SCATTERING OF ELASTIC WAVES BY INCLINED SUBSURFACE CRACKS

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INTRODUCTION

Recently analytical techniques using the T-matrix of an isolated flaw were developed to study the problem of ultrasonic wave scattering from a subsurface flaw in an elastic half space that interfaces with a fluid half space.[1,2] The scattered far field in the fluid was expressed in orders of multiple scattering between the flaw and the fluid-solid interface. It was concluded from both the theoretical study and the experimental study[3] that for incident wave angles oblique to the interface multiple scattering effects are negligible for depths greater than the flaw diameter. At incidence normal to the interface, it was found that first order multiple scattering modulated the scattered field of the isolated flaw. The modulation wavelength could be related directly to the depth of the flaw and its amplitude became smaller as the depth increased. At oblique angles of incidence, mode conversion at the fluid-solid interface is the major cause of the observed interference pattern in the frequency spectrum. At depths below one crack diameter, multiple scattering effects are significant and the full scattering series must be retained.

In previous studies, the flaw with a rotational axis of symmetry was assumed to be in an orientation such that its rotational axis of symmetry is normal to the fluid-solid interface. This rendered the representations of the T-matrix of the flaw and the reflection matrix of the fluid-solid interface in a basis of vector spherical functions block diagonal in the azimuthal index. This resulted in very compact storages for these matrices which would otherwise be very large. In the present study the penny shaped crack modelled as an oblate spheroid of high aspect ratio is assumed to be oriented at an angle 'α' with respect to the interface. The depth to diameter ratio is, however, greater than two, so that multiple scattering between the flaw and the interface can be neglected. Under these conditions, the computations can still be done for a general angle of incidence.

Two particular cases are considered, one of a crack oriented at an angle of 63° in a plastic block located at different depths and a crack in a glass sample which is normal to the fluid-solid interface and at different depths below the interface. These two cases were chosen because samples have been fabricated to these specifications and measurements of the ultrasonic backscattering from these flaws for a given depth will be reported by T.A. Gray and R. B. Thompson.[4] The calculated spectra are studied as a function of depth, crack diameter and crack orientation.

FORMULATION

Consider a flaw, void or inclusion where the largest diameter is '2a' located at a depth 'd' and at an angle 'α' w.r.t. the fluid-solid interface (see Fig. 1). We assume that $d/2a>1$, in which case multiple scattering effects between the flaw and the fluid-solid interface can be neglected. The source of incident waves is

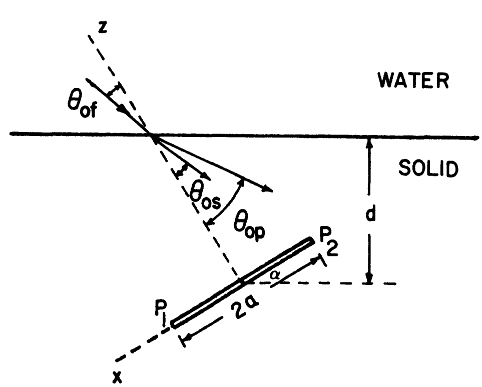

Fig. 1. Scattering geometry

located in the fluid at a distance much greater than either d or 2a. The waves are plane waves when they strike the interface. At the interface there is mode conversion in general and both P- (longitudinal) and S- (transverse) waves are launched in the solid which in turn are scattered by the flaw.

In order to simplify the computations and render the T-matrices block diagonal in the azimuthal index it is necessary to orient the z-axis along the rotational axis of symmetry of the flaw. All angles are measured with respect to this axis and this only requires a small modification in expressions for the transmission coefficients for both the incident and scattered waves. In the literature expressions for the transmission coefficients are given in terms of angles that are measured w.r.t. the normal to the interface.

In Refs. 1 and 2 a self-consistent multiple scattering formulation was presented and the details are not repeated here. The final expression for the scattered field in the fluid half space for a wave incident at an angle θ_o w.r.t. the symmetry axis (z-axis) of the flaw is given by

$$\vec{u}_f^{\,s}(\vec{r}) \xrightarrow[k_f r \to \infty]{} \hat{r}\, \frac{e^{ik_f r}}{k_f r} \sum_{\ell m \sigma} i^{-\ell} \Bigg(P_\ell^m(\theta_p)\, T_{fp}(\theta)\, f_{1\ell m\sigma} \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix}$$

$$- \frac{i}{\sqrt{\ell(\ell+1)}} \frac{mP_\ell^m(\theta_s)}{\sin\theta_s}\, T_{fs}(\theta)\, f_{2\ell m\sigma} \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix} \tag{1}$$

$$+ \frac{1}{\sqrt{\ell(\ell+1)}} \frac{dP_\ell^m(\theta)}{d\theta}\bigg|_{\theta=\theta_s} T_{fs}(\theta)\, f_{3\ell m\sigma} \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \Bigg)$$

where $k_f = \omega/c_f$ is the wavenumber in the fluid, (θ,ϕ) specify the direction of observation in the fluid, θ_p and θ_s are related to θ by Snell's law i.e.

$$\frac{\sin\theta}{c_f} = \frac{\sin\theta_p}{c_p} = \frac{\sin\theta_s}{c_s} ;$$

c_p and c_s being the P- and S-wave speeds in the solid. In Eq. (1) P_ℓ^m are normalized associate Legendre polynomials, T_{fp}

and T_{fs} are the transmission factors for P- and S- waves from the solid to the fluid writtern w.r.t. the symmetry axis of the flaw and $f_{\tau\ell m\sigma}$ are scattered field coefficients in the solid given by

$$f = T(1-RT)^{-1}a \approx Ta \tag{2}$$

where 'T' is the T-matrix of the isolated flaw in the solid, R is the reflection matrix at the fluid-solid interface, 'RT' being much less than unity for large values of d/a and 'a' is the incident field coefficient in the solid. This is made up of both P- and S- waves due to mode conversion of the original acoustic wave in the fluid. These coefficients are

$$a_{1\ell m\sigma} = \pi i^{\ell-1} Y_{\ell m\sigma}(\theta_{op},\phi_o)\, T_{pf}(\theta o) \tag{3}$$

$$a_{2\ell m\sigma} = \frac{\pi i^{\ell}}{\sqrt{\ell(\ell+1)}} \frac{d}{d\phi_o}\left(Y_{\ell m\sigma}\{\theta_{os},\phi_o\}\right) T_{sf}(\theta_o) \tag{4}$$

$$a_{3\ell m\sigma} = \frac{\pi i^{\ell-1}}{\sqrt{\ell(\ell+1)}} \frac{d}{d\theta_{os}}\left(Y_{\ell m\sigma}\{\theta_{os},\phi_o\}\right) T_{sf}(\theta_o) \tag{5}$$

where θ_o,ϕ_o specify the incident wave direction in the fluid, $Y_{\ell m\sigma}$ are normalized spherical harmonics, θ_{op} and θ_{os} are related to θ_o by Snell's law and T_{pf} and T_{sf} are the transmission factors for P- and S- waves from the fluid to the solid. Details of evaluating the T-matrix of the isolated flaw have been discribed by Varadan and Varadan[5].

In the calculations and results that are described in the next section, the approximate form of 'f' as given in Eq. (2) was used in Eq. (1) to compute the scattered field in the fluid. On examining Eq. (1) along with Eqs, (2) and (3) it is seen that four terms contribute to the scattered field in the fluid, namely P-P, P S, S P and S S terms which contain products of $T_{fp}\,T_{pf}$, $T_{fs}T_{pf}$, $T_{fp}T_{sf}$, and $T_{fs}T_{sf}$ respectively. Since the P- and S-wave speeds are different in the fluid and the points P_1 and P_2 of the crack edge (see Fig. 1) serve as strong centers of scattering (flash points), the difference in path length from the 4 rays scattered from P_1 and P_2 results in a strong interference pattern as evidenced by the maxima and minima in the scattered spectrum. Since the path difference depends on both the crack depth and orientation as well as the crack diameter, the four contributions were calculated separately in order to understand their relative magnitude.

RESULTS AND DISCUSSION

The spectrum of the back scattered far field in the fluid are presented for two different samples containing penny shaped cracks. The cracks are modelled as oblate spheroidal voids of aspect ratio $b/a = 0.2$. Previous studies indicate that if $b/a < 0.25$, the results for the oblate spheroid do not differ appreciably from the exact calculations for a penny shaped crack. As mentioned in the introduction, two sets of calculations are presented. In one set a crack embedded in plastic inclined at an angle of 63°to the free surface is considered. In the actual sample used in Ref. 3, $d/a=4.0$. In the calculations presented here d/a ranged from 3.0 all the way up to 10.0. The back scattered field is plotted for several different angles of incidence. In the second set a crack in a glass sample oriented normal to the surface is considered. The depth in the actual sample is 7.0a, in the calculations d/a was allowed to vary from 3.0 to 10.0. Several different angles of incidence were considered.

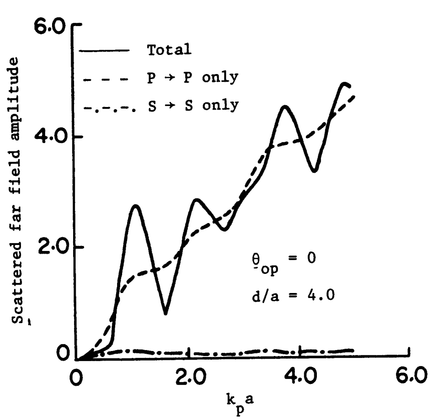

Fig. 2. Back scattered far field amplitude as a function of non-dimensional wavenumber $k_p a = \omega a/c_p$ for a penny shaped void in a plastic sample inclined at an angle 63° to the surface (c_p = 2590m/s, c_s = 1250m/s, ρ = 1.2g/cc)

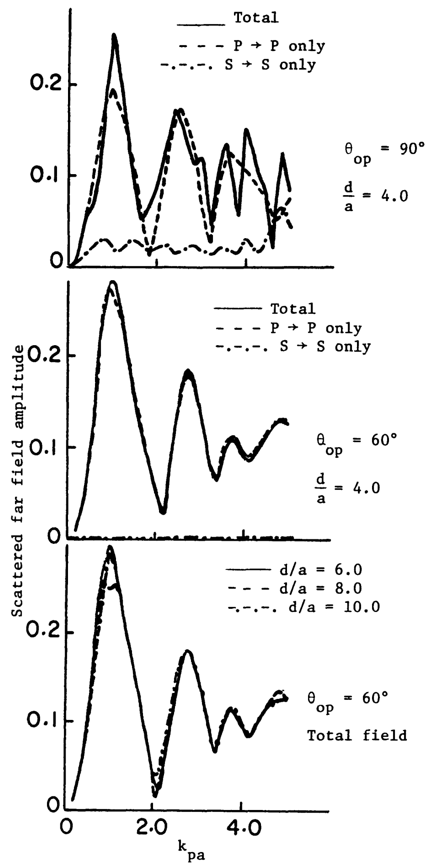

Fig. 3. Scattered far field amplitude as a function of non-dimensional wavenumber $k_pa = \omega a/c_p$ for a penny shaped void in a plastic sample inclined at an angle of 63° to the surface.

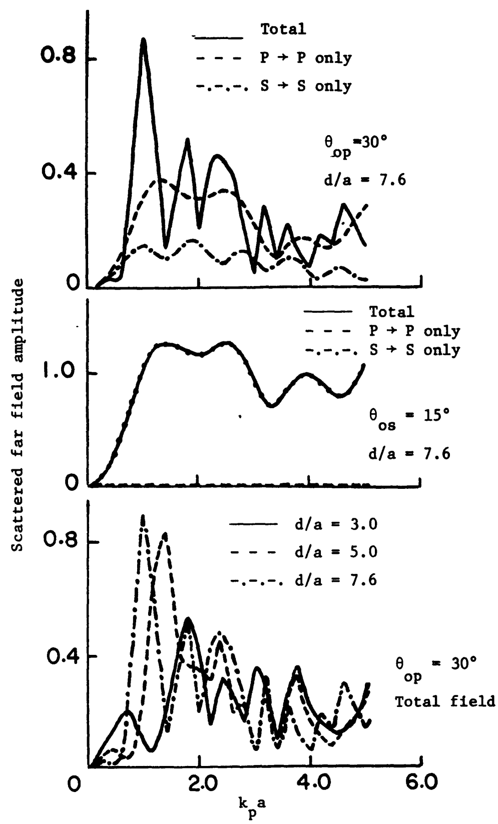

Fig. 4. Scattered far field amplitude as a function of non-dimensional wavenumber $k_p a = \omega a/c_p$ for a penny shaped void in a glass sample oriented normal to the surface (c_p = 6000m/s, c_s = 3780m/s ρ = 2.2g/cc)

For the plastic sample, mode conversion to S- waves is very small and hence the S - S contributions are negligible compared to the P - P contribution which is nearly equal to the total field. It can be seen that the total back scattered field in the fluid is almost independent of the depth (Figs. 2,3,).

Some obvious conclusions can be drawn. In the glass sample mode converted scattered waves contribute comparably to the non-mode converted waves and the composite spectrum including all contributions is fairly complex. A critical angle exists above which P- waves are not transmitted into the solid. The spectrum is considerably smooth above the critical angle of incidence since only the S - S wave contributes to the field in the fluid. The spectra due to S - S or P - P contributions only are independent of depth and depend only on the crack diameter and orientation. If the S - S and P - P contributions are comparable and allowed to interfere with one another, then the results depend on the depth of the crack (Fig. 4).

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