	5167	195									1656	97		11		14	150	7731
3	1480	443	4908		34							33						21	6919
4	490	278		4402	218	29						27			65				5509
5	400	6941	11177	29	18715	522						40					33		37857
6					133	8985						4					55		9177
7						862	1414	2861	33						1724	193			7087
8						2926		3968						44		41	12		6991
9					182	402		213	19644	66	2271						27		22805
10		267	1717		771	2963			725	2953							227		9623
11									296		3384								3680
14		53																	53
15			316		526	2284		454											3580
16						1869	242	2234	463										4808
17							36												36
18		2244				87													2331
19								264											264
Total	2811	15393	18313	4431	20579	20929	1692	9994	21161	3019	5655	1760	97	44	1800	234	368	171	128451

[a]No data available for area 12. No shipments out of the Southeast into areas 19, 20, 21 and 22. No shipments into the Southeast from areas 1, 13, 20, 21 and 22.

[b]Destination unknown.

Table 39. Shipments of slaughter hogs within, into and out of the Southeast areas delineated in Figure 1, August 14-20, 1960 (12)[a]

Origin	Destination																
	1	2	3	4	5	6	7	8	9	10	11	13	16	17	18	Unk.[b]	Total
									(head)								
2	502	5929		140								1195	23		17	84	7890
3	405	1193	3408		467							45				103	5621
4		418		3091	688	80							69				4346
5	262	4242	10634	133	14250	623											30144
6					262	7850									47		8159
7						563	1167	2480					1256	44			5510
8						671		3727	79					124			4601
9						561		746	16798		2500[c]				57		20662
10					1776	2301			1136	2561					97		7871
11									465		3000[c]						3465
15		224	1981		1794	1905		342									6246
16					321	1867	223	1469	328								4208
17							1		52								53
18		3737	1364	727	444	707			289								7268
Total	1169	15743	17387	4091	20002	17128	1391	8764	19147	2561	5500	1240	1348	168	218	187	116044

[a]No data available for area 12. No shipments out of the Southeast areas into areas 14, 15, 19, 20, 21 and 22. No shipments into the Southeast from areas 1, 13, 14, 19, 20, 21 and 22.

[b]Destination unknown.

[c]Shipments estimated.

Table 40. Shipments of total pork by packers in the Southeast areas delineated in Figure 1, except areas 1, 3 and 12, November 15-21, 1959 (37)

Origin	Destination															
	1	2	3	4	5	6	7	8	9	10	11	12	13	a	b	Total
	(thousand pounds)															
2	340.1	1783.7	466.9	305.6	168.9								114.6	55.7		3235.5
4				864.1	42.3											904.6
5	48.5	19.5	47.5	1109.4	1375.5	251.3			57.4	36.8	62.9	61.3	273.7			3343.8
6				31.3	283.2	2425.3			225.3	197.3						3162.4
7							291.7	27.4								319.1
8			12.1	26.8	15.2	155.9	55.2	1419.0	126.5		20.6	14.4			98.7	1944.4
9			68.6	28.1	163.2	644.8		133.1	1522.7	354.6	404.6	1074.4			67.3	4461.4
10						102.2			39.1	616.0						757.3
11									116.9		826.3	488.2				1431.4

[a]Includes shipments to the states of Tennessee, Kentucky and West Virginia.

[b]Includes shipments to the states of Mississippi, Louisiana and Arkansas.

Table 41. Shipments of total pork by packers in the Southeast areas delineated in Figure 1, except areas 1, 3 and 12, February 14-20, 1960 (37)

Origin	Destination																
	1	2	3	4	5	6	7	8	9	10	11	12	13	a	b	c	Total
	(thousand pounds)																
2	627.3	1240.6	313.9	324.0	145.5								89.2	36.9			2777.4
4				922.1	42.8												964.9
5	119.0	32.9	144.8	914.5	1499.2	263.5				75.7	13.5	34.4	243.4		41.5		3882.4
6		2.5		25.1	270.6	2279.3			241.1	169.9		23.0	89.8				3101.3
7							350.0	62.6									412.6
8			18.6	25.1	6.2	115.3	28.5	968.1	103.1					19.1	13.1		1297.1
9	79.1		115.9	30.2	171.6	850.2		97.1	1704.5	328.2	476.9	813.2			28.9	43.0	4738.8
10						78.0			51.9	575.0							704.9
11									32.2		392.4	68.6					493.2

[a]Includes shipments to the states of Tennessee, Kentucky and West Virginia.

[b]Includes shipments to the states of Mississippi, Louisiana and Arkansas.

[c]Includes shipments to the states of Missouri, Illinois, Indiana, Ohio, Michigan, Wisconsin, Iowa and Minnesota.

Table 42. Shipments of total pork by packers in the Southeast areas delineated in Figure 1, except areas 1, 3 and 12, May 15-21, 1960 (37)

Origin	Destination																
	1	2	3	4	5	6	7	8	9	10	11	12	13	a	b	c	Total
	(thousand pounds)																
2	522.1	1350.1	371.2	244.2	305.1								207.9	45.7			3046.3
4				709.9	20.9												730.8
5	557.8	59.2	100.9	658.7	1235.1	187.4				61.7	15.7	34.0	305.9	7.7			3224.1
6				29.7	332.9	2077.9			223.0	192.0		36.1	131.3				3022.9
7							271.4	22.6									294.0
8		5.3	21.0	23.7	40.8	157.9	46.0	1215.9	115.3			30.8	29.2	84.1	18.1		1788.1
9	127.5			23.0	100.7	531.0		89.1	1087.3	298.6	172.1	863.0	56.3		298.5	16.2	3663.3
10						91.3			41.8	342.6							475.7
11									71.6		229.5	62.9					364.0

[a] Includes shipments to the states of Tennessee, Kentucky and West Virginia.

[b] Includes shipments to the states of Mississippi, Louisiana and Arkansas.

[c] Includes shipments to the states of Missouri, Illinois, Indiana, Ohio, Michigan, Wisconsin, Iowa and Minnesota.

Table 43. Shipments of total pork by packers in the Southeast areas delineated in Figure 1, except areas 1, 3 and 12, August 14-20, 1960 (37)

Origin	Destination															
	1	2	3	4	5	6	7	8	9	10	11	12	13	a	b	Total
	(thousand pounds)															
2	479.5	1327.8	336.3	231.5	155.3								148.9	51.5		2730.8
4				777.4	19.6											797.0
5	221.3	111.7	91.8	848.8	1320.5	282.0				61.6	8.3	28.7	113.6	3.4		3091.7
6	26.9				257.6	2322.6			257.8	204.7			301.8			3371.4
7							284.8	30.6								315.4
8			17.0	31.1	20.2	138.3	59.0	1192.3	141.0						16.2	1615.1
9				23.3	47.2	711.2		148.8	1258.6	302.1	240.2	651.0			35.3	3417.7
10						87.4			53.6	407.6						548.6
11									28.2		329.2	63.1				420.5

[a] Includes shipments to the states of Tennessee, Kentucky and West Virginia.

[b] Includes shipments to the states of Mississippi, Louisiana and Arkansas.