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Undergraduate Faculty program

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(Communicated by Darren A. Narayan)

The Research Experiences for Undergraduate Faculty (REUF) program of the American Institute of Mathematics prepares faculty to engage in research with undergraduate students, encourages long-term research collaborations among some of its faculty, and builds a network of faculty who supervise undergraduate research. Participants of each REUF workshop are faculty members from undergraduate colleges interested in mentoring students in research mathematics at their home institutions. During a workshop, senior mathematicians with experience supervising undergraduate research present open problems suitable for undergraduates. The REUF program also includes several follow-up activities.

1. Overview

Participating in research in mathematics as an undergraduate can be a pivotal experience that contributes to a student's decision to pursue a career in the mathematical sciences. More than 60% of undergraduate mathematics degrees are awarded by colleges and universities that do not have doctoral programs. Faculty at primarily undergraduate institutions typically have more teaching responsibilities than their counterparts at research universities. Such faculty often have limited time to invest in their own research, and engaging in research with students can be challenging. The Research Experiences for Undergraduate Faculty (REUF) program addresses these issues with a series of activities designed to

- prepare faculty to engage in research with undergraduate students at their home institutions,
- involve some faculty in long-term research collaborations,

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- provide faculty the opportunity to have a research experience investigating open questions in the mathematical sciences, and thereby rekindle or further fuel a love of doing original mathematics, and
- establish a network of faculty at primarily undergraduate institutions together with faculty at research universities who support collaboration and undergraduate research.

The REUF program was developed by the American Institute of Mathematics (AIM) under the direction of Leslie Hogben, Roselyn Williams, and Ulrica Wilson. The REUF program continues to grow and develop, and in 2011 began a partnership with the Institute for Experimental and Computational Research in Mathematics (ICERM).

The core activity of REUF is a series of annual workshops. Each workshop involves a new group of faculty members who investigate problems suitable for subsequent research with undergraduates; each participant receives full funding for travel and local expenses. Four senior mathematicians (here called group leaders) who have experience supervising undergraduate research lead the research activities at each workshop. The organizers (Hogben, Williams, and Wilson) and Brianna Donaldson, AIM's Director of Special Projects, manage the program and facilitate the large group discussions. Several types of activities provide support beyond the workshop:

- The continuation of a research project begun at the workshop through a small research group meeting approximately a year after the workshop.
- Regular discussion gatherings of REUF participants (from any prior REUF workshop) who are attending national meetings such as Joint Mathematics Meetings or MAA MathFest.
- An online discussion group (the REUF Forum) and a REUF Resources webpage for participants.

More information about the REUF program can be found at reuf.aimath.org.

2. Recruitment

About half the participants for each annual REUF workshop are invited by the organizers and the other half are selected through an open application process. Recruitment for REUF participants targets faculty who want to direct undergraduate research projects, especially those in departments that serve a substantial percentage of undergraduates underrepresented in the mathematical sciences, or faculty who are themselves a member of an underrepresented group. No previous experience with undergraduate research is required. Participants come from colleges and universities throughout the United States. In selecting applicants, in addition to broadening

the mathematical workforce, preference may be given to groups of two or three applicants at the same college or in close geographic proximity. Such circumstances provide local support for continuing research activities after the workshop. Fliers advertising the workshop are distributed through email, at professional mathematics meetings, on the AIM web page, and through professional organization newsletters and websites.

3. The workshop

The five-day workshop takes place in the summer and includes approximately 22 participants and four group leaders. Table 1 gives a typical schedule of activities for the week.

Participants are mathematicians at primarily undergraduate institutions interested in conducting research with undergraduates. The group leaders are mathematicians with an established record of excellence in doing research with undergraduates. Prior to a workshop, each of the four group leaders chooses a topic/problem that will be investigated by participants during the course of the workshop. A sampling of previous research topics is given in Section 3.2. A small amount of background reading material for each of the four topics is provided to every participant before the workshop begins. This allows participants to become more familiar with the general

| | |
|-----------|---|
| Monday | <p>morning: Introductions followed by presentations of topics and open problems to be investigated during workshop.</p> <p>afternoon: Divide into groups and begin working.</p> |
| Tuesday | <p>morning: Introduction to <i>Sage</i> and work in research groups.</p> <p>afternoon: Work in research groups.</p> |
| Wednesday | <p>morning: Preliminary reports from each group; break out into research groups.</p> <p>afternoon: Short discussion on publishing undergraduate research, including an introduction to the research journal <i>Involve</i>; participants work in research groups.</p> |
| Thursday | <p>morning: Full group discussion: topics related to undergraduate research.</p> <p>afternoon: Participants work in research groups.</p> |
| Friday | <p>morning: Participants work in research groups and plan activities to continue project.</p> <p>afternoon: Groups present final reports.</p> |

Table 1. Typical schedule of a REUF workshop.



Figure 1. REUF research group at AIM in 2008.

area of all four topics prior to the workshop without committing to a specific topic. Typically, participants will work in a mathematical area outside their primary field of expertise. This creates a dynamic similar to that of most undergraduate research experiences and allows the group leader to model best practices in supervising research projects with students.

On day 1, each group leader presents a 30-minute overview of his/her topic including a list of open problems, and subsequently the participants break up into groups — a working research group for each topic/group leader. Most of the week is designated for groups to make progress on their research problem (see [Figure 1](#)). On day 2 or 3, groups give very brief oral reports on the progress/plan for the remainder of the week; and on the last day a more detailed final report is presented including plans for continuing the work started. Along with the progress reports, there are also a few other full group sessions scheduled throughout the week, including an introduction to *Sage* — a free open source mathematics software system. Often undergraduate research includes experimentation and construction of examples using computer software. Since it is free and available for download, the *Sage* computer mathematics system is extremely accessible to students, so it was selected for use in the workshop. Participants receive a quick introduction and are encouraged (but not required) to practice using the software throughout the week as it relates to their problem. There is also a full group session scheduled during the workshop to discuss topics related to supervising undergraduate research.



Figure 2. REUF full group discussion at ICERM in 2012.

Some of the topics that have been covered in the group discussion on undergraduate research are listed in [Section 3.1](#). Research groups are also encouraged to plan activities to continue the research started during the workshop; more information on continuation activities is given in [Section 4](#).

3.1. Group discussion. A goal of the REUF program is for participants to increase their capacity to supervise undergraduate research as part of an academic year experience and/or a summer program. To this end, we schedule a group discussion (see [Figure 2](#)) on topics related to undergraduate research. The discussion may include attributes of a good undergraduate research problem and/or practical issues related to a productive undergraduate research experience, including how to structure these experiences during the academic year versus the summer. There is an introduction to some of the resources available to support faculty involvement in undergraduate research, and this information is also shared through the REUF Forum and on the REUF Resources website, which are discussed in [Section 4.3](#).

The group discussion varies with participant interest, and begins by surveying the participants for the issues/questions that concern them the most. Each question to be discussed is written on the board, and then the moderator goes through each topic soliciting comments from all participants. Discussion topics from four REUF workshops have included:

- Characteristics of a good undergraduate research problem.
- Sources for good research problems for undergraduates.
- Mentoring undergraduate research.
- Fostering student collaborations.
- Selecting/recruiting students.
- Recruiting underrepresented students.

- Establishing expectations.
- Getting students to use \LaTeX .
- Getting students to write-up results.
- Publishing student results.
- Student incentives: graduation requirement, course credit, compensation, etc.
- Faculty incentives: concurrent teaching credit, accrued teaching credit, compensation, etc.
- Balancing supervising undergraduate research with your own research agenda.
- Building a team of faculty on campus to supervise undergraduate research projects.
- Funding for summer research programs.

3.2. *Research topics.* The research problems for the workshop are chosen by the group leaders and are expected to be suitable for research with undergraduates. Some of the problems at each workshop are also suitable for faculty research. Problems have been chosen in algebra, linear algebra, graph theory, operator theory, and number theory. Examples of previous topics include:

- The linear algebra of the *Lights Out!* game.
- The distribution of eigenvalues of the $n \times n$ unitary matrices given certain restrictions on the eigenvalues.
- The relationship between large gaps between zeros of the Riemann zeta function and large values of the zeta function on the critical line.
- Sphere-of-influence graphs.
- Minimum rank of a graph.
- Cyclotomy using representation theory.
- Groups of perfect shuffles.
- Exponential graph domination.
- Structure of symmetric k -varieties.
- Symmetry of numerical range.
- Prime and coprime labeling of graphs.
- Dessins d'enfant graphs.

In response to participant feedback, REUF 2013 will include one or more group leaders who will focus on more applied topics.

4. Continuation activities

The workshop only starts the process of involvement with undergraduate research or faculty collaborations in new research areas. Continuing the work initiated is necessary to realize the full effectiveness of the workshop. As such, time is designated during the workshop for participants to develop a plan to continue workshop activities. Such continuation can be facilitated by electronic dialog, but in-person meetings are usually necessary.

For example, participants at the same institutions or in close proximity to each other will be encouraged to make plans together for undergraduate research. Those who wish to continue their research collaboration will have the opportunity to begin preparing an application for a small research group meeting at AIM (see [Section 4.1](#)) and/or identify professional meetings that several group members are attending and make plans to work there. AIM directors and workshop organizers are available to consult with participants throughout the workshop to provide information about AIM programs and other funding sources.

4.1. *Small research groups.* Each year participants in the REUF workshop who wish to continue work on their research project at a level needed for publication are invited to apply for REUF funding to go to AIM for an additional week of research that takes place a year or more after the REUF workshop. This an attractive opportunity for participants eager to continue their work started during the workshop. It is modeled on the AIM SQuaREs program, in which a small group comes to AIM for two or more weeks a year apart to collaborate on a long-term research project. During the interval between the REUF workshop and the small research group meeting, group members are expected to continue work and communicate electronically and/or through personal contact, for example, at professional meetings.

4.2. *REUF at meetings.* We have experimented with various strategies to bring REUF alumni together to provide additional support to faculty participants who are actively mentoring undergraduate student research. We have tried to bring together participants from the most recent REUF workshop for a day-long meeting at a national conference such as Joint Mathematics Meetings (JMM) or MAA MathFest for group discussions on their experience supervising undergraduate research: What have participants done? What worked? What were the challenges? We have found this costly and the majority of participants are not able to attend.

We now host shorter gatherings of all REUF alumni at national meetings such as JMM and MathFest. By including participants from several REUF workshops, we have a larger group with a broad perspective, including faculty with varying amounts of experience mentoring student research, and are building a network supporting undergraduate research. The first of these gatherings was held at MathFest 2012

and produced a good discussion of successes and challenges of doing research with undergraduates.

At such national meetings many REUF alumni bring their undergraduates. One of the suggestions from MathFest 2012 that we are implementing at JMM 2013 is to gather information about undergraduate talks/posters by students of REUF alumni and disseminate it to all REUF alumni attending the meeting.

4.3. REUF online. To provide continuing support for REUF participants, AIM maintains the private REUF Resources website and a listserv (the REUF Forum) for all previous REUF participants to alert REUF alumni of relevant opportunities and facilitate exchanges of advice and information among faculty. To support the broader community of faculty interested in doing research with undergraduate students, AIM maintains the public REUF Undergraduate Research Resources website: reuf.aimath.org/resources/undergraduate-research. This page includes information contributed by REUF participants.

5. Outcomes

In this section we provide data showing that REUF is achieving its goals.

Research with undergraduate students. With the exception of the first REUF workshop, more than half of participants mentor undergraduate research, although it often takes some time for this to happen (see Tables 2 and 3). Continuation activities that have taken place since the second REUF workshop in 2009 appear to play an important role in supporting this.

Faculty research outcomes of REUF. As the first REUF workshop was envisioned, the expected outcomes were research with undergraduates and fostering faculty engagement with mathematics, but it was not expected that faculty research would be an outcome. However, a few successful long-term faculty collaborations were established (see Table 4). Some of the continuation activities, including the small

| REUF workshop | # participants | # participants mentoring undergraduate research | % participants mentoring undergraduate research |
|---------------|----------------|---|---|
| 2008 | 20 | 6 | 30 |
| 2009 | 20 | 12 | 60 |
| 2011 | 24 | 11 | 46 [†] |

Table 2. REUF participants mentoring undergraduate research, as of June 2012. [†]Two more participants in REUF 2011 reported plans to work with specific undergraduates, bringing the total for REUF 2011 to 54%.

| | | 2008 | 2009 | 2011 |
|--------------------------------|-------------------------------|------|----------------|------|
| Students mentored | | 23 | 53 | 23 |
| Senior theses | completed | 1 | 6 | 1 |
| | planned | 0 | 0 | 1 |
| Presentations (national) | completed | some | 10 | 1 |
| | planned | 0 | 0 | 4 |
| Presentations (regional/local) | completed | some | some | 3 |
| | planned | 0 | 0 | 4 |
| Publications | accepted/appeared | 1 | 9 | 1 |
| | under review | 0 | 2 | 0 |
| | in preparation | 0 | 5 | 1 |
| Other | grants | 2 | 1 [†] | 0 |
| | national student prize | 1 | 0 | 0 |
| | started math research seminar | 0 | 0 | 1 |

Table 3. Undergraduate student research outcomes of REUF, as of June 2012 (“some” indicates that exact data are not available and [†] indicates a NSF REU site grant).

| | | 2008 | 2009 | 2011 | 2012 |
|---|-------------------|------|------|-------------------|--------------------|
| Publications | accepted/appeared | 2 | 1 | 0 | 0 |
| | under review | 1 | 0 | 0 | 0 |
| | in preparation | 0 | 0 | 2 | 2 |
| Continuing research groups | | 0 | 3 | 4, 2 [†] | 3, 2 ^{††} |
| Long-term individual collaborations | | 2 | 0 | 0 | 0 |
| Participated in other AIM workshop or CBMS conference | | 3 | 5 | 0 | 0 |

Table 4. Faculty research outcomes of REUF. The last formal data collection was June 2012, but data known to the authors through December 2012 is included.

[†] Each of these two will meet for a week at AIM in summer 2013.
^{††} One of these two will meet for a week at ICERM in summer 2013 (with REUF funding), and the other will meet at a university in the summer of 2013 (with funding from the university of the group leader and colleges of some participants).

research group meeting, have been established to foster continuing faculty research collaborations. The first of the small research groups to be funded through REUF will meet at AIM in summer 2013.

Research experience for faculty. Responses to questions about the experience in participant surveys immediately after the REUF workshops has been overwhelmingly positive (85–100%). Here is a sample of comments:

- *Being here makes me miss doing math so much that I have experienced a bit of heartache, but I have had to make the choices I made [due] to the demands of my time. It was fun being here!*
- *Fantastic! I had a great time working in groups.*
- *This was a great workshop all around (great people, great format, flexible schedule, intense and exciting collaboration opportunities, wonderful ideas for future work).*
- *The format of AIM workshops is the best model — organizers and staff greatly succeeded in creating the best environment for discussion and research work!*
- *It was the most useful workshop I've ever attended. It proved to me that faculty with high teaching loads that haven't done research in a long time still have it in them. They just need some open problems, some collaborators, and some free time!*

Network of faculty to support undergraduate research. The establishment of the REUF Network is still a work in progress. The regular gatherings of all prior REUF participants who are at attending national meeting such as MathFest and Joint Mathematics Meetings is described in [Section 4.2](#). The REUF Forum and REUF Resources website for REUF participants is described in [Section 4.3](#).

6. Growth and development

The REUF program continues to grow and evolve. While the workshop itself is an established and successful model, we continue to explore options for faculty research continuation and supporting research with undergraduates after the workshop, including building a community to support undergraduate research (for information about the latter, see [Sections 4.2](#) and [4.3](#)).

This summer the first four REUF small research groups will meet, two from the 2011 workshop and two from the 2012 workshop (see [Section 4.1](#)). After we receive feedback from participants we will analyze the outcomes and perhaps revise this part of the program. We are also gathering information about three smaller collaborations that grew out of REUF, and how these might be replicated. We plan

to explore these ideas with both alumni at the REUF gatherings at meetings and participants in each future REUF workshop.

One small collaboration involved three members of a 2009 REUF group at two colleges thousands of miles apart; they continued work on the problem presented at the workshop, and published a paper in *Journal of Number Theory*. Two of the collaborators (at the same college) later produced several related papers with undergraduates. Much of the work on the faculty paper was done during the REUF workshop at AIM and most of the rest was done electronically, although all three did meet briefly in person at a national meeting.

Two pairs of researchers from the 2008 REUF workshop began collaborating on separate topics different from the workshop problems, and produced papers that appeared in *Linear Algebra and its Applications* and *Electronic Journal of Linear Algebra*; a second paper from one of the teams is also under review. For both of these, multiple week-long research visits (totaling at least a month) were necessary for the development of shared background between the collaborators as well as for the advancement of the research itself (much of the communication about writing was done via e-mail); in addition there were numerous in-person consultations when both participants were at the same meeting.

We are exploring ways to expand the REUF program, since demand substantially exceeds capacity (by a factor of 2 in 2012). This has led to a partnership between AIM and ICERM. The 2012 and 2013 REUF workshops were held at ICERM; in 2014 we will be back at AIM, and are exploring the possibility of holding two REUF workshops per year, one each at AIM and ICERM.

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