77-29,845

JENSEN, Ronald Eugene, 1949-
THE ANALYSIS OF LIFE TABLES WITH SPECIFIC GROUPS OF CAUSES OF DEATH ELIMINATED.

Iowa State University, Ph.D., 1977
Sociology, demography

**Xerox University Microfilms**, Ann Arbor, Michigan 48106

The analysis of life tables with specific groups of causes of death eliminated

by

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A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Department: Sociology and Anthropology
Major: Sociology

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

For the Major Department

Signature was redacted for privacy.

For the Graduate College

Iowa State University
Ames, Iowa

1977

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# CHAPTER 1. INTRODUCTION

There is a universal value attached to human life. By virtue of this universal value, further improvement in mortality conditions remains a major human goal. Osborne (1958:xi) notes that

> Expectation of a long life is the most tangible end product of western European civilization. It distinguishes the people of the United States, of Canada, of western Europe, of Australia and New Zealand, and the people of western European descent all over the world. Whatever history may ultimately say about our achievements, it will record that we were the first in all the long history of man to live out our allotted span. Beside this gift the material things of our civilization seem insignificant. Who would exchange 30 years of life for all the automobiles, radios, television sets, telephones, or even all the bathtubs in the United States?

Thus, the search for longer life seems to be almost universal throughout history and in most societies. This search is related to the basic drive for self-preservation inherent in individual and group survival. Many ancient writings show the high value placed on long life in early societies. The Old Testament, for example, promises long life as a reward for obeying the Commandments. Ponce de Leon is only the most famous of a long line of men who spent their lifetimes seeking a longer life. Medical science is dedicated to preserving longer life by combatting disease and death.

One of the ways of prolonging life is to determine the factors that contribute to longer life and attempt to manipulate these factors to increase longevity. In this context, it is useful to distinguish between prolongevity and longevity. According to Gruman (1966:6), prolongevity may be defined as the "significant extension of the length of life by human action." The prefix "pro" is used to indicate a "moving forward" while longevity retains its customary reference to "length of life." The belief that prolongevity is possible <u>and</u> desirable is referred to as prolongevitism.

Dublin et al. (1949:27-28) note that the term "length of life" (i.e., longevity) may refer to either of two phenomena. One meaning of the phrase refers to the average number of years a person can expect to live, commonly referred to as life expectancy. Life expectancy has increased greatly during the course of history with the most striking advances in the past century. Historians have estimated that life expectancy in ancient Greece and Rome was about 20 years. Only a very gradual improvement occurred over the fourteen centuries between the decline of the Roman Empire and the beginning of the 18th century. Life expectancy at the beginning of the 1700's has been estimated to be 30 years. By 1800 life expectancy in the more advanced countries had reached 35 years and by 1900 it was nearly 50 years in England, Sweden, and the United States. In the 1970's, life expectancy in the most industrialized nations stands above 70 years.

Secondly, "length of life" refers to the concept of "life span." In contrast to life expectancy which refers to the length of life of the average person, life span refers to the longevity of the most long-lived persons. Life span, then, is the extreme limit of age in human life. It is the age beyond which virtually no one can expect to survive. Although there is no known exact value for this concept, life span is estimated between 100 and 110 years at the present time. Life span, unlike life expectancy, has not increased noticeably during the course of history (Dublin et al., 1949:27-29; Shryock and Siegel, 1973:433).

In describing the idea of prolongevitism, Gruman (1966:6) distinguishes between two philosophical traditions, apologism and meliorism. Apologism condemns any attempt to basically alter earthly conditions by human action. Within the context of prolongevitism, apologism may be defined as the belief that prolongevity is neither possible nor desirable. Meliorism, however, may be thought of as the antonym of apologism. Meliorism implies that human efforts can and should be applied to improving the world. In the narrower context of prolongevitism, meliorism appears to be an indispensable element in modern society, for a community based on industry, technology, and science must continue to progress or face disaster. The most relevant example is the meliorist efforts of public health reformers and medical researchers in the 19th century to bring infectious diseases under control, resulting in an increase in the average length of life. Concomitantly, a new problem has been created, the problem of an aging population and a society burdened with larger numbers of disabled, indigent, and chronically ill persons. Consequently, the community is necessarily diverting money into research on the nature of degenerative diseases and the process of aging itself. It can be predicted that these efforts will further increase the length of life. From this experience it is apparent that meliorism is inherent in the structure of modern society with its emphasis on progress and improved well-being.

Prolongevitist thinkers, whether moderate or radical, visualize not merely an increase in time _per se_ but an extension of the healthy and productive period of life. Prolongevitists advocate the search for long life in conjunction with the quest for a vital life.

It is against this basic framework of the value of human life and the ideas of prolongevitism and meliorism that the present study is undertaken. The basic problem to be addressed is the consequences resulting from improvement in mortality conditions. In the present study, this improvement constitutes a complete elimination of a given group of causes of death. All persons must eventually die. It appears likely, in the absence of violent or exogenous forces of destruction, that the continuous process of cellular renewal which occurs within the human body must ultimately breakdown and fail to support life. The average length of life has increased so dramatically in the past century that nearly everyone can foresee the possibility of a certain degree of mortality improvement and further extension of human life. All that is necessary is something like a more powerful drug against cancer, more effective measures to prevent motor vehicle accidents, or the provision of better medical facilities to some segments of the population.

Science can be expected to influence human longevity in two distinct ways: by suppressing causes of premature death and by postponing or slowing the process of aging itself. The first of these influences has already meant that more persons in the developed nations have achieved or approached the natural life span but has not altered those ages appreciably. The second influence is in the stage of active research (Comfort, 1970:157).

In this study, hypothetical improvements in mortality conditions are analyzed through the method known as the life table. The life table is one of the oldest, most useful, and best known topics in the fields of demography, actuarial science, and statistics. The statistical analysis of death is as old as demography itself. In fact, it has been argued very convincingly that the origin of demography can be traced to Graunt's analysis of mortality in England and Wales in the mid-17th century (Bourgeois-Pichat, 1963:194). The subject matter to which life table methods have been applied is by no means limited to human beings. Zoologists, biologists, physicists, manufacturers, and investigators in many fields have found the life table method useful in analyzing and presenting data (Chiang, 1960a:618).

The data and analysis techniques of this study emphasize age. Perhaps the most fundamental feature of any population is the distribution of its members by age. Stockwell (1972:1) asserts that almost any aspect of human behavior from subjective to physiological to objective characteristics may be expected to vary with age. Furthermore, the present and future needs of a society are in large part determined by the age structure of its population.

Since mortality varies by age and by sex, it is crucial to relate deaths at each age by sex to the number of persons at that age and sex and, thus, obtain age-sex-specific mortality rates. To evaluate the impact of certain diseases on human longevity, these rates may be translated through life table methods into probabilities of dying and surviving and life tables may be generated to compare the mortality, survival, and longevity experience of the current population with the hypothetical experience of the same population under improved mortality conditions resulting from the elimination of a certain group of causes of death. Such comparisons are the purpose of this study.

The life table is one method of illustrating a set of age-specific mortality rates. In general, the life table follows a generation or cohort of births as it progresses through life, subjecting the cohort to the rates of mortality at successive ages, and observing their survival. Perhaps the most familiar life table value is life expectancy. However, the other life table functions are of equal importance. The purpose of this study is to compare and analyze the life table due to all causes and life tables resulting from the hypothetical elimination of certain groups of causes of death. This study distinguishes two interpretations of life tables and describes methods appropriate to each interpretation that may be used to compare life tables due to all causes with those in which a cause of death has been hypothetically eliminated.

Chapter 2 describes the method of constructing abridged life tables due to all causes. The method of constructing special life tables in which a specific group of causes of death has been hypothetically eliminated is presented in Chapter 3. Neither Chapters 2 nor 3 present a formal mathematical discussion of life table methods[1] but instead present a commonsense explanation of the life table understandable to those unfamiliar with the life table method. Sources of, and adjustments to, the data are described in Chapter 4. The methods used to compare the main life tables due to all causes with the special life tables due to elimination of causes of death, and the results of these comparisons, are presented in Chapters 5 and 6.

The life table may be interpreted in one of two ways. First, the life table may be viewed as a single cohort of 100,000 births which is subject to a set of age-specific mortality rates as it passes through successive ages until the terminal age group in which none of the original cohort survives. Methods of comparison appropriate to this interpretation of the life table are presented in Chapter 5. Such comparisons are based on competing risk theory, conditional probabilities and joint probabilities of survival, gains in life expectancy, alternative measures of longevity, and descriptive statistical analysis of life table deaths.

The second interpretation of the life table is that of the stationary population. The stationary population is a

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[1]Those readers wishing a more mathematical explanation of the life table and its properties may refer to Wilson (1938), Greville (1943), Chiang (1960a, 1960b, 1972), and Keyfitz (1966a, 1966b, 1968a, 1970).

special case of stable population theory. Methods appropriate to this life table interpretation are presented in Chapter 6. These methods are primarily demographic in nature and are directed toward comparing the age distributions of stationary populations.

In Chapter 6, a distinction is made between individual measures of age distribution (summary measures of each distribution) and comparative measures of age distribution (summary measures of comparison between two or more distributions). Individual measures of age distribution include median age, proportion of young and old persons, and index of aging. Comparative measures of age distribution include index of dissimilarity, age-specific indexes, and goodness-of-fit tests. Chapter 6 includes a discussion of the general uses of methods of comparing life tables and their relevance to determining the effect of mortality improvements on the individual and the larger social system.

Chapter 7 describes special methods used to compare and analyze gains in life expectancy. This chapter focuses on Crosson's (1963) method of analyzing changes in life expectancy due to improved mortality at a given age and beyond and improved mortality prior to that age, and regression analysis of gains in life expectancy.

Chapter 8 presents a summary of the present study and conclusions.

# CHAPTER 2. METHOD OF CONSTRUCTING ABRIDGED LIFE TABLES

> Suppose we imagine a thousand babes to start together along the bridge or crossway of life. The length of that bridge shall represent the maximum duration of life, and our cohort shall march slowly across it, completing the journey in something perhaps over the hundred years. No, - not the cohort completing the journey, the veriest remnant of the thousand who started together! At each step Death, the marksman, takes his aim, and one by one individuals fall out of the ranks - terribly many in early infancy, many in childhood, fewer in youth, more again in middle age, but many more still in old age. At every step forward the target alters; those who fall at twenty cannot be aimed at, at sixty, and the long line of life which serves Death as a target reduces almost to nothing at the extreme end of the Bridge of Life (Pearson, 1897:25).

Although Pearson did not intend it as such, the preceding analogy presents a vivid description of the basic idea underlying the construction of the life table. The life table represents a method of combining a set of age-specific mortality rates of a population into a single probability model. The entire life table is generated from age-specific mortality rates. Resulting values are used to measure mortality, survivorship, and life expectancy.

The life table is one of the most useful and versatile of the demographer's tools. It is designed as a measure of the level of mortality of a given population. It is, however, employed in a number of ways to areas other than the study of mortality. It is used by public health specialists, demographers, actuaries, and others in the study of longevity, fertility, migration, and population growth. It is also an important tool in making population projections and in studies of widowhood, orphanhood, and length of married, working, and disability-free life.

## Historical Development of Life Tables

The concept of the life table originated in studies of human longevity. Long before the development of modern probability and statistics, scholars concerned with the length of life constructed tables to measure longevity. A crude table was constructed as early as the middle of the third century A.D. by the Praetorian Praefect Ulipianus (Chiang, 1968:189). Not until the 17th century, however, was the study of longevity undertaken in a manner that may be regarded as the predecessor to modern life table methods. It is the work of Graunt (1662) and Halley (1693) that is generally recognized as leading to the construction of the life table.

The first outline of what later came to be called the life table is found in John Graunt's (1662) book, _Natural and Political Observations on the Bills of Mortality_.[1] Its contents were strictly conjectural but its form set the precedent for the death and survivorship columns of all future life tables. Graunt's work was based on his analysis

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[1]There is some speculation that Sir William Petty, not Graunt, was the author of this work (Greenwood, 1928; Glass, 1950; Hull, 1963; Ronan, 1969).

of weekly lists of christenings and burials in the city of London.

Graunt's calculations are regarded as crude and the data imperfect. Dublin et al. (1949:32) note that Graunt's table is too rough and lacking in detail to allow the computation of the average length of life with any degree of accuracy.[2] However, despite the imperfections in his data and methods, Graunt opened up a very important field of scientific investigation.

> The idea which he presented was a group of births followed through life and gradually reduced in number by deaths. This was not a life table in the proper sense of the term and it was not correctly calculated, but it represented a tremendous leap forward from the simple death rate to a new and graphic method of representing the age pattern of mortality: . . . (Benjamin, 1963:38).

It was Graunt's crude calculations that prompted Sir Edmond Halley, the noted astronomer, some thirty years later to construct what is generally regarded as the first modern life table for the city of Breslaw in Silesia for the years 1687 through 1691. His work was published in 1693 under the title "An estimate of the degrees of the mortality of mankind, drawn from curious tables of the births and funerals at the city of Breslaw." The data for the table were taken

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[2]Dublin et al.'s (1949:32) approximate calculations yield an average length of life of 18.2 years. The accuracy of the figure is difficult to determine because of the many sources of errors in the data and in the method of approximation.

from the monthly records of births by sex and deaths by age and sex.[3]

Although the construction of the life table itself was a monumental contribution, Halley's further insights into the uses of the table were equally valuable. Halley recognized that data from Breslaw contained information that would yield, under given assumptions, an estimate of the age distribution of the population essential in the determination of age-specific mortality. He further noted that the rates had a variety of uses including the calculation of annuity prices, proportion of men of military age, probability of survival between successive ages, life expectancy, and probability of joint survival. The uses of the table described by Halley indicate the scope of demographic and social problems that he recognized as depending for their solution on a knowledge of the mortality table.

During the years following Halleys's work, several life tables were constructed including Kresseboom's Dutch tables in 1738, the French tables of Deparcieux in 1746, of Buffen in 1749, and of Mourgue and of Duvillard in the 1790's, Price's Northhampton table in 1783, and Wiggleworth's Massachusetts and New Hampshire table in 1793. The first

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[3]Halley's table indicates an average length of life of 33.5 years (Dublin et al., 1949:32).

official English life table was published in 1843 during the term of William Farr as Compiler of Abstracts (Chiang, 1968:189). The first official United States life tables were prepared for the years 1890, 1901, 1910, and 1901 to 1910 for the death registration states (Glover, 1921) although unofficial tables for 1901 to 1910 appeared in 1914 (Forsyth, 1914). Subsequent official publications have presented life tables for decennial years from 1910 to the present (U. S. Bureau of the Census, 1936; Greville, 1946; U. S. Department of Health, Education, and Welfare, 1954, 1964; Greville, 1975). Thus, since the middle of the 17th century, the life table has come to occupy a central place as a descriptive and analytical tool in the study of population.

## Types of Life Tables

Life tables differ according to the reference year of the table and the age detail involved. Two types of life tables may be distinguished according to the reference year of the table. Current, period, or synthetic life tables are based on the mortality experience of a population over a short period of time in which mortality has remained relatively unchanged. This type of table does not represent the mortality experience of an actual cohort but, instead, reflects the combined mortality experience by age of the population in a particular period of time. In this type of table, mortality is treated synthetically or viewed cross-sectionally. The current life table assumes a hypothetical cohort subject to the age-specific death rates observed during a particular period.

Cohort, generation, or longitudinal life tables are based on the mortality experience of an actual cohort of births; that is, all persons born in a given year. In this type of table, the mortality experience of the members of the cohort is observed from birth through each successive age in successive calendar years until the cohort has been completely depleted by death. Such tables, obviously, require data over a large number of years to complete a single table. It is not possible to construct a cohort life table for generations born in this century on the basis of actual data alone.

Life tables resulting from these two types of data, period and generation, will, except under unusual circumstances, be different in form. Lack of generational data, however, has forced life tables to be constructed from period data, requiring the assumption that the age-specific mortality rates for a particular year will prevail throughout the lifetime of the population covered by the life table. Dublin and Spiegelman (1941) demonstrated the weakness of this assumption by showing with life tables for the period 1871 to 1931 that there was a much greater savings of lives during this period than would have been expected if the 1871 death rates applied in later years. The difference between life tables resulting from the two methods of construction is due to the fact that as a cohort moves through life, conditions change and those survivors of the initial cohort enjoy the benefits of improved mortality conditions. Period life tables do not take later mortality improvements into account since they are based on mortality over a short duration of time.

Jacobsen (1964:50) presents life table values for U. S. cohorts from 1840 to 1960 based on actual and projected mortality. He notes that poor sanitary conditions and other health hazards persisted throughout the 1800's. Thus, most persons born in the middle 19th century experienced during their lifetime much the same mortality levels that prevailed at the time of their birth. Consequently, the life expectancy of a cohort of persons born at this time did not differ appreciably from life expectancy at birth based on a current life table for the calendar year of their birth. Jacobsen found that persons born in 1850 lived, on the average, about one year longer than would have been expected if no changes in mortality in their lifetime had occurred. Later generations fared much better. Jacobsen's calculations show that the 1900 birth cohort will exceed its life expectancy at birth by five years among males and almost nine years among females. For the 1930 birth cohort, corresponding gains may be as much as 8.6 and 11.5 years, respectively.

Moriyama and Gustavus (1972:1) note that because of the decline in mortality over the years, differences in cohort and period survivorship reveal that past period life tables have not represented the real-life mortality of a cohort. However, they suggest that because the rate of decline in mortality rates is slowing, future period life tables should become better predictors of mortality in a cohort than were past period life tables. Despite these limitations, and because of the lack of complete cohort data, the period life table is generally regarded as the best summary of mortality in a given area (Young, 1974:427).

Life tables may also be distinguished according to the length of the age interval in which data are presented. When data are presented for every single year of age from birth to the last applicable age, the table is referred to as a complete life table. An abridged life table presents data for broader age intervals, generally five years. Construction of abridged life tables inherently assumes that deaths are uniformly distributed over the age interval.

The life tables constructed for the present study are abridged period life tables based on a 3-year average of mortality data for the period 1969 to 1971 and 1970 midyear population. Nineteen age intervals were considered: under 1 year, 1 to 4 years, and 5-year age intervals thereafter until the terminal age interval, 85 and over. These life tables due to all causes are presented in Table 2.1.

## Alternative Interpretations of Life Tables

The life table and its basic functions are, in general, subject to two interpretations. First, the life table may be viewed as depicting the lifetime mortality of a single cohort of newborn infants subject to the age-specific mortality rates on which the life table is based. This is the more common interpretation of the life table and will be referred to in the discussion of the methods of constructing abridged life tables.

The second interpretation views the life table as a stationary population resulting from an unchanging schedule of age-specific mortality rates and a constant annual number of births and deaths.

When the life table is interpreted as depicting the mortality experience of a cohort of newborn infants, the initial size of the cohort, called the radix of the table, is generally assumed to be 100,000. The life table, then, traces the depletion of this initial cohort of 100,000 from birth through successive ages until the cohort is entirely depleted by death. In this case, the interpretation of the life table functions is as follows:

(1) x to x + n is the period of life between two exact ages. For example, "15-19" refers to the 5-year interval including the ages 15 through 19 or, more precisely,

Table 2.1. Life tables by sex due to all causes of death, United States, 1969-1971.

| Age | $_nm_x$ | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|---|
| | | | | **Males** | | | |
| < 1 | .023340 | .023071 | 100000 | 2307 | 97920 | 6695190 | 66.95190 |
| 1- 4 | .000930 | .003715 | 97693 | 362 | 390046 | 6597270 | 67.53062 |
| 5- 9 | .000500 | .002498 | 97331 | 243 | 486047 | 6207224 | 63.77437 |
| 10-14 | .000507 | .002530 | 97088 | 245 | 484825 | 5721177 | 58.92773 |
| 15-19 | .001588 | .007909 | 96843 | 765 | 482300 | 5236352 | 54.07053 |
| 20-24 | .002251 | .011194 | 96078 | 1075 | 477701 | 4754052 | 49.48117 |
| 25-29 | .002047 | .010185 | 95003 | 967 | 472595 | 4276351 | 45.01279 |
| 30-34 | .002283 | .011353 | 94036 | 1067 | 467511 | 3803756 | 40.45000 |
| 35-39 | .003134 | .015548 | 92969 | 1445 | 461231 | 3336245 | 35.88556 |
| 40-44 | .004780 | .023615 | 91524 | 2161 | 452216 | 2875014 | 31.41267 |
| 45-49 | .007461 | .036623 | 89363 | 3272 | 438633 | 2422798 | 27.11186 |
| 50-54 | .011692 | .056798 | 86091 | 4889 | 418230 | 1984165 | 23.04729 |
| 55-59 | .018347 | .087710 | 81202 | 7122 | 388204 | 1565935 | 19.28442 |
| 60-64 | .027666 | .129383 | 74080 | 9584 | 346438 | 1177731 | 15.89810 |
| 65-69 | .040925 | .185632 | 64496 | 11972 | 292548 | 831293 | 12.88906 |
| 70-74 | .059326 | .258318 | 52524 | 13567 | 228700 | 538745 | 10.25712 |
| 75-79 | .087839 | .306113 | 38957 | 14028 | 159712 | 310045 | 7.95865 |
| 80-84 | .129368 | .488764 | 24929 | 12184 | 94184 | 150333 | 6.03045 |
| 85+ | .226983 | 1.000000 | 12745 | 12745 | 56149 | 56149 | 4.40557 |

## Females

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| < 1 | .017996 | .017835 | 100000 | 1785 | 98403 | 7461325 | 74.61325 |
| 1- 4 | .000766 | .003058 | 98217 | 300 | 392267 | 7362922 | 74.96585 |
| 5- 9 | .000349 | .001743 | 97917 | 170 | 489158 | 6970655 | 71.18942 |
| 10-14 | .000302 | .001507 | 97747 | 147 | 488367 | 6481497 | 66.30890 |
| 15-19 | .000622 | .003104 | 97600 | 302 | 487243 | 5993130 | 61.40501 |
| 20-24 | .000746 | .003721 | 97298 | 362 | 485584 | 5505887 | 56.58786 |
| 25-29 | .000865 | .004315 | 96936 | 418 | 483633 | 5020303 | 51.78987 |
| 30-34 | .001189 | .005929 | 96518 | 572 | 481159 | 4536670 | 47.00334 |
| 35-39 | .001811 | .009014 | 95946 | 864 | 477567 | 4055511 | 42.26868 |
| 40-44 | .002751 | .013663 | 95082 | 1299 | 472162 | 3577944 | 37.63008 |
| 45-49 | .004153 | .020554 | 93783 | 1927 | 464096 | 3105782 | 33.11668 |
| 50-54 | .006115 | .030113 | 91856 | 2766 | 452364 | 2641686 | 28.75899 |
| 55-59 | .008946 | .043751 | 89090 | 3897 | 435705 | 2189322 | 24.57426 |
| 60-64 | .013109 | .063467 | 85193 | 5406 | 412447 | 1753617 | 20.58405 |
| 65-69 | .020220 | .096237 | 79787 | 7678 | 379738 | 1341170 | 16.80937 |
| 70-74 | .032259 | .149259 | 72109 | 10762 | 333637 | 961432 | 13.33304 |
| 75-79 | .053809 | .237144 | 61347 | 14548 | 270364 | 627795 | 10.23351 |
| 80-84 | .089494 | .365660 | 46799 | 17112 | 191213 | 357431 | 7.63758 |
| 85+ | .178603 | 1.000000 | 29687 | 29687 | 166218 | 166218 | 5.59902 |

the interval between exact age 15.0 to exact age 19.99 . . . ;

(2) ${}_nq_x$ refers to the proportion of persons in the cohort alive at the beginning of the interval at age x who will die before reaching age x + n, the end of the interval. This value is generally referred to as the probability of dying, the probability that a person age x will die before reaching age x + n;

(3) $l_x$ is the number of persons living at the beginning of an age interval, age x, out of the total number of births assumed in the radix. This function is often referred to as the number of life table survivors;

(4) ${}_nd_x$ indicates the number of persons who would die in an age interval, x to x + n, out of the total number of survivors to the beginning of that interval. This function is referred to as the number of life table deaths;

(5) ${}_nL_x$ is referred to as the number of person-years that would be lived within an age interval, x to x + n, by the cohort of births assumed in the radix of the table. If the initial cohort of 100,000 survived the first year of life, the value of ${}_nL_x$ would be 100,000. If, on the other hand, 500 of the initial 100,000 died before their first birthday, the 99,500 survivors would have lived one year each and the 500 who died would each have lived varying periods of time less than one year.

Thus, in this case, assuming that those who died before age 1 lived, on the average, one-half a year,

$_nL_x$ = 99,500 + .5(500) = 99,750 person-years;

(6) $T_x$ is the total number of person-years that would be lived by the cohort after the beginning of the age interval, x to x + n. This value represents the cumulation of the $_nL_x$ values from the end of the table forward through the indicated age interval;

(7) $e_x$ or life expectancy at age x, refers to the average remaining lifetime for a person who survives to the beginning of the age interval, x to x + n.

A stationary population is one whose total number and distribution by age does not change with time. Such a hypothetical population could be obtained if the number of births per year, assumed to be 100,000, remained constant for a long period and each cohort of births experienced the same set of currently observed mortality rates throughout life. In the stationary population, the annual numbers of births and deaths are equivalent (i.e., 100,000), resulting in no change in the size or age distribution of the population. In the case of the stationary population interpretation, the x to x + n, $_nq_x$, and $e_x$ functions are interpreted as before. The other life table functions, however, are interpreted as follows:

(1) $l_x$ indicates the number of persons who reach the be-

ginning of the age interval, x to x + n. each year;

(2) $_nd_x$ refers to the number of persons that die within the age interval, x to x + n, each year;

(3) $_nL_x$ is the number of persons in the population living at any moment within the age interval, x to x + n. This function is indicative of the age distribution of the stationary population;

(4) $T_x$ is the number of persons in the population at any moment living within the age interval, x to x + n, and all subsequent age intervals.

Each interpretation, of course, has particular associated applications. The cohort interpretation of the life table is applied in public health studies and mortality analysis, and in the calculation of survival rates for projecting populations and estimating net migration, fertility, and reproductivity. The stationary population is used in the comparative measurement of mortality and in studies of population structure.

## Life Table Construction

The life table is designed essentially as a measure of mortality. The entire life table is generated from age-specific mortality rates and life table values are used to measure mortality, survivorship, and life expectation. The life table makes use of estimated probabilities of dying to obtain measures of mortality by exposing a hypothetical cohort of fixed initial size, the radix, assumed to be born on the same day to these probabilities and observing the consequences of the particular pattern of age-specific mortality. The life table, then, describes the history of an artificial population as it is gradually diminished by exposure to age-specific mortality conditions. The life table assumes that the observed period mortality conditions to which the cohort of births is exposed remain unchanged and are unaffected by migration. Changes in the composition of the cohort, thus, occur only as a result of losses due to death (Rogers, 1971:41).

The crucial value to be calculated in the construction of any life table is ${}_nq_x$, the probability of dying. All other life table functions are derived either directly or indirectly from this function. The main problem in life table construction is to derive a formula that expresses the probability of death, ${}_nq_x$, in terms of the corresponding age-specific mortality rate, ${}_nm_x$; that is, some mathematical relation between ${}_nm_x$ and ${}_nq_x$ must be assumed. There is no exact answer to the question of how to construct a life table and several methods have been proposed (King, 1908, 1914; Reed and Merrell, 1939; Greville, 1943; Sirken, 1964; Coale and Demeny, 1966; Keyfitz, 1966a, 1968a, 1970; Chiang, 1960b, 1968, 1972; Keyfitz and Frauenthal, 1975). The most popular methods are those suggested by King, Reed and Merrell, and

Greville. In general, these methods require the same amount and type of data but differ in the method through which observed mortality rates are converted into probabilities of dying. The life tables due to all causes constructed for the present study represent a synthesis of King's simple method for calculating ${}_nq_x$ and Greville's method for computing the remaining functions.

King's (1914) method of calculating the probability of dying will be discussed in greater detail later. However, for purposes of comparison, a few comments are necessary. The method of converting observed mortality rates to probabilities of dying suggested by King is based on the assumption that deaths in any year are rectangularly distributed by age and time; that is, those persons who die within an age interval live, on the average, for one-half the interval.

The Reed-Merrell method is perhaps the most popular and frequently used method of calculating abridged life tables. Under this method, probabilities of dying are read from standard conversion tables associated with various observed mortality rates. Tables are available for 1, 3, 4, 5, and 10-year age intervals. The tables were prepared on the assumption that the exponential equation

$$ {}_nq_x = 1 - e^{-n\,{}_nm_x - an^3\,{}_nm_x^2} \qquad (2.1) $$

holds, where n is the length of the age interval, ${}_nm_x$ is the observed mortality rate, e is the base of the natural logarithm system, and a is an arbitrary constant empirically determined by Reed and Merrell to be .008. This constant corrective term has the effect of lifting the curve of probability of dying at older age intervals to provide a better fit to observed mortality (Reed and Merrell, 1939:39).

Greville's (1943) method of calculating the probability of dying is based on the assumption that values of ${}_nm_x$ follow an exponential curve. The observed mortality rates, ${}_nm_x$, are converted to the needed values of ${}_nq_x$ by use of the formula

$$ {}_nq_x = \frac{{}_nm_x}{1/n + {}_nm_x[1/2 + n/12({}_nm_x - \log_e c)]} \qquad (2.2)$$

where c is a constant that is based on the assumption that values of ${}_nm_x$ follow the exponential curve. The value of c has been found empirically to lie between 1.08 and 1.10.

Most of the alternative methods of deriving the probability of dying are mathematically and conceptually complex, with the result that most users cannot appreciate the value of life tables as a means of summarizing mortality experience. A comparison of methods (Shryock and Siegel, 1973; Keyfitz and Frauenthal, 1975) reveals that the values of ${}_nq_x$ do not vary significantly with the method of caluculation.*

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*Keyfitz and Frauenthal (1975:898), comparing several methods of calculating ${}_nq_x$ on the basis of values of $l_x$, conclude that differences in values produced by different methods are trivial.

Thus, the simpler and more easily understood method of King was utilized in the present study.

The assumption employed in the construction of the life tables presented was that $l_x$, the number of persons living at the beginning of the indicated age interval out of the total number of births assumed in the radix of the table, can be regarded as a linear function in the age interval (Greville, 1943:34; 1967:7). Specifically, it was assumed that deaths were evenly distributed over time and over years within the age interval. Thus, it was assumed that deaths within the age interval, x to x + n, occurred, on the average, at age x + 1/2*n. The equation for ${}_nq_x$ may be derived from the basic equations for ${}_nm_x$, the observed mortality rate, and ${}_nq_x$, the probability of dying where

$${}_nm_x = {}_nD_x / {}_nP_x \tag{2.3}$$

and

$${}_nq_x = (n \, {}_nD_x) / [{}_nP_x + (n/2) \, {}_nD_x] \tag{2.4}$$

where ${}_nD_x$ is the 3-year average of deaths for the interval beginning at age x, ${}_nP_x$ is the midpoint population of the 3-year period, and n is the length of the age interval. Dividing both the numerator and the denominator of (2.4) by ${}_nP_x$ yields

$${}_nq_x = (n \, {}_nm_x) / [1 + (n/2) \, {}_nm_x]. \tag{2.5}$$

This equation is the standard approximation for deriving values of ${}_nq_x$ from the values of ${}_nm_x$. For the terminal age group (85 and over), ${}_nq_x$ was set to unity since the probability of dying for that age interval is a certain event.

It is convenient to calculate the probability of surviving a given age interval, ${}_np_x$, although this value does not conventionally appear in the life table. A person must either die in or survive the age interval. Thus, death and survival are not only mutually exclusive events but also complementary. From probability theory, it follows from the addition rule of mutually exclusive events and the law of complementation that

$$ {}_np_x = 1 - {}_nq_x. \tag{2.6} $$

Once ${}_nq_x$ has been calculated, all other life table functions may be derived. By successive application of a particular set of probabilities of dying to a cohort of a given radix, the life table indicates how this initial cohort is diminished over time by calculating the number of survivors to the beginning of each age interval by the equation

$$ l_{x+n} = (1 - {}_nq_x)\, l_x \tag{2.7} $$

where x and x + n represent adjacent age intervals.

The number of deaths to persons in the indicated age interval out of the total number of births assumed in the radix is

$$ {}_nd_x = l_x - l_{x+n}, \tag{2.8} $$

the difference between the number of survivors in successive age intervals. In the life table, the sum of deaths over all ages is equal to the size of the original cohort or radix; that is,

$$d_1 + d_2 + \ldots + d_\alpha = l_0. \tag{2.9}$$

Equations which give results equivalent to (2.7) and (2.8) are

$$l_{x+n} = l_x - {}_nd_x \tag{2.10}$$

and

$${}_nd_x = l_x {}_nq_x. \tag{2.11}$$

For the terminal age interval, $l_x = {}_nd_x$ since ${}_nq_x = 1.0$; that is, all persons alive at the beginning of this interval must die during the interval.

The number of person-years that would be lived within the indicated age interval by the assumed cohort of 100,000 births, ${}_nL_x$, cannot be calculated directly but may be approximated on the basis of an assumption about the relation between the mortality rates in the life table and the observed mortality rates in the population. The method suggested by Greville (1943:39) is based on the assumption that ${}_nm_x$, the observed mortality rate, has the same value in the actual population and the life table population. Thus, the life table age-specific mortality rate, ${}_nM_x$, is defined as

$${}_nM_x = {}_nd_x/{}_nL_x. \tag{2.12}$$

This rate, however, cannot be calculated until ${}_nL_x$ is known. If ${}_nm_x$ is assumed equal to ${}_nM_x$, then

$${}_nm_x = {}_nM_x = {}_nd_x/{}_nL_x. \tag{2.13}$$

Thus, simple algebraic manipulation yields

$$ {}_nL_x = {}_nd_x / {}_nm_x. \qquad (2.14) $$

National separation factors were applied to the calculation of the ${}_nL_x$ value for the first age interval, under 1 year. Separation factors are used to control for sharp fluctuations in the number of births and infant deaths between and within calendar years. Separation factors were calculated by sex for 1970 according to procedures suggested by Shryock and Siegel (1973:412-414). Separation factors represent the proportion of infant deaths in a given year which occurred to infants born in that year and the proportion of infant deaths in a given year which occurred to infants born in the previous year.

The total number of person-years lived after the beginning of the indicated age interval by the assumed cohort of 100,000 births is given by $T_x$. Thus, $T_x$ is the sum of the number of person-years lived by persons aged x and all subsequent ages. If values of ${}_nL_x$ are obtained, $T_x$ may be calculated by cumulating the ${}_nL_x$ values from the end of the table forward,

$$ T_x = \sum_{i=\alpha}^{x} {}_nL_x \qquad (2.15) $$

where the summation begins at the end of the life table ($\alpha$) and cumulates to and including the age interval beginning with age x.

The expectation of life is given by

$$e_x = T_x/l_x. \tag{2.16}$$

The value of $e_0$ is referred to as the expectation of life at birth and is given by

$$e_0 = T_0/l_0. \tag{2.17}$$

Life expectancy indicates the average remaining lifetime in years for a person who survives to the beginning of the indicated age interval.

The terminal age interval in the life table is a half-open interval. It refers to persons aged 85 and over. Thus, for this interval, as noted previously, ${}_nq_x$ is set to unity and the remaining life table functions refer to the interval aged 85 and over. The length of the interval is infinite. If z denotes the age interval 85 and over and x is set equal to z, then

$$L_z = d_z/M_z. \tag{2.18}$$

Since all of the $l_z$ persons aged 85 and over will die, the number of survivors to the beginning of the terminal age interval is equivalent to the number of persons who die in the same interval, or

$$l_z = d_z, \tag{2.19}$$

$$L_z = l_z/M_z, \tag{2.20}$$

and, by definition,

$$T_z = L_z. \tag{2.21}$$

Thus,

$$e_z = T_z / l_z = L_z / l_z \qquad (2.22)$$

(Rogers, 1971: 44-45).

## Summary

This chapter presented a general description of the life table as a measure of mortality. The points discussed were:

1. Historically, the concept of the life table originated in studies of human longevity. Although crude methods may be traced back to the third century A.D., the modern life table is generally attributed to the work of Graunt and Halley in the 17th century.

2. Life tables may be distinguished on the basis of reference year of the table and age detail involved. Current life tables are based on the mortality experience of a population over a short period of time in which mortality has remained relatively unchanged while cohort life tables are based on the mortality experience of an actual birth cohort. When data are presented by single years of age from birth, the life table is referred to as a complete life table. Abridged life tables present data for broader age intervals. Life tables constructed for the present study are abridged, current life tables for the period 1969 to 1971.

3. Two interpretations of the life table were distinguished. The cohort interpretation views the life table as depicting the mortality experience of a cohort of infants from birth until the cohort has been depleted by death. The

stationary population interpretation views the life table as depicting the age distribution of a population subject to unchanging mortality rates and a constant annual number of births and deaths. The meaning of life table functions under alternative interpretations was also discussed.

4. The methods of life table construction were described. Although several alternative methods of life table construction are available, methods used in the present study are due primarily to King and Greville.

# CHAPTER 3. METHOD OF CONSTRUCTING ABRIDGED LIFE TABLES ELIMINATING CAUSES OF DEATH

Like the main life table itself, life tables eliminating causes of death arose, historically, from the investigation of longevity and cause-specific mortality. The initial development of methods for measuring the effect of eliminating certain causes of death was stimulated by the early 18th century controversy over the value of small pox inoculation. The mathematicians Bernoulli, D'Alembert, and Laplace each derived a method of determining the change in the composition of the population that would result from the elimination of small pox as a cause of death.

Bernoulli's (1760) method of determining the influence of small pox on the duration of life was the comparison of Halley's life table with a hypothetical life table presenting the number of survivors to each age if mortality due to small pox was entirely eliminated. Bernoulli compared the age distributions and the mean durations of life of the two tables. The mean duration of life for Halley's table by Bernoulli's calculations was 26 years and 7 months. The mean duration of life for the special table eliminating small pox as a cause of death was 29 years and 9 months. Thus, in this way Bernoulli attempted to illustrate the advantages of eliminating small pox as a cause of death. The method was later generalized by Laplace (Greville, 1948b:283).

D'Alembert (1768) developed a comparable formula independently by use of a geometrical representation. D'Alembert unfavorably criticized the work of Bernoulli on the grounds of inadmissible assumptions and offered his geometric method as a means of determining how a population would be affected by the elimination of small pox as a cause of death.

It was not, however, until over a century later that Makeham (1866, 1867, 1874) first formulated the law of composition of decremental forces and applied it to the problem of analyzing causes of death. The law of composition of decremental forces states that the total force of mortality is equal to the sum of the several partial forces. This more direct and elemental approach to the problem of eliminating a cause of death has served as the basis for the subsequent development of procedures for constructing life tables eliminating causes of death.

A number of these special life tables have appeared. An early effort in this direction was taken by Mendenhall and Castle (1911) who used life table methods to measure the effects of typhoid fever as a cause of death in the death registration states of the United States in 1900. The results were taken as an indication of the minimum community loss due to impure water. Forsyth (1915) compared two life tables for the registration states for the period 1900 to

1910 to determine the effects of preventable deaths on the average length of life. Karn (1931, 1933) restated the formulas of Bernoulli, D'Alembert, and others and applied them to United States data for 5-year periods grouped around 1891, 1901, 1911, and 1921 to determine the increase in life expectancy due to the elimination of various causes of death. Dublin et al. (1949), in their lengthy treatment of the life table, prepared life tables based on 1939-1941 data for the United States eliminating, in turn, eight major groups of causes of death.

More recently, life tables eliminating causes of death have been prepared for the United States (Woodhall and Jablon, 1957; Metropolitan Life Insurance Company, 1967; Bayo, 1968; Cohen, 1975), individual states (Park and Scott, 1971; Schoen and Collins, 1973), Canada (Silens and Zayachkowski, 1968a, 1968b; Pressat, 1974) and other countries (Sekar, 1949; Gupta and Rao, 1973; Hemminski et al., 1974; Madeira, 1974). By far the most extensive series of life tables with causes of death eliminated are those of Preston et al. (1972) who present life tables for 180 populations covering a period of 103 years for 48 nations.

The seminal article describing methods of constructing abridged life tables with causes of death eliminated is Greville's (1948b) article. This article describes the relation of the various life table functions to the notion of multiple decrements as explicated originally by Makeham. The life tables used in the present study were constructed according to methods recently suggested by Greville (1948b, 1973, n.d.), Bayo (1968), and Chiang (1968). Life tables for United States males and females, 1969-1971, were generated eliminating, in turn, malignant neoplasms (cancer), diseases of heart, and motor vehicle accidents (Tables 3.1, 3.2, and 3.3).

These three groups of causes were selected because of the age pattern of mortality due to each cause. Each cause is most prevalent in certain age groups. Motor vehicle accidents occur most frequently among younger persons, malignant neoplasms are primarily a disease of middle ages for females, and diseases of heart are more prevalent among older females and middle-aged to elderly males.

## Life Table Construction for the Special Tables

According to Greville (1948b:284), the basic feature of a life table eliminating groups of causes of death is the subdivision of the ${}_nd_x$ values of the main table due to all causes into a number of parts corresponding to certain groups of causes of death. Thus, if ${}_nd_x$ denotes the total life table deaths in the main table between ages x to x + n and m denotes the number of causes, and ${}_nd_x^i$ denotes the number of life table deaths due to cause i in the same interval, then

Table 3.1. Life tables by sex eliminating malignant neoplasms as a cause of death, United States, 1969-1971.

| Age | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|
| | | | Males | | | |
| < 1 | .023026 | 100000 | 2302 | 97924 | 6926272 | 69.26272 |
| 1- 4 | .003392 | 97698 | 331 | 390128 | 6828348 | 69.89240 |
| 5- 9 | .002113 | 97367 | 205 | 486322 | 6438220 | 66.12321 |
| 10-14 | .002241 | 97162 | 217 | 485265 | 5951898 | 61.25746 |
| 15-19 | .007479 | 96945 | 725 | 482910 | 5466633 | 56.38901 |
| 20-24 | .010622 | 96220 | 1022 | 478543 | 4983723 | 51.79507 |
| 25-29 | .009486 | 95198 | 903 | 473730 | 4505180 | 47.32431 |
| 30-34 | .010357 | 94295 | 976 | 469033 | 4031450 | 42.75359 |
| 35-39 | .013815 | 93319 | 1289 | 463371 | 3562417 | 38.17461 |
| 40-44 | .020243 | 92030 | 1862 | 455493 | 3099046 | 33.67430 |
| 45-49 | .030356 | 90168 | 2737 | 443995 | 2643553 | 29.31807 |
| 50-54 | .045569 | 87431 | 3984 | 427192 | 2199558 | 25.15764 |
| 55-59 | .068942 | 83447 | 5753 | 402851 | 1772366 | 21.23941 |
| 60-64 | .101874 | 77694 | 7915 | 368680 | 1369515 | 17.62703 |
| 65-69 | .148357 | 69779 | 10352 | 323013 | 1000835 | 14.34293 |
| 70-74 | .213175 | 59427 | 12668 | 265462 | 677822 | 11.40596 |
| 75-79 | .308822 | 46759 | 14440 | 197691 | 412360 | 8.81884 |
| 80-84 | .438384 | 32319 | 14168 | 126173 | 214669 | 6.64219 |
| 85+ | 1.000000 | 18151 | 18151 | 88496 | 88496 | 4.87554 |

Table 3.1. (continued)

| Age | ${}_nq_x$ | $l_x$ | ${}_nd_x$ | ${}_nL_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|
| | | | Females | | | |
| < 1 | .017790 | 100000 | 1779 | 98406 | 7721296 | 77.21296 |
| 1- 4 | .002790 | 98221 | 274 | 392335 | 7622890 | 77.60957 |
| 5- 9 | .001454 | 97947 | 142 | 489378 | 7230555 | 73.82109 |
| 10-14 | .001280 | 97805 | 125 | 488712 | 6741177 | 68.92465 |
| 15-19 | .002833 | 97680 | 276 | 487708 | 6252465 | 64.00967 |
| 20-24 | .003380 | 97404 | 329 | 486196 | 5764757 | 59.18398 |
| 25-29 | .003740 | 97075 | 363 | 484465 | 5278561 | 54.37610 |
| 30-34 | .004797 | 96712 | 463 | 482401 | 4794096 | 49.57085 |
| 35-39 | .006793 | 96249 | 653 | 479610 | 4311695 | 44.79729 |
| 40-44 | .009495 | 95596 | 907 | 475712 | 3832085 | 40.08624 |
| 45-49 | .013469 | 94689 | 1275 | 470256 | 3356373 | 35.44627 |
| 50-54 | .019521 | 93414 | 1823 | 462511 | 2886117 | 30.89597 |
| 55-59 | .029106 | 91591 | 2665 | 451290 | 2423606 | 26.46117 |
| 60-64 | .045257 | 88926 | 4024 | 434567 | 1972316 | 22.17929 |
| 65-69 | .073430 | 84902 | 6234 | 408923 | 1537749 | 18.11205 |
| 70-74 | .122165 | 78668 | 9610 | 369312 | 1128826 | 14.34924 |
| 75-79 | .205059 | 69058 | 14160 | 309889 | 759514 | 10.99820 |
| 80-84 | .332353 | 54898 | 18245 | 228875 | 449625 | 8.19019 |
| 85+ | 1.000000 | 36653 | 36653 | 220750 | 220750 | 6.02270 |

Table 3.2. Life tables by sex eliminating diseases of heart as a cause of death, United States, 1969-1971.

| Age | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|
| | | | Males | | | |
| < 1 | .022933 | 100000 | 2293 | 97932 | 7330380 | 73.30380 |
| 1- 4 | .003645 | 97707 | 356 | 390114 | 7232448 | 74.02179 |
| 5- 9 | .002458 | 97351 | 239 | 486156 | 6842334 | 70.28519 |
| 10-14 | .002478 | 97112 | 240 | 484957 | 6356178 | 65.45203 |
| 15-19 | .007770 | 96872 | 752 | 482477 | 5871221 | 60.60802 |
| 20-24 | .010962 | 96120 | 1053 | 477966 | 5388744 | 56.06267 |
| 25-29 | .009746 | 95067 | 926 | 473017 | 4910778 | 51.65596 |
| 30-34 | .010243 | 94141 | 964 | 468293 | 4437761 | 47.13951 |
| 35-39 | .012496 | 93177 | 1164 | 462973 | 3969468 | 42.60136 |
| 40-44 | .016709 | 92013 | 1537 | 456221 | 3506495 | 38.10869 |
| 45-49 | .023246 | 90476 | 2103 | 447121 | 3050274 | 33.71362 |
| 50-54 | .033703 | 88373 | 2978 | 434418 | 2603153 | 29.45642 |
| 55-59 | .050921 | 85395 | 4348 | 416104 | 2168735 | 25.39650 |
| 60-64 | .074703 | 81047 | 6054 | 390098 | 1752631 | 21.62486 |
| 65-69 | .108040 | 74993 | 8102 | 354708 | 1362533 | 18.16879 |
| 70-74 | .152135 | 66891 | 10176 | 309013 | 1007825 | 15.06668 |
| 75-79 | .217779 | 56715 | 12351 | 252694 | 698812 | 12.32147 |
| 80-84 | .304651 | 44364 | 13515 | 188031 | 446118 | 10.05586 |
| 85+ | 1.000000 | 30849 | 30849 | 258087 | 258087 | 8.36614 |

Table 3.2. (continued)

| Age | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|
| | | | Females | | | |
| < 1 | .017715 | 100000 | 1771 | 98413 | 8096523 | 80.96523 |
| 1- 4 | .002995 | 98229 | 294 | 392326 | 7998110 | 81.42310 |
| 5- 9 | .001699 | 97935 | 166 | 489258 | 7605784 | 77.66154 |
| 10-14 | .001458 | 97769 | 142 | 488489 | 7116526 | 72.78918 |
| 15-19 | .003010 | 97627 | 293 | 487400 | 6628037 | 67.89143 |
| 20-24 | .003577 | 97334 | 348 | 485798 | 6140637 | 63.08830 |
| 25-29 | .004061 | 96986 | 393 | 483945 | 5654839 | 58.30571 |
| 30-34 | .005427 | 96593 | 524 | 481654 | 5170894 | 53.53279 |
| 35-39 | .007964 | 96069 | 765 | 478429 | 4689240 | 48.81116 |
| 40-44 | .011581 | 95304 | 1103 | 473762 | 4210811 | 44.18294 |
| 45-49 | .016744 | 94201 | 1577 | 467061 | 3737049 | 39.67101 |
| 50-54 | .023217 | 92624 | 2150 | 457744 | 3269988 | 35.30389 |
| 55-59 | .031493 | 90474 | 2849 | 445245 | 2812244 | 31.08344 |
| 60-64 | .041891 | 87625 | 3670 | 428947 | 2366999 | 27.01282 |
| 65-69 | .059232 | 83955 | 4972 | 407343 | 1938052 | 23.08441 |
| 70-74 | .086841 | 78983 | 6858 | 377768 | 1530709 | 19.38022 |
| 75-79 | .135918 | 72125 | 9803 | 336116 | 1152941 | 15.98532 |
| 80-84 | .212001 | 62322 | 13212 | 278578 | 816825 | 13.10653 |
| 85+ | 1.000000 | 49110 | 49110 | 538247 | 538247 | 10.96003 |

Table 3.3. Life tables by sex eliminating motor vehicle accidents as a cause of death, United States, 1969-1971.

| Age | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|
| | | | Males | | | |
| < 1 | .022970 | 100000 | 2297 | 97928 | 6788455 | 67.88455 |
| 1- 4 | .003203 | 97703 | 312 | 390186 | 6690527 | 68.47821 |
| 5- 9 | .001830 | 97391 | 178 | 486509 | 6300341 | 64.69119 |
| 10-14 | .001907 | 97213 | 185 | 485600 | 5813832 | 59.80508 |
| 15-19 | .004672 | 97028 | 453 | 484005 | 5328232 | 54.91437 |
| 20-24 | .006941 | 96575 | 670 | 481199 | 4844227 | 50.16025 |
| 25-29 | .007423 | 95905 | 711 | 477745 | 4363028 | 45.49323 |
| 30-34 | .009219 | 95194 | 877 | 473776 | 3885283 | 40.81436 |
| 35-39 | .013663 | 94317 | 1288 | 468363 | 3411507 | 36.17064 |
| 40-44 | .021856 | 93029 | 2033 | 460061 | 2943144 | 31.63684 |
| 45-49 | .034898 | 90996 | 3175 | 447040 | 2483083 | 27.28781 |
| 50-54 | .055062 | 87821 | 4835 | 427014 | 2036043 | 23.18401 |
| 55-59 | .085926 | 82986 | 7130 | 397103 | 1609029 | 19.38916 |
| 60-64 | .127631 | 75856 | 9681 | 355075 | 1211926 | 15.97667 |
| 65-69 | .183815 | 66175 | 12163 | 300465 | 856851 | 12.94826 |
| 70-74 | .256386 | 54012 | 13847 | 235439 | 556386 | 10.30116 |
| 75-79 | .357965 | 40165 | 14377 | 164879 | 320947 | 7.99071 |
| 80-84 | .486742 | 25788 | 12552 | 97558 | 156068 | 6.05196 |
| 85+ | 1.000000 | 13236 | 13236 | 58510 | 58510 | 4.42052 |

Table 3.3. (continued)

| Age | $_nq_x$ | $l_x$ | $_nd_x$ | $_nL_x$ | $T_x$ | $e_x$ |
|---|---|---|---|---|---|---|
| | | | Females | | | |
| < 1 | .017736 | 100000 | 1773 | 98411 | 7502388 | 75.02388 |
| 1- 4 | .002643 | 98227 | 259 | 392389 | 7403977 | 75.37617 |
| 5- 9 | .001338 | 97968 | 131 | 489510 | 7011588 | 71.57018 |
| 10-14 | .001176 | 97837 | 115 | 488897 | 6522078 | 66.66269 |
| 15-19 | .001949 | 97722 | 190 | 488133 | 6033181 | 61.73820 |
| 20-24 | .002716 | 97532 | 264 | 486999 | 5545048 | 56.85362 |
| 25-29 | .003626 | 97268 | 352 | 485458 | 5058049 | 52.00116 |
| 30-34 | .005306 | 96916 | 514 | 483294 | 4572591 | 47.18097 |
| 35-39 | .008389 | 96402 | 808 | 479987 | 4089297 | 42.41920 |
| 40-44 | .013019 | 95594 | 1244 | 474859 | 3609310 | 37.75665 |
| 45-49 | .019913 | 94350 | 1878 | 467053 | 3134451 | 33.22151 |
| 50-54 | .029389 | 92472 | 2717 | 455566 | 2667398 | 28.84546 |
| 55-59 | .043005 | 89755 | 3859 | 439124 | 2211832 | 24.64299 |
| 60-64 | .062673 | 85896 | 5383 | 416019 | 1772708 | 20.63783 |
| 65-69 | .095337 | 80513 | 7675 | 383375 | 1356689 | 16.85056 |
| 70-74 | .148201 | 72838 | 10794 | 337201 | 973314 | 13.36272 |
| 75-79 | .236076 | 62044 | 14647 | 273601 | 636113 | 10.25261 |
| 80-84 | .364800 | 47397 | 17290 | 193757 | 362512 | 7.64842 |
| 85+ | 1.000000 | 30107 | 30107 | 168755 | 168755 | 5.60517 |

$$\sum_{i=1}^{m} {}_nd_x^i = {}_nd_x. \tag{3.1}$$

Likewise, if $l_x^i$ is the number of persons in the life table population at age x who will eventually die from cause i, then

$$\sum_{i=1}^{m} l_x^i = l_x. \tag{3.2}$$

Since ${}_nd_x^i$ values indicate the distribution of the ${}_nd_x$ life table deaths by cause, the $l_x^i$ values reflect the distribution of the $l_x$ survivors according to the causes of their future deaths.

Following the same argument,

$$\sum_{i=1}^{m} {}_nL_x^i = {}_nL_x \tag{3.3}$$

and

$$\sum_{i=1}^{m} T_x^i = T_x. \tag{3.4}$$

Finally, if $u_x^i$ denotes the instantaneous death rate from cause i at exact age x, then, according to Greville (1948b:285), the numerator of $u_x^i$ is that rate of decrease of that part of $l_x$ represented by $l_x^i$ since deaths resulting from cause i can affect only that part of $l_x$. Thus, following Makeham's (1867) law of composition of decremental forces that the total force of mortality is equal to the sum of the several partial forces, Greville (1948b:285) concludes that

$$\sum_{i=1}^{m} u_x^i = u_x. \tag{3.5}$$

According to Greville (1948b:285-286), the relation between the values of the partial force of mortality, $u_x^i$, in a given age interval and the number of life table deaths is given by

$$r_x^i = u_x^i/u_x \tag{3.6}$$

or

$$u_x^i = r_x^i u_x. \tag{3.7}$$

Consequently,

$$_nd_x^i = r_{x\,n}^i d_x \tag{3.8}$$

where $r_x^i$ is the proportion of the average number of deaths observed in the population during the 3-year period in the age interval, x to x + n, due to cause i, $_nd_x$ is the number of deaths in the same interval in the life table, and $_nd_x^i$ is an estimate of the number of life table deaths in the interval due to cause i.

In constructing life tables with groups of causes of death eliminated, the total force of mortality, $u_x^{(-i)}$, is taken as equal to $u_x^{-i}$ where superscript (-i) refers to the situation in which cause i has been completely eliminated as a cause of death and superscript -i without parentheses refers to the aggregate of all causes except cause i in the main table; that is,

$$_nd_x^{-i} = {}_nd_x - {}_nd_x^i. \tag{3.9}$$

This assumption means that causes of death are completely independent of one another. This, of course, is generally not true since a given disease may leave an individual with increased resistance to some other disease. These factors are extremely difficult, if not impossible, to take into account.

The assumption made in the calculations is that a group of causes of death has been eliminated. This assumption does not mean that the disease or condition has been eliminated. The disease or condition is assumed to continue at the same level that prevailed during the period of observation but that it is not possible to die from the disease or condition.

The construction of life tables eliminating groups of causes of death rests on the determination of three basic values. First, if ${}_nD_x$ denotes the number of observed deaths occurring between ages x to x + n and ${}_nD_x^i$ denotes the number of observed deaths due to cause i in the same interval, then

$$ {}_nD_x^i = r_x^i {}_nD_x \qquad (3.10) $$

or

$$ r_x^i = {}_nD_x^i / {}_nD_x. \qquad (3.11) $$

The values to the right of the equal sign in (3.11) are available from published data. Values of $r_x^i$ are given in Table 3.4.

The second basic value to be calculated is ${}_nq_x^{(-i)}$, the probability of dying in the age interval x to x + n when cause i is eliminated as a cause of death. The formula initially proposed by Greville (1948b:291) received considerable use (Bayo, 1968). More recently, however, the so-called "actuarial method" has received extensive use (Chiang, 1968; Greville, 1973, n.d.). The actuarial method was proposed by Chiang (1968:242-268) as an approximate formula for

Table 3.4. Proportion of total deaths due to specific causes by age and sex, United States, 1969-1971.

| | Cause | | | | | |
|---|---|---|---|---|---|---|
| Age | MN[a] | DH | MVA | MN | DH | MVA |
| | | Males | | | Females | |
| < 1 | .001964 | .006061* | .004408 | .002530 | .006811* | .005611 |
| 1- 4 | .087010 | .018689 | .138021* | .087757 | .020724 | .135800* |
| 5- 9 | .154331 | .016142 | .267717* | .165884 | .025250 | .232237* |
| 10-14 | .114434 | .020435 | .246331* | .150454 | .032425 | .219520* |
| 15-19 | .054524 | .017591 | .410295* | .087148 | .030349 | .372330* |
| 20-24 | .051329 | .020854 | .381270* | .091798 | .038959 | .270505* |
| 25-29 | .069024 | .043315 | .272265* | .133367 | .059051 | .160040* |
| 30-34 | .088132 | .098262 | .188810* | .191377* | .084945 | .105429 |
| 35-39 | .112222 | .197557* | .122100 | .247207* | .116911 | .069626 |
| 40-44 | .144286 | .294929* | .075321 | .306502* | .153310 | .047488 |
| 45-49 | .173799 | .369599* | .047960 | .347035* | .186927 | .031479 |
| 50-54 | .202380 | .413684* | .031434 | .355254* | .231735 | .024410 |
| 55-59 | .221834 | .430670* | .021289 | .339749* | .284719 | .017432 |
| 60-64 | .224521 | .439637* | .014515 | .293689 | .347373* | .012924 |
| 65-69 | .217957 | .443204* | .010857 | .246293 | .396581* | .009839 |
| 70-74 | .197721 | .447745* | .008706 | .193946 | 438007* | .007695 |
| 75-79 | .172706 | .449859* | .007508 | .152202 | .460304* | .005171 |
| 80-84 | .140087 | .458447* | .005884 | .112430 | .476551* | .002979 |
| 85+ | .096396 | .473406* | .003398 | .070352 | .489143* | .001102 |

[a]Abbreviations used here and in all subsequent tables are: MN--malignant neoplasms; DH--diseases of heart; MVA--motor vehicle accidents.

* Denotes the largest age-specific value here and in all subsequent tables.

expressing the value of ${}_nq_x^{(-i)}$ and is based on competing risk theory.

Given an individual alive at age x, his probability of dying in the interval, x to x + n, from cause i, ${}_nQ_x^i$, and his probability of surviving the interval, ${}_np_x$, satisfy the conditions

$$ {}_nq_x = \sum_{i=1}^{m} {}_nQ_x^i \tag{3.12}$$

and

$$ 1 = \sum_{i=1}^{m} {}_nQ_x^i + {}_np_x. \tag{3.13}$$

For the observed deaths,

$$ {}_nD_x = \sum_{i=1}^{m} {}_nD_x^i. \tag{3.14}$$

It follows , then, that the numerator of the equations for calculating ${}_nq_x$ [equations (2.3), (2.4), and (2.5)] may be partitioned according to cause and, thus, so also may the value of ${}_nq_x$. The denominator of these equations is assumed to be unaffected by such partitioning.

The probability of dying in a given age interval when a cause of death has been eliminated is given by

$$ {}_nq_x^{(-i)} = 1 - {}_np_x^{({}_nq_x - {}_nQ_x^i)/{}_nq_x} \tag{3.15}$$

which is estimated by

$$ {}_nq_x^{(-i)} = 1 - {}_np_x^{({}_nD_x - {}_nD_x^i)/{}_nD_x} = 1 - {}_np_x^{1 - r_x^i} \tag{3.16}$$

where ${}_np_x$ is the probability of surviving the age interval (i.e., the complement of ${}_nq_x$), ${}_nD_x^i$ is the number of deaths due to cause i in the age interval, and ${}_nD_x$ is the total number of deaths in the age interval (Chiang, 1968:257; Greville, 1973:114; n.d.:7).[1] As in the main life table, the value of ${}_nq_x^{(-i)}$ for the terminal age interval is set at unity.

As in the main life table, once the values of ${}_nq_x^{(-i)}$ have been determined, all other life table values may be calculated in the same manner as those in main life table, except for ${}_nL_x^{(-i)}$.

The third basic value requiring an additional method of calculation is ${}_nL_x^{(-i)}$. Greville (1948b, 1954) earlier suggested alternative methods for calculating this value. However, in his later work, Greville (n.d.) utilized the formula below.

Under the assumption that the average number of years lived by those who die in an age interval is the same in the life table eliminating cause i as in the main life table due to all causes, the values of ${}_nL_x^{(-i)}$ may be calculated by

$$ {}_nL_x^{(-i)} = (n - {}_nf_x)\, l_x^{(-i)} + {}_nf_x\, l_{x+n}^{(-i)} \qquad (3.17) $$

where

$$ {}_nf_x = (nl_x - {}_nL_x)/(l_x - l_{x+n}), \qquad (3.18) $$

where the values are taken from the main life table. The value $(n - {}_nf_x)$ is the average number of years lived by those

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[1]The mathematical derivation of the estimation formula is given by Chiang (1968:249).

who die in the age interval.

Thus, the value of ${}_nL_x^{(-i)}$ is made up of two components. First, all the survivors to the beginning of the age interval, $l_x$, live, on the average, $(n - {}_nf_x)$ years in the age interval, x to x + n. Those who die in the age interval live, on the average, only $(n - {}_nf_x)$ years. Second, those persons who survive to the next age interval, $l_{x+n}$, live an additional ${}_nf_x$ years. Thus, those $l_{x+n}$ persons who survive the age interval, x to x + n, live a total of $[(n - {}_nf_x) + {}_nf_x]$ or n years, the length of the age interval.

For the terminal age group, the value of ${}_nL_x^{(-i)}$ is taken as equal to

$$ {}_nL_x^{(-i)} = [e_{85} l_{85}^{(-i)}]/(1 - r_{85}^{i}) \qquad (3.19) $$

where $e_{85}$ is the life expectancy at age 85 from the main life table, $l_{85}^{(-i)}$ is the number of survivors to the terminal age group from the cause-eliminated table, and $r_{85}^{i}$ is the proportion of deaths due to cause i in the terminal age interval.

The expectation of life for the cause-eliminated table was calculated in the usual manner.

## Summary

This chapter presented a general description of life tables with causes of death eliminated. The points discussed were:

1. Cause-eliminated life tables arose, historically, from the study of longevity and cause-specific mortality, es-

pecially the study of deaths due to small pox. The major impetus for the development of these special cause-eliminated tables came from the work of Bernoulli, Laplace, D'Alembert, and Makeham.

2. Methods of constructing life tables with causes of death eliminated were described. Methods used to construct the special tables in the present study were due primarily to Greville and Chiang.

# CHAPTER 4. DATA: SOURCES AND ADJUSTMENTS

The construction of 1969-71 life tables due to all causes and life tables with groups of causes eliminated required three sets of data:

(1) 3-year average of deaths by age, sex, and cause including deaths due to all causes;

(2) estimated July 1, 1970 population (i.e., the midpoint of the 3-year period, 1969-71) by age and sex;

(3) separation factors by sex for the under 1 year age interval.

The 3-year average of deaths and the July 1 population were the input data used to generate values of ${}_nq_x$ and, consequently, the remaining functions in the life table. The WATFIV program written to construct the main and special life tables is given in Appendix A. Separation factors were applied to adjust the value of ${}_nL_x$ for the first age interval.

## 3-Year Average of Deaths

A 3-year average of deaths centered on July 1, 1970, was used to minimize the possible effects of unusual fluctuation of deaths in a given year. Data were taken from the _Vital Statistics of the United States_ for respective years (U. S. Department of Health, Education, and Welfare, 1973, 1974b, 1974d). Three-year averages were computed for all causes and for specific groups of causes of death by the required age detail for the 3-year period. Three-year averages of deaths due to all causes and due to specific groups of causes are presented in Table 4.1.

## Estimates of July 1, 1970 Population

Estimates of the July 1, 1970 populations by age, and sex were derived using the procedure suggested by Tarver and Black (1966). The program developed to carry out this procedure is given in Appendix B.

There are no census data or estimates of sufficient detail for years not ending in zero. Consequently, the 3-year average population cannot be calculated in the same manner as average yearly deaths (i.e., the simple average of the sum of deaths during the 3-year period). Instead, an estimate of the average population is taken as the midpoint population; that is, July 1 of the middle or second year of the 3-year period. Since census data are given for April 1 for years ending in zero, these figures can be adjusted to allow for 3 months of fertility, mortality, migration, and other factors influencing population change.

The general procedure suggested by Tarver and Black (1966:17-27) involves comparing the population of cohorts classified by age at two censuses, taking into account births and deaths which add to and subtract from the younger cohorts, and aging these cohorts by three months to derive an estimate of the July 1 populations for the two census years.

Table 4.1. Three-year average of deaths due to all causes and to specific causes by age and sex, United States, 1969-1971.

| | Cause | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Age | All | MN | DH | MVA | All | MN | DH | MVA |
| | Males | | | | Females | | | |
| < 1 | 41742 | 82 | 253 | 184 | 30832 | 78 | 210 | 173 |
| 1- 4 | 6528 | 568 | 122 | 901 | 5162 | 453 | 107 | 701 |
| 5- 9 | 5080 | 784 | 82 | 1360 | 3406 | 565 | 86 | 791 |
| 10-14 | 5383 | 616 | 110 | 1326 | 3084 | 464 | 100 | 677 |
| 15-19 | 15406 | 840 | 271 | 6321 | 5898 | 514 | 179 | 2196 |
| 20-24 | 17986 | 923 | 375 | 6856 | 6340 | 582 | 247 | 1715 |
| 25-29 | 13575 | 937 | 588 | 3696 | 5961 | 795 | 352 | 954 |
| 30-34 | 12833 | 1131 | 1261 | 2423 | 6981 | 1336 | 593 | 736 |
| 35-39 | 17109 | 1920 | 3380 | 2089 | 10384 | 2567 | 1214 | 723 |
| 40-44 | 28000 | 4040 | 8258 | 2109 | 17057 | 5228 | 2615 | 810 |
| 45-49 | 43953 | 7639 | 16245 | 2108 | 26176 | 9084 | 4893 | 824 |
| 50-54 | 62862 | 12722 | 26005 | 1976 | 35355 | 12560 | 8193 | 863 |
| 55-59 | 87935 | 19507 | 37871 | 1872 | 46811 | 15904 | 13328 | 816 |
| 60-64 | 111749 | 25090 | 49129 | 1622 | 60353 | 17725 | 20965 | 780 |
| 65-69 | 128126 | 27926 | 56786 | 1391 | 78569 | 19351 | 31159 | 773 |
| 70-74 | 136799 | 27048 | 61251 | 1191 | 101229 | 19633 | 44339 | 779 |
| 75-79 | 136127 | 23510 | 61238 | 1022 | 122797 | 18690 | 56524 | 635 |
| 80-84 | 109282 | 15309 | 50100 | 643 | 124549 | 14003 | 59354 | 371 |
| 85+ | 97992 | 9446 | 46390 | 333 | 153401 | 10792 | 75035 | 169 |

The difference between the population of cohorts for July 1 of the census years is an estimate of 10-year change in population. This estimate is inflated to account for 10 years and 3 months of change and this inflated value, under the assumption of stability of trends, is added to the census population for the previous census to estimate the July 1 population for the later census year. These estimated cohort populations on July 1 may be adjusted to an estimate of the total population. The results of the adjustment procedure using 1960 and 1970 data for U. S. males and females are given in Table 4.2.

The adjustment procedure requires population data by age and sex and births and deaths by sex for selected age intervals. The age intervals utilized must match those employed in the life table (i.e., under 1, 1-4, 5-9, 10-14, . . ., 80-84, 85 and over).

Column (1) of Table 4.2 shows the April 1, 1970 population aged to July 1, 1970. Each enumerated age interval on April 1, 1970, was a cohort. Those persons, for example, enumerated in the 20 to 24 age interval were born between April 1, 1945, and April 1, 1950. However, in the three months between April 1, and July 1, 1970, some of the cohort reached their 25th birthday and, thus, contributed to the 25 to 29 age interval. Assuming that births occurred uniformly throughout each year in the age interval, it was estimated

Table 4.2. Estimated July 1 population by age and sex, United States, 1970.

| Age | April 1, 1970 population aged to July 1 (1) | April 1, 1960 population aged to July 1 (2) | Adjusted 1960 population classified by age July 1, 1970 (3) | 10-year change (1)-(3) (4) | Change April 1, 1960 to July 1, 1960 (3)*1.025 (5) | Estimated July 1 population (April 1, 1960 population + column 4) (6) | Estimated July 1 population adjusted to census estimates (7) |
|---|---|---|---|---|---|---|---|
| | | | | Males | | | |
| < 1 | 1777833 | 0 | 0 | 0 | 0 | 1777833 | 1788436 |
| 1- 4 | 6974964 | 10342900 | 0 | 0 | 0 | 6974964 | 7016564 |
| 5- 9 | 10094280 | 9545635 | 0 | 0 | 0 | 10094280 | 10154480 |
| 10-14 | 10569620 | 8573292 | 10342900 | 226721 | 232389 | 10562110 | 10625110 |
| 15-19 | 9681690 | 6728191 | 9545635 | 136055 | 139456 | 9643824 | 9701342 |
| 20-24 | 8003097 | 5340405 | 8573292 | -570195 | -584450 | 7939839 | 7987194 |
| 25-29 | 6686351 | 5330038 | 6728191 | -41840 | -42886 | 6590775 | 6630084 |
| 30-34 | 5647078 | 5820565 | 5340405 | 306673 | 314340 | 5586679 | 5619999 |
| 35-39 | 5421590 | 6067847 | 5330038 | 91552 | 93841 | 5426915 | 5459282 |
| 40-44 | 5798493 | 5696061 | 5820565 | -22072 | -22624 | 5823600 | 5858333 |
| 45-49 | 5849707 | 5373822 | 6067847 | -218140 | -223593 | 5855918 | 5890844 |
| 50-54 | 5373086 | 4765983 | 5696061 | -322975 | -331049 | 5344831 | 5376709 |
| 55-59 | 4794924 | 4157623 | 5373822 | -578898 | -593370 | 4764554 | 4792971 |
| 60-64 | 4063914 | 3445215 | 4765983 | -702069 | -719620 | 4015208 | 4039155 |
| 65-69 | 3167327 | 2954998 | 4157623 | -990296 | -1015053 | 3112192 | 3130753 |
| 70-74 | 2355353 | 2222509 | 3445215 | -1089862 | -1117108 | 2292211 | 2305882 |
| 75-79 | 1598377 | 1400712 | 2954998 | -1356621 | -1390536 | 1540552 | 1549740 |
| 80-84 | 909838 | 699810 | 2222509 | -1312671 | -1345487 | 839729 | 844737 |
| 85+ | 586158 | 395531 | 2496051 | -1909892 | -1957638 | 429155 | 431715 |

Females

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| < 1 | 1704866 | 0 | 0 | 0 | 0 | 1704866 | 1713314 |
| 1- 4 | 6708209 | 9999959 | 0 | 0 | 0 | 6708209 | 6741450 |
| 5- 9 | 9716368 | 9227599 | 0 | 0 | 0 | 9716368 | 9764515 |
| 10-14 | 10178180 | 8296112 | 9999959 | 178222 | 182677 | 10173840 | 10224260 |
| 15-19 | 9474611 | 6668762 | 9227599 | 247012 | 253187 | 9440599 | 9487379 |
| 20-24 | 8502889 | 5581278 | 8296112 | 206777 | 211946 | 8461149 | 8503076 |
| 25-29 | 6935341 | 5535667 | 6668762 | 266579 | 273243 | 6858825 | 6892812 |
| 30-34 | 5885684 | 6074615 | 5581278 | 304406 | 312016 | 5840437 | 5869378 |
| 35-39 | 5701438 | 6386665 | 5535667 | 165771 | 169915 | 5705964 | 5734238 |
| 40-44 | 6138754 | 5948222 | 6074615 | 64139 | 65742 | 6168704 | 6199271 |
| 45-49 | 6259481 | 5541699 | 6386665 | -127184 | -130364 | 6271233 | 6302308 |
| 50-54 | 5781526 | 4903646 | 5948222 | -166696 | -170863 | 5753498 | 5782008 |
| 55-59 | 5234651 | 4331044 | 5541699 | -307048 | -314724 | 5206835 | 5232636 |
| 60-64 | 4620681 | 3761607 | 4903646 | -282965 | -290039 | 4581086 | 4603786 |
| 65-69 | 3905553 | 3347136 | 4331044 | -425491 | -436128 | 3866491 | 3885650 |
| 70-74 | 3165866 | 2592371 | 3761607 | -595741 | -610634 | 3122498 | 3137970 |
| 75-79 | 2316905 | 1737113 | 3347136 | -1030231 | -1055986 | 2270836 | 2282088 |
| 80-84 | 1451998 | 953799 | 2592371 | -1140373 | -1168881 | 1384835 | 1391697 |
| 85+ | 1038958 | 612718 | 3303629 | -2264670 | -2321285 | 854660 | 858895 |

that 3/12 or .25 of the persons born each year had a birthday between April 1 and July 1. Thus, in the 20 to 24 age cohort, .25 of the 24 year olds (or 3/60 or .05 of the 5-year age cohort) were assumed to have attained their 25th birthday. As a result of this aging process, the composition of the 20 to 24 age interval changed; that is , on July 1 it was vacated by 1/20 of the cohort (or 1/4 of the 24 year olds) and 1/4 of the 19 year olds (or 1/20 of the 15 to 19 age interval on April 1) entered the 20 to 24 age interval. This procedure was used to calculate the loss and gain for each age interval from 10 to 14 through 80 to 84.

Under the assumption of the uniform distribution of births throughout a year, the expected number of births in any 3-month period is 1/4 or .25 of the total births for the year. Thus, .25 of all persons of a particular age are expected to "age" into the next age interval. With 5-year age intervals, 3/60 (i.e., 60 months = 5 years) or .05 of all persons in the interval are expected to "age" into the next 5-year age interval. On the other hand, 57/60 or .95 of all persons in the interval are expected to remain in the interval even while aging 3 months. In general, for 5-year age intervals,

$$ {}_5P'_x = (.05)\,{}_5P_{x-5} + (.95)\,{}_5P_x \qquad (4.1) $$

where x denotes the 5-year age interval of interest, x - 5 denotes the previous 5-year age interval, and ${}_5P'_x$ denotes the adjusted population.

Special procedures were used to calculate the population for age intervals under 10 years and for the terminal age interval. For the population under 1 year of age, the July 1 population was expected to consist of 9/12 or .75 of the persons in the interval on April 1 plus the number of births in the 3-month period and minus the number of infant deaths in the 3-month period. In general,

$$ {}_{1}P'_{0-1} = (.75)\ {}_{1}P_{0-1} + B^{(a-j)} - D^{(a-j)}_{0-1} \qquad (4.2) $$

where (a-j) denotes April through June.

The 1 to 4 age interval on July 1 was expected to contain .25 of the under 1 year age interval persons who aged in the 3-month period plus 45/48 (i.e., 48 months = 4 years) or .9375 of the cohort that remained in the 1 to 4 age interval and minus the number of deaths in the 3-month period to persons aged 1 to 4 years. In general,

$$ {}_{4}P'_{1-4} = (.25)\ {}_{1}P_{0-1} + (.9375)\ {}_{4}P_{1-4} - D^{(a-j)}_{1-4}. \qquad (4.3) $$

The 5 to 9 age interval on July 1 was expected to contain 3/48 or .0625 of the 1 to 4 age interval persons who aged in the 3-month period plus .95 of the cohort that remained in the 5 to 9 age interval and minus the number of deaths to persons aged 5 to 9 years in the 3-month period. In general,

$$ {}_{5}P'_{5-9} = (.0625)\ {}_{4}P_{1-4} + (.95)\ {}_{5}P_{5-9} - D^{(a-j)}_{5-9}. \qquad (4.4) $$

The terminal age interval (85 years and older) received .05 of the persons in the 80 to 84 age interval while no one exited this interval. In general,

$$P'_{85} = (.05)\,{}_5P_{80-84} + P_{85}. \qquad (4.5)$$

The population by age and sex at the previous census (i.e., April 1, 1960) was "aged" by the same procedure as described above. These values are given in column (2) in Table 4.2. The only difference in the procedures lies in the calculations for the youngest age intervals. The value in (2) refers to the combined age interval 0 to 4 years. It was expected that this age interval would contain .95 of the cohort that remained in the interval even while aging in the 3-month period plus the number of births in April through June, 1960, and minus the number of infant deaths in the same 3-month period. In general,

$$ {}_5P'_{0-4} = (.95)\,{}_5P_{0-4} + B^{(a-j)} - D^{(a-j)}_{0-4}. \qquad (4.6)$$

Since the adjusted July 1 , 1960, population was classified by age on July 1, 1970, it was unnecessary to carry out similar calculations for subsequent age intervals. For the terminal age interval, the procedure for "aging" the population was the same as described above.

Column (3) is the adjusted July 1, 1960 population classified by age on July 1, 1970; that is, it is the July 1, 1960 population shown 10 years later in 1970 assuming no loss or gain due to death, migration or other factors. The first three age intervals were given values of zero since persons in these age intervals in 1970 were not yet born in 1960. The value for the terminal age interval was calculated as the sum of the last three age intervals on July 1, 1970; that is,

$$ {}_5P'_{75-79} + {}_5P'_{80-84} + P'_{85}. \tag{4.7} $$

The July 1, 1960 population classified by age on July 1, 1970 (3), was subtracted from the adjusted July 1, 1970 population (2) to yield an estimate of the amount of change for the decade between the two July 1 dates. These values are shown in column (4). These values represent the differences between the "observed" (adjusted) 1970 population and the expected 1970 population under the assumption of no change due to death, migration, or other factors. Consequently, the positive and negative differences indicate growth and decline. The first three age intervals were again omitted because those persons were not yet born in 1960.

Column (5) allows for an additional three months of mortality, migration, and other factors by extrapolating the changes found in the 10-year period, July 1, 1960, to July 1, 1970. It represents the changes in the population in 10 1/4 years. Three months of 10 years or 120 months is 3/120 or .025 of the total period. Thus, the change for the 10-year and 3-month period is 1.025 of the change for the 10 years.

The estimated July 1, 1970 population [column (6)] was obtained by adding the change for 10 1/4 years, (5) to the

April 1, 1960 population for each age interval. These values were adjusted to Bureau of the Census July 1, 1970 estimates of the total resident population of the United States [column (7)].

The July 1 population adjustment procedure required four sets of data:

(1) population by age and sex, 1970;

(2) births, April through June, by sex, 1960 and 1970;

(3) infant deaths, April through June, by sex, 1960 and 1970, and deaths, April through June, by sex, 1970, for persons aged 1 to 4 years and 5 to 9 years;

(4) estimates of the total July 1 resident population of the United States by sex, 1970.

Census data by age and sex for 1970 were taken from Table 52, Characteristics of the Population (U. S. Bureau of the Census, 1973) and for 1960 from Table 46, Characteristics of the Population (U. S. Bureau of the Census, 1964).

For 1970, Table 1-49, Vital Statistics of the United States 1970 (U. S. Department of Health, Education, and Welfare, 1975) reported births by month and sex. The number of births, April through June, by sex was summed for use in adjusting the under 1 year population to July 1, 1970. The number of infant deaths by sex, April through June, 1970, was taken from Table 2-10, Vital Statistics of the United States 1970 (U. S. Department of Health, Education, and Welfare, 1974a). Deaths for the 1 to 4 and 5 to 9 age intervals were derived from Table 7-5, Vital Statistics of the United States 1970 (U. S. Department of Health, Education, and Welfare, 1974b). These deaths were not reported by month. Under the assumption that deaths were uniformly distributed throughout the year, 1/4 or .25 of annual deaths were expected to occur in any 3-month period.

The derivation of births, April through June, 1960, by sex involved a two-step procedure. The data were reported by month but not by sex. Thus, the sum of births, April through June (U. S. Department of Health, Education, and Welfare, 1962: Table 2.3), was multiplied by the proportion of male and female births of total births for the year (U. S. Department of Health, Education, and Welfare, 1962: Table 2-4) to derive estimates of births by sex for the 3-month period. This procedure assumed that the sex ratio at birth was uniform throughout the year. The number of infant deaths, April through June, by sex was taken from Table 3-2, Vital Statistics of the United States 1960 (U. S. Department of Health, Education, and Welfare, 1963).

July 1, 1970 estimates by age and sex were adjusted to sum to an estimate of the total resident population of the United States, July 1, 1970. Estimates of the July 1, 1970 resident population of the United States were obtained from Table 3, Current Population Reports, Series P-25, Number 520

(U. S. Bureau of the Census, 1974). These estimates were not given by sex. Thus, it was necessary to derive estimates by sex. Assuming that the sex distribution of the population did not change radically in three months, the July 1, 1970 estimate of the total population was multiplied by the percentage distribution by sex on April 1, 1970 (U. S. Bureau of the Census, 1973: Table 52) to produce the required estimates.

Separation Factors by Sex, 1970

Separation factors for infant deaths reflect the proportion of infant deaths in a given year which occurred to infants born in that year. Separation factors for 1970 by sex were calculated by the procedure suggested by Shryock and Siegel (1973:412-414). This method assumes that accurate estimates of separation factors can be made on the basis of tabulations of deaths by detailed age at death under 1 year (i.e., days, under 1 week, weeks, under 1 month, months, under 1 year). According to Shryock and Siegel (1973:413), the method consists of assuming that within each tabulation cell the deaths are rectangularly (uniformly) distributed over time and age, determining the proportions for separating deaths in each cell according to year of birth, estimating the number of deaths in each cell that occurred to births of the previous year, cumulating those numbers over all cells, and dividing the result by the total number of infant deaths in that year. Calculations for U. S. males and females, 1970, are presented in Tables 4.3, 4.4, 4.5, and 4.6 using data from Table 2-10, Vital Statistics of the United States 1970 (U. S. Department of Health, Education, and Welfare, 1974a).

The procedure for calculating separation factors, using male data, may be described as follows. Table 4.3 arrays infant male deaths by detailed age and month of death. The heavy lines designate four groups of deaths. Deaths falling below the diagonal lines occurred wholly to infants born in 1969. Monthly columns were totaled over all rows below the diagonal. These column totals were summed across all columns (months) (i.e., 3584). Deaths falling above the diagonal occurred wholly to infants born in 1970. As before, monthly columns were totaled over all rows above the diagonal. These column totals were summed across all columns (months) (i.e, 35,815).

Deaths falling within the diagonal lines and within the vertical lines occurred partly to infants born in 1969 and partly to infants born in 1970. Under the assumption of a rectangular distribution, deaths falling between the diagonals from February through December belong equally to each year. Thus, one-half of the sum of the diagonal deaths (i.e., 826) were allocated to each year.

Table 4.3. Deaths to males under 1 year of age by age and month, United States, 1970.[a]

| Age at death | Jan. | Feb. | March | April | May | June |
|---|---|---|---|---|---|---|
| Under 1 year | 3749 | 3173 | 3546 | 3365 | 3692 | 3606 |
| Sum | | 2190 | 2853 | 2871 | 3352 | 3356 |
| Under 28 days | 2622 | 2190 | 2627 | 2514 | 2891 | 2875 |
| Under 1 hour | 270 | 273 | 274 | 292 | 318 | 297 |
| 1 to 23 hours | 1249 | 987 | 1245 | 1199 | 1389 | 1449 |
| 1 day | 350 | 315 | 401 | 362 | 402 | 397 |
| 2 days | 243 | 201 | 233 | 232 | 253 | 235 |
| 3 days | 98 | 87 | 116 | 104 | 103 | 118 |
| 4 days | 58 | 61 | 59 | 56 | 65 | 59 |
| 5 days | 41 | 41 | 44 | 41 | 44 | 52 |
| 6 days | 37 | 22 | 23 | 21 | 41 | 29 |
| 7 to 13 days | 137 | 92 | 114 | 97 | 139 | 132 |
| 14 to 20 days | 60 | 53 | 60 | 61 | 64 | 63 |
| 21 to 27 days | 79 | 58 | 58 | 49 | 73 | 44 |
| 28 to 59 days | 239 | 202 | 226 | 205 | 194 | 197 |
| 2 months | 235 | 186 | 184 | 152 | 154 | 138 |
| 3 months | 157 | 154 | 135 | 131 | 113 | 75 |
| 4 months | 131 | 107 | 87 | 104 | 72 | 71 |
| 5 months | 97 | 78 | 88 | 58 | 66 | 59 |
| 6 months | 75 | 72 | 40 | 51 | 50 | 47 |
| 7 months | 44 | 55 | 48 | 47 | 42 | 36 |
| 8 months | 33 | 44 | 38 | 32 | 31 | 31 |
| 9 months | 39 | 31 | 30 | 31 | 32 | 24 |
| 10 months | 44 | 31 | 23 | 25 | 20 | 33 |
| 11 months | 33 | 23 | 20 | 15 | 27 | 20 |
| Sum | 1127 | 781 | 509 | 363 | 268 | 191 |

[a]U. S. Department of Health, Education, and Welfare (1974a: Table 2-10).

| July | Aug. | Sept. | Oct. | Nov. | Dec. | Sum |
|---|---|---|---|---|---|---|
| 3624 | 3670 | 3523 | 3622 | 3525 | 3752 | |
| 3475 | 3533 | 3453 | 3544 | 3467 | 3721 | 35815 |
| 3012 | 2923 | 2802 | 2770 | 2556 | 2705 | |
| 313 | 311 | 327 | 296 | 276 | 270 | |
| 1430 | 1385 | 1323 | 1257 | 1204 | 1269 | |
| 456 | 414 | 405 | 416 | 368 | 367 | |
| 251 | 246 | 229 | 268 | 227 | 247 | |
| 113 | 117 | 92 | 106 | 102 | 136 | |
| 75 | 82 | 57 | 72 | 61 | 73 | |
| 56 | 52 | 53 | 41 | 50 | 44 | |
| 38 | 44 | 40 | 41 | 25 | 33 | |
| 133 | 147 | 143 | 151 | 102 | 128 | |
| 85 | 62 | 67 | 63 | 79 | 69 | |
| 62 | 63 | 66 | 59 | 62 | 69 | |
| 181 | 209 | 228 | 246 | 232 | 263 | |
| 105 | 136 | 151 | 173 | 216 | 221 | |
| 82 | 95 | 90 | 118 | 142 | 178 | |
| 57 | 79 | 57 | 72 | 105 | 123 | |
| 38 | 54 | 57 | 56 | 61 | 70 | |
| 35 | 37 | 42 | 48 | 49 | 50 | |
| 28 | 34 | 26 | 35 | 50 | 31 | |
| 23 | 34 | 22 | 26 | 34 | 23 | |
| 28 | 21 | 17 | 24 | 22 | 29 | |
| 14 | 25 | 22 | 28 | 32 | 28 | |
| 21 | 23 | 9 | 26 | 26 | 31 | 826 |
| 114 | 103 | 48 | 54 | 26 | 0 | 3584 |

Table 4.4. Deaths to females under 1 year of age by age and month, United States, 1970.[a]

| Age at death | Jan. | Feb. | March | April | May | June |
|---|---|---|---|---|---|---|
| Under 1 year | 2766 | 2416 | 2586 | 2522 | 2504 | 2639 |
| Sum | | 1695 | 1994 | 2154 | 2227 | 2465 |
| Under 28 days | 1904 | 1695 | 1825 | 1863 | 1931 | 2123 |
| Under 1 hour | 233 | 200 | 255 | 252 | 261 | 284 |
| 1 to 23 hours | 874 | 782 | 855 | 847 | 909 | 996 |
| 1 day | 253 | 249 | 241 | 250 | 244 | 291 |
| 2 days | 151 | 148 | 138 | 132 | 136 | 185 |
| 3 days | 72 | 71 | 57 | 70 | 84 | 66 |
| 4 days | 40 | 45 | 47 | 42 | 42 | 47 |
| 5 days | 30 | 26 | 36 | 37 | 37 | 40 |
| 6 days | 30 | 19 | 23 | 29 | 26 | 24 |
| 7 to 13 days | 100 | 82 | 76 | 109 | 105 | 97 |
| 14 to 20 days | 66 | 36 | 53 | 54 | 44 | 56 |
| 21 to 27 days | 55 | 37 | 44 | 41 | 43 | 37 |
| 28 to 59 days | 205 | 177 | 169 | 165 | 134 | 123 |
| 2 months | 158 | 135 | 125 | 126 | 87 | 89 |
| 3 months | 119 | 112 | 125 | 87 | 75 | 76 |
| 4 months | 109 | 66 | 78 | 71 | 84 | 54 |
| 5 months | 82 | 56 | 61 | 39 | 47 | 42 |
| 6 months | 42 | 40 | 46 | 43 | 44 | 45 |
| 7 months | 38 | 32 | 43 | 35 | 30 | 28 |
| 8 months | 33 | 36 | 26 | 27 | 20 | 18 |
| 9 months | 32 | 17 | 29 | 19 | 21 | 10 |
| 10 months | 20 | 18 | 32 | 20 | 15 | 15 |
| 11 months | 24 | 32 | 27 | 27 | 16 | 16 |
| Sum | 862 | 544 | 467 | 281 | 193 | 132 |

[a]U. S. Department of Health, Education, and Welfare (1974a: Table 2-10).

| July | Aug. | Sept. | Oct. | Nov. | Dec. | Sum |
|---|---|---|---|---|---|---|
| 2764 | 2724 | 2616 | 2813 | 2730 | 2740 | |
| 2601 | 2614 | 2538 | 2744 | 2693 | 2709 | 26434 |
| 2205 | 2185 | 2068 | 2142 | 1969 | 1882 | |
| 269 | 261 | 242 | 246 | 242 | 223 | |
| 1074 | 1018 | 947 | 1009 | 876 | 852 | |
| 293 | 301 | 310 | 285 | 299 | 275 | |
| 183 | 170 | 147 | 178 | 160 | 168 | |
| 70 | 87 | 71 | 75 | 73 | 75 | |
| 49 | 46 | 68 | 48 | 53 | 41 | |
| 45 | 42 | 49 | 37 | 25 | 21 | |
| 31 | 26 | 30 | 25 | 33 | 23 | |
| 92 | 106 | 106 | 115 | 111 | 108 | |
| 58 | 73 | 53 | 76 | 55 | 51 | |
| 41 | 55 | 45 | 48 | 42 | 45 | |
| 151 | 179 | 163 | 183 | 181 | 210 | |
| 102 | 97 | 101 | 155 | 145 | 190 | |
| 57 | 57 | 74 | 82 | 116 | 136 | |
| 49 | 50 | 32 | 47 | 69 | 92 | |
| 37 | 25 | 30 | 46 | 75 | 57 | |
| 30 | 21 | 35 | 38 | 42 | 43 | |
| 37 | 29 | 35 | 26 | 33 | 32 | |
| 29 | 24 | 18 | 25 | 31 | 20 | |
| 18 | 22 | 17 | 24 | 32 | 28 | |
| 18 | 21 | 21 | 24 | 17 | 19 | |
| 31 | 14 | 22 | 21 | 20 | 31 | 664 |
| 133 | 81 | 60 | 45 | 20 | 0 | 2818 |

Table 4.5. Proportion of deaths under 2 months of age in January, by age, assumed to occur to births of the previous year (calculated on the basis of 31 days and 744 hours in the month).

| Age at death | Proportion occurring in previous year |
|---|---|
| 28 to 59 days | (28/31)+(3/31) (28/32)+(1/2) (3/31) (4/32)=(493/496)=.9940=1.0000 |
| 21 to 27 days | (21/31)+(1/2) (7/31)=(49/62)=.7903 |
| 14 to 20 days | (14/31)+(1/2) (7/31)=(35/62)=.5645 |
| 7 to 13 days | (7/31)+(1/2) (7/31)=(21/62)=.3381 |
| 6 days | (6/31)+(1/2) (1/31)=(13/62)=.2097 |
| 5 days | (5/31)+(1/2) (1/31)=(11/62)=.1774 |
| 4 days | (4/31)+(1/2) (1/31)=(9/62)=.1452 |
| 3 days | (3/31)+(1/2) (1/31)=(7/62)=.1129 |
| 2 days | (2/31)+(1/2) (1/31)=(5/62)=.0806 |
| 1 day | (1/31)+(1/2) (1/31)=(3/62)=.0484 |
| 1 to 23 hours | (1/744)+(1/2) (23/744)=(25/1488)=.0168 |
| Under 1 hour | (1/2) (1/744)=(1/1488)=.0007=.0000 |

Table 4.6. Separation factors by sex, United States, 1970.

| | Males | | Females | |
|---|---|---|---|---|
| | D" | D' | D" | D' |
| Total | 4231 | 38616 | 3328 | 28492 |
| Sum of deaths below diagonal | 3584 | X | 2818 | X |
| Sum of deaths above diagonal | X | 35815 | X | 26434 |
| Sum of .5 deaths each month, Feb. through Dec., within diagonal | 413 | 413 | 332 | 332 |
| Sum of Jan. deaths under 1 year | (234) | (2388) | (178) | (1726) |
| 21 to 27 days | 62 | 17 | 43 | 12 |
| 14 to 20 days | 34 | 26 | 37 | 29 |
| 7 to 13 days | 46 | 91 | 34 | 66 |
| 6 days | 8 | 29 | 6 | 24 |
| 5 days | 7 | 34 | 5 | 25 |
| 4 days | 8 | 50 | 6 | 34 |
| 3 days | 11 | 87 | 8 | 64 |
| 2 days | 20 | 223 | 12 | 139 |
| 1 day | 17 | 333 | 12 | 241 |
| 1 to 23 hours | 21 | 1228 | 15 | 859 |
| Under 1 hour | 0 | 270 | 0 | 233 |
| Separation factors: | | | | |
| f'=D'/(D'+D") | .90125329 | | .89541169 | |
| f"=D"/(D'+D") | .09874670 | | .10458830 | |

The deaths within the vertical lines must be allocated to the years according to the proportion of deaths under one month of age in January, by age, assumed to occur to births in the respective years. Table 4.5 presents these proportions. Table 4.5 was calculated on the assumption of 31 days (January) and 744 hours in a month. This procedure also assumes a rectangular distribution; that is, deaths at any age were assumed to belong equally to each year. For example, of all deaths at 3 days of age in January, all deaths on January 1, 2, and 3 (3/31 of the total), one-half of those on January 4 (1/2 * 1/31), and none of the deaths later in the year occurred to births of the previous, year, 1969. The resulting proportion, .1129, is multiplied by January deaths of 3 days, 98, to yield an estimate of 11 deaths to infant males born in 1969. Table 4.6 presents the distribution of 1970 infant male deaths by year of birth. Summing each column and dividing by the total infant male deaths yields the proportion of 1970 infant deaths to infant males born in 1970, f', and the proportion of 1970 infant male deaths to infant males born in 1969, f". f' and f" are the separation factors.

## Summary

This chapter described the sources of, and adjustments to, data required to construct life tables for the present study. Three types of data were required.

1. Three-year averages of deaths by age and sex were

derived from Vital Statistics of the United States for the years 1969, 1970, and 1971.

2. Estimates of the July 1, 1970 population by age and sex were derived from procedures suggested by Tarver and Black. The general procedure involves comparing the population of cohorts classified by age at two censuses, taking into account births and deaths for younger cohorts, and aging these cohorts by three months to derive an estimate of the July 1 population for the two census years, 1960 and 1970. The difference between these two July 1 populations is an estimate of the 10-year change in population. This estimate is inflated to account for 10 years and 3 months of change and this inflated value is added to the 1960 census count to estimate the July 1 population for 1970.

3. Separation factors for infant deaths were calculated by sex using procedures suggested by Shryock and Siegel. Separation factors for infant deaths reflect the proportion of infant deaths in a given year that occurred to infants born in that year.

# CHAPTER 5. COMPARISONS BASED ON THE LIFE TABLE AS A COHORT

As noted in Chapter 1, the life table is subject to two interpretations. The most common interpretation is that of the life table viewed as a method of tracing the mortality experience of a cohort of 100,000 persons from birth until the cohort has been depleted by death. This chapter describes methods of comparison which are appropriate to this interpretation of the life table. Under the cohort interpretation of the life table, each function has a particular importance. However, the most often utilized functions, and those which are the focus of the present discussion, are probabilities of dying and surviving, joint probabilities, life expectancy, and life table deaths.

## Competing Risk Theory

Death is not a repetitive event and is usually attributed to a single cause. Various risks compete for the life of an individual. Therefore, these competing risks must be considered in cause-specific mortality studies. The distinction between risk and cause is a temporal one. Both terms may refer to the same condition. However, prior to death the condition is referred to as a risk while after death the same condition is referred to as the cause (Chiang, 1968:243). Thus, for example, cancer is a risk of dying to which an individual is exposed, but is also the cause of death if it is the disease to which the individual eventually succumbs.

Competing risk theory (Chiang, 1968, 1970; Moeschberger and David, 1971; Gail, 1975) depends on the relationship between three types of probability of death from a specific cause. The crude probability is the probability of death from a specific cause in the presence of all other risks acting in a population or

$_{n}Q_{x}^{i}$ = Pr[an individual alive at age x will die in the interval (x, x + n) from cause i in the presence of all other risks of death in the population].

The net probability refers to the probability of death if a specific risk of death is eliminated from the population.[1] This probability of dying is the probability calculated in constructing the special life tables in Chapter 3 with causes of death eliminated. The net probability is

$_{n}q_{x}^{(-i)}$ = Pr[an individual alive at age x will die in the interval (x, x + n) if cause i is eliminated as a risk of death].

The partial crude probability is the probability of death from a specific cause when another risk is eliminated from the population. Thus,

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[1]Chiang (1968:243) delineates two types of net probability. The other type of net probability refers to the probability of death if a specific risk is the only risk operating in a population.

$_{n}Q_{x}^{i \cdot z}$ = Pr[an individual alive at age x will die in the interval (x, x + n) from cause i if another disease is eliminated as a risk of death].

The study of the relations between these three types of probability constitutes the problem of competing risk or multiple decrement.[2] Net and partial crude probabilities can be estimated only through their relations with the crude probability in human populations.

The crude probability of dying from a particular cause is derived from the assumption that the total force of mortality in a given age interval is the sum of the risk-specific forces of mortality; that is

$$_{n}q_{x} = {}_{n}Q_{x}^{1} + {}_{n}Q_{x}^{2} + \cdot \cdot \cdot + {}_{n}Q_{x}^{z}, \quad i = 1, \ldots, z \qquad (5.1)$$

where z is the number of risks acting simultaneously on each individual in the population. Thus, $_{n}Q_{x}^{i}$ is that portion of the total probability of dying which is due to cause i.

The age-specific death rate is given by

$$_{n}m_{x} = {}_{n}D_{x}/{}_{n}P_{x} \qquad (5.2)$$

where $_{n}D_{x}$ is the number of observed deaths and $_{n}P_{x}$ is the midyear population. The probability of dying is estimated from

$$_{n}q_{x} = {}_{n}D_{x}/{}_{n}N_{x} \qquad (5.3)$$

---

[2]Mathematical derivations of the equations for these three types of probability are given by Chiang (1968: Chapter 11).

where

$${}_nN_x = ({}_nP_x + .5n\,{}_nD_x)/n. \qquad (5.4)$$

where ${}_nN_x$ is the population exposed to the risk of dying. Dividing both numerator and denominator of (5.3) by ${}_nP_x$ yields

$${}_nq_x = n\,{}_nm_x/(1 + .5n\,{}_nm_x). \qquad (5.5)$$

If deaths are further divided by cause such that

$${}_nD_x = {}_nD_x^1 + {}_nD_x^2 + \cdots + {}_nD_x^z, \qquad (5.6)$$

the estimator of the crude probability of dying from cause i in the presence of of competing risks is given by

$${}_nQ_x^i = {}_nD_x^i/{}_nN_x. \qquad (5.7)$$

Substituting in equation (5.5) yields

$${}_nQ_x^i = n\,{}_nm_x^i/(1 + .5n\,{}_nm_x). \qquad (5.8)$$

Thus, the crude probability can be estimated by multiplying the probability of dying due to all causes by the proportion of deaths due to cause i or

$${}_nQ_x^i = {}_nq_x({}_nD_x^i/{}_nD_x) = {}_nq_x r_x^i. \qquad (5.9)$$

Chiang (1968:246) shows that the probability of dying when risk i is eliminated[3] is given by

$${}_nq_x^{(-i)} = 1 - {}_np_x^{({}_nq_x - {}_nQ_x^i)/{}_nq_x} \qquad (5.10)$$

which may be estimated by

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[3]These are the values which appear in the special life tables with causes eliminated in Chapter 3.

$$ {}_nq_x^{(-i)} = 1 - {}_np_x^{({}_nD_x - {}_nD_x^i)/{}_nD_x} \qquad (5.11) $$

where $({}_nD_x - {}_nD_x^i)/{}_nD_x$ is equivalent to $(1 - r_x^i)$.

The partial crude probability is given by

$$ {}_nQ_x^{i\cdot z} = [{}_nQ_x^i/({}_nq_x - {}_nQ_x^z)]\,{}_nq_x^{(-z)} $$

$$ = [{}_nQ_x^i/({}_nq_x - {}_nQ_x^z)][1 - {}_np_x^{({}_nq_x - {}_nQ_x^z)/{}_nq_x}] \qquad (5.12) $$

where ${}_nq_x^{(-z)}$ is the net probability of dying when cause z is eliminated as a cause of death. The partial crude probability is estimated by

$$ {}_nQ_x^{i\cdot z} = [{}_nD_x^i/({}_nD_x - {}_nD_x^z)][1 - {}_np_x^{({}_nD_x - {}_nD_x^z)/{}_nD_x}] \quad (5.13) $$

where ${}_nD_x^z$ is the number of observed deaths due to cause z.

Table 5.1 presents the absolute and relative changes in the probability of dying when a given cause of death is eliminated. The absolute change represents the difference between ${}_nq_x$ from the life table due to all causes and ${}_nq_x^{(-i)}$ from the life table eliminating cause i as a cause of death. The relative change is calculated as

$$ [{}_nq_x - {}_nq_x^{(-i)}]/{}_nq_x. \qquad (5.14) $$

Examination of Table 5.1 reveals that decreases, absolute and relative, follow the same pattern as the proportion of deaths, $r_x^i$, due to a given cause. The greatest absolute and relative changes by eliminated cause occur for those causes which show the greatest incidence at given ages. This is expected since the value $r_x^i$ is crucial in the calculation of the probability of dying with a given cause eliminated.

Table 5.1. Absolute and relative changes in probability of dying due to the elimination of causes of death by sex, United States, 1969-1971.

| | Cause eliminated | | | | | |
|---|---|---|---|---|---|---|
| | MN | | DH | | MVA | |
| Age | Change | % | Change | % | Change | % |
| | | | **Males** | | | |
| < 1 | .000045 | .1942 | .000138 | .5990* | .000101 | .4356 |
| 1- 4 | .000323 | 8.6873 | .000069 | 1.8656 | .000512 | 13.7808* |
| 5- 9 | .000385 | 15.4151 | .000040 | 1.6092 | .000668 | 26.7473* |
| 10-14 | .000289 | 11.4273 | .000052 | 2.0396 | .000623 | 24.6097* |
| 15-19 | .000430 | 5.4320 | .000139 | 1.7512 | .003237 | 40.9321* |
| 20-24 | .000572 | 5.1055 | .000232 | 2.0735 | .004253 | 37.9943* |
| 25-29 | .000700 | 6.8697 | .000439 | 4.3101 | .002763 | 27.1254* |
| 30-34 | .000995 | 8.7681 | .001110 | 9.7767 | .002134 | 18.7941* |
| 35-39 | .001733 | 11.1437 | .003052 | 19.6317* | .001885 | 12.1258 |
| 40-44 | .003373 | 14.2814 | .006906 | 29.2449* | .001759 | 7.4494 |
| 45-49 | .006267 | 17.1130 | .013377 | 36.5259* | .001725 | 4.7112 |
| 50-54 | .011228 | 19.7688 | .023094 | 40.6605* | .001735 | 3.0552 |
| 55-59 | .018768 | 21.3979 | .036789 | 41.9441* | .001784 | 2.0345 |
| 60-64 | .027509 | 21.2616 | .054681 | 42.2625* | .001753 | 1.3545 |
| 65-69 | .037276 | 20.0804 | .077592 | 41.7988* | .001817 | .9790 |
| 70-74 | .045143 | 17.4759 | .106184 | 41.1058* | .001932 | .7480 |
| 75-79 | .051292 | 14.2433 | .142334 | 39.5248* | .002148 | .5966 |
| 80-84 | .050380 | 10.3077 | .184113 | 37.6691* | .002022 | .4137 |
| 85+ | .000000 | .0000 | .000000 | .0000 | .000000 | .0000 |

Females

| | | | | | | |
|---|---|---|---|---|---|---|
| < 1 | .000045 | .2512 | .000120 | .6751* | .000099 | .5562 |
| 1- 4 | .000268 | 8.7633 | .000063 | 2.0698 | .000415 | 13.5635* |
| 5- 9 | .000289 | 16.5787 | .000044 | 2.5250 | .000404 | 23.2067* |
| 10-14 | .000227 | 15.0365 | .000049 | 3.2382 | .000331 | 21.9376* |
| 15-19 | .000270 | 8.7031 | .000094 | 3.0288 | .001154 | 37.1967* |
| 20-24 | .000341 | 9.1640 | .000145 | 3.8860 | .001005 | 27.0135* |
| 25-29 | .000574 | 13.3103 | .000254 | 5.8915 | .000689 | 15.9733* |
| 30-34 | .001132 | 19.0916* | .000502 | 8.4715 | .000623 | 10.5139 |
| 35-39 | .002221 | 24.6361* | .001050 | 11.6446 | .000625 | 6.9340 |
| 40-44 | .004168 | 30.5043* | .002083 | 15.2416 | .000645 | 4.7178 |
| 45-49 | .007085 | 34.4689* | .003810 | 18.5354 | .000641 | 3.1167 |
| 50-54 | .010592 | 35.1756* | .006896 | 22.9018 | .000724 | 2.4046 |
| 55-59 | .014645 | 33.4746* | .012258 | 28.0181 | .000746 | 1.7053 |
| 60-64 | .018210 | 28.6919 | .021577 | 33.9965* | .000794 | 1.2509 |
| 65-69 | .022806 | 23.6983 | .037005 | 38.4518* | .000900 | .9353 |
| 70-74 | .027094 | 18.1523 | .062419 | 41.8189* | .001059 | .7094 |
| 75-79 | .032085 | 13.5299 | .101226 | 42.6853* | .001069 | .4506 |
| 80-84 | .033307 | 9.1087 | .153659 | 42.0224* | .000861 | .2354 |
| 85+ | .000000 | .0000 | .000000 | .0000 | .000000 | .0000 |

For males, the greatest decreases in the probability of dying occur through and including the 30 to 34 age interval when motor vehicle accidents are eliminated, the lone exception being the first age interval (infants) where the elimination of diseases of heart results in the largest decrease. This result regarding diseases of heart is due primarily to the high incidence among infants of deaths due to congenital malformations. The largest increases due to elimination of motor vehicle accidents are found in the 15-19 and 20-24 age intervals. Beginning with the interval 35 to 39 and all subsequent age intervals, the elimination of diseases of heart results in the greatest changes, again consonant with the age pattern of mortality by cause.

The values for the female population show different results. Like males, the greatest change for infants occurs when diseases of heart are eliminated. For the age intervals 1 to 4 through 25 to 29, the elimination of motor vehicle accidents results in the largest change. However, for females ages 30 through 59, the greatest decrease in the probability of dying results when malignant neoplasms are eliminated, reflecting the increased incidence of cancer, especially cervical and breast cancer, as a cause of death among women in these ages. From age 60 through the terminal age interval, the elimination of diseases of heart precipitates the greatest decreases in the probability of dying among females.

In the comparison of the types of life tables described in the present study, the analysis of the probability of dying is an important, and perhaps obvious, step. An equally important comparison is that of the crude and partial crude probability of dying from a given cause.

The absolute differences between the crude probability of dying from cause i in the presence of all other risks, ${}_nQ_x^{i}$ , and the partial crude probability of dying from cause i in the absence of risk z, ${}_nQ_x^{i \cdot z}$, are presented in Tables 5.2, 5.3, and 5.4. The difference between these values is found by subtracting the crude probability from the partial crude probability. The partial crude probability is always greater than or equal to the crude probability of dying from a given cause. This seeming anomaly arises from the fact that those who would have died from the eliminated cause have greater exposure to the remaining causes (Spiegelman, 1957:302). Thus, when a cause is eliminated, allowance must be made for slight increases in the rates for remaining causes.

Differences between partial crude and crude probabilities of dying relative to the crude probability are given in Table 5.5. These results reveal an interesting finding. When a given cause of death is eliminated, the relative difference between the partial crude probability and

Table 5.2. Crude and partial crude probabilities of dying from malignant neoplasms and absolute differences due to elimination of causes of death by age and sex, United States, 1969-1971.

| | ${}_nQ_x^{MN}$ | ${}_nQ_x^{MN \cdot DH}$ | Change | ${}_nQ_x^{MN \cdot MVA}$ | Change |
|---|---|---|---|---|---|
| | | | **Males** | | |
| < 1 | .000045 | .000045 | .000000 | .000045 | .000000 |
| 1- 4 | .000323 | .000323 | .000000 | .000323 | .000000 |
| 5- 9 | .000386 | .000386 | .000000 | .000386 | .000000 |
| 10-14 | .000290 | .000290 | .000000 | .000290 | .000000 |
| 15-19 | .000431 | .000431 | .000000 | .000432 | .000001 |
| 20-24 | .000575 | .000575 | .000000 | .000576 | .000001 |
| 25-29 | .000703 | .000703 | .000000 | .000704 | .000001 |
| 30-34 | .001001 | .001001 | .000001 | .001002 | .000001 |
| 35-39 | .001745 | .001748 | .000003 | .001746 | .000002 |
| 40-44 | .003407 | .003419 | .000012 | .003410 | .000003 |
| 45-49 | .006365 | .006409 | .000044 | .006371 | .000006 |
| 50-54 | .011495 | .011634 | .000139 | .011505 | .000011 |
| 55-59 | .019457 | .019841 | .000384 | .019476 | .000019 |
| 60-64 | .020949 | .029931 | .000882 | .029078 | .000029 |
| 65-69 | .040460 | .042292 | .001832 | .040504 | .000044 |
| 70-74 | .051075 | .054468 | .003393 | .051138 | .000063 |
| 75-79 | .062194 | .068368 | .006174 | .062291 | .000097 |
| 80-84 | .068470 | .078806 | .010337 | .068590 | .000120 |
| 85+ | .096396 | .183055 | .086659 | .096724 | .000329 |

Table 5.2. (continued)

| | ${}_nQ_x^{MN}$ | ${}_nQ_x^{MN \cdot DH}$ | Change | ${}_nQ_x^{MN \cdot MVA}$ | Change |
|---|---|---|---|---|---|
| | | | <u>Females</u> | | |
| < 1 | .000045 | .000045 | .000000 | .000045 | .000000 |
| 1- 4 | .000268 | .000268 | .000000 | .000268 | .000000 |
| 5- 9 | .000289 | .000289 | .000000 | .000289 | .000000 |
| 10-14 | .000227 | .000227 | .000000 | .000227 | .000000 |
| 15-19 | .000271 | .000271 | .000000 | .000271 | .000000 |
| 20-24 | .000342 | .000342 | .000000 | .000342 | .000000 |
| 25-29 | .000575 | .000576 | .000000 | .000576 | .000000 |
| 30-34 | .001135 | .001135 | .000000 | .001135 | .000000 |
| 35-39 | .002228 | .002229 | .000001 | .002229 | .000001 |
| 40-44 | .004188 | .004192 | .000004 | .004189 | .000001 |
| 45-49 | .007133 | .007147 | .000014 | .007135 | .000002 |
| 50-54 | .010698 | .010736 | .000038 | .010702 | .000004 |
| 55-59 | .014865 | .014959 | .000094 | .014870 | .000006 |
| 60-64 | .018640 | .018851 | .000212 | .018647 | .000008 |
| 65-69 | .023702 | .024176 | .000474 | .023714 | .000012 |
| 70-74 | .028948 | .029969 | .001021 | .028966 | .000018 |
| 75-79 | .036094 | .038331 | .002237 | .036118 | .000024 |
| 80-84 | .041111 | .045535 | .004424 | .041137 | .000026 |
| 85+ | .070352 | .137713 | .067361 | .070429 | .000078 |

Table 5.3. Crude and partial crude probabilities of dying from diseases of heart and absolute differences due to elimination of causes of death by age and sex, United States, 1969-1971.

| | ${}_nQ_x^{DH}$ | ${}_nQ_x^{DH \cdot MN}$ | Change | ${}_nQ_x^{DH \cdot MVA}$ | Change |
|---|---|---|---|---|---|
| | | | **Males** | | |
| < 1 | .000140 | .000140 | .000000 | .000140 | .000000 |
| 1- 4 | .000069 | .000069 | .000000 | .000069 | .000000 |
| 5- 9 | .000040 | .000040 | .000000 | .000040 | .000000 |
| 10-14 | .000052 | .000052 | .000000 | .000052 | .000000 |
| 15-19 | .000139 | .000139 | .000000 | .000139 | .000000 |
| 20-24 | .000233 | .000234 | .000000 | .000234 | .000001 |
| 25-29 | .000441 | .000441 | .000000 | .000442 | .000001 |
| 30-34 | .001116 | .001116 | .000001 | .001117 | .000001 |
| 35-39 | .003072 | .003074 | .000003 | .003075 | .000003 |
| 40-44 | .006965 | .006977 | .000012 | .006971 | .000006 |
| 45-49 | .013536 | .013580 | .000044 | .013548 | .000012 |
| 50-54 | .023496 | .023635 | .000138 | .023518 | .000021 |
| 55-59 | .037774 | .038156 | .000381 | .037811 | .000036 |
| 60-64 | .056882 | .057755 | .000873 | .056938 | .000056 |
| 65-69 | .082273 | .084078 | .001805 | .082362 | .000089 |
| 70-74 | .115661 | .118971 | .003310 | .115804 | .000143 |
| 75-79 | .162000 | .167929 | .005928 | .162252 | .000252 |
| 80-84 | .224073 | .233716 | .009644 | .224466 | .000394 |
| 85+ | .473406 | .523908 | .050503 | .475020 | .001614 |

Table 5.3. (continued)

| | ${}_nQ_x^{DH}$ | ${}_nQ_x^{DH\cdot MN}$ | Change | ${}_nQ_x^{DH\cdot MVA}$ | Change |
|---|---|---|---|---|---|
| | | | **Females** | | |
| < 1 | .000122 | .000122 | .000000 | .000122 | .000000 |
| 1- 4 | .000063 | .000063 | .000000 | .000063 | .000000 |
| 5- 9 | .000044 | .000044 | .000000 | .000044 | .000000 |
| 10-14 | .000049 | .000049 | .000000 | .000049 | .000000 |
| 15-19 | .000094 | .000094 | .000000 | .000094 | .000000 |
| 20-24 | .000145 | .000145 | .000000 | .000145 | .000000 |
| 25-29 | .000255 | .000255 | .000000 | .000255 | .000000 |
| 30-34 | .000504 | .000504 | .000000 | .000504 | .000000 |
| 35-39 | .001054 | .001055 | .000001 | .001054 | .000000 |
| 40-44 | .002095 | .002099 | .000004 | .002095 | .000001 |
| 45-49 | .003842 | .003856 | .000014 | .003843 | .000001 |
| 50-54 | .006978 | .007016 | .000038 | .006981 | .000003 |
| 55-59 | .012457 | .012551 | .000094 | .012462 | .000005 |
| 60-64 | .022047 | .022258 | .000211 | .022056 | .000009 |
| 65-69 | .038166 | .038637 | .000472 | .038184 | .000019 |
| 70-74 | .065377 | .066384 | .001008 | .065416 | .000040 |
| 75-79 | .109158 | .111335 | .002176 | .109231 | .000073 |
| 80-84 | .174256 | .178446 | .004190 | .174365 | .000109 |
| 85+ | .489143 | .526159 | .037016 | .489682 | .000540 |

Table 5.4. Crude and partial crude probabilities of dying from motor vehicle accidents and absolute differences due to elimination of causes of death by age and sex, United States, 1969-1971.

| | ${}_nQ_x^{MVA}$ | ${}_nQ_x^{MVA\cdot MN}$ | Change | ${}_nQ_x^{MVA\cdot DH}$ | Change |
|---|---|---|---|---|---|
| | | | <u>Males</u> | | |
| < 1 | .000102 | .000102 | .000000 | .000102 | .000000 |
| 1- 4 | .000513 | .000513 | .000000 | .000513 | .000000 |
| 5- 9 | .000669 | .000669 | .000000 | .000669 | .000000 |
| 10-14 | .000623 | .000623 | .000000 | .000623 | .000000 |
| 15-19 | .003245 | .003246 | .000001 | .003245 | .000000 |
| 20-24 | .004268 | .004269 | .000001 | .004268 | .000001 |
| 25-29 | .002773 | .002774 | .000001 | .002774 | .000001 |
| 30-34 | .002144 | .002145 | .000001 | .002145 | .000001 |
| 35-39 | .001898 | .001900 | .000002 | .001901 | .000003 |
| 40-44 | .001779 | .001782 | .000003 | .001785 | .000006 |
| 45-49 | .001757 | .001762 | .000006 | .001769 | .000012 |
| 50-54 | .001785 | .001796 | .000011 | .001807 | .000022 |
| 55-59 | .001867 | .001886 | .000019 | .001904 | .000037 |
| 60-64 | .001878 | .001907 | .000029 | .001935 | .000057 |
| 65-69 | .002015 | .002060 | .000044 | .002107 | .000091 |
| 70-74 | .002249 | .002313 | .000064 | .002398 | .000149 |
| 75-79 | .002704 | .002803 | .000099 | .002972 | .000268 |
| 80-84 | .002876 | .003000 | .000124 | .003310 | .000434 |
| 85+ | .003398 | .003761 | .000363 | .006453 | .003055 |

Table 5.4. (continued)

| | ${}_nQ_x^{MVA}$ | ${}_nQ_x^{MVA \cdot MN}$ | Change | ${}_nQ_x^{MVA \cdot DH}$ | Change |
|---|---|---|---|---|---|
| | | | **Females** | | |
| < 1 | .000100 | .000100 | .000000 | .000100 | .000000 |
| 1- 4 | .000415 | .000415 | .000000 | .000415 | .000000 |
| 5- 9 | .000405 | .000405 | .000000 | .000405 | .000000 |
| 10-14 | .000331 | .000331 | .000000 | .000331 | .000000 |
| 15-19 | .001156 | .001156 | .000000 | .001156 | .000000 |
| 20-24 | .001007 | .001007 | .000000 | .001007 | .000000 |
| 25-29 | .000691 | .000691 | .000000 | .000691 | .000000 |
| 30-34 | .000625 | .000626 | .000000 | .000625 | .000000 |
| 35-39 | .000628 | .000628 | .000001 | .000628 | .000000 |
| 40-44 | .000649 | .000650 | .000001 | .000650 | .000001 |
| 45-49 | .000647 | .000649 | .000002 | .000648 | .000001 |
| 50-54 | .000735 | .000739 | .000004 | .000738 | .000003 |
| 55-59 | .000763 | .000768 | .000006 | .000768 | .000005 |
| 60-64 | .000820 | .000826 | .000006 | .000830 | .000009 |
| 65-69 | .000947 | .000959 | .000012 | .000966 | .000019 |
| 70-74 | .001149 | .001166 | .000018 | .001189 | .000041 |
| 75-79 | .001226 | .001251 | .000025 | .001302 | .000076 |
| 80-84 | .001089 | .001115 | .000026 | .001206 | .000117 |
| 85+ | .001102 | .001185 | .000083 | .002157 | .001055 |

Table 5.5. Relative differences between crude and partial crude probabilities of dying from remaining causes of death when a given cause is eliminated, United States, 1969-1971.

| | Cause eliminated | | | | | |
|---|---|---|---|---|---|---|
| | Males | | | Females | | |
| Age | MN | DH | MVA | MN | DH | MVA |
| < 1 | .0022 | .0071* | .0052 | .0018 | .0061* | .0049 |
| 1- 4 | .0149 | .0033 | .0247* | .0135 | .0031 | .0191* |
| 5- 9 | .0213 | .0051 | .0333* | .0116 | .0002 | .0222* |
| 10-14 | .0182 | .0039 | .0311* | .0105 | .0045 | .0185* |
| 15-19 | .0216 | .0080 | .1651* | .0129 | .0064 | .0578* |
| 20-24 | .0289 | .0122 | .2145* | .0175 | .0104 | .0507* |
| 25-29 | .0351 | .0224 | .1390* | .0304 | .0144 | .0366* |
| 30-34 | .0495 | .0549 | .1071* | .0569* | .0251 | .0324 |
| 35-39 | .0884 | .1545* | .0959 | .1125* | .0526 | .0308 |
| 40-44 | .1720 | .3517* | .0895 | .2103* | .1056 | .0326 |
| 45-49 | .3231 | .6886* | .0891 | .3594* | .1934 | .0322 |
| 50-54 | .5882 | 1.2074* | .0910 | .5425* | .3536 | .0372 |
| 55-59 | 1.0094 | 1.9724* | .0964 | .7578* | .6345 | .0385 |
| 60-64 | 1.5352 | 3.0359* | .0983 | .9585 | 1.1351* | .0420 |
| 65-69 | 2.1933 | 4.5289* | .1078 | 1.2353 | 1.9992* | .0491 |
| 70-74 | 2.8621 | 6.6431* | .1238 | 1.5412 | 3.5264* | .0606 |
| 75-79 | 3.6593 | 9.9267* | .1554 | 1.9938 | 6.1982* | .0669 |
| 80-84 | 4.3039 | 15.0965* | .1758 | 2.4046 | 10.7608* | .0627 |
| 85+ | 10.6679 | 89.8996* | .3409 | 7.5676 | 95.7495* | .1103 |

the crude probability of dying from a given cause is the same for all remaining causes. For example, when diseases of heart are eliminated as a cause of death, the relative difference between the partial crude probability and the crude probability of dying from malignant neoplasms is identical to the relative difference between the corresponding probabilities for motor vehicle accidents.

This finding is best explained by a comparison of the formulas for estimating ${}_nQ_x^i$ and ${}_nQ_x^{i \cdot z}$. The crude probability of dying from cause i is calculated by multiplying the ratio of deaths due to cause i to total deaths in a given age group, $({}_nD_x^i / {}_nD_x)$, by the probability of dying, ${}_nq_x$. The calculation of the partial crude probability of dying essentially involves the substitution of two new values. First, ${}_nq_x^{(-z)}$ replaces ${}_nq_x$. Second, a new ratio is formed. This ratio retains the numerator of the previous ratio, ${}_nD_x^i$, but adds a new denominator; that is, total deaths less the deaths due to the eliminated cause, $({}_nD_x - {}_nD_x^z)$. This constant change in the denominator, coupled with the constant difference between ${}_nq_x$ and ${}_nq_x^{(-z)}$ at each age accounts for the constant relative difference between crude and partial crude probabilities for remaining causes.

The relative differences between crude and partial crude probabilities, like those between probabilities of dying, reflect the age-cause pattern of mortality. Among males, the elimination of motor vehicle accidents results in the largest relative increases in the partial crude probabilities of the remaining causes in the earlier age intervals while the elimination of diseases of heart produces the largest gains among males 35 and over. Among females, the largest relative gains in the probability of dying from the remaining causes results from the elimination of motor vehicle accidents at the younger ages, malignant neoplasms at the middle ages, and diseases of heart at the older ages.

## Probability of Survival

There are two values commonly referred to as probabilities of survival which must be distinguished. The first probability of survival refers to the probability of surviving a given age interval and is equal to the complement of the probability of dying, ${}_nq_x$. The second type of survival probability is a conditional probability and refers to the probability that an individual who has survived to a given age will survive to some specified subsequent age. Such probabilities may be calculated from the $l_x$ values of the life table. Thus, the probability that a person alive at age x will survive to age x + z where z is some specified number of years is

$$ {}_np_x^{x+z} = l_{x+z}/l_x. \tag{5.15}$$

Of the $l_x$ persons in the original cohort of size $l_0$ who survive to age x, $l_{x+z}$ will survive to age x + z. There are a large number of combinations of ages which could be examined. However, Table 5.6 presents the conditional probabilities of survival from and to selected ages based on life tables due to all causes and life tables with causes eliminated.

In general, the results presented in Table 5.6 correspond to those presented earlier in the chapter; that is, the results follow the age-cause pattern of mortality. Among males, the largest conditional probability of survival from ages 0, 5, and 20 to ages 5, 20, and 45 occur when motor vehicle accidents are eliminated. For all other combinations, the elimination of diseases of heart produces the largest conditional survival probabilities.

Among females, the elimination of motor vehicle accidents results in the largest survival probabilities from ages 0 and 5 to ages 5 and 20. The conditional probabilities of survival to ages 45 and 65 from ages 0, 5, 20, and 45 are greatest when the risk of death from malignant neoplasms is eliminated. The probability of survival to age 85 from all other ages considered is largest due to the elimination of diseases of heart.

The results for the female population depart, perhaps, somewhat from those expected. Based on the age-cause pattern of mortality, it may be expected that the greatest probability of survival to age 65 would be incurred when diseases of

Table 5.6. Conditional probabilities of survival from and to selected ages under varying mortality conditions by sex, United States, 1969-1971.

| From age and cause eliminated | | To age 5 | 20 | 45 | 65 | 85 |
|---|---|---|---|---|---|---|
| | | | | Males | | |
| 0 | No | .97331 | .96078 | .89363 | .64496 | .12745 |
| | MN | .97367 | .96220 | .90168 | .69779 | .18151 |
| | DH | .97351 | .96120 | .90476 | .74993* | .30849* |
| | MVA | .97391* | .96575* | .90996* | .66175 | .13236 |
| 5 | No | | .98713 | .91814 | .66265 | .13094 |
| | MN | | .98822 | .92606 | .71666 | .18642 |
| | DH | | .98736 | .92380 | .77034* | .31688* |
| | MVA | | .99162* | .93434* | .67948 | .13591 |
| 20 | No | | | .93011 | .67129 | .13265 |
| | MN | | | .93903 | .72670 | .18903 |
| | DH | | | .94128 | .78020* | .32094* |
| | MVA | | | .94223* | .68522 | .13705 |
| 45 | No | | | | .72173 | .14262 |
| | MN | | | | .77388 | .20130 |
| | DH | | | | .82887* | .34096* |
| | MVA | | | | .72723 | .14546 |
| 65 | No | | | | | .19761 |
| | MN | | | | | .26012 |
| | DH | | | | | .41136* |
| | MVA | | | | | .20002 |

Table 5.6. (continued)

| From age and cause eliminated | | To age 5 | 20 | 45 | 65 | 85 |
|---|---|---|---|---|---|---|
| | | | | Females | | |
| 0 | No | .97917 | .97298 | .93783 | .79787 | .29687 |
| | MN | .97947 | .97404 | .94689* | .84902* | .36653 |
| | DH | .97935 | .97334 | .94201 | .83955 | .49110* |
| | MVA | .97968* | .97532* | .94350 | .80513 | .30107 |
| 5 | No | | .99368 | .95778 | .81484 | .30318 |
| | MN | | .99446 | .96674* | .86682* | .37421 |
| | DH | | .99386 | .96187 | .85766 | .50146* |
| | MVA | | .99555* | .96370 | .82183 | .30731 |
| 20 | No | | | .96387 | .82003 | .30511 |
| | MN | | | .97213* | .87165* | .37630 |
| | DH | | | .96781 | .86255 | .50455* |
| | MVA | | | .96737 | .82550 | .30869 |
| 45 | No | | | | .85706 | .31655 |
| | MN | | | | .89664* | .38709 |
| | DH | | | | .89123 | .52133* |
| | MVA | | | | .85334 | .31910 |
| 65 | No | | | | | .37208 |
| | MN | | | | | .43171 |
| | DH | | | | | .58496* |
| | MVA | | | | | .37394 |

heart are eliminated as a cause of death. However, survival probabilities associated with the elimination of particular causes are not determined solely by the most prevalent cause in a given interval, but also by the age-cause pattern of previous age intervals. There are two sources of change in survival probabilities: one associated with the numerator and one associated with the denominator of the calculation equation. The age-cause pattern may affect one of these sources to a greater degree than the other depending on the extent and location of improvements in mortality due to the elimination of a cause of death. For example, among females, the elimination of malignant neoplasms as a cause of death increases the number of survivors to the middle and subsequent age intervals, thus, affecting both the numerator and denominator in the calculation of survival probabilities from middle to later ages.

## Joint Probabilities

Thus far, only life tables for each sex have been considered. However, there are a number of values combining functions from life tables for males and females. Such values are joint probabilities and include the joint probability of survival of a married couple, probability of widowhood, and probability of orphanhood. The problem of joint survival has been addressed by many scholars (Dublin et al., 1949; Spiegelman, 1957; Pollard et al., 1974; Preston,

1974) including Halley (1693:604).

The joint probability of survival for a specified period is calculated by forming the product of the individual chances of survival for the same period. The product of the individual survival chances is used because the joint probability of survival is conditional on the survival of both spouses. Survival of husband and wife are regarded as independent events and the joint probability of the occurrence of two independent events is the product of the probability of their individual occurrence. This method can be easily used to build survivorship columns for any combination of ages of husband and wife. In other words, the joint probability of survival is based on the type of calculations presented in the previous section. For example, the joint probability of survival for 20 years of a husband and wife both age 25 at marriage is given by the product

$$(l^m_{45}/l^m_{25})(l^f_{45}/l^f_{25}) \qquad (5.16)$$

where m and f denote sex.

Table 5.7 presents the survivorship experience of couples married at specified ages based on life tables due to all causes and life tables eliminating specific causes. The factor of divorce is ignored. Thus, these probabilities may properly be viewed as the chance that both partners will be alive after a given number of years whether they are still married or not. The joint probability of survival is less

Table 5.7. Joint probabilities that both husband and wife are alive after specified number of years under various mortality conditions and age at marriage, United States, 1969-1971.

| Age at marriage and eliminated cause of death | Duration of marriage 10 | 25 | 40 | 50 |
|---|---|---|---|---|
| Both spouses age 20 | | | | |
| No | .97090 | .89651 | .67511 | .40515 |
| MN | .97303 | .91098 | .73718 | .49882 |
| DH | .97196 | .91098 | .75908* | .56471* |
| MVA | .97947* | .91149* | .69175 | .41768 |
| Both spouses age 25 | | | | |
| No | .96860 | .85870 | .55878 | .25951 |
| MN | .97192 | .88378 | .64107 | .34942 |
| DH | .97085 | .88778* | .68285* | .44365* |
| MVA | .97469* | .87056 | .57115 | .26714 |

than either survival probability for husband or wife singly since the chance of both surviving jointly from year to year is formed by the product of probabilities less than unity. A large number of combinations of ages of husband and wife and duration of marriage could be generated. However, only two ages at marriage, 20 and 25, and four durations of marriage, 10, 25, 40, and 50 years, are examined in Table 5.7. These ages at marriage were selected because they had the highest marriage rates for males and females in 1970 (U. S. Department of Health, Education, and Welfare, 1974c: Table 1-8). The durations of marriage selected represent significant milestones in the married career.

As in the case of other conditional probabilities, joint survival probabilities associated with the elimination of causes of deaths are not determined solely by the most prevalent cause at a given age, but also by the age-cause pattern of previous age intervals. In the current situation, the magnitude of the joint probability of surviving a given duration depends also on the age-sex-cause pattern of mortality.

Table 5.7 shows that among spouses both age 20 at marriage, the joint probability of survival for 10 years is greatest when motor vehicle accidents are eliminated as a cause of death. This result is expected since such deaths are highly prevalent during this age group for both sexes.

Table 5.8 gives the individual survival probabilities used to calculate the joint probabilities of survival of a married pair. Table 5.8 shows that elimination of motor vehicle accidents produces the largest probability of survival to age 30 for both sexes. However, this table reveals that for durations of 25, 40, and 50 years different age-sex-cause-specific patterns of death produce the results in Table 5.7. Thus, the joint probability of survival to age 45 (25 years duration) is greatest when motor vehicle accidents are eliminated although the individual probabilities of survival are greatest for females when malignant neoplasms are eliminated. This discrepancy may be explained by the wider disparity between survival probabilities for males with causes eliminated which outweigh the effect of rather homogeneous survival probablities for females. A similar pattern develops for the joint probability of survival for 40 years although the causes involved are malignant neoplasms for females and diseases of heart for males. For the 50-year survival probability, the elimination of diseases of heart produced the largest individual survival probabilities by sex and, thus, the largest joint probability.

For spouses both age 25 at marriage, the joint probability of survival for 10 years is greatest when motor vehicle accidents are eliminated and for 25, 40, and 50 years when diseases of heart are eliminated. As above, these overall

Table 5.8. Survival probabilities to certain ages by sex due to all causes and causes eliminated, United States, 1969-1971.

| Cause eliminated | Survival probability $l_{30}/l_{20}$ | $l_{45}/l_{20}$ | $l_{60}/l_{20}$ | $l_{70}/l_{20}$ |
|---|---|---|---|---|
| No | | | | |
| Males | .97875 | .93011 | .77104 | .54668 |
| Females | .99198 | .96387 | .87559 | .74111 |
| MN | | | | |
| Males | .97999 | .93710 | .80746 | .61762 |
| Females | .99290 | .97213* | .91296* | .80765 |
| DH | | | | |
| Males | .97941 | .94128 | .84319* | .69591* |
| Females | .99239 | .96781 | .90025 | .81146* |
| MVA | | | | |
| Males | .98570* | .94223* | .78546 | .55928 |
| Females | .99368* | .96737 | .88070 | .74681 |
| | $l_{35}/l_{25}$ | $l_{50}/l_{25}$ | $l_{65}/l_{25}$ | $l_{75}/l_{25}$ |
| No | | | | |
| Males | .97859 | .90619 | .67888 | .41006 |
| Females | .98979 | .94759 | .82309 | .63286 |
| MN | | | | |
| Males | .98026 | .91841 | .73299 | .49118 |
| Females | .99149* | .96229* | .87460* | .71139 |
| DH | | | | |
| Males | .98012 | .92959* | .78884* | .59658* |
| Females | .99054 | .95502 | .86564 | .74664* |
| MVA | | | | |
| Males | .98344* | .91571 | .69001 | .41880 |
| Females | .99110 | .95069 | .82774 | .63787 |

results prevailed even though the largest individual survival probabilities may have been greater with another cause of death eliminated (Table 5.8).

There are two other types of joint probability which may be considered. Although these values are not calculated in the present study because they are not of central interest, they deserve attention because of their possible implications for family structure. The probability of widowhood is computed by multiplying the probability that a husband and wife will survive jointly by the probability that a specified spouse, either husband or wife, will die within the ensuing year or some other period of time. The probability of widowhood must be calculated relevant to one spouse since probabilities of survival differ by sex (Dublin et al., 1949; Spiegelman, 1957; Preston, 1974).

A second type of joint probability is the probability of orphanhood. The probability of orphanhood refers to the chances that a child of a given sex will be orphaned by both parents after a given period of time. For example, the probability that a male child just born to a mother aged 25 and a father age 30 will 20 years later be orphaned by both parents is computed from three values: (1) the probability of survival by the child to age 20, $l^m_{20}/l^m_0$; (2) the probability of the mother dying in the 20-year period, $(l^f_{25} - l^f_{45})/l^f_{25}$; and (3) the probability of the father dying in the 20-year period, $(l^m_{30} - l^m_{50})/l^m_{30}$. The probability of orphanhood is the product of these three components (Spiegelman, 1957; Pollard et al., 1974; Preston, 1974).

## Measures of Longevity

### Life expectancy

Perhaps the best known and most usual measure of human longevity is life expectancy. The observed expectation of life summarizes the mortality experience of a cohort of births from a given age to the end of the life span. Thus, at any given age, life expectancy or average remaining lifetime expresses the average number of years of life remaining to each individual surviving to that age if all individuals are subject to the mortality conditions and estimated probabilities of death on which the life table is based. Life expectancy at birth is often referred to as average duration of life. It refers to the average number of years a newborn infant can expect to live if he is subjected to the current mortality conditions throughout his lifetime. At ages beyond infancy, life expectancy expresses the average number of years a person age x may expect to live in addition to those which he has already lived. Thus, life expectancy at age x, when added to current age x, yields the average age of death of those surviving to age x.

Expectation of life at birth is often used to assess

comparative health conditions in two or more populations.*
Life expectancy as an index of social progress appears to possess some validity since the desire to keep on living as long as possible and to have loved ones preserved from death is almost universal. The longer the life expectancy of a community, the more adequately these basic values are being fulfilled. Furthermore, high expectation of life is generally coincidental with a number of other widely desired social conditions. The prevalence of sicknesses that kill usually means that sicknesses that do not kill are more prevalent. Low death rates reflect the effectiveness of hospitals, health departments, doctors, nurses, and medical research agencies. High life expectation reflects good working conditions, relative freedom from bereavement, high standards of living, efficient government, and effective education (Hart and Hertz, 1944:609-610).

Davis (1961:510) warns, however, that caution should be exercised when utilizing life expectancy values for comparative purposes. She notes that when comparing $e_x$ values for different populations that a population which exhibits a high expectation of life at birth may have a high proportion of

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*There is some objection to the use of average duration of life as a standard of comparison because its calculation gives great weight to the large number of infant deaths. This influence may be eliminated by considering the average remaining lifetime of the survivors to age 1.

chronically ill persons, particularly at older ages. Spiegelman (1957:301) notes that there is no single index appropriate for assessing the health conditions of communities and that the use of life expectancy or other life table values for this purpose places undue emphasis on mortality.

In the comparative context of the current study, the most relevant contrast between the two types of life table examined is the gain in expectation of life by age produced by the hypothetical eliminaton of causes of death. Table 5.9 presents gains in life expectancy resulting from such elimination of causes. Similar results appear for males and females. The elimination of diseases of heart produces the largest gains in life expectancy at all ages for both sexes. The elimination of malignant neoplasms produces moderate gains in life expectancy for males and females while elimination of motor vehicle accidents results in only slight gains for both sexes.

Caution, however, should be exercised in interpreting these values. Because of the manner in which the components of the calculation formula for life expectancy are computed, the value of $e_x$ is dependent on the age-cause pattern of mortality. The numerator of the calculation formula for $e_x$, $T_x$, is the sum of the $_nL_x$ values cumulated from the end of the life table forward. Thus, for example, the elimination of diseases of heart, which are highly prevalent among both

Table 5.9. Gains in life expectancy by age and sex due to elimination of causes of death, United States, 1969-1971.

| Age | MN | DH | MVA | MN | DH | MVA |
|---|---|---|---|---|---|---|
| | Cause eliminated | | | | | |
| | | <u>Males</u> | | | <u>Females</u> | |
| < 1 | 2.31802 | 6.35190 | 0.93265 | 2.59971 | 6.35198 | 0.41063 |
| 1- 4 | 2.36177 | 6.49117 | 0.94759 | 2.64372 | 6.45724 | 0.41032 |
| 5- 9 | 2.34885 | 6.51082 | 0.91682 | 2.63167 | 6.47212 | 0.38075 |
| 10-14 | 2.32973 | 6.52429 | 0.87735 | 2.61575 | 6.48029 | 0.35379 |
| 15-19 | 2.31848 | 6.53749 | 0.84384 | 2.60466 | 6.48642 | 0.33319 |
| 20-24 | 2.31390 | 6.58150 | 0.67908 | 2.59612 | 6.50044 | 0.26576 |
| 25-29 | 2.31152 | 6.64317 | 0.48044 | 2.58623 | 6.51584 | 0.21129 |
| 30-34 | 2.30359 | 6.68951 | 0.36436 | 2.56750 | 6.52945 | 0.17763 |
| 35-39 | 2.28905 | 6.71581 | 0.28508 | 2.52861 | 6.54248 | 0.15053 |
| 40-44 | 2.26163 | 6.69601 | 0.22417 | 2.45616 | 6.55286 | 0.12657 |
| 45-49 | 2.20621 | 6.60176 | 0.17595 | 2.32959 | 6.55432 | 0.10483 |
| 50-54 | 2.11035 | 6.40913 | 0.13672 | 2.13698 | 6.54491 | 0.08647 |
| 55-59 | 1.95499 | 6.11208 | 0.10474 | 1.88690 | 6.50917 | 0.06873 |
| 60-64 | 1.72893 | 5.72677 | 0.07857 | 1.59525 | 6.42877 | 0.05379 |
| 65-69 | 1.45386 | 5.27973 | 0.05920 | 1.30267 | 6.27504 | 0.04118 |
| 70-74 | 1.14884 | 4.80956 | 0.04403 | 1.01620 | 6.04718 | 0.02969 |
| 75-79 | 0.86019 | 4.36282 | 0.03207 | 0.76470 | 5.75181 | 0.01910 |
| 80-84 | 0.61175 | 4.02541 | 0.02152 | 0.55261 | 5.46895 | 0.01084 |
| 85+ | 0.46997 | 3.96057 | 0.01495 | 0.42368 | 5.36101 | 0.00616 |

sexes at older ages, makes a large contribution to $T_x$ in the terminal age interval and that contribution is carried forward to the first age interval. On the other hand, the contributions of the elimination of malignant neoplasms and motor vehicle accidents at older ages is relatively minor. Thus, those components have smaller effects on the gain in life expectancy. Although the contributon of the elimination of these causes may be greater at earlier ages, the impact of these contributions to $T_x$ is not as great as the contribution of a large number of deaths postponed in the later ages.

For these reasons, perhaps the best summary measure of mortality improvement due to the elimination of causes of death is the gain in life expectancy at birth. This value takes into account the pattern of mortality improvements at all subsequent ages.

The sex differential is one of the most prominent mortality differentials. Life tables for 1969-1971 indicate that at every age from birth to the end of the life table the age-specific death rate and probability of dying for males is higher than for females. This, of course, translates into greater life expectancy at all ages for females. The same relation holds when life tables eliminating causes of death are examined. According to Bogue (1969:594), the sex differential in mortality is a development of recent origin. There was a sex differential in mortality in 1900 but it was quite small. However, the differential has widened since that time because there has been greater improvement in the reduction of deaths among females than males.

A common comparison of sex differences in mortality is the differences between life expectancy at birth for males and females. Life tables due to all causes show that females, on the average, can expect to live 7.66136 years longer than males (Table 5.10). This difference remains virtually unchanged when comparisons are based on life tables eliminating diseases of heart as a cause of death. However, elimination of malignant neoplasms causes the difference to expand slightly while elimination of motor vehicle accidents results in a slight contraction of the difference.

Probable lifetime

Greville (1946) notes that the life table may be viewed as a frequency distribution of the ages at death of the hypothetical life table cohort. The arithmetic mean of this distribution is the average age at death of the members of the hypothetical cohort or, more specifically, the average duration of life or life expectancy at birth. Greville, however, suggests that an alternative standard for comparing longevity is the median length of life or probable lifetime. Probable lifetime is the age at which exactly half the original members of the cohort have died and half are still alive. It is, in other words, the age to which a newborn infant has an

Table 5.10. Sex differentials in life expectancy at birth, United States, 1969-1971.

| Cause eliminated | Life expectancy at birth Females | Males | Difference |
|---|---|---|---|
| No causes | 74.61325 | 66.95189 | 7.66136 |
| MN | 77.21296 | 69.26272 | 7.95024 |
| DH | 80.96523 | 73.30380 | 7.66143 |
| MVA | 75.02386 | 67.88455 | 7.13931 |

even chance of surviving. Probable lifetime is found by dividing the number of survivors to any age by 2 and finding by interpolation at what age the number of survivors is equal to half those living at the age in question. Probable lifetime at birth is the age at which 50,000 of the original 100,000 births assumed in the radix of the life table survive. Probable lifetimes at birth for life tables due to all causes and life tables with causes eliminated are presented in Table 5.11. Table 5.11 also presents the difference between probable lifetime and life expectancy at birth.

Comparison of probable lifetime and life expectancy at birth shows the former to exceed the latter for each cause-eliminated, including no causes eliminated, life table. Since the distribution of ages at death in a life table cohort is always characterized by greater dispersion below the median than above it, the median always exceeds the mean (Greville, 1946:23). These differences are, in large part, due to the high toll of mortality during infancy which skews the distribution of deaths in such a manner that the median exceeds the mean.

The appropriate measure of longevity, mean or median, is left to the user's discretion. In view of the pronounced skewedness of the distribution, it may be thought that the mean is not sufficiently representative. The layman probably has it in his mind that life expectancy at birth refers to

Table 5.11. Probable lifetime at birth and excess years over life expectancy at birth by sex, United States, 1969-1971.

| | Probable lifetime at birth | | | |
|---|---|---|---|---|
| Cause eliminated | Males | Excess | Females | Excess |
| No causes | 70.93020 | 3.97830 | 78.89985 | 4.28660 |
| MN | 73.72080 | 4.45808 | 81.34229 | 4.12933 |
| DH | 77.71840 | 4.41460 | 84.66318 | 3.69795 |
| MVA | 71.44869 | 3.56414 | 79.11142 | 4.08754 |

the age to which an infant has a reasonably good chance of surviving. Thus, he may be told that life expectancy at birth for males is 66.95 years, but may be surprised to find that more than 59 percent of male infants outlive their expectation while approximately 40 percent die before reaching that age. The alternative statement that 70.93 years is the probable lifetime, the age to which the infant has a 50 - 50 chance of surviving, is probably a more satisfactory answer to the layman's question (Greville, 1946:23).

On the other hand, probable lifetime is not sufficiently sensitive to changes in the ages at death of the members of the life table cohort. The value of probable lifetime is unaffected by any change in which the age at death of an individual is not actually shifted from one side to the other of the median itself. The value of expectation of life is affected to some degree by any change in the rate of mortality at any age or in the ages at death in the life table cohort (Greville, 1946:24).

The median length of life or probable lifetime is a special case of percentile analysis of life table survivorship. Probable lifetime is the age to which exactly 50 percent of the radix has survived. As an extension of this analysis, Table 5.12 shows the percentage of the original cohort surviving to ages 1, 20, 65, and 85 years in life tables due to all causes and with causes eliminated.

Table 5.12. Percent of cohort surviving to specified ages under various mortality conditions by sex, United States, 1969-1971.

| Cause eliminated | Percent surviving to age 1 | 20 | 65 | 85 |
|---|---|---|---|---|
| No causes | | | | |
| Males | 97.693 | 96.078 | 64.496 | 12.745 |
| Females | 98.217 | 97.298 | 79.787 | 29.687 |
| MN | | | | |
| Males | 97.698 | 96.220 | 69.779 | 18.151 |
| Females | 98.221 | 97.404 | 84.902 | 36.653 |
| DH | | | | |
| Males | 97.707 | 96.120 | 74.993 | 30.849 |
| Females | 98.229 | 97.334 | 83.955 | 49.110 |
| MVA | | | | |
| Males | 97.703 | 96.575 | 66.175 | 13.236 |
| Females | 98.227 | 97.532 | 80.513 | 30.107 |

These results follow the age-cause pattern of mortality. Among infants, diseases of heart are the most prevalent cause of death of those examined. Thus, the elimination of this cause results in a greater proportion of infants surviving the first year of life. A similar age-cause interpretation may be placed on the results for ages 20 and 85 for both sexes; that is, the age pattern of the incidence of eliminated causes results in a higher proportion of survivors to those age where the cause is eliminated.

The results for age 65 depart somewhat from the expected pattern. Among males, the results are as expected. However, among females, diseases of heart are most prevalent at age 65 but the elimination of malignant neoplasms produces a higher proportion of survivors to age 65. Elimination of a large number of female deaths due to malignant neoplasms during the middle years carries over to later years, producing a larger proportion of survivors when malignant neoplasms are eliminated. The percentage of survivors to age 65 due to the elimination of malignant neoplasms is less than .1 percent greater than the percentage of survivors to age 65 when diseases of heart are eliminated.

<u>Age at which expectation of life equals 10 years</u>

Rather than examine the proportion of survivors to a given age, it may be suggested that some arbitrary length of time be selected to determine at what age the expectation of life is a given number of years, that age to be considered the point of entry into old age. Ryder (1975:16) suggests 10 years as the criteria. Accordingly, this procedure substitutes for the fixed age 65, a lower limit for "old age" which depends on the level of survival. Ages of entry into "old age" using an average remaining lifetime of 10 years are given in Table 5.13. The values were obtained through linear interpolation.

These results reflect those found when gains in life expectancy were analyzed in Table 5.9. Because diseases of heart are degenerative diseases highly prevalent among older age groups, their elimination produces an immediate large contribution to the increase in life expecancy. The results shown in Table 5.13 are consistent with those in Table 5.9 because, in essence, the broad age intervals have been broken into infinitely small intervals.

## Descriptive Analysis of Life Table Deaths

The life table provides several functions which may be analyzed through descriptive statistical methods. As the term implies, descriptive statistics consists of methods used

Table 5.13. Age at which average remaining lifetime is 10 years under various mortality conditions by sex, United States, 1969-1971.

| | Average remaining lifetime is 10 years at age | |
|---|---|---|
| Cause eliminated | Males | Females |
| No causes | 70.559 | 75.450 |
| MN | 72.717 | 76.777 |
| DH | 80.165 | 85.+ |
| MVA | 70.652 | 75.485 |

to describe collections of statistical observations.[5] Although descriptive methods could be applied to all life table functions, in this section several descriptive methods are applied to life table deaths, ${}_nd_x$ (cf. Cox, 1957:127).

This column of the life table was selected for three reasons. First, measures of the distribution of life table deaths are substantively meaningful. Many of the values in other columns in the life table are cumulative in nature and descriptive measures of location, concentration, and form are substantively meaningless when applied to that type of data. In addition, if the number of deaths in each age interval was equal to the mean, the age pattern of mortality would conform closely to the observed pattern; that is, a positively linear relationship between age and mortality. This results from the fact that the number of deaths remains constant at each age as the number of survivors diminishes with age.

Second, although all life table values are essentially derived from the probability of dying, life table deaths are more directly calculated from ${}_nq_x$ and are, thus, more direct-

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[5]Inferential statistics deal with the logic and procedures for evaluating risks of inference from descriptions of samples to descriptions of populations. The difference between descriptive and inferential statistics does not lie in the techniques themselves but in the manner in which the techniques are used. If techniques are used to summarize data, they are descriptive. If they are used to estimate parameters of a population from which the data are a sample, they are inferential (Loether and McTavish, 1974:8).

ly influenced by changes in age-specific probabilities of dying.

Third, and perhaps most important, the ${}_nd_x$ column of the life table possesses a characteristic which makes it attractive for comparative purposes. For any two life tables similarly constructed (i.e., with the same initial radix, $l_0$, and the same number of age intervals), the means of life table deaths are identical. This result derives from the identity given in equation (2.9) that the sum of deaths over all ages in the life table is equal to the size of the original cohort and from the equality of the number of age intervals in a given set of life tables. Thus, in the case of life tables constructed in the same manner, both the numerator and the denominator of the calculation formula for the mean number of life table deaths are constants, yielding a constant value for the average.

These results, then, make the comparison of distributions of life table deaths with identical means more meaningful in terms of other descriptive measures. In the present study, assuming $l_0$ = 100,000 and 19 age intervals, the average number of life table deaths is

$$\bar{X} = 100{,}000/19 = 5263.16.$$

The number of life table deaths is treated as a continuous variable associated with each age interval in the present study. Summary statistics are presented in Table

Table 5.14. Descriptive measures of distribution of life table deaths due to all causes and with causes of death eliminated by sex, United States, 1969-1971.

| Cause eliminated | Median | Standard Deviation | Beta 1 | Beta 2 |
|---|---|---|---|---|
| No causes | | | | |
| Males | 2307 | 5281.3828 | .3737 | 1.6644 |
| Females | 1783 | 7804.8828 | 3.7469 | 6.1482 |
| MN | | | | |
| Males | 2302 | 5836.1992 | .9090 | 2.4535 |
| Females | 1275 | 9174.3164 | 5.8392 | 8.3739 |
| DH | | | | |
| Males | 2103 | 7495.4219 | 5.2591 | 8.1806 |
| Females | 1577 | 11210.8280 | 11.5075 | 13.7583 |
| MVA | | | | |
| Males | 2297 | 5497.3750 | .3834 | 1.6707 |
| Females | 1773 | 7922.2695 | 3.7865 | 6.1928 |

5.14.

There are three features characteristic of any distribution: central tendency, variation, and form of distribution.

The location or central tendency of a distribution refers to the place on the scale of values where a particular distribution is centered. One measure of central tendency is the mean which, as noted above, is equal for all distributions of life table deaths for life tables constructed with the same radix and same number of age intervals. The median, an alternative measure of central tendency, is the value below which, and above which, half the values in the distribution fall. Thus, the mean is determined by summing all values and dividing by the number of elements while the median is the middle value in magnitude. For an infinite population, the mean is the center of gravity of the density function and the median is the value that divides the density function into two equal parts. The mean is the center of mass of the distribution function. The median is the center of area of the distribution function (i.e., area under the curve) (Gibbons, 1976:90).

For males and females, Table 5.14 shows that the median number of life table deaths is considerably less than the mean number of life table deaths for life tables due to all causes and life tables with causes eliminated, indicating that the distributions are skewed to the right (positive).

Since the distribution of deaths by age is, in general, J-shaped, such a relationship between the median and mean implies that the relatively larger number of deaths at older ages tends to pull the mean away from the median.

A second characteristic of a distribution refers to dispersion, relative concentration, or, more commonly, variation. Although a number of measures of variation[6] exist, the preferred measure is variance or its square root, standard deviation, because of its mathematical uses in other areas of statistics. Variance and standard deviation are measures of variation which describe the extent to which values in a distribution differ from a single value, the mean.

Variance is an average squared deviation of values from the mean,

$$s^2 = \Sigma(X_i - \overline{X})^2/N \tag{5.17}$$

and standard deviation is the square root of variance,

$$s = \sqrt{s^2} \tag{5.18}$$

where $X_i$ is the ith observation of $X$, $i = 1, \ldots, N$.

The standard deviations shown in Table 5.14 indicate that for both sexes the elimination of causes of death results in greater variation in the number of life table deaths. The greatest increase in variability as measured by

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[6]Other suggested measures of variation include range, interquartile range, and average absolute deviation (Loether and McTavish, 1974:144-146).

the standard deviation occurs when diseases of heart are eliminated. This result, in conjunction with the diminishing value of the median accompanying the elimination of causes of death, indicates that the effect of postponing deaths to later years is to increase the variability among life table deaths and to extend the tail of the distribution toward extreme values associated with older intervals.

There are two summary measures which may be used to describe the form of the distribution of life table deaths. These are measures of skewness and kurtosis. Skewness refers to the trailing off of extreme values in one direction away from the majority of cases. Positively skewed distributions trail off to the right. Negatively skewed distributions trail off to the left. The normal curve is symmetrical (non-skewed). Kurtosis refers to the degree to which cases are distributed across the categories of the distribution. A leptokurtic distribution is unusually concentrated around the mean. A platykurtic distribution is unusually distributed across all categories. The normal curve is described as mesokurtic.

Summary measures of skewness and kurtosis are based on the moment system.[7] The moment system is a system for de-

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[7]The moment system is based on normal theory and refers to the normal curve.

scribing data in terms of its balance around some central point, specifically, the mean. The mean is the point at which the algebraic sum of values is zero. Deviation of values from the mean of a distribution is expressed as

$$x = (X_i - \overline{X}).$$

The first moment about the mean is the average of the first power of deviations from the mean,

$$m_1 = \Sigma x/N. \qquad (5.19)$$

The sum of deviations from the mean is always zero. Thus, the first moment is always zero.

The second moment is the variance of the distribution,

$$m_2 = \Sigma x^2/N. \qquad (5.20)$$

The third moment is the average of the third power of deviations about the mean,

$$m_3 = \Sigma x^3/N. \qquad (5.21)$$

The fourth moment is the average of the fourth power of deviations about the mean,

$$m_4 = \Sigma x^4/N. \qquad (5.22)$$

The third moment provides an index of skewness. It is an odd moment and if high and low scores around the mean do not balance, it will be nonzero. It is also a higher moment and, thus, tends to emphasize extreme deviations.

The fourth moment is an even moment. Consequently, it does not distinguish between deviations above and below the mean. It is useful as an index of kurtosis because it is a higher moment that emphasizes the deviation of values falling in the tails of a distribution.

Two measures of skewness and kurtosis have been derived from the moment system. The skewness measure, Beta 1, is the ratio of the square of the third moment to the cube of the second moment or

$$\beta_1 = m_3^2/m_2^3. \qquad (5.23)$$

If the distribution is symmetrical, the third moment and Beta 1 will be zero. Positive values of Beta 1 indicate skewness to the right while negative values indicate left skewness. The magnitude of Beta 1 expresses the relative skewness and may be compared across distributions.

Table 5.14 shows that values of Beta 1 for the distributions of life table deaths for all life tables considered in the present study are positive, indicating skewness to the right. This result is as expected since in all cases the mean is greater than the median, indicating a trailing off of extreme values in the direction of the mean. However, a comparison across tables shows that the distribution of life table deaths with diseases of heart eliminated is more positively skewed for both males and females than that of all other life tables. This indicates that there is more trailing off of extreme values in these distributions due to the postponement of deaths associated with the elimination of causes of death. Values of Beta 1 also show that the distribution of life table deaths for females is more positively skewed than for males. This result is consistent with observed sex differentials in mortality and life expectancy.

Beta 2 is a measure of peakedness or kurtosis of a distribution. Beta 2 is the ratio of the fourth moment to the square of the second moment or

$$\beta_2 = m_4/m_2^2. \tag{5.24}$$

This measure is usually compared to a normal distribution for which Beta 2 = 3. Distributions which are flatter at the center than the normal distribution have values of Beta 2 less than 3. Such platykurtic distributions have observations widely scattered about the mean. Values of Beta 2 greater than 3 indicate distributions which are more peaked than the normal distribution. Thus, a leptokurtic distribution has many values close to the mean.

Examination of values of Beta 2 in Table 5.14 reveals interesting results. For males, values of Beta 2 indicate that the distributions of life table deaths due to all causes and with malignant neoplasms and motor vehicle accidents eliminated are less peaked than the normal distribution. Compared to the distribution of life table deaths resulting from the elimination of diseases of heart which is highly peaked, these life tables have a relatively more homogeneous distribution of deaths around the mean. The value of Beta 2 for life table deaths due to the elimination of malignant neoplasms, while slightly less peaked than the normal curve, is nearly normal, an indication of the prevalence of cancer as a cause of death in the middle ages. The high value of Beta 2 resulting from elimination of diseases of heart is consistent with previous results. The elimination of this cause postpones a large number of deaths to older ages. However, for the remaining age intervals, life table deaths are more concentrated around the mean relative to the distribution resulting from the elimination of diseases of heart.

For females, Table 5.14 shows that all values of Beta 2 are greater than 3, indicating unusual peakedness for all distributions. As in the case of males, the life table death distribution associated with elimination of diseases of heart produces the most peaked distribution. The same interpretation of this result applies in the female case.

Comparison of distributions of life table deaths for males and females shows than in every instance the female distributions are more peaked than those for males. This result, taken in conjunction with values of Beta 1, confirms the greater mortality among males and the more pronounced postponement of female deaths (i.e., greater life expectancy) to later years.

The results of this descriptive statistical analysis reveal several important findings. For both males and females, the elimination of causes of death results in large discrepancies between mean and median. The larger values of the mean indicate a skewness or trailing off of extreme values to the right. For both sexes, the elimination of causes of death produce greater variability in the number of life table deaths among age intervals. Measures of skewness and kurtosis also reveal a change in the forms of life table death distributions with elimination of causes of death. In every instance, elimination of causes of death results in more positively skewed and more peaked distributions than those associated with life tables due to all causes.

## Summary

This chapter presented methods of comparing life tables appropriate to the cohort interpretation of the life table. The following points were discussed:

1. Competing risk theory provides a framework for comparing life tables in terms of three types of probability: crude probability of death in the presence of all other risks, net probability of death with a cause of death eliminated, and partial crude probability of death from a given cause when another cause is eliminated. Results of the comparison of these values derived from life tables due to all causes and due to elimination of causes reflected the age-cause pattern of mortality. It was shown that the elimination of a cause of death produces constant relative changes between crude and partial crude probabilities for the

remaining causes.

2. Conditional probabilities of surviving from one age interval to a subsequent age interval were also examined. These results also reflected age-cause patterns of mortality. It was shown, however, that slight departures from expected patterns were due to the particular age-cause patterns of mortality of prior age intervals which may affect either, or both, the numerator or the denominator of the calculation formula for the conditional probability of survival.

3. Analysis of gains in life expectancy due to elimination of causes of death showed that for both sexes the elimination of diseases of heart produced the greatest gains at all ages while elimination of malignant neoplasms and motor vehicle accidents produced moderate and slight gains in life expectancy, respectively.

4. Alternative measures of longevity were examined. Analysis of probable lifetime, the age to which half the initial radix survives, showed that for each life table considered, probable lifetime at birth exceeded life expectancy at birth. The relative merits of life expectancy and probable lifetime as measures of longevity were discussed. Other measures of longevity examined were percent of original cohort surviving to specified ages under different mortality regimes and the age at which life expectancy equals 10 years. These results also reflected age-cause patterns of

mortality.

5. Descriptive measures of distributions of life table deaths were analyzed. These measures, in general, indicated that elimination of causes of death resulted in distributions of life table deaths that were more postively skewed, more peaked, and more variable than that associated with life tables due to all causes.

# CHAPTER 6. COMPARISONS BASED ON THE LIFE TABLE AS A STATIONARY POPULATICN

The second interpretation of the life table is that of the stationary population. Under this interpretation, the $_nL_x$ function of the life table represents the age distribution of the stationary population under a given set of mortality and assumed fertility and migration conditions. Thus, the comparisons in this chapter focus on measures of the age distributions of stationary populations associated with various mortality regimes and the implications of those results.

## Stable Population Theory

The stationary population is a special case of stable population theory. Among Alfred Lotka's (1907, 1922, 1929, 1939a, 1939b) contributions to the mathematical theory of human populations was the stable population model. Lotka's model defines the age compositon implicit in a given regime of vital rates. The model deals with the dynamic behavior of a population which is closed to migration and subjected to unchanging schedules of age-specific fertility and mortality rates. Under these conditions, a stable population structure is generated and determined (cf. Coale, 1968). Lotka stressed the fact that the shape of the stable population distribution is a function of prevailing vital rates and independent of the age structure possessed by the population at the initialization of the regime of mortality and fertility rates.

Dublin and Lotka (1925) proved that a closed population with constant age-specific mortality and fertility rates will eventually have a constant rate of natural increase. Lotka called this rate the true rate of natural increase. Subsequently, this value has been referred to as Lotka's r. A life table or stationary population is a stable population with natural increase of zero. A stable population is a population whose relative composition remains unchanging although the size of the population as a whole and of each age group changes at a constant rate. A stationary population, as a result of a zero rate of natural increase, remains unchanging in size and composition.

The basic assumptions underlying the stable population model are:

(1) The human population under study is closed to migration;

(2) The demographic process is studied for each sex separately and the problem of reconciling discrepancies between the two resulting processes is not considered in the standard version of the model;

(3) There is a fixed probability, p(a), that a newborn female will survive to age a. The function p(a) is a

continuous function of age and is sufficient to characterize mortality conditions;

(4) There is a probability, $m(a)da$, that a female alive at age a will bear a female child between age a and age a + da. The function $m(a)$ is continuous and independent of time;

(5) There is an age $\omega$ such that $p(a) = 0$ for $a > \omega$ and ages $\alpha$ and $\beta$ such that $0 < \alpha < \beta < \omega$ for which the function $m(a)$ vanishes beyond ages outside the limits of $(\alpha, \beta)$ (Lopez, 1961:9).

Given these assumptions, the basic theorem of stable population theory states that a closed population of one sex subject to unchanging vital rates eventually attains a fixed age composition and a constant rate of increase. This eventual age composition and rate of increase are completely determined by the fixed mortality and fertility rates independently of the initial age structure of the population, provided that the population includes some members who are in the childbearing ages $(\alpha, \beta)$ (Coale, 1968:396).

A number of stable population values may be generated from the model including the intrinsic rate of natural increase (r), mean length of a generation (T), intrinsic birth rate (b), and intrinsic death rate (d). However, the values of particular importance in the current study are stable age distributions. The stable age distribution is given by the formula

$$c(a) = be^{-ra}p(a) \qquad (6.1)$$

where c(a) is the proportion at age a, b is the constant intrinsic birth rate, r is the constant rate of increase, p(a) is the proportion surviving to age a, e is the base of the natural logarithm system, and a is the midpoint of the age interval.

The stationary population is the result of the following conditions: a fixed survival function for females, p(a); a fixed maternity function for females, m(a); a value of unity for the net reproduction rate[1], the product sum of these two functions; a fixed survival function for males; and a population closed to migration. Under these conditions, the growth rate of the population will be zero. The combination of a net reproduction rate of unity and a fixed length of life guarantees that the person-years of life ($_nL_x$) in the generation of daughters will be identical to the number and age distribution of $_nL_x$ in the female parent cohort. The same relation holds true for males (Ryder, 1975:3-4).

The stationary population is a population in which there

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[1]Net reproduction rate measures the number of daughters that a cohort of newborn females will bear in their lifetime assuming a fixed schedule of age-specific fertility rates and a fixed set of mortality rates. Thus, it measures the extent to which a cohort of newborn females will replace themselves under a fixed schedule of vital rates. A net reproduction rate of unity indicates exact replacement.

is a constant number of annual births and an equally constant number of annual deaths, resulting in natural increase of zero. Thus, ${}_nL_x$ values of the life table may be viewed as the resulting age distribution of a population experiencing 100,000 annual births and 100,000 annual deaths. Because the annual numbers of births and deaths are constant and equal, the stationary population model describes the survival chances of a cohort of births with age and these chances are determined by the unchanging age-specific mortality risks to which the population is subject. The number of persons at any age in a stationary population does not vary from year to year. Thus, the numbers of 40-year olds in successive years are the survivors of the same numbers of births, $l_0$, and, under constant mortality conditions, the same proportion of the annual numbers of births survive to specified ages. In a stationary population, the number of persons 40 years old would be exactly the same as the number of infants in the absence of mortality before age 40 (Coale, 1972:592).

If a census of a stationary population, assuming no migration, was taken, then the count of that census would show a number of persons equal to ${}_nL_x$ for respective age intervals. If another census was taken 10 years later, it would reveal the same size and age composition since a stationary population is a population that retains constant size and composition. The total size of a stationary population is $T_0$, the sum of the $_nL_x$ values over all ages. The proportion older than age x is $T_x/T_0$. The number of annual births is $l_0$. Thus, the crude birth rate is $l_0/T_0$, the reciprocal of the expectation of life at birth. The number of life table deaths by age is $_nd_x$ and the age-specific death rate is $_nd_x/_nL_x$. Since the sum of $_nd_x$ over all ages is $l_0$, the crude death rate is $l_0/T_0$, which is equivalent to the crude birth rate, yielding zero natural increase (Keyfitz and Flieger, 1971:133).

The stationary population model is useful in comparing mortality experiences in two or more populations because the factors of fertility and migration are disregarded. Elimination of these factors minimizes the number of variables which may limit comparability of mortality experiences (Davis, 1961:509). Furthermore, a number of studies (Lorimor, 1951; Sauvy, 1954; United Nations, 1954; Coale, 1956; Stolnitz, 1956; Osborne, 1958; Hermalin, 1966; Keyfitz, 1968b) have shown that past increases in the proportion of aged persons in the United States and other Western countries were due almost entirely to declines in fertility rates and virtually not at all to declines in mortality rates. Past reductions in mortality rates giving rise to greater life expectancy have been heavily concentrated at younger ages. Thus, these mortality declines had an effect similar to that which would have resulted from increases in fertility.

Mortality declines at younger ages tended to increase the proportion of younger persons and to retard the aging of population.[2] If it is assumed that fertility has reached, or is near, its lower limit, then the stationary population is a useful model for analyzing the effect of further declines in mortality on the age structure of a population.

The stationary population model is of more than mere academic interest. In recent years, there has been increased interest in population growth and the notion of stationarity. According to Mayer (1970:83), a sense that population growth is becoming a burden has arisen from several interrelated concerns dealing with crowding, pollution, and deterioration of the environment. Ryder (1973:45) suggests that there is a substantial probability that the United States is approaching the end of its era of population growth and that important sectors of professional and public opinion are advocating that government take actions to ensure or hasten that process. Lobbying groups, such as Zero Population Growth, have arisen to advocate the stabilization of the United States' population while others advocate less than zero growth (Center for the Study of Democratic Institutions, 1970; Notestein, 1975).

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[2]Aging of a population refers to an increase in the proportion of old persons and a decrease in the proportion of young people.

The importance, according to Mayer (1971:81), of the zero growth idea is that is has taken root in the thinking and vocabulary of large numbers of educated people and has achieved recognition by government.[3] The Commission on Population Growth and the American Future (1972:110) concluded on the basis of their studies that no substantial benefits would result from continued growth of the United States' population. They further suggest that stabilization of the population would contribute significantly to the ability of the nation to solve its problems. Other recent studies of stationarity or zero growth deal with economic growth and welfare (Enke, 1971; Eilenstine and Cunningham, 1972), population policy (Coale, 1970), and demographic paths to stationarity (Frejka, 1968, 1972, 1973).

## Measures of Age Distribution

The stationary population model is a useful model for investigating the effect of changing mortality on the age structure of a population. Examination of the ${}_nL_x$ function of the life table indicates the age distribution of a stationary population associated with a given regime of mortality. Thus, this column of the life table may be used

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[3]There are now official Bureau of the Census population projections based on zero population growth assumptions as well as less than zero growth assumptions (U. S. Bureau of the Census, 1975).

to answer questions involving the proportion of persons in a stationary population who are in a given age group. Specifically, values of ${}_nL_x$ may be used to determine whether a decline in mortality tends to make an age distribution older or younger. Declines in mortality will undoubtedly enable individuals to grow older*, but for population aggregates the effect depends on the ages at which mortality improvements occur. When relative changes in survival rates are the same at all ages, age distribution is unaffected since the total population will change in the same proportion as the number at each age. In general, a change in mortality will increase the size of those age groups in which deaths have been postponed. If mortality is reduced at all ages, the effect will be an extension of the age distribution upward toward older ages and, thus, to increase the proportion of persons in older age groups. If improved mortality occurs primarily at younger ages, the population becomes younger. If declines in mortality occur at older ages, there will be an increase in the proportion of older persons. Table 6.1 presents proportional age distributions of stationary populations by sex based on life tables due to all causes and with causes of death eliminated.

The aging of a population refers to an increase in the

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*See results presented in Chapter 5.

Table 6.1. Percent distribution of stationary populations due to various mortality conditions by sex, United States, 1969-1971.

| | Cause eliminated | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Males | | | | Females | | | |
| Age | No | MN | DH | MVA | No | MN | DH | MVA |
| < 1 | 1.46 | 1.41 | 1.34 | 1.44 | 1.32 | 1.27 | 1.22 | 1.31 |
| 1- 4 | 5.83 | 5.63 | 5.32 | 5.75 | 5.26 | 5.08 | 4.85 | 5.23 |
| 5- 9 | 7.26 | 7.02 | 6.63 | 7.17 | 6.56 | 6.34 | 6.04 | 6.52 |
| 10-14 | 7.24 | 7.00 | 6.62 | 7.15 | 6.55 | 6.33 | 6.03 | 6.52 |
| 15-19 | 7.20 | 6.97 | 6.58 | 7.13 | 6.53 | 6.32 | 6.02 | 6.51 |
| 20-24 | 7.14 | 6.91 | 6.52 | 7.09 | 6.51 | 6.30 | 6.00 | 6.49 |
| 25-29 | 7.06 | 6.84 | 6.45 | 7.04 | 6.48 | 6.27 | 5.98 | 6.47 |
| 30-34 | 6.98 | 6.77 | 6.39 | 6.98 | 6.45 | 6.25 | 5.95 | 6.44 |
| 35-39 | 6.89 | 6.69 | 6.32 | 6.90 | 6.40 | 6.21 | 5.91 | 6.40 |
| 40-44 | 6.75 | 6.58 | 6.22 | 6.78 | 6.33 | 6.16 | 5.85 | 6.33 |
| 45-49 | 6.55 | 6.41 | 6.10 | 6.59 | 6.22 | 6.09 | 5.77 | 6.23 |
| 50-54 | 6.25 | 6.17 | 5.93 | 6.29 | 6.06 | 5.99 | 5.65 | 6.07 |
| 55-59 | 5.80 | 5.82 | 5.68 | 5.85 | 5.84 | 5.84 | 5.50 | 5.85 |
| 60-64 | 5.17 | 5.32 | 5.32 | 5.23 | 5.53 | 5.63 | 5.30 | 5.55 |
| 65-69 | 4.37 | 4.66 | 4.84 | 4.43 | 5.09 | 5.30 | 5.03 | 5.11 |
| 70-74 | 3.42 | 3.83 | 4.22 | 3.47 | 4.47 | 4.78 | 4.67 | 4.49 |
| 75-79 | 2.39 | 2.85 | 3.45 | 2.43 | 3.62 | 4.01 | 4.15 | 3.65 |
| 80-84 | 1.41 | 1.82 | 2.57 | 1.44 | 2.56 | 2.96 | 3.44 | 2.58 |
| 85+ | .84 | 1.28 | 3.52 | .86 | 2.23 | 2.86 | 6.65 | 2.25 |

proportion of old persons and a decrease in the proportion of young persons. Several techniques may be used to measure the "age" of a population. Two types of measures may be distinguished. First, individual measures of age distribution refer to measures which provide summary measures of each age distribution. These individual measures may be compared and include median age, proportion of aged persons, proportion of young persons, and index of aging. These values are presented in Table 6.2.

Second, comparative measures of age distributions refer to measures which provide summary measures of comparisons between two or more age distributions. Included among this type of measure are index of dissimilarity, age-specific indexes, and goodness-of-fit tests. These values are presented in Tables 6.3, 6.4, and 6.5.

Individual measures of age distribution

Median age   The most commonly used summary measure of the age of a population is median age. Median age is that age which divides the population into two equal-size groups, one which is younger and one which is older than the median. The formula for calculating median age from grouped data is

$$Md = l_{Md} + [(N/2 - \Sigma f_x)/f_{Md}]i \qquad (6.2)$$

where $l_{Md}$ = lower limit of interval containing the middle or N/2th item, N = sum of all frequencies, $\Sigma f_x$ = sum of all frequencies in all classes preceding the class containing the

N/2th item, $f_{Md}$ = frequency of the median class, and i = size of class interval containing the N/2th item (Downey, 1975:79-80).

Median ages for stationary populations based on life tables in this study are presented in Table 6.2. When median age rises, a population is described as "aging." When it falls the population is described as "rejuvenating." Values of median age under various mortality conditions indicate that in all cases elimination of causes of death results in aging of the population. The largest increases occur for males and females when diseases of heart are eliminated while elimination of malignant neoplasms produces smaller increases and elimination of motor vehicle accidents results in only minor changes.

Proportion of aged and young persons    Median age is useful as a general measure of age distribution of a population. However, in more detailed analysis it is of limited value. Thus, when greater precision is desired, demographers generally examine the proportions of a population in particular age groups. In general, three broad age groups are designated which correspond roughly to three major stages of the life cycle : youth (under 15 years), adulthood (15 to 64 years), and old age (65 years and older). These groups represent biological and economic security, working and reproductive ages, and superannuation, respectively (Keyfitz

Table 6.2. Proportions of stationary populations in given age intervals, median ages, and indexes of aging by sex under various mortality conditions, United States, 1969-1971.

| Proportion of population | Cause eliminated: No | MN | DH | MVA |
|---|---|---|---|---|
| | | *Males* | | |
| Under 15 | .21789 | .21074 | .19906 | .21510 |
| 65 and over | .12416 | .14450 | .18587 | .12622 |
| 15 to 64 | .65795 | .64476 | .61507 | .65868 |
| Median age | 34.87861 | 36.07129 | 38.28613 | 35.18447 |
| Index of aging | 56.98301 | 68.56775 | 93.37778 | 57.28438 |
| | | *Females* | | |
| Under 15 | .19677 | .19023 | .18137 | .19279 |
| 65 and over | .17975 | .19916 | .23937 | .18083 |
| 15 to 64 | .62348 | .61061 | .57926 | .62638 |
| Median age | 38.40108 | 39.70223 | 41.71552 | 38.52200 |
| Index of aging | 91.34841 | 104.69166 | 131.97682 | 93.79992 |

and Flieger, 1971:49).

The proportion of aged persons is defined as the ratio of population 65 years and older to total population. The proportion of young persons is given by the ratio of population under age 15 to total population. The proportion of persons in the adult or active population is simply the residual proportion. These proportions are given in Table 6.2.

For males, elimination of causes of death results in all cases in a reduction in youth proportion, indicating a displacement of population to older age groups. This result is confirmed by increases in proportions of old persons when causes of death are eliminated. Elimination of diseases of heart and malignant neoplasms result in decreasing proportions of active persons. However, elimination of motor vehicle accidents results in increasing proportions of aged and active persons, suggesting that while elimination of this cause shifts the age distribution upward, this change is felt by both the aged and active populations.

For females, elimination of causes of death results in all cases in a reduction in proportion of youths and an increase in proportion of old persons. Elimination of malignant neoplasms and diseases of heart produces a decrease in proportion of active population. However, when motor vehicle accidents are eliminated the size of proportion of active population increases slightly (i.e., .003). This may be explained by examining the magnitude of changes in proportion of young and old persons. The increase in proportion of persons 65 years and over due to elimination of motor vehicle accidents is very slight. The decline in proportion of young persons is greater, leading to a slight increase in proportion of active persons. This result indicates that although elimination of motor vehicle accidents as a cause of death displaces the distribution of population upward and results in a larger proportion of old persons, the greatest effect, as with males, occurs in the active population.

<u>Index of aging</u> Stockwell (1972:3) notes that a useful technique for depicting more precisely changes that have taken place with respect to the older segment of a population is the index of aging.[5] The index of aging takes numbers and changes at both ends of a distribution into account and is defined as the number of persons aged 65 years and over per 100 persons under 15 years. Values of this index are presented in Table 6.2.

Values of the index of aging for stationary populations representing various mortality conditions show that for both sexes and for all causes considered, elimination of causes of death results in a greater proportion of old persons relative

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[5]This index is also referred to as aged-child ratio.

to young persons. Only minor increases result from elimination of motor vehicle accidents for both sexes. Elimination of malignant neoplasms and diseases of heart, however, results in substantial increases in values of the index of aging for males and females.

Comparing sexes, the age distribution for males shows a greater concentration of young persons based on the index of aging. Only when diseases of heart are eliminated does the index approach 100, indicating equality of proportions of young and old persons. For females, the value of the index of aging associated with the stationary population based on the life table due to all causes is already high and exceeds 100 when malignant neoplasms and diseases of heart are eliminated, indicating a higher proportion of old persons than young persons.

Comparative measures of age distribution

Index of dissimilarity The index of dissimilarity as a summary measure of the difference between two age distributions was developed by Duncan and Duncan (1955) and is based on absolute differences between percent distributions at each age. Under this procedure, absolute differences between percentages for corresponding ages groups in two populations are summed and one-half the sum is taken.[6] The general formula is

$$ID = (1/2)\Sigma|r_{1a} - r_{2a}| \qquad (6.3)$$

where $r_{2a}$ refers to percent distribution of persons age a in the second distribution and $r_{1a}$ refers to percent distribution of persons age a in the first distribution taken as a base. The index of dissimilarity indicates how far the age distribution resulting from elimination of a cause of death departs from the age distribution of the stationary population resulting when no causes of death are eliminated. The index of dissimilarity may be interpreted as a measure of displacement. It indicates the proportion of the age distribution of one population that would have to be displaced from one age group to other age groups in order to make the distribution identical to that of a second population.

Table 6.3 presents indexes of dissimilarity of age distributions associated with stationary populations resulting from elimination of causes of death with the age distribution of the stationary population due to elimination of no causes as a base. These results show that slightly over 2 percent of males and females in stationary populations resulting from no elimination of causes would have to be displaced to yield

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[6]Taking one-half the sum of absolute differences is equivalent to taking the sum of positive differences or the sum of negative differences.

Table 6.3. Indexes of dissimilarity for stationary populations by sex under various mortality conditions, United States, 1969-1971.

| Sex | Cause eliminated MN | DH | MVA |
|---|---|---|---|
| Males | 2.200 | 6.319 | 0.424 |
| Females | 2.046 | 6.035 | 0.173 |

stationary age distributions identical to those resulting from elimination of malignant neoplasms as a cause of death. When diseases of heart are eliminated, over 6 percent of the original stationary populations must be displaced. Elimination of motor vehicle accidents necessitates only minor displacement.

Age-specific indexes     The index of dissimilarity offers a measure of differences between two age distributions. However, since it is based on absolute differences, this index does not indicate the direction of differences in distributions or at what point distributions differ. Age-specific indexes provide a method of comparing age distributions that allows such comparison (Shryock and Siegel, 1973:230). Age-specific indexes are derived by dividing the proportion at a given age in one distribution, $r_{2a}$, by the proportion at the same age in another distribution, $r_{1a}$, chosen as a standard and multiplying by 100, or

$$r_{2a}/r_{1a} * 100. \qquad (6.4)$$

In the present study, the stationary age distributions based on life tables due to all causes were selected as the standards. Thus, index values greater than 100 indicate a higher proportion of persons in an age group in a stationary population due to elimination of a cause of death. Age-specific index values are given in Table 6.4.

Table 6.4. Age-specific indexes for stationary populations due to elimination of causes of death by sex, United States, 1969-1971.

| | Cause eliminated | | | | | |
|---|---|---|---|---|---|---|
| | Males | | | Females | | |
| Age | MN | DH | MVA | MN | DH | MVA |
| < 1 | 96.67 | 91.35 | 98.63 | 96.64 | 92.16 | 99.46 |
| 1- 4 | 96.68 | 91.35 | 98.66 | 96.65 | 92.17 | 99.48 |
| 5- 9 | 96.72 | 91.36 | 98.72 | 96.68 | 92.17 | 99.52 |
| 10-14 | 96.75 | 91.36 | 98.78 | 96.70 | 92.18 | 99.56 |
| 15-19 | 96.79 | 91.37 | 98.97 | 96.73 | 92.18 | 99.63 |
| 20-24 | 96.83 | 91.39 | 99.35 | 96.75 | 92.20 | 99.74 |
| 25-29 | 96.90 | 91.42 | 99.70 | 96.80 | 92.21 | 99.83 |
| 30-34 | 96.98 | 91.49 | 99.94 | 96.88 | 92.25 | 99.89 |
| 35-39 | 97.11 | 91.68 | 100.15 | 97.05 | 92.32 | 99.96 |
| 40-44 | 97.36 | 92.14 | 100.34 | 97.36 | 92.47 | 100.02 |
| 45-49 | 97.85 | 93.10 | 100.52 | 97.92 | 92.74 | 100.09 |
| 50-54 | 98.74 | 94.87 | 100.70 | 98.80 | 93.25 | 100.16 |
| 55-59 | 100.31 | 97.90 | 100.89 | 100.09 | 94.17 | 100.23 |
| 60-64 | 102.87 | 102.84 | 101.08 | 101.82 | 95.84 | 100.31 |
| 65-69 | 106.73 | 110.74 | 101.30 | 104.06 | 98.85 | 100.41 |
| 70-74 | 112.65 | 123.41 | 101.53 | 106.10 | 104.34 | 100.52 |
| 75-79 | 119.65 | 144.51 | 101.82 | 110.76 | 114.57 | 100.64 |
| 80-84 | 129.49 | 182.34 | 102.16 | 115.66 | 134.26 | 100.78 |
| 85+ | 152.35 | 419.82 | 102.77 | 128.34 | 298.42 | 100.97 |

Results presented in Table 6.4 are consistent with those shown in Table 6.2 with respect to proportions of persons in young, active, and aged population groups. For both sexes, elimination of causes of death results in shifts in age distributions of stationary populations toward older ages.

When motor vehicle accidents are eliminated, proportional distributions of stationary populations by age begin very near equality (i.e., 100) and shifts to values of age-specific indexes greater than 100 occur earlier than those associated with other eliminated causes. For males, this shift occurs at age 35 while for females it occurs at age 40. This confirms results noted earlier that elimination of motor vehicle accidents produces slightly larger proportions of persons in the active population for both sexes.

When malignant neoplasms are eliminated as a cause of death, values of age-specific indexes greater than 100 occur at the same age for both sexes, age 55.

Elimination of diseases of heart as a cause of death results in the most marked shifts in age distribution of stationary populations toward older ages for both sexes. Age-specific index values for younger age groups are well below 100 and shifts to values greater than 100 do not occur until age 60 for males and age 70 for females. Values of indexes are quite high for terminal age intervals, indicating shifts toward aging populations.

Generally higher values of age-specific indexes for males in older age groups are reflective of sex differentials in mortality. Comparatively, shifts toward older ages are more dramatic for males than females, indicating, as shown previously in Table 6.2, that the original female stationary population is "older" than the original male stationary population.

Goodness-of-fit tests Goodness-of-fit tests may be used to test similarity between two or more distributions. The two best known statistical tests of goodness-of-fit are the chi-square test and the Kolmogorov-Smirnov test. Both tests are based on comparison of two or more distributions. However, chi-square measures of incompatibility are based on vertical deviations between observed and expected histograms whereas Kolmogorov-Smirnov procedures are based on vertical deviations between observed and expected cumulative frequency distributions (Gibbons, 1976:75).

Goodness-of-fit tests are sensitive to differences throughout the entire distribution, not just to differences in location or variability. These tests, then, compare entire distributions and provide summary measures of compatibility of two sets of relative frequencies or empirical distribution functions. The chi-square test is appropriate if data to be analyzed are count data. The Kolmogorov-Smirnov test is applicable if data are measured at least on an ordinal scale. There are, however, several reasons for preferring the Kolmogorov-Smirnov test.

First, the exact sampling distribution of the Kolmogorov-Smirnov test is known and tabulated for population distributions that are continuous. The sampling distribution of Q, the chi-square measure of goodness-of-fit, is only approximate for any finite sample size. Thus, when the hypothesis test concerns a continuous population, Kolmogorov-Smirnov is the preferred statistic especially when the number of groups is small (Gibbons, 1976:76-77).

Second, for the Kolmogorov-Smirnov test, the population distributions, though unspecified, should be continuous. If not, the test may be performed, but it is conservative; that is, the test will tend to support the null hypothesis more often than a less conservative procedure (Gibbons, 1976:250, 258).

Third, unlike the chi-square test, the Kolmogorov-Smirnov test is more flexible. Specifically, Kolmogorov-Smirnov techniques provide one-sided statistics to test for deviations in a particular direction (Gibbons, 1976:76, 254).

Kolmogorov-Smirnov one-sided tests were used to test the following hypothesis set for compatibility of age distributions of stationary populations from life tables due to all causes with stationary age distributions due to life tables with causes eliminated:

$$H: F_1(x) = F_2(x) \text{ for all } x$$

$$A+: F_1(x) > F_2(x) \text{ for some } x$$

where $F_1(x)$ is the cumulative age distribution of the stationary population due to all causes and $F_2(x)$ is a cumulative age distribution due to the elimination of a cause of death.

The two empirical distribution functions may be defined as

$S_1(x)$ = (number of observations in the first distribution that are less than or equal to x)/m,

$S_2(x)$ = (number of observations in the second distribution that are less than or equal to x)/n

where m and n are the total number of observations in each distribution, respectively. In the present instance, m and n are total sizes of the respective stationary populations. Thus, for any x, $S_1(x)$ is the proportion of observations in the first distribution that do not exceed the value x, age, and similarly for $S_2(x)$ in the second distribution. If two distributions are identical, there should be reasonable agreement between $S_1(x)$ and $S_2(x)$ for all values of x.

In the present study, a one-sided hypotheses set was used to test the hypothesis of identical distributions of life table stationary populations. With elimination of causes of death, it is expected that deaths will be postponed and that age distributions of stationary populations will shift upward toward older ages. Consequently, the cumulative frequency distribution of a stationary population resulting from elimination of a cause of death will accumulate less rapidly than the stationary age distribution due to all causes. The test statistic is defined as

$$D+ = \text{maximum}\,[S_1(x) - S_2(x)].$$

A large value of D+ supports the alternative hypothesis, A+.

Table 6.5 presents cumulative relative frequency distributions of stationary populations with values of D+ and P.[7] P-values were derived from Harter and Owens (1970). Kolmogorov-Smirnov tests of the hypothesis sets show that for both males and females elimination of malignant neoplasms and diseases of heart result in life table age distributions which are significantly different than the stationary distribution due to all causes at a level of statistical significance less than .005. Elimination of motor vehicle accidents for both sexes does not, however, result in significantly different stationary age distributions (i.e., $P > .10$). This result suggests that elimination of motor vehicle accidents as a cause of death results in what Keyfitz (1968b:237)

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[7]P-value is the probability of obtaining a value of D+ which is equal to or more extreme than its observed value, given that the null hypothesis, H, is true. Small P-values indicate that a result this extreme occurs rarely by chance and leads to the conclusion that the null hypothesis is discredited.

Table 6.5. Cumulative relative frequency distributions of stationary populations due to various mortality conditions, United States, 1969-1971.

| | Cause eliminated | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Males | | | | Females | | | |
| Age | No | MN | DH | MVA | No | MN | DH | MVA |
| < 1 | .01463 | .01414 | .01336 | .01443 | .01319 | .01274 | .01215 | .01312 |
| 1- 4 | .07288 | .07046 | .06658 | .07190 | .06576 | .06356 | .06061 | .06542 |
| 5- 9 | .14548 | .14068 | .13290 | .14357 | .13132 | .12694 | .12104 | .13067 |
| 10-14 | .21789 | .21074 | .19906 | .21510 | .19677 | .19023 | .18137 | .19583 |
| 15-19 | .28993 | .28046 | .26488 | .28640 | .26208 | .25340 | .24157 | .26090 |
| 20-24 | .36128 | .34955 | .33008 | .35729 | .32716 | .31636 | .30157 | .32581 |
| 25-29 | .43187 | .41795 | .39461 | .42766 | .39198 | .37911 | .36134 | .39052 |
| 30-34 | .50170 | .48567 | .45849 | .49745 | .45646 | .44158 | .42083 | .45493 |
| 35-39 | .57059 | .55257 | .52165 | .56645 | .52047 | .50370 | .47992 | .51891 |
| 40-44 | .63813 | .61833 | .58389 | .63422 | .58375 | .56531 | .53844 | .58221 |
| 45-49 | .70364 | .68243 | .64488 | .70007 | .64595 | .62621 | .59612 | .64446 |
| 50-54 | .76611 | .74411 | .70414 | .76298 | .70658 | .68611 | .65266 | .70518 |
| 55-59 | .82409 | .80227 | .76091 | .82147 | .76497 | .74456 | .70765 | .76371 |
| 60-64 | .87584 | .85550 | .81413 | .87378 | .82025 | .80084 | .76063 | .81917 |
| 65-69 | .91953 | .90214 | .86251 | .91804 | .87114 | .85380 | .81094 | .87027 |
| 70-74 | .95369 | .94046 | .90467 | .95272 | .91586 | .90163 | .85760 | .91521 |
| 75-79 | .97755 | .96901 | .93914 | .97701 | .95210 | .94177 | .89911 | .95168 |
| 80-84 | .99161 | .98722 | .96480 | .99138 | .97772 | .97141 | .93352 | .97751 |
| 85+ | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| D+ | | .02200 | .06318 | .00425 | | .02047 | .06020 | .00156 |
| P | | <.005 | <.005 | >.100 | | <.005 | <.005 | >.100 |

refers to as a neutral change, a mortality change that leaves the age distribution virtually unaffected.

Discussion

This section discusses general uses of the methods described in Chapters 5 and 6 for comparing life tables due to all causes with special life tables due to the hypothetical elimination of causes of death and the importance to policy- and decision-makers values obtained from such methods may have. Under most circumstances, these methods will not reveal values which possess direct policy implications in and of themselves, but, when used to supplement research in other areas, will be useful in policy- and decision-making and in planning further research.

Methods described in Chapter 5 reveal that, based on 1969-1971 mortality data, elimination of causes of death allows individuals to live longer on the average although results vary between eliminated causes. These methods, then, produce values important from an individual perspective. Methods described in Chapter 6 are based on the stationary population model and indicate changes in age distribution of a population resulting from changes in mortality and, thus, are important from a system perspective. Methods presented are neither mutually exclusive nor collectively exhaustive. Instead, they represent alternatives that may be used separately or collectively to determine the impact of mortality on the individual and the system.

Methods presented in this study have special relevance to two policy areas: health policy and population policy. Health policy is concerned with public health measures, medical research, and provision and distribution of, access to, and knowledge of, health services. Population policy is concerned with the relation between the demographic processes of fertility, mortality, and migration as these processes contribute to the growth and distribution of the population. Improvements in mortality conditions have implications for both health and population policy.

The most obvious use of special life tables and methods of comparison associated with each interpretation of life tables is in the study of the impact of mortality by cause. Measures of longevity and probabilities of dying and surviving may be used to compare and contrast the impact of causes of death by region, community, subareas within a community, sex, race, and any number of socioeconomic characteristics for which mortality and population data are available. These values may be used as evaluative benchmarks against which improvement in mortality conditions resulting from public health and other governmental programs, federal, state, and local, may be measured. The values of these measures will be influenced by the organization of medical services and by the form of public health and other programs.

These measures may be used to evaluate improvements in mortality conditions resulting from programs or policies aimed at improving socioeconomic conditions as well as biomedical conditions.

Although other measures of mortality such as changes in age-specific or cause-specific mortality could be examined to determine the extent of mortality improvements, the life table allows such rates to be translated into more meaningful overall measures of the impact of mortality and improvement in mortality conditions on the individual. Thus, for example, an increase in life expectancy at a given age accompanied by a decrease in the gain in life expectancy at that age due to elimination of a cause of death would be indicative of the effectiveness of programs designed to reduce that disease as a cause of death.

Of special interest to health planners should be the analysis of differences between crude and partial crude probabilities of dying derived from competing risk theory. The effect of total or partial elimination of a cause of death results in a redistribution of the probability of dying from remaining causes. Historically, reduction in deaths due to infectious diseases resulted in a realignment of the principal causes of death to the presently numerically important causes (cf. Moriyama, 1964; Preston et al., 1973). Persons responsible for developing policies and programs directed toward reduction or elimination of specific diseases or conditions should be aware that such improvements will affect the relative incidence of other diseases or conditions as well.

The stationary population model allows determination of changes in age structure resulting from changes in mortality uncontaminated by the effects of fertility and migration. It is, sociologically speaking, an ideal type used to analyze the age structure implicit in a given set of mortality rates and assumed fertility and migration conditions. Although such a population may never be empirically approximated, there is some indication that a stationary population is a viable alternative as a population policy in the United States. Furthermore, the Commission on Population Growth and the American Future (1972:136-140) recommended organizational changes to improve the government's capacity to develop and implement population-related policies and to evaluate the interaction between public policies, programs, and population trends. Thus, measures of age distribution of stationary populations under varying mortality conditions may gain added importance.

In broader perspective, cause-eliminated life tables may be useful in the preparation of population projections based on elimination of specific causes of death. Preston (1974), for example, has shown that population projections based on cause-eliminated life tables may be used to determine the impact of mortality improvements on population size, growth rate, age and sex composition, kinship ties, retirement age, and per capita income and that such projections may fill in part of the background against which policy decisions may be made. Preston was able to show, for example, that a decline in fertility to replacement level coupled with major progress against certain degenerative diseases results in a larger proportion of persons aged 65 and over than a decline in fertility alone.

The uses of cause-eliminated life tables and methods of comparing life tables noted above are particular to public health and other health planning applications and to population policy. However, these methods may have wider application because of the implications which mortality improvements may have for the larger social system.

The length of a person's life is of utmost importance to the individual. However, the individual is a unit of a social system. Anything that affects his longevity also affects the aggregate social system. Smith and Evers (1977:74-75) suggest that a sizable increase in the proportion of older people may make the elderly individual less important than in the past while the aggregate of elderly persons becomes more important. Thus, changes in human longevity have definite social implications. Since further improvements in mortality are likely to lead to an older population, an aging population will have consequences socially, medically, and economically. Recently, there has been renewed interest in the problems of a mature society due to the general recognition of the relationship between population growth and environmental problems and the acceptance of the proposition that a zero rate of population growth is the only equilibrium rate that can be sustained (Eilenstine and Cunningham, 1972:223).

Decisions concerning health and population policy do not exist in a vacuum. Instead, they are part of a system of interrelated parts. Changes in one part of the system may initiate changes in other parts of the system. Thus, policy- and decision-makers must be aware not only of the intended consequences of their policies and decisions but also of unintended consequences for other parts of the system. Consequently, methods suggested in this study take on further importance when used to supplement research in other areas as a means of determining implications of certain policies for other parts of the system. Methods based on either interpretation of life tables may be brought to bear on such considerations. Illustrative applications are considered below.

If major medical discoveries are made that affect any of the degenerative diseases prevalent in the older population, the effects on the vitality of remaining life could be dramatic. In past decades, the relationship between the onset of biological old age and death has been pushed back. Increased longevity must be viewed in terms of the quality of survival. Postponement of death does not imply cessation of the process of degeneration. Benjamin (1964:226) notes that medical care may preserve the body but can do little to prevent wearing out of the brain. Thus, a critical issue facing decision-makers pursuing policies advocating prolongation of life is the allocation of funds directly or indirectly connected with prolongation of life. The major consideration appears to be the relationship between prolongation of life and prolongation of vital life and its effect on other areas of decision-making. For example, life tables eliminating diseases of heart as a cause of death indicate that such an elimination would add a substantial number of years of life at all ages. Furthermore, results show that elimination of this cause would result in a larger proportion of persons aged 65 and over. Policy-makers must consider increases of both kinds in light of the relationship between length of life and length of vital life.

Elimination of causes of death results in increased probabilities of survival for the individual and the married pair. One almost certain consequence of improved joint survival is an increase in the proportion of marriages ending in divorce. Such an increase follows on formal grounds alone. The situation is one of competing risks. When one risk function declines, the proportion of the cohort ultimately succumbing to other risks must increase if those risk functions themselves are unchanged.

Despite reducing proportions of widowed at each age, elimination of causes of death is unlikely to alter the proportion of widows over all ages combined. The age-specific reduction is largely offset by an older age structure in the population. Furthermore, life expectancy at birth under improved mortality conditions indicates that while both males and females can expect to live longer on the average, sex differences in life expectancy change only slightly. Thus, female widowhood may be postponed but the period of life spent as a widow remains virtually unchanged.

Increases in probabilities of survival are also salient because of the responsibilities which one generation undertakes on behalf of another. As the relationship moves through time and age, responsibility typically shifts from older to younger generations. With improvements in mortality at older ages, the proportion of children with one or more generations surviving to old age can only increase with increases in longevity (Preston, 1974:155). Illness is a strong function of age. Thus, the likelihood of institutional care increases with age. With elimination of causes of death comes increased probability that one or more generations of a family may require institutional care, thus placing severe financial and emotional strain on younger family members. Furthermore, it is likely that much of the financial burden may shift more and more to all levels of government. Thus, society must be prepared to face the inevitable surge in institutional care for the eldrly that may accompany increases in their number.

Work organizations tend to be more or less hierarchical in their organizational structure and the hierarchy of power and responsibility tends to be correlated with age. If the age structure of an organization reflects that of the labor force as a a whole, a consequence of an older age structure is a probable decline in the rate of upward mobility through the ranks of the organization (cf. Keyfitz, 1973).

Assuming two hierarchical organizations with a fixed proportion of occupations at each rank, and assuming a prominent role of seniority in job promotion, the organization with an older age distribution will have more of the higher status jobs filled by older workers. If the establishment is not expanding, upward mobility will take place only as the more senior workers leave either through death, retirement, or other employment. Thus, it will require more time for a young person in the older organization to achieve a responsible position. This leads to the possibility of greater job dissatisfaction and lower productivity due to the age structure of industry, the new pattern of mobility, and the discouragement and loss of interest in work due to the frustration of the desire to advance. Day (1972:667) suggests that one way of opening up positions of power and prestige no longer created demographically is the adoption of more stringent retirement practices forcing earlier retirement.

By way of retirement pensions and other plans underwritten by former employers and governmental measures including social security and medicare, the elderly are recipients of so-called transfer payments. Under such schemes, moneys are transferred from current workers to former workers. With a shift toward a significantly older population, support which depends primarily on transfer payments will prove more and more costly to the working population. Sauvy (1948:115) suggests that the burden on the worker may become so heavy that it will be resented and evaded.

One method of avoiding such problems is postponement of retirement age. Preston (1974:152) suggests that one way to determine the extent of adjustment required is to calculate the age, x, that satisfies the equation

$$P_{x+}^{(-i)} / P_{15+}^{(-i)} = P_{65+} / P_{15+} \qquad (6.5)$$

where $P_{x+}^{(-i)}$ = number of persons above age x in the population with cause of death i eliminated and $P_{15+}^{(-i)}$ = number of persons age 15 and over in the same population. X is the age in the population with cause i eliminated that maintains the same ratio as on the right side of equation (6.5). It is close to the new retirement age. Calculations based on values taken from stationary male populations resulting from elimination of causes of death show the new retirement age to be: malignant neoplasms - 67.06 years; diseases of heart - 71.23 years; and motor vehicle accidents - 65.18 years.

A dilemma results. On the one hand, a stationary population results in deflated rates of mobility within work organizations. The solution appears to be to open up positions of power and prestige by forcing earlier retirement. On the other hand, the balance of transfer payments requires postponement of retirement under different levels of stationarity. This dilemma accentuates the need for consideration of unintended consequences of programs and policies and reinforces the use of special life tables to make informed decisions concerning possible implications of pursuing a given policy with respect to health and population.

This section considered a number of uses for policy- and decision-makers of special life tables with causes eliminated and methods of comparing main and special life tables. This discussion by no means exhausts the possible uses of these methods, but serves to illustrate their value to policy- and decision-makers. These methods have particular relevance to health and population policy- and decision-makers who need to consider both intended and unintended consequences of certain lines of action for the individual and the system.

## Summary

This chapter presents methods of comparing life tables based on the stationary population model. The major points discussed in this chapter were:

1. The stationary population model is a special case of stable population theory developed by Lotka. The stationary model is especially adapted to the study of changes in mortality conditions because the stationary age distribution is determined by mortality and assumed fertility and migration conditions.

2. Several individual summary measures of stationary age distributions were compared for each sex including median age, proportion of aged persons, proportion of young persons, and index of aging. These measures indicated that elimination of causes of death considered in this study resulted in older stationary populations.

3. A number of measures directly comparing age distributions were calculated. Included were index of dissimilarity, age-specific indexes, and Kolmogorov-Smirnov goodness-of-fit tests. These measures also showed that elimination of causes of death resulted in older stationary populations. Goodness-of-fit tests revealed that only the elimination of

malignant neoplasms and diseases of heart produced significantly different age distributions at conventional levels of significance.

4. The general uses of methods of comparing life tables due to all causes with life tables due to elimination of causes of death were described. These methods have special relevance to health and population policy but are also useful in a broader systems perspective in examining the intended and unintended consequences of health and population policies and programs.

# CHAPTER 7. SPECIAL METHODS OF ANALYZING GAINS IN LIFE EXPECTANCY

More than any other life table value, life expectancy receives considerable attention. This is true, perhaps, because life expectancy is a value which is easily understood and which has meaning to both layman and scientist. Furthermore, when life tables are constructed eliminating causes of death, gains in life expectancy are nearly always reported. Again, such values are readily understood by the lay and scientific communities.

This chapter presents a special method of analyzing gains in life expectancy. The method is proposed by Crosson (1963) and is based on improvements in survival probabilities. However, unlike previous chapters which described methods appropriate to different interpretations of the life table, the second section of this chapter discusses the use of multiple regression, a method which may yield uninterpretable results, to analyze gains in life expectancy. A discussion of multiple regression is included because it receives considerable use in the social sciences and because its use in the current context may be tempting in light of the fact that apparently valid indicators of mortality improvements before and after a given age may be generated from life table functions.

Crosson's Method of Analyzing Gains in Life Expectancy

Crosson proposed his method of analyzing gains in life expectancy in 1963. At that time it was well-received and acclaimed by many actuaries as a valid method of analyzing expectation of life at any age into its components, reflecting improvements in mortality rates before and after any given higher age (Amer, 1963; Campbell, 1963; Greville, 1963). However, the method has received little use.

The method addresses itself to the problem of the proportion of increase in life expectancy at a given age which can be attributed to changes in mortality rates at and over some higher age. Although Crosson suggested use of the method in analyzing changes in life expectancy over time, it is easily adapted to analysis of hypothetical increases in life expectancy due to elimination of causes of death.

According to Crosson (1963:386, 388), if x denotes the younger age whose gain in life expectancy is to be analyzed and y denotes the older age involved, then complete analysis of the gain in life expectancy at age x involves three components.

(1) The increase due solely to mortality improvements at ages y and over is

$$ {}_{y-x}p_x(e'_y - e_y) \tag{7.1} $$

where the primed function denotes mortality after improvement (i.e., elimination of a cause of death),

${}_{y-x}p_x$ is the conditional probability of survival from age x to age y in the life table due to all causes, and $e'_y$ and $e_y$ are life expectancy at age y from cause-eliminated and main life tables, respectively. Here, the excess in life expectancy at age x is attributed to improvements in mortality conditons at age y and over, $(e'_y - e_y)$, under the assumption of no improvements in mortality below age y, ${}_{y-x}p_x$.

(2) The increase due solely to improvements in mortality at ages under y is

$$(e'_y - e_y) - {}_{y-x}p'_x(e'_y - e_y). \qquad (7.2)$$

Here, the excess in life expectancy at age x is attributed only to mortality improvements under age y. The term ${}_{y-x}p'_x(e'_y - e_y)$ gives the portion of improvement in life expectancy at age x under the assumption of improvements in mortality at all ages. The difference given by (7.2) is that portion of the gain in life expectancy at age x due to improvements in mortality under age y.

(3) The additional increase due to the increased probability of survival to age y to participate in increased expectancy at that age is

$$({}_{y-x}p'_x - {}_{y-x}p_x)(e'_y - e_y). \qquad (7.3)$$

The third component is needed because improvements in mortality at ages under y produce a greater number of survivors to age y. Greville (1963:394) describes this component as the interaction of the first two components.

These three components account for the entire gain in life expectancy at age x. Table 7.1 presents the analysis of gains in life expectancy at birth and ages 25, 45, and 65 due to elimination of causes of death. Values presented in Table 7.1 reflect age-cause patterns of mortality for both sexes. For example, elimination of motor vehicle accidents, which are highly prevalent in earlier age intervals for both sexes, is reflected in high proportions of gains due to improvements in mortality condtions before ages 25, 45, and 65. These changes are, of course, cumulative, accounting for the extension of improvements due to prior ages beyond the ages where motor vehicle accidents are most prevalent.

Improvements due to elimination of diseases of heart present the opposite picture. These deaths are most prevalent at older ages. Thus, as expected, the largest proportional increases in life expectancy due to mortality improvements beyond age y are found at younger ages. Again, changes are cumulative, but in the opposite manner as those associated with elimination of motor vehicle accidents as a cause of death. Here, improvements extend from the oldest

Table 7.1. Analysis of gains in life expectancy due to elimination of causes of death by Crosson's method, United States, 1969-1971.

| Eliminated cause | Age x | Gain in $e_x$ | Age y | Increase due solely to mortality improvement: At ages y and over | | At ages under y | | Increase due to increased probability of survival to age y | |
|---|---|---|---|---|---|---|---|---|---|
| MN | | | | | | | | | |
| Males | 0 | 2.31082 | 25 | 2.19601 | (95.03) | .11030 | ( 4.77) | .00451 | ( 1.95) |
| | 25 | 2.31152 | 45 | 2.07523 | (89.78) | .22188 | ( 9.60) | .01441 | ( 6.24) |
| | 45 | 2.20621 | 65 | 1.04930 | (47.56) | 1.08110 | (49.00) | .07581 | ( 3.44) |
| | 65 | 1.45386 | 85 | .09829 | ( 6.39) | 1.33161 | (91.59) | .02939 | ( 2.02) |
| Females | 0 | 2.59971 | 25 | 2.50699 | (96.43) | .08913 | ( 3.43) | .00359 | ( .14) |
| | 25 | 2.58623 | 45 | 2.25382 | (87.15) | .31390 | (12.14) | .01851 | ( .71) |
| | 45 | 2.32959 | 65 | 1.10826 | (47.57) | 1.16156 | (49.86) | .05977 | ( 2.57) |
| | 65 | 1.30267 | 85 | .15764 | (12.10) | 1.11976 | (85.96) | .02526 | ( 1.94) |
| DH | | | | | | | | | |
| Males | 0 | 6.35190 | 25 | 6.31121 | (99.36) | .03644 | ( .57) | .00425 | ( .07) |
| | 25 | 6.64317 | 45 | 6.20984 | (93.84) | .36022 | ( 5.42) | .07311 | ( 1.10) |
| | 45 | 6.60176 | 65 | 3.81005 | (57.72) | 2.22554 | (33.71) | .56617 | ( 8.57) |
| | 65 | 5.27973 | 85 | .78260 | (14.82) | 3.65052 | (69.14) | .84657 | (16.03) |
| Females | 0 | 6.35198 | 25 | 6.12072 | (96.36) | .03253 | ( .53) | .19873 | ( 3.13) |
| | 25 | 6.64317 | 45 | 6.34113 | (97.32) | .14973 | ( 2.30) | .02498 | ( .38) |
| | 45 | 6.55432 | 65 | 5.33856 | (81.45) | .96180 | (14.67) | .25406 | ( 3.88) |
| | 65 | 6.27504 | 85 | 1.99470 | (31.78) | 3.31908 | (50.02) | 1.14124 | (18.19) |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| MVA | | | | | | | | | | |
| Males | 0 | .93265 | 25 | .45643 | (48.94) | .47188 | (50.60) | .00433 | ( .46) |
| | 25 | .48044 | 45 | .16550 | (34.44) | .31350 | (65.25) | .00144 | ( .30) |
| | 45 | .17595 | 65 | .04273 | (24.28) | .04305 | (24.47) | .90171 | (51.25) |
| | 65 | .05920 | 85 | .00295 | ( 4.99) | .00299 | ( 5.05) | .05326 | (89.96) |
| Females | 0 | .41063 | 25 | .19848 | (48.33) | .20511 | (49.95) | .007046 | ( 1.72) |
| | 25 | .21129 | 45 | .10142 | (48.00) | .10960 | (51.87) | .00027 | ( .13) |
| | 45 | .10483 | 65 | .03415 | (32.58) | .06969 | (66.48) | .00099 | ( .94) |
| | 65 | .04118 | 85 | .00229 | ( 5.57) | .00230 | ( 5.59) | .03658 | (88.84) |

ages downward.[1]

Finally, values associated with elimination of malignant neoplasms reflect the fact that this cause of death is most prevalent among middle age intervals. Specifically, large values appear at opposite ends of columns for gains due solely to improvements at age y and over and due solely to improvements under age y. Large proportional values appear for younger age intervals in the column indicating increases due to improvements at age y and over while large proportional values appear for the older age intervals in the column indicating increases due to improvements at ages under y.

Crosson has provided a valuable method for analyzing increases in life expectancy into their component parts. The value of the method lies in its ability to detect the source of gains in life expectancy and could readily be included in studies of effects of improvements in mortality at given ages. It could become more valuable in studies of historical improvements in life expectancy in which the method, if carried out in greater detail, could detect the source and degree of changes in life expectancy at given ages due to improvements in mortality conditions at other ages.

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[1]This result confirms the need for caution when examining gains in life expectancy noted in Chapter 5.

## Regression Analysis

It may be appealing to use multiple regression analysis to analyze gains in life expectancy. Multiple regression analysis has come into extensive use in the social sciences in recent years. Multiple regression analysis is a method of analyzing collective and separate contributions of two or more independent variables to variation in a dependent variable.

Various functions in the life table provide information about survivorship of the radix assumed in the construction of the life table. For example, the $l_x$ function of the life table indicates the number of survivors to the beginning of each age interval. If there is an improvement in mortality conditions in an age interval, there will be more survivors to all subsequent intervals (assuming that mortality conditions in subsequent intervals at least remain constant or, in any event, do not worsen). Even a minor improvement in mortality conditons in a given age interval will have an effect on older age intervals. Thus, differences in age-specific values of $l_x$ in the two types of life tables considered in this study may be taken as indicators of improvements in mortality before a given age.

Values of $T_x$ indicate the number of person-years lived in a given age interval and all subsequent intervals by survivors to the beginning of the given age interval. If improvements in mortality occur in a given age interval, those improvements will be reflected in larger values of $T_x$ in that age interval and all prior age intervals. Thus, differences in age-specific values of $T_x$ in the two types of life tables may be taken as indicators of improvements in mortality at or beyond a given age.

These two indicators appear to possess validity in terms of the life table model. Therefore, it may be tempting to regress gains in life expectancy on changes in values of $l_x$ and $T_x$. The proposed regression equation is

$$g_x = a + b_1(l'_x - l_x) + b_2(T'_x - T_x) + e_x \qquad (7.4)$$

where primed functions indicate values taken from cause-eliminated life tables, a is the intercept, and $e_x$ is the error term. However, such an analysis is fraught with statistical and substantive problems.

One of the basic assumptions underlying the general linear model is that the independent variables are uncorrelated. When independent variables are highly correlated, the problem is referred to as multicollinearity. The extreme case of multicollinearity exists when independent variables are perfectly correlated. A less extreme but still serious case arises when independent variables are highly but not perfectly correlated. Table 7.2 presents the correlation matrix of the proposed dependent and independent variables. For both males and females and for all eliminated causes,

Table 7.2. Correlations between gains in life expectancy and changes in $l_x$ and $T_x$ due to elimination of causes of death, United States, 1969-1971.[a]

| | $X_l$ (MN) | $X_l$ (DH) | $X_l$ (MVA) | $X_T$ (MN) | $X_T$ (DH) | $X_T$ (MVA) |
|---|---|---|---|---|---|---|
| Males | | | | | | |
| $X_g$ (MN) | -.94852 | | | .98670 | | |
| $X_g$ (DH) | | -.99172 | | | .96048 | |
| $X_g$ (MVA) | | | -.83822 | | | .87094 |
| $X_l$ (MN) | | | | -.88762 | | |
| $X_l$ (DH) | | | | | -.94245 | |
| $X_l$ (MVA) | | | | | | -.52527 |
| Females | | | | | | |
| $X_g$ (MN) | -.99122 | | | .95835 | | |
| $X_g$ (DH) | | -.98442 | | | .96758 | |
| $X_g$ (MVA) | | | -.95073 | | | .88467 |
| $X_l$ (MN) | | | | -.93273 | | |
| $X_l$ (DH) | | | | | -.99062 | |
| $X_l$ (MVA) | | | | | | -.77098 |

[a] $X_l = (l'_x - l_x)$; $X_T = (T'_x - T_x)$; $X_g = (e'_x - e_x)$.

correlations between gains in life expectancy and the independent variables are quite high. Furthermore, the matrix reveals that correlations between independent variables are also high, indicating the presence of multicollinearity (Johnston, 1972:163).

One of the major consequences of multicollinearity is that the precision of estimation falls so that it becomes difficult, if not impossible, to disentangle the relative influences of independent variables. Consequently, estimates of the parameters of the model may have very large errors, these errors may be highly correlated, and the sampling variance of regression coefficients will be very large. This is especially critical in the present context since multiple regression is proposed as a method to address the question of the relative contributions of mortality improvements before and after a given age to gains in life expectancy.

Farrar and Glauber (1967:98) provide a rule of thumb for determining if the degree of multicollinearity is harmful. Accordingly, multicollinearity is not a problem unless it is high relative to the overall multiple correlation. Thus, if $r_{ij}$ is the zero-order correlation between two independent variables and $R_y$ is the multiple correlation between dependent and independent variables, multicollinearity is said to be harmful if $r_{ij} \geq R_y$.

Regression of gains in life expectancy on changes in the number of survivors and number of person-years lived yields high values of multiple correlation ranging from .979 to .999. While the criteria proposed by Farrar and Glauber's rule of thumb is satisfied, multicollinearity still causes problems in determining relative contributions of independent variables. For example, use of methods for determining relative contributions of independent variables such as commonality analysis (Mayeske et al., 1969; Mood, 1969, 1971) and Englehart's (1936) path analytic approach yield unreliable results when a high degree of multicollinearity exists because of the large proportion of variance common to independent variables due to their high intercorrelation.

Among sociologists, Blalock has devoted more attention to the problem of multicollinearity than anyone else. Blalock (1964:179) notes that when independent variables are highly correlated controls for other independent variables may be misleading. It will, thus, be difficult to assess relative contributions of various independent variables. Blalock (1963:237) further notes that he has failed in several attempts to arrive at satisfactory formulas for breaking variation in the dependent variable into distinct components attributed to each of the correlated independent variables. Tukey (1954:45) suggests that the problem is complex and perhaps not capable of yielding a satisfactory solution.

More important, however, than the statistical problems created by multicollinearity are the substantive problems. Substantive interpretation of regression coefficients and portions of variance attributed directly to each independent variable is difficult and dangerous when multicollinearity exists. In the presence instance, for example, when gains in life expectancy are regressed on changes in $l_x$ and $T_x$, changes in $l_x$ generally produce the greatest direct relative contribution to variation in gains in life expectancy although differences between relative contributions of independent variables are not large. The magnitudes of these unique contributions are insignificant, however, when compared to the proportion of common variance shared by independent variables due to their correlation.

These results point to two cautionary notes. First, it is dangerous to apply regression methods to values generated by well-defined mathematical models especially, as is the case with the life table model, when values generated by the model are highly interdependent. Life table values are derived from one basic value, ${}_nq_x$. Furthermore, these derived values are highly interdependent. For example, values of $T_x$ are derived in part from values of $l_x$. Consequently, these values should be highly correlated.

Second, social scientists must be careful to avoid what Kaplan (1964:28) calls the "law of the instrument"; that is, "Give a boy a hammer and he will find that everything needs pounding." Thus, more complex statistical techniques should be used judiciously and when they are capable of providing meaningful statistical and substantive results. Thus, multiple regression techniques may not be used satisfactorily in the present context.

## Summary

This chapter presented a further anaysis of gains in life expectancy. This analysis showed that:

1. Crosson's method of analyzing gains in life expectancy based on conditional probabilities of survival presents results which are easily understood by laymen and scientists. As applied in the present study, this method gave results consistent with those found in previous chapters concerning age patterns of mortality improvements due to elimination of causes of death. It was suggested that Crosson's method is an invaluable tool in analysis of hypothetical and historical changes in mortality affecting life expectancy.

2. Unlike Crosson's technique which yields interpretable results, any attempt to apply multiple regression techniques to the analysis of gains in life expectancy is likely to yield results which are unreliable. The major problem is that of multicollinearity resulting for the interrelationship of functions inherent in the life table model. High interde-

pendence between functions creates substantive as well as statistical problems in the use of multiple regression.

# CHAPTER 8. SUMMARY

This study was undertaken against the basic framework of the universal value attached to human life and the ideas of prolongevitism and meliorism. By virtue of the universal value attached to human life, further extensions of life resulting from improvements in mortality conditions remain a major human goal. Prolongevity refers to the "significant extension of the length of life by human action." Meliorism implies that human efforts can and should be applied to improving the world. These ideas are inherent in the structure of modern society with its emphasis on progress and improved well-being.

The basic problem addressed by this study is the population consequences resulting from improvements in mortality conditions. In this study, these improvements constituted complete elimination of a given group of causes of death. These hypothetical improvements in mortality conditions were analyzed through the method known as the life table. Life tables were constructed to compare the mortality, survival, and longevity experience of the current population with hypothetical experiences of the same population under improved mortality conditions resulting from elimination of selected causes of death. Such comparisons were the purpose of this study.

Chapter 2 presented a general description of life tables due to all causes of death. This discussion included the historical development of life table methods, types of life tables, and methods of constructing life tables due to all causes. Life tables were differentiated according to reference year and age detail involved. Two types of life tables were distinguished in terms of reference year. Current life tables are based on the mortality experience of a population over a short period of time in which mortality has remained relatively unchanged. Cohort life tables are based on the mortality experience of an actual cohort of births. Two types of life tables were distinguished according to the length of age intervals in which data are presented. When data are presented for single years of age from birth to the last applicable age, the life table is referred to as a complete life table. An abridged life table presents data for broader age intervals. Life tables constructed for the present study were abridged current life tables based on a 3-year average of deaths for the period 1969 to 1971 and 1970 midyear population. Life tables by sex due to all causes were presented in Chapter 2 and life tables by sex due to elimination of causes of death were presented in Chapter 3.

However, in terms of the present study, the most important distinction made in Chapter 2 was the distinction between two alternative interpretations of life tables. These distinct interpretations were the basis for differentiating appropriate methods of comparing life tables presented in Chapters 5 and 6. First, the life table may be viewed as depicting the mortality experience of a cohort of newborn infants from birth until the cohort has been depleted by death. Second, the life table may be viewed as a stationary population whose total number and distribution by age does not change. The meaning of life table functions under alternative interpretations was discussed.

Chapter 3 presented a general description of life tables due to elimination of causes of death. This discussion included the historical development of such life table methods as well as a description of methods used to construct cause-eliminated life tables in the present study. Life tables for United States males and females, 1969-1971, were constructed eliminating, in turn, malignant neoplasms, diseases of heart, and motor vehicle accidents. These three groups of causes were selected because of age patterns of mortality associated with each cause. Each cause is most prevalent in certain age groups. Motor vehicle accidents occur most frequently among younger persons, malignant neoplasms are primarily a disease of middle ages for females, and diseases of heart are most prevalent among older females and middle-aged to elderly males.

Construction of 1969-1971 life tables due to all causes and life tables with causes of death eliminated required three sets of data: 3-year average of deaths by age, sex, and cause, including deaths due to all causes; estimated July 1, 1970 population by age and sex; and separation factors by sex for the population under one year of age. The sources of, and adjustments to, these data were described in Chapter 4.

Comparisons based on the life table as a cohort were presented in Chapter 5. This chapter focused on probabilities of dying and surviving, joint probabilities, life expectancy, and life table deaths.

Three types of probabilities from competing risk theory were used to compare life tables due to all causes with life tables eliminating causes of death. Changes in these probabilities due to elimination of causes of death reflected age-cause patterns of mortality. An important finding not usually presented in discussions of competing risk theory was that elimination of a cause of death results in constant relative changes between crude and partial crude probabilities of dying from remaining causes.

Examination of conditional probabilities of survival from one age to specified subsequent ages reflected age-cause patterns of mortality. Joint probabilities of survival of a married pair were also presented and, again, results reflected, in general, age-cause patterns of mortality. It was shown, however, that for both conditional and joint probabilities slight departures from expected patterns were due to age-cause patterns of mortality of prior age intervals which may affect either, or both, numerator or denominator of the calculation formula for the conditional probability of survival.

Analysis of gains in life expectancy due to elimination of causes of death revealed that for both sexes, elimination of diseases of heart produced greatest gains at all ages. Elimination of malignant neoplasms and motor vehicle accidents produced moderate and slight gains in life expectancy, respectively.

Analysis of alternative measures of longevity reflected age-cause patterns of mortality. Alternative measures of longevity included probable lifetime, percent of original cohort surviving to specified ages under various mortality conditions, and age at which average remaining lifetime is 10 years.

Finally, Chapter 5 presented a descriptive statistical analysis of life table deaths. Several descriptive measures were utilized including median, standard deviation and measures of skewness and kurtosis and these measures revealed that elimination of causes of death resulted in distributions of life table deaths that were more positively skewed, more peaked, and more variable that those distributions associated with life tables due to all causes. This section also pointed out an important characteristic of distributions of life table deaths not described in previous literature that makes comparisons of life table death distributions more meaningful. Specifically, means of life table death distributions for similarly constructed life tables are equal.

Comparisons based on the life table as a stationary population were presented in Chapter 6. The emphasis in this chapter was on changes in age distributions of stationary populations due to eliminating causes of death. A distinction was made between individual measures of age distribution (measures which provide summary measures of each distribution) and comparative measures of age distribution (summary measures of comparison between two or more distributions).

Comparisons of individual measures of age distributions showed that elimination of causes of death resulted in an older population for each eliminated cause. Elimination of causes of death produced age distributions with greater median ages, indexes of aging, and proportions of old persons, and smaller proportions of young persons. An exception to these results in regards to proportions of young and old persons occurred when motor vehicle accidents were eliminated. Elimination of motor vehicle accidents produced larger proportions of active persons for both sexes. This result was accounted for by the prevalence of motor vehicle accidents as a cause of death among younger persons whose elimination postpones a larger proportion of deaths to later ages.

Three comparative measures of age distributions of stationary populations were proposed. Indexes of dissimilarity revealed that displacements of the original stationary populations of approximately 2 percent, 6 percent, and less than 1 percent would be required to make those distibutions identical to stationary age distributions resulting from elimination of malignant neoplasms, diseases of heart, and motor vehicle accidents, respectively.

Age-specific indexes revealed that elimination of diseases of heart produced the most marked shifts in stationary age distributions toward older ages. Furthermore, examination of age-specific indexes with elimination of motor vehicle accidents confirmed results from the analysis of proportions of young and old persons that elimination of this cause produces slightly larger proportions of active population.

The equality of stationary age distributions under various mortality conditions was tested using Kolmogorov-Smirnov goodness-of-fit procedures. These tests revealed that elimination of motor vehicle accidents did not produce stationary age distributions significantly different from stationary age distributions due to all causes for either males or females.

Chapter 6 also included a discussion of the uses of methods of comparing life tables with particular attention to examining the intended and unintended consequences of health and population policies directed at reducing mortality, either general or specific.

Chapter 7 presented a further analysis of gains in life expectancy due to elimination of causes of death. Two methods of analysis were examined: Crosson's method based on conditional probabilities of survival and multiple regression analysis, an appealing though inappropriate method. It was shown that Crosson's method allows analysis of gains in life expectancy into three components: gains due solely to improvements in mortality conditions at ages y and over; gains due solely to improvements in mortality conditions at ages under y; and gains due to increased probability of survival to age y to participate in increased expectancy. This method gave results consistent with those found in previous chapters concerning age patterns of mortality improvements due to elimination of causes of death.

The application of multiple regression techniques to the analysis of gains in life expectancy is plagued with statistical and substantive problems. Statistically, the problem is multicollinearity or high intercorrealtion among independent variables. Substantively, multicollinearity produces unreliable results which make interpretation difficult. Regression of gains in life expectancy on changes in the number of survivors and person-years lived results in large values of $R^2$. However, multicollinearity produces regression coefficients which are unreliable and unique contributions to variance which cannot be disentangled in a meaningful manner.

## Discussion

The present study addressed the question of the consequences of mortality improvements in a population using the life table model as a method of analysis. One question raised in the introduction and which cannot be answered by this study is "Can and should mortality conditions be improved?"

It was noted that prolongevitists advocate not only the search for a long life but also the search for a vital life as well. Most people can foresee the possibility of at least some degree of mortality improvement and further extension of life. The problem becomes, then, whether improvements in mortality will lead to a more vital life or, more specifically, the question of quantity of life versus quality of life. Hauser (1976:82) notes that in the past the extension of life has been accompanied by greater incidence of chronic illness and physical impairments which were largely precluded when death occurred at earlier ages. Benjamin (1964:221) argues that it is not easy to interpret increased longevity in terms of prolonged activity. Postponement of death in an elderly person by improved medical care does not imply arrest of degenerative process and though some retardations may occur, it may be sufficient only to maintain life without permitting normal activity.

Hakulinen and Teppo (1976:433) suggest that deaths from cancer, for example, are often preventable through treatment but the quality of the additional person-years achieved may vary. They further suggest that although an increase in life expectancy resulting from elimination of a cause of death is of primary importance to the individual, society may also be interested in the ages at which person-years are saved. Some authors (Greville, 1948a:419; Sutton, 1971:369-370) have argued that greater importance may be attached to the loss of a year of life falling in the active ages that to a year lost during the latter part of the life span. Comfort (1970:158, 160) favors switching a sizable part of medical research to controlling the rate of degeneration rather than to disease control. He argues that patching-up of single age-dependent diseases is both expensive and of limited use, judged by the length of further vigorous life. He advocates rate-control because longer vigorous life can be achieved in no other way, it appears to be feasible on the basis of present evidence, and it ought to be easier to affect the rate of degeneration due to a disease than to prevent or cure the disease when established.

The importance of the present study is threefold. First, this study presents a general discussion of the origin, uses, and construction of life tables due to all causes of death and life tables due to elimination of causes of death which attempts to avoid much of the confusion stemming from mathematical derivations of equations used in life table procedures. Furthermore, it presents practical procedures for making necessary adjustments to data used in the construction of life tables. This study is a study in formal demography. However, certain aspects of social demography, the social consequences of an aging population, were included. This study, then, represents a work which could readily be used as a reference or text for preparing life tables.

Second, this study is unique in its delineation of methods of comparison appropriate to the two general interpretations of life tables. Although these interpretations are generally recognized, they are seldom mentioned when life tables are compared. Specification of methods appropriate to alternative interpretations of life tables makes these comparisons more meaningful historically, hypothetically, and cross-sectionally. The further delineation of methods appropriate to the stationary population interpretation of life tables into individual and comparative measures of age distributions is also an unique contribution of this study.

Most measures applied to the analysis of life tables in the present study yielded results consistent with age-cause patterns of mortality. This, of course, raises the question of which measures are "best." No attempt was made in the present study to discern which of the proposed measures for comparing life tables are "best." The selection of the most appropriate measure is left to the user and is dictated by the research problem.

If there is a shortcoming of the specification of alternative interpretations of life tables, it is that there exists at present no model capable of integrating methods appropriate to the cohort interpretation of life tables. The stationary population model provides a method of integrating methods appropriate to that interpretation of life tables. No such model exists for the cohort interpretation due, in large part, to interest in several rather than a single function of the life table.

Third, the most important contribution of this study is that, to the author's knowledge, it is the first of its kind. A large amount of sociological, demographic, actuarial, and statistical literature was reviewed in the preparation of this study and no similar studies were found. Admittedly, this study takes an elementary approach to the problem of the effects of mortality improvements, but as a first study it deserves attention. Most previous studies of life tables simply report life table values. This study is unique in that it compares life tables in terms of a number of methods and these methods have been defined as appropriate to one or the other interpretation of life tables. The methods used in this study, with the exception of tests of goodness-of-fit, are descriptive, but demography as a science has been characterized by descriptive studies. However, application of more sophisticated procedures calls for borrowing techniques from other areas. Methods used in medical follow-up studies (Elveback, 1958; Mantel and Haenszel, 1959; Mantel, 1963, 1966; Gehan, 1965) and reliability theory (cf. Gross and Clark, 1975) may prove fruitful in the study of life table survivorship.

Furthermore, studies similar to the present study may gain increased significance in light of renewed interest in population growth and stationarity. If maintenance of a stationary population becomes an official goal, studies similar to the present study may prove indispensable. The stationary population model is an oversimplified model. A stationary population occurs when the birth rate and death rate are identical, resulting in zero population growth. If there is a drop in mortality, fertility must also decline commensurately in order to maintain stationarity. Since these vital processes are unlikely to change simultaneously, incorporating stable population theory into studies taking the present approach may represent a viable area for future research. Furthermore, complete elimination of a cause of death is a purely hypothetical situation. However, partial elimination of a cause of death could be easily incorporated into life table models, indicating another area of further research.

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# ACKNOWLEDGEMENTS

Special thanks to Dr. H. C. Chang and Dr. Richard Warren, co-major professors, and Dr. Willis Goudy, Dr. Gerald Klonglan, and Dr. Earl Morris, committee members, for their assistance and guidance throughout my graduate program. Also, thanks to Dr. Mary Winter for helping me with my problems with Thesis 3.5.

Finally, love to my wife for her patience and assistance.

# APPENDIX A.

## WATFIV PROGRAM FOR CONSTRUCTING MAIN AND SPECIAL LIFE TABLES

```
C      THIS PROGRAM WAS REVISED JANUARY 1977 BY RONALD E. JENSEN
C      MAIN PROGRAM FOR LIFE TABLE
C      KAGE=AGE
C      NOD=NUMBER OF DEATH
C      NOP=NUMBER OF MID-YEAR POPULATION
C      MAGE=AGE
C      MNOD=NUMBER OF DEATH
C      CE=COMPLETE EXPECTATION OF LIFE AT AGE X
C      MNOP=NUMBER OF MID-YEAR POPULATION
C      DR=DEATH RATE ANNUAL 1970
C      NS=NUMBER SURVIVING TO EXACT AGE X OUT OF 100000 BORN ALIVE
C      ND=NUMBER DYING AT AGE X TO X+N-1
C      NYL=YEARS OF LIFE LIVED AT AGE X TO X+N-1
C      NYLO=YEARS OF LIFE LIVED AT AGE X AND OVER
C      QX=THE PROBABILITY OF DEATH
C      NN=LENGTH OF AGE INTERVAL
C      NN-F=AVERAGE NUMBER OF YEARS LIVED BY THOSE WHO DIE WITHIN AN
C      AGE INTERVAL
C      NDNEO=NUMBER OF DEATHS DUE TO MALIGNANT NEOPLASMS
C      NDCDV=NUMBER OF DEATHS DUE TO MAJOR CARDIOVASCULAR DISEASES
C      NDDSH=NUMBER OF DEATHS DUE TO DISEASES OF HEART
C      NDCBV=NUMBER OF DEATHS DUE TO CEREBROVASCULAR DISEASES
C      NDACC=NUMBER OF DEATHS DUE TO ACCIDENTS
C      NDMVA=NUMBER OF DEATHS DUE TO MOTOR VEHICLE ACCIDENTS
C      QXMIN=THE PROBABILITY OF DEATH WITH A GIVEN CAUSE ELIMINATED
C      NODI=NUMBER OF DEATHS DUE TO A GIVEN CAUSE
C      NSI=NUMBER SURVIVING TO EXACT AGE X OUT OF 100000 BORN ALIVE ELIMINATING
C      A GIVEN CAUSE
C      NDI=NUMBER OF LIFE TABLE DEATHS WITH A GIVEN CAUSE ELIMINATED
C      NYLMIN=SAME AS NYL FOR A GIVEN CAUSE ELIMINATED
C      NYLSUM=SAME AS NYLO, FOR A GIVEN CAUSE ELIMINATED
C      CEI=SAME AS CE,FOR A GIVEN CAUSE ELIMINATED
C      GAIN=GAIN IN EXPECTATION OF LIFE WITH A GIVEN CAUSE ELIMINATED
       DIMENSION KAGE(20),NOD(20),NOP(20),MAGE(20),MNOD(20)
       DIMENSION MNOP(20),DR(20),NS(20),ND(20),NYL(20),NYLO(20)
```

```
      DIMENSION QX(20),CE(20),KK(20),LL(20)
      DIMENSION NDNEO(20),NDCDV(20),NDDSH(20),NDCBV(20),NDACC(20)
      DIMENSION NDMVA(20),D(20),NN(20),F(20)
C     READ AGE, TOTAL NUMBER OF DEATHS, MID-YEAR POPULATION, AND
C       DEATHS DUE TO PARTICULAR CAUSES
      DO 9 I=1,19
    9 READ(5,1) KAGE(I),NOD(I),NOP(I),NDNEO(I),NDCDV(I),NDDSH(I),
     1NDCBV(I),NDACC(I),NDMVA(I)
    1 FORMAT(9X,I2,I6,I8,6(I6))
C     ORDER THE AGE SPECIFIC GROUP FROM UNDER 1 YEAR TO 85-AND-OVER
      DO 10 II=1,19
      KSMAL=100
      DO 11 I=1,19
      IF(KAGE(I).GE.KSMAL) GO TO 11
      KSMAL=KAGE(I)
      MAGE(II)=KAGE(I)
      MNOD(II)=NOD(I)
      MNOP(II)=NOP(I)
      LOCK=I
   11 CONTINUE
   10 KAGE(LOCK)=100
      DO 12 I=1,19
C     CALCULATE AGE SPECIFIC DEATH RATE
      DR(I)=FLOAT(MNOD(I))/FLOAT(MNOP(I))
      IF(I-2) 4,5,6
    4 AN=1.
      GO TO 12
    5 AN=4.
      GO TO 12
    6 AN=5.
C     CALCULATE THE PROBABILITY OF DEATH
   12 QX(I)=((AN*DR(I))/(1. +.5*AN*DR(I)))
      QX(19)=1.
C     CALCULATE NUMBER OF SURVIVING AND NUMBER DYING
      NS(1)=100000
```

```
      D(1)=FLOAT(NS(1))*QX(1)
      ND(1)=D(1)
      DO 13 I=1,18
      NS(I+1)=NS(I)-ND(I)
      D(I+1)=FLOAT(NS(I+1))*QX(I+1)
   13 ND(I+1)=D(I+1)
      KSUM=0
      DO 14 I=1,18
C     CALCULATE YEARS OF LIFE LIVED AT AGE X TO X+N-1
      NYL(1)=.0987467*FLOAT(NS(1))+.90125329*FLOAT(NS(2))
      NYL(I+1)=D(I+1)/DR(I+1)
   14 KSUM=KSUM+NYL(I+1)
      NYLO(1)=KSUM+NYL(1)
      DO 15 I=1,18
C     CALCULATE YEARS OF LIFE LIVED AT AGE X AND OVER
   15 NYLO(I+1)=NYLO(I)-NYL(I)
      DO 16 I=1,19
C     CALCULATE COMPLETE EXPECTATION OF LIFE AT AGE X
   16 CE(I)=FLOAT(NYLO(I))/FLOAT(NS(I))
C     PRINT THE TITLE OF TABLE
      WRITE(6,100)
  100 FORMAT(///////1H1,15X,'TABLE 1. ABRIDGED LIFE TABLE FOR UNITED STA
     1TES MALES, 1969-71')
      WRITE(6,101)
  101 FORMAT(/32X,'PROBABI-    NUMBER')
      WRITE(6,102)
  102 FORMAT(32X,'LITY OF',5X,'SURVIVING')
      WRITE(6,103)
  103 FORMAT(20X,'DEATH',7X,'A PERSON',4X,'TO EXACT  NUMBER',2X,
     1'YEARS OF LIFE LIVED',19X,'ESTIMATED',3X,'NUMBER')
      WRITE(6,104)
  104 FORMAT(20X,'RATE',8X,'AGE X DY-',3X,'AGE X OUT DYING',2X,21('-'),
     14X,'COMPLETE',6X,'JULY 1',8X,'OF')
      WRITE(6,105)
  105 FORMAT(20X,'1969-71',5X,'ING',9X,'OF 100000 AT AGE',3X,'AGE X',
```

```
     118X,'EXPECTATION',3X,'POPULATION',2X,'ANNUAL')
      WRITE(6,106)
 106  FORMAT(32X,'BEFORE',6X,'BORN',6X,'X TO',5X,'TO',9X,'AT AGE X'
     1,4X,'OF LIFE',19X,'DEATHS')
      WRITE(6,107)
 107  FORMAT(32X,'AGE X+N',5X,'ALIVE',5X,'X+N-1',4X,'X+N-1',6X,'AND OVER
     1',4X,'AT AGE X',18X,'1969-71')
      WRITE(6,108)
 108  FORMAT(21X,'NMX',11X,'NQX',8X,'X',8X,'NDX',6X,'NLX',10X,'TX',10X,
     1'EX',13X,'N',9X,'ND')
      WRITE(6,109)
 109  FORMAT(12X,'AGE',6X,'(1)',11X,'(2)',7X,'(3)',7X,'(4)',6X,'(5)',
     110X,'(6)',9X,'(7)',11X,'(8)',8X,'(9)')
      DO 20 I=1,19
      IF(I-2) 33,44,55
  33  KK(1)=0
      LL(1)=1
      GO TO 20
  44  KK(2)=1
      LL(2)=4
      GO TO 20
  55  KK(2)=0
      LL(I)=LL(I-1)+5
      KK(I)=KK(I-1)+5
  20  CONTINUE
      KK(2)=1
      NN(1)=1
      NN(2)=4
      DO 18 I=3,18
  18  NN(I)=(LL(I)-KK(I))+1
      NN(19)=5
      DO 19 I=1,19
  19  F(I)=((NN(I)*FLOAT(NS(I)))-NYL(I))/FLOAT(ND(I))
      DO 21 I=1,19
  21  WRITE (6,200) KK(I),LL(I),DR(I),QX(I),NS(I),ND(I),NYL(I),NYLO(I),
```

```
     1CE(I),MNOP(I),MNOD(I)
 200 FORMAT(/11X,I2,'-',I2,2X,F11.9,3X,F9.7,3X,I6,3X,I5,5X,I6,6X,I7,5X,
     1F8.5,4X,I8,5X,I6)
      WRITE(6,400)
 400 FORMAT('1'///////1H1,11X,'TABLE 2. ABRIDGED LIFE TABLE ELIMINATING
     1 MALIGNANT NEOPLASMS AS A CAUSE OF DEATH,',
     2/' ',20X,'UNITED STATES MALES, 1969-71')
C     SUBROUTINE SUBTAB CALCULATES THE OUTPUT FOR TABLES 2 THROUGH 7
      CALL SUBTAB(NDNEO,MNOD,QX,NN,F,KK,LL,CE)
      WRITE(6,401)
 401 FORMAT('1'///////1H1,11X,'TABLE 3. ABRIDGED LIFE TABLE ELIMINATING
     1 MAJOR CARDIOVASCULAR-RENAL DISEASES AS A CAUSE OF DEATH,',
     2/' ',20X,'UNITED STATES MALES, 1969-71')
      CALL SUBTAB(NDCDV,MNOD,QX,NN,F,KK,LL,CE)
      WRITE(6,402)
 402 FORMAT('1'///////1H1,11X,'TABLE 4. ABRIDGED LIFE TABLE ELIMINATING
     1 DISEASES OF HEART AS A CAUSE OF DEATH,',
     2/' ',20X,'UNITED STATES MALES, 1969-71')
      CALL SUBTAB(NDDSH,MNOD,QX,NN,F,KK,LL,CE)
      WRITE(6,403)
 403 FORMAT('1'///////1H1,11X,'TABLE 5. ABRIDGED LIFE TABLE ELIMINATING
     1 CEREBROVASCULAR DISEASE AS A CAUSE OF DEATH,',
     2/' ',20X,'UNITED STATES MALES, 1969-71')
      CALL SUBTAB(NDCBV,MNOD,QX,NN,F,KK,LL,CE)
      WRITE(6,404)
 404 FORMAT('1'///////1H1,11X,'TABLE 6. ABRIDGED LIFE TABLE ELIMINATING
     1 ACCIDENTS AS A CAUSE OF DEATH,',
     2/' ',20X,'UNITED STATES MALES, 1969-71')
      CALL SUBTAB(NDACC,MNOD,QX,NN,F,KK,LL,CE)
      WRITE(6,405)
 405 FORMAT('1'///////1H1,11X,'TABLE 7. ABRIDGED LIFE TABLE ELIMINATING
     1 MOTOR VEHICLE ACCIDENTS AS A CAUSE OF  DEATH,',
     2/' ',20X,'UNITED STATES MALES, 1969-71')
      CALL SUBTAB(NDMVA,MNOD,QX,NN,F,KK,LL,CE)
      WRITE(6,406)
```

```
406 FORMAT('1'///1H1,15X,'END OF TABLES')
    STOP
    END
    SUBROUTINE SUBTAB(NODI,MNDI,QS,NI,FI,KI,LI,CI)
    DIMENSION PDI(20),QXMIN(20),NYLMIN(20),NDI(20),NYLSUM(20),CEI(20)
    DIMENSION NSI(20),GAIN(20),DN(20),NODI(20)
    DIMENSION MNDI(20),QS(20),NI(20),FI(20),KI(20),LI(20),CI(20)
    DO 25 I=1,19
 25 PDI(I)=FLOAT(NODI(I))/FLOAT(MNDI(I))
    DO 21 I=1,18
 21 QXMIN(I)=1-((1-QS(I))**(1-PDI(I)))
    QXMIN(19)=1.
    NSI(1)=100000
    DN(1)=FLOAT(NSI(1))*QXMIN(1)
    NDI(1)=DN(1)
    DO 13 I=1,18
    NSI(I+1)=NSI(I)-NDI(I)
    DN(I+1)=FLOAT(NSI(I+1))*QXMIN(I+1)
 13 NDI(I+1)=DN(I+1)
    DO 31 I=1,18
 31 NYLMIN(I)=((FLOAT(NI(I))-FI(I))*FLOAT(NSI(I)))+FI(I)*FLOAT(NSI(I+1
   1))
    NYLMIN(19)=(CI(19)*FLOAT(NSI(I)))/(1-PDI(19))
    NSUM=0
    DO 22 I=1,19
 22 NSUM=NSUM+NYLMIN(I)
    NYLSUM(1)=NSUM
    DO 23 I=1,18
 23 NYLSUM(I+1)=NYLSUM(I)-NYLMIN(I)
    DO 24 I=1,19
    CEI(I)=FLOAT(NYLSUM(I))/FLOAT(NSI(I))
 24 GAIN(I)=CEI(I)-CI(I)
    WRITE(6,301)
301 FORMAT(/22X,'PROBABI-     NUMBER')
    WRITE(6,302)
```

```
302 FORMAT(22X,'LITY OF',5X,'SURVIVING')
     WRITE(6,303)
303 FORMAT(22X,'A PERSON',4X,'TO EXACT  NUMBER',3X,
    1'YEARS OF LIFE LIVED')
     WRITE(6,304)
304 FORMAT(22X,'AGE X DY-',3X,'AGE X OUT DYING',3X,21('-'),3X,
    1'COMPLETE',6X,'NUMBER OF')
     WRITE(6,305)
305 FORMAT(22X,'ING',9X,'OF 100000 AT AGE',3X,'AGE X',18X,
    1'EXPECTATION',3X,'ANNUAL')
     WRITE(6,306)
306 FORMAT(22X,'BEFORE',6X,'BORN',6X,'X TO',5X,'TO',9X,'AT AGE X',
    14X,'OF LIFE',7X,'DEATHS')
     WRITE(6,307)
307 FORMAT(22X,'AGE X+N',5X,'ALIVE',5X,'X+N-1',4X,'X+N-1',6X,'AND OVER
    1',4X,'AT AGE X',6X,'1969-1971')
     WRITE(6,308)
308 FORMAT(25X,'NQX',8X,'X',8X,'NDX',6X,'NLX',9X,'TX',11X,'EX',10X,
    1'ND',9X,'GAIN')
     WRITE(6,309)
309 FORMAT(13X,'AGE',9X,'(1)',7X,'(2)',7X,'(3)',6X,'(4)',9X,'(5)',
    110X,'(6)',9X,'(7)',9X,'(8)')
     DO 321 I=1,19
320 FORMAT(/12X,I2,'-',I2,5X,F9.7,3X,I6,3X,I5,4X,I7,5X,I8,4X,F9.5,
    14X,I6,5X,F8.5)
321 WRITE(6,320) KI(I),LI(I),QXMIN(I),NSI(I),NDI(I),NYLMIN(I),
    1NYLSUM(I),CEI(I),NODI(I),GAIN(I)
     RETURN
     END
$ENTRY
```

# APPENDIX B.

## WATFIV PROGRAM FOR ADJUSTING APRIL 1 POPULATION TO JULY 1

```
C     THIS PROGRAM WAS DEVELOPED BY RONALD E. JENSEN, MARCH 1976
C     THIS PROGRAM ADJUSTS APRIL 1 POPULATION TO JULY 1
C     THIS PROGRAM IS A MODIFIED VERSION OF THE PROCEDURE FOR ADJUSTING
C     APRIL 1 CENSUS ENUMERATIONS TO JULY 1 DESCRIBED BY JAMES D. TARVER
C     AND THEREL R. BLACK, "MAKING COUNTY POPULATION PROJECTIONS-A DETAILED
C     EXPLANATION OF A THREE-COMPONENT METHOD, ILLUSTRATED BY REFERENCE TO
C     UTAH COUNTIES". UTAH AGRICULTURAL EXPERIMENT STATION, UTAH STATE
C     UNIVERSITY, LOGAN, UTAH. JUNE 1966:17-27.
C     KAGE=AGE
C     KCEN=POPULATION BY AGE GROUPS, APRIL 1,1970
C     MCEN=POPULATION BY AGE GROUPS, APRIL 1,1960
C     NAT=BIRTHS, APRIL THRU JUNE,1970
C     MORT=DEATHS, APRIL THRU JUNE,1970
C     NUMB=STATE AND COUNTY NUMBER
C     NEST=CENSUS ESTIMATE, TOTAL POPULATION, JULY 1,1970
C     JUST=APRIL 1,1970 POPULATION ADJUSTED TO JULY 1
C     MUST=APRIL 1,1960 POPULATION ADJUSTED TO JULY 1
C     JANG=10-YEAR CHANGE, JULY 1,1960 TO JULY 1,1970
C     LANG=CHANGE, APRIL 1,1960 TO JULY 1,1970 (10.25 YEARS)
C     JULE=ESTIMATED JULY 1,1970 POPULATION
C     JULA=ESTIMATED JULY 1,1970 POPULATION ADJUSTED TO CENSUS ESTIMATE
C     KSUM=SUM OF ESTIMATED JULY 1,1970 POPULATION ACROSS AGE GROUPS
C     MAT(2)=BIRTHS, APRIL THRU JUNE, 1960
C     MAT(3)=INFANT(0-1) DEATHS, APRIL THRU JUNE, 1960
C     LUST=APRIL 1,1960 POPULATION CLASSIFIED BY AGE ON JULY 1,1970
C     KK=LOWER BOUND OF AGE INTERVAL
C     LL=UPPER BOUND OF AGE INTERVAL
C     NCEN=POPULATION BY AGE GROUP,APRIL 1,1960 CLASSIFIED BY AGE ON JULY 1,1970
      DIMENSION NUMB(19),KAGE(19),KCEN(19),MCEN(19),NAT(19),MORT(19)
      DIMENSION NEST(19),JUST(19),MUST(19),JANG(19),LANG(19),JULE(19)
      DIMENSION JULA(19),MAGE(19),KK(19),LL(19),MNUMB(19),KKCEN(19)
      DIMENSION MMCEN(19),MNAT(19),MMORT(19),MNEST(19),MAT(19),MMAT(19)
      DIMENSION LUST(19),LLUST(19),NCEN(19)
      INTEGER KK,LL,NUMB,KCEN,MCEN,NAT,MAT,MORT,NEST
      REAL JUST,MUST,LUST,JANG,LANG,JULE,JULA,KSUM,NCEN
```

```
 8 DO 9 I=1,19
 9 READ(5,1,END=300) NUMB(I),KAGE(I),KCEN(I),MCEN(I),NAT(I),MORT(I),
  1NEST(I),MAT(I)
 1 FORMAT(7X,I5,I2,2(I6),2(I5),I8,I5)
   DO 10 II=1,19
   KSMAL=100
   DO 11 I=1,19
   IF(KAGE(I).GE.KSMAL) GO TO 11
   KSMAL=KAGE(I)
   MAGE(II)=KAGE(I)
   MNUMB(II)=NUMB(I)
   KKCEN(II)=KCEN(I)
   MMCEN(II)=MCEN(I)
   MNAT(II)=NAT(I)
   MMORT(II)=MORT(I)
   MNEST(II)=NEST(I)
   MMAT(II)=MAT(I)
   LOCK=I
11 CONTINUE
10 KAGE(LOCK)=100
   JUST(1)=((.75*FLOAT(KCEN(1)))+NAT(1)-MORT(1))
   JUST(2)=((.25*FLOAT(KCEN(1)))+(.9375*FLOAT(KCEN(2)))-MORT(2))
   JUST(3)=((.0625*FLOAT(KCEN(2)))+(.95*FLOAT(KCEN(3)))-MORT(3))
   DO 12 I=4,19
12 JUST(I)=((.05*FLOAT(KCEN(I-1)))+(.95*FLOAT(KCEN(I))))
   JUST(19)=((.05*FLOAT(KCEN(18)))+KCEN(19))
   MUST(1)=0
   MUST(2)=((.95*FLOAT(MCEN(2)))+MAT(2)-MAT(3))
   DO 13 I=3,19
13 MUST(I)=((.05*FLOAT(MCEN(I-1)))+(.95*FLOAT(MCEN(I))))
   MUST(19)=((.05*FLOAT(MCEN(18)))+MCEN(19))
   LUST(1)=0
   LUST(2)=0
   LUST(3)=0
   DO 22 I=4,19
```

```
 22  LUST(I)=MUST(I-2)
     LUST(19)=MUST(17)+MUST(18)+MUST(19)
     JANG(1)=0
     JANG(2)=0
     JANG(3)=0
     DO 14 I=4,19
 14  JANG(I)=JUST(I)-LUST(I)
     DO 15 I=1,19
 15  LANG(I)=(JANG(I)*1.025)
     NCEN(1)=0
     NCEN(2)=0
     NCEN(3)=0
     DO 27 I=4,19
 27  NCEN(I)=MCEN(I-2)
     NCEN(19)=MCEN(17)+MCEN(18)+MCEN(19)
     JULE(1)=JUST(1)
     JULE(2)=JUST(2)
     JULE(3)=JUST(3)
     DO 16 I=4,19
 16  JULE(I)=NCEN(I)+LANG(I)
     KSUM=0
     DO 17 I=1,19
 17  KSUM=KSUM+JULE(I)
     DO 18 I=1,19
 18  JULA(I)=(JULE(I)*(FLOAT(NEST(1))/KSUM))
     WRITE(6,100)
100  FORMAT(/2X,'AGE',4X,'NUMB',4X,'KCEN',4X,'MCEN',5X,'JUST',6X,'MUST'
    1,6X,'LUST',6X,'JANG',6X,'LANG',6X,'JULE',6X,'JULA',5X,'NAT',4X,'MA
    2T',3X,'MORT',4X,'SUM')
     DO 20 I=1,19
     IF(I-2)33,44,55
 33  KK(1)=0
     LL(1)=1
     GO TO 20
 44  KK(2)=1
```

```
      LL(2)=4
      GO TO 20
   55 KK(2)=0
      LL(I)=LL(I-1)+5
      KK(I)=KK(I-1)+5
   20 CONTINUE
      KK(2)=1
      DO 21 I=1,19
   21 WRITE(6,200) KK(I),LL(I),NUMB(I),KCEN(I),MCEN(I),JUST(I),MUST(I),
     1LUST(I),JANG(I),LANG(I),JULE(I),JULA(I),NAT(I),MAT(I),MORT(I),
     2KSUM
  200 FORMAT(/1X,I2,'-',I2,2X,I5,2X,I6,2X,I6,2X,F8.0,2X,F8.0,2X,F8.0,2X,
     1F8.0,2X,F8.0,2X,F8.0,2X,F8.0,2X,I5,2X,I5,2X,I5,2X,F8.0)
      GO TO 8
  300 STOP
      END
$ENTRY
```