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# Gully bank erosion: Mechanics and simulation by digital computer

by

John Colin Taylor

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
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DOCTOR OF PHILOSOPHY

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# INTRODUCTION, SELECTED LITERATURE REVIEW AND OBJECTIVES

## Introduction

Erosion of gullies has long been recognized as a problem in the loess hills of the Missouri River valley. Deep gullies with sheer banks are characteristic of the deep loess soils in western Iowa. These gullies are periodically inclined to rapid growth rates caused by flash floods from localized but severe rainstorms and by high water tables which result from sustained rainfalls.

Material that has accumulated in the channel over a period of time is suddenly sluiced out by heavy rains. Both during and immediately following this event bank caving is observed. The headward advance of the gully can be spectacular, perhaps 20 to 30 feet in a few hours, which for a gully cross section that may be 600 square feet, amounts to 670 cubic yards of material eroded. In addition to the advance of the gully head we observe lateral widening along the banks that also contributes vast quantities of material to the sediment load of the stream.

Loess, popularly considered Aeolian in derivation, is largely silt with low cohesion and of particle size easily transported by running water. Thus previously sloughed material is moved away easily. Bank caving is observed immediately after the peak flow stage and continually after the peak Q has long passed until temporarily stable talus profiles are formed. This suggests that instability is induced by a combination of factors: the removal of talus from the toe of the banks giving a steeper and less stable profile, the rapid drawdown effect of water recharged into the banks at high stage and left as extra weight after the peak flow passes, and continuous groundwater seepage through the banks.

Very little has been reported to explain the mechanics of this caving process in quantitative terms. The conceptualization, formulation and simulation of the process on a digital computer is presented in this thesis.

## Selected Literature Review

Because there are few articles that relate explicitly to the approach taken in this thesis only a selected review of literature related to gullying is presented. Articles from subject areas such as soil physics and soil mechanics that deal with physical principles or techniques used in the synthesis and formulation of the computer models are cited appropriately in later sections.

Gully studies of the past have taken several forms that are conveniently classified as: historical documentations, statistical analyses of growth; reports on control measures or detailed studies of a particular gully.

Historical documentations

A most interesting study was conducted by Daniels (1960) of the entrenchment of the Willow drainage ditch in southwestern Iowa. The man-made ditch was constructed in the 1920s; the modification of its cross section and 28 mile longitudinal profile by erosion since then has been extreme. At one point the cross sectional change was from 11 feet depth and 30 feet top width in 1920 to 42 feet depth and 110 to 120 feet top width in 1958. Although the Willow drainage ditch was man-made, the processes inducing bank caving are the same as in a natural gully and the sudden change in cross section below Daniel's "knick point" that erodes its way upstream is very similar to a natural gully head where we observe a natural stream of much smaller cross section flowing into it (Bradford et al., 1973).

A similar historical documentation was part of the study by Beer and Johnson (1963). Original survey notes, aerial photography and interviews with local inhabitants were the basis for estimating the stages of gully development in Steer Creek Watershed in southwest Iowa from 1851 to 1961.

Another very detailed study of several gullies in the Piedmont Region of South Carolina was made by Ireland et al. (1939). Qualitative observations of the mechanics of gullying processes such as caving were made in addition to many quantitative surveys of gully geometries and their local geologic formations.

The United States is not the only place where gullies have been studied; in fact, there are many examples of historical studies in literature from the Third World and Australasia. One author, Bishop (1962), speculated that the cause of gully erosion in the Queen Elizabeth National Park of Uganda was the rapid build-up of "perched" water tables during heavy rain causing increases in subterranean flow into the gully head.

### Statistical analyses of growth

Attempts have been made to quantify gully growth through statistical correlation to hydrologic and watershed parameters. The studies of Beer and Johnson (1963), and Thompson (1954) in the United States and Seginer (1966) in Israel are examples. The objectives of this type of study are two-fold: to derive a quantitative prediction equation and to isolate the important parameters or parameter combinations. The equations derived are not related to any hypothesized physical process but isolation of important parameters may give the researcher an indication of where to start formulating his physical model. Such equations are of limited value for prediction or simulation modeling where prediction of the effect of short-term rainfall events is required. The quantity and accuracy of data available to develop the prediction model determines the accuracy of such equations.

Reports on control measures

Because of the great inconvenience of gullies to man and resource loss much has been done to develop suitable control measures. The literature, particularly Soviet and Polish, is abundant with reports on the effectiveness of different types of control measures: For example, Kobezskii (1959), Asatryan (1965), and Zaitsev (1968). Examples similar to this type of report but from the Western World are those of Woolhizer and Miller (1963) in the United States, Hudson (1963) in Rhodesia and Thompson (1962) and Glass (1966) in New Zealand. Many types of control measures have been tried; concrete and earth structures, rock bolsters and weirs have been designed to plug up the gully by providing a sediment trap and thereby preventing headward advance. Stabilization of the bank and head soil by planting vegetation such as willow trees is another approach.

All the control measures reported seem variously successful depending on the particular geographic location. Appropriateness of a control measure, if not limited by economics or raw materials, is left to the experience of the individual worker. Lack of understanding of the processes that control erosion in the individual cases has prevented the formulation of scientifically based guidelines.

Detailed studies

There have been few detailed studies of individual gullies, that of Piest and Spomer (1968) near Treynor, Iowa, being an exception. The headward advance of the gully was measured periodically during 1965 and compared to the corresponding runoff quantities. There was no direct correlation between runoff and erosion quantities but significant runoff was needed for erosion to occur. They measured gully erosion during a storm by taking sediment samples above and below the gully head and subtracting the contribution of sheet and rill erosion.

Figure 1 shows the sediment concentration curve and runoff hydrograph obtained. The initial high peak coexistent with the rising stage of the hydrograph was attributed to the clean-out of talus and debris accumulated since the last runoff event. There is a drop in sediment concentration during the peak stages followed by a rise coexistent with the recession limb of the hydrograph. This was attributed to erosion induced by the present runoff event such as bank caving.

The above study prompted Bradford et al. (1973) to consider an approach similar to this study. They attempted to evaluate the stability of gully banks through soil mechanics by use of the Simplified Bishop Method of Slices. They used an available standard computer program to evaluate the factor of safety of the banks against shear failure. They considered

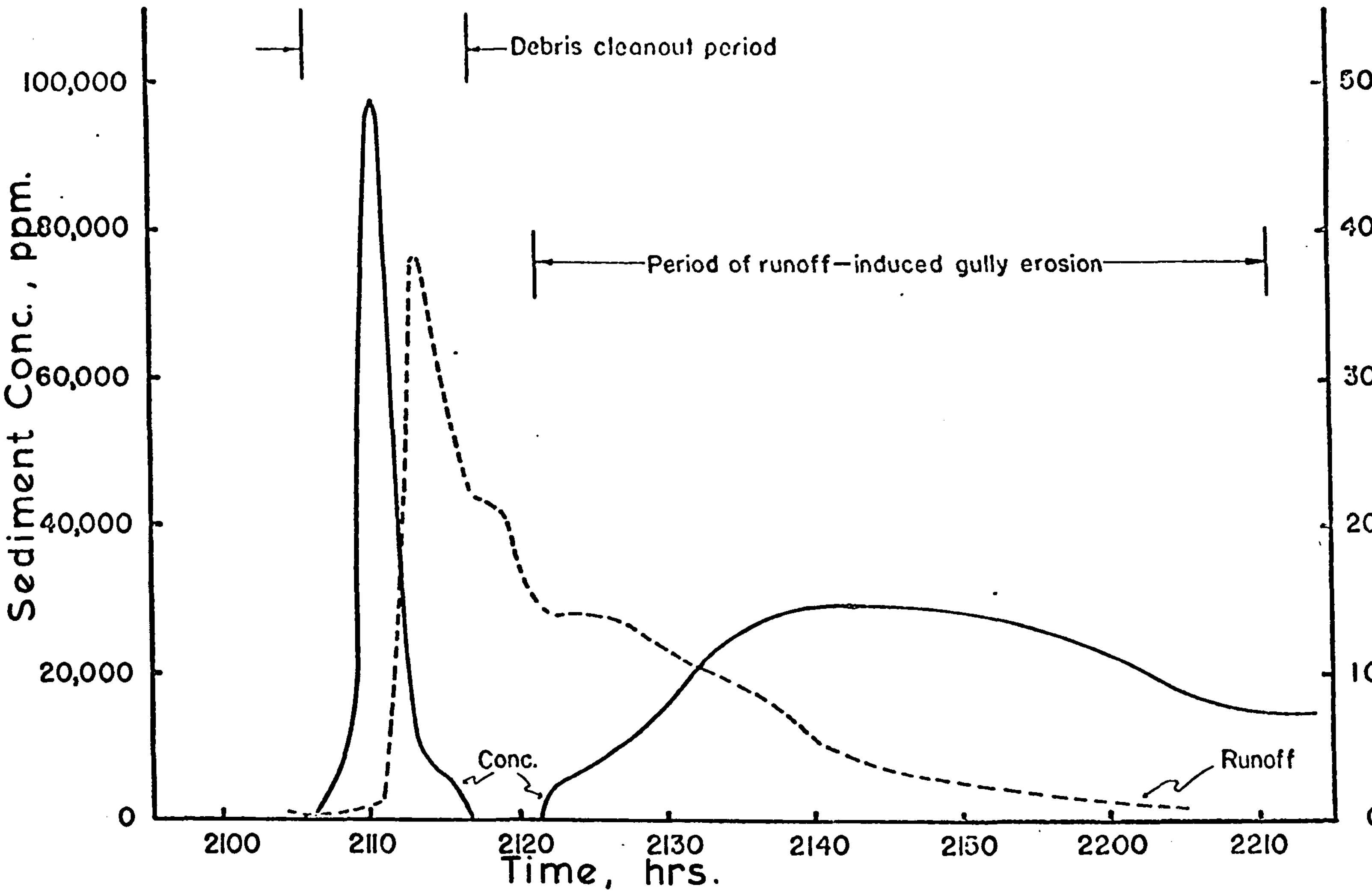



Figure 1. The process of gully **cleanout** and the erosive effect of runoff. (Piest and Spomer, 1968)

the situation where the phreatic surface of groundwater was at the toe of the gully wall and the true cohesion was zero. The banks were assumed saturated or nearly saturated with negative pore water pressure at a point equal to its height above the phreatic surface. This gave the soil some apparent cohesion. For angles of soil internal friction $\leq$35 degrees their model indicated instability of vertical banks. They also considered cases with water table positions below the base of the slope, that is, completely beneath the bed of the gully. This is equivalent to increasing the apparent cohesion of the bank soil because the increased height of the soil in the failure zone above the water table means increased soil water suction and an increase in effective stress. For an angle of internal shearing resistance of 25 degrees, a 300 centimeter vertical bank, and water table located less than 110 centimeters below the gully bed, their model predicted instability.

For the groundwater conditions they assumed, there would be no gully base flow; indeed, water would be imbibed into the soil under the suction head. Predictions of instability for high water tables was inconsistent with known field observations where the water table was above the toe of the gully bank and there existed a seepage face. They concluded that the assumption of zero true cohesion for the soils considered was invalid.

The model of Bradford et al. (1973) did not extend to sloping banks which in the loess, according to Lohnes and Handy (1968), could commonly be anywhere from 69 degrees to nearly 90 degrees to the horizontal. Their model did not account for desaturation of the soil under the suction heads above the water table. Loess is very porous and desaturates easily with consequent reductions in both soil wet unit weight and apparent cohesion.

## Objectives

The objectives of this study are now summarized:

I. To develop concepts of the mechanics of gully bank failures. The effect of groundwater on soil shear strengths and the interaction with gravitational forces on the soil is the main theme of this work.

II. To express the concepts in the form of a digital simulation model. Groundwater flow systems are modeled through techniques developed in the area of soil physics and these flow systems are then incorporated into a slope stability model developed from the area of soil mechanics.

III. To examine future developments of the model and other problems where it may be applied.

# GROUNDWATER FLOW MODELS

## Background

In this section systems that describe flow of groundwater into a gully through its bed and banks are defined in mathematical terms. Laplace's equation for hydraulic potential is solved in the resulting boundary value problems using the method of Powers et al. (1967a), described in Kirkham and Powers (1972). Hereafter this is referred to as the PKS method. Many conceptualized groundwater flow systems have been modeled by this approach. Boast (1970), Khan (1970) and Van der Ploeg et al. (1971) modeled systems of flow toward wells; Powers et al. (1967b) and Khan (1973) modeled flow between ditches with unequal water levels, and Powers et al. (1967a) and Selim and Kirkham (1972a, b) modeled flow through various shapes of soil bedding that was saturated to the surface.

Other approaches to the simulation of groundwater flow are the numerical techniques of Freeze (1972) and Kirkham and Gaskell (1950). However, we consider that the PKS method gives solutions that are more conveniently applied to slope stability analyses. This is because the hydraulic potential function $\phi(x,y)$ is obtained for any point of cartesian coordinates $(x,y)$ in the flow region. The numerical techniques evaluate $\phi$ at discrete points at nodes of a grid pattern over the flow region. Values of $\phi$ between the nodes must be found by interpolation. Computer programs that include numerical techniques are less general than those using the PKS method. That is, they are less easily applied to different geometrical configurations of similar flow systems and they tend to be more costly to run because convergence to the solution may be hampered by a bad initial "guess" of $\phi$ at each node. Another advantage of the PKS method is that the stream function $\psi(x,y)$ is obtained analytically from the potential function; thus only one solution for the constants of the flow region is required to draw a flow net.

Groundwater flow systems are generally analyzed in an approximate way for slope stability analyses where the chief objective is to calculate pore water pressures along a trial failure surface in the soil. Usually the boundaries of the flow region are defined in terms of equipotentials and streamlines. The flow net is sketched in by trial and error and then used for the calculation of pore water pressures. Solutions obtained by the trial and error method (Lambe and Whitman, 1969) yield practical results and are relatively insensitive to error. However, because human judgment is needed to draw the flow net the method is unsuitable for computer simulation. In fact, the complete flow nets presented later are superfluous to the actual analysis of bank stability but they allow the comparison of flow systems visually and are the ultimate check that the mathematical problem is solved. Once the potential function $\phi(x,y)$ is obtained by the PKS method we merely substitute in the x and y coordinates of points on the trial failure plane to evaluate $\phi$ from which we then calculate the pore water pressure.

## Groundwater Flow

The soil material that comprises the porous flow medium is assumed to be isotropic and homogeneous. Flow velocities are assumed small so that the hydraulic potential at (x,y) is given by

$$\phi(x,y) = y + P'/\gamma_w \qquad (1)$$

where $P'$ is the gage pressure, y is the height above the reference level, $\gamma_w$ is the unit weight of water and velocity head is neglected.

We assume laminar flow and the validity of Darcy's law

$$v_s = -K\partial\phi/\partial s \qquad (2)$$

where $v_s$ is the flow per unit cross section of soil, known as the Darcy velocity, K is the saturated hydraulic conductivity and $\partial\phi/\partial s$ is the hydraulic gradient in the s direction. The minus sign is required to make velocity positive because flow is positive in the direction of negative hydraulic potential gradient.

For steady state flow, using Darcy's law and continuity of flow, the equation that describes hydraulic potential in the flow region is

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{3}$$

which is Laplace's equation in two dimensions. The expression $\phi(x,y)$ that we seek must satisfy equation 3 and the boundary conditions of the particular flow region...

A general solution but not all solutions to equation 3 is given in Kirkham and Powers (1972, p. 96) and is the starting point for obtaining $\phi(x,y)$.

## The Recharge Model

This flow system is termed the "recharge" case because water is supplied to the system from the soil above the water table. The flow region 0abcde0 is shown in Figure 2 and represents a semi-cross section of the groundwater flow region perpendicular to the longitudinal profile of the gully. Flow through unit thickness of the flow region perpendicular to the plane of the paper is equivalent to flow per unit distance along the gully bank. The flow system is symmetrical about the center line of the gully which is the y-axis. There is no flow across 0a which is assumed to be a boundary streamline.

The length ab represents the surface and half-width T of the horizontal gully bed and bc the gully bank that slopes at $\theta$ degrees to the horizontal. Water is seeping into the gully through its bed and banks along the saturated face abc where c is the upper extremity of the seepage face $H_s$ above the

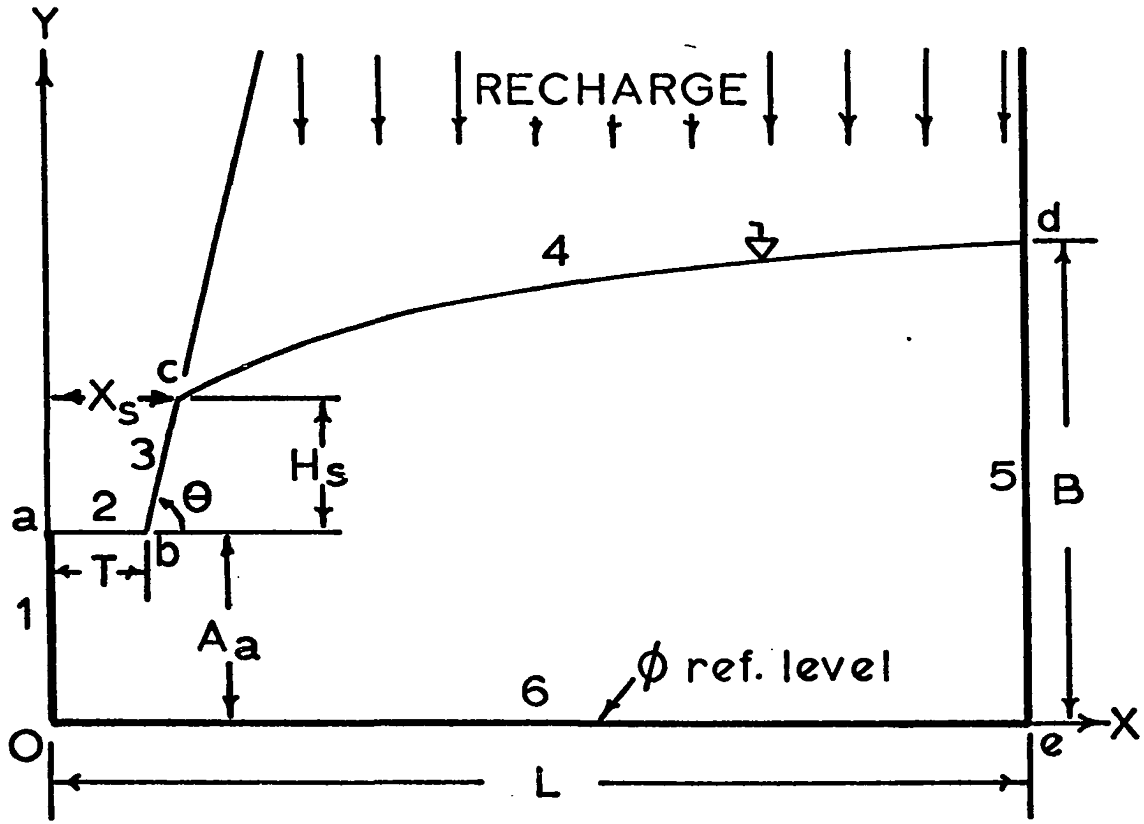



Figure 2. Diagrammatic representation of the flow region for the recharge model

gully bed. We assume water is transported away as fast as it seeps, thus the depth of water in the gully is zero. The water table cd extends to the right into the medium and is recharged through overlying soil from the surface. Our solution is general for any water table that is of known shape. The seepage recharge need not be known for the solution of the problem. The significance of recharge rate in real problems is discussed in later sections.

The right-hand limit of the flow region de is the groundwater divide, taken to be a vertical boundary streamline. The whole region is underlain by a horizontal impermeable barrier at depth $A_a$ below the gully bed, which also represents a boundary streamline. This is taken as the x-axis and reference level for $\phi$. The flow system of Figure 2 could represent real cases in western Iowa where Kansan Till is overlain by permeable loess through which the gully has partially eroded.

The boundary conditions (BC's) are summarized and expressed mathematically in Table 1. The BC's are mathematically similar to those in the problems of Selim and Kirkham (1972a, b). Therefore, we use the same solution of Laplace's equation which is our notation

$$\phi = \sum_{m=0,1}^{N\to\infty} A_{Nm} \frac{\cosh(m\pi y/L)}{\cosh(m\pi B/L)} \cos(m\pi x/L) \qquad (4)$$

Equation 4 satisfied the BC's 1, 5 and 6 for all $A_{Nm}$'s. The subscript N of $A_{Nm}$ indicates the value of $m$ at which the series

Table 1. Summary and mathematical definition of boundary conditions for the flow region of the recharge model.

| Boundary Number | Letter Range | Boundary Condition | Coordinate Ranges x | y |
|---|---|---|---|---|
| 1 | 0a | $K\partial\phi/\partial x = 0$ | $x = 0$ | $0 \leq y \leq A_a$ |
| 2[a] | ab | $\phi = y = f_1(x) = A_a$ | $0 \leq x \leq T$ | $y = A_a$ |
| 3 | bc | $\phi = y = f_1(x) = A_a + (x-T)\tan\theta$ | $T \leq x \leq x_s$ | $A_a \leq y \leq y_s$ |
| | | | where $x_s = T + H_s \cot\theta$ | |
| | | | | $y_s = A_a + H_s$ |
| 4[b] | cd | $\phi = y = f_2(x)$ | $x_s \leq x \leq L$ | $y_s \leq y \leq B$ |
| 5 | de | $K\partial\phi/\partial x = 0$ | $x = L$ | $0 \leq y \leq B$ |
| 6 | e0 | $K\partial\phi/\partial y = 0$ | $0 \leq x \leq L$ | $y = 0$ |

[a] $f_1(x)$ is any function describing the soil surface. The function used in this problem is defined in the table.

[b] $f_2(x)$ is any function describing water shape. A series of points are taken from an arbitrary curve representing the water table.

approximation is truncated. The $A_{Nm}$'s are calculated by the PKS method so that BC's 2, 3 and 4 are satisfied. For BC's 2, 3 and 4 along abcd we write

$$\phi = F(x) \tag{5}$$

where

$$F(x) = \begin{cases} A_a & 0 \leqq x \leqq T \\ f_1(x) = A_a + (x - T)\tan\theta & T \leqq x \leqq x_s \\ f_2(x) & x_s \leqq x \leqq L \end{cases} \tag{6}$$

Equation 4 is now rewritten without super and subscripts on the summation sign as

$$F(x) = \sum A_{Nm} u_m(x) \tag{7}$$

wherein

$$u_m(x) = \begin{cases} \dfrac{\cosh m\pi f_1(x)/L}{\cosh m\pi B/L} \cos m\pi x/L & 0 \leqq x \leqq x_s \\[2ex] \dfrac{\cosh m\pi f_2(x)/L}{\cosh m\pi B/L} \cos m\pi x/L & x_s \leqq x \leqq L \end{cases} \tag{8}$$

Equation 7 is the correct form for application to the PKS method described later.

To obtain the stream function $\psi(x,y)$ the potential function $\phi(x,y)$ is first converted to velocity potential

$$\Phi = K\phi \tag{9}$$

where K is the saturated hydraulic conductivity. The Cauchy-

Riemann relations shown in Kirkham and Powers (1972, p. 105) could now be used to obtain $\psi(x,y)$. Instead we make use of the convenient Table 3-1, (Kirkham and Powers, 1972, p. 106) and get from Line 13

$$\psi = -K \sum A_{Nm} \frac{\sinh m\pi y/L}{\cosh m\pi B/L} \sin m\pi x/L \qquad (10)$$

This function is used to calculate streamlines for flow nets of the system.

## The Rapid Drawdown Model

The model described in this section is for cases where water table fluctuations are important only in the vicinity of the gully bank. The flow region is 0abcd0 of Figure 3. The bank face 0b slopes at $\theta$ degrees to the horizontal and the water level in the gully is at any point a between 0 and b and ab represents the seepage face. The gully water level at a is the reference level for $\phi$ in this model. The water table of any, but known, shape extends to the right into the soil and is represented by bc. The flow region in a real situation would extend back to the groundwater divide and be supplied by seepage recharge from overlying soil. We assume that distance L from the toe of the bank, the water table elevation is not affected by changes in the water level in the gully, but is held steady by groundwater supply from the right. The flow system

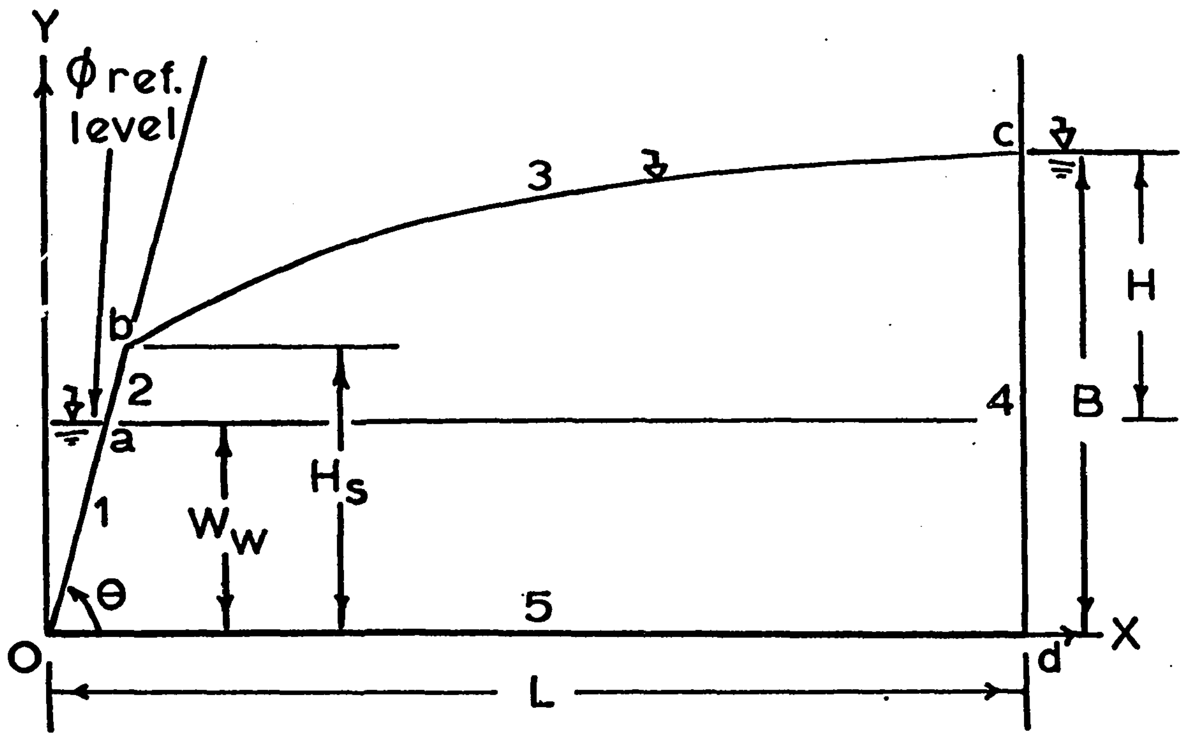



Figure 3. Diagrammatic representation of the flow region for the rapid drawdown model

for $x > L$ is thus replaced by a fictitious constant head source at $x = L$. It is implicitly assumed that flow enters the region horizontally from the fictitious source because cd is a vertical equipotential. The position of the soil surface relative to the water table is immaterial since seepage recharge from above for $0 \leq x \leq L$ is neglected. The gully bank is shown extending upward from b in Figure 3. The lower horizontal boundary d0 along the x-axis is impermeable and passes through the toe of the gully bank and is a boundary streamline. The flow region 0abcd0 is equivalent to real cases in western Iowa where gullies have completely eroded through surface loess to the relatively impermeable Kansan Till below.

The use of the fictitious source limits the flow region to the area of interest near the gully bank and eliminates superfluous calculation.

The boundary conditions are summarized and defined mathematically in Table 2.

We chose the solution* of equation 3 as

$$\frac{\phi}{H} = \frac{x}{L} + \sum_{m=1,2}^{N\to\infty} A_{Nm} \frac{\cosh m\pi y/L}{\cosh m\pi B/L} \sin m\pi x/L \qquad (11)$$

*The author is indebted to Dr. Sami Selim for advice given in the selection of this solution. Iowa State University, Ames, Iowa, 1973.

Table 2. Summary and mathematical definition of boundary conditions for the flow region of the rapid drawdown model.

| Boundary Number | Letter Range | Boundary Condition | Coordinate Ranges x | y |
|---|---|---|---|---|
| 1 | 0a | $\phi = 0$ | $0 \leq x \leq x_w$ | $0 \leq y \leq W_w$ |
| | | | where $x_w = W_w \tan\theta$ | |
| 2[a] | ab | $\phi = f_1(x) - W_w$ | $x_w \leq x \leq x_s$ | $W_w \leq y \leq H_s$ |
| | | where $f_1(x) = x \tan\theta$ | where $x_s = H_s \tan\theta$ | |
| 3[b] | bc | $\phi = f_2(x) - W_w$ | $x_s \leq x \leq L$ | $H_s \leq y \leq B$ |
| 4 | cd | $\phi = H = B - W_w$ | $x = L$ | $0 \leq y \leq B$ |
| 5 | d0 | $K\partial\phi/\partial y = 0$ | $0 \leq x \leq L$ | $y = 0$ |

[a] $f_1(x)$ is any function describing the soil surface. The function used in this work is defined in the table.

[b] $f_2(x)$ is any function describing the water table shape. A series of points are taken from an arbitrary curve representing the water table.

Equation 11 satisfied BC's 4 and 5 for all $A_{Nm}$'s. BC's 1, 2 and 3 are satisfied by choice of the $A_{Nm}$'s using the PKS method. Rearranging equation 11 and dropping super and subscripts from the summation sign

$$\frac{\phi}{H} - \frac{x}{L} = \sum A_{Nm} \frac{\cosh \alpha_m y}{\cosh \alpha_m B} \sin \alpha_m x \tag{12a}$$

where

$$\alpha_m = m\pi/L \qquad m = 1, 2, \ldots \tag{12b}$$

Substituting BC's 1, 2 and 3 into equations 12a and b for BC 1; $\phi = 0$, $y = f_1(x) = x \tan \theta$

$$- \frac{x}{L} = \sum A_{Nm} \frac{\cosh \alpha_m f_1 (x)}{\cosh \alpha_m B} \sin \alpha_m x \tag{13a}$$

for BC 2; $\phi = f_1(x) - W_w$, $y = f_1(x) = x \tan \theta$

$$\frac{f_1(x) - W_w}{H} - \frac{x}{L} = \sum A_{Nm} \frac{\cosh \alpha_m f_1 (x)}{\cosh \alpha_m B} \sin \alpha_m x \tag{13b}$$

for BC 3; $\phi = f_2(x) - W_w$, $y = f_2(x)$

$$\frac{f_2(x) - W_w}{H} - \frac{x}{L} = \sum A_{Nm} \frac{\cosh \alpha_m f_2 (x)}{\cosh \alpha_m B} \sin \alpha_m x \tag{13c}$$

From equations 13a, b and c we define

$$F(x) = \begin{cases} -x/L & \text{over BC 1} \\ \dfrac{f_1(x)-W_w}{H} - x/L & \text{over BC 2} \\ \dfrac{f_2(x)-W_w}{H} - x/L & \text{over BC 3} \end{cases} \tag{14}$$

and

$$u_m(x) = \begin{cases} \dfrac{\cosh \alpha_m f_1(x)}{\cosh \alpha_m B} \sin \alpha_m x & \text{over BC's 1 and 2} \\ \dfrac{\cosh \alpha_m f_2(x)}{\cosh \alpha_m B} \sin \alpha_m x & \text{over BC 3} \end{cases} \tag{15}$$

Equations 13a, b and c can now be written in the form

$$F(x) = \sum A_{Nm} u_m(x) \tag{16}$$

where $F(x)$ and $u_m(x)$ are defined in equations 14 and 15. The $A_{Nm}$'s can now be calculated by the PKS method.

By use of the same procedure as that for the recharge model, from Table 3-1, Kirkham and Powers (1972), lines 2 and 11, the stream function is from equations 11 and 12b

$$\phi = KH \left( y/L + \sum A_{Nm} \frac{\sinh \alpha_m y}{\cosh \alpha_m B} \cos \alpha_m x \right) \tag{17}$$

This stream function is used in the flow net calculations for the rapid drawdown model.

Choice of $A_{Nm}$'s by the PKS Method

The solutions of the two boundary value problems defined rest on the correct choice of the $A_{Nm}$'s to satisfy BC's 2, 3 and 4 of the recharge model and BC's 1, 2 and 3 of the rapid drawdown model. The $A_{Nm}$'s are selected by the PKS method using Table A2-1 (Kirkham and Powers, 1972, pp. 502,503). An abbreviated derivation of the table is given in Chapter 4 (Kirkham and Powers, 1972). For the full derivation refer to Van der Ploeg (1972).

To use Table A2-1, the following integrals are necessary:

$$w_m = \int_0^L F(x)u_m(x)dx \tag{18}$$

$$u_{mn} = \int_0^L u_m(x)u_n(x)dx \tag{19}$$

where

$m = 0, 1, 2 \ldots . N$
$n = 0, 1, 2 \ldots . N \leqq m$
for the recharge model

and

$m = 1, 2 \ldots . N$
$n = 1, 2 \ldots . N \leqq m$
for the rapid drawdown model.

The functions $F(x)$, $u_m(x)$ and $u_n(x)$ are given by equations 6 and 8 for the recharge model and equations 14 and 15 for the rapid drawdown model. As an example, consider the rapid drawdown case:

$$w_m = \int_0^{x_w} -(x/L) \frac{\cosh \alpha_m f_1(x)}{\cosh \alpha_m B} \sin \alpha_m x dx$$

$$+ \int_{x_w}^{x_s} \frac{f_1(x)-W_w}{H} -x/L \frac{\cosh \alpha_m f_1(x)}{\cosh \alpha_m B} \sin \alpha_m x dx \qquad (20)$$

$$+ \int_{x_s}^{L} \frac{f_2(x)-W_w}{H} -x/L \frac{\cosh \alpha_m f_2(x)}{\cosh \alpha_m B} \sin \alpha_m x dx$$

$$u_{mn} = \int_0^{x_s} \frac{\cosh \alpha_m f_1(x)}{\cosh \alpha_m B} \sin \alpha_m x \frac{\cosh \alpha_n f_1(x)}{\cosh \alpha_n B} \sin \alpha_n x dx$$

$$(21)$$

$$+ \int_{x_s}^{L} \frac{\cosh \alpha_m f_2(x)}{\cosh \alpha_m B} \sin \alpha_m x \frac{\cosh \alpha_n f_2(x)}{\cosh \alpha_n B} \sin \alpha_n x dx$$

The constants $w_m$ and $u_{mn}$ of equations 20 and 21 are evaluated by numerical integration. With these constants known, a

FORTRAN subroutine ORTH developed by Boast (1969) is used to perform the calculations shown in Table A2-1 (Kirkham and Powers, 1972).

In the PKS method, one way to check that the accuracy of the solution is increasing with the addition of extra terms to the series approximation is through Bessel's inequality (Kirkham and Powers, 1972, p. 499).

$$\sum A^2_{NN} D_N \leq \int_0^L F(x)^2 dx \qquad N = 1, 2 \ldots . \tag{22}$$

wherein the $A_{NN}$ are the last $A_{Nm}$ of each successive approximation as N increases and the $D_N$ are defined in the Table A2-1 and must be positive. For example, in the rapid drawdown case the right-hand side of Bessel's inequality (BRHS) is given by

$$\begin{aligned} BRHS = & \int_0^{x_w} (-x/L)^2 dx \\ & + \int_{x_w}^{x_s} \left( \frac{f_1(x)-W_w}{H} - x/L \right)^2 dx \\ & + \int_{x_s}^{L} \left( \frac{f_2(x)-W_w}{H} - x/L \right)^2 dx \end{aligned} \tag{23}$$

A similar expression was used for the recharge model.

## Summary

In this section the basic equations governing groundwater flow were briefly reviewed. Two groundwater flow systems termed recharge and rapid drawdown models were then defined as boundary value problems with hydraulic potential functions $\phi(x,y)$'s given by solutions to Laplace's equation. The solutions are in the form of an infinite series of functions. Each term of the series satisfies as many of the boundary conditions as possible, regardless of the value of its constant coefficient $A_{Nm}$. Remaining boundary conditions of each model are to be approximated by the respective series through careful choice of the $A_{Nm}$ values. The $A_{Nm}$'s are calculated by the PKS method.

# BANK STABILITY ANALYSIS

## Background

Standard texts on soil mechanics such as Lambe and Whitman (1969) and Harr (1966) are referred to for more detailed discussions of background material than is presented in this section.

### Limit design

The objective of a bank stability analysis is to compare the forces required for limiting equilibrium of soil in the bank with its available strength. This is referred to as "limit design" (Lambe and Whitman, 1969, Chapter 13) because design against total collapse of the bank is required and smaller strains are ignored.

There are two common approaches to limit design. The first is the solution procedure of Sokolovsky in Harr (1966), and the second is the method of slices. Sokolovsky's method involves the numerical solution of Kotter's equation which is derived from the differential equations of soil equilibrium and the Mohr-Coulomb Failure Law. The limits of the failure zone are calculated from the solution so that trial failure surfaces do not have to be assumed as in the method of slices. Although mathematically less rigorous, the method of slices is more commonly used in the United States because of its simplicity. A version of the method is used for this work.

Methods of slices

According to Lambe and Whitman (1969), the main assumption in all methods of slices is that the normal stress on an assumed failure surface is predominantly influenced by the weight of overlying soil. The soil is considered semi-infinite so that stability is evaluated by taking a representative two-dimensional cross section through the bank and considering it to be of unit thickness. The body of soil above the trial surface is divided into slices and the equilibrium of each slice is considered. If all the forces on the two-dimensional slice have to be considered, the system is statically indeterminate. Indeterminacy is usually overcome by making simplifying assumptions regarding the resultant of side forces on the slice. In the Ordinary Method of Slices it is assumed that side forces have no resultant perpendicular to the failure plane at the base of the slice, whereas the Simplified Bishop Method assumed a zero resultant in the vertical direction. Bradford et al. (1973) used the Simplified Bishop Method. However, they point out that scatter in calculated factors of safety caused by errors due to soil parameter variation is greater than that caused by choice of method of slices. The side force assumption of the Ordinary Method of Slices is used for this work.

When the failure surface is assumed to be an arc of a circle, factor of safety F is often defined as the ratio of moments about the circle center of disturbing forces to available soil strength. When the failure surface is assumed to be other than circular another definition is the ratio of disturbing forces along the surface to available shear strength along the surface. The failure surface we use is part of a cycloid; therefore, the second definition is followed.

Once F is determined for one trial failure, the trial failure surfaces are systematically cnanged until a minimum value of F is found. If $F < 1$, the bank is unstable. If $F = 1$ the bank is in limiting equilibrium, and if $F > 1$, the bank is stable.

## The effective stress principle

Effective stress $\bar{\sigma}$ on a plane in saturated soil is defined as the total normal stress $\sigma$ minus the pore water pressure u on the plane so that

$$\bar{\sigma} = \sigma - u \tag{24}$$

Physically this is interpreted as being approximately the force per unit area of the plane carried by the soil skeleton. Deformation or failure in soil therefore results from changes in effective rather than total stresses.

In a partially saturated soil with degree of saturation S, assuming pore air is at atmospheric pressure and the soil is isotropic

$$\bar{\sigma} = \sigma - Su \tag{25}$$

In such cases u is negative; therefore, $\bar{\sigma}$ is greater than $\sigma$. This results in an increase in mobilizable shear strength. However, as u becomes more negative, proportional increases in $\bar{\sigma}$ are smaller because the soil desaturates and the area of water in the soil pores is reduced.

The purpose of the groundwater flow models previously described is to enable the calculation of positive pore water pressures below the water table so that effective stresses may be evaluated. The water table is a phreatic surface; that is, the pore water pressure is at atmospheric or zero gage pressure. Above the water table, the pore water pressures become negative and are assumed to be equivalent to minus the elevation head above the water table.

## The Mohr-Coulomb Failure Law

The familiar Mohr envelope is obtained by plotting a series of Mohr stress circles representing failure conditions for soil at different confining stresses and drawing an envelope that is tangent to them. The equation of this envelope describes the shear stress $\tau_{ff}$ at failure as a function of normal effective stress $\bar{\sigma}_{ff}$ on the failure plane.

$$\tau_{ff} = f(\bar{\sigma}_{ff}) \tag{26}$$

Over a large range of confining stresses, this function is curvilinear. However, over limited ranges of confining stress a straight line approximation is

$$\tau_{ff} = c + \bar{\sigma}_{ff} \tan \phi_s \tag{27}$$

where c is the cohesion intercept with units of force per unit area and $\phi_s$ is the angle of internal shearing resistance of soil. The cohesion intercept is referred to as "true cohesion" in this work to distinguish it from "apparent cohesion" derived from negative pore pressures. Our definition of true cohesion does not reflect the strength of chemical bonds between individual soil particles as does the usual definition. The linear approximation of equation 27 was first made by Coulomb and though not a physical law, is often referred to as the Mohr-Coulomb Failure Law. Because the true Mohr envelope is curvilinear, the values of c and $\phi_s$ vary depending on the stress range; therefore, it is most important to match the stress range of the problem to the laboratory tests that determine c and $\phi_s$.

## Bank Stability Analysis Using Cycloidal Failure Surfaces

### Generation of cycloidal failure surfaces

Using portions of a cycloid as the shape of a trial failure surface is not new. Ellis (1973) used them to develop a simple stability analysis for trench walls and referred to a 19th century article where they had been used for the description of embankment failure surfaces.

The basis for choosing cycloids in this work was their visual similarity to observed failure surfaces though there was no verification of this hypothesis by measurement. A contributing factor influencing the choice is the easy mathematical generation and parametric representation of such curves.

Figure 4 shows how the cycloidal shape is generated. The line aa' is the cycloidal surface that is the locus of a point on the circumference of a circle of radius R when it is rolled along the x-axis through angle of rotation $\theta'$ radians. The parametric representation of the cycloid with respect to the x and y axes shown is

$$y = R(1 - \cos\theta') \tag{28}$$

$$x = R(\theta' - \sin\theta') \tag{29}$$

The soil surface in the model is assumed to be horizontal and coincident with the x-axis. Consistent with field observation, the failure plane is assumed to pass through the toe of the gully bank.

With the end points of the failure surface fixed in this way its shape is completely defined by the radius R of the generating circle. Cycloidal failure surfaces are generated in the computer program by the following sequence of calculations that also define the bounds of n slices of equal thickness $\Delta x$. Referring to Figure 5 that shows the bank cross section, and the failure surface $O_s$ is the origin of the cycloid with coordinate axes $x_s$; $y_s$. The x and y axes are the axes for the flow system and eventually coordinates of the failure plane are transformed from the $(x_s, y_s)$ to the $(x, y)$.

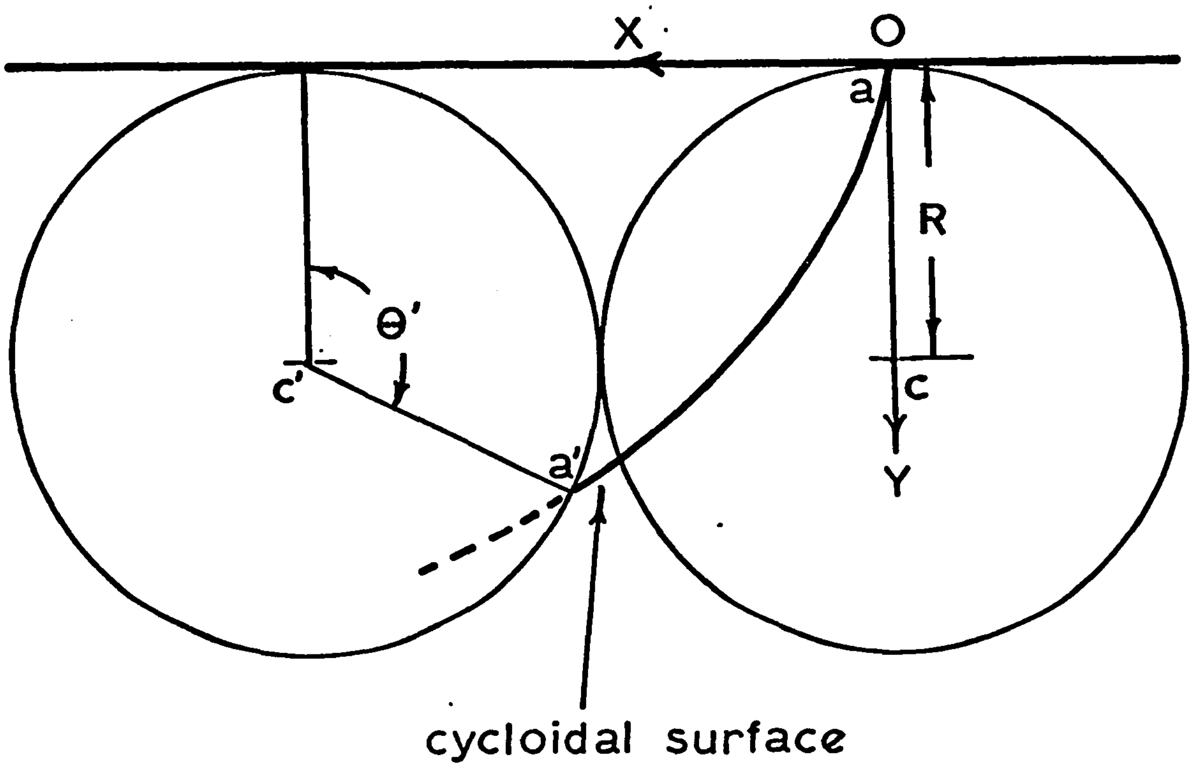



Figure 4. Generation of cycloidal failure surfaces

The maximum angle of rotation $\theta'_m$ of the generating circle, radius R, is found by substituting $B_h$, the bank height, as the maximum value of $y_s$ in equation 28 which on rearrangement gives

$$\theta'_m = \cos^{-1} (1 - B_h/R) \tag{30}$$

Substituting $\theta'_m$ into equation 29, the maximum horizontal distance $x_{sm}$ is given by

$$x_{sm} = R(\theta'_m - \sin \theta'_m) \tag{31}$$

The thickness of each slice in the x or $x_s$ direction is

$$\Delta x = x_{sm}/n \tag{32}$$

The surface is generated backwards from the toe of the slope where $x_s = x_{sm}$, $y_s = B_h$ and $\theta' = \theta'm$. Differentiating equation 29 and rearranging

$$d\theta' = dx/R(1 - \cos \theta') \tag{33}$$

wherein $d\theta'$, the angle of rotation required to generate the horizontal distance dx, is dependent on the total angle $\theta'$.

For the first slice, the generating circle has to be rolled backwards from $(x_{sm}, B_h)$ through Angle $\Delta\theta'_1$ given by

$$\Delta\theta'_1 = \Delta x/R(1 - \cos \theta'_m) \tag{34}$$

Differentiating equation 28

$$dy = R(1 - \cos \theta')d\theta' \tag{35}$$

which is in finite difference form for the first slice

$$\Delta y_1 = R(1-\cos \theta'_m)\Delta\theta'_1 \tag{36}$$

The coordinates of the bottom of the first slice are therefore $(x_{sm}, B_h)$ and $(x_{s1}, y_{s1})$ where

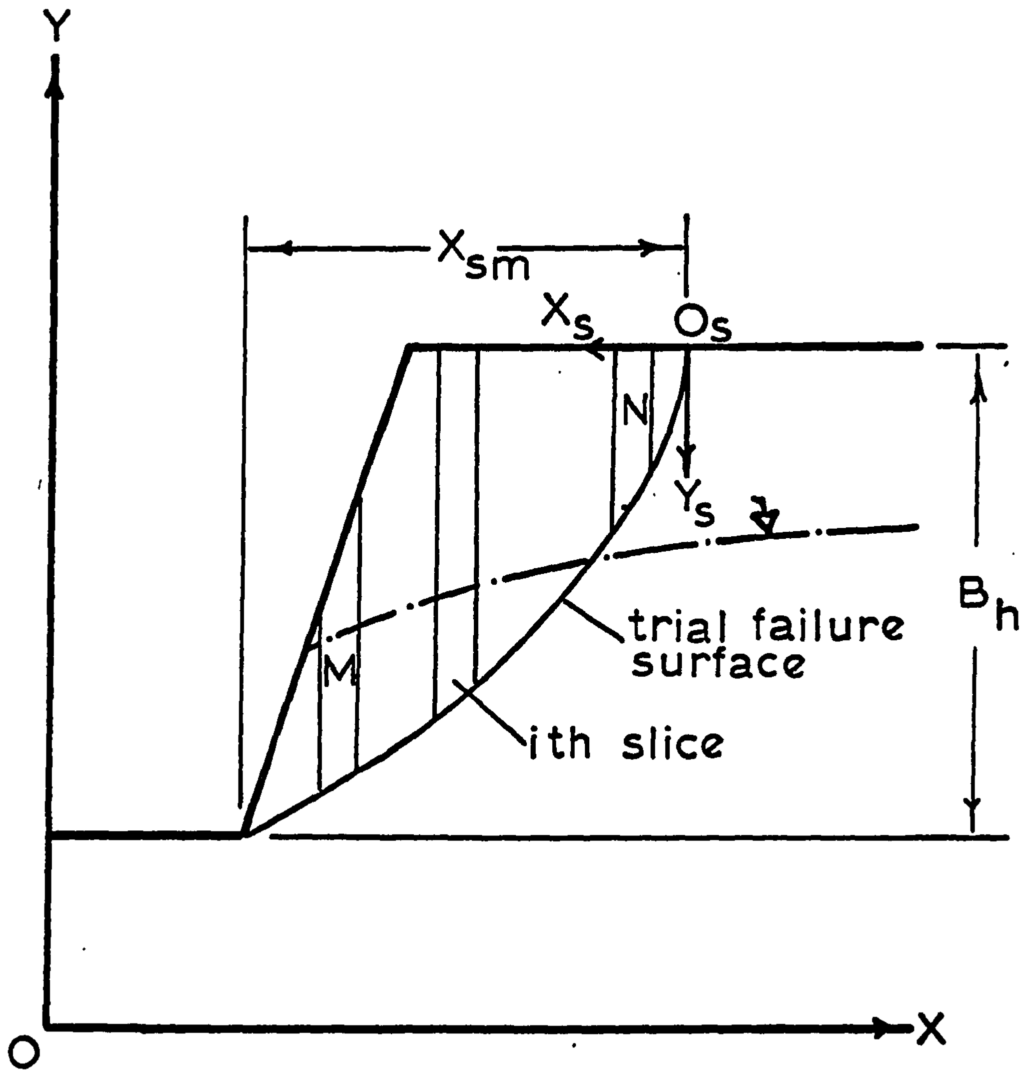



Figure 5. Cross section through bank showing a trial cycloidal failure surface, a hypothetical water table position and location of sample slices

$$y_{s1} = y_{sm} - \Delta y_1 \tag{37}$$

and

$$x_{s1} = x_{sm} - \Delta x \tag{38}$$

and the total angle of rotation to this point is

$$\theta'_1 = \theta'_m - \Delta\theta'_1 \tag{39}$$

for the i th slice

$$\Delta\theta'_i = \Delta x / R(1 - \cos\theta'_{i-1}) \tag{40}$$

$$\Delta y_i = R(1 - \cos\theta'_{i-1})\Delta\theta'_i \tag{41}$$

$$y_{s(i)} = y_{s(i-1)} - \Delta y_i \tag{42}$$

$$x_{s(i)} = x_{s(i-1)} - \Delta x \tag{43}$$

$$\theta'_i = \theta'_{i-1} - \Delta\theta'_i \tag{44}$$

Cycling through equations 40 through 44 generates the coordinates of the failure plane at the bounds of n slices working backwards from the toe of the bank. The finite-difference algorithm of equations 40 to 44 assumes $\cos\theta'_{i-1} \approx \cos\theta'_i$. Thus the $\Delta\theta'_i$ must be small; therefore n must be relatively large so that $\Delta x$ is small. Even for small $\Delta x$, as the failure plane approaches the $x_s$-axis $\Delta\theta'$ becomes larger for the fixed increment $\Delta x$. Therefore, errors can occur. For the last slice the coordinate, $y_{sn}$ is known to be zero. Therefore, $\Delta y_{sn}$ is given directly from

$$\Delta y_{sn} = y_{s(n-1)} \tag{45}$$

The finite difference approximation and the procedure for calculating $\Delta y_{sn}$ give rise to the discontinuities in the generated curves for the last slice as seen in the curves of Figure 20. These errors were ignored.

Evaluation of bank factor of safety

Figure 6 is a diagram of the i th slice isolated from the overall view of the bank shown in Figure 5. The small segment of the trial failure surface CD is assumed linear as is the water table segment BE. Disturbing forces on the slice act from D to C and restoring forces are in the reverse direction.

The resultant of any side forces on ABC and DEF is assumed to be zero in the direction perpendicular to CD. Thus the total normal force on CD is given by

$$F_{ni} = W_i \cos \beta_i \tag{46}$$

where $W_i$ is the weight of the i th slice and $\beta_i = \tan^{-1}(\Delta y_i/\Delta x)$.

The resultant of side forces acting in the direction CD is neglected in this analysis so that the disturbing force on the slice is the component of $W_i$ acting along DC

$$F_{ti} = W_i \sin \beta_i \tag{47}$$

The total weight of the i th slice $W_i$ is equal to the weight of soil plus the weight of water. Wet unit weight of soil $\gamma_t$ is calculated from

$$\gamma_t = \frac{G + Se}{1 + e} \gamma_w \tag{48}$$

where G is the specific gravity of soil particles, e is the voids ratio, S is the degree of saturation and $\gamma_w$ is the unit

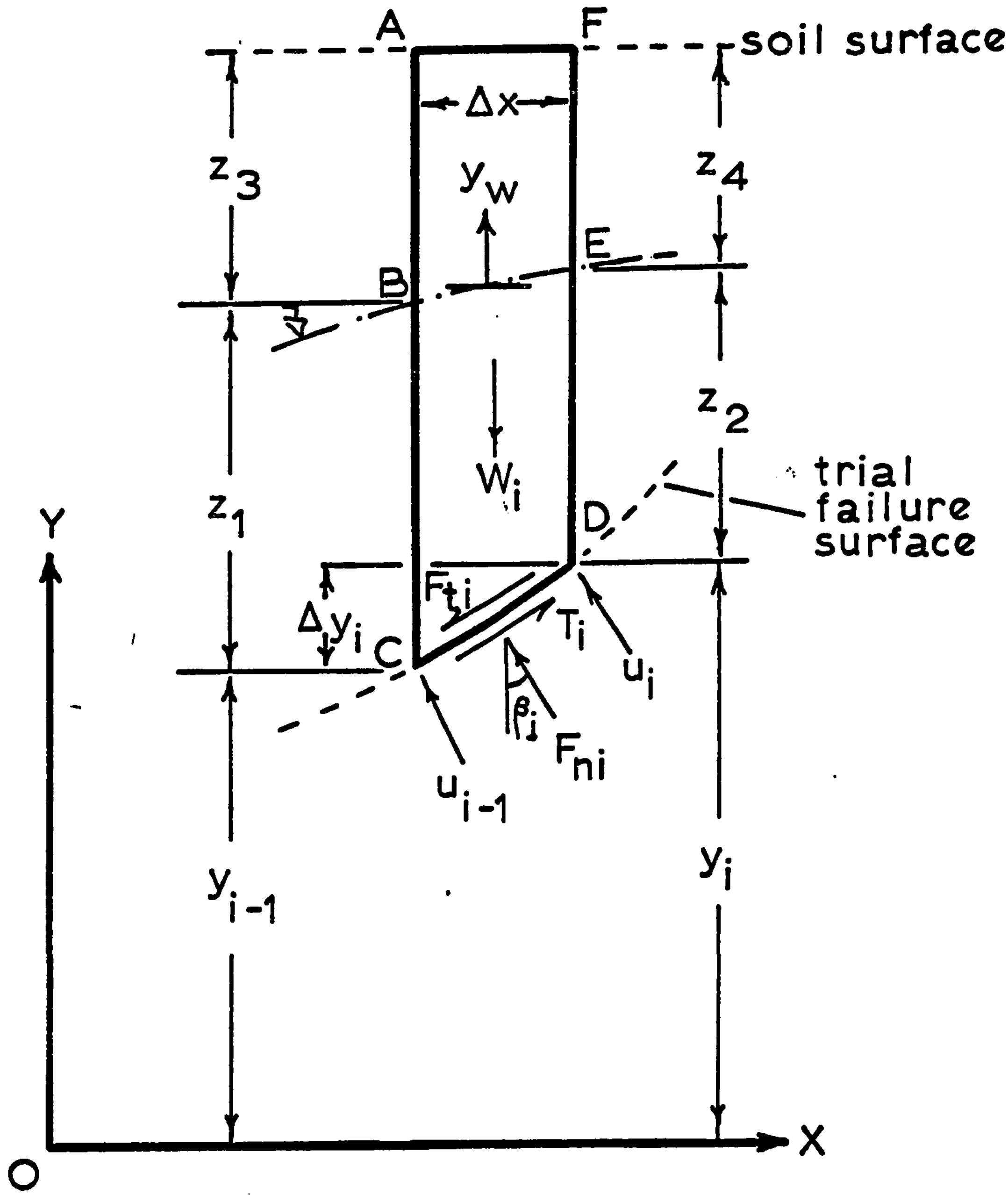



Figure 6. Free body diagram of the i th slice showing forces and dimensions

weight of water. For saturated soil S = 1. Using equation 48 and dimensions given in Figure 6, the weight of soil below the water table $W_1$ is

$$W_1 = (z_1 + z_2)\Delta x \gamma_t / 2 \tag{49}$$

For soil above the water table S < 1 and is a function of the height above the water table $y_w$ which is the suction head. The unit weight of soil distance $y_w$ above the water table is given by

$$\gamma_t(y_w) = \frac{G + S(y_w)e}{1 + e}\gamma_w \tag{50}$$

Therefore the weight of the column $W_2$ above the water table up to point $y_{w1}$ is given by

$$W_2 = \Delta x \int_0^{y_{w1}} \gamma_t(y_w)dy_w$$

$$= \frac{\Delta x G \gamma_w}{(1 + e)}(y_{w_1} - 0) + \frac{\Delta x e \gamma_w}{(1 + e)}\int_0^{y_{w_1}} S(y_w)dy_w \tag{51}$$

= wt of dry soil + wt of water in column

From Figure 6 the average distance from the water table to the soil surface is

$$y_{w1} = (z_3 + z_4)/2 \tag{52}$$

To evaluate the integral in equation 51 it is necessary to know the function $S(y_w)$. This function is obtained by fitting a polynomial to experimental data

$$S(y_w) = Q_1 + Q_2 y_w + \ldots + Q_{p+1} y_w^p \qquad (53)$$

where p is the order of the polynomial and the Q's are the coefficients that give the best least squares fit. Experimental data for loess soil were obtained from Figure 43, Melvin (1970) and were replotted with different units in Figure 7. Points from Melvin's curve and values back calculated using a fitted 5th order polynomial are almost coincident. The integral of equation 51 is now evaluated by integrating equation 53

$$\int_0^{y_w} S(y_w)dy_w = Q_1 y_{w_1} + \frac{Q_2 y_{w_1}^2}{2} + \ldots + \frac{Q_{p+1} y_{w_1}^{p+1}}{p+1} \qquad (54)$$

Therefore from equations 49, 51 and 54 the weight of the i th slice $W_i$ is

$$W_i = W_1 + W_2 \qquad (55)$$

The available stabilizing force $T_i$ per unit slice thickness is derived from the shear strength of the soil along CD. Multiplying each side of the Mohr-Coulomb Failure Law of equation 27 by the length CD gives

$$T_i = cA_{bi} + \bar{F}_{ni} \tan \phi_s \qquad (56)$$

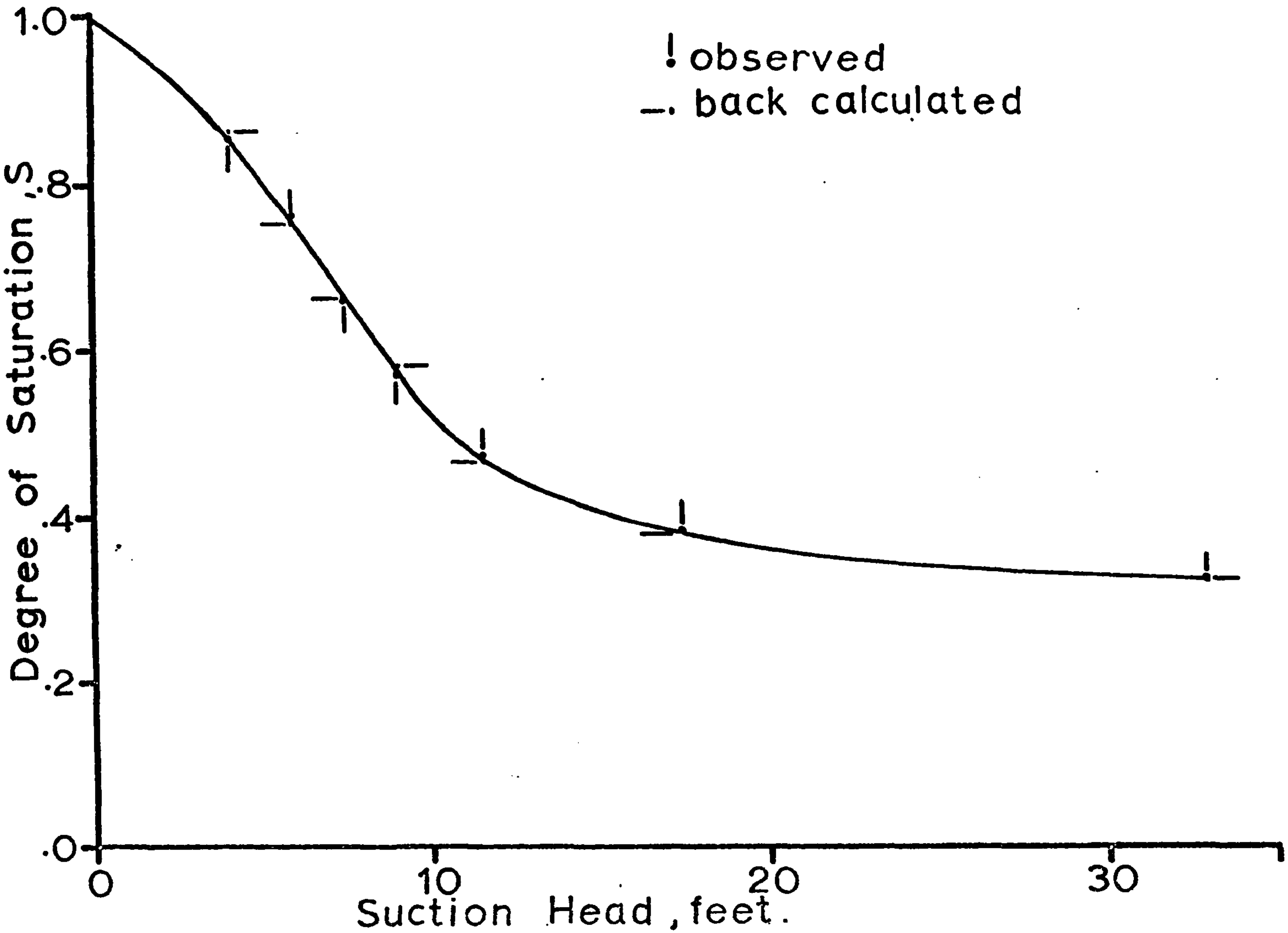
where $A_{bi}$ is the length of $CD = (\Delta x^2 + \Delta y_i^2)^{\frac{1}{2}}$ and $\bar{F}_{ni}$ is the effective normal force which is

$$\bar{F}_{ni} = F_{ni} - U_i \qquad (57)$$

where the average boundary pore water force on CD

$$U_i = A_{bi}\ (u_{i-1} + u_i)/2 \qquad (58)$$

Figure 7. Desaturation curve for loess soil redrawn from Melvin (1970)


observed
back calculated
Degree of Saturation, S
1.0
.8
.6
.4
.2
.0
0
10
20
30
Suction Head, feet.

and the pore water pressures $u_{i-1}$ and $u_i$ at C and D, respectively, are given by

$$\begin{aligned} u_{i-1} &= \phi(x_{i-1}, y_{i-1}) - y_{i-1} \\ u_i &= \phi(x_i, y_i) - y_i \end{aligned} \qquad (59)$$

where $(x_i, y_i)$ is the potential function derived from the flow system models.

The factor of safety F of the bank is calculated by using equations 47 and 56 so that

$$F = \frac{\sum_{i-1}^{n} T_i}{\sum_{i-1}^{n} F_{ti}} \qquad (60)$$

which is the sum over n slices of restoring forces along the failure plane divided by the sum of the disturbing forces along the failure plane.

The procedure for calculating the weight of the i th slice shows the general principles used in the model. However, the computer programs for calculating factor of safety take account of eleven specific varieties of slice that can occur in the analysis; for example, slices along the sloping bank face or completely above the water table, labeled M and N in Figure 5.

As an example, consider the slice N completely above the water table, shown in Figure 8. In this case component $W_1$ of equation 55 is zero and for component $W_2$ given by equation 51, the lower limit of the integral is changed from zero to $y_{w2}$, the average distance of the slice bottom above the water table

$$y_{w2} = (z_5 + z_6)/2 \tag{61}$$

The pore water pressures $u_{i-1}$ and $u_i$ for such a case are negative and equal to the elevation heads above the water table

$$\begin{aligned} u_{i-1} &= - z_5\gamma_w \\ u_i &= - z_6\gamma_w \end{aligned} \tag{62}$$

so that the pore water force for equation 59 is given by

$$U_i = A_{bi}S(y_{w2})\ (u_{i-1})/2 \tag{63}$$

where the area of pore water is proportionately reduced by $S(y_{w2})$ because of desaturation under the suction head and $S(y_{w2})$ is given by equation 53.

Once a value of factor of safety F has been found for a given failure plane, the computer program systematically changes the radius of the generating circle R until the minimum value of F has been determined.

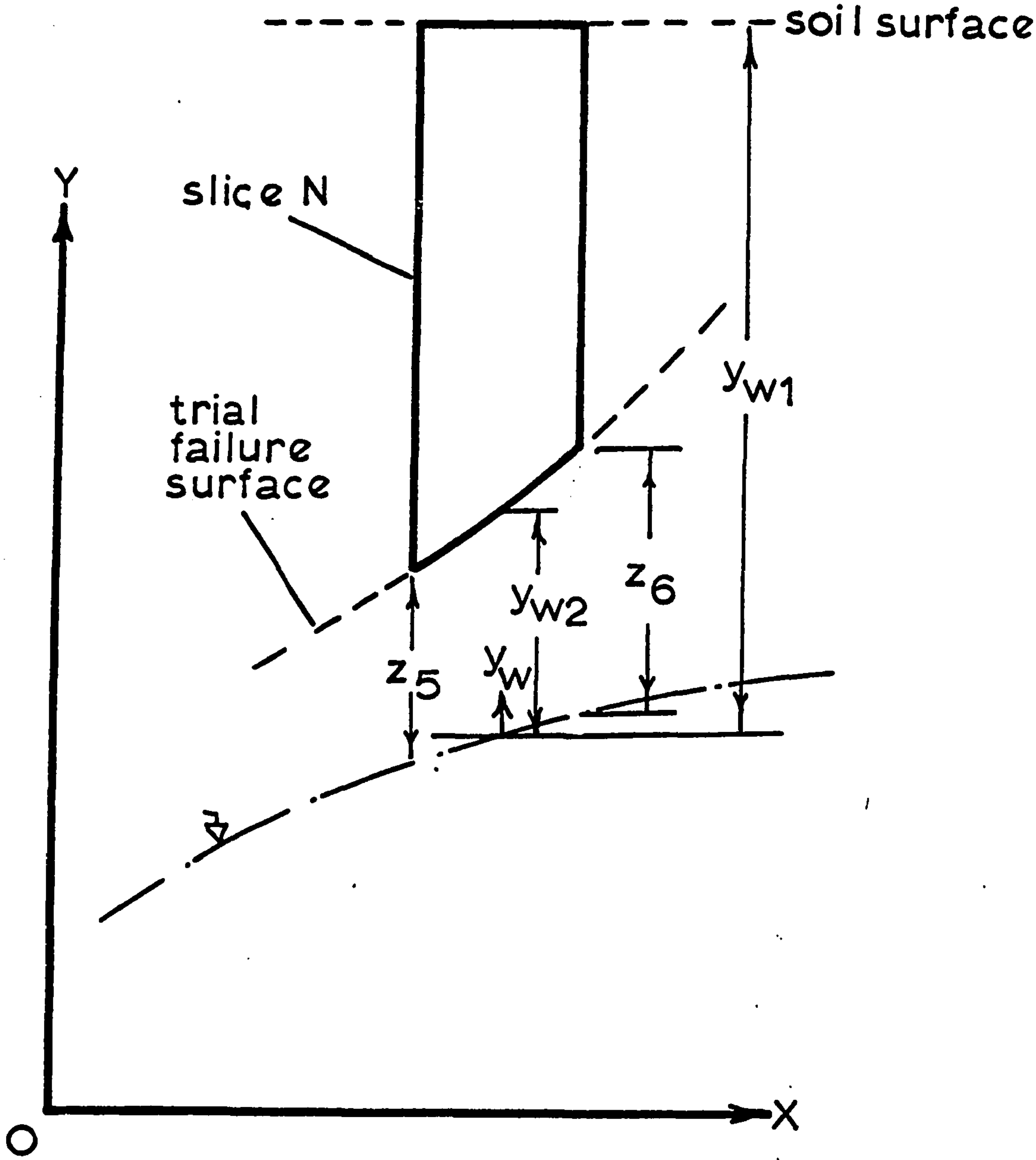



Figure 8. Free body diagram for slice N, completely above the water table

# RESULTS AND DISCUSSION

Two flow systems and a bank stability model have been formulated. This section deals with the representation of these models by computer programs and their ensuing performance in the analysis of gully bank stability.

## Computer Programs

The computer programs written for the flow system models, flow net calculations and the bank stability analysis are listed in FORTRAN in Appendix A. All the programs were run on the I.S.U. 360 computer in both WATFIV and FORTG. The storage requirements and running times of the programs are so dependent on declared array sizes and the convergence properties of a given solution that details of run costs and times may be misleading. The runs required to obtain results for this thesis cost from $20.00 to $30.00 to solve each flow system and $5.00 to $6.00 for each bank stability analysis. All subprograms required but not listed in Appendix A are standard and available from the IBM Scientific Subroutine Package. The programs also use the I.S.U. Computation Center Calcomp Plotter options for graphical output.

### Flow systems

Figure 9 is a simplified flow chart showing the sequence of major calculations in programs 1 and 2 that calculate the $A_{Nm}$'s up to a pre-set maximum value of N using the PKS method.

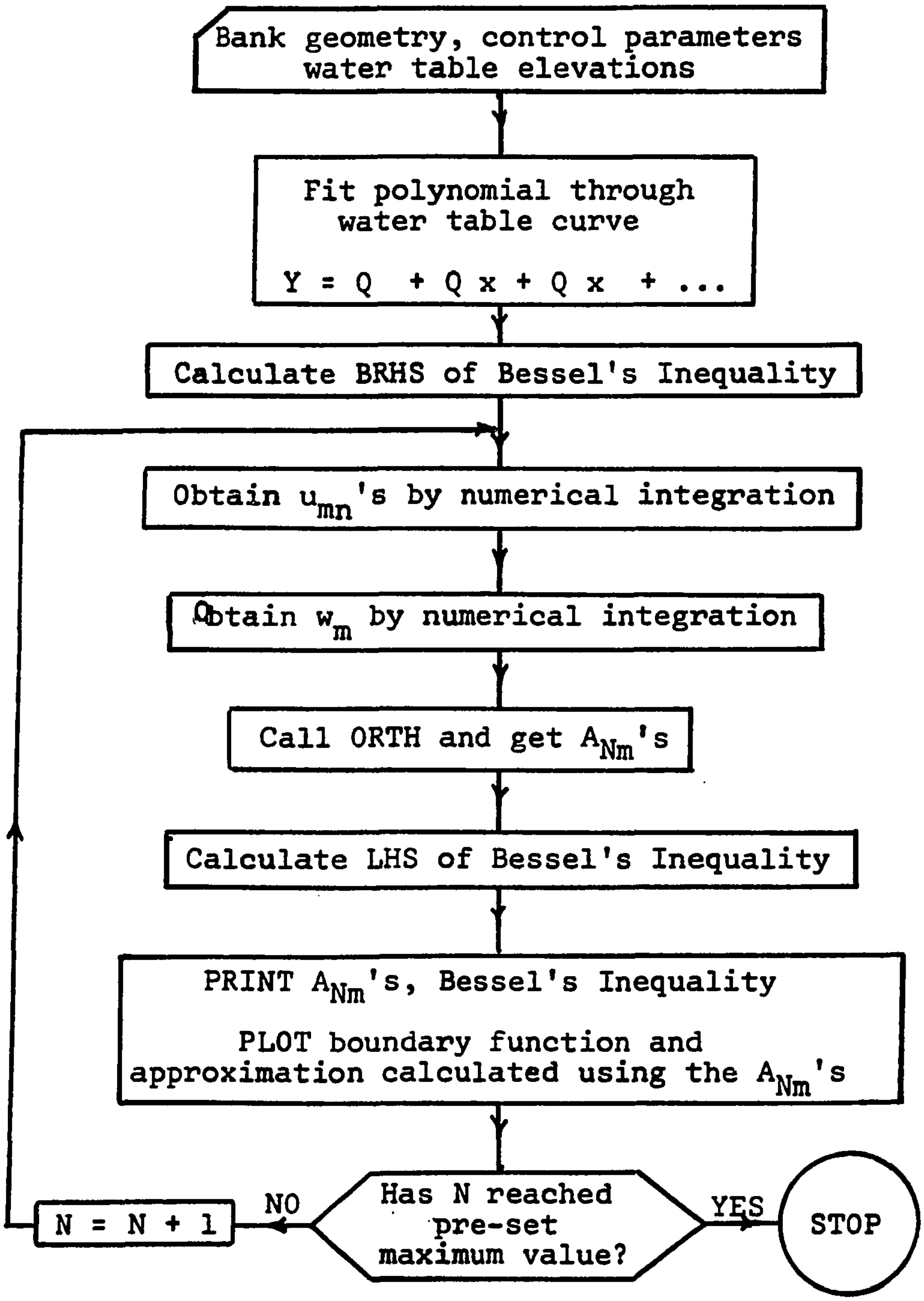



Figure 9. Flow chart showing the sequence of major calculations in the flow system computer programs

Program 1 is for the recharge model and program 2 is for the rapid drawdown model, but the sequence of calculations presented in Figure 9 is the same for both.

Examples of input data for programs 1 and 2 are given in Appendix B and consist of flow region geometry, control parameters that fix the maximum value of N, the order of polynomial approximations, the number of bisections for the ranges of numerical integrations et cetera, and a table of points that describe the water table shape.

For ease of manipulation in the program, a polynomial curve is fitted by least squares to the water table data. BRHS of Bessel's Inequality is then calculated from equation 22. The $u_{mn}$'s of equation 19 are obtained by numerical integration using Simpson's Rule as is the $w_m$ of equation 18. These are then passed to Boast's subroutine ORTH that calculates the $A_{Nm}$'s. The left-hand side LHS of Bessel's Inequality is then calculated and divided into BRHS and should give a number less than but approaching unity as N increases. This number we call Bessel's check BSCHK.

In addition to being printed, the $A_{Nm}$'s are punched onto cards and the boundary function and its approximation using the $A_{Nm}$'s are graphed. The cycle is repeated as shown in Figure 9 until the maximum pre-set value of N is reached.

<u>Flow nets</u>

Programs 3 and 4 were used to calculate $\phi$ and $\psi$ at regular intervals of x and y within the flow regions. The flow system geometry and the $A_{Nm}$'s from programs 1 and 2 were the inputs. Calculated values of $\phi$ or $\psi$ were then graphed for one cartesian coordinate fixed, for example, $\phi(0,y)$ versus y with x fixed at zero. The y coordinates for required values of $\phi$ or $\psi$ were interpolated from a series of these graphs drawn for different values of x. Streamlines or equipotentials for the flow net were then drawn through the interpolated points representing equal values of $\phi$ or $\psi$ respectively.

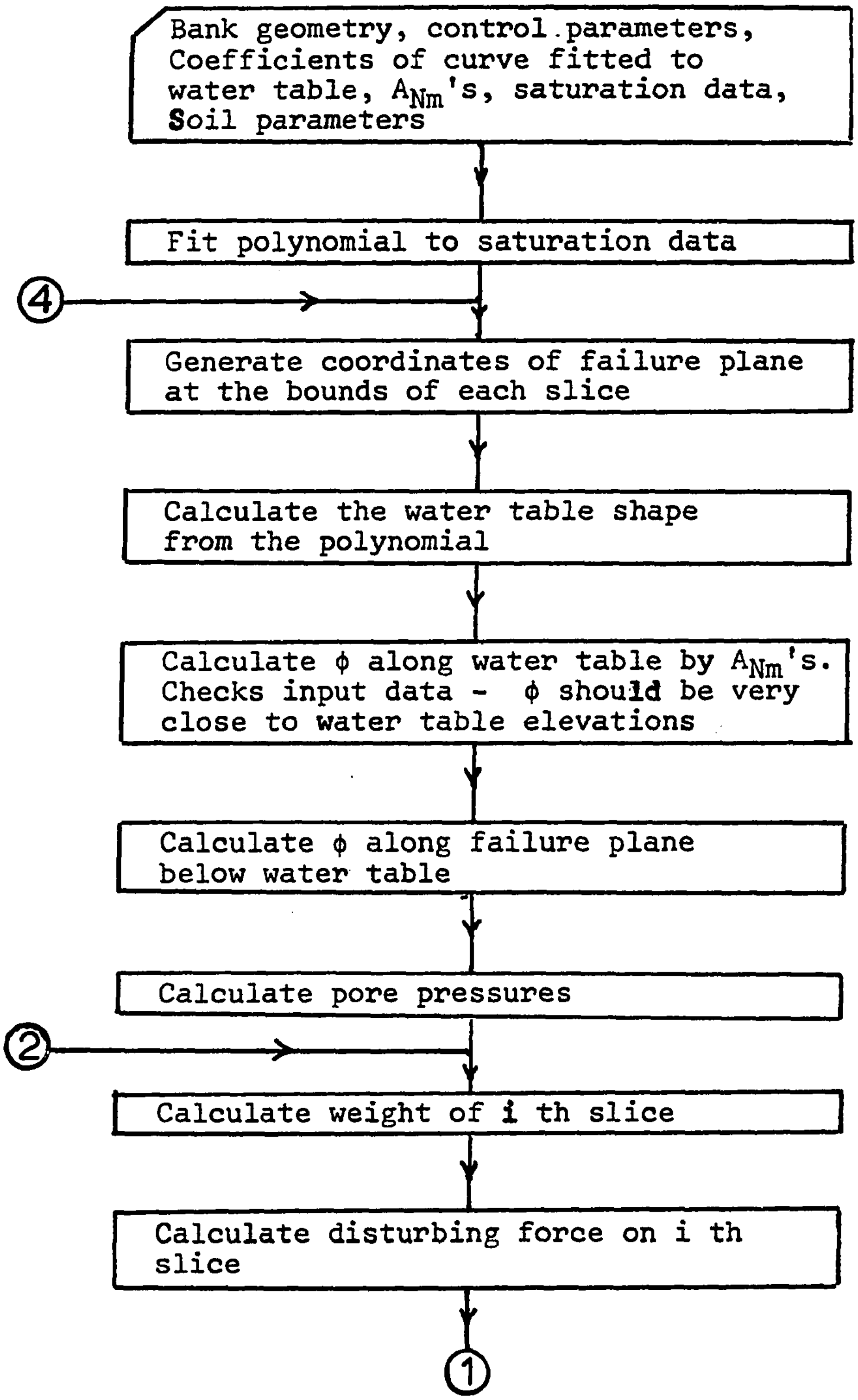
<u>Bank stability</u>

Programs 5 and 6 are the listings for bank stability analysis using the recharge and rapid drawdown models respectively. The sequence of the major calculations is the same for both and is shown in Figure 10. Sample input is listed in Appendix B. Details of the calculations are described in the previous section on BANK STABILITY ANALYSIS.

## The Flow Systems

The convergence of the PKS series approximation is demonstrated in Figure 11 for the recharge model and Figure 12 for the rapid drawdown model. As N increases, Bessel's check BSCHK approaches unity and the approximation of the boundary function F(x) gets visually better. In Figures 11 and 12 the

Figure 10. Flow chart showing the sequence of major calculations for the bank stability computer programs


Bank geometry, control parameters, Coefficients of curve fitted to water table, $A_{Nm}$'s, saturation data, Soil parameters
Fit polynomial to saturation data
4
Generate coordinates of failure plane at the bounds of each slice
Calculate the water table shape from the polynomial
Calculate $\phi$ along water table by $A_{Nm}$'s. Checks input data - $\phi$ should be very close to water table elevations
Calculate $\phi$ along failure plane below water table
Calculate pore pressures
2
Calculate weight of i th slice
Calculate disturbing force on i th slice
1

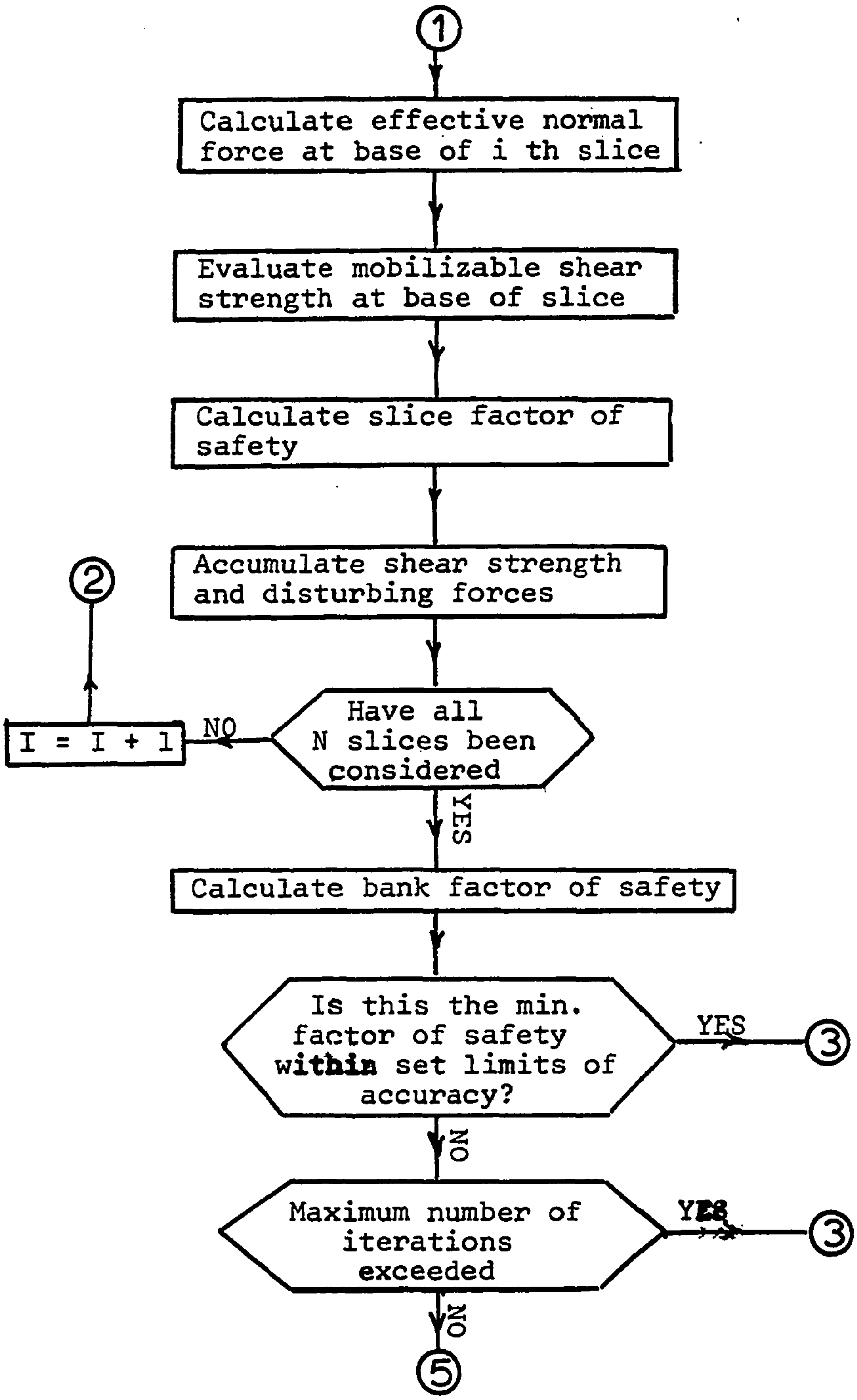



Figure 10 continued

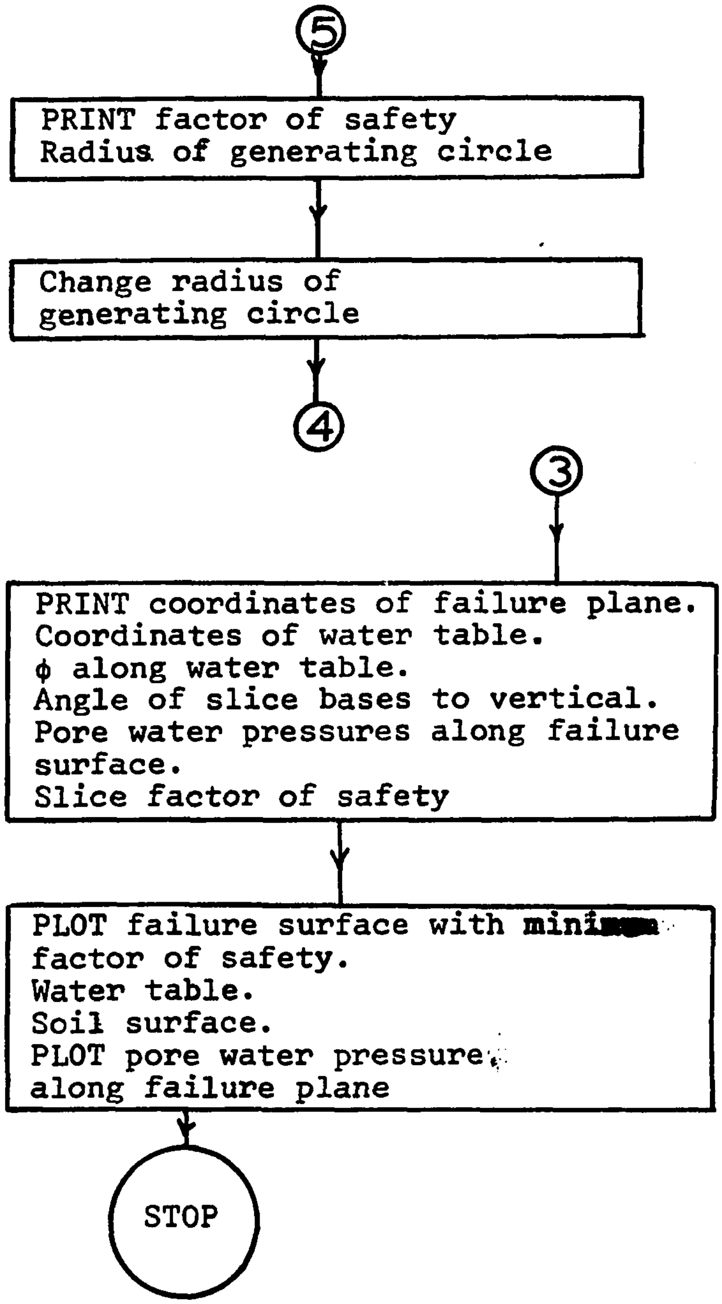



Figure 10 continued

Figure 11. Required boundary function $F(x)$ for the recharge model and successive approximations by the PKS method using N terms

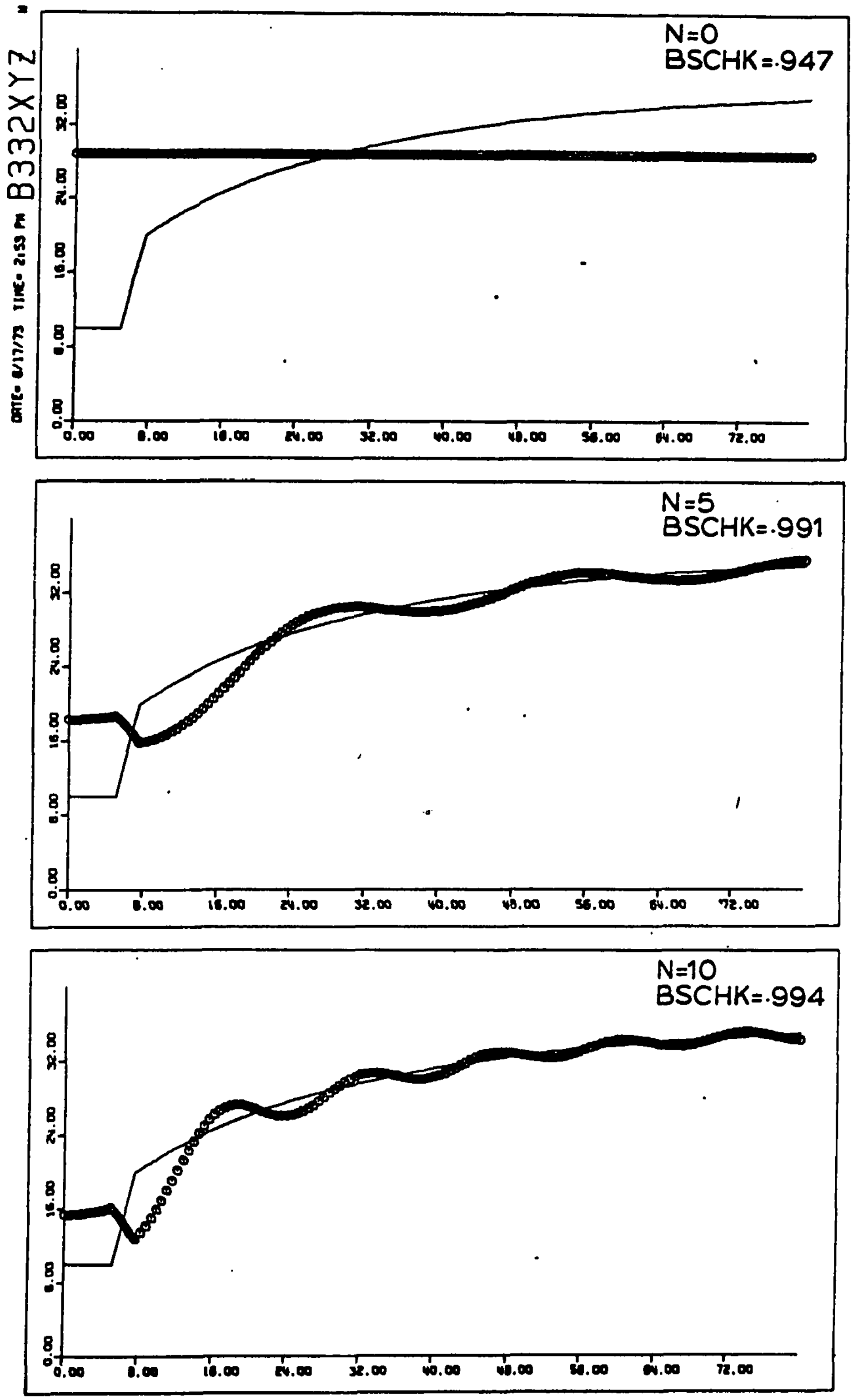
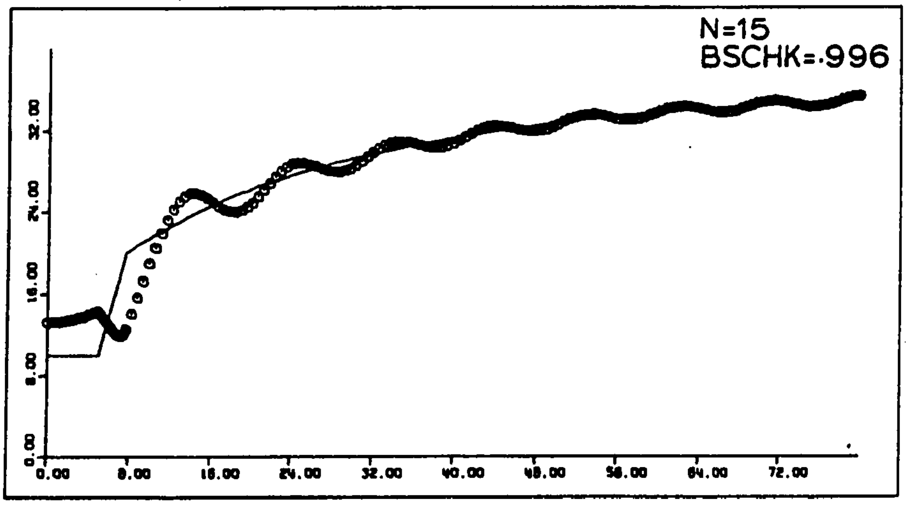
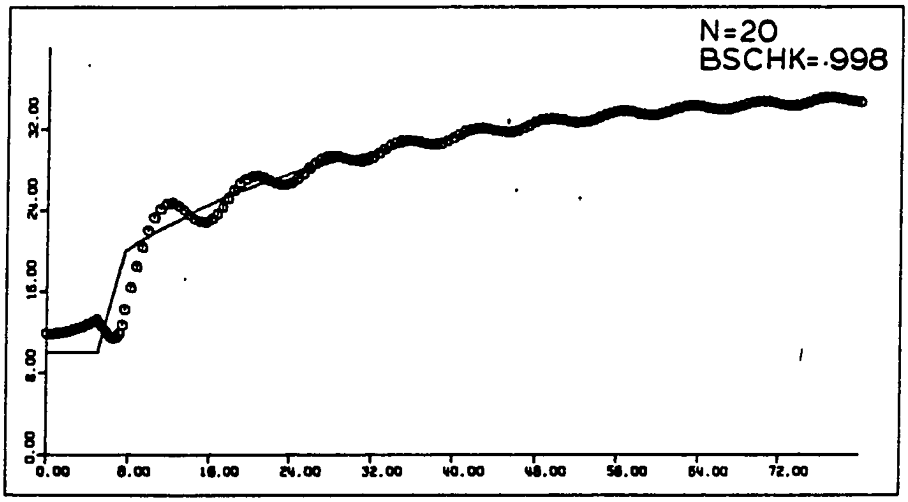

B332XYZ
N=0
BSCHK=.947
N=5
BSCHK=.991
N=10
BSCHK=.994
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00

Figure 11 continued


N=15
BSCHK=.996
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00


N=20
BSCHK=.998
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00

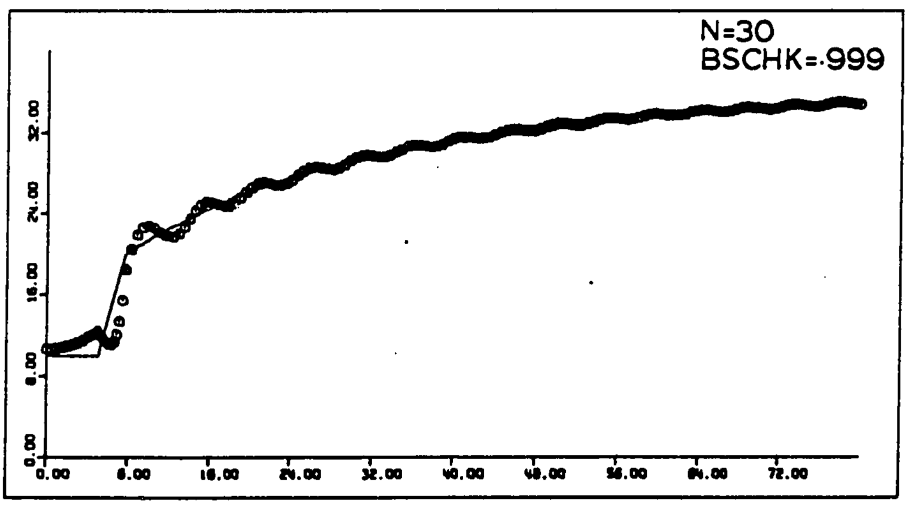

N=30
BSCHK=.999
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00

Figure 12. Required boundary function F(x) for an extreme case of rapid drawdown and its successive approximations by the PKS method using N terms

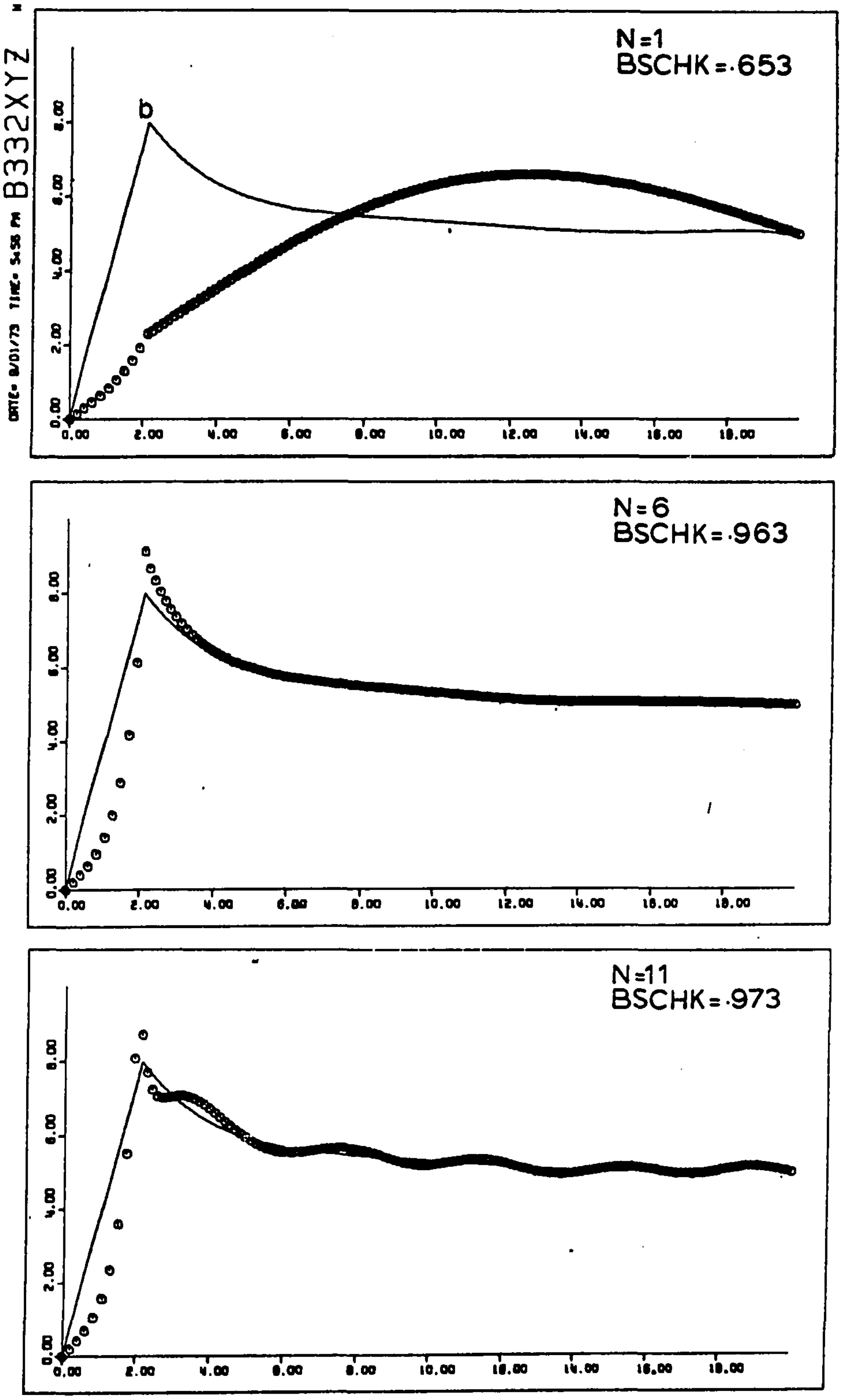
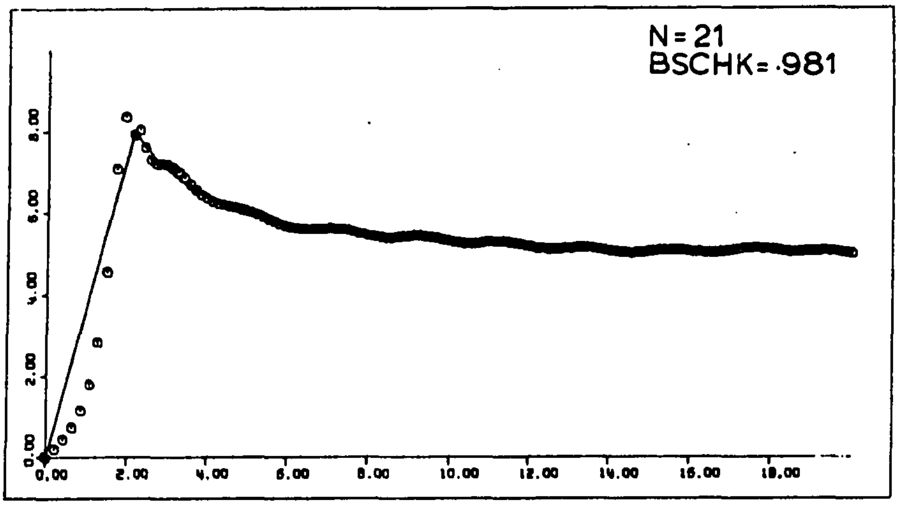
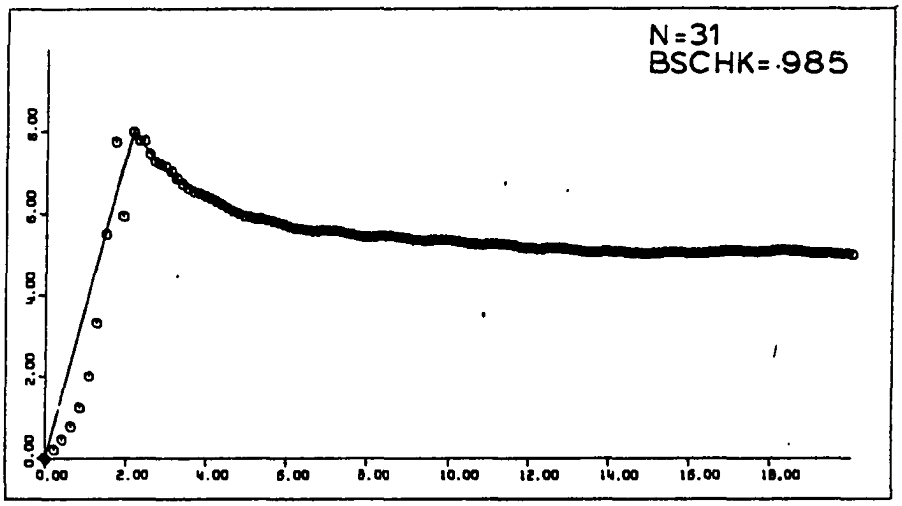
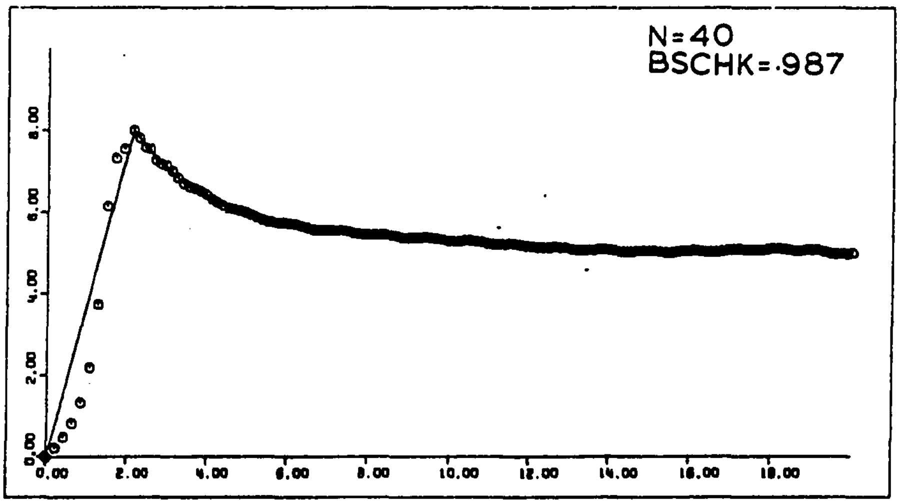

B332XYZ
N=1
BSCHK=.653
b
N=6
BSCHK=.963
N=11
BSCHK=.973

Figure 12 continued


N=21
BSCHK=.981


N=31
BSCHK=.985


N=40
BSCHK=.987

abscissae are horizontal distance x in feet and the ordinates are hydraulic potential $\phi$ in feet. The solid line represents the required shape of F(x) and the circles are the approximation given by the PKS method for the indicated value of N. Flow region dimensions and all other parameters were selected as "typical" values that could occur in a field situation in the deep loess soils of western Iowa. They are listed in Appendix B.

The flow nets of Figures 13 and 14 are for the "typical" geometries of the recharge model and rapid drawdown models using the $A_{Nm}$'s for N = 30, and N = 40, respectively. The flow patterns in each case are intuitively reasonable and are the basis for assuming the $A_{Nm}$'s are correctly calculated. The arithmetic of all programs was checked as far as possible by electronic desk calculator.

Figures 15 a and b show two flow nets that represent two instances in time of an extreme case of rapid drawdown. The lower flow net Figure 15 b, applies when the gully water level is high and the direction of the streamlines indicates that water is being recharged into the banks. The water table shape was arbitrarily chosen. The fact that the streamlines intersect the water table indicates that it is rising. All the flow systems analyzed are quasi-steady state, that is, we assume that water velocities are negligible compared to the speed of sound which is the velocity of pressure waves in the

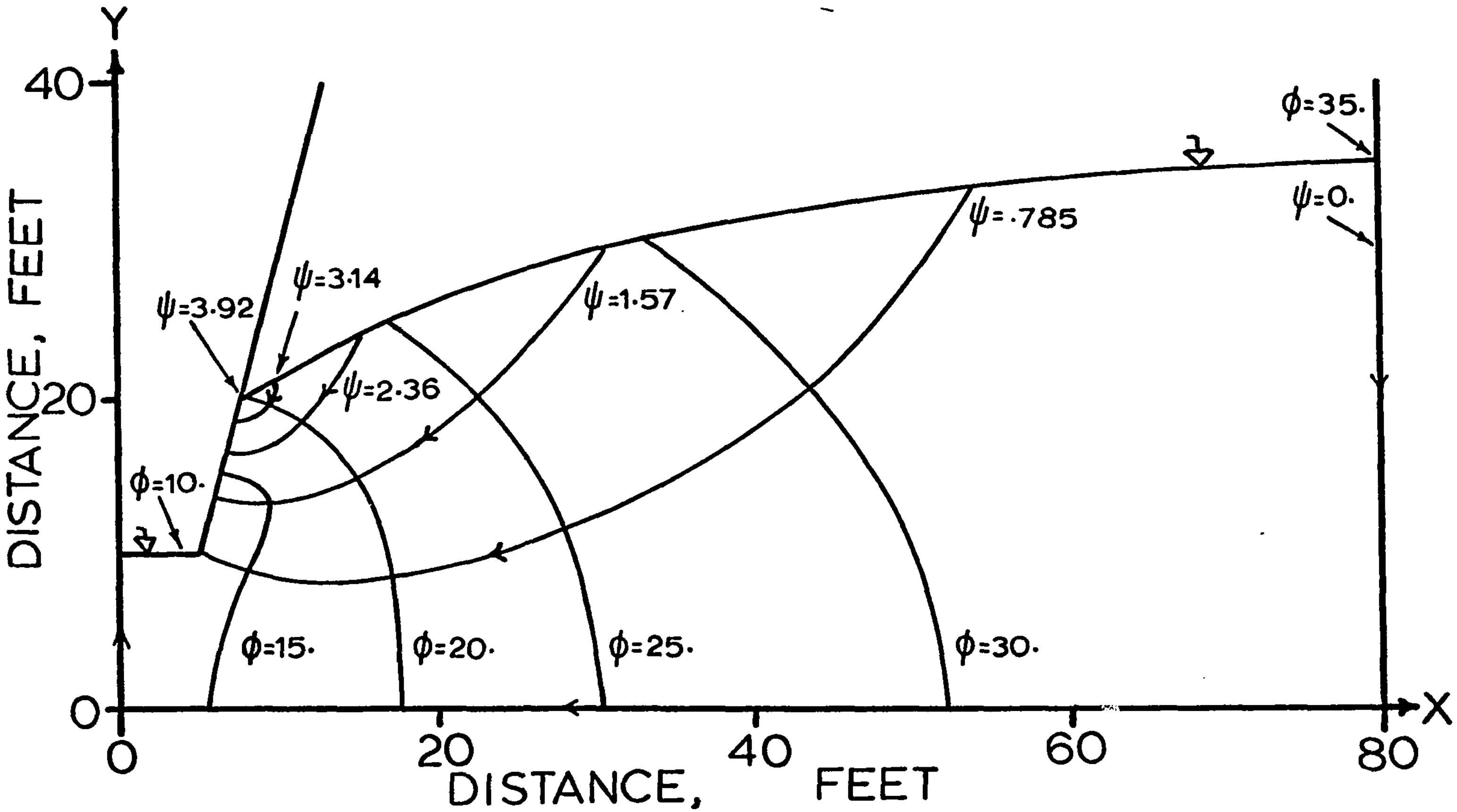



Figure 13. Flow net obtained using the recharge model for an arbitrary but typical flow region geometry

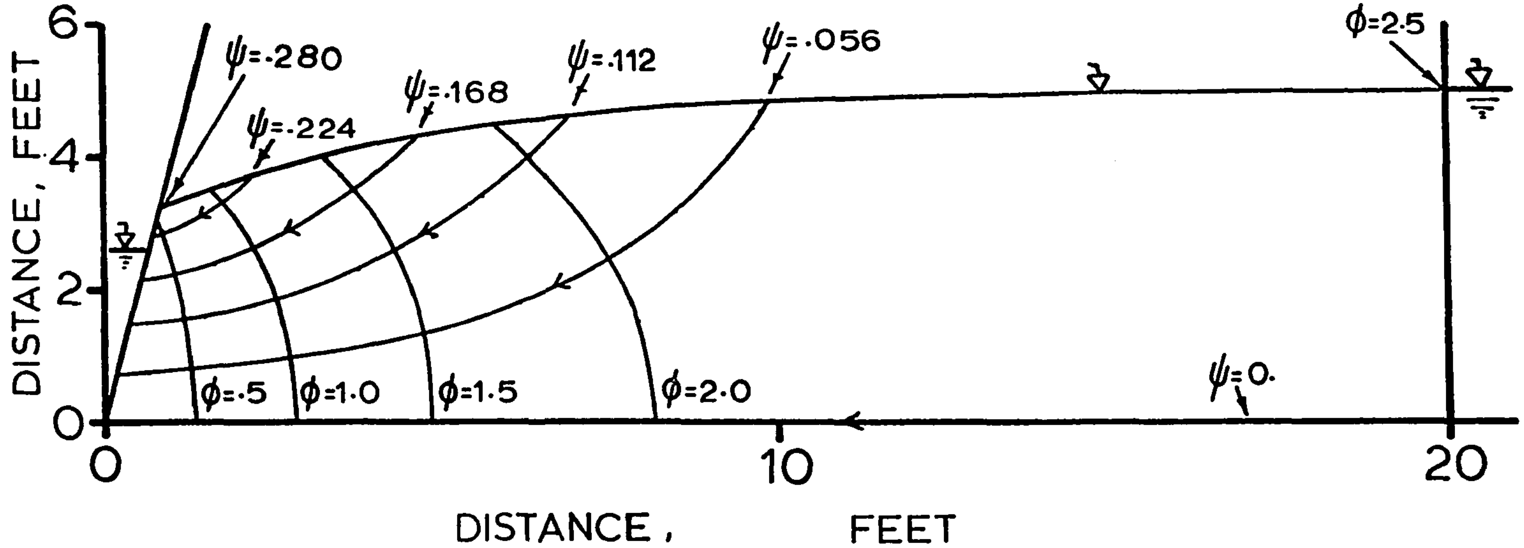
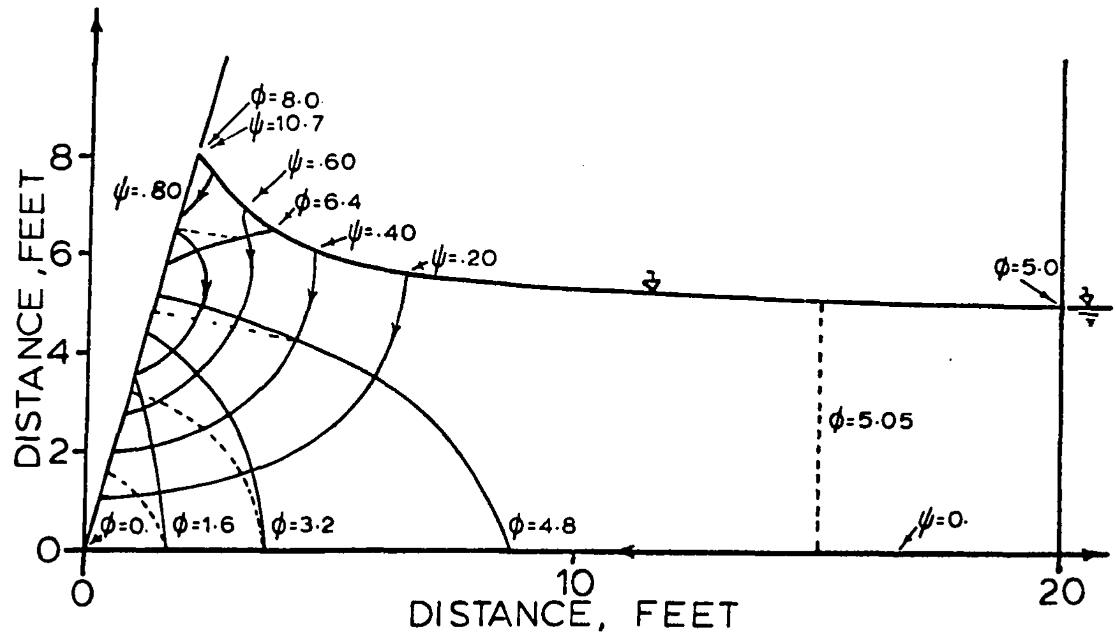
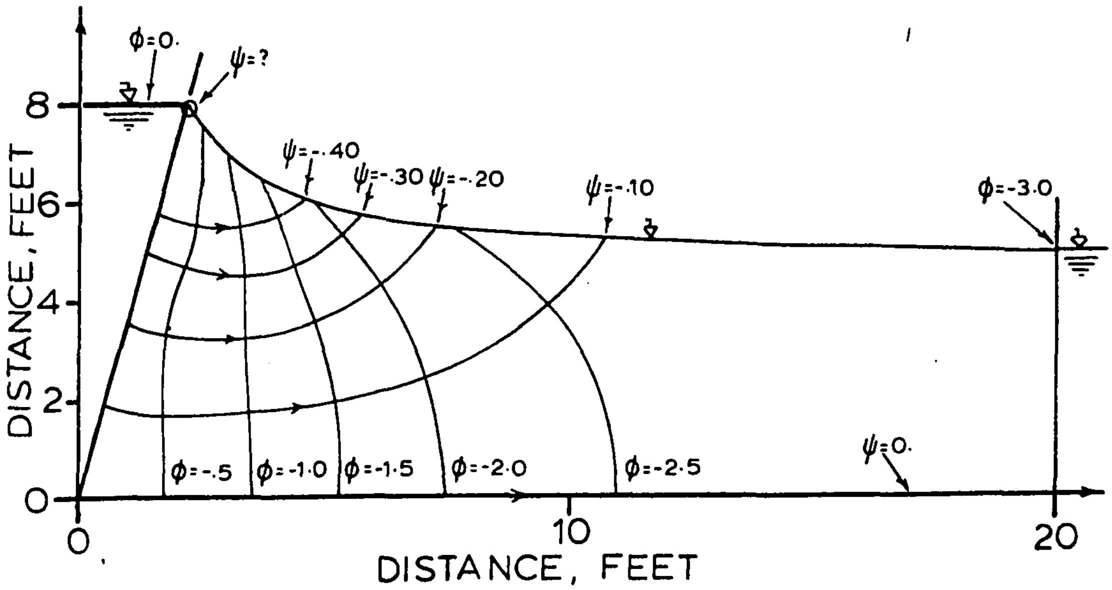



Figure 14. Flow net obtained using the rapid drawdown model for an arbitrary but typical flow region geometry

Figure 15a. Flow net obtained using the rapid drawdown model for an extreme case of rapid drawdown. Water level in the gully has fallen to zero

Figure 15b. Flow net for extreme rapid drawdown. High water level in the gully


ϕ=8.0
ψ=10.7
ψ=.80
ψ=.60
ϕ=6.4
ψ=.40
ψ=.20
ϕ=5.0
ϕ=5.05
ϕ=0.
ϕ=1.6
ϕ=3.2
ϕ=4.8
ψ=0.
DISTANCE, FEET
DISTANCE, FEET


ϕ=0.
ψ=?
ψ=-.40
ψ=-.30
ψ=-.20
ψ=-.10
ϕ=-3.0
ϕ=-.5
ϕ=-1.0
ϕ=-1.5
ϕ=-2.0
ϕ=-2.5
ψ=0.
DISTANCE, FEET
DISTANCE, FEET

medium. Thus a transient flow system can be modeled as a series of such quasi-steady states. For Figure 15b we assume there is no seepage face so that the gully water and the water table are the same level at the bank. The value of $\psi$ at this point is uncertain as the solution seemed to break down, probably because of the cusp in the flow region. The units of $\psi$ in all the flow nets are $ft^2$/day for hydraulic conductivity K = 0.2 ft/day. The difference in the value of $\psi$ at two streamlines gives the flow between them in $ft^3$/day per foot thickness of the flow region perpendicular to the plane of the paper.

The flow net of Figure 15a is for an instant of time later when the gully water level has fallen to zero. The whole bank face below the water table is now a seepage face. The streamlines show the change in direction of flow so that water is in this case moving out of the bank as base flow and the water table is falling.

While the flow nets shown in Figures 15a and b were again intuitively reasonable, the approximation by the PKS method of $\phi$ along the bank face for the case in Figure 15a was not good. This is shown by the deviation of the circles from the required function between the origin and point b in the first graph of Figure 12. These errors are reflected by errors in the flow net. In Figure 15a, we know that along the seepage face $\phi$ is equal to the height above the x axis y. The dotted equipotential lines were sketched in to show the discrepancy between known and calculated values of $\phi$ at the bank face. However, their true paths back in the flow region are only surmised.

For all cases considered, the solutions gave good approximations to $\phi$ along the water table and relatively inaccurate approximations along the gully bed and bank segments of the boundary. The case in Figures 12 and 15a was the worst case encountered. Reference to equations 20 and 21 shows that the integrals $w_m$ and $u_{mn}$ for the PKS method are broken into parts corresponding to segments of the boundary being represented, that is, the gully bed or portion of the gully face below the water table, the seepage face and the water table. The integrals representing segments of the boundary around the gully bed and the banks are over small ranges of x and are small in comparison to the integrals representing the water table segment. This is one reason for the relatively inaccurate approximation of $\phi$ at the gully. Another is the discontinuities in the gradient of the boundary function F(x) where the water table intersects the bank face and where the bank face intersects the gully bed.

There are several possible ways of improving the solutions. The first is to increase N. This may prove to be impractical because the integrals of equations 18 and 19 involve sine and cosine functions, for example, equations 20 and

21. The number of bisections of the range $0 \leq x \leq L$ for the numerical integrations must be great enough so that there is an accurate representation of the harmonics produced by the sine or cosine functions. The number of harmonics over the range increases with N; therefore, so must the number of bisections. For large values of N the cost to run the program may be prohibitive.

A second improvement would simply be to choose upper boundary functions that have no discontinuities of gradient. However, such boundaries may not match practical cases.

A third possibility is to make the relative sizes of the integrals for each boundary segment more equal. For cases with less steep banks this would automatically happen, though another approach for the rapid drawdown case would be to move the fictitious source nearer to the gully bank. For the rapid drawdown case of Figure 15a the effect of moving the fictitious source towards the bank 5 feet and changing its height to 5.05 feet is assessed in Figure 16. The solid line shows $\phi(15,y)$. For the solution with the fictitious source at $x = 20$ feet moving the source in would make $\phi(15,y)$ a vertical equipotential as indicated by the dotted line. Figure 16 shows that there would be very small changes in the potential distribution for this case.

In spite of the described difficulties in matching the boundary conditions along the gully bed and banks, calculated

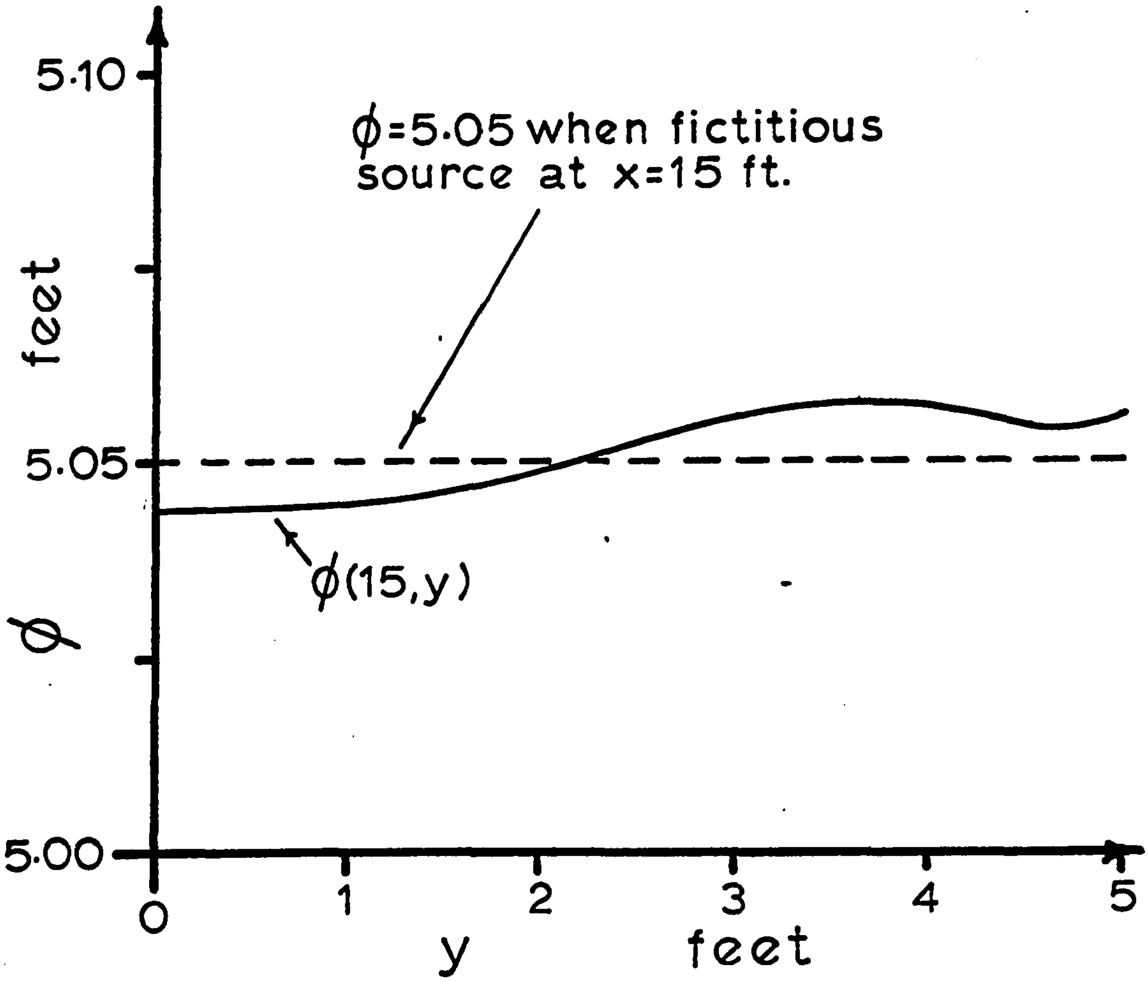



Figure 16. The effect on $\phi(15,y)$ of moving the fictitious source of Figure 15a from $x = 20$ to $x = 15$

values of $\phi$ further into the soil mass were assumed accurate enough for the calculation of pore water pressures.

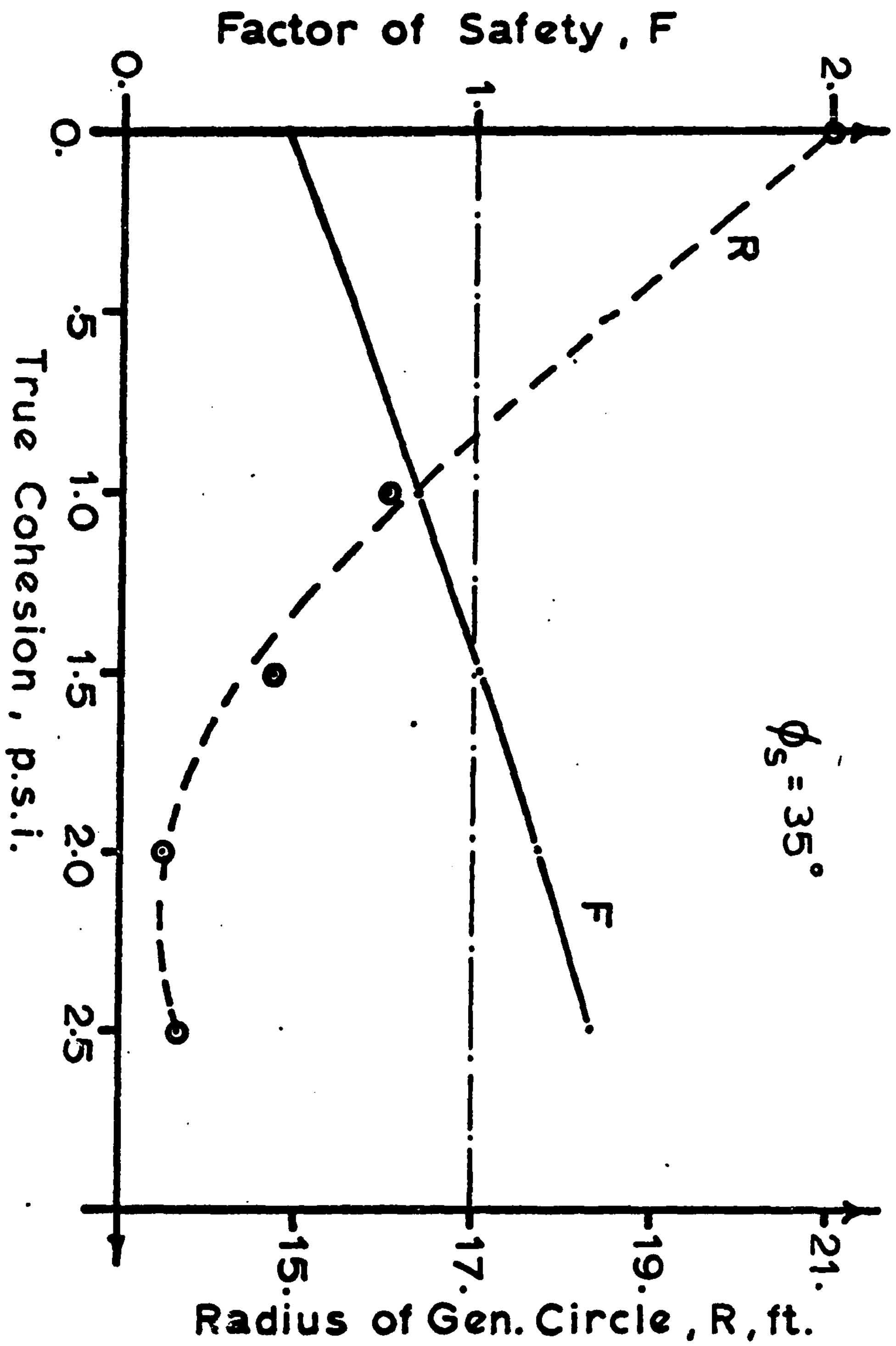
## Bank Stability Analyses

Stability analyses were made for banks of typical dimensions and soil properties using the flow systems of the recharge model in Figure 13 and the rapid drawdown model in Figures 14 and 15a.

The bank height chosen for the case in Figure 13 was 20 feet, which was twice the height of the seepage face above the gully bed. For the flow systems of Figures 14 and 15a, a 15 foot high bank was considered. All the banks sloped at 75 degrees to the vertical. Figure 17 shows how the predicted factor of safety and radius of failure plane generating circle change with true cohesion c, assuming the soil angle of internal shearing resistance is constant at 35 degrees. For cohesion greater than around 1.5 psi the model predicted $F > 1$, that is, a stable condition.

Figure 18 shows similar graphs obtained using the rapid drawdown flow system. Curves $F_1$ and $R_1$ were for the extreme rapid drawdown case in Figure 15a and $F_2$ and $R_2$ were for the less severe case of Figure 14. As would logically be expected, the extreme case of Figure 15 required a higher value of cohesion to stabilize the slope. The radii of the generating circles were also somewhat higher so the size of the failure zone at $F = 1$ would be larger. The values of true soil

Figure 17. The effect of true cohesion on factor of safety F and radius of generating circle R. Using the recharge model in Figure 13 for a bank height of 20 feet, slope angle of 75 degrees and angle of internal shearing resistance of 35 degrees.


Factor of Safety , F
0.
1.
2.
0.
.5
1.0
1.5
2.0
2.5
True Cohesion , p.s.i.
R
F
$\phi_s = 35°$
15.
17.
19.
21.
Radius of Gen. Circle , R , ft.

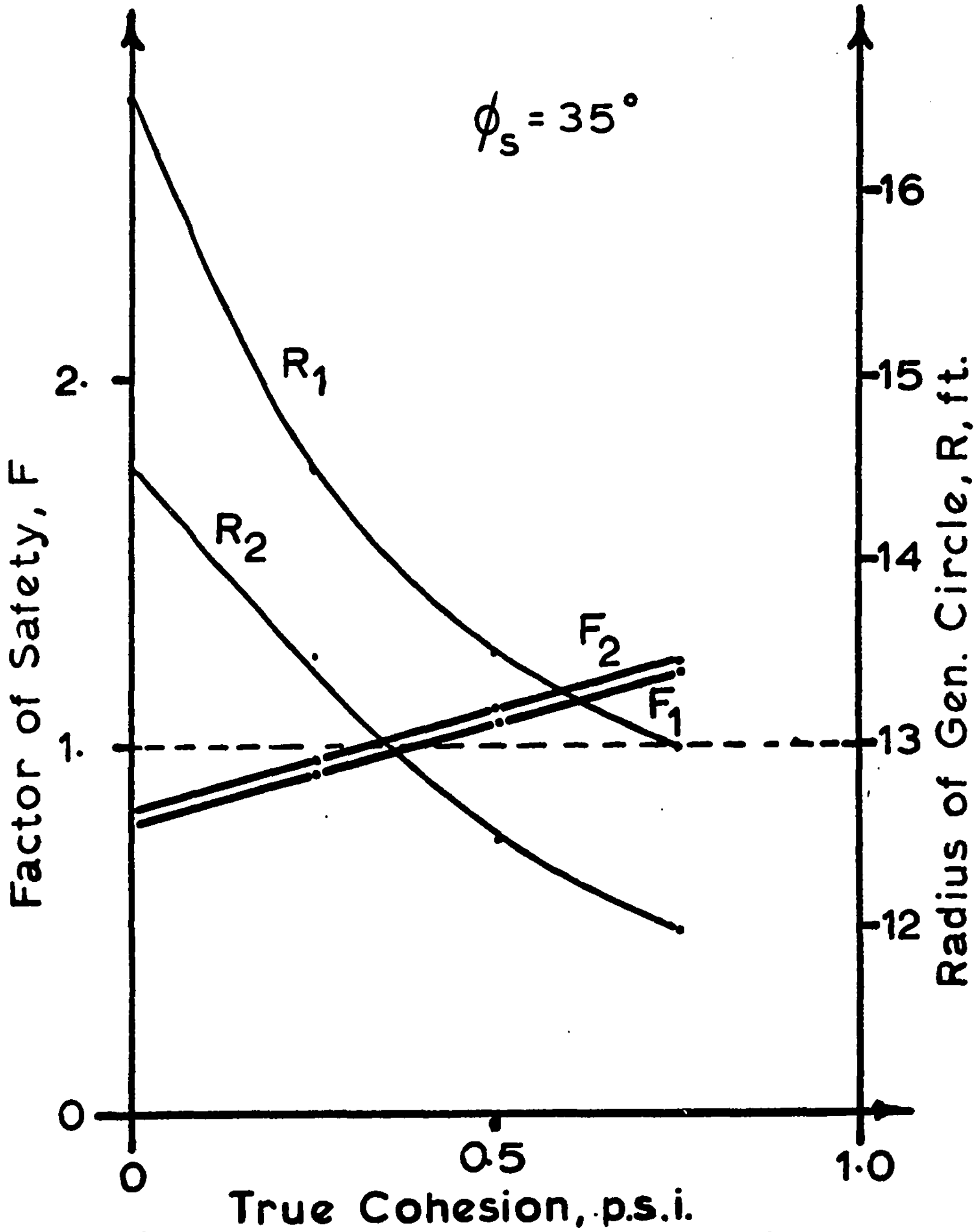



Figure 18. The effect of true cohesion of factor of safety F and radius of generating circle R. $F_1$ and $R_2$ are for the extreme rapid drawdown case in Figure 15a. $F_2$ and $R_2$ correspond to the rapid drawdown case of Figure 14. The bank height was 15 feet and the slope angle 75 degrees.

cohesion required for F = 1 were less than 0.5 psi in each case. The height of the bank relative to the water table was greater for the cases in Figure 18 than for the case in Figure 17.

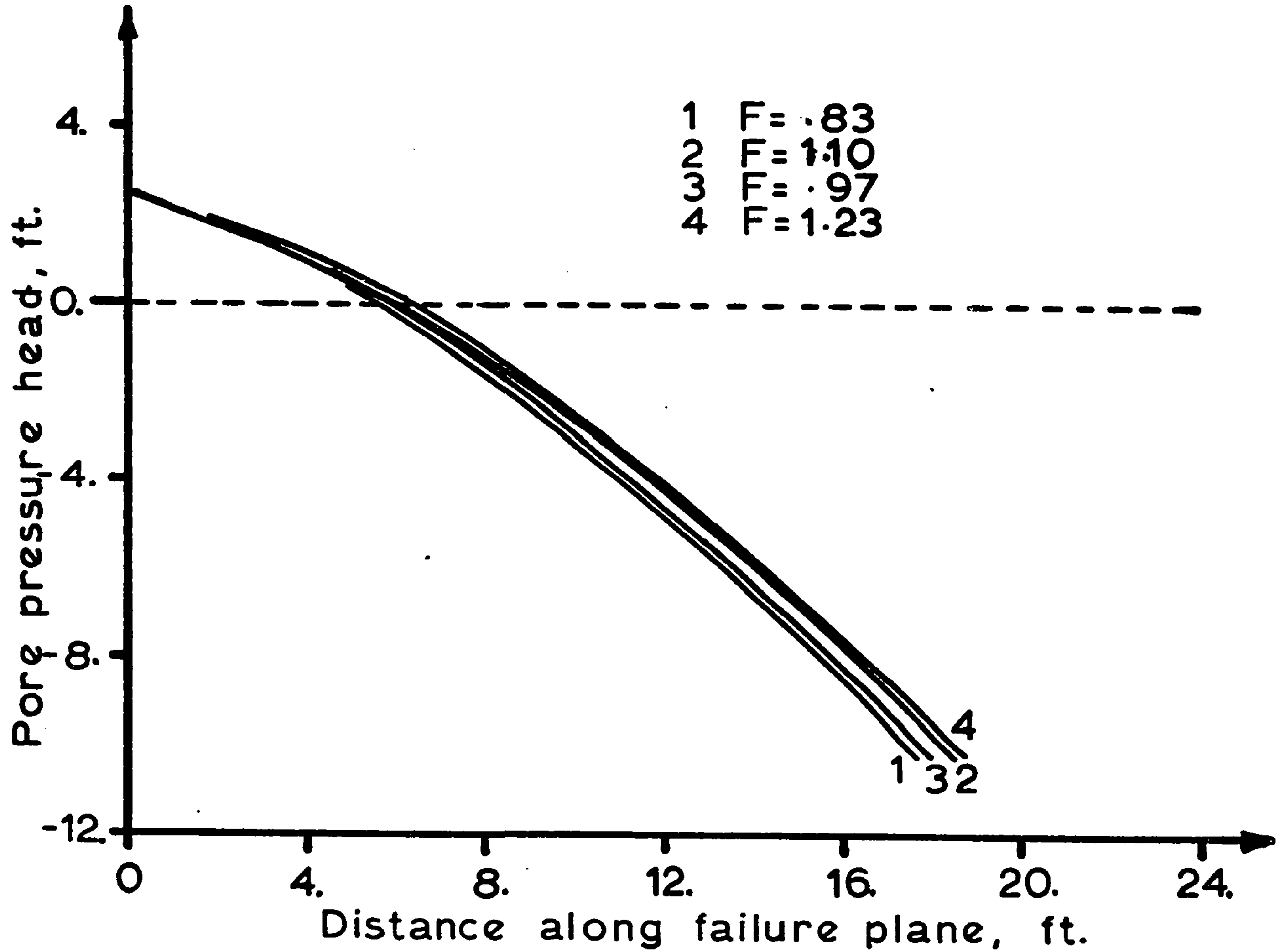
It is evident that much of the stabilizing strength of the bank is obtained from apparent cohesion through negative pore pressures as illustrated by the typical plots of pressure head versus distance along the failure plane in Figure 19. Therefore, the case in Figure 17 was relatively more severe because there was proportionately less soil above the water table. This is one reason why a greater value of soil cohesion was required to stabilize the bank. The shapes and proportions of the trial failure surfaces do not vary greatly and are similar to the geometries of failure planes observed in the field. Figure 20 shows the failure planes corresponding to the relationship $R_2$ versus C in Figure 18.

## Summary and Conclusions

Computer programs were written to simulate the two groundwater flow systems and perform bank stability analyses using cycloidal failure surfaces. Typical soil parameters for deep loess soils of western Iowa and typical groundwater and bank geometries were used as input data.

The reaction of the model to variations in true cohesion was explored, all other data remaining constant. Where seepage faces were high relative to the total bank height, higher

Figure 19. Calculated pore water pressure head distribution along the trial failure surfaces for the runs used to generate $F_2$ versus c in Figure 18


1 F= ·83
2 F= 1·10
3 F= ·97
4 F= 1·23
Pore pressure head, ft.
4.
0.
-4.
-8.
-12.
0
4.
8.
12.
16.
20.
24.
Distance along failure plane, ft.
1
3
2
4

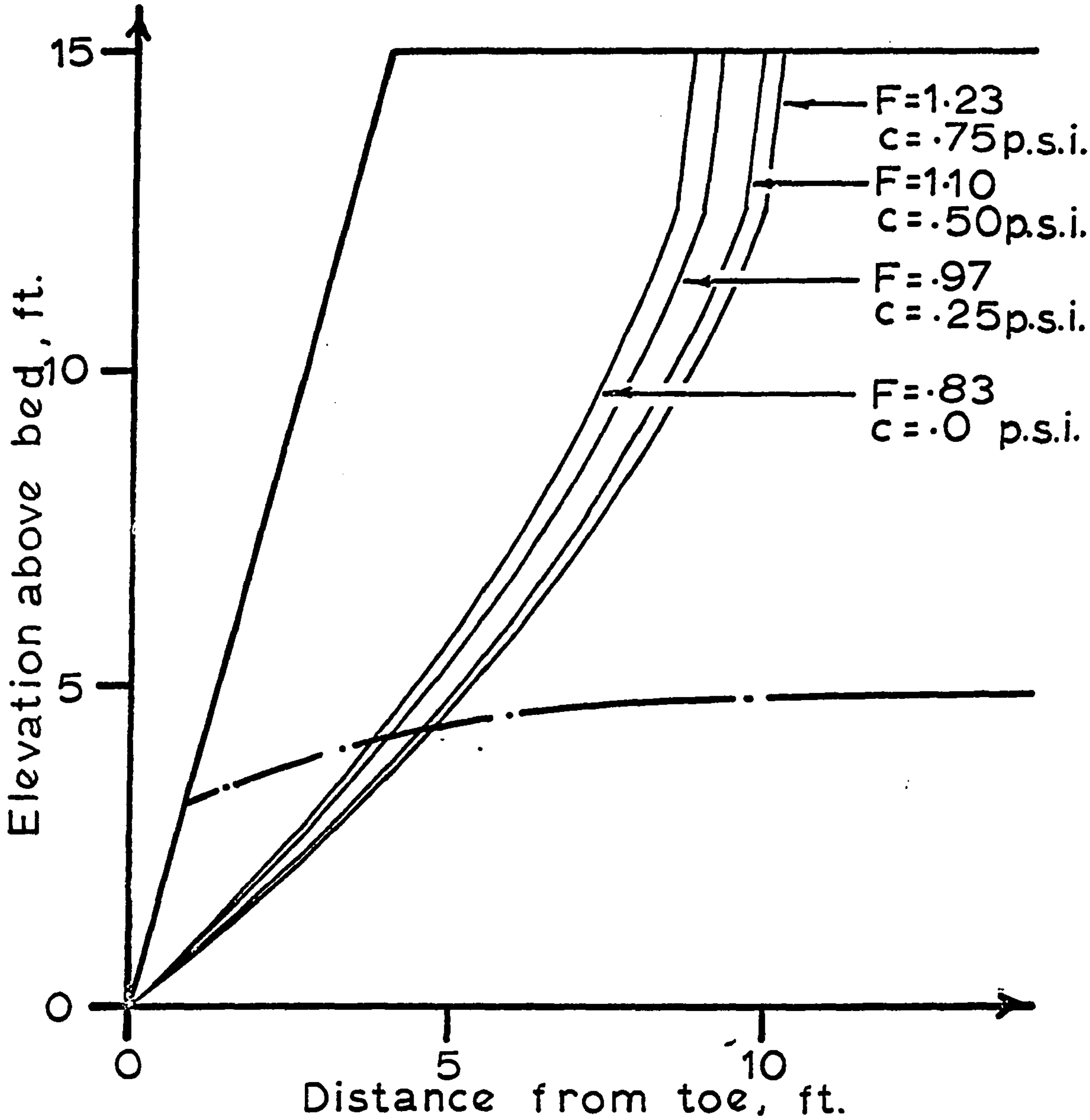



Figure 20. Generated cycloidal failure surfaces giving minimum values of F corresponding to the runs used to generate the curve $F_2$ versus c in Figure 18

values of true cohesion were required to stabilize the bank. Small increases in true cohesion changed the model's prediction from failure to stability. Low values of true cohesion were required for the prediction of stable banks. Low experimental accuracy when determining true cohesion in field cases may thus cause difficulty when evaluating the model's predictions in specific cases.

The accuracy of predicted results could not be assessed because comparison with field cases was not attempted. However, the predictions using the "typical" values were again intuitively reasonable. All the performance characteristics, such as reactions to changes in internal shearing resistance $\phi_s$ and bank steepness $\theta$, could not be explored because of limitations in available time and money.

# FUTURE DEVELOPMENTS AND OTHER APPLICATIONS

The computer programs written for this thesis form only the first stage in the development of a bank erosion model. Many other processes have been ignored, such as the transportation of talus away from the toe of the banks and the effect of freeze - thaw cycles. Additionally, the model has not been field tested. In this section we consider some possibilities for future development of the model and additional applications.

## Time Dependent Solutions

When the water table in the rapid drawdown case is steady-state, it becomes a boundary streamline. When the water table of the recharge model is steady-state, the seepage through the gully bed and banks is equal to the recharge to the water table from the soil above. The streamlines intersecting the water tables in Figures 14, 15a and 15b indicate that those cases were not steady-state. Recharge was zero for the case in Figure 13, therefore, it also cannot be steady-state.

The models presented in this thesis can be extended to calculate steady-state conditions by using an equation derived by Kirkham and Gaskell (1950), which in our notation is

$$\Delta y_{wt} = \frac{\Delta t\ K}{f_p} (\partial\phi/\partial y - \partial\phi/\partial x \tan \theta_{wt}) \qquad (64)$$

where $\Delta y_{wt}$ is the fall of the water table in time $\Delta t$. The soil has saturated hydraulic conductivity K and drainable pore space $f_p$. The partial derivatives at points (x,y) along the water table are obtained by partial differentiation of the potential function $\phi(x,y)$ which is easily performed term by term in our series solutions.

When the new water table position is found a new set of $A_{Nm}$'s must be calculated. By repeatedly adjusting the water table levels through equation 64 and recalculating a new set of $A_{Nm}$'s, equilibrium of the water table is eventually reached. This procedure, first used by Boast (1970) to simulate falling water tables around wells, assumed that movement of the water table is slow compared to the velocity of sound so that each water table position may be considered a quasi-steady state; that is, steady-state for the time increment $\Delta t$.

Such an extension of the recharge model would be particularly valuable because the effect of different recharge rates on water table shape and the height of the seepage face could be evaluated. Field observations by Saxton and Spomer (1968) show how base flows from small experimental watersheds were increased by the application of conservation measures, especially level terraces. The modified recharge model could be applied to a field case such as theirs to simulate the effects on the water table of increasing the recharge rate by encouraging infiltration.

## Collapsible Soil Failure

Our bank stability analysis assumes that soil fails in shear. Handy (1973) proposed that loessal soils of western Iowa can be susceptible to collapsible failure. Collapsible failure occurs when soil grains suddenly collapse into the voids as a result of overburden pressure. This is precipitated by the loss of apparent cohesion when the soil moisture content increases.

Soil below the water table has probably collapsed already and is normally consolidated. Should the water table rise some small increment, then collapse could occur within the newly wetted layer. At gully banks and heads, the collapse could cause unfavorable stress distributions that could make the banks fail.

Such a process is not considered in the existing model. However, a model for the collapse process should be formulated and combined with a time dependent recharge model to evaluate any changes in bank stability brought about by changes in recharge rate.

## Other Applications

The models developed in this thesis can be applied to any similar stability problems. For example, the stability of excavation banks could be particularly suited because regular man-made soil boundaries can be approximated more closely in the model. Also, intensive soil and groundwater surveys are economically feasible so that a more accurate assessment of the flow region could be made. For excavations, short-term groundwater fluctuations are of interest nearer to the site, therefore the rapid drawdown model would be suitable.

## Final Conclusions

Two groundwater flow systems and a bank stability model were formulated and represented by computer programs. Data generated by the groundwater programs was used in the bank stability program for calculating pore water pressures along trial cycloidal failure arcs. The results generated by use of "typical" soil parameters and bank geometries showed that the model reacted in an "intuitively reasonable" fashion; that is, in accordance with general field experience. However, the absolute accuracy of the predictions for a specific case was not tested with field data.

Three-dimensional cases, such as the gully head, were not considered because three-dimensional methods for bank stability analysis are not available.

The mathematical models we have presented are flexible in that they may be applied to a wide range of stability problems. But they are not fully developed and we have proposed that they be extended to include time dependent solutions and to consider collapsible soil failures. We conclude that the objectives of this thesis have been met although more testing and development of the models is required.

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# APPENDIX A

```
C*********************************************************************
C
C     PROGRAM 1 CALCULATION OF A-NMS FOR RECHARGE MODEL
C
C*********************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 J(780)
      REAL*4 XB(151),YB(151),XLAB(5),YLAB(5),GLAB(5),DATLAB(5)
      REAL*4 FXS(129),XXS(129),WT(129)
      DIMENSION DFX(129),PHIWT(129)
      DIMENSION XX(129),YFIT(129)
      DIMENSION Q(11)
      DIMENSION UU(40),C(40),D(40),G(40),A(40)
      DIMENSION X(5),Y(129),Z(129),FX(129)
      INTEGER TOINT,TOINP1
      COMMON X,Y,Z,YFIT
      READ(5,985)XLAB,YLAB,GLAB,DATLAB
  985 FORMAT(20A4)
      READ(5,1005)INT1,INT2,INTS,MAX,NN,INTX
 1005 FORMAT(6I3)
      WRITE(6,1006)INT1,INT2,INTS,INTX,MAX,NN
 1006 FORMAT(' NUMBER OF INTERVALS IN INTEGRATIONS = ',I3,3X,I3,3X,I3/,
     X' NUMBER OF WATERTABLE INTERVALS READ IN = ',I3,/,
     X' MAX. NO. OF A-NMS TO BE CALCULATED = ',I3,/,
     X' ORDER OF POLYNOMIAL TO BE FITTED TO WATERTABLE SHAPE = ',I3)
      READ(5,1015)WW,AA,B,THETA,DL,HS,T
 1015 FORMAT(8F10.0)
      NBPTS=INTS+4
      NCPTS=INTS+23
      NNP1=NN+1
      NPTS=INTS+1
      IXP1=INTX+1
      IP1=INTS+1
      PI=3.141592653589793
      X(1)=0.D0
      TTHETA=DTAN(THETA*PI/180.D0)
      X(2)=T
```

```
      X(3)=T+(WW/TTHETA)
      X(4)=T+(HS/TTHETA)
      X(5)=DL
      WRITE(6,1016)WW,HS,AA,B,DL,THETA,T
 1016 FORMAT(' DEPTH OF WATER IN GULLY = ',F10.4/,
     X' HEIGHT OF INITIAL SEEPAGE FACE = ',F10.4,/,
     X' DEPTH TO IMPERMEABLE BARRIER = ',F10.4,/,
     X' MAXIMUM DRIVING HEAD = ',F10.4,/,
     X ' LENGTH OF FLOW REGION = ',F10.4,/,
     X' SLOPE OF BANK TO HORIZONTAL = ',F10.4,/,
     X' HALF WIDTH OF GULLY = ',F10.4,/)
C  READ IN WATERTABLE ELEVATIONS ABOVE THE X-AXIS
      READ(5,1025)(FX(I),I=1,IXP1)
 1025 FORMAT(8F10.0)
      WRITE(6,1026)
 1026 FORMAT(//' INITIAL WATERTABLE HEIGHTS '/)
      WRITE(6,1036)(FX(I),I=1,IXP1)
 1036 FORMAT(10F10.4,3X)
      KA=MAX
      KAM1=KA-1
      KADIAG=(KA*KAM1)/2
C  FIT CURVE TO WATERTABLE
C  CALCULATE X'S FOR WATERTABLE HEIGHTS
      DELX=DABS(X(4)-X(5))/INTX
      XX(1)=X(4)
      XX(IXP1)=X(5)
      DO 100 I=2,INTX
      XX(I)=XX(I-1)+DELX
  100 CONTINUE
      WRITE(6,1046)
 1046 FORMAT(//' DISTANCES ALONG X-AXIS CORRESPONDING TO FED IN
     X WATERTABLE HEIGHTS '/)
      WRITE(6,1036)(XX(I),I=1,IXP1)
C  WEIGHT VALUES OF INDEPENDANT VARIABLE TO BE FITTED
  101 CONTINUE
      BESLHS=0.D0
      DO 99 I=1,IXP1
```

```
      FXS(I)=FX(I)
      XXS(I)=XX(I)
      WT(I)=1.0
      IF(I.LE.10)WT(I)=10.
      IF(I.GE.30)WT(I)=10.
   99 CONTINUE
C
C  FIT YFIT(I)=Q(1)+Q(2)*X(I)+Q(3)*X(I)*X(I)+.......
C
      CALL OPLSPA(NN,IXP1,XXS,FXS,WT,Q,0.)
      WRITE(6,1076)
 1076 FORMAT(//' COEFFICIENTS OF EQUATION YFIT '/)
      WRITE(6,1086)(Q(I),I=1,NNP1)
      WRITE(7,1087)(Q(I),I=1,NNP1)
 1087 FORMAT(3D24.17)
 1086 FORMAT(4(6X,D24.17))
      DELX=DABS(X(4)-X(5))/INTS
      XX(1)=X(4)
      YFIT(1)=FX(1)
      XX(IP1)=X(5)
      YFIT(IP1)=FX(IXP1)
      DO 200 JJ=2,INTS
      XX(JJ)=XX(JJ-1)+DELX
      YFIT(JJ)=Q(1)
      XDUM=1.D0
      DO 300 K=1,NN
      XDUM=XDUM*XX(JJ)
  300 YFIT(JJ)=YFIT(JJ)+(Q(K+1)*XDUM)
  200 CONTINUE
      WRITE(6,1056)
 1056 FORMAT(//' CALCULATED WATERTABLE HEIGHTS '/)
      WRITE(6,1036)(YFIT(I),I=1,IP1)
      WRITE(6,1066)
 1066 FORMAT(//' DISTANCES ALONG X-AXIS CORRESPONDING TO CALCULATED
     X WATERTABLE HEIGHTS '/)
      WRITE(6,1036)(XX(I),I=1,IP1)
C  CALCULATE THE RHS OF BESSEL'S INEQUALITY
```

```
      BCKDUM=0.
      CALL BRHS(INTS,TTHETA,AA,DL,BESRHS,WW)
C     CALCULATE THE UMN'S
      M=0
      MCOU=5
    1 CONTINUE
      MP1=M+1
      DO 10 I=1,MP1
      N=I-1
      CALL UMN(NN,INT1,INT2,INTS,U,M,N,DL,TTHETA,B,AA,WW)
      UU(I)=U
   10 CONTINUE
C     CALCULATE THE WM'S
      CALL WM(NN,INT1,INT2,INTS,M,W,DL,TTHETA,B,AA,WW)
C     CALCULATE THA ANM'S
      CALL ORTH(UU,W,C,D,G,J,A,MP1,KA,KAM1,KADIAG,IER)
C     CALCULATE THE LHS OF BESSELS INEQUALITY
      BESLHS=BESLHS+(A(MP1)*A(MP1)*D(MP1))
      BESCHK=BESLHS/BESRHS
      WRITE(6,1096)M,BESCHK
 1096 FORMAT(/,' M = ',I3,5X,' BESSELS CHECK = ',D24.17)
      BCKDUM=BESCHK
      IF(BESCHK.GE..9999)GOTO 500
      IF(MP1.GE.MAX)GOTO 500
      IF(MCOU.LT.5)GOTO 98
      MCOU=0
  500 CONTINUE
      XB(1)=0.
      YB(1)=0.
      XB(2)=0.
      YB(2)=AA
      XB(3)=T
      YB(3)=AA
      XB(4)=T+HS/TTHETA
      YB(4)=AA+HS
      DO 97 I=5,NBPTS
      XB(I)=XX(I-3)
```

```
      YB(I)=YFIT(I-3)
   97 CONTINUE
      CALL GRAPH(NBPTS,XB,YB,1,4,10., 5.,8.,0.,8.,0.,XLAB,YLAB,GLAB,
     XDATLAB)
C     EVALUATE THE PHI ALONG BOUNDARIES 1. 2. AND 3.
   20 XP=X(1)
      WRITE(6,1106)
 1106 FORMAT(//,' A-NM VALUES '/)
      WRITE(6,1086)(A(I),I=1,MP1)
      WRITE(7,1087)(A(I),I=1,MP1)
      WRITE(6,1116)
 1116 FORMAT(//,' ALONG GULLY BED AND BANK '/,'         XP         YP
     X    PHIXY ')
      DELXP=DABS(X(2)-X(1))/10
      YP=AA
      DO 30 I=1,11
      CALL PHI(XP,YP,PHIXY,MP1,DL,H,A,KA,B)
      WRITE(6,1126)XP,YP,PHIXY
 1126 FORMAT(3(F10.4,3X))
      YB(I)=PHIXY
      XB(I)=XP
      XP=XP+DELXP
      IF(I.EQ.INTS)XP=X(2)
   30 CONTINUE
C     EVALUATE PHI ALONG THE BANK
      DELXP=DABS(X(4)-X(2))/10
      XP=X(2)
      DO 40 I=1,11
      YP=AA+(XP-X(2))*TTHETA
      CALL PHI(XP,YP,PHIXY,MP1,DL,H,A,KA,B)
      WRITE(6,1126)XP,YP,PHIXY
      YB(I+11)=PHIXY
      XB(I+11)=XP
      XP=XP+DELXP
      IF(I.EQ.INTS)XP=X(4)
   40 CONTINUE
      WRITE(6,1146)
```

```
 1146 FORMAT(//' HYDRAULIC HEAD ALONG WATERTABLE '/)
C     EVALUATE PHI ALONG THE WATERTABLE
      XPT=X(4)
      DELX=DABS(X(4)-X(5))/INTS
      DO 59 I=1,NPTS
      YPT=YFIT(I)
      CALL PHI(XPT,YPT,PHIXY,MP1,DL,H,A,KA,B)
      PHIWT(I)=PHIXY
      YB(I+22)=PHIWT(I)
      XB(I+22)=XPT
      XPT=XPT+DELX
   59 CONTINUE
      WRITE(6,1086)(PHIWT(I),I=1,NPTS)
      CALL GRAPHS(NCPTS,XB,YB,1,7,';')
   98 CONTINUE
      MCOU=MCOU+1
      IF(BESCHK.GE..9999)STOP
      IF(MP1.GE.MAX)STOP
      M=M+1
      GOTO 1
      END
C
C
C
      SUBROUTINE WM(NN,INTX1,INTX2,INTX3,M,W,DL,TTHETA,B,AA,WW)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON X(5),Y(129),Z(129),FX(129)
      WY(CAPFX,AM1,XX,Y1,B)=(DCOSH(AM1*Y1)/DCOSH(AM1*B))*CAPFX*DCOS(AM1*
     XXX)
      IP1=INTX1+1
      PI=3.141592653589793
      AM1=M*PI/DL
      DELTAX=DABS(X(1)-X(2))/INTX1
      XX=X(1)
      DO 10 I=1,IP1
      CAPFX=AA+WW
      Y1=AA
```

```
      Y(I)=WY(CAPFX,AM1,XX,Y1,B)
      XX=XX+DELTAX
      IF(I.GE.INTX1)XX=X(2)
   10 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      WI1=Z(IP1)
      DELTAX= DABS(X(2)-X(4))/INTX2
      IP1=INTX2+1
      DO 30 I=1,IP1
      CAPFX=AA+(XX-X(2))*TTHETA
      Y1=AA+(XX-X(2))*TTHETA
      Y(I)=WY(CAPFX,AM1,XX,Y1,B)
      XX=XX+DELTAX
      IF(I.GE.INTX2)XX=X(4)
   30 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      WI2=Z(IP1)
      DELTAX=DABS(X(4)-X(5))/INTX3
      IP1=INTX3+1
      DO 40 I=1,IP1
      CAPFX=FX(I)
      Y1=FX(I)
      Y(I)=WY(CAPFX,AM1,XX,Y1,B)
      XX=XX+DELTAX
      IF(I.GE.INTX3)XX=X(5)
   40 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      WI3=Z(IP1)
      W=WI1+WI2+WI3
      RETURN
      END
C
C
C
      SUBROUTINE UMN(NN,INTX1,INTX2,INTX3,U,M,N,DL,TATHET,B,AA,WW)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON X(5),Y(129),Z(129),FX(129)
```

```
C
C   FUNCTION FOR INTEGRATION
C
      ABCY(AM1,AN1,X1,Y1,DL,B)=(DCOSH(AM1*Y1)/DCOSH(AM1*B))
     X*(DCOSH(AN1*Y1)/DCOSH(AN1*B))*DCOS(AM1*X1)*DCOS(AN1*X1)
      PI=3.141592653589793
      AM1=M*PI/DL
      AN1=N*PI/DL
      DELTAX=DABS(X(1)-X(2))/INTX1
      IP1=INTX1+1
      XX=X(1)
      DO 10 I=1,IP1
      Y(I)=ABCY(AM1,AN1,XX,AA,DL,B)
      XX=XX+DELTAX
      IF(I.GE.INTX1 )XX=X(2)
   10 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      AI1=Z(IP1)
      DELTAX= DABS(X(2)-X(4))/INTX2
      IP1=INTX2+1
      DO 20 I=1,IP1
      Y1=AA+((XX-X(2))*TATHET)
      Y(I)=ABCY(AM1,AN1,XX,Y1,DL,B)
      XX=XX+DELTAX
      IF(I.GE.INTX2)XX=X(4)
   20 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      AI2=Z(IP1)
      DELTAX=DABS(X(4)-X(5))/INTX3
      IP1=INTX3+1
      DO 30 I=1,IP1
      Y(I)=ABCY(AM1,AN1,XX,FX(I),DL,B)
      XX=XX+DELTAX
      IF(I.GE.INTX3)XX=X(5)
   30 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      AI3=Z(IP1)
```

```
      U = AI1+AI2+AI3
      RETURN
      END
C
C
C
      SUBROUTINE PHI(X,Y,PHIXY,M,DL,H,A,KA,B)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(KA)
      UM(AM1,X,Y,B)=(DCOSH(AM1*Y)/DCOSH(AM1*B))*DCOS(AM1*X)
      PI=3.141592653589793
      MM=0
      PHIXY=0.
    1 AM1=MM*PI/DL
      MP1=MM+1
      PHIXY=PHIXY+A(MP1)*UM(AM1,X,Y,B)
      IF(MP1.EQ.M)GOTO 2
      MM=MM+1
      GOTO 1
    2 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE BRHS(INTS,TTHETA,AA,DL,BESRHS,WW)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON X(5),Y(129),Z(129),FX(129)
      IP1=INTS+1
      BI1=AA*AA*X(2)
      DELX=DABS(X(2)-X(4))/INTS
      XX=X(2)
      DO 10 I=1,IP1
      Y(I)=(AA+(XX-X(2))*TTHETA)*(AA+(XX-X(2))*TTHETA)
      XX=XX+DELX
      IF(I.GE.INTS)XX=X(4)
   10 CONTINUE
```

```
      CALL DQSF(DELX,Y,Z,IP1)
      BI2=Z(IP1)
      DELX=DABS(X(4)-X(5))/INTS
      DO 20 I=1,IP1
      Y(I)=FX(I)*FX(I)
   20 CONTINUE
      CALL DQSF(DELX,Y,Z,IP1)
      BI3=Z(IP1)
      BESRHS=BI1+BI2+BI3
      RETURN
      END
```

```
C**********************************************************************
C
C     PROGRAM 2 CALCULATION OF A-NMS FOR RAPID DRAWDOWN MODEL
C
C**********************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 J(780)
      REAL*4 XB(153),YB(153),XLAB(5),YLAB(5),GLAB(5),DATLAB(5)
      REAL*4 FXS(129),XXS(129),WT(129)
      DIMENSION DFX(129),PHIWT(129)
      DIMENSION XX(129),YFIT(129)
      DIMENSION Q(11)
      DIMENSION UU(40),C(40),D(40),G(40),A(40)
      DIMENSION X(5),Y(129),Z(129),FX(129)
      INTEGER TOINT,TOINP1
      COMMON X,Y,Z,YFIT
      READ(5,985)XLAB,YLAB,GLAB,DATLAB
  985 FORMAT(20A4)
      READ(5,1005)INT1,INT2,INTS,MAX,NN,INTX
 1005 FORMAT(6I3)
      WRITE(6,1006)INT1,INT2,INTS,INTX,MAX,NN
 1006 FORMAT(' NUMBER OF INTERVALS IN INTEGRATIONS = ',I3,3X,I3,3X,I3/,
     X' NUMBER OF WATERTABLE INTERVALS READ IN = ',I3,/,
     X' MAX. NO. OF A-NMS TO BE CALCULATED = ',I3,/,
     X' ORDER OF POLYNOMIAL TO BE FITTED TO WATERTABLE SHAPE = ',I3)
      READ(5,1015)WW,AA,B,THETA,DL,HS,T
 1015 FORMAT(8F10.0)
      NBPTS=INTS+4
      NCPTS=INTS+23
      NNP1=NN+1
      NPTS=INTS+1
      IXP1=INTX+1
      IP1=INTS+1
      PI=3.141592653589793
      X(1)=0.D0
      TTHETA=DTAN(THETA*PI/180.D0)
      X(2)=(WW-AA)/TTHETA
```

```
      IF((WW.EQ.0.D0).AND.(AA.EQ.0.D0))X(2)=0.D0
      X(3)=0.
      X(4)=(HS-AA)/TTHETA
      X(5)=DL
      H=B-WW
      WRITE(6,1016)WW,HS,AA,B,DL,THETA,H
 1016 FORMAT(' DEPTH OF WATER IN GULLY = ',F10.4/,
     X' HEIGHT OF INITIAL SEEPAGE FACE = ',F10.4,/,
     X' HEIGHT OF VERTICAL SECTION OF BANK = ',F10.4,/,
     X' HEIGHT OF FICTICIOUS SOURCE = ',F10.4,/,
     X ' LENGTH OF FLOW REGION = ',F10.4,/,
     X' SLOPE OF BANK TO HORIZONTAL = ',F10.4,/,
     X' MAXIMUM DRIVING HEAD = ',F10.4,/)
      IF(HS.GT.B)B=HS
C  READ IN WATERTABLE ELEVATIONS ABOVE THE X-AXIS
      READ(5,1025)(FX(I),I=1,IXP1)
 1025 FORMAT(8F10.0)
      WRITE(6,1026)
 1026 FORMAT(//' INITIAL WATERTABLE HEIGHTS '/)
      WRITE(6,1036)(FX(I),I=1,IXP1)
 1036 FORMAT(10F10.4,3X)
      KA=MAX
      KAM1=KA-1
      KADIAG=(KA*KAM1)/2
C  FIT CURVE TO WATERTABLE
C  CALCULATE X'S FOR WATERTABLE HEIGHTS
      DELX=DABS(X(4)-X(5))/INTX
      XX(1)=X(4)
      XX(IXP1)=X(5)
      DO 100 I=2,INTX
      XX(I)=XX(I-1)+DELX
  100 CONTINUE
      WRITE(6,1046)
 1046 FORMAT(//' DISTANCES ALONG X-AXIS CORRESPONDING TO FED IN
     X WATERTABLE HEIGHTS '/)
      WRITE(6,1036)(XX(I),I=1,IXP1)
C  WEIGHT VALUES OF INDEPENDANT VARIABLE TO BE FITTED
```

```
  101 CONTINUE
      BESLHS=0.D0
      DO 99 I=1,IXP1
      FXS(I)=FX(I)
      XXS(I)=XX(I)
      WT(I)=1.0
      IF(I.LE.10)WT(I)=10.
      IF(I.GE.30)WT(I)=10.
   99 CONTINUE
C
C   FIT YFIT(I)=Q(1)+Q(2)*X(I)+Q(3)*X(I)*X(I)+.......
C
      CALL OPLSPA(NN,IXP1,XXS,FXS,WT,Q,0.)
      WRITE(6,1076)
 1076 FORMAT(//' COEFFICIENTS OF EQUATION YFIT '/)
      WRITE(6,1086)(Q(I),I=1,NNP1)
 1086 FORMAT(4(6X,D24.17))
      WRITE(7,1087)(Q(I),I=1,NNP1)
 1087 FORMAT(3D24.17)
      DELX=DABS(X(4)-X(5))/INTS
      XX(1)=X(4)
      YFIT(1)=FX(1)
      XX(IP1)=X(5)
      YFIT(IP1)=FX(IXP1)
      DO 200 JJ=2,INTS
      XX(JJ)=XX(JJ-1)+DELX
      YFIT(JJ)=Q(1)
      XDUM=1.D0
      DO 300 K=1,NN
      XDUM=XDUM*XX(JJ)
  300 YFIT(JJ)=YFIT(JJ)+(Q(K+1)*XDUM)
  200 CONTINUE
      WRITE(6,1056)
 1056 FORMAT(//' CALCULATED WATERTABLE HEIGHTS '/)
      WRITE(6,1036)(YFIT(I),I=1,IP1)
      WRITE(6,1066)
 1066 FORMAT(//' DISTANCES ALONG X-AXIS CORRESPONDING TO CALCULATED
```

```
     X WATERTABLE HEIGHTS '/)
      WRITE(6,1036)(XX(I),I=1,IP1)
C  CALCULATE THE RHS OF BESSEL'S INEQUALITY
      BCKDUM=0.
      CALL BRHS(INTS,TTHETA,AA,DL,BESRHS,WW,H)
C     CALCULATE THE UMN'S
      M=1
      MCOU=5
    1 CONTINUE
      MP1=M
      DO 10 I=1,MP1
      N=I
      CALL UMN(NN,INT1,INT2,INTS,U,M,N,DL,TTHETA,B,AA,WW)
      UU(I)=U
   10 CONTINUE
C     CALCULATE THE WM'S
      CALL WM(NN,INT1,INT2,INTS,M,W,DL,TTHETA,B,AA,WW,H)
C     CALCULATE THA ANM'S
      CALL ORTH(UU,W,C,D,G,J,A,MP1,KA,KAM1,KADIAG,IER)
C     CALCULATE THE LHS OF BESSELS INEQUALITY
      BESLHS=BESLHS+(A(MP1)*A(MP1)*D(MP1))
      BESCHK=BESLHS/BESRHS
      WRITE(6,1096)M,BESCHK
 1096 FORMAT(/,' M = ',I3,5X,' BESSELS CHECK = ',D24.17)
      BCKDUM=BESCHK
      IF(BESCHK.GE..9999)GOTO 500
      IF(MP1.GE.MAX)GOTO 500
      IF(MCOU.LT.5)GOTO 98
      MCOU=0
  500 CONTINUE
      XB(1)=0.
      YB(1)=0.
      XB(2)=(WW-AA)/TTHETA
      YB(2)=0.
      XB(3)=(WW-AA)/TTHETA
      YB(3)=0.
      XB(4)=(HS-AA)/TTHETA
```

```
      YB(4)=HS-WW
      DO 97 I=5,NBPTS
      XB(I)=XX(I-3)
      YB(I)=YFIT(I-3)-WW
   97 CONTINUE
      CALL GRAPH(NBPTS,XB,YB,1,4,10., 5.,2.,0.,2.,0.,XLAB,YLAB,GLAB,
     XDATLAB)
C     EVALUATE THE PHI ALONG BOUNDARIES 1. 2. AND 3.
   20 XP=X(1)
      WRITE(6,1106)
 1106 FORMAT(//,' A-NM VALUES '/)
      WRITE(6,1086)(A(I),I=1,MP1)
      WRITE(6,1116)
      WRITE(7,1087)(A(I),I=1,MP1)
 1116 FORMAT(//,' ALONG GULLY BED AND BANK '/,'          XP          YP
     X    PHIXY ')
      DELXP=DABS(X(2)-X(1))/10
      YP=AA
      DO 30 I=1,11
      CALL PHI(XP,YP,PHIXY,MP1,DL,H,A,KA,B)
      WRITE(6,1126)XP,YP,PHIXY
 1126 FORMAT(3(F10.4,3X))
      YB(I)=PHIXY
      YB(I)=DABS(PHIXY)
      XB(I)=XP
      XP=XP+DELXP
      YP=AA+XP*TTHETA
      IF(I.EQ.INTS)XP=X(2)
   30 CONTINUE
C     EVALUATE PHI ALONG THE BANK
      DELXP=DABS(X(4)-X(2))/10
      XP=X(2)
      DO 40 I=1,11
      YP=AA+XP*TTHETA
      CALL PHI(XP,YP,PHIXY,MP1,DL,H,A,KA,B)
      WRITE(6,1126)XP,YP,PHIXY
      YB(I+11)=PHIXY
```

```
      YB(I+11)=DABS(PHIXY)
      XB(I+11)=XP
      XP=XP+DELXP
      IF(I.EQ.INTS)XP=X(4)
   40 CONTINUE
      WRITE(6,1146)
 1146 FORMAT(//' HYDRAULIC HEAD ALONG WATERTABLE '/)
C     EVALUATE PHI ALONG THE WATERTABLE
      XPT=X(4)
      DELX=DABS(X(4)-X(5))/INTS
      DO 59 I=1,NPTS
      YPT=YFIT(I)
      CALL PHI(XPT,YPT,PHIXY,MP1,DL,H,A,KA,B)
      PHIWT(I)=PHIXY
      YB(I+22)=PHIWT(I)
      YB(I+22)=DABS(PHIWT(I))
      XB(I+22)=XPT
      XPT=XPT+DELX
   59 CONTINUE
      WRITE(6,1086)(PHIWT(I),I=1,NPTS)
      CALL GRAPHS(NCPTS,XB,YB,1,7,';')
   98 CONTINUE
      MCOU=MCOU+1
      IF(BESCHK.GE..9999)STOP
      IF(MP1.GE.MAX)STOP
      M=M+1
      GOTO 1
      END
C
C
C
      SUBROUTINE WM(NN,INTX1,INTX2,INTX3,M,W,DL,TTHETA,B,AA,WW,H)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON X(5),Y(129),Z(129),FX(129)
      WY(CAPFX,AM1,XX,Y1,B)=(DCOSH(AM1*Y1)/DCOSH(AM1*B))*CAPFX*DSIN(AM1*
     XXX)
      IP1=INTX1+1
```

```
      PI=3.141592653589793
      AM1=M*PI/DL
      WI1=0.
      DELTAX=DABS(X(1)-X(2))/INTX1
      XX=X(1)
      IF(X(1).EQ.X(2))GOTO 11
      DO 10 I=1,IP1
      CAPFX=-XX/DL
      Y1=AA+XX*TTHETA
      Y(I)=WY(CAPFX,AM1,XX,Y1,B)
      XX=XX+DELTAX
      IF(I.GE.INTX1)XX=X(2)
   10 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      WI1=Z(IP1)
      WI2=0.
      IF(X(2).EQ.X(4))GOTO 12
   11 DELTAX=DABS(X(2)-X(4))/INTX2
      IP1=INTX2+1
      DO 30 I=1,IP1
      CAPFX=((AA+XX*TTHETA-WW)/H-XX/DL)
      Y1=AA+XX*TTHETA
      Y(I)=WY(CAPFX,AM1,XX,Y1,B)
      XX=XX+DELTAX
      IF(I.GE.INTX2)XX=X(4)
   30 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      WI2=Z(IP1)
   12 DELTAX=DABS(X(4)-X(5))/INTX3
      IP1=INTX3+1
      DO 40 I=1,IP1
      CAPFX=((FX(I)-WW)/H-XX/DL)
      Y1=FX(I)
      Y(I)=WY(CAPFX,AM1,XX,Y1,B)
      XX=XX+DELTAX
      IF(I.GE.INTX3)XX=X(5)
   40 CONTINUE
```

```
      CALL DQSF(DELTAX,Y,Z,IP1)
      WI3=Z(IP1)
      W=WI1+WI2+WI3
      RETURN
      END
C
C
C
      SUBROUTINE UMN(NN,INTX1,INTX2,INTX3,U,M,N,DL,TATHET,B,AA,WW)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON X(5),Y(129),Z(129),FX(129)
C
C   FUNCTION FOR INTEGRATION
C
      ABCY(AM1,AN1,X1,Y1,DL,B)=(DCOSH(AM1*Y1)/DCOSH(AM1*B))
     X*(DCOSH(AN1*Y1)/DCOSH(AN1*B))*DSIN(AM1*X1)*DSIN(AN1*X1)
      PI=3.141592653589793
      AM1=M*PI/DL
      AN1=N*PI/DL
      AI1=0.
      DELTAX=DABS(X(1)-X(2))/INTX1
      IP1=INTX1+1
      XX=X(1)
      IF(X(1).EQ.X(2))GOTO 11
      DO 10 I=1,IP1
      Y1=AA+XX*TATHET
      Y(I)=ABCY(AM1,AN1,XX,Y1,DL,B)
      XX=XX+DELTAX
      IF(I.GE.INTX1)XX=X(2)
   10 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      AI1=Z(IP1)
      AI2=0.
      IF(X(2).EQ.X(4))GOTO 12
   11 DELTAX=DABS(X(2)-X(4))/INTX2
      IP1=INTX2+1
      DO 20 I=1,IP1
```

```
      Y1=AA+XX*TATHET
      Y(I)=ABCY(AM1,AN1,XX,Y1,DL,B)
      XX=XX+DELTAX
      IF(I.GE.INTX2)XX=X(4)
   20 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      AI2=Z(IP1)
   12 DELTAX=DABS(X(4)-X(5))/INTX3
      IP1=INTX3+1
      DO 30 I=1,IP1
      Y(I)=ABCY(AM1,AN1,XX,FX(I),DL,B)
      XX=XX+DELTAX
      IF(I.GE.INTX3)XX=X(5)
   30 CONTINUE
      CALL DQSF(DELTAX,Y,Z,IP1)
      AI3=Z(IP1)
      U=AI1+AI2+AI3
      RETURN
      END
C
C
C
      SUBROUTINE PHI(X,Y,PHIXY,M,DL,H,A,KA,B)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(KA)
      UM(AM1,X,Y,B)=(DCOSH(AM1*Y)/DCOSH(AM1*B))*DSIN(AM1*X)
      PI=3.141592653589793
      MM=1
      PHIXY=X/DL
    1 AM1=MM*PI/DL
      MP1=MM
      PHIXY=PHIXY+A(MP1)*UM(AM1,X,Y,B)
      IF(MP1.EQ.M)GOTO 2
      MM=MM+1
      GOTO 1
    2 PHIXY=PHIXY*H
      RETURN
```

```
      END
C
C
C
      SUBROUTINE BRHS(INTS,TTHETA,AA,DL,BESRHS,WW,H)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON X(5),Y(129),Z(129),FX(129)
      IP1=INTS+1
      BI1=X(2)*X(2)*X(2)/(3.*DL*DL)
      DELX=DABS(X(2)-X(4))/INTS
      XX=X(2)
      DO 10 I=1,IP1
      Y(I)=(((AA+XX*TTHETA-WW)/H)-XX/DL)*(((AA+XX*TTHETA-WW)/H)-XX/DL)
      XX=XX+DELX
      IF(I.GE.INTS)XX=X(4)
   10 CONTINUE
      CALL DQSF(DELX,Y,Z,IP1)
      BI2=Z(IP1)
      DELX=DABS(X(4)-X(5))/INTS
      DO 20 I=1,IP1
      Y(I)=(((FX(I)-WW)/H)-XX/DL)*(((FX(I)-WW)/H)-XX/DL)
      XX=XX+DELX
   20 CONTINUE
      CALL DQSF(DELX,Y,Z,IP1)
      BI3=Z(IP1)
      BESRHS=BI1+BI2+BI3
      RETURN
      END
```

```
C**********************************************************************
C
C     PROGRAM 3 CALCULATION OF PHI AND PSI AT NODES
C                    RECHARGE CASE
C
C**********************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION XX(101),YY(101),PHIWT(101),PSIWT(101)
      REAL*8 A(40),Q(11)
  995 FORMAT(8I3)
 1015 FORMAT(3D24.17)
 1025 FORMAT(8F10.0)
 1036 FORMAT(/,' Y = ',F10.4)
 1046 FORMAT(10(2X,F10.4))
      READ(5,995)NA,NN,NWT,NFL,NPHIL,NPTS,INTX,INTY
      NNP1=NN+1
      PRINT,NA,NN,NWT,NFL,NPHIL,NPTS,INTX,INTY
      READ(5,1015)(A(I),I=1,NA)
      READ(5,1015)(Q(I),I=1,NNP1)
      PRINT,A
      PRINT,Q
      READ(5,1025)WW,AA,B,THETA,DL,HS,T,HYDCON
      PRINT,WW,AA,B,THETA,DL,HS,T,HYDCON
      PI=3.141592653589793
      TTHETA=DTAN(THETA*PI/180.)
C   GRAPH THE FLOW REGION
      XX(1)=0.
      YY(1)=0.
      XX(2)=0.
      YY(2)=AA
      XX(3)=T
      YY(3)=AA
      XX(4)=T+HS/TTHETA
      YY(4)=AA+HS
      PHIMAX=YY(4)
      DELX=DABS(XX(4)-DL)/NWT
      NWTP4=NWT+4
```

```
      DO 10 I=5,NWTP4
      XX(I)=XX(I-1)+DELX
      CALL HITWT(XX(I),Q,NN,YY(I))
      CALL PHI(XX(I),YY(I),PHIWT(I),NA,DL,H,A,NA,B)
      CALL PSI(XX(I),YY(I),PSIWT(I),NA,DL,A,NA,B,HYDCON)
      IF(YY(I).GT.PHIMAX)PHIMAX=YY(I)
   10 CONTINUE
      XX(NWTP4+1)=XX(NWTP4)
      YY(NWTP4+1)=0.
      NBPTS=NWTP4+1
      PRINT,XX
      PRINT,YY
      PRINT,PHIWT
      PRINT,PSIWT
C     CALCULATE PSIMAX
      CALL PSI(XX(4),YY(4),PSIMAX,NA,DL,A,NA,B,HYDCON)
C    CALCULATE PSIMIN
      CALL PSI(XX(NWTP4),YY(NWTP4),PSIMIN,NA,DL,A,NA,B,HYDCON)
      DELPSI=(PSIMAX-PSIMIN)/(NFL+1)
      PRINT,PSIMAX,PSIMIN,DELPSI
C    CALCULATE STREAMLINES
      DELX=DL/INTX
      NX=INTX+1
      DELY=(B+1)/INTY
      NY=INTY+1
C    CALCULATE PSI
      PRINT,DELX,DELY
      Y=0.
      DO 30 K=1,NY
      X=0.
      DO 31 J=1,NX
      CALL PSI(X,Y,PSIXY,NA,DL,A,NA,B,HYDCON)
      YY(J)=PSIXY
      XX(J)=X
      X=X+DELX
   31 CONTINUE
      WRITE(6,1036)Y
```

```
      IF(K.EQ.1)WRITE(6,1046)(XX(I),I=1,NX)
      WRITE(6,1046)( YY(I),I=1,NX)
      Y=Y+DELY
   30 CONTINUE
C    CALCULATE EQUIPOTENTIAL LINES
      PHIMIN=AA
      DELPHI=(PHIMAX-PHIMIN)/(NPHIL-1)
      PRINT,PHIMAX,PHIMIN,DELPHI
C    CALCULATE PHI
      Y=0.
      DO 40 K=1,NY
      X=0.
      DO 41 J=1,NX
      CALL PHI(X,Y,PHIXY,NA,DL,H,A,NA,B)
      YY(J)=PHIXY
      XX(J)=X
      X=X+DELX
   41 CONTINUE
      WRITE(6,1036)Y
      IF(K.EQ.1)WRITE(6,1046)(XX(I),I=1,NX)
      WRITE(6,1046)(YY(I),I=1,NX)
      Y=Y+DELY
   40 CONTINUE
      STOP
      END
C
C
C
      SUBROUTINE PHI(X,Y,PHIXY, M,DL,H,A,KA,B)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 A(KA)
      UM(AM1,X,Y,B)=(DCOSH(AM1*Y)/DCOSH(AM1*B))*DCOS(AM1*X)
      PI=3.141592653589793
      MM=0
      PHIXY=0.
    1 AM1=MM*PI/DL
      MP1=MM+1
```

```
      PHIXY=PHIXY+A(MP1)*UM(AM1,X,Y,B)
      IF(MP1.EQ. M)GOTO 2
      MM=MM+1
      GOTO 1
    2 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE PSI(X,Y,PSIXY,M,DL,A,KA,B,CON)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 A(KA)
      PI=3.141592653589793
      PSIXY=0.
      DO 100 I=2,M
      EM=I-1
      D1=EM*PI*X/DL
      D2=EM*PI*Y/DL
      D3=EM*PI*B/DL
      CSH=DCOSH(D3)
      APSI=-DSIN(D1)*(DSINH(D2)/CSH)
      PSIXY=PSIXY+CON*A(I)*APSI
  100 CONTINUE
      RETURN
      END
C
C
C
      SUBROUTINE HITWT(X,Q,NN,HTWT)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 Q(11)
      HTWT=Q(1)
      XDUM=1.
      DO 300 K=1,NN
      XDUM=XDUM*X
  300 HTWT=HTWT+Q(K+1)*XDUM
```

```
RETURN
END
```

```
C***********************************************************************
C
C      PROGRAM 4 CALCULATION OF PHI AND PSI AT NODES
C                RAPID DRAWDOWN CASE
C
C***********************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION XX(101),YY(101),PHIWT(101),PSIWT(101)
      REAL*8 A(40),Q(11)
  995 FORMAT(8I3)
 1015 FORMAT(3D24.17)
 1025 FORMAT(8F10.0)
 1036 FORMAT(/,' Y = ',F10.4)
 1046 FORMAT(10(2X,F10.4))
 1136 FORMAT(/,' DISTANCES ALONG X-AXIS')
 1146 FORMAT(/,' PSI VALUES AT GRID NODES ')
 1156 FORMAT(/,' PHI VALUES AT GRID NODES ')
      READ(5,995)NA,NN,NWT,NFL,NPHIL,NPTS,INTX,INTY
      NNP1=NN+1
      WRITE(6,1056)NA,NN,     INTX,INTY
 1056 FORMAT('1NUMBER OF A-NMS READ IN = ',I3,/,
     X' ORDER OF POLYNOMIAL FITTED TO WATERTABLE = ',I3,/,
     X' X-DIMENSION OF GRID = ',I3,/,
     X' Y-DIMENSION OF GRID = ',I3)
      READ(5,1015)(A(I),I=1,NA)
      READ(5,1015)(Q(I),I=1,NNP1)
      WRITE(6,1066)
 1066 FORMAT(/,' A-NM VALUES ')
      WRITE(6,1076)(A(I),I=1,NA)
 1076 FORMAT(4(6X,D24.17))
      WRITE(6,1086)
 1086 FORMAT(/,' POLYNOMIAL COEFFICIENTS ')
      WRITE(6,1076)(Q(I),I=1,NNP1)
      READ(5,1025)WW,AA,B,THETA,DL,HS,T,WTMAX,HYDCON
      WRITE(6,1096)WW,AA,B,THETA,DL,HS,T,WTMAX,HYDCON
 1096 FORMAT(/,' DEPTH OF WATER IN GULLY = ',F10.4,/,
     X' HEIGHT OF VERTICAL PORTION OF GULLY FACE = ',F10.4,/,
```

```
     X' HEIGHT OF FICTITIOUS SOURCE = ',F10.4,/,
     X' ANGLE OF BANK TO HORIZONTAL = ',F10.4,/,
     X' LENGTH OF FLOW REGION = ',F10.4,/,
     X' HEIGHT OF SEEPAGE FACE ABOVE GULLY BOTTOM = ',F10.4,/,
     X' HALF WIDTH OF GULLY = ',F10.4,/,
     X' MAX. HEIGHT OF WATERTABLE = ',F10.4,/,
     X' HYDRAULIC CONDUCTIVITY = ',F10.4)
      PI=3.141592653589793
      TTHETA=DTAN(THETA*PI/180.)
      XX(1)=0.
      YY(1)=0.
      XX(2)=0.
      YY(2)=AA
      XX(3)=(WW-AA)/TTHETA
      YY(3)=WW
      XX(4)=(HS-AA)/TTHETA
      YY(4)=HS
      H=B-WW
      IF(HS.GT.B)B=HS
      PHIMAX=WTMAX-WW
C     CALCULATE PSIMAX
      CALL PSI(XX(4),YY(4),PSIMAX,NA,DL,A,NA,B,HYDCON,H)
C     CALCULATE PSIMIN
      CALL PSI(XX(1    ),YY(1    ),PSIMIN,NA,DL,A,NA,B,HYDCON,H)
      DELPSI=(PSIMAX-PSIMIN)/(NFL+1)
      WRITE(6,1106)PSIMAX,PSIMIN,DELPSI
 1106 FORMAT(/,' PSIMAX = ',F10.4,3X,' PSIMIN = ',F10.4,3X,' DELPSI = '
     X,F10.4)
C     CALCULATE STREAMLINES
      DELX=DL/INTX
      NX=INTX+1
      DELY=(B+1)/INTY
      NY=INTY+1
C     CALCULATE PSI
      WRITE(6,1116)DELX,DELY
 1116 FORMAT(/,' DELX = ',F10.4,3X,' DELY = ',F10.4)
      Y=0.
```

```
      DO 30 K=1,NY
      X=0.
      DO 31 J=1,NX
      CALL PSI(X,Y,PSIXY,NA,DL,A,NA,B,HYDCON,H)
      YY(J)=PSIXY
      XX(J)=X
      X=X+DELX
   31 CONTINUE
      IF(K.EQ.1)WRITE(6,1146)
      IF(K.EQ.1)WRITE(6,1136)
      IF(K.EQ.1)WRITE(6,1046)(XX(I),I=1,NX)
      WRITE(6,1036)Y
      WRITE(6,1046)(YY(I),I=1,NX)
      Y=Y+DELY
   30 CONTINUE
C   CALCULATE EQUIPOTENTIAL LINES
      PHIMIN=0.
      DELPHI=(PHIMAX-PHIMIN)/(NPHIL+1)
      WRITE(6,1126)PHIMAX,PHIMIN,DELPHI
 1126 FORMAT(/,' PHIMAX = ',F10.4,3X,' PHIMIN = ',F10.4,3X,' DELPHI = '
     X,F10.4)
C   CALCULATE PHI
      Y=0.
      DO 40 K=1,NY
      X=0.
      DO 41 J=1,NX
      CALL PHI(X,Y,PHIXY,NA,DL,H,A,NA,B)
      YY(J)=PHIXY
      XX(J)=X
      X=X+DELX
   41 CONTINUE
      IF(K.EQ.1)WRITE(6,1156)
      IF(K.EQ.1)WRITE(6,1136)
      IF(K.EQ.1)WRITE(6,1046)(XX(I),I=1,NX)
      WRITE(6,1036)Y
      WRITE(6,1046)(YY(I),I=1,NX)
      Y=Y+DELY
```

```
   40 CONTINUE
      STOP
      END
C
C
C
      SUBROUTINE PHI(X,Y,PHIXY, M,DL,H,A,KA,B)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 A(KA)
      UM(AM1,X,Y,B)=(DCOSH(AM1*Y)/DCOSH(AM1*B))*DSIN(AM1*X)
      PI=3.141592653589793
      MM=1
      PHIXY=X/DL
    1 AM1=MM*PI/DL
      MP1=MM
      PHIXY=PHIXY+A(MP1)*UM(AM1,X,Y,B)
      IF(MP1.EQ. M)GOTO 2
      MM=MM+1
      GOTO 1
    2 PHIXY=PHIXY*H
      RETURN
      END
C
C
C
      SUBROUTINE PSI(X,Y,PSIXY,M,DL,A,KA,B,CON,H)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 A(KA)
      PI=3.141592653589793
      PSIXY=Y*CON/DL
      DO 100 I=1,M
      EM=I
      D1=EM*PI*X/DL
      D2=EM*PI*Y/DL
      D3=EM*PI*B/DL
      CSH=DCOSH(D3)
      APSI=DCOS(D1)*(DSINH(D2)/CSH)
```

```
      PSIXY=PSIXY+CON*A(I)*APSI
  100 CONTINUE
      PSIXY=PSIXY*H
      RETURN
      END
C
C
C
      SUBROUTINE HITWT(X,Q,NN,HTWT)
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 Q(11)
      HTWT=Q(1)
      XDUM=1.
      DO 300 K=1,NN
      XDUM=XDUM*X
  300 HTWT=HTWT+Q(K+1)*XDUM
      RETURN
      END
```

```
C***********************************************************************
C
C     PROGRAM 5 BANK STABILITY ANALYSIS USING CYCLOIDAL ARCS
C               RECHARGE CASE
C
C***********************************************************************
      IMPLICIT LOGICAL*1($)
      REAL*8 A(40),Q(11),QS(11)
      DIMENSION S(20),HEAD(20),WEIGHT(20),SATINT(20)
      DIMENSION XFP(65),YFP(65),YWT(65),PORPR(65),PHIWT(65)
      DIMENSION DELYS(64),SPSI(64),F(64)
      DIMENSION XL(5),YL(5),GL(5),DDL(5)
      DIMENSION SX(5),SY(5),XXWT(65),YYWT(65),SWT(65)
10105 FORMAT(3F10.0,I3)
20005 FORMAT(6F10.0)
20105 FORMAT(4I3)
20205 FORMAT(3D24.17)
20305 FORMAT(8F10.0)
30005 FORMAT(7F10.0,I3)
20216 FORMAT(4(3X,D24.17))
20246 FORMAT(10(2X,F10.4))
10095 FORMAT(20A4)
      READ(5,10105)BANKHT,ARAD,BTHETA,NSLCS
      WRITE(6,10106)BANKHT,ARAD,BTHETA,NSLCS
10106 FORMAT(/,'1HEIGHT OF BANK = ',F10.4,/,
     X' RADIUS OF INITIAL GENERATING CIRCLE = ',F10.4,/,
     X' ANGLE OF BANK TO HORIZONTAL = ',F10.4,/,
     X' NUMBER OF SLICES USED IN ANALYSIS = ',I3)
      READ(5,20005)AA,T,WW,DL,HS,B
      WRITE(6,20006)AA,T,WW,DL,HS,B
20006 FORMAT(/,' DEPTH TO IMPERMEABLE BARRIER = ',F10.4,/,
     X' HALF WIDTH OF GULLY = ',F10.4,/,
     X' DEPTH OF WATER IN GULLY =',F10.4,/,
     X' LENGTH OF FLOW REGION = ',F10.4,/,
     X' HEIGHT OF SEEPAGE FACE ABOVE GULLY BOTTOM = ',F10.4,/,
     X' HEIGHT OF GROUNDWATER DIVIDE = ',F10.4)
      READ(5,20105)NS,NN,NA,NSPTS
```

```
      WRITE(6,20106)NS,NN,NA,NSPTS
20106 FORMAT(/,' ORDER OF POLYNOMIAL TO BE FITTED TO SATURATION CURVE =
     X',I3,/,' ORDER OF POLYNOMIAL TO BE FITTED TO WATERTABLE DATA = ',I
     X3,/,' NUMBER OF A-NMS TO BE READ IN = ',I3
     X,/,' NUMBER OF POINTS ON SATURATION CURVE READ IN = ',I3)
      READ(5,10095)XL,YL,GL,DDL
      NNP1=NN+1
      NSP1=NS+1
      READ(5,20205)(Q(I),I=1,NNP1)
      WRITE(6,20206)
20206 FORMAT(/,' COEFFICIENTS OF POLYNOMIAL FITTED TO WATERTABLE DATA ')
      WRITE(6,20216)(Q(I),I=1,NNP1)
      READ(5,20205)(A(I),I=1,NA)
      WRITE(6,20226)
20226 FORMAT(/,' A-NMS READ IN ')
      WRITE(6,20216)(A(I),I=1,NA)
      READ(5,20305)(S(I),HEAD(I),I=1,NSPTS)
      WRITE(6,20236)
20236 FORMAT(/,' SATURATION VALUES ')
      WRITE(6,20246)(S(I),I=1,NSPTS)
      WRITE(6,20256)
20256 FORMAT(/,' CORRESPONDING SUCTION HEAD VALUES ')
      WRITE(6,20246)(HEAD(I),I=1,NSPTS)
      READ(5,30005)GAMAW,GS,PORSTY,DRAPOR,TRUCOH,SPHI,FTOL,MAXCOU
      WRITE(6,30006)GAMAW,GS,PORSTY,DRAPOR,TRUCOH,SPHI,FTOL,MAXCOU
30006 FORMAT(/,' UNIT WT. OF WATER = ',F10.4,/,
     X' SPECIFIC GRAVITY OF SOIL PARTICLES  = ',F10.4,/,
     X' SOIL POROSITY = ',F10.4,/,' DRAINABLE POROSITY = ',F10.4,/,
     X' TRUE SOIL COHESION = ',F10.4,/,' ANGLE OF INTERNAL SHEARING RESI
     XSTANCE = ',F10.4,/,' TOLERANCE ON FACTOR OF SAFETY = ',F10.4,/,
     X' MAXIMUM NUMBER OF ITERATIONS = ',I3)
C     THIS SECTION FITS CURVE TO SATURATION-SUCTION HEAD DATA
      DO 100 I=1,NSPTS
  100 WEIGHT(I)=1.
      CALL OPLSPA(NS,NSPTS,HEAD,S,WEIGHT,QS,0.)
      WRITE(6,30016)
30016 FORMAT(/,' COEFFICIENTS OF CURVE FITTED TO SATURATION DATA ')
```

```
      WRITE(6,20216)(QS(I),I=1,NS)
C     SUBSTITUTE BACK INTO FITTED EQUATION FOR CHECK
      DO 101 I=1,NSPTS
      CALL SINT(HEAD(I),QS,NS,SATINT(I))
  101 CALL SAT(HEAD(I),QS,NS,S(I))
      WRITE(6,30026)
30026 FORMAT(/,' BACK CALCULATED SATURATION VALUES ')
      WRITE(6,20246)(S(I),I=1,NSPTS)
      WRITE(6,30036)
30036 FORMAT(/,' INTEGRALS OF S.DHEAD UP TO HEAD(I)')
      WRITE(6,20246)(SATINT(I),I=1,NSPTS)
C     THIS SECTION INITIALIZES CONSTANTS FOR THE PROGRAM
      PI=3.1415926
      SPHI=SPHI*PI/180.
      TRUCOH=TRUCOH*144.
      BTHETA=BTHETA*PI/180.
      NPTS=NSLCS+1
      NSLM1=NSLCS-1
      VOIDR=PORSTY/(1.-PORSTY)
C
C     THIS SECTION COMPUTES X,Y COORDINATES OF FAILURE PLANE FOR
C     THE BOUNDS OF EACH SLICE
C
      DELA=4.
      NCOUN=1
      $COUN1=.FALSE.
      $COUN2=.FALSE.
      $COUN3=.FALSE.
      $RITYP=.FALSE.
      $RITYP=.TRUE.
      XSMIN=BANKHT/(TAN(BTHETA))
      TATHET=TAN(BTHETA)
      XSMINT=XSMIN+T
      GAMAT=((GS+VOIDR)/(1.+VOIDR))*GAMAW
      GAMAD=GS*GAMAW/(1.+VOIDR)
      WCON=VOIDR*GAMAW/(1.+VOIDR)
      XHS=(HS/(TAN(BTHETA)))+T
```

```
      WRITE(6,30136)GAMAT,GAMAD,VOIDR,WCON,XHS
30136 FORMAT(/,' WET UNIT WT. OF SOIL = ',F10.4,/,
     X' DRY UNIT WT. OF SOIL = ',F10.4,/,
     X' VOIDS RATIO OF SOIL = ',F10.4,/,
     X' CONSTANT TERMS FOR INT S.DY = ',F10.4,/,
     X' X-COORDINATE OF TOP OF SEEPAGE FACE = ',F10.4)
   99 DUMMY=1.-(BANKHT/ARAD)
      IF(ABS(DUMMY).GE.1.)GOTO 97
      THETAP=ARCOS(DUMMY)
      XSMAX=ARAD*(THETAP-SIN(THETAP))
      IF(XSMAX.LT.XSMIN)GOTO 96
      $THETA=.FALSE.
      GOTO 98
   97 ARAD=BANKHT/2.
      THETAP=PI
      XSMAX=ARAD*PI
      $THETA=.TRUE.
      WRITE(6,30046)
30046 FORMAT(//,' RADIUS OF GENERATING CIRCLE LESS THAN BANK HEIGHT/2.')
      GOTO 98
   96 WRITE(6,30056)
30056 FORMAT(//,' FAILURE PLANE ENDS ON BANK SLOPE')
      ARAD=ARAD+DELA
      IF($COUN3)STOP
      $COUN3=.TRUE.
      GOTO 99
   98 DELXS=XSMAX/NSLCS
      YS=BANKHT
      XS=XSMAX
      THETA=THETAP
      XFP(NPTS)=XSMAX+T
      YFP(NPTS)=BANKHT+AA
      XFP(1)=T
      YFP(1)=AA
      DO 102 I=1,NSLM1
      XS=XS-DELXS
      XFP(I+1)=T+XSMAX-XS
```

```
      IF($THETA)GOTO 94
      DETH=DELXS/(ARAD*(1.-COS(THETA)))
      GOTO 95
   94 DETH=DELXS/(ARAD*2.)
   95 THETA=THETA-DETH
      YSN=ARAD*(1.-COS(THETA))
      SPSI(I)=ATAN((YS-YSN)/DELXS)
      DELYS(I)=YS-YSN
      YS=YSN
      YFP(I+1)=BANKHT-YS+AA
  102 CONTINUE
      SPSI(NSLCS)=ATAN((BANKHT+AA-YFP(NSLCS))/DELXS)
      DELYS(NSLCS)=BANKHT-YFP(NSLCS)+AA
C     THE ABOVE DO LOOP ALSO CALCULATES SPSI FOR EACH SLICE
C
C     CALCULATE X,Y COORDINATES OF WATERTABLE
C     CALCULATE PORE PRESSURES FOR THE BOUNDS OF EACH SLICE
      DO 200 I=1,NPTS
      CALL HITWT(XFP(I),Q,NN,YWT(I))
      CALL PHI(XFP(I),YWT(I),PHIWT(I),NA,DL,H,A,NA,B)
C     SEE IF WE ARE ABOVE OR BELOW THE WATERTABLE
      IF(YWT(I)-YFP(I))201,202 ,203
  201 PORPR(I)=-(YFP(I)-YWT(I))
      GOTO 200
  202 PORPR(I)=0.
      GOTO 200
  203 CALL PHI(XFP(I),YFP(I),PORPR(I),NA,DL,H,A,NA,B)
C     SUBTRACT ELEVATION HEAD YFP(I)
      PORPR(I)=PORPR(I)-YFP(I)
  200 CONTINUE
C     THIS SECTION EVALUATES THE FACTOR OF SAFETY FOR CHOSEN FAILURE
C     PLANE
      TSFS=0.
      TTFS=0.
      DO 300 I=1,NSLCS
      IF(XFP(I).GE.XSMINT)GOTO 301
C     SLICE ENDS ON BANK FACE
```

```
      IF(XFP(I+1).GT.XSMINT)GOTO 302
      IF(XFP(I).GE.XHS)GOTO 303
      IF(XFP(I+1).GT.XHS)GOTO 304
C     CALCULATE WT. OF SLICE IN SATURATED REGION ON BANK FACE
C     AVG. LENGTH OF SLICE
      SLI=(XFP(I)-T)*TATHET+AA-YFP(I)
      SLIP1=(XFP(I+1 )-T)*TATHET+AA-YFP(I+1)
      AVGL=(SLI+SLIP1)/2.
C     VOLUME OF SLICE PER UNIT LENGTH OF CHANNEL
      SLVOL=AVGL*DELXS
      SLWT=SLVOL*GAMAT
      IF($RITYP)GOTO 305
      WRITE(6,40006) SLWT
40006 FORMAT(/,' TYPE 1 ',F10.4)
      GOTO 305
C     FOR SLICE THAT INCLUDES HS
  304 SL1=(XFP(I)-T)*TATHET-YFP(I)+AA
      DEL=XHS-XFP(I)
      YFPDEL=YFP(I)+(YFP(I+1)-YFP(I))*DEL/DELXS
      SL2=HS+AA-YFPDEL
      W1=(SL1+SL2)*DEL*GAMAT/2.
      SL3=YWT(I+1)-YFP(I+1)
      W2=(SL2+SL3)*(DELXS-DEL)*GAMAT/2.
C     FIND WT. OF UNSAT. PART
      SL4=(XFP(I+1)-T)*TATHET-YWT(I+1)+AA
C     WT. OF DRY SOIL
      WDS3=SL4*(DELXS-DEL)*GAMAD/2.
C     WT. OF WATER IN UNSAT REGION
      CALL SINT(SL4,QS,NS,SATIN )
      WW3=(DELXS-DEL)*WCON*SATIN /2.
      SLWT=W1+W2+WDS3+WW3
      IF($RITYP)GOTO 305
      WRITE(6,40016)W1,W2,WDS3,WW3,SLWT
40016 FORMAT(/,' TYPE 2 ',5(2X,F10.4))
      GOTO 305
C     FOR SLICE ABOVE HS BUT STILL ON BANK FACE
  303 SL1=YWT(I+1)-YFP(I+1)
```

```
      SL2=YWT(I)-YFP(I)
      IF((SL1.GE.0.).AND.(SL2.GT.0.))GOTO 313
C     WT. OF DRY SOIL
      SL3=(XFP(I+1)-T)*TATHET-YFP(I+1)+AA
      SL4=(XFP(I)-T)*TATHET-YFP(I)+AA
      W1=(SL3+SL4)*DELXS*GAMAD/2.
      IF((SL1.LT.0.).AND.(SL2.GE.0.))GOTO 323
C     WT. OF WATER
      CALL SINT(-SL2,QS,NS,SAT2)
      SL5=SL4-SL2
      CALL SINT(SL5,QS,NS,SAT4)
      CALL SINT(-SL1,QS,NS,SAT1)
      SL6=SL3-SL1
      CALL SINT(SL6,QS,NS,SAT3)
      WW=DELXS*WCON*((SAT4-SAT2)+(SAT3-SAT1))/2.
      SLWT=W1+WW
      IF($RITYP)GOTO 305
      WRITE(6,40026)W1,WW,SLWT
40026 FORMAT(/,' TYPE 3 ',3(2X,F10.4))
      GOTO 305
C     WT. OF WATER
  323 CALL SINT(SL3,QS,NS,SAT3)
      CALL SINT(SL4,QS,NS,SAT4)
      WW=DELXS*WCON*(SAT4+SAT3)/2.
      SLWT=W1+WW
      IF($RITYP)GOTO 305
      WRITE(6,40036)W1,WW,SLWT
40036 FORMAT(/,' TYPE 4 ',3(2X,F10.4))
      GOTO 305
C     BELOW WATERTABLE
  313 AVGL1=(SL1+SL2)/2.
      W1=AVGL1*DELXS*GAMAT
C     WT. OF SOIL  ABOVE WATERTABLE
      SL1=(XFP(I+1)-T)*TATHET-YWT(I+1)+AA
      SL2=(XFP(I)-T)*TATHET-YWT(I)+AA
      AVGL2=(SL2+SL1)/2.
      WDS2=AVGL2*DELXS*GAMAD
```

```
C     WT. OF WATER IN SOIL ABOVE THE WATERTABLE
      CALL SINT(SL2,QS,NS,SAT2)
      WW21=DELXS*WCON*SAT2
      CALL SINT(SL1,QS,NS,SAT3)
      WW22 =DELXS*WCON*(SAT3-SAT2)/2.
      SLWT=W1+WDS2+WW21+WW22
      IF($RITYP)GOTO 305
      WRITE(6,40046)W1,WDS2,WW21,WW22,SLWT
40046 FORMAT(/,' TYPE 5 ',5(2X,F10.4))
      GOTO 305
C     FOR SLICE AT TOP OF BANK
  302 SL1=YWT(I+1)-YFP(I+1)
      SL2=YWT(I)-YFP(I)
      IF((SL1.GE.0.).AND.(SL2.GT.0.))GOTO 312
C     WT. OF DRY SOIL
      DEL =XSMINT-XFP(I)
      YFPDEL=YFP(I)+(YFP(I+1)-YFP(I))*DEL/DELXS
      SL3=(XFP(I)-T)*TATHET-YFP(I)
      SL4=YFP(NPTS)-YFPDEL
      SL5=YFP(NPTS)-YFP(I+1)
      W1=((SL3+SL4)*DEL*GAMAD/2.)+((SL4+SL5)*(DELXS-DEL)*GAMAD/2.)
C     WT. OF WATER
      IF((SL1.LT.0.).AND.(SL2.GE.0.))GOTO 322
      YWTDEL=YWT(I)+(YWT(I+1)-YWT(I))*DEL/DELXS
      SL6=YFPDEL-YWTDEL
      CALL SINT(-SL1,QS,NS,SAT1)
      SDUM1=SL3-SL1
      CALL SINT(SDUM1,QS,NS,SAT2)
      CALL SINT(SL6,QS,NS,SAT3)
      SDUM2=SL4+SL6
      CALL SINT(SDUM2,QS,NS,SAT4)
      WW1=DEL*WCON*((SAT2-SAT1)+(SAT4-SAT3))/2.
      CALL SINT(-SL2,QS,NS,SAT5)
      SDUM3=SL5-SL2
      CALL SINT(SDUM3,QS,NS,SAT6)
      WW2=(DELXS-DEL)*WCON*((SAT6-SAT5)+(SAT4-SAT3))/2.
      SLWT=W1+WW1+WW2
```

```
      IF($RITYP)GOTO 305
      WRITE(6,40056)W1,WW1,WW2,SLWT
40056 FORMAT(/,' TYPE 6 ',4(2X,F10.4))
      GOTO 305
  322 CALL SINT(SL3,QS,NS,SAT3)
      CALL SINT(SL4,QS,NS,SAT4)
      CALL SINT(SL5,QS,NS,SAT5)
      WW1=DEL*WCON*( SAT4+SAT3)/2.
      WW2=(DELXS-DEL)*WCON*(SAT5+SAT4)/2.
      SLWT=W1+WW1+WW2
      IF($RITYP)GOTO 305
      WRITE(6,40066)W1,WW1,WW2,SLWT
40066 FORMAT(/,' TYPE 7 ',4(2X,F10.4))
      GOTO 305
  312 AVGL1=(SL1+SL2)/2.
      W1=AVGL1*DELXS*GAMAT
C     WT. OF DRY SOIL ABOVE WATERTABLE
      DEL =XSMINT-XFP(I)
      SL1=(XFP(I)-T)*TATHET-YWT(I)+AA
      YWTDEL=YWT(I)+(YWT(I+1)-YWT(I))*DEL/DELXS
      SL2=YFP(NPTS)-YWTDEL
      SL3=YFP(NPTS)-YWT(I+1)
      WDS1=(SL1+SL2)*DEL*GAMAD/2.
      WDS2=(SL2+SL3)*(DELXS-DEL)*GAMAD/2.
C     WT. OF WATER ABOVE WATERTABLE
      CALL SINT(SL1,QS,NS,SAT1)
      CALL SINT(SL2,QS,NS,SAT2)
      CALL SINT(SL3,QS,NS,SAT3)
      WW1=DEL*WCON*( SAT1+SAT2)/2.
      WW2=(DELXS-DEL)*WCON*(SAT2+SAT3)/2.
      SLWT=W1+WDS1+WDS2+WW1+WW2
      IF($RITYP)GOTO 305
      WRITE(6,40076)W1,WDS1,WDS2,WW1,WW2,SLWT
40076 FORMAT(/,' TYPE 8 ',6(2X,F10.4))
      GOTO 305
  301 SL1=YWT(I)-YFP(I)
      SL2=YWT(I+1)-YFP(I+1)
```

```
      IF((SL1.GT.0.).AND.(SL2.GE.0.))GOTO 311
C     WT. OF DRY SOIL
      SL3=YFP(NPTS)-YFP(I)
      SL4=YFP(NPTS)-YFP(I+1)
      W1=DELXS*GAMAD*(SL3+SL4)/2.
C     WT. OF WATER
      IF((SL1.GE.0.).AND.(SL2.LT.0.))GOTO 321
      CALL SINT(-SL1,QS,NS,SAT1)
      CALL SINT(-SL2,QS,NS,SAT2)
      SDUM1=SL3-SL1
      CALL SINT(SDUM1,QS,NS,SAT3)
      SDUM2=SL4-SL2
      CALL SINT(SDUM2,QS,NS,SAT4)
      WW=DELXS*WCON*((SAT4-SAT2)+(SAT3-SAT1))/2.
      SLWT=W1+WW
      IF($RITYP)GOTO 305
      WRITE(6,40086)W1,WW,SLWT
40086 FORMAT(/,' TYPE 9 ',3(2X,F10.4))
      GOTO 305
  321 CALL SINT(SL3,QS,NS,SAT1)
      CALL SINT(SL4,QS,NS,SAT2)
      WW=DELXS*WCON*(SAT1+SAT2)/2.
      SLWT=W1+WW
      IF($RITYP)GOTO 305
      WRITE(6,40096)W1,WW,SLWT
40096 FORMAT(/,' TYPE 10 ',3(2X,F10.4))
      GOTO 305
C     WT. OF WET SOIL
  311 W1=DELXS*GAMAT*(SL1+SL2)/2.
      SL3=YFP(NPTS)-YWT(I)
      SL4=YFP(NPTS)-YWT(I+1)
C     WT. OF DRY SOIL
      W2=DELXS*GAMAD*(SL3+SL4)/2.
C     WT. OF WATER
      CALL SINT(SL3,QS,NS,SAT1)
      CALL SINT(SL4,QS,NS,SAT2)
      WW=DELXS*WCON*(SAT1+SAT2)/2.
```

```
      SLWT=W1+W2+WW
      IF($RITYP)GOTO 305
      WRITE(6,40106)W1,W2,WW,SLWT
40106 FORMAT(/,' TYPE 11 ',4(2X,F10.4))
  305 CONTINUE
C     AREA OF SLICE BOTTOM
      ASLIBO =SQRT((DELXS*DELXS)+(DELYS(I)*DELYS(I)))
C     AVG. PORE WATER FORCE ON EACH SLICE
      PORAV=(PORPR(I)+PORPR(I+1))/2.
      IF(PORAV.LT.O.)GOTO 315
      UAVG=PORAV*GAMAW*ASLIBO
      GOTO 316
  315 CALL SAT(-PORAV,QS,NS,SPOR)
      UAVG=SPOR*PORAV*GAMAW*ASLIBO
  316 CONTINUE
C     TANGENTIAL WEIGHT FORCE ON SLICE
      TFS=SLWT*SIN(SPSI(I))
C     NORMAL FORCE
      ANFS=SLWT*COS(SPSI(I))
C     EFFECTIVE NORMAL FORCE
      ENF=ANFS-UAVG
      IF(ENF.LT.O.)ENF=0.
C     COHESIVE FORCE
      FCOH=ASLIBO*TRUCOH
C     MOBILIZABLE SHEAR FORCE AVAILABLE TO SLICE
      IF(ABS(ENF).LT.1.E-10)GOTO 604
      SFS=FCOH+ENF*TAN(SPHI)
      GOTO 605
  604 SFS=FCOH
C     SLICE FACTOR OF SAFETY
  605 F(I)=SFS/TFS
C     TOTAL FACTOR OF SAFETY
      TSFS=TSFS+SFS
      TTFS=TTFS+TFS
  300 CONTINUE
      TF=TSFS/TTFS
      WRITE(6,50016)TF
```

```
50016 FORMAT(/,' FACTOR OF SAFETY FOR THIS TRIAL FAILURE SURFACE = '
     X,F10.4)
      WRITE(6,50026)ARAD,NCOUN
50026 FORMAT(/,' RADIUS OF GENERATING CIRCLE = ',F10.4,/,
     X' NUMBER OF ITERATIONS = ',I3)
      IF(NCOUN.EQ.MAXCOU)GOTO 804
      IF($COUN1)GO TO 800
      ARAD=ARAD+DELA
      $COUN1=.TRUE.
      TFO =TF
      NCOUN=NCOUN+1
      GOTO 99
  800 IF($COUN2)GOTO 802
      IF(TF.GT.TFO)GOTO 801
      ARAD=ARAD+DELA
      $COUN2=.TRUE.
      TFO =TF
      NCOUN=NCOUN+1
      GOTO 99
  801 DELA=-DELA
      ARAD=ARAD+2.0*DELA
      $COUN2=.TRUE.
      NCOUN=NCOUN+1
      GOTO 99
  802 IF(TF.GT.TFO)GOTO 803
      ARAD=ARAD+DELA
      NCOUN=NCOUN+1
      TFO =TF
      GOTO 99
  803 DELF=TF-TFO
      IF(ABS(DELF).LE.FTOL)GOTO 804
      DELA=-DELA/2.0
      ARAD=ARAD+DELA
      NCOUN=NCOUN+1
      TFO =TF
      GOTO 99
  804 CONTINUE
```

```
      WRITE(6,30066)
30066 FORMAT(/,' X-COORDINATES OF FAILURE PLANE ')
      WRITE(6,20246)(XFP(I),I=1,NPTS)
      WRITE(6,30076)
30076 FORMAT(/,' Y-COORDINATES OF FAILURE PLANE')
      WRITE(6,20246)(YFP(I),I=1,NPTS)
      WRITE(6,30086)DELXS
30086 FORMAT(/,' DELTA Y INCREMANTS ALONG FAILURE PLANE FOR DELTA X = ',
     XF10.4)
      WRITE(6,20246)(DELYS(I),I=1,NSLCS)
      WRITE(6,30096)
30096 FORMAT(/,' ANGLE OF SLICE BOTTOM TO VERTICAL ALPHA = TAN(DELTAY(I
     X/DELTAX)')
      WRITE(6,20246)(SPSI(I),I=1,NSLCS)
      WRITE(6,30106)
30106 FORMAT(/,' WATERTABLE ELEVATIONS ')
      WRITE (6,20246)(YWT(I),I=1,NPTS)
      WRITE(6,30116)
30116 FORMAT(/,' HYDRAULIC POTENTIAL ALONG WATERTABLE ')
      WRITE(6,20246)(PHIWT(I),I=1,NPTS)
      WRITE(6,30126)
30126 FORMAT(/,' PORE WATER PRESSURES ALONG TRIAL FAILURE SURFACE ')
      WRITE(6,20246)(PORPR(I),I=1,NPTS)
      WRITE(6,50006)
50006 FORMAT(/,' SLICE FACTORS OF SAFETY ')
      WRITE(6,20246)(F(I),I=1,NSLCS)
C
C     PLOTTING SECTION
C
C     PLOT AXES AND SOIL SURFACE
      SX(1)=0.
      SY(1)=AA
      SX(2)=T
      SY(2)=AA
      SX(3)=XSMINT
      SY(3)=YFP(NPTS)
      SX(4)=XFP(NPTS)
```

```
      SY(4)=YFP(NPTS)
      YSCALE=YFP(NPTS)/5.25
      XSCALE=XFP(NPTS)/7.25
      IF(YSCALE.GT.XSCALE)GOTO 1000
      SCALE=XSCALE
      GOTO 1001
 1000 SCALE=YSCALE
 1001 CONTINUE
      CALL GRAPH(4,SX,SY,0,4,7.25,5.25,SCALE,0.,SCALE,0.,XL,YL,GL,DDL)
C     PLOT WATERTABLE
      XXWT(1)=XHS
      YYWT(1)=HS+AA
      DO 1002 I=2,NPTS
      II=I
      IF(XFP(I).GT.XHS)GOTO 1003
 1002 CONTINUE
 1003 CONTINUE
      NWTPT=NPTS-II+1
      IIM2=II-2
      DO 1004 I=2,NWTPT
      XXWT(I)=XFP(IIM2+I)
      YYWT(I)=YWT(IIM2+I)
 1004 CONTINUE
      CALL GRAPHS(NWTPT,XXWT,YYWT,0,2,';')
C     PLOT FAILURE PLANE
      CALL GRAPHS(NPTS,XFP,YFP,0,2,';')
C     PLOT PORE WATER PRESSURES
      SWT(1)=0.
      DO 1005 I=1,NSLCS
      ASLIBO=SQRT(DELXS*DELXS+(DELYS(I)*DELYS(I)))
 1005 SWT(I+1)=SWT(I)+ASLIBO
      CALL GRAPH(NPTS,SWT,PORPR,0,2,7.25,5.25,0.,0.,0.,0.,XL,YL,GL,DDL)
      STOP
      END
C
C
C
```

```
      SUBROUTINE SAT(HEAD,QS,NS,S)
C     THIS SUBROUTINE CALCULATES THE PERCENT SATURATION FOR A GIVEN
C     SUCTION HEAD USING FITTED CURVE
      REAL*8 QS(11),DS,XDUM,DBLE
      DS=QS(1)
      XDUM=1.D0
      DO 300 K=1,NS
      XDUM=XDUM*(DBLE(HEAD))
  300 DS=DS+QS(K+1)*XDUM
      S=SNGL(DS)
      RETURN
      END
C
C
C
      SUBROUTINE HITWT(X,Q,NN,HTWT)
C     THIS SUBROUTINE CALCULATES WATERTABLE ELEVATIONS
      REAL*8 Q(11),DHTWT,XDUM,DBLE
      DHTWT=Q(1)
      XDUM=1.D0
      DO 300 K=1,NN
      XDUM=XDUM*(DBLE(X))
  300 DHTWT=DHTWT+Q(K+1)*XDUM
      HTWT=SNGL(DHTWT)
      RETURN
      END
C
C
C
      SUBROUTINE PHI(X,Y,PHIXY,M,DL,H,A,KA,B)
C     THIS SUBROUTINE CALCULATES PHI AT X,Y
      REAL*8 A,PI,AM1,DPHIXY,DBLE,DCOSH,DCOS
      DIMENSION A(KA)
      UM(AM1,X,Y,B)=(DCOSH(AM1*Y)/DCOSH(AM1*B))*DCOS(AM1*X)
      PI =3.141592653589793
      MM=0
      DPHIXY=0.D0
```

```
      Y1=DBLE(Y)
      X1=DBLE(X)
      B1=DBLE(B)
    1 AM1=MM*PI/DBLE(DL)
      MP1=MM+1
      DPHIXY=DPHIXY+A(MP1)*UM(AM1,X1,Y1,B1)
      IF(MP1.EQ.M)GOTO 2
      MM=MM+1
      GOTO 1
    2 PHIXY=SNGL(DPHIXY)
      RETURN
      END
C
C
C
      SUBROUTINE SINT(HEAD,QS,NS,SATINT)
C     THIS SUBROUTINE CALCULATES THE INTEGRAL OF S DY UP TO SUCTION
C     OF HEAD
      REAL*8 QS(11),DSATIN,XDUM,DHEAD,DBLE,DFLOAT
      DHEAD=DBLE(HEAD)
      DSATIN=QS(1)*DHEAD
      XDUM=DHEAD
      DO 300 K=1,NS
      XDUM=XDUM*DHEAD
  300 DSATIN=DSATIN+(QS(K+1)*XDUM/DFLOAT(K+1))
      SATINT=SNGL(DSATIN)
      RETURN
      END
```

```
C**********************************************************************
C
C      PROGRAM 6 BANK STABILITY ANALYSIS USING CYCLOIDAL ARCS
C                RAPID DRAWDOWN CASE
C
C**********************************************************************
       IMPLICIT LOGICAL*1($)
       REAL*8 A(40),Q(11),QS(11)
       DIMENSION S(20),HEAD(20),WEIGHT(20),SATINT(20)
       DIMENSION XFP(65),YFP(65),YWT(65),PORPR(65),PHIWT(65)
       DIMENSION DELYS(64),SPSI(64),F(64)
       DIMENSION XL(5),YL(5),GL(5),DDL(5)
       DIMENSION SX(5),SY(5),XXWT(65),YYWT(65),SWT(65)
10105  FORMAT(3F10.0,I3)
20005  FORMAT(6F10.0)
20105  FORMAT(4I3)
20205  FORMAT(3D24.17)
20305  FORMAT(8F10.0)
30005  FORMAT(7F10.0,I3)
20216  FORMAT(4(3X,D24.17))
20246  FORMAT(10(2X,F10.4))
10095  FORMAT(20A4)
       READ(5,10105)BANKHT,ARAD,BTHETA,NSLCS
       WRITE(6,10106)BANKHT,ARAD,BTHETA,NSLCS
10106  FORMAT(/,'1HEIGHT OF BANK = ',F10.4,/,
      X' RADIUS OF INITIAL GENERATING CIRCLE = ',F10.4,/,
      X' ANGLE OF BANK TO HORIZONTAL = ',F10.4,/,
      X' NUMBER OF SLICES USED IN ANALYSIS = ',I3)
       READ(5,20005)AA,T,WWTBL,DL,HS,B
       WRITE(6,20006)AA,T,WWTBL,DL,HS,B
20006  FORMAT(/,' DEPTH TO IMPERMEABLE BARRIER = ',F10.4,/,
      X' HALF WIDTH OF GULLY = ',F10.4,/,
      X' DEPTH OF WATER IN GULLY =',F10.4,/,
      X' LENGTH OF FLOW REGION = ',F10.4,/,
      X' HEIGHT OF SEEPAGE FACE ABOVE GULLY BOTTOM = ',F10.4,/,
      X' HEIGHT OF FICTITIOUS SOURCE = ',F10.4)
       H=B-WWTBL
```

```
      IF(HS.GT.B)B=HS
      READ(5,20105)NS,NN,NA,NSPTS
      WRITE(6,20106)NS,NN,NA,NSPTS
20106 FORMAT(/,' ORDER OF POLYNOMIAL TO BE FITTED TO SATURATION CURVE =
     X',I3,/,' ORDER OF POLYNOMIAL TO BE FITTED TO WATERTABLE DATA = ',I
     X3,/,' NUMBER OF A-NMS TO BE READ IN = ',I3
     X,/,' NUMBER OF POINTS ON SATURATION CURVE READ IN = ',I3)
      READ(5,10095)XL,YL,GL,DDL
      NNP1=NN+1
      NSP1=NS+1
      READ(5,20205)(Q(I),I=1,NNP1)
      WRITE(6,20206)
20206 FORMAT(/,' COEFFICIENTS OF POLYNOMIAL FITTED TO WATERTABLE DATA ')
      WRITE(6,20216)(Q(I),I=1,NNP1)
      READ(5,20205)(A(I),I=1,NA)
      WRITE(6,20226)
20226 FORMAT(/,' A-NMS READ IN ')
      WRITE(6,20216)(A(I),I=1,NA)
      READ(5,20305)(S(I),HEAD(I),I=1,NSPTS)
      WRITE(6,20236)
20236 FORMAT(/,' SATURATION VALUES ')
      WRITE(6,20246)(S(I),I=1,NSPTS)
      WRITE(6,20256)
20256 FORMAT(/,' CORRESPONDING SUCTION HEAD VALUES ')
      WRITE(6,20246)(HEAD(I),I=1,NSPTS)
      READ(5,30005)GAMAW,GS,PORSTY,DRAPOR,TRUCOH,SPHI,FTOL,MAXCOU
      WRITE(6,30006)GAMAW,GS,PORSTY,DRAPOR,TRUCOH,SPHI,FTOL,MAXCOU
30006 FORMAT(/,' UNIT WT. OF WATER = ',F10.4,/,
     X' SPECIFIC GRAVITY OF SOIL PARTICLES  = ',F10.4,/,
     X' SOIL POROSITY = ',F10.4,/,' DRAINABLE POROSITY = ',F10.4,/,
     X' TRUE SOIL COHESION = ',F10.4,/,' ANGLE OF INTERNAL SHEARING RESI
     XSTANCE = ',F10.4,/,' TOLERANCE ON FACTOR OF SAFETY = ',F10.4,/,
     X' MAXIMUM NUMBER OF ITERATIONS = ',I3)
C     THIS SECTION FITS CURVE TO SATURATION-SUCTION HEAD DATA
      DO 100 I=1,NSPTS
  100 WEIGHT(I)=1.
      CALL OPLSPA(NS,NSPTS,HEAD,S,WEIGHT,QS,0.)
```

```
      WRITE(6,30016)
30016 FORMAT(/,' COEFFICIENTS OF CURVE FITTED TO SATURATION DATA ')
      WRITE(6,20216)(QS(I),I=1,NS)
C     SUBSTITUTE BACK INTO FITTED EQUATION FOR CHECK
      DO 101 I=1,NSPTS
      CALL SINT(HEAD(I),QS,NS,SATINT(I))
  101 CALL SAT(HEAD(I),QS,NS,S(I))
      WRITE(6,30026)
30026 FORMAT(/,' BACK CALCULATED SATURATION VALUES ')
      WRITE(6,20246)(S(I),I=1,NSPTS)
      WRITE(6,30036)
30036 FORMAT(/,' INTEGRALS OF S.DHEAD UP TO HEAD(I)')
      WRITE(6,20246)(SATINT(I),I=1,NSPTS)
C     THIS SECTION INITIALIZES CONSTANTS FOR THE PROGRAM
      PI=3.1415926
      SPHI=SPHI*PI/180.
      TRUCOH=TRUCOH*144.
      BTHETA=BTHETA*PI/180.
      NPTS=NSLCS+1
      NSLM1=NSLCS-1
      VOIDR=PORSTY/(1.-PORSTY)
C
C     THIS SECTION COMPUTES X,Y COORDINATES OF FAILURE PLANE FOR
C     THE BOUNDS OF EACH SLICE
C
      DELA=4.
      NCOUN=1
      $COUN1=.FALSE.
      $COUN2=.FALSE.
      $COUN3=.FALSE.
      $RITYP=.FALSE.
      $RITYP=.TRUE.
      XSMIN=BANKHT/(TAN(BTHETA))
      TATHET=TAN(BTHETA)
      XSMINT=XSMIN+T
      GAMAT=((GS+VOIDR)/(1.+VOIDR))*GAMAW
      GAMAD=GS*GAMAW/(1.+VOIDR)
```

```
      WCON=VOIDR*GAMAW/(1.+VOIDR)
      XHS=(HS/(TAN(BTHETA)))+T
      WRITE(6,30136)GAMAT,GAMAD,VOIDR,WCON,XHS
30136 FORMAT(/,' WET UNIT WT. OF SOIL = ',F10.4,/,
     X' DRY UNIT WT. OF SOIL = ',F10.4,/,
     X' VOIDS RATIO OF SOIL = ',F10.4,/,
     X' CONSTANT TERMS FOR INT S.DY = ',F10.4,/,
     X' X-COORDINATE OF TOP OF SEEPAGE FACE = ',F10.4)
   99 DUMMY=1.-(BANKHT/ARAD)
      IF(ABS(DUMMY).GE.1.)GOTO 97
      THETAP=ARCOS(DUMMY)
      XSMAX=ARAD*(THETAP-SIN(THETAP))
      IF(XSMAX.LT.XSMIN)GOTO 96
      $THETA=.FALSE.
      GOTO 98
   97 ARAD=BANKHT/2.
      THETAP=PI
      XSMAX=ARAD*PI
      $THETA=.TRUE.
      WRITE(6,30046)
30046 FORMAT(//,' RADIUS OF GENERATING CIRCLE LESS THAN BANK HEIGHT/2.')
      GOTO 98
   96 WRITE(6,30056)
30056 FORMAT(//,' FAILURE PLANE ENDS ON BANK SLOPE')
      ARAD=ARAD+DELA
      IF($COUN3)STOP
      $COUN3=.TRUE.
      GOTO 99
   98 DELXS=XSMAX/NSLCS
      YS=BANKHT
      XS=XSMAX
      THETA=THETAP
      XFP(NPTS)=XSMAX+T
      YFP(NPTS)=BANKHT+AA
      XFP(1)=T
      YFP(1)=AA
      DO 102 I=1,NSLM1
```

```
      XS=XS-DELXS
      XFP(I+1)=T+XSMAX-XS
      IF($THETA)GOTO 94
      DETH=DELXS/(ARAD*(1.-COS(THETA)))
      GOTO 95
   94 DETH=DELXS/(ARAD*2.)
   95 THETA=THETA-DETH
      YSN=ARAD*(1.-COS(THETA))
      SPSI(I)=ATAN((YS-YSN)/DELXS)
      DELYS(I)=YS-YSN
      YS=YSN
      YFP(I+1)=BANKHT-YS+AA
  102 CONTINUE
      SPSI(NSLCS)=ATAN((BANKHT+AA-YFP(NSLCS))/DELXS)
      DELYS(NSLCS)=BANKHT-YFP(NSLCS)+AA
C     THE ABOVE DO LOOP ALSO CALCULATES SPSI FOR EACH SLICE
C
C     CALCULATE X,Y COORDINATES OF WATERTABLE
C     CALCULATE PORE PRESSURES FOR THE BOUNDS OF EACH SLICE
      DO 200 I=1,NPTS
      CALL HITWT(XFP(I),Q,NN,YWT(I))
      CALL PHI(XFP(I),YWT(I),PHIWT(I),NA,DL,H,A,NA,B,WWTBL)
C     SEE IF WE ARE ABOVE OR BELOW THE WATERTABLE
      IF(YWT(I)-YFP(I))201,202 ,203
  201 PORPR(I)=-(YFP(I)-YWT(I))
      GOTO 200
  202 PORPR(I)=0.
      GOTO 200
  203 CALL PHI(XFP(I),YFP(I),PORPR(I),NA,DL,H,A,NA,B,WWTBL)
C     SUBTRACT ELEVATION HEAD YFP(I)
      PORPR(I)=PORPR(I)-YFP(I)
  200 CONTINUE
C     THIS SECTION EVALUATES THE FACTOR OF SAFETY FOR CHOSEN FAILURE
C     PLANE
      TSFS=0.
      TTFS=0.
      DO 300 I=1,NSLCS
```

```
      IF(XFP(I).GE.XSMINT)GOTO 301
C     SLICE ENDS ON BANK FACE
      IF(XFP(I+1).GT.XSMINT)GOTO 302
      IF(XFP(I).GE.XHS)GOTO 303
      IF(XFP(I+1).GT.XHS)GOTO 304
C     CALCULATE WT. OF SLICE IN SATURATED REGION ON BANK FACE
C     AVG. LENGTH OF SLICE
      SLI=(XFP(I)-T)*TATHET+AA-YFP(I)
      SLIP1=(XFP(I+1)-T)*TATHET+AA-YFP(I+1)
      AVGL=(SLI+SLIP1)/2.
C     VOLUME OF SLICE PER UNIT LENGTH OF CHANNEL
      SLVOL=AVGL*DELXS
      SLWT=SLVOL*GAMAT
      IF($RITYP)GOTO 305
      WRITE(6,40006)SLWT
40006 FORMAT(/,' TYPE 1 ',F10.4)
      GOTO 305
C     FOR SLICE THAT INCLUDES HS
  304 SL1=(XFP(I)-T)*TATHET-YFP(I)+AA
      DEL=XHS-XFP(I)
      YFPDEL=YFP(I)+(YFP(I+1)-YFP(I))*DEL/DELXS
      SL2=HS+AA-YFPDEL
      W1=(SL1+SL2)*DEL*GAMAT/2.
      SL3=YWT(I+1)-YFP(I+1)
      W2=(SL2+SL3)*(DELXS-DEL)*GAMAT/2.
C     FIND WT. OF UNSAT. PART
      SL4=(XFP(I+1)-T)*TATHET-YWT(I+1)+AA
C     WT. OF DRY SOIL
      WDS3=SL4*(DELXS-DEL)*GAMAD/2.
C     WT. OF WATER IN UNSAT REGION
      CALL SINT(SL4,QS,NS,SATIN )
      WW3=(DELXS-DEL)*WCON*SATIN /2.
      SLWT=W1+W2+WDS3+WW3
      IF($RITYP)GOTO 305
      WRITE(6,40016)W1,W2,WDS3,WW3,SLWT
40016 FORMAT(/,' TYPE 2 ',5(2X,F10.4))
      GOTO 305
```

```
C       FOR SLICE ABOVE HS BUT STILL ON BANK FACE
  303 SL1=YWT(I+1)-YFP(I+1)
      SL2=YWT(I)-YFP(I)
      IF((SL1.GE.0.).AND.(SL2.GT.0.))GOTO 313
C       WT. OF DRY SOIL
      SL3=(XFP(I+1)-T)*TATHET-YFP(I+1)+AA
      SL4=(XFP(I)-T)*TATHET-YFP(I)+AA
      W1=(SL3+SL4)*DELXS*GAMAD/2.
      IF((SL1.LT.0.).AND.(SL2.GE.0.))GOTO 323
C       WT. OF WATER
      CALL SINT(-SL2,QS,NS,SAT2)
      SL5=SL4-SL2
      CALL SINT(SL5,QS,NS,SAT4)
      CALL SINT(-SL1,QS,NS,SAT1)
      SL6=SL3-SL1
      CALL SINT(SL6,QS,NS,SAT3)
      WW=DELXS*WCON*((SAT4-SAT2)+(SAT3-SAT1))/2.
      SLWT=W1+WW
      IF($RITYP)GOTO 305
      WRITE(6,40026)W1,WW,SLWT
40026 FORMAT(/,' TYPE 3 ',3(2X,F10.4))
      GOTO 305
C       WT. OF WATER
  323 CALL SINT(SL3,QS,NS,SAT3)
      CALL SINT(SL4,QS,NS,SAT4)
      WW=DELXS*WCON*(SAT4+SAT3)/2.
      SLWT=W1+WW
      IF($RITYP)GOTO 305
      WRITE(6,40036)W1,WW,SLWT
40036 FORMAT(/,' TYPE 4 ',3(2X,F10.4))
      GOTO 305
C       BELOW WATERTABLE
  313 AVGL1=(SL1+SL2)/2.
      W1=AVGL1*DELXS*GAMAT
C       WT. OF SOIL  ABOVE WATERTABLE
      SL1=(XFP(I+1)-T)*TATHET-YWT(I+1)+AA
      SL2=(XFP(I)-T)*TATHET-YWT(I)+AA
```

```
      AVGL2=(SL2+SL1)/2.
      WDS2=AVGL2*DELXS*GAMAD
C     WT. OF WATER IN SOIL ABOVE THE WATERTABLE
      CALL SINT(SL2,QS,NS,SAT2)
      WW21=DELXS*WCON*SAT2
      CALL SINT(SL1,QS,NS,SAT3)
      WW22 =DELXS*WCON*(SAT3-SAT2)/2.
      SLWT=W1+WDS2+WW21+WW22
      IF($RITYP)GOTO 305
      WRITE(6,40046)W1,WDS2,WW21,WW22,SLWT
40046 FORMAT(/,' TYPE 5 ',5(2X,F10.4))
      GOTO 305
C     FOR SLICE AT TOP OF BANK
  302 SL1=YWT(I+1)-YFP(I+1)
      SL2=YWT(I)-YFP(I)
      IF((SL1.GE.0.).AND.(SL2.GT.0.))GOTO 312
C     WT. OF DRY SOIL
      DEL =XSMINT-XFP(I)
      YFPDEL=YFP(I)+(YFP(I+1)-YFP(I))*DEL/DELXS
      SL3=(XFP(I)-T)*TATHET-YFP(I)
      SL4=YFP(NPTS)-YFPDEL
      SL5=YFP(NPTS)-YFP(I+1)
      W1=((SL3+SL4)*DEL*GAMAD/2.)+((SL4+SL5)*(DELXS-DEL)*GAMAD/2.)
C     WT. OF WATER
      IF((SL1.LT.0.).AND.(SL2.GE.0.))GOTO 322
      YWTDEL=YWT(I)+(YWT(I+1)-YWT(I))*DEL/DELXS
      SL6=YFPDEL-YWTDEL
      CALL SINT(-SL1,QS,NS,SAT1)
      SDUM1=SL3-SL1
      CALL SINT(SDUM1,QS,NS,SAT2)
      CALL SINT(SL6,QS,NS,SAT3)
      SDUM2=SL4+SL6
      CALL SINT(SDUM2,QS,NS,SAT4)
      WW1=DEL*WCON*((SAT2-SAT1)+(SAT4-SAT3))/2.
      CALL SINT(-SL2,QS,NS,SAT5)
      SDUM3=SL5-SL2
      CALL SINT(SDUM3,QS,NS,SAT6)
```

```
      WW2=(DELXS-DEL)*WCON*((SAT6-SAT5)+(SAT4-SAT3))/2.
      SLWT=W1+WW1+WW2
      IF($RITYP)GOTO 305
      WRITE(6,40056)W1,WW1,WW2,SLWT
40056 FORMAT(/,' TYPE 6 ',4(2X,F10.4))
      GOTO 305
  322 CALL SINT(SL3,QS,NS,SAT3)
      CALL SINT(SL4,QS,NS,SAT4)
      CALL SINT(SL5,QS,NS,SAT5)
      WW1=DEL*WCON*(SAT4+SAT3)/2.
      WW2=(DELXS-DEL)*WCON*(SAT5+SAT4)/2.
      SLWT=W1+WW1+WW2
      IF($RITYP)GOTO 305
      WRITE(6,40066)W1,WW1,WW2,SLWT
40066 FORMAT(/,' TYPE 7 ',4(2X,F10.4))
      GOTO 305
  312 AVGL1=(SL1+SL2)/2.
      W1=AVGL1*DELXS*GAMAT
C     WT. OF DRY SOIL ABOVE WATERTABLE
      DEL =XSMINT-XFP(I)
      SL1=(XFP(I)-T)*TATHET-YWT(I)+AA
      YWTDEL=YWT(I)+(YWT(I+1)-YWT(I))*DEL/DELXS
      SL2=YFP(NPTS)-YWTDEL
      SL3=YFP(NPTS)-YWT(I+1)
      WDS1=(SL1+SL2)*DEL*GAMAD/2.
      WDS2=(SL2+SL3)*(DELXS-DEL)*GAMAD/2.
C     WT. OF WATER ABOVE WATERTABLE
      CALL SINT(SL1,QS,NS,SAT1)
      CALL SINT(SL2,QS,NS,SAT2)
      CALL SINT(SL3,QS,NS,SAT3)
      WW1=DEL*WCON*(SAT1+SAT2)/2.
      WW2=(DELXS-DEL)*WCON*(SAT2+SAT3)/2.
      SLWT=W1+WDS1+WDS2+WW1+WW2
      IF($RITYP)GOTO 305
      WRITE(6,40076)W1,WDS1,WDS2,WW1,WW2,SLWT
40076 FORMAT(/,' TYPE 8 ',6(2X,F10.4))
      GOTO 305
```

```
  301 SL1=YWT(I)-YFP(I)
      SL2=YWT(I+1)-YFP(I+1)
      IF((SL1.GT.0.).AND.(SL2.GE.0.))GOTO 311
C     WT. OF DRY SOIL
      SL3=YFP(NPTS)-YFP(I)
      SL4=YFP(NPTS)-YFP(I+1)
      W1=DELXS*GAMAD*(SL3+SL4)/2.
C     WT. OF WATER
      IF((SL1.GE.0.).AND.(SL2.LT.0.))GOTO 321
      CALL SINT(-SL1,QS,NS,SAT1)
      CALL SINT(-SL2,QS,NS,SAT2)
      SDUM1=SL3-SL1
      CALL SINT(SDUM1,QS,NS,SAT3)
      SDUM2=SL4-SL2
      CALL SINT(SDUM2,QS,NS,SAT4)
      WW=DELXS*WCON*((SAT4-SAT2)+(SAT3-SAT1))/2.
      SLWT=W1+WW
      IF($RITYP)GOTO 305
      WRITE(6,40086)W1,WW,SLWT
40086 FORMAT(/,' TYPE 9 ',3(2X,F10.4))
      GOTO 305
  321 CALL SINT(SL3,QS,NS,SAT1)
      CALL SINT(SL4,QS,NS,SAT2)
      WW=DELXS*WCON*(SAT1+SAT2)/2.
      SLWT=W1+WW
      IF($RITYP)GOTO 305
      WRITE(6,40096)W1,WW,SLWT
40096 FORMAT(/,' TYPE 10 ',3(2X,F10.4))
      GOTO 305
C     WT. OF WET SOIL
  311 W1=DELXS*GAMAT*(SL1+SL2)/2.
      SL3=YFP(NPTS)-YWT(I)
      SL4=YFP(NPTS)-YWT(I+1)
C     WT. OF DRY SOIL
      W2=DELXS*GAMAD*(SL3+SL4)/2.
C     WT. OF WATER
      CALL SINT(SL3,QS,NS,SAT1)
```

```
      CALL SINT(SL4,QS,NS,SAT2)
      WW=DELXS*WCON*(SAT1+SAT2)/2.
      SLWT=W1+W2+WW
      IF(IPRITYP)GOTO 305
      WRITE(6,40106)W1,W2,WW,SLWT
40106 FORMAT(/,' TYPE 11 ',4(2X,F10.4))
  305 CONTINUE
C     AREA OF SLICE BOTTOM
      ASLIBO =SQRT((DELXS*DELXS)+(DELYS(I)*DELYS(I)))
C     AVG. PORE WATER FORCE ON EACH SLICE
      PORAV=(PORPR(I)+PORPR(I+1))/2.
      IF(PORAV.LT.0.)GOTO 315
      UAVG=PORAV*GAMAW*ASLIBO
      GOTO 316
  315 CALL SAT(-PORAV,QS,NS,SPOR)
      UAVG=SPOR*PORAV*GAMAW*ASLIBO
  316 CONTINUE
C     TANGENTIAL WEIGHT FORCE ON SLICE
      TFS=SLWT*SIN(SPSI(I))
C     NORMAL FORCE
      ANFS=SLWT*COS(SPSI(I))
C     EFFECTIVE NORMAL FORCE
      ENF=ANFS-UAVG
      IF(ENF.LT.0.)ENF=0.
C     COHESIVE FORCE
      FCOH=ASLIBO*TRUCOH
C     MOBILIZABLE SHEAR FORCE AVAILABLE TO SLICE
      IF(ABS(ENF).LT.1.E-10)GOTO 604
      SFS=FCOH+ENF*TAN(SPHI)
      GOTO 605
  604 SFS=FCOH
C     SLICE FACTOR OF SAFETY
  605 F(I)=SFS/TFS
C     TOTAL FACTOR OF SAFETY
      TSFS=TSFS+SFS
      TTFS=TTFS+TFS
  300 CONTINUE
```

```
      TF=TSFS/TTFS
      WRITE(6,50016)TF
50016 FORMAT(/,' FACTOR OF SAFETY FOR THIS TRIAL FAILURE SURFACE = '
     X,F10.4)
      WRITE(6,50026)ARAD,NCOUN
50026 FORMAT(/,' RADIUS OF GENERATING CIRCLE = ',F10.4,/,
     X' NUMBER OF ITERATIONS = ',I3)
      IF(NCOUN.EQ.MAXCOU)GOTO 804
      IF($COUN1)GO TO 800
      ARAD=ARAD+DELA
      $COUN1=.TRUE.
      TFO =TF
      NCOUN=NCOUN+1
      GOTO 99
  800 IF($COUN2)GOTO 802
      IF(TF.GT.TFO)GOTO 801
      ARAD=ARAD+DELA
      $COUN2=.TRUE.
      TFO =TF
      NCOUN=NCOUN+1
      GOTO 99
  801 DELA=-DELA
      ARAD=ARAD+2.0*DELA
      $COUN2=.TRUE.
      NCOUN=NCOUN+1
      GOTO 99
  802 IF(TF.GT.TFO)GOTO 803
      ARAD=ARAD+DELA
      NCOUN=NCOUN+1
      TFO =TF
      GOTO 99
  803 DELF=TF-TFO
      IF(ABS(DELF).LE.FTOL)GOTO 804
      DELA=-DELA/2.0
      ARAD=ARAD+DELA
      NCOUN=NCOUN+1
      TFO =TF
```

```
      GOTO 99
  804 CONTINUE
      WRITE(6,30066)
30066 FORMAT(/,' X-COORDINATES OF FAILURE PLANE ')
      WRITE(6,20246)(XFP(I),I=1,NPTS)
      WRITE(6,30076)
30076 FORMAT(/,' Y-COORDINATES OF FAILURE PLANE')
      WRITE(6,20246)(YFP(I),I=1,NPTS)
      WRITE(6,30086)DELXS
30086 FORMAT(/,' DELTA Y INCREMANTS ALONG FAILURE PLANE FOR DELTA X = ',
     XF10.4)
      WRITE(6,20246)(DELYS(I),I=1,NSLCS)
      WRITE(6,30096)
30096 FORMAT(/,' ANGLE OF SLICE BOTTOM TO VERTICAL ALPHA = TAN(DELTAY(I
     X/DELTAX)')
      WRITE(6,20246)(SPSI(I),I=1,NSLCS)
      WRITE(6,30106)
30106 FORMAT(/,' WATERTABLE ELEVATIONS ')
      WRITE (6,20246)(YWT(I),I=1,NPTS)
      WRITE(6,30116)
30116 FORMAT(/,' HYDRAULIC POTENTIAL ALONG WATERTABLE ')
      WRITE(6,20246)(PHIWT(I),I=1,NPTS)
      WRITE(6,30126)
30126 FORMAT(/,' PORE WATER PRESSURES ALONG TRIAL FAILURE SURFACE ')
      WRITE(6,20246)(PORPR(I),I=1,NPTS)
      WRITE(6,50006)
50006 FORMAT(/,' SLICE FACTORS OF SAFETY ')
      WRITE(6,20246)(F(I),I=1,NSLCS)
C
C     PLOTTING SECTION
C
C     PLOT AXES AND SOIL SURFACE
      SX(1)=0.
      SY(1)=AA
      SX(2)=T
      SY(2)=AA
      SX(3)=XSMINT
```

```
      SY(3)=YFP(NPTS)
      SX(4)=XFP(NPTS)
      SY(4)=YFP(NPTS)
      YSCALE=YFP(NPTS)/5.25
      XSCALE=XFP(NPTS)/7.25
      IF(YSCALE.GT.XSCALE)GOTO 1000
      SCALE=XSCALE
      GOTO 1001
 1000 SCALE=YSCALE
 1001 CONTINUE
      CALL GRAPH(4,SX,SY,0,4,7.25,5.25,SCALE,0.,SCALE,0.,XL,YL,GL,DDL)
C     PLOT WATERTABLE
      XXWT(1)=XHS
      YYWT(1)=HS+AA
      DO 1002 I=2,NPTS
      II=I
      IF(XFP(I).GT.XHS)GOTO 1003
 1002 CONTINUE
 1003 CONTINUE
      NWTPT=NPTS-II+1
      IIM2=II-2
      DO 1004 I=2,NWTPT
      XXWT(I)=XFP(IIM2+I)
      YYWT(I)=YWT(IIM2+I)
 1004 CONTINUE
      CALL GRAPHS(NWTPT,XXWT,YYWT,0,2,';')
C     PLOT FAILURE PLANE
      CALL GRAPHS(NPTS,XFP,YFP,0,2,';')
C     PLOT PORE WATER PRESSURES
      SWT(1)=0.
      DO 1005 I=1,NSLCS
      ASLIBO=SQRT(DELXS*DELXS+(DELYS(I)*DELYS(I)))
 1005 SWT(I+1)=SWT(I)+ASLIBO
      CALL GRAPH(NPTS,SWT,PORPR,0,2,7.25,5.25,0.,0.,0.,0.,XL,YL,GL,DDL)
      STOP
      END
C
```

```
C
C
      SUBROUTINE SAT(HEAD,QS,NS,S)
C     THIS SUBROUTINE CALCULATES THE PERCENT SATURATION FOR A GIVEN
C     SUCTION HEAD USING FITTED CURVE
      REAL*8 QS(11),DS,XDUM,DBLE
      DS=QS(1)
      XDUM=1.D0
      DO 300 K=1,NS
      XDUM=XDUM*(DBLE(HEAD))
  300 DS=DS+QS(K+1)*XDUM
      S=SNGL(DS)
      RETURN
      END
C
C
C
      SUBROUTINE HITWT(X,Q,NN,HTWT)
C     THIS SUBROUTINE CALCULATES WATERTABLE ELEVATIONS
      REAL*8 Q(11),DHTWT,XDUM,DBLE
      DHTWT=Q(1)
      XDUM=1.D0
      DO 300 K=1,NN
      XDUM=XDUM*(DBLE(X))
  300 DHTWT=DHTWT+Q(K+1)*XDUM
      HTWT=SNGL(DHTWT)
      RETURN
      END
C
C
C
      SUBROUTINE PHI(X,Y,PHIXY,M,DL,H,A,KA,B,WW)
C     THIS SUBROUTINE CALCULATES PHI AT X,Y
      REAL*8 A,PI,AM1,DPHIXY,DBLE,DCOSH,DSIN
      REAL*8 Y1,X1,B1,DDL,DWW,DH,UM
      DIMENSION A(KA)
      UM(AM1,X1,Y1,B1)=(DCOSH(AM1*Y1)/DCOSH(AM1*B1))*DSIN(AM1*X1)
```

```
      PI =3.141592653589793
      MM=1
      Y1=DBLE(Y)
      X1=DBLE(X)
      B1=DBLE(B)
      DDL=DBLE(DL)
      DWW=DBLE(WW)
      DH=DBLE(H)
      DPHIXY=(X1/DDL)
    1 AM1=MM*PI/DDL
      MP1=MM
      DPHIXY=DPHIXY+A(MP1)*UM(AM1,X1,Y1,B1)
      IF(MP1.EQ.M)GOTO 2
      MM=MM+1
      GOTO 1
    2 PHIXY=SNGL((DPHIXY*DH)+DWW)
      RETURN
      END
C
C
C
      SUBROUTINE SINT(HEAD,QS,NS,SATINT)
C     THIS SUBROUTINE CALCULATES THE INTEGRAL OF S DY UP TO SUCTION
C     OF HEAD
      REAL*8 QS(11),DSATIN,XDUM,DHEAD,DBLE,DFLOAT
      DHEAD=DBLE(HEAD)
      DSATIN=QS(1)*DHEAD
      XDUM=DHEAD
      DO 300 K=1,NS
      XDUM=XDUM*DHEAD
  300 DSATIN=DSATIN+(QS(K+1)*XDUM/DFLOAT(K+1))
      SATINT=SNGL(DSATIN)
      RETURN
      END
```

```
C*************************************************************************
C
C      SUBROUTINE ORTH WRITTEN BY BOAST(1969). CALCULATES A-NMS FROM
C                          U-MNS AND W-M
C
C      COMMON TO PROGRAMS 1,2
C
C*************************************************************************
       SUBROUTINE ORTH(U,W,C,D,G,J,A,NCAPP1,KA,KAM1,KADIAG,IER)
       IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION U(KA),C(KAM1),D(KA),G(KA),A(KA)
       REAL*8 J(KADIAG),JTEMP
       IF(NCAPP1-1) 1,2,2
1      IER=1
       RETURN
2      IF(NCAPP1-KA) 4,4,3
3      IER=2
       RETURN
4      IF(KA-1-KAM1) 5,6,5
5      IER=3
       RETURN
6      IF((KA*KAM1)/2-KADIAG) 7,8,7
7      IER=4
       RETURN
8      CONTINUE
       IER=0
       NCAP = NCAPP1-1
       NCAPM1 = NCAP-1
       IF(NCAPM1) 10,20,30
10     D(1) = U(1)
       G(1) = W
       E = G(1)/D(1)
       A(1) = E
       RETURN
20     C(1) = U(1)/D(1)
       D(2) = U(2)-C(1)*C(1)*D(1)
       G(2) = W-C(1)*G(1)
```

```
      E = G(2)/D(2)
      J(1) = C(1)
      A(1) = A(1)-E*J(1)
      A(2) = E
      RETURN
 30   C(1) = U(1)/D(1)
      NFORJ = 0
      DO 120 N = 2,NCAP
      CTEMP = U(N)
      NM1 = N-1
      DO 110 NN = 1,NM1
      NFORJ = NFORJ+1
 110  CTEMP = CTEMP-U(NN)*J(NFORJ)
 120  C(N) = CTEMP/D(N)
      DTEMP = U(NCAPP1)
      GTEMP = W
      DO 140 N = 1,NCAP
      CTEMP = C(N)
      DTEMP = DTEMP-CTEMP*CTEMP*D(N)
 140  GTEMP = GTEMP-CTEMP*G(N)
      D(NCAPP1) = DTEMP
      G(NCAPP1) = GTEMP
      E = GTEMP/DTEMP
      NSTART = 0
      DO 180 N = 1,NCAPM1
      JTEMP = C(N)
      NSTART = NSTART+N
      NFORJ = NSTART
      NP1 = N+1
      DO 170 NN = NP1,NCAP
      JTEMP = JTEMP-C(NN)*J(NFORJ)
 170  NFORJ = NFORJ+NN-1
      J(NFORJ) = JTEMP
 180  A(N) = A(N)-E*JTEMP
      NFORJ = NFORJ+1
      J(NFORJ) = C(NCAP)
      A(NCAP) = A(NCAP)-E*J(NFORJ)
```

```
A(NCAPP1) = E
RETURN
END
```

```
C*********************************************************************
C
C     SUBROUTINE OPLSPA FITS POLYNOMIAL THROUGH DATA BY LEAST
C                       SQUARES
C
C     COMMON TO PROGRAMS 1,2,5,6.
C
C*********************************************************************
      SUBROUTINE OPLSPA (NDEG,NPTS,X,Y,W,Q,TUWYLO)                      OPLSP001
      DIMENSION  X(1), Y(1), W(1)                                       OPLSP002
      DOUBLE PRECISION Q(1), PN(11), PN1(10), SUM(4), B, C, PNX, TMP    OPLSP003
      IF (TUWYLO)  2,1,2                                                OPLSP004
    1 N=0                                                               OPLSP005
      C=0.                                                              OPLSP006
      PN(1)=1.0                                                         OPLSP007
      GO TO 6                                                           OPLSP008
    2 C=-SUM(3)/SUM(4)                                                  OPLSP009
    3 B=-SUM(1)/SUM(3)                                                  OPLSP010
      SUM(4)=SUM(3)                                                     OPLSP011
      N=N+1                                                             OPLSP012
      PN1(N)=0.                                                         OPLSP013
      PN(N+1)=0.                                                        OPLSP014
      DO 4   J=1,N                                                      OPLSP015
      TMP=PN(J)                                                         OPLSP016
      PN(J)=B*PN(J)+C*PN1(J)                                            OPLSP017
    4 PN1(J)=TMP                                                        OPLSP018
      DO 5   J=1,N                                                      OPLSP019
    5 PN(J+1)=PN(J+1)+PN1(J)                                            OPLSP020
    6 DO 7   K=1,3                                                      OPLSP021
    7 SUM(K)=0.0                                                        OPLSP022
      DO 11  I=1,NPTS                                                   OPLSP023
      PNX=1.0                                                           OPLSP024
      J=N                                                               OPLSP025
    8 IF (J)  10,10,9                                                   OPLSP026
    9 PNX=PN(J)+PNX*X(I)                                                OPLSP027
      J=J-1                                                             OPLSP028
      GO TO 8                                                           OPLSP029
```

```
10 SUM(1)=SUM(1)+W(I)*X(I)*PNX*PNX                                OPLSP030
   SUM(2)=SUM(2)+W(I)*Y(I)*PNX                                    OPLSP031
11 SUM(3)=SUM(3)+W(I)*PNX*PNX                                     OPLSP032
   Q(N+1)=SUM(2)/SUM(3)                                           OPLSP033
   IF (N)  3,3,12                                                 OPLSP034
12 DO 13  J=1,N                                                   OPLSP035
13 Q(J)=Q(J)+Q(N+1)*PN(J)                                         OPLSP036
   IF (N-NDEG)  2,14,14                                           OPLSP037
14 RETURN                                                         OPLSP038
   END                                                            OPLSP039
```

# APPENDIX B

```
C*********************************************************************
C
C       EXAMPLE INPUT DATA
C
C*********************************************************************
C
C       NOTE.  REMOVE COMMENT CARDS UNLESS OTHERWISE STATED
C
C*********************************************************************
C
C*********************************************************************
C
C       PROGRAM 1
C
C*********************************************************************
C
C       REPLACE WITH BLANK CARD
C       INT1,INT2,INTS,MAX,NN,INTX
 20 20128 31  5 40
C       WW,AA,B,THETA,DL,HS,T
0.0       10.0      35.0      75.0      80.0      10.0      5.0
C       WATER TABLE ELEVATIONS
20.0      21.2      22.25     23.25     24.2      25.0      25.7      26.4
27.0      27.6      28.2      28.7      29.1      29.6      30.0      30.3
30.7      31.1      31.4      31.7      32.0      32.2      32.4      32.6
32.8      33.1      33.3      33.4      33.6      33.75     33.9      34.1
34.2      34.3      34.4      34.5      34.6      34.7      34.8      34.9
35.0
```

```
C************************************************************************
C
C      PROGRAM 2
C
C************************************************************************
C
C      REPLACE WITH BLANK CARD
C      INT1,INT2,INTS,MAX,NN,INTX
 20 20128 40  5 20
C      WW,AA,B,THETA,DL,HS,T
2.5       0.0       5.0       75.0      20.0      3.25      0.0
C      WATER TABLE ELEVATIONS
3.25      3.50      3.85      4.15      4.40      4.5       4.6       4.70
4.75      4.80      4.85      4.90      4.92      4.93      4.94      4.95
4.96      4.97      4.98      4.99      5.0
```

```
C*********************************************************************
C
C      PROGRAM 3
C
C*********************************************************************
C
C      NA,NN,NWT,NFL,NPHIL,NPTS,INTX,INTY
 31  5 10  4  4  0 20  9
C      A-NM VALUES
 0.25729509508989850D 02-0.16570660110813910D 02-0.17856227083176280D 02
-0.26125905735572700D 02-0.41020013635109650D 02-0.66585788291730770D 02
-0.10882957410417420D 03-0.17760034923845590D 03-0.28717316565169740D 03
-0.45841929239784980D 03-0.71991977625957000D 03-0.11094159419041650D 04
-0.16732425125816770D 04-0.24637228703295620D 04-0.35319805429481520D 04
-0.49155880506206620D 04-0.66197810676262680D 04-0.85944345772344970D 04
-0.10711267996656450D 05-0.12751076880229810D 05-0.14413153165405920D 05
-0.15359207168614220D 05-0.15295022992673850D 05-0.14076391172729260D 05
-0.11802504962143940D 05-0.88449554694885070D 04-0.57687338161464800D 04
-0.31479937535696010D 04-0.13491393822892750D 04-0.40392122735571010D 03
-0.63298627516304000D 02
C      COEFFICIENTS OF POLYNOMIAL FITTED TO WATER TABLE Q-S
 0.13517086031231160D 02 0.10268580201549250D 01-0.26833492308425730D-01
 0.44586750632442200D-03-0.40896591521771240D-05 0.15350789309272290D-07
C      WW,AA,B,THETA,DL,HS,T,HYDCON
0.          10.         35.         75.         80.         10.         5.          .2
```

```
C*************************************************************************
C
C       PROGRAM 4
C
C*************************************************************************
C
C       NA,NN,NWT,NFL,NPHIL,NPTS,INTX,INTY
 31  5 10  4  4  0 20  6
C       A-NM VALUES
 0.507609527748573l0D 00 0.199964833078437400 00 0.12017895991658940D 00
 0.79754703506975800D-01 0.85064659425183500D-01 0.80175657387105200D-01
 0.85170424864755800D-01 0.89776395122907800D-01 0.96265786946715600D-01
 0.10450976170403540D 00 0.11220902523199800D 00 0.12193391867258370D 00
 0.12979009527244290D 00 0.13909629673824140D 00 0.14534321881141390D 00
 0.15193430169928130D 00 0.15420324254842350D 00 0.15549560887627820D 00
 0.15135547004373010D 00 0.14523337602127670D 00 0.13329567824707500D 00
 0.11941495125637450D 00 0.10074925170973970D 00 0.81789182658954300D-01
 0.60704776061945640D-01 0.42288563214693900D-01 0.25073004556234820D-01
 0.13113120949183880D-01 0.42449669081079190D-02 0.67700672077578340D-03
-0.11837276201770870D-02
C       COEFFICIENTS OF POLYNOMIAL FITTED TO WATER TABLE Q-S
 0.28607521691292110D 01 0.42018922882232030D 00-0.15741200989563220D-01
-0.23840099637375740D-02 0.22367553276290370D-03-0.52277051353505090D-05
C       WW,AA,B,THETA,DL,HS,T,WTMAX
2.5        0.          5.0        75.        20.        3.25       0.         5.0
C       HYDCON
0.2
```

```
C*****************************************************************
C
C      PROGRAM 5
C
C*****************************************************************
C
C      BANKHT,ARAD,BTHETA,NSLCS
20.        15.        75.         32
C      AA,T,WW,DL,HS,B
10.        5.         0.          80.         10.         35.
C      NS,NN,NA,NSPTS
  5  5 31  8
C      REPLACE WITH BLANK CARD
C      COEFFICIENTS OF POLYNOMIAL FITTED TO WATER TABLE Q-S
 0.13517086031231160D 02 0.10268580201549250D 01-0.26833492308425730D-01
 0.44586750632442200D-03-0.40896591521771240D-05 0.15350789309272290D-07
C      A-NM VALUES
 0.25729509508989850D 02-0.16570660110813910D 02-0.17856227083176280D 02
-0.26125905735572700D 02-0.41020013635109650D 02-0.66585788291730770D 02
-0.10882957410417420D 03-0.17760034923845590D 03-0.28717316565169740D 03
-0.45841929239784980D 03-0.71991977625957000D 03-0.11094159419041650D 04
-0.16732425125816770D 04-0.24637228703295620D 04-0.35319805429481520D 04
-0.49155880506206620D 04-0.66197810676262680D 04-0.85944345772344970D 04
-0.10711267996656450D 05-0.12751076880229810D 05-0.14413153165405920D 05
-0.15359207168614220D 05-0.15295022992673850D 05-0.14076391172729260D 05
-0.11802504962143940D 05-0.88449554694885070D 04-0.57687338161464800D 04
-0.31479937535696010D 04-0.13491393822892750D 04-0.40392122735571010D 03
-0.63298627516304000D 02
C      EXPERIMENTAL SATURATION VS SUCTION HEAD DATA IN PAIRS (S,HEAD)
1.         0.         .859        4.1         .763        5.9         .668        7.4
.573       9.0        .477        11.5        .382        17.4        .324        32.8
C      GAMAW,GS,PORSTY,DRAPOR,TRUCOH,SPHI,FTOL,MAXCOU
62.4       2.7        .524        .33         2.5         35.         .001         15
```

```
C*****************************************************************
C
C      PROGRAM 6
C
C*****************************************************************
C
C      BANKHT,ARAD,BTHETA,NSLCS
15.        10.        75.        32
C      AA,T,WWTBL,DL,HS,B
0.0        0.0        2.5        20.0       3.25       5.0
C      NS,NN,NA,NSPTS
  5  5 31  8
C      REPLACE WITH BLANK CARD
C      COEFFICIENTS OF POLYNOMIAL FITTED TO WATER TABLE Q-S
 0.2860752169129211OD 01 0.4201892288223203OD 00-0.1574120098956322OD-01
-0.2384009963737574OD-02 0.2236755327629037OD-03-0.5227705135350509OD-05
C      A-NM VALUES
 0.5076095277485731OD 00 0.1999648330784374OD 00 0.1201789599165894OD 00
 0.7975470350697580OD-01 0.8506465942518350OD-01 0.8017565738710520OD-01
 0.8517042486475580OD-01 0.8977639512290780OD-01 0.9626578694671560OD-01
 0.1045097617040354OD 00 0.1122090252319980OD 00 0.1219339186725837OD 00
 0.1297900952724429OD 00 0.1390962967382414OD 00 0.1453432188114139OD 00
 0.1519343016992813OD 00 0.1542032425484235OD 00 0.1554956088762782OD 00
 0.1513554700437301OD 00 0.1452333760212767OD 00 0.1332956782470750OD 00
 0.1194149512563745OD 00 0.1007492517097397OD 00 0.8178918265895430OD-01
 0.6070477606194564OD-01 0.4228856321469390OD-01 0.2507300455623482OD-01
 0.1311312094918388OD-01 0.4244966908107919OD-02 0.6770067207757834OD-03
-0.1183727620177087OD-02
C      EXPERIMENTAL SATURATION VS SUCTION HEAD DATA IN PAIRS (S,HEAD)
1.         0.         .859       4.1        .763       5.9        .668       7.4
.573       9.0        .477       11.5       .382       17.4       .324       32.8
C      GAMAW,GS,PORSTY,DRAPOR,TRUCOH,SPHI,FTOL,MAXCOU
62.4       2.7        .524       .33        0.0        35.        .001        15
```

```
C***********************************************************************
C
C     DICTIONARY OF INPUT VARIABLE NAMES
C
C***********************************************************************
C
C***********************************************************************
C
C     NAME                FUNCTION                                 PROGRAM
C
C***********************************************************************
C
C     A        A-NMS                                               3-6
C     AA       DEPTH TO IMP. BARRIER FT.                           1-6
C     ARAD     RADIUS OF GEN. CIRCLE FT.                           5,6
C     B        HEIGHT OF WATER TABLE AT X=DL FT.                   1-6
C     BANKHT   HEIGHT OF GULLY BANK FT.                            5,6
C     BTHETA   ANGLE OF BANK TO HORIZ. DEG.                        5,6
C     DL       LENGTH OF FLOW REGION FT.                           1-6
C     DRAPOR   DUMMY=ZERO                                          5,6
C     FTOL     TOLERANCE ON FACTOR OF SAFETY                       5,6
C     FX       WATER TABLE HEIGHTS FT.                             1,2
C     GAMAW    UNIT WT. OF WATER P.C.F.                            5,6
C     GS       SPEC. GRAV. OF SOIL PARTICLES                       5,6
C     HEAD     SUCTION HEAD  FT.                                   5,6
C     HS       HEIGHT OF SEEPAGE FACE FT.                          1-6
C     HYDCON   HYDRAULIC CONDUCTIVITY FT./DAY                      3,4
C     INTS     BISECTIONS ALONG WATER TABLE                        1,2
C     INTX     NO. OF WATER TABLE POINTS-1                         1,2
C              BISECTIONS ALONG X-AXIS                             3,4
C     INTY     BISECTIONS ALONG Y-AXIS                             3,4
C     INT1     BISECTIONS ALONG GULLY BED OR BELOW WATERTABLE      1,2
C     INT2     BISECTIONS ALONG SEEPAGE FACE                       1,2
C     MAX      MAX. NO. OF A-NMS TO BE CALCULATED                  1,2
C     MAXCOU   MAX. NO. OF FAILURE PLANES TO BE GENERATED          5,6
C     NA       NUMBER OF A-NMS TO BE READ                          3-6
C     NFL      DUMMY                                               4,5
```

```
C      NFL      DUMMY                                                 3,4
C      NN       ORDER OF POLYNOMIAL FITTED TO WATER TABLE DATA        1-6
C      NPHIL    DUMMY                                                 3,4
C      NPTS     DUMMY                                                 3,4
C      NS       ORDER OF POLYNOMIAL FITTED TO SATURATION DATA         5,6
C      NSLCS    NUMBER OF SLICES                                      5,6
C      NSPTS    NO. OF SATURATION DATA POINTS                         5,6
C      NWT      DUMMY                                                 3,4
C      PORSTY   SOIL POROSITY                                         5,6
C      Q        COEFFICIENTS OF POLYNOMIAL FITTED TO WATER TABLE      5,6
C      S        DEGREE OF SATURATION                                  5,6
C      SPHI     ANGLE OF INTERNAL SHEARING RESISTANCE DEG.            5,6
C      T        HALF WIDTH OF GULLY FT.                               1-6
C      THETA    ANGLE OF BANK TO HORIZONTAL DEG.                      1-4
C      TRUCOH   TRUE COHESION P.S.I.                                  5,6
C      WTMAX    MAX. HEIGHT OF WATER TABLE FT.                        4
C      WW       DEPTH OF WATER IN GULLY FT.                           1-5
C      WWTBL    DEPTH OF WATER IN GULLY FT.                           6
C
C*************************************************************************
```