A METHOD FOR THE PREDICTION OF THE OFF-DESIGN

PERFORMANCE OF AXIAL-FLOW COMPRESSORS

by

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INTRODUCTION

The axial-flow turbomachine has, during the last two decades, become a strong competitor in the field of compression of gases. It has demonstrated higher values of peak efficiency than other compressor types and the ability to handle large volumes of working fluid without undue increase in dimensions. Also, during this period important gains have been made in the attainment of high pressure ratios per stage and in the reliability of the machine. The preeminence of the axial compressor in gas turbine applications and its increased use in steel manufacture, wind tunnels and chemical processes are based on these developments.

A great deal of analytical and experimental research has provided the foundation for the performance improvements which have been obtained. The demand by designers for improved information and techniques led to primary emphasis on the so-called "inverse" compressor problem. In this problem, the conditions of compressor operation or performance are given at a "design point", and the results required are the flow passage dimensions and a blading arrangement which will yield these performance conditions.

In order to obtain the required performance with a minimum number of stages, and with minimum over-all compressor dimensions, studies directed toward determining the optimum flow pattern through the machine were conducted. To provide blading to produce the desired flow pattern, blade section information was obtained by analytical means and by experimental investigation in two-dimensional cascades, annular cascades, and in single-stage compressor tests.

In the early phases of the development of solutions for the "inverse"

or design problem, performance requirements were necessarily not severe. Multistage compressors which provided moderate compressor performance were built and tested. When these compressors failed to achieve the design point, the failure was often attributed to "three-dimensional effects" and to "boundary layer effects". As time went on increased confidence on the part of designers led to more stringent design point values. Unfortunately, it soon became obvious that higher design point pressure ratios were, in general, accompanied by poor performance at flows and rotative speeds other than design. The reasons for poor off-design performance and the nature of the flow phenomena which occur under off-design operation became the objects of intensive study.

As the investigation of the items which determine the compressor over-all performance map has progressed, the "direct" compressor problem has been recognized as being at least equal in importance to the "inverse" problem. In the "direct" problem, in contrast to the "inverse" problem, the compressor blading and flow passage dimensions are completely defined; and it is required that the performance be computed not only at design conditions but over the entire expected range of compressor operation.

Most of the early off-design performance studies resulted from a problem encountered with a particular multi-stage compressor design, and methods for analysis of operating difficulties were largely semi-empirical in nature. Successful procedures were developed, and are still frequently used, in which individual stage performance curves were assumed and assembled into a calculated performance map by a "stage-stacking" technique. These methods are in some respects not completely satisfactory.

One important reason for this is that such methods require a set of
assumed stage performance curves. The selection of appropriate curves is
not easy, particularly for the unusual or advanced stage design. A
second objection to "stage-stacking" methods arises because of the limited
extent of off-design performance improvement that results from their use.
Also, there is no possibility of discovering new stage designs; i. e.,
new velocity diagram and blading arrangements within individual stages
that may offer improvement.

It may be concluded then, that a reasonably simple solution of the
"direct" or analysis problem for axial-flow compressors is highly
desirable. Moreover, such a solution is necessary before a complete study
of the off-design performance problem can be made. If the solution could
be considered as sufficiently accurate, costly multi-stage compressor
tests would be at least partially eliminated. The present investigation
was begun to determine whether present-day knowledge of blade-element
performance and current techniques and assumptions for the simplification
of the equations governing the three-dimensional flow through axial-flow
compressors are adequate for the development of a reasonably accurate
solution to the "direct" problem. An additional object was the study of
possible improvement of off-design performance by proper velocity
diagram selection.

HISTORICAL BACKGROUND OF AXIAL-FLOW COMPRESSOR THEORY AND EXPERIMENT

Early Development of the Axial-Flow Compressor—to 1930

The axial-flow compressor is one of a group of machine types which in fairly recent times has been classed under the broad designation of turbomachinery. As a member of this group, it must look back toward the reaction turbine of Hero for its earliest historical background. This phase of turbomachinery development has been recorded in detail elsewhere and will not be explored herein.

In the construction of the earlist forms of turbomachinery there was undoubtedly some "design" involved. It is, however, necessary to look to the 18th century for the appearance of the first ideas which led toward the methods which we now employ in turbomachinery design. These ideas were presented in the papers of Euler, and consisted of the application of the laws of motion to the transfer of energy between a rotor and a fluid (1).

It is difficult to determine by the examination of published information exactly when the use of a form of gas compression device similar to the axial-flow compressor was first suggested, but in 1847 a French engineer, Burdin, proposed the construction of a gas turbine in which air was to be compressed by "a series of blowers" arranged in a manner similar to a turbine (2). Later, in 1872, Dr. F. Stolze, of Charlottenburg, near Berlin, applied for a patent on a gas turbine similar to that of Burdin, and drawings of this scheme show a "multiple turbine compressor" of 10 stages (3). The Stolze machine was actually built and

tested with little success, but not until the period 1900 - 1904. Prior to this time, however, Sir C. A. Parsons (4, 5) had become interested in axial-flow compressor possibilities. In his earliest steam turbine patent (1884) he stated, "-and if such an apparatus be driven, it becomes a pump and can be used for actuating a fluid column or producing pressure in a fluid." In a 1901 patent titled "Improvements in Compressors and Pumps of the Turbine Type", Parsons wrote, "My invention consists in a compressor or pump of the turbine type, operating by the motion of sets of movable vanes or blades between sets of fixed blades, the movable blades being more widely spaced than in my steam turbines, and constructed with curved surfaces on the delivery side and set at a suitable angle to the axis of rotation. The fixed blades may have a similar configuration and be similarly arranged on the containing casing at any suitable angle." About 30 compressors were built by Parsons during the period 1901 - 1907. The first, which was completed in 1901, had a delivery pressure of about 1.75 psig. with a capacity of 3000 cfm free air at 4000 rpm. This compressor had 19 stages. The remaining compressors were of larger capacity and were built to produce pressures up to 15 psig. An experimental unit was built in which two compressors were placed in tandem with an intercooler between to produce 80 psig. The blades in all of Parsons' early compressors were of "plano-convex section" with the rotor blades "set in rows at an angle similar to that of a ship's propeller." The stationary or guide blades were "set with their plane surfaces parallel to the axis" and their purpose was "to stop the rotation of the air after being acted on by the moving blades." It may be noted that

these blades were of very short span. Parsons, himself, in 1906 gave the tip velocity of the rotating blades as "about 400 feet per second." The Parsons compressors of this era had an efficiency of about 55 per cent. This disappointing performance along with the necessity for rapid development of other products caused Parsons to drop the axial compressor as a commercial product.

During this period Parsons' organization was not the only one to build and test axial-flow compressors. References to work of Rateau and to experiments run by the General Electric Company appeared in the literature, and the Westinghouse Machine Company in 1905-7 (6) built an experimental unit to Parsons' patents. This compressor had an efficiency similar to those of the other Parsons compressors. There can be no doubt that the reason for the failure of these machines to be commercially successful was the lack of application of the principles of aerodynamics to their design. This cannot be considered a criticism of those responsible, because much of the aerodynamic knowledge now in use was non-existent or in its earliest stages of development in the early 1900's.

Bauersfeld (7), in 1922, reported the first published work on the application of airfoil theory to the design of turbomachinery. This paper was probably not responsible for the subsequent renewal of interest in the axial compressor but its existence provided the basis for the practical design of such machines.

In 1926, an unpublished proposal was made by Dr. A. A. Griffith (8, 9, 10) to representatives of the British Aeronautical Research Committee. He had worked out a method for the aerodynamic design of

compressors and turbines which was based on airfoil theory. As a result of his proposal the first British cascade experiments were begun (11) and Griffith himself designed a single-stage compressor and single-stage turbine on the same shaft for testing. The blading was of free-vortex type and tests on the unit run in 1929 showed compressor efficiency of 91 per cent. Unfortunately, proposals to continue and extend Griffith's work were not carried out and a lapse of six years followed in British government research efforts.

For test purposes the Swiss firm of Brown-Boveri built in 1927 a small 4-stage axial-flow compressor, and during the following years, several single- and two-stage fans were built and tested (12).

Also, just prior to 1930, several other industrial organizations had built successful single- and two-stage blowers for the ventilation of turbo-generators. These machines were in general of very low pressure ratio and were not built as compressors. In addition, extensive work was proceeding in many countries on the design of axial-flow or propeller pumps and turbines (13-16). These developments cannot be discounted as contributors to the background of ideas and logic relating to axial-compressor design.

Attempts to achieve solutions to the design problem, which would provide satisfactory performance at the design point, occupied the attention of nearly all of the investigators of this period. Off-design performance was not considered as critical and was probably regarded as something about which little could be done.

Development during the Period 1930-45

The years between 1930 and 1945 were marked by the development of two distinct schools of thought concerning axial-flow compressor blade design. One group proceeded by basing their approach on isolated airfoil theory, with appropriate methods of correction for the presence of adjacent blades in compressor blade rows. The other proceeded by using a more experimental approach. Investigations of two-dimensional cascades of blade sections were made, and correlations of these data were made for use by designers. Although differences did exist in other aspects of the approach to the design problem, the principal division was in the method used for blade selection.

During the early 1930's, the Swiss turbomachinery manufacturers, Brown-Boveri and Escher-Wyss, were both particularly active in compressor work. Design of the Velox steam generator, construction of a supersonic wind tunnel, and stationary gas-turbine research at Brown-Boveri required a highly compact, reliable compressor (12). A systematic investigation of the axial-flow machine followed. This led to some very efficient multistage units with, however, a very low pressure ratio per stage. In the Escher-Wyss organization a similar program of development was undertaken in which numerous cascade investigations and single-stage tests accompanied the analytical work reported by Keller (17, 18). Although cascade tests were performed by both Swiss groups, the corrected isolated airfoil approach was developed at this time. It should be mentioned that the strict adherance of most Continental designers to the so-called free-vortex design stems from this beginning. It should also be

noted that the pioneer work of Keller was first reported in his Doctoral thesis directed by Prof. Ackeret of the Swiss Federal Institute of Technology, Zurich.

German development programs prior to and during the Second World War were influenced greatly by the Swiss findings and procedures. The German efforts were, of course, aimed primarily at the production of workable turbo-jet aircraft engines. The most effective German work was probably performed at the Aerodynamic Research Institute (AVA) of Goettingen. The general course of this work and the concurrent industrial program is discussed by Schlaifer and Heron (9), and important results are reported in several NACA translations (19-26).

An examination of British efforts between 1930 and 1945 indicates that British study and development were essentially independent of work on the European continent. Research begun in 1936 produced a workable framework of design equations based largely on incompressible flow and backed by a comprehensive cascade testing program. These equations yielded multistage compressors which were efficient and reliable (8). The British were aware of the Swiss techniques and results, but the design procedure set forth by Howell in 1945 (27, 28) introduced many new and different ideas and provided a guide for British and many American designers. Recognition and pioneer analysis of many basic problems must be credited to Constant, Howell, and their associates.

In the United States, during the early 1930's, papers dealing with the design and performance of fans showed little original design work (29, 30). The influence of Keller's publications and the existence of a

licensing agreement between Brown-Boveri and the Allis-Chalmers Company resulted in a more organized approach after 1936.

The first systematic investigation of axial-flow compressor problems in the United States was begun in 1938 by Jacobs and Wasielewski of the National Advisory Committee for Aeronautics (N. A. C. A.). Their work resulted in the construction and testing of an 8-stage axial-flow compressor which produced a pressure ratio of 3.4:1 with an efficiency of 87 per cent. The detailed procedure used for design and the test results were published in 1943 (31). Even before final publication of the N. A. C. A. results work was begun by several manufacturers on axial-flow units, probably encouraged by the favorable performance of the 8-stage machine. The design philosophies of this period, including that of Jacobs, were undoubtedly affected by the earlier Keller results.

Although many satisfactory axial-flow compressors were designed and constructed prior to 1945, their performance at off-design operating points was often not critical in the specific application. However, both German and British gas-turbine designers had encountered situations in which engines could not be started. This indication of possible inferior off-design compressor performance, or of poor compressor-turbine matching, was obviously not fully understood. Some unpublished work on compressor stage matching done during World War II in Germany was the only attempt at detailed analysis of the problem during this time.

Recent Advances in Axial-Flow Compressor Design Methods

At the conclusion of the European phase of World War II an

accelerated rate of research began which has continued until the present. There was a general realization that extensive cascade, single-stage, and multi-stage compressor testing was necessary in order to reveal the relative importance of the major design variables, and that the experimental studies must be accompanied by the systematic analysis and correlation of available data. With the progress made in all parts of the world in the compressor area, workers tended toward a more nearly common approach. The uniformity of approach was, of course, influenced by the availability of funds and research equipment.

Numerous investigations have been made of the effects of velocity diagram variation on compressor design performance. The free-vortex flow patterns used in successful early compressors were limited in usefulness as increased performance was demanded. The limitations imposed by high blade-element losses at high relative Mach numbers and for high diffusion rates made the use of other types of flow distribution necessary.

Experimental results, showing that good efficiencies could be obtained with a wide variety of velocity diagrams, prompted optimization studies, which attempted, for example, to show which diagram types offered maximum power input, pressure ratio, or flow per unit area within the limitations imposed by Mach number and diffusion (32-38). Also accompanying the trend away from the "two-dimensional" free-vortex flow pattern were the problems associated with radial flows, and streamline curvature effects, both manifestations of the existence of "three-dimensional" flow.

Formulations of the equations of motion, continuity, and of the

applicable laws of thermodynamics were introduced (39-41) to describe the flow in compressors. These equations in their most general form have proved impractical to solve, and for the average compressor designer are useless unless simplified by assumption of a more easily treated flow model. The assumptions used in nearly all design methods are those of axisymmetric, time-steady flow. These assumptions are applied to the flow at stations between blade rows where blade forces are not present, and have proved adequate for the design of numerous high-performance compressors.

The selection of compressor blading has been an area in which a division of opinion exists between investigators in different geographical regions. The airfoil theory school of thought, for which an able spokesman has been Schlichting (42), has attacked the problem of blade selection by first stating that theoretical methods must be developed because of the infinite possible variation in compressor blade-row geometry. The need for verification by experiment is recognized. However, the majority of reported British and American design philosophy is based on the correlation of the available two- and three-dimensional cascade data (43-45). The large amount of systematic two-dimensional cascade data which has been obtained has been analyzed and used as the basis for the formulation of empirical design rules. These rules have been used in the design of many good compressors.

Experimental test programs have, since 1945, been used as a means for verification of design methods. Single stage tests under conditions approaching incompressible flow have verified and extended two-dimensional

cascade test results (46). In addition high-speed single-stage results
have been obtained to provide design limit information and to check the
trends indicated by velocity diagram analysis.

Recently, the availability of equipment for full-range testing of
multistage compressors having high pressure ratios and weight flows has
not only made possible an increased confidence in design procedures
but has also brought about the realization that a family of off-design
problems remained to be understood and investigated.

RESEARCH RELATED TO OFF-DESIGN PERFORMANCE PROBLEMS

Off-design performance problems occurring in axial-flow compressors were not investigated in detail until the early 1950's. The effects of the deterioration of compressor performance at speeds and flow rates different from the design values were noted by German investigators (9), and by the British (8) as they developed the first turbojet aircraft engines using axial-flow compressors. The phenomenon known as surge or pumping was well-known to every experimenter who had been able to test a compressor over its operating range.

The attention of investigators was first drawn by the lack of understanding of compressor surge. The typical multistage compressor performance map of Figure 1 shows that surging occurs when a compressor operating at constant speed is throttled so that a sufficiently low weight flow is obtained. Studies of the surge phenomenon were made by Pearson and Bowmer (47), Bullock et al. (48) and others prior to 1950. However, these papers give only a qualitative description of the nature of surging, although quantitative information was presented relating to the surge characteristics of some specific machines. After 1950, the use of appropriate instrumentation enabled personnel of several research groups to study in detail the nature of the surge phenomenon. As a result the second important manifestation of deterioration of off-design performance was discovered and attacked intensively by several investigators (49-54). This phenomenon was called "rotating" or "propagating" stall.

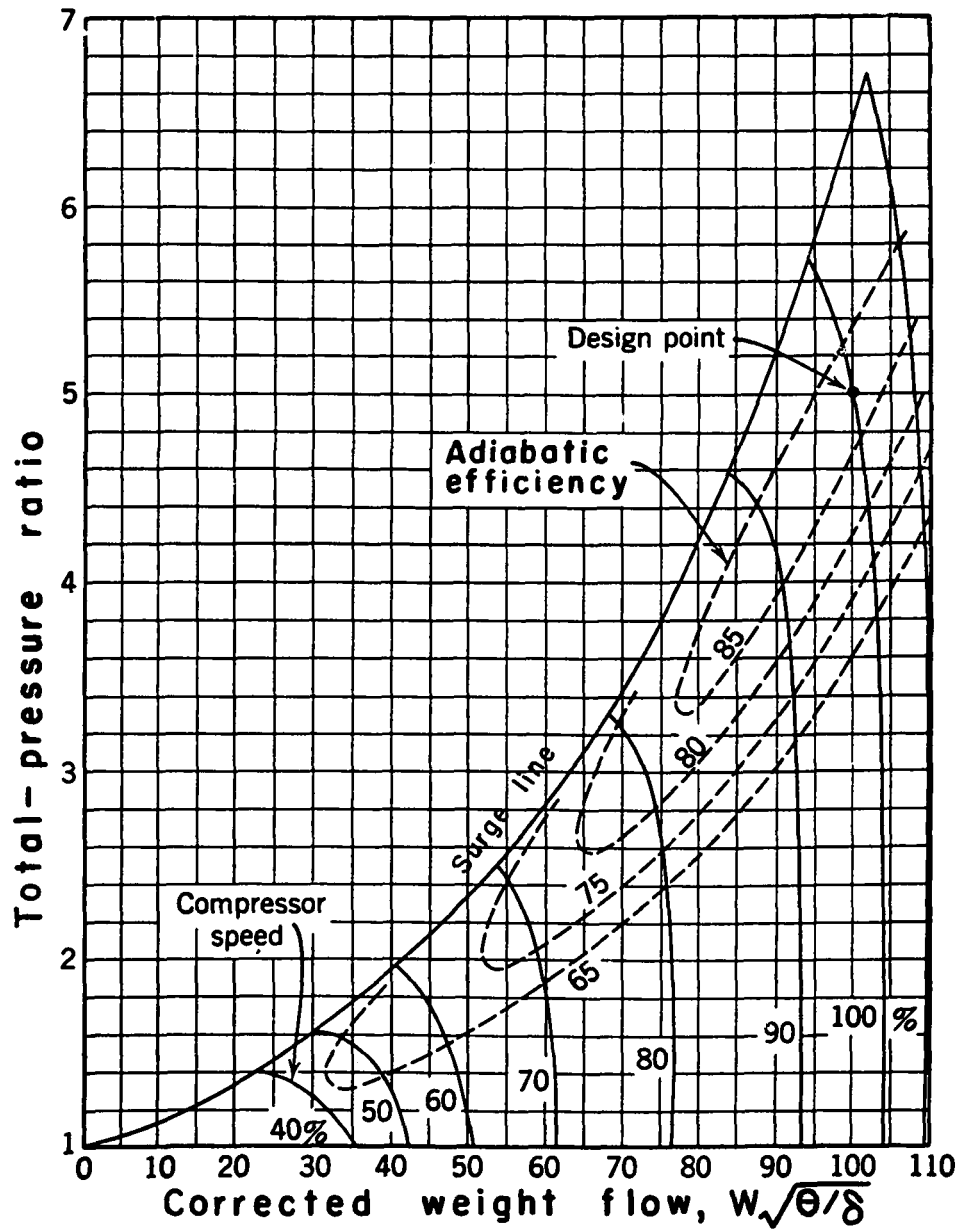The most important single fact which may be noted in connection with

Figure l. Variation of total—pressure ratio and adiabatic efficiency with corrected weight flow at constant corrected tip speeds for a typical multistage axial—flow compressor

compressor surging and stall propagation is that they only occur in a blade row or in a multistage compressor when some of the blade elements have been forced to operate at angles of incidence which are far from optimum values. The realization that this was true was the basis for increased study of compressor stage matching and the nature of stage performance curves (55-57). These stage matching studies provided the first means by which a compressor with poor off-design performance might be improved. At this same time the improvement of off-design compressor performance by blade adjustment was revived (58, 59), although it had been available for several years due to the pioneer work of Sinnette (60). It should also be noted that the blade adjustment technique had been in use for many years by manufacturers of pumps and blowers.

In connection with stage-matching studies it is obvious that the success of the method requires that the nature of the performance of individual stages must be known before the stages may be "stacked" in a matching analysis. This requires either an extensive single-stage testing program or the construction and testing of numerous multistage compressors. Neither procedure is economically desirable. The ideal approach to the off-design performance problem would be one in which an accurate and complete solution of the "direct" compressor problem was available. Such a method would mean that most costly construction and testing of multistage compressor prototypes would be avoided. However, the lack of sufficient background information makes this approach so difficult that few investigators have seriously attempted to use it.

The work of Louis and Horlock on the "direct" problem has been published recently (61, 62). The assumptions used by Louis and Horlock are such (incompressible flow, validity of the actuator disc solution) that although the investigations make a valuable contribution, there is still much room for improvement. Experimental checks (62) of the method point this out rather well.

The current investigation then was begun with two principal objectives. First, it was hoped that a framework of equations could be set up and used with a correlation of blade-element performance based on the best available two-dimensional and annular cascade data to yield a more satisfactory solution to the compressor analysis or direct problem. Second, if the direct problem solution was sufficiently good, it was believed that specific stage designs should be investigated to determine whether significant improvement in off-design performance of a multistage compressor would result from proper selection of design velocity diagrams within the individual stages.

ANALYSIS

Preliminary Study of the Off-Design Performance Problem

As stated in the previous section, compressor surging and stall
propagation can only occur in a multistage compressor when some of the
blade elements are forced to operate at angles of incidence which are far
from the optimum values. Although these phenomena, which occur in a
compressor stage when the angles of incidence on certain blade elements
become excessive in a positive sense, are obviously important, performance
also deteriorates in a very objectionable manner when blade elements
operate at excessive negative incidence angles or when flow velocities in
the blade passages are so high that "choking" occurs. Finger and Dugan
(56) and Benser (57) demonstrate the existence of both types of off-design
performance problem in their stage-matching studies.

Considering for the moment only the situation in which blade-element
operation at excessive positive angles of incidence results in stalling
and flow separation, a simple analysis seems possible. As pointed out
by Louis and Horlock (61), the occurrence of blade-element stall is
governed by, first, the loading of the various blade elements assigned
for the design operating condition and, second, the rate at which the
angle of incidence changes on each blade element when changes in the flow
rate take place. In considering the first condition, it should be
remembered that the limiting blade loading is set by the amount of
diffusion and turning which occur in a cascade of blade elements and
also by the flow conditions at the inlet to the cascade, such as relative

Mach number and Reynolds number. Conditions which are conducive to the attainment of high pressure ratios in compressor stages, i. e. high relative inlet Mach numbers, combined with high blade cambers, will, in general, place the design blade-element operation at a condition where only a small change in angle of incidence may be sufficient to increase losses considerably. A study of the second factor affecting the occurrence of blade-element stall, the rate of change of angle of incidence with flow rate, indicates that it is also dependent on the velocity and angle distribution at the blade-row inlet. Because of this dependency on the inlet flow conditions, a study of the two factors was made on the basis of the blade-element inlet velocity diagram. The trigonometric relationships among certain important velocity diagram variables were set up and used in the preparation of Figures 2, 3, and 4. Reference should be made to APPENDIX B - SYMBOLS AND NOTATION as use is made of these figures. In a compressor rotor blade row each blade element operates at a varying incidence angle as the flow rate through the row changes. To illustrate this, it is known the absolute flow angle β entering a rotating blade row remains nearly constant as flow rate changes over a reasonable range. Assuming a constant blade-element speed U the ratio V_z/U will change as the flow rate changes. Accompanying the change in V_z/U, a variation in the rotor relative inlet angle β' occurs, changing the incidence angle by the same amount. The nature of this variation is shown in Figure 2. It would appear from this figure that for a given β, the value of V_z/U should be as high as possible at the design point. Furthermore absolute flow angles which are negative

appear to be most desirable because there is a tendency for the rate of change of β' to be reduced as β is reduced.

Figures 3 and 4 demonstrate that for operating conditions which appear to be desirable from the point of view of Figure 2, serious disadvantages may occur if design velocities are established in the ranges ordinarily used to obtain high flow rates per unit passage area and high pressure ratios. If the value of axial velocity and the flow angles are known at any station in the compressor where the flow is about to enter a rotor, Figures 3 and 4 permit the rapid determination of the relative inlet Mach number to the blade row. In this procedure, the ratio of the axial velocity component to the velocity of sound based on the local stagnation conditions and the absolute flow angle are used with Figure 3 to determine the local absolute Mach number. Then the values of absolute flow angle and the flow angle relative to the rotating blades are used with Figure 4 to find the ratio of the relative inlet Mach number to the absolute Mach number. The conclusion may be drawn from Figures 2, 3, and 4 that the conditions which lead to minimum rate of change of angle of incidence with flow (negative absolute flow angles and high V_z/U ratios) are also the conditions which lead to increased relative Mach numbers. Inasmuch as high relative inlet Mach numbers are invariably accompanied by reduction in the efficient operating incidence angle range of a blade row, a careful analysis for each proposed design may be necessary to balance reductions in incidence angle change rate against possible reductions in the available range for the blade type to be used.

It is apparent that although the charts presented give a preliminary
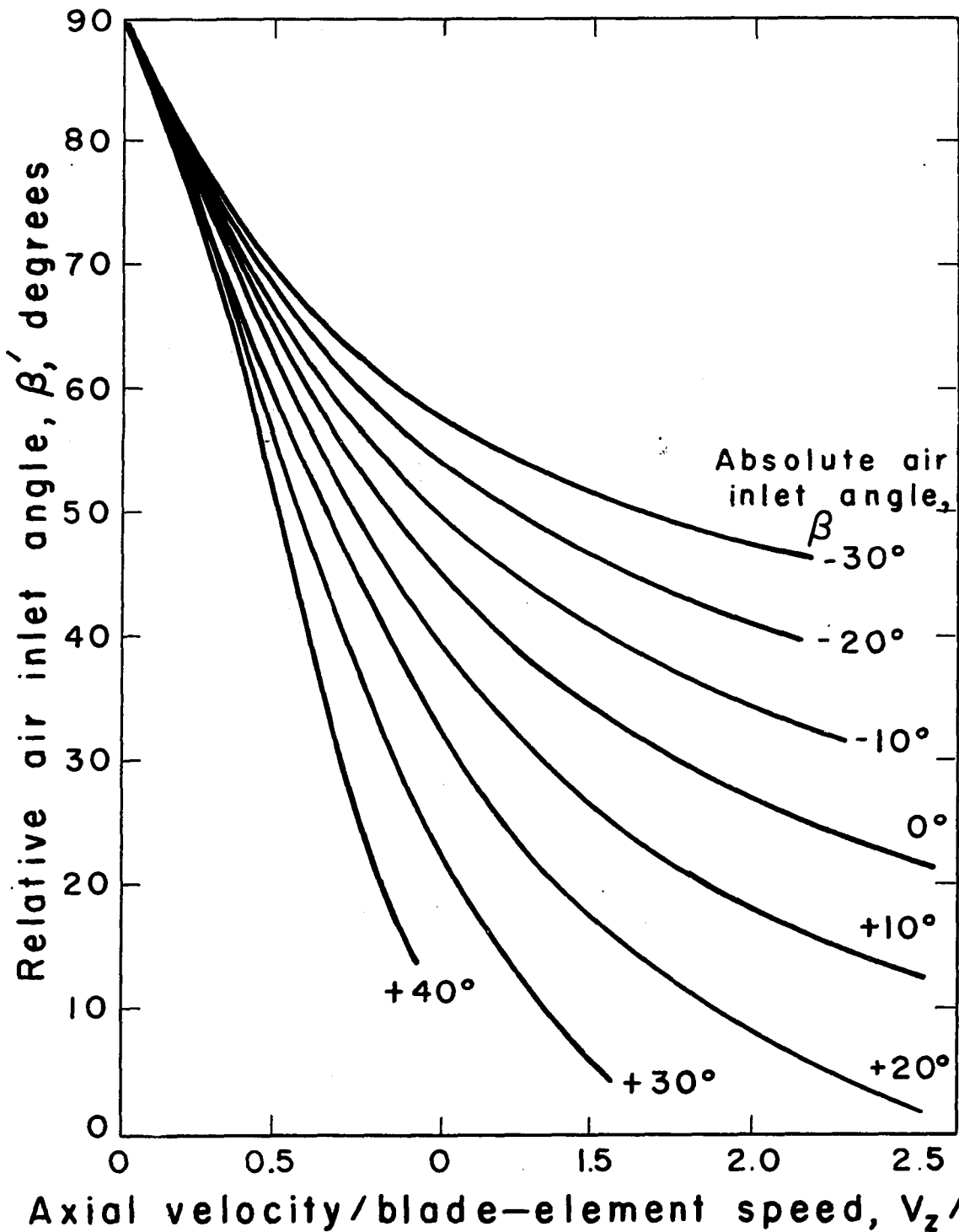
Figure 2. Variation of relative air inlet angle with ratio of axial velocity to blade-element speed for a range of values of absolute air angle
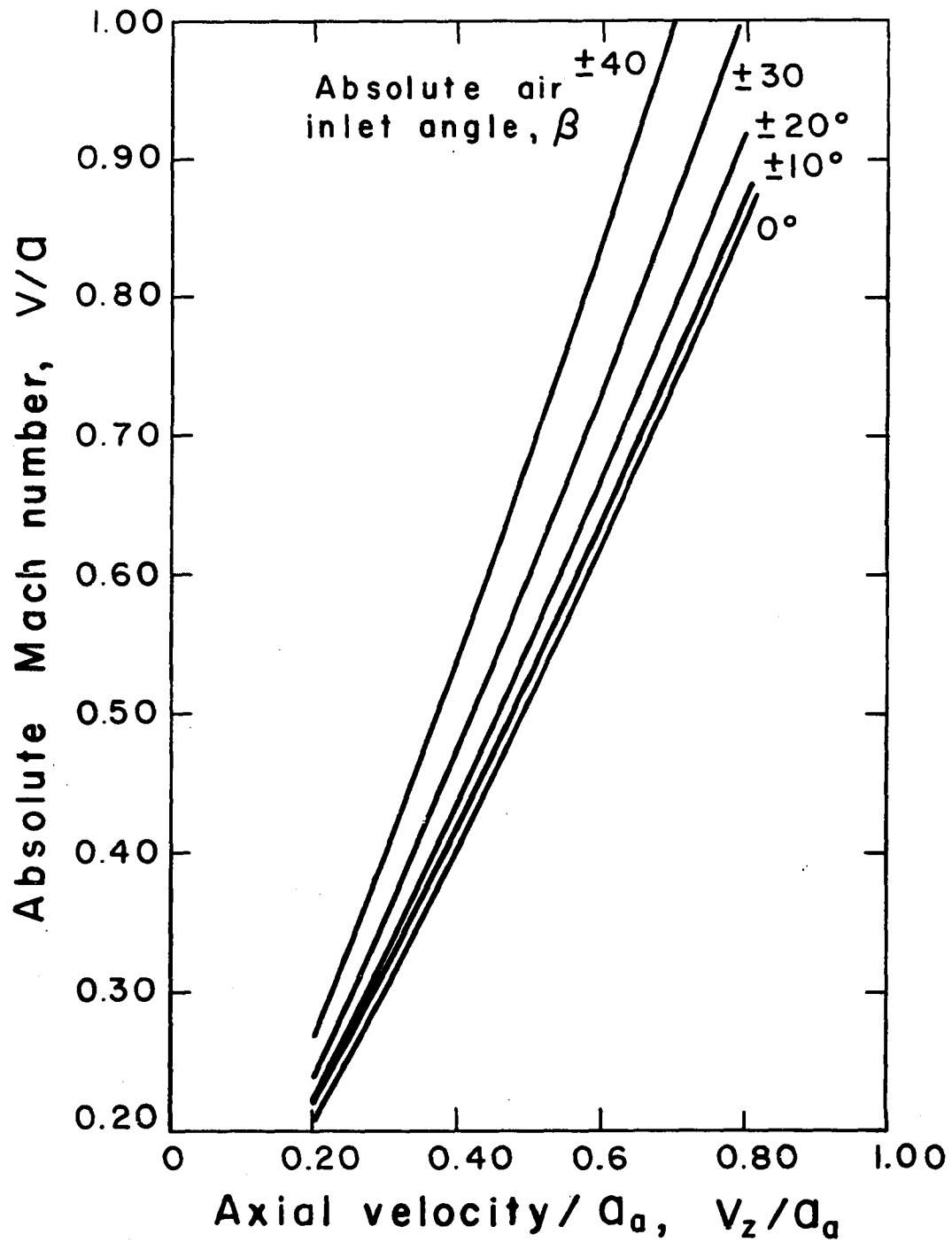
Figure 3. Variation of absolute Mach number with ratio of axial velocity to stagnation speed of sound for a range of values of absolute air angle
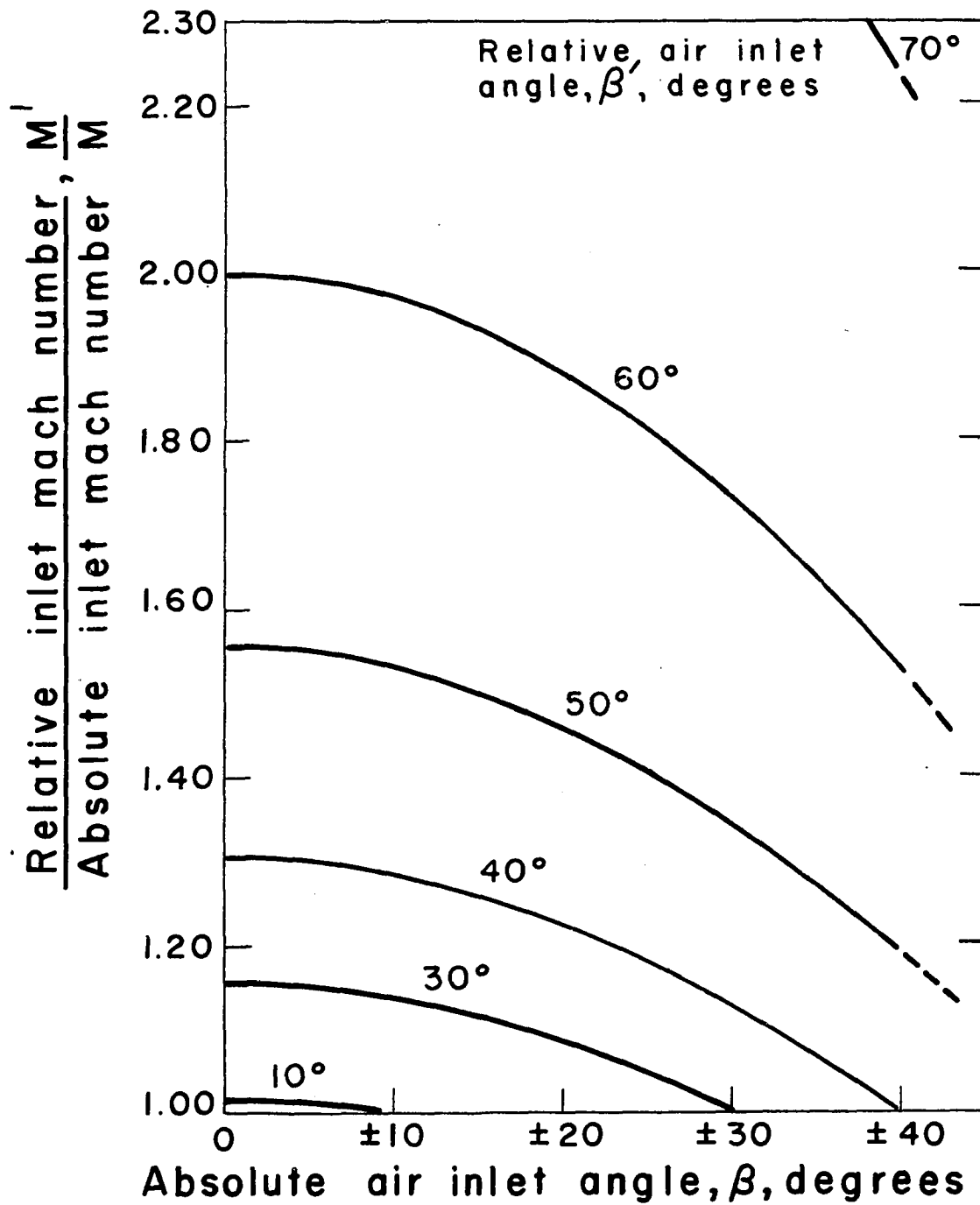
Figure 4. Variation of ratio of relative
inlet Mach number to absolute inlet
Mach number with absolute air inlet
angle for a range of values of
relative air inlet angle

view of some phases of the off-design performance problem, there is a need
to know what will occur on the downstream side of a blade row which is
being studied. For example, radial-equilibrium conditions dictate the
distribution of axial velocity that exists at any axial station in a
compressor. It is possible that even though conditions favorable to the
off-design performance may exist upstream of a rotor-blade row, the energy
addition distribution through the rotor-blade row might set up extremely
undesirable conditions at the entrance to the stators downstream.
Therefore, it is believed that the only really satisfactory means for
studying the improvement of compressor off-design performance must be
based on the availability of a method for solution of the "direct"
compressor problem. That is, it must be possible to calculate with
reasonable accuracy the performance of a series of compressor blade rows
over at least a limited range of flows.

Basic Equations for Performance Computation

A method for performance computation must be consistent with
satisfactory design procedures. In addition, it must be possible to
perform any numerical work in a reasonable time, because a number of
solutions corresponding to various rotative speeds and flow rates are
necessary for each compressor studied.

The purpose of this section is to present a framework of equations
basic to the method used herein, emphasizing the assumptions made and
limitations which may exist. Reference should again be made to the
section entitled SYMBOLS AND NOTATION, in which the terminology used is

defined.

At the inlet to the first blade row, it was believed appropriate to assume uniform flow throughout the annulus. For cases where inlet flow distortions exist because of upstream struts, excessive wall boundary layer thicknesses, and irregularities in the flow through the engine inlet configuration, it is extremely difficult to assess the effect on compressor blade-row performance. However where the inlet is well designed it has been found that under normal operating conditions the flow is in fact quite uniform, but that for inlet stages having very low hub-tip diameter ratios (on the order of 0.4), streamline curvature may produce a non-uniform radial distribution of axial velocity which should be considered (63).

A complete solution to the "direct" compressor problem would permit the calculation of flow conditions at any point in the machine. The equations which would permit this type of solution are available (40, 41). However, the limited nature of the background data available for performing the solution indicates that a more simple flow model must be provided for present purposes. In this analysis, provision was made for the calculation of flow conditions at axial stations located at the exit of each blade row. The radial distribution of velocities, flow angles, pressures, and temperatures was determined in the meridional plane at each station assuming steady, axisymmetric flow at each station. Blade-element performance in the circumferential or blade-to-blade plane was used in the determination; and because the data used were circumferentially-averaged experimental information, it was considered that some of the effects of

lack of axial symmetry may be accounted for.

The general outline of the performance calculation procedure for any blade row is as follows:

A. It was assumed that the flow at the entrance to the blade row was given and that the velocity (magnitude and direction), stagnation pressure, and stagnation temperature at each of eleven equally spaced radial positions were known. At the inlet to the first blade row the flow was defined as discussed above. For all other blade rows these conditions were available from computations for the preceding row.

B. From the given inlet conditions, parameters essential to the determination of blade-element turning and loss characteristics were obtained. For rotating blade rows, relative entrance conditions were calculated, using the given absolute flow conditions and the blade-element speed. For stationary blade rows the absolute quantities and relative quantities are identical. The entrance parameters necessary for blade-element performance determination were: β' and M' for rotating blade rows and β and M for stationary rows.

C. With the passage dimensions known at the exit from the given blade row, eleven equally spaced radial positions were established at the exit calculation station, and the relative flow angles at the exit station were estimated assuming flow through cascades of blade elements intersected by assumed conical stream surfaces connecting corresponding radial positions at blade inlet and exit. The required cascade geometry was assumed known (γ^o, ϕ, σ, blade section).

D. The velocity distribution at any point in an axial-flow

compressor must satisfy the condition of radial equilibrium which, as derived by Wu and Wolfenstein (39) for steady, adiabatic, axially symmetric flow, where H is the stagnation enthalpy in Btu/lb$_m$, is

$$gJ\frac{\partial H}{\partial r} = gF_r + gJt\frac{\partial S}{\partial r} + \frac{V_\theta}{r} \frac{\partial (rV_\theta)}{\partial r} + V_z \frac{\partial V_z}{\partial r} - V_z \frac{\partial V_r}{\partial z} \quad (1)$$

As presented, the equation neglects the local effects of viscosity (shearing stress at the designated point in the fluid) but does account for the cumulative effects of viscosity upstream from the calculation station by means of the entropy gradient term gJt($\partial S/\partial r$).

Because calculation stations for this analysis were located between blade rows, the radial component of blade force F_r was zero. To further simplify the analysis the axial component of velocity was assumed equal to the complete meridional velocity ($\sqrt{V_z{}^2 + V_r{}^2}$) and the term $V_z \partial V_r/\partial z$ was neglected. This term represents the effect of streamline curvature in the meridional plane and is ordinarily neglected in design and analysis of compressor stages with hub-tip diameter ratios greater than about 0.5 (63). On the basis of these assumptions and conditions the radial-equilibrium equation becomes

$$gJ\frac{\partial H}{\partial r} = gJt \frac{\partial S}{\partial r} + \frac{V_\theta}{r} \frac{\partial (rV_\theta)}{\partial r} + V_z \frac{\partial V_z}{\partial r} \quad (2)$$

This equation, although simplified from the complete radial-equilibrium condition, remained difficult to use in a practical analysis because of the presence of the entropy gradient term. The isentropic simple-radial-equilibrium equation as used in this step was then (assuming

isentropic flow through the blade row)

$$gJ\frac{\partial H}{\partial r} = \frac{V_\theta}{r}\frac{\partial(rV_\theta)}{\partial r} + V_z\frac{\partial V_z}{\partial r} \qquad (3)$$

A first approximation to the exit flow conditions (velocity, stagnation pressure, stagnation temperature) was made using the isentropic simple-radial-equilibrium equation and the continuity equation. The continuity equation was set up on the basis of the previously assumed steady, axially symmetric flow. An empirical boundary layer correction or "blockage" factor was used as a means for including hub and outer casing boundary effects in the continuity equation.

E. The velocity distributions obtained from the first approximation to the outlet conditions were used to estimate blade-element losses. The blade-element geometry and the flow parameters used in step B and C and a blade-element diffusion rate parameter D_{ref} were necessary for this estimation.

F. The exit velocity, pressure, and temperature distributions were determined by a second approximation using the non-isentropic simple-radial-equilibrium equation and the continuity equation. The blade-element losses from step E were used in estimating the radial distribution of entropy at the blade exit station for solution of the equilibrium equation.

G. Additional improvements in exit condition values were made as necessary by continued iteration of the loss, equilibrium, and continuity equations.

The method used for the blade-row performance calculation was made

consistent insofar as possible with the axial-flow compressor design procedures outlined in reference 63.

Determination of Blade-Element Performance for NACA 65-(A_{10}) Sections

A great deal of experimental performance data has been obtained for NACA 65-(A_{10}) compressor-blade sections operating in both two-dimensional cascades and in single-stage axial-flow compressors (three-dimensional cascades). Typical of the extensive information available is that reported in references 43 and 44. The primary objective of recent investigators has been to put the available data in a form useful to compressor designers. Probably the most comprehensive of such efforts were reported by Lieblein (45) and by Robbins, Jackson, and Lieblein (46). These references present methods for the determination for 65-series blades at given entrance conditions of a reference incidence angle and of deviation angle and blade-element loss corresponding to the reference incidence conditions. Values are obtained from curves constructed from two-dimensional cascade data and are corrected on the basis of three-dimensional cascade results.

In the solution of the "direct" compressor problem, it is not sufficient to know how the blade elements will operate at the reference incidence. It must be possible to approximate with reasonable accuracy the deviation angles and losses which occur over a wide range of incidence. Reference 45 suggests a procedure for the calculation of off-design deviation angles based on the slope $d\delta^o/di^o$ of the deviation-incidence curve at the reference incidence. The methods allow

a reasonable value of deviation angle to be estimated for incidence values when the blade element is not stalled. There is, however, no suggested method for approximation of the blade-element losses.

For this investigation, two-dimensional low speed cascade loss data for 65-series compressor blade sections originally reported in reference 43 were correlated on the basis of a single curve using as parameters $\bar{\omega}/\bar{\omega}_{ref}$ and $(i - i_{ref})/(i_{PS} - i_{NS})$. This curve is shown in Figure 5. The values of the angles of incidence for positive stall and for negative stall were defined for a given cascade as suggested by Lieblein (45) as the incidence angles for which the measured blade-element loss reached twice the minimum or reference value. The difference between the positive and negative stalling incidence angles $i_{PS} - i_{NS}$ was then considered to define the most effective operating range of the given cascade. It was necessary next to devise a method for finding the effective operating range $(i_{PS} - i_{NS})_{LS}$ for a cascade as a function of its geometry and operating conditions. For the low speed data a reasonably good correlation of values of $(i_{PS} - i_{NS})_{LS}$ was obtained by plotting $(i_{PS} - i_{NS})_{LS}$ against the blade-element diffusion parameter, D_{ref}, with relative inlet angle β' and solidity, σ, as parameters. This correlation is shown in Figure 6. Although very limited data were available for determining the effect of relative inlet Mach number M' on operating range Figure 7 was prepared on the basis of references 45 and 64. In this figure the ratio $(i_{PS} - i_{NS})/(i_{PS} - i_{NS})_{LS}$ is shown as a function of inlet Mach number. Although the data which were used in the preparation of Figure 7 were not obtained in studies of NACA
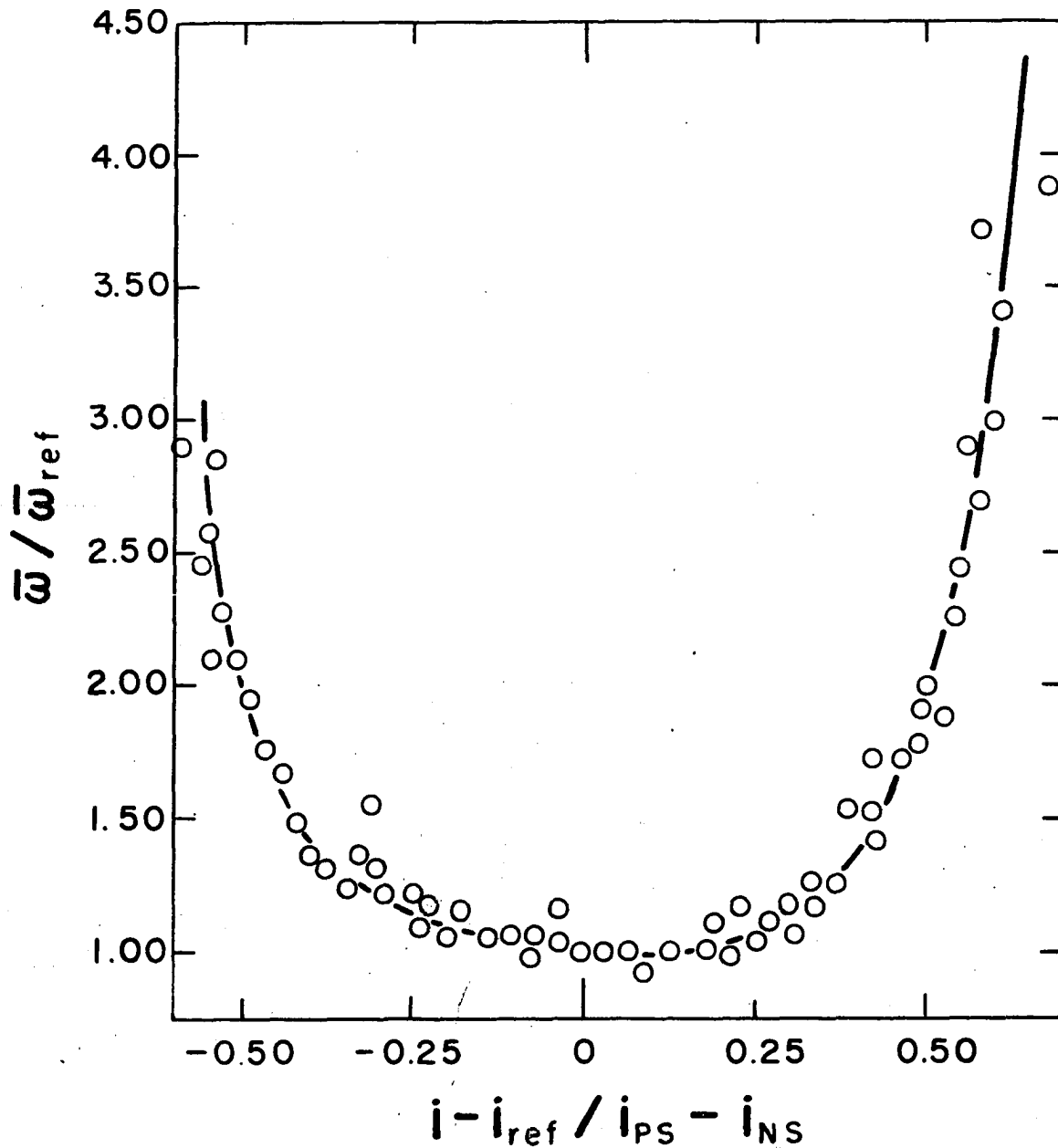
Figure 5. Correlation of two-dimensional cascade
losses with departure from reference operating
incidence angle for a large number of N.A.C.A.
$65-(A_{10})$ blade-row configurations $(\sigma \geq 1.0)$
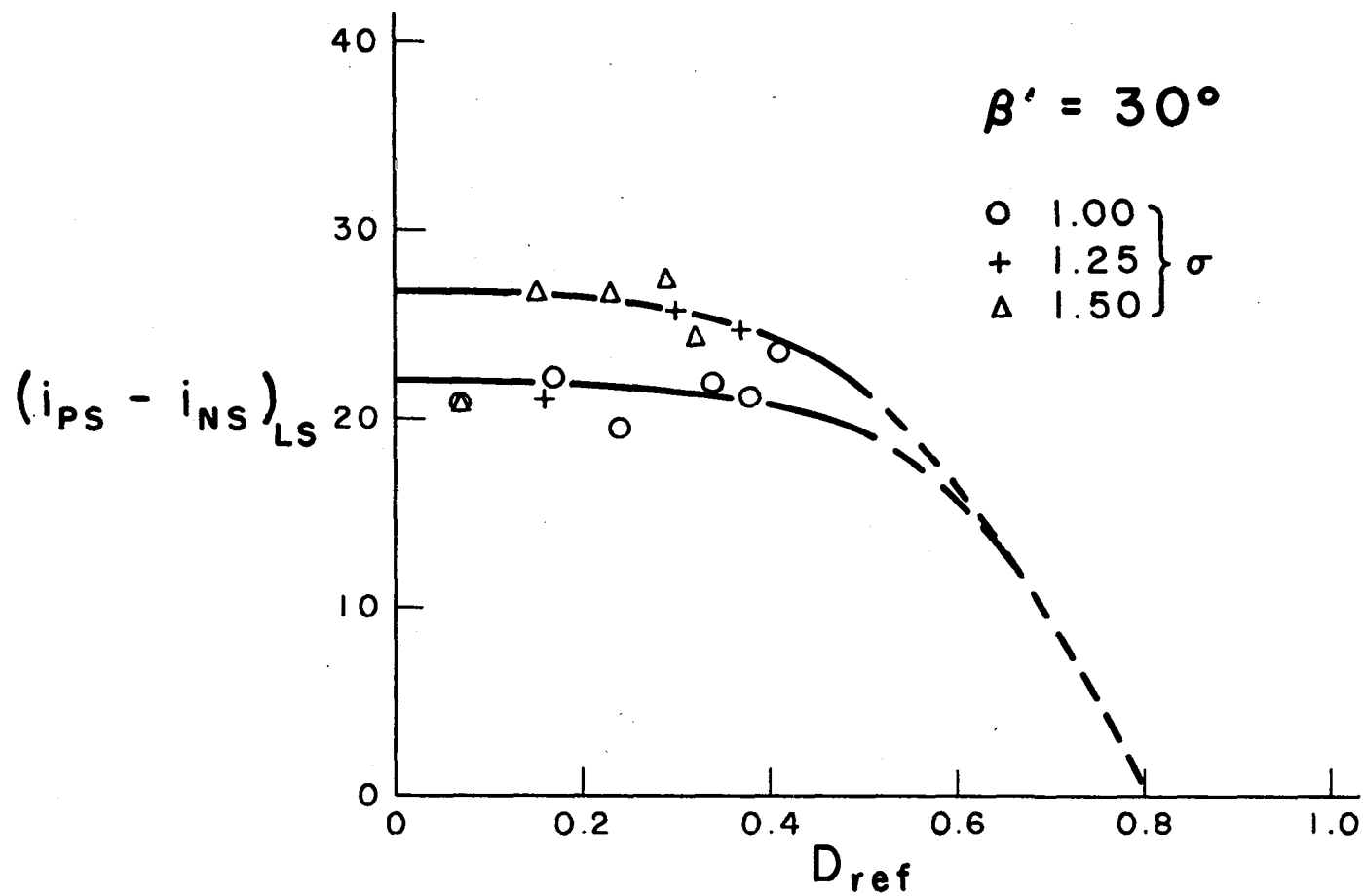
(a) Relative air inlet angle, 30°

Figure 6. Correlation of two-dimensional cascade operating range with value of diffusion parameter at reference operating condition for N.A.C.A. 65-(A₁₀) blade rows with solidities of 1.00, 1.25, and 1.50
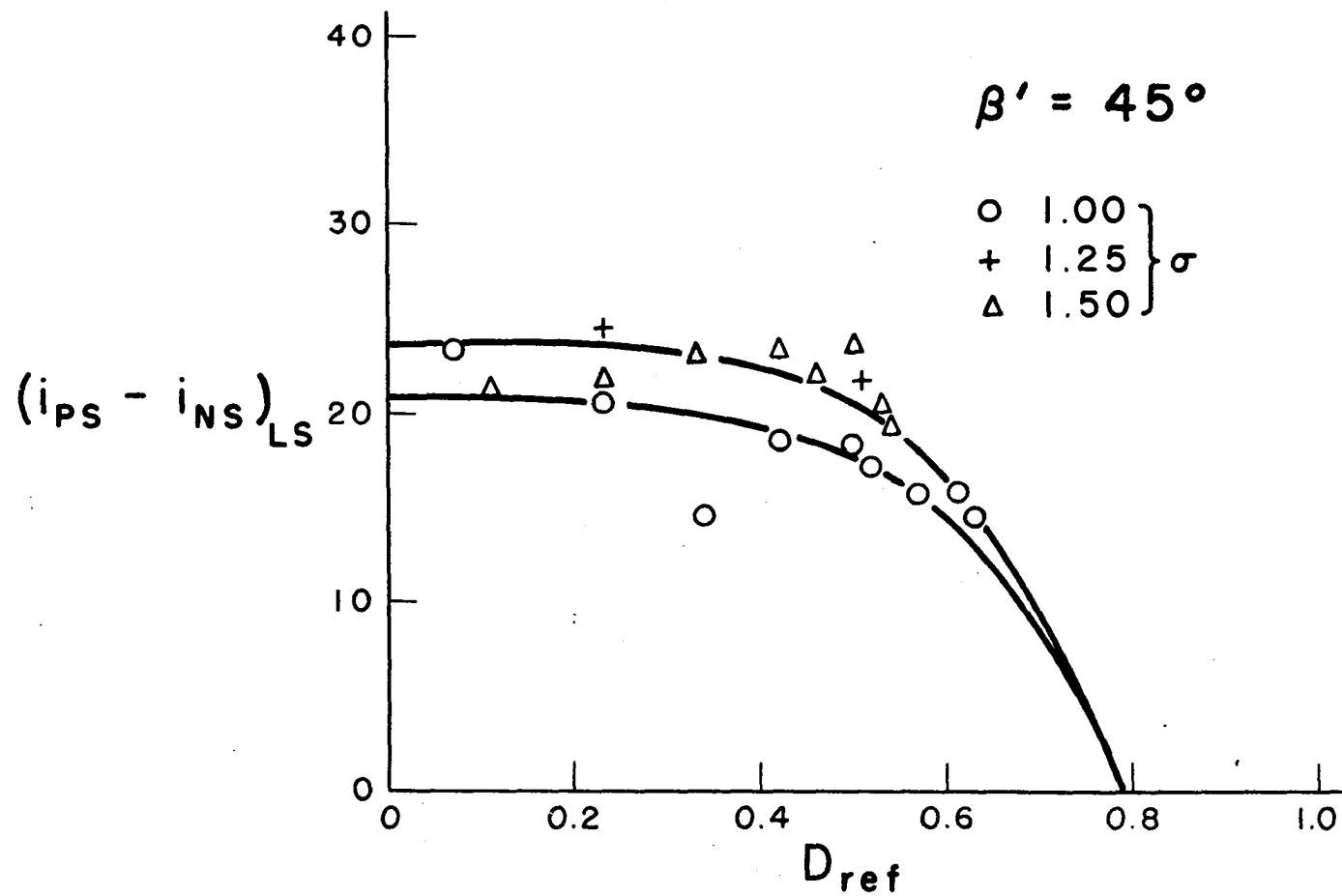
(b) Relative air inlet angle, 45°

Figure 6. Continued
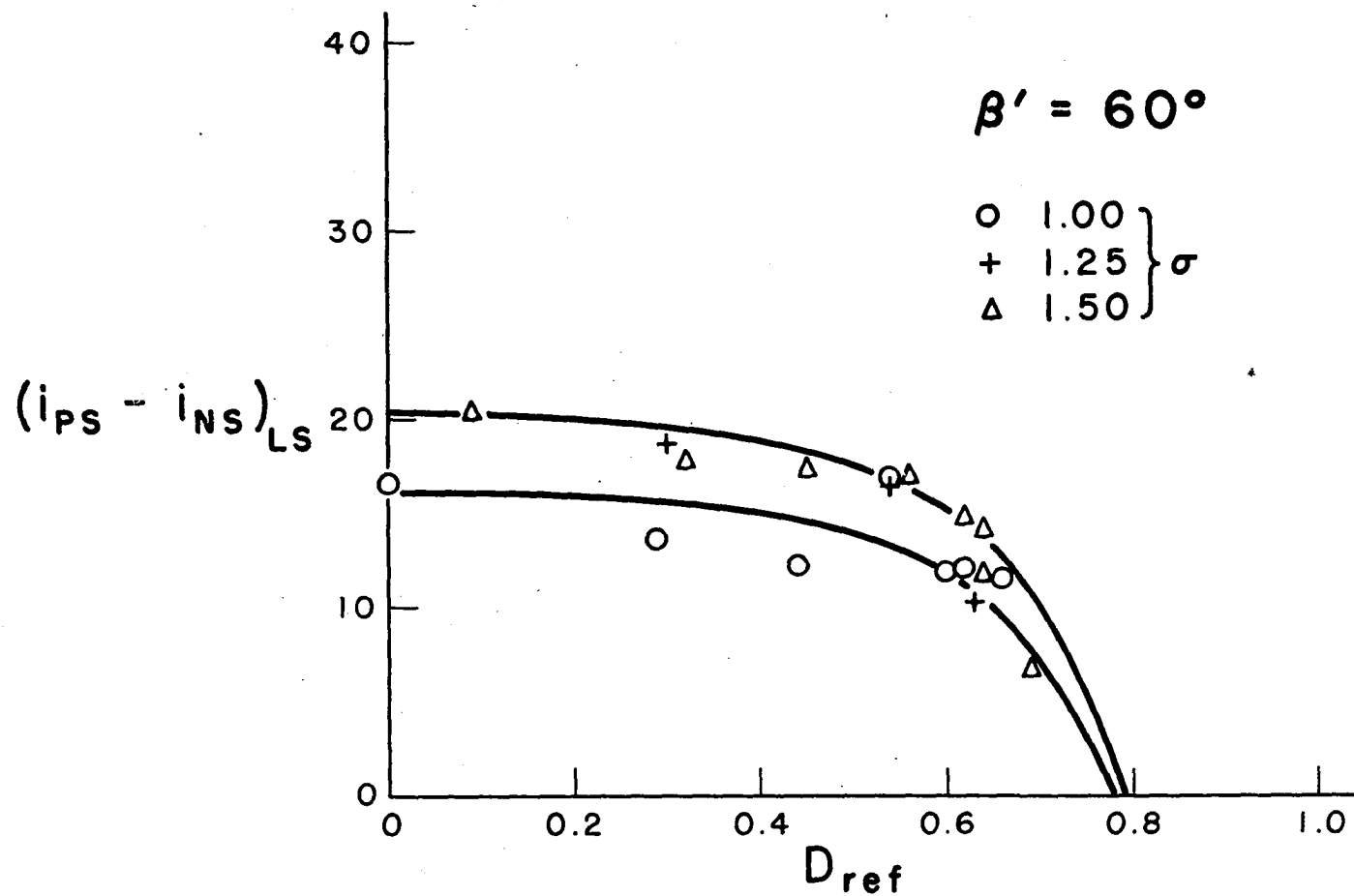
(c) Relative air inlet angle, 60°

Figure 6. Continued
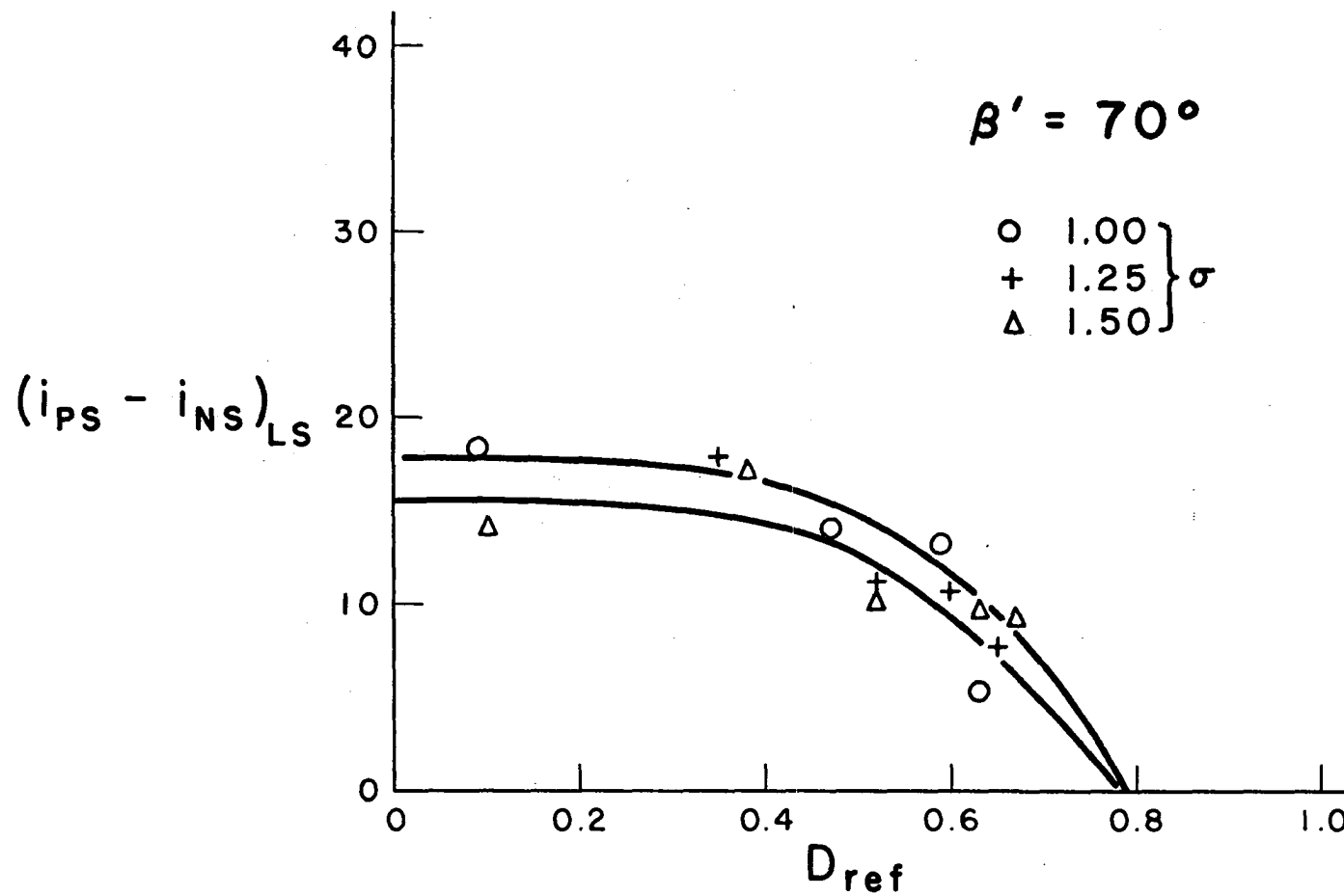
(d) Relative air inlet angle, 70°

Figure 6. Continued
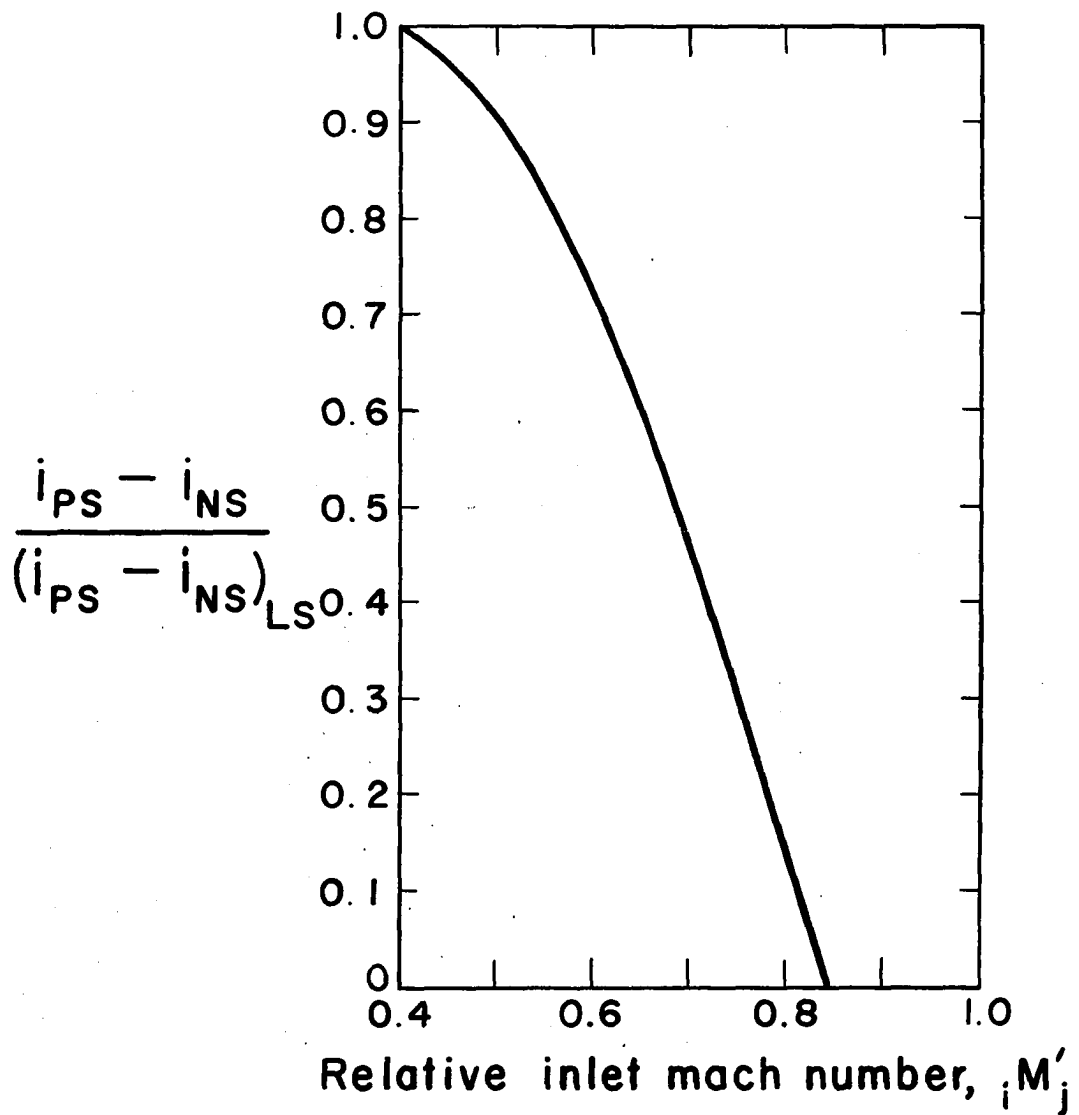
Figure 7. Estimated variation of useful operating incidence range with relative inlet Mach number

65-series sections, the sections were of similar geometry and show similar operating characteristics in other respects to those of 65-series section. Figures 5, 6 and 7, then, provide a method for estimation of blade-element losses for NACA 65-(A_{10}) or similar compressor blade sections when inlet conditions and the blade-element diffusion rate for the given application are known.

APPLICATION OF ANALYTICAL PROCEDURE—CALCULATION
OF THE PERFORMANCE OF A ROTOR-BLADE ROW

Computation of the Rotor Entrance Conditions

In order to determine the accuracy of the proposed performance
computation method, and to define any areas in which improvements were
necessary, it was decided to apply the procedure first to a single
rotating blade row. The experimental performance of such a blade row
was reported for a range of rotative speeds and flow rates in references
65 and 66. The 29 blades used employed NACA 65-(A_{10})-series compressor
sections and were designed for a rotor inlet hub-tip ratio of 0.8.
Details on the design of the stage are given in reference 65, and
Table 1 gives blade design information pertinent to the current
investigation. This conservative stage was selected for study because
the relatively high hub-tip ratio would minimize streamline curvature
effects on the computations and because the stage performance had been
reasonably well defined experimentally. Figure 8 shows a meridional
cross-section through the stage and the calculation stations referred to
in the following paragraphs.

It was necessary to establish operating points for which performance
was desired. Five rotative speeds were selected to provide information
on the value of the methods for including the effect of relative inlet
Mach number on rotor performance. Weight flows were selected at each
speed to cover approximately the same range for which experimentally
determined performance was available. The selected speeds and flow
rates are tabulated in Table 2. The first step in the calculation of

Table 1 - Blade design data for rotor-blade row

Percent of passage from tip	Equivalent circular-arc camber angle, ϕ degrees	Thickness/chord ratio	Blade-element solidity, σ	Blade-chord angle, γ^o degrees
10	19.4	.082	1.20	49.1
30	21.4	.086	1.20	47.2
50	24.2	.090	1.20	44.8
70	27.5	.094	1.20	41.9
90	31.7	.098	1.20	38.9

Casing inner diam.

Inlet calculation station, i = 1

Outlet calculation station, i = 2

Radial calculation positions, j = 0 - 10

Hub contour

C. G. of rotor blades

For inlet station

$_1 r_0 = 7.00$ in.

$_1 r_{10} = 5.58$ in.

For outlet station

$_2 r_0 = 7.00$ in.

$_2 r_{10} = 5.73$ in.
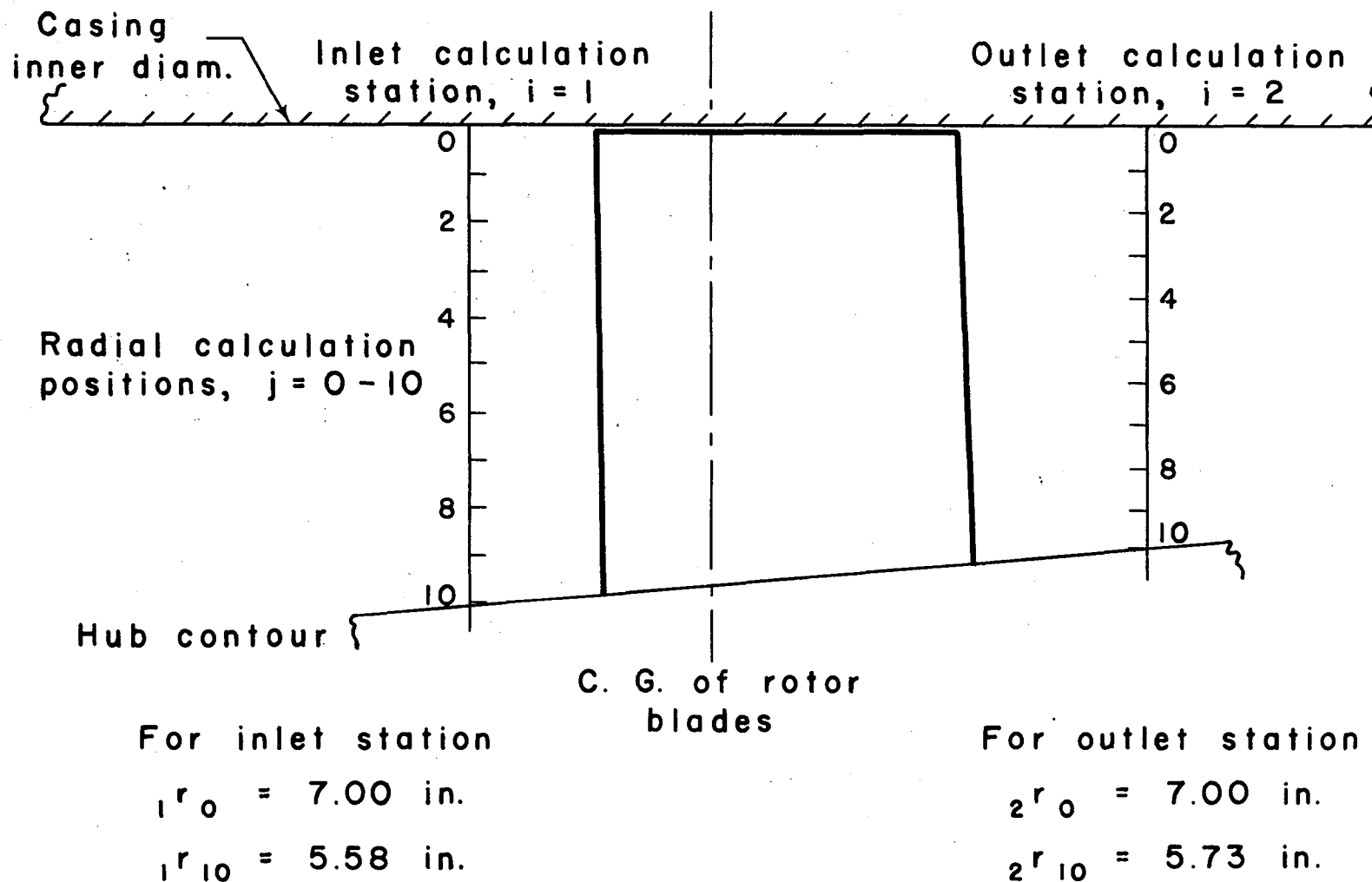
Figure 8. Meridional cross-section of axial-flow compressor rotor-blade row configuration used in analytical investigation

Table 2 - Operating points for performance computation

Point number	Corrected rotor tip speed, $U_0/\sqrt{\theta}$ ft/sec	Corrected flow rate, $W\sqrt{\theta}/\delta$ lb/sec
1	600	12.0
2	600	11.0
3	600	10.0
4	600	9.0
5	600	8.0
6	600	7.0
7	798	14.0
8	798	13.0
9	798	12.0
10	798	11.0
11	798	10.0
12	798	9.0
13	915	14.5
14	915	13.5
15	915	12.5
16	915	11.5
17	915	10.5
18	736	11.0
19	736	13.5

Table 2 (Continued)

Point number	Corrected rotor tip speed, $_1U_0/\sqrt{\theta}$ ft/sec	Corrected flow rate, $W\sqrt{\theta}/\delta$ lb/sec
20	736	13.0
21	736	12.0
22	736	10.0
23	736	9.0
24	450	10.0
25	450	9.0
26	450	8.0
27	450	7.0
28	450	6.0
29	450	5.0

performance was the determination of rotor entrance conditions. As mentioned in the previous section, it was assumed that uniform flow existed in the annulus at the rotor entrance. No correction was made for boundary-layer blockage in velocity calculation at this station. This was done because the convergent passage shape would produce an accelerating flow which would not permit the development of thick wall boundary layers. Inlet stagnation (total) temperature and pressure were assumed equal to NACA standard sea-level values (29.92 in. Hg absolute and

518.7 °R) so that computed performance would be equivalent to corrected values ordinarily used as a basis for performance comparison. Figure 9 was prepared to show the variation of inlet axial velocity with flow per unit annulus area for standard entrance conditions. Once this velocity was known, the absolute entrance conditions were completely determined.

The next step in the procedure was the calculation of relative flow angles and velocities at the entrance station. The typical velocity diagram of Figure 10 shows both the relative and absolute entrance quantities. Similar velocity diagrams were calculated for five equally spaced radial positions at station 1.

Use of the Blade-Element Performance Correlation

From the known blade-row geometry and the correlations of references 45 and 46, the reference values of incidence angle and deviation angle were found for the relative entrance flow conditions corresponding to each radial position at each flow rate. From the same references, it was possible to find the predicted deviation angle for each case and from this to calculate a relative exit angle. For these calculations it was assumed that flow would proceed along conical streamlines through the blade row so that the flow angles at blade exit were determined for the same number of equally spaced radial stations as used at the inlet. This is the basis for compressor design and analysis whenever cascade data rather than a general through-flow method are used for determining blade-element performance. With the relative exit angle distribution known for the rotor blades, a first approximation to the velocity
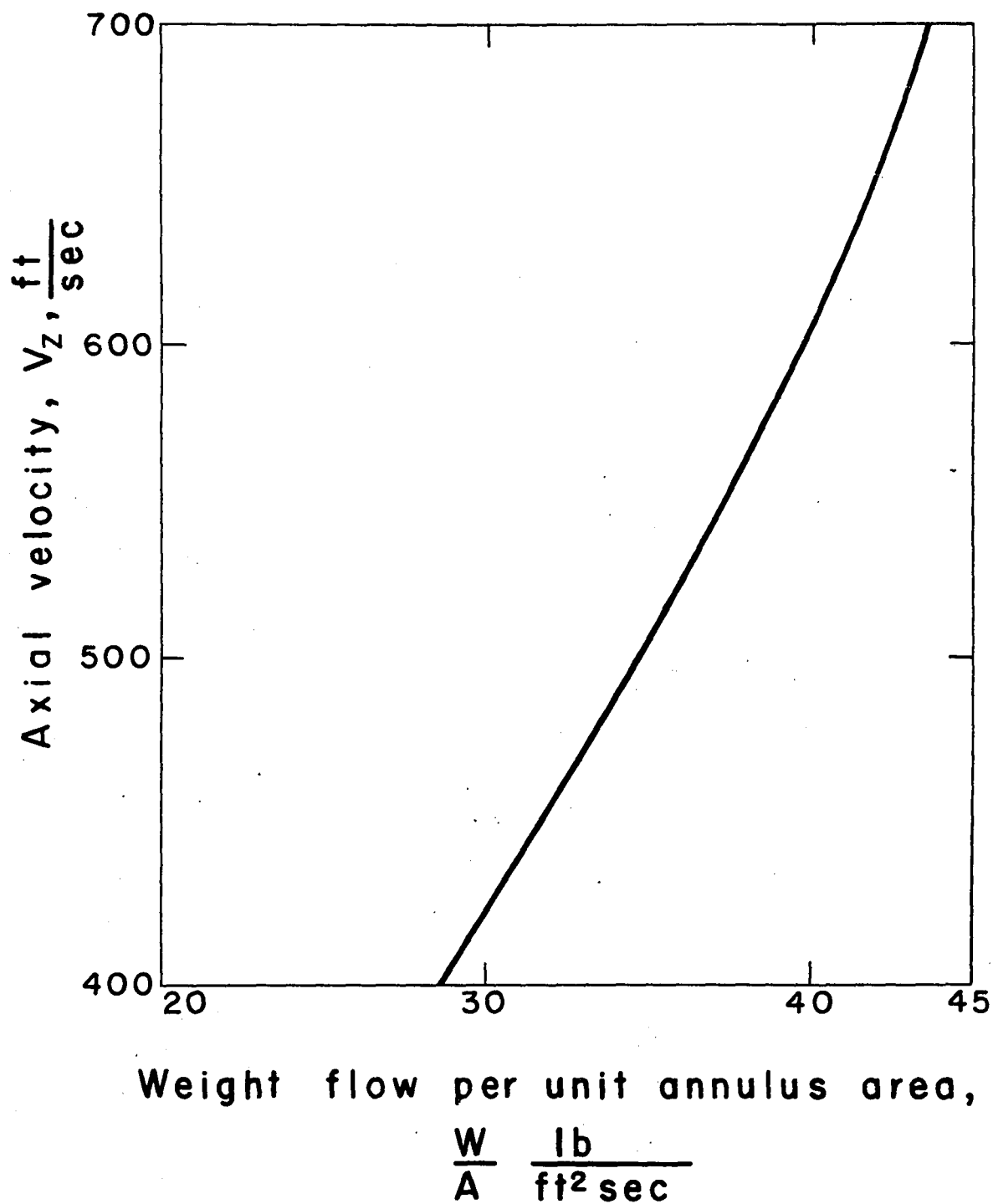
Figure 9. Variation of axial velocity with weight flow per unit annulus area for blade row with N.A.C.A. standard sea-level total pressure and temperature at entrance
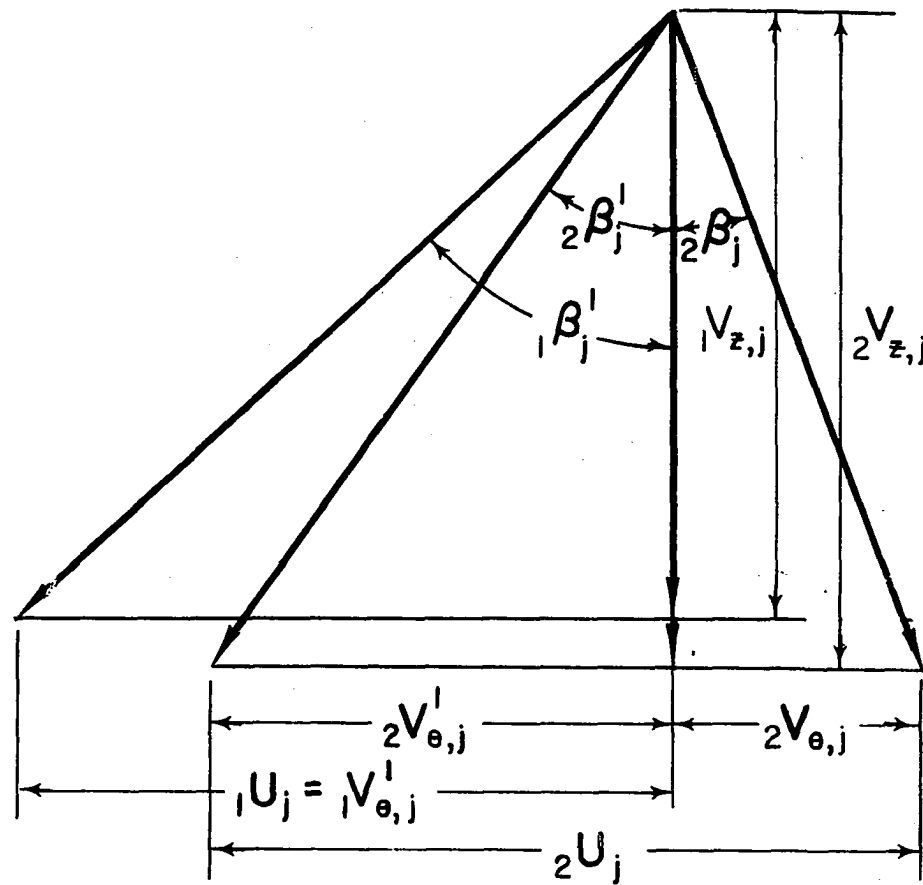
Figure 10. Typical velocity diagram for axial-flow compressor rotor-blade row

distribution at the exit calculation station was computed assuming

isentropic flow through the blade row with simple radial equilibrium of

static pressure at the calculation station. This computation is

discussed in detail in the following section. Using the above-mentioned

radial distribution of axial velocity at the exit station, an

approximation to the losses through the blade row was made in the following

manner. Values of the blade-element diffusion factor D (67) were computed

for each radial position at each flow rate. Then, for a given speed, D

was plotted against $i - i_{ref}$ for each radial position and the value of

D for $i = i_{ref}$ was determined. This value was designated at D_{ref} and

was used with Figure 23 of reference 46 to estimate $\overline{\omega}_{ref}$ for each blade

element. It was then possible to calculate values of $\overline{\omega}$ for the range

of incidence covered by each blade element at a given speed using the

method discussed under ANALYSIS.

<div align="center">

Numerical Solution of the Isentropic
Simple-Radial-Equilibrium Equation

</div>

In the previous section the first approximation to the axial

velocity distribution at the exit calculation station was mentioned.

The isentropic simple-radial-equilibrium equation as developed for

numerical solution is presented in APPENDIX A as equation A-3. To permit

the numerical solution it was necessary to assume that certain quantities

varied linearly between adjacent radial stations. This was believed

justified by the close spacing of the radial stations. For the first

approximation it was assumed that at any station between blade rows the

flow distribution must satisfy this equation and the equation of

continuity (equation A-6). The numerical solution of these equations was set up for the International Business Machine Model 650 digital computing machine. Using the program written for the solution in the Bell Laboratories interpretive routine reported in reference 68, it was possible to perform the simultaneous solution of the equations in less than 15 minutes for each flow rate. A block diagram showing the essentials of the program is shown in Figure 11. Eleven equally spaced radial stations were employed as shown in Figure 8 and a boundary layer blockage factor K_{bk} of 0.98 was used for all flow rates as suggested in reference 63.

<div align="center">

Numerical Solution of the Non-Isentropic
Simple-Radial-Equilibrium Equation

</div>

With the radial distribution of losses for the rotor blade elements obtained on the basis of the first approximation of the velocity distribution at the exit station, it was possible to calculate the radial entropy distribution required for a solution of the non-isentropic simple-radial-equilibrium equation using equations A-10 through A-13. Inclusion of this entropy distribution in the equation permitted a second and improved approximation to the velocity pattern existing at the exit calculation station. Again it seemed necessary to assume a linear variation of certain terms between adjacent radii for integration purposes. The simultaneous solution of the non-isentropic simple-radial-equilibrium equation and the continuity equation was again programmed for the IBM 650 machine in a similar fashion to the solution of the isentropic simple-radial-equilibrium equation.

| 1 | 2   1131 STOP | 3 | 4   1120 STOP |

Box 1:
Read input and compute all quantities which do not change with iteration

Box 2 — 1131 STOP:
Feed in assumed $_2V_{z,5}$ through storage entry switches

Box 3:
Machine calculates
$$\frac{\Delta w}{w}$$
$\Delta w = w$ calculated $- w$

Box 4 — 1120 STOP:
$\dfrac{\Delta w}{w}$ displayed on console

Box 15 — 1120 STOP:
$(\Delta w/w)_o$ from 13 displayed on console

YES

Box 13:
Machine inquires whether $(\Delta w/w)_o < 10^6$

Box 12:
Machine designates $\Delta w/w$ from 10 as $(\Delta w/w)_o$

Box 11 — 1131 STOP:
Feed in through storage entry switches $+02020(50+\delta)084$ where w is desired to accurac $\Delta w/w < 10^6$

NO

Box 16 — 1131 STOP:
Feed in a $+$ or $-$ sign

Box 17:
Machine transfers storage locations so that output cards punched may be used as input for next blade row

Box 18:
Punch out from storage locations $_2V_{z,j},\ _2U_j,\ _2^{\Upsilon}j$ $_2f_j,\ _2e_j,\ w,$ and $\dfrac{J}{R}(_2S_j)$

Figure 11. Block diagram of IBM 650 digital computer program fo

4 1120 STOP	5 1131 STOP	6 1131 STOP	7
$\dfrac{\Delta w}{w}$ displayed on console	Feed in a + or - sign through storage entry switches	Feed in a $\Delta_2 V_{z,5}$ value through storage entry switches	Machine designates last computed value of $\Delta w/w$ from 3 or 10 as $(\Delta w/w)_o$

NO

11 1131 STOP	10	9	8
Feed in through storage entry switches +02020(50+δ)084 where w is desired to accuracy $\Delta w/w<10^6$	**YES** Machine determines whether $(\Delta w/w)(\Delta w/w)_o$ <0 Has $\Delta w/w$ changed sign?	Machine calculates a new $\Delta w/w$	Machine calculates a new $_2 V_{z,5}$ value

14   Machine calculates a new $\Delta_2 V_{z,5}$ from $(\Delta_2 V_{z,5})_{new} = -(\Delta_2 V_{z,5})_{old}/e$

18	19 9999 STOP	20
Punch out from storage locations $_2 V_{z,j}, _2 U_j, _2 T_j,$ $_2 f_j, _2 e_j,$ w, and $\dfrac{J}{K}(_2 S_j)$	Program complete	To begin next run, feed in through storage entry switches +7019511333 Reset computer

digital computer program for solution of simple - radial - equilibrium equations

Computation of Over-All Performance from Blade-Element Results

The results of the simultaneous solution of the non-isentropic radial-equilibrium equation and the continuity equation for the blade-row outlet station make up the basic output for the performance computation method. The flow angles, velocities and temperatures determined would be the input for the following blade row; and a row-by-row solution could give the performance for a compressor consisting of any desired number of stages.

For the blade row under study in the current investigation, however, the only available experimental performance data were not in the form of blade-element results but were published in reference 66 as curves showing the variation of mass-weighted average total-pressure ratio and adiabatic efficiency with corrected weight flow at several rotor-blade tip speeds. For comparison with these results, the calculated blade-element performance was used to estimate the mass-weighted average performance parameters. The equations used were equation A-16 and A-18 presented in APPENDIX A. The indicated integrations were performed by plotting the various functions and determining the magnitude of the integrals by means of a polar planimeter. The correction factors K_e and K_p were both assigned values of 1.0 as indicated in reference 63.

RESULTS AND DISCUSSION

As indicated in the previous section, the performance of the rotor-blade row of reference 66 was predicted for a range of flow rates at several rotative speeds. A calculation method was used which was based on a framework of equations similar to those used currently in compressor design and on correlations of experimental two- and three-dimensional blade-element performance. The results of these calculations are presented for comparison with experimental results in Figure 12.

In this figure, the experimentally-determined performance of the rotor-blade row is shown for corrected tip speeds of 450, 600, and 736 feet per second. The points plotted represent the performance measured for three different blade trailing-edge thicknesses, 0.045-inch, 0.030-inch, and 0.015-inch. In these tests the same set of blades was modified in successive steps by hand-finishing the aft section of the blades on both the suction and pressure surfaces to decrease the trailing-edge thickness. To minimize experimental discrepencies, extreme precautions were taken to eliminate differences in blade-row geometry other than trailing-edge thickness amongst the three series of tests. The results of the tests showed that blade trailing-edge thickness was essentially a minor factor in its effect on performance. For this reason no differentiation is made amongst the three trailing-edge thicknesses in plotting the data in Figure 12.

Three sets of calculated performance values are shown in Figure 12. At corrected tip speeds of 450, 600, and 736 feet per second, the mass-averaged total-pressure ratio and adiabatic efficiency are plotted
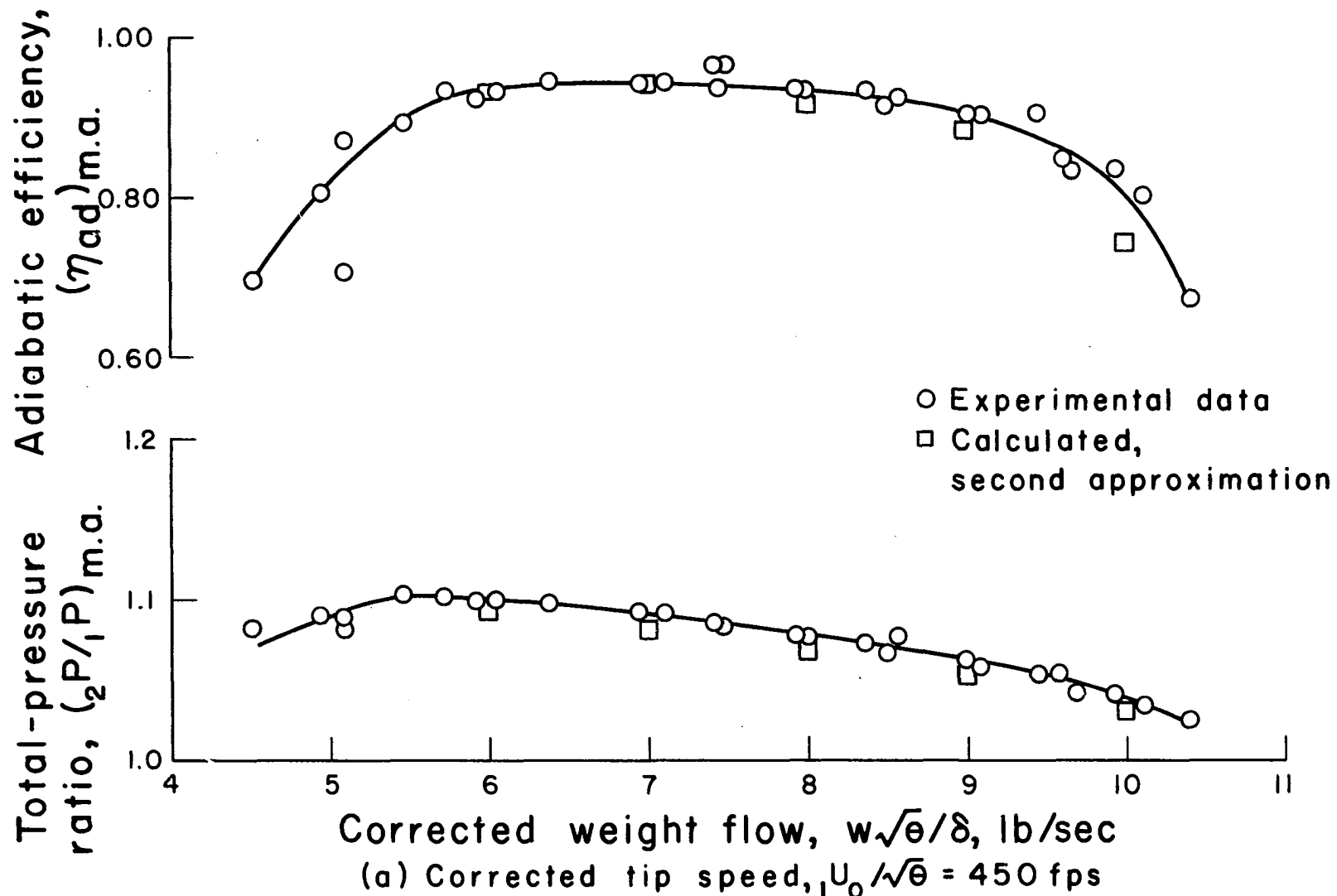
Figure 12. Variation of calculated and experimentally-measured total-pressure ratio and adiabatic efficiency with corrected weight flow.

Adiabatic efficiency, (η_{ad})m.a.

Total-pressure ratio, $(_2P/_1P)$m.a.

1.00

0.80

0.60

○ Experimental data
+ Calculated, first approximation
□ Calculated, second approximation
◇ Calculated, third approximation

1.2

1.1

1.0

6 7 8 9 10 11 12 13

Corrected weight flow, $w\sqrt{\theta}/\delta$, lb/sec
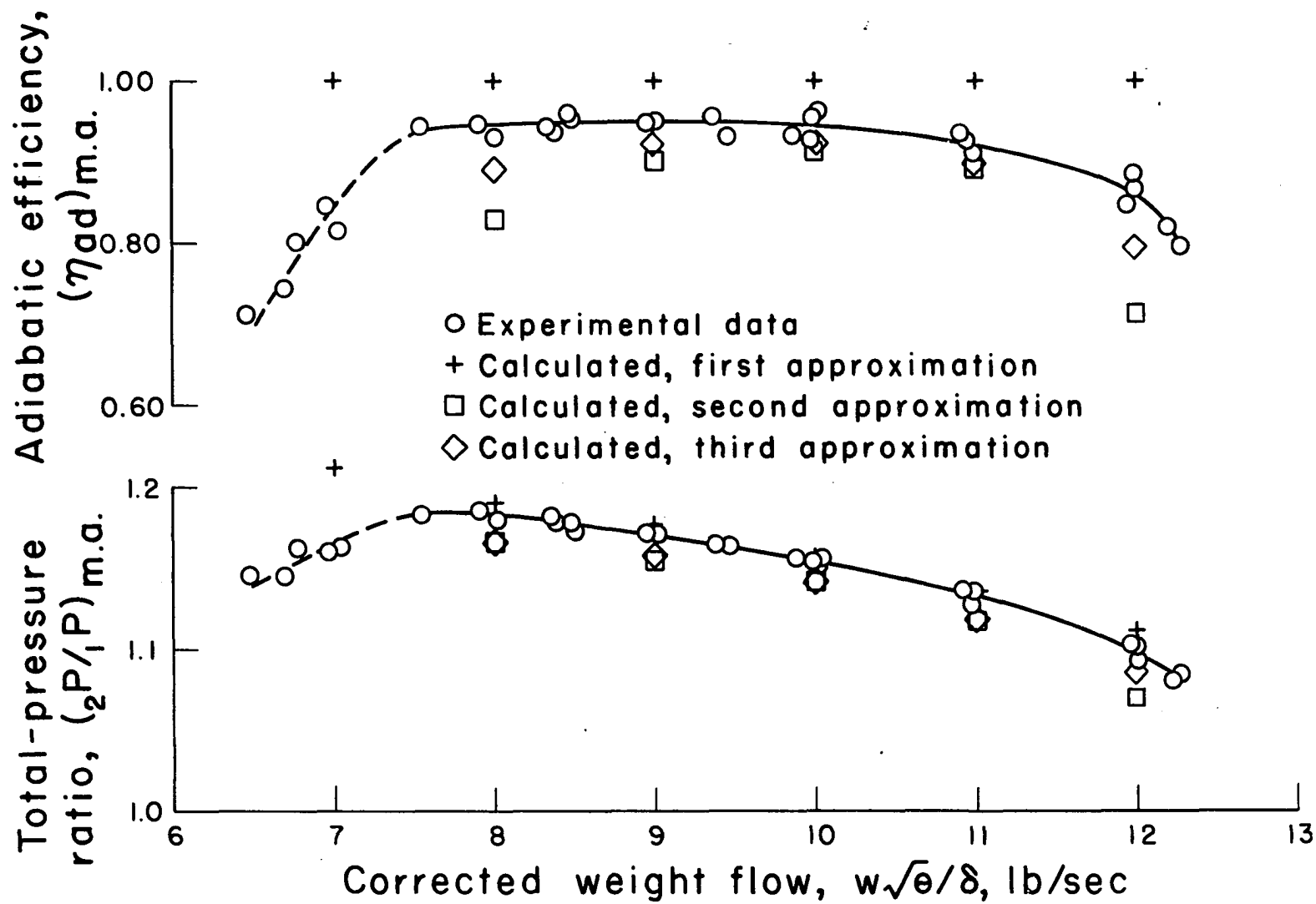
(b) Corrected tip speed, $_1U_o/\sqrt{\theta}$ = 600 fps

Figure 12. Continued.
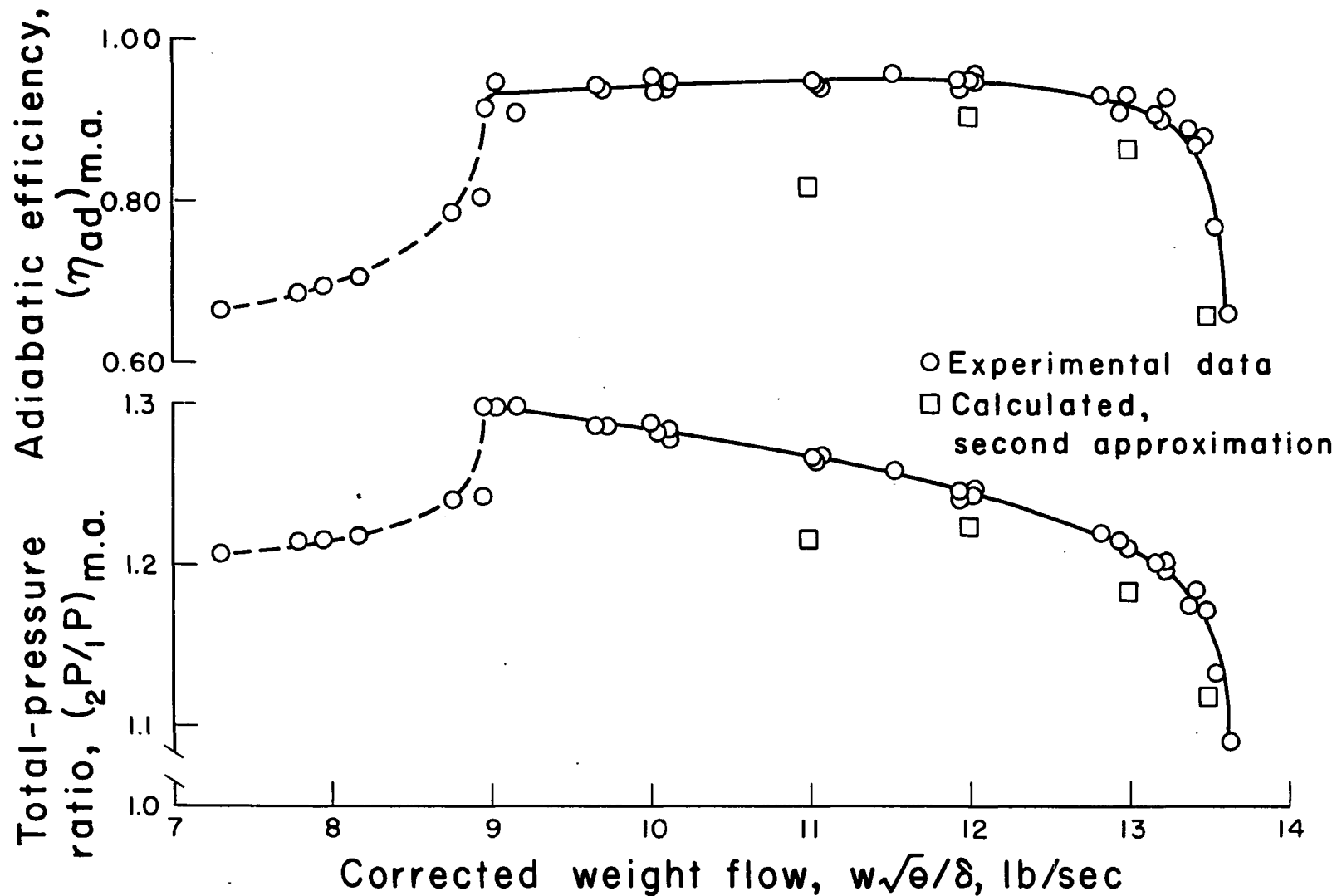
(c) Corrected tip speed, $U_0/\sqrt{\theta} = 736$ fps

Figure 12. Continued.

as calculated from the results of the second approximation calculations. At an equivalent tip speed of 600 feet per second, the results of the first and third approximations are also shown. It should be remembered that the difference amongst the various approximations lies primarily in the treatment of and the values estimated for the radial distribution of the losses occurring in flow through the blade row. In the first approximation, isentropic flow was assumed as a means for obtaining an approximate distribution of velocities, pressures and temperatures along the radius at the blade-row outlet. This, of course, led to adiabatic efficiencies of 1.00 for all points. In the second approximation, an improvement in these radial distributions was attempted by estimating the blade-element losses on the basis of the previously obtained first trial values of velocities, pressures and temperatures. This permitted the inclusion of the radial entropy gradient terms in the simple-radial-equilibrium equation. The improved distributions obtained from the second approximation were used as the basis for another loss estimation step and a third approximation of the blade-row performance.

At an equivalent tip speed of 450 fps, calculated performance for points 24 through 28 (Table 2) is shown. The computed total-pressure ratios and efficiencies are both slightly below experimental values, indicating the possibility that losses in the rotor tests were slightly below those predicted by the techniques used herein. Because blade-element total-pressure ratios and efficiencies were both weighted by the flow rate through the respective blade elements in the mass-averaging process, it was also considered possible that radial distributions of flow were

present in the calculations which would tend to overemphasize high-loss regions near the hub and outer casing. Experimental blade-element performance was not available to permit further investigation of this possibility. In general, however, the calculated performance at this speed was in good agreement with experimental data. It may be noted that point 29 was not calculated because of the extremely high losses indicated to exist at the assigned flow rate, particularly at the rotor tip sections. The magnitude of these losses may be indicated by the fact that the correlation of Figure 5 was not of sufficient range to permit accurate prediction. The decrease in performance which would occur because of these losses is confirmed by the trend in the experimental data toward lower efficiency below a corrected weight flow of about six pounds per second. The magnitude of the decrease, as measured experimentally, was, however, not as great as would have been obtained had it been possible to complete the calculation of performance at this point.

At equivalent tip speeds of 600 and 736 feet per second, similar trends in the calculated performance existed for values of flow rate where incidence angles were near the reference values. Slightly lower values of total-pressure ratio and efficiency were calculated than had been previously measured. It became apparent at these speeds, however, that the performance calculated for low and very high flow rates was significantly lower than measured, indicating the prediction of high losses due to positive and negative angle of incidence before such losses actually occurred in the rotor performance. This in turn is an indication that the prediction of the variation of operating range $i_{PS} - i_{NS}$ on the

basis of the two-dimensional low- and high-speed cascade data is not too satisfactory. It should be noted that the discrepencies between experimental and calculated performance increased with the equivalent tip speed. Although only speeds up to 736 feet per second are shown in the plots, the differences continued to increase at higher speeds, making it impossible to satisfactorily calculate performance for points 7 through 17 (798 and 915 feet per second). The major difference amongst flow conditions in the rotor for the various tip speeds lies in the fact that the absolute and relative Mach numbers increase as tip speed increases. For example, at an equivalent tip speed of 450 feet per second, the rotor relative inlet Mach number at radial station 5 for reference (minimum loss) conditions was about 0.44. The corresponding Mach number at a tip speed of 915 feet per second was 0.91. This fact makes it likely that largest errors in operating range prediction were not in the curves of Figure 6 used for range estimation at low speed but in Figure 7, which permitted the correction of operating range for increased relative Mach numbers. Further verification of this indication is provided by the fact that the calculated third approximation to the rotor performance, although slightly improved over the second trial, still showed the operating incidence angle range error.

CONCLUDING REMARKS

The analytical method for prediction of the performance of axial-flow compressor blade rows has been demonstrated to be quite satisfactory for low-speed performance of a simple axial-inlet rotor-blade row. This represents a necessary first step in a rather complicated sequence of applications which will be required before the generality of the procedure is clearly shown.

The rotor-blade row to which the method was first applied was selected because of the availability of design and experimental performance information. The principal difficulty encountered was the lack of a satisfactory method for estimating the effect of high relative Mach numbers on the low-loss operating range of a compressor blade element. The availability of more complete high-speed two-dimensional cascade data or a study of blade-element performance in rotating blade rows (three-dimensional annular cascades) will be required to provide the necessary method. An approach to this problem might be made by correlating the operating range variation with relative inlet Mach number as measured in rotating compressor cascades (single-stage tests) instead of on the basis of two-dimensional stationary cascades. Such a study should emphasize the determination of those factors existing in compressor flow situations which cause the discrepencies in operating range prediction noted in the present investigation.

It was also found that the lack of experimentally measured velocity, angle, pressure, and temperature distributions both upstream and downstream from the blade row left some doubt as to the accuracy of

performance prediction. This is true because of the necessity for accurate radial distributions as a means for computing inlet conditions to downstream blade rows. The next compressor geometry considered should be one for which the required radial distributions have been published. Examination of the results reported herein, however, shows that the trends and magnitudes in the distributions calculated are correct and that with no more than three approximations, good values may be obtained. It is likely that the third approximation makes a worthwhile improvement in radial distributions, even for flows where the mass-weighted average performance is not significantly improved. The simple-radial-equilibrium and continuity equations programmed for the IBM 650 digital computer may be used without alteration for the computation of the radial distribution of velocities and gas properties at any station located between blade rows in an axial-flow compressor where streamline-curvature is not a major consideration in determining the flow pattern. All other procedures involved in the manual portion of the performance prediction calculations are general and may be applied to any blade row in which $65-(A_{10})$ series blade sections are used. An attempt to extend the calculation procedure to compressor configurations involving more than one stage should be made to establish the value of the method in the prediction of multistage compressor performance.

A consideration of the factors involved in the use of the analytical performance prediction method, including the simplicity of application, the computation time required, and the accuracy of results achieved, indicates that the development of the method should be continued and extended as discussed above.

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APPENDIX A - SUMMARY OF ANALYSIS EQUATIONS

The following equations were used in the computation of compressor

blade-row performance:

1. Energy transfer between rotor and working fluid (reference 32)

$$J g c_p \left({}_2 T_j - {}_1 T_j \right) = \left[\left({}_2 U_j \right) \left({}_2 V_{\theta,j} \right) - \left({}_1 U_j \right) \left({}_1 V_{\theta,j} \right) \right] \qquad \text{(A-1)}$$

2. Isentropic simple-radial-equilibrium equation (references 39 and 63)

$${}_i V_{z,j\pm1}^2 = {}_i V_{z,j}^2 + 2 g J c_p \left({}_i T_{j\pm1} - {}_i T_j \right)$$

$$- \left({}_i V_{\theta,j\pm1}^2 - {}_i V_{\theta,j}^2 \right) - \int_{{}_i r_j}^{{}_i r_{j\pm1}} \frac{2 \, {}_i V_\theta^2}{{}_i r} \, d({}_i r) \qquad \text{(A-2)}$$

As programmed for numerical computation, assuming V_θ^2/r varies

linearly between adjacent radial stations

$${}_i V_{z,j\pm1}^2 = {}_i V_{z,j}^2 + 2 g J c_p \left({}_i T_{j\pm1} - {}_i T_j \right)$$

$$- \left({}_i V_{\theta,j\pm1}^2 - {}_i V_{\theta,j}^2 \right) - \left(\frac{{}_i V_{\theta,j\pm1}^2}{{}_i r_{j\pm1}} + \frac{{}_i V_{\theta,j}^2}{{}_i r_j} \right) \left({}_i r_{j\pm1} - {}_i r_j \right) \qquad \text{(A-3)}$$

3. Non-isentropic simple-radial-equilibrium equation (reference 63)

$${}_i V_{z,j\pm1}^2 = {}_i V_{z,j}^2 + 2 g J c_p \left({}_i T_{j\pm1} - {}_i T_j \right) - \left({}_i V_{\theta,j\pm1}^2 - {}_i V_{\theta,j}^2 \right)$$

$$- \int_{{}_i r_j}^{{}_i r_{j\pm1}} \frac{2 \, {}_i V_\theta^2}{{}_i r} \, d({}_i r) - 2 g J R \int_{{}_i r_j}^{{}_i r_{j\pm1}} {}_i T \frac{\partial \left(\frac{S}{R} \right)}{\partial ({}_i r)} \, d({}_i r) \qquad \text{(A-4)}$$

As programmed for numerical computation assuming that V_θ^2/r, T, V^2 and S/R vary linearly between the adjacent radii of integration

$$_iV_{z,j\pm1}^2 = {}_iV_{z,j}^2 + 2gJc_p({}_iT_{j\pm1} - {}_iT_j) - ({}_iV_{\theta,j\pm1}^2 - {}_iV_{\theta,j}^2)$$

$$- \left(\frac{{}_iV_{\theta,j\pm1}^2}{{}_ir_{j\pm1}} + \frac{{}_iV_{\theta,j}^2}{{}_ir_j}\right)({}_ir_{j\pm1} - {}_ir_j) - \left[JgR({}_iT_{j\pm1} + {}_iT_j)\right.$$

$$\left. - \frac{J(\gamma-1)}{2\gamma}({}_iV_{j\pm1}^2 + {}_iV_j^2)\right] \times \left(\frac{{}_iS_{j\pm1}}{R} - \frac{{}_iS_j}{R}\right) \qquad (A-5)$$

4. Continuity equation

$$w = 2\pi K_{bk} \int_{{}_ir_{10}}^{{}_ir_0} ({}_i\rho_j)({}_iV_{z,j})({}_ir_j)d({}_ir) \qquad (A-6)$$

5. Relationship between static and stagnation temperatures

$$Jgc_p({}_it_j) = Jgc_p({}_iT_j) - \frac{{}_iV_{z,j}^2}{2} - \frac{{}_iV_{\theta,j}^2}{2} \qquad (A-7)$$

6. Density change along assumed streamline through blade row

   a. for isentropic flow

$$_i\rho_j = {}_{i-1}\rho_j \left(\frac{{}_it_j}{{}_{i-1}t_j}\right)^{\frac{1}{\gamma-1}} \qquad (A-8)$$

   b. for non-isentropic flow

$$_i\rho_j = {}_{i-1}\rho_j \left(\frac{{}_it_j}{{}_{i-1}t_j}\right)^{\frac{1}{\gamma-1}} e^{-\frac{J}{R}({}_iS_j - {}_{i-1}S_j)} \qquad (A-9)$$

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7. Relationships defining relative total-pressure loss coefficient for a blade element (reference 67)

$$\overline{\omega}'_j = \frac{(_iP'_j)_{ideal} - (_iP'_j)_{actual}}{_{i-1}P'_j - _{i-1}P_j}$$ (A-10)

in which all pressures are circumferential mass or area averages

8. Entropy change equations for a perfect gas expressed in logarithmic form

a. across a blade element

$$\frac{J}{R}\left(_iS_j - _{i-1}S_j\right) = \ln\left[\left(\frac{_iT_j}{_{i-1}T_j}\right)^{\frac{\gamma}{\gamma-1}} \bigg/ \frac{_iP_j}{_{i-1}P_j}\right]$$ (A-11)

b. from the reference state to the entrance to a blade row

$$\frac{J}{R}\left(_{i-1}S_j - _oS\right) = \ln\left[\left(\frac{_{i-1}T_j}{_oT}\right)^{\frac{\gamma}{\gamma-1}} \bigg/ \frac{_{i-1}P_j}{_oP}\right]$$ (A-12)

9. Equations relating entropy change across a blade element to relative total pressure loss coefficient (reference 67)

$$\frac{\left(\frac{_iT_j}{_{i-1}T_j}\right)^{\frac{\gamma}{\gamma-1}}}{\frac{_iP_j}{_{i-1}P_j}} = \frac{1}{1 - \frac{\overline{\omega}'_j}{\left(\frac{_iP'_j}{_{i-1}P'_j}\right)_{ideal}}\left\{1 - \left[\frac{1}{1 + \frac{\gamma-1}{2}\left(_{i-1}M'_j\right)^2}\right]^{\frac{\gamma}{\gamma-1}}\right\}}$$ (A-13)

in which

$$\left(\frac{{}_iP'_j}{{}_{i-1}P'_j}\right)_{ideal} = \left\{1 + \frac{\gamma-1}{2}\frac{({}_iU^2_j)}{\gamma g R({}_{i-1}T'_j)}\left(1 - \frac{{}_{i-1}r^2_j}{{}_ir^2_j}\right)\right\}^{\frac{\gamma}{\gamma-1}} \qquad \text{(A-14)}$$

10. Blade-element diffusion parameter (reference 67)

$$D = 1 - \frac{{}_iV'_j}{{}_{i-1}V'_j} + \frac{{}_{i-1}V'_{\theta,j} - {}_iV'_{\theta,j}}{2\sigma\,{}_{i-1}V'_j} \qquad \text{(A-15)}$$

This parameter is one of several which have been developed to provide an indication of the rate of diffusion of the working fluid as it passes through an axial-flow compressor blade element. The parameter D has been found to give a good indication of the tendency toward flow separation from or stalling of blade elements operating at incidence angles near the reference values.

11. Equations defining mass-weighted average total-pressure ratio and adiabatic efficiency for a rotor-blade row (reference 63)

$$\left(\frac{{}_iP}{{}_{i-1}P}\right)_{m.a.} = \left\{\frac{K_pK_{bk}\displaystyle\int_{{}_ir_{10}}^{{}_ir_0}\left[\left(\frac{{}_iP_j}{{}_{i-1}P_j}\right)^{\frac{\gamma-1}{\gamma}}-1\right]({}_i\rho_j)({}_iV_{z,j})({}_ir_j)\,d({}_ir)}{K_{bk}\displaystyle\int_{{}_ir_{10}}^{{}_ir_0}({}_i\rho_j)({}_iV_{z,j})({}_ir_j)\,d({}_ir)}+1\right\}^{\frac{\gamma}{\gamma-1}} \qquad \text{(A-16)}$$

$$\left(\frac{_iT}{_{i-1}T}\right)_{m.a.} = \frac{K_e K_{bk} \int_{_ir_{lo}}^{_ir_o} \left(\frac{_iT_j}{_{i-1}T_j} - 1\right)(_i\rho_j)(_iV_{z,j})(_ir_j)\,d(_ir)}{K_{bk} \int_{_ir_{lo}}^{_ir_o} (_i\rho_j)(_iV_{z,j})(_ir_j)\,d(_ir)} + 1 \qquad \text{(A-17)}$$

$$\left(\eta_{ad}\right)_{m.a.} = \frac{\left(\dfrac{_iP}{_{i-1}P}\right)_{m.a.}^{\frac{\gamma-1}{\gamma}} - 1}{\left(\dfrac{_iT}{_{i-1}T}\right)_{m.a.} - 1} \qquad \text{(A-18)}$$

It should be noted that only the equations which are basic to the particular analytical procedure used in this dissertation are given here. The velocity diagram relationships, some compressible flow equations, and certain other items (e. g. equation of state for a perfect gas) used commonly in turbomachinery design and analysis are not included.

APPENDIX B - SYMBOLS AND NOTATION

The following symbols are used in this dissertation:

A cross-sectional area of annulus, ft^2

a speed of sound, ft/sec

c_p specific heat at constant pressure, Btu/(lb)(°R)

D blade-element diffusion parameter

F blade force, lb_f/lb_m

f $2gJc_p(_it_j)$

g gravitational constant, 32.17 (ft)(lb_m)/(lb_f)(sec^2)

i incidence angle, angle between inlet-air direction and tangent to blade mean camber line at leading edge, deg

J mechanical equivalent of heat, 778.26 (ft)(lb_f)/Btu

K_{bk} weight-flow blockage factor

K_e energy-addition correction factor

K_p pressure-correction factor

M Mach number

P total or stagnation pressure, lb_f/ft^2

p static pressure, lb_f/ft^2

R gas constant, 53.35 (ft)(lb_f)/(lb_m)(°R) for air

r radius, ft

S entropy, Btu/(lb_m)(°R)

T total or stagnation temperature, °R

t static temperature, °R

U blade element speed, ft/sec

V air velocity, ft/sec

W weight flow rate, lb_m/sec

z axial distance, ft

β air angle, angle between air velocity and axial direction, deg

γ ratio of specific heats, 1.40 for air

γ^o blade-chord angle, angle between blade chord and axial

       direction, deg

δ ratio of compressor-inlet stagnation pressure to N. A. C. A.

       standard sea-level pressure of 2116 lb_f/ft^2

δ^o deviation angle, angle between outlet-air direction and tangent

       to blade mean camber line at trailing edge, deg

η efficiency

θ ratio of compressor-inlet stagnation temperature to N. A. C. A.

       standard sea-level temperature of 518.7 ^{o}R

ρ density, lb_m/ft^3

σ blade-element solidity, ratio of chord to spacing

τ $2gJc_p(_iT_j)$

ϕ blade camber angle, difference between angles of tangents to mean

       camber line at leading and trailing edges, deg

$\overline{\omega}$ blade-element total pressure - loss coefficient

Presubscripts:

i axial station (numerical values indicate various axial stations

       as in Figure 8)

Postsubscripts

a   based on stagnation conditions

ad   adiabatic

j   radial station (numerical values indicate various radial stations as in Figure 8)

m.a.   mass-weighted average

PS   value measured for stall due to positive incidence (see reference 45)

NS   value measured for stall due to negative incidence

r   radial direction

ref   reference

z   axial direction

Θ circumferential direction

Superscripts:

$'$ measured relative to a reference frame rotating at blade element velocity
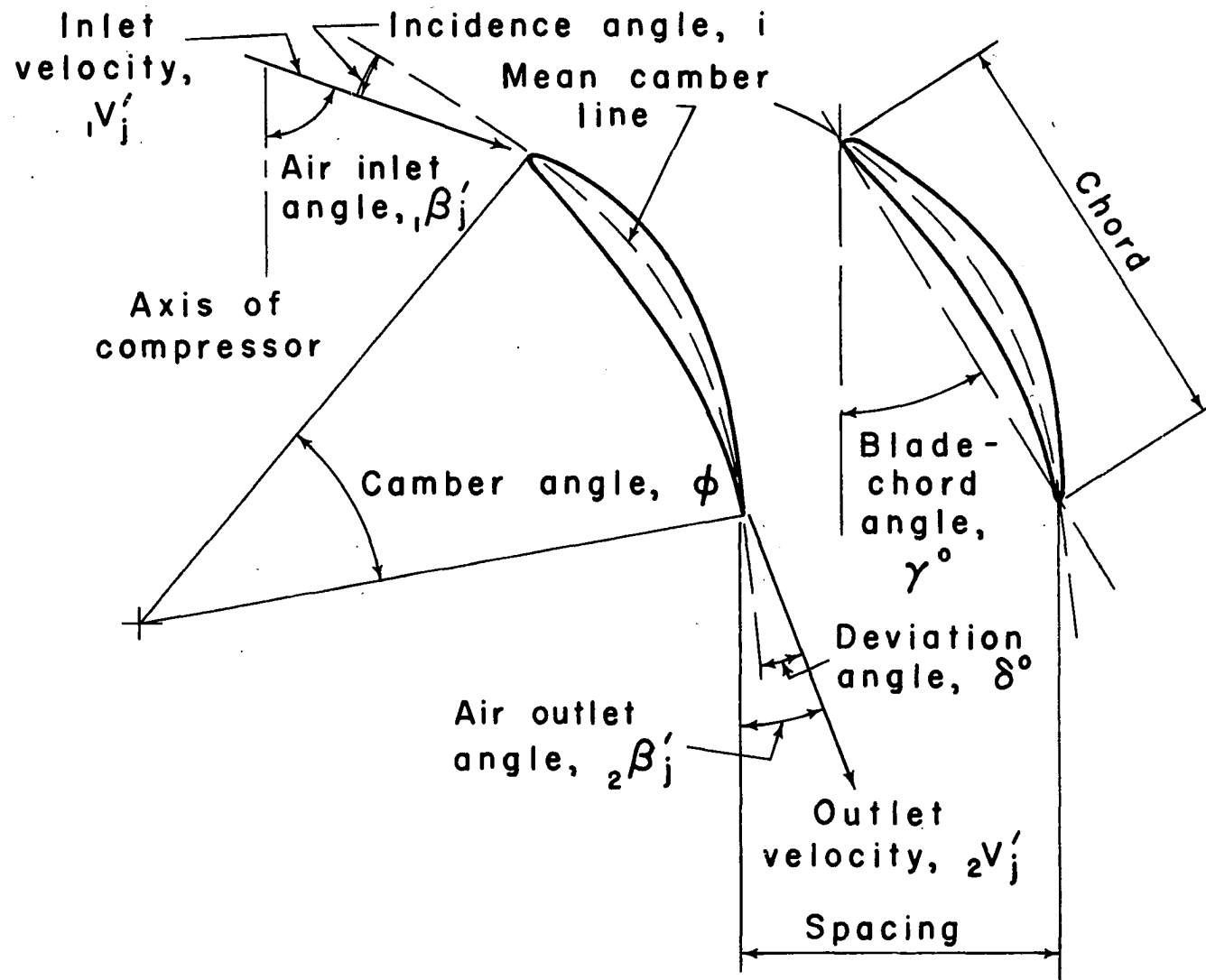
The notation used in defining blade-element and cascade performance is shown in Figure 13.

Figure 13. Compressor cascade notation