A study of closed-loop supply chain models with governmental incentives and fees

by

Karla B. Valenzuela

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Industrial Engineering

Program of Study Committee:

Jo Min, Major Professor
Juan Gaytán
John K. Jackman
Mervyn Marasinghe
Sigurdur Olafsson
Marco Serrato
Lizhi Wang

Iowa State University

Ames, Iowa

2009

To Valeria, who gives meaning to it all

Table of Contents

List of Tables v
List of Figures vi
Acknowledgement vii
Abstract viii
1. Introduction 1
1.1 Research Objectives and Overview 1
1.2 Environmental Laws 4
2. Literature Review 8
3. Closed-loop Supply Chain with One Manufacturer 11
3.1 Description of the Supply Chain and Assumptions 11
3.2 Basic Model 14
3.3 Extended Model with Governmental Incentives and Fees 16
3.3.1 Special Cases of Extended Model 23
3.3.2 Revenue Neutrality Constraint 25
3.3.3 Optimal Incentive and Fee for the Extended Model 27
3.4 Centrally Coordinated Model 30
3.5 Alternative Financial Instruments 31
3.5.1 Allocation Mechanism 32
3.6 Entry and Exit Implications of Governmental Participation 33
3.6.1 The Case of the Unprofitable Manufacturer only 34
3.6.2 The Case of the Unprofitable Collector only 37
3.6.3 The Case of the Unprofitable Collector and Unprofitable Manufacturer 38
3.6.4 Impact of Centrally Coordinated Model on Unprofitability 40
3.7 Revenue-Sharing Contract Model 41
3.8 Comparative Examinations 44
3.9 Distributive Issues 45
3.10 Discussion of Results 49
4. Closed-loop Supply Chain with Multiple Manufacturers 50
4.1 Description of the Supply Chain and Assumptions 50
4.2 Basic Model 52

4.2.1 The Case of Homogeneous Products with Symmetric Cost 54
4.3 Extended Model with Governmental Incentives and Fees 56
4.3.1Homogeneous Products with Symmetric Costs 61
4.3.2 Multiple Manufacturers 63
4.4 Centrally Coordinated Model: Extended Model 66
4.4.1 Centrally Coordinated Model: Homogeneous products with symmetric cost 67
4.4.3 Approximation for Extended Model 69
4.5 Discussion of the Results 73
5. Extension to Some Current Policies Mexico 74
6. Concluding Remarks and Future Research 76
Appendix A. Concavity and Optimality of Profit Functions in Basic and Extended Model 81
Appendix B. Expression for α^*, its Uniqueness and Bounds 82
Appendix C. Relationships among the Total Surpluses and the Collection Rates of Basic, Extended, Alternative Financial Instruments, and Revenue-Sharing Contract Models 84
Appendix D. Relationships among Collector Profits, Manufacturer Profits, and Consumer Surpluses for Basic, Extended, Alternative Financial Instrument, and Revenue-Sharing Contract Models 85
Appendix E. Optimality of the Total Surplus Function in Centrally Coordinated Model and Derivation of Financial Instruments 87
Appendix F. Concavity and Optimality of the Forward only Supply Chain and the Revenue-Sharing Contract Model 88
Appendix G. Concavity and Optimality of Profit Functions in Basic and Extended Model with Competition 89
Appendix H. Expression for α^* its Uniqueness and Bounds for Homogeneous Products 93
Appendix I. Relationships among the Total Surpluses as well as the Collection Rates of Basic Model, Extended Model, and Centrally Coordinated Model for the case of Homogeneous Products 95
Appendix J. Derivation of Boundaries for α_M^* 97
Appendix K. Optimality of Total Surplus in Centrally Coordinated Model with Competition and Derivation of Instruments 98
References 100

List of Tables

Table 1. Comparison of Environmental Legislation in US…………………………………………......5

Table 2. Extended Model with Governmental Incentives and Fees………………………………..23

Table 3. Special Cases Equilibrium Solutions of Extended Model………………………………...24

Table 4. Extended Model Results with Revenue Neutrality……………………………………….25

Table 5. Special Cases Equilibrium Solutions of Extended Model with Revenue Neutrality………..26

Table 6. Results for Basic and Extended Models…………………………………………………..29

Table 7. Profits and Decision Variables for the Allocation Mechanism…………………………..…..32

Table 8. Unprofitability with TS^*_{CC} …………………………………………………………………..40

Table 9. Ranges for Fixed Costs under Alternative Financial Instruments where $A = \delta - \beta c_m$ and $B = \beta k \Delta^2$ ……………………………………………………………………………………….41

Table 10. Initial Allocation Results and the Distributive Actions…………………………………….46

Table 11. Results for numerical example for Basic, Extended, AFI, Revenue-Sharing Contract…...49

Table 12. Results for Basic and Extended model with Competition...………………………………..60

Table 13. Results for the new Scheme of Incentives and Fees in the Model with Competition….…..68

Table 14. Results of the numerical example for the Alternative Incentives and Fees in the Model with Competition..……………………………………………………………………………….…69

List of Figures

Figure 1. Basic Model………………………………………………………………………………...11

Figure 2. Extended Model with Governmental Incentives and Fees (E)……………………………...17

Figure 3. Special Cases of Extended Model………………………………………………………..24

Figure 4. TS_E vs. α……………………………………………………………………………...29

Figure. 5. Total surplus for the Basic Model, Extended Model, Alternative Financial Instruments, Revenue- Sharing contract……………………………………………………..……………………48

Figure 6. Basic Model with Competition…...……………………………………………………...50

Figure 7. Extended Model with Governmental Incentives and Fees (G) and Competition…………..56

Figure 8. TS vs α for Basic Model and Extended Model with competition…………………………..60

Figure 9. TS for Basic, Extended and Centrally Coordinated Model with Homogeneous Products....65

Figure 10. Ratio vs Δ_2 ……………………………………………………………………………...71

Figure 11. Ratio vs β_2……………………………………………………………………………71

Figure 12. Ratio vs k_2……………………………………………………………………………72

Figure 13. Ratio vs δ_2……………………………………………………………………………72

Acknowledgement

"To accomplish great things, we must not only act, but also dream, not only plan, but also believe."
Anatole France.

I feel truly blessed with this accomplishment in my life, and I'd like to take this opportunity to thank each and everyone who made it possible. I would like to thank God for all His blessings.

I would like to thank Dr. Min for all his support. From the beginning of my journey he helped me in every possible way. He taught me what research is and how it must be done. His guidelines will help me through my entire future career. I could not have asked for a better advisor.

I would like to thank my committee members Dr. Gaytan, Dr. Jackman, Dr. Marasinghe, Dr. Wang, Dr. Olafsson and Dr. Serrato for their suggestions and thoughtful comments. I would also like to thank José Carlos Miranda and Roberto Villaseñor for their support for the last years.

I would like to thank Lori Bushore for everything she has done for me. Her kindness shown since the very first emails made me feel confident about starting this adventure.

I would like to thank Carlos, he has encouraged me in so many ways to achieve this goal: First with his passion to teach and to learn; then by encouraging me to start my studies at Tec de Monterrey and by pushing me to continue at ISU. And finally, by giving me the most beautiful present I could ever ask for.

I would like to thank Karla for her friendship and support; for all the Starbucks coffee and talks that made this journey more enjoyable.

I would like to thank my parents and siblings for their support, prayers and unconditional love through all my life and during my studies. Specially, I would like to thank my mother, for loving my daughter as her own and for taking care of her while I was working.

Last, but definitely not the least I would like to thank Valeria, my daughter. Her laughs, her smile and her hugs gave me the courage and the strength to go through the long nights and weekends of endless work. Her love was what kept me going and made me finish. This is for you my baby.

Abstract

A rich mixture of government incentives and fees to encourage the collection of used products and the subsequent remanufacturing has been increasingly utilized both domestically and internationally. In this paper, toward a fuller understanding of such government participation in closed-loop supply chains (CLSC's), we construct and analyze a series of game-theoretic CLSC models with remanufacturing. Specifically, we investigate a basic decentralized CLSC model, two government participation models of linear incentives and fees as well as of central coordination via alternative financial instruments, and a revenue-sharing contract model without the government participation. We also analyze the impact of competition among manufacturers in our results. A key differentiating feature in our government participation models is the incorporation of the revenue neutrality requirement from a government's perspective whose financial sources for such incentives must eventually reconcile with the financial sinks for such fees. By comparing and contrasting the equilibrium solutions and the economic consequences of these models, managerial insights and economic implications relevant to academics and practitioners including decision and policy makers are obtained. For example, we show how the government participation can induce an entry or prevent an exit of a CLSC when one or more members are unprofitable.

1. Introduction

1.1 Research Objectives and Overview

This research is motivated by the concurrent practices and plans of a heterogeneous and sometimes seemingly unrelated mixture of incentives and fees that are theoretically difficult to collectively analyze, compare, and produce policy and decision implications and guidelines for. They are often practically difficult to implement due to the complexity and ambiguity of the legislations as well. This research provides guidelines to those involved in the process of designing and analyzing the environmental laws and regulations.

To our best knowledge, however, there have been few papers that address the implications from a government's perspective whose financial sources for such incentives must eventually reconcile with the financial sinks for such fees. Under these circumstances, as a first step toward a fuller understanding of such government participation, in this paper, we construct four CLSC models with remanfuacturing, and utilize these models to obtain insights and implications including policy and decision guidelines. The specific research objectives are:

(1) To analyze the impact of governmental participation via incentives and fees in the economic efficiency of a closed-loop supply chain.

(2) To investigate alternative mechanism that can improve the economic efficiency of a closed-loop supply chain.

(3) To analyze the impact of competition among manufacturers in the efficiency of a closed-loop supply chain with governmental participation.

The outlines and the background information of each model derived in this paper are as follows:

First, we construct a basic decentralized CLSC model without government participation consisting of a manufacturer who manufactures as well as remanufactures her product of a single kind, customers who directly purchase from the manufacturer, and a collector who collects the used products from customers and sells them back to the manufacturer. In the basic model as well as all the other subsequent models, we assume that the manufacturer sells directly to the customers. For example, Chiang et al. (2003) report that 42% of top suppliers like IBM, Estee Lauder, and Nike have begun to sell directly to customers over the Internet. Furthermore, companies such as Dell, Sony, and HP have long been offering their products directly to their customers via the Internet.

Also, we assume that the manufacturer has sufficient channel power over the collector to act as a Stackelberg leader (hence, the collector is the follower; see e.g., Savaskan et al. 2004). This is a reasonable assumption as major manufacturers who also remanufactures such as Kodak, BMW, IBM, DEC, and Xerox (Atasu et al. 2008b) do seem to have more than sufficient channel power relative to collectors of their products. In addition, in this paper, by remanufacturing we mean restoring used products to their original performance standards based on Section 3102-e(1)(b)(5) of the Public Authorities Law in the state of New York (New York State Department of Taxation and Finance 1999b). The scope in this paper is restricted to the products described in this law, such as: facsimile machines, photocopiers, printers, duplication equipment, magnetic ink cartridges, toner cartridges, inkjet cartridges and printer cartridges.

Following the formulation of the basic model, our analyses of the basic model include the economic efficiency and the collection rate at the equilibrium, which also serve as the benchmark reference points later. By economic efficiency, we mean the total surplus or the sum of all profits of the manufacturer, collector, and customers (i.e., consumer surplus), and it is often referred to as the social welfare from a social planner or a government perspective. Also, we note that the collection rate is an important environmental measure because some critical legislations such as the Waste Electrical and Electronic Equipment (WEEE) Directive contains the minimum target that must be met (Atasu et al. 2009) even though there certainly are other environmental measures.

Based on the basic model, we next propose two new models of the government participation with the revenue neutrality requirement. By revenue neutrality, we mean that all the incentives must be financed by all the fees without any external financial source or sink (i.e., all the fees and incentives are endogenously raised and spent, and the net gain to the government is zero). This revenue neutrality requirement allows us to concentrate on the economic efficiency of the incentives and fees without the distraction of financial sources to fund the incentives or financial sinks to apply the fees that are external to the CLSC models (Mrozek 2000).

In the first extension, we simultaneously consider a remanufacturing incentive for the manufacturer and a collecting incentive for the collector as well as a manufacturing fee for the manufacturer and a consumption fee for the customers where all incentives and fees are linear in product quantity. We note that the linear incentives and fees are easy to understand and implement, and have been widely used in the literature (e.g. Baker 1992). Also they are of importance on their own right and as a first order approximation - especially in the absence of more sophisticated previous studies in the context of this paper. In this way, fairly large classes of linear incentives and fees are incorporated within a single framework and we are able to concurrently address a wide variety of scenarios, which may first appear different and separate, and provide a unified analysis on the equilibrium solutions and the economic efficiencies. We also note that, a priori, the improvement of the economic efficiency may be unclear due to the revenue neutrality requirement. For this extended model, through the analysis of various bounds, we demonstrate that the economic efficiency does improve – if the government sets the incentives and fees at the optimal levels. Otherwise, the economic efficiency may deteriorate relative to the decentralized model.

In the second extension, we assume that there exists a social planner (see e.g., Carraro and Topa 1995) who centrally coordinates the forward and the reverse flows of products so as to maximize the total surplus. This leads to the upper bound of the total surplus that is theoretically achievable. We then show how the government can achieve this theoretically maximal total surplus via alternative financial instruments under the revenue neutrality requirement.

With these government participation models, we proceed to the basic model cases where one or more of the CLSC members are unprofitable. In the context of recycling, there have been numerous documents indicating that government incentives should be utilized when the recycling efforts are not profitable (Evans 1994). On the other hand, in the context of remanufacturing, there seem to be few quantitative papers addressing either the cases of unprofitable CLSC's or the government participation in such cases. As the same logic of unprofitability and incentives is easily extendable to the case of remanufacturing, under the extended and centrally coordinated model framework, we will show how the government participation may induce an entry of a new CLSC or prevent an exit of an existing CLSC. We will also provide concrete guidelines for the government efforts regarding these aspects.

In this paper, in addition, we examine how the manufacturer and the collector can coordinate themselves without the government by constructing a revenue-sharing contract model (see e.g., Cachon and Lariviere 2005). In this model, the sum of the CLSC member profits is maximized, but not the total surplus. Next, we proceed to compare and contrast all the models studied in this paper focusing on the economic efficiencies as well as the collection rate. Finally, we show how the economic distribution issues may be addressed in order to implement the proposed CLSC models of the linear incentives and fees, central coordination, and revenue-sharing contract as some initial allocation may not be agreeable to one or more members of the CLSC's. We then extend our analysis to the case where two manufacturers act as the leaders in a Stackelberg-Cournot game.

1.2 Environmental Laws

In recent years, the usage of government incentives and fees to facilitate the collection of used products and the subsequent remanufacturing has significantly increased at all of local, state, national, as well as international levels. At the same time, the types of incentives and fees vary greatly from a community to a community. For example, in New York, remanufacturers receive tax credits that are commensurate with the number of employees and/or the durability of capital investment (New York State Department of Taxation and Finance 1999a). Also, in 2008, a federal bill was introduced to allow a credit against income tax for remanufacturing or recycling equipment. It, however, is unclear if the federal definition of the remanufacturing or recycling is consistent with those of states (H.R. 5659-Government Relations: The Remanufacturing Institute, 2009). Furthermore, in the context of recycling, in California, customers pay an advance recovery fee for numerous video display devices (Electronic Waste Recycling Act California 2003) while collectors are reimbursed for the costs of collection per pound (Council of State Governments 2009). In addition, in Minnesota, certain electronics manufacturers pay a fee that is commensurate with their sales quantities. Table 1 shows some of the states that currently have an environmental legislation as well as their characteristics.

Table 1. Comparison of environmental legislation in US.

State	Incentive	Fee	Product
California	A key element of the act that affects e-waste collectors and recyclers is the availability of recovery and recycling payments to approved participants for certain collection and recycling activities	ARF (Advance Recycling Fee) charged to customers at point of sale. The fee depends on the size of the device: • between 4 and 15 inches $8; • between 15 and 35 inches $16; • 35 inches and larger $25	Cathode ray tube, televisions and computer monitors; LCD desktop monitors, laptop computers with LCD displays; LCD televisions; plasma televisions; portable DVD
Connecticut	The state will use the fees to administer a recycling program.	Manufacturers must register with the state Department of Environmental Protection and pay an annual fee.	Desktop or personal computers, computer monitors, portable computers, televisions.
Illinois	All registration fees are deposited in the Electronics Recycling Fund	Manufacturers must register with the state and pay an annual fee of $5,000.	Computer, computer monitor, television, printer.
Maryland	Fees are deposited in a fund to make grants to counties and municipalities to implement local collection plans.	The manufacturer registration fee is$10,000 for the initial registration	Computer or video display devices with a screen greater than four inches.
Minnesota	As an incentive to increase collection. Recyclers, collectors, and manufacturers can multiply the actual weight collected by 1.5.	Manufacturer pays a registration fee for year one. • $1,250 for companies manufacturing fewer than 100 units per year for sale •$5000 for manufacturers producing more than 100 units per year	Regulated video display devices (VDDs):televisions, laptop computers and computer monitors with displays larger than nine inches, measured diagonally.
New Jersey	The department shall make payments to the county or municipality,based upon the costs incurred municipality	IT companies will include payments with their annual fees, calculated as weight times 50 cents per pound of product sold.	Desktop or personal computer, computer monitor, television

Considering these examples, it is highly desirable to understand what the managerial insights and economic implications of the government participation through incentives and fees may be on closed-loop supply chains (CLSC's).

To our knowledge, this is the first paper that, in the context of CLSC's, formulates the government participation models subject to the revenue neutrality requirement via active and creative pricing of incentives and fees, and analyzes the economic efficiencies and collection rates relative to benchmark models without the government. From a broader perspective, this paper contributes to a fuller understanding of the impact of the government incentives and fees in the context of CLSC's that are nowadays ubiquitous across the U.S. and certain international regions such as the European Union. That is, this paper strongly argues that the government incentives and fees do have major ramifications on CLSC's in terms of economic efficiency and collection rate. Accordingly, various guidelines with respect to economic parameters are provided for the decision and policy makers to determine deliberate and purposeful choices on the kinds of financial instruments and the levels of incentives and fees to each member of CLSC's.

The rest of this paper is organized as follows. In Chapter 2, we provide a brief review of the relevant literature. Next, in Chapter 3, we elaborate on the major assumptions, and present the basic decentralized CLSC model with one manufacturer and one collector. We then extend the basic model by incorporating the government participation subject to the revenue neutrality requirement via the linear incentives and fees. Additionally we construct a centrally coordinated model, and show how the theoretically maximal economic efficiency can be achieved via the alternative financial instruments. We also examine the cases where one or more members of the CLSC are unprofitable, and the government participation and its policy and decision implications on the entry and exit of CLSC's. Finally, we present the revenue-sharing contract model without the government as well as the economic comparisons, and address the distributive issues of all the models. In Chapter 4, we derive the basic model that presents competition among manufacturers. We extend the basic model by incorporating the governmental participation via incentives and fees. We show how the optimal levels of the incentives and fees can be determined under the requirement of revenue neutrality when the firms have symmetric costs and they sell a homogeneous product. We derive a centrally coordinated model and present the theoretical upper bound of the total surplus. We generate a mechanism that achieves the upper bound for the case of homogeneous products and symmetric costs and then present an approximation for the case of heterogeneous products. In Chapter 5, we discuss the implications of our findings for some current policies in Mexico. Finally, in Chapter 6, we make concluding remarks and comment on future research.

2. Literature Review

In this section, we briefly review the most pertinent publications to this paper in the existing literature. The study of CLSC’s is a relatively new field of research, and there exist numerous challenging managerial problems (Atasu et al. 2008a, Guide and Van Wassenhove 2006). Within this study of CLSC’s, a game-theoretic framework has been widely utilized so as to gain insights about the decisions of CLSC members and their economic consequences (see e.g., Webster and Mitra 2007). Moreover, within the game-theoretic framework for collection and remanufacturing, Savaskan et al. (2004) examine various collection strategies with respect to channel structures under a Stackelberg game for a manufacturer/remanufacturer, a retailer, customers, and a collector. In contrast, in this paper, the channel structure is simpler and fixed, but the increasingly ubiquitous governmental participation via incentives and fees is investigated for economic and environmental consequences. In both cases, we note that the fundamental driver of remanufacturing is the cost savings for the maximization of profits, and not a requirement by the government or corporate wishes for better image.

In the context of closed-loop supply chains with competition, Savaskan and Wassenhove (2006) described a supply chain consisting of one manufacturer that sells his product to two competing retailers. The objective was to analyze the reverse channel that would improve the collection rate and total profit. In contrast, in this paper, we examine the competition among manufacturers while they sell directly to the customer. Ferguson and Toktay (2006) analyze the competition that the original manufacturer faces from the remanufacturer. In this paper the game is played in a two-stage period, where the new product is introduced in the first period and the remanufactured product enters the market in the second period. In this paper we analyze competition among manufacturers which are also remanufacturers, so the competition appears at the initial stage of the game. Choi (1991) analyzed a channel structure with two competing manufacturers and one intermediary (a common retailer) that sells both manufacturers' products. This paper studied different noncooperative games of different power structures between the two manufacturers and the retailer, i.e., two Stackelberg and one Nash games. This is the approach in this paper.

In the area of governmental participation as a social planner, the use of taxes and subsidies as a tool to improve social welfare has been widely studied (Galiana et al. 2003). Carraro and Topa (1995) analyze the impact of environmental regulation in the form of taxation on the innovation activity of firms. In that paper, the regulation problem is modeled as a two-stage game. In the first stage, the government sets its policy instruments. In the second stage, the firms decide whether and when to innovate. The regulator's objective function is the sum of the consumer surplus and industry profits, which is the total surplus.

The governmental participation through a scheme of incentives and fees that aims to align different objectives is similar to the principal's problem in the economic theory of agency (Ross 1973). As in Mirrlees (1976), the members in our CLSC models have different objectives, and behave in accordance with their own interest. Hence, we can view that the government in our models will need an incentive contract in the form of incentives and fees so as to maximize the total surplus, which is the principal's objective.

Once the government participates through incentives and fees, for cash flow issues in practice and for completeness issues in theory, it is highly desirable to address how these incentives are financed or the collected fees get utilized eventually. For this, Mrozek (2000) argues that deposit/refund systems must be implemented in a way that the revenue is neutral. That is, the amount of incentives disbursed is equal to the amount of fees collected within a closed system without a source or a sink for cash flow that is external to the model under consideration.

In this paper, as in Carraro and Topa (1995), the economic efficiency is measured in total surplus while, as in Mrozek (2000), the revenue neutrality is exploited in several applicable sections.

In the area of environmental regulations on supply chains, Webster and Mitra (2007) examine the impact of take-back laws when a manufacturer and a remanufacturer compete. In their study, two possible implementations of a take-back law are analyzed based on the degree of control that the manufacturer has on returns sold to the remanufacturer. Also, in Mitra and Webster (2008), the impact of governmental subsidies for remanufacturing is examined and a mechanism to distribute subsidies to benefit both manufacturer and remanufacturer is presented. Furthermore, Hammond and Beullens (2007) investigate a CLSC network with governmental participation through the Waste Electrical and Electronic Equipment (WEEE) directive. This investigation shows how the manufacturer's and consumer's behavior under different schemes of environmental regulations can be quantitatively modeled and analyzed. Also, Atasu et al. (2009) examine a series of efficient take-back legislation models with subsidies.

We note that these papers explicitly or implicitly assume that the government financial instruments are exogenous factors or external to their models subject to no limitation (i.e., no upper bound or budget). In addition, the need of government participation to create a viable environmental supply chain has been well described in the literature. For example, Evans (1994) analyzes the impact of the Resource Recovery Act that encourages private recycling operators to undertake operations that were first unprofitable. In this paper, in the context of CLSC's, we formalize various conditions under which the government can influence the entry and exit of CLSC's, clarifying the relevance of the government participation.

3. Closed-loop Supply Chain with One Manufacturer

3.1 Description of the Supply Chain and Assumptions

In this section, we formulate and analyze a basic closed loop supply chain consisting of a manufacturer who manufactures as well as remanufactures her product of a single kind, customers who directly purchase from the manufacturer and a collector who collects the used products from customers and sells them back to the manufacturer. We incorporate the participation of a collector since a third party collector can be more technically advanced in collection and recovery of returned products (see Hamza et al. 2007). We assume that there are no governmental incentives or fees. Figure 1 depicts the configuration of the basic model.

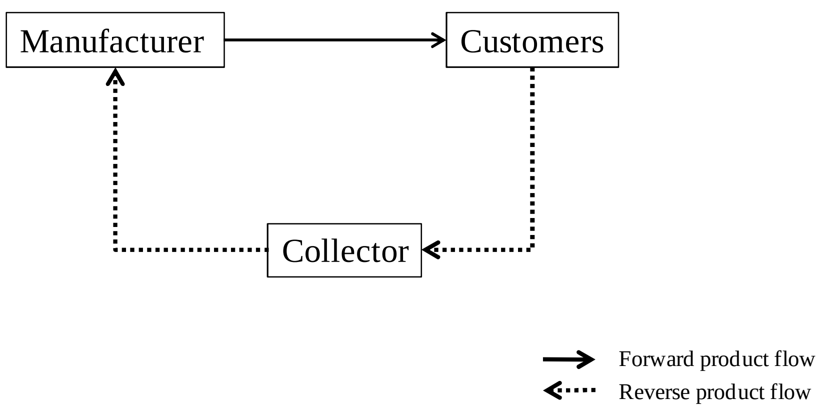

Figure 1. Basic Model

We denote the manufacturer's per unit selling price as *w*. Given *w*, we assume that the manufacturer faces a demand of $D(w) = \delta - \beta w$ where $\delta > 0$ is the maximal demand of the product and $\beta > 0$ denotes the decrease in demand for a unit increase in price *w*. We note that the linear demand functions have been widely utilized in economic models (Dobbs 1991, Gutiérrez-Alcaraz and Sheblé 2005). In order for the manufacturer to manufacture and remanufacture she incurs in a fixed cost of k_M .

We also assume that, after consuming the products, the customers are willing to return them to the collector and that the collector incurs a cost *e* per unit collected representing his level of collection efforts. *e* specifically represents the expenses per unit collected of additional collection bins, business hours, advertising, promotions and campaigns, etc. The collector then sells the collected used products to the manufacturer at a buyback price of *d* per unit. The relation between *d* and *e* is given by e = d - λ where λ represents the per unit profit margin for the collector.

We assume that the used product collection rate from the customers is commensurate with the level of collection efforts represented by *e*. φ(e) denotes the collection rate, and we assume that φ(e)=ke where *e* ϵ [0,1/k] and k > 0. We note that the linear collection rate functions are of importance in and of themselves as well as a first order approximation of more sophisticated functions (Qiaolun et al. 2008). The collector incurs in a fixed cost of k_C to implement the system for collecting the products. In our models the per unit collection cost is represented by e (collection efforts). The total cost of collection to the collector can be expressed as $\left(\delta-\beta w\right)*ke*e$. Atasu et al. (2009) presents, in one section, a per unit cost times demand which is increasing quadratically in collection rate. This is consistent with our case since we can rewrite the total cost of collection as $\left(\delta-\beta w\right)\frac{\varphi(e)^2}{k}$. Our total collection cost structure is different from Savaskan et al. (2004) in the sense that we only have a variable cost of collection, while they presented a variable cost and an investment cost. However, as in our case, we can see that the total collection cost is increasing quadratically in collection rate.

Finally, we denote the manufacturing cost per product as c_m while the remanufacturing cost per product as c_r . We assume that remanufacture cost c_r is less than the manufacturing cost c_m by an amount of Δ. As in Savaskan et al. (2004) the remanufacture savings are the main driver for the reverser flow. In what follows, we elaborate on the five key assumptions that are applicable throughout this paper.

ASSUMPTION 1: *The planning horizon is a static single period.* The selling price w, the buyback price d, and the profit margin λ are decided in a static single period. By this simplifying assumption, we imply a steady-state type model (as in Guide et al. 2003, Savaskan et al. 2004). In this way, we can concentrate on the aspect of the governmental incentives and fees without being distracted by dynamic ramifications that are beyond the scope of this paper.

ASSUMPTION 2: *The customers make no distinction between the newly manufactured and the remanufactured products.* The validity of this assumption depends on the nature of the product, but there are examples for which the new and remanufactured versions are perceived as having no difference. e.g., a single-use camera or a copy machine (Hammond and Beullens 2007).

ASSUMPTION 3: *We assume that the parameters of the model are such that the optimal quantity demanded is positive and the optimal collection rate $\varphi(e)$ satisfies $0 < \varphi(e) < 1$.* We make this assumption to focus on much more relevant and interesting cases of feasible interior solutions without being distracted by technical and perhaps pathological boundary solutions.

ASSUMPTION 4: *All products collected are sold back to the manufacturer and remanufactured*. We note that this assumption is often found in the literature for simplification (Savaskan and Van Wassenhove 2006). It can be relaxed by considering a parametric level of fractions. On the other hand, such a relaxation does not seem to yield much additional insights or implications in our models.

ASSUMPTION 5: *While optimizing their objective functions, all supply chain members have access to the same information without uncertainty*. This assumption allows us to bypass confounding factors such as inefficiencies related to asymmetry of information and risks due to uncertainties (Nagarajan and Bassok 2005, He et al. 2006), which are beyond the scope of this paper.

Throughout this paper we will assume that both the collector and the manufacturer will operate as long as their profit level is nonnegative. In this section we assume that the manufacturer and collector have nonnegative profit level.

3.2 Basic Model

We now proceed to formulate the basic model and derive the equilibrium solution as follows. The profit maximization problem for the collector (the follower) given *w* and *d* by the manufacturer (the leader) is:

$$Max_{\lambda}\Pi_B^C = (\delta - \beta w)\, d\varphi(e) - (\delta - \beta w)\, e\varphi(e) - k_C \tag{1}$$

In (1), the profit is expressed as the difference between the revenue from selling the collected products to the manufacturer and the variable cost of collecting the used products and the fixed cost for the operation of the system. With the linear collection rate, we simplify (1) to $Max_{\lambda}\Pi_B^C = (\delta - \beta w)\,\lambda\varphi(e) - k_C$. Because the objective function is strictly concave in λ (see Appendix A), the collector's first order condition characterizes the unique best response, $\lambda_B = \dfrac{d}{2}$

The manufacturer's problem, on the other hand, is to maximize her profit over *w* and *d*. That is,

$$Max_{w,d}\Pi_B^M = (\delta - \beta w)\big(w - c_m + (\Delta - d)\,\varphi(e)\big) - k_M \tag{2}$$

where the first part of the right hand side represents the quantity demanded, the second part of the right hand side represents the profit per unit and the third part shows the fixed costs to operate the production system.

Substituting the collector's best response function $\lambda_B(d)$ into (2), the manufacturer's profit maximization problem is given by

$$Max_{w,d}\Pi_B^M = (\delta - \beta w)\left(w - c_m + \frac{1}{2}kd(\Delta - d)\right) - k_M \tag{3}$$

It can be verified (see Appendix A) that the unique optimal equilibrium solution is:

$$w_B^* = \frac{8(\delta + \beta c_m) - \beta k\Delta^2}{16\beta}, d_B^* = \frac{\Delta}{2}$$

Given the manufacturer's solution, the collector's equilibrium solution is $\lambda_B^* = \dfrac{\Delta}{4}$. Hence, the level of collection efforts is $e_B^* = \dfrac{\Delta}{4}$ and the corresponding collection rate is given by $\varphi_B^* = k\left(\dfrac{\Delta}{4}\right)$. In addition, the profits for the collector and the manufacturer at the equilibrium are given by

$$\Pi_B^{C^*} = \frac{\Delta^2 k\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{256} - k_C \text{ and } \Pi_B^{M^*} = \frac{\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{256\beta} - k_M .$$

As for the customers, we measure their "profits" from their business transactions with the consumer surplus (CS). CS can be defined as the triangular area under the linear demand curve and above the rectangle representing the total purchase cost to the customers. Hence,

$$CS = \frac{1}{2}\left(\frac{\delta}{\beta} - w\right)\left(\delta - \beta w\right) = \frac{\left(\delta - \beta w\right)^2}{2\beta} \tag{4}$$

At the equilibrium, it can be verified that $CS_B^* = \dfrac{\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{512\beta}$.

As mentioned earlier, the economic efficiency will be measured by the total surplus (TS), which is the sum of the profits of the collector and manufacturer as well as the consumer surplus. Hence,

$$TS_B^* = \frac{\Delta^2 k\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{256} - k_C + \frac{\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{256\beta} - k_M + \frac{\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{512\beta} \tag{5}$$

We simplify (5) to express the equilibrium total surplus as

$$TS_B^* = \frac{\left(24\left(\delta-\beta c_m\right)+5\beta k\Delta^2\right)\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{512\beta} - k_C - k_M \tag{6}$$

In the basic model, there exist some straightforward and managerial insights. For example, we observe that as Δ increases, w_B^* decreases while d_B^* increases. That is, the manufacturer will pass along some of the manufacturing cost savings from the increase in Δ to the customers in the form of a lowered w_B^*. She will also increase the buyback price so as to encourage the collector to provide more used products to her. From the collector's perspective, the profit margin and the level of the collection efforts both increase as Δ increases, which is intuitive. It is also interesting to observe that the social welfare related measures such as the profits and the surpluses all increase with respect to Δ which implies the substantial importance of the remanufacturing cost saving on the social welfare.

In this section, we have derived the equilibrium solution as well as economic efficiency and collection rate of the basic model without governmental participation. This model can be considerer as conventional model since it does not include any external mechanisms (see Cachon and Lariviere 2005).

In what follows, we will contrast and compare these results with the ones obtained with various governmental linear incentives and fees.

3.3 Extended Model with Governmental Incentives and Fees

In this section, we extend the basic model by simultaneously incorporating a remanufacturing incentive for the manufacturer and a collecting incentive for the collector as well as a manufacturing fee for the manufacturer and a consumption fee for the customers. In this way, we can concurrently address a wide variety of scenarios and provide a unified analysis on the equilibrium solutions and the economic efficiency and collection rate. For example, in California, customers pay an advance recovery fee for numerous video display devices (Electronic Waste Recycling Act California 2003). Also, in Minnesota, certain electronics manufacturers pay a fee that is commensurate with their sales quantities (Council of State Governments 2009). In addition, in California (Council of State Governments 2009), collectors are reimbursed for the costs of collection per pound. Furthermore, in New York, remanufacturers receive tax credits that are commensurate with the number of employees and/or the durability of capital investment (New York State Department of Taxation and Finance 1999a). Also, on March 31, 2008, U.S. Rep. Phil English (R-Pa.). introduced H.R. 5659, a bill designed to amend the Internal Revenue Code of 1986 to allow a credit against income tax for recycling or remanufacturing equipment. H.R. 5659 will establish a tax credit equal to 20 percent of the amount of a taxpayer's expenditures on certain equipment purchases (Government Relations: The remanufacturing institute, 2009)

Also, a key reason that we consider a multiple number of incentives or fees simultaneously is to investigate the impact of the relative weights for the incentives or fees. e.g., does a remanufacturing incentive contribute to a higher level of economic efficiency than a collecting incentive? In addition, a key reason that we consider both incentives and fees simultaneously is to lay the groundwork on the financial source to fund the incentives and the financial sink for the usage of the fees that will be examined in later sections. Figure 2 depicts the participation of the government through the incentives and fees in the free market model.

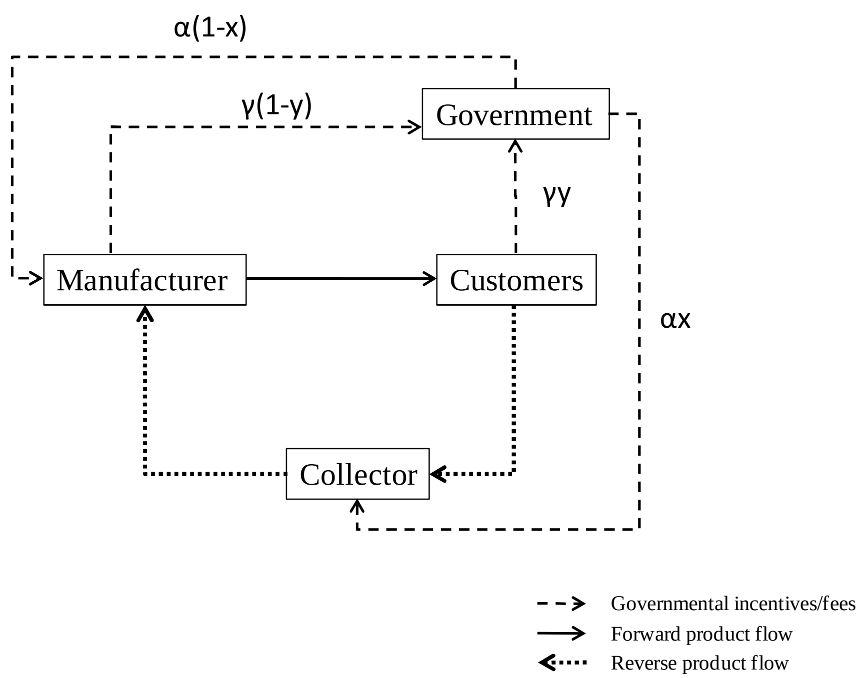

Figure 2. Extended Model with Governmental Incentives and Fees (E)

In this extended model, we incorporate fairly large classes of linear incentives and fees within a single framework. Linear incentives and fees have been widely used in the literature (e.g. Baker 1992, Fershtman and Judd 1987). Also they are of importance on their own right and as a first order approximation - especially in the absence of more sophisticated previous studies as in the case of this paper. We now elaborate on the linear incentives and fees as follows.

Let us first begin with the incentives. We assume that the government provides an incentive total of α dollars per unit collected or remanufactured ($/unit). That is, the government provides αx to the collector per unit collected where x represents the fraction of α that goes to the collector per unit collected while $(1-x)$ represents the fraction of α that goes to the manufacturer per unit remanufactured. In this paper, α represents the overall incentive per unit collected or remanufactured, and will be modeled as a passive parameter in this section and an active decision variable in the next section. On the other hand, x , $x \in [0,1]$, will be utilized as a what-if type parameter that will generalize numerous particular policies.

Let us now proceed to the fees. We assume that the government receives a fee total of γ dollars per unit assembled or purchased ($/unit). That is, the government receives $\gamma(1-y)$ from the manufacturer per unit assembled where $(1-y)$ represents the fraction of γ that is charged to the manufacturer while the government receives γy from the customers per unit purchased where y represents the fraction of γ that is charged to the customers. As in the case of α, in this paper, γ represents the overall fee per unit assembled or purchased, and will be modeled as a passive parameter in this section and an active decision variable in the next section. On the other hand, y , $y \in [0,1]$, will be utilized as a what-if type parameter that will generalize numerous particular policies.

With the introduction of these incentives and fees, the basic model from the previous section is extended as follows. The collector's profit expression is given by

$$Max_{\lambda}\Pi_E^C = \left(\delta-\beta\left(w+\gamma\right)\right)\left(d+\alpha x\right)\varphi(e)-\left(\delta-\beta\left(w+\gamma\right)\right)e\varphi(e)-k_C \tag{7}$$

In (7), $\delta-\beta\left(w+\gamma y\right)$ represents the adjusted demand function given γy (see e.g., Binger and Hoffman 1998) as the demand is now $D\left(w,\gamma y\right)=\left(\delta-\beta\left(w+\gamma y\right)\right)$ while $\left(\delta-\beta\left(w+\gamma y\right)\right)\varphi\left(e\right)$ represents the collection amount given γy. At the same time, $(d+\alpha x)$ represents the revenue per unit collected given αx. Hence, for the collector, the first and second terms represent the revenue and the cost, respectively.

As $e=d-\lambda$, we simplify (7) to $Max_{\lambda}\Pi_{E}^{C}=\left(\delta-\beta w\right)\left(\lambda+\alpha x\right)\varphi(e)-k_{C}$, which will be maximized by the collector over λ. Since this objective function is strictly concave in λ, from the first order condition, the best unique response is given by $\lambda_{E}=\dfrac{d-\alpha x}{2}$.

On the other hand, the manufacturer's profit maximizing problem can be formulated as

$$Max_{w,d}\Pi_{E}^{M}=\left(\delta-\beta\left(w+\gamma y\right)\right)\left(w-c_{m}-\gamma(1-y)+\left(\Delta-d+\alpha(1-x)\right)\varphi(e)\right)-k_{M} \tag{8}$$

Given λ_{E}, it can be verified (see Appendix A) that the unique optimal equilibrium solution is:

$w_{E}^{*}=\dfrac{8\left(\delta+\beta\left(c_{m}+\gamma\left(1-2y\right)\right)\right)-\beta k\left(\alpha+\Delta\right)^{2}}{16\beta}, d_{E}^{*}=\dfrac{\Delta+\alpha\left(1-2x\right)}{2}$ Also, for the collector, the corresponding equilibrium solution is: $\lambda_{E}^{*}=\dfrac{\Delta+\alpha\left(1-4x\right)}{4}$.

Given these expressions for the decision variables at the equilibrium, we proceed to obtain relevant managerial insights of the pricing, and to compare and contrast with the expressions derived in the basic model. We first analyze from the collector's perspective the impact of the government's participation through the incentives and fees. Specifically, we focus on the collector's profit margin relative to that ($\lambda_{B}^{*}=\dfrac{\Delta}{4}$) in the basic model.

If $x > 1/4$, then the collector's profit margin is decreased at the equilibrium due to the incentive of αx. This implies that, when the incentive for collection is sufficiently large relative to the incentive for remanufacturing, the collector finds it more beneficial to reduce the profit margin. An intuitive reasoning is as follows: by reducing the profit margin, the collector produces an effect to increase the collection effort level and the collection rate, which will benefit from the governmental incentive. At the same time, the manufacturer produces an effect to decrease the buyback price, which will lead to the lower selling price and higher quantity demanded from the customers. This will, in turn, lead to the higher quantity collected, which once again will benefit the collector from the governmental incentive.

If $x < 1/4$, then the collector's profit margin is increased at the equilibrium due to the incentive of αx. This implies that, when the incentive for collection is sufficiently small relative to the incentive for remanufacturing, the collector finds it more beneficial to increase the profit margin. Managerial insights for this case can be obtained as in the previous case in a similar way.

If $x = 1/4$, then the collector's profit margin remains the same. This is an important threshold value for regulatory policies given the definitions of our α, γ, *x*, and *y* as any change of the fraction at this point will change the direction of the collector's pricing strategy. In turn, this change of the direction will alter the manufacturer's pricing strategy differently.

Let us now proceed to analyze from the manufacturer's perspective the impact of the government's participation through the incentives and fees. Specifically, we focus on the manufacturer's buyback price and selling price relative to those ($d_B^* = \dfrac{\Delta}{2}$, $w_B^* = \dfrac{8(\delta + \beta c_m) - \beta k \Delta^2}{16\beta}$) in the basic model.

For d_E^*, if $x < 1/2$, then the manufacturer's buyback price is increased at the equilibrium due to the incentive of $\alpha(1-x)$. This implies that, when the incentive for remanufacturing is sufficiently large relative to the incentive for collecting, the manufacturer finds it more beneficial to increase the buyback price. An intuitive reasoning is as follows: by increasing the buyback price, the manufacturer produces an effect to induce the collector to increase the collection rate. The increased collection rate will lead to the increased quantity to be remanufactured, which will benefit from the governmental incentive.

If x > 1/2, the manufacturer's buyback price is decreased at the equilibrium due to the incentive of $\alpha(1-x)$. This implies that, when the incentive for remanufacturing is sufficiently small relative to the incentive for collection, the manufacturer finds it more beneficial to decrease the purchase price for the used product from the collector. Managerial insights for this case can be obtained as in the previous case in a similar way.

If x = 1/2, the manufacturer's buyback price remains the same. This is also an important threshold value for regulatory policies given the definitions of our α, γ, x, and y as any change of the fraction at this point will change the direction of the manufacturer's buyback pricing strategy. In turn, this change of the direction will alter the collector's pricing strategy differently.

For w_E^*, if y < 1/2 (i.e., $1-2y$ > 0), then the manufacturer's selling price contains a component that contributes to a higher level of the selling price due to the manufacturing fee of $\gamma(1-y)$. This implies that, when the fee for manufacturing is sufficiently large relative to the fee for consumption, there exists an inducement for passing the increased cost of manufacturing to the customers. This pressure to raise the selling price is counter-balanced by another component in the selling price that contributes to a lower level of the selling price due to the incentive total of α dollars per unit collected or remanufactured. This incentive, as addressed in the cases of λ_E^* and d_E^*, implies a reduction in the cost of manufacturing, and there exists a counter-inducement for passing the cost savings of manufacturing to the customers in the form of a reduced selling price. Hence, the exact direction and magnitude comparisons between w_E^* vs. w_B^* will depend on a particular set of the remaining parameter values.

If y > 1/2 (i.e., $1-2y$ < 0), then the manufacturer's selling price contains a component that contributes to a lower level of the selling price due to the manufacturing fee of $\gamma(1-y)$. This implies that, when the fee for manufacturing is sufficiently small relative to the fee for consumption, there actually exists an inducement for the manufacturer to lower the selling price so as to alleviate the magnitude of reduction in the customers demand due to the relatively significant consumption fee that the customers now pay. At the same time, there exists another component in the selling price that contributes to a lower level of the selling price due to the incentive total of α dollars per unit collected or remanufactured. Hence, in this case, the selling price will decrease regardless of any particular set of the remaining parameter values.

If y = 1/2, then the manufacturer's selling price contains no component that contributes to a change of the selling price due to the manufacturing fee of $\gamma(1-y)$. On the other hand, there still exists the incentive of α, which implies that the selling price will decrease.

Let us next examine the impact of the governmental incentives and fees on the environmental efficiency represented by the collection rate. From our definition of the collection rate, at the equilibrium, we have $\varphi_E(e) = k(d_E - \lambda_E) = k\left(\frac{\Delta+\alpha}{4}\right)$ Compared to the basic model's φ_B^* $= k\left(\frac{\Delta}{4}\right)$, for α > 0, the collection rate is strictly higher in the extended model.

Finally, let us examine the impact of the governmental incentives and fees on the economic efficiency represented by the levels of the profits, consumer surplus, and total surplus. Such values and the corresponding decision variables are summarized in Table 2 as follows.

Table 2. Extended Model with Governmental Incentives and Fees

	Objective Function Value	Decision Variable(s)
Collector Profit	$\Pi_E^C = \frac{(\alpha+\Delta)^2 k\left(8\left(\delta-\beta\left(c_m+\gamma\right)\right)+\beta k\left(\alpha+\Delta\right)\right)}{256} - k_C$	$\lambda_E = \frac{\Delta+\alpha(1-4x)}{4}$
Manufacturer Profit	$\Pi_E^M = \frac{\left(8\left(\delta-\beta\left(c_m+\gamma\right)\right)+\beta k\left(\alpha+\Delta\right)^2\right)^2}{256\beta} - k_M$	$d_E = \frac{\Delta+\alpha(1-2x)}{2}$ $w_E = \frac{8\left(\delta+\beta\left(c_m+\gamma(1-2y)\right)\right)-\beta k\left(\alpha+\Delta\right)^2}{16\beta}$
Consumer Surplus	$CS_E = \frac{\left(8\left(\delta-\beta\left(c_m+\gamma\right)\right)+\beta k\left(\alpha+\Delta\right)^2\right)^2}{512\beta}$	NA
Total Surplus	$TS_E = \frac{\left(24\left(\delta-\beta\left(c_m+\gamma\right)\right)+5\beta k\left(\alpha+\Delta\right)^2\right)\left(8\left(\delta-\beta\left(c_m+\gamma\right)\right)+\beta k\left(\alpha+\Delta\right)^2\right)}{512\beta}$ $-k_C-k_M$	NA

So far, we have modeled fairly large classes of incentives and fees within a single framework. In what follows, we further examine some special cases by fixing the values of the policy parameters *x* and *y*. In this way, some of the current practices may be better emulated as they are often simpler than the extended model with the four financial instruments in a single setting. Furthermore, in the next section, we revisit the extended model with the aforementioned revenue neutrality requirement, and derive conclusive results with respect to the economic efficiency of the linear incentives and fees described.

3.3.1 Special Cases of Extended Model

In this subsection, we will examine three special cases of the extended model. Namely, the cases of the manufacturing-fee collecting-incentive (MFCI), consuming-fee collecting-incentive (CFCI), and consuming-fee remanufacturing-incentive (CFRI). These represent the extreme cases of ($x = 1$,

$y = 0$), ($x = 1, y = 1$), and ($x = 0, y = 1$), respectively. Figure 3 depicts the configurations of these three cases.

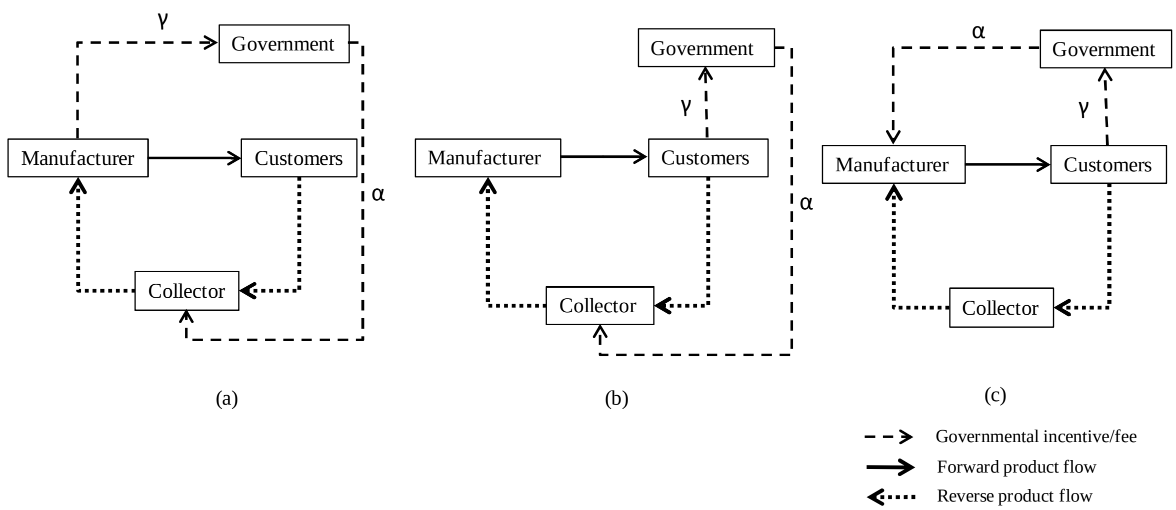

Figure 3. Special Cases of Extended Model

The derivation of the equilibrium solutions are straightforward, and the results are summarized in the following table.

Table 3. Special Cases Equilibrium Solutions of Extended Model

Variable	MFCI	CFCI	CFRI
λ	$\dfrac{\Delta - 3\alpha}{4}$	$\dfrac{\Delta - 3\alpha}{4}$	$\dfrac{\alpha + \Delta}{4}$
d	$\dfrac{\Delta - \alpha}{2}$	$\dfrac{\Delta - \alpha}{2}$	$\dfrac{\Delta + \alpha}{2}$
w	$\dfrac{8(\delta+\beta(c_m+\gamma))-\beta k(\alpha+\Delta)^2}{16\beta}$	$\dfrac{8(\delta+\beta(c_m-\gamma))-\beta k(\alpha+\Delta)^2}{16\beta}$	$\dfrac{8(\delta+\beta(c_m-\gamma))-\beta k(\alpha+\Delta)^2}{16\beta}$

We note that, relative to the case of CFRI (the collector receives no incentive), the buyback price d and the profit margin λ will decrease in the cases of MFCI and CFCI. The reason is that, with the governmental incentive, the collector does not need to achieve as high profit margin as in CFRI and the manufacturer does not need to pay as high buyback price as in CFRI. Likewise, relative to the case of MFCI (the manufacturer pays the fee), the selling price w will decrease in the cases of CFCI and CFRI. The reason is that, the manufacturer does not need to charge as much as in MFCI where the cost increase due to the fee gets passed along to the customers in the form of an increased selling price.

3.3.2 Revenue Neutrality Constraint

We now assume the revenue neutrality. That is, all the incentives provided within the model are financed through all the fees received within the same model without any external financial source or sink. In the context of the extended model, the revenue neutrality is given by

$$\left(\delta-\beta\left(w+\gamma y\right)\right)\gamma y+\left(\delta-\beta\left(w+\gamma y\right)\right)\gamma\left(1-y\right)=\left(\delta-\beta\left(w+\gamma y\right)\right)\alpha x k\left(d-\lambda\right)+\left(\delta-\beta\left(w+\gamma y\right)\right)\alpha\left(1-x\right)k\left(d-\lambda\right) \quad (9)$$

where the left hand side of (9) represents the total amount of the fees and the right hand side of (9) represents the total amount of the incentives. The equation (9) is simplified to become:

$$\gamma=\alpha k\left(d-\lambda\right) \quad (10)$$

We have the corresponding values for the objective functions and decision variables as in Table 4

Table 4. Extended Model Results with Revenue Neutrality

	Objective Function Value	Decision Variable(s)
Collector Profit	$\Pi_E^C=\frac{(\alpha+\Delta)^2 k\left(8\left(\delta-\beta c_m\right)-\beta k\left(\alpha^2-\Delta^2\right)\right)}{256}-k_C$	$\lambda_E=\frac{\Delta+\alpha\left(1-4x\right)}{4}$
Manufacturer Profit	$\Pi_E^M=\frac{\left(8\left(\delta-\beta c_m\right)+\beta k\left(\Delta^2-\alpha^2\right)\right)^2}{256\beta}-k_M$	$w_E=\frac{8\left(\delta+\beta c_m\right)-\beta k\left(\alpha+\Delta\right)\left(\alpha\left(4y-1\right)+\Delta\right)}{16\beta}$ $d_E=\frac{\Delta+\alpha\left(1-2x\right)}{2}$
Consumer Surplus	$CS_E=\frac{\left(8\left(\delta-\beta c_m\right)+\beta k\left(\Delta^2-\alpha^2\right)\right)^2}{512\beta}$	NA
Total Surplus	$TS_E=\frac{\left(24\left(\delta-\beta c_m\right)-\beta k\left(\alpha-5\Delta\right)\left(\alpha+\Delta\right)\right)\left(8\left(\delta-\beta c_m\right)-\beta k\left(\alpha-\Delta\right)\left(\alpha+\Delta\right)\right)}{512\beta}-k_C-k_M$	NA

By examining the objective function values and the corresponding decision variables, we observe the followings:

a. The equilibrium decision variable values critically depend on the policy parameters *x* and *y*. That is, the government can profoundly influence the business strategies of the manufacturer and the collector by simply adjusting a parameter value or two. For example, if *x* is sufficiently high, λ_E^* may result in a negative number (which can be considered a loss-leader type pricing strategy; see e.g., Lal and Matutes 1994, Chevalier et al. 2003).

b. The equilibrium values of the objective functions and the collection rate do not depend on the policy parameters *x* and *y*. This implies that, if the governmental focus is on the economic efficiency and collection rate, the question of what fraction of the incentives are allocated to the remanufacturer vs. the collector or what fraction of the fees are levied on the manufacturer or the customers may be irrelevant.

c. The level of the collection rate increases with the incentives. Hence, this collection rate is higher in the extended model than in the basic model.

Substituting (10) into Table 3, we can obtain the equilibrium solutions as functions of α only shown in Table 5.

Table 5. Special Cases Equilibrium Solutions of Extended Model with Revenue Neutrality

Variable	MFCI	CFCI	CFRI
λ	$\frac{\Delta-3\alpha}{4}$	$\frac{\Delta-3\alpha}{4}$	$\frac{\alpha+\Delta}{4}$
d	$\frac{\Delta-\alpha}{2}$	$\frac{\Delta-\alpha}{2}$	$\frac{\Delta+\alpha}{2}$
w	$\frac{8(\delta+\beta c_m)+\beta k(\alpha^2-\Delta^2)}{16\beta}$	$\frac{8(\delta+\beta c_m)-\beta k(\alpha+\Delta)(3\alpha+\Delta)}{16\beta}$	$\frac{8(\delta+\beta c_m)-\beta k(\alpha+\Delta)(3\alpha+\Delta)}{16\beta}$

Without the revenue neutrality, for example, the difference in selling prices (MFCI – CFCI) is a function of γ only and independent of α. On the other hand, with the revenue neutrality, any increase in α will lead to the increase in the difference. The reason is that any increase in incentive α must be financed by a proportional increase in γ. In the next section, we will expand on this observation, by modeling and analyzing an optimization version of the extended model with respect to α and γ subject to the revenue neutrality requirement.

3.3.3 Optimal Incentive and Fee for the Extended Model

In the previous section, the incentive and fee, α and γ, are treated as passive parameters. In this section, we treat the incentive and fee as active decision variables for the government to maximize the total surplus. A similar role for the government can be observed in the literature (Mrozek 2000, Calcott and Walls 2005, Becker and Schechter 1996).

Therefore, the optimization problem now becomes $Max_{\alpha,\gamma} TS_E = CS_E + \Pi_E^M + \Pi_E^C$ with the equilibrium expressions for *w*, *d*, and λ appropriately substituted into the relevant profit and consumer surplus expressions. Also, we impose the aforementioned neutrality of revenue in this section.
Let us now examine this extended model. The government's optimization problem is:

$$Max_\alpha TS_E = \frac{\left(24\left(\delta-\beta c_m\right)-\beta k\left(\alpha-5\Delta\right)\left(\alpha+\Delta\right)\right)\left(8\left(\delta-\beta c_m\right)-\beta k\left(\alpha-\Delta\right)\left(\alpha+\Delta\right)\right)}{512\beta}-k_C-k_M \qquad (11)$$

The first order necessary condition leads to a unique optimal α* as shown in Appendix B. Even though α* can be easily programmed for computing, it is quite unwieldy for analytic uses. Hence, for analysis, we rely on indirect bounds of α*. Specifically, we show in Appendix B that $0 < \alpha^* < \frac{\Delta}{2}$.

This implies that $0 < \gamma^* < \frac{3\Delta^2 k}{16}$ while $k(\frac{\Delta}{4}) < \varphi_E^* < k(\frac{3\Delta}{8})$. This in turn implies that the optimal collection rate is strictly greater than that in the basic model $(\varphi_B^* = k(\frac{\Delta}{4}))$. As for the economic efficiency measured in total surplus, $TS^*_E = TS_{E(\alpha^*)} > TS^*_B$ as shown in Appendix C.

The increase in the total surplus relative to the basic model is generated by the subsidy of collection rate and the fact that the fees that are charged as just a fraction of the incentive given (i.e. $\gamma = \alpha\varphi(e)$)

Therefore, in so far as the collection rate as well as the economic efficiency measured by the total surplus level, the governmental participation through the incentive and fee is well justified relative to the free market model without the governmental participation.

For the extended model and the basic model, the profits and consumer surplus are compared as follows: $\Pi_E^{C^*} > \Pi_B^{C^*}, \Pi_E^{M^*} < \Pi_B^{M^*}$, and $CS_E^* < CS_B^*$ (see Appendix D). This means that the manufacturer and the customers are better off without the governmental participation. However, the collector increases his profit with the optimal levels of the incentive and fee.

With the levels of profits and surpluses, it is relatively easy to induce the manufacturer and the customers to voluntarily switch from the free market configuration of the basic model to the optimal governmental participation of the extended model. To do so, from the increase in the collector's profit, a lump sum incentive will be provided to the manufacturer so that she is no worse off. At the same time, a lump sum incentive to the customers will be provided from the same source (for example, at the end of the year) assuming that such an incentive is not significant enough to alter the customers purchase quantities (see Pratt et al. 2004, Atkinson et al. 1999).

To illustrate the key features of our models so far, we now present a numerical example. For this example, the hypothetical values of the parameters are as follows: δ = 1,000, c_m = \$5/unit, Δ = \$4/unit, β = 20, and k = 0.45. Also, we will assume that the fractions x = 0.5 and y = 0.5 and $k_C = k_M = 0$. With these parameter values, we can plot the behavior of TS_E as a function of α as in Figure 4.

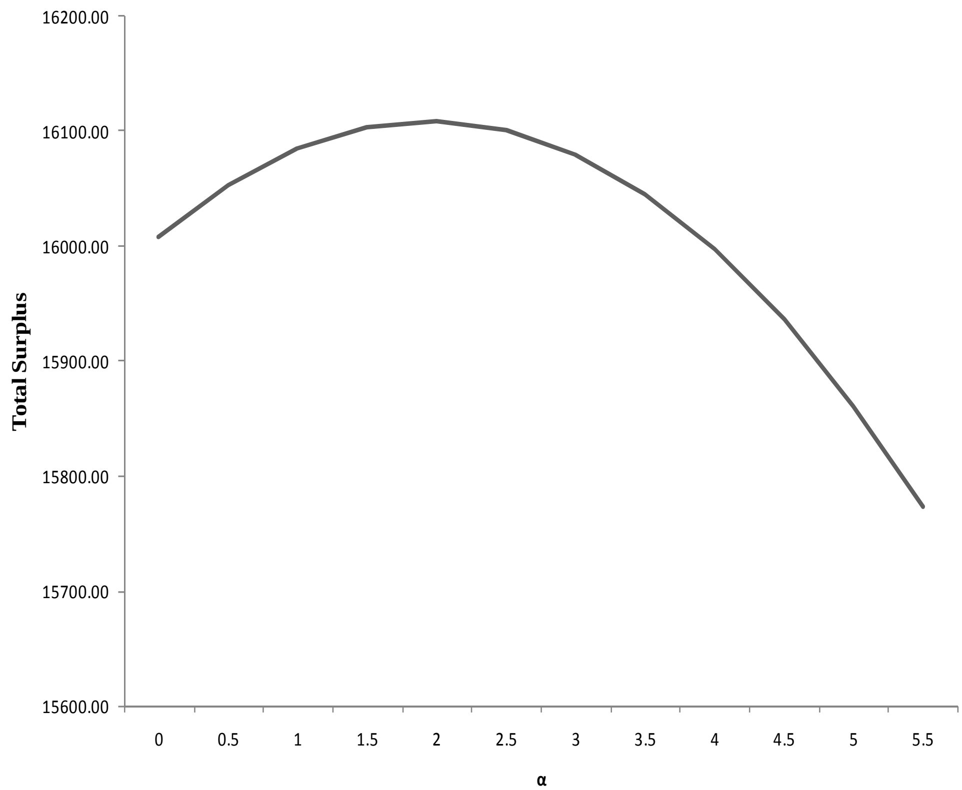

Figure 4. TS_E vs. α

We note that with the optimal value of α, the total surplus will increase. On the other hand, with any other choice of α value, it is actually possible that the total surplus will decrease with the governmental participation.

Furthermore, Table 6 summarizes the resulting equilibrium values for the variables and the economic and environmental consequences relative to the basic model.

Table 6. Results for Basic and Extended Models

Model	λ	d	w	α	γ	Π^C	Π^M	CS	TS	$\varphi(e)$
Basic	1	2	27.05	NA	NA	206.55	10534.05	5267.03	16007.63	0.45
Extended	0.51	2	27.17	1.96	1.31	455.98	10435.4	5217.69	16109.1	0.67

Relative to the basic model, since x > 1/4, the collector decreases his profit margin and still increases his profit due to the sufficient amount of the incentive he receives. For the manufacturer, since x=1/2, there is no impact of the incentive α on her buyback price, *d*. However, α induces an increase in the selling price, which contributes to the decline of the profit level. For the customers, the overall effect of the incentives and fees is a decrease in their consumer surplus. We also note that the governmental participation in this example increases the total surplus as well as the collection rate.

3.4 Centrally Coordinated Model

In the previous section, we examined how the government maximizes the total surplus over the linear incentives and fees under the neutrality of revenue. In this section, we revisit the problem from a social planner's perspective. Specifically, we assume that there exists a social planner who centrally coordinates the forward and the reverse flows of products so as to maximize the total surplus. This leads to the upper bound of the total surplus that is theoretically achievable denoted by TS_{CC}^{*}. We then provide an allocation mechanism that achieves this theoretical bound via pricing, and present a distribution mechanism that will make each party better off than in the basic model without governmental participation (Campbell 2006).

From the social planner's perspective, the profits of the collector and the manufacturer as well as the consumer surplus are given by $(\delta-\beta w)(d-e)\varphi(e)-k_C$,

$(\delta-\beta w)\left(w-c_m+(\Delta-d)\varphi(e)\right)-k_M$, and $\dfrac{(\delta-\beta w)^2}{2\beta}$, respectively.

Hence, the total surplus under the central coordination is:

$$TS_{CC}=(\delta-\beta w)(d-e)\varphi(e)+(\delta-\beta w)\left(w-c_m+(\Delta-d)\varphi(e)\right)+\frac{(\delta-\beta w)^2}{2\beta}-k_C-k_M \quad (12)$$

and (12) can be simplified to

$$TS_{CC}=-c_m(\delta-\beta w)+(\delta-\beta w)(\Delta-e)\varphi(e)+\frac{(\delta-\beta w)(\delta+\beta w)}{2\beta}-k_C-k_M \quad (13)$$

We note that, with the social planner as the sole decision maker, the selling price *w* and the cost of collection efforts *e* are the only relevant variables in his maximization of TS_{CC}. It can be verified (see

Appendix E) that the unique optimal solution is: $w_{CC}^{*}=\frac{4c_m-k\Delta^2}{4}$ and $e_{CC}^{*}=\frac{\Delta}{2}$. For further analysis, we assume that $4c_m-k\Delta^2>0$. For example, c_m is sufficiently high. The corresponding optimal total surplus is given by

$$TS_{CC}^{*}=\frac{\left(4\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{32\beta}-k_C-k_M \tag{14}$$

while the corresponding optimal collection rate is given by $\varphi_{CC}^{*}=k\frac{\Delta}{2}$

Given (14), we have the following proposition (see Appendix C for the proof).

Proposition 1. For the models described in this paper, the relationship among the levels of the total surpluses is given by $TS_{CC}^{*}>TS_{E}^{*}>TS_{B}^{*}$. The collection rate is related as $\varphi_{CC}^{*}>\varphi_{E}^{*}>\varphi_{B}^{*}$.

3.5 Alternative Financial Instruments

From Proposition 1, we see that the centrally coordinated model provides a higher level of economic efficiency and collection rate. However, the question arises as to how an allocation mechanism can be designed via pricing to achieve TS_{CC}^{*} and φ_{CC}^{*} as any direct attempt by the social planner to enforce the optimal selling price on the manufacturer and the optimal level of collection efforts on the collector is unrealistic.

To answer this question, let us examine this issue mathematically. First, we note that a degree of operational flexibility represented by three control variables is necessary to satisfy the following three requirements: the cost of collection efforts must be equal to e_{CC}^{*}, the selling price must be equal to w_{CC}^{*}, and the revenue neutrality must be met. The details are elaborated in the next subsection.

3.5.1 Allocation Mechanism

Let us first introduce the following financial instruments of α, ε, and η. Incentive α is as defined in the previous two sections. That is, x of α is given to the collector per unit collected while $(1-x)$ of α is given to the manufacturer per unit remanufactured. Fee ε is charged to the manufacturer per unit revenue (i.e., per dollar) for selling the product to the customers. Similar approaches have been used to analyze the behavior of the supply chain under incentives and fees (Kouvelis and Rosenblatt 2002). Finally, we utilize η as an incentive (i.e., η is negative) or as a fee (i.e., η is positive) provided or charged to the manufacturer per unit sold. It can be shown that the conditions under which η can be an incentive or fee are determined by the parameters k, Δ, and c_m.

Proposition 2. Theoretical total surplus $TS_{CC}^{*} = \dfrac{\left(4\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{32\beta} - k_C - k_M$ can be achieved with α_{AM}, η_{AM} and ε_{AM} defined as follows: $\alpha_{AM} = \Delta$,

$$\eta_{AM} = \frac{\Delta^2 k\left(3\delta-5\beta c_m\right)+\beta\Delta^4 k^2-\left(\delta-\beta c_m\right)4c_m}{4\left(\delta-\beta c_m\right)+\beta\Delta^2 k} \text{ and } \varepsilon_{AM} = 2-\frac{4\left(\delta-\beta c_m\right)}{4\left(\delta-\beta c_m\right)+\beta\Delta^2 k}.$$

It can be shown (see Appendix E) that the scheme defined in Proposition 2 achieves TS_{CC}^{*}.Table 7 summarizes the levels of profits as well as the corresponding decision variables for the allocation mechanism described above.

Table 7. Profits and Decision Variables for the Allocation Mechanism

	Objective Function Value	Decision Variable(s)
Collector's profit	$\Pi_{AM}^{C^*} = \dfrac{\Delta^2 k\left(4\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{16} - k_C$	$\lambda_{AM}^{*} = \dfrac{\left(\Delta - 2\Delta x\right)}{2}$
Manufacturer's profit	$\Pi_{AM}^{M^*} = -\dfrac{\Delta^2 k\left(4\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{16} - k_M$	$w_{AM}^{*} = \dfrac{1}{4}\left(4c_m - \Delta^2 k\right)$ $d_{AM}^{*} = \Delta - \Delta x$

The corresponding consumer and total surpluses are given by $CS^{*}_{AM} = \frac{\left(4\left(\delta - \beta c_m\right) + \beta k \Delta^2\right)^2}{32\beta}$ and

$TS^{*}_{AM} = \frac{\left(4\left(\delta - \beta c_m\right) + \beta k \Delta^2\right)^2}{32\beta} - k_C - k_M$, respectively.

So far we have presented the scheme of incentives and fees that will achieve the maximal total surplus. For implementation purposes, however, the resolution of some distributive issues is necessary. For example, as the initially allocated profit of the manufacturer is negative, she has no incentive to operate without a lump sum payment from the collector and/or the customers, which may be relatively hard to implement (see Section 3.9 for details).

3.6 Entry and Exit Implications of Governmental Participation

In this section, we examine the implications of the governmental incentives and fees on the entry and exit aspects of the CLSC and its members. In the context of recycling, there have been numerous documents indicating that governmental incentives should be utilized when the recycling efforts are not profitable. On the other hand, in the context of remanufacturing, there seem to be few quantitative papers addressing either the cases of unprofitable CLSC's or the governmental participation in such cases.

As the same logic of unprofitability and incentives is easily extendable to the case of remanufacturing, under the extended and centrally coordinated model framework, we will show how the governmental participation may induce an entry of a new CLSC or prevent an exit of an existing CLSC. We will also provide concrete guidelines for the government efforts regarding these aspects.

Specifically, in this section, we will mathematically characterize the conditions under which one or more members of the CLSC in the basic model are unprofitable. Then we will derive the extent to which such a CLSC can be viable in the long run (i.e., when it is currently viable only in the short run as they are not covering the fixed cost) or the extent to which the governmental participation induces a viable CLSC to materialize (as its members currently do not see themselves viable).

We now proceed to examine the three possible cases for the basic model of (A) only the manufacturer is unprofitable, (B) only the collector is unprofitable, and (C) both manufacturer and collector are unprofitable. In the first three subsections, under the framework of the extended model, we will sequentially examine the cases of (A), (B), and (C). In the fourth subsection, under the framework of the centrally coordinated model, we will re-examine the cases of (A), (B), and (C).

3.6.1 The Case of the Unprofitable Manufacturer only

As in case (A), if the manufacturer of the basic model is unprofitable, then the manufacturer would not be able to operate. This implies that the CLSC is not economically viable, and the corresponding total surplus is zero (TS = 0). Meanwhile from Section 3.3.3, we have

$$TS_E^* = \frac{\left(24(\delta-\beta c_m)-\beta k\left(\alpha^*-5\Delta\right)\left(\alpha^*+\Delta\right)\right)\left(8(\delta-\beta c_m)-\beta k\left(\alpha^*-\Delta\right)\left(\alpha^*+\Delta\right)\right)}{512\beta}-k_C-k_M$$

Hence, if the CLSC in the extended model is economically viable, then the difference in the total surplus levels is given by $\Delta TS = TS_E^*$. Then, the mathematical conditions corresponding to the case (A) when the governmental participation will enable the manufacturer to make nonnegative profit will be:

a. $\Pi_B^{M^*} = \dfrac{\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{256\beta}-k_M<0$ b. $\Pi_B^{C^*} = \dfrac{\Delta^2 k\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{256}-k_C\geq 0$

c. $TS_E^* + \Pi_B^{M^*} \geq 0$

Conditions a and b imply that, under the basic model, the profits from the manufacturer and the collector are negative and nonnegative, respectively. Condition c implies that the difference, ΔTS , is sufficiently large to afford a lump sum subsidy to the manufacturer so that her resulting overall profit is nonnegative. At the same time, if Condition c is not met, then under the extended model, ΔTS is too small to induce the manufacturer to operate in the CLSC. In what follows, let us derive the specific conditions, in terms of the fixed costs of the manufacturer and the collector, under which the government is able to induce an entry of a new CLSC or prevent an exit of an existing CLSC. First, we note that $\alpha*$ in the extended model does not have a closed-form solution, and the corresponding TS_E^* is difficult to manipulate. Hence, we will utilize a lower bound $TS_E\left(\alpha=\frac{\Delta}{2}\right)$

where $TS_E^* > TS_E\left(\alpha=\frac{\Delta}{2}\right)$ and $TS_E\left(\alpha=\frac{\Delta}{2}\right)=\frac{\left(24(\delta-\beta c_m)+\frac{27}{4}\beta k\Delta^2\right)\left(8(\delta-\beta c_m)+\frac{3}{4}\beta k\Delta^2\right)}{512\beta}-k_C-k_M$.

Also, for notational simplicity, we utilize the notations, $A=\delta-\beta c_m$ and $B=\beta k\Delta^2$.

Now, conditions a and b result in $k_M > \frac{(8A+B)^2}{256\beta}$ and $k_C \le \frac{\Delta^2 k(8A+B)}{256}$, respectively.

Condition c on the other hand results in

$$k_M \le \frac{\left(24A+\frac{27}{4}B\right)\left(8A+\frac{3}{4}B\right)}{1024\beta}+\frac{(8A+B)^2}{512\beta}-\frac{k_C}{2}$$

Utilizing the upper bound of k_C in condition b, a sufficient condition for the upper bound of k_M can be obtained as follows:

$$k_M \le \frac{\left(24(\delta-\beta c_m)+\frac{27}{4}\beta k\Delta^2\right)\left(8(\delta-\beta c_m)+\frac{3}{4}\beta k\Delta^2\right)}{1024\beta}+\frac{\left(8(\delta-\beta c_m)+\beta k\Delta^2\right)^2}{512\beta}-\frac{\Delta^2 k\left(8(\delta-\beta c_m)+\beta k\Delta^2\right)}{512}$$
$$=\frac{5120A^2+1408AB+81B^2}{16384\beta}$$

The corresponding ranges of k_C and k_M (i.e., the differences between the upper and lower bounds of k_C and k_M) are given by $\frac{\Delta^2 k(8A+B)}{256}$ and $\frac{1024A^2+384AB+17B^2}{16384\beta}$, respectively.

From our analysis of the bounds, we observe that

1. Given that $k_C \le \frac{\Delta^2 k(8A+B)}{256}$ and $\frac{(8A+B)^2}{256\beta} < k_M \le \frac{5120A^2+1408AB+81B^2}{16384\beta}$, the government will be able to induce an entry of a new CLSC or prevent an exit of an existing CLSC by implementing the linear incentives and fees of the extended model without external financial source or sink.

2. As the remanufacturing cost saving Δ or the collection effectiveness coefficient k increases, the range of k_C (k_M based on the sufficient condition) also increases. That is, the applicability of the governmental participation with respect to the collector's fixed cost k_C (the manufacturer's fixed cost k_M based on the sufficient condition) for inducing entries and preventing exits increases with Δ and k. Conversely, as Δ or k decreases, the justification for the governmental participation with respect to the collector's fixed cost k_C will be less persuasive.

3. As the remanufacturing cost saving Δ or the collection effectiveness coefficient k increases, the range based on the exact value of TS_E^* and k_C for the upper bound of k_M will also increase when the measurement intervals are sufficiently large in Δ or k (showing a monotonic increase in Δ or k is technically difficult).

From these observations, if the government wishes to induce an entry of a new CLSC or prevent an exit of an existing CLSC by implementing the linear incentives and fees of the extended model when only the manufacturer is not profitable, the accurate estimation of Δ and k will be highly critical. In the case of larger Δ and/or k, the governmental participation may be worthwhile with wider areas of applicability with respect to k_C and k_M. In the case of smaller Δ and/or k, on the other hand, the governmental participation may not be enough to induce an entry or prevent an exit, making such governmental efforts ineffective.

3.6.2 The Case of the Unprofitable Collector only

As in case (B), if the collector of the basic model is unprofitable, then the collector would not be able to operate and the reverse flow of products will be inexistent. This implies that the forward channel is the only channel that is economically viable. The manufacturer's objective function in the forward supply chain is stated in (15)

$$\Pi_F^M = (\delta - \beta w)(w - c_m) - k_M \tag{15}$$

In expression (15) the first term represents the demand and the second term represents the profit of selling only new products, since remanufacture is not allowed. It can be shown (see Appendix F) that the unique optimal solution for (15) is $w_F^* = \dfrac{\delta + \beta c_m}{2\beta}$ This optimal solution results in

$$\Pi_F^{M*} = \frac{(\delta - \beta c_m)^2}{4\beta} - k_M \text{ , } CS_F^* = \frac{(\delta - \beta c_m)^2}{8\beta} \text{ and } TS_F^* = \frac{3(\delta - \beta c_m)^2}{8\beta} - k_M$$

In this case, if the CLSC in the extended model is economically viable, the increase in the total surplus is given by $\Delta TS = TS_E^* - TS_F^*$. Then, the mathematical conditions corresponding to the case (B) when the governmental participation will enable the collector to make nonnegative profit will be:

a. $\Pi_B^{M^*} = \dfrac{\left(8(\delta - \beta c_m) + \beta k \Delta^2\right)^2}{256\beta} - k_M \geq 0$ b. $\Pi_B^{C^*} = \dfrac{\Delta^2 k \left(8(\delta - \beta c_m) + \beta k \Delta^2\right)}{256} - k_C < 0$

c. $TS_E^* + \Pi_B^{C^*} \geq TS_F^*$

Conditions a and b imply that, under the basic model, the profits from the manufacturer and the collector are nonnegative and negative, respectively. Condition c implies that the difference ΔTS is sufficiently large to afford a lump sum subsidy to the collector so that his resulting profit will be nonnegative. If condition c is not met, then ΔTS is too small to induce the collector to operate in the CLSC. Following the approach described in subsection 3.6.1, we now derive the specific conditions, in terms of the fixed costs of the manufacturer and the collector, under which the government is able to induce the creation of the reverse flow or prevent the disappearance of it. We will utilize again the lower bound $TS_E\left(\alpha=\frac{\Delta}{2}\right)$. For notational simplicity we utilize $A=\delta-\beta c_m$ and $B=\beta k\Delta^2$

Conditions a and b result in $k_C>\frac{\Delta^2 k\left(8A+B\right)}{256}$ and $k_M\le\frac{\left(8A+B\right)^2}{256\beta}$. Condition c results in

$$k_C\le\frac{\left(24\left(\delta-\beta c_m\right)+\frac{27}{4}\beta k\Delta^2\right)\left(8\left(\delta-\beta c_m\right)+\frac{3}{4}\beta k\Delta^2\right)}{1024\beta}+\frac{\Delta^2 k\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{512}-\frac{3\left(\delta-\beta c_m\right)^2}{16\beta}$$

$$=\frac{2048A^2+1920AB+145B^2}{16384\beta}$$

From the previous analysis we can conclude that:

1. Given that $k_M\le\frac{\left(8A+B\right)^2}{256\beta}$ and $\frac{\Delta^2 k\left(8A+B\right)}{256}<k_C\le\frac{2048A^2+1920AB+145B^2}{16384\beta}$ the government will be able to induce an entry of the reverse flow of products through the implementation of the linear incentives and fees described before.
2. We notice that the observation from case (A) is also true in case (B) so the applicability of the governmental participation increases with Δ and k.
3. The applicability of the governmental participation does not depend on the actual fixed cost for the manufacturer. This implies that the government needs to focus only on the collector in order to see if he can afford a lump sum subsidy to the collector to make him operate in the CLSC.

3.6.3 The Case of the Unprofitable Collector and Unprofitable Manufacturer

In case (C) as in previous case (A) if the manufacturer of the basic model is unprofitable, then the manufacturer would not be able to operate and the CLSC would not be economically viable. This implies that the total surplus is zero. As in case (A) we have that $\Delta TS=TS_E^*$. Then, the mathematical conditions when the governmental participation will enable the manufacturer and the collector to make nonnegative profit will be:

a. $\Pi_B^{M^*} = \frac{\left(8\left(\delta - \beta c_m\right) + \beta k \Delta^2\right)^2}{256\beta} - k_M < 0$ b. $\Pi_B^{C^*} = \frac{\Delta^2 k\left(8\left(\delta - \beta c_m\right) + \beta k \Delta^2\right)}{256} - k_C < 0$

c. $TS_E^* + \Pi_B^{C^*} + \Pi_B^{M^*} \geq 0$

Conditions a and b imply that, under the basic model, the profits from the manufacturer and the collector have negative profits. Condition c implies that the difference ΔTS is sufficiently large to afford a lump sum subsidy to the manufacturer and the collector so that their resulting overall profit is nonnegative. We now derive the specific conditions, in terms of the fixed costs of the manufacturer and the collector, under which the government is able to induce an entry of a CLSC.

Conditions a and b result in $k_C > \frac{\Delta^2 k\left(8A + B\right)}{256}$ and $k_M > \frac{\left(8A + B\right)^2}{256\beta}$, respectively. With the lower bound $TS_E\left(\alpha = \frac{\Delta}{2}\right)$ we can rewrite condition c as $k_C + k_M \leq \frac{5120A^2 + 2176AB + 177B^2}{16384\beta}$

We notice that condition c is stated as the sum of the fixed cost of the supply chain, rather that of the individual costs. The corresponding range for $k_C + k_M$ is given by $\frac{1024A^2 + 640AB + 49B^2}{16384\beta}$

From our analysis of the bounds, we observe that:

1. The government can induce the creation of a CLSC when $k_C + k_M \leq \frac{5120A^2 + 2176AB + 177B^2}{16384\beta}$

2. As the remanufacturing cost saving Δ or the collection effectiveness coefficient k increases the range of the sum of the fixed costs increases. This means that the applicability of the governmental participation with respect to the collector's fixed cost and the manufacturer's fixed cost for inducing entries and preventing exits increases. When both manufacture and collector are unprofitable the government should considerer both fixed costs to analyze the convenience of his participation.

3.6.4 Impact of Centrally Coordinated Model on Unprofitability

We will now show how the governmental participation through the use of the alternative financial instruments described in section 3.5 can induce the entry or prevent the exit of a CLSC. In this subsection follow the same approach as in previous subsection to derive the extent to which such a CLSC can be viable in the long run.

From section 3.5 we have that the government, through the alternative financial instruments can now achieve $TS_{CC}^{*}=\frac{\left(4\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{32\beta}-k_C-k_M$ Following the same analysis that we present in subsections 3.6.1, 3.6.2 and 3.6.3 we show the results for case (A) only the manufacturer is unprofitable, (B) only the collector is unprofitable, and (C) both manufacturer and collector are unprofitable. Table 8 presents the mathematical conditions corresponding to the case (A) (B) and (C) when the governmental participation will enable the manufacturer to make nonnegative profit.

Table 8. Unprofitability with TS_{CC}^{*}

Case	Conditions
Only the manufacturer is unprofitable	$\Pi_B^{M^*}=\frac{\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{256\beta}-k_M<0$ $\Pi_B^{C^*}=\frac{\Delta^2 k\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{256}-k_C>0$ $TS_{CC}^{*}+\Pi_B^{M^*}\geq 0$
Only the collector is unprofitable	$\Pi_B^{M^*}=\frac{\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{256\beta}-k_M>0$ $\Pi_B^{C^*}=\frac{\Delta^2 k\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{256}-k_C<0$ $TS_{CC}^{*}+\Pi_B^{C^*}\geq TS_F^{*}$
The manufacturer and the collector are unprofitable	$\Pi_B^{M^*}=\frac{\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)^2}{256\beta}-k_M<0$ $\Pi_B^{C^*}=\frac{\Delta^2 k\left(8\left(\delta-\beta c_m\right)+\beta k\Delta^2\right)}{256}-k_C<0$ $TS_{CC}^{*}+\Pi_B^{C^*}+\Pi_B^{M^*}\geq 0$

The analysis of condition c and the ranges for the fixed costs are shown in Table 9.

Table 9. Ranges for Fixed Costs under Alternative Financial Instruments where $A = \delta - \beta c_m$ and $B = \beta k \Delta^2$.

Case	Exact conditions as a function of other fixed cost	Exact conditions	Ranges
A	$k_M > \frac{(8A+B)^2}{256\beta}$ $k_M \le \frac{192A^2+80AB+9B^2}{512\beta} - \frac{k_C}{2}$ $k_C \le \frac{\Delta^2 k(8A+B)}{256}$	$k_M > \frac{(8A+B)^2}{256\beta}$ $k_M \le \frac{24A^2+9AB+B^2}{64\beta}$	For k_M $\frac{32A^2+20AB+3B^2}{256\beta}$
B	$k_C > \frac{\Delta^2 k(8A+B)}{256}$ $k_C \le \frac{B(1408A+113B)}{16384\beta}$ $k_M \le \frac{(8A+B)^2}{256\beta}$	The same conditions as in previous column	For k_C $\frac{864A^2+1432AB+179B^2}{13824\beta}$
C	$k_C > \frac{\Delta^2 k(8A+B)}{256}$ $k_M > \frac{(8A+B)^2}{256\beta}$ $k_C + k_M \le \frac{96A^2+44AB+5B^2}{256\beta}$	The same conditions as in previous column	For $k_C + k_M$ $\frac{4640A^2+1444AB+1355B^2}{6912\beta}$

From Table 8 and 9 we notice that observations 2 and 3 from subsection 3.6.1 also applies with the alternative financial instruments. Since this instruments achieve the maximum total surplus the governmental participation may be worthwhile with wider areas of applicability with respect to k_C and k_M .

3.7 Revenue-Sharing Contract Model

In the previous sections, we have formulated and analyzed the decentralized model, the governmental participation model with linear incentives and fees, and the centrally coordinated model with alternative financial instruments by the government. In this section, we first address the question of how the manufacturer and the collector can coordinate themselves without the government by constructing a revenue-sharing contract model. We then examine the impact of such coordination on the economic efficiency and the collection rate.

In the literature, in the absence of the government, there are a multiple number of coordination mechanisms that maximize the total profit of the members of a supply chain. A particular mechanism that has been extensively studied recently is a revenue-sharing contract model by Cachon and Lariviere's (2005). In the context of our paper, the revenue-sharing contract is a coordination mechanism offered by the manufacturer to the collector. This contract modifies the collector's profit function so as to align his own profit maximization to the total profit maximization of the CLSC.

Before the collector decides the level of his collection efforts, *e*, the manufacturer and the collector agree to the following revenue-sharing contract that has two parameters. The first is *d* representing the buyback price per unit paid by the manufacturer to the collector and the second is χ representing the fraction of the collector's revenue that he keeps for himself while $(1-\chi)$ of his revenue is delivered to the manufacturer.

The corresponding mathematical development is as follows: For the CLSC described in Figure 1, the total profit of the manufacturer and the collector, TP, is given by

$$TP = (\delta - \beta w)\left(w - c_m + (\Delta - e)\varphi(e)\right) - k_C - k_M \tag{16}$$

If the manufacturer and the collector behave as a single decision maker, TP can be maximized with respect to the decision variables *w* and *e*. Since TP is concave in *w* and *e* (see Appendix F), the optimal solution is $w_{TP}^* = \dfrac{4(\delta + \beta c_m) - \beta k \Delta^2}{8\beta}$ and $e_{TP}^* = \dfrac{\Delta}{2}$. The corresponding optimal total profit is given by $TP^* = \dfrac{\left(4(\delta - \beta c_m) + \beta k \Delta^2\right)^2}{64\beta} - k_C - k_M$.

One way to induce the collector to set the level of his collection efforts to e^*_{TP} is for the manufacturer and the collector to agree on a contract that results in the collector's profit Π^C being equal to χTP. In order to produce such a contract, let us consider

$$d_{RSC} = \frac{\chi(w-c_m)+ke(e+\chi(\Delta-e))+\dfrac{(1-\chi)k_C-\chi k_M}{\delta-\beta w}}{ke} \text{ and } \chi\in(0,1].$$

From the basic model of Section 3.2, the collector's profit function is given by $\Pi^C_B = (\delta-\beta w)dke-(\delta-\beta w)eke-k_C$. Given the values of the parameters, d_{RSC} and χ, the collector's profit function can be expressed as:

$$\Pi^C_{RSC} = \chi\left[(\delta-\beta w)\left(w-cm+(\Delta-e)ke\right)-k_C-k_M\right] \tag{17}$$

From this, we observe that the contract between the manufacturer and the collector transforms the collector's profit function as a fraction of the total profit of the CLSC. It can be verified (see Appendix F) that the unique optimal solution for (17) is $e^*_{RSC}=\dfrac{\Delta}{2}$. Meanwhile, the manufacturer's profit function is given by

$$\Pi^M_B = (\delta-\beta w)\left(w-c_m+(\Delta-d)ke\right)-k_M \tag{18}$$

Substituting d_{RSC} and e^*_{RSC} into (18) leads to

$$\Pi^M_{RSC} = (1-\chi)\left[(\delta-\beta w)\left(w-c_m+\frac{\Delta^2 k}{4}\right)-k_C-k_M\right] \tag{19}$$

It can also be verified (see Appendix F) that the unique optimal solution for (19) is $w^*_{RSC}=\dfrac{4(\delta+\beta c_m)-\beta k\Delta^2}{8\beta}$. With these results, it can be easily verified that the sum of the collector's and manufacturer's profits is $TP^*=\dfrac{\left(4(\delta-\beta c_m)+\beta k\Delta^2\right)^2}{64\beta}-k_C-k_M$.

In this type of contracts, the actual value of χ depends on the firm's relative bargaining power.

3.8 Comparative Examinations

In the following propositions we state relationship between the individual profits, consumer surplus and the total surplus presented throughout this paper. For proposition 3 and proposition 4 we take the approach of pursuing the goal of having the two partners gain one-half each of the total profit (see Giannoccaro and Pontrandolfo 2002). See Appendix C and D for proof of propositions 3 and 4 respectively.

Proposition 3. With $\chi = 0.5$ if

a. $0 < k_M < \dfrac{16A^2 - 24AB - 7B^2 + 64\beta k_C}{64\beta}$ then the collector's profit is related as

$\Pi_B^{C^*} < \Pi_E^{C^*} < \Pi_{AFI}^{C^*} < \Pi_{RSC}^{C^*}$.

b. $\dfrac{16A^2 - 24AB - 7B^2 + 64\beta k_C}{64\beta} < k_M < \dfrac{512A^2 - 32AB + 5B^2 + 2048\beta k_C}{2048\beta}$ then the collector's profit is related as $\Pi_B^{C^*} < \Pi_E^{C^*} < \Pi_{RSC}^{C^*} < \Pi_{AFI}^{C^*}$.

c. $\dfrac{512A^2 - 32AB + 5B^2 + 2048\beta k_C}{2048\beta} < k_M < \dfrac{(8A + B)^2}{256\beta}$ then the collector's profit is related as

$\Pi_B^{C^*} < \Pi_{RSC}^{C^*} < \Pi_E^{C^*} < \Pi_{AFI}^{C^*}$.

For the manufacturer's perspective, with $\chi = 0.5$ if

a. $0 < k_M < \dfrac{512A^2 - 64AB - 23B^2 + 2048\beta k_C}{2048\beta}$ then the relationship between her profit is

$\Pi_{AFI}^{M^*} < \Pi_{RSC}^{M^*} < \Pi_E^{M^*} < \Pi_B^{M^*}$

b. $\frac{512A^2 - 64AB - 23B^2 + 2048\beta k_C}{2048\beta} < k_M < \frac{32A^2 - B^2 + 128\beta k_C}{128\beta}$ then the relationship

between her profit is $\Pi_{AFI}^{M^*} < \Pi_E^{M^*} < \Pi_{RSC}^{M^*} < \Pi_B^{M^*}$

c. $\frac{32A^2 - B^2 + 128\beta k_C}{128\beta} < k_M < \frac{(8A+B)^2}{256\beta}$ then the relationship between her profit is

$\Pi_{AFI}^{M^*} < \Pi_E^{M^*} < \Pi_B^{M^*} < \Pi_{RSC}^{M^*}$

Proposition 4. The relationship between consumer surplus is $CS_E^* < CS_B^* < CS_{RSC}^* < CS_{AFI}^*$. The relationship between total surplus is $TS_B^* < TS_E^* < TS_{RSC}^* < TS_{AFI}^*$ and for collection rate is $\varphi_B^* < \varphi_E^* < \varphi_{AFI}^* = \varphi_{RSC}^*$

From proposition 3 we observe that the scheme preferred by the collector and the manufacturer depends on the actual value of the fixed costs. However, from the perspective of maximizing the total surplus, the alternative financial instruments are the scheme that the government should incorporate. From Proposition 4, we note that although the economic efficiency of the revenue-sharing contract is less than of the centrally coordinated model, the revenue-sharing contract achieves the maximum collection rate for the CLSC.

3.9 Distributive Issues

Let us address the economic distribution issues that may be necessary to implement the proposed CLSC models of the linear incentives and fees, central coordination, and revenue-sharing contract as follows: In Section 3.5, we have already noted that, under the alternative financial instruments, the manufacturer’s profit would be negative without any distributive action. Hence, let us first address the distribution issues of the negative profit among the various models. We note that, for notational simplicity, throughout the rest of this section, we utilize the notations, $A = \delta - \beta c_m$ and $B = \beta k \Delta^2$, and all variables and objective function values are at their optimal values without *.

As the baseline, we will assume that the profits of the basic model are nonnegative. That is, $\Pi_B^C = \dfrac{B(8A+B)}{256\beta} - k_C \geq 0$ and $\Pi_B^M = \dfrac{(8A+B)^2}{256\beta} - k_M \geq 0$. The initial allocation results are summarized in Table 10. We note that, and $CS_B = \dfrac{(8A+B)^2}{512\beta} > 0$ without further assumption.

Table 10. Initial Allocation Results and the Distributive Actions

Linear incentives and fees	Alternative instruments	Revenue-sharing contract
$\Pi_E^C = \dfrac{(\alpha+\Delta)^2 k\left(8A - \beta k\left(\alpha^2 - \Delta^2\right)\right)}{256} - k_C \geq 0$ $\Pi_E^M = \dfrac{\left(8A + \beta k\left(\Delta^2 - \alpha^2\right)\right)^2}{256\beta} - k_M$ $CS_E = \dfrac{\left(8A + \beta k\left(\Delta^2 - \alpha^2\right)\right)^2}{512\beta} > 0$	$\Pi_{AFI}^C = \dfrac{B(4A+B)}{16\beta} - k_C \geq 0$ $\Pi_{AFI}^M = -\dfrac{B(4A+B)}{16\beta} - k_C \leq 0$ $CS_{AFI} = \dfrac{(4A+B)^2}{32\beta} > 0$	$\Pi_{RSC}^C = \chi\left[\dfrac{(4A+B)^2}{64\beta} - k_C - k_M\right] \geq 0$ $\Pi_{RSC}^M = (1-\chi)\left[\dfrac{(4A+B)^2}{64\beta} - k_C - k_M\right] \geq 0$ $CS_{RSC} = \dfrac{(4A+B)^2}{128\beta} > 0$

The distributive actions that will be necessary to overcome the negative profits so that each member has a sufficient incentive to operate in the various models are as follows:

1. For the case of linear incentives and fees, if Π_E^M is negative, then the gain in the total surplus will be used to make it nonnegative. Specifically, a lump sum coming from the additional profit of the collector will be paid to the manufacturer so that her overall profit is now zero as only the collector gains additional profit relative to the basic model (i.e., $\Pi_B^C < \Pi_E^C$ and $CS_B > CS_E$).

2. For the case of alternative financial instruments, as Π_{AFI}^M is negative, the gain in the total surplus will be used to make it nonnegative. Specifically, a lump sum will be paid to the manufacturer so that her overall profit is now zero. This payment will first come from the additional profit of the collector

(since $\Pi^C_{AFI} > \Pi^C_B$). If the additional profit is not sufficient, then the balance of the payment will come from customers (since $CS_{AFI} > CS_B$). We note that in this case, a tax such as a community-based one that does not influence the customers' purchase decision may be necessary (unless such a tax is negligible to the customers).

3. For the case of revenue-sharing contracts, the nonnegative profits assumption in the basic model implies that the profits from this model are nonnegative as well.

As all unprofitable cases are now addressed, let us proceed to examine what distributive actions are necessary for each party of the basic model to move to the other models without making anybody worse off.

4. For the case of linear incentives and fees, a lump sum coming from the additional profit of the collector will be paid to the manufacturer so that her overall profit is now no less than that in the basic model. Also, a payment such as a year-end rebate that does not influence the customers' purchase decision from the same source will be made to the customers so that the consumer surplus is now no less than that in the basic model (unless such a payment is negligible to the customers).

5. For the case of alternative financial instruments, a lump sum will be paid to the manufacturer so that her overall profit is now no less than that in the basic model. This sum will come from the additional gains of the collector and customers (similar to Case 2).

6. For the case of revenue-sharing contracts, with $\chi = 0.5$, only when $k_M > \dfrac{32A^2 - B^2 + 128\beta k_C}{128\beta}$, a lump sum will be paid to the manufacturer so that her profit is no less than that in the basic model. This sum will come from the additional gains of the collector and customers (similar to Case 2).

We note that there are other distributive issues beyond what we have addressed. For example, the remaining gain in the total surplus after making nobody worse off can be distributed based on the bargaining power of each member of the CLSC. Likewise, the fraction χ may be determined based on the market power of the manufacturer vs. the collector. For both cases, further studies are highly desirable.

The following numerical example shows the difference between the models presented in this paper with respect to the total surplus obtained as well as other parameters.

For this example, the hypothetical values of the parameters are as follows: δ = 1,000, c_m = \$5/unit, Δ = \$4/unit, β = 20, k = 0.45, $k_C = 100$, $k_M = 1000$ and $x = y = 0.5$ and $\chi = 0.5$ The graph in figure 5 shows the total surplus in the different models presented in this paper.

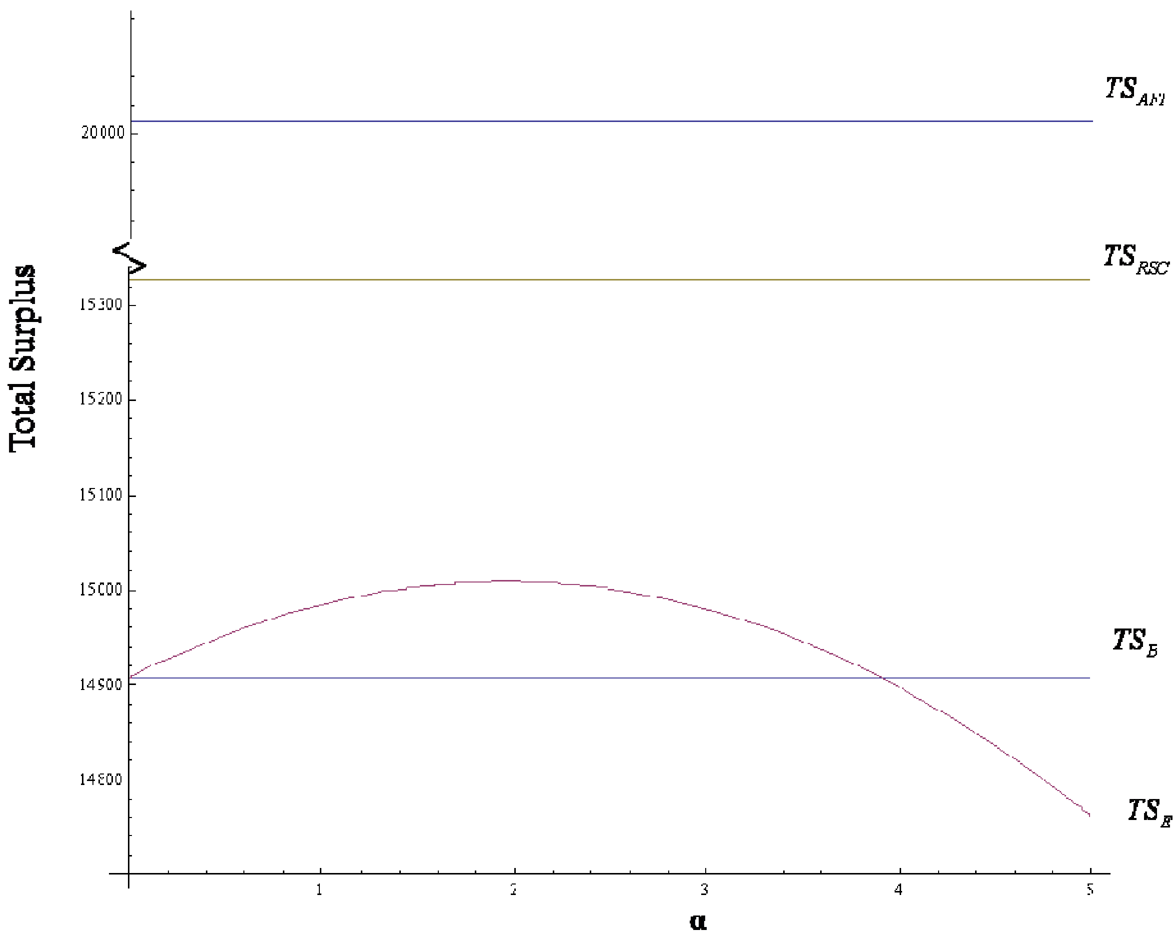

Figure 5. Total surplus for the Basic Model, Extended Model, Alternative Financial Instruments, Revenue- Sharing contract

We can see that relative to the basic model and the extended model, the revenue-sharing contract increases the total surplus. However, there is still a great range of improvement that can actually be achieved with the alternative financial instruments. Table 11 shows the rest of the results for the example.

Table 11. Results for numerical example for Basic, Extended, AFI, Revenue-Sharing Contract.

	Basic Model	Extended Model	Alternative Financial Instruments	Revenue-sharing contract
λ	1	0.51	0	11.9316
d	2	2	2	13.9316
w	27.05	26.501	3.2	26.6
α	NA	1.96	4	NA
ε	NA	NA	1.0385	NA
η	NA	NA	0.2769	NA
Π^C	106.55	356.403	1584.8	4925.6
Π^M	9534.05	9435.1	-2684.8	4925.6
CS	5267.03	5217.55	21902.4	5475.6
TS	14907.6	15009.1	20802.4	15326.8
$\varphi(e)$	0.45	0.6705	0.9	0.9

3.10 Discussion of Results

The results for this chapter show that relative to the basic model, the linear incentives and fees can improve the economic efficiency and collection rate if they are set at the optimal level. This result was derived with the constraint of revenue neutrality. It is important to notice that the analysis without the revenue neutrality will set α as high as possible and $\gamma=0$. The question now arises as to how the incentive is financed. A complete analysis should be done taking into account the source of this incentive, because it might imply a tax charged to other activities or products.

It is also necessary to notice that the management of any program of incentives and fees will generate costs that are not taking into account in any of our models. Further research must be done to derive an approximation of the cost of management per product, since it is possible these costs are for a set of products.

4. Closed-loop Supply Chain with Multiple Manufacturers

4.1 Description of the Supply Chain and Assumptions

In this section, we formulate and analyze a basic closed loop supply chain consisting of two manufacturers who manufactures as well as remanufactures their products of a single kind, customers who directly purchase from the manufacturers and a collector who collects the used products from customers and sells them back to the manufacturers. We assume that the collector consolidates every product collected, classifies the product by brand and then returns the product to its original manufacturer. Figure 6 depicts the configuration of the basic model with competition.

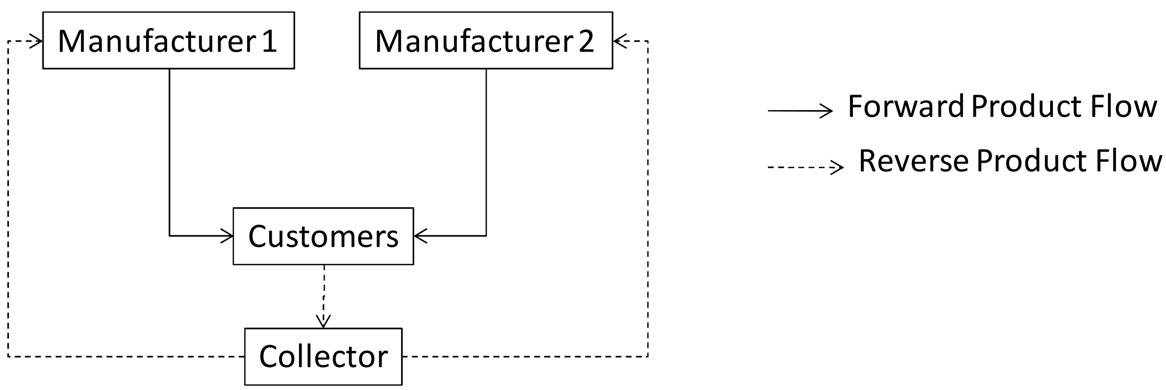

Figure 6. Basic Model with Competition

The two manufacturers produce substitute products, with manufacturer 1 producing an amount q_1 units and manufacturer 2 producing an amount of q_2.

Following Singh and Vives (1984) the representative customer maximizes the consumer surplus that is expressed as $U(q_1, q_2) - \sum_{i=1}^{2} w_i q_i$ where q_i is the amount of good *i* and w_i its price. The utility function is defined as $U(q_1, q_2) = \delta_1 q_1 + \delta_2 q_2 - (\beta_1 q_1^2 + 2\theta q_1 q_2 + \beta_2 q_2^2)/2$ where δ_i and β_i are positive, $\beta_1\beta_2 - \theta^2 > 0$ and $\delta_i\beta_j - \delta_j\theta > 0$ for $i \neq j$. This utility function gives rise to a linear demand structure. The inverse demands are given by $w_1 = \delta_1 - \beta_1 q_1 - \theta q_2$ and $w_2 = \delta_2 - \beta_2 q_2 - \theta q_1$

We also assume that, after consuming the products, the customers are willing to return them to the collector and that the collector incurs a cost e_1 and e_2 per unit collected of product 1 and product 2 respectively. e_i *i=1,2* specifically represents the expenses per unit collected of additional collection bins, business hours, advertising, promotions and campaigns, etc. The collector then sells the collected used products to the manufacturer *i* at a buyback price of d_i per unit. The relation between d_i and e_i is given by $e_i = d_i - \lambda_i$ where λ_i represents the per unit profit margin for the collector.

We assume that the used product collection rate for product *i* is commensurate with the level of collection efforts represented by e_i . $\varphi_i(e_i)$ denotes the collection rate for product *i*, and we assume that $\varphi_i(e_i) = k_i e_i$ for *i=1,2* where $e_i \in \left[0, \frac{1}{k_i}\right]$ and $k_i > 0$. Finally, we denote the manufacturing cost per manufacturer as c_{mi} the manufacturing cost for manufacturer i and c_{ri} the remanufacturing cost per product. We assume that remanufacture cost c_{ri} is less than the manufacturing cost c_{mi} by an amount of Δ_i . In this chapter we do not include fixed costs since our intention is to gain insights about the impact of competition among manufacturers not to derive entry and exit implications that were analyzed before.

ASSUMPTION 6: *All products collected are sold back to their original manufacturer and remanufactured*. We note that this assumption is often found in the literature for simplification (Savaskan and Van Wassenhove 2006). In our model the collector sorts the products and returns each product to its original manufacturer. This assumption is consistent with products as single-used camera's supply chain, where all collected products are consolidated in a single facility and there they are sorted and retuned to their original manufacturer (Kodak One-Time-Use Camera). The reason is that a manufacturer may not have the technology to remanufacture a product other than his (Goldstein, 1994)

4.2 Basic Model

We now proceed to formulate the basic model and derive the equilibrium solution as follows: each manufacturer chooses the quantity to sell and the buyback price using the response function of the collector, conditional on the observed quantity and buyback price of the competitor's product. The collector determines the profit margin of each product so as to maximize total profit from both brands given the respective quantities and buyback prices. The manufacturers take the collector's reaction function into consideration for their respective decisions.

The profit maximization problem for the collector (the follower) given q_1, q_2 and d_1, d_2 by the manufacturers (the leaders) is:

$$Max_{\lambda_1,\lambda_2}\Pi_B^C = q_1k_1(d_1-\lambda_1)\lambda_1 + q_2k_2(d_2-\lambda_2)\lambda_2$$

Because the objective function is strictly concave in λ_i (see Appendix G), the collector's first order condition characterizes the unique best response $\lambda_1 = \frac{d_1}{2}$ and $\lambda_2 = \frac{d_2}{2}$

The manufacturers' problems, on the other hand, are to maximize their profit over w_1, d_1 and w_2, d_2. That is,

$$Max_{q_1,d_1}\Pi_B^{M_1} = \left(\delta_1 - \beta_1 q_1 - \theta q_2 - c_{m1} + \left(\Delta_1 - d_1\right)k_1\left(d_1 - \lambda_1\right)\right)q_1$$

$$Max_{q_2,d_2}\Pi_B^{M_2} = \left(\delta_2 - \beta_2 q_2 - \theta q_1 - c_{m2} + \left(\Delta_2 - d_2\right)k_2\left(d_2 - \lambda_2\right)\right)q_2 \quad (20)$$

Substituting the collector's best response function into (20), the manufacturer's profit maximization problem is given by

$$Max_{q_1,d_1}\Pi_B^{M_1} = \left(\delta_1 - \beta_1 q_1 - \theta q_2 - c_{m1} + \left(\Delta_1 - d_1\right) k_1 \left(d_1 - \frac{\Delta_1}{2} \right) \right) q_1$$

$$Max_{q_2,d_2}\Pi_B^{M\,2} = \left(\delta_2 - \beta_2 q_2 - \theta q_1 - c_{m2} + \left(\Delta_2 - d_2\right) k_2 \left(d_2 - \frac{\Delta_2}{2} \right) \right) q_2$$

The simplest and the most widely used technique for demonstrating the existence of Nash equilibrium is through verifying concavity of the players' payoffs, which implies continuous best response (or reaction) functions. With this approach it can be verified (see Appendix G) that the unique optimal equilibrium solution is: $q^*_{B1} = \dfrac{16c_{m1}\beta_2 - 16\beta_2\delta_1 - 2k_1\beta_2\Delta_1^2 - 8c_{m2}\theta + 8\delta_2\theta + k_2\Delta_2^2\theta}{8(-4\beta_1\beta_2 + \theta^2)}$,

$q^*_{B2} = \dfrac{-16c_{m2}\beta_1 + 16\beta_1\delta_2 + 2k_2\beta_1\Delta_2^2 + 8c_{m1}\theta - 8\delta_1\theta - k_1\Delta_1^2\theta}{32\beta_1\beta_2 - 8\theta^2}$, $d^*_{B1} = \dfrac{\Delta_1}{2}$ and $d^*_{B2} = \dfrac{\Delta_2}{2}$

Given the manufacturers' solution and the collector's equilibrium solution the level of collection efforts is $e^*_{B1} = \dfrac{\Delta 1}{4}$ and $e^*_{B2} = \dfrac{\Delta 2}{4}$ and the corresponding collection rate is given by $\varphi^*_{B1} = k\left(\dfrac{\Delta 1}{4}\right)$ and $\varphi^*_{B2} = k\left(\dfrac{\Delta 2}{4}\right)$. The profits for the collector and the manufacturers at the equilibrium are given by

$$\Pi_B^{C^*} = \frac{k_1\beta_2\Delta_1^2(-8c_{m1} + 8\delta_1 + k_1\Delta_1^2) + k_2\beta_1\Delta_2^2(-8c_{m2} + 8\delta_2 + k_2\Delta_2^2) + (4k_1\Delta_1^2(c_{m2} - \delta_2) + k_2(4c_{m1} - 4\delta_1 - k_1\Delta_1^2)\Delta_2^2)\theta}{64(4\beta_1\beta_2 - \theta^2)}$$

and $\Pi_B^{M^*_1} = \dfrac{\beta_1(16c_{m1}\beta_2 - 2\beta_2(8\delta_1 + k_1\Delta_1^2) + (-8c_{m2} + 8\delta_2 + k_2\Delta_2^2)\theta)^2}{64(-4\beta_1\beta_2 + \theta^2)^2}$,

$$\Pi_B^{M^*_2} = \frac{\beta_2(16c_{m2}\beta_1 - 2\beta_1(8\delta_2 + k_2\Delta_2^2) + (-8c_{m1} + 8\delta_1 + k_1\Delta_1^2)\theta)^2}{64(-4\beta_1\beta_2 + \theta^2)^2}$$

As for the representative customer, we measure his "profits" from his business transactions with the consumer surplus (CS). CS can be defined as the difference between the utility and the total purchase cost to the customer. Hence,

$$CS = U\left(q_1, q_2\right) - \sum_{i=1} w_i q_i$$

At the equilibrium, it can be verified that

$$CS^*{}_B = \frac{1}{128(-4\beta_1\beta_2+\theta^2)^2}(4\beta_1\beta_2(\beta_2(8(\delta_1-c_{m1})+k_1\Delta_1^2)^2+\beta_1(8(\delta_2-c_{m2})+k_2\Delta_2^2)^2)$$
$$-3(\beta_2(8(\delta_1-c_{m1})+k_1\Delta_1^2)^2+\beta_1(8(\delta_2-c_{m2})+k_2\Delta_2^2)^2)\theta^2$$
$$+2(8(\delta_1-c_{m1})+k_1\Delta_1^2)(8(\delta_2-c_{m2})+k_2\Delta_2^2)\theta^3)$$

The economic efficiency will be measured by the total surplus (TS), which is the sum of the profits of the collector and manufacturer as well as the consumer surplus. Hence,

$$TS^*{}_B = \frac{1}{128(-4\beta_1\beta_2+\theta^2)^2}(4\beta_1\beta_2(\beta_2(24(\delta_1-c_{m1})+5k_1\Delta_1^2)(8(\delta_1-c_{m1})+k_1\Delta_1^2)+$$
$$\beta_1(24(\delta_2-c_{m2})+5k_2\Delta_2^2)(8(\delta_2-c_{m2})+k_2\Delta_2^2))-8\beta_1\beta_2(4(32(\delta_1-c_{m1})+5k_1\Delta_1^2)(\delta_2^2-c_{m2})+$$
$$k_2(20(\delta_1-c_{m1})+3k_1\Delta_1^2)\Delta_2^2)\theta-(\beta_2(8(\delta_1-c_{m1})+3k_1\Delta_1^2)(8(\delta_1-c_{m1})+k_1\Delta_1^2)+\beta_1(8(\delta_2-c_{m2})+3k_2\Delta_2^2)$$
$$(8(\delta_2-c_{m2})+k_2\Delta_2^2))\theta^2+4(2(16(\delta_1-c_{m1})+3k_1\Delta_1^2)(\delta_2-c_{m2})+k_2(6(\delta_1-c_{m1})+k_1\Delta_1^2)\Delta_2^2)\theta^3)$$

4.2.1 The Case of Homogeneous Products with Symmetric Cost

To get some insights about the behavior of the supply chain under competition, we now assume that the products are perfect substitutes and that the firms have symmetric cots. This implies that the selling price is the same for the two manufacturers and they only compete on the quantity to sell. We also assume that the $k_1 = k_2 = k$ since the products are indistinguishable for the customer.

We define now $Q = a - bw$ as the total quantity demanded of both products (i.e. $Q = q_1 + q_2$).

Solving for w gives $w = \frac{a}{b} - \frac{Q}{b} = A - BQ = A - B(q1 + q2)$. Finally, we have $c_{m1} = c_{m2} = c_m$ and

$\Delta_1 = \Delta_2 = \Delta$

With this assumptions we the objective function for the collector expressed as

$$\Pi^{C}_{B-H} = q_1 k(d_1 - \lambda_1)\lambda_1 + q_2 k(d_2 - \lambda_2)\lambda_2$$

From concavity of the objective function the unique best response (see Appendix G) for the collector is the set his margins to: $\lambda_{B-H1} = \dfrac{d_1}{2}$ $\lambda_{B-H2} = \dfrac{d_2}{2}$

As for the manufacturers their profits are expressed as

$$\Pi^{M_1}_{B-H} = (A - Bq_1 - Bq_2 - c_m + (\Delta - d_1)k(d_1 - \lambda_1))q_1$$

$$\Pi^{M_2}_{B-H} = (A - Bq_2 - Bq_1 - c_m + (\Delta - d_2)k(d_2 - \lambda_2))q_2$$

As shown in Appendix G, the unique best response is: $q^{*}_{B-H1} = q^{*}_{B-H2} = \dfrac{8(A - c_m) + k(\alpha + \Delta)^2}{24B}$

$d^{*}_{B-H1} = d^{*}_{B-H2} = \dfrac{1}{2}\Delta$ At equilibrium we have that $\Pi^{C^*}_{B-H} = \dfrac{k\Delta^2\left(8\left(A - c_m\right) + k\Delta^2\right)}{192B}$ and

$$\Pi^{M_1^*}_{B-H} = \Pi^{M_2^*}_{B-H} = \frac{\left(8\left(A - c_m\right) + k\Delta^2\right)^2}{576B}$$

The consumer surplus and the total surplus are expressed as $CS^{*}_{B-H} = \dfrac{\left(8\left(A - c_m\right) + k\Delta^2\right)^2}{288B}$ and

$TS^{*}_{B-H} = \dfrac{\left(8\left(A - c_m\right) + k\Delta^2\right)\left(32\left(A - c_m\right) + 7k\Delta^2\right)}{576B}$ respectively. Given the manufacturers' solution and the collector's equilibrium solution the corresponding collection rate is given by

$$\varphi^{*}_{B-H1}(e) = \varphi^{*}_{B-H2}(e) = \frac{k\Delta}{4}.$$

We observe that as Δ increases d^{*}_{B-Hi} increases. That is, the manufacturers will increase the buyback price so as to encourage the collector to provide more used products to each one. From the collector's perspective, the profit margin and the level of the collection efforts both increase as Δ increases, which is intuitive. It is also interesting to observe that the social welfare related measures such as the profits and the surpluses all increase with respect to Δ, which implies the substantial importance of the remanufacturing cost saving on the social welfare.

4.3 Extended Model with Governmental Incentives and Fees

In this section, we extend the basic model by incorporating an incentive α given to the collector per unit collected and a fee to the manufacturers of γ dollars per unit assembled. Figure 7 depicts the participation of the government through the incentives and fees in the basic model.

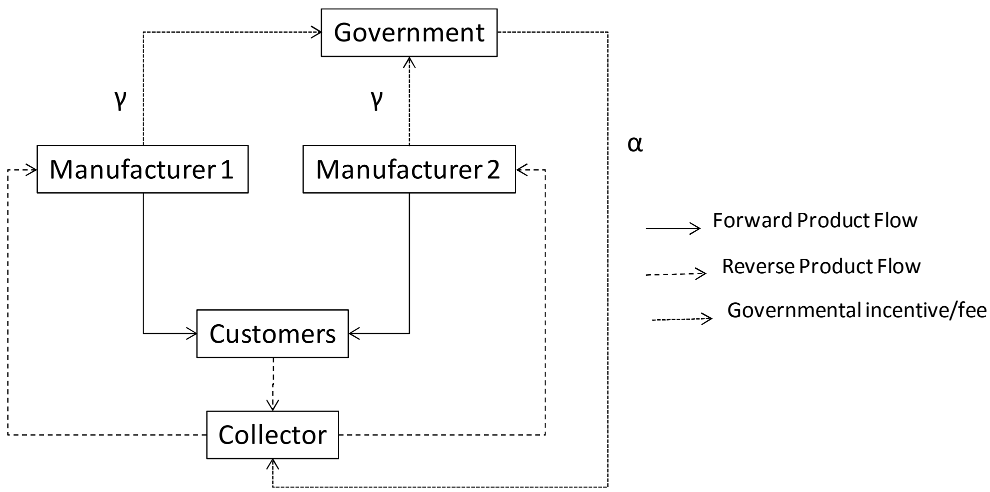

Figure 7. Extended Model with Governmental Incentives and Fees (G) and Competition

With the introduction of these incentives and fees, the basic model from the previous section is extended as follows. The collector's profit expression is given by

$$Max_{\lambda_1,\lambda_2}\Pi_G^C = q_1k_1(d_1-\lambda_1)(\lambda_1+\alpha)+q_2k_2(d_2-\lambda_2)(\lambda_2+\alpha)$$

Since the objective function is strictly concave in λ_i (See Appendix G), from the first order condition, the best unique response is given by $\lambda_{G1}=\dfrac{d_1+\alpha}{2}$ $\lambda_{G2}=\dfrac{d_2+\alpha}{2}$

The manufacturers' problem, on the other hand, is to maximize their profit over w_1, d_1 and w_2, d_2. That is,

$$Max_{q_1,d_1}\Pi_G^{M_1} = \left(\delta_1 - \beta_1 q_1 - \theta q_2 - c_{m1} - \gamma + \left(\Delta_1 - d_1\right)k_1\left(d_1 - \lambda_1\right)\right)q_1$$

$$Max_{q_2,d_2}\Pi_G^{M_2} = \left(\delta_2 - \beta_2 q_2 - \theta q_1 - c_{m2} - \gamma + \left(\Delta_2 - d_2\right)k_2\left(d_2 - \lambda_2\right)\right)q_2 \qquad (21)$$

It can be verified (see AppendixG) that the unique optimal equilibrium solution for (21) is:

$$q_{G1}^* = \frac{16c_{m1}\beta_2 - 2k_1\alpha^2\beta_2 + 16\beta_2\gamma - 16\beta_2\delta_1 - 4k_1\alpha\beta_2\Delta_1 - 2k_1\beta_2\Delta_1^2 - 8c_{m2}\theta + k_2\alpha^2\theta - 8\gamma\theta + 8\delta_2\theta + 2k_2\alpha\Delta_2\theta + k_2\Delta_2^2\theta}{8(-4\beta_1\beta_2 + \theta^2)}$$

$$q_{G2}^* = \frac{-16c_{m2}\beta_1 + 2k_2\alpha^2\beta_1 - 16\beta_1\gamma + 16\beta_1\delta_2 + 4k_2\alpha\beta_1\Delta_2 + 2k_2\beta_1\Delta_2^2 + 8c_{m1}\theta - k_1\alpha^2\theta + 8\gamma\theta - 8\delta_1\theta - 2k_1\alpha\Delta_1\theta - k_1\Delta_1^2\theta}{32\beta_1\beta_2 - 8\theta^2}$$

$$d^*_{G1} = \frac{1}{2}(\Delta_1 - \alpha) \quad d^*_{G2} = \frac{1}{2}(\Delta_2 - \alpha)$$

Let us next examine the impact of the governmental incentives and fees on the environmental efficiency represented by the collection rate. From our definition of the collection rate, at the equilibrium, the level of collection efforts is $e_{G1}^* = \frac{\Delta_1 + \alpha}{4}$ and $e_{G2}^* = \frac{\Delta_2 + \alpha}{4}$ and the corresponding collection rate is given by $\varphi_{G1}^* = k_1\left(\frac{\Delta_1 + \alpha}{4}\right)$ and $\varphi_{G2}^* = k_2\left(\frac{\Delta_2 + \alpha}{4}\right)$.

Compared to the basic model's $\varphi_{B1}^* = k_1\left(\frac{\Delta_1}{4}\right)$ and $\varphi_{B2}^* = k_2\left(\frac{\Delta_2}{4}\right)$, for $\alpha > 0$, the collection rate is strictly higher in the extended model with governmental intervention.

The profits for the collector and the manufacturers at the equilibrium are given by

$$\begin{aligned}\Pi_G^{C^*} = \frac{1}{64(4\beta_1\beta_2 - \theta^2)}&(-8c_{m2}k_2\beta_1(\alpha + \Delta_2)^2 + k_2^2\beta_1(\alpha + \Delta_2)^4 + 4c_{m2}k_1(\alpha + \Delta_1)^2\theta \\ &-k_2(\alpha + \Delta_2)^2(8\beta_1(\gamma - \delta_2) + (-4c_{m1} - 4\gamma + 4\delta_1 + k_1(\alpha + \Delta_1)^2)\theta) \\ &+k_1(\alpha + \Delta_1)^2(-8c_{m1}\beta_2 + \beta_2(-8\gamma + 8\delta_1 + k_1(\alpha + \Delta_1)^2) + 4(\gamma - \delta_2)\theta))\end{aligned}$$

$$\Pi_G^{M_1^*} = \frac{\beta_1(2\beta_2(8c_{m1} + 8\gamma - 8\delta_1 - k_1(\alpha + \Delta_1)^2) + (-8c_{m2} - 8\gamma + 8\delta_2 + k_2(\alpha + \Delta_2)^2)\theta)^2}{64(-4\beta_1\beta_2 + \theta^2)^2}$$

$$\Pi_G^{M_2^*} = \frac{\beta_2(2\beta_1(8c_{m2} + 8\gamma - 8\delta_2 - k_2(\alpha + \Delta_2)^2) + (-8c_{m1} - 8\gamma + 8\delta_1 + k_1(\alpha + \Delta_1)^2)\theta)^2}{64(-4\beta_1\beta_2 + \theta^2)^2}$$

Finally we have that the consumer surplus is expressed as:

$$
\begin{aligned}
CS_G^* = {} & \frac{1}{128(-4\beta_1\beta_2+\theta^2)^2}(256{c_{m1}}^2\beta_1\beta_2^2 - 64c_{m1}k_1\alpha^2\beta_1\beta_2^2 + 4k_1^2\alpha^4\beta_1\beta_2^2 \\
& +512c_{m1}\beta_1\beta_2^2\gamma - 64k_1\alpha^2\beta_1\beta_2^2\gamma + 256\beta_1^2\beta_2\gamma^2 + 256\beta_1\beta_2^2\gamma^2 - 512c_{m1}\beta_1\beta_2^2\delta_1 \\
& +64k_1\alpha^2\beta_1\beta_2^2\delta_1 - 512\beta_1\beta_2^2\gamma\delta_1 + 256\beta_1\beta_2^2\delta_1^2 - 128c_{m1}k_1\alpha\beta_1\beta_2^2\Delta_1 \\
& +16k_1^2\alpha^3\beta_1\beta_2^2\Delta_1 - 128k_1\alpha\beta_1\beta_2^2\gamma\Delta_1 + 128k_1\alpha\beta_1\beta_2^2\delta_1\Delta_1 - 64c_{m1}k_1\beta_1\beta_2^2\Delta_1^2 \\
& +24k_1^2\alpha^2\beta_1\beta_2^2\Delta_1^2 - 64k_1\beta_1\beta_2^2\gamma\Delta_1^2 + 64k_1\beta_1\beta_2^2\delta_1\Delta_1^2 + 16k_1^2\alpha\beta_1\beta_2^2\Delta_1^3 \\
& +4k_1^2\beta_1\beta_2^2\Delta_1^4 - 512\beta_1^2\beta_2\gamma\delta_2 + 256\beta_1^2\beta_2\delta_2^2 - 3(64{c_{m1}}^2\beta_2 - 16k_1\beta_2(\gamma-\delta_1)(\alpha+\Delta_1)^2 \\
& +k_1^2\beta_2(\alpha+\Delta_1)^4 - 16c_{m1}\beta_2(-8\gamma+8\delta_1+k_1(\alpha+\Delta_1)^2)+64(\beta_2(\gamma-\delta_1)^2 \\
& +\beta_1(\gamma-\delta_2)^2))\theta^2 + 16(8c_{m1}+8\gamma-8\delta_1-k_1(\alpha+\Delta_1)^2)(\gamma-\delta_2)\theta^3 \\
& +64{c_{m2}}^2\beta_1(4\beta_1\beta_2-3\theta^2)+k_2^2\beta_1(\alpha+\Delta_2)^4(4\beta_1\beta_2-3\theta^2)+16c_{m2}(-4\beta_1^2\beta_2(-8\gamma \\
& +8\delta_2+k_2(\alpha+\Delta_2)^2+3\beta_1(-8\gamma+8\delta_2+k_2(\alpha+\Delta_2)^2)\theta^2+(8c_{m1}+8\gamma-8\delta_1-k_1(\alpha+\Delta_1)^2)\theta^3) \\
& +2k_2(\alpha+\Delta_2)^2(32\beta_1^2\beta_2(-\gamma+\delta_2)+24\beta_1(\gamma-\delta_2)\theta^2+(-8c_{m1}-8\gamma+8\delta_1+k_1(\alpha+\Delta_1)^2)\theta^3))
\end{aligned}
$$

We analyze now the incentive and fee as active decision variables for the government to maximize the total surplus.

Therefore, the optimization problem now becomes $Max_{\alpha,\gamma} TS_G = CS_G + \Pi_G^{M_1} + \Pi_G^{M_2} + \Pi_G^{C}$ with the equilibrium expressions for $q_{G1}, q_{G2}, d_{G1}, d_{G2}$ and $\lambda_{G1}, \lambda_{G2}$ appropriately substituted into the relevant profit and consumer surplus expressions. Also, we impose the neutrality of revenue in this section. The revenue neutrality constraint is expressed as:

$$(q_1 k_1(d_1-\lambda_1)+q_2k_2(d_2-\lambda_2))\alpha = (q_1+q_2)\gamma$$

Solving for γ gives two possible solutions: $\gamma_1 = C - D$ and $\gamma_2 = C + D$ where

$$
\begin{aligned}
C = {} & -\frac{1}{32(\beta_1+\beta_2-\theta)}(16c_{m2}\beta_1 - 6k_2\alpha^2\beta_1 + 16c_{m1}\beta_2 - 6k_1\alpha^2\beta_2 - 16\beta_2\delta_1 - 8k_1\alpha\beta_2\Delta_1 - \\
& 2k_1\beta_2\Delta_1^2 - 16\beta_1\delta_2 - 8k_2\alpha\beta_1\Delta_2 - 2k_2\beta_1\Delta_2^2 - 8c_{m1}\theta - 8c_{m2}\theta + 3k_1\alpha^2\theta + 3k_2\alpha^2\theta + \\
& 8\delta_1\theta + 4k_1\alpha\Delta_1\theta + k_1\Delta_1^2\theta + 8\delta_2\theta + 4k_2\alpha\Delta_2\theta + k_2\Delta_2^2\theta)
\end{aligned}
$$

$$D = \frac{1}{32(\beta_1+\beta_2-\theta)}\sqrt{(}(-2(\beta_2(-8c_{m1}+8\delta_1+k_1(\alpha+\Delta_1)(3\alpha+\Delta_1))+8\beta_1\delta_2+k_2\beta_1(\alpha+$$
$$\Delta_2)(3\alpha+\Delta_2))+8c_{m2}(2\beta_1-\theta)+(-8c_{m1}+k_1(\alpha+\Delta_1)(3\alpha+\Delta_1)+8(\delta_1+\delta_2)+$$
$$k_2(\alpha+\Delta_2)(3\alpha+\Delta_2))\theta)^2+16\alpha(\beta_1+\beta_2-\theta)(16c_{m2}k_2\beta_1(\alpha+\Delta_2)-2k_2^2\beta_1(\alpha+\Delta_2)^3-$$
$$8c_{m2}k_1(\alpha+\Delta_1)\theta-2k_1(\alpha+\Delta_1)(\beta_2(-8c_{m1}+8\delta_1+k_1(\alpha+\Delta_1)^2)-4\delta_2\theta)+k_2(\alpha+\Delta_2)$$
$$(-16\beta_1\delta_2+(-8c_{m1}+8\delta_1+k_1(\alpha+\Delta_1)(2\alpha+\Delta_1+\Delta_2))\theta))))$$

It is reasonable to assume that the government would prefer to maximize the total surplus with the minimal intervention, which implies that he can select the minimal incentive and fee that meet the objective of maximizing the total surplus. This approach is consistent with the one of minimal intrusion by the government (Skipper and Klein, 1999) With this criterion we select γ_1 as the optimal fee to be charged.

To illustrate the key features of the two models with heterogeneous products described so far, we now present a numerical example. For this example, the hypothetical values of the parameters are as follows: $\delta_1 = \delta_2 = 1000$, $c_{m1} = c_{m2}$ = \$5/unit, $\Delta_1 = \Delta_2$ = \$4/unit, $\beta_1 = \beta_2 = 20$, $\theta = 19$, and $k_1 = k_2 = 0.45$. With these parameter values, we can plot the behavior of TS_G as a function of α and compare it to TS_B as in Figure 8.

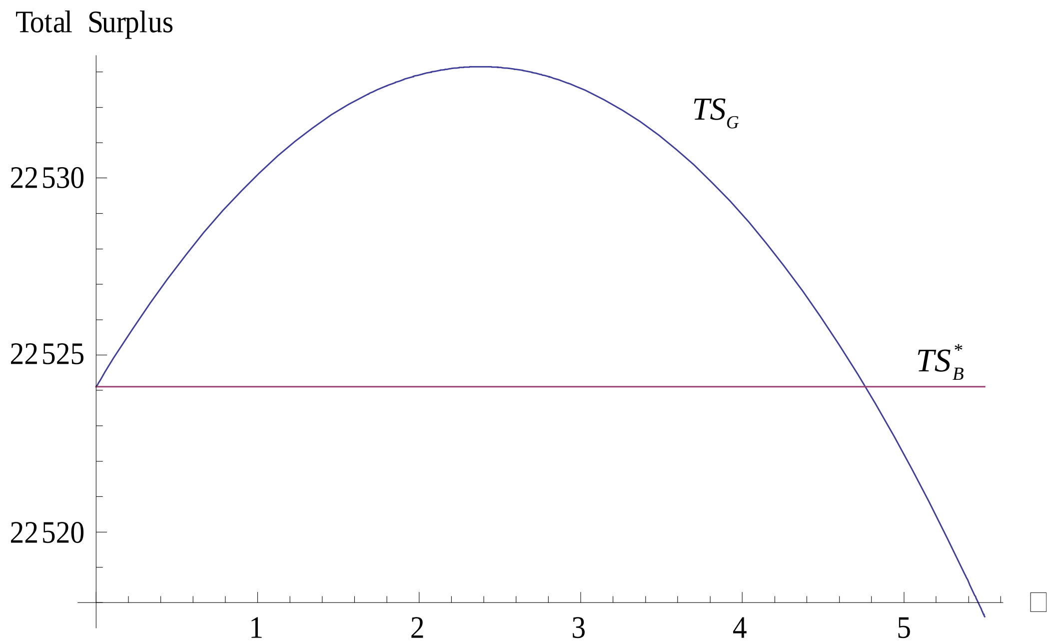

Figure. 8. TS vs α for Basic Model and Extended Model with Competition

We note that with the optimal value of α, the total surplus will increase. On the other hand, with any other choice of α value, it is actually possible that the total surplus will decrease with the governmental participation.

Furthermore, Table 12 summarizes the resulting equilibrium values for the variables and the economic and environmental consequences relative to the basic model.

Table 12. Results for Basic and Extended Model with Competition

Model	λ_i	d_i	q_i	α	γ	Π^C	Π^{M_i}	CS	TS	$\varphi_i(e_i)$
Basic	1	2	16.88	NA	NA	15.19	5698.46	11112	22524.1	0.45
Extended	-0.78	0.81	16.87	2.38	1.71	38.64	5694.81	11104.9	22533.1	0.718

Relative to the basic model the collector decreases his profit margin and still increases his profit due to the sufficient amount of the incentive he receives. For the manufacturer, α induces a decrease in the buy-back price, which contributes to the decline of the profit level. For the customers, the overall effect of the incentives and fees is a decrease in their consumer surplus. We also note that the governmental participation in this example increases the total surplus as well as the collection rate.

From the previous numerical example we can also notice that the optimal α is greater than Δ/2. This is a clear effect of the competition among manufacturers. In Chapter 3, we derived the optimal solution for the incentive α when the supply chain consists of only one manufacturer. In that case $0 < \alpha^* < \frac{\Delta}{2}$, we can see from the numerical example that in the case of two competing manufacturers, the optimal incentive is greater than $\frac{\Delta}{2} = \frac{4}{2} = 2$ which implies that competition increases the flexibility in α*.

4.3.1 Homogeneous Products with Symmetric Costs

We now revise section 4.3 for the case of homogeneous products with symmetric costs. Recall that in the case of homogeneous products, each firm decides only the quantity to sell, given the fact that the selling price should be the same, and its costs are symmetric.

With this assumptions the objective function for the collector is expressed as

$$\Pi^C_{G-H} = q_1 k(d_1 - \lambda_1)(\lambda_1 + \alpha) + q_2 k(d_2 - \lambda_2)(\lambda_2 + \alpha)$$

From concavity of the objective function the unique best response (see Appendix G) for the collector is the set his margins to: $\lambda_{G-H1} = \frac{d_1 - \alpha}{2}$ $\lambda_{G-H2} = \frac{d_2 - \alpha}{2}$

As for the manufacturers their profits are expressed as

$$\Pi^{M_1}_{G-H} = (A - Bq_1 - Bq_2 - c_m - \gamma + (\Delta - d_1)k(d_1 - \lambda_1))q_1$$

$$\Pi^{M_2}_{G-H} = (A - Bq_2 - Bq_1 - c_m - \gamma + (\Delta - d_2)k(d_2 - \lambda_2))q_2$$

As shown in Appendix G, the unique best response is:

$$q^*_{G-H1} = q^*_{G-H2} = \frac{8A - 8c_m + k\alpha^2 - 8\gamma + 2k\alpha\Delta + k\Delta^2}{24B}$$

$$d^*_{G-H1} = d^*_{G-H2} = \frac{1}{2}(-\alpha + \Delta)$$

The consumer surplus is expressed as $CS^*_{G-H} = \frac{(8A - 8c_m - 8\gamma + k(\alpha+\Delta)^2)^2}{288B}$

Given the manufacturers' solution and the collector's equilibrium solution the level of collection efforts is $e^*_{G-H1} = e^*_{G-H2} = \frac{\Delta + \alpha}{4}$ and the corresponding collection rate is given by $\varphi^*_{G-H1} = \varphi^*_{G-H2}$ $= k\left(\frac{\Delta + \alpha}{4}\right)$.

We analyze now the incentive and fee as active decision variables for the government to maximize the total surplus.

As in previous section the optimization problem for the government becomes

$Max_{\alpha,\gamma} TS_{G-H} = CS_{G-H} + \Pi^{M_1}_{G-H} + \Pi^{M_2}_{G-H} + \Pi^{C}_{G-H}$ with the revenue neutrality constraint

$(q_1 k(d_1 - \lambda_1) + q_2 k(d_2 - \lambda_2))\alpha = (q_1 + q_2)\gamma$ which can be simplified as $\gamma = \frac{1}{4}\alpha k(\alpha + \Delta)$

We have now the total surplus

$$TS_{G-H} = \frac{(32(A - c_m) - k(\alpha - 7\Delta)(\alpha + \Delta))(8(A - c_m) - k(\alpha - \Delta)(\alpha + \Delta))}{576B}$$

The first order necessary condition leads to a unique optimal α* as shown in Appendix H. We again rely on indirect bounds of α*. Specifically, we show in Appendix H that $0 < \alpha^* < \frac{3\Delta}{5}$. This implies that $0 < \gamma^* < \frac{6\Delta^2 k}{25}$ while $k(\frac{\Delta}{4}) < \varphi^*_{G-H_i} < k(\frac{2\Delta}{5})$. This in turn implies that the optimal collection rate is strictly greater than that in the basic model $(\varphi^*_{B_i} = k(\frac{\Delta}{4}))$. As for the economic efficiency measured in total surplus, $TS_{G-H} = TS_{G-H(\alpha^*)} > TS_B$ as shown in Appendix I.

Therefore, in so far as the collection rate as well as the economic efficiency measured by the total surplus level, the governmental participation through the incentive and fee is well justified in the case for homogeneous products with symmetric cost relative to the basic model without the governmental participation.

For the extended model and the basic model with homogeneous products, the profits and consumer surplus are compared as follows: $\Pi^C_{G-H} > \Pi^C_{B-H}, \Pi^{M_i}_{G-H} < \Pi^{M_i}_{B-H}$, and $CS_{G-H} < CS_{B-H}$ (see Appendix I). This means that the manufacturers and the customers are better off without the governmental participation. However, the collector increases his profit with the optimal levels of the incentive and fee. As in the case with no competition we can provide a lump sum payment to the manufacturer and customer to encourage them to participate in this scheme.

4.3.2 Multiple Manufacturers

We consider the case for the homogeneous product and symmetric costs when we have *n* manufacturers that compete in a Cournot-game.

The objective function for the collector is expressed as: $Max_{\lambda_i} \Pi^C_M = \sum_{i=1}^{n} q_i k(d_i - \lambda_i)(\lambda_i + \alpha)$

Since the objective function is concave in λ_i the unique optimal solution is $\lambda_{Mi} = \frac{d_i - \alpha}{2}$

Manufacturer *i* maximize

$$\Pi^{M_i}_M = \left(A - B\sum_{i=1}^{n} q_i - c_m - \gamma + (\Delta - d_i)k(d_i - \lambda_i) \right) q_i$$

It can be seen that the unique optimal solution to the variables are:

$$d_{Mi} = \frac{1}{2}\left(\Delta - \alpha\right) \text{ and } q_{Mi} = \frac{-8c_m + k\alpha^2 - 8\gamma + 8A + 2k\alpha\Delta + k\Delta^2}{8(n+1)B}$$

The corresponding expression for the consumer surplus, total surplus and collection rate is expressed as

$$CS_M = A\sum_{i=1}^{n} q_i - \frac{B}{2}\left(\sum_{i=1}^{n} q_i\right)^2 - \left(A - B\sum_{i=1}^{n} q_i\right)\sum_{i=1}^{n} q_i$$

$$TS_M = \frac{\left(8\left(A - c_m\right) + \left(\Delta^2 - \alpha^2\right)k\right)n\left(8\left(2+n\right)\left(A - c_m\right) - \left(\alpha + \Delta\right)k\left(\alpha - 2\Delta n - 3\Delta\right)\right)}{128B\left(1+n\right)^2}$$

$$\varphi_{M_i}(e_i) = k\frac{\Delta + \alpha}{4}$$

As in Chapter 3, the analysis of the optimal α relies on bounds.

Proposition 5. In the case with homogeneous products and symmetric cost where there are n manufacturers that compete in a Cournot-game, the bounds for the optimal incentive is

$0 < \alpha_M^* < \left(\frac{n+1}{n+3}\right)\Delta$ so when $n \to \infty$ $0 < \alpha_M^* < \Delta$

From Proposition 5 (see proof in Appendix J), we observe that as the number of competitors in the supply chain increases, the upper bound for the optimal α also increases, however, the government the incentive should never be set equal to the remanufacturing savings Δ. Let us now compare the models with homogeneous products and symmetric cost described so far with a numerical example. For this example, the parameters are as follows: $A = 100$, c_m = \$5/unit, Δ = \$4/unit, $B = 0.05$, and $k = 0.45$. With these parameter values, we can plot the behavior of TS_G as a function of α, and TS_B^* and compare its behavior. The graph is shown in Figure 9

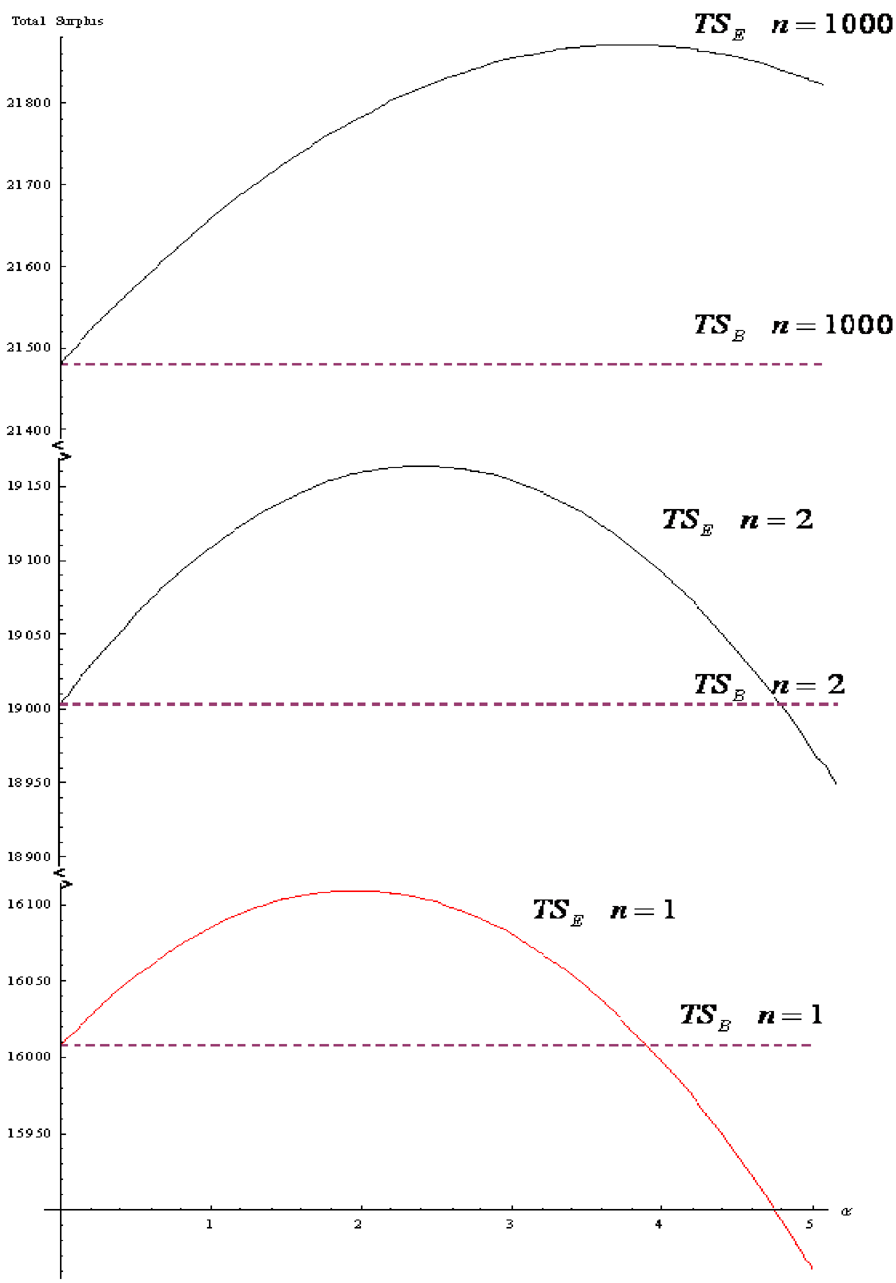

Figure 9. TS for Basic and Extended Model with Homogeneous Products

As we can see from Fig. 9, the total surplus under both the free market and the linear incentive and fee is improved with competition. We also notice that the optimal incentive α* increases as the number of competitors increases and it tends to Δ as $n \to \infty$. The government should be aware of the increase in the size of the program as the incentive that must be given will increase.

4.4 Centrally Coordinated Model: Extended Model

We now analyze the problem from a social planner's perspective. Specifically, we assume that there exists a social planner who centrally coordinates the forward and the reverse flows of products so as to maximize the total surplus. This leads to the upper bound of the total surplus that is theoretically achievable denoted by TS_{CC}^{*}.

From the social planner's perspective, he wants to maximize

$$TS_{CC} = \Pi^{C} + \Pi^{M_2} + \Pi^{M_1} + CS$$

Hence, the total surplus under the central coordination is:

$$TS_{CC} = \frac{1}{2}(-q_2(2c_{m2} + q_2\beta_2 - 2\delta_2 + 2e_2k_2(e_2 - \Delta_2)) - q_1(2c_{m1} + q_1\beta_1 - 2\delta_1 + 2e_1k_1(e_1 - \Delta_1) + 2q_2\theta))$$

We note that, with the social planner as the sole decision maker, the quantity to sell q_i and the cost of collection efforts e_i are the only relevant variables in his maximization of TS_{CC}. It can be verified (see Appendix K) that the unique optimal solution is:

$$q_{CC1} = \frac{4c_{m1}\beta_2 - 4\beta_2\delta_1 - k_1\beta_2\Delta_1^2 - 4c_{m2}\theta + 4\delta_2\theta + k_2\Delta_2^2\theta}{4(-\beta_1\beta_2 + \theta^2)}$$

$$q_{CC2} = \frac{-4c_{m2}\beta_1 + 4\beta_1\delta_2 + k_2\beta_1\Delta_2^2 + 4c_{m1}\theta - 4\delta_1\theta - k_1\Delta_1^2\theta}{4(\beta_1\beta_2 - \theta^2)} \quad e_1 = \frac{\Delta_1}{2} \quad e_2 = \frac{\Delta_2}{2}$$

which implies a maximal theoretical upper bound for the total surplus of

$$TS_{CC} = \frac{\beta_2(-4c_{m1} + 4\delta_1 + k_1\Delta_1^2)^2 + (4c_{m2} - 4\delta_2 - k_2\Delta_2^2)(4c_{m2}\beta_1 - \beta_1(4\delta_2 + k_2\Delta_2^2) + 2(-4c_{m1} + 4\delta_1 + k_1\Delta_1^2)\theta)}{32(\beta_1\beta_2 - \theta^2)}$$

while the corresponding optimal collection rate is given by $\varphi_{CC_i}^{*} = k\dfrac{\Delta_i}{2}$

4.4.1 Centrally Coordinated Model: Homogeneous products with symmetric cost

Let us now revise the model described in previous section for the case of homogeneous products with symmetric cost.

From the social planner's perspective, he wants to maximize

$$TS_{CC-H} = Aq_1 - c_m q_1 - e_1 k q_1 - \frac{Bq_1^2}{2} + Aq_2 - c_m q_2 - e_2^2 k q_2 - Bq_1 q_2 - \frac{Bq_2^2}{2} + e_1 k q_1 \Delta + e_2 k q_2 \Delta$$

We note that, with the social planner as the sole decision maker, the quantity to sell q_i and the cost of collection efforts e_i are the only relevant variables in his maximization of TS_{CC-H}. As for the effort the unique optimal solution (see Appendix K) is: $e_{CC-H1} = e_{CC-H2} = \frac{\Delta}{2}$ As for the quantity to sell, the social planner is not interested in q_i separately, since the products are perfect substitutes any linear combination that results in $q_1 + q_2 = \frac{4A - 4c_m + k\Delta^2}{4B}$ will maximize TS_{CC-H}.

The optimal values for *e* and $q_1 + q_2$ implies a maximal theoretical upper bound for the total surplus of $TS^*_{CC-H} = \frac{(4A - 4c_m + k\Delta^2)^2}{32B}$ while the corresponding optimal collection rate is given by

$$\varphi^*_{CC_i-H} = k\frac{\Delta_i}{2}$$

Proposition 6. For the models with homogeneous products and symmetric cost, the relationship among the levels of the total surpluses and the levels of the collection rates are given by

$TS^*_{CC-H} > TS^*_{G-H} > TS^*_{B-H}$ and $\varphi^*_{CC_i-H} > \varphi^*_{G_i-H} > \varphi^*_{B_i-H}$.

From Proposition 6 (see proof Appendix K), we see that the centrally coordinated model increases the total surplus and the collection rate relative to the basic and extended model. We can now provide a set of instruments that will achieve TS^*_{CC-H} and $\varphi^*_{CC_i-H}$

Let us introduce the following financial instruments of α , ε, and F .

α : incentive given to the collector per unit collected

ε: fee/ incentive that is charged/provided to the manufacturer per unit sold

F : fixed fee that is charged to the manufacturer to enter the market.

It can be shown that the conditions under which ε can be an incentive or fee are determined by the parameters k, Δ , and c_m .

The scheme proposed consists on a two-part fee which can be found in the literature for coordination of the supply chain (De Borger, 2001)

Proposition 7. Theoretical total surplus $TS^*_{CC-H} = \frac{(4A-4c_m+k\Delta^2)^2}{32B}$ can be achieved with α, ε and F defined as follows: $\alpha = \Delta$, $\varepsilon = \frac{1}{8}\left(-4\left(A-c_m\right)+\Delta^2 k\right)$ and $F = \frac{\left(4\left(A-c_m\right)+\Delta^2 k\right)\left(4\left(A-c_m\right)+3\Delta^2 k\right)}{64B}$

It can be shown (see Appendix K) that the scheme defined in Proposition 7 achieves TS^*_{CC-H}

Table 13 summarizes the levels of profits and surpluses as well as the corresponding decision variables for the allocation mechanism described above.

Table 13. Results for the New Scheme of Incentives and Fees in the Model with Competition

	Objective Function	Decision Variable(s)
Collector's profit	$\Pi^C = \frac{\Delta^2 k\left(4\left(A-c_m\right)+k\Delta^2\right)}{16B}$	$\lambda_i = -\frac{\Delta}{2}$
Manufacturers' profit	$\Pi^{M_i} = -\frac{\Delta^2 k\left(4\left(A-c_m\right)+k\Delta^2\right)}{32B}$	$q_i = \frac{1}{8B}\left(4\left(A-c_m\right)+\Delta^2 k\right)$ $d_i = 0$
Consumer Surplus	$CS = \frac{\left(4\left(A-c_m\right)+k\Delta^2\right)^2}{32B}$	N/A
Total Surplus	$TS = \frac{(4(A-c_m)+k\Delta^2)^2}{32B}$	N/A

We now present a numerical example that illustrates the key features of the proposed allocation mechanisms. For this example, the hypothetical values of the parameters are as follows: A = 1,000, c_m = \$5/unit, Δ = \$4/unit, B = 20, and k = 0.45. Given these data, $\alpha = \Delta = 4$, $F = 12465$, and $\varepsilon = -496.6$. This leads to Table 14, which summarizes the results for all variables, profits, and surpluses.

Table 14. Results of the Numerical Example for the Alternative Incentives and Fees in the Model with Competition.

α	F	ε	q	d	λ	$\varphi(e)$	Π^M	Π^C	CS	TS
4	12465	-496.6	24.92	0	-2	0.9	-44.856	89.712	24890.3	24840.3

Given Table 14, the increase in total surplus $TS_{\Delta} = 2785$ and the ratio of improvement with respect to the basic model is 0.1262. To induce the manufacturers' optimal behavior a lump sum of \$6250.72 each that can be financed with a lump sum payment of \$72.89 and \$12428.54 from the collector and representative customer respectively. Similar approaches in the use of a lump sum payment or fee can be seen in Zhao and Wang (2002) and Kim and El Ouardighi (2007).

After the lump sum payment are given to the manufacturers, we can see that each member of the supply chain increases his profit in 12.62%

4.4.3 Approximation for Extended Model

With the results obtained for the centrally coordinated model with homogeneous products and symmetric cost, we approximate the incentives and fees defined before to capture the differentiation between firms.

Let approximate the parameters as $A = \frac{\delta_1 + \delta_2}{2}$ $B = \frac{\beta_1 + \beta_2}{2}$ $cm = \frac{c_{m1} + c_{m2}}{2}$ $k = \frac{k_1 + k_2}{2}$

$\Delta = \frac{\Delta_1 + \Delta_2}{2}$

The approximate incentives and fees that will be applied to the extended model with governmental intervention are:

$$\alpha' = \left(\frac{\Delta_1+\Delta_2}{2}\right)\ \varepsilon' = \frac{1}{8}\left(-4\left(\left(\frac{\delta_1+\delta_2}{2}\right)-\left(\frac{c_{m1}+c_{m2}}{2}\right)\right)+\left(\frac{\Delta_1+\Delta_2}{2}\right)^2\left(\frac{k_1+k_2}{2}\right)\right)$$ and

$$F' = \rho_1\left(\frac{1}{8}k_1\left(3\Delta_1+\Delta_2\right)\alpha+\varepsilon\right)+\rho_2\left(\frac{1}{8}k_2\left(3\Delta_2+\Delta_1\right)\alpha+\varepsilon\right)$$ where

$$\rho_1 = \frac{-2\beta_2\left(-80\left(\delta_1-c_{m1}\right)-16\left(\delta_2-c_{m2}\right)+k_2\left(\Delta_1+\Delta_2\right)-k_1\left(17\Delta_1^2+10\Delta_1\Delta_2+\Delta_2^2\right)\right)}{64\left(4\beta_1\beta_2-\theta^2\right)}+\frac{\left(-16\left(\delta_1-c_{m1}\right)-80\left(\delta_2-c_{m2}\right)+k_1\left(\Delta_1+\Delta_2\right)^2-k_2\left(\Delta_1^2+10\Delta_1\Delta_2+17\Delta_2^2\right)\right)\theta}{64\left(4\beta_1\beta_2-\theta^2\right)}$$

and

$$\rho_2 = \frac{-2\beta_1\left(-80\left(\delta_2-c_{m2}\right)-16\left(\delta_1-c_{m1}\right)+k_1\left(\Delta_1+\Delta_2\right)-k_2\left(17\Delta_2^2+10\Delta_1\Delta_2+\Delta_1^2\right)\right)}{64\left(4\beta_1\beta_2-\theta^2\right)}+\frac{\left(-16\left(\delta_2-c_{m2}\right)-80\left(\delta_1-c_{m1}\right)+k_2\left(\Delta_1+\Delta_2\right)^2-k_1\left(\Delta_2^2+10\Delta_1\Delta_2+17\Delta_1^2\right)\right)\theta}{64\left(4\beta_1\beta_2-\theta^2\right)}$$

We now compare the efficiency that can be achieved with the approximation for the incentives and fees described previously (i.e. α', ε' and F')

We define the *ratio* as the quotient between the TS_{approx} (the TS that is achieved with α', ε' and F') and TS_{CC}^{*} (the theoretical upper bound).

Because of the complexity of the expression, we analyze the ratio with numerical examples. The hypothetical parameters values are $\delta_1 = \delta_2 = 1000$ $\beta_1 = \beta_2 = 20$ $c_{m1} = c_{m2} = 5$ $k_1 = k_2 = 0.45$ $\Delta_1 = \Delta_2 = 3$ $\theta = 10$

Figure 10 shows the change in ratio when the manufacturers differ only with respect to the remanufacture savings.

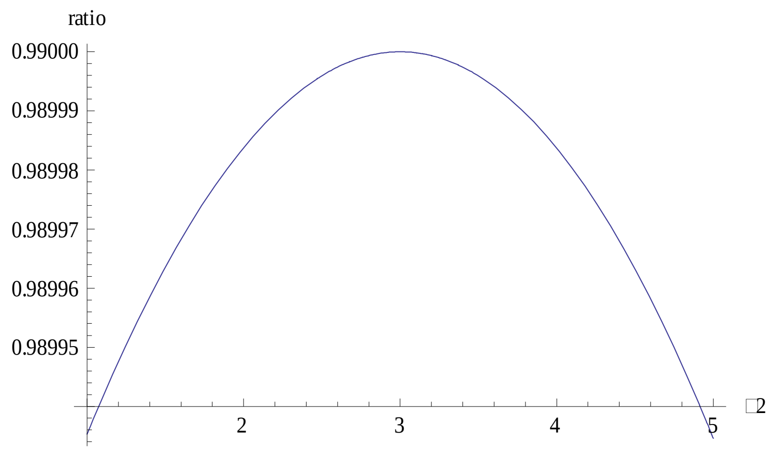

Figure 10. Ratio vs Δ_2

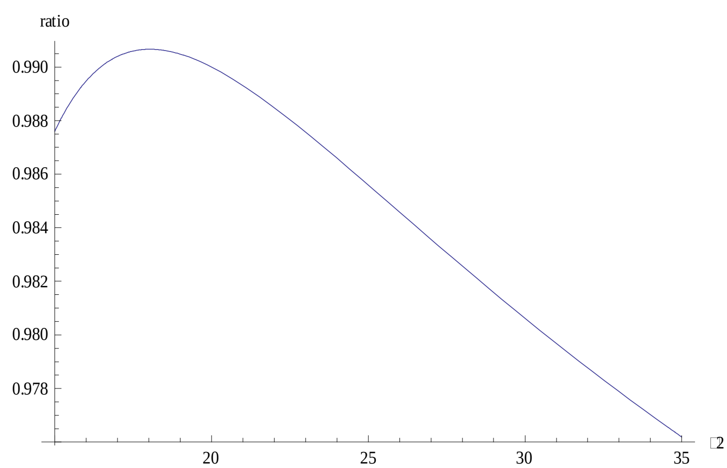

Figure 11. Ratio vs β_2

In Figure 11 we show the behavior of the ratio when the manufacturers differ only in parameter β. We notice that, in this case, the effect of changing β is not symmetric, as oppose to the case of the remanufacture savings.

Figures 12 and 13 show the change in the ratio when the manufacturers are not symmetric in k and δ respectively. Again, we notice that the change in k produces a symmetric behavior for the ratio, will changing a parameter in the inverse demand function produces a different effect as it increases or decreases.

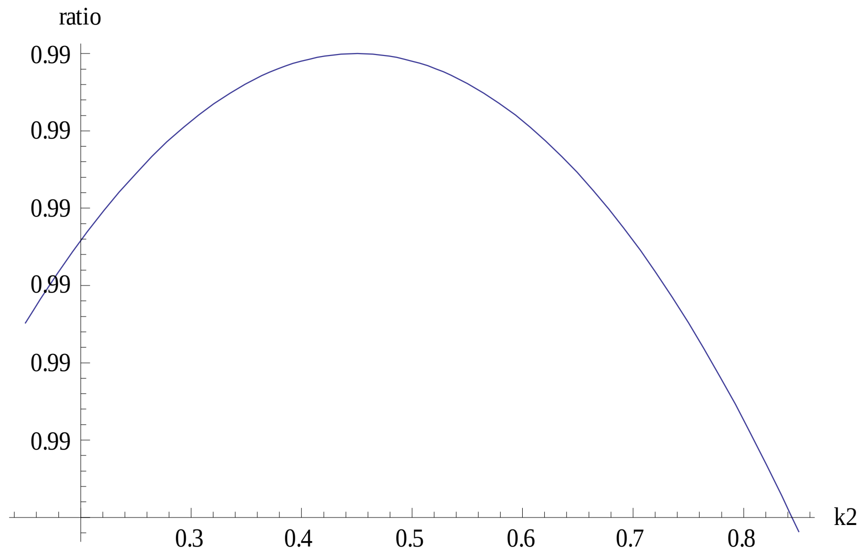

Figure 12. Ratio vs k_2

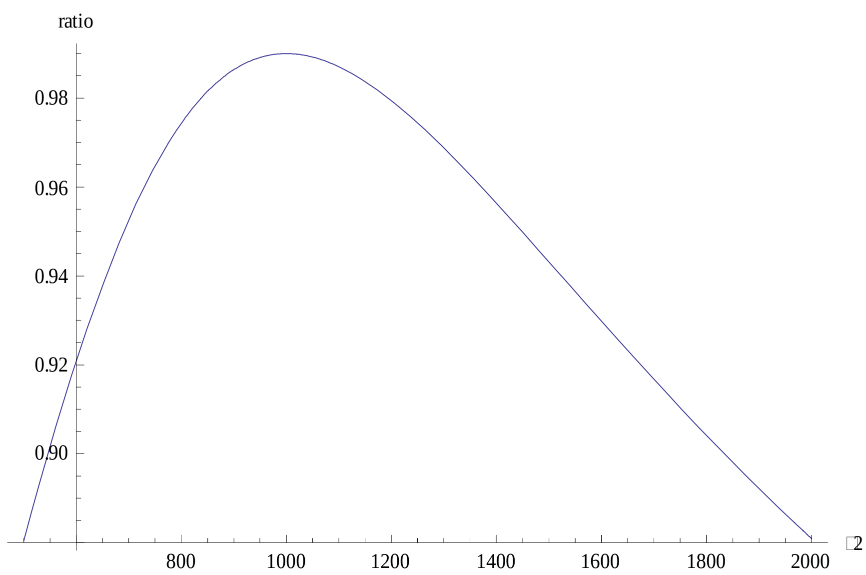

Figure 13. Ratio vs δ_2

From the numerical examples we see that as the parameters become symmetric the ratio increases. This is intuitively since, as the parameters become symmetric, we are approximating the case of homogeneous products with symmetric cost. Conversely, as the parameters become more different the ratio decreases which implies that the $TS_{approx} << TS_{CC}$

We can see that, although the approximation of financial instruments does not achieve the theoretical upper bound, it can still increase the total surplus in cases where the manufacturers are similar in their cost structure and the products that they sell.

4.5 Discussion of the Results

In the case of multiple manufacturers, the similarity of the products and the firms represent a major aspect to considerate. Governmental incentives and fees must account these differences since a uniform incentive can be less effective as the companies are less similar.

As the number of competitors increases, the flexibility of the incentive also increases. However, it is important to notice that the incentive provided has a limit, even though the number of competitors increases up to infinity.

The numerical analysis presented in this chapter and in previous one, where derived with hypothetical data. Further analysis must be done with real data, to derive conclusions for particular supply chains.

5. Extension to Some Current Policies Mexico

According to the study “Diagnóstico sobre la generación de basura electrónica en México”, each year Mexico generates between 150,000 and 180,000 tons of electronic waste (Urgen a controlar basura electrónica, 2008). Although there have been juridical and environmental efforts in the subject, Mexico still does not have a regulation in this matter.The country does not have clear politics of how to recover, recycle or remanufacture the electronic waste. Just a small percentage of the televisions, sound systems, computers or cellular phones that are discarded in the country goes through controlled process for their recycle or remanufacture. Approximately only 5 percent is recycle when most of the products can be reuse or recycle up to 100% (Organizan Reciclón Querétaro 2009)

A major reason for this low collection rate is that there are few companies in Mexico that can accept this waste for further use. For example, HP has a program for collecting their products at the end of their life that is currently operating in USA, Europe and China, but there is no program implemented in Mexico (Green Peace México). The environmental law that is active and that attempts to control the e-waste generation is Ley General para la prevención y gestión integral de los residuos 2007. This law classifies the electronic waste in the group of waste of special treatment. However there is no specific scheme that defines the responsibility of each member of the supply chain in the recovery of the products.

In the case of Mexico, our findings suggest that:

a. There is a need of an environmental law that clearly defines the responsibility of each member of the supply chain, in order to actually improve the collection rate and the economic efficiency. Little can be done to analyze the effect of such incentives and fees, if there is no clear definition of who is responsible of what.

b. As shown in Chapter 3, there is the possibility that the governmental participation can be useless if there is a high fixed cost for the collector and/or the manufacturer. This is probably the case for Mexico, since there is no structure for the reverse flow (Román, 2007). The government must analyze the characteristics of a particular supply chain to see if the approach presented in this paper is appropriated or there is a need of alternative incentives that will first make viable the supply chain and then improve the economic efficiency.

c. The case of the unprofitable collector is very plausible in the context of Mexico’s culture. The current collection rates are low, since there is no participation of the customers. An analysis of the variables that affect the collection rate should be done in order to fully understand the behavior of the customers and derive appropriate guidelines. A possible approximation for the current collection rates can be model in a dynamic model which might me able to capture the dynamics of the market and the behavior of the customers.

d. The government must quantify the economic benefit of the reusing the electronic waste to be in position of setting the values of the optimal incentives and fees.

e. About the scheme of incentives and fees, through this study we show that a linear approach can improve the economic efficiency and the collection rate. Although there are alternative schemes that could provide better results, since the linear incentives are quite intuitive, our suggestion is that it could be used as a first approach.

f. As discuss in section 3.10, the cost of managing the incentives and fees can be quite complex to account for. This is a particular problem in the context of Mexico, since our governmental structures have shown incapability of managing other kinds of taxations.

g. A uniform incentive and fee can increase the efficiency of the supply chain and could allow companies to start developing their network structure for collection. So an approach of letting each state implement its own scheme could generate more economic and logistics complications for the members of the supply chain.

6. Concluding Remarks and Future Research

In this paper, we have investigated the economic efficiency of a series of CLSC's subject to the governmental participation via incentives and fees. Specifically, this study had the following research objectives:

(1) To analyze the impact of governmental participation via incentives and fees in the economic efficiency of a closed-loop supply chain.

(2) To investigate alternative mechanism that can improve the economic efficiency of a closed-loop supply chain.

(3) To analyze the impact of competition among manufacturers in the economic efficiency of a closed-loop supply chain with governmental participation.

To achieve these objectives first, we formulated and analyzed a basic model of a CLSC consisting of a manufacturer/remanufacturer, customers, and a collector for a product of a single kind in a Stackelberg game. We then extended the basic model by incorporating fairly large classes of governmental incentives and fees in a single modeling framework, and showed how the government could determine the optimal levels of the incentives and fees that would maximize the total surplus under the revenue neutrality requirement. Next, we revisited the extended model from a social planner perspective of central coordination, and showed how the theoretical upper bound of the total surplus could be achieved utilizing three financial instruments, and presented the allocation and distribution mechanisms for the centrally coordinated model. In addition, we derived the quantitative conditions under which the government participation induces an entry or prevents an exit of a CLSC when one or two members of the CLSC are unprofitable. Moreover, through the revenue-sharing contract model, we showed how the economic efficiency compared to the basic model improves without the government participation and how the collection rate is greater than or equal to that in any other model. However, we note that when one or two members of a CLSC are unprofitable, it is difficult to imagine that a revenue-sharing contract induces an entry or prevents an exit as the credibility of the government far exceeds that of an unprofitable supply chain member. We also note that, even though the revenue-sharing contract maximizes the sum of the member profits, the corresponding economic efficiency is strictly less than that in the centrally coordinated model with the alternative financial instruments (in fact, it can be verified that the revenue-sharing contract achieves at most 75% of the theoretically maximal economic efficiency).

In addition, we note that there may be aforementioned distributive challenges when the models of the government participation and the revenue-sharing contract are implemented. e.g., in some cases, taxes and rebates could influence the customers' purchase decisions. We also note that, conceptually, by taking a representative customer approach (see e.g., Mas-Colell (1995); cf. the assumption of the numerous customers in our paper), the narrative and exposition could be simplified. On the other hand, the challenges in practice will more or less remain the same. Additional challenges in implementing relatively sophisticated contracts such as the information requirements are extensively illustrated in Cachon and Lariviere (2005).

To investigate the impact of competition, we formulated and analyzed a basic model of a CLSC consisting of two manufacturers/remanufacturer, customers, and a collector for two products of a single kind in a Stackelberg-Cournot game. We extended the basic model by incorporating a scheme of governmental incentives and fees and examined the implications of the governmental intervention. We then analyze the symmetric case for the firms, where the manufacturers sell a homogeneous product that is indistinguishable to the customer. With this model we showed how the government could determine the optimal levels of the incentives and fees that would maximize the total surplus under the revenue neutrality requirement. Next, we revisited the model of homogeneous products in the case of multiple manufacturers with symmetric costs. Finally, we analyze the models from a social planner perspective of central coordination, and showed how the theoretical upper bound of the total surplus for the symmetric case could be achieved utilizing three financial instruments and present a numerical analysis for the case of heterogeneous products.

There are a multiple number of critical managerial insights and economic implications relevant to the academics and practitioners including policy and decision makers of governmental incentives and fees for environmental purposes on one or more members of CLSC's:

(1) Relative to the basic model of a free market CLSC, the extended model clearly demonstrates that the linear incentives and fees can improve the economic efficiency and collection rate. Also, in the extended model, the economic and environmental consequences of the profits, surpluses, and the collection rate are independent of the fractions which determine how much each party would receive incentives and/or pay fees. Hence, for example, the policy and decision makers can focus on improving the economic efficiency and collection rate, and perhaps leave the fractions to a political settlement. On the other hand, we note that if the governmental incentives and fees are not at the optimal level, there is no guarantee that the economic efficiency is improved. In fact, the total surplus level of the basic model can be strictly greater than that of the extended model with non-optimal levels of incentives and fees.

(2) Theoretically, it is quite possible to achieve the upper bound of the total surplus by utilizing a combination of conventional as well as more creative financial instruments. In such a case, the allocation mechanism is straightforward in theory. On the other hand, in practice, the distribution mechanism could become a challenge as a community that includes the customers may not be amenable to a lump sum tax.

(3) For a given range of the fixed costs, the government will be able to induce an entry of a new CLSC or prevent an exit of an existing CLSC under the extended model. Furthermore, the applicability of the governmental participation for inducing entries and preventing exits with respect to k_C at given k_M (or k_M at given k_C) increases with Δ and k. This implies that when the government aims to induce an entry of a new CLSC or prevent an exit of an existing CLSC, the accurate estimation of Δ and k will be highly critical. In the case of smaller Δ and/or k, the

governmental participation may not be enough to induce an entry or prevent an exit, making such governmental efforts ineffective.

(4) Although the manufacturer and the collector can agree on a coordination mechanism such as a revenue sharing contract without any governmental participation, this contract can only achieve the maximum total profit of the supply chain, but not the maximum total surplus of the CLSC.

(5) In the case, where multiple manufacturers, the upper bound for the optimal incentive increases as the number of manufacturers increase. This implies that the government must provide more flexibility in the setting of the optimal incentives and fees as the competition increases. However, with the scheme of intervention described, the government should not provide an incentive higher than the remanufacturing savings.

(6) In the case of heterogeneous products, there is no straightforward derivation of the optimal financial instruments, since a uniform incentive and fee would not be able to achieve the theoretical upper bound. So the government must analyze different approximations in order to improve as much as possible the economic efficiency and collection rate.

To our knowledge, this study is the first paper that, in the context of CLSC's, game-theoretically models the governmental participation via active and somewhat creative pricing of incentives and fees, and analyzes the economic and environmental impacts. As such, there are numerous opportunities for further studies. For example, a study of relaxing one or more of the simplifying assumptions is also desirable as it could generalize our findings and expand their applicability. Some other environmental measures can be incorporated to reinforce the environmental impact of governmental incentives and fess. The assumption that customers make no distinction between new and remanufactured products can be relaxed by treating them as imperfect substitute products. An analysis of a two period game can be used for this relaxation, perhaps with the use of simulation. The assumption that the demand and the collection rate functions, as well as the structure of the incentives and fees, are linear can also be relaxed by considering various nonlinear relationships such as a polynomial function of a higher degree or an exponential/natural log function. Further analysis can be done to analyze the government's problem focusing in the collection rate (i.e. maximize collection rate) .In this paper the remanufacturing savings are the main driver for collection the used products, different analysis must be done to consider the case where Δ is negative.

This study analytically and quantitatively demonstrates that the governmental incentives and fees have major impacts on CLSC's from both economic and environmental perspectives. Accordingly, it is hoped that the government will make deliberate and purposeful choices of the kinds of financial instruments to be utilized and the levels of incentives and fees to each member of CLSC's.

Appendix A. Concavity and Optimality of Profit Functions in Basic and Extended Model

For Basic Model: For Π_B^C to be strictly concave in λ, we have $\frac{d^2\Pi_B^C}{d\lambda^2} = -2k(\delta - \beta w) < 0$. For Π_B^M, $\frac{\partial \Pi_B^M}{\partial w} = \delta + \frac{1}{2}\beta\left(2c_m + d(d-\Delta)k - 4w\right) = 0$ and $\frac{\partial \Pi_B^M}{\partial d} = (\delta - \beta w)k\frac{1}{2}(\Delta - 2d) = 0$. The only solution that satisfies the first order necessary conditions (FONC's) is $w_B = \frac{8(\delta + \beta c_m) - \beta k \Delta^2}{16\beta}$ and $d_B = \frac{\Delta}{2}$. For the second order sufficient conditions (SOSC's), we have $\frac{\partial^2 \Pi_B^M}{\partial w^2} = -2\beta < 0$ and $\frac{\partial^2 \Pi_B^M}{\partial d^2} = -k(\delta - \beta w) < 0$. At $d_B^* = \frac{\Delta}{2}$, $\left(\frac{\partial^2 \Pi_B^M}{\partial w^2}\right)\left(\frac{\partial^2 \Pi_B^M}{\partial d^2}\right) > \left(\frac{\partial \Pi_B^M}{\partial w \partial d}\right)^2$ is met because $2\beta k(\delta - \beta w) > 0$. Hence, $w_B = \frac{8(\delta + \beta c_m) - \beta k \Delta^2}{16\beta}, d_B = \frac{\Delta}{2}$ are the unique optimal solution.

For Extended Model: For Π_E^C to be strictly concave in λ, we have $\frac{d^2\Pi_E^C}{d\lambda^2} = -2k\left(\delta - \beta(w + \gamma y)\right)$ < 0. For Π_E^M, $\frac{\partial \Pi_E^M}{\partial w} = \delta - \beta\left(-c_m - \gamma(1-y) + \left(\Delta - d + \alpha(1-x)\right)k\left(\frac{d + \alpha x}{2}\right) + 2w - \gamma y\right) = 0$ and $\frac{\partial \Pi_E^M}{\partial d} = \left(\delta - \beta(w + \gamma y)\right)k\frac{1}{2}\left(\Delta - 2d + \alpha(1-2x)\right) = 0$. The only solution that satisfies the FONC's is $w_E = \frac{8\left(\delta + \beta\left(c_m + \gamma(1-2y)\right)\right) - \beta k(\alpha + \Delta)^2}{16\beta}$ and $d_E = \frac{\Delta + \alpha(1-2x)}{2}$. For the SOSC's, we have $\frac{\partial^2 \Pi_E^M}{\partial w^2} = -2\beta < 0$ and $\frac{\partial^2 \Pi_E^M}{\partial d^2} = -k\left(\delta - \beta(w + \gamma y)\right) < 0$. At $d_E^* = \frac{\Delta + \alpha(1-2x)}{2}$, $\left(\frac{\partial^2 \Pi_E^M}{\partial w^2}\right)\left(\frac{\partial^2 \Pi_E^M}{\partial d^2}\right) > \left(\frac{\partial \Pi_E^M}{\partial w \partial d}\right)^2$ is met because $2\beta k\left(\delta - \beta(w + \gamma y)\right) > 0$. Hence, the unique optimal solution is $w_E = \frac{8\left(\delta + \beta\left(c_m + \gamma(1-2y)\right)\right) - \beta k(\alpha + \Delta)^2}{16\beta}$ and $d_E = \frac{\Delta + \alpha(1-2x)}{2}$.

Appendix B. Expression for α^*, its Uniqueness and Bounds

For the extended model described in Section 3.3.3, with the help of *Mathematica*, we have:

$$\alpha^* = \frac{1}{2}\left[-S + 3\Delta + \frac{\sqrt{\beta k\left(64(\delta - \beta c_m) - 3\beta k\left(S^2 - 2S\Delta - 7\Delta^2\right)\right)}}{\beta k}\right]$$

where

$$S = \Delta - \left(2^{1/3}\left(48\beta\left(\beta c_m k - \delta k\right) - 18\beta^2\Delta^2k^2\right)\right) / \left[3\beta k\left(-216\beta^3 c_m \Delta k^2 + 216\beta^2\Delta\delta k^2 + 108\beta^3\Delta^3k^3 + \sqrt{4\left(48\beta^2 c_m k - 48\beta k\delta - 18\beta^2\Delta^2k^2\right)^3 + \left(-216\beta^3 c_m\Delta k^2 + 216\beta^2\Delta\delta k^2 + 108\beta^3\Delta^3k^3\right)^2}\right)^{1/3}\right] +$$

$$\frac{1}{3*2^{1/3}\beta k}\left[-216\beta^3 c_m\Delta k^2 + 216\beta^2\Delta\delta k^2 + 108\beta^3\Delta^3k^3 + \sqrt{4\left(48\beta^2 c_m k - 48\beta k\delta - 18\beta^2\Delta^2k^2\right)^3 + \left(-216\beta^3 c_m\Delta k^2 + 216\beta^2\Delta\delta k^2 + 108\beta^3\Delta^3k^3\right)^2}\right]^{1/3}$$

As for the uniqueness of α^*, we exploit the polynomial properties of the quartic objective function and the cubic equation for the FONC. That is, as 1) TS_E is strictly concave between $0 < \alpha^* < \frac{\Delta}{2}$ and 2) α is bounded from below and above the optimal value of α must be unique and is equal to α^*.

Derivation of boundaries for α^*: $0 < \alpha^* < \frac{\Delta}{2}$

From the optimal values of *w*, *d*, and λ with the revenue neutrality, we have

$$TS_E = \frac{\left(24(\delta - \beta c_m) - \beta k(\alpha - 5\Delta)(\alpha + \Delta)\right)\left(8(\delta - \beta c_m) - \beta k(\alpha - \Delta)(\alpha + \Delta)\right)}{512\beta} - k_C - k_M \text{ and}$$

$$\frac{dTS_E}{d\alpha} = \frac{1}{128}k\left(-8(2\alpha - \Delta)(\delta - \beta c_m) + \beta k(\alpha + \Delta)\left(\alpha^2 - 4\alpha\Delta + \Delta^2\right)\right) = 0.$$

By our definition, α is an *incentive* given to the collector/manufacturer. Hence, $\alpha \geq 0$ (Also, if the optimal incentive α* is negative, we risk the collection rate (i.e. $k(\frac{\alpha + \Delta}{4})$) to become negative). In addition, we observe that as $\alpha \to \infty$, the demand function will become negative. That is, α is bounded from below as well as above. We now analyze the first order condition at the minimum possible value of α*, which is 0. For $\alpha = 0$, $\left.\frac{dTS_E}{d\alpha}\right|_{\alpha=0} = \frac{1}{128}k\left(8\Delta(\delta - \beta c_m) + \beta k\Delta^3\right)$ >0. We then analyze the first order condition at $\alpha = \frac{\Delta}{2}$, which shows $\left.\frac{dTS_E}{d\alpha}\right|_{\alpha=\frac{\Delta}{2}} = -\frac{9}{1024}\beta k^2\Delta^3$ <0. Combining the results of the first order conditions at $\alpha = 0, \frac{\Delta}{2}$, we have $0 < \alpha^* < \frac{\Delta}{2}$ if TS_E is strictly concave. To prove the strict concavity, we need $\frac{d^2 TS_E}{d\alpha^2} = \frac{1}{128} k\left(-16\left(\delta - \beta c_m\right) + 3\beta k\left(\alpha^2 - 2\alpha\Delta - \Delta^2\right)\right)$<0. It can be shown that the range of α that satisfies this condition is $0 \le \alpha < \Delta + \sqrt{\frac{2}{3}}\sqrt{\frac{8\left(\delta - \beta c_m\right) + 3\beta k \Delta^2}{\beta k}}$. Since TS_E is strictly concave in the range $\alpha \in \left[0, \Delta\right]$, we conclude that $0 < \alpha^* < \frac{\Delta}{2}$.

Appendix C. Relationships among the Total Surpluses and the Collection Rates of Basic, Extended, Alternative Financial Instruments, and Revenue-Sharing Contract Models

Throughout Appendix C, let $A=\delta-\beta c_m$ and $B=\beta k\Delta^2$. *For* $TS_B^* < TS_E^* < TS_{RSC}^* < TS_{AFI}^*$

(i)To prove $TS_E^* > TS_B^*$ we need to show that

$\frac{\left(24A-\beta k\left(\alpha^*-5\Delta\right)\left(\alpha^*+\Delta\right)\right)\left(8A-\beta k\left(\alpha^*-\Delta\right)\left(\alpha^*+\Delta\right)\right)}{512\beta}-k_C-k_M>\frac{\left(24A+5B\right)\left(8A+B\right)}{512\beta}-k_C-k_M$ From Appendix B, we know that TS_E is strictly concave in the range of α of our interest. This implies that

$TS_E^* > TS_E\left(\alpha=\frac{\Delta}{2}\right)$. Hence, we have $TS_E\left(\alpha=\frac{\Delta}{2}\right)=\frac{\left(24A+\frac{27}{4}B\right)\left(8A+\frac{3}{4}B\right)}{512\beta}-k_C-k_M$ and

$TS_E\left(\alpha=\frac{\Delta}{2}\right)-TS_B^*=\frac{8Ak\Delta^2+\frac{1}{16}\beta k^2\Delta^4}{512}>0$. Therefore, $TS_E^* > TS_B^*$.

(ii)To prove $TS_{RSC}^* > TS_E^*$, we need to show that

$\frac{3\left(4A+B\right)^2}{128\beta}-k_C-k_M>\frac{\left(24A-\beta k\left(\alpha^*-5\Delta\right)\left(\alpha^*+\Delta\right)\right)\left(8A-\beta k\left(\alpha^*-\Delta\right)\left(\alpha^*+\Delta\right)\right)}{512\beta}-k_C-k_M$. After simplification, it is shown that

$\frac{1}{512\beta}\left[12(4A+B)^2-(24A-\beta k(\alpha^*-5\Delta)(\alpha^*+\Delta))(8A+\beta k\left(\Delta^2-\alpha^{*2}\right))\right]>0$ for $\alpha^*\in\left(0,\frac{\Delta}{2}\right)$. Hence, $TS_{RSC}^* > TS_E^*$.

(iii)To prove $TS_{AFI}^* > TS_{RSC}^*$ we need to show that $\frac{\left(4A+B\right)^2}{32\beta}-k_C-k_M>\frac{3\left(4A+B\right)^2}{128\beta}-k_C-k_M$. After simplification, it is shown that $\frac{\left(4A+B\right)^2}{128\beta}>0$. Hence, $TS_{AFI}^* > TS_{RSC}^*$

From (i) (ii) and (iii) we have that $TS_B^* < TS_E^* < TS_{RSC}^* < TS_{AFI}^*$.

For the collection rate $\varphi(e)_B^* < \varphi(e)_E^* < \varphi(e)_{AFI}^* = \varphi(e)_{RSC}^*$

(i)To prove that $\varphi(e)_E^* > \varphi(e)_B^*$, we need $k\left(\frac{\Delta+\alpha^*}{4}\right)>k\left(\frac{\Delta}{4}\right)$. From Appendix B, $\alpha^*>0$. Hence, $\varphi(e)_E^* > \varphi(e)_B^*$.

(ii)To prove that $\varphi(e)_{AFI}^* > \varphi(e)_E^*$, we need $k\left(\frac{\Delta}{2}\right)>k\left(\frac{\Delta+\alpha^*}{4}\right)$. From Appendix B, $\alpha^*<\frac{\Delta}{2}$.

Hence, $\varphi(e)_E^* < k\frac{3\Delta}{8}$, and $\varphi(e)_{AFI}^* > \varphi(e)_E^*$.

From (i) and (ii), and since $\varphi(e)_{AFI}^* = \varphi(e)_{RSC}^*$, we have $\varphi(e)_B^* < \varphi(e)_E^* < \varphi(e)_{AFI}^* = \varphi(e)_{RSC}^*$.

Appendix D. Relationships among Collector Profits, Manufacturer Profits, and Consumer Surpluses for Basic, Extended, Alternative Financial Instrument, and Revenue-Sharing Contract Models

Throughout Appendix D, let $A=\delta-\beta c_m$ and $B=\beta k\Delta^2$. First, we show that $\Pi_B^{C^*}<\Pi_E^{C^*}<\Pi_{AFI}^{C^*}$.

(i) $\Pi_E^{C^*}>\Pi_B^{C^*}$: We note that $\dfrac{d\Pi_E^C}{d\alpha}=\dfrac{1}{128}\left(\alpha+\Delta\right)k\left(8A-\beta k\left(2\alpha-\Delta\right)\left(\alpha+\Delta\right)\right)$, $\left.\dfrac{d\Pi_E^C}{d\alpha}\right|_{\alpha=0}=\dfrac{1}{128}\Delta k\left(8A+B\right)$, and $\left.\dfrac{d\Pi_E^C}{d\alpha}\right|_{\alpha=0}>0$. Next, we have $\left.\dfrac{d\Pi_E^C}{d\alpha}\right|_{\alpha=\frac{\Delta}{2}}=\dfrac{1}{128}\left(\dfrac{3\Delta k}{2}\right)8A>0$. It can be verified that Π_E^C is convex in the range of $\alpha\in\left(0,\dfrac{\Delta}{2}\right)$, which implies that Π_E^C is an increasing in α .

(ii) $\Pi_{AFI}^{C^*}>\Pi_E^{C^*}$: We note that

$\Pi_{AFI}^{C^*}-\Pi_E^{C^*}=\dfrac{56AB+15B^2-8A\alpha^{*2}k\beta+\alpha^{*4}k^2\beta+16A\alpha^*\Delta k\beta+2B\alpha^*\Delta k\beta-2\alpha^{*3}\Delta k^2\beta^2}{256\beta}>0$ for $\alpha^*\in\left(0,\dfrac{\Delta}{2}\right)$. From (i) and (ii), $\Pi_B^{C^*}<\Pi_E^{C^*}<\Pi_{AFI}^{C^*}$. We now focus on the relationships between $\Pi_{RSC}^{C^*}$ and the collector's profit in the other three models.

(iii) $\Pi_{AFI}^{C^*}\le\Pi_{RSC}^{C^*}$: For $\Pi_{AFI}^{C^*}\le\Pi_{RSC}^{C^*}$, we have $k_M\le\dfrac{16A^2-24AB-7B^2}{64\beta}+k_C=k_1$

(iv) $\Pi_E^{C^*}\le\Pi_{RSC}^{C^*}<\Pi_{AFI}^{C^*}$: We now address the case where $\Pi_{RSC}^{C^*}<\Pi_{AFI}^{C^*}$, but $\Pi_E^{C^*}\le\Pi_{RSC}^{C^*}$. From (iii) we know that if $k_M>\dfrac{16A^2-24AB-7B^2}{64\beta}+k_C$, then $\Pi_{RSC}^{C^*}<\Pi_{AFI}^{C^*}$. Hence, we will focus on solving the inequality, $\Pi_E^{C^*}\le\Pi_{RSC}^{C^*}$. After simplification, we have $k_M\le\dfrac{\left(4A+B\right)^2}{64\beta}-\dfrac{\left(\alpha^*+\Delta\right)^2k\left(8A-\beta k\left(\alpha^{*2}-\Delta^2\right)\right)}{128}+k_C=k_2$. It can be verified that $k_2>k_1$ by utilizing a minimum bound produced at $\alpha=\Delta/2$.

(v) $\Pi_B^{C^*}<\Pi_{RSC}^{C^*}<\Pi_E^{C^*}$: We now address the case where $\Pi_{RSC}^{C^*}<\Pi_E^{C^*}$, but $\Pi_B^{C^*}<\Pi_{RSC}^{C^*}$. From (iv) we know that if $k_M>k_2$, then $\Pi_{RSC}^{C^*}<\Pi_E^{C^*}$. Hence, we will focus on solving inequality $\Pi_B^{C^*}<\Pi_{RSC}^{C^*}$. After simplification, we have $k_M<\dfrac{32A^2+8AB+B^2}{128\beta}+k_C=L_1$. From our assumption of nonnegative profit for the manufacturer in the basic model we have an upper bound for

$k_M \le \frac{(8A+B)^2}{256\beta} = k_3$. We can show that $L_1 > k_3$ since this implies showing that $\frac{32A^2+8AB+B^2}{128\beta} + k_C < \frac{(8A+B)^2}{256\beta} + k_C$. This is equivalent to $B^2 + 256\beta k_C > 0$ after simplification. Hence, for $k_C \ge 0$, we have $L_1 > k_3$, which implies that the upper bound for k_M is k_3 .Since the upper bound for k_M is k_3 and not L_1, $\Pi_B^{C^*} = \Pi_{RSC}^{C^*}$ cannot occur.

To prove that $k_3 > k_2$, after simplification and using the approximation at $\alpha = \frac{\Delta}{2}$, we need $k_c < \frac{B(160A+B)}{2048\beta} = L_3$. From our assumption of nonnegative profit in the basic model for the collector we have $k_c < \frac{B(8A+B)}{256\beta} = L_4$. It can be verified that $L_3 > L_4$ under our assumption that $A > B$ (i.e. $\delta > \beta\left(k\Delta^2 + c_m\right)$). Hence, $k_C < L_3$, implying $k_3 > k_2$.

We note that the relationship among the four profits of the manufacturer can be proven with the steps and techniques we just utilized for the collector's relationship.

For consumer surplus, we want to show that $CS_E^* < CS_B^* < CS_{RSC}^* < CS_{AFI}^*$

(vi) $CS_E^* < CS_B^*$: $CS_E^* = \frac{\left(8A+\beta k\left(\Delta^2 - \alpha^{*2}\right)\right)^2}{512\beta} < CS_B^* = \frac{\left(8A+\beta k\Delta^2\right)^2}{512\beta}$ since $\alpha^* > 0$.

(vii) $CS_B^* < CS_{RSC}^*$: $CS_{RSC}^* - CS_B^* = \frac{B(16A+3B)}{512\beta} > 0$, implying $CS_{RSC}^* > CS_B^*$.

(viii) $CS_{AFI}^* > CS_{RSC}^*$: $CS_{AFI}^* - CS_{RSC}^* = \frac{3(4A+B)^2}{128\beta} > 0$, implying $CS_{AFI}^* > CS_{RSC}^*$.

Appendix E. Optimality of the Total Surplus Function in Centrally Coordinated Model and Derivation of Financial Instruments

For TS_{CC}, we have $\frac{\partial TS_{CC}}{\partial w} = \beta\left(c_m + (e-\Delta)ke - w\right) = 0$ and $\frac{\partial TS_{CC}}{\partial e} = (\delta - \beta w)k(\Delta - 2e) = 0$. The only solution that satisfies the FONC's is $w_{CC} = \frac{4c_m - k\Delta^2}{4}$ and $e_{CC} = \frac{\Delta}{2}$. For the SOSC's, we have $\frac{\partial^2 TS_{CC}}{\partial w^2} = -\beta < 0$ and $\frac{\partial^2 TS_{CC}}{\partial e^2} = -2k(\delta - \beta w) < 0$. At $e^*_{CC} = \frac{\Delta}{2}$, $\left(\frac{\partial^2 TS_{CC}}{\partial w^2}\right)\left(\frac{\partial^2 TS_{CC}}{\partial e^2}\right) > \left(\frac{\partial TS_{CC}}{\partial w \partial e}\right)^2$ is met because $2\beta k(\delta - \beta w) > 0$. Hence, $w_{CC} = \frac{4c_m - k\Delta^2}{4}$ $e_{CC} = \frac{\Delta}{2}$ is the unique optimal solution.

Derivation of the incentives and fees for the allocation mechanism

The collector's profit function is expressed as $\Pi^C_{AM} = (\delta - \beta w)(\lambda + \alpha x)k(d - \lambda) - k_C$, and the unique best response is $\lambda_{AM} = \frac{d - \alpha x}{2}$. Given λ_{AM}, the manufacturer's profit maximizing problem becomes $Max\ \Pi^M_{AM} = (\delta - \beta w)\left(w - c_m - \eta + \left(\Delta - d + \alpha(1-x)\right)k\left(d - \frac{d - \alpha x}{2}\right)\right) - (\delta - \beta w)w\varepsilon - k_M$. From $\frac{\partial \Pi^M_{AM}}{\partial d} = 0$, we have $d_{AM} = \frac{\Delta + \alpha(1 - 2x)}{2}$. Hence, $e_{AM} = \frac{\alpha + \Delta}{4}$. To satisfy the first requirement ($e^*_{AM} = e^*_{CC} = \frac{\Delta}{2}$), we set $\alpha_{AM} = \Delta$. For the second requirement, we need $w^*_{AM} = w^*_{CC}$. With $\lambda^*_{AM} = \frac{(\Delta - 2\Delta x)}{2}$ and $d^*_{AM} = \Delta - \Delta x$, the first order condition leads to the unique solution $w_{AM} = \frac{-8(\delta + \beta c_m) - 8\beta\eta + 8\delta\varepsilon + 4\beta k\Delta^2}{16\beta(\varepsilon - 1)}$. Hence, the second requirement is:

$$\frac{-8(\delta + \beta c_m) - 8\beta\eta + 8\delta\varepsilon + 4\beta k\Delta^2}{16\beta(\varepsilon - 1)} = \frac{1}{4}\left(4c_m - \Delta^2 k\right)$$

. The last requirement of the revenue neutrality implies that $(\delta - \beta w)\alpha x k(d - \lambda) + (\delta - \beta w)\alpha(1-x)k(d - \lambda) = (\delta - \beta w)\eta + (\delta - \beta w)w\varepsilon$. By simultaneous solution of the last two expressions with respect to η and ε, we have:

$$\eta_{AM} = \frac{\Delta^2 k(3\delta - 5\beta c_m) + \beta\Delta^4 k^2 - (\delta - \beta c_m)4c_m}{4(\delta - \beta c_m) + \beta\Delta^2 k} \text{ and } \varepsilon_{AM} = 2 - \frac{4(\delta - \beta c_m)}{4(\delta - \beta c_m) + \beta\Delta^2 k}$$

Hence, with the values of α, η, and ε specified as above, the pricing of the incentives and fees for the allocation mechanism is complete.

Appendix F. Concavity and Optimality of the Forward only Supply Chain and the Revenue-Sharing Contract Model

For Forward Only Supply Chain: For Π_F^M to be strictly concave in w, we need $\frac{d^2\Pi_F^M}{dw^2} < 0$.

$\frac{d^2\Pi_F^M}{dw^2} = -2\beta$ < 0.

For Revenue-Sharing Contract: For TP, we have $\frac{\partial TP}{\partial w} = \delta + \beta\left(c_m + e\left(e-\Delta\right)k - 2w\right) = 0$ and $\frac{\partial TP}{\partial e} = \left(\delta - \beta w\right)\left(\left(\Delta - e\right)k - ek\right) = 0$. The only solution that satisfies the FONC's is $w_{TP}^* = \frac{4\left(\delta + \beta c_m\right) - \beta k \Delta^2}{8\beta}$ and $e_{TP}^* = \frac{\Delta}{2}$. For the SOSC's, we have $\frac{\partial^2 TP}{\partial w^2} = -2\beta$ < 0 and $\frac{\partial^2 TP}{\partial e^2} = -2k\left(\delta - \beta w\right)$ < 0. At $e_{TP}^* = \frac{\Delta}{2}$, $\left(\frac{\partial^2 TP}{\partial w^2}\right)\left(\frac{\partial^2 TP}{\partial e^2}\right) > \left(\frac{\partial TP}{\partial w \partial e}\right)^2$ is met because $2\beta k\left(\delta - \beta w\right) > 0$. Hence, $w_{TP}^* = \frac{4\left(\delta + \beta c_m\right) - \beta k \Delta^2}{8\beta}$ and $e_{TP}^* = \frac{\Delta}{2}$ are the unique optimal equilibrium solution.

For Π_{RSC}^C to be strictly concave in e, we need $\frac{d^2\Pi_{RSC}^C}{de^2} < 0$. $\frac{d^2\Pi_{RSC}^C}{de^2} = -2\chi k\left(\delta - \beta w\right)$ < 0.

For Π_{RSC}^M to be strictly concave in w, we need $\frac{d^2\Pi_{RSC}^M}{dw^2} < 0$. $\frac{d^2\Pi_{RSC}^M}{dw^2} = -2\beta\left(1-\chi\right)$ < 0.

Appendix G. Concavity and Optimality of Profit Functions in Basic and Extended Model with Competition

For Basic Model:

For Π_B^C to be strictly concave in λ_1 and λ_2, we need (i) $\frac{\partial^2 \Pi_B^C}{\partial \lambda_1^2} < 0$, (ii) $\frac{\partial^2 \Pi_B^C}{\partial \lambda_2^2} < 0$, and (iii) det(H)>0, where H is the hessian matrix. Now, $\frac{\partial^2 \Pi_B^C}{\partial \lambda_1^2} = -2k_1 q_1 < 0$ since $k_1 > 0$ and we asume positive demand. Likewise, $\frac{\partial^2 \Pi_B^C}{\partial \lambda_2^2} = -2k_2 q_2 < 0$ since we assume positive demand and $k_2 > 0$. For condition (iii), we calculate the determinant of the Hessian matrix which results in $Det(H) = 4k_1 k_2 q_1 q_2 > 0$ from the previous assumptions. For the solution of the Cournot-game, we need to solve first order necessary conditions simultaneously for both manufacturers. The only solution that satisfies the first order necessary conditions and does not violate our assumption of the positive demand is $q_{B1} = \frac{16c_{m1}\beta_2 - 16\beta_2\delta_1 - 2k_1\beta_2\Delta_1^2 - 8c_{m2}\theta + 8\delta_2\theta + k_2\Delta_2^2\theta}{8(-4\beta_1\beta_2 + \theta^2)}$,
$q_{B2} = \frac{-16c_{m2}\beta_1 + 16\beta_1\delta_2 + 2k_2\beta_1\Delta_2^2 + 8c_{m1}\theta - 8\delta_1\theta - k_1\Delta_1^2\theta}{32\beta_1\beta_2 - 8\theta^2}$, $d_{B1} = \frac{\Delta_1}{2}$ and $d_{B2} = \frac{\Delta_2}{2}$

To verify the second order sufficient conditions for manufacturer 1, we need (i) $\frac{\partial^2 \Pi_B^{M1}}{\partial q_1^2} < 0$, (ii) $\frac{\partial^2 \Pi_B^{M1}}{\partial d_1^2} < 0$, and (iii) $\left(\frac{\partial^2 \Pi_B^{M1}}{\partial q_1^2}\right)\left(\frac{\partial^2 \Pi_B^{M1}}{\partial d_1^2}\right) > \left(\frac{\partial \Pi_B^{M1}}{\partial q_1 \partial d_1}\right)^2$ Conditions (i) and (ii) are satisfy since $\frac{\partial^2 \Pi_B^{M1}}{\partial q_1^2} = -2\beta_1 < 0$ and $\frac{\partial^2 \Pi_B^{M1}}{\partial d_1^2} = -k_1 q_1 < 0$ since we assume positive demand. Condition (iii) is expressed as $(2\beta_1 k_1 q_1) > \left(\frac{1}{2}k_1(\Delta_1 - 2d_1)\right)^2$. At $d_{B1} = \frac{\Delta_1}{2}$, condition (iii) can be expressed as $(2\beta_1 k_1 q_1) > 0$, which holds since we assume the positive demand and $\beta_1, k_1 > 0$. To verify the second order sufficient conditions for manufacturer 2, we need (i) $\frac{\partial^2 \Pi_B^{M2}}{\partial q_2^2} < 0$, (ii) $\frac{\partial^2 \Pi_B^{M2}}{\partial d_2^2} < 0$, and (iii) $\left(\frac{\partial^2 \Pi_B^{M2}}{\partial q_2^2}\right)\left(\frac{\partial^2 \Pi_B^{M2}}{\partial d_2^2}\right) > \left(\frac{\partial \Pi_B^{M2}}{\partial q_2 \partial d_2}\right)^2$ Conditions (i) and (ii) are satisfy since $\frac{\partial^2 \Pi_B^{M2}}{\partial q_2^2} = -2\beta_2 < 0$ and $\frac{\partial^2 \Pi_B^{M2}}{\partial d_2^2} = -k_2 q_2 < 0$ since we assume positive demand. Condition (iii) is expressed as

$\left(2\beta_2 k_2 q_2\right) > \left(\frac{1}{2}k_2(\Delta_2 - 2d_2)\right)^2$. At $d_{B2} = \frac{\Delta_2}{2}$, condition (iii) can be expressed as $\left(2\beta_2 k_2 q_2\right) > 0$, which holds since we assume the positive demand and $\beta_2, k_2 > 0$.

Hence, $q_{B1} = \frac{16c_{m1}\beta_2 - 16\beta_2\delta_1 - 2k_1\beta_2\Delta_1^2 - 8c_{m2}\theta + 8\delta_2\theta + k_2\Delta_2^2\theta}{8(-4\beta_1\beta_2 + \theta^2)}$,

$q_{B2} = \frac{-16c_{m2}\beta_1 + 16\beta_1\delta_2 + 2k_2\beta_1\Delta_2^2 + 8c_{m1}\theta - 8\delta_1\theta - k_1\Delta_1^2\theta}{32\beta_1\beta_2 - 8\theta^2}$, $d_{B1} = \frac{\Delta_1}{2}$ and $d_{B2} = \frac{\Delta_2}{2}$

are the unique optimal equilibrium solution.

For Basic Model with homogeneous products:

For Π_{B-H}^{C} to be strictly concave in λ_1 and λ_2, we need (i) $\frac{\partial^2\Pi_{B-H}^{C}}{\partial\lambda_1^2} < 0$, (ii) $\frac{\partial^2\Pi_{B-H}^{C}}{\partial\lambda_2^2} < 0$, and (iii) det(H)>0, where H is the hessian matrix. Now, $\frac{\partial^2\Pi_{B-H}^{C}}{\partial\lambda_1^2} = -2kq_1 < 0$ since $k > 0$ and we asume positive demand. Likewise, $\frac{\partial^2\Pi_{B-H}^{C}}{\partial\lambda_2^2} = -2kq_2 < 0$ since we assume positive demand and $k_2 > 0$. For condition (iii), we calculate the determinant of the Hessian matrix which results in $Det\left(H\right) = 4k^2 q_1 q_2 > 0$ from the previous assumptions. For the solution of the Cournot-game among manufacturers, we need to solve first order necessary conditions simultaneously for both manufacturers. The only solution that satisfies the first order necessary conditions and does not violate our assumption of the positive demand is $q_{B-H1} = q_{B-H2} = \frac{8(A - c_m) + k(\alpha + \Delta)^2}{24B}$

$d_{B-H1} = d_{B-H2} = \frac{1}{2}\Delta$ To verify the second order sufficient conditions for manufacturer 1, we need (i) $\frac{\partial^2\Pi_{B-H}^{M1}}{\partial q_1^2} < 0$, (ii) $\frac{\partial^2\Pi_{B-H}^{M1}}{\partial d_1^2} < 0$, and (iii) $\left(\frac{\partial^2\Pi_{B-H}^{M1}}{\partial q_1^2}\right)\left(\frac{\partial^2\Pi_{B-H}^{M1}}{\partial d_1^2}\right) > \left(\frac{\partial\Pi_{B-H}^{M1}}{\partial q_1\,\partial d_1}\right)^2$ Conditions (i) and (ii) are satisfy since $\frac{\partial^2\Pi_{B}^{M1}}{\partial q_1^2} = -2B < 0$ and $\frac{\partial^2\Pi_{B-H}^{M1}}{\partial d_1^2} = -kq_1 < 0$ since we assume positive demand. Condition (iii) is expressed as $\left(2Bkq_1\right) > \left(\frac{1}{2}k(\Delta - 2d_1)\right)^2$. At $d_{B-H1} = \frac{\Delta}{2}$, condition (iii) can be expressed as $\left(2Bkq_1\right) > 0$, which holds since we assume the positive demand and $B, k > 0$. To verify the second order sufficient conditions for manufacturer 2, we need (i) $\frac{\partial^2\Pi_{B-H}^{M2}}{\partial q_2^2} < 0$, (ii) $\frac{\partial^2\Pi_{B-H}^{M2}}{\partial d_2^2} < 0$, and (iii) $\left(\frac{\partial^2\Pi_{B-H}^{M2}}{\partial q_2^2}\right)\left(\frac{\partial^2\Pi_{B-H}^{M2}}{\partial d_2^2}\right) > \left(\frac{\partial\Pi_{B-H}^{M2}}{\partial q_2\partial d_2}\right)^2$ Conditions (i) and (ii) are satisfy since

$\frac{\partial^2 \Pi_{B-H}^{M2}}{\partial q_2^2} = -2B < 0$ and $\frac{\partial^2 \Pi_{B-H}^{M2}}{\partial d_2^2} = -kq_2 < 0$ since we assume positive demand. Condition (iii) is expressed as $\left(2Bkq_2\right) > \left(\frac{1}{2}k(\Delta - 2d_2)\right)^2$. At $d_{H-B2} = \frac{\Delta}{2}$, condition (iii) can be expressed as $\left(2Bkq_2\right) > 0$, which holds since we assume the positive demand and $\beta_2, k_2 > 0$.

Hence, is $q_{B-H1} = q_{B-H2} = \frac{8(A - c_m) + k(\alpha + \Delta)^2}{24B}$ $d_{B-H1} = d_{B-H2} = \frac{1}{2}\Delta$ are the unique optimal equilibrium solution.

For Extended Model:

For Π_G^C to be strictly concave in λ_1 and we need (i) $\frac{\partial^2 \Pi_G^C}{\partial \lambda_1^2} < 0$, (ii) $\frac{\partial^2 \Pi_G^C}{\partial \lambda_2^2} < 0$, and (iii) det(H)>0, where H is the hessian matrix. Now, $\frac{\partial^2 \Pi_G^C}{\partial \lambda_1^2} = -2k_1 q_1 < 0$ since $k_1 > 0$ and we asume positive demand. Likewise, $\frac{\partial^2 \Pi_G^C}{\partial \lambda_2^2} = -2k_2 q_2 < 0$ since we assume positive demand and $k_2 > 0$. For condition (iii), we calculate the determinant of the Hessian matrix which results in $Det\left(H\right) = 4k_1 k_2 q_1 q_2 > 0$ from the previous assumptions.

So the collector's first order condition characterizes the unique best response, $\lambda_1 = \frac{\Delta_1 + \alpha}{2}, \lambda_2 = \frac{\Delta_2 + \alpha}{2}$.

For the solution of the Cournot-game among manufacturers, we need to solve first order necessary conditions simultaneously for both manufacturers. The only solution that satisfies the first order necessary conditions and does not violate our assumption of the positive demand is

$$q_{G1} = \frac{16c_{m1}\beta_2 - 2k_1\alpha^2\beta_2 + 16\beta_2\gamma - 16\beta_2\delta_1 - 4k_1\alpha\beta_2\Delta_1 - 2k_1\beta_2\Delta_1^2 - 8c_{m2}\theta + k_2\alpha^2\theta - 8\gamma\theta + 8\delta_2\theta + 2k_2\alpha\Delta_2\theta + k_2\Delta_2^2\theta}{8(-4\beta_1\beta_2 + \theta^2)}$$

$$q_{G2} = \frac{-16c_{m2}\beta_1 + 2k_2\alpha^2\beta_1 - 16\beta_1\gamma + 16\beta_1\delta_2 + 4k_2\alpha\beta_1\Delta_2 + 2k_2\beta_1\Delta_2^2 + 8c_{m1}\theta - k_1\alpha^2\theta + 8\gamma\theta - 8\delta_1\theta - 2k_1\alpha\Delta_1\theta - k_1\Delta_1^2\theta}{32\beta_1\beta_2 - 8\theta^2}$$

$$d_{G1} = \frac{1}{2}(\Delta_1 - \alpha) \; d_{G2} = \frac{1}{2}(\Delta_2 - \alpha)$$

To verify the second order sufficient conditions for manufacturer 1, we need (i) $\frac{\partial^2 \Pi_G^{M1}}{\partial q_1^2} < 0$, (ii) $\frac{\partial^2 \Pi_G^{M1}}{\partial d_1^2} < 0$, and (iii) $\left(\frac{\partial^2 \Pi_G^{M1}}{\partial q_1^2}\right)\left(\frac{\partial^2 \Pi_G^{M1}}{\partial d_1^2}\right) > \left(\frac{\partial \Pi_G^{M1}}{\partial q_1 \partial d_1}\right)^2$ Conditions (i) and (ii) are satisfy since $\frac{\partial^2 \Pi_G^{M1}}{\partial q_1^2} = -2\beta_1 < 0$ and $\frac{\partial^2 \Pi_G^{M1}}{\partial d_1^2} = -k_1 q_1 < 0$ since we assume positive demand. Condition (iii) is expressed as $\left(2\beta_1 k_1 q_1\right) > \left(\frac{1}{2}k_1(\Delta_1 - 2d_1 - \alpha)\right)^2$. At $d_{G1} = \frac{1}{2}(\Delta_1 - \alpha)$, condition (iii) can be expressed as $\left(2\beta_1 k_1 q_1\right) > 0$, which holds since we assume the positive demand and $\beta_1, k_1 > 0$.

To verify the second order sufficient conditions for manufacturer 2, we need (i) $\frac{\partial^2 \Pi_G^{M2}}{\partial q_2^2} < 0$, (ii) $\frac{\partial^2 \Pi_G^{M2}}{\partial d_2^2} < 0$, and (iii) $\left(\frac{\partial^2 \Pi_G^{M2}}{\partial q_2^2}\right)\left(\frac{\partial^2 \Pi_G^{M2}}{\partial d_2^2}\right) > \left(\frac{\partial \Pi_G^{M2}}{\partial q_2 \partial d_2}\right)^2$ Conditions (i) and (ii) are satisfy since $\frac{\partial^2 \Pi_G^{M2}}{\partial q_2^2} = -2\beta_2 < 0$ and $\frac{\partial^2 \Pi_G^{M2}}{\partial d_2^2} = -k_2 q_2 < 0$ since we assume positive demand. Condition (iii) is expressed as $\left(2\beta_2 k_2 q_2\right) > \left(\frac{1}{2}k_2(\Delta_2 - 2d_2 - \alpha)\right)^2$. At $d_{G2} = \frac{1}{2}(\Delta_2 - \alpha)$, condition (iii) can be expressed as $\left(2\beta_2 k_2 q_2\right) > 0$, which holds since we assume the positive demand and $\beta_2, k_2 > 0$.

Hence, solution

$$q_{G1} = \frac{16c_{m1}\beta_2 - 2k_1\alpha^2\beta_2 + 16\beta_2\gamma - 16\beta_2\delta_1 - 4k_1\alpha\beta_2\Delta_1 - 2k_1\beta_2\Delta_1^2 - 8c_{m2}\theta + k_2\alpha^2\theta - 8\gamma\theta + 8\delta_2\theta + 2k_2\alpha\Delta_2\theta + k_2\Delta_2^2\theta}{8(-4\beta_1\beta_2 + \theta^2)}$$

$$q_{G2} = \frac{-16c_{m2}\beta_1 + 2k_2\alpha^2\beta_1 - 16\beta_1\gamma + 16\beta_1\delta_2 + 4k_2\alpha\beta_1\Delta_2 + 2k_2\beta_1\Delta_2^2 + 8c_{m1}\theta - k_1\alpha^2\theta + 8\gamma\theta - 8\delta_1\theta - 2k_1\alpha\Delta_1\theta - k_1\Delta_1^2\theta}{32\beta_1\beta_2 - 8\theta^2}$$

$d_{G1} = \frac{1}{2}(\Delta_1 - \alpha)$ $d_{G2} = \frac{1}{2}(\Delta_2 - \alpha)$ is the unique best response.

For Extended Model: Homogeneous products.

A similar analysis can be made with the simplifications of symmetric costs and unique selling price.

Appendix H. Expression for α^* its Uniqueness and Bounds for Homogeneous Products

For the extended model described in Section 4.3.1, with the help of *Mathematica*, we can solve :

$$\frac{dTS_{G-H}}{d\alpha}=\frac{1}{288B}k\Big(8\big(5\alpha-3\Delta\big)\big(-A+c_m\big)+k\left(\alpha+\Delta\right)\Big(2\alpha^2-11\alpha\Delta+3\Delta^2\Big)\Big)=0$$

As for the uniqueness of α^*, we exploit the polynomial properties of the quartic objective function and the cubic equation for the first order necessary condition. That is, as 1) TS_G is strictly concave between 0 and $\frac{3\Delta}{5}$ and $0<\alpha^*<\frac{3\Delta}{5}$ and 2) α is bounded from below as well as above, the optimal value of α must be unique and is equal to α^*.

Derivation of boundaries for α^* *:* $0<\alpha^*<\frac{3\Delta}{5}$

From the optimal values of the decision variables with the revenue neutrality, we have

$$TS_{G-H}=\frac{(32(A-c_m)-k(\alpha-7\Delta)(\alpha+\Delta))(8(A-c_m)-k(\alpha-\Delta)(\alpha+\Delta))}{576B}$$

The first order condition states that

$$\frac{dTS_{G-H}}{d\alpha}=\frac{1}{288B}k\Big(8\big(5\alpha-3\Delta\big)\big(-A+c_m\big)+k\left(\alpha+\Delta\right)\Big(2\alpha^2-11\alpha\Delta+3\Delta^2\Big)\Big)=0$$

By our definition, α is an *incentive* given to the collector/manufacturer. Hence, $\alpha\geq 0$ (Also, if the optimal incentive α* is negative, we risk the collection rate (i.e. $k(\frac{\alpha+\Delta}{4})$) to become negative). In addition, we observe that as $\alpha\rightarrow\infty$, the demand function will become negative. That is, α is bounded from below as well as above.

We now analyze the first order condition at the minimum possible value of α*, which is 0. For $\alpha=0$, $\left.\frac{dTS_{G-H}}{d\alpha}\right|_{\alpha=0}=\frac{1}{288B}k\Delta\Big(24\big(A-c_m\big)+3k\Delta^2\Big)$. From our assumption of the positive selling price and $k,B,\Delta>0$, $\left.\frac{dTS_{G-H}}{d\alpha}\right|_{\alpha=0}>0$. We then analyze the first order condition at $\alpha=\frac{3\Delta}{5}$, which shows $\left.\frac{dTS_{G-H}}{d\alpha}\right|_{\alpha=\frac{3\Delta}{5}}=-\frac{2}{125B}k^2\Delta^3$. Since $k,B,\Delta>0$, $\left.\frac{dTS_{G-H}}{d\alpha}\right|_{\alpha=\frac{3\Delta}{5}}<0$. Combining the results of the first order conditions at $\alpha=0,\frac{3\Delta}{5}$, we have $0<\alpha^*<\frac{3\Delta}{5}$ if TS_{G-H} is strictly concave.

To prove the strict concavity, we can verify the second order condition for TS_{G-H} as follows: $\frac{d^2TS_{G-H}}{d\alpha^2}=\frac{1}{288B}k\Big(-40\big(A-c_m\big)+k\left(4\alpha-11\Delta\right)\left(\alpha+\Delta\right)+k\Big(2\alpha^2-11\alpha\Delta+3\Delta^2\Big)\Big)$. For TS_{G-H} to be strictly concave, we need $\frac{d^2TS_{G-H}}{d\alpha^2}<0$. From our assumptions, $\alpha\geq 0, A-c_m>0$ and $k,\Delta>0$. It can be shown that the range of α that satisfies the conditions described above is

$0 \le \alpha < \frac{3\Delta}{2} + \frac{\sqrt{\frac{80(\delta - \beta c_m)}{k} + 43\Delta^2}}{2\sqrt{3}}$. Since TS_{G-H} is strictly concave in the range $\alpha \in [0, \Delta]$, we conclude that $0 < \alpha^* < \frac{3\Delta}{5}$.

Appendix I. Relationships among the Total Surpluses as well as the Collection Rates of Basic Model, Extended Model, and Centrally Coordinated Model for the case of Homogeneous Products

Proof of Proposition 6.

(i) To prove $TS^*_{CC-H} > TS^*_{G-H}$, we need to show that

$$\frac{(4A-4c_m+k\Delta^2)^2}{32B} > \frac{(32(A-c_m)-k(\alpha-7\Delta)(\alpha+\Delta))(8(A-c_m)-k(\alpha-\Delta)(\alpha+\Delta))}{576B}$$

After simplification, this reduces to

$$\frac{1}{576B}\left[32(A^2+cm^2)+8k\left(A-c_m\right)\left(5\alpha^2-6\alpha\Delta+7\Delta^2\right)+k^2\left(-\alpha^4+6\alpha^3\Delta+8\alpha^2\Delta^2-6\alpha\Delta^3+11\Delta^4\right)-64Ac_m\right]>0$$

It can be verified that the three terms inside the bracket are positive since $\alpha^* \in \left(0,\frac{3\Delta}{5}\right)$. Hence,

$TS^*_{CC-H} > TS^*_{G-H}$.

(ii) To prove $TS^*_{G-H} > TS^*_{B-H}$ we need to show that

$$\frac{(32(A-c_m)-k(\alpha-7\Delta)(\alpha+\Delta))(8(A-c_m)-k(\alpha-\Delta)(\alpha+\Delta))}{576B} > \frac{\left(8\left(A-c_m\right)+k\Delta^2\right)\left(32\left(A-c_m\right)+7k\Delta^2\right)}{576B}$$

From Appendix H, we know that TS_{G-H} is strictly concave in the range of α of our interest. We take a value for α in the range of solution (i.e. 0,3Δ/5) We have

$$TS_{G-H}\left(\alpha=\frac{\Delta}{2}\right)=\frac{\left(8\left(A-c_m\right)+\frac{3}{4}k\Delta^2\right)\left(32\left(A-c_m\right)+\frac{39}{4}k\Delta^2\right)}{576B}$$ and

$TS_{G-M}\left(\alpha=\frac{\Delta}{2}\right)-TS^*_{B-H}=\frac{k\Delta^2\left(224\left(A-c_m\right)+5k\Delta^2\right)}{9216B}>0$. Therefore, $TS^*_{EM} > TS^*_B$.

(i) To prove that $\varphi^*_{G-H} > \varphi^*_{B-H}$, we need $k\left(\frac{\Delta+\alpha^*}{4}\right) > k\left(\frac{\Delta}{4}\right)$. From Appendix H, $\alpha^* > 0$. Hence,

$\varphi^*_{G-H} > \varphi^*_{B-H}$.

(ii) To prove that $\varphi^*_{CC-H} > \varphi^*_{G-H}$, we need $k\left(\frac{\Delta}{2}\right) > k\left(\frac{\Delta+\alpha^*}{4}\right)$. From Appendix H, $\alpha^* < \frac{3\Delta}{5}$.

Hence, $\varphi^*_{G-H} < k\frac{2\Delta}{5}$, and $\varphi^*_{CC-H} > \varphi^*_{G-H}$.

Relationships between the collector profits, manufacturers profits, and consumer surpluses for Basic Model and Extended Model with homogeneous products

(i) $\Pi^{C^*}_{G-H} > \Pi^{C^*}_{B-H}$:

It can be verified that $\Pi^{C^*}_{G-H}=\frac{\left(\alpha^*+\Delta\right)^2 k\left(8\left(A-c_m\right)-k\left(\alpha^{*2}-\Delta^2\right)\right)}{192B}$ is convex in the range of

$\alpha \in \left(0,\frac{3\Delta}{5}\right)$, which implies that Π^{C}_{G-H} is an increasing function of α. Hence, $\Pi^{C^*}_{G-H} > \Pi^{C^*}_{B}$.

(ii) $\Pi_{G-H}^{M^*} < \Pi_{B-H}^{M^*}$:

For the manufacturers profits, it is easy to see that

$$\Pi_{G-H}^{M^*} = \frac{\left(8\left(A-c_m\right)+k\left(\Delta^2-\alpha^{*2}\right)\right)^2}{576B} < \Pi_{B-H}^{M^*} = \frac{\left(8\left(A-c_m\right)+k\Delta^2\right)^2}{576B} \text{ since } \alpha^* > 0 .$$

(iii) $CS_{EM}^* < CS_B^*$:

For the consumer surplus, it is easy to see that

$$CS_{G-H}^* = \frac{\left(8\left(A-c_m\right)+k\left(\Delta^2-\alpha^{*2}\right)\right)^2}{288B} < CS_{B-H}^* = \frac{\left(8\left(A-c_m\right)+k\Delta^2\right)^2}{288B} \text{ since } \alpha^* > 0 .$$

Appendix J. Derivation of Boundaries for α_M^*

Proof of Proposition 5.
For the case of multiple manufacturers, we have

$$TS_M = \frac{\left(8\left(A-c_m\right)+\left(\Delta^2-\alpha^2\right)k\right)n\left(8\left(2+n\right)\left(A-c_m\right)-\left(\alpha+\Delta\right)k\left(\alpha-2\Delta n-3\Delta\right)\right)}{128B\left(1+n\right)^2}$$

We now analyze the first order condition at the minimum possible value of α*, which is 0. For $\alpha = 0$, $\left.\frac{dTS_M}{d\alpha}\right|_{\alpha=0} = \frac{1}{64B(1+n)} k\Delta n\left(8\left(A-c_m\right)+k\Delta^2\right) > 0$ We then analyze the first order condition at $\alpha = \frac{(n+1)\Delta}{(n+3)}$, which shows Combining the results of the first order conditions at $\alpha = 0, \frac{(n+1)\Delta}{(n+3)}$, we have $0 < \alpha^* < \frac{(n+1)\Delta}{(n+3)}$ since TS_M is strictly concave in the range of interest.

Appendix K. Optimality of Total Surplus in Centrally Coordinated Model with Competition and Derivation of Instruments

For TS_{CC} , the only solution that satisfies the first order necessary conditions, and does not violate our assumption of the positive demand is $q_{CC1} = \dfrac{4c_{m1}\beta_2 - 4\beta_2\delta_1 - k_1\beta_2\Delta_1^2 - 4c_{m2}\theta + 4\delta_2\theta + k_2\Delta_2^2\theta}{4(-\beta_1\beta_2 + \theta^2)}$

$q_{CC2} = \dfrac{-4c_{m2}\beta_1 + 4\beta_1\delta_2 + k_2\beta_1\Delta_2^2 + 4c_{m1}\theta - 4\delta_1\theta - k_1\Delta_1^2\theta}{4(\beta_1\beta_2 - \theta^2)}$ $e_1 = \dfrac{\Delta_1}{2}$ $e_2 = \dfrac{\Delta_2}{2}$

To verify the second order sufficient conditions, we need (i) $\dfrac{\partial^2 TS_{CC}}{\partial q_i^2} < 0$, (ii) $\dfrac{\partial^2 TS_{CC}}{\partial e_i^2} < 0$, and (iii) $\left(\dfrac{\partial^2 TS_{CC}}{\partial q_i^2}\right)\left(\dfrac{\partial^2 TS_{CC}}{\partial e_i^2}\right) > \left(\dfrac{\partial TS_{CC}}{\partial q_i\,\partial e_i}\right)^2$ Conditions (i) and (ii) are satisfy since $\dfrac{\partial^2 TS_{CC}}{\partial q_i^2} = -\beta_i < 0$ and $\dfrac{\partial^2 TS_{CC}}{\partial e_i^2} = -2k_i q_i < 0$ since we assume positive demand. Condition (iii) is expressed as $\left(2\beta_i k_i q_i\right) > \left(k_i(\Delta_i - 2e_i)\right)^2$. At $e_i = \dfrac{\Delta_i}{2}$, condition (iii) can be expressed as $\left(2\beta_i k_i q_i\right) > 0$, which holds since we assume the positive demand and $\beta_i, k_i > 0$. Hence,

$q_{CC1} = \dfrac{4c_{m1}\beta_2 - 4\beta_2\delta_1 - k_1\beta_2\Delta_1^2 - 4c_{m2}\theta + 4\delta_2\theta + k_2\Delta_2^2\theta}{4(-\beta_1\beta_2 + \theta^2)}$

$q_{CC2} = \dfrac{-4c_{m2}\beta_1 + 4\beta_1\delta_2 + k_2\beta_1\Delta_2^2 + 4c_{m1}\theta - 4\delta_1\theta - k_1\Delta_1^2\theta}{4(\beta_1\beta_2 - \theta^2)}$ $e_1 = \dfrac{\Delta_1}{2}$ $e_2 = \dfrac{\Delta_2}{2}$ is the unique optimal solution.

A similar approach can be made for the homogeneous case to see that $e_{CC-H1} = e_{CC-H2} = \dfrac{\Delta}{2}$

$q_1 + q_2 = \dfrac{4A - 4c_m + k\Delta^2}{4B}$ is the unique optimal solution.

Derivation of the incentives and fees for homogeneous product model

Proof of Proposition 6.
Under the proposed mechanism, the collector's profit function is expressed as $\Pi^C = q_1 k(d_1 - \lambda_1)(\lambda_1 + \alpha) + q_2 k(d_2 - \lambda_2)(\lambda_2 + \alpha)$. Because the objective function is strictly concave in λ_1 and λ_2 , from the first order conditions, the unique best response is $\lambda_i = \dfrac{d_i - \alpha}{2}$. The manufacturers' profit maximizing problem is formulated as

$$\Pi_{G-H}^{M_1} = (A - Bq_1 - Bq_2 - c_m - \varepsilon + (\Delta - d_1)k(d_1 - \lambda_1))q_1 - F$$

$$\Pi_{G-H}^{M_2} = (A - Bq_2 - Bq_1 - c_m - \varepsilon + (\Delta - d_2)k(d_2 - \lambda_2))q_2 - F$$

Given λ_i , we have $d_i = \frac{\Delta - \alpha}{2}$. Hence, $e_i = \frac{\alpha + \Delta}{4}$. To satisfy the first requirement ($e_{CC}^* = \frac{\Delta}{2}$), we set $\alpha = \Delta$. For the second requirement, we need $q_1 + q_2 = \frac{4A - 4c_m + k\Delta^2}{4B}$. Hence, the second requirement is: $\frac{4A - 4c_m - 4\varepsilon + 2k\Delta^2}{6B} = \frac{4A - 4c_m + k\Delta^2}{4B}$. The last requirement of the revenue neutrality implies that $q_1 \alpha k(d_1 - \lambda_1) + q_2 \alpha k(d_2 - \lambda_2) = (q_1 + q_2)\varepsilon + 2F$. By simultaneous solution of the last two expressions with respect to F and ε, we have:

$\varepsilon = \frac{1}{8}\left(-4(A - c_m) + \Delta^2 k\right)$ and $F = \frac{\left(4(A - c_m) + \Delta^2 k\right)\left(4(A - c_m) + 3\Delta^2 k\right)}{64B}$ Hence, with the values of α , F , and ε specified as above, the pricing of the incentives and fees is complete.

References

Atasu, A., D. Guide, L. N. Van Wassenhove. 2008a. Product reuse economics in closed-loop supply chain research. *Production and Operations Management* **17**(5) 483-496

Atasu, A., M. Sarvary, L. N. Van Wassenhove. 2008b. Remanufacturing as a marketing strategy. *Management Science* **54**(10) 1731-1746

Atasu, A., L. N. Van Wassenhove, M. Sarvary. 2009. Efficient Take-Back Legislation. *Production and Operations Management* 18(3) 243-258

Atkinson, M. E., J. Creedy, D. M. Knox. 1999. Alternative retirement income arrangements and lifetime income inequality: Lessons from Australia. *International Tax and Public Finance*. **6**(1) 103-117.

Baker, G.P. 1992. Incentive Contracts and Performance Measurement. *The Journal of Political Economy* **100**(3) 598-614

Becker, N., M. Schechter. 1996. Decentralized economic incentives under technological indivisibilities: a cooperative game approach. *Ecological Economics* **17** 9-20.

Binger, B. R., Hoffman, E. 1998. *Microeconomics with Calculus*. Addison-Wesley Educational Publishers Inc. Reading, MA.

Cachon, G.P., M. A. Lariviere. 2005. Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations. *Management Science*. 51(1) 30-44.

Calcott, P., M. Walls. 2005. Waste, recycling, and "Design for Environment": Roles for market and policy instruments. *Resource and Energy Economics*. **27** 287-305.

Campbell, D. E. 2006. *Incentives. Motivation and the Economics of Information*. Cambridge University Press, NY.

Carraro, C., G. Topa. 1995. Taxation and environmental innovation. In *Control and Game-Theoretic Models of the Environment*. *Annals of the International Society of Dynamic Games,* Vol. 2, ed. C. Carraro, J. A. Filar. Birkhäuser, Boston.

Chevalier, J., A. Kashyap, P. Rossi. 2003. Why don't prices rise during periods of peak demand? Evidence from scanner data. *The American Economic Review* **93**(1) 15-37.

Chiang, W. K., D. Chhajed, J. D. Hess. 2003. Direct Marketing, Indirect Profits: A Strategic Analysis of Dual-Channel Supply-Chain Design. *Management Science* **49** (1) 1-20.

Choi, S. C. 1991. Price competition in a channel structure with a common retailer. *Marketing Science* **10**(4) 271-296

Council of State Governments 2009. Easter Regional Conference. www.csgeast.org (accessed 01.20.09)

De Borger, B. 2001. Discrete choice models and optimal two-part tariffs in the presence of externalities: optimal taxation of cars. *Regional Science & Urban Economics*. **31** 271-504.

Dobbs, I. 1991. Litter and Waste Management: Disposal Taxes versus User Charges. *The Canadian Journal of Economics*, **24** (1) 221-227.

Electronic Waste Recycling Act California. 2003. http://www.ciwmb.ca.gov/electronics/Act2003/Retailer/Fee/ (accessed 03.17.09)

Evans, D. 1994. A Rationale for Recycling. *Environmental Management*. 18(3) 321-329.

Ferguson, M.E., L.B. Toktay. 2006. The effect of competition on recovery strategies. *Production and operations management.* **15**(3) 351-368.

Fershtman, C., K. L. Judd. 1987. Equilibrium Incentives in Oligopoly. *The American Economic Review* **77**(5) 927-940

Galiana, D., A. Motto, F. Bouffard. 2003. Reconciling Social Welfare, Agent Profits, and Consumer Payments in Electricity Pools. *IEEE Transactions On Power Systems* **18**(2) 452-459.

Giannoccaro, I., P. Pontrandolfo. 2004. Supply Chain Coordination by Revenue Sharing Contracts. *International Journal of Production Economics*. 89 131-139

Goldstein. L. 1994. The Strategic Management of Environmental issues: A case study of Kodak's single use cameras. Master's dissertation.

Government Relations: The Remanufacturing Institute. http://www.reman.org/GovRelations_main.htm (accessed 08.01.09)

Green Peace México. Acopio y reciclaje de basura electrónica en México. http://www.greenpeace.org/mexico/campaigns/t-xicos/acopio-y-reciclaje-de-basura-e (accessed 11.04.09)

Guide, D., L. N. Van Wassenhove. 2006. Closed-Loop Supply Chains: An Introduction to the Feature Issue (Part 1). *Production and Operations Management* **5**(3) 345-350.

Guide, D., R. H. Teunter, L. N. Van Wassenhove. 2003. Matching demand and supply to maximize profits from remanufacturing. *Manufacturing & Service Operations Management* **5**(4) 303-316.

Gutiérrez-Alcaraz, G., G. B. Sheblé. 2005. Electricity market dynamics: Oligopolistic competition. *Electric Power Systems Research* **76** 695-700.

Hammond, D., P. Beullens. 2007. Closed-loop supply chain network equilibrium under legislation. *European Journal of Operational Research* **183** 895-908.

Hamza, H., Y. Wang, B. Bidanda. 2007. Modeling total cost of ownership utilizing interval-based reliable simulation technique in reverse logistics management. *Proceedings of the 2007 Industrial Engineering Research Conference.*

He, J., K. S. Chin, J. B. Yang, D. L. Zhu. 2006. Return Policy Model of Supply Chain Management for Single-Period Products. *Journal of Optimization Theory and Applications* **129**(2) 293-308.

Kim, B., F. El Ouardighi. 2007. Supplier-Manufacturer collaboration on new product development. In *Advances in Dynamic Game Theory,* ed S. Jørgensen, M. Quincampoix, T. Vincent. Birkhäuser, Boston.

Kodak One-Time-Use Camera http://www.kodak.com/US/en/corp/HSE/oneTimeUseCamera.jhtml?pq-path=7225 (accessed 09.08.09)

Kouvelis, P., M. Rosenblatt. 2002. A mathematical programming model for global supply chain management: conceptual approach and managerial insights. In *Supply chain management: Models, Applications, and Research Directions.* Vol 62, ed J. Geunes, P. Pardalos. Kluwer Academic Publishers, Netherlands. 245-277.

Lal, R., C. Matutes. 1994. Retail pricing and advertising strategies. *The Journal of Business* **67**(3) 345-370.

Ley General para la Prevención y Gestión integral de los Residuos. Diario Oficial de la Federación. 2003. Last update June 2007

Mas-Colell, A. 1995. *Microeconomic theory*. Oxford University Press, NY 116-118.

Mirrlees, J. 1976. The optimal structure of incentives and authority within an organization. *The Bell Journal of Economics* 7(1) 105-131.

Mitra, S., S. Webster. 2008. Competition in remanufacturing and the effects of government subsidies. *International Journal of Production Economics* 111(2) 287-299.

Mrozek, J. 2000. Revenue Neutral Deposit/Refund Systems. *Environmental and Resource Economics* **17** (2) 183-193.

Nagarajan, M., Y. Bassok. 2005. Bargaining and alliances in supply chains. In *Consumer Driven Electronic Transformation,* ed G. J. Doukidis, A. P. Vrechopoulos. Springer Berlin Heidelberg, NY 39-51.

New York State Department of Taxation and Finance. 1999a. Recent Income Tax Changes Affecting Tax Years 2000 and After. http://www.tax.state.ny.us/pdf/memos/income/m99_8i.pdf (accessed 03.20.09)

New York State Department of Taxation and Finance. 1999b. 1999 Summary of Corporation Tax Legislative Changes Taking Effect in 2000 and After. http://www.tax.state.ny.us/pdf/memos/corporation/m99_4c.pdf (accessed 03.20.09)

Organizan "Reciclón Querétaro 2009".2009. http://www.libertaddepalabra.com/2009/02/organizan-reciclon-queretaro-2009/ (accessed 11.04.09)

Pratt, M., C. Macera, J. Sallis, M. O'Donnell, L. Frank. 2004. Economic interventions to promote physical activity. Application of the SLOTH model. *American Journal of Preventive Medicine* **27**(35) 136-145.

Qiaolun, G., J. Jianhua, G. Tiegang. 2008. Pricing management for a closed-loop supply chain. *Journal of Revenue and Pricing Management* **7**(1) 45-60.

Román, G.J. 2007.Diagnóstico sobre la generación de Residuos Electrónicos en México. Instituto Nacional de Ecología. Instituto Politécnico Nacional. Mexico DF.

Ross, S. 1973.The economic theory of agency: the principal's problem. *The American Economic Review* **63**(2) 134-139.

Savaskan, R., S. Bhattacharya, L. N. Van Wassenhove. 2004. Closed-Loop Supply Chain Models with Product Remanufacturing. *Management Science* **50** (2) 239-252.

Savaskan, R., L. N. Van Wassenhove. 2006. Reverse Channel Design: The case of competing retailers. *Management Science* **52** (1) 1-14.

Singh, N.,X. Vives. 1984. Price and Quantity Competition in a Differentiated Duopoly *The RAND Journal of Economics*, **15**(4) pp. 546-554

Skipper, H.D., R.W. Klein. 1999. Insurance Regulation in the Public Interest. The Path towards Solvent, Competitive Markets. Center for Risk Management and Insurance Research. Georgia State University. Atlanta Georgia.

Urgen a controlar basura electrónica. 2008. http://www.planetaazul.com.mx/www/2008/01/05/urgen-a-controlar-basura-electronica/
México, D.F. (accessed 11.04.09)

Webster, S., S. Mitra. 2007. Competitive strategy in remanufacturing and the impact of take-back laws. *Journal of Operations Management* **25** 1123-1140.

Zhao, W., Y. Wang. 2002. Coordination of joint pricing-production decisions in a supply chain. *IIE Transactions* **34** 701-715.