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Revalidation of Iowa type survivor curves

by

John George Russo

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Department: Industrial Engineering
Major: Engineering Valuation

Approved:

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In Charge of Major Work

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For the Major Department

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For the Graduate College

Iowa State University
Ames, Iowa

1978

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INTRODUCTION

Depreciation estimation is a critical function of all organizations, both private and public, that maintain capital and thus depreciable equipment and facilities. Life analysis and life forecasting procedures and techniques play important roles in the depreciation estimation function.

Depreciation gained recognition and importance in the United States primarily as a result of income tax applications in the early 1900's. It was not until the Treasury Department ruled in 1934 (16) that the best available evidence of average service lives should be the basis for depreciation rates, however, that depreciation practices and thus life analysis and life forecasting became an area of prime concern.

Notions of Life Analysis and Life Forecasting

Life analysis and life forecasting are considered to be parts of the overall area of life estimation for use in depreciation activities. In the end, determination of the probable service life a group property at any age is critical in itself and also allows the expectancy to be determined (Age + Expectancy = Probable Life). Figure 1 reflects the graphical relationships between age, expectancy, and probable life as they relate to the survivor, probable life, and frequency curves. The expectancy becomes the basis for the number of years used in the present worth calculations. The basic measure of value being the present worth of <u>future</u> returns. Life estimation techniques also provide the capability of continual reassessment of probable average lives as group properties age. Thus annual as well as depreciation reserve amounts, for example, can be monitored and adjusted as the properties are affected by economics,

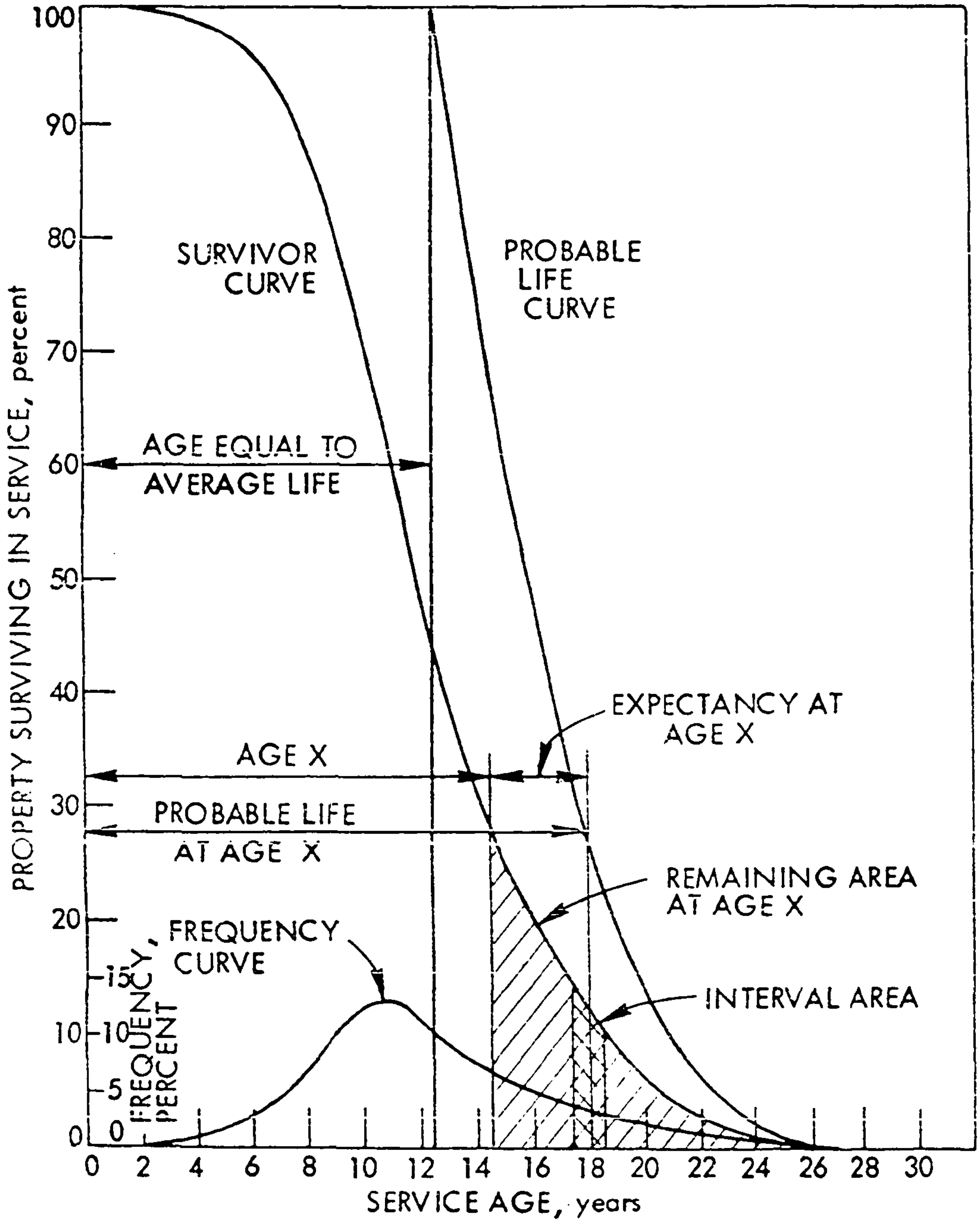

Figure 1. The survivor curve, frequency curve, and probable life curve and their relationships (15)

obsolescence, or a variety of other changes or decisions that affect probable average service lives over a period of time.

The basis of the life analysis aspect of life estimation is a review of the past and the application of mathematical or graphical procedures to give evidence of future direction. Since property data are generally of less than a full life cycle, a reliable system of extrapolation, by statistical or graphical means as noted above, is an important part of the life analysis process. Although seemingly identical properties are installed for the same use, a wide variety of service lives result from quality, condition, or management decision variations. Thus the groups of properties must be analyzed by reflecting these variations through the use of one composite retirement or survivor curve for the entire property.

The initial phase of life analysis of related group properties involves expressing the composite retirement or survivor characteristics of the properties with respect to age. Thus a representation of the past performance of the properties is produced that may be used to draw worthwhile inferences about the future. This phase makes use of one of the three commonly accepted methods of documenting past performance. These three are covered well by the Edison Electric Institute (6) and by Marston, Winfrey, and Hempstead (15) and include actuarial methods, turnover methods, and the simulated plant balance method. Choice of method is primarily based on the availability of aged mortality data, while quality and format of data as well as results required are also important criteria. The most commonly used and those related to this study are the acturial methods.

Three principal actuarial methods are commonly recognized (15): the individual unit method, the original group method, and the retirement rate method. The retirement rate method is considered the best (15) since it reflects both in service and retired properties. The individual unit method and the original group method consider only retirement experience of a single vintage or groups of vintages.

The partial survivor curves (percent surviving vs. age) resulting from the application of actuarial methods are then used for the final phase of life analysis. This final phase called fitting involves the extension to zero percent surviving and smoothing of the stub curves and estimation of average service life, probable average life, expectancy, and the mortality dispersion of the properties being studied. The extension and smoothing process can be accomplished either by mathematical means or by graphical means. The most common methods of fitting actuarial data are:

1. The Iowa Type Curves (25)
2. The Gompertz-Makeham distribution (1)
3. The Truncated Normal Distribution or h curves (9)
4. The Fitted Polynominals (4)
5. The Weibull Distribution (23)

Of the several other methods introduced over the years, the Patterson series (17) is probably the only other worthy of mention. Of the five main methods listed above, the first three involve fitting to an observed survivor curve or its life table, while polynomials are fit to observed retirement ratios, and Weibull distributions are fit to cumulative retirement curves.

Whether mathematical or graphical in nature, the fitting methods assume that given actuarial aged data for property in a format prescribed by that method, the mathematical or graphical bases of the method will extend and predict the remaining retirement or survivor characteristics of the property. The very fact that there are many methods of completing or extending survivor curves leads to the second and possibly most important aspect of life estimation, that of life forecasting.

All of the processes described to this point are life analysis processes. If the results of all possible fitting methods were available, a great variety of evidence as to average service life, probable average life, expectancy, and dispersion would be available with very little if any agreement between methods. Life forecasting would then be needed to come to a decision based on this evidence. Life forecasting relies solely on the judgment of a person's experience in this area. All evidences produced in the life analysis stage must be considered. These evidences not only include fitting results but also consideration of such things as overall economic trends, management policies, development of new technology, future potential direction, and an additional large variety of influences on the properties being treated. This concept was stressed by Robley Winfrey (25) as follows:

> "While the author strongly recommends the development and use of the retirement data and survivor curves as the basis of estimating the probable life of property units, he does not mean to infer that the expert judgment should be done away with in favor of pure statistical treatment. Each individual item, each group of items, and each property or company must be dealt with in the light of its present condition, its character and amount of service or production and its relation to the present and probable future economic trends, art of manufacture, and management policy. Tables of probable service lives, type survivor curves, and statistical methods are simply means of recording past experience to use in predicting what the future service might be."

While few properties are analyzed by the use of more than one fitting method, much less all methods, the use of at least one sound method does provide good evidence for use in the life forecasting process. The entire process of life estimation is thus a tool that provides basic information critical to the treatment of depreciation of industrial properties. Without it, depreciation accounting and planning would be baseless systems.

The Concept and Uses of Standard Curves

As previously discussed, standard curves are used in the life analysis phase of life estimation. Although details as to how each type of standard curve system operates will be handled in the next section, a review of the overall concept of standard curves as well as why they are needed and used is first needed.

Even though the family of normal curves is probably the best known for a variety of statistical applications, within the depreciation field only two major standard curve systems exist that are used to any degree:

1. The Iowa type curves
2. The h-curves (truncated normal distribution)

The normal family (13) of curves establish the basis for the life estimation related h-curves. The concept of the normal (frequency) curves is based on the probability of occurrence within a population. Thus an infinite variety of normal or nonskewed curves have been classified based on their population mean, representing curve location, and population variance, representing dispersion. Thus once the mean and variance of a sample is known, a fit can be made to the appropriate normal curve and a variety of data relative to the normal curve obtained. Although the normal family is basically a set of nonskewed frequency curves, much work has been accomplished over the years, and the development of standardized systems for skewed curves such as the chi-square family has evolved.

The basis of standard curve sets used in life estimation is that an array of curves (survivor or retirement based) can be developed, based on statistics or actual data that closely encompass all property mortality situations that may be encountered. Thus the concept is that an array developed from a large enough sample of the past will accurately give evidence as to the future, not at all unlike the probability based normal curve concept.

The availability of precalculated data and the simplicity of use of standard curves have led to their widespread use. The fitting process with standard curves involves manual or computer matching of account data or survivor curve shape to that of the property account in question to find the standard curve that best represents the survivor characteristics of property. This best fit curve is then considered to represent the survivor curve throughout its entire range of years for that property. This procedure compares to the mathematical systems which involve burdensome and extensive computations of mathematical functions that are fit to the survivor curve property data.

Standard curve systems also have precalculated data available for each curve within the array. Thus once the best fit curve has been chosen, a large variety of precalculated service life, renewal, depreciation reserve, etc., data can be quickly assumed for the property under study. This situation has led to the extensive use of the Iowa Curves since a large variety of tables of information and data are readily available.

The advantages of standard curve use over mathematical systems have led to a much more widespread application of sound life estimation practices. The economic feasibility of standard curve systems has allowed many small industrial and utility companies to use them, when the mathematically based systems would be beyond their scope technically as well as financially.

The use of standard curve systems, however, must be based on the fact that the array of curves available within a standard curve set can truly represent the property accounts being treated as well or better than the mathematical methods. The fact that the Iowa Curves and the h-curves systems produced at least equal if not slightly better results was brought out by Henderson (9) in 1968. There, in fact, have been no known studies accomplished that have shown the use of the graphical standard curves, on the whole, to be inferior to the mathematical approaches. Over the years, the availability of computers, while simplifying the mathematical approaches, have also made the standard curve systems much easier to apply. The relative ease with which several hundred property accounts were computer documented and fitted in this study is a good example of the increased capability of standard curve systems.

The Major Standard Curve Systems

It is important to understand how each of the standard curve systems functions, and in that regard three major standard curve systems will be discussed in detail.

1. The normal family of curves

2. The Iowa Type Curves

3. The h-Curves

The normal curves (13)

The normal frequency distribution was first developed in 1733 by DeMoivre, then later independently derived by Gauss. Each normal distribution within the infinite family can be represented by a symmetric bell-shaped curve (see Figure 2) that is related to the standard normal curve denoted: $Z \sim N(0,1)$, where the 0 denoted a zero mean and the 1 denoted a variance of one.

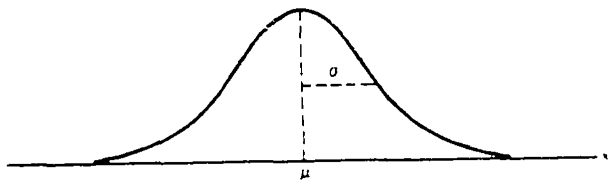

Figure 2. The normal frequency distribution with μ and σ denoted (22)

Any symmetrical sample curve can be denoted, $X \sim N(\mu,\sigma^2)$, and identified to its population normal curve by its relative population mean (μ) and variance (σ^2). The mean locates the curve within the family, while the variance indicates its dispersion. The standardized mathematical equation for normal curves:

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-1}{2\sigma^2}(x-\mu)^2\right], -\infty < x < \infty$$

where $\exp[a] = e^a$, and $e = 2.71828$

leads to the ability to obtain certain probability information for any normal distribution. This information is based on the following facts true of any normal distribution:

68.3% of its elements fall in the interval $\pm \sigma$

95.4% of its elements fall in the interval $\pm 2 \sigma$

99.7% of its elements fall in the interval $\pm 3 \sigma$

Since all normal distributions have a relationship to the standard normal curve, this curve has been very extensively tabulated. Few other normal distributions have probability tables developed since the number of distributions is nearly infinite. The relationship of all normal distributions to the standard normal allows a simple conversion of any normal distribution to a standard normal base. Thus with the tabulation of standard normal probabilities, extensive information can easily be acquired for any normal curve. The ease of this conversion has discouraged any attempt to tabulate even a small array of normal distributions.

The normal family of curves is somewhat unrealistic in that few samples or populations are unskewed. Normal distributions adequately approximate the actual distributions of many continuous variables, however. From a life estimation standpoint, however, they are far from adequate although the h-curve system to be discussed later is totally based on the normal family of curves.

Systems for dealing with skewed distributions based on the normal family have been developed and expand the application of the normal curves. The chi-square distribution is one of those extensions. The chi-square distribution is basically a sum of squares of independent standard normal variable distribution. This distribution was developed by Karl Pearson around 1900, and much of Pearson's chi-square research leads to his

Pearsonian type curve equations utilized by Winfrey (25) and Winfrey and Kurtz (26) in the development of the Iowa type curves.

The Iowa type curves

By far the most used standard curve system in the industrial property field, the Iowa curves, were empirically developed specifically for industrial property and have evolved over the years into an array of 31 curves that are accompanied by a great variety of tables. They were developed to conform to Pearson type frequency functions on a modified basis. History and development of the Iowa curves will be handled in a later portion of this paper. The intent at this stage is to present the concepts and procedures of the system.

The Iowa curve system is well set up to give good evidence concerning the extension of stub survivor curves based on aged mortality data and thus probable average lives of the properties represented by the stub curves. The underlying concept is that since the curves were empirically developed and form an array of survivor experience, partial or stub curves can be graphically fit to the best Iowa curve in the array and precalculated information from the Iowa curve assumed for the property represented by the stub curve. The actual fitting procedure can be accomplished manually or by computer. The manual procedures were described by Winfrey in 1935 (25) as follows:

> The probable average life and type of distribution are selected without computation other than the calculation and plotting of the stub curve for which the probable average life is wanted. The method involves simply plotting the survivor curve (stub or completed curve) to the same scale that the 18 type curves are plotted using the ordinates in percent of the total number of units and the abcissa in years. For this method the type curves need to be drawn for definite average lives, say, for each 5-year

interval from 5 to 50, making about 10 patterns of the same type curve on a sheet.

Of these type survivor curves and the stub survivor curve for which the probable average life is wanted are each drawn on transparent graphs, the individual stub curve can be superimposed on each of the 18 type sheets in turn until a satisfactory agreement is found. The stub curve is classified by the type curve which it fits best, and the probable average life estimated according to the position the individual curve occupies when superimposed upon the type sheets.

Although most of the development of the Iowa curves was done with survivor curves with age expressed as a percent of average life, the fitting process cannot be accomplished this way because the stub curves being fitted do not have an average service life available. But since the relative distribution for any average life for each type curve is the same, graphs for a number of average lives (as described above) can be plotted and used in the fitting process.

Fitting of stub curves to the Iowa type curves can also easily be accomplished by computer. Generally a method of least squares is used to find the best conformance Iowa curve (9).

The h-curves

Although the h-curve system of survivor curves is not extensively used across the United States, they have been a primary system for many of the utilities in New York State. The system as it now is applied was developed by Kimball (11) in 1947 and is based on the truncated normal distribution. The name "h-curves" was chosen because of the single parameter h. Kimball (11) states that truncating the normal curve, in effect, accounts for chance retirements, and that smaller and smaller values of h allow greater and greater influences of chance retirements. As the h value goes into the negative range and becomes more negative, the survivor curve approaches exponential, thus the ultimate indication of chance retirements.

It can be seen in Figure 3 that as the h value becomes larger, the remaining nontruncated frequency curve (noncrosshatched portion) moves closer to a normal bell shaped curve and becomes less skewed to the left--less left model in Iowa curve nomenclature. As long as the value of h is not $+\infty$ (normal), thus between $+\infty$ and $-\infty$, the modal point will always be to the left of the mean value of the truncated frequency curve. This creates a situation where nearly all h-curves are origin or left modal. Right modal curves cannot exist and symmetrical modal curves exist only at $h = +\infty$. Left modal curves exist when h is positive (except at $h = +\infty$) and origin modal curves exist when h is negative.

In the h-curve system, the vertical axis is set at the point of truncation, a distance of $\pm$ h from the normal mode. The centroidal t value or $\bar{t}$ as well as the mean of the new truncated distribution then move to the right based on the value of h. The h-curve system thus produced reflects an infinite array of frequency and survivor curves, some of which are shown in Figures 4 and 5. The family of curves is classed based on the h value selected and generally the range of h values required to assure a good fit is from h = +7.0 to h = -1.5.

The fitting process for h-curves is very similar to that used for the Iowa curves. A manual matching process could be accomplished by checking the stub curve against the average life variations of the array of h curves, considered practical. Thus the best fit could be selected. Generally this process is accomplished by computer. Since the h-curves are all of the same form and uniformly move from a symmetrical frequency form

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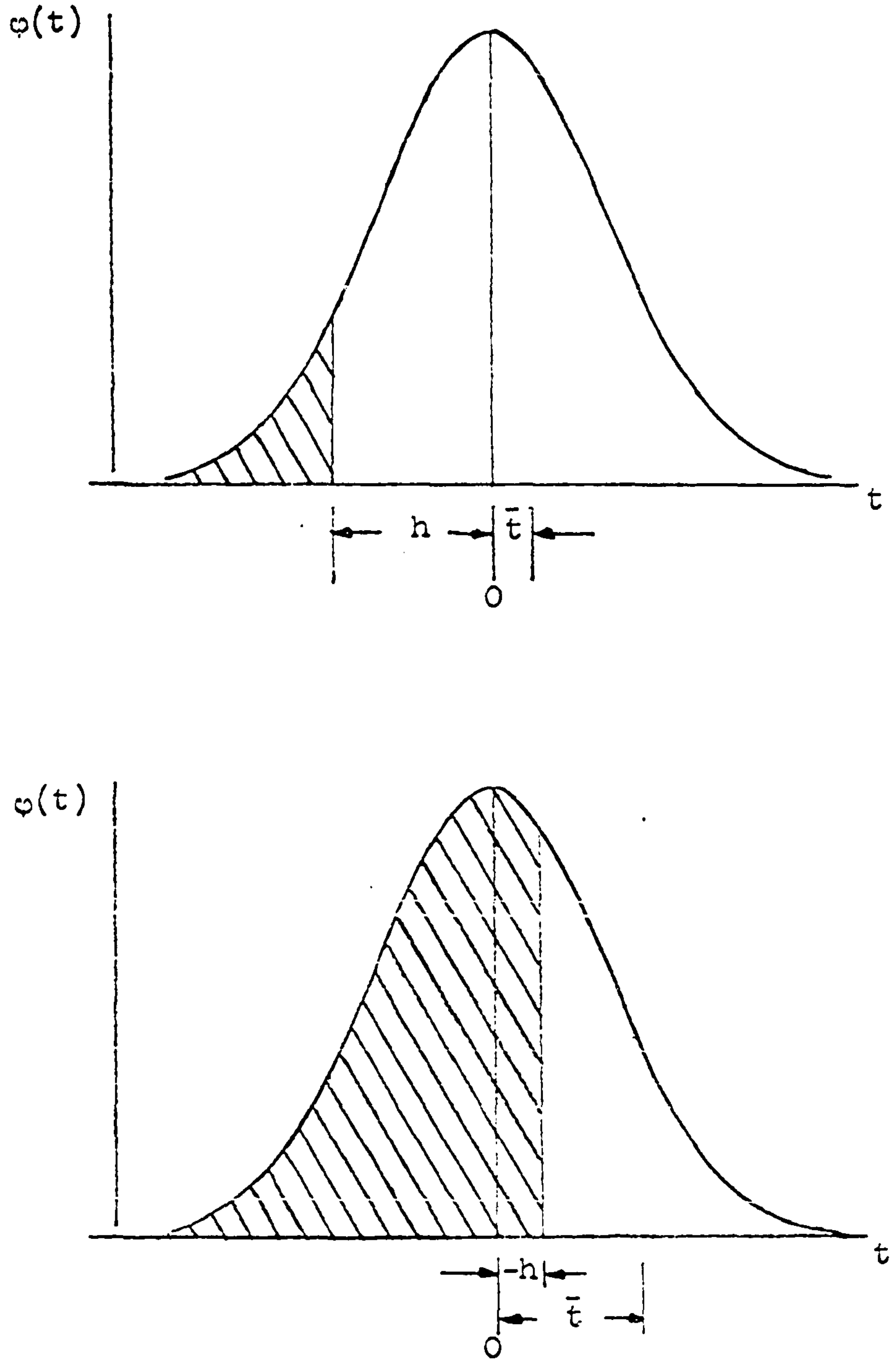


Figure 3. The standard normal distribution showing negative and positive h values (9)

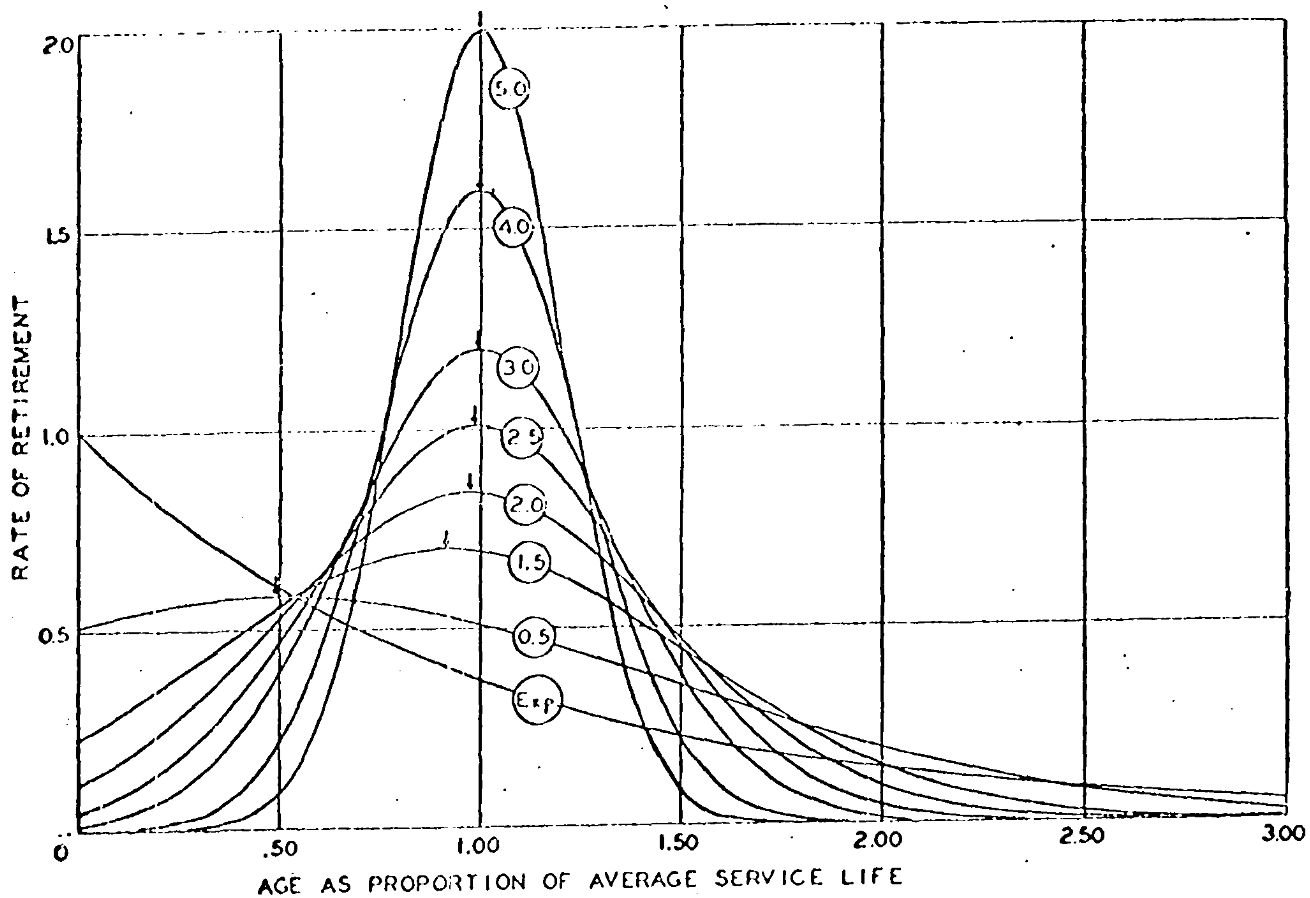
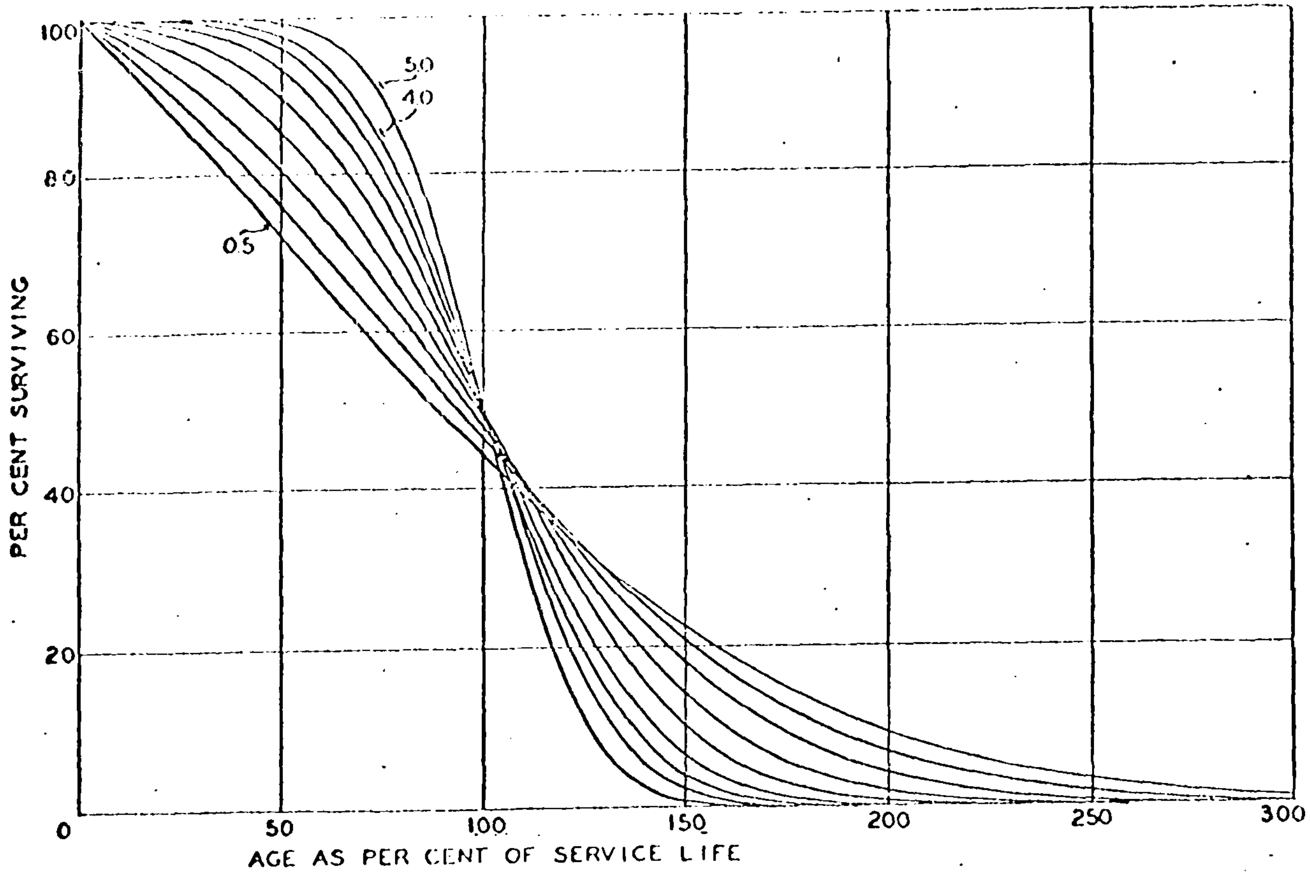


Figure 4. Retirement frequency curves for h-curve system (11)



Note: Curves intermediate between $h = 0.5$ and $h = 4.0$ are the life-table curves: $h = 1.0$, 1.5, 2.0, 2.5, 3.0, and 3.5.

Figure 5. Survivor curves for h-curve system (11)

($h = +\infty$) to an exponential frequency form ($h = -\infty$), a systematic process of starting with the largest h value being used and working towards lower h values until the best fit is obtained can be used. The best fit is generally indicated by a zero value of the algebraic sum of the vertical deviations between the h-curve points and stub curve points. Other best fit statistical criterion can also be used in computer aided fitting.

NEED FOR THE STUDY AND ITS OBJECTIVES

Although the Iowa type survivor curves have been added to since their initial formation, much of the account data was collected between 1916 and 1935. Since the use of Iowa curves is widespread, the re-establishment of their validity under current economic, technological, and managerial conditions is a need that cannot easily be challenged.

The need for this study has been informally discussed by proponents and opponents of the Iowa curve system for nearly two decades. Since a large variety of aged mortality data appeared available and since the use of computers presented a means of simplifying the required procedures, it seemed an appropriate time to initiate the Iowa type survivor curve revalidation project.

Fears have been expressed by some individuals from industrial and utility organizations that if the results of the study show the Iowa curves no longer valid, serious accounting and rate making consequences could be created for organizations using the Iowa curve system. A vast majority, however, felt that a comprehensive examination of the Iowa curves was in order and that, no matter what the result, a current and soundly based system would be established or re-established.

The objective of the study is to revalidate or reject the existing set of Iowa curves; to recommend development of a new set and add to or subtract from the present array based on valid and proven need for the actions determined. The intention is not to bias the study by assuming the existing curves, as they stand, are as valid today as when they were formulated.

The objectives of the study specifically are:

1. To collect a large number (400-500) of current industrial account mortality data and analyze the data.
2. To formulate an array of survivor curves from the data collected that represent the survivor characteristics of current industrial properties.
3. To test this array of curves against the existing Iowa curves to find if the new array does truly fall into the spectrum of the Iowa curves.
4. To test a separate set of 40-60 industrial property accounts against the new array established and the Iowa curves to see which produces the best results.
5. To declare the existing Iowa curves valid or invalid as they stand and to recommend additions to or deletions from or re-establishment of the existing Iowa curves if partial validation exists.

While the objectives of research center around revalidation of the Iowa curves, it is understood that the formulation of an improved array may cause rejection of the existing Iowa curves. Further, a failure to find an improved array will only result in a conclusion that, with the procedures of this research, no improved array was found.

Since the principal objective is not to abandon the empirical basis of the standard survivor curves, but rather to see if a noticeably different set of curves could be produced from summarizing modern property retirement experience, it was deemed appropriate to substantially follow the procedures adopted originally by Winfrey (25) but to incorporate the use of computers whenever possible. The computers will permit the treatment of large quantities of data efficiently and objectively. The test of validity of the present Iowa curves will be made within the context only; that is, with the same procedures but with modern data, will the resulting new array vary to a great extent from the present Iowa curves and will the new array represent present day retirement experience significantly better?

HISTORY AND DEVELOPMENT OF THE IOWA CURVE SYSTEM

History of the Iowa Curve System

Statistical compilation and treatment of human births and deaths and thus human survivor curves have been used for over 200 years for insurance rate determination. Yet it was not until the early 1900's that actuarial work was accomplished for service lives of depreciable physical properties. It was 1931 before a standard set of accepted survivor curves was produced. These original 13 Iowa type curves were a product of nearly 15 years of data collection and analysis work.

Work on the original sets of the currently existing 31 Iowa curves was begun in 1916 by Edwin B. Kurtz; he was joined in 1922 by Robley Winfrey of the Iowa State College, Iowa Engineering Experiment Station (now Iowa State University, Engineering Research Institute). Up to 1931 many other people were also involved to a lesser degree in collecting data, calculating curves, and analyzing data for 65 property groups. From this research and partially based on Kurtz's earlier University of Wisconsin thesis, the Iowa Engineering Experiment Station Bulletin 103 (26) was produced in 1931 that grouped the 65 survivor curves into 13 types.

Between 1931 and 1935 Winfrey continued data collection and added 111 property group curves to the 65 exhibited in 1931. From these 176 total curves studied, 18 standard Iowa type curves (see Figures 6, 7, and 8) emerged in 1935 when Winfrey authored the still existent Iowa Engineering Experiment Station Bulletin 125 (25). This bulletin not only presented the 18 Iowa type curves but also presented detail concerning the procedures for analyzing historical property retirement data on a statistical basis. Thus

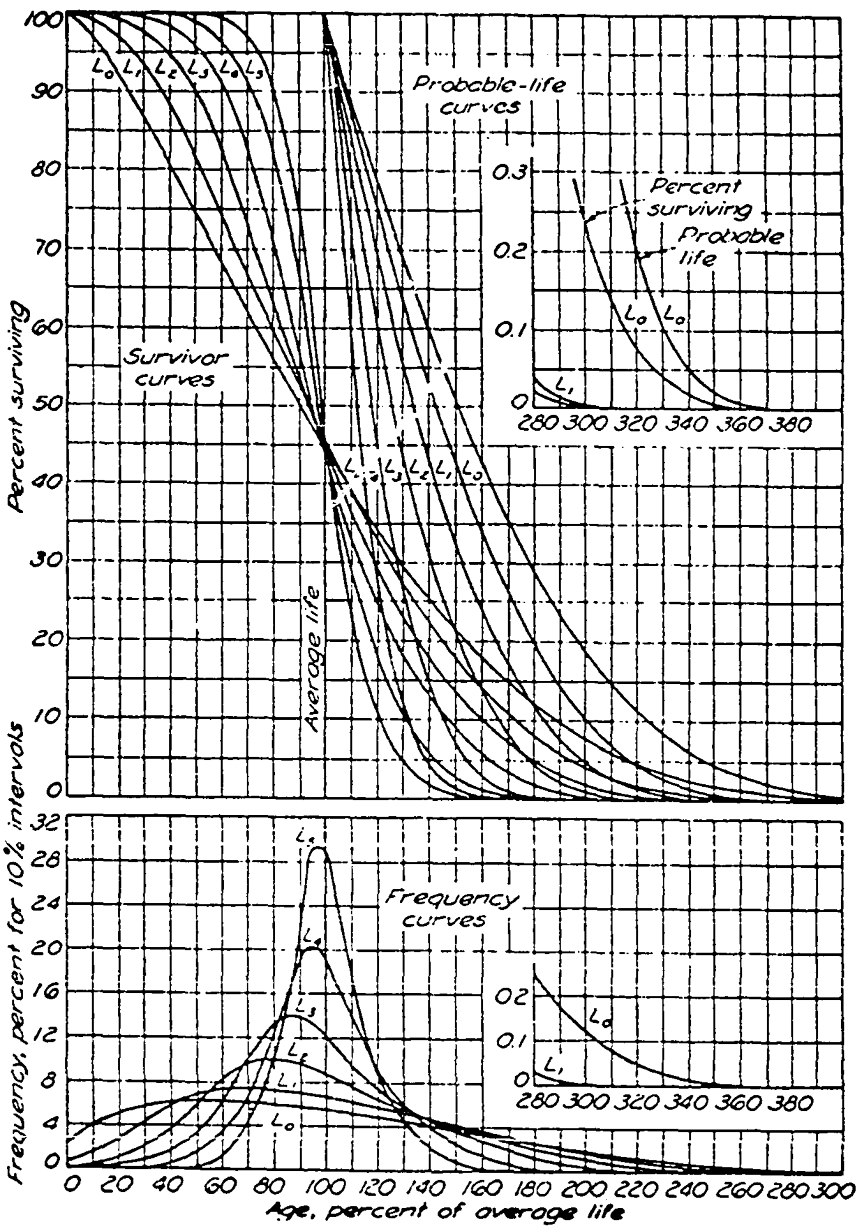


Figure 6. Final survivor, probable life, and frequency curves for the left-modal types (25)

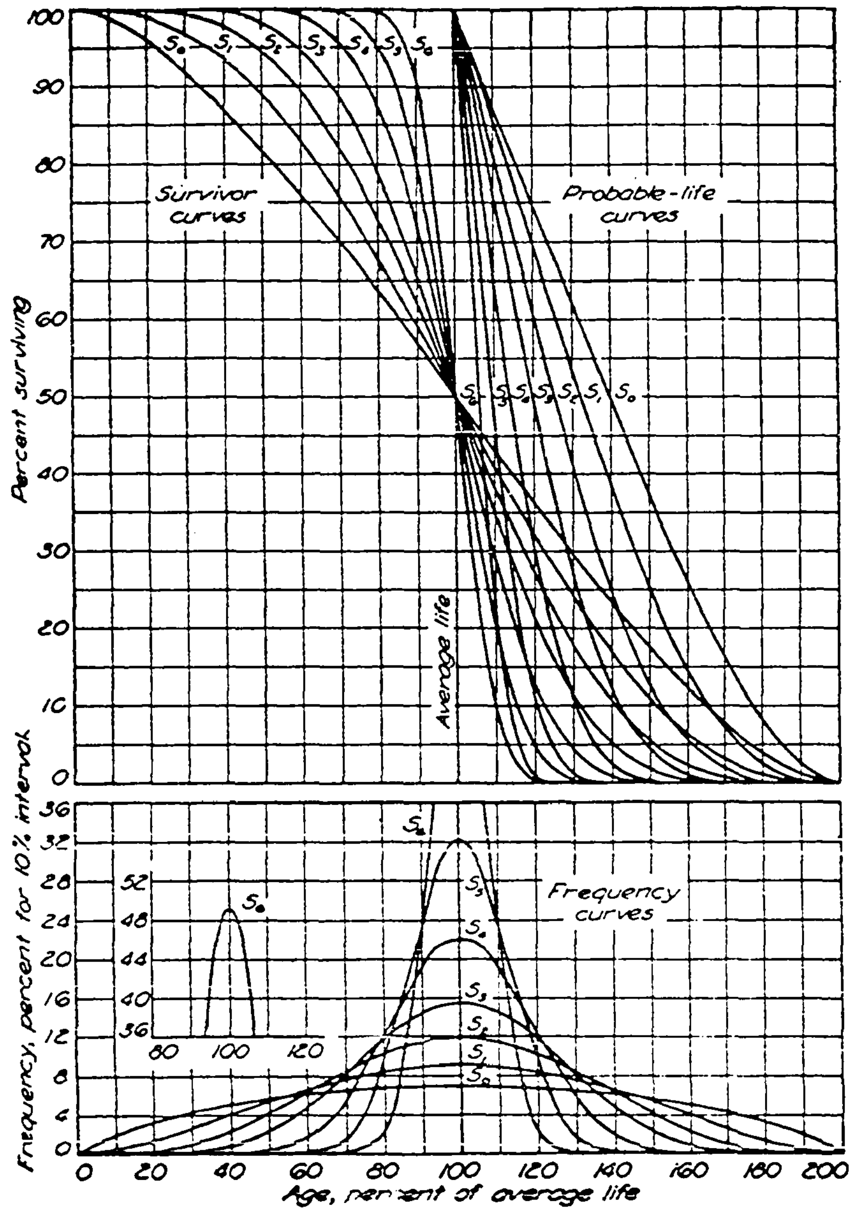


Figure 7. Final survivor, probable life, and frequency curves for the symmetrical-modal types (25)

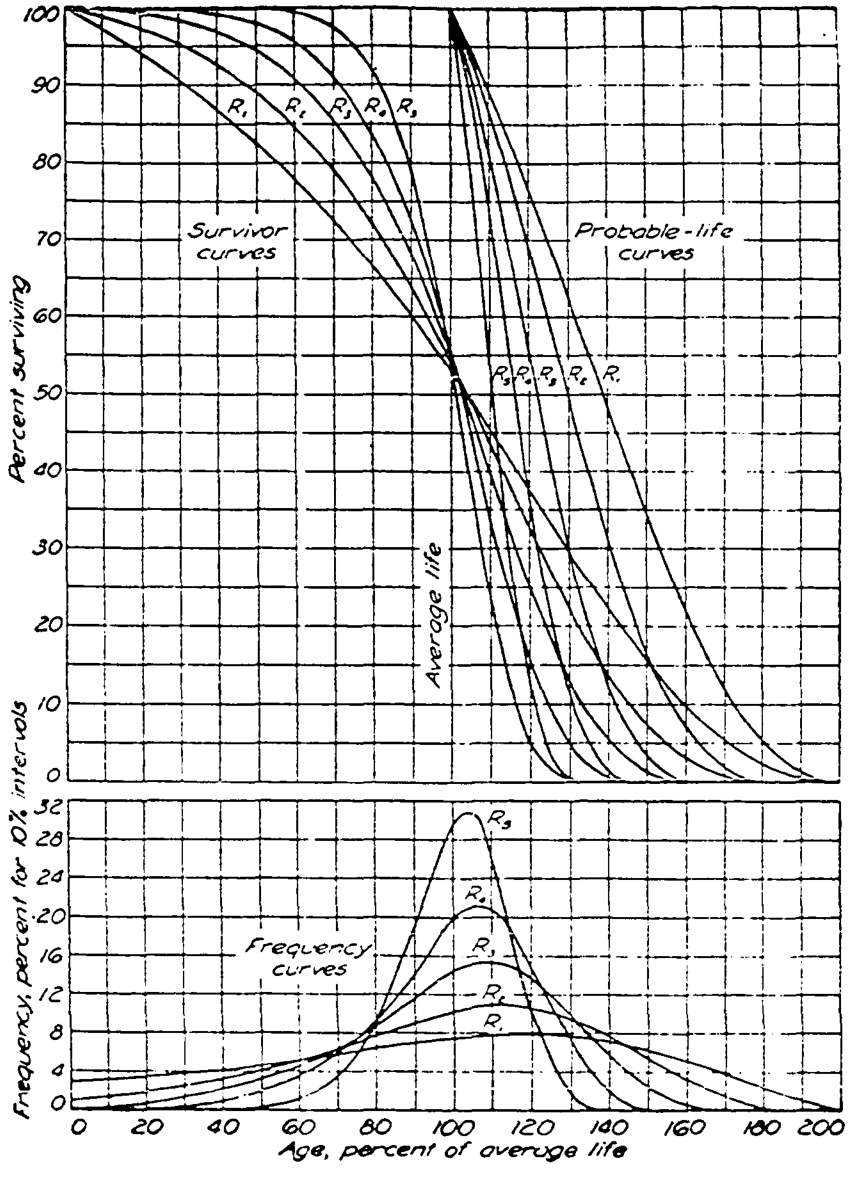


Figure 8. Final survivor, probable life, and frequency curves for the right-modal types (25)

a readily available aid for making estimations of the lives of physical property was established. Depreciation systems and methods were given a solid boost due to the newly developed capability of more accurate prediction of service lives.

A revised edition of Bulletin 125 was made available in 1967 by the Iowa State University Engineering Research Institute. This revision included minor corrections in the 18 existent Iowa curves as well as the addition of four origin modal curves (see Figure 9) developed by Frank Couch (3), formerly of Iowa State University. These curves were developed based on empirical methods similar to those used for the original 18 curves. Development was initiated in response to industry feeling that although additional curve shapes were infrequently encountered, they nevertheless existed and should be represented in Iowa curve array. Thus the standard Iowa curve set increased to 22.

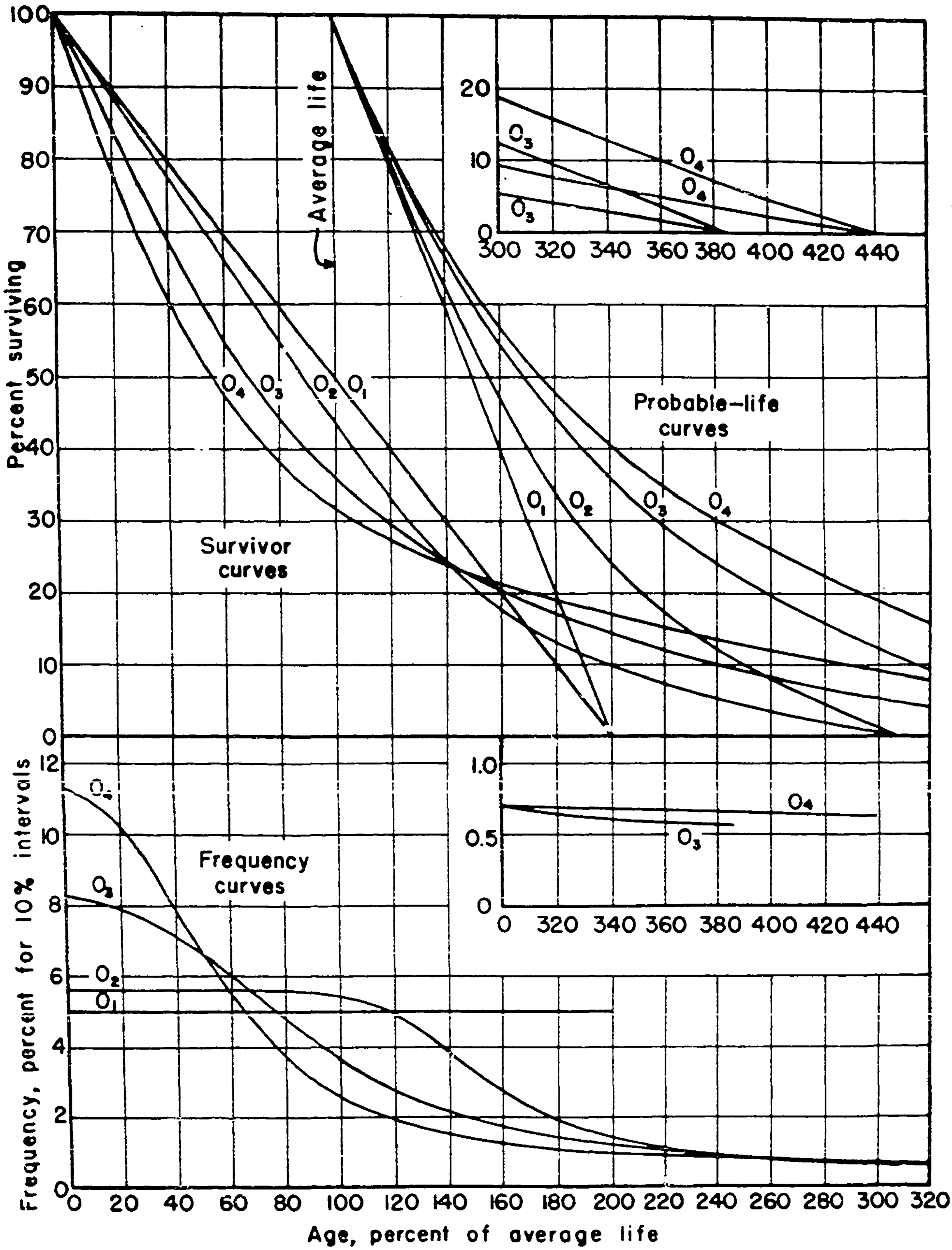
Nine additional standard curves have since been recognized, although their development did not necessitate work with actual property data. Number 23 was identified as the standard square curve and numbers 24 through 31 as "half curves" that simply provide intermediate steps between those original 18 curves where large percentages of industrial property experience fall. Work outlined in this report concerns itself with the entire Iowa curve array, including the half curves and the origin modal curves.

At the present time, the following are the full array of Iowa curves:

Left Modal -- L type -- (8 total)

L_0, $L_{.5}$, L_1, $L_{1.5}$, L_2, L_3, L_4, L_5,

Figure 9. Final survivor, probable life, and frequency curves for the origin-modal types (3)


Average life
Percent surviving
Survivor curves
Probable-life curves
O4 O3 O2 O1
Frequency curves
Frequency, percent for 10% intervals
Age, percent of average life

Symmetrical Modal -- S type -- (10 total)

$S_{-.5}$, S_0, $S_{.5}$, S_1, $S_{1.5}$, S_2, S_3, S_4, S_5, S_6

Right Modal -- R type -- (8 total)

$R_{.5}$, R_1, $R_{1.5}$, R_2, $R_{2.5}$, R_3, R_4, R_5

Origin Modal -- O type -- (4 total)

O_1, O_2, O_3, O_4

Square -- SQ type (1 total)

SQ

Iowa curve identifications are made as reflected by the following example: L_3-20; "L" indicates the location of the frequency curve mode as left of the mean or average life (R indicates mode location right; S indicates mode location at the mean or symmetrical; and O indicates mode, location at the origin); "3" indicates the relative magnitude (height) of the frequency curve mode for each modal group; and "20" indicates the average service life of the property. Thus within each curve type a large variety of mode location, mode height, and average service life flexibility exists.

Development of the Iowa Curve System

Although some historical background will be included because of the nature of the discussion, the major intention of this subsection was to review the details involved in the development of the existing Iowa curve set. This developmental review was meant to give a better understanding of how the system was developed and where it stands today, before the revalidation procedures and results are discussed. All information used was extracted directly from Iowa State College Bulletin 103 (26), Iowa State

University Bulletin 125 (25), and Frank Couch's 1957 Iowa State University thesis (3).

The original 13 Iowa curves, as well as additional Iowa curves set forth later, were empirically developed. Kurtz and Winfrey collected data for 65 property accounts in formulating the original 13 curves. Each survivor curve was calculated by the individual unit method and smoothed as explained by Winfrey and Kurtz in 1931 (26):

> It will be seen that the original data curves are irregular in shape, and therefore typical of the curves which are secured by ordinary methods of assembling mortality data on industrial properties. Should a sufficiently large number of units be observed at regular frequent intervals throughout the period their service lives and all units removed from service at the time they were worn out, the resulting data would no doubt work up into a smooth curve. In order to simulate this condition for the purpose of arriving at a theoretical mortality curve, a smooth curve was drawn through the points of original data. The same characteristics of the original curve were retained in the adjusted curve by shifting the smooth curve until it had the same area underneath as the original curve, the same maximum life, and the same general shape.
>
> In addition, 11 of the first 65 frequency curves were drawn from the smoothed survivor curves and adjusted for smoothness. The remaining 54 frequency curves were drawn from the unsmoothed survivor curves and thus showed more irregularities; they were, however, somewhat adjusted for smoothness.

After curve formulation, work was first done with the renewal patterns of the property and renewal tables built for analysis. Next, several relationships were plotted and studied--these relationships included:

1. Average life to maximum life---It was found that the range of the maximum life in terms of average life ran from about 115% to slightly over 290%, with about 80% of the groups falling with 125% to 250%, the average being 200%.

2. Magnitude of mode to maximum life---It was found that the magnitude (horizontal) of the mode was longer for longer maximum lives than for short maximum lives.
3. Magnitude of mode to average life---The same relationship as in 2 (above) was found.
4. Length of time required for renewals to become constant to several characteristics and ratios including the ratio of maximum life to location of the mode, the ratio of maximum life to the magnitude of the mode, and maximum life---It was found that curves of the same general shape could be drawn for either the maximum life cycles or the average life cycles, the ordinate in the latter case being approximately twice the former.
5. Modal year to the year of maximum total renewals---It was found that no correlation existed between location of the mode and the year of maximum total renewals.
6. Magnitude of mode to maximum total renewals---Again, as in 5 (above), no correlation was found.

After having studied and analyzed the 65 property groups as outlined above, enough similarities and relationships were noted to conclude that a standard set of curves might well be feasible. It was also felt that such a standard set might eliminate much of the calculation required for determining extended data information concerning individual property data that could be matched to the standard curve set. The first step in this standardization process was to convert the abcissa of the curves from age to age as a percent of average life by setting the average life equal to 100%, Grouping of survivor curves was tried on several bases including class of physical property, maximum life, inspection, and the slope of the central portion of the curves. The final basis attempted was that of using characteristics of the frequency curve--in the end this proved to be the most satisfactory. Three distinguishing characteristics were noted: location of the mode relative to the average life, magnitude of the mode, and the maximum age in percent of average life. Three groups of curves were then formulated based on whether the mode was left of, coincident with, or right of the average life. These three groups, called L type, S type, and R type, were then further subgrouped based on the magnitude of the mode and thus the array of 13 curves produced by averaging the frequency curves of each subgroup and extracting the survivor curves of each. Of the 13 curves, four were left modal, five symmetrical modal, and four right modal. Each subgroup was also adjusted slightly so that a more or less equally spaced array resulted and so that the area under each survivor curve was held constant at 10,000 percent units. It is interesting to note that the curves within each subgroup were subjectively rated as to "goodness of fit" simply by designating good, fair, or poor. The 65 curves were compared to the standard curve of their subgroup as well as to the 13 curve array in general.

At the time the 13 curve set was complete, the developers noted that additional research was necessary and that more than 13 Iowa curves might, in the end, emerge. An attempt was also made at that time to find mathematical equations that fit the 13 Iowa curves. Although the symmetrical modal curves were fitted to Pearson's type II with good success, neither the left modal nor right modal curves were fit with any success to any general equations during the initial study. A detailed coverage of the final equations for Iowa curves can be found in Appendix A of this report.

Additional work was also done on renewal calculations for all 13 curves and general equations as well as calculated data formulated.

After publication of the 13 original Iowa curves, Winfrey continued property data collection and acquired 59 additional property groups from which 111 survivor curves were calculated. Thus a total of 176 curves were now included in the overall study. The second study resulted in the addition in 1935 of five curves to the original 13 for a total of 18 in Iowa curve array. The 13 original curves were basically unchanged, with only minor modifications being made when equations were fit to them.

The second set of survivor and frequency curves (66-176) were calculated, drawn, analyzed, and grouped in the same manner as described for the original 65 curves, except that calculations were done by other than just the individual-unit method. For the 111 curves, the individual-unit method was used 41 times, the annual-rate method 36 times, the original-group method 26 times, and the multiple original-group method 8 times. The group averages resulted in general curves very much like the original 13 Iowa curves, except that new shapes outside the original 13 were evident. Additional testing and grouping were accomplished with all 176 curves, and the result was the identification and addition of two new L type curves, two new S type curves, and one new R type curve to the existing 13, for a total of 18 Iowa curves within the set.

Final survivor, frequency, and probable life curves, as previously shown in Figures 6, 7, and 8 of this report, were then completed by adjusting spacing and area of the full array for each group as was done with the original 13 curves. As reflected by Winfrey in Bulletin 125 (25), the 18 curve Iowa set with number of curves included in each was then:

	L_0 - 7	S_0 - 6		
	L_1 - 15	S_1 - 13	R_1 - 5	
	L_2 - 17	S_2 - 13	R_2 - 5	
	L_3 - 15	S_3 - 7	R_3 - 19	
	L_4 - 8	S_4 - 9	R_4 - 24	
	L_5 - 3	S_5 - 3	R_5 - 6	
		S_6 - 1		
Total No. of Curves	65	52	59	--- 176

It should be noted that six curves were found that were of unusual shapes that did not easily group with the other curves. It also should be noted that of the 18 Iowa curves formulated, 9 of them were represented by 132 data account curves or 75% of the 176 total data curves used.

After development of the five additional Iowa curves, Winfrey spent a considerable amount of effort to find mathematical expressions to fit the left and right modal curves. The symmetrical modal curves had already been fit to Pearson's type II as previously noted; the two new S type curves were also found to successfully fit to Pearson's type II expression. Winfrey was able to identify equations for all of the 18 curves, many involving complicated mathematical models; to this day little improvement has been found. Winfrey basically investigated three types of expressions in trying to fit the L type and R type curves: Pearsonian; Gram-Charlier; and Gompertz-Makeham. In the end compounding of curves and the use of modified Pearsonian expressions were used for the L type and R type fit. A detailed coverage of the final equations for all Iowa curves can be found in Appendix A of this report.

In 1957 Couch (3) researched 24 curves of "unusual shape" as noted by Winfrey and Kurtz in their research. The curves were calculated by the annual-rate method and all other empirical procedures followed in the formulation of the previous 18 Iowa curves were also followed by Couch. The result was the addition of four origin modal or O type curves to the Iowa set as previously shown in Figure 9. These four additional curves and the number of curves included in each were:

	O_1 - 4
	O_2 - 6
	O_3 - 6
	O_4 - 8
Total No. of Curves	24

Although the O_1 type curve was classed as origin modal, it was simply the straight line curve. While this curve had been recognized for some time, it had not been assigned an Iowa type classification.

Mathematically, the O type curves (except O_1, which is straight line and requires no fitting) were all fitted best by the trial and error adjustment of Pearson's type VIII curve. Equations for these Iowa curves can be found in Appendix A as well.

As previously noted, the intermediate step half curves have also been added to the Iowa curve set. While they were included in the Iowa curve set used in the testing portion of this research, they will not be discussed further.

PROCEDURES OF THE STUDY

The General Approach

It was decided that the study should parallel the original development of the Iowa curves as closely as possible while being flexible enough to include improvements in procedures and techniques whenever it became evident that they were needed. The availability of computers, of course, was the biggest change from the circumstances under which the original curves were developed. Although a large amount of manual work was required, the use of computers for calculations, statistical procedures, and graphing saved thousands of hours and allowed a larger sample of data to be included in the study. The statistical methods and procedures, although kept as uncomplicated as possible, were more advanced and more sound than those used originally. The clustering procedure, specifically, was one that was not available until a few years ago and also could not have feasibly been done without computer assistance.

The overall approach, as with the original development of the Iowa curves, is empirical in nature. After considerable thought, it was not felt that a better method existed than the analysis of a large number of actual industrial property accounts.

Account data were collected from a wide variety of applications, then the data for the study selected to represent the best possible cross-section of industrial groups and property within each group. Data were then converted to survivor curve and frequency curve format so that statistical work could be accomplished. After segmenting the accounts into modal groups, each group was additionally segmented into survivor curve shape related clusters. These clusters were then averaged and used as the new array of curves empirically developed from the data collected. The final step involved testing a group of property accounts against the new array and the existing Iowa curves and testing the new array against the existing Iowa curves.

The procedures used were designed to be basic and statistically sound. Great care was taken so that involved statistical and mathematical procedures did not interfere with what was considered the objective of the study--an empirical revalidation of the Iowa curves.

Data Collection

The objective of the data collection procedure was to collect aged mortality data on as many industrial property accounts as possible so that 400-500 of these accounts could be used in the study. Data were sought and received from the following types of industries:

-Gas transmission

-Electric

-Gas

-Water

-Telephone

-ICC (highway & rail vehicles)

-Oil pipeline

-Roads and highways

-Commercial

Data were solicited by mail from all members of the Depreciation Accounting Committee of the American Gas Association--Edison Electric

Institute as well as from the Interstate Commerce Commission, American Telephone and Telegraph Company, and from the private files of Robley Winfrey, who also provided assistance in acquiring other data.

Instructions-Specifications for data collection and transmittal (see Appendix B) were formulated to give direction to those volunteering data for the project. The major points covered in the instructions-specifications were:

1. Only aged mortality data would be used.
2. The latest experience year should be between 1965 and 1976 so that recent data with recent retirement years could be used.
3. Survivor curve data from different experience bands of the same account could be sent and would possibly be used.
4. Only accounts with survivor curves extending down to between 20% and 0% surviving would be used, so that extensions could be done without significant distortion.
5. The preferred form of the data when transmitted was tabular with dollars exposed, dollars retired, retirement or survivor ratios, and observed percent surviving.
6. Other information to be submitted with each data set:
   A. Name, position, company, and address of sender.
   B. Account number and name or class of property and geographical location of equipment.
   C. Name and address of company owning equipment (if not own equipment) or code number if owner name was confidential.
   D. Placement band, experience band, ASL, and Iowa Type Curve (if available).

E. Method used in calculating survivor curve, retirement rate preferred.

In the end over 2,000 data sets were received from companies or organizations. The individual number of sets received ranged from 10 to several hundred. Data were provided from every state in the U.S.; however, 12 states were represented only by "national" data from road and ICC accounts--thus 38 states are directly represented. Figure 10 shows the industry type and location of all data collected. A complete list of all those contributing data is included in Appendix C.

Of the 2,000 data sets collected, 490 were actually used in the study. Figure 11 shows the industry type and location of this data, and Table 1 reflects the number of each equipment type used, by industry.

In reducing the number of data sets down to the number used in the study, several factors were considered. These factors (listed below) included as few biases as possible and were considered as an attempt to use the broadest, most comprehensive data for the study.

1. As many zero percent surviving accounts as were possible were incorporated. This eliminated potential distortion that might occur extending nonzero accounts.
2. After zero accounts, those close to zero (rather than close to 20%) were considered with more favor, for the same reasons listed in #1.
3. Nearly all accounts received with percent surviving above 20% were eliminated. Most of the 14 accounts used that were above the 20% cutoff were within industry or equipment types that were not plentiful and were very close to 20%.

Figure 10. Locations of all documented data

- ✕ Gas
- ● Electric
- E Industrial
- △ Water
- C Commercial
- T Telephone
- P Oil pipeline
- G Gas transmission

<u>Note</u>: Roads and highways data are multi-state; Interstate Commerce Commission is widespread national data

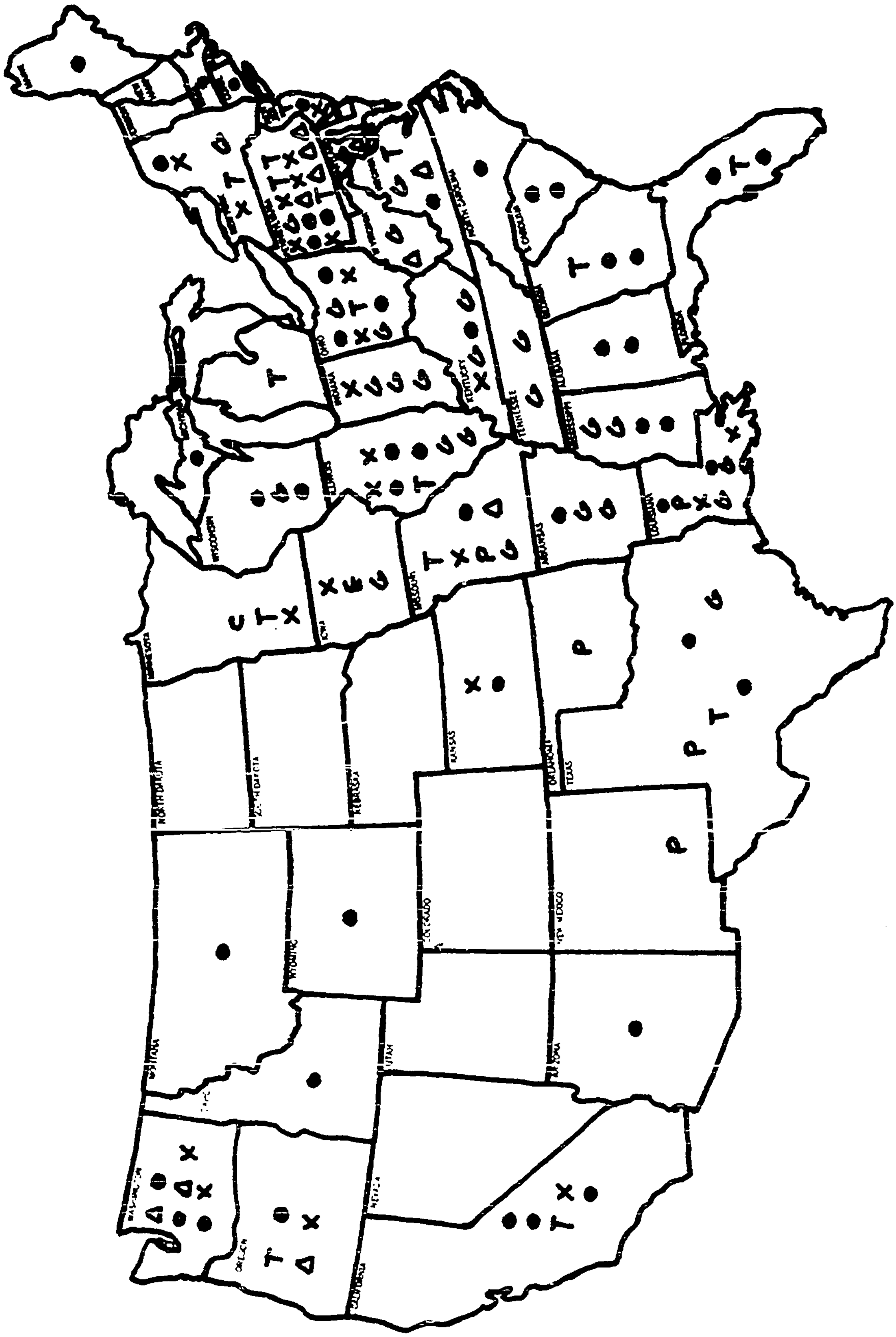


Figure 11. Locations of data used in the study

- X Gas
- ● Electric
- E Industrial
- Δ Water
- C Commercial
- T Telephone
- P Oil pipeline
- G Gas transmission

Note: Roads and highways data are multi-state; Interstate Commerce Commission is widespread national data

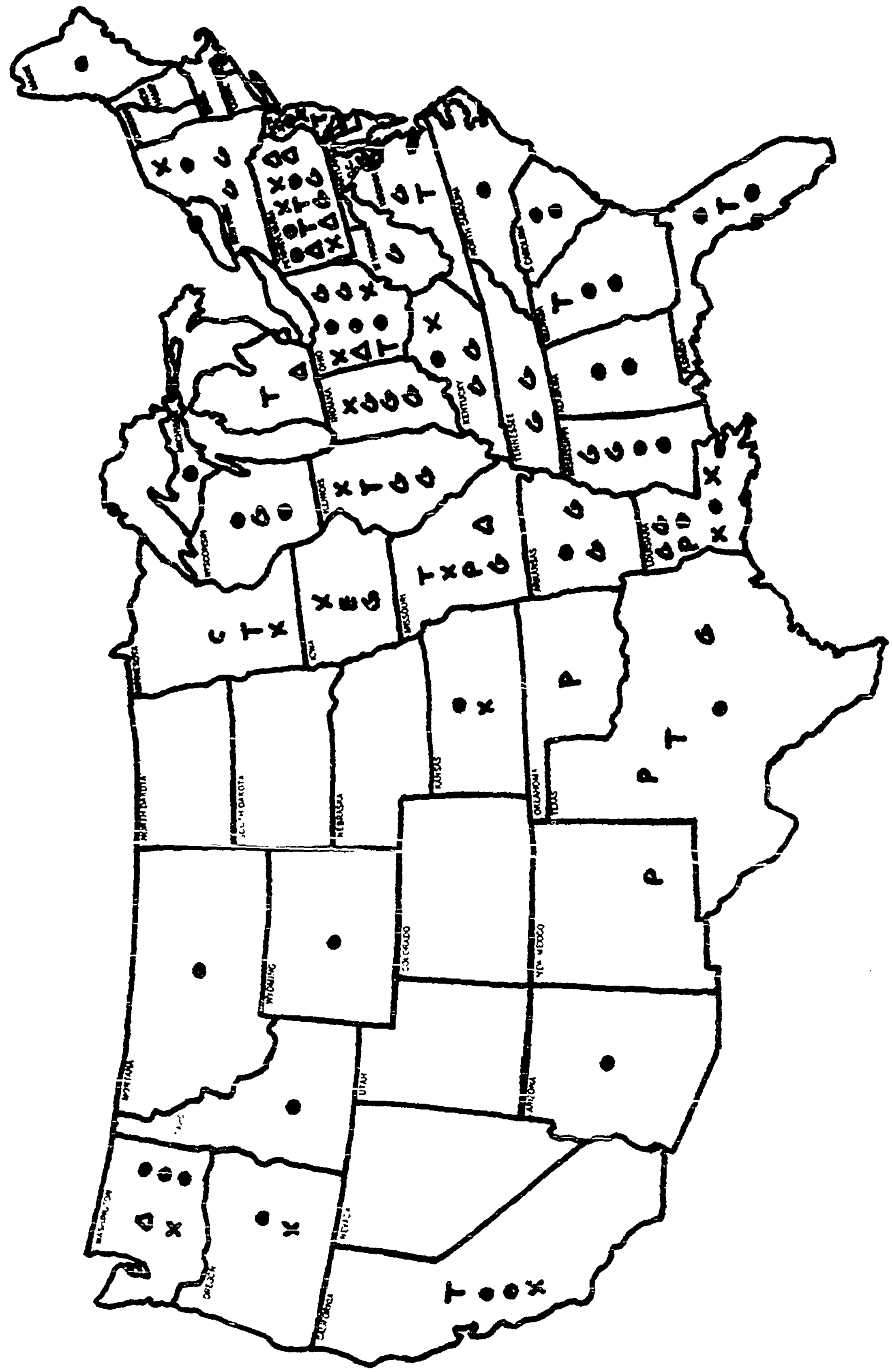


Table 1. Number of each equipment type used by industry--490 total

No. used	Acct. no.	Description
01 - Gas Transmission - 66 Total		
		Production Plant
1	305.0	Structures & Improvements
1	306.0	Boiler Plant Equipment
1	311.0	Liquid Petroleum Gas Plant Equipment
1	317.0	Purification Equipment
2	328.0	Field Measuring & Regulating Station Structures
1	329.0	Other Structures
1	329.1	Purification Structures
1	331.0	Gas Well Equipment
1	332.1	Field Lines
1	333.0	Field Compressor Station Equipment
2	334.0	Field Regulating Measuring Station Equipment
1	335.1	Drilling & Cleaning Equipment
1	335.2	Drilling & Cleaning Equipment
1	336.0	Purification Equipment
		Pipeline
1	343.1	Pipeline - Pipe, Coupling & Fittings
		Underground Storage
1	350.2	Rights of Way
1	351.2	Compressor Station Structures
1	352.02	Wells - Equipment
1	353.01	U/G Storage Lines
1	353.2	Storage Tributary Lines
2	354.0	Compressor Station Equipment
2	355.0	Measuring Regulating Station Equipment
1	357.1	Drilling & Cleaning Equipment
		Transmission Plant
1	365.2	R.O.W.
2	366.1	Compressor Station Structures
2	366.2	Measuring & Regulating Station Structures
3	366.3	Other Structures
1	367.0	Mains
1	368.0	Compressor Station Equipment
2	369.0	Measuring & Regulating Station Equipment
1	369.12	Measuring & Regulating Station Equipment
2	370.1	Communication Equipment - Telephone
2	370.2	Communication Equipment - Radio

Table 1. (continued)

No. used	Acct. no.	Description
1	370.3	Communication Equipment - Microwave
1	370.5	Communication Equipment - Telemetering
1	371.1	Other Equipment
		General Plant
1	391.0	Office Furniture & Equipment
2	391.1	Office Furniture & Equipment
3	392.11	Transportation Equipment - Auto
1	392.12	Transportation Equipment - Light Trucks
1	392.13	Transportation Equipment - Heavy Trucks
2	392.21	Transportation Equipment - Trailers
1	393.0	Stores Equipment
2	394.2	Garage Equipment
1	394.3	Shop Equipment
1	395.0	Lab Equipment
2	396.0	Power Operated Tools & Equipment
2	398.0	Miscellaneous Equipment
02 - Electric - 122 Total		
		Steam Production
2	312.0	Boiler Plant Equipment
2	314.0	Turbogenerator Units
1	315.0	Access Electric Equipment
2	316.0	Miscellaneous Power Plant Equipment
		Transmission Plant
3	352.0	Structures & Improvements
2	353.0	Station Equipment
3	354.0	Towers & Fixtures
5	355.0	Poles & Fixtures
3	356.0	O/H Conductors & Devices
1	357.0	U/G Conduit
1	358.0	U/G Conductors & Devices
		Distribution Plant
4	361.0	Structures & Improvements
10	362.0	Station Equipment
3	364.0	Poles, Towers & Fixtures
4	365.0	O/H Conductors & Devices
1	366.0	U/G Conduit
2	367.0	U/G Conductors & Devices

Table 1. (continued)

No. used	Acct. no.	Description
3	368.0	Line Transformers
3	368.1	Line Transformers
1	368.2	Line Transformers
3	369.0	Services
7	370.0	Meters
1	370.1	Meters
3	373.0	Street Lighting & Signal Systems
1	373.1	Street Lighting & Signal Systems
1	373.2	Street Lighting & Signal Systems
		<u>General Plant</u>
1	390.0	Structures & Improvements
2	390.1	Structures & Improvements
2	390.2	Structures & Improvements
5	391.0	Office Furniture & Equipment
1	391.1	Office Furniture & Equipment
2	391.2	Computers
1	391.4	Mechanical Equipment
5	392.0	Transportation Equipment
2	392.1	Transportation Equipment - Cars, Station Wagons, ½ Ton Trucks
1	392.2	Transportation Equipment - 3/4 Ton Trucks
2	392.3	Transportation Equipment - 3/4, 1, 1½ Ton Trucks
2	392.4	Transportation Equipment - 1½-2 Ton + Trucks
1	392.9	Trailers
2	393.0	Stores Equipment
3	394.0	Tools Shop & Garage Equipment
2	394.2	Shop-Tools & Equipment
1	394.4	Construction Tools & Work Equipment
1	394.8	Transportation Garage Equipment
2	395.0	Laboratory Equipment
2	396.0	Power Operated Equipment
5	397.0	Communication Equipment
5	398.0	Miscellaneous Equipment
<u>03 - Gas - 72 Total</u>		
		<u>Production Plant</u>
1	306.0	Boiler Plant Equipment
1	311.0	Liquid Petroleum Gas Equipment
1	320.0	Other Equipment
1	326.0	Gaswell Structures

Table 1. (continued)

No. used	Acct. no.	Description
1	327.0	Field Compressor Stations
1	328.0	Field Measuring & Regulating Station Structures
1	329.0	Other Structures
1	331.0	Gas Wells - Pipe
1	332.0	Field Lines
1	333.0	Field Compressor Station Equipment
1	334.0	Field Measuring Regulating Station Equipment
1	335.0	Drilling & Cleaning Equipment
2	338.0	Gas Services
		Natural Gas Storage Plant
1	351.2	Storage Structures
1	355.1	Measuring & Regulating Station Equipment
1	362.0	Gas Holders
		Transmission Plant
1	366.0	Compressor Station Structures
2	367.0	Gas Mains
1	369.0	Measuring & Regulating Station Equipment
		Distribution Plant
1	375.0	Structures & Improvements
7	376.0	Gas Mains
1	376.5	Gas Mains
2	377.0	Compressor Station Equipment
4	378.0	Measuring & Regulating Station Equipment
3	380.0	Services
8	381.0	Meters
1	382.0	Meter Installations
1	383.0	House Regulators
1	384.0	House Regulator Installation
3	385.0	Industrial Measuring & Regulating Station Equipment
1	387.0	Other Equipment
		General Plant
1	390.0	Structures & Improvements
3	391.0	Office Furniture & Equipment
5	392.0	Transportation Equipment
1	393.0	Stores Equipment
3	394.0	Tools, Shop & Garage Equipment
1	395.0	Lab Equipment

Table 1. (continued)

No. used	Acct. no.	Description
1	396.0	Power Operated Equipment
1	397.0	Communications Equipment
2	398.0	Miscellaneous Equipment
04 - Water - 36 Total		
		Production
1	320.0	Purification System - Chemical Treatment Plan
1	321.0	Laboratory Equipment
4	322.0	Mains & Accessories
2	323.0	Services
2	324.0	Pumping Equipment & Meters
3	329.0	Transportation Equipment
1	330.0	Stores Equipment
1	331.0	Shop Equipment
1	332.0	Tools & Work Equipment
1	334.0	Miscellaneous Equipment
		Transmission
1	342.0	Distribution Reservoir & Standpipes
6	343.0	Valves & Boxes
2	346.0	Meters
6	348.0	Fire Hydrants
		General
1	390.0	Structures & Improvements
1	392.0	Transportation Equipment
1	395.0	Lab Equipment
1	396.0	Power Operated Equipment
05 - Telephone - 65 Total		
1	212.1	Buildings
1	221.0	Central Office Equipment
10	221.57	Central Office Equipment - Circuits
1	231.2	Small PBX - Station
13	234.0	Large PBX - Equipment
10	241.0	Pole Lines
6	242.1	Aerial Cable - Exchange
4	242.1	Aerial Cable - Toll
3	242.2	U/G Cable - Exchange
3	242.2	U/G Cable - Toll
5	242.3	Buried Cable - Exchange

Table 1. (continued)

No. used	Acct. no.	Description
2	242.3	Buried Cable - Toll
6	264.1	Vehicles (Cars, Light & Heavy Trucks)
06 - ICC - 101 Total		
3	058	Auto
3	058	Truck
17	058	Miscellaneous
1	058	Trailers
27	052	Locomotives
50	053	Railroad Cars of Various Types
07 - Oil Pipeline - 17 Total		
		Gathering
1	1020.0	R.O.W.
1	1040.0	Pipe Fittings
1	1050.0	Installation
1	1060.0	Buildings
1	1080.0	Pumping Equipment
1	1100.0	Other Station Equipment
1	1110.0	Oil Tanks
1	1130.0	Communication System
1	1140.0	Office Furniture & Equipment
1	1150.0	Vehicles & Work Equipment
		Truck Line
1	1560.0	Buildings
1	1580.0	Pumping Equipment
1	1600.0	Other Station Equipment
1	1610.0	Oil Tanks
1	1630.0	Communication System
1	1640.0	Office Furniture & Equipment
1	1650.0	Vehicles & Other Work Equipment
08 - Roads and Highways - 7 Total		
2	Type F	Bituminous Surface Treated
1	Type G-1	Bituminous - Thin Mixed
2	Type H-1	Thin Bituminous Penetrating
1	Type I	Bituminous Concrete
1	(Various)	Various Bituminous Mixes

Table 1. (continued)

No. used	Acct. no.	Description
10 - Other - 4 Total		
1		Education Research Equipment
3		Large Retail Store Equipment & Buildings

4. Experience bands that were representative of current economic and industrial experience were more strongly considered. Bands ending between 1965 and 1975 and 5-15 years in length were viewed most favorably.

5. A good spread among the various industry types was sought so that as many curve dispersion patterns as possible could be included in the study. Since considerably more data sets were available for gas and electric accounts, more care had to be taken to reduce these account numbers, whereas, for example, few water accounts were made available and thus a large percentage actually used.

6. Representation of as many types of equipment as possible for each industry was also sought. In most cases, the availability of certain equipment types was good (for example, electric poles, fixtures, electric meters, all industries general plant) from all data sources, while other equipment types were scarce or completely void. An attempt was made to fill in as many equipment types as were available.

7. Placement bands extending to the old possible age were also viewed favorably. Some of the accounts reflected start of placement band as far back as the 1870's.

8. Geographical location was given consideration in an attempt to represent conditions and operations in as broad an area of the U.S. as possible. Because data from the eastern U.S. were very plentiful, more data were filtered and reduced, while a relatively large percentage of western and central data was used since it was not plentiful.

9. Some data were rejected because of readability problems created by blurred copies.

10. The format of some of the data provided was not as specified and thus the data were rejected.

11. A small amount of the data was received after the study was well under way and thus not usable.

No one of the above listed factors, by itself, determined whether or not a data set would or would not be picked and used from all accounts received. Normally two to six of these factors entered into the decision for each account. Thus judgment played an important role in data selection as it does with many aspects of valuation work and statistical analysis.

Data Documentation

In order to simplify choices of which accounts would be used in the study, a general screening of data was done based on the criteria outlined in the previous section. Those accounts that appeared to be good candidates were documented on a master data sheet with a variety of information recorded. Over 900 accounts had the following information transferred to the master data sheet:

-Account Number

-Equipment Owner & Industry Type

-Equipment Type

-Location (general) of equipment

-Number of years of data available

-Placement band used

-Experience band used

-End percent surviving

-Average service life noted (if available)

-Iowa Type Curve (if available)

-Project code number

-Other comments

  -Multiple experience bands of some accounts

  -Units or dollars base for data

  -Quality of data (readability)

  -Other notable items

It was felt that the master data sheet would not only serve as a guide when selecting the final accounts to be used for the study but would also provide quick reference when analysis work was accomplished as the study progressed. In the end it proved invaluable and saved many hours of work that would have been required digging back into original data.

After all candidate accounts were documented, final selections were made and accounts appropriately marked (yellow lined) on the master.

In addition, all original data for those accounts used were separated and stacked in sequential order by project account code so that it could easily be located if review was necessary. Data from accounts not used were stacked with owner identification evident but without any name or number sequence. Since substantial amounts of good quality data existed in the nonused group, future researchers will have this data available for other project studies.

In order to review where data accounts were located and thus to best consider geographical location as a criterion for account selection, a state and industry type documentation was made on U.S. outline maps of all data received (see Figure 10) and also of data used in the study (see Figure 11).

Since the best possible cross section of equipment type for each industry was one of the criteria for selecting accounts, a running tabulation of equipment types was kept as to number of each equipment type used (by industry account number and/or name) and is reflected in Table 1. While not all of the equipment types for each industry are represented, given the data available, the largest possible spread was included. Discussions with industry representatives verified that for many of the accounts not represented, such as electrical nuclear production equipment, little or no aged mortality data existed--or the data collection process had only started relatively recently.

Data Coding

A relatively descriptive but yet simple coding system for accounts used in the study was developed. This coding system was designed to accomplish several things:

1. To provide project code numbers for easy location and identification of accounts.
2. To allow easy determination and location of industry type for each account.
3. To allow easy determination and location of industry account number (and thus type of equipment) for each account.
4. To identify those accounts included in the study that had more than one treatment of base data--i.e., those accounts that were used more than once but with different experience or placement bands.

The project code for each account consists of 14 places (including decimal point) in the form of:

0474	03	00	3	7	6	0	.	0
1234	56	78	9	10	11	12	13	14

-The first four places (0474) represent the chronological project number for that data set and provide a means of easily locating the set.

-The fifth and sixth places (03) represent the industry type for the account. Industry type codes used:

01 Gas Transmission

02 Electric

03 Gas

04 Water

05 Telephone

06 ICC

07 Oil Pipeline

08 Roads and Highways

10 Other (includes retail and educational equipment)

-The seventh and eighth places (00) represent a special coding for ICC accounts as to the type of transportation equipment involved. Transportation type equipment codes used:

01 Misc (a mixture of various types)

02 Bus

03 Auto

04 Trailer

05 Truck

06 Locomotive

07 Caboose

08 Flatcar

09 Gondola car

10 Hopper car

11 Refrigerator car

12 Stock car

13 Freight car

14 Box car

15 Tank car

-The ninth through the twelfth places (3760) represent the industry account number for each account. Uniform system of account numbers was used for those industry for which they exist.

-The thirteenth and fourteenth places indicate whether more than one treatment of any one account was used. The thirteenth place is a decimal in every case; the fourteenth place code used:

0 one treatment only

1 first treatment of more than one treatment

2 second treatment of more than one treatment

3 third treatment of more than one treatment

Very few accounts used had more than one treatment, but to balance data amongst the industries a few instances of multiple treatment of data accounts were included.

Data Preparation and Vertification

Once accounts were identified, documented, and coded on the master sheets, preparation of raw data for transfor to computer cards and eventually to tape was begun. Each set of raw data was closely reviewed for errors or problems, code numbers applied, and cut off points identified.

Key punching included a master card for each data set containing the 14 place code for that set, plus an appropriate number of cards to hold alternating exposure and retirement data values. Exposures and retirements were punched in alternating eight place fields. The exposure and/or retirement values that exceeded eight places were reduced to eight by eliminating an equal amount of numbers from the one, ten, and hundred positions of both the exposure and retirement values to maintain the eight place field.

Key punched data was not verified by the technicians but rather was handled by the author personally. The verification procedure was handled in two ways. First, a random check of key punched data against actual data was done for approximately 10% of the accounts. This served to give a general feel for the amount and seriousness of errors. Second, a print out of all data, in the form of survivor curves, was accomplished (by computer and plotter). These survivor curves were then reviewed in some detail by comparing the changes in observed life table from the original data with the shapes and changes in each curve. A sample of original plotted survivor curves is included in Appendix D for reference. As errors were spotted, exact locations on the life tables and on the computer cards were determined and corrections made on the cards. Locations of errors were then noted in the card decks by insertion of "flag sheets" at each error point noting the account number and corrections required. The cards were then sent back for repunching, returned for reverification, and inserted back into the deck.

As a final verification of punched data, another print out of survivor curves was made, and the entire process of checking curves against observed life tables was repeated in detail.

It might be noted that the total key punch operation, not including corrections, took high speed technicians nearly 110 hours. It was found very early in the key punch operation that large numbers of errors were being made. The length and volume of the number sets were identified as a problem for even highly skilled key punch technicians. To cut down on these errors, each technician had to limit the number of accounts handled in one sitting so as not to tire and increase error possibilities. Transposition of critical (left place) numbers and confusion in adjusting greater than eight place numbers were found to be the most common errors. In the end, errors had to be identified and corrected in approximately one-half of all accounts and with multiple errors per accounts included, over 500 corrections were made.

Curve Extensions to Zero

Of all data accounts selected for use in the study, 293 or 59.8% had observed life tables extending to 0% surviving. The end percent surviving of the remaining 197 accounts was as reflected in Table 2.

Table 2. Detail concerning non-0% surviving accounts

Range	No. of accounts	% of total accounts
0-5%	102	20.8
5-10%	32	6.5
10-15%	26	5.3
15-20%	22	4.5
> 20%	15	3.1

Of the accounts with end percent surviving greater than 20%, four were greater than 30% (with the maximum being 33.53%) and included two water and two telephone accounts. Since data were not plentiful in these areas, they were left in the study.

It was decided that to accomplish average service life study, each account would have to be at 0% surviving before the analysis process began. Thus each nonzero account was manually extended by eye. This process involved review of the plotted survivor curves (calculated by the retirement rate method) for each nonzero account, determining the general trend of each curve, and by eye extending the curves to 0% surviving. Since raw data were used, the percent surviving values for the beginning of each extended age interval were then documented. The survivor rate was then calculated for each age interval by dividing the percent surviving at the end of each interval (being the percent surviving for the beginning of the next interval in each case) by the percent surviving at the beginning of that interval. The retirement rate was then determined for each interval by subtracting the calculated survivor rate from unity. Since it was necessary to have all data in the form of exposures and retirements for each age interval, the retirement rate had to be converted. This was done by setting beginning exposures for each interval at 1000 and multiplying each three place retirement rate by 1000 to produce the desired retirement value during each interval. Since the retirement rate is always less than 1.000 except at 0% surviving, the retirement values were always less than 1000, except for the final value, which was always 1000, resulting in a final percent surviving of zero.

The exposures and retirements data thus produced for all nonzero accounts were key punched and transferred to computer tape. Survivor curves were again plotted and individually reviewed for final verification of all data. After some corrections of key punch errors in extension data, all accounts data were considered ready for analysis and treatment.

Conversion to Frequency Curves and Identification of Mean and Mode

When the original Iowa Type Curves were developed, a majority of the analysis and treatment work was accomplished using frequency curves rather than survivor curves because, as Robley Winfrey put it, ". . . frequency curves (which) have more distinguishing characteristics than do survivor curves." Basically two important characteristics are of interest: (1) The position of the mode along the abcissa and (2) the height of the mode. In the original study, the frequency curve abcissa was converted to age as a percent of average life (mean), thereby reducing all curves to the same equivalent average life and allowing comparison analysis of modal position (age) and modal height (frequency) of all curves no matter what their average life. This conversion made the two characteristics noted above important tools in establishing patterns of retirements required for producing standard curves.

It was decided that, as Winfrey had concluded over 40 years ago, the frequency curves do indeed offer the best distinguishing characteristics for classifying retirement data. Thus as was done originally, frequency curves, with age as a percent of average life, were used. Required average service lives had previously been determined by computer calculation of areas under each survivor curve.

The procedures used to convert to frequency curves, convert from age to age as a percent of average life, reflect abcissa values of the frequency curve at 1% intervals and select and locate the means and modes were as follows:

1. The raw data frequency curves were determined by computer by observing slopes between successive points on the previously computed survivor curves. This raw data curve information was stored by the computer for future use.
2. The raw data mean and mode (location and height) of the frequency curves were determined by computer and stored for future use. The mean being the average service life. The modes were located by using a three point moving average (center point, plus left and right of each center point) of raw data frequency values for all

but the first point. For the first point only the center and right points were used since the origin resulted in no left point existing. This three point moving average procedure was used so that no single frequency value could establish the mode, but rather the mode would be established by the stronger evidence of the values of three points.

3. The curves were reduced to the same equivalent average life by converting the abcissa (age) to age as a percent of average life (mean). This was done by:
   a. Setting the mean at 100%.
   b. Linearly interpolating the actual number of years of data left of the mean from 0 to 100%.
   c. Linearly interpolating an equivalent number of years right of the mean from 100% to 200%. If the number of years of data to the right of the mean was less than the number of years to the left (as with a right modal curve), interpolation ended at less than 200%, and the remaining values to 200% (and subsequently to 300%) were set at zero.
   d. Linearly interpolating an equivalent number of years equal to the number left of the mean from 200% to 300%. If the operation described in item c resulted in a zero application prior to or at 200%, zeros were then extended from 200% to 300%. If the number of years remaining after 200% was less than the number of years left of the mean, interpolation ended at less than 300%, and the remaining values were set at zero. If the number of years remaining after 200% was more than the number

of years left of the mean, as with extreme left modal, interpolation was done to 300% and cut off at that point.

4. The modal locations and values that were carried by the computer from step 2 were then identified by the computer on the 0 to 300% spread data. The means were already identified by step 3 as always being at 100%.

5. Data were then interpolated by using cubic interpolation so that points existed at every 1% interval between 0 and 300%. This "standardized" the curves such that all curves could be compared equally at every 1% on the abcissa. Curves with many years of data required a minimum of interpolation, while short lived curves required extensive interpolation. The mode values were not affected by the interpolating and thus were left at exactly the same height and position for each curve.

Plotting of the resultant frequency curves was not done at this point, since classification into modal groups had not yet been accomplished. It was decided that plotting of frequency curves could be done more effectively once modal classification was known.

A decision also had to be made during this phase of conversion to and working with frequency curves as to whether or not the curves should be smoothed. The question of when and how much to smooth data is one that has always challenged statisticians. It was decided that the data should be left in as raw a condition as possible and that the spreading and interpolation procedures used would be statistically valid.[1]

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[1]Dr. H. T. David and Mr. Neil Werner provided counseling throughout the research in matters concerning statistical procedures and analysis.

Thus the account data were in the form of standardized frequency curves and ready for treatment, comparison, and grouping.

Classification of Frequency Curves into Modal Groups

Although the original study only classified the frequency curves into three modal groups, it was decided to identify four groups in this study. This decision was later modified as will be covered later in this section.

The four frequency curve modal group classifications were identified and boundaries established as follows:

Origin Modal - Mode located between 0% and 4.99%

Left Modal - Mode located between 5% and 94.99% on the abcissa

Symmetrical Modal - Mode located between 95% and 104.99% on the abcissa

Right Modal - Mode located at any point 105% or greater on the abcissa

The above boundaries were considered to be a trial run set to see if a reasonable number of curves fell into each group. They were established for the left, right, and symmetrical groups by referring to how it appeared Winfrey had set his boundaries in the original study. The origin group boundaries were set subjectively at what were felt to be reasonable points. The result was that the number of curves identified in the left (231), right (173), and symmetrical (84) groups by the computer were balanced well-enough to maintain the boundary conditions as initially established. There were, however, only two curves identified in the Origin Modal group. A decision was then made to transfer those two curves into the left group and make the determination later in the study as to whether a full origin modal group existed or whether the data reflected only new shapes in the extreme left area of the left modal group.

After computer classification of all frequency curves, mechanical plotting was accomplished so that each curve could be analyzed. Curves were plotted two to a graph, with each set being from the same modal group. Each curve was then manually reviewed to see that its overall tendency agreed with the classification given it by the computer. Since for some curves some single point values were extreme compared to the overall tendency of the curve, decisions were made on each curve whether to leave it in the group previously assigned or to reclassify it. The initial computer classification turned out to be very accurate in that few changes were made from the original groups assigned. Table 3 shows the results of the original classification by computer and the final results after manual changes.

Table 3. Number of curves--original and modified classifications

	Number originally in group	Number change to			Changes	Final number in group
		Left	Right	Symmetrical		
Origin Modal	2	2	0	0	-2	0
Left Modal	231	0	1	3	-4	239
Symmetrical Modal	84	2	2	0	-4	84
Right Modal	173	8	0	1	-9	167
Totals	490	+12	+3	+4		490

As the manual inspection of frequency curves was being accomplished, a particular curve shape became evident that was not included in the initial classification. This shape was the multi-modal curve. These curves reflected more than one well-defined modal location. Single data points or "spikes" were not considered to be modal locations in themselves, rather a reasonable bulk of data supporting these spikes had to be evident for them to be considered multi-modal. After it became evident that a substantial number of these curves were being encountered, a second manual review of all frequency curves was done to specifically identify all curves with multi-modal tendencies. It was found that 57 curves fell into this category with 44 showing strong tendencies and 13 weak but definable tendencies.

Although it appeared that a "new" curve shape had been uncovered, it was decided that these curves be left in the group assigned and treatment and analysis proceed with only the three modal groups. This decision was based on two things: (1) the original Winfrey study had obviously left all curves with multi-modal tendencies in the three modal groups, since there was no indication of recognition of this separate shape and (2) the job of classifying and analyzing multi-modal curves appeared to be a major undertaking on its own. It was felt that a separate study should be initiated to consider only multi-modal curves. The identification of the multi-modal shapes in themselves was not considered to be a "new" discovery, since these types of shapes had been sporatically noted in the past. However, the uncovering of the amount of data that reflected multi-modal tendencies was considered a substantial discovery.

Clustering of Curves within Modal Groups

Procedures were established for clustering of the curves of each modal group into subgroups of curves that were very similar in shape. The initial intention was to cluster the frequency curves since the analysis, for the most part, had used these curves. It was decided, however, that the frequency curves contained many more erratic changes than the survivor curves and that it would be much simpler to analyze the smoother survivor curves in the clustering stage. It also became evident that the clustering procedures would work much better on smoother curves since major changes in percent surviving were reflected only by slope changes in the survivor curves rather than by extreme spikes or valleys that result on frequency curves.

The initial clustering procedure was accomplished by computer. The basis of the clustering was to group curves together by minimizing the area between curves. Thus the curves with the minimum areas between, no matter whether the area was above or below, were considered to be related the closest.

The procedure involved calculation of the area between each curve and every other curve within each modal group. This was accomplished by measuring the vertical distances at every 1% abcissa interval between curves and adding these distances together to get area (since 1% intervals were used, the summation of vertical linear distances approximated areas with very little error). Since for each curve the ordinate represented "percent surviving" and the abcissa represented "age as a percent of average life," all area values computed were in percent squared units. This also held true in later stages of the research when area, in percent squared units, was used as a criteria for fitting curves. At the beginning of the actual clustering procedure, each curve within a modal group represented a cluster of its own. Then, based on the area calculations, the two closest curves were selected to form a new cluster, thus reducing the total number of clusters in the group by one. The two curve cluster was then converted to a single representative curve for that cluster by averaging the two curves. The next two closest curves were then selected and the same procedure repeated. In this case, the two closest curves could have been two single curves or a single curve and the representative curve of the two curve cluster. If the two curve cluster was involved, the averaging procedure to form the new three curve cluster representative curve would have involved weighted averaging of the three curves. The two curve cluster representative curve would have been weighted twice and the single curve being added weighted once to produce the new three curve cluster representative curve. This procedure was then repeated until the clustering boundary conditions were met.

The intention was to produce approximately 10-12 substantial clusters within each group. It was feared that specifying 12 clusters would produce two or three very large and many one or two curve clusters. Thus some testing was necessary to determine a percent squared area boundary for each cluster that would produce a sufficient number of substantial clusters. Two tests on the left modal curves were run: the first specifying simply 20 clusters, the second setting the maximum area between any two curves in a cluster at 1,300% squared units. The former produced four large clusters and 20 one or two curve clusters. The latter produced six large clusters with 70 one or two curve clusters. The decision was then made to try a boundary of 1,650% squared units as maximum. This produced eight substantial clusters and 20 sets of clusters with from one to three curves included.

Based on these boundary tests, clustering of all three modal groups was done with 1,650% squared units as a maximum boundary condition. The computer clustering procedure resulted in 28 left modal clusters, 7 symmetrical modal clusters, and 21 right modal clusters.

Since the 1,650% squared units boundary condition was simply a means of allowing the computer to formulate a reasonable number of substantial clusters, manual comparisons of clusters were then attempted to see if additional clustering of the computer subgroups could be accomplished. The result of the manual clustering for each of the modal groups was as follows:

<u>Left Modal</u> - The 28 original computer clustered subgroups were manually combined to produce 15 subgroup clusters:

Subgroup 1 - 1 curve - same original cluster
2 - 2 curves - two combined clusters
3 - 3 curves - two combined clusters
4 - 1 curve - same original cluster
5 - 4 curves - same original cluster
6 - 4 curves - same original cluster
7 - 6 curves - four combined clusters
8 - 5 curves - four combined clusters
9 - 2 curves - two combined clusters
10 - 13 curves - three combined clusters
11 - 12 curves - two combined clusters
12 - 6 curves - same original cluster

13 - 88 curves - same original cluster

14 - 3 curves - two combined clusters

15 - 88 curves - same original cluster

Symmetrical Modal - The 7 original computer clustered subgroups were manually combined, and no additional manual combinations were found. Thus 7 subgroup clusters remained:

Subgroup 16 - 1 curve

17 - 7 curves

18 - 5 curves

19 - 7 curves

20 - 16 curves

21 - 46 curves

22 - 2 curves

Right Modal - The 21 original computer clustered subgroups were manually combined to produce 11 subgroup clusters:

Subgroup 23 - 1 curve - same original cluster

24 - 1 curve - same original cluster

25 - 1 curve - same original cluster

26 - 4 curves - three combined clusters

27 - 1 curve - same original cluster

28 - 9 curves - four combined clusters

29 - 9 curves - four combined clusters

30 - 29 curves - same original cluster

31 - 27 curves - same original cluster

32 - 45 curves - three combined clusters

33 - 40 curves - same original cluster

It should be noted that the manual comparisons of clustered subgroups involved the use of computer prepared clustered survivor curve graphs of each subgroup. These graphs were carefully cross checked on a light table to determine if close shape relationships existed between clusters. For the most part, one or two curve clusters were combined with each other or with larger clusters. It was estimated that the maximum boundary conditions after manual comparisons and reclustering of subgroups were in the range of 2,400% squared units for the widest clusters. This was still considered to reflect good curve relationship. A sample cluster graph (with the 1,650% squared units boundary) from each of the three modal groups is included in Appendix E.

Thus a total of 33 "Russo curve clusters" was established as a basis for testing the validity of the existing Iowa curve set.

Averaging of Clustered Subgroup Curves

Since all clustered subgroups (except for outliers with only one curve) contained more than one curve, the subgroup curves were averaged to produce a representative curve for each subgroup. The averaging procedure involved addition of the ordinate percent value for each curve within the cluster at each 1% point along the abcissa. This total at each 1% point was then divided by the total number of curves within the cluster to get a representative average value for each one percent increment. Thus an average curve representing all of the curves within each cluster was produced. These 33 subgroup curves were then denoted as the "Russo curves." All 33 Russo curves, with cluster boundaries, are shown in Appendix F.

No further smoothing or adjusting of the Russo curves was done, and the numbering system used was random. Thus the numbering system in no way reflected any pattern of increasing or decreasing dispersion as was done with the current Iowa curve set.

Preparation of Test Curves

The actual testing of the Iowa curves was accomplished in two ways. The first procedure was the fitting of a group of test curves against the Russo curves and the Iowa curves to determine which produced the best fit. The second procedure was the fitting of the Russo curves against the Iowa curves to see how well the Russo curve array was fit by the Iowa curves. Details of both of these procedures are handled in future subsections of this report.

Just as with the choice of the original data for the study, the test curves were chosen to represent as great a cross section of industries and industrial property types as possible. In addition, whenever possible, accounts were chosen from companies that did not have data represented among the 490 accounts used to formulate the Russo curves. Since no additional data were available from gas transmission or ICC accounts, seven gas transmission and five ICC accounts were used from the same sources but not duplicates of account data previously used. Data from totally separate companies were used for the remaining 44 accounts as follows:

13 electric utility

12 gas utility

16 telephone

3 steam utility

Since no additional data from the areas of oil pipeline, water, or roads were available, they were not represented in the test account group.

Thus a total of 56 test accounts from ten separate companies, all with 0% surviving, were prepared for testing. Preparation involved formulation of survivor curves by the retirement rate method so that graphic comparisons with Russo and Iowa curve sets could be accomplished.

Testing

The first test involved computer fitting of the 56 test curves to both the Russo curve array and the Iowa curve array to determine which produced the best fit for each test curve. The test curves were held constant in each case, and the Russo and Iowa curves adjusted until the closest fit was found. Since the average service life of each test curve could be easily determined by the area under the survivor curve, a starting point for each curve of the curve sets was established. Thus with this average service life starting point, curve set shapes determined by average service lives, adjacent to the starting point average service life, could be investigated until the least area between each test curve and the Russo and Iowa curve was found. The test curve average service life was used simply as a device to start the investigation of which standard curve best fit the test curve. The final criteria of best fit was the total area between the curves. All area was considered to be positive in sign, so full variation was indicated. Once the best fit based on areas was determined, the average service life of the standard curve chosen was projected as the fit average service life.

Thus for each test curve the actual calculated average service life as well as the average service lives produced by the fit curves of each set were projected. In addition, the area between the test curve and the fit curves of each set were projected. The results of fitting the test curves to each set are reflected in Table 4 (see Results and Discussion) and are discussed in a forthcoming section of this report. Samples of the graphs showing the fitting of several test curves to Russo and Iowa curves are also shown in Appendix G.

The second test involved fitting of each Russo curve to the Iowa curve set to determine how well the Iowa curves fit each curve of the Russo set. This was accomplished as outlined above for the test curves with the Russo curves being treated as the test curves. Actual average service life percentages as compared to Iowa curve predicted average service life percentages were produced. In addition, the areas between Russo curves and their three best fit Iowa curves were produced. The results of this fitting are reflected in Table 5 (see Results and Discussion) and discussed in a forthcoming section of this report. Samples of the graphs showing the fitting of several Russo curves to Iowa curves are also shown in Appendix H.

Statistical Treatment of Test Results

Test curves vs. Russo and Iowa curve sets

After considerable investigation into appropriate statistical procedures to apply to results provided by this research, three statistical procedures were chosen for analysis of the 56 test curves fit to the Russo and Iowa curve sets. The procedures were chosen so that conclusions could be drawn as to which curve set best fit the test curves. Each of the three procedures was applied to both area differences as a reflection of dispersion and average service lives resulting from the fitting tests as outlined in Table 4. The three statistical procedures are outlined and discussed below as they apply to this study.

1. Sign test (22)

This test is nonparametric. While it does not reflect relative or absolute magnitudes of differences in population means, it does reflect direction in that the larger or smaller member of a pair of observations is identified.

The sign test was applied to areas as follows:

Define $$A_I - A_R = AD_{I-R}$$

where: A_I = the area between each test curve and its best fit Iowa curve

A_R = the area between each test curve and its best fit Russo curve

Let p equal the probability that $AD_{I-R} \geq 0$ (i.e.: Russo does better); then test the hypotheses:

$$H_0: \; p \leq .5$$

$$H_1: \; p > .5$$

The test statistic:

$$z = \frac{x - n(.5)}{\sqrt{n(.5)(.5)}} \doteq N(0,1)$$

where: n = sample size

x = number of times $AD_{I-R} > 0$ in a sample of size n

Reject H_0 if z value is substantially > 0; the amount > 0 depends on the level of significance desired.

The sign test was applied to the average service lives as follows:

Define $$|E_I - E_A| - |E_R - E_A| = ED_{I-R}$$

where: E_A = average service life calculated for each test curve

E_I = average service life predicted for each test curve by the Iowa curve set

E_R = average service life predicted for each test curve by the Russo curve set

Let p equal the probability that $ED_{I-R} \geq 0$ (i.e.: Russo does better); then test the hypotheses:

$$H_0: \; p \leq .5$$

$$H_1: \; p > .5$$

The test statistic:

$$z = \frac{x - n(.5)}{\sqrt{n(.5)(.5)}} \stackrel{.}{\sim} N(0,1)$$

where: n = sample size

x = number of times $ED_{I-R} \geq 0$ in a sample of size n

Reject H_0 if z value is substantially > 0; the amount > 0 depends on the level of significance desired.

2. Wilcoxon test for paired observations (22)

This test is nonparametric and is more sensitive than the sign test in that it reflects the relative magnitude of differences in population central tendencies. This is evident due to the use of the magnitude of differences within the test statistic. It is accomplished by ranking the differences between paired values and applying the rankings in the statistic as outlined below. Direction, as described for the sign test, is also reflected in this test.

The Wilcoxon test for paired observations was applied to area differences and average service life prediction errors as follows: Let CT_{I-R} be the true central tendency for the paired differences for the population between Iowa and Russo curves; then test the hypotheses:

$$H_0: \; CT_{I-R} \leq 0$$

$$H_1: \; CT_{I-R} > 0$$

The test statistic:

$$z = \frac{w - CT_w}{\sqrt{\sigma_w^{\,2}}} \;\; \underset{n > 30}{\dot{\sim}} \;\; N(0,1)$$

where: w = the total of the rank numbers with positive difference when all differences are ranked from one (smallest) to n (largest) and when zero differences are discarded before ranking

$$CT_w = \frac{n(n + 1)}{4} \;; \; n = \text{sample size after zero differences are discarded}$$

$$\sigma_w^{\,2} = \frac{n(n + 1)(2n + 1)}{24} \;; \; n = \text{sample size after zero differences are discarded}$$

Reject H_0 if z value is substantially > 0; the amount > 0 depends on the level of significance desired.

3. Parametric test for paired observations (paired t test) (22)

This test is parametric and reflects the actual magnitude of differences in population means. It concentrates on and compares mean optimal areas of Russo and Iowa curves and also mean optimal average service life prediction errors of Russo and Iowa curves and has somewhat better discriminating powers than the other two tests for rejecting H_0 if H_0 truly deserves to be rejected when normality can be assumed.

The parametric test for paired observations was applied to areas and average service lines as follows: Let μ_I and μ_R be the true means for the population when the Iowa and Russo curves, respectively, are used; then test the hypotheses:

$$H_0: \ \mu_I - \mu_R = \mu_{I-R} \leq 0$$

$$H_1: \ \mu_I - \mu_R = \mu_{I-R} > 0$$

The test statistic:

$$T = \frac{\overline{ED}_{I-R} \text{ or } \overline{AD}_{I-R}}{\sqrt{S_d^{\,2}/n}}$$

where: S_d = sample standard deviation

$\overline{ED}_{I-R}$ = the mean of the differences of average service life prediction errors

$\overline{AD}_{I-R}$ = the mean of the differences of areas

n = sample size

Reject H_0 if T value is substantially > 0; the amount > 0 depends on the level of significance desired.

It was felt that if all three tests provided evidence in one direction, valid conclusions could easily be drawn in that direction. If the three tests provided conflicting evidence, it was felt that an analysis of the underlying assumptions of each test would be critical in drawing conclusions. Results of the statistical testing are covered in the following section of this report.

<u>Russo curves vs. Iowa curve set</u>

After considerable investigation into possible statistical procedures to apply, it was concluded that analysis of the Russo curves fit to the

Iowa curve set by formal statistical procedures was not appropriate. Since the fitting resulted in a simple comparison of relative average service lives and areas, a variety of comparative measures and relationships not involving formal statistics was deemed the most appropriate means of analysis. Results of these procedures are covered in the following section.

RESULTS AND DISCUSSION

The results of the study are reflected graphically in Tables 4 and 5. Results were analyzed and considered both subjectively, by direct review and comparison of test outputs, and objectively, by statistical treatment of test outputs. While statistical methods were applied only to the test curves fit to Russo and Iowa curve sets, as previously noted, a considerable amount of evidence and information was also obtained by nonstatistical analysis of the Russo curves fit to the Iowa curve set.

Subjective Analysis

Subjective analysis involved a thorough review of both test situations in themselves as well as review of the relationships evident from cross-checking the two.

The following was evident concerning the 56 test curves fit to the Iowa and Russo sets:

1. Based on the combination of average service life predictions, area differences and visual review of the fits, the Russo set produced 10 best fits, the Iowa set 20 best fits, and both fit equally as well 26 times. This is reflected in the remarks column of Table 4.

2. Considering only area differences, the Russo and Iowa sets, each had 28 test curves where their fit curve had the least area.

3. Considering only average service life predictions, the Russo set had 22 and the Iowa set 34 best fit curves that predicted average service lives closest to the actual average service lives of the test curves.

Table 4. Results of fitting 56 test account curves to Iowa and Russo curve sets

Test curve		Russo curves			Iowa curves			
Number	ASL (yrs.)	Curve No.	Area diff. (sq. % units)	ASL (yrs.)	Curve type	Area diff. (sq. % units)	ASL (yrs.)	Remarks
1	14.0	33	1900	14.7	R_4	2343	14.9	Tie
2	24.1	2	704	23.2	$R_{1.5}$	1026	23.4	R
3	17.3	33	799	17.2	R_3	960	16.9	R
4	3.0	8	1014	2.9	O_3	1338	3.0	R
5	6.9	28	525	7.1	R_5	730	7.2	R
6	9.1	20	543	9.4	S_6	159	9.2	I
7	8.5	24	953	8.8	O_1	1014	8.9	Tie
8	20.1	13	525	20.5	L_0	599	20.0	Tie
9	4.8	7	584	4.9	O_3	712	4.8	Tie
10	9.8	29	949	10.2	$R_{0.5}$	1560	10.1	R
11	11.1	21	592	11.2	R_2	671	10.9	Tie
12	13.4	21	637	13.4	L_3	883	13.9	R
13	18.0	10	733	18.3	L_1	754	17.2	Tie
14	24.5	19	374	24.8	$L_{0.5}$	250	24.7	Tie
15	10.5	13	1205	10.6	O_2	1256	10.3	Tie
16	9.4	14	1483	10.2	O_1	1603	10.5	Tie
17	30.9	22	666	30.9	R_1	564	31.1	Tie
18	46.4	28	598	48.4	R_5	540	47.3	Tie
19	11.3	19	787	11.1	L_1	350	11.1	I
20	9.3	32	696	9.2	$S_{1.5}$	634	9.2	Tie
21	24.8	21	431	24.6	L_3	229	25.5	I
22	27.3	15	622	28.5	L_2	495	27.9	I
23	17.9	18	410	18.1	$R_{1.5}$	444	18.4	Tie
24	24.1	17	640	24.3	R_3	453	24.4	I
25	15.0	14	1620	16.0	L_0	1729	15.3	Tie
26	54.1	27	859	50.6	$L_{1.5}$	686	51.9	I
27	29.2	19	712	28.3	L_1	287	28.3	I
28	36.0	21	480	34.1	$S_{1.5}$	239	37.7	I
29	23.2	13	288	23.1	L_0	475	22.5	R
30	18.8	31	699	17.9	$L_{1.5}$	472	18.3	I
31	27.5	20	383	26.8	R_5	169	27.9	I
32	41.1	29	831	42.6	R_1	959	39.7	Tie
33	50.4	2	1135	48.2	R_3	1365	49.8	Tie
34	52.3	20	726	49.8	S_6	235	55.2	I
35	28.5	14	1012	28.7	L_0	1097	29.6	Tie
36	10.4	7	1372	9.8	O_2	1398	10.0	Tie
37	8.8	11	410	8.8	L_4	343	8.7	Tie
38	12.4	33	376	12.3	R_4	316	12.4	Tie
39	26.5	3	1145	28.0	L_2	1241	26.5	Tie

Table 4. (continued)

Test curve Number	Test curve ASL (yrs.)	Russo curves Curve No.	Russo curves Area diff. (sq. % units)	Russo curves ASL (yrs.)	Iowa curves Curve type	Iowa curves Area diff. (sq. % units)	Iowa curves ASL (yrs.)	Remarks
40	34.9	22	728	34.0	S_0	739	34.6	Tie
41	27.2	31	445	25.5	S_0	322	26.1	I
42	26.1	21	486	25.1	L_2	705	26.1	R
43	31.4	10	792	31.9	L_2	941	29.7	R
44	40.6	18	458	42.3	R_2	203	37.8	I
45	23.0	31	852	23.9	O_1	304	22.9	I
46	25.7	29	1328	24.5	R_1	1343	25.0	Tie
47	33.4	31	626	31.2	$R_{0.5}$	637	33.8	Tie
48	27.9	15	688	27.2	$L_{1.5}$	307	28.4	I
49	30.2	26	693	28.6	$R_{2.5}$	185	30.8	I
50	25.8	13	559	27.3	L_0	324	26.6	I
51	29.8	15	319	29.5	L_3	225	30.7	Tie
52	12.0	33	642	11.9	R_3	408	12.1	I
53	28.3	18	666	28.4	$R_{1.5}$	842	28.6	R
54	6.5	11	620	6.2	L_3	533	6.3	Tie
55	9.7	11	399	9.7	L_4	145	9.6	I
56	31.0	31	747	29.9	L_1	475	30.4	I

4. Of the 10 test curves best fit by the Russo set, 9 were best fit by Russo curves of considerable bulk (five clustered curves or more), while 1 was best fit by a Russo curve representing a two-curve cluster.

5. Considering the average service lives predicted by Russo and Iowa fit curves, whether best fit curves or not, they were generally within 0% to 6% of the actual average service lives of the 56 test curves.

6. Of the 46 test curves best fit by the Iowa set, 6 were origin modal, 19 left modal, 6 symmetrical modal, and 15 right modal.

7. Of the 10 test curves best fit by the Russo set, 4 were Russo left modal, 3 symmetrical modal, and 3 right modal.

Table 5. Results of fitting 33 Russo curves to Iowa curve sets

Russo curves		Three closest Iowa curves								
Curve No.	ASL %	Curve type	ASL %	Area between (sq. % units)	Curve type	ASL %	Area between (sq. % units)	Curve type	ASL %	Area between (sq. % units)
* 1	163.6	S_0	161.1	2353	$S_{0.5}$	162.3	2430	L_1	165.8	2487
* 2	126.9	$R_{1.5}$	126.1	855	R_2	128.8	980	$S_{0.5}$	128.0	1051
3	119.2	L_2	119.0	1441	S_0	119.0	1454	$L_{1.5}$	119.8	1461
* 4	69.0	L_2	62.0	884	$L_{1.5}$	62.4	994	L_3	62.8	1013
5	110.7	O_3	103.6	1218	O_2	104.4	1312	L_0	101.0	1459
6	106.0	$R_{0.5}$	106.2	834	$S_{-.5}$	106.9	941	O_1	104.2	964
7	110.2	O_3	112.1	488	O_4	116.6	1111	O_2	116.3	1407
8	93.0	O_3	91.7	504	O_4	95.1	906	O_2	94.6	1775
* 9	82.3	O_3	73.2	2187	O_4	75.5	2268	L_2	62.4	2507
10	95.9	$L_{1.5}$	89.0	944	L_2	88.2	950	L_1	90.1	1132
11	98.9	L_4	97.6	317	S_3	96.4	468	S_4	95.6	528
12	117.5	O_1	114.4	851	$S_{-.5}$	116.7	880	$R_{0.5}$	116.3	982
13	103.0	L_0	102.5	322	$L_{0.5}$	101.8	350	O_2	100.7	618
14	138.3	O_3	143.7	796	O_4	138.2	1230	O_2	139.1	1381
15	102.5	$S_{1.5}$	103.4	362	S_1	100.3	374	L_3	103.4	436
*16	122.7	L_2	122.9	2372	L_3	114.7	2436	$L_{1.5}$	121.7	2537
17	122.4	R_3	123.3	807	S_3	123.0	840	R_4	123.7	940
18	112.5	R_2	113.2	379	$R_{1.5}$	111.2	488	S_1	112.4	515
19	107.1	L_0	105.2	452	$L_{0.5}$	107.6	479	L_1	108.1	778
20	102.6	L_5	101.9	279	S_5	101.9	344	R_5	101.5	358
21	103.0	$S_{1.5}$	101.5	456	L_3	105.4	461	R_2	101.1	484
*22	129.2	S_0	130.4	699	$S_{-.5}$	128.8	759	$R_{0.5}$	129.2	787
*23	223.7	R_1	195.2	3483	$R_{0.5}$	191.5	3489	$S_{-.5}$	190.7	3523
*24	137.1	O_1	136.6	917	$R_{0.5}$	139.8	1222	$S_{-.5}$	140.5	1249

*25	198.4	S_6	200.3	1902	S_0	201.3	1995	R_5	199.9	2076
26	123.2	$R_{2.5}$	125.3	811	R_3	126.1	849	R_2	123.0	883
*27	128.0	L_2	119.0	1463	$L_{1.5}$	122.5	1551	L_1	123.2	1741
28	116.5	R_4	117.5	396	S_4	119.4	521	R_5	119.8	576
29	122.0	R_1	123.7	818	$R_{0.5}$	123.3	833	$S_{-.5}$	122.2	960
30	109.2	$R_{1.5}$	109.7	522	R_2	111.6	564	$R_{2.5}$	113.2	732
31	105.0	$R_{0.5}$	103.8	238	$S_{-.5}$	103.4	293	S_0	106.9	447
32	104.2	R_2	102.8	324	S_1	103.4	394	$R_{1.5}$	102.3	417
33	105.5	R_4	106.6	336	R_3	105.4	544	S_4	108.1	546

Note: --Russo curves 1-15 are left modal; curves 16-22 symmetrical modal; curves 23-33 right modal.

--Russo curves marked with asterisks (*) are from one or two curve Russo clusters.

8. Of the total array of 31 Iowa curves, 10 curves did not produce any fits, no matter whether best fits or not. These nonfit curves included 7 symmetrical modals ($S_{-.5}$, $S_{.5}$, S_1, S_2, S_3, S_4, and S_5), 1 left modal (L_5), 1 origin modal (O_4), and the square curve.

9. Of the total array of 33 Russo curves, 10 curves did not produce any fits, no matter whether best fits or not. These nonfit curves included 1 symmetrical modal, 6 left modal, and 3 right modal curves.

In fitting the Russo curves to the Iowa curve set, an average service life calculation for each Russo curve and a predicted average service life from best fit Iowa curves could not be reflected on a year's basis since the Russo curves represented the average of a variety of clustered curves that had been converted to age as a percent of average service life. Thus the "ASL - %" columns in Table 5 reflect relative average service lives. These average service lives are relative in that they represent the point along a 300 point (points or years or percents) scale where the calculated and predicted average service lives occur. Although this system does not easily lead to statistical analysis of the Russo curves fit to the Iowa set, conclusions can validly be drawn based on comparisons of the relative average service lives and area differences.

The following was evident concerning the Russo curves fit to the Iowa set:

1. Of the 8 Russo curves that had an area difference in excess of 1000 square percent units, 6 represented one or two curve clusters. The other 2 Russo curves represented three curve and four curve clusters, respectively. Thus, of the 490 curves used to create the Russo curves, all

but 14 were within Russo curves that had area differences of 944 square percent units or less when fit to the Iowa curve set.

2. Of the 7 Russo curves for which the predicted Iowa curve average service life was substantially (5% or greater) different from the calculated Russo curve average service life, 4 represented one or two curve clusters. The other 3 represented 3, 4, and 13 curve clusters, respectively. Thus of the 490 curves used to create the Russo curves, all but 25 were within Russo curves that had average service life predictions within 5% when fit to the Iowa set. Four of the 7 curves were also curves that had large area differences as reflected in number 1 above.

3. Of the 33 Russo curves created by clustering 490 original curves, 22 Russo curves represented by 459 original curves were fit well by the Iowa set when considering relative area differences and average service lives predicted.

The following was evident when crosschecking of the two tests was accomplished:

1. Of the 10 test curves best fit by the Russo set, 9 were fit by Russo curves that were fit well by the Iowa set. The 10th curve was moderately well fit by the Iowa set, its area difference being just under the range and average service life percentage difference being just outside the range of what was considered a good fit.

2. The 11 Russo curves that had area differences in excess of 1000 square percent units and/or average service life differences in excess of 5% created best fits, considering curves only, seven times out of the 56 tests curves fit. Considering both Russo and Iowa curves, these 11 curves created best fit twice. Thus in two cases out of the 56 test curves fit

did Russo best fits occur from a Russo curve that was not well fit by an Iowa curve.

3. Of the 56 test curves fit by the Iowa set, 12 were fit by Iowa half curves. Of the 33 Russo curves fit by the Iowa set, 8 were fit by Iowa half curves.

Statistical Objective Analysis

As outlined in the previous section of this report, statistical tests were performed on the data that resulted from fitting the 56 test curves to Russo and Iowa curve sets. The calculations for each of these statistical tests are included in Appendix I of this report. An analysis of each test follows.

1. Sign tests--areas and average service lives

These were basic tests that considered only direction in that only the number of fit curves of each curve set that produced minimum areas or closest average service life predictions between the fit curves and the test curves were considered. These tests failed to adduce evidence that the Russo curves produced better fits than the Iowa curves, since the z value calculated for areas was 0 and the z value calculated for average service lives was -1.60. The value of z in each case would have to have been substantially greater than zero to reject the null hypotheses that indicated that the Iowa curves produced at least as good or better fits than the Russo curves. Although not conclusive in themselves, these two tests provide evidence that the Iowa curves would produce at least as good fits for a population of industrial property accounts when considering areas and average service lives.

2. Wilcoxon tests for paired observations--areas and average service lives

These tests were more sensitive than the sign tests in reflecting relative magnitudes of differences in population central tendencies. These relative magnitudes of differences produced a z value calculated for areas of -7.65 and a z value calculated for average service lives of -7.77. Thus the tests failed to adduce evidence that the Russo curves produced better fits than the Iowa curves. The value of z, in each case, would have to have been substantially greater than zero to reject the null hypotheses that indicated that the Iowa curves produced at least as good or better fits than the Russo curves. Although not conclusive in themselves, those two tests provide strong evidence that the Iowa curves would produce at least as good fits for a population of industrial property accounts when considering areas and average service lives.

3. Parametric tests for paired observations (paired t tests)--areas and average service lives

These tests concentrated on and compared mean optimal areas of Russo and Iowa curves and also mean optimal average service life prediction errors of Russo and Iowa curves and, as previously discussed, had somewhat better discriminating powers than the other two tests.

CONCLUSIONS

The study's objectives of data collection, formulation of a new set of curves, testing, and analysis were accomplished and presented in previous sections of this report. This section will accomplish the last objective, that of concluding whether or not the existing Iowa survivor curve set is valid as it stands or needs replacement or revision.

Conclusions concerning the validity of the Iowa curves are as follows:

1. That no evidence was found to conclude that the Iowa curve set, as it stands, is not a valid system of standard curves. This conclusion was based on the following major evidences provided by the study:

    a. The soundness of procedures used in the study. This included the large number of data sets used in formulating the new array, the actual testing of accounts and of the Russo set against the Iowa set, and the analysis procedures accomplished.

    b. The results of the statistical treatment and analysis of the test curves fit to the Russo and Iowa sets. Virtually all statistical evidence indicated that the Iowa set produced better results.

    c. The fact that all but one of the 10 Russo curves that produced best fits were well fit by Iowa curves.

    d. The fact that the Iowa curves produced good service life predictions for all 56 test curves, whether Iowa curves were best fit curves or not.

    e. The fact that the Russo curves were basically fit well by the Iowa curve set.

2. That no evidence was found to conclude that new curve shapes, not now represented in the Iowa curve set, are necessary.

3. That although some Iowa curves were not utilized to produce fits during fitting of the 56 test curves, no evidence was found that the number of curves within the Iowa curve set should be reduced. A larger sample of test curves might well include these nonused curves.

No attempt was made during the course of this study to determine whether or not the Iowa curve system was better than other graphical or mathematical systems in use. The attempt was purely to find whether or not a new or altered array of standard empirically developed curves should replace the current set of 31 Iowa type survivor curves. As reflected in conclusions 1, 2, and 3, above, no evidence was found to conclude that any alterations in the current Iowa set are necessary at this time. Industrial organizations using the Iowa set should feel confident in the evidences produced by their use of the system.

Evidence was found that a small percentage of industrial property account data could not be well fit by Iowa curves. These occasional curves appeared to reflect random management or economic situations that did not fall within the norms of any particular industry. By far the most prevalent types of accounts that could not be easily fit were the multi-modal accounts. Some documentation of multi-modal accounts was accomplished during preparation of the 490 accounts used to produce the Russo curve array. Because of the diversity of locations and magnitudes of the modes within each account, no discernable patterns were found that might lead to formulation of a set of standard multi-modal curves. Additional study, however, might determine some patterns to make standardization possible.

It was also noted that Iowa half curves and Iowa O type curves played important roles in the tests involving fitting, thus both were verified as significant groups within the Iowa curve array. Without these half curves and O type curves, different conclusions as to the validity of the Iowa curves may well have been readied.

Additional suggestions drawn from the study are as follows:

1. That additional study be accomplished concerning multi-modal curves to determine if patterns exist that might lead to the development of standardized multi-modal (M type) Iowa curves. Sufficient account data, including both that used in this study and additional data not used in the study, are readily available.

2. That additional study be accomplished to determine if "custom curve sets" for individual industries would produce the best possible fits for those industries. The manual and especially computer techniques used in this study could be duplicated to produce these custom sets with a reasonable and feasible effort by each industry.

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APPENDIX A:

FINAL EQUATIONS FOR IOWA TYPE CURVES (25)

In the following 22 equations, "x" is measured from the mean, or average life, negative values of "x" being to the left of 100% of average life and positive values to the right. An age interval of 10% of average life is equal to "x." Y_x is the ordinate to the frequency curve at age "x."

Left Modal No. 0

$$Y_x = 6.24256418\left(1 - \frac{(x+5.06)^2}{24.60758105}\right)^{0.4411811}$$ for x values to left of 49.4 percent of average life

$$Y_x = 6.24256418\left(1 - \frac{(x+5.06)^2}{1569.183739}\right)^{7.75906308}$$ for x values to right of 49.4 percent of average life

Left Modal No. 1

$$Y_x = 7.45095687\left(1 - \frac{(x+4)^2}{85.49500000}\right)^{4.77742941}$$ for x values to left of 60 percent of average life

$$Y_x = 7.45095687\left(1 - \frac{(x+4)^2}{697.8983268}\right)^{4.74147112}$$ for x values to right of 60 percent of average life

Left Modal No. 2

$$Y_x = 6.2\left(1 + \frac{x - 0.56632298}{10.56632298}\right)^{2.00691507}\left(1 - \frac{x - 0.56632298}{18.11962398}\right)^{4.15639835}$$

$$+ 4.03141046\left(1 + \frac{x + 1.98831766}{4.90258200}\right)^{2.73360830}\left(1 - \frac{x + 1.98831766}{12.07825433}\right)^{8.19831032}$$

Left Modal No. 3

$$Y_x = 6.12\left(1 + \frac{x - 0.69997304}{9.94997304}\right)^{2.51767682} \left(1 - \frac{x - 0.69997304}{13.35543784}\right)^{3.72163230}$$

$$+ 8.19722280\left(1 + \frac{x + 1.22119072}{6.98766177}\right)^{10.15754029} \left(1 - \frac{x + 1.22119072}{16.85048078}\right)^{25.90598437}$$

Left Modal No. 4

$$Y_x = 10.811999434\left(1 - \frac{(x + 0.600)^2}{51.8400}\right)^{25.300}$$

$$+ 9.901828065\left(1 - \frac{(x + 0.300)^2}{56.2500}\right)^{3.650} \qquad -10 \leqq x \leqq -0.6$$

$$Y_x = 10.811999434\left(1 - \frac{(x + 0.600)^2}{184.9600}\right)^{62.000}$$

$$+ 9.901828065\left(1 - \frac{(x + 0.300)^2}{56.2500}\right)^{3.650} \qquad -0.6 \leqq x \leqq -0.3$$

$$Y_x = 10.811999434\left(1 - \frac{(x + 0.600)^2}{184.9600}\right)^{62.000}$$

$$+ 9.901828065\left(1 - \frac{(x + 0.300)^2}{176.8900}\right)^{8.350} \qquad -0.3 \leqq x \leqq (13.6 - 0.6)$$

Left Modal No. 5

$$Y_x = 12.76925713\left(1 + \frac{x - 0.088051975}{5.9500}\right)^{4.7715} \left(1 - \frac{x - 0.088051975}{10.7500}\right)^{9.4275}$$

$$+ 16.28938438\left(1 + \frac{x + 0.161460055}{4.0000}\right)^{11.8000} \left(1 - \frac{x + 0.161460055}{5.7000}\right)^{17.2400}$$

Symmetrical No. 0

$$Y_x = 6.95219904\left(1 - \frac{x^2}{100}\right)^{0.74857140}$$

Symmetrical No. 1

$$Y_x = 9.08025966\left(1 - \frac{x^2}{100}\right)^{1.82839970}$$

Symmetrical No. 2

$$Y_x = 11.91103882\left(1 - \frac{x^2}{100}\right)^{3.70009374}$$

Symmetrical No. 3

$$Y_x = 15.61048797\left(1 - \frac{x^2}{100}\right)^{6.9015918}$$

Symmetrical No. 4

$$Y_x = 22.32936082\left(1 - \frac{x^2}{81}\right)^{11.93537940}$$

Symmetrical No. 5

$$Y_x = 33.22051575\left(1 - \frac{x^2}{64}\right)^{21.43782170}$$

Symmetrical No. 6

$$Y_x = 52.47259169\left(1 - \frac{x^2}{49}\right)^{41.63414220}$$

Right Modal No. 1

$$Y_x = 4.87234751\left(1 + \frac{x + 2.1173}{19.08200310}\right)^{2.16036988}\left(1 - \frac{x + 2.1173}{12.2}\right)^{1.02056945}$$

$$+ 2.95921394\left(1 + \frac{x - 2.03848}{9.25013197}\right)^{2.69374074}\left(1 - \frac{x - 2.03848}{6.76380495}\right)^{1.69831583}$$

Right Modal No. 2

$$Y_x = 6.89465710\left(1 + \frac{x + 0.470}{30.05448169}\right)^{9.16816044} \left(1 - \frac{x + 0.470}{9.05171312}\right)^{2.06241419}$$

$$+ 3.34428110\left(1 + \frac{x - 0.470}{91.60465100}\right)^{100.000} \left(1 - \frac{x - 0.470}{7.80000000}\right)^{7.600}$$

Right Modal No. 3

$$Y_x = 9.4035297069\left(1 + \frac{x + 0.235}{17.61801370}\right)^{7.950} \left(1 - \frac{x + 0.235}{7.18500000}\right)^{2.650}$$

$$+ 5.5945716839\left(1 + \frac{x - 0.698}{17.31323077}\right)^{27.800} \left(1 - \frac{x - 0.698}{6.25200000}\right)^{9.400}$$

Right Modal No. 4

$$Y_x = 15.20129316\left(1 + \frac{x + 0.11}{17.92683200}\right)^{14.05850860} \left(1 - \frac{x + 0.11}{5.41801100}\right)^{3.55112010}$$

$$+ 5.85667821\left(1 + \frac{x - 0.70}{2.56783700}\right)^{3.66879450} \left(1 - \frac{x - 0.70}{3.45398750}\right)^{5.27997721}$$

Right Modal No. 5

$$Y_x = 14.99330391\left(1 + \frac{x + 0.12869}{7.0000000}\right)^{5.79473520} \left(1 - \frac{x + 0.12869}{3.8764409}\right)^{2.76276990}$$

$$+ 15.44614441\left(1 + \frac{x - 0.2086}{4.2350000}\right)^{6.05833400} \left(1 - \frac{x - 0.2086}{2.4150000}\right)^{3.02500040}$$

Origin Modal No. 1

$Y_x = 5$ for all values of "x" from -10 to +10.

Origin Modal No. 2

$$Y_x = 5.1\left(1 + (0.1x + 0.6)^6\right)^{-0.6} \times 0.52468485$$

for all values of "x" from -10 to +20.9

Origin Modal No. 3

$$Y_x = 8\left(1 + (0.067x + 1.23)^6\right)^{-0.6} + 0.43923460$$

for all values of "x" from -10 to +28.6

Origin Modal No. 4

$$Y_x = 11\left(1 + (0.1x + 1.6)^6\right)^{-0.6} + 0.59863384$$

for all values of "x" from -10 to +34.0

APPENDIX B:

INSTRUCTIONS - SPECIFICATIONS - SAMPLE

INSTRUCTIONS - SPECIFICATIONS
Data Collection and Transmittal
Iowa Type Survivor Curve Project

I. Our objectives are to:

1. Collect 350-450 new data sets that represent a variety of utility and industrial properties.

2. Classify survivor curves for these properties with procedures similar to those used in formulating the original Iowa Type Survivor Curves published in the 1935 E.R.I. Bulletin #125 (and revised in 1967). This work will determine whether additions to or deletions from the current set of Iowa Type Curves should be made.

3. Create a new "revalidated" set of Iowa Type Survivor Curves.

4. Publish results, including the new set of curves, in a bulletin similar to the existing E.R.I. Bulletin #125.

II. Please keep the following in mind when selecting life data:

1. Do not send SPR or Turnover Method data; only "aged mortality data".

2. The latest year (latest vintage or experience year) should be from 1965 though 1976. 1965 is a general designation to indicate that we desire rather recent data that includes recent retirement years. If possible, also choose life data for accounts of relatively long average service lives; such as 15-20 years plus.

3. Send as many separate accounts, survivor curves, inventory tables, etc., as you feasibly can. Since we are seeking 350-450 accounts, we can use all that you send.

4. Send all of the survivor curves that you have for each account survivor table, calculated from different experience bands. Since retirement dispersions and average service lives may vary widely for these different experience bands, they may all be of full use.

5. Choose accounts that have survivor curves extending down to between 20 percent and 0 percent surviving. The lower 20 percent can then be extended without significant distortion.

III. When transmitting data, please consider the following:

1. You may send information in batches as you get it ready. This will allow treatment at an early date and also provide some indication of the amount of data that is being sent.

2. The information sent can be in any one of three forms; with (a) the preferred form.

a) Tabular form with dollars exposed, dollars retired, retirement or survivor ratios, observed percent surviving.

b) A complete survivor inventory table that shows the property surviving for each vintage of additions for a span of calendar years.

c) The final or original survivor curves expressed in tabular form (columns) or plotted on graph paper; or both table and curve.

Data may be a computer printout, duplicated copies from your originals, longhand sheets, or other forms that provide readable data without uncertainties as to numbers. It is critical that we have available dollars or units exposed and retired, as well as percent surviving.

3. Identify the data as to:

a) Name, position, company and address of sender.

b) Account number and name or class of property; and geographical location where property is in service.

c) Industrial manufacturer, model, or type that applies to account property, if available and relevant.

d) Description of property within account if account name is not fully descriptive.

e) Name and address of company owning property (if not your own company). If you cannot disclose owner, please code each account so that it can be identified if questions arise.

f) Whether data is expressed in dollars or physical units.

g) If survivor curve precent surviving, or table of calculation of the survivor curve are sent, furnish the following:

1 - Vintage band (original groups) included.

2 - Retirement band (experience band) of years included.

3 - Average service life and Iowa Type Curve determined (if available).

4 - Dollar or unit volume surviving within the calculations for each vintage or total of all vintages as of the latest year; to give a feel as to the relative mass of data.

5 - Method used in calculating survivor curve -- retirement rate, original group, composite group, etc. It is anticipated that most data will reflect the retirement rate method.

APPENDIX C:

LIST OF ALL FIRMS AND INDIVIDUALS CONTRIBUTING DATA FOR THE STUDY

The cooperation and support provided by individuals and firms within the industries represented in the study were outstanding. Substantially more data than was expected were received. This allowed the best quality and most representative data to be selected and used and increased the overall quality of the study. Special acknowledgments are due Robley Winfrey, who collected and made available data from several sources that would otherwise not have been available, Paul Peterson, Director of the ISU Engineering Research Institute, who provided both moral and financial support for the project, and Harold Cowles of ISU, who provided the kind of insight and direction during the project that could not have been gotten from any other source.

Several other firms not listed below that did not have aged mortality data available were kind enough to write and indicate their situation. Most also indicated an interest in receiving a copy of the final report.

In addition, data were received from several consulting firms, some of which had no company identification included. Unfortunately no acknowledgment could be made in these cases.

Allegheny Power Systems

American Appraisal Company

American Telephone and Telegraph

Baltimore Gas & Electric Company

Burgess & Niple, Ltd.

Carolina Power & Light

Central Hudson Gas & Electric

Central Illinois Public Service Company

Citizens Gas & Coke Company

The Cleveland Electric Illuminating Company

Columbia Gas Distribution Companies

Columbia Gas Transmission Corporation

Consolidated Edison of New York

Dallas Power & Light

Dayton Hudson Corporation

Duquesne Light

Ebasco Services

Equitable Gas Company

Florida Power Corporation

Gannett Fleming Corddry & Carpenter, Inc.

Gilbert Management Consultants

GTE Service Corporation

Interstate Commerce Commission

Iowa State University

Kansas Gas & Electric Company

Laclede Gas Company

Long Island Lighting Company

Middle West Service Company

Natural Fuel Gas Distribution Corp.

Natural Fuel Gas Supply Corp.

Natural Gas Pipeline Co. of America

New Orleans Public Service, Inc.

New York Water Co.

Northern Illinois Gas Co.

Pacific Power & Light Co.

Pennsylvania Power & Light Co.

The Peoples Gas Light & Coke Co.

Philadelphia Electric Co.

Philadelphia Suburban Water Co.

Quaker State Telephone Co.

Rochester Gas & Electric Corp.

San Diego Gas & Electric Co.

Shenango Valley Water Co.

Southern Co. Services, Inc.

St. Louis County Water Co.

Texas Gas Transmission Corp.

UGI Corporation

Union Electric Co.

United Engineers

Washington Water Power

West Pennsylvania Power Co.

Wisconsin Electric Power Co.

APPENDIX D:

ORIGINAL PLOTTED SURVIVOR CURVES - SAMPLES

Figure 12. Sample of original plotted survivor curve for data account 0106

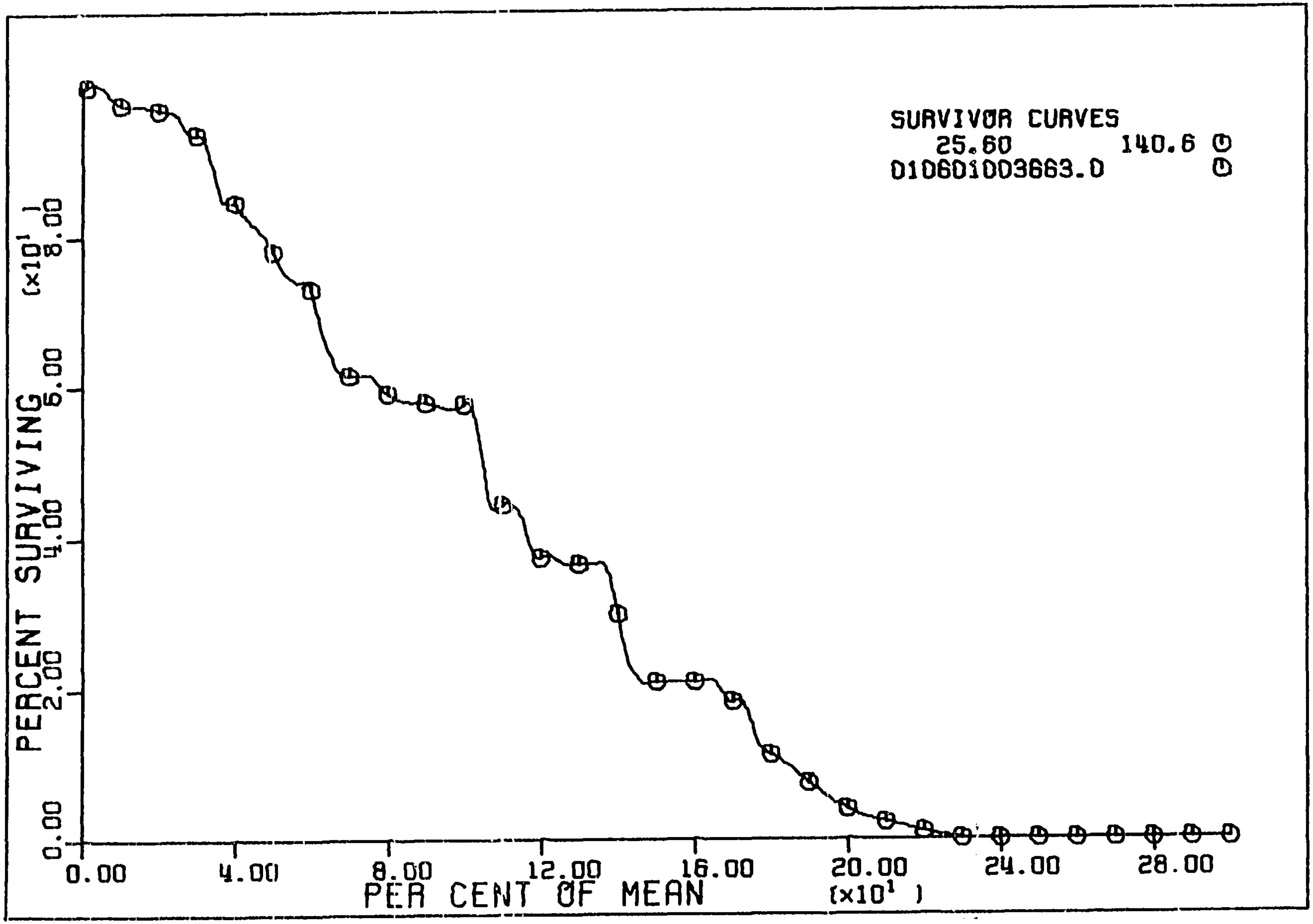

SURVIVOR CURVES
25.60 140.6
010601003663.0
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PER CENT OF MEAN
(x10¹)

Figure 13. Sample of original plotted survivor curve for data account 0385

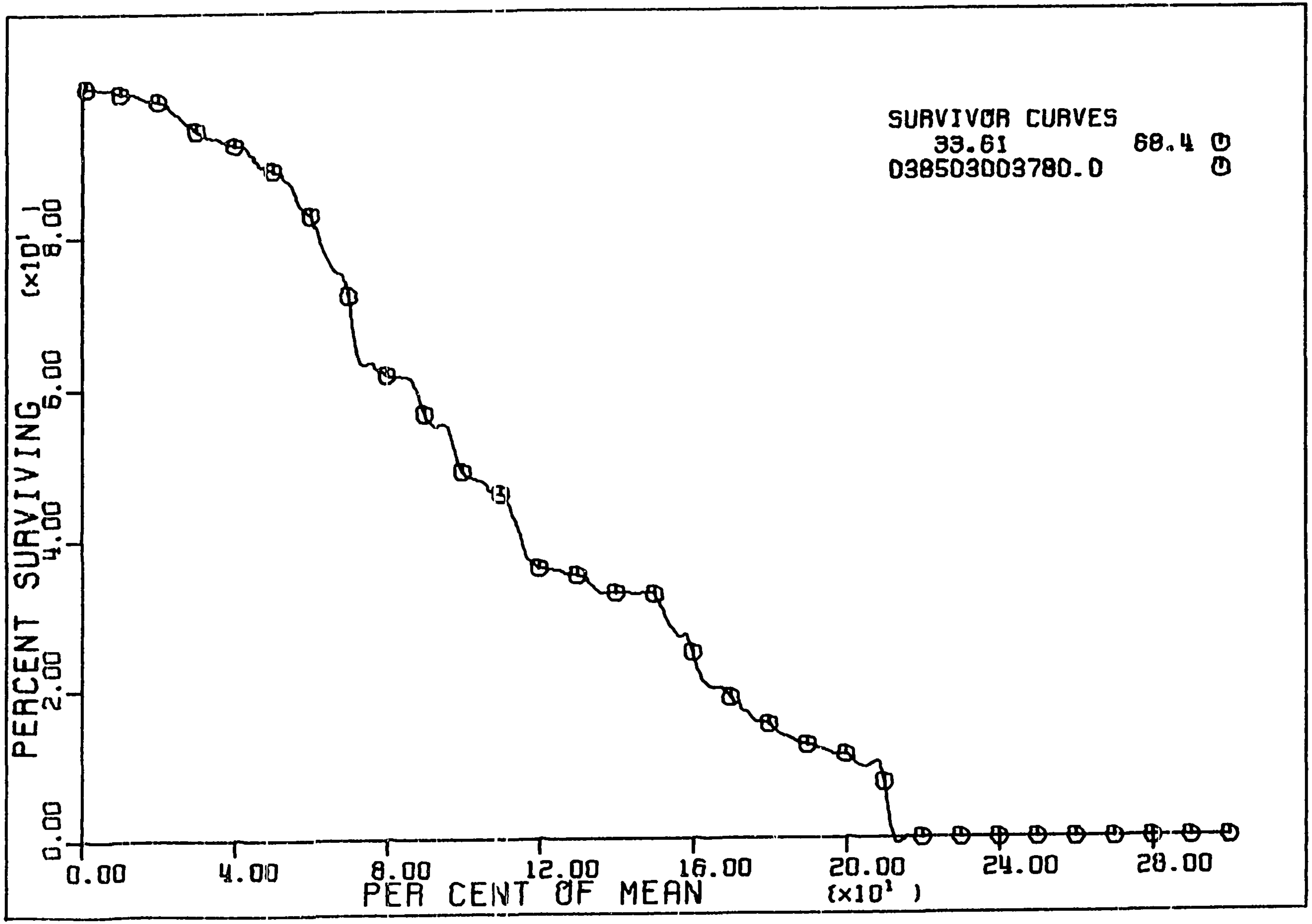
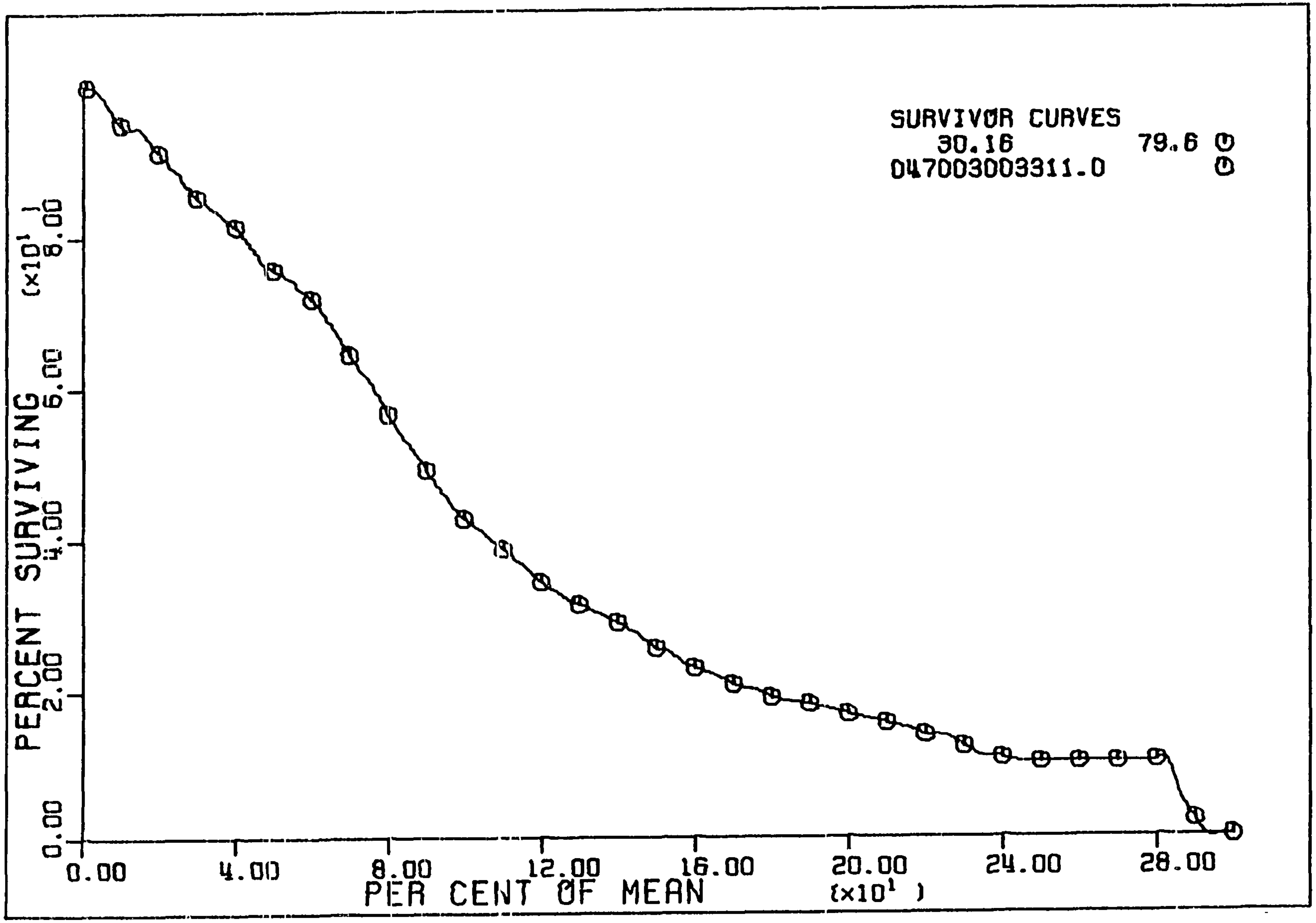

SURVIVOR CURVES
33.61 68.4
038503003780.0
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
PER CENT OF MEAN (x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 14. Sample of original plotted survivor curve for data account 0470


SURVIVOR CURVES
30.16 79.6
047003003311.0
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
4.00
12.00
16.00
20.00
24.00
28.00
PER CENT OF MEAN
(x10¹)

APPENDIX E:

CLUSTERED CURVE GROUPS - SAMPLES

Figure 15. Sample of Russo clustered curve group 10

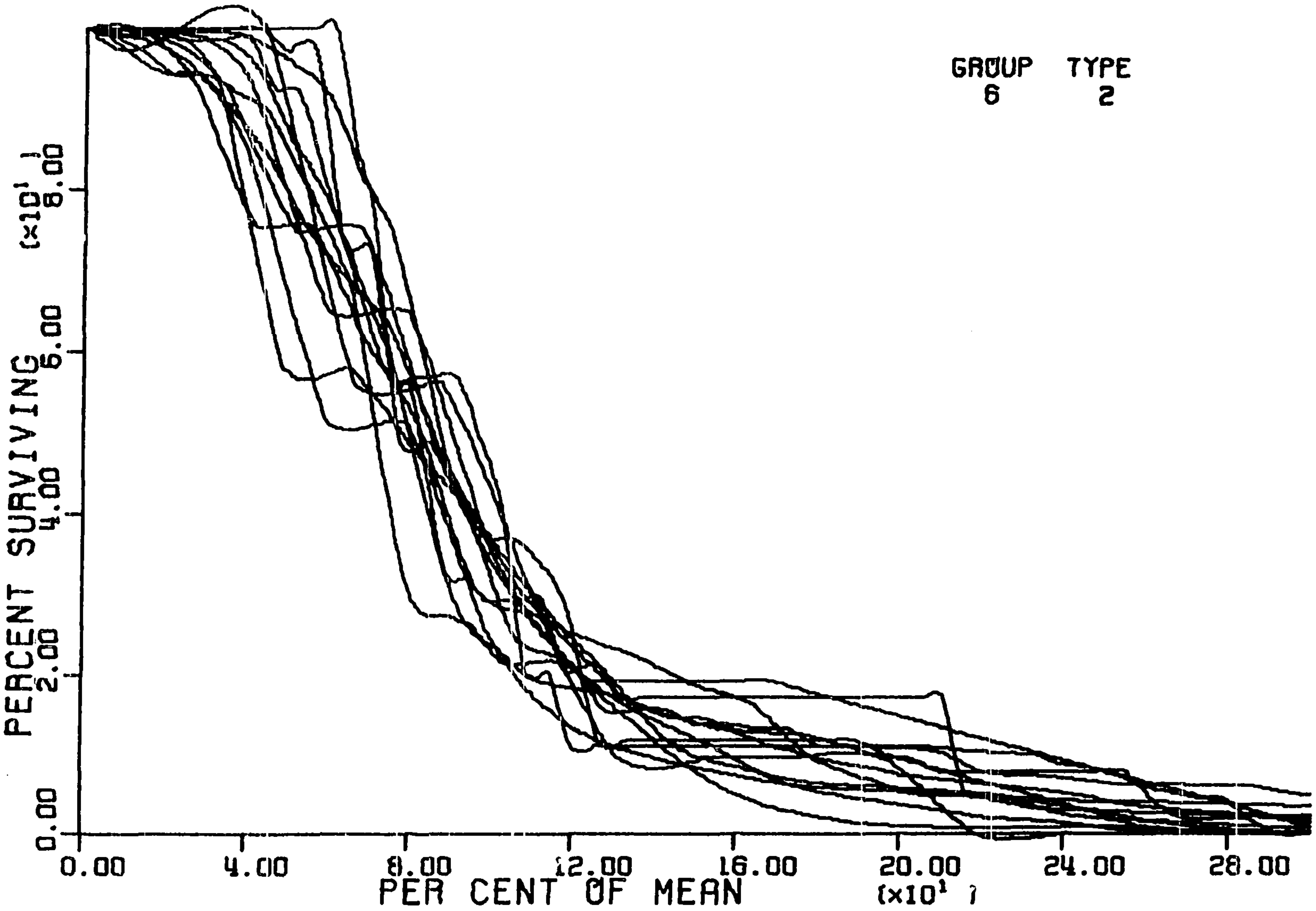
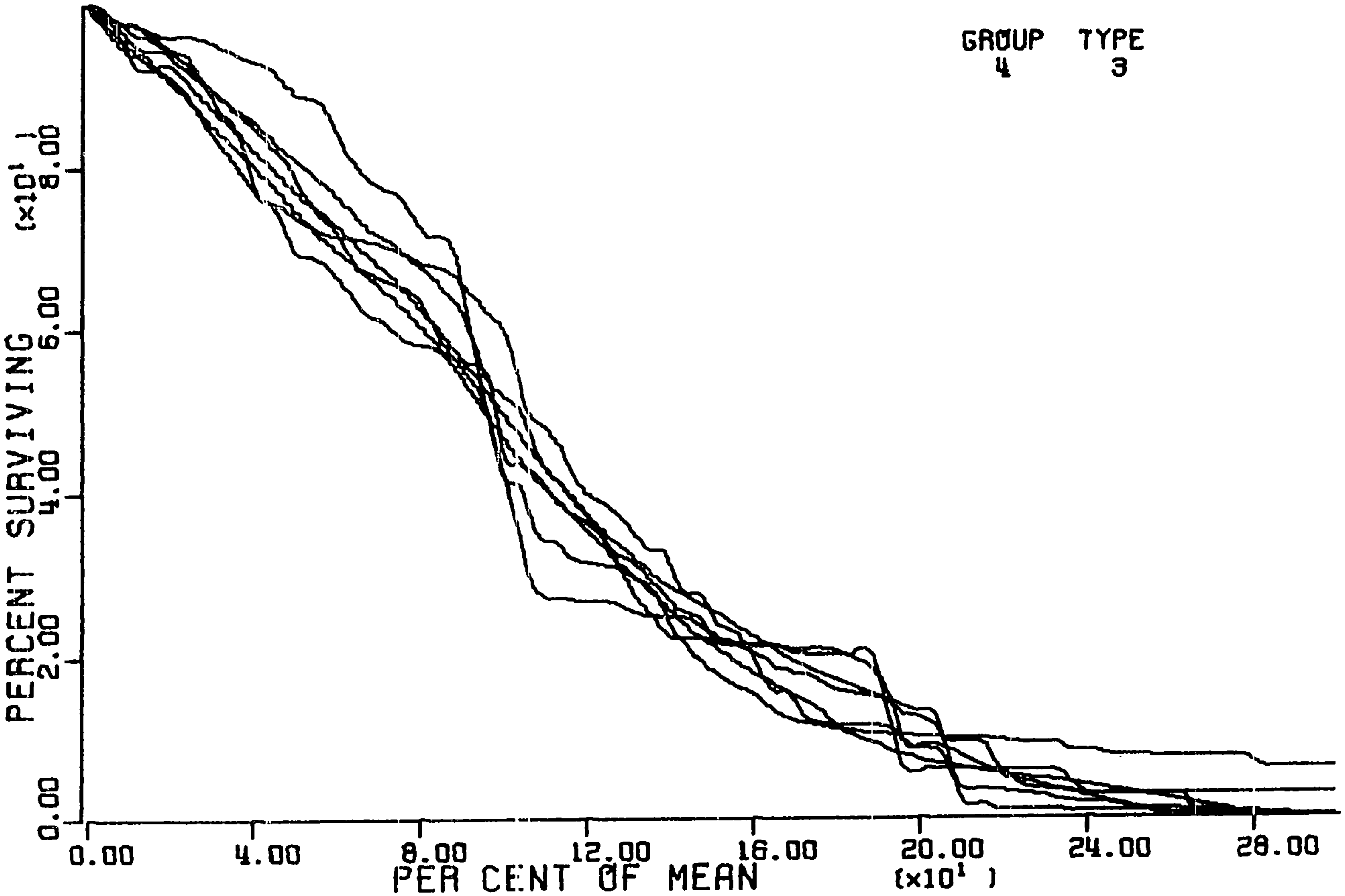
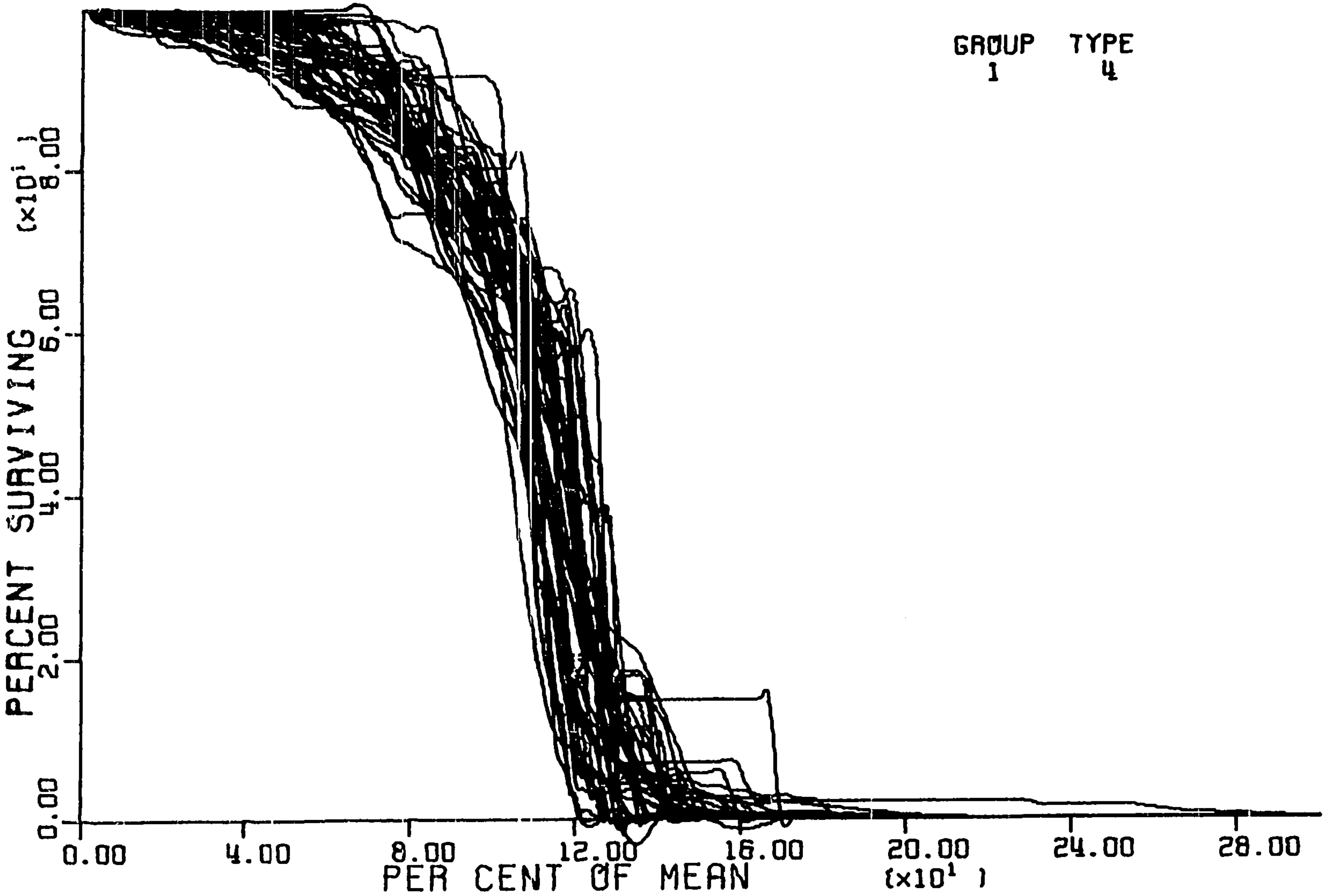

GROUP TYPE
6 2
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PER CENT OF MEAN
(x10¹)

Figure 16. Sample of Russo clustered curve group 19



GROUP TYPE
4 3
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PER CENT OF MEAN
(x10¹)

Figure 17. Sample of Russo clustered curve group 33


GROUP 1
TYPE 4
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PER CENT OF MEAN (x10¹)

APPENDIX F:

ALL AVERAGED RUSSO CURVES

Figure 18. Averaged Russo curve number 1--single curve cluster, no boundaries

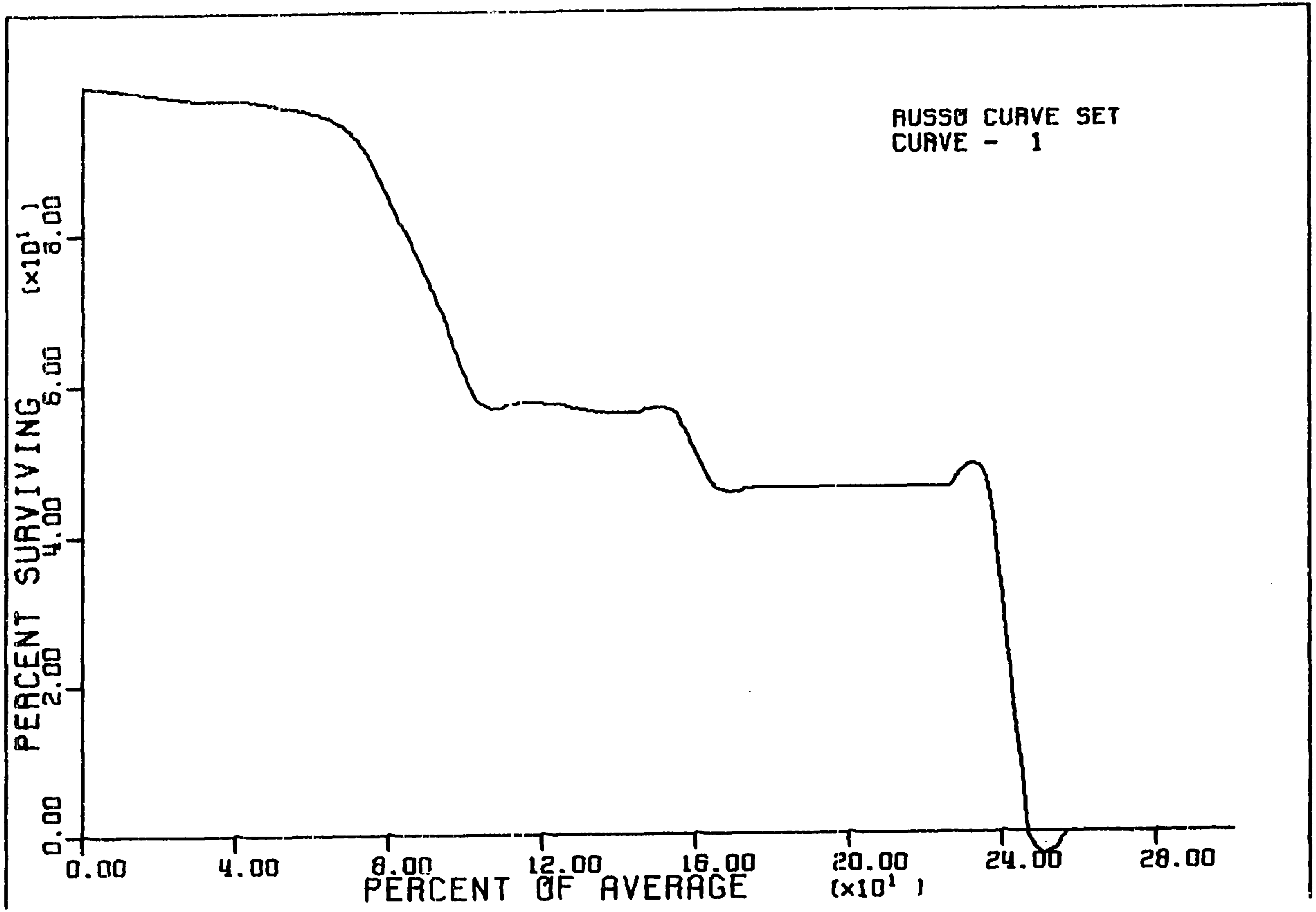

RUSSO CURVE SET
CURVE - 1
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE (x10¹)

Figure 19. Averaged Russo curve number 2 with boundaries

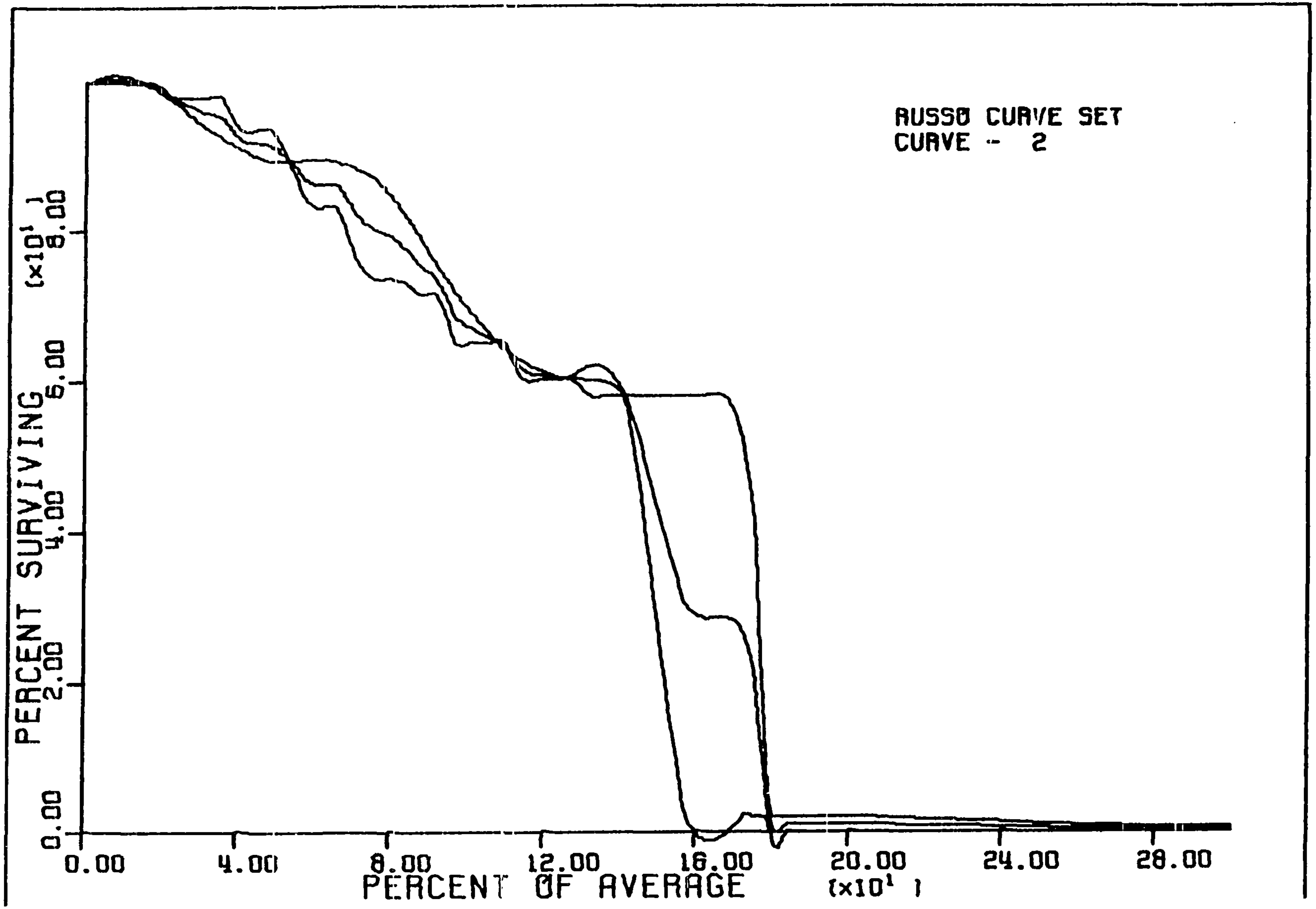

RUSSO CURVE SET
CURVE - 2
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 20. Averaged Russo curve number 3 with boundaries

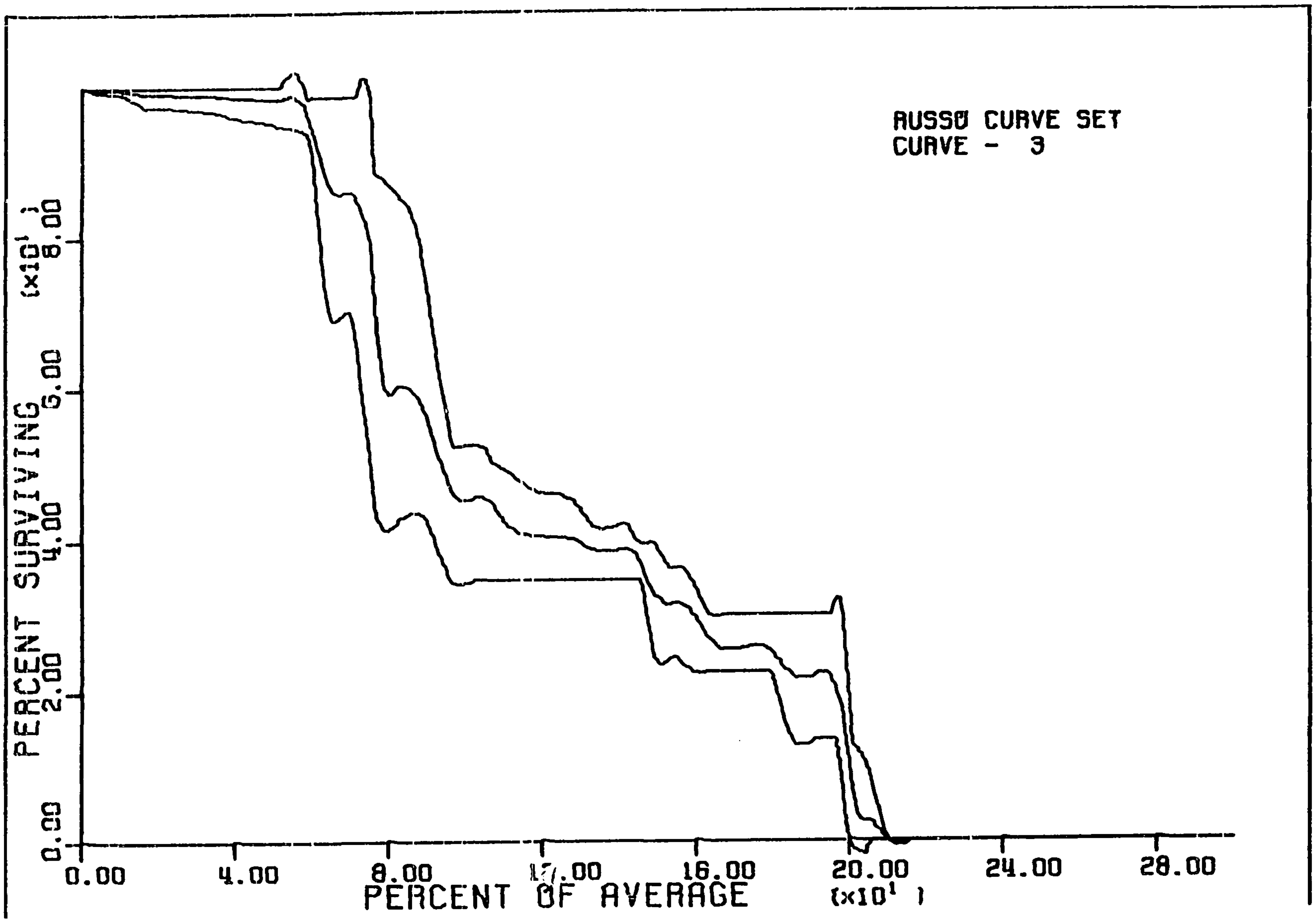

RUSSO CURVE SET
CURVE - 3
PERCENT SURVIVING
(x10[1])
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10[1])

Figure 21. Averaged Russo curve number 4--single curve cluster, no boundaries

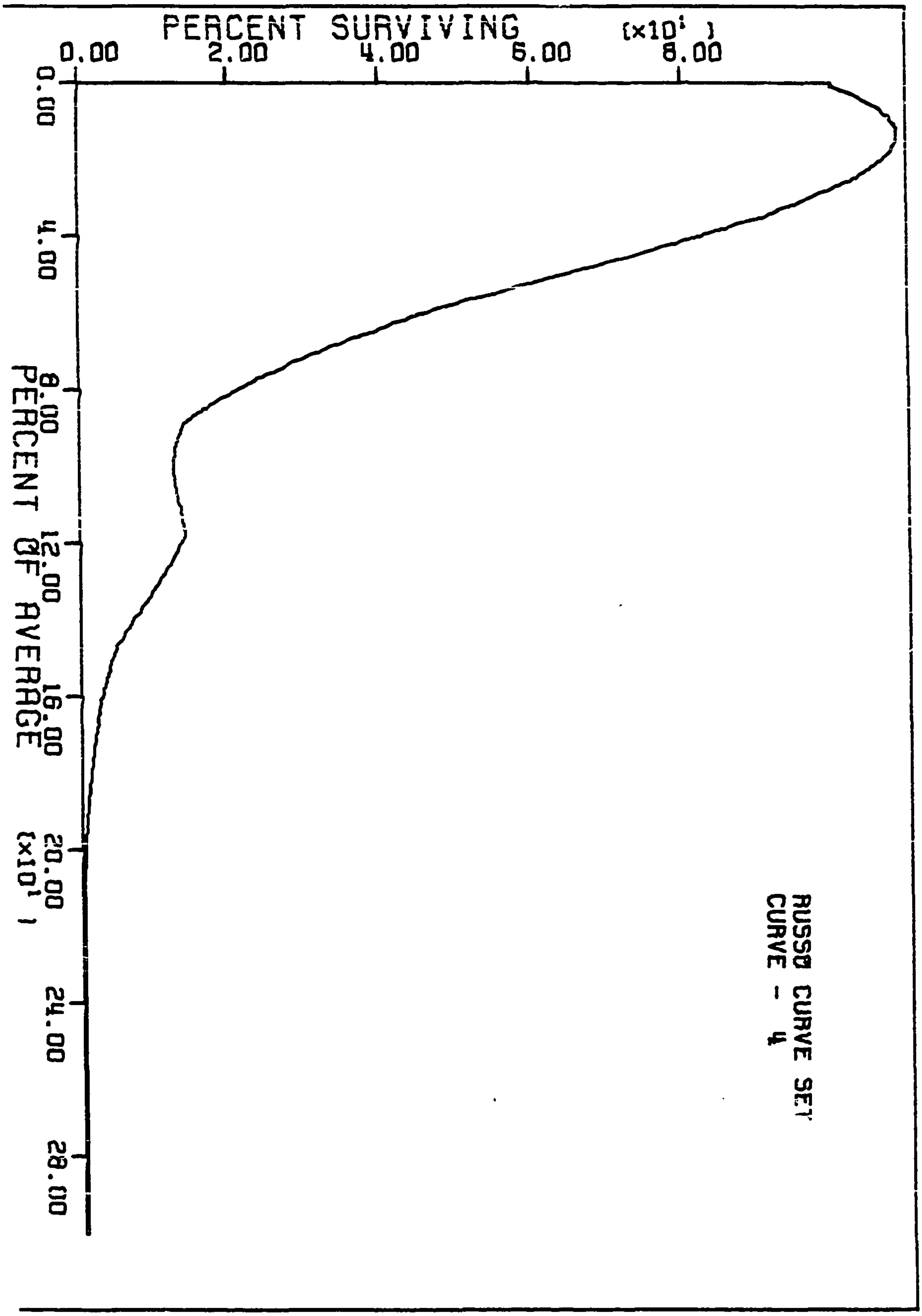

PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE (x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
RUSSO CURVE SET
CURVE - 4

Figure 22. Averaged Russo curve number 5 with boundaries

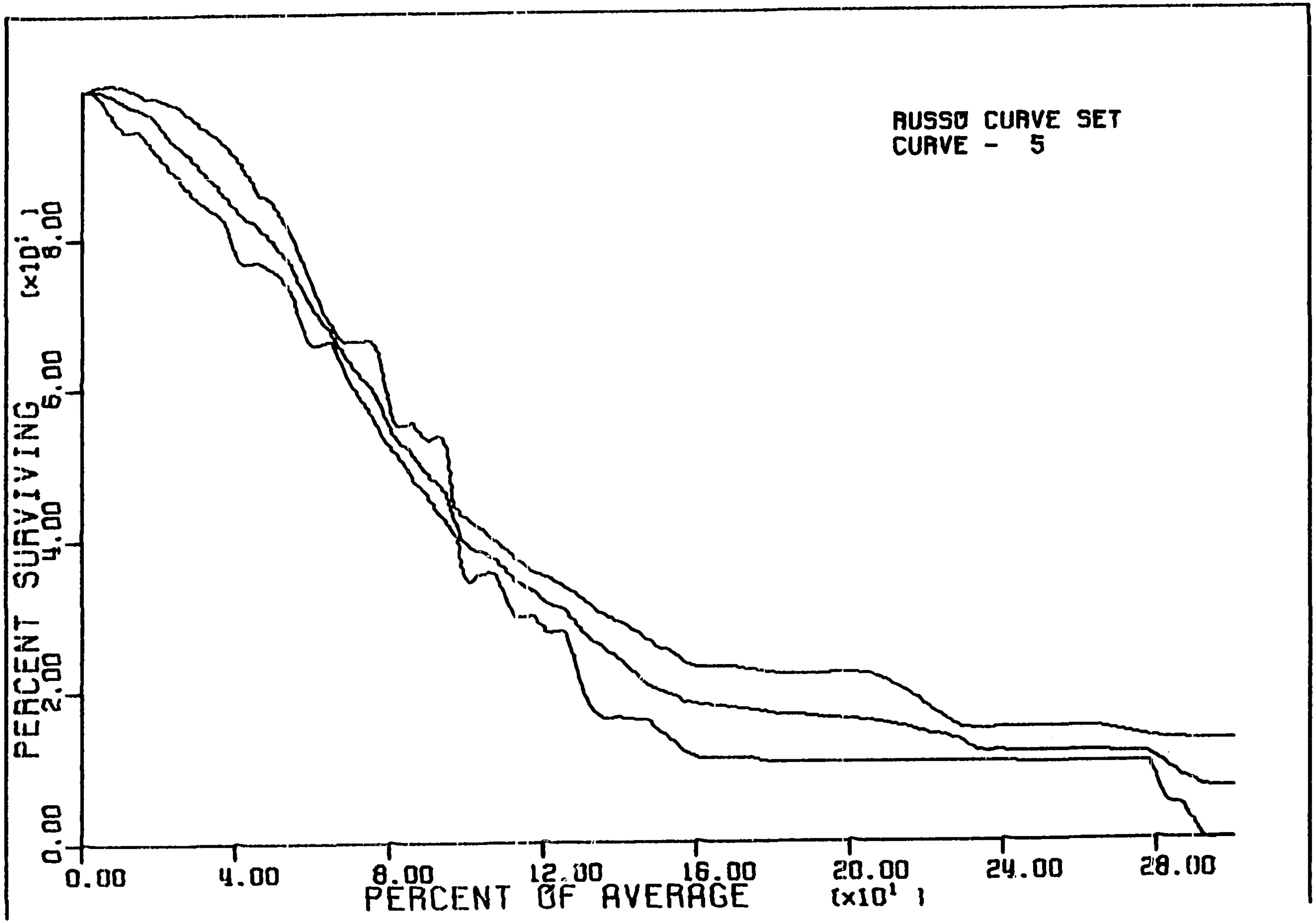
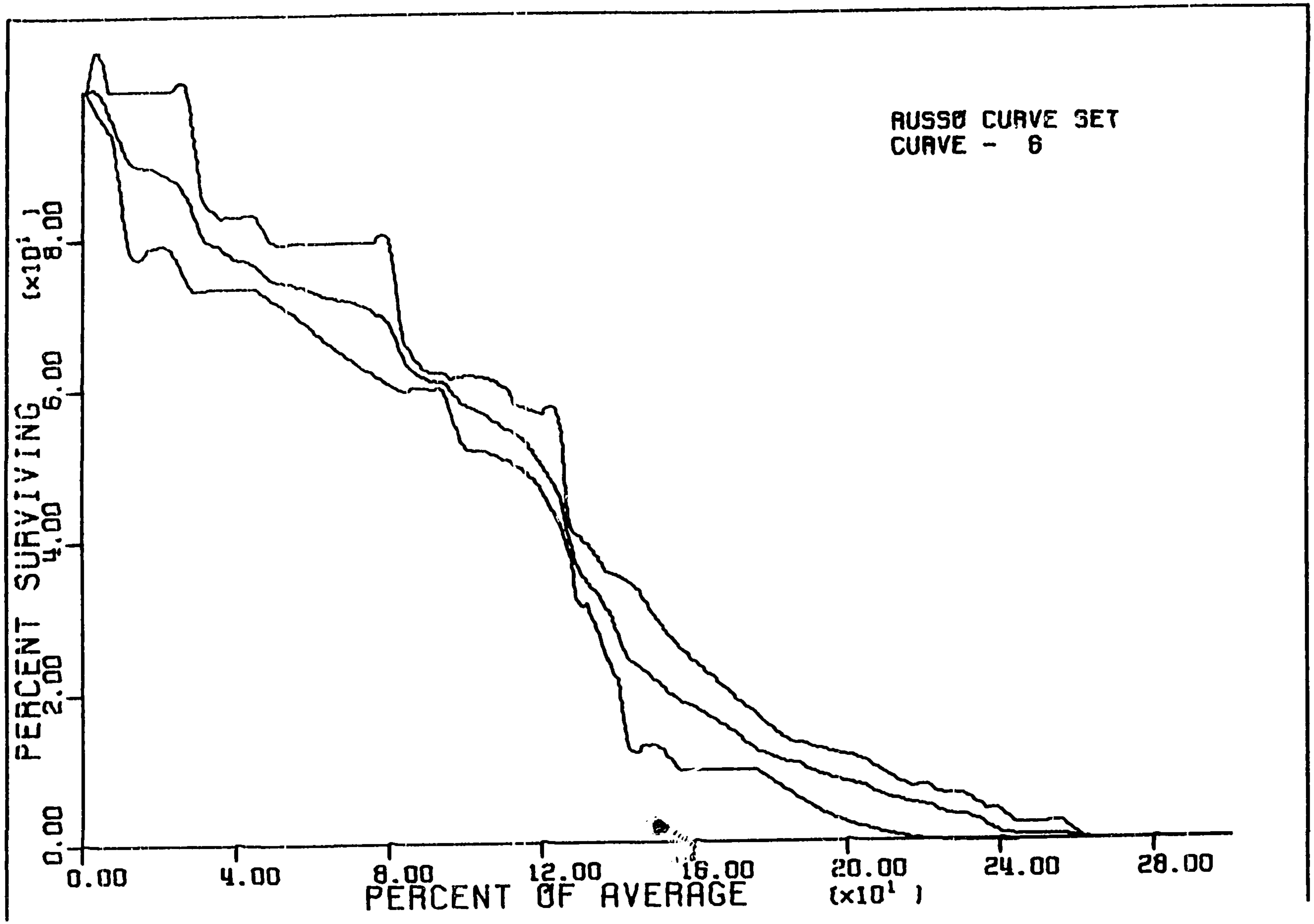

RUSSO CURVE SET
CURVE - 5
PERCENT SURVIVING (x10^1)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE (x10^1)

Figure 23. Averaged Russo curve number 6 with boundaries



RUSSO CURVE SET
CURVE - 6
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE (x10¹)

Figure 24. Averaged Russo curve number 7 with boundaries

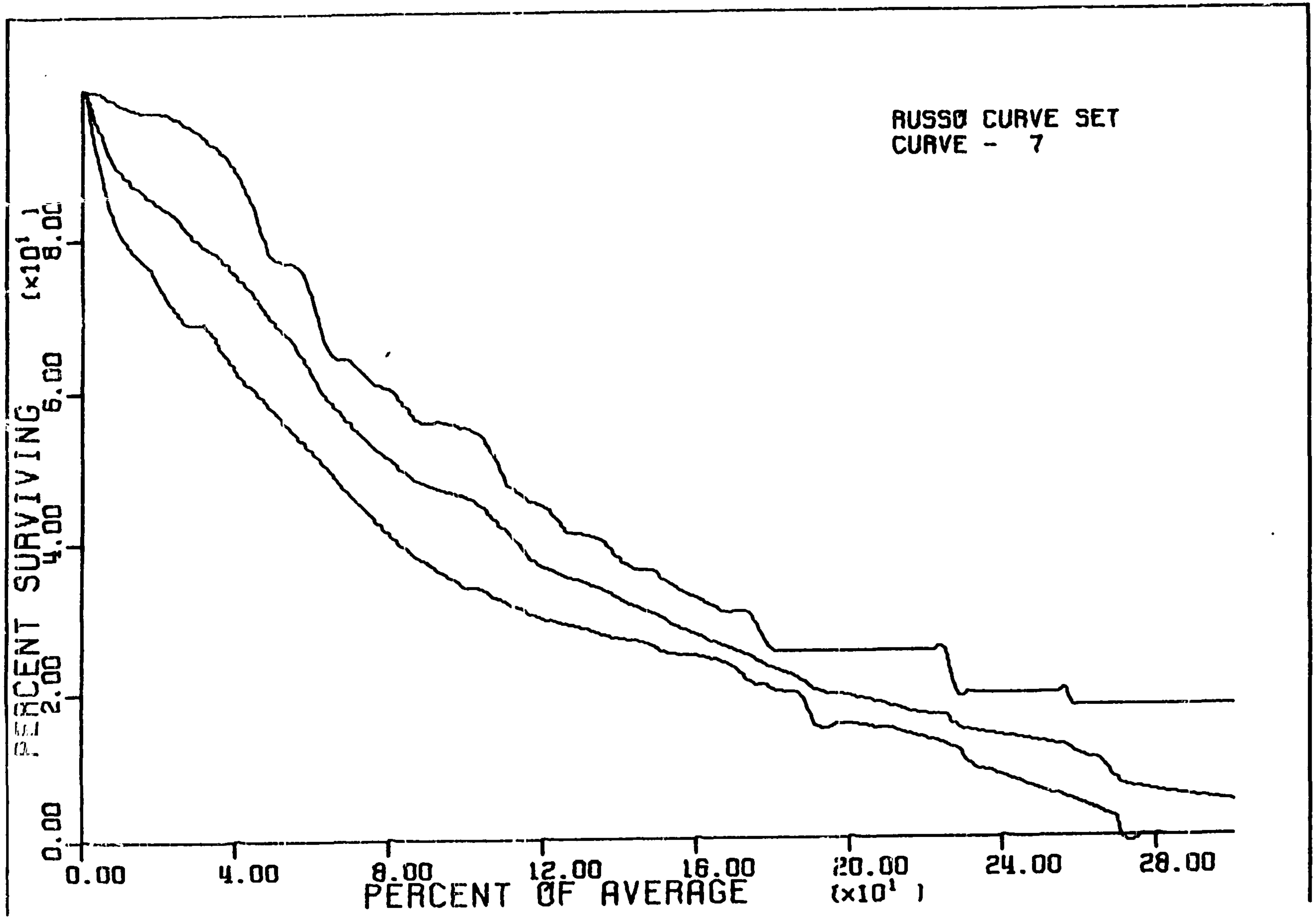

RUSSO CURVE SET
CURVE - 7
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE (x10¹)

Figure 25. Averaged Russo curve number 8 with boundaries

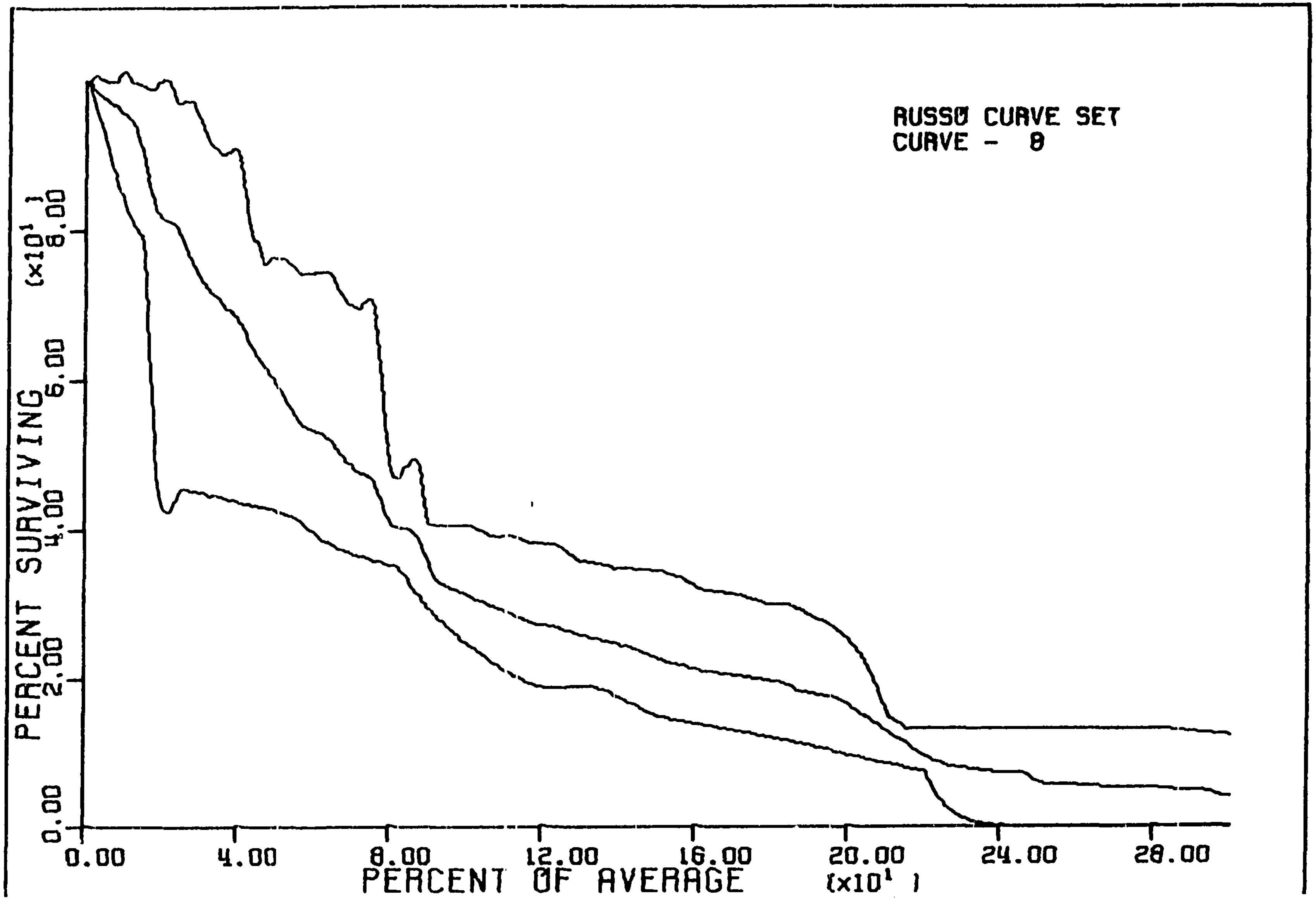

RUSSO CURVE SET
CURVE - 8
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE
(x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 26. Averaged Russo curve number 9 with boundaries

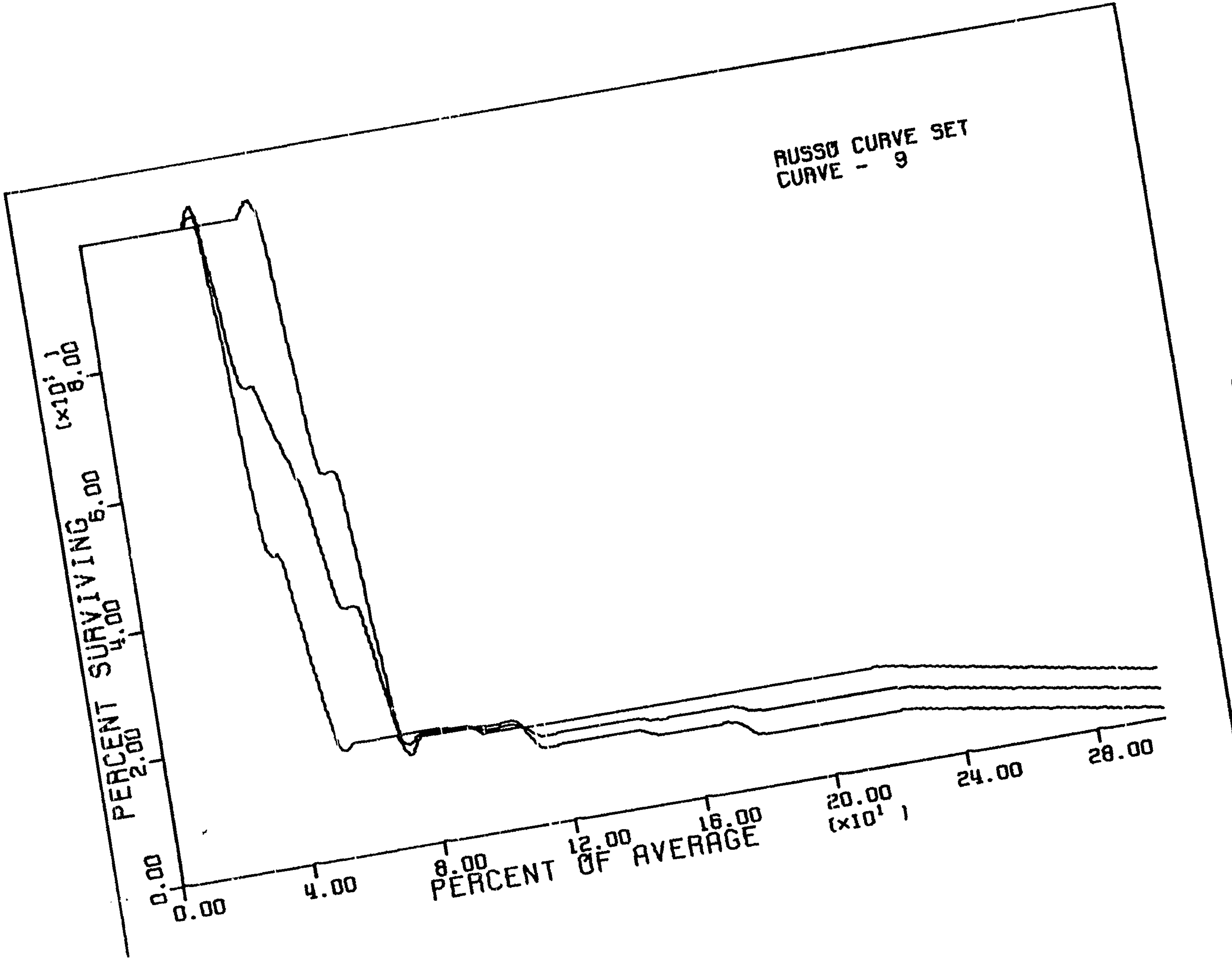
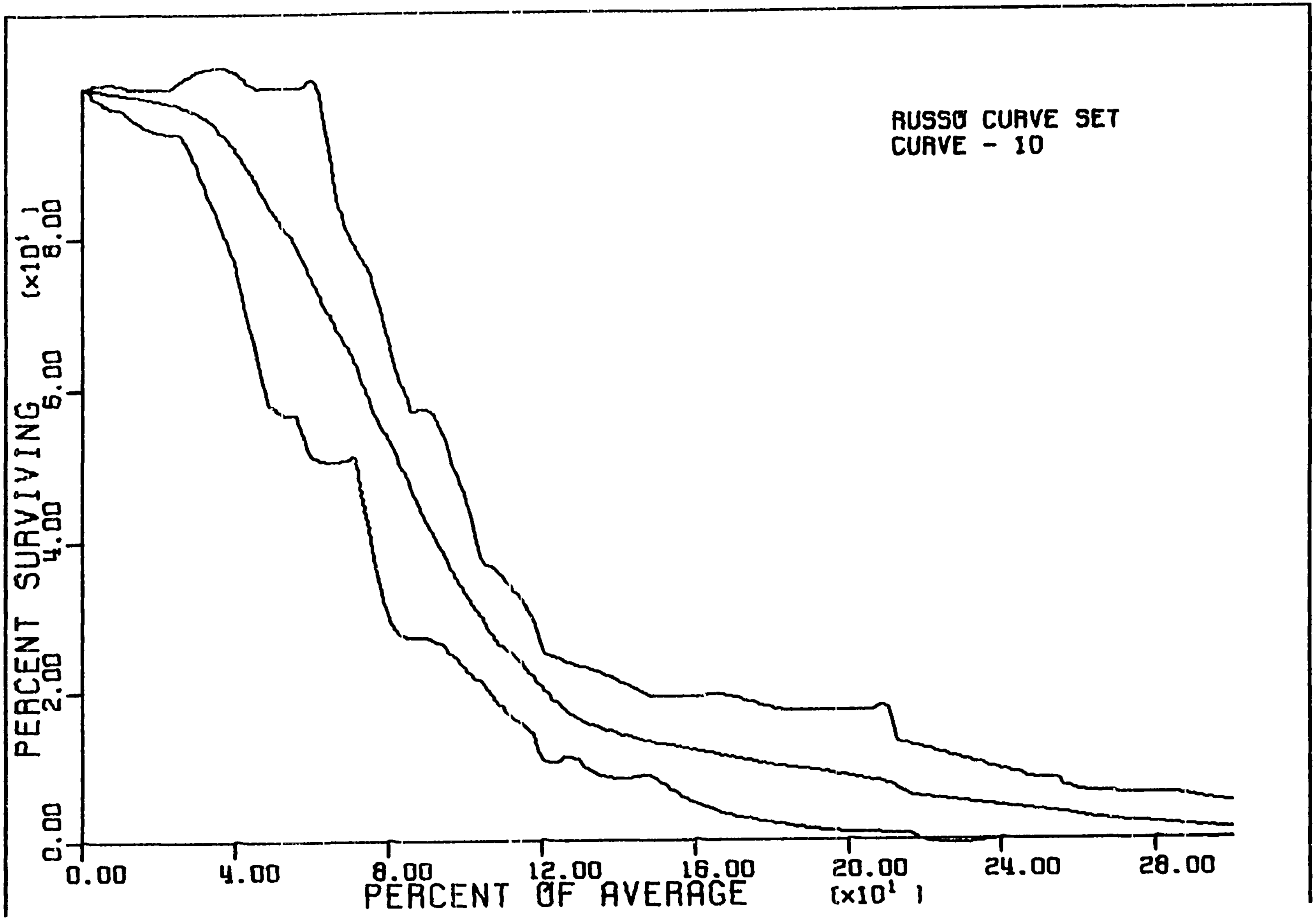

RUSSØ CURVE SET
CURVE - 9
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE
(x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 27. Averaged Russo curve number 10 with boundaries



RUSSØ CURVE SET
CURVE - 10
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT ØF AVERAGE
(x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 28. Averaged Russo curve number 11 with boundaries

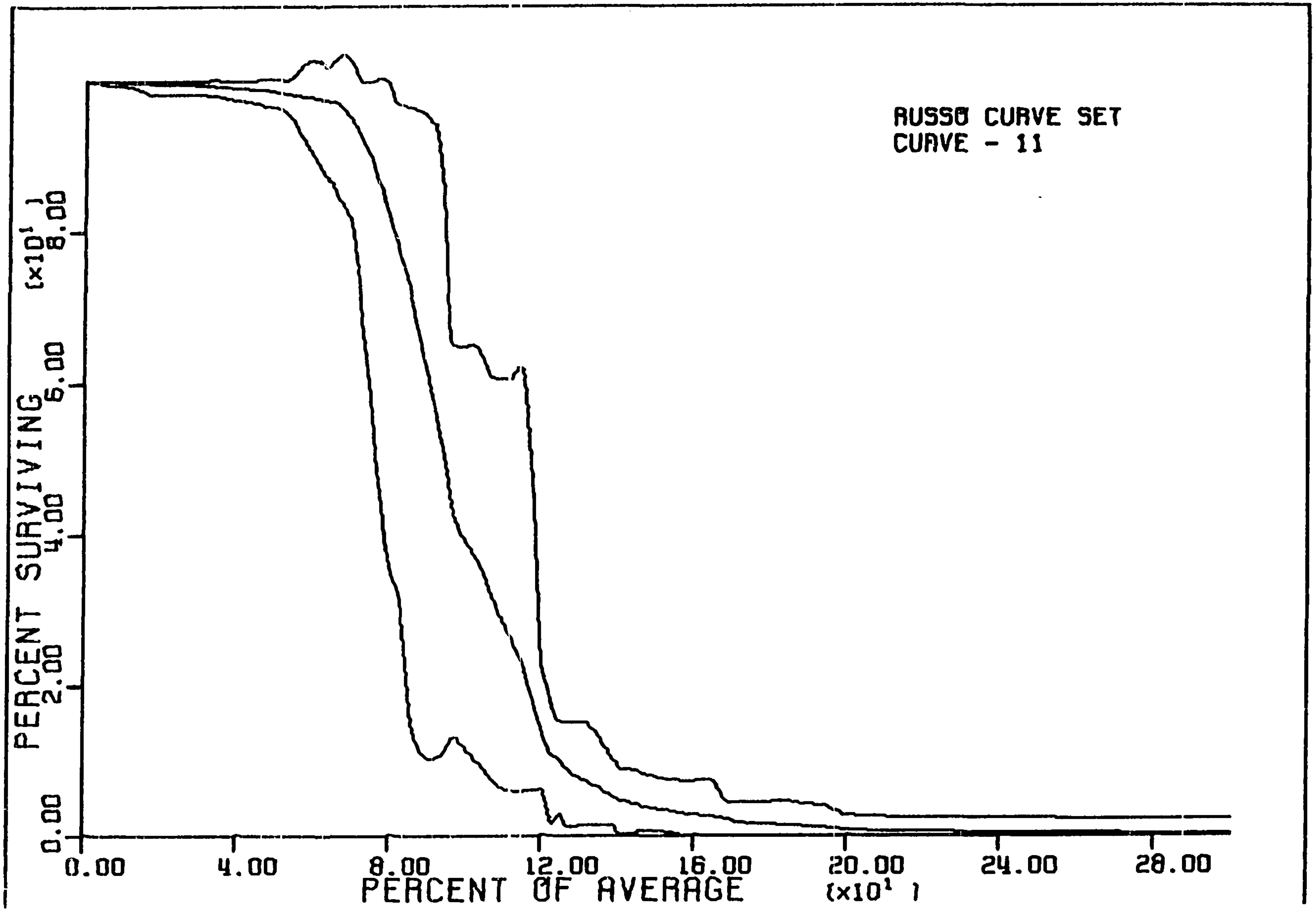

RUSSO CURVE SET
CURVE - 11
PERCENT SURVIVING
(x10[1])
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE
(x10[1])
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 29. Average Russo curve number 12 with boundaries

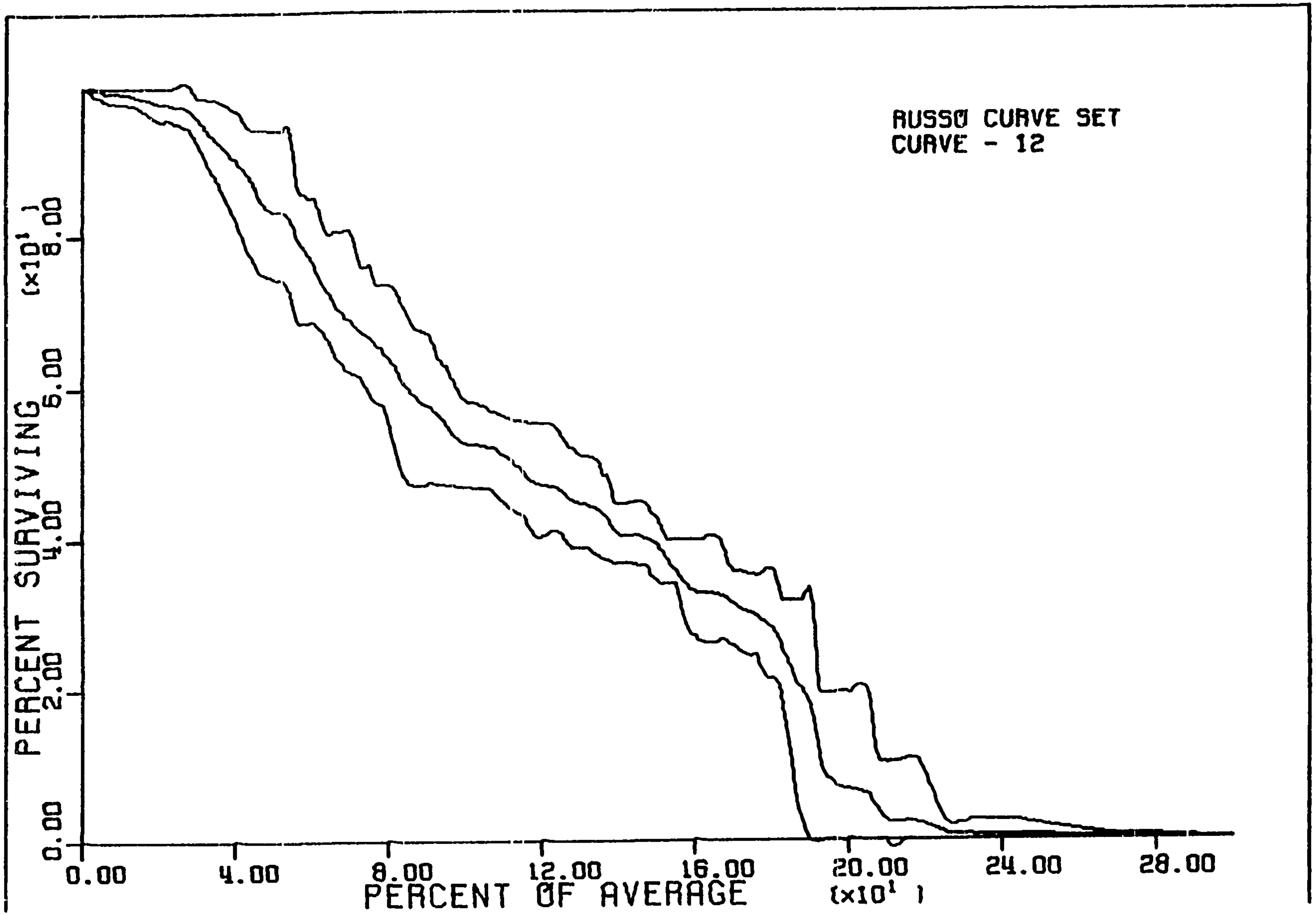

RUSSO CURVE SET
CURVE - 12
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE
(x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 30. Averaged Russo curve number 13 with boundaries

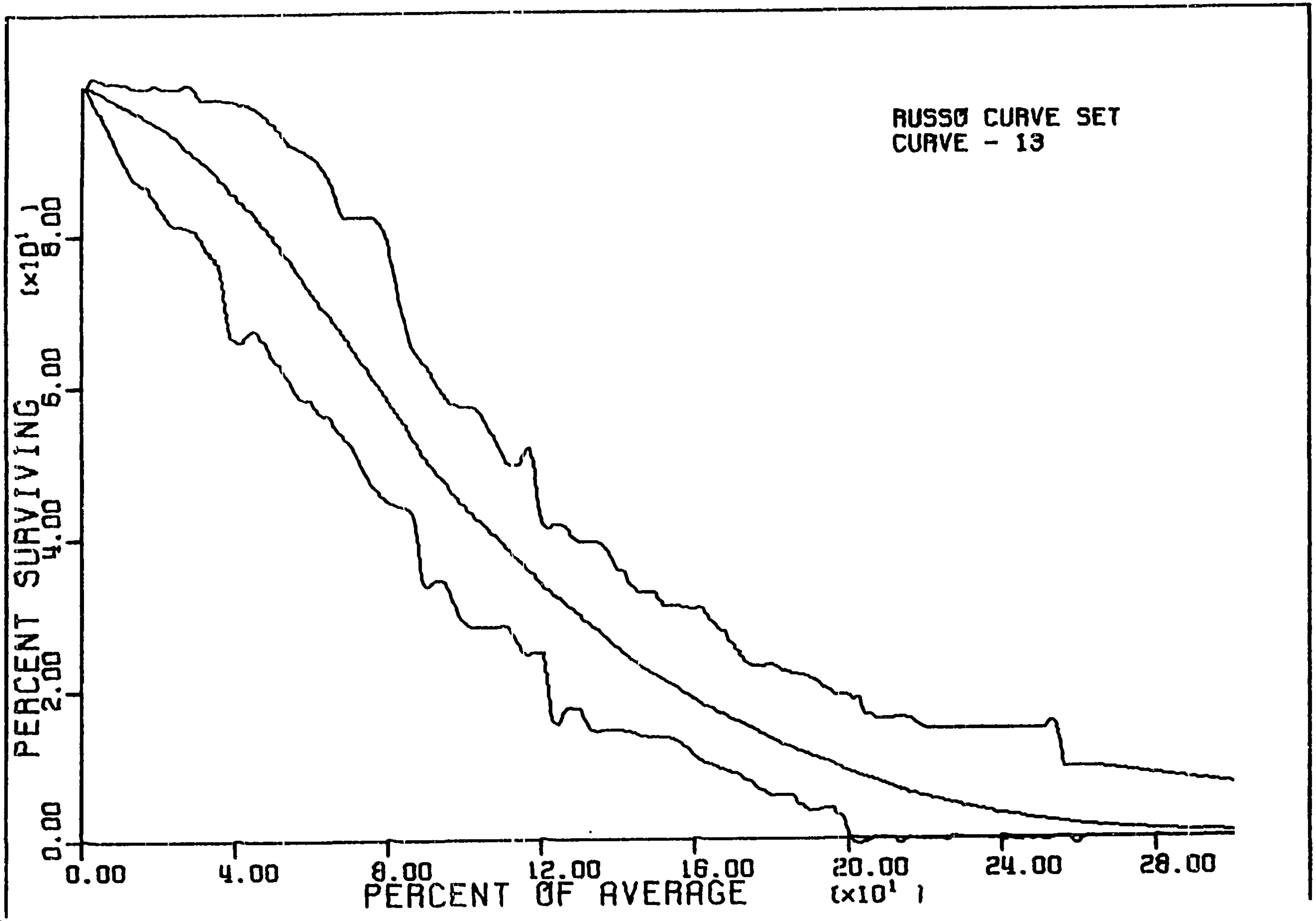

RUSSØ CURVE SET
CURVE - 13
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT ØF AVERAGE
(x10¹)

Figure 31. Averaged Russo curve number 14 with boundaries

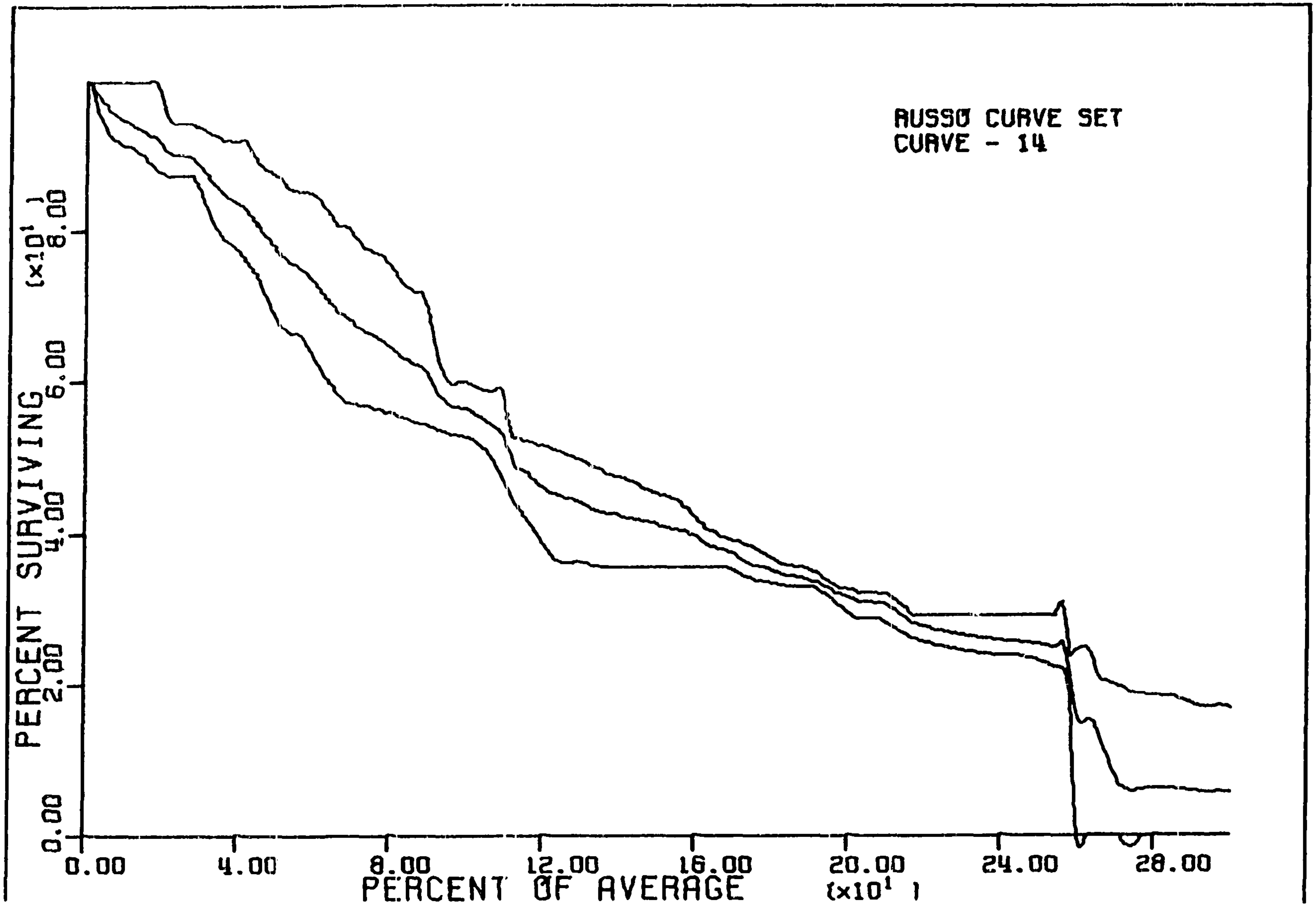

RUSSO CURVE SET
CURVE - 14
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 32. Averaged Russo curve number 15 with boundaries

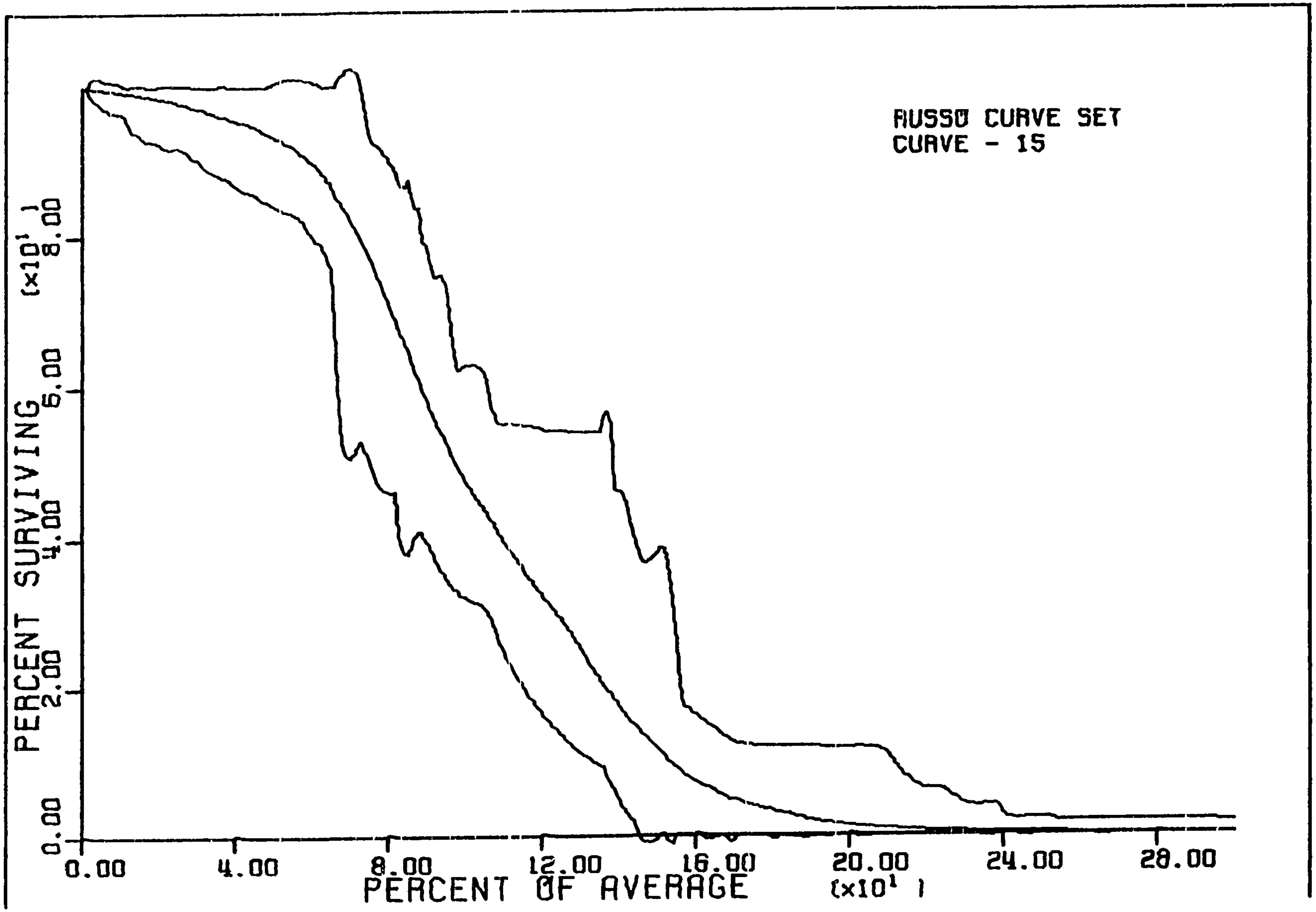

RUSSO CURVE SET
CURVE - 15
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE (x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 33. Averaged Russo curve number 16--single curve cluster, no boundaries

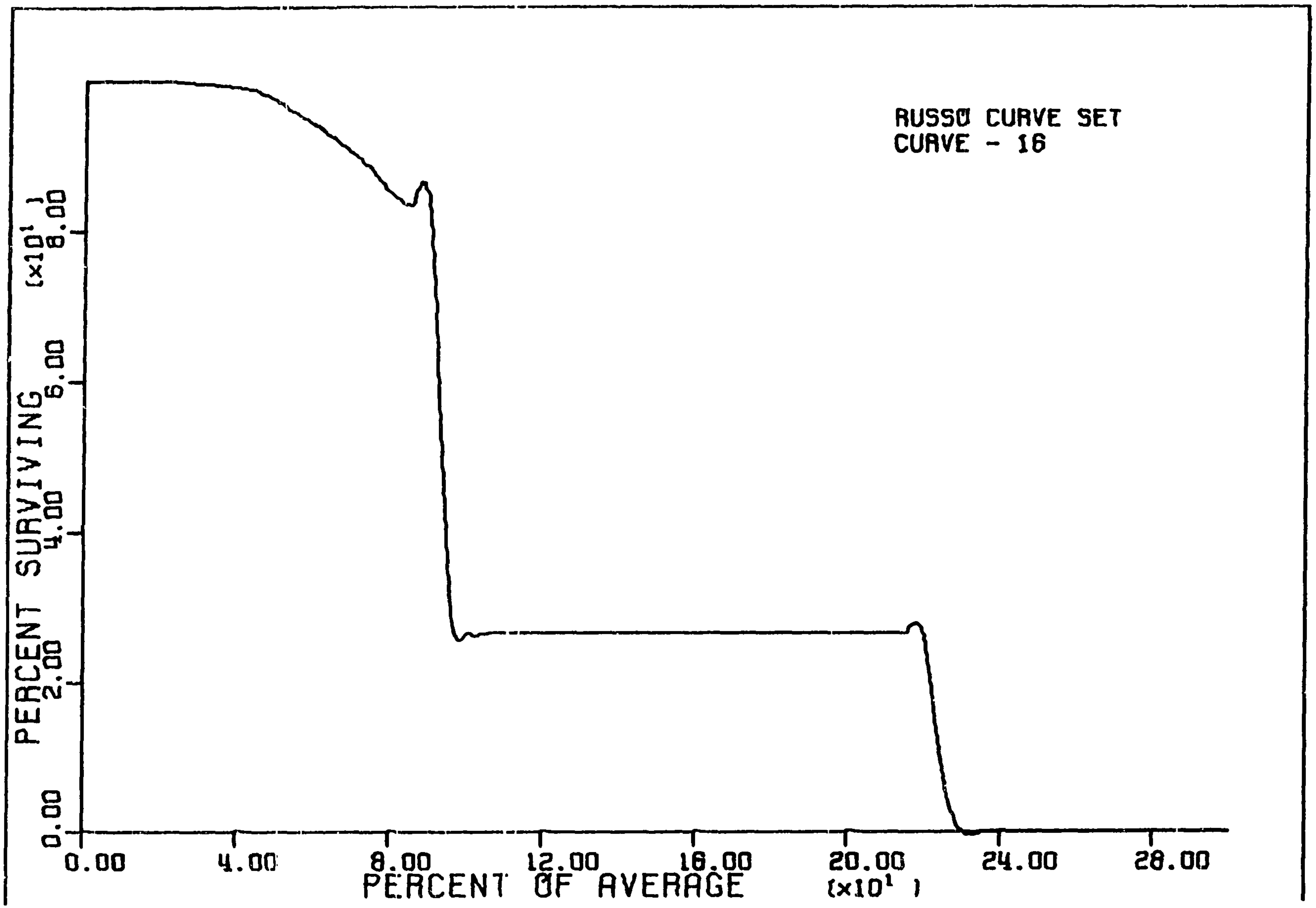

RUSSO CURVE SET
CURVE - 16
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 34. Averaged Russo curve number 1.7 with boundaries

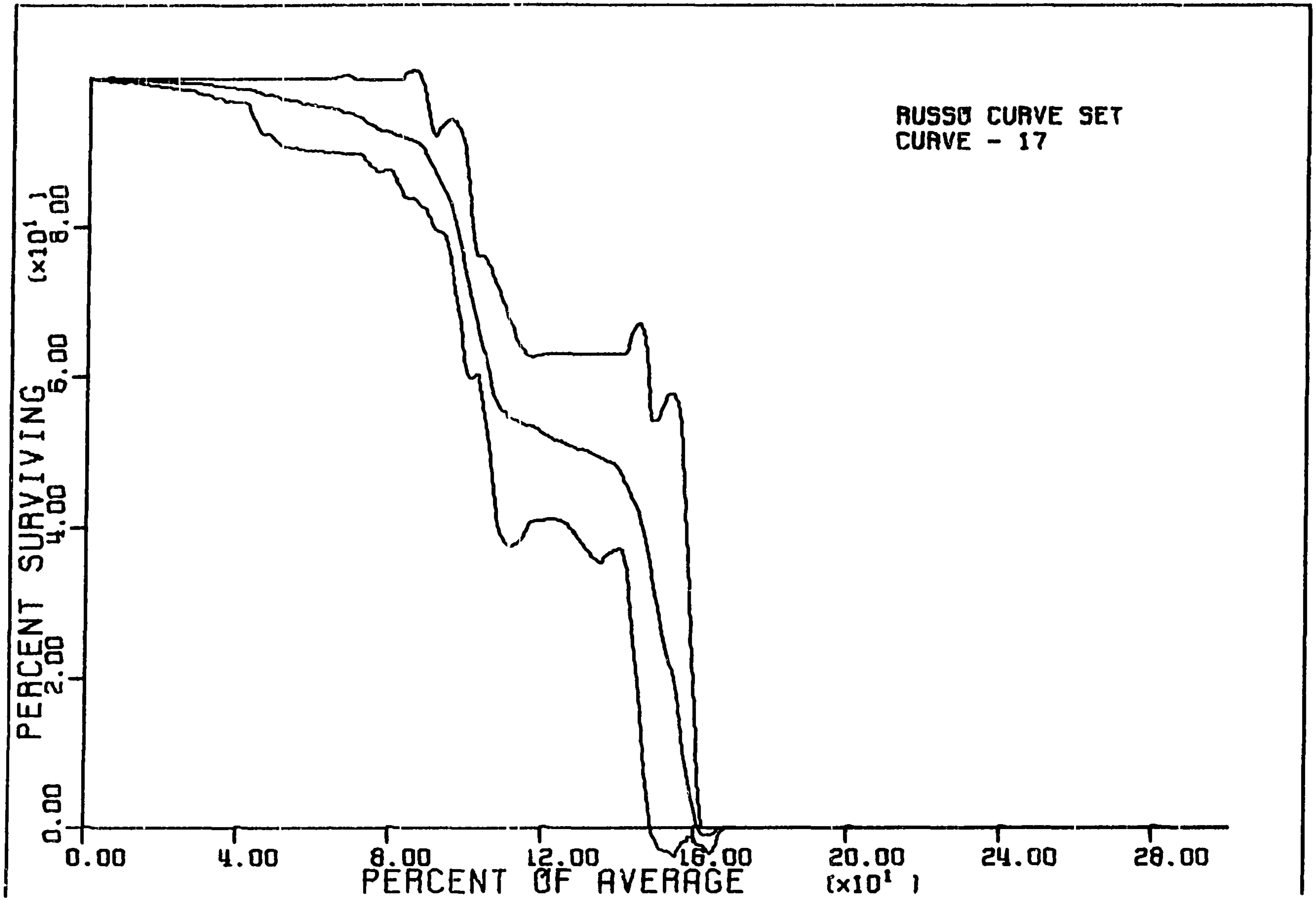

RUSSO CURVE SET
CURVE - 17
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE (x10¹)

Figure 35. Averaged Russo curve number 18 with boundaries

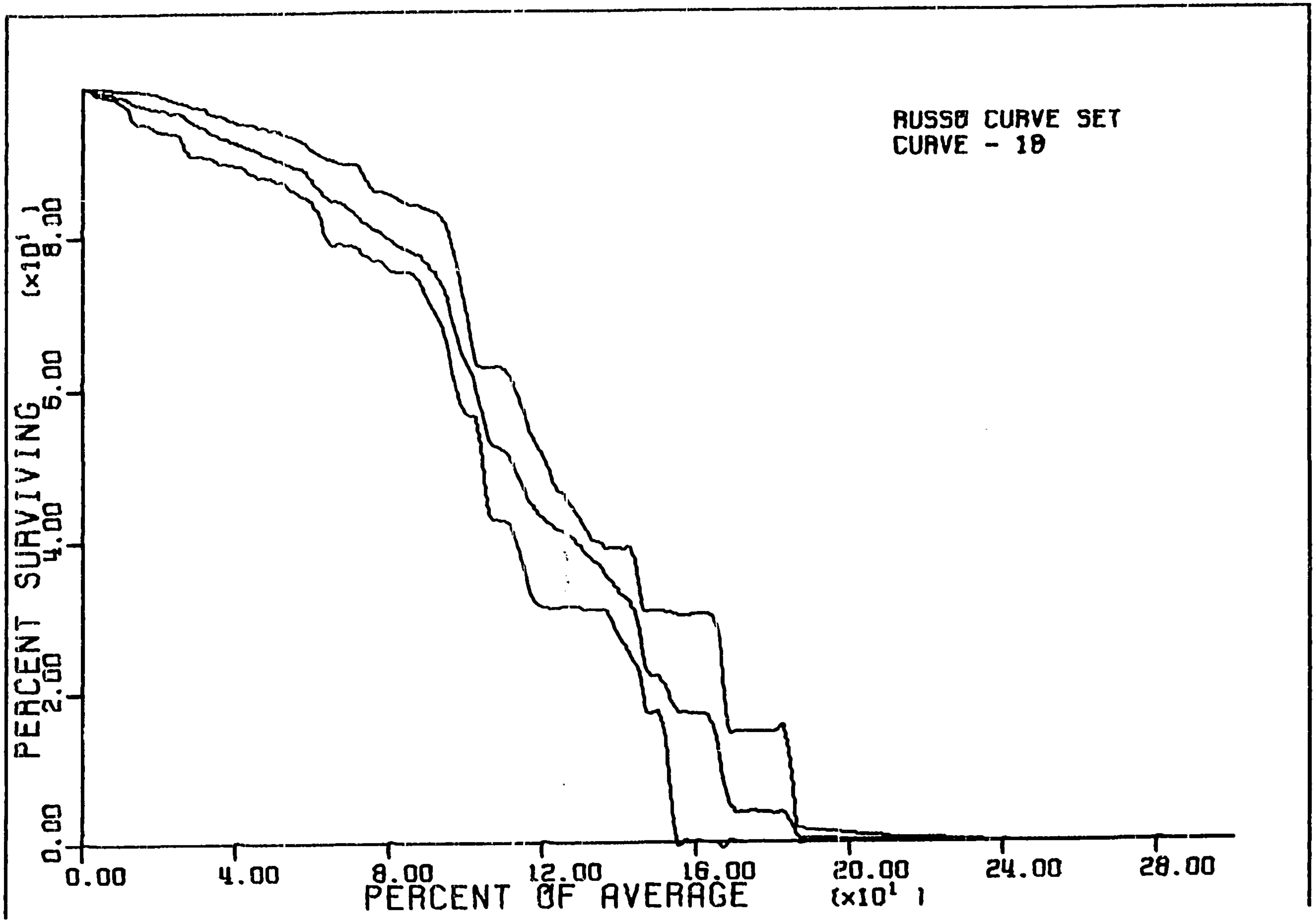

RUSSO CURVE SET
CURVE - 10
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 36. Averaged Russo curve number 19 with boundaries

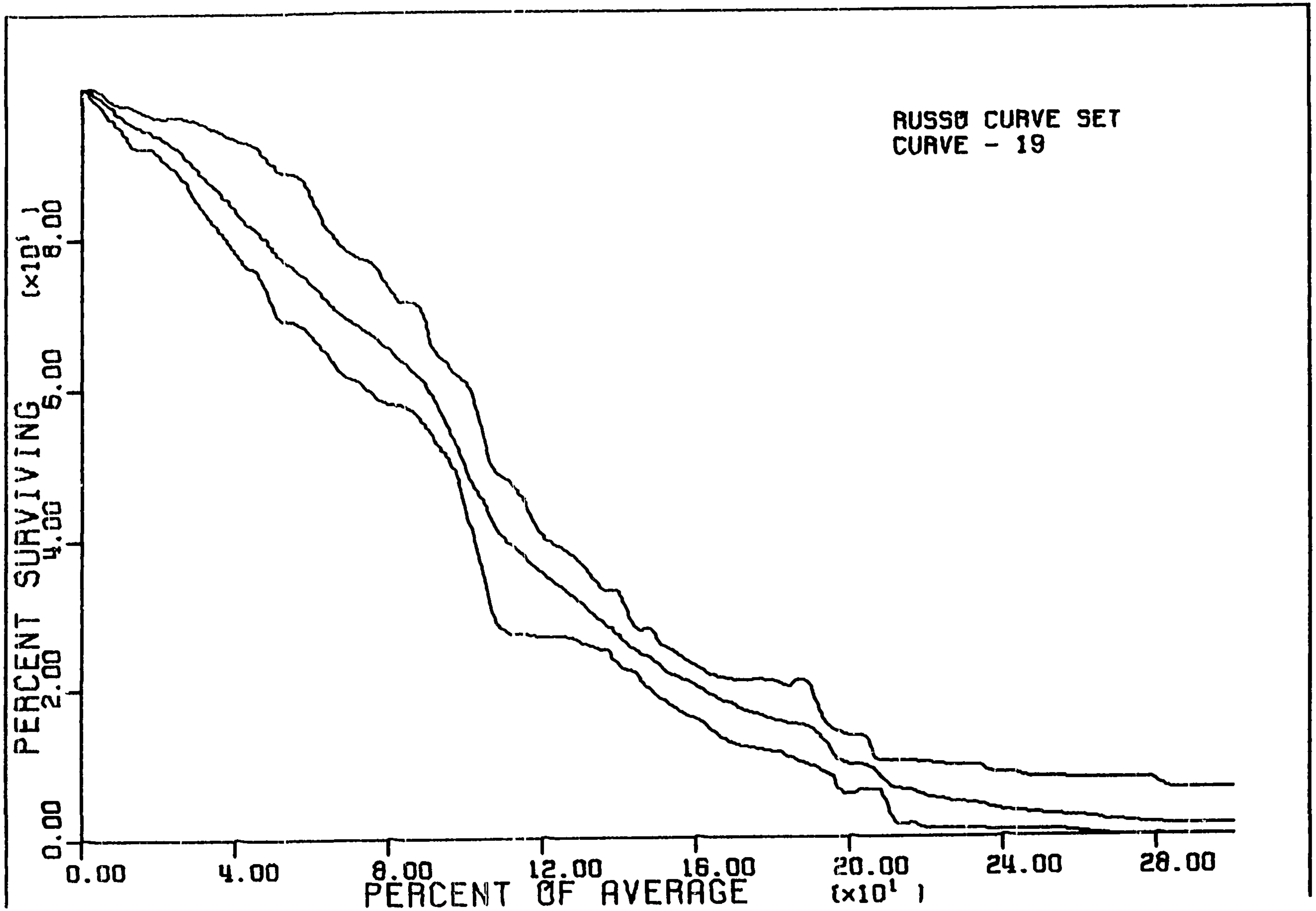

RUSSO CURVE SET
CURVE - 19
PERCENT SURVIVING (x10[1])
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE (x10[1])
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 37. Averaged Russo curve number 20 with boundaries

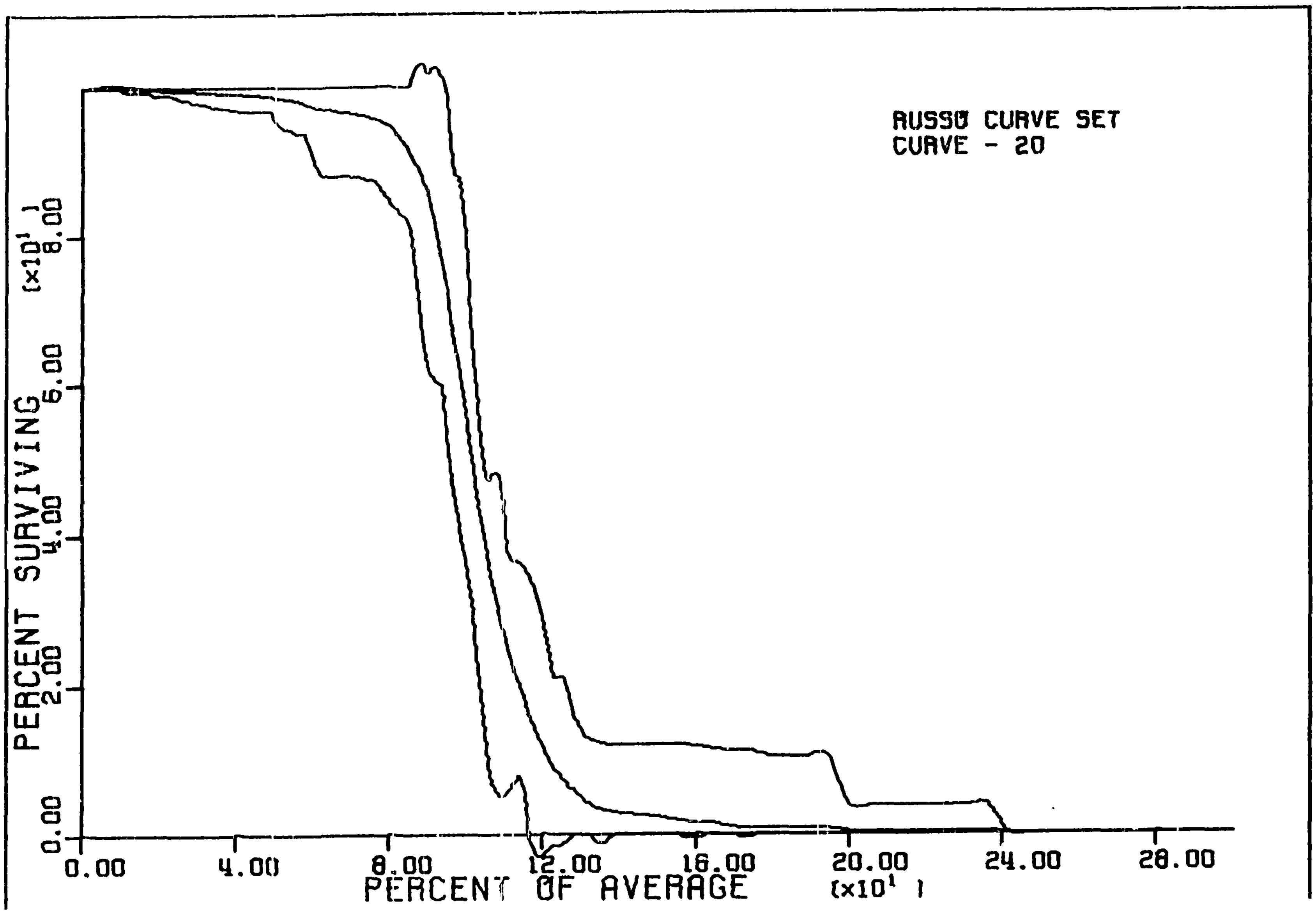

RUSSO CURVE SET
CURVE - 20
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 38. Averaged Russo curve number 21 with boundaries

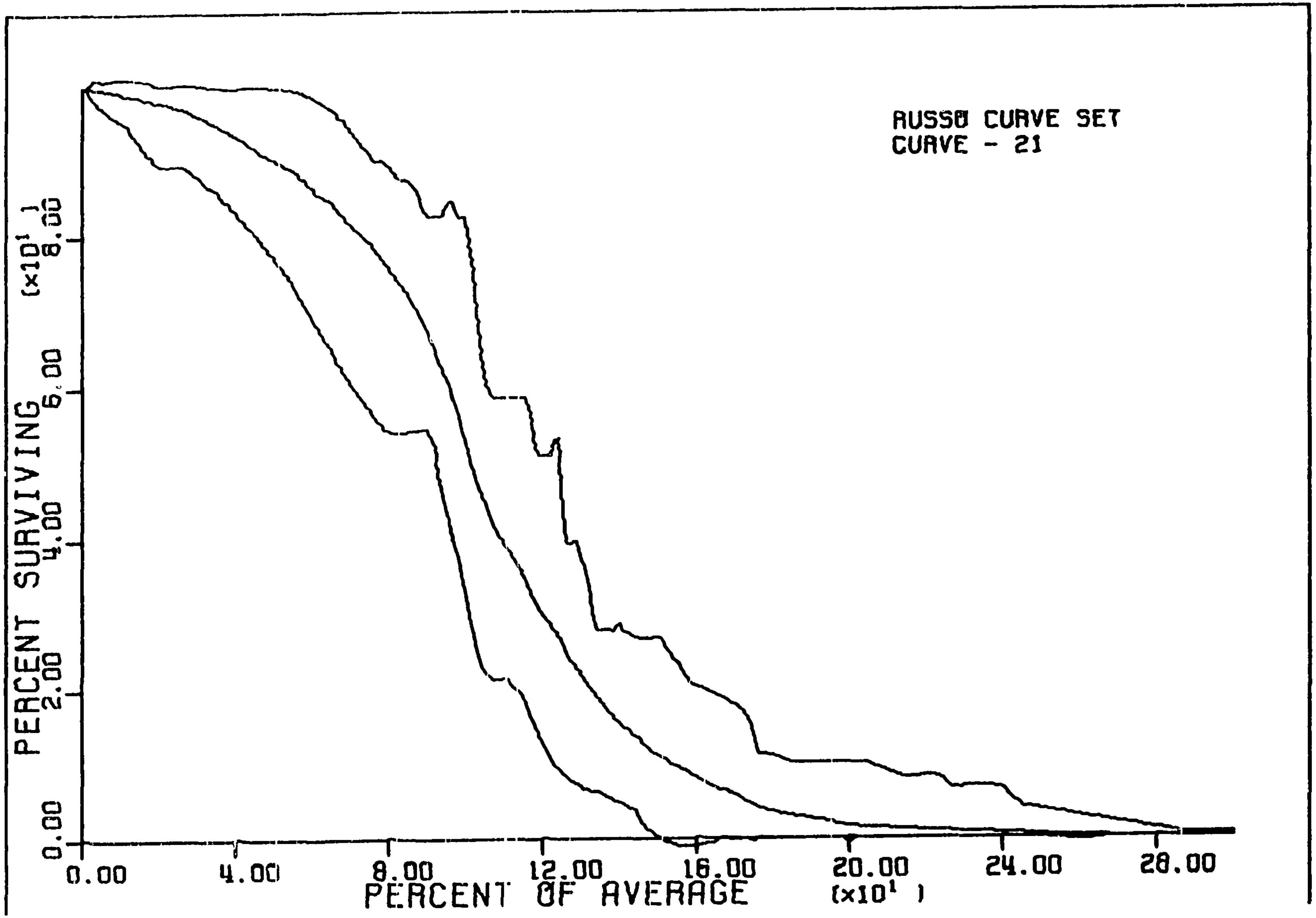

RUSSO CURVE SET
CURVE - 21
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 39. Averaged Russo curve number 22 with boundaries

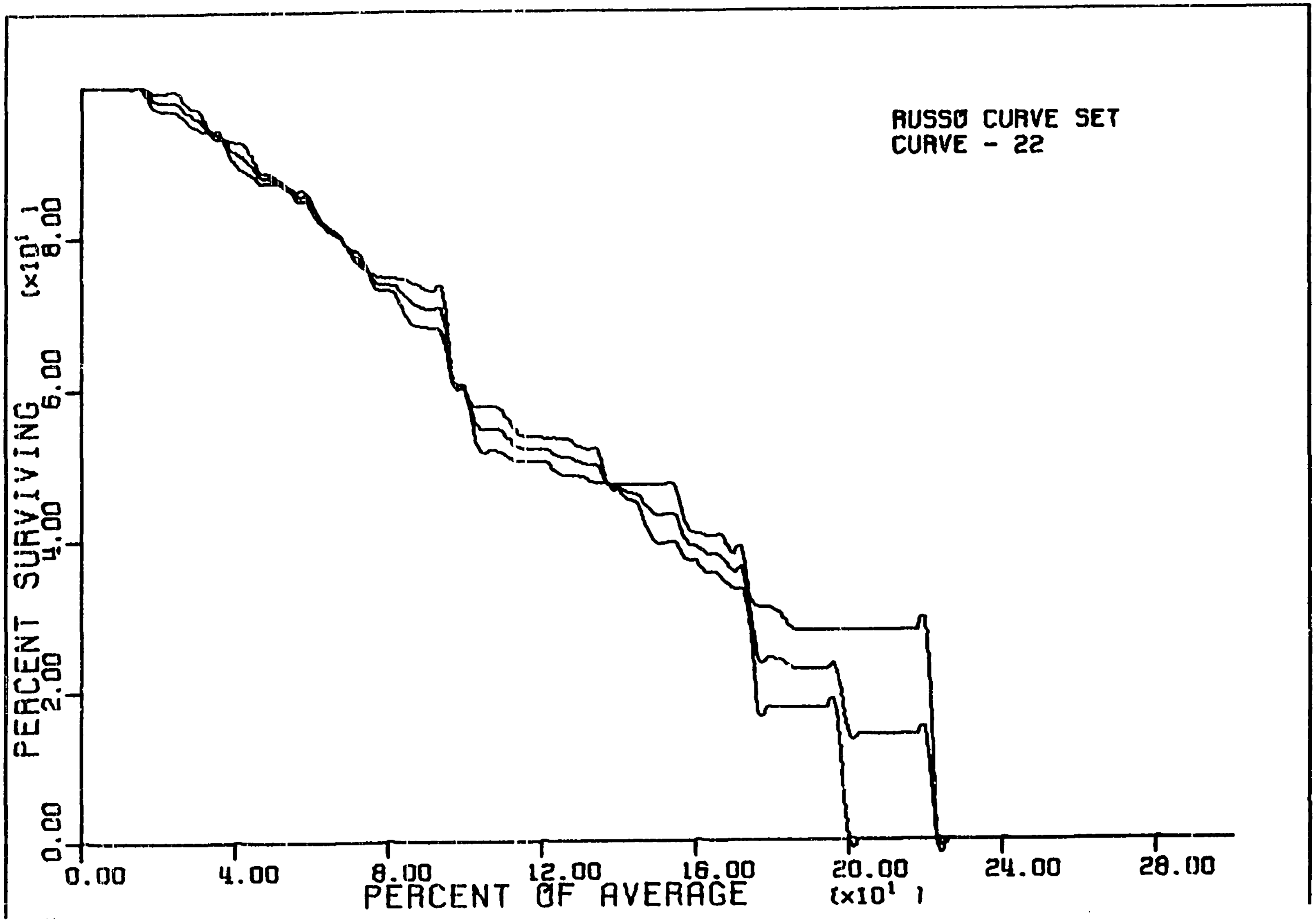

RUSSO CURVE SET
CURVE - 22
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 40. Averaged Russo curve number 23--single curve cluster, no boundaries

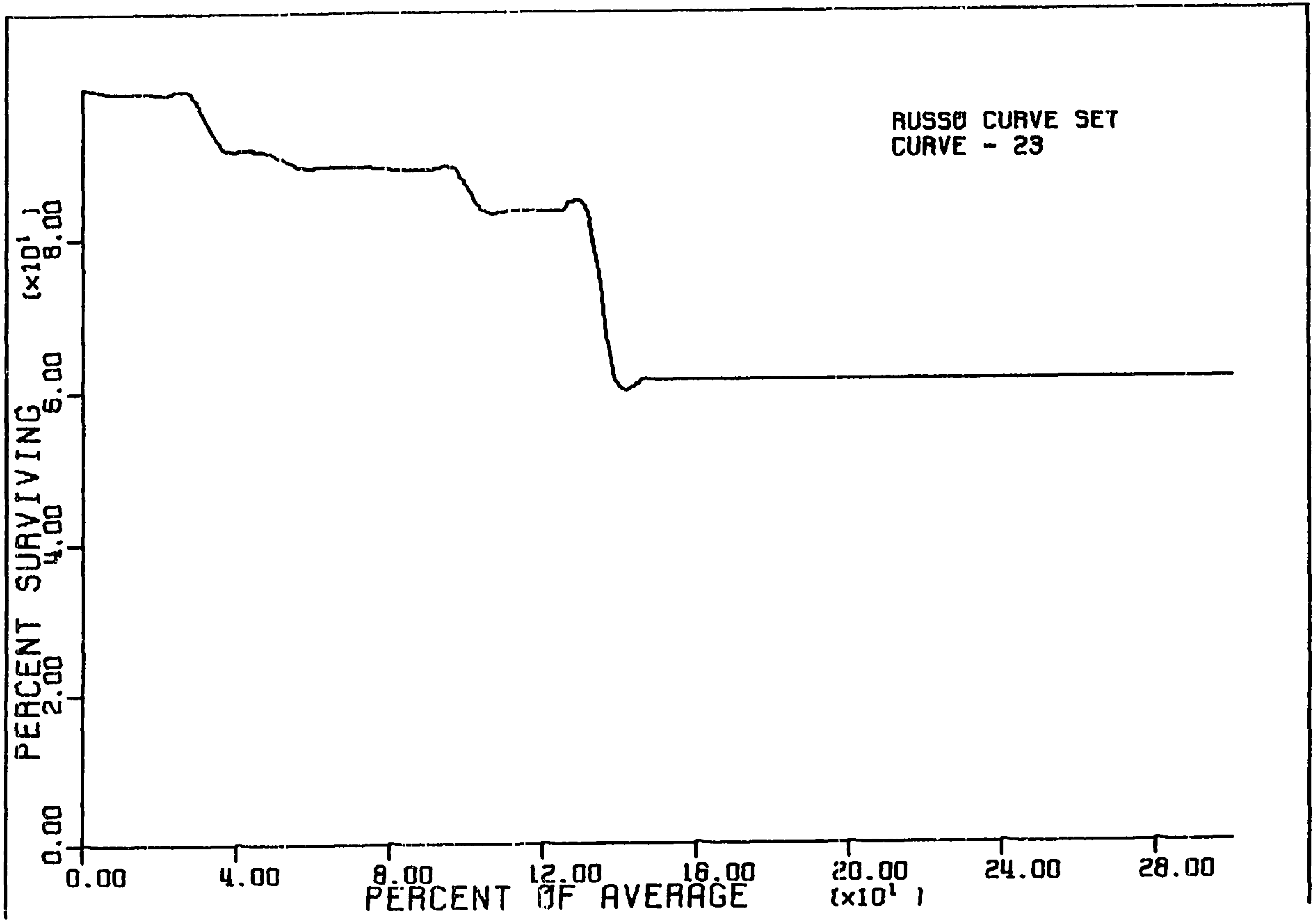

RUSSO CURVE SET
CURVE - 23
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE
(x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 41. Averaged Russo curve number 24--single curve cluster, no boundaries

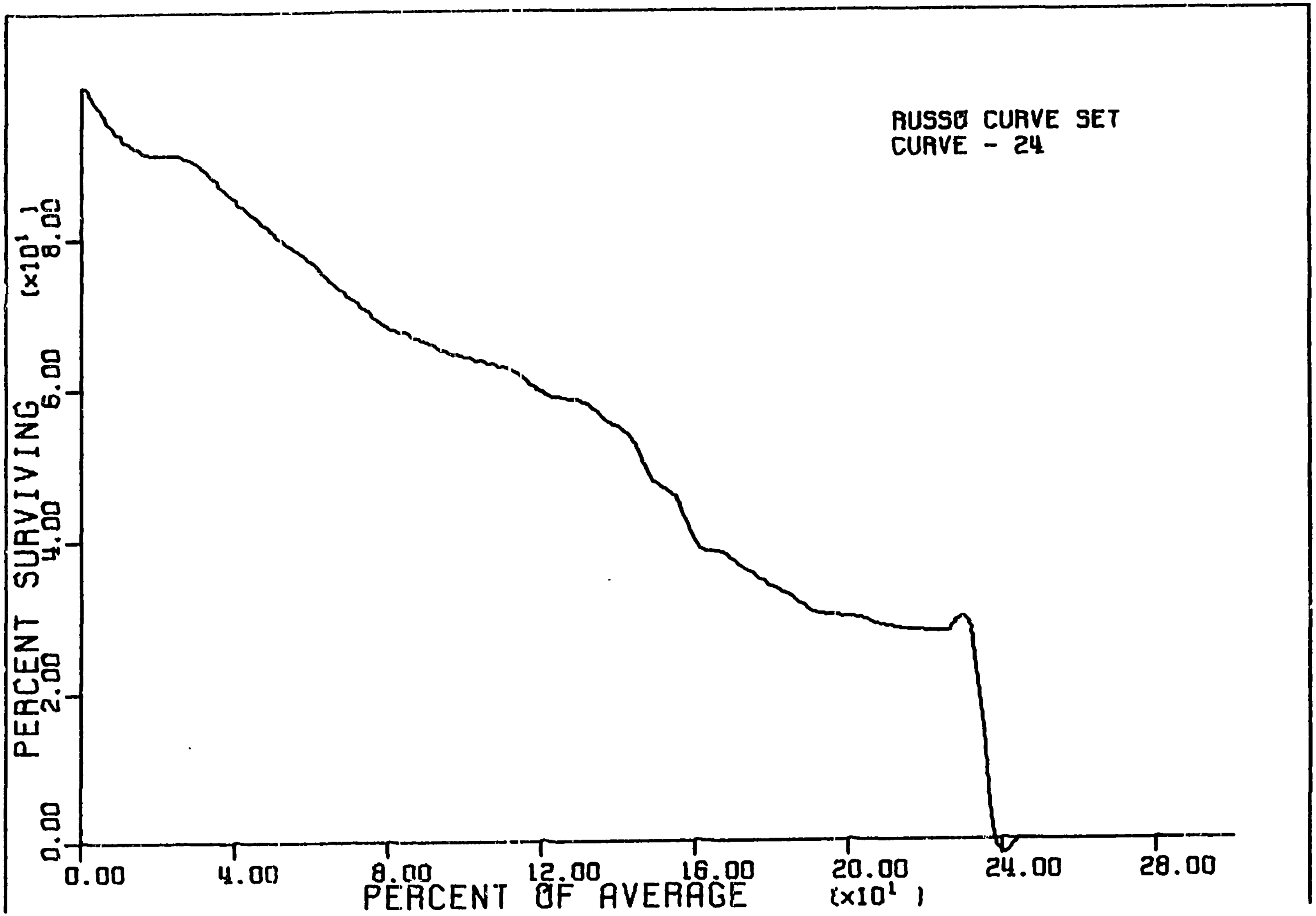

RUSSO CURVE SET
CURVE - 24
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE (x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 42. Averaged Russo curve number 25--single curve cluster, no boundaries

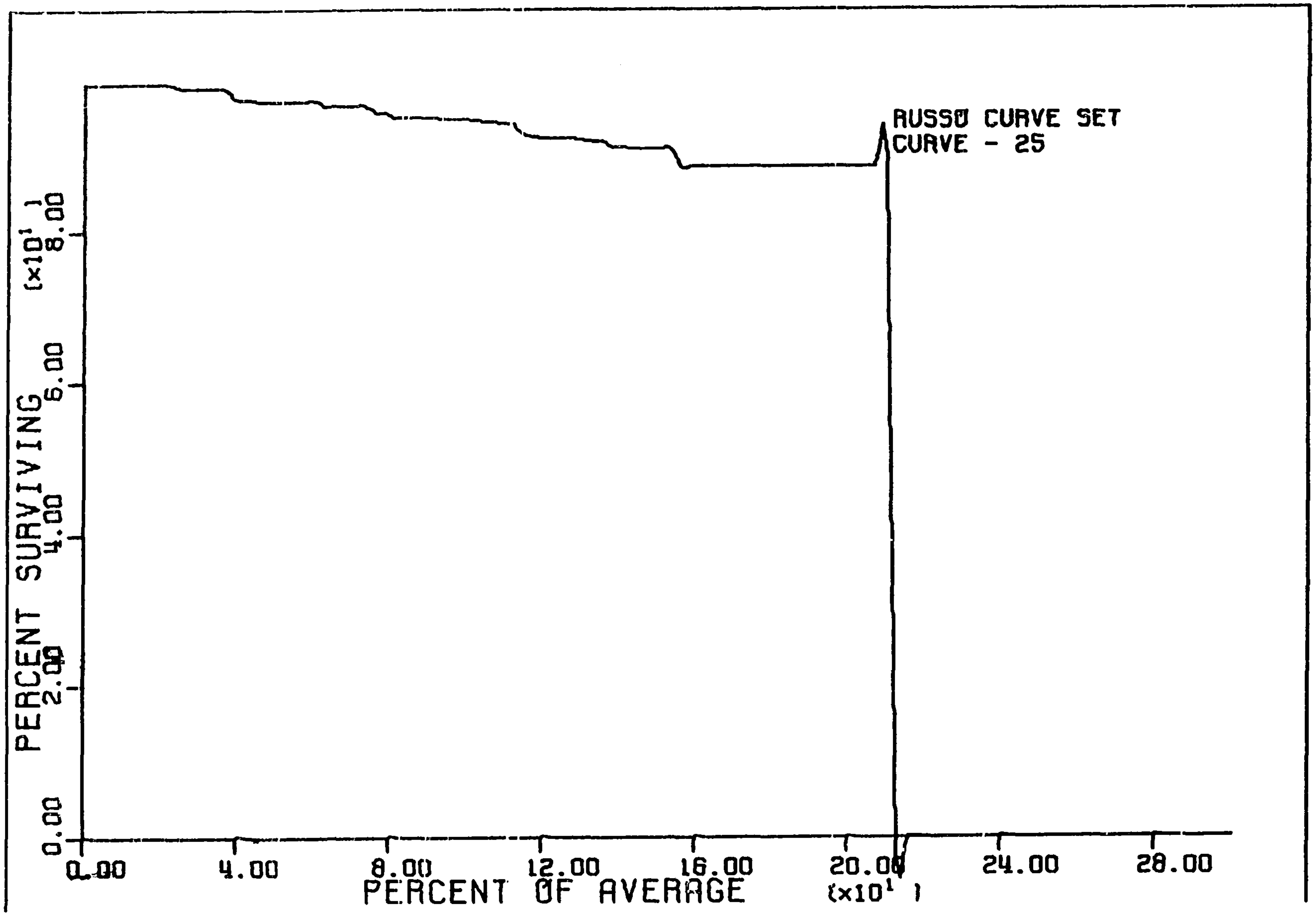
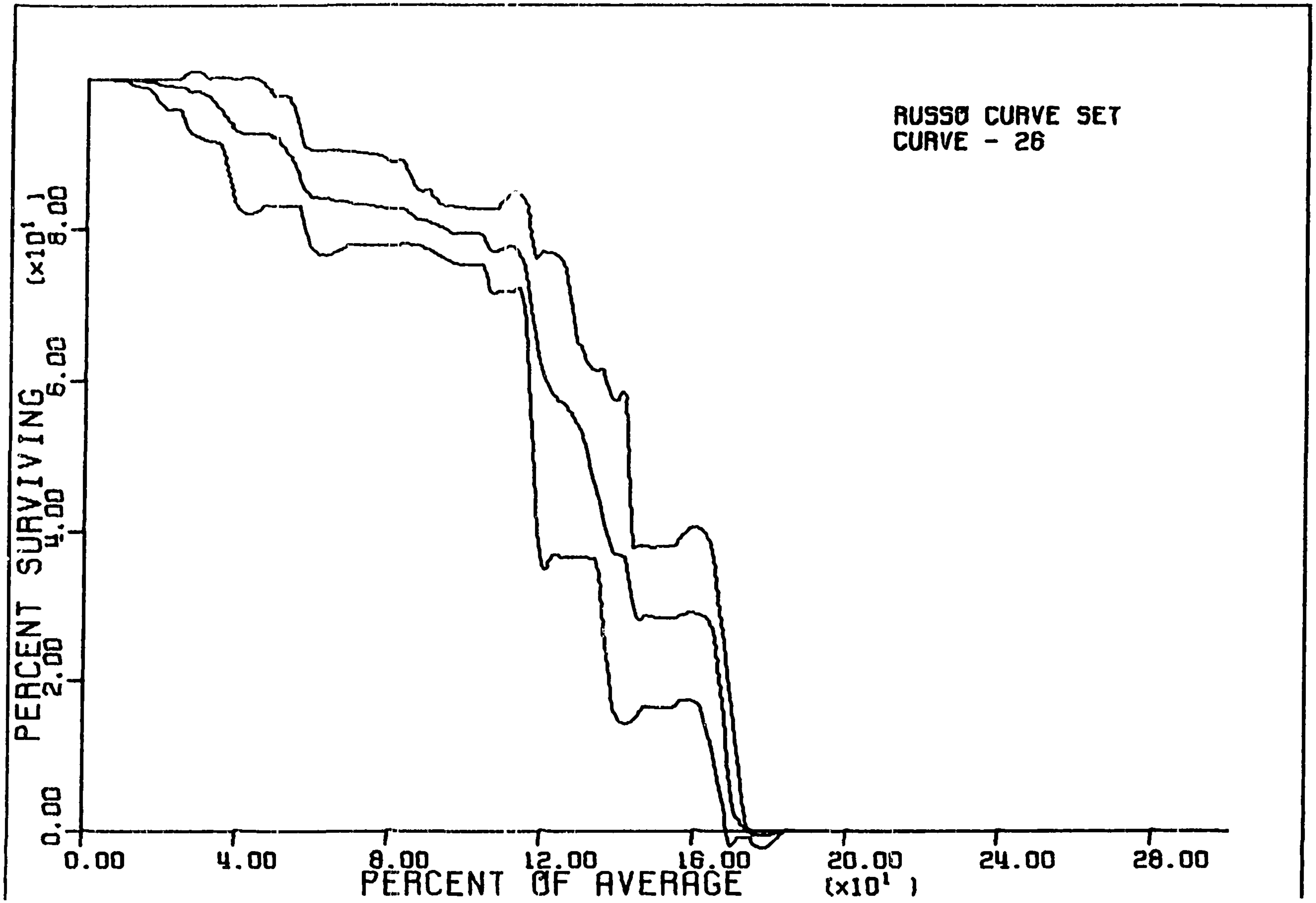

RUSSO CURVE SET
CURVE - 25
PERCENT SURVIVING
(x10^1)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE
(x10^1)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 43. Averaged Russo curve number 26 with boundaries



RUSSO CURVE SET
CURVE - 26
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE
(x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 44. Averaged Russo curve number 27--single curve cluster, no boundaries

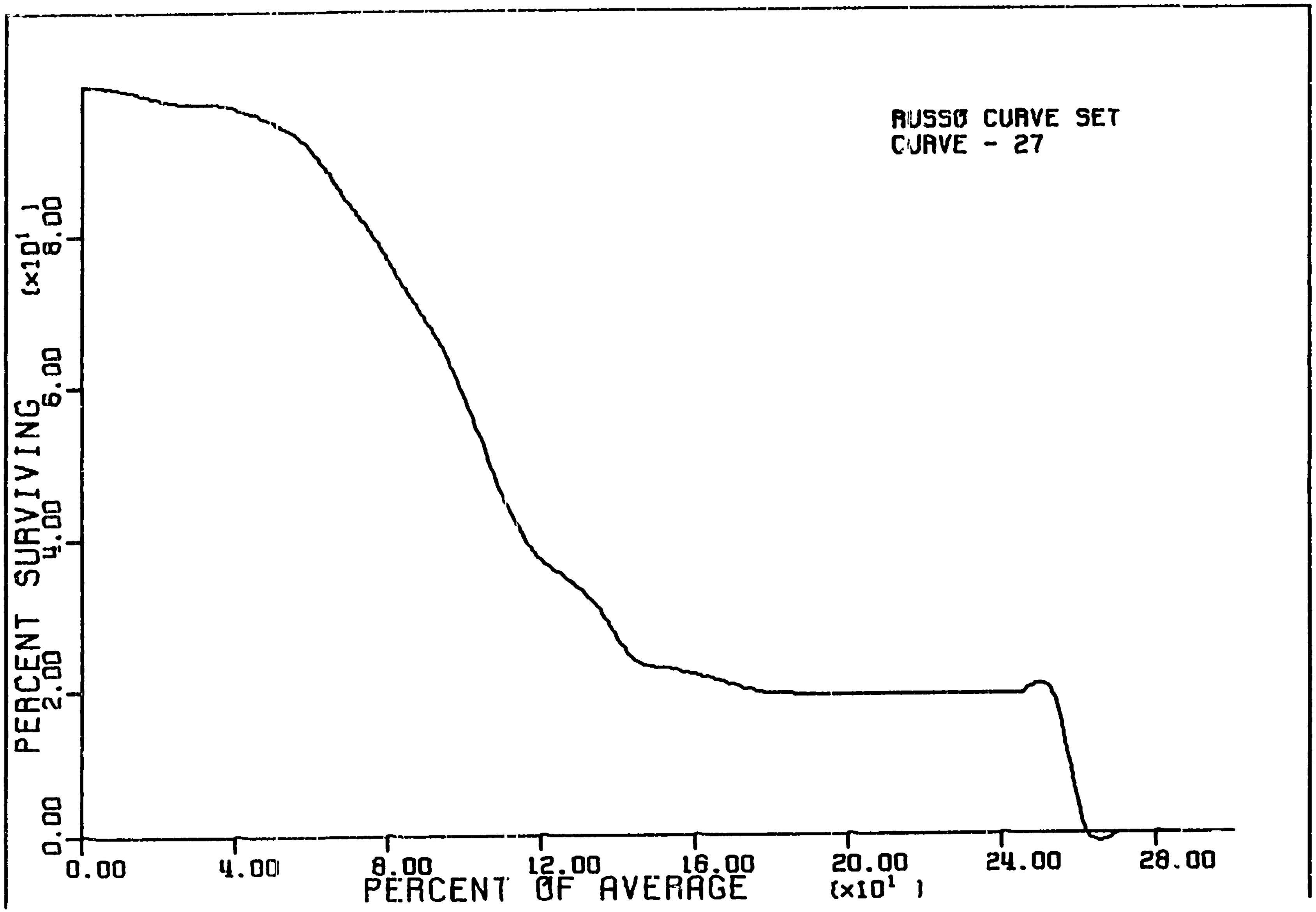
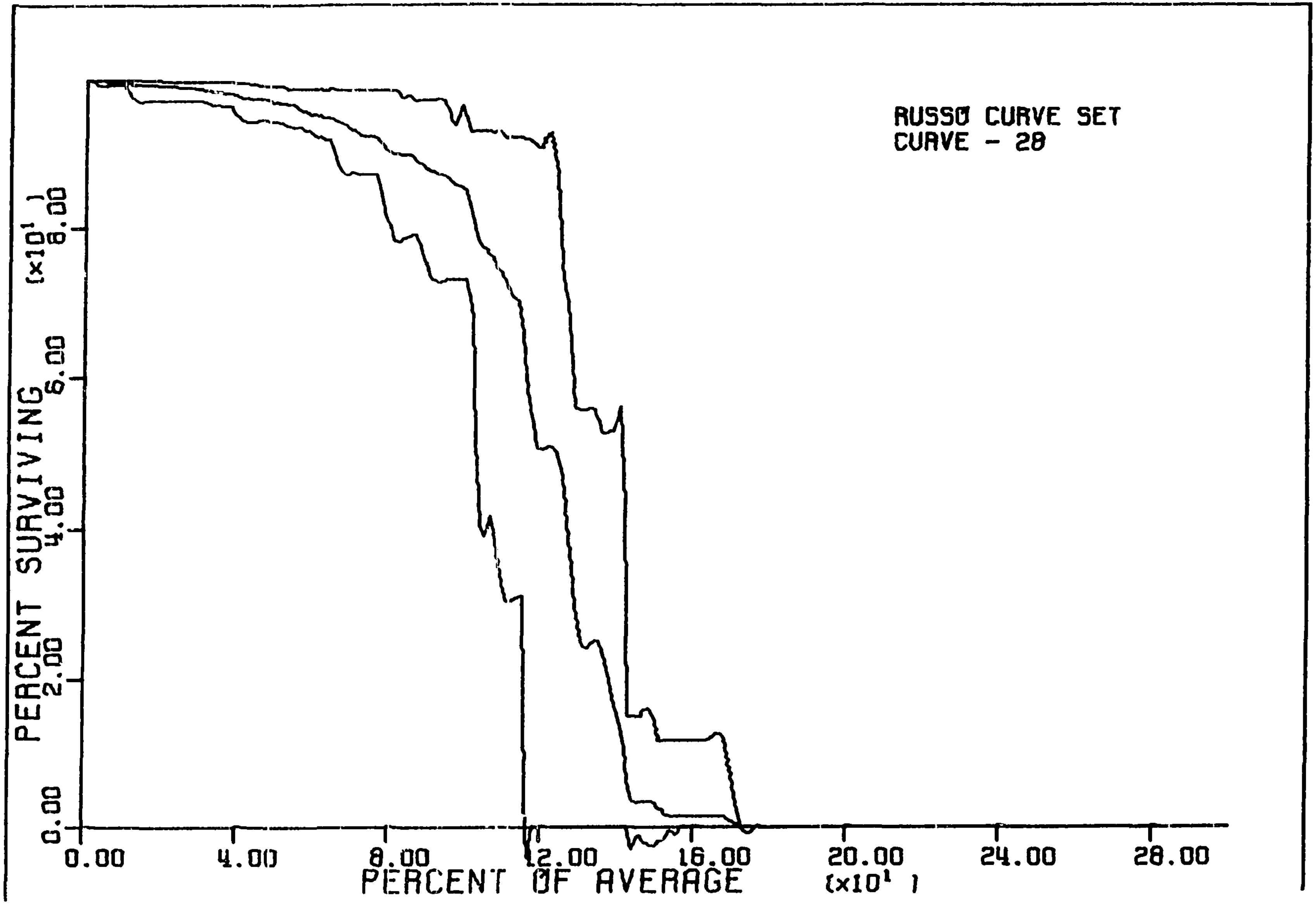

RUSSØ CURVE SET
CURVE - 27
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT ØF AVERAGE
(x10¹)

Figure 45. Averaged Russo curve number 28 with boundaries



RUSSO CURVE SET
CURVE - 28
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE (x10¹)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 46. Averaged Russo curve number 29 with boundaries

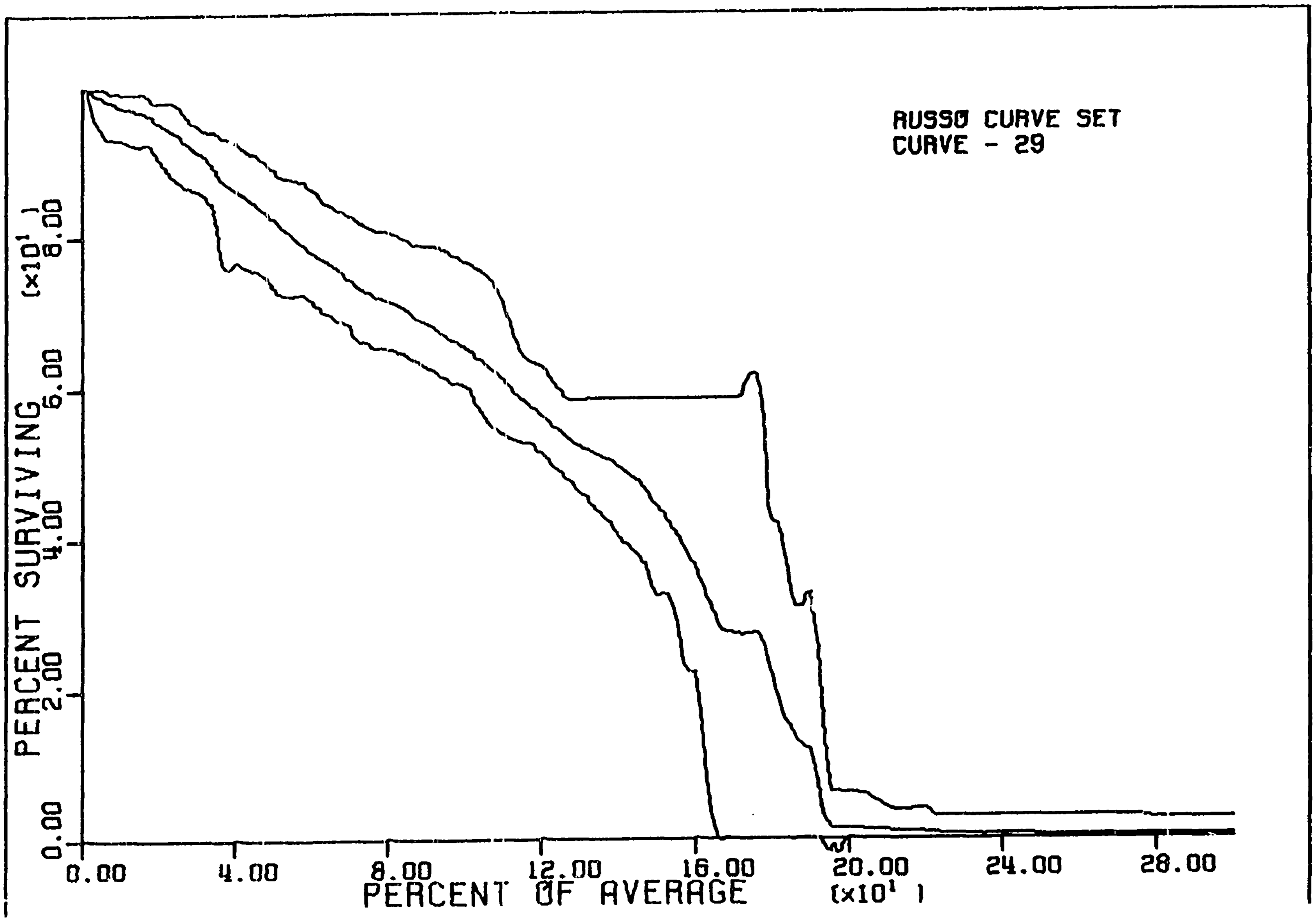

RUSSO CURVE SET
CURVE - 29
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 47. Averaged Russo curve number 30 with boundaries

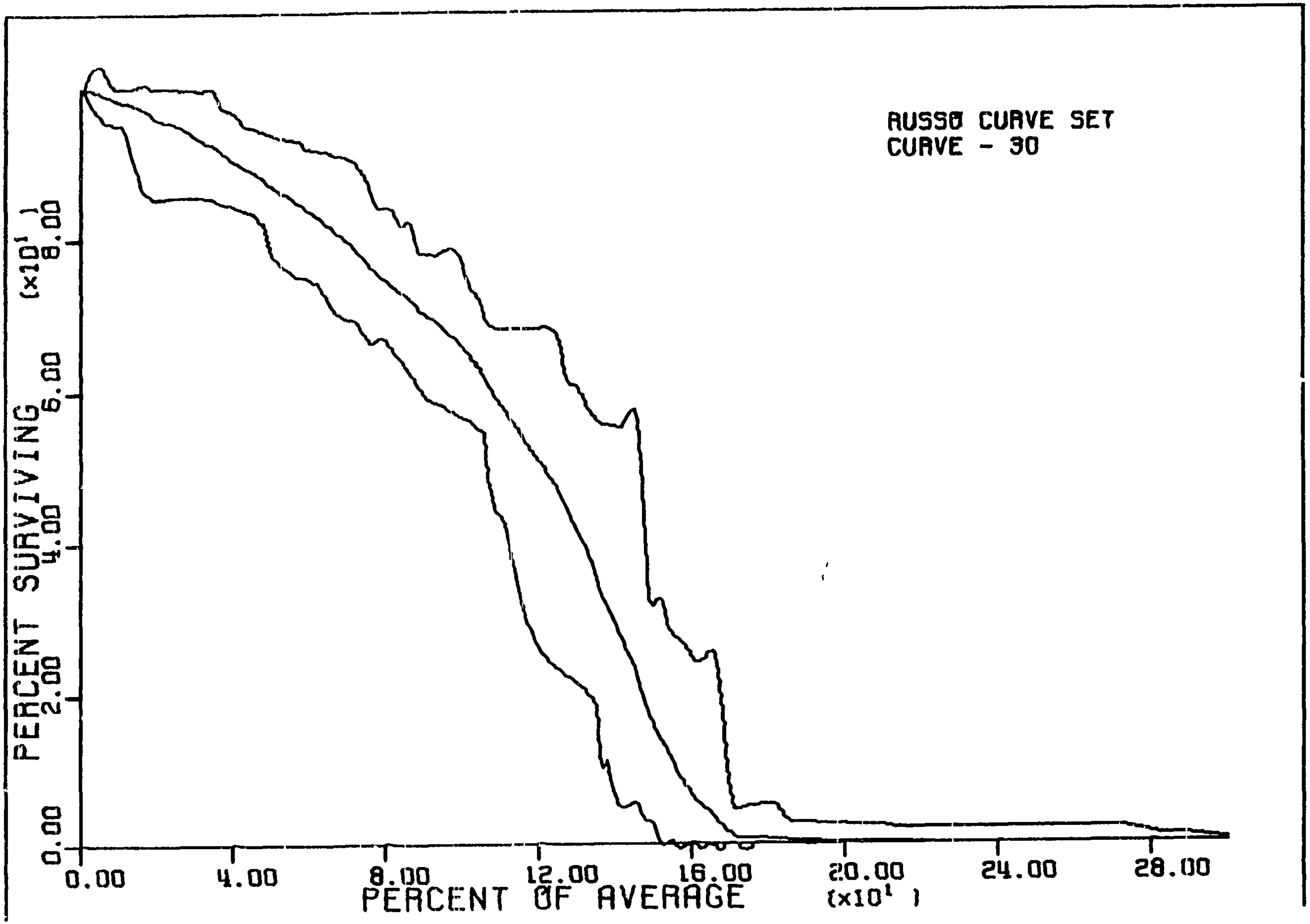

RUSSO CURVE SET
CURVE - 30
PERCENT SURVIVING (x10^1)
0.00
2.00
4.00
6.00
8.00
PERCENT OF AVERAGE (x10^1)
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00

Figure 48. Averaged Russo curve number 31 with boundaries

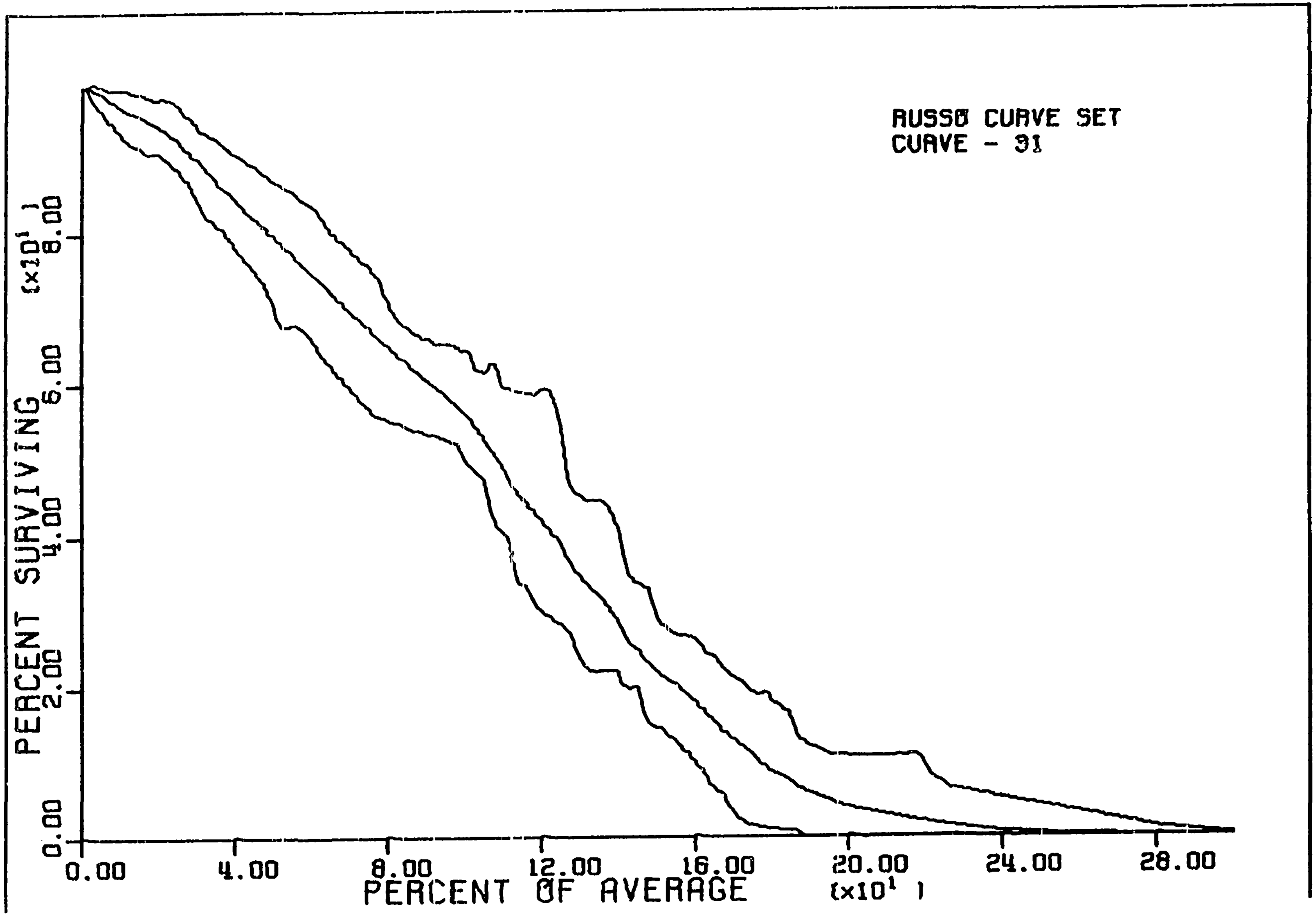

RUSSO CURVE SET
CURVE - 31
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 49. Averaged Russo curve number 32 with boundaries

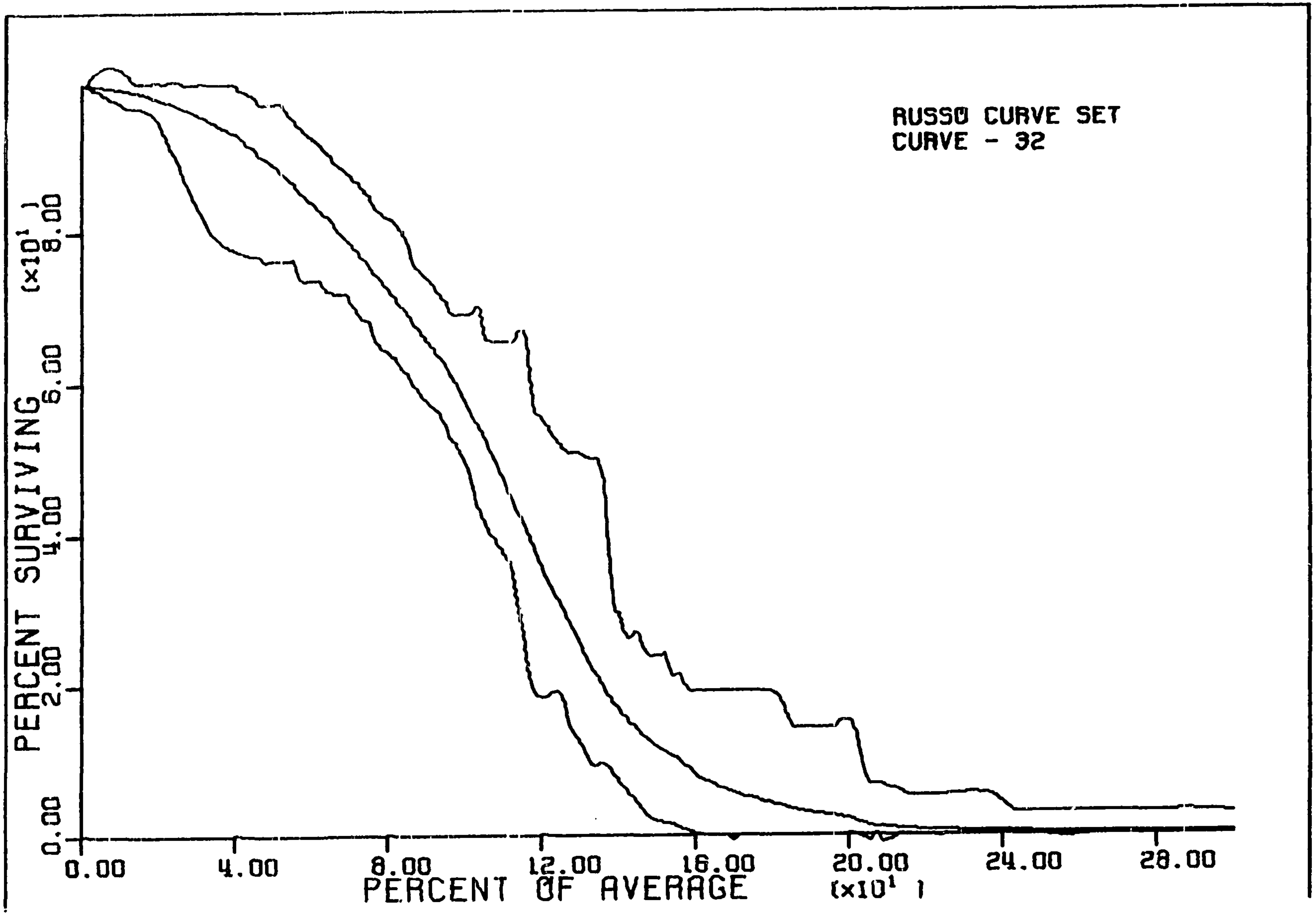
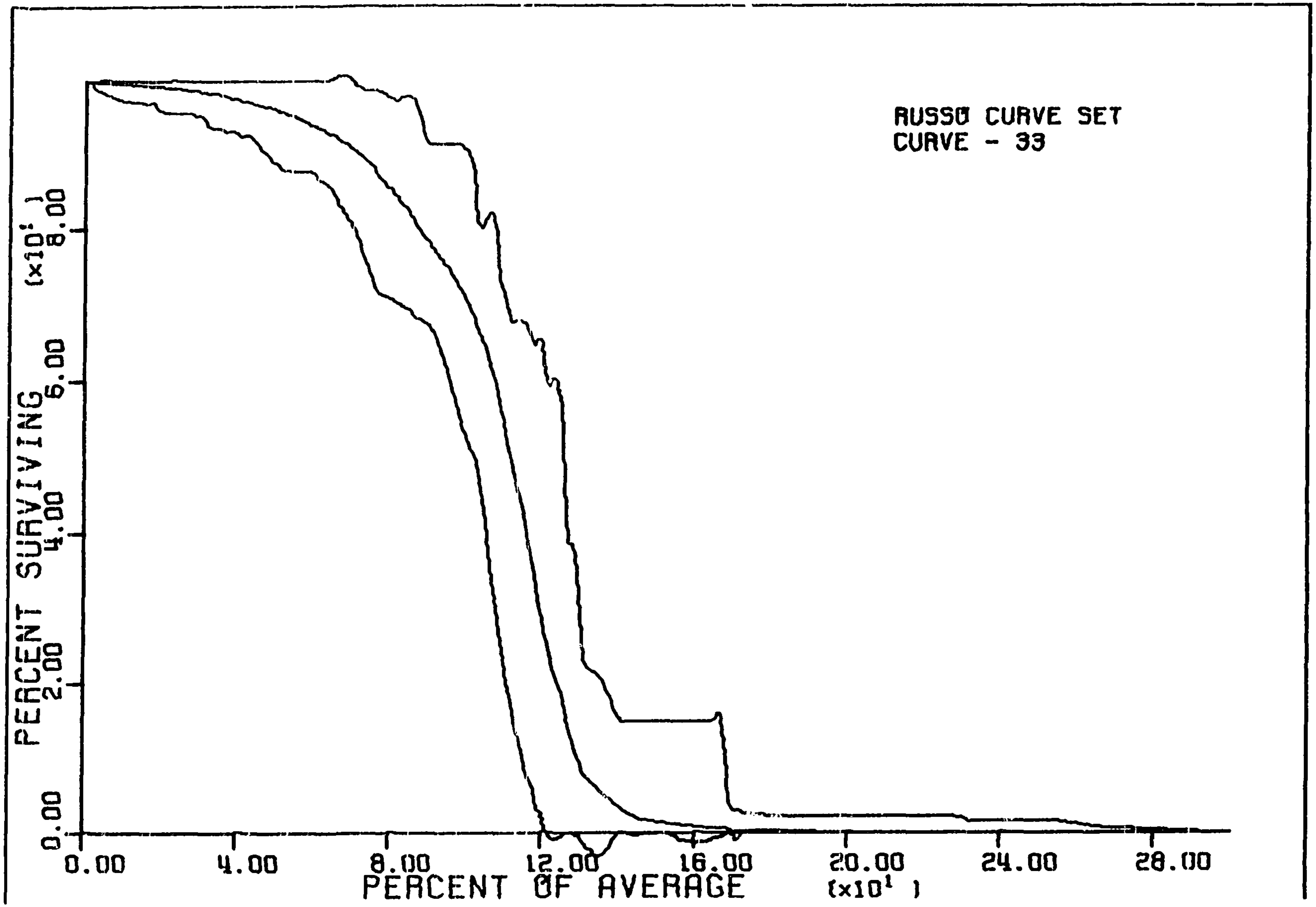

RUSSO CURVE SET
CURVE - 32
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

Figure 50. Averaged Russo curve number 33 with boundaries


RUSSO CURVE SET
CURVE - 33
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
PERCENT OF AVERAGE
(x10¹)

APPENDIX G:

TEST CURVES FIT TO RUSSO AND IOWA CURVES - SAMPLES

Figure 51. Sample of test curve fit to Russo and Iowa curves--test curve 29

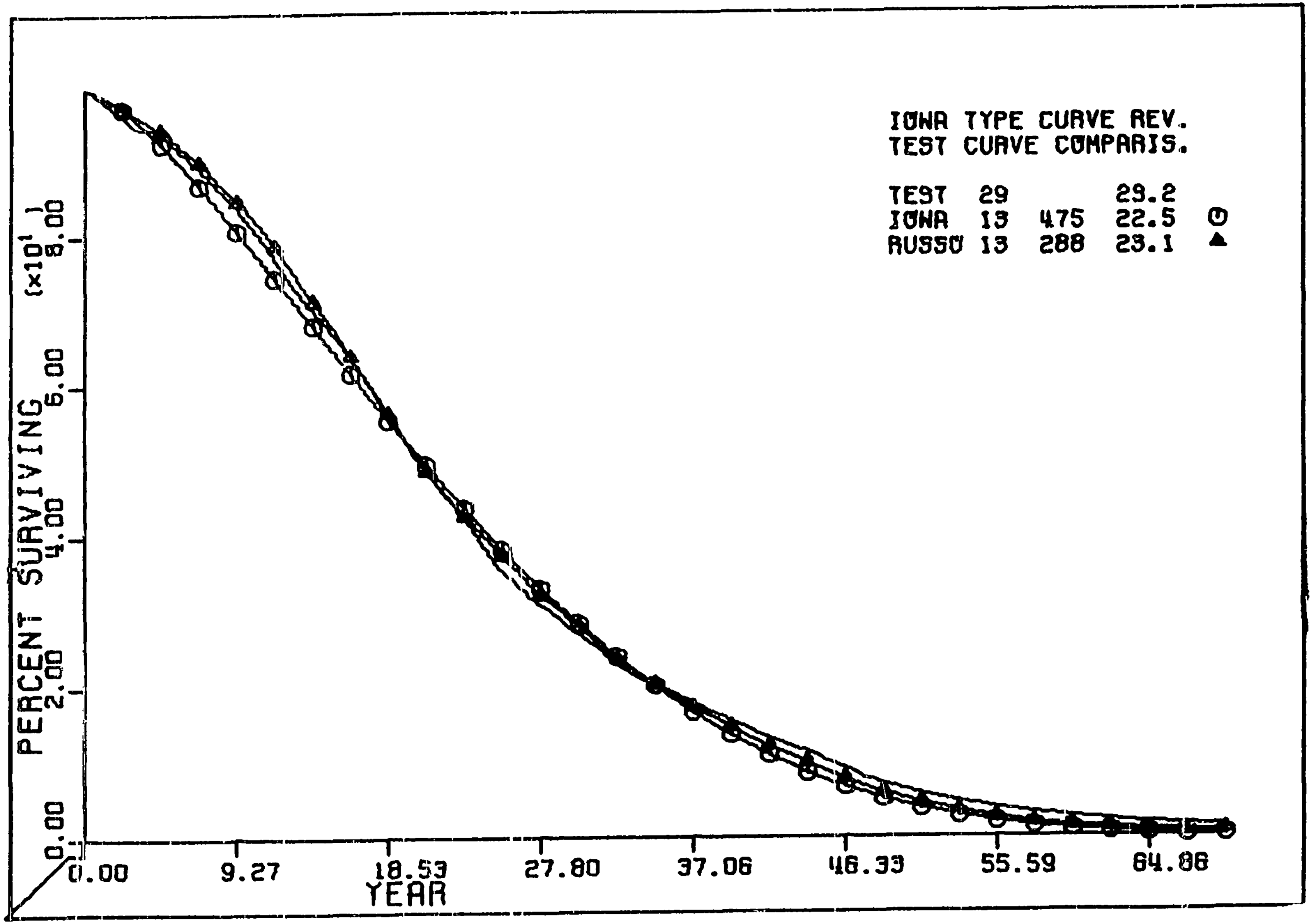

IOWA TYPE CURVE REV.
TEST CURVE COMPARIS.
TEST 29 29.2
IOWA 13 475 22.5
RUSSO 13 288 23.1
PERCENT SURVIVING (x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
9.27
18.53
27.80
37.06
46.33
55.59
64.86
YEAR

Figure 52. Sample of test curve fit to Russo and Iowa curves--test curve 39

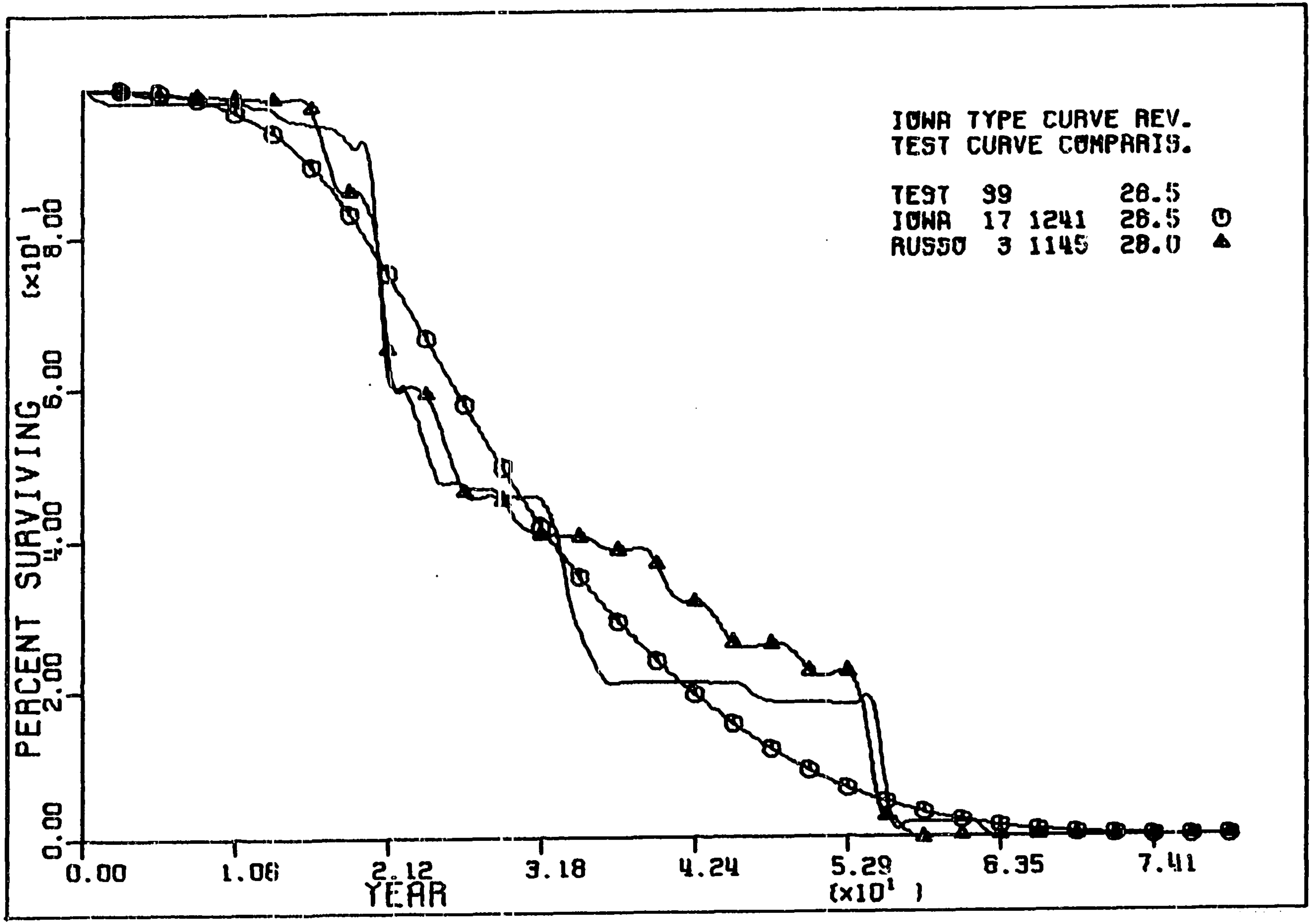
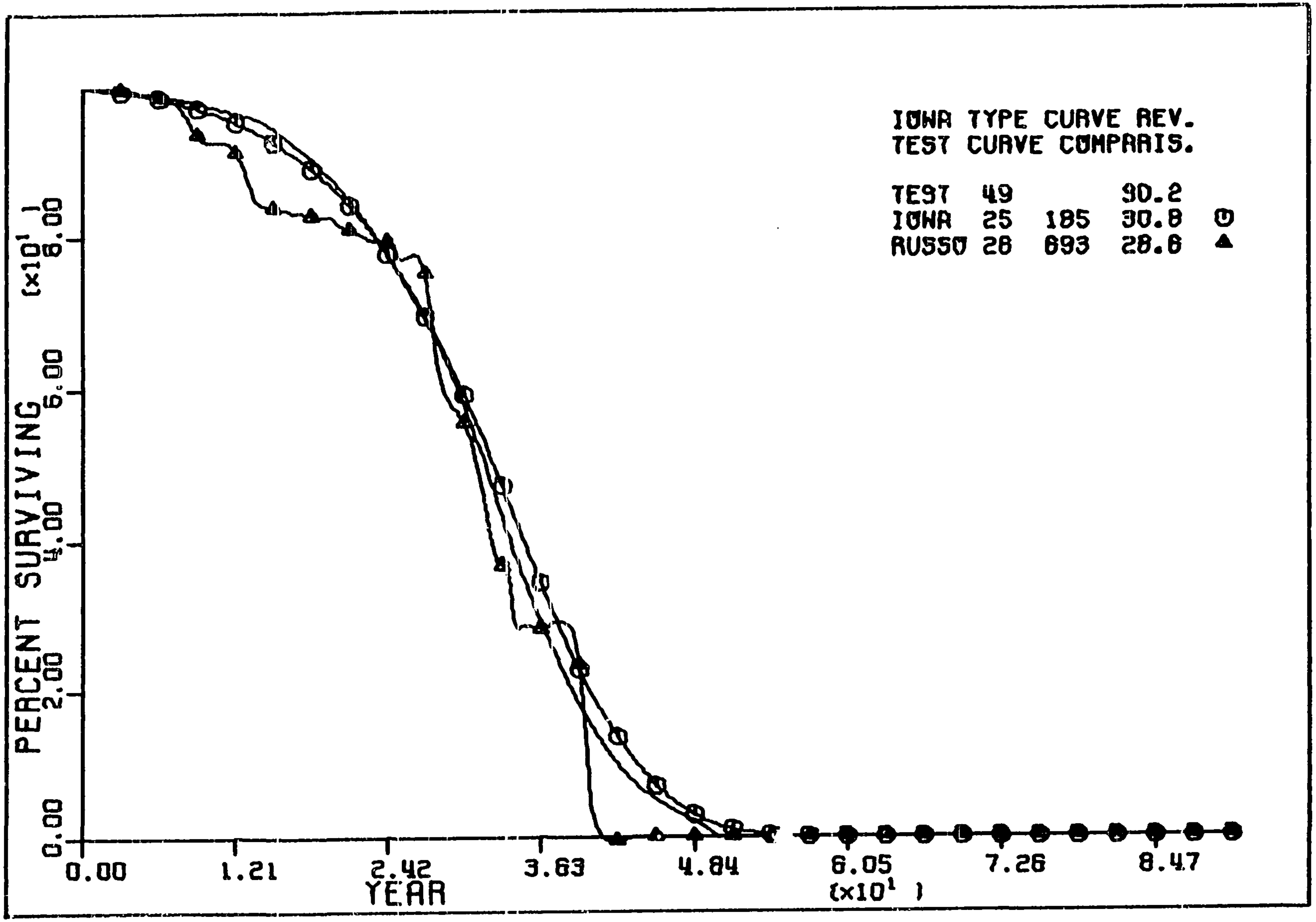

IOWA TYPE CURVE REV.
TEST CURVE COMPARIS.
TEST 99 28.5
IOWA 17 1241 28.5
RUSSO 3 1145 28.0
PERCENT SURVIVING
(x10[1])
0.00
2.00
4.00
6.00
8.00
YEAR
(x10[1])
0.00
1.06
2.12
3.18
4.24
5.29
6.35
7.41

Figure 53, Sample of test curve fit to Russo and Iowa curves--test curve 49


IOWA TYPE CURVE REV.
TEST CURVE COMPARIS.
TEST 49 90.2
IOWA 25 185 30.8
RUSSO 28 893 28.6
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
1.21
2.42
3.63
4.84
6.05
7.26
8.47
(x10¹)
YEAR

APPENDIX H:

RUSSO CURVES FIT TO IOWA CURVES - SAMPLES

Figure 54. Sample of Russo curve fit to Iowa curves--Russo curve 13

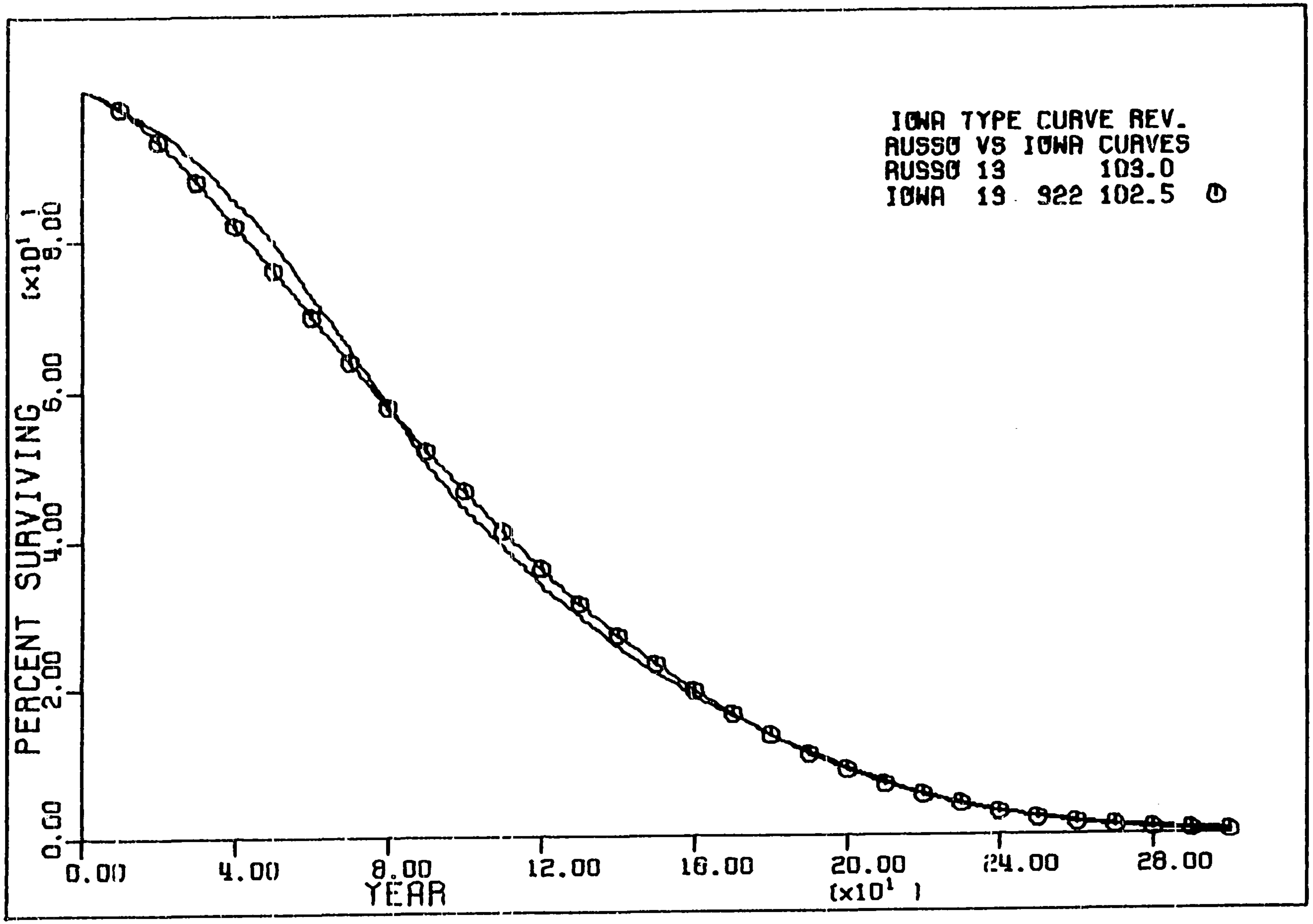

IOWA TYPE CURVE REV.
RUSSO VS IOWA CURVES
RUSSO 13 103.0
IOWA 19 922 102.5
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
YEAR
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
(x10¹)

Figure 55. Sample of Russo curve fit to Iowa curves--Russo curve 21

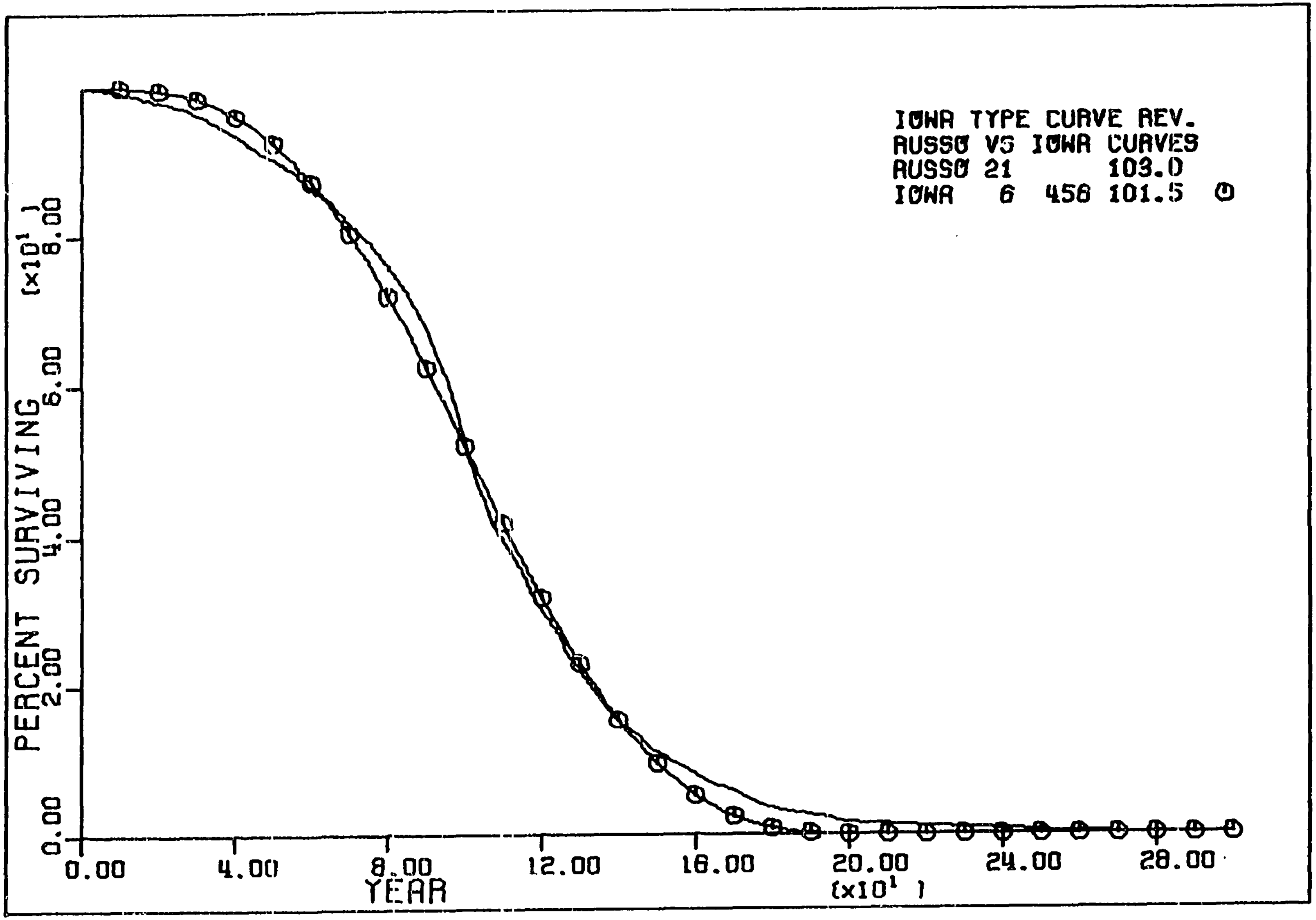
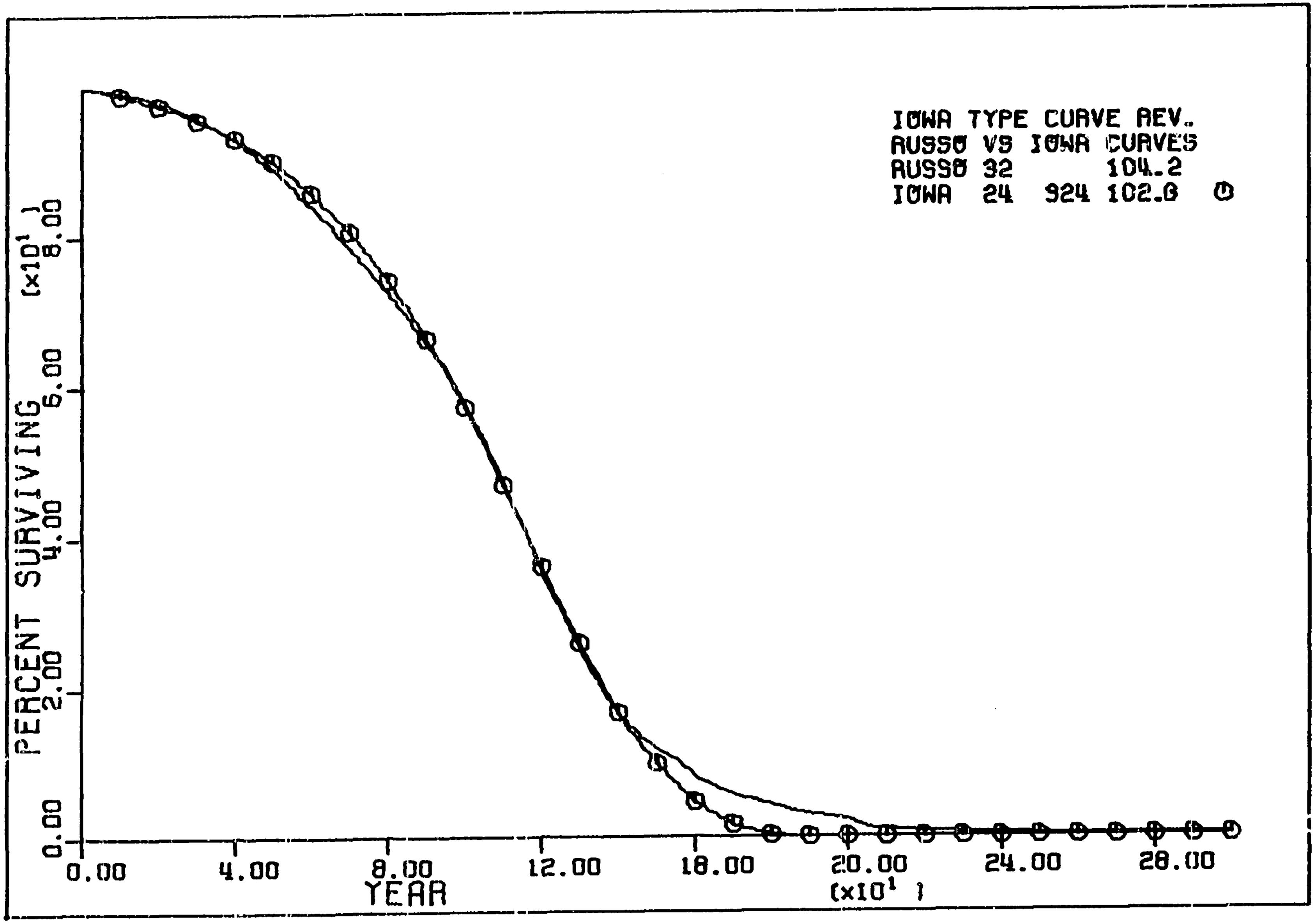

IOWA TYPE CURVE REV.
RUSSO VS IOWA CURVES
RUSSO 21 103.0
IOWA 6 456 101.5
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
YEAR
(x10¹)

Figure 56. Sample of Russo curve fit to Iowa curves--Russo curve 32


IOWA TYPE CURVE REV.
RUSSO VS IOWA CURVES
RUSSO 32 104.2
IOWA 24 924 102.0
PERCENT SURVIVING
(x10¹)
0.00
2.00
4.00
6.00
8.00
YEAR
0.00
4.00
8.00
12.00
18.00
20.00
24.00
28.00
(x10¹)

APPENDIX I:

STATISTICAL CALCULATIONS FOR TEST CURVES VS. RUSSO AND IOWA CURVES DATA

1. Sign test--areas

   Let p = probability that $AD_{I-R} \geq 0$

   H_0: $p \leq .5$

   H_1: $p > .5$

   $$z = \frac{x - n(.5)}{\sqrt{n(.5)(.5)}} \doteq N(0,1)$$

   n = 56

   $$z = \frac{28 - 56(.5)}{\sqrt{56(.5)(.5)}} = 0$$

   x = 28

   $\therefore$ Fail to reject H_0

2. Sign test--average service lives

   Let p = probability that $ED_{I-R} \geq 0$

   H_0: $p \leq .5$

   H_1: $p > .5$

   $$z = \frac{x - n(.5)}{\sqrt{n(.5)(.5)}} \doteq N(0,1)$$

   n = 56

   $$z = \frac{22 - 56(.5)}{\sqrt{56(.5)(.5)}} = -1.60$$

   x = 22

   $\therefore$ Fail to reject H_0

3. Wilcoxon test for paired observations--areas

Let CT_{I-R} be the true central tendency for the paired differences for the population between Iowa and Russo curves

H_0: $CT_{I-R} \leq 0$

H_1: $CT_{I-R} > 0$

$$z = \frac{w - CT_w}{\sqrt{\sigma_w^2}} \overset{\dot{\sim}}{n > 30} N(0,1)$$

$$z = \frac{657.5 - 1596}{\sqrt{15,028}} = -7.65$$

$\therefore$ Fail to reject H_0

$n = 56$

$w = 657.5$

$\sigma^2 = 15,028$

$CT_w = 1596$

4. Wilcoxon test for paired observations--average service lives

Let CT_{I-R} be the true central tendency for the paired differences for the population between Iowa and Russo curves

H_0: $CT_{I-R} \leq 0$

H_1: $CT_{I-R} > 0$

$$z = \frac{w - CT_w}{\sqrt{\sigma_w^2}} \overset{\dot{\sim}}{n > 30} N(0,1)$$

$$z = \frac{524.5 - 1378}{\sqrt{12,056}} = -7.77$$

$\therefore$ Fail to reject H_0

$n = 52$; four differences were zero

$\sigma_w^2 = 12,056$

$CT_w = 1378$

$w = 524.5$

5. Parametric test for paired observations (paired t test)--areas

Let μ_I and μ_R be the true means for the population when the Iowa and Russo curves, respectively, are used

H_0: $\mu_I - \mu_R = \mu_{I-R} \leq 0$

H_1: $\mu_I - \mu_R = \mu_{I-R} > 0$

$$T = \frac{\overline{AD}_{I-R}}{\sqrt{S_d^2/n}}$$

$$\overline{AD}_{I-R} = -40.18$$

$$S_d^2 = 58{,}564$$

$$T = \frac{-40.18}{\sqrt{58{,}564/56}} = -1.24 \qquad n = 56$$

$\therefore$ Fail to reject H_0

6. Parametric test for paired observations (paired t test)--average service lines

Let μ_I and μ_R be the true means for the population when the Iowa and Russo curves, respectively, are used

H_0: $\mu_I - \mu_R = \mu_{I-R} \leq 0$

H_1: $\mu_I - \mu_R = \mu_{I-R} > 0$

$$T = \frac{\overline{ED}_{I-R}}{\sqrt{S_d^2/n}}$$

$$\overline{ED}_{I-R} = -.152$$

$$S_d^2 = .382$$

$$T = \frac{-.152}{\sqrt{.382/56}} = -1.84 \qquad n = 56$$

$\therefore$ Fail to reject H_0