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DEVELOPMENT AND EVALUATION OF SELF-INSTRUCTIONAL MATHEMATICS MATERIALS DESIGNED FOR STUDENTS IN EDUCATIONAL STATISTICS

by

Gerald Joseph Wisnieski

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Dean of Graduate College

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INTRODUCTION

This study developed out of the investigator's belief that many of the students who encounter difficulty in educational statistics do so, not because of the difficulty of statistics, but because they lack the necessary mathematical competencies to successfully perform the work in statistics. In this era of specialization it is not uncommon for a person who is a scholar in one field to be a layman, sometimes an all but illiterate layman, in another field.

Many graduate students are mature, intelligent men and women who have had no intimate acquaintance with mathematics for years. These people are now discovering that in order to understand the modern world, and in order to read the rapidly expanding literature of their own fields, they need to become conversant with statistical method. But statistical method, even in its simpler phases, is a branch of applied mathematics. Refusal to admit this fact will not lessen the difficulties which await the student who is unable to think competently in numerical and symbolic forms.

In regard to this problem, Bashaw (11, p. viii) states:

> Many of the problems in the teaching of basic statistics arise from student heterogenity with respect to mathematical background. It is common for the teacher to spend considerable time on mathematical notation definitions. He also usually spends countless unplanned hours in and out of the classroom reviewing elementary algebraic manipulations and clarifying elementary mathematical concepts. This time can be better spent directly on statistics topics if all the students have a fairly good mathematics preparation. It is not rare to find that the majority of the students in a first course in statistics have very weak backgrounds. These students find it necessary to struggle through numeric and algebraic manipulations, often preventing them from focusing on the statistical concept that is being taught. Moreover, the inability to perform algebraic operations increases the memory component of learning. Instead of trying to understand how several special-case formulas relate to each other, these students

> frequently attempt to memorize the entire set of formulas. Thus a knowledge of algebra is seen to help the student use his study time in learning basic concepts rather than in memorizing. The algebra-wise student has no need for extensive memorization because he can readily derive the various special-case formulas from the general forms.

These views are supported by L. E. Christ (20) who found that students who scored well on mathematics tests based on arithmetic and basic algebra performed significantly better in elementary statistics than did those who scored poorly on the tests. H. M. Walker (83) provided her students with units in mathematics which they used voluntarily. She found that her classes as a whole were able to cover more ground in statistics than was possible before the units in mathematics were available; and the weaker students who formerly felt a distinct dislike for statistics achieved a new sense of confidence and a better comprehension of the statistical concepts studied. By spending time on definite background work, the students saved the time which might otherwise have been fruitlessly consumed by erroneous computation, slowness, haziness of thinking, and by inability to read a text because of failure to comprehend symbolic language.

With this as a basis, it seems logical that there is a need to provide some means for the students who score poorly on mathematics tests, based on arithmetic and basic algebra, to improve their mathematical competencies in these areas.

The reasons why students do or do not possess the mathematical competencies necessary for success in educational statistics were not of major importance in this study. What was considered important, was first to identify those students who do not possess the mathematical competencies necessary for success in educational statistics, and second to provide assistance in mathematics for those students who require it. If one can provide a means for students to improve their chances for success in educational statistics, one should also effect a favorable change in students' attitudes toward educational statistics.

The Problem

In view of the fact that every student who plans to complete the requirements for a graduate degree in education at Iowa State University is required to successfully complete one or more courses in educational statistics, some awareness of the mathematical competencies possessed by these students seems desirable. If a student does not possess the mathematical competencies necessary for success in these educational statistics courses, an effort should be made to equip the student with those competencies he will need to successfully complete the courses. The effort to accomplish this goal should be made with a minimum sacrifice of scheduled class time.

The problem then, included the following: (1) to identify the mathematical competencies necessary for success in educational statistics, (2) to identify those students who do or do not possess these mathematical competencies, (3) to develop mathematics materials to be used to improve mathematical competencies, and (4) to do an experimental evaluation of the effectiveness of the mathematics materials.

Purpose of the Study

The specific purposes of this study may be outlined as follows:

1. To develop a test, based on arithmetic and basic algebra, to determine whether or not students possess the mathematical competencies which the investigator has identified as being necessary

for success in educational statistics.

2. To develop self-instructional mathematics materials which students can use to improve their mathematical competencies.
3. To determine if the use of the self-instructional mathematics materials has a favorable influence on the mathematical competencies of students enrolled in an educational statistics course.
4. To determine if the use of the self-instructional mathematics materials has a favorable influence on achievement of students enrolled in an educational statistics course.
5. To determine if the use of the self-instructional mathematics materials will cause a favorable change in the attitudes of students toward educational statistics.

The following hypotheses were stated for investigation:

1. There is no significant difference in mathematical competencies (as measured by the mean change in students' scores on pre- and post-administrations of the mathematics test) of students who have used self-instructional mathematics materials from those who have not used the materials.
2. There is no significant difference in achievement in educational statistics (as measured by three tests developed and administered by the instructors of the educational statistics 590X classes) of students who have used self-instructional mathematics materials from those who have not used the materials.
3. There is no significant difference in mean change in attitude toward statistics (as measured by the mean change in students' responses on pre- and post-administrations of a questionnaire

designed to determine attitude toward statistics) of students who have used self-instructional mathematics materials from those who have not used the materials.

The following assumptions were made relative to this study:

1. Those enrolling in educational statistics 590X are a representative group of those students who enroll in an educational statistics class.
2. The students in the experimental group used the self-instructional mathematics materials as directed.
3. The students in the control group did not have access to the self-instructional mathematics materials.
4. The treatment received in class was the same for both experimental and control groups.
5. The responses given by the students to the attitudinal questionnaires were an honest evaluation of their attitudes.
6. That basic mathematical competencies from the areas of arithmetic and algebra were related to achievement in educational statistics.
7. That the attitude questionnaires measured true attitudes.

Sample for the Study

Students who take the course Education 590X, Statistical Interpretation for Education, at Iowa State University form the sample for the study. This course was designed to be a terminal course for students planning to take a single statistics course. The only prerequisites for this course are 15 credits in education.

The topics for study include introduction to measurement, frequency distributions and their graphs, percentile ranks, statistical notation, measures of central tendency, measures of dispersion, standard scores, normal distributions, normalized standard score distributions, Pearson product moment correlation, principles of prediction, basic sampling theory, the t-distribution and tests, the chi-square distribution and tests, the F-distribution, and the simple analysis of variance. The textbook used for the course was Fundamental Research Statistics for the Behavorial Sciences by John T. Roscoe, published by Holt, Rinehart and Winston, Inc., in 1969. The book was written primarily for students in education and psychology and assumes no background other than elementary high school algebra.

Delimitations

The scope of this investigation was confined to the study of the effect of the use of self-instructional mathematics materials on the mathematical competencies, achievement in statistics, and attitude toward statistics of students enrolled in educational statistics 590X for the 1970 summer session at Iowa State University. The mathematical competencies considered were those that the investigator identified as being necessary for success in educational statistics 590X. These competencies were based on arithmetic and basic algebra. Achievement in statistics was based on scores obtained on three tests developed and administered by the instructors of educational statistics 590X. Attitude was based on an attitudinal questionnaire developed by the investigator.

Organization of the Study

The material for this study was organized and presented in six chapters. The first chapter includes the rationale, the problem, purposes of the study, sample for the study, delimitations, and organization of the study. The second chapter contains a summary and analysis of related literature and research. The review of literature includes sections on recognition of the problem, related studies, and a study investigating "help" in mathematics.

The methods and procedures used for the study are discussed in the third chapter. This chapter includes sections on the development of the materials, experimental procedure and collection of the data, and treatment of the data.

The findings of the data assimilated from the study are presented in the fourth chapter. The fifth chapter includes the discussion and recommendations. The final chapter contains the summary and conclusions.

REVIEW OF LITERATURE

The literature reviewed for this study has been grouped into three categories: (1) recognition of the problem, (2) related studies, and (3) a study investigating "help" in mathematics.

Recognition of the Problem

Many educators agree that a student's mathematical background is important to success in educational statistics. Quinn McNemar (57, p. v) states, "... I think it is fruitless to try explaining statistics to those who are not prepared to do some thinking in mathematical language." C. W. Odell (65, p. vi) adds, "... it is impossible to discuss educational statistics without some use of mathematical formulae ...."

It is also commonly accepted that the lack of mathematical preparation of students in education who take educational statistics is a cause of frustration to both students and instructors alike. In reporting opinions of 85 teachers of educational statistics from throughout the country, Brown (18, p. 48) states: "There is a rather general feeling among teachers of elementary statistics that insufficient mathematical training is the reason why many students fail to do satisfactory work in the subject." Gregory and Renfrow (33, p. i) agree by saying:

> Most students in education and psychology are not primarily mathematicians. They do not think in terms of mathematical symbols and formulas. They have been away from their college mathematics so long that many of the simple formulas and symbols they once knew are unfamiliar.

Most writers of statistics textbooks are aware of the inadequate mathematical background of students. A casual reading of textbook prefaces verifies this observation. Cornell (22, p. vi) began his text with simple arithmetic and algebra reviews "... in view of the limited mathematical training of most education students ...." He recognized that the student whose arithmetic skills are weak "... spends his time on computation and not on learning statistics." Guest (34) demands of the student no more mathematical training than a knowledge of simple arithmetic and the rudiments of elementary algebra. Even so, he provides a review of simple arithmetic. Brown (18, p. 48) examined the statistics book used in the elementary statistics classes of 26 of the 85 teachers that he polled, and concluded, "The only mathematical skills required in the solution of the exercises in the most generally used text were skills of arithmetic and of elementary algebra."

Brown (18) felt that the first thing necessary was to determine the extent to which beginning students possess or lack the skills necessary for success in educational statistics. He surmised that this could be determined accurately only by means of an actual test. He constructed a test based on arithmetic and basic algebra and administered it to 990 students in 31 classes of beginning statistics. With the results of this test as a basis, he suggested that students of statistics be given special "help" in the following areas:

1. Conventional order of algebraic operations.
2. Principles of computation with approximate numbers.
3. Rational method of locating the decimal point in division and in square root.
4. Systematic checking of numerical and algebraic operations.
5. Use of tables, including reading in reverse order and linear interpolation.

6. Acquaintance with a greater variety of symbols, including initial letters, Greek letters, and subscripts.
7. Practice in translating from symbols to words and from words to symbols.
8. Evaluation of formulas with attention to algebraic signs.
9. Practice in distinguishing between variables and unknowns and between independent and dependent variables.
10. Co-ordinates and plotting of points and lines.
11. Equation of a straight line.
12. Development of the pure number concept.
13. Symbol of algebraic summation, Σ.
14. Manipulation of formulas, including formulas with radicals.

In regard to those students who need this special "help", Brown (18, p. 51) states:

> Those students who have forgotten the facts and skills of elementary mathematics may be asked to review on their own time selected topics in the textbooks of arithmetic and algebra, or, better yet, they may be given carefully prepared self-teaching materials.

Baggaley (9) in supporting Brown's viewpoint notes that the problem becomes one of bringing those students who need "help" back to the necessary level of mathematical sophistication in the period of a few weeks. He feels that the techniques of programmed instruction afford an opportunity for this goal to be attained with maximal efficiency.

Fusco (30, pp. 87-88) pointed out that:

> Self-instructional techniques are tailored to the learning speed of the individual student. This method provides the student with as much practice as he needs, permits the rapid learner to cover the material quickly and the slower student to proceed at his own pace. As a consequence, the classroom teacher may be

freed from the burdensome and time-consuming tasks of presenting material and taking precious class time to repeat material for students who didn't get it the first time. Students working with self-instructional materials will be enabled to work privately and independently and, as a consequence, recognize the fruits of their own labor.

Related Studies

The following studies, although not directly related, were helpful in providing this investigation an added perspective.

Bendig and Hughes (12) constructed a 30 item test to measure students' attitudes toward an introductory statistics course designed for psychology and related biological and social science seniors. For two samples, the product-moment correlation coefficients between this test and the quantified letter grade (A being assigned a numerical value of 4, B a value of 3, and so on) were 0.24 and 0.21. Furthermore, this test correlated 0.31 with Kuder Reference Record Computational, 0.22 with Persuasive Scale, 0.32 (biserial) with amount of high school mathematics, and 0.40 (biserial) with amount of college mathematics. On the basis of these results, Bendig and Hughes (12, p. 274) surmise that, "... increased exposure to mathematics courses in high school and college, plus an interest in computational pursuits, should dispose a student toward a statistics course."

O'Connor (64), acknowledging the fear of students in statistics, gives many helpful suggestions. Of 13 suggestions, the four to which 13 outstanding educators working with statistics students agreed most were:

1. Provide adequate time to thoroughly review the basic principles of mathematics and algebra.
2. Remove the fear of statistics. Start and work always with easy numbers.

3. Stress understanding rather than rote memorization of material.
4. Exhibit patience in retracing, repeating, and rephrasing concepts and information.

The seemingly important place given to mathematics in the above articles was contradicted by the results of at least one other study.

Averill (7) examined "the mathematical background necessary to succeed in a course in elementary statistics" as a part of a study of elementary statistics in selected Michigan Community Colleges. Elementary statistics was one of the several general mathematics courses offered in the community colleges. Students were mostly freshmen pursuing studies in many fields; about half were terminal students. Thus elementary statistics in this context is different from the idea of statistics in this paper. Nevertheless, the statistics of these two contexts are similar in two important respects, (1) the mathematics required to succeed, and (2) the expected outcome from the course. Averill listed the expected outcomes as follows:

1. An increased knowledge of
   a. Principles of statistics
   b. Statistical methods, and
   c. Statistical analysis;
2. Increased skill in
   a. Statistical computation
   b. Statistical presentation, and
   c. Interpretation and reporting;
3. Improved attitude toward
   a. Professional responsibility, and
   b. Progress in the field of statistics.

Averill felt that a normal eighth-grade education in general mathematics would be adequate to succeed in statistics. In view of this analysis, he used the students' scores on the quantitative portion of the ACE or SCAT tests as a predictor for students' success in statistics. For first correlation, the ACE or SCAT test scores and course grades were grouped into frequency distributions. This was done on a two-way frequency table and resulted in a correlation coefficient of 0.19. Because grouping errors might have led to a lower coefficient of correlation than was correct, the product moment was applied to the original data, which produced a coefficient correlation of 0.10. These results are in agreement in that the relationship between the ACE or SCAT test scores and grades in statistics is very small.

Averill further checked the possibility that the students with lowest test scores might be sufficiently bad risks to warrant dropping them from the course. He found that the overall performance of the 19 students whose scores were less than the 30th percentile was almost as high as the overall performance of the 120 students as a group.

Despite the low correlation between the mathematics tests and success in elementary statistics, Averill recommends, on the basis of personal teaching experience, that an elementary statistics course be preceded by a review of mathematics. His feelings can be summarized as follows:

> While elementary statistics requires the knowledge of nothing more advanced than the first part of elementary algebra, it does require a certain type of quantitative reasoning and a sound background in arithmetic as well as a familiarity with numerous common mathematical terms. (7, p. 48)
>
> ... Some refresher work, particularly in algebra might prove worthwhile; prior to introducing any new subject of this type. Possibly the results of some other test in mathematics might be

> used to eliminate students who are not ready, mathematically speaking, to do this kind of work. (7, pp. 51-52)

A Study Investigating "Help" in Mathematics

In spite of all the statements supporting the opinion that a good mathematics background is important to success in educational statistics, there was only one study found that investigated the effect of "help" in mathematics in relation to achievement in educational statistics.

Christ (20) felt that the main problem of students in educational statistics was lack of understanding of elementary mathematics. She divided this main problem into four parts, each part corresponding to a subproblem: (1) whether elementary mathematical skill correlates as well as some of the other measurable skills, with the statistics course grade, (2) how mathematics achievement and success in elementary statistics are influenced by various factors and by each other, (3) what specifically is the level of mathematics achievement of students entering elementary statistics, and do they improve their mathematical knowledge during the course in elementary statistics, and (4) whether extra "help" sessions in mathematics improves poor students' mathematical knowledge and their performance in elementary statistics.

In an attempt to gather information concerning the students, Christ used several measuring instruments. First, the Cooperative Mathematics Tests, arithmetic and algebra I, were used for measuring mathematical achievement. Second, the Miller Analogies Test was used for measuring general aptitude. Third, the Doppelt Mathematical Reasoning Test was used for measuring ability in mathematical reasoning. Fourth, the Quantitative Evaluation Device was used for measuring symbolic manipulation.

Grading was done on a nine point basis, with the following assignment; nine for A+, eight for A, seven for A-, ..., one for C-. A successful grade was defined to be a grade greater than B+, a satisfactory grade was defined to be a grade equal to B+, and an unsuccessful grade was defined to be a grade less than B+. Christ made many interesting observations. Among those most directly related to this study include:

1. The students who had three or more years of mathematics in high school did significantly better (5 per cent level) than those students who had less than three years of mathematics in high school. This was true in regard to both the mathematics tests and achievement in statistics.
2. The students who had a combined score of equal to or greater than 65 on the mathematics tests (out of a possible 90) did significantly better (1 per cent level) than those students who had a combined score of less than 65 on the mathematics tests.
3. A regression equation using only the mathematics tests as predictors had a standard error of "about 1". Therefore about 66 per cent of the students predicted to receive B+ would have received grades between B and A-.
4. The weakest area in arithmetic was the type of problem involving percentage.
5. The weakest area in algebra was the type of problem involving graphs of linear functions.
6. When the top 27 per cent of the students were compared with the 7th, 8th, and 9th grade norm groups, they did better than the norm groups in every item of the 50 item arithmetic test and all but

two items of the 40 item algebra test.

7. When the bottom 27 per cent of the students were compared with the norm groups, they did worse than the norm groups in 17 items of the arithmetic test and in all but one item of the algebra test.
8. Students who received better than a B+ grade did better than the norm group in every item of both the arithmetic and algebra tests. On the other hand, the students who received less than a B+ grade did worse than the norm group in 15 items of the arithmetic test and all but one item of the algebra test.
9. The mathematics post-test correlated higher with statistics achievement than did the mathematics pre-test. The difference in correlations was highly significant (1 per cent level).
10. Many students said that because of the mathematics "help" they did not feel as panicked with the regular statistics course nor did they require as much time for homework assignments.

The following conclusions were drawn by Christ in regard to the four sub-problems:

1. Sub-problem 1. Is there a relationship between some of the measurable skills of students and success in elementary statistics?

   The simple mathematical achievement which was measured by the direct sum of arithmetic and algebra I of the Cooperative Mathematics Tests correlated best with the grade in the statistics class. A correlation of 0.634 was reported.
2. Sub-problem 2. Are there significant differences in mathematics achievement and success in statistics in terms of various

factors--sex, mathematical background in high school, and degrees for which students are working--and in terms of each other?

There were significant differences in mathematics achievement and in success in statistics in terms of mathematical background in high school. There were no significant differences in terms of the other factors listed.

3. Sub-problem 3. What is the mathematical background of students entering the elementary statistics course? Do students improve their mathematical background during the course in elementary statistics?

   Given that the mathematics necessary to succeed in elementary statistics was eighth-grade level of arithmetic achievement and ninth-grade level of algebraic achievement, students entering the elementary statistics course did not have a satisfactory amount of mathematical knowledge. Left alone, students did not significantly improve their mathematical knowledge in elementary statistics.

4. Sub-problem 4. Will extra "help" sessions in mathematics improve the performance of poor students in a statistics course?

   The limited "help" sessions of this study (four laboratory periods spent in learning how to use calculators and doing worksheets of math review problems) were not sufficient to help students in overcoming their deficiencies in mathematics, and thus did not help them to succeed in statistics.

In summarizing the findings of her study, Christ (20, p. 105) states:

> The summaries of the findings of this study point to the close relationship between mathematical background and success in elementary statistics. One of the most encouraging outcomes is

> that a simple knowledge of elementary mathematics plays a key role in success in elementary statistics. While a high mathematics test score does not quarantee success in elementary statistics, it would be difficult for a low achiever in mathematics to do very well in elementary statistics. There is also an indication, not yet proven, that the lack of mathematical knowledge can be overcome by most students given the proper opportunity.

As a result of her findings, Christ recommends that a non-credit course such as "Mathematics Essential for Statistics" be offered. Possible content would include:

Unit I. Basic operations
Substitutions

Unit II. Linear equations
Percentages

Unit III. Graphs
Square roots
Interpolations

In unit I, the review of basic operations would include fractions and signed numbers almost immediately. The idea of variables in equations should be clarified, and the basic operations could be practiced by substitutions.

In unit II, solving linear equations could be taught as applications of fundamental properties of basic operations. Problems involving percentage should be pointed out as a particular example of linear equations.

In unit III, plotting points and making graphs of linear equations should be emphasized. There should be many exercises in reading tables and interpolating for values.

Each unit should consist of about five sessions of one hour each. Students should be allowed to enter any one of the units, depending upon their individual needs.

In regard to further research, Christ (20, p. 126) suggests that more experiments should be conducted to determine whether extra "help" sessions in mathematics indeed improve the performance of poor students in a statistics course. She feels that at least one of the following two conditions should be met in the new experiments.

1. "Help" in mathematics should be given for at least ten sessions.
2. "Help" should be given before the students take elementary statistics.

In the literature reviewed for this study there was general agreement supporting the thesis that mathematical background, at least in the areas of arithmetic and algebra, is important to success in educational statistics. Suggestions for ways to improve mathematical competencies in these areas centered around the use of programmed or self-instructional materials. However, very little experimental research has been conducted on the effectiveness of "help" in mathematics in relation to achievement in statistics.

METHODS AND PROCEDURES

This study was concerned with the mathematical competencies of the students at Iowa State University who enroll in educational statistics 590X. The majority of these students were enrolled in a non-thesis masters degree program. Attention was focused on two areas: (1) The developmental aspect--a test based on arithmetic and algebra I (Appendix A) was developed to assess the mathematical competencies of students who enroll in educational statistics 590X. Self-instructional mathematics materials (Appendix B) were then developed to be used by the students who scored poorly on the mathematics test. The purpose of the self-instructional mathematics materials was to provide a means for students to improve their mathematical competencies through individual study. Attitudinal questionnaires (Appendix C) were developed to determine students' attitudes toward statistics and toward the use and usefulness of the self-instructional mathematics materials. (2) The evaluation aspect--an experimental evaluation was done to determine if the use of the self-instructional mathematics materials had a favorable effect on improving students' mathematical competencies, and on their achievement in educational statistics 590X. An attempt was also made to determine if the use of the self-instructional mathematics materials improved students' attitudes toward statistics.

This chapter describes the methods and procedures that were used in developing the materials, and in gathering and analyzing the required data for the study. The chapter has been divided into three parts: (1) development of the materials, (2) experimental procedure and collection of the data, and (3) treatment of the data.

Development of the Materials

The first and most important aspect in the development of the materials was to determine the mathematical competencies necessary for success in educational statistics 590X. A careful analysis of the textbook used, personal observations and experiences, and discussions with the instructors of the educational statistics 590X classes, resulted in the list of mathematical competencies which it was felt were necessary for success in educational statistics. The mathematical competencies selected were from seventh and eighth-grade arithmetic and ninth-grade algebra. This was in agreement with previous studies, authors of educational statistics textbooks, and instructors of educational statistics classes at other institutions. The topics selected from arithmetic included: (1) symbols and definitions, (2) signed numbers, (3) symbols of grouping, (4) order of arithmetic operations, (5) fractions, (6) decimals, (7) percentage, and (8) significant figures and rounding. The topics selected from algebra I included: (1) summation, (2) absolute value, (3) equations with one unknown, (4) inequalities, (5) exponents, (6) radicals, (7) graphing, (8) solving pairs of equations by addition and subtraction, (9) linear interpolation, (10) factorials, and (11) square root calculation. Although there were additional topics that it was felt could be included, it was recognized that most of the students would be enrolled in other classes as well, and would probably not have available time to spend on additional topics.

Utilizing the list of selected mathematical competencies, the two part mathematics test was developed. The first part was based on arithmetic, and the second part was based on algebra. These were administered to two educational statistics 590X classes during the 1970 spring quarter in an attempt to perfect the instrument. After administering the test, an item analysis was completed on each part in an attempt to determine poor items. Utilization was made of a process described in a booklet published by the student counseling center at Iowa State University (80, p. 6).

> Examination of responses of a class to a given question or set of questions is called an item analysis. From an item analysis, easy questions, difficult questions and non-discriminating questions can be identified. These items can be revised for future use.

The item analysis was divided into three parts, decoy or distractor analysis, difficulty analysis and discrimination analysis.

By examining the option numbers for a given test item, it was possible to determine the plausibility of the distractors. If an option was not chosen by any student, a lack of plausibility was indicated. This impairs the measurement characteristics of the test because of improving the possibility of guessing the correct answer.

A difficulty index was computed for each item by dividing the total number attempting an item into the number who answered it correctly and subtracting that from one. ($1 - \frac{NR}{TA}$ where NR is the number right and TA is the total answering). The item difficulty could range from zero per cent, everyone had the correct answer, to 100 per cent, everyone had the wrong answer.

> Items which are approximately 50% difficult provide the greatest dispersion of test scores. The greater the score dispersion, the greater the reliability. In general, items with difficulty greater than 70% are too hard for the achievement level of the group testing and should be omitted or revised; items with difficulty less than 10% are too easy and should be omitted or revised. (80, p. 7)

"An ideal test is composed of items answered correctly by more high scoring students than low scoring students" (80, p. 7). A discrimination analysis can be used to determine this. If the number of correct responses to a given item made by students in the lower half of the distribution is subtracted from the number of correct responses made by students in the upper half, an indication of discrimination is obtained.

> If the result is positive, the item discriminates against the poor students and should be retained. If the result is negative, the item discriminates against the good students, and should be revised or rejected. (80, p. 7)

As a result of the item analysis, several of the items were replaced and several others were reworded in an attempt to improve them. These administrations were also used in determining satisfactory time limits for the tests. The final forms resulted in a 30 item arithmetic test with a 20 minute time limit, and a 40 item algebra test with a 50 minute limit. New tests were developed rather than using those that had been used in other studies because of the investigator's belief that many of the test items used in other studies did not apply.

The list of mathematical competencies was also used in the development of the self-instructional mathematics materials. One self-instructional unit was developed for arithmetic, and one for algebra. The manner in which the individual topics were presented in the self-instructional units was determined by the investigator's personal teaching experience, an examination of several arithmetic and algebra textbooks, and discussion with the instructors of the educational statistics 590X classes. Each topic was presented in a manner which the investigator felt the individual students would be able to understand without the assistance of an instructor. For each topic, several explanatory examples, and problems for the student to work were provided. A solution key was also included at the end of each self-instructional unit.

Three self-inventories were also developed to be administered by the investigator as the individual students completed designated topics. One self-inventory was provided for use with the arithmetic unit, and the other two were used with the algebra unit. The purposes of these self-inventories were to determine student understanding of the mathematical topics presented, and to ascertain if the students in the experimental group were actually using the self-instructional mathematics materials.

An attitudinal questionnaire was developed to determine the attitudes of students toward statistics. This questionnaire was based on an attitudinal scale used by Aiken and Dreger (1) in a study with college freshmen enrolled in general mathematics at a southeastern college.

> Preliminary investigation using this scale attested to its reliability ($r = 0.94$ for test-retest). In addition a test of independence between the scores on the attitude scale and scores on four items designed to measure attitudes toward academic subjects in general suggested that attitudes specific to mathematics were being measured. (1, p. 20)

Several of the items for the present study were obtained by simply replacing the word "mathematics" by the word "statistics" in items from the scale used by Aiken and Dreger. A tentative list of items, and the type of format to use were determined after consultation with specialists in the counseling center, the statistics department, and others who had had previous experience with attitudinal questionnaires. The scale used was essentially an eleven point Likert scale (63). The list of items was administered to one of the educational statistics 590X classes that were in session the 1970 spring quarter. Recommendations of the students and an examination of their responses, which indicated that items which had been developed to measure specific feelings were consistent in doing so, resulted in the final form of 26 items. There were 13 positive items and 13 negative items. A positive item was a statement expressing a favorable opinion of statistics, while a negative item was a statement expressing an unfavorable opinion of statistics. Integral values from positive five to negative five were assigned to the student responses, with positive five being assigned to a response indicating complete agreement with a positive item or complete disagreement with a negative item. This resulted in a range of possible total values for an individual student from -130 to +130.

A second attitudinal questionnaire was developed to be administered to the students in the experimental group after they completed the self-instructional mathematics units. The purpose of this questionnaire was to determine students' attitudes toward the use and usefulness of the self-instructional mathematics materials. This questionnaire was based on an attitude scale used by Eigen (26). Many of the items used in the present study resulted from replacing the words "automated teaching" by the words "self-instructional materials" in items used by Eigen. Twenty items were selected for use, with two of the items not being used in determining individual student response totals, because they were included in an attempt to determine if the students felt the self-instructional mathematics materials would have been more useful had they been available before the class started. This resulted in nine positive and nine negative items being assigned values. The same method of assignment was used as had been employed with the attitude toward statistics questionnaire. The range of possible total values for an individual student's responses was from -90 to +90.

The choice of the mathematical topics and the method of presentation of these topics were of major concern in the use of this questionnaire.

Experimental Procedure and Collection of the Data

The sample for the study consisted of the students enrolled in the two sections of educational statistics 590X that met the first 1970 summer session. Class sessions were 60 minutes in length. The first class period was devoted to explaining the structure of the class, collecting personal information, and administering the 20 minute arithmetic test. A 60 minute testing period was arranged in addition to the regular class period for the second day of classes in order to administer the 50 minute algebra test.

The results of the arithmetic and algebra tests were then used to determine the experimental and control groups. Students with scores of less than 32 on the algebra test and less than 24 on the arithmetic test, or a combined total score of less than 56, were considered to be in need of "help" in mathematics. In each class section, students with the same, or as close as possible to the same, total scores were paired. Where it was possible, educational background was also taken into consideration in the pairing. The purpose of the pairing was to make the experimental and control groups as similar as possible, and to insure that both groups would contain students representing the entire range of mathematics scores. A table of random numbers was then used to randomly assign one student from each pair to the experimental group with the other student being assigned to the control group. One class section had 27 students with three of the students not requiring "help" in mathematics, while the other class section had 30 students with two of the students not requiring "help" in mathematics. The result was experimental and control groups of 12 students each for the first section, and of 14 students each for the second section, with a total of 26 students in the experimental group and 26 students in the control group.

The self-instructional mathematics materials, along with instructions for their use, were distributed to the students in the experimental group the fourth day of classes. A classroom was provided the investigator to assist students in the experimental group with any problems they encountered in regard to the self-instructional mathematics materials. The students in the experimental group were instructed to work on the materials by themselves, and to seek necessary assistance from the investigator when they encountered difficulty with the materials. No assistance was provided by the investigator in regard to the statistical topics discussed in the statistics classes.

The students in the experimental group were instructed to complete the use of the self-instructional mathematics materials by the end of the fourth week of classes. This included utilizing the three self-inventories which were administered by the investigator. The first self-inventory was administered to individual students when they completed the arithmetic unit, the second was administered when they completed the first six topics of the algebra unit, and the third was administered when they completed the last five topics of the algebra unit.

The students in the experimental group were also provided time sheets to record the amount of time spent on each topic presented in the self-instructional materials. These times were used to determine the mean time spent on each topic. When the students completed use of the materials, they were also asked to complete the attitude questionnaire which had been developed to determine students' attitudes with regard to the use and usefulness of the materials.

Fifteen minutes of the sixth class period were used to administer the attitudinal questionnaire designed to determine attitude toward statistics.

Christ (20) had found in discussing elementary statistics with professors of elementary statistics that one often mentioned skill or ability which seemed to be important in elementary statistics was reasoning power. This led to the decision to use scores obtained on the test, Logical Reasoning, Form A, by A. F. Hertzka and J. P. Guilford (40), as a covariate in the analysis of the data. This test was administered by the investigator during a special testing session the second week of classes.

During the fifth week of classes another special testing session was arranged for the purpose of again administering the arithmetic and algebra tests to determine what changes had occurred in the students' mathematical competencies.

Fifteen minutes of the second last day of classes were devoted to having the students again complete the questionnaire with regard to attitude toward statistics to determine what changes in attitudes had taken place during the five weeks of classes.

The instructors of the two sections of educational statistics 590X made arrangements to discuss the same topics, and to give the same assignments and tests on the same day throughout the duration of the course.

The three tests used to determine achievement in statistics were developed by the instructors of the educational statistics 590X classes, and were administered at the end of the second, fourth, and sixth week of classes. Scores for each test were converted to T-scores with a mean of

50 and a standard deviation of 10.

Treatment of the Data

The primary objectives of the evaluation aspect of this study were to assess the effectiveness of the self-instructional mathematics materials with regard to: (1) increasing the mathematical competencies of the students in educational statistics 590X, as measured by the mean change in students' scores on the mathematics tests from the first administration to the second administration, (2) having a favorable influence on achievement in statistics for the students in the educational statistics 590X classes, as measured by the three tests developed and administered by the instructors of the classes, and (3) effecting a favorable change in students' attitudes toward statistics while enrolled in educational statistics 590X, as measured by the mean change in students' responses on the attitude toward statistics questionnaire from the first administration to the second administration.

To check the first objective an analysis of covariance procedure with equal numbers was employed. The scores obtained on the logical reasoning test were used as the covariate. This procedure was applied to the adjusted means for the experimental and control groups with regard to: (1) mean gain in arithmetic, (2) mean gain in algebra, and (3) mean total gain in mathematics. The sources of variability and the effects isolated in each analysis can be shown by means of the following model:

$$Y_{ij} = \mu + A_i + B(X_{ij} - \overline{X}..) + E_{ij}$$

$$i = 1, 2$$

$$j = 1, 2, \ldots, 26$$

where

Y_{ij} = the appropriate score for the j^{th} student in the i^{th} group.

μ = the overall grand mean.

A_i = the effect of the i^{th} treatment.

B = the regression coefficient of Y on X.

X_{ij} = the score on the logical reasoning test for the j^{th} person in the i^{th} group.

$\overline{X}..$ = the overall mean for the logical reasoning test.

E_{ij} = the residual associated with the j^{th} student in the i^{th} group.

The hypothesis associated with this portion of the study was:

There is no significant difference in mathematical competencies (as measured by mean gain in arithmetic, mean gain in algebra, and mean total gain in mathematics) of experimental and control groups when the experimental group has used self-instructional mathematics materials and the control group has not, and initial differences in logical reasoning have been controlled.

To check the second objective an analysis of covariance procedure with equal numbers utilizing a factorial design with repeated measures was employed. Scores obtained on the logical reasoning test were again used as the covariate. The three tests given in the statistics class were used as the repeated measures. The sources of variability and the effects isolated in the analysis can be shown by means of the following model:

$$Y_{ijk} = \mu + A_i + C_{ij} + D_k + AD_{ik} + B(X_{ijk} - \overline{X}...) + E_{ijk}$$

$i = 1, 2$

$j = 1, 2, \ldots, 26$

$k = 1, 2, 3$

where

Y_{ijk} = the k^{th} test score for the j^{th} person in the i^{th} group.

μ = the overall grand mean.

A_i = the effect of the i^{th} treatment.

C_{ij} = the effect of the j^{th} person within the i^{th} group.

D_k = the effect of the k^{th} statistics test.

AD_{ik} = the interaction of the k^{th} test with the i^{th} treatment group.

B = the regression coefficient of Y on X.

X_{ijk} = the score on the logical reasoning test for the k^{th} test for the j^{th} person in the i^{th} group.

$\overline{X}...$ = the overall mean for the logical reasoning test.

E_{ijk} = residual associated with the k^{th} test for the j^{th} person in the i^{th} group.

The hypotheses associated with this portion of the study were:

1. There is no significant difference in achievement in educational statistics (as measured by the overall means on the three statistics tests) of experimental and control groups when the experimental group has used self-instructional mathematics materials and the control group has not, and initial differences in logical reasoning have been controlled.
2. There is no interaction between group membership and group performance on the three statistics tests.

Using the assumption that if no change was made in mathematical competencies there would be no difference in statistics achievement, an analysis was set up to investigate the second objective using an experimental group composed of those students in the original experimental group who increased their mathematics total at least 10 points, and a control group composed of those students who had been matched with the students in the experimental group in the original balancing of the groups. The result was experimental and control groups of 18 students each. The same tests and procedures were used as had been used in checking the objective with all 26 students in each group.

To check the third objective an analysis of variance procedure with equal numbers was used. This procedure was applied to the means for the experimental and control groups with regard to mean change in total attitude scores. The following model can be used to show the analysis and its source of variation:

$$Y_{ij} = \mu + A_i + E_{ij}$$

$$i = 1, 2$$

$$j = 1, 2, \ldots, 26$$

where

Y_{ij} = total change in attitude for the j^{th} student in the i^{th} group.

μ = overall grand mean.

A_i = the effect of the i^{th} treatment.

E_{ij} = the residual associated with the j^{th} student in the i^{th} group.

The hypothesis associated with this portion of the study was:

There is no significant difference in attitude toward educational statistics (as measured by mean change in attitude totals) of experimental and control groups when the experimental group has used self-instructional mathematics materials and the control group has not.

Student responses to the attitude toward statistics questionnaire were also used to determine mean responses to items by the experimental and control groups, and also to determine mean change in responses by the groups.

In regard to the relationship between previous mathematics training and achievement in statistics, it was desired to ascertain whether or not there was a difference in statistics achievement between those students who had three or more years of high school mathematics and those who had less than three years of high school mathematics. Achievement in this case was determined by adding the three T-scores obtained on the statistics tests for each individual student. Since the students in the experimental group received a treatment in mathematics, only the scores for the 26 students in the control group were used, resulting in groups of 12 and 14. An analysis of variance procedure with unequal n was used in this investigation.

The hypothesis associated with this question was:

> There is no significant difference in achievement in educational statistics (as measured by mean totals of the three statistics test scores) of students who have had three or more years of high school mathematics from those who have had less than three years of high school mathematics.

Responses by the students in the experimental group to the attitude questionnaire designed to ascertain how students felt about the use and usefulness of the self-instructional mathematics materials were used to find mean responses to each of the items. The first 18 items were then used to determine overall attitude. The last two items were for the purpose of determining whether or not the students were of the opinion that the materials would have been more beneficial had they been used before the class started.

The time sheets that the students in the experimental group used to record the amount of time they had spent on each topic in the self-instructional mathematics materials were used to calculate the mean time spent on each topic, and to determine the mean time spent on the units.

A correlation matrix was used to determine the degree of relationship between several different combinations of the variables. Of particular interest were the following combinations: (1) first arithmetic score with first algebra score, (2) first mathematics total with logical reasoning, (3) first mathematics total with achievement, (4) logical reasoning with achievement, and (5) first attitude total with achievement. For these combinations, achievement for each student was determined by the sum of the T-scores on the three statistics tests.

Several combinations of the variables were also used to obtain regression equations to determine which combination of variables could be used to account for the largest percentage of variance in regard to achievement.

FINDINGS

The balancing that was done to equate the experimental and control groups in terms of mathematical competencies, and to insure that the entire range of mathematics scores was represented in each group, was successful. The sum of the mathematics scores for all students in each group was 660. The range of scores in the experimental group was from 7 to 46, while the range in the control group was from 9 to 48.

Several null hypotheses were set forth to be tested in attempting to assess the effectiveness of the self-instructional mathematics materials. The hypotheses were stated in accordance with the purposes of the study as reported previously. The findings give evidence to test the null hypotheses. Other findings that provided meaning to the study were also presented.

Null hypothesis number 1: There is no significant difference in mathematical competencies (as measured by mean gain in arithmetic, mean gain in algebra, and mean total gain in mathematics) of experimental and control groups when the experimental group has used self-instructional mathematics materials and the control has not, and initial differences in logical reasoning have been controlled.

The unadjusted and adjusted mean gains for arithmetic are presented in Table 1. The difference between the experimental and control groups increased from 2.19 to 2.27 in favor of the experimental group when logical reasoning was used as a covariate. An analysis of covariance was computed on these mean gains and is reported in Table 2. A significant F-value of 5.87 was obtained and the null hypothesis was rejected. The mean gain in

Table 1. Unadjusted and adjusted mean gains for arithmetic when logical reasoning was used as a covariate

Group	Mean gain	
	Unadjusted	Adjusted
Experimental	4.77	4.81
Control	2.58	2.54
Difference	2.19	2.27

Table 2. Analysis of covariance for mean gain in arithmetic when logical reasoning was used as a covariate

Source of variation	Residuals		
	d.f.	S.S.	M.S.
Total	50	614.90	
Within subgroups	49	549.09	11.21
Difference	1	65.81	65.81

$$F_{1,49} = \frac{65.81}{11.21} = 5.87^{*}$$

*Significant beyond the five per cent level.

arithmetic for the experimental group was significantly different from that of the control group when logical reasoning was used as a covariate.

Table 3 contains the unadjusted and adjusted mean gains in algebra for the experimental and control groups. The difference between the experimental and control groups increased from 4.65 to 5.21 in favor of the ex-

Table 3. Unadjusted and adjusted mean gains in algebra when logical reasoning was used as a covariate

Group	Mean gain	
	Unadjusted	Adjusted
Experimental	8.27	8.55
Control	3.62	3.34
Difference	4.65	5.21

perimental group when logical reasoning was used as a covariate. A highly significant F-value of 15.40 was obtained on the analysis of covariance reported in Table 4. The null hypothesis was rejected. There were significant differences in mean gain in algebra for the experimental and control groups when logical reasoning has been controlled.

Data in Table 5 present the unadjusted and adjusted mean gains in the

Table 4. Analysis of covariance for mean gain in algebra when logical reasoning was used as a covariate

Source of variation	Residuals		
	d.f.	S.S.	M.S.
Total	50	1443.42	
Within subgroups	49	1099.01	22.43
Difference	1	344.41	344.41

$$F_{1,49} = \frac{344.41}{22.43} = 15.40^{**}$$

**Significant beyond the one per cent level.

Table 5. Unadjusted and adjusted mean gains in mathematics total when logical reasoning was used as a covariate

Group	Mean gain	
	Unadjusted	Adjusted
Experimental	13.04	13.36
Control	6.20	5.88
Difference	6.84	7.48

mathematics total for the experimental and control groups. When the mean gains were adjusted, it was revealed that the experimental mean gain exceeded that of the control mean gain by 7.48. The experimental and control mean gains were then analyzed using the analysis of covariance and is reported in Table 6. A highly significant F-value of 20.30 was obtained and the null hypothesis was rejected. There is a significant difference in mathematics mean gain for the experimental and control groups when logical reasoning is used as a covariate.

The reliabilities of the measuring instruments used in obtaining the scores utilized in the analysis of null hypothesis 1 were determined by the Kuder-Richardson formula 20 (63). These were as follows: (1) logical reasoning--0.87, (2) first application of the arithmetic test--0.84, (3) first application of the algebra test--0.88, and (4) first combination of arithmetic and algebra--0.92.

Null hypothesis number 2: There is no significant difference in achievement in educational statistics (as measured by the overall group means on the three statistics tests) of experimental and control groups

Table 6. Analysis of covariance of mean gain in mathematics total when logical reasoning was used as a covariate

Source of variation	Residuals		
	d.f.	S.S.	M.S.
Total	50	2425.47	
Within subgroups	49	1714.13	34.98
Difference	1	711.34	711.34

$$F_{1,49} = \frac{711.34}{34.98} = 20.30^{**}$$

**Significant beyond the one per cent level.

when the experimental group has used self-instructional mathematics materials and the control group has not, and initial differences in logical reasoning have been controlled.

Null hypothesis number 3: There is no interaction between group membership and group performance on the three statistics tests.

The two previous hypotheses were tested using an analysis of covariance procedure utilizing a factorial design with repeated measures. The three tests given in the statistics class were used as the repeated measures. Table 7 presents the unadjusted and adjusted overall group means of the three statistics tests. The difference in the adjusted means reveals a 1.71 point difference in favor of the experimental group. Table 8 was used to present the experimental and control group means on the individual statistics tests. The difference in group means for the first test was 2.60 in favor of the control group. For the second test the difference was

Table 7. Unadjusted and adjusted overall achievement means when logical reasoning was used as a covariate

Group	Overall means	
	Unadjusted	Adjusted
Experimental	48.80	49.59
Control	48.67	47.88
Difference	0.13	1.71

Table 8. Means for the experimental and control groups for the three statistics tests

Test	Experimental	Control	Difference
Test I	47.59	50.19	2.60
Test II	48.85	48.78	0.07
Test III	49.95	47.04	2.91

0.07 in favor of the experimental group, and for the third test the difference was 2.91 in favor of the experimental group. The analysis for null hypotheses 2 and 3 is presented in Table 9. An adjustment was made in the main-plot analysis due to logical reasoning, however, an adjustment is not made in the sub-plot analysis due to the fact that the covariate measure is constant for all criterion measures on the same subject. Two separate error terms were used in the analysis as suggested by Winer (87). Null hypothesis number 2 was not rejected when a non-significant F-value of 0.83

Table 9. Analysis of covariance of achievement using the three statistics test scores as repeated measures in a factorial design with logical reasoning used as a covariate

Source of variation	Residuals			
	d.f.	S.S.	M.S.	F
Groups	1	107.21	107.21	0.83
Error (a)	49	6303.94	128.65	
Tests	2	4.62	2.31	
Groups x tests	2	197.24	98.62	2.82
Error (b)	100	3495.10	34.95	

was obtained. There was no difference in group achievement means when logical reasoning was used as a covariate. Null hypothesis number 3 was also not rejected when a non-significant F-value of 2.82 was obtained. There was no interaction between group membership and group performance on the three statistics tests.

Employing the assumption that if no change was made in mathematical competencies there would be no difference in statistics achievement, an analysis was set up to test null hypotheses two and three using an experimental group composed of those students in the original experimental group who increased their mathematics total at least 10 points, and a control group composed of those students who had been matched with the students in the experimental group in the original balancing of the groups. The same method of analysis was used as had been used in testing the hypotheses with the full groups. Table 10 presents the unadjusted and adjusted overall

Table 10. Unadjusted and adjusted overall achievement means for the reduced groups when logical reasoning was used as a covariate

Group	Overall means	
	Unadjusted	Adjusted
Experimental	52.03	51.84
Control	48.94	49.13
Difference	3.09	2.71

group means on the three tests for the reduced groups. Using logical reasoning as a covariate decreases the difference in means from 3.09 to 2.71 in favor of the experimental group. The experimental and control group means on the three statistics tests for the reduced groups are presented in Table 11. The difference in reduced group means for the first test was 0.33, for the second test was 2.55, and for the third test was 6.35, with all of the differences in favor of the reduced experimental group. The null hypotheses were analyzed as presented in Table 12. Neither null

Table 11. Means for the reduced experimental and control groups for the three statistics tests

Test	Experimental	Control	Difference
Test I	50.70	50.37	0.33
Test II	52.16	49.61	2.55
Test III	53.21	46.86	6.35

Table 12. Analysis of covariance of achievement for the reduced groups using the three statistics test scores as repeated measures in a factorial design with logical reasoning used as a covariate

Source of variation	Residuals			
	d.f.	S.S.	M.S.	F
Groups	1	191.40	191.40	1.79
Error (a)	33	3520.47	106.68	
Tests	2	13.16	6.58	
Groups x tests	2	166.94	83.47	2.53
Error (b)	68	2235.37	32.87	

hypothesis was rejected. There was no significant difference in achievement means for the reduced groups when logical reasoning was used as a covariate. A non-significant F-value of 2.53 was found for null hypothesis number 3. There was no interaction between group membership and group performance for the reduced groups on the three statistics tests.

Null hypothesis number 4: There is no significant difference in attitude toward educational statistics (as measured by change in attitude totals) of experimental and control groups when the experimental group has used self-instructional mathematics materials and the control group has not.

The experimental group had a decrease of 175 points in their attitude total while the control group had a decrease of 159 points. This change was for 26 students on 26 items, resulting in an overall decrease of 0.26 per student per item for the experimental group, and an overall decrease of

0.24 per student per item for the control group. In Table 13 the experimental and control totals for the first and second applications of the attitude questionnaire were presented. The overall student per item responses and changes were also included. The analysis of variance on the change in attitude totals was presented in Table 14. The null hypothesis

Table 13. Experimental and control group totals for the two applications of the attitudinal questionnaire

Group	1st application total	Student average per item	2nd application total	Student average per item	Average change per item
Experimental	-245	-0.36	-420	-0.62	-0.26
Control	-158	-0.23	-317	-0.47	-0.24

was not rejected. A non-significant F-value of 0.00 was found. There is no difference in attitude toward educational statistics for the experimental and control groups.

The student responses to the measuring instrument used in the analysis of null hypothesis number 4 were also used to determine individual mean

Table 14. Analysis of variance for change in attitude totals

Source of variation	d.f.	S.S.	M.S.	F
Groups	1	4.92	4.92	0.00
Within	50	108,429.77	2168.60	

responses to individual items by the experimental and control groups. These are reported in Table 15 along with change in response means.

The investigator's interest in the relationship between previous mathematics training and achievement in statistics led to the test of the following null hypothesis.

Null hypothesis number 5: There is no significant difference in achievement in educational statistics (as measured by mean totals of the three statistics test scores) of students who have had three or more years of high school mathematics from those who have had less than three years of high school mathematics.

Due to the fact that the students in the experimental group received a treatment in mathematics, only the scores for the 26 students in the control group were used for the analysis. The students who had three or more years of high school mathematics had a mean total of 157.67, while the students who had less than three years of high school mathematics had a mean total of 136.14. The analysis of variance is presented in Table 16. A highly significant F-value of 8.79 was found and the null hypothesis was rejected. There is a significant difference in achievement in educational statistics dependent upon previous training in mathematics.

The mean responses of the students in the experimental group to items in the attitude questionnaire designed to ascertain how students felt about the use and usefulness of the self-instructional mathematics materials are presented in Table 17. The overall mean response per item considering items one through eighteen was 1.93.

The mean times that the students in the experimental group spent on each topic in the self-instructional mathematics materials were reported in

Table 15. Group means for individual students on individual items for attitude toward statistics[a]

Items	Experimental			Control		
	1st	2nd	Change	1st	2nd	Change
1. It scares me to have to take statistics	-1.69	-0.92	0.77	-2.50	-1.46	1.04
2. The feeling I have toward statistics is a good feeling	-1.57	-1.46	0.11	-1.31	-1.58	-0.27
3. Statistics can be made understandable to almost every college student	1.54	1.96	0.42	1.58	1.77	0.19
4. I can't see where statistics will ever help me	2.23	2.12	-0.11	2.88	2.42	-0.46
5. I don't think I can ever do well in statistics	0.62	0.50	-0.12	0.58	0.27	-0.31
6. Only people with a very special talent can learn statistics	1.96	1.88	-0.08	2.12	1.88	-0.24
7. Statistics is fascinating and fun	-0.50	-1.50	-1.00	-0.19	-1.27	-1.08
8. I feel a sense of insecurity when attempting statistics	-2.31	-2.96	-0.65	-1.62	-3.15	-1.53
9. I feel at ease in statistics	-2.42	-3.04	-0.62	-1.81	-2.54	-0.73

[a]Items 2, 3, 7, 9, 10, 13, 14, 17, 19, 22, 23, 25, and 26 were scored as positive items; items 1, 4, 5, 6, 8, 11, 12, 15, 16, 18, 20, 21, and 24 were scored as negative items.

Table 15 (Continued)

Items	Experimental			Control		
	1st	2nd	Change	1st	2nd	Change
10. Statistics is something which I enjoy a great deal	-1.65	-2.64	-0.99	-2.00	-2.42	-0.42
11. When I hear the word statistics I have a feeling of dislike	0.58	-0.65	-1.23	-0.08	-0.46	-0.38
12. I do not like statistics	0.46	-0.27	-0.73	0.12	-0.31	-0.43
13. Statistics makes me feel secure	-2.88	-3.62	-0.74	-2.38	-2.42	-0.04
14. Statistics is stimulating	0.73	1.54	0.81	1.23	0.92	-0.31
15. Very few people can learn statistics	2.65	2.69	0.04	2.62	2.88	0.26
16. It makes me nervous to even think about having to do a statistics problem	-0.69	-0.88	-0.19	-0.23	-1.04	-0.81
17. Statistics is enjoyable	-0.85	-1.62	-0.77	-1.00	-1.62	-0.62
18. I approach statistics with a feeling of hesitation--hesitation from a fear of not being able to do statistics	-1.81	-2.38	-0.57	-2.31	-2.00	0.31
19. I really like statistics	-0.85	-1.19	-0.34	-1.04	-1.69	-0.65
20. I wish I were not required to study any statistics	-0.81	-0.23	0.58	-0.81	-0.46	0.35

Table 15 (Continued)

Items	Experimental			Control		
	1st	2nd	Change	1st	2nd	Change
21. Statistics makes me feel uncomfortable, restless, irritable and impatient	-1.08	-1.77	-0.69	-0.88	-1.35	-0.47
22. Any person of average intelligence can learn to understand a good deal of statistics	1.85	1.96	0.11	2.31	2.16	-0.15
23. I would like to study more statistics whether or not it is required for my program	-2.88	-3.27	-0.39	-1.58	-2.42	-0.84
24. Statistics makes me feel as though I'm lost in a jungle of numbers and can't find my way out	-1.65	-1.38	0.27	-1.19	-0.23	0.96
25. Almost anyone can learn statistics if he is willing to study	1.46	1.42	-0.04	1.23	1.69	0.46
26. Statistics is very interesting to me	0.15	-0.62	-0.77	0.19	0.27	0.08

Table 16. Analysis of variance of achievement comparing students with three or more years of high school mathematics with those with less than three years

Source of variation	d.f.	S.S.	M.S.	F
Groups	1	2,993	2,993.00	8.79**
Within	24	8,173	340.54	
Total	25	11,166		

**Significant beyond the one per cent level.

Table 18. Individual total times for set I varied from 33 minutes to 345 minutes, while individual total times for set II varied from 120 minutes to 590 minutes.

Table 19 contains the coefficients of correlation for the several combinations of variables which were of interest to this study. Scores for all 57 of the students in the two sections of educational statistics 590X were used in determining the coefficients of correlation.

The regression equation which proved to be the most meaningful for this study in regard to achievement in statistics was the one in which achievement was regressed on the first mathematics total and logical reasoning. Achievement in this instance was defined to be the sum of the T-scores for the three statistics tests. This combination of variables yielded a multiple R^2 of 0.57, indicating that this combination was able to account for 57 per cent of the variance in regard to achievement. The resulting equation took the form:

Table 17. Mean responses for attitude toward use and usefulness of the self-instructional mathematics materials

Item	Mean response
1. The use of the self-instructional mathematics materials helped me to learn the mathematics I need for statistics.	3.00
2. Probably the best way to learn the mathematics I need for statistics is with a teacher.	-0.40
3. With the self-instructional mathematics materials I know exactly how I am doing all the time.	0.36
4. There is no thinking involved in learning with the self-instructional mathematics materials.	4.64
5. The self-instructional mathematics materials are a boring method of learning.	3.16
6. There is no pressure on me when I use the self-instructional mathematics materials.	1.68
7. The self-instructional mathematics materials offered no challenge to me.	3.16
8. The use of the self-instructional mathematics materials has helped to reduce my anxiety with regard to statistics.	0.16
9. The self-instructional mathematics materials did not provide a clear explanation of the topics presented.	2.40
10. The mathematical topics presented in the self-instructional mathematics materials were not sufficient to enable me to do the work in statistics.	1.94
11. With the use of the self-instructional mathematics materials, I never get left behind the class.	-1.16
12. The level of difficulty of the self-instructional mathematics materials was appropriate.	3.20
13. The self-instructional mathematics materials are more trouble than they are worth.	3.36

Table 17 (Continued)

Item	Mean response
14. The use of the self-instructional mathematics materials helped to reduce the amount of time required to do my statistics assignments.	-1.14
15. With the self-instructional mathematics materials, good students are not held back by the class.	1.68
16. The time I spent using the self-instructional mathematics materials would have been better spent studying statistics.	1.94
17. The use of the self-instructional mathematics materials is a good way to learn.	3.36
18. The self-instructional mathematics materials do not contain things that I can use in statistics.	3.40
19. The self-instructional mathematics materials would have been more useful if I could have used them before I started statistics.	3.44
20. The self-instructional mathematics materials would have reduced my anxiety with regard to statistics if I could have used them before I started statistics.	1.84

$$\hat{Y} = 86.77 + 0.85\, X_1 + 1.53\, X_2$$

where

$\hat{Y}$ = predicted total for the three statistics tests

X_1 = first mathematics total

X_2 = logical reasoning score.

Table 18. Reported mean time spent on the self-instructional mathematics materials (in minutes)

Set I	
15.2	1. Symbols and definitions
16.7	2. Signed numbers
20.4	3. Symbols of grouping
23.0	4. Order of arithmetic operations
27.8	5. Fractions
16.6	6. Decimals
14.4	7. Percentage
13.3	8. Significant figures and rounding
147.4	
Set II	
31.6	1. Summation
19.3	2. Absolute value
37.2	3. Equations with one unknown
23.2	4. Inequalities
27.2	5. Exponents
37.1	6. Radicals
28.4	7. Graphing
28.0	8. Solving pairs of equations by addition and subtraction
21.0	9. Linear interpolation
13.5	10. Factorials
19.9	11. Square root calculation
286.4	
433.8	Mean time for set I + set II

Table 19. Coefficients of correlation considering all 57 students

Combination	r
1. First arithmetic score with first algebra score	0.77
2. First mathematics total with logical reasoning	0.53
3. First mathematics total with achievement in statistics	0.67
4. Logical reasoning with achievement in statistics	0.66
5. First attitude total with achievement in statistics	0.20

DISCUSSION

The students in the experimental group were instructed to seek necessary assistance in regard to the self-instructional mathematics materials from the investigator. Very little assistance was requested, and the results on the self-inventories indicated that some of the students in the experimental group may not have been spending as much time on the self-instructional mathematics materials as they reported. Some of the students were unable to answer questions that even a very brief inspection of the materials should have allowed them to answer. The students were aware that what they did with the materials would have no effect on their grade in statistics, and consequently some of them may have felt they were being imposed on. Although the difference in mean gain in mathematical competencies for the two groups was significantly different, there might have been an even larger difference had all the students in the experimental group used the materials as directed.

The largest gains on the arithmetic unit were in the areas of fractions, symbols of grouping, and order of operations. The largest gains on the algebra unit were in the areas of graphing, radicals, summation, linear interpolation and absolute value. Greater increases may have occurred in these areas because students may have been able to see a direct need in relation to their work in statistics.

The reliabilities of the measuring instruments for the second application as determined by the Kuder-Richardson formula 20 were as follows: arithmetic test--0.86, algebra test--0.89, and mathematics total--0.93. The slight increases in reliabilities over those found for the first application (0.84 for arithmetic, 0.88 for algebra, and 0.92 for mathematics total) served to indicate the consistency of the instruments.

Although there was no significant difference in achievement in statistics for the two groups, an inspection of the three test means for the reduced groups when those students in the experimental group who increased their mathematics total less than 10 points were eliminated, along with the students in the control group with whom they had been paired, would seem to indicate that the students in the reduced experimental group may have been starting to take advantage of their increased mathematical competencies. The differences in means for the three tests were all in favor of the reduced experimental group and increased from 0.33 on the first test, to 2.55 on the second test, and to 6.35 on the third test.

If the students' responses to the attitude toward statistics questionnaire were a true indication of their feelings, it would seem that the objective of improving students' attitudes toward educational statistics is not being met in the educational statistics 590X classes. One of the few positive changes in attitude was a 0.91 increase in responses to the item "it scares me to have to take statistics." The overall change per student per item (-0.25) was insignificant, other than to indicate that attitudes toward statistics were not improved.

The results in testing the significance of previous mathematics training in regard to achievement in statistics were in agreement with the results found in other studies. There was a highly significant difference in achievement depending upon whether the student had had three or more years or less than three years of high school mathematics. This would indicate that considerable attention should be shown a student's previous mathematics training when he enrolls in an educational statistics class.

As a group, the students in the experimental group felt the self-instructional mathematics materials (1) contained things they could use in statistics, (2) presented the topics in an understandable, appropriate manner, (3) were worth the time spent on them, (4) did not reduce their anxiety toward statistics, and (5) would have been more useful had they been available before the class started.

The coefficient of correlation found for first attitude total with achievement in statistics ($r = 0.20$) was similar to that found in other studies. How students felt about educational statistics did not have a large influence on their achievement in the class. The highest coefficient of correlation found was for the first mathematics total with achievement in statistics ($r = 0.67$).

The multiple R for the first mathematics total and logical reasoning with achievement in statistics ($r = 0.75$), indicated that the two variables in combination would allow for a fairly good prediction of achievement in educational statistics 590X.

The following recommendations for further study are made:

1. Replicate the study just completed. This would give the opportunity for comparison of findings and would provide further validation of the data.
2. Duplicate the study when the educational statistics 590X class is in session a full quarter rather than during a 5½ week summer session. This would provide an opportunity to investigate whether or not the additional time that would be available for students to take advantage of an increase in mathematical competencies would

be beneficial in regard to achievement in educational statistics.

3. Duplicate the study when it is possible to have the students use the self-instructional mathematics materials before the class starts. This would necessitate having the students available about six weeks before the class meets for the first time in order to provide time for testing and for the experimental group to complete use of the materials.
4. Complete another study where a record of the mathematical competencies actually used in educational statistics 590X could be maintained. How often these competencies are used would also be of interest.
5. Additional ways might be found to check and improve the reliability and validity of the tests and questionnaires used in the study.

The following recommendations for practice are made:

1. For those students who enroll in educational statistics 590X, the arithmetic and algebra tests used in this study, or some other suitable mathematics test, should be used to determine who needs "help" in mathematics.
2. Since the results of this and previous studies indicate that mathematics background is important to success in educational statistics, some attempt should be made to provide "help" in mathematics for those students who require it. This could consist of providing these students with the materials developed for this study or some other suitable self-instructional materials. If students are allowed to enroll in educational statistics 590X without pos-

sessing the necessary mathematical competencies, and no attempt is made to provide these students with the necessary competencies, they have been done a disservice; and the instructor of the class is presented with the unenviable task of trying to teach mathematics and statistics at the same time.

3. If the attitude questionnaire used in this study to determine students' attitudes toward statistics accurately reflected how the students felt about statistics, ways and means should be found to improve students' attitudes toward statistics. This should be one of the purposes of educational statistics 590X.

SUMMARY

The two general purposes of this study were:

1. Developmental--to develop an instrument to assess the mathematical competencies of students who enroll in educational statistics 590X, and then to develop some means of self-instruction for students who need "help" in mathematics to enable them to increase their mathematical competencies.
2. Evaluation--to determine the effect of the self-instructional mathematics materials with regard to improving students' mathematical competencies, achievement in educational statistics, and attitude toward statistics.

The Development Aspect

The mathematical competencies that were of interest were those that the investigator identified as being necessary for success in educational statistics 590X. These competencies were based on seventh and eighth grade arithmetic, and ninth grade algebra. This was in agreement with specialists in the field of educational statistics.

The list of mathematical competencies was utilized in the development of the instrument used to assess the mathematical competencies of the students, and in the development of the self-instructional mathematics materials. The method of assessment of the mathematical competencies of the students consisted of a 30 item arithmetic test with a 20 minute time limit, and a 40 item algebra test with a 50 minute time limit. The time limits and final items were arrived at after first administering the tests to the educational statistics 590X classes that were in session the 1970 spring quarter, and then performing an item analysis on each test in an effort to perfect the instruments.

The self-instructional mathematics materials consisted of two units: one for arithmetic and one for algebra. The topics in the units were presented in a manner which would require a minimum of assistance for understanding. Examples, problems, and a solution key were provided for each topic presented. Three self-inventories were included to be used to determine individual student understanding of the topics presented. These were administered by the investigator to individual students after they had completed designated topics.

The attitude questionnaire that was developed to determine students' attitudes toward statistics was based on an attitudinal scale used by Aiken and Dreger (1) in their work. The items and type of format used were determined after consultation with specialists in the counseling center, the statistics department, and others who had had previous experience with attitudinal questionnaires. Thirteen positive items and thirteen negative items were employed. The scale utilized was essentially an eleven point Likert scale.

The second attitudinal questionnaire that was developed was administered to the students in the experimental group after they completed the self-instructional mathematics units. The purpose of this questionnaire was to determine students' attitudes toward the use and usefulness of the materials. Nine positive items and nine negative items were used in determining individual totals. Two additional items were included to determine if the students felt the materials would have been more useful had they been available before the class started.

The Evaluation Aspect

The two sections of educational statistics 590X that met the first session of the 1970 summer quarter had a total enrollment of 57 students. Fifty-two of these students had scores of less than 32 on the algebra test and less than 24 on the arithmetic test, or a combined total of less than 56 on the two tests. These students were considered to be in need of "help" in mathematics. Students in each section with the same scores, or as close as possible to the same scores, were paired. One of each pair was then randomly assigned to the experimental group with the other member of each pair being assigned to the control group. This resulted in experimental and control groups each containing 26 students. The purpose of the pairing was to equate the groups in regard to mathematical competencies, and to insure that the entire range of mathematics scores was represented in each group.

Special sessions were arranged for testing and for the distribution of the materials to insure that a minimum of class time was used in regard to the study. The students in the experimental group were instructed to complete the self-instructional mathematics materials and to utilize the three self-inventories by the end of the fourth week of classes. Necessary assistance in regard to the self-instructional mathematics materials was provided by the investigator.

The attitude toward statistics questionnaire was administered the sixth day of classes, and again the second last day of classes. A special session was utilized the second week of classes to obtain scores on the logical reasoning test. These scores were used as a covariate in the analysis of the data. Another special session was arranged the fifth week of classes for the second administration of the arithmetic and algebra tests.

The students in the experimental group recorded the amount of time they spent on each topic in the self-instructional mathematics materials, and when they completed the materials, they were asked to respond to the attitudinal questionnaire which was designed to determine students attitudes toward the use and usefulness of the materials.

The three tests which were used to determine achievement in statistics were developed by the instructors of the course, and were administered at the end of the second, fourth, and sixth week of classes. The instructors of the two sections of educational statistics 590X discussed the same topics and gave the same assignments and tests on the same day throughout the duration of the course.

The results of the analysis of covariance procedure that was used to test the null hypothesis in regard to mean gain in mathematical competencies revealed that the mean gains in arithmetic, algebra, and mathematics total were significantly higher for the group that had used the self-instructional mathematics materials when differences in logical reasoning were controlled.

In checking achievement in statistics for the two groups, it was found that there was no difference in achievement when logical reasoning was used as a covariate. There was no significant difference in the means of the two groups when considering the averages for the three tests, and there was no group by test interaction.

Employing the assumption that if no change was made in mathematical competencies there would be no difference in statistics achievement, an analysis on achievement was performed using an experimental group composed of those students in the original experimental group who increased their mathematics totals at least 10 points. The control group was made up of those students who had been paired with the students in the experimental group in the original balancing of the groups. The analysis using logical reasoning as a covariate revealed that there was no significant difference in the means of the two reduced groups when considering the averages for the three tests, and there was no group by test interaction.

An analysis of variance procedure was used to determine if there was a difference between the two groups in regard to mean change in attitude totals from the first administration to the second administration of the attitude toward statistics questionnaire. The result of the analysis was that there was no difference in the mean change in attitudes of the two groups. The means per item for the two groups on the two administrations of the questionnaire disclosed that both groups were slightly negative in their attitude toward statistics when the course started, and at the conclusion of the course there had been a slight decrease in the means per item of their original attitudes (on a scale of -5 to +5, the mean per item for the experimental group decreased from -0.36 to -0.62, while the mean per item for the control group decreased from -0.23 to -0.47).

An investigation of the relationship between previous mathematics training and achievement in statistics was performed utilizing the students in the control group. The results of an analysis of variance on total achievement revealed that those students who had three or more years of high school mathematics achieved significantly more in statistics (beyond the one per cent level) than did those students who had less than three years of high school mathematics.

Responses by the students in the experimental group to the attitude questionnaire designed to ascertain how students felt about the use and usefulness of the self-instructional mathematics materials revealed an overall mean per item of 1.93 (on the scale of -5 to +5). Four of the 26 students had negative totals on the questionnaire. The mean on the item that was included to determine whether or not the students felt the materials would have been more useful had they been available before the class started was 3.44.

The times recorded by the students in the experimental group as having been spent on the self-instructional mathematics materials disclosed a mean total time of 433.8 minutes for the two sets. The mean reported time for the first set was 147.4 minutes, and for the second set was 286.4 minutes.

The coefficients of correlation that were of particular interest were: (1) first mathematics total with logical reasoning--$r = 0.53$, (2) first mathematics total with achievement--$r = 0.67$, (3) logical reasoning with achievement--$r = 0.66$, and (4) first attitude total with achievement--$r = 0.20$.

A multiple regression equation in which achievement was regressed on the first mathematics totals and logical reasoning scores exhibited a multiple R of 0.75. This would indicate that these two variables in combination account for 57 per cent of the variance in regard to achievement.

Conclusions

If the assumption was made that the results on the tests and questionnaires that were used in the study accurately reflected the abilities and attitudes of the students in the educational statistics 590X classes, then the following conclusions could be drawn:

1. While enrolled in educational statistics 590X, students who used the self-instructional mathematics materials developed for this study increased their mathematical competencies significantly more than did those students who did not use the materials. These mathematical competencies were in the areas of arithmetic and algebra.

2. Those students who used the self-instructional mathematics materials did not achieve significantly more in educational statistics than did those students who did not use the materials when logical reasoning was used as a covariate. The adjusted mean achievement score (when considering T-scores with a mean of 50 and a standard deviation of 10) for the experimental group was 1.71 points higher than the adjusted mean for the control group. There was no interaction between group membership and group performance on the three statistics tests.

3. The use of the self-instructional mathematics materials did not cause a favorable change in students' attitudes toward educational statistics. Both experimental and control groups were slightly negative in their attitudes toward educational statistics when the class started, and a slight decrease in mean attitude per item was noted for both groups at the conclusion of the course.

4. An analysis of variance procedure utilizing the students in the control group disclosed that the students who had three or more years of high school mathematics achieved significantly more in educational statistics 590X than did those students who had less

than three years of high school mathematics. The mean total on the three statistics tests for the students who had three or more years of high school mathematics was 157.67, while for the students who had less than three years it was 136.14. This would indicate that previous mathematics training is important in regard to achievement in educational statistics.

5. The responses of the students in the experimental group to the questionnaire designed to determine students' attitudes toward the use and usefulness of the self-instructional mathematics materials revealed that as a group the students felt that the topics were clearly and appropriately presented, the topics presented were sufficient to do the work in statistics, and that the materials were worth the time spent on them. However, the group as a whole strongly agreed that the materials would have been more useful had they been available for use before the class started.

6. The coefficients of correlation that were determined indicated that there is a high degree of relationship between mathematics ability (in terms of arithmetic and algebra) and achievement in educational statistics ($r = 0.67$), and between logical reasoning and achievement in statistics ($r = 0.66$). Only about four per cent of the variability in achievement in educational statistics 590X can be accounted for by attitude toward statistics ($r = 0.20$).

7. The first mathematics total and logical reasoning in combination were able to account for 57 per cent of the variance present in achievement in educational statistics 590X. These two variables

in combination would provide a meaningful regression equation for this study in regard to achievement in statistics.

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APPENDIX A: MATHEMATICS TESTS

In this test, there are 30 items to be done in 20 minutes. For each item, there are five alternatives, only one of which is correct. You are to blacken the space on the answer sheet corresponding to your desired answer. Please do <u>not</u> write on the test. Use only a #2 pencil as these papers will be machine scored. There is no penalty for guessing.

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1. $\frac{4 + 9}{2(3)} = ?$

A) 5
B) 2 3/5
C) 2 1/6
D) 6
E) None of these

2. 5/3 + 3/2 - 3/4 = ?

A) 5
B) - 45/24
C) 7/4
D) 29/12
E) None of these

3. 3 x 4 + 12 ÷ 4 + 2 = ?

A) 4
B) 8
C) 14
D) 16
E) None of these

4. 3.1416(0.041) = ?

A) 0.1288056
B) 0.01288056
C) 1.288056
D) 0.001288056
E) None of these

5. 62.832 ÷ .006 = ?

A) 10.472
B) 104.72
C) 1047.2
D) 10472
E) None of these

6. 0.84 ÷ 3/7 = ?

A) 0.196
B) 1.96
C) 0.36
D) 3.6
E) None of these

7. 3(P + 2N) - (N - 4 - 3P) = ?

A) 5N - 4
B) 5N + 4
C) 6P + 5N + 4
D) 6P + 5N - 4
E) None of these

8. The result of sbutracting -4 from -3 is?

A) 1
B) -1
C) 7
D) -7
E) None of these

9. $\frac{1}{a} + \frac{1}{b} = ?$

A) $\frac{2}{ab}$

B) $\frac{a + b}{ab}$

C) $\frac{2}{a + b}$

D) $\frac{1}{a + b}$

E) None of these

10. $\frac{1}{a} \div \frac{1}{b} = ?$

A) $\frac{1}{ab}$

B) $\frac{ab}{1}$

C) $\frac{b}{a}$

D) $\frac{a}{b}$

E) None of these

11. $\frac{3}{a} + \frac{2}{b} = ?$

A) $\frac{5}{a + b}$

B) $\frac{3b + 2a}{a + b}$

C) $\frac{6}{ab}$

D) $\frac{2a + 3b}{ab}$

E) None of these

12. Rounded to the nearest hundredth 4.5994 is?

A) 4.599
B) 4.60
C) 4.59
D) 4.600
E) None of these

13. How many significant figures has 0.0038010?

A) 4
B) 5
C) 7
D) 8
E) None of these

14. $\frac{9a + 21b}{3} = ?$

A) 3a + 21b
B) 9a + 7b
C) 10ab
D) 3a + 7b
E) None of these

15. As a percent, $\frac{420}{35,000}$ is ?

A) 0.012%
B) 0.12%
C) 1.2%
D) 12%
E) None of these

16. As a decimal, 2.7% is?

A) 0.027
B) 2.70
C) 27.0
D) 270
E) None of these

17. As a percent .007 is?

A) 0.07%
B) 0.7%
C) 7%
D) 70%
E) None of these

18. Find the value of $\frac{a - bc}{df}$ when $a = 43$, $b = -3$, $c = 4$, $d = -2$, $f = 5$

A) 31/10
B) -31/10
C) 11/2
D) -11/2
E) None of these

19. Divide 3 1/5 by 2 2/7. The result is

A) 1 1/5
B) 32/35
C) 35/32
D) 7
E) None of these

20. If $P = \frac{a(b - c)}{d}$, find P when $a = 1/2$, $b = 2/3$, $c = 1/6$, $d = 3/4$

A) 1/8
B) 1/6
C) 1/4
D) 1/2
E) None of these

21. $\frac{9a + 8b}{3 + 2} = ?$

A) 3a + 4b

B) $\frac{17ab}{5}$

C) $\frac{72ab}{6}$

D) $\frac{17(a + b)}{5}$

E) None of these

22. A student has an average of 80 for three tests. What must he score on the next test in order to obtain an average of 84?

A) 96
B) 92
C) 88
D) 84
E) 80

23. What is the smallest number which can be divided evenly by each of the following numbers: 4, 6, 8?

A) 48
B) 32
C) 24
D) 16
E) 12

24. Simplify $\frac{1\ 1/2 + 1/6}{1/3 - 1/4}$. The result is?

A) 1/20
B) 1/12
C) 10/6
D) 10/72
E) 20

25. Which of the following is the smallest?

A) $(1/2)^2$
B) $\sqrt{1/4}$
C) $\sqrt{0.16}$
D) $(0.4)^2$
E) $\sqrt{4/9}$

26. Which of the following describes how to find the average of a group of scores?

A) Find the sum of the scores and divide by 2.
B) Arrange the scores from lowest to highest and select the middle one.
C) Take half the difference between the highest score and the lowest score.
D) Find the sum of the scores and divide by the number of scores.
E) None of these

27. A boy saves 18 dollars in 8 weeks. If he continues to save at the same rate, how many weeks will it take him to save 81 dollars?

A) 13
B) 36
C) 40
D) 71
E) 182 1/4

28. $48 \div 2 + 6 \times 6 - 7 = ?$

A) 18
B) 41
C) 53
D) 171
E) None of these

29. $a^2 - 2a(a - b + 3c) + a(b - c) = ?$

A) $-a^2 + 3ab - 7ac$
B) $-a^2 - ab - 7ac$
C) $-a^2 - ab + 5ac$
D) $-a^2 + 3ab + 5ac$
E) None of these

30. The result of subtracting $2a + 3b - 7c$ from $-3a + 9b - 6c$ is?

A) $5a - 6b - c$
B) $-5a - 3b + c$
C) $a - 12b - 13c$
D) $-5a + 6b - c$
E) None of these

In this test, there are 40 items to be done in 50 minutes. For each item, there are five alternatives, only <u>one</u> of which is correct. You are to blacken the space on the answer sheet corresponding to your desired answer. Please do <u>not</u> write on the test. Use only a #2 pencil as these papers will be machine scored. There is no penalty for guessing.

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1. If C = 2W - 4, then W =

   A) $\frac{C}{2} - 4$

   B) $\frac{C}{2} + 4$

   C) $\frac{C}{2} - 2$

   D) $\frac{C}{2} + 2$

   E) None of these

2. $9^{-1/2} =$

   A) 3
   B) −1/3
   C) 1/3
   D) $-\sqrt{9}$
   E) None of these

3. If i = prt, what is the value of t when p = 200, r = 0.04, and i = 12?

   A) 96
   B) 9.6
   C) 4
   D) 1.5
   E) None of these

4. If X is a whole number greater than one, which one of the following is the smallest?

   A) $\frac{7X}{8}$

   B) $\frac{9X}{10}$

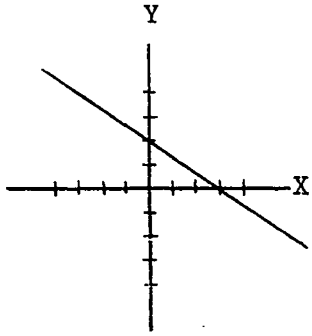
   C) $\frac{17X}{19}$

   D) $\frac{11X}{13}$

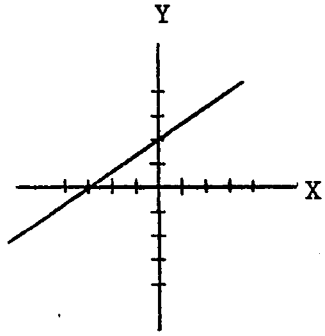
   E) $\frac{9X}{11}$

5. The graph of the equation 2X + 3Y = 6 is?

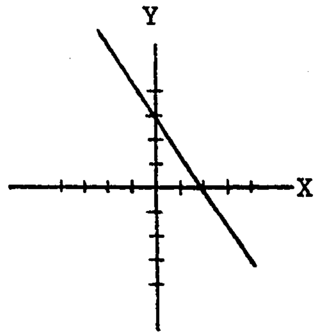
A)



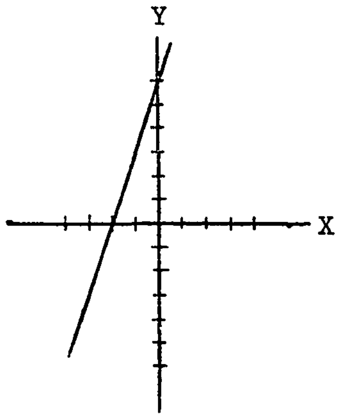
B)



C)



D)



E) None of these

6. If X = -3, the absolute value of X - 5, represented by $|X - 5|$, is =

A) 2
B) -2
C) 8
D) -8
E) None of these

7. $\frac{\sqrt{153}}{\sqrt{17}}$ to the nearest $\frac{1}{10}$ is?

A) 2.9
B) 3.0
C) 3.1
D) 3.3
E) None of these

8. If $Y = \frac{1}{X}$ and X is greater than 0, which of the following is true?

A) When X is greater than 1, Y is greater than 1
B) When X is less than 1, Y is less than 1
C) As X increases, Y increases
D) As X decreases, Y descreases
E) As X increases, Y descreases

9. Which one of the following is not equivalent to $\sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})}$?

A) $s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

B) $s\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$

C) $\sqrt{\frac{s^2(n_2 + n_1)}{n_1 n_2}}$

D) $\dfrac{s}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

E) $\dfrac{\sqrt{s^2(n_1 + n_2)}}{\sqrt{n_1 n_2}}$

10. If $4 - 2X < 5$, then

A) $X > 9/2$
B) $X < -9/2$
C) $X > 1/2$
D) $X < -1/2$
E) None of these

11. If $X_1 = 2$, $X_2 = 3$, $X_3 = 4$, then $\sum_{i=1}^{3} X_i^2 =$

A) 29
B) 27
C) 81
D) 9
E) None of these

12. $\dfrac{7}{\sqrt{5}} = ?$

A) $\dfrac{\sqrt{35}}{5}$

B) $\dfrac{7\sqrt{5}}{5}$

C) $\dfrac{\sqrt{35}}{\sqrt{25}}$

D) $\dfrac{7\sqrt{5}}{25}$

E) None of these

13. $a^{-3} \div a^{-5} = ?$

A) a^2

B) a^{-2}

C) a^8

D) a^{-8}

E) None of these

14. If $\frac{Y-6}{3} - \frac{Y-8}{5} = 0$, then Y = ?

A) -3
B) 3
C) 27
D) -21/2
E) None of these

15. Which one of the following ordered pairs does not lie on the line $5X - 3Y = 11$?

A) (1, -2)
B) (2, - 1/3)
C) (-1, -16/3)
D) (-11/5, 0)
E) (0, -11/3)

16. If Y = 15 corresponds to X = 34, and Y = 33 corresponds to X = 79, then Y = 23 corresponds to X = ?

X	Y
34	15
	23
79	33

A) 42
B) 54
C) 64
D) 69
E) None of these

17. If a factorial is represented by a!, then $\frac{5!}{3!} = ?$

A) 2!
B) 9
C) 5!(4!)
D) 20
E) None of these

18. $(3/2 - 7/4)^0 = ?$

A) 1
B) -1
C) 1/4
D) -1/4
E) None of these

19. $\sqrt{\frac{4}{27}} = ?$

A) $2/3\ \sqrt{3}$

B) $\frac{2}{9\sqrt{3}}$

C) $2/9\ \sqrt{3}$

D) $3/2\ \sqrt{3}$

E) None of these

20. If Y has a value greater than one, which one of the following would decrease as Y increased?

A) $Y - \frac{1}{Y}$

B) $Y^2 + Y$

C) $\frac{10 - Y^2}{Y}$

D) $\frac{2Y^3 - Y^2}{Y}$

E) $Y^3 - \frac{1}{3Y}$

21. If $Y^{**} = \frac{1}{Y}$, what is the value of $ab^{**} + cd^{**}$

A) $\frac{ac}{bd}$

B) $\frac{a + c}{bd}$

C) $\frac{ab + cd}{bd}$

D) $\frac{ad + bc}{bd}$

E) $(a - b) + (c - d)$

22. $(3 + X)^{-1} = ?$

A) $3^{-1} + X^{-1}$

B) $\frac{1}{3 + X}$

C) $-1/3 + \frac{-1}{X}$

D) $2 + X$

E) None of these

23. To the nearest $\frac{1}{10}$, $\sqrt{88.1} = ?$

A) 8.9
B) 9.1
C) 9.2
D) 9.3
E) 9.4

24. Which one of the following is not equivalent to $X\sqrt{\frac{B}{S - (S - R^2)}}$?

A) $XR\sqrt{\frac{B}{R^4}}$

B) $X\sqrt{\frac{B}{R^2}}$

C) $\frac{X\sqrt{B}}{R}$

D) $\sqrt{\frac{BX^2}{R^2}}$

E) $X\sqrt{\frac{B}{S-S-R^2}}$

25. If $X_1 = 2$, $X_2 = 3$, $X_3 = 5$, then $(\sum_{i=1}^{3} X_i)^2 = ?$

A) 100
B) 38
C) 12
D) 10
E) None of these

26. The equation of the straight line which crosses the Y - axis 2 units above the origin and has a slope of +5 is ?

A) $X = 5Y + 2$
B) $X = 2Y + 5$
C) $Y = 5X + 2$
D) $Y = 2X + 5$
E) None of these

27. If $\frac{3a}{2} - \frac{X}{3} + a = 3$, then $X = ?$

A) $\frac{15a}{2} - 9$

B) $\frac{15a - 9}{2}$

C) $15a - 18$

D) $15a - 9$

E) None of these

28. If $5 - 2X < 3X - 10$, then

A) $-5 < X$
B) $X > 3$
C) $X > 15$
D) $3 > X$
E) None of these

29. If the absolute value of a is represented by $|a|$, then $|-7| - |-2| = ?$

A) -9
B) 9
C) -5
D) 5
E) None of these

30. $(4a^3)^2 \; (2b^2)^3 = ?$

A) $48a^5b^5$

B) $8^5(a^3b^2)^5$

C) $128a^6b^6$

D) $8^6a^6b^6$

E) None of these

31. The ordered pair which satisfies the system $\begin{matrix} 3X + Y = 6 \\ 2X - 3Y = 15 \end{matrix}$ is ?

A) (1,3)
B) (2,0)
C) (3,-3)
D) (4, -7/3)
E) (6,1)

32. A number b equals twice the average of three numbers 5, 11, and c. What is b in terms of c?

A) c + 16
B) c/3 + 48
C) 1/2(c + 16)
D) 1/3(c + 16)
E) 2/3(c + 16)

33. The probability of getting a six on two consecutive rolls of a single die is?

A) 1/6
B) 1/36
C) 6
D) 36
E) None of these

34. If $r_1 = \dfrac{r_2 - r_3 r_4}{\sqrt{1 - r_3^2}\,\sqrt{1 - r_4^2}}$, find r_1 when $r_2 = .72$, $r_3 = .60$, $r_4 = .80$

A) .5

B) $\dfrac{.24}{\sqrt{.8}}$

C) .4

D) $\dfrac{.24}{\sqrt{.48}}$

E) None of these

35. If K is a constant, and X varies inversely as Y, this can be expressed as ?

A) $X = \dfrac{K}{Y}$

B) $X = K - Y$

C) $X = \dfrac{Y}{K}$

D) $\dfrac{X}{Y} = K$

E) None of these

36. If $\dfrac{a(b + c)}{d} = f$, then b = ?

A) $\dfrac{df + ac}{a}$

B) $\dfrac{df - ac}{a}$

C) $\dfrac{df - c}{a}$

D) $df - c - a$

E) None of thse

37. $(Y - aX)^2 = ?$

A) $Y^2 - a^2X^2$

B) $Y^2 + a^2X^2$

C) $Y^2 - 2aXY + a^2X^2$

D) $Y^2 - aXY + a^2X^2$

E) None of these

38. If $2\sqrt{n} = 4 - 3a$, then $n = ?$

A) $\frac{16 - 9a^2}{2}$

B) $\frac{16 - 9a^2}{4}$

C) $\frac{16 - 24a + 9a^2}{-2}$

D) $\frac{16 - 24a + 9a^2}{4}$

E) None of these

39. $3\sqrt{2/3} = ?$

A) $\sqrt{2}$

B) $\sqrt{2/9}$

C) $\sqrt{2/27}$

D) $\sqrt{6}$

E) None of these

40. $a\sqrt{\frac{b}{c} - d} = ?$

A) $\sqrt{\frac{ab - acd}{c}}$

B) $\sqrt{\frac{ab}{c} - d}$

C) $\sqrt{\frac{a^2b}{c} - a^2d}$

D) $\sqrt{\frac{a^2bc - a^2cd}{c}}$

E) None of these

APPENDIX B: SELF-INSTRUCTIONAL MATHEMATICS MATERIALS.

The purpose of this material is to provide a means for students to increase their understanding, and to improve their computational competencies in the areas of:

SYMBOLS AND DEFINITIONS

The operations of addition, subtraction, multiplication, and division are indicated by the symbols +, −, x, and ÷ respectively. The symbols x, and ÷ are not often used in statistical work since parentheses and brackets are commonly used to indicate multiplication, and a bar to indicate division. For example, 5(13) means 5 multiplied by 13 and $\frac{X}{Y}$ or X/Y means X divided by Y. Sometimes there is no need for a multiplication sign at all, and thus, XY simply means X multiplied by Y. Likewise a raised dot is often used to designate multiplication. For example, $X \cdot Y$ means X multiplied by Y.

The familiar minus sign "−" has two distinctly different meanings.

1. When the "−" stands between two numerals, as in "a − b", it indicates that the operation of subtraction is to be performed.
2. When the "−" is part of a numeral, as in "(−b)", the minus sign means "the negative of."

To subtract a real number (all positive and negative numbers and zero are real numbers) Y from a real number X, we add the negative of Y to X. The result of subtracting Y from X is written (X − Y); thus $X - Y = X + (-Y)$. The expression 7 − 4 − 2 may be read either, "from seven subtract four and then two to get a result of one," or "to seven add a negative four and a negative two to get a result of one." The result is the same by the two methods. A number with no sign before it is treated as if it had a plus sign before it.

Symbols pertaining to equalities and inequalities are often used in statistics. Some of these include:

1. The symbol ≠ means "is not equal to."
2. The symbol < means "is less than."

3. The symbol > means "is greater than."

4. The symbol $\leqq$ or $\leq$ means "is equal to or is less than."

5. The symbol $\geqq$ or $\geq$ means "is equal to or greater than."

"Cancellation" is a term of ambiguous meaning which is rather carelessly used--sometimes to connote division, sometimes subtraction, sometimes merely "getting rid of." In general, if there is any doubt, it is safer to avoid the use of this word and to go back to the fundamental principles. "Cancellation" is the source of a large number of arithmetic errors made by persons who are not sure why they are "canceling."

There are many instances, however, such as in fraction multiplication, where "canceling" can simplify and speed up the operation. For example:

a. $\frac{1}{\cancel{3}}(\frac{\cancel{3}}{7}) = \frac{1}{7}$

b. $\frac{\cancel{11}}{52}(\frac{7}{\cancel{11}}) = \frac{7}{52}$ (instead of $\frac{77}{572}$)

c. $\frac{2}{\cancel{3}}(\frac{\cancel{3}}{11}) = \frac{2}{11}$ (instead of $\frac{6}{33}$)

"Canceling" usually is done automatically--i.e., if a numerator and a denominator are equal, they are stricken out. However, there is a rationale for doing so based on two facts: multiplication is commutative (that is, ab = ba), and a number divided by itself is one. So for the previous three examples we have:

a. $\frac{1}{3}(\frac{3}{7}) = \frac{3}{3}(\frac{1}{7}) = 1(\frac{1}{7}) = \frac{1}{7}$

b. $\frac{11}{52}(\frac{7}{11}) = \frac{11}{11}(\frac{7}{52}) = 1(\frac{7}{52}) = \frac{7}{52}$

c. $\frac{2}{3}(\frac{3}{11}) = \frac{3}{3}(\frac{2}{11}) = 1(\frac{2}{11}) = \frac{2}{11}$

Similar to the obvious canceling of identical numerals, one can often perform other elementary divisions by canceling. For example:

d. $\frac{7}{8}\left(\frac{4}{14}\right) = \frac{4}{8}\left(\frac{7}{14}\right) = \frac{\cancel{4}\cdot 1}{\cancel{4}\cdot 2}\left(\frac{\cancel{7}\cdot 1}{\cancel{7}\cdot 2}\right) = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

or simply $\frac{\cancel{7}^{1}}{\cancel{8}_{2}}\left(\frac{\cancel{4}^{1}}{\cancel{14}_{2}}\right) = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

e. $\frac{\cancel{2}^{1}}{\cancel{8}_{1}}\left(\frac{\cancel{18}^{2}}{\cancel{30}_{15}}\right) = 1\left(\frac{2}{15}\right) = \frac{2}{15}$

f. $\frac{\cancel{7}^{1}}{\cancel{8}_{1}}\left(\frac{\cancel{64}^{8}}{\cancel{7}_{1}}\right) = 1\cdot 8 = 8$

Notice that canceling applies to multiplication of fractions, but does not apply to addition and subtraction of fractions. For example,

g. $\frac{2}{3} + \frac{3}{8} \neq \frac{\cancel{2}^{1}}{\cancel{3}_{1}} + \frac{\cancel{3}^{1}}{\cancel{8}_{4}} = 1 + \frac{1}{4} = \frac{5}{4}$

we must instead find the lowest common denominator (which will be explained later) to get

$\frac{2}{3} + \frac{3}{8} = \frac{16}{24} + \frac{9}{24} = \frac{25}{24}$

h. $\frac{30 - 5}{5} \neq \frac{\cancel{30}^{6} - 5}{\cancel{5}} = 6 - 5 = 1$

we must instead subtract first, and then divide to get

$\frac{30 - 5}{5} = \frac{25}{5} = 5$

When one number can be divided by another number without a remainder, the second number is said to be a <u>factor</u> of the first number. Thus, 6 is a factor of 18; so are 2, 3, and 9. But 4 is not a factor of 18, since 18 cannot be divided by 4 without a remainder; that is, there is no integer which, when multiplied by 4, equals 18.

A <u>prime number</u> is a positive integer that is not equal to 1 and has no factors other than 1 and itself. Thus, the prime factors of 18 are 2, 3, and 3, because 18 = 2(3)(3), and 2 and 3 have no factors other than 1 and themselves.

Every integer is made up of prime factors in only one way (not considering the order in which the factors are written); of course, some of the factors may appear more than once. For example:

a. $105 = 3 \cdot 5 \cdot 7$

b. $180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

c. $144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

d. $1776 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 37$

e. $5423 = 11 \cdot 17 \cdot 29$

SIGNED NUMBERS

The rules for combining signed numbers (numbers with a plus or minus in front of them) in the fundamental operations are:

a. To add numbers with the same sign, add their values and affix the common sign.

$4 + 3 = 7$ $\qquad$ $(-4) + (-3) = -7$ <u>or</u> $-4 - 3 = -7$

b. To add two numbers with unlike signs, subtract the smaller from the larger and affix the sign of the larger number to the answer.

$-5 + 2 = -3$ $\qquad$ $5 + (-4) = 1$ <u>or</u> $5 - 4 = 1$

c. To subtract a positive number proceed as in ordinary subtraction.

$5 - (+2) = 3$ <u>or</u> $5 - 2 = 3$

$-6 - (+4) = -10$ <u>or</u> $-6 - 4 = -10$

d. To subtract a negative number change its sign and add.

$5 - (-3) = 5 + 3 = 8$

$-7 - (-2) = -7 + 2 = -5$

e. The product or quotient of two numbers with the same sign is positive.

$(+5)(+3) = +15$ $\qquad$ $(-5)(-3) = +15$

$\frac{20}{5} = +4$ $\qquad$ $\frac{-20}{-5} = +4$

f. The product or quotient of two numbers with different signs is negative.

$(-5)(+3) = -15$ $\qquad$ $(+5)(-3) = -15$

$\frac{-10}{+2} = -5$ $\qquad$ $\frac{+10}{-2} = -5$

Problems

Simplify the following and compare answers with the key.

1. $18 - (-5) =$
2. $20 - (-10) - (3) =$
3. $(-7)(-4) =$
4. $\frac{12}{-3} =$
5. $(-2)(3)(-4) =$
6. $-5 + (-2) - (-3) =$
7. $4(-2)(-3)(-1) =$
8. $\frac{-5(3)}{-10} =$
9. $-7 - (6) =$
10. $-8 - (-2) - (4) =$

SYMBOLS OF GROUPING

Grouping two or more terms (numbers or letters) to indicate that they are to be considered as one term or quantity may be accomplished by the use of parentheses ( ), brackets [ ], braces { }, or a variety of other symbols. The symbols of grouping may be removed (or inserted) by the following basic rules.

1. If the symbol of grouping is preceded by a plus sign, it may be removed by leaving the enclosed terms with their original signs.
2. If the symbol of grouping is preceded by a minus sign, each sign of the enclosed terms must be changed.

Examples of the above:

a. $$(3X - 5Y) + 2(X + 4Y) = 1(3X) + 1(-5Y) + 2(X) + 2(4Y)$$
$$= 3X - 5Y + 2X + 8Y$$
$$= 5X + 3Y$$

b. $$(3X - 5Y) - 2(X + 4Y) = 1(3X) + 1(-5Y) - 2(X) - 2(4Y)$$
$$= 3X - 5Y - 2X - 8Y$$
$$= X - 13Y$$

c. $$-(7X - 2Y) - 3(2X - 3Y) = -1(7X) - 1(-2Y) - 3(2X) - 3(-3Y)$$
$$= -7X + 2Y - 6X + 9Y$$
$$= -13X + 11Y$$

Notice, as in example c, $-3(2X - 3Y) = -3(2X) - 3(-3Y) = -6X + 9Y$, that each term in the grouping symbol must be multiplied by the coefficient of the grouping symbol.

In expressions where symbols of grouping are included within others, the procedure is to remove one set at a time, starting with the innermost one. For example: $a - [4Y + 3(2X - 1) + 5] = a - [4Y + 6X - 3 + 5] = a - 4Y - 6X - 2$.

Problems

Simplify the following and compare your answer with the key.

1. $-4(a - b) - 3a =$

2. $2(4X - 3) -4(X - 5) =$

3. $(Y - 5) - (3Y - 4) =$

4. $4(2h^2 - 3) - 3(h^2 + 1) =$

5. $X(X - 5) + 3X(X + 4) - (X^2 - 7X + 1) =$

6. $2(3X - 4) -3[2X - (X - 4)] =$

7. $-3(2Y + 4) + 2[3(2Y - 4) - 3] =$

8. $5(2X - 1) - [5(X - 3) + 4] =$

ORDER OF ARITHMETIC OPERATIONS

1. The order in which numbers are added does not affect the result. Thus $8 + 13 + 12 = 13 + 8 + 12 = 12 + 8 + 13$ etc. In general, $a + b + c = b + a + c = c + b + a$, etc.

2. The order in which numbers are multiplied does not affect the result. Thus $abc = bac = acb = bca = cab = cba$. Sometimes it is more convenient to multiply in one order than in another. For example:

$$\left(\frac{2}{3}\right)(8)(5)\left(\frac{1}{4}\right)(9) = \left(\frac{2}{\cancel{3}_1}\right)\left(\cancel{9}^{3}\right)\left(\cancel{8}^{2}\right)\left(\frac{1}{\cancel{4}_1}\right)(5) = 2(3)(2)(1)(5)$$

3. In an example like $18 + 3 \times 10 + 64 \div 3 - 1$, the multiplication and division should be performed first, in the order in which they occur, and then the addition and subtraction should be performed. Thus:

$$\begin{aligned} 18 + 3 \times 10 + 64 \div 3 - 1 &= 18 + (3 \times 10) + (64 \div 3) - 1 \\ &= 18 + 30 + 21\tfrac{1}{3} - 1 \\ &= 68\tfrac{1}{3} \end{aligned}$$

4. An expression enclosed in a grouping symbol is to be treated as a single term. If convenient, the value of the total set may be worked out first, as $3(22 - 2) = 3(20) = 60$. This is not always convenient, as in $35(\frac{1}{5} + \frac{1}{7})$, where it is easier to multiply each part of the parenthesis separately by 35, giving $35(\frac{1}{5} + \frac{1}{7}) = 35\ (\frac{1}{5}) + 35\ (\frac{1}{7}) = 7 + 5 = 12$. In any case, the terms within the parenthesis must all be treated in the same way. For example:

$$\begin{aligned} 2(3X - 4) - 3(2Y - 5) &= 2(3X) + 2(-4) - 3(2Y) - 3(-5) \\ &= 6X - 8 - 6Y + 15 \end{aligned}$$

5. The bar of a fraction has the same effect as a parenthesis, the numerator being treated as a single number and the demoninator as a single number. For example:

$\frac{4 + 11}{10 + 5} = \frac{15}{15}$

$\frac{28 - 3}{4} = \frac{25}{4}$ <u>and</u> <u>not</u> $\frac{\overset{7}{\cancel{28}} - 3}{\cancel{4}_1} = 7 - 3$

Notice, however, that $\frac{5 \times 9}{2 \times 8} = \frac{5}{2} \times \frac{9}{8} = \frac{5}{8} \times \frac{9}{2}$

Notice also that $\frac{4}{7} = \frac{3}{7} + \frac{1}{7}$, and that in general, $\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$,

hence $\frac{5 + 9}{2 \times 8} = \frac{5}{2 \times 8} + \frac{9}{2 \times 8}$. However, $\frac{c}{a + b} \neq \frac{c}{a} + \frac{c}{b}$

6. A radical sign has the same effect as a symbol of grouping, the expression under the radical sign being treated as a single number.

Thus: $\sqrt{36 - 9} = \sqrt{27}$ <u>and</u> <u>not</u> $6 - 3$

$\sqrt{1 - (0.7)^2} = \sqrt{1 - .49} = \sqrt{.51}$

<u>Problems</u>

Perform the indicated operations, writing your answers in the simplest possible form, and compare your answers with the key.

1. $13 + 2 \div 5 + 6 - 4 =$
2. $18 + 72 \div 3 + 2 \times 4 - 2 =$
3. $56 \div 2 + 6 \times 5 - 3 =$
4. $49 \div (3 + \frac{6}{5} \times \frac{10}{3}) - 5 =$
5. $14 + 42\ (\frac{1}{6} + \frac{1}{7}) - 14 \div 4 =$
6. $\frac{28 - 16}{7 - 4} =$
7. $\sqrt{64 - 36} =$
8. $72 \div (24 \div 2) - 6 \times 3 - 2 =$

FRACTIONS

In combining fractions which do not have the same denominator, it is necessary to find the lowest common denominator. The smallest whole number that is exactly divisible by each denominator is called the lowest common denominator.

Example A. $\frac{1}{3} + \frac{5}{6} - \frac{7}{12} =$

SOLUTION. The smallest whole number which can be exactly divided by the denominators 3, 6, and 12 is 12. The solution can be written as follows: $\frac{\overset{4}{\cancel{12}}}{12}\left(\frac{1}{\cancel{3}}\right) + \frac{\overset{2}{\cancel{12}}}{12}\left(\frac{5}{\cancel{6}}\right) - \frac{\overset{1}{\cancel{12}}}{12}\left(\frac{7}{\cancel{12}}\right) = \frac{4}{12} + \frac{10}{12} - \frac{7}{12} = \frac{7}{12}$

A common denominator can always be found by finding the product of the denominators.

Example B. $\frac{2}{3} + \frac{1}{4} + \frac{3}{5} =$

SOLUTION. A common denominator is 3(4)(5) = 60.

$$\frac{\overset{20}{\cancel{60}}}{60}\left(\frac{2}{\cancel{3}}\right) + \frac{\overset{15}{\cancel{60}}}{60}\left(\frac{1}{\cancel{4}}\right) + \frac{\overset{12}{\cancel{60}}}{60}\left(\frac{3}{\cancel{5}}\right) = \frac{40}{60} + \frac{15}{60} + \frac{36}{60} = \frac{91}{60}$$

Example C. $\frac{2}{X} + \frac{3}{Y} =$

SOLUTION. A common denominator is X(Y) = XY

$$\frac{\cancel{X}Y}{XY}\left(\frac{2}{\cancel{X}}\right) + \frac{X\cancel{Y}}{XY}\left(\frac{3}{\cancel{Y}}\right) = \frac{2Y}{XY} + \frac{3X}{XY} = \frac{2Y + 3X}{XY}$$

Example D. $\frac{3}{4} + \frac{1}{6} - \frac{3}{10} =$

SOLUTION. A common denominator is 4(6)(10) = 240. This is not, however, the lowest common denominator. The lowest common denominator is found by factoring each denominator into its prime factors and using each prime factor the maximum number of times that it appears in any one denominator. In the above example:

$4 = (\underline{2})(\underline{2})$ $\quad$ $6 = (2)(\underline{3})$ $\quad$ $10 = (2)(\underline{5})$

Using each prime factor the maximum number of times that it appears in any one denominator we have (2)(2)(3)(5) = 60. Finishing the solution of our example we have:

$$\frac{\overset{15}{\cancel{60}}}{60}\left(\frac{3}{\cancel{4}}\right) + \frac{\overset{10}{\cancel{60}}}{60}\left(\frac{1}{\cancel{6}}\right) - \frac{\overset{6}{\cancel{60}}}{60}\left(\frac{3}{\cancel{10}}\right) = \frac{45}{60} + \frac{10}{60} - \frac{18}{60} = \frac{37}{60}$$

We can reduce a fraction such as $\frac{18}{30}$ by factoring the numerator and denominator and dividing out the common factors. Thus: $\frac{18}{30} = \frac{(2)(3)(3)}{(2)(3)(5)} = \frac{3}{5}$. This applies to all fractions which have common factors in the numerator and denominator.

Example E. $\frac{2X + 6XY}{4XZ + 6XZ^2} = \frac{\cancel{2X}(1 + 3Y)}{\cancel{2X}(2Z + 3Z^2)} = \frac{1 + 3Y}{2Z + 3Z^2}$

Notice: $\frac{8 + 15}{12 + 18} = \frac{23}{30}$ <u>and not</u> $\frac{\overset{2}{\cancel{8}} + \overset{5}{\cancel{15}}}{\cancel{12}_3 + \cancel{18}_6} = \frac{2 + 5}{3 + 6} = \frac{7}{9}$

In the <u>multiplication</u> of fractions, all the numerators are multiplied together and likewise all the denominators.

Example F. $\frac{1}{4}\left(\frac{2}{3}\right) = \frac{1(2)}{4(3)} = \frac{2}{12} = \frac{1}{6}$

Example G. $\frac{a}{b}\left(\frac{c}{d}\right)\left(\frac{e}{f}\right) = \frac{ace}{bdf}$

In the <u>division</u> of fractions, the divisor must be inverted (interchange the numerator and denominator) and the remaining procedure is the same as in the multiplication of fractions.

Example H. $\frac{1}{4} \div \frac{2}{3} = \frac{1}{4}\left(\frac{3}{2}\right) = \frac{3}{8}$

Example I. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b}\left(\frac{d}{c}\right) = \frac{ad}{bc}$

The reciprocal of any number is the fraction resulting when 1 is divided by the number.

Example J. The reciprocal of 3 is $\frac{1}{3}$

Example K. The reciprocal of a is $\frac{1}{a}$

Example L. The reciprocal of $\frac{X}{Y}$ is $\frac{1}{\frac{X}{Y}} = \frac{Y}{X}$

Problems

Simplify the following and check your answers with the key.

1. $\frac{5}{4} - \frac{7}{6} + \frac{2}{3} =$

2. $\frac{7}{20} + \frac{5}{6} - \frac{2}{3} =$

3. $\frac{1}{3a} + \frac{1}{2a} =$

4. $\frac{2}{3a} - \frac{3}{2b} =$

5. $\frac{98}{140} =$

6. $\frac{16X^3 - 20X^2 + 8X + 4}{4} =$

7. $\frac{9y + 12X^2Y}{6XY + 3YZ} =$

8. $(\frac{6a}{9b})(\frac{18b}{30}) =$

9. $\frac{2X}{5Y} \div \frac{10Z}{15W} =$

10. $\frac{-3a}{4b} \div \frac{18aX}{16bZ} =$

11. The reciprocal of XY is?

12. The reciprocal of $\frac{2a}{3b}$ is ?

DECIMALS

<u>Multiplication</u>. In multiplying decimals, multiply as with whole numbers; then beginning at the right, point off as many decimal places in the product as there are in the multiplier and the multiplicand together.

Example:

```
   23.42   ---- 2 decimal places
   0.013   ---- 3 decimal places
   -----
    7026
   2342
  -------
 0.30446   ---- 5 decimal places
```

<u>Division</u>. In dividing by a decimal, count as many places to the right of the decimal point in the dividend as there are decimal places in the divisor and place the decimal point in the quotient directly over this point. Be careful always to align your quotient figure with the corresponding figure in the dividend, as shown in the example.

Example:

```
              0.46
 6.32^ √2.93^61    ---- Moving decimal two places in the
         2 52 8         divisor and dividend
         ------
           40 81
           37 92
           -----
            2 890
```

<u>Fractions and Decimals</u>.

Examples: a. $\overset{4.08}{\cancel{32.64}}\left(\frac{3}{\cancel{8}}\right) = 4.08(3) = 12.24$

b. $2.01\left(\frac{4}{7}\right) = \frac{2.01(4)}{7} = \frac{8.04}{7} = 1.15$

c. $13.42 \div \frac{4}{9} = \overset{3.355}{\cancel{13.42}}\left(\frac{9}{\cancel{4}}\right) = 3.355(9) = 30.195$

Problems

Solve and compare your answers with the key.

1. $4.3(6.15) =$

2. $0.023(1.34) =$

3. $203(0.11) =$

4. $43.2 \div 2.4 =$

5. $0.765 \div 0.03 =$

6. $0.06 \div 7.5 =$

7. $28.64(\frac{7}{4}) =$

8. $63.63 \div \frac{9}{2} =$

9. $36.42 \div \frac{6}{5}(0.10) =$

PERCENTAGE

"Percent" means hundredths. It is indicated by the symbol %. Thus, 6% of a number means $\frac{6}{100}$ of the number or 0.06 of the number. Also, 25% of a number means $\frac{25}{100}$ of the number, or 0.25 of the number.

Since a percent expresses a number of hundredths, it can be easily changed to a common fraction or to a decimal by dropping the % sign and dividing by 100.

Thus: $7\% = \frac{7}{100} = 0.07$

$$18.3\% = \frac{18.3}{100} = 0.183$$

$$\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{.5}{100} = 0.005$$

$$.6\% = \frac{.6}{100} = 0.006$$

A fraction or decimal is changed to a percent by finding how many hundredths it contains, that is, by multiplying it by 100. (Note that dividing by $\frac{1}{100}$ is equivalent to multiplying by 100.)

Thus: $\frac{3}{4}(100) = 75$ then $\frac{3}{4} = 75\%$

$$.35(100) = 35 \text{ then } .35 = 35\%$$

$$\frac{240}{3600}(100) = 6\frac{2}{3} \text{ then } \frac{240}{3600} = 6\frac{2}{3}\%$$

It is sometimes convenient to change the fraction to a decimal and then change the decimal to a percent.

Thus: $\frac{1}{4} = 0.25$ and $0.25 = 25\%$

$$\frac{1}{200} = 0.005 \text{ and } 0.005 = 0.5\%$$

$$\frac{7}{8} = 0.875 \text{ and } 0.875 = 87.5\%$$

Problems

Solve and check your answers with the key.

1. Change 36% to a decimal.
2. Change 13.3% to a decimal.
3. Change 112% to a decimal.
4. Change 0.04% to a decimal.
5. Change 1.3 to a percent
6. Change 0.034 to a percent.
7. Change 0.111 to a percent.
8. Change $\frac{44}{110}$ to a percent.
9. Change $\frac{305}{152,500}$ to a percent.
10. Change $\frac{86.075}{27.5}$ to a percent.

SIGNIFICANT FIGURES AND ROUNDING

All nonzero digits are significant. Also, a zero between two nonzero digits is always significant, as in 304. However, the leading zero(s) preceding the first nonzero digit in a decimal number can never be significant, as in 0.0304. The first two zeros are not significant while the zero between 3 and 4 is significant. All zeros following a nonzero digit after a decimal are significant as in 0.0340. The zero following the 4 is significant because it indicates that the number has been rounded off to the nearest $\frac{1}{10,000}$th. However, whenever a measurement is given by an integer which ends in zeros, such as 200, you cannot tell without additional information, which if any, of the final zeros are significant. The 200 may have been rounded to the nearest hundred, ten, or one.

Examples:	30.4020 ----	6 significant digits
	0.03010 ----	4 significant digits
	0.0300 ----	3 significant digits
	3,400 ----	need additional information, the number could have been rounded to the nearest hundred, ten, or unit.
	220.0 ----	4 significant digits, here we know it has been rounded to the nearest $\frac{1}{10}$th.

In rounding use this rule:

To round a decimal, add 1 to the last digit retained if the first digit dropped is 5 or more, otherwise, leave the retained digits unchanged.

For example: 2.347 rounded to 3 significant figures is 2.35

0.025607 rounded to 3 significant figures is 0.0256

36.295 rounded to 3 significant figures is 36.3

26.489 rounded to the nearest hundredth is 26.49

1.449 rounded to the nearest tenth is 1.4

Problems

Solve and check your answers with the key.

1. 1.0320 has how many significant figures?
2. 0.003020 has how many significant figures?
3. 1330.0 has how many significant figures?
4. 2300 has how many significant figures?
5. 26.649 rounded to the nearest hundredth is?
6. 204.053 rounded to the nearest tenth is?
7. 27.049 rounded to the nearest tenth is?
8. 203.0604 rounded to six significant figures is?
9. 0.043059 rounded to three significant figures is?

KEY

Signed Numbers

1. 23
2. 27
3. 28
4. −4
5. 24
6. −4
7. −24
8. $\frac{3}{2}$
9. −13
10. −10

Symbols of Grouping

1. $-7a + 4b$
2. $4X + 14$
3. $-2Y - 1$
4. $5h^2 - 15$
5. $3X^2 + 14X - 1$
6. $3X - 20$
7. $6Y - 42$
8. $5X + 6$

Order of Arithmetic Operations

1. $15\frac{2}{5}$
2. 48
3. 55
4. $49 \div (3 + \frac{\cancel{6}^{2}}{\cancel{5}} \times \frac{\cancel{10}^{2}}{\cancel{3}}) - 5 = 49 \div (3 + 4) - 5$

   $= 49 \div 7 - 5$

   $= 7 - 5$

   $= 2$
5. $\frac{47}{2}$
6. 4
7. $\sqrt{28}\ \sqrt{4(7)} = 2\sqrt{7}$
8. −14

Fractions

1. $\frac{3}{4}$

2. $\frac{31}{60}$

3. $\frac{5}{6a}$

4. $\frac{4b - 9a}{6ab}$

5. $\frac{7}{10}$

6. $\frac{\cancel{4}(4X^3 - 5X^2 + 2X + 1)}{\cancel{4}} = 4X^3 - 5X^2 + 2X + 1$

7. $\frac{3 + 4X^2}{2X + Z}$

8. $\frac{2}{5}a$

9. $\frac{3XW}{5YZ}$

10. $\frac{-\cancel{3}a}{\cancel{4}b}\left(\frac{\overset{4}{\cancel{160}}Z}{\underset{6}{\cancel{180}}X}\right) = \frac{-4Z}{6X}$

11. $\frac{1}{XY}$

12. $\frac{3b}{2a}$

Decimals

1. 26.445
2. 0.03082
3. 22.33
4. 18
5. 25.5
6. 0.008
7. 50.12
8. 14.14
9. 3.035

Percentage

1. 0.36	6. 3.4%
2. 0.133	7. 11.1%
3. 1.12	8. 40%
4. 0.0004	9. 0.2%
5. 130%	10. 313%

Significant Figures and Rounding

1. 5	6. 204.1
2. 4	7. 27.0
3. 5	8. 203.060
4. Need more information	9. 0.0431
5. 26.65	

The purpose of this material is to provide a means for students to increase their understanding, and to improve their computational competencies in the areas of:

SUMMATION

In order to simplify the many formulas that involve great quantities of numerical data, we use the summation symbol Σ (capital Greek sigma). We have by definition:

$$\sum_{i=1}^{n} X_i = X_1 + X_2 + X_3 + \ldots + X_n,$$

which is read as: "the summation of X_i, i going from 1 to n." It means that we take the sum of the X's that have in succession the subscripts 1, 2, . . . , n. For example:

Let's define a variable X_i (i=1, 2, . . ., 6) as follows:

$X_1 = 2$ $\quad$ $X_4 = 8$

$X_2 = 4$ $\quad$ $X_5 = 10$

$X_3 = 6$ $\quad$ $X_6 = 12$

The symbol to designate the command, "find the sum of all of the X_i starting with X_1 and going to X_6" is: $\sum_{i=1}^{6} X_i$

The symbol means:

$$\sum_{i=1}^{6} X_i = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 2 + 4 + 6 + 8 + 10 + 12 = 42$$

We might also have for example:

$$\sum_{i=3}^{6} X_i = X_3 + X_4 + X_5 + X_6 = 6 + 8 + 10 + 12 = 36$$

$$\sum_{i=1}^{3} X_i^2 = X_1^2 + X_2^2 + X_3^2 = (2)^2 + (4)^2 + (6)^2 = 4 + 16 + 36 = 56$$

$$\left(\sum_{i=4}^{6} X_i\right)^2 = (X_4 + X_5 + X_6)^2 = (8 + 10 + 12)^2 = (30)^2 = 900$$

Fundamental Theorems about Summations

Theorem 1. The summation of the sum (or difference) of two or more variables or terms is equal to the sum (or difference) of their separate summations. Symbolically we can write this theorem for three variables as:

$$\sum_{i=1}^{n} (X_i + Y_i + Z_i) = \sum_{i=1}^{n} X_i + \sum_{i=1}^{n} Y_i + \sum_{i=1}^{n} Z_i$$

Example: $X_1 = 2$ $Y_1 = 4$ $Z_1 = 3$

$X_2 = 5$ $Y_2 = 6$ $Z_2 = 8$

$$\sum_{i=1}^{2} (X_i + Y_i + Z_i) = \sum_{i=1}^{2} X_i + \sum_{i=1}^{2} Y_i + \sum_{i=1}^{2} Z_i$$

$$= (X_1 + X_2) + (Y_1 + Y_2) + (Z_1 + Z_2)$$

$$= (2 + 5) + (4 + 6) + (3 + 8)$$

$$= 7 + 10 + 11$$

$$= 28$$

Theorem 2. The summation of a constant multiplied by a variable is equal to the constant multiplied by the summation of the variable. Symbolically this can be written as:

$$\sum_{i=1}^{n} KX_i = K \sum_{i=1}^{n} X_i, \text{ where K is a constant}$$

Example: $K = 5$ $X_1 = 2$ $X_2 = 3$ $X_3 = 6$

$$\sum_{i=1}^{3} 5X_i = 5 \sum_{i=1}^{3} X_i = 5(2 + 3 + 6) = 5(11) = 55$$

Theorem 3. The summation of a constant K, from i=1 to n, equals the product of K and n. This means that $\sum_{i=1}^{n} K = nK$.

Examples: if K = 4, n = 3

$$\sum_{i=1}^{3} K = K + K + K = nK = 3K = 3(4) = 12$$

$$\sum_{i=1}^{4} 5 = 5 + 5 + 5 + 5 = 4(5) = 20$$

When the summation symbol is used without explicit limits, it is understood that the summation runs over all possible values of the subscript or variable. Thus if the only numbers under discussion are X_1, X_2, X_3 then: ΣX_i means $\sum_{i=1}^{3} X_i$.

Double Subscripts and Double Summation

In many instances we may be considering more than one observation on each of several individuals. Suppose each of six students was measured in regard to some characteristic by each of four investigators. We could call the measurements M_1, M_2, M_3, and M_4. If we use the subscript "j" to represent the measurements we would have j = 1, 2, 3, 4. Now denote each of the six students by the subscript "i", and we have i = 1, 2, 3, 4, 5, 6.

"M_{ij}" will mean, "the jth measurement for student i." "M_{34}" will mean, "the fourth measurement for the third student."

We can define the average measurement for the third student as one-fourth the sum of the four measurements. If we call the average "A", we can write $A_3 = \frac{1}{4} (M_{31} + M_{32} + M_{33} + M_{34})$

or, in summation notation,

$$A_3 = \frac{1}{4} \sum_{j=1}^{4} M_{3j}$$

In general, for any student we have:

$$A_i = \frac{1}{4} \sum_{j=1}^{4} M_{ij}$$

We can also talk about the sum of the first measurements for all students. This is: $\sum_{i=1}^{6} M_{i1}$

also, we have $\sum_{i=1}^{6} M_{i2}$ as the sum of the second measurements for all students.

We can also calculate a double sum, that is the grand total of all measurements added over all students. This is written with a double Σ as:

$$\sum_{i=1}^{6} \sum_{j=1}^{4} M_{ij}$$

or equivalently,

$$\sum_{j=1}^{4} \sum_{i=1}^{6} M_{ij}$$

Notice that the order of summing is not important. However, the order of the subscripts of M is important. The first, i, refers to the student's identification, while the second, j, refers to which measurement it is. Suppose for our example we have the following values for the measurements.

	Student					
Investigator	1	2	3	4	5	6
1	7	5	2	3	8	7
2	5	4	0	2	9	5
3	8	3	1	4	6	5
4	4	5	3	2	8	6

We have: $M_{53} = 6$, the measurement for the fifth student by the third investigator.

$\sum_{j=1}^{4} M_{2j} = 5 + 4 + 3 + 5 = 17$, the sum of all measurements for the second student.

$$\sum_{i=1}^{6} M_{i3} = 8 + 3 + 1 + 4 + 6 + 5 = 27,$$ the sum of the third measurements for all students.

$$\sum_{i=1}^{6} \sum_{j=1}^{4} M_{ij} = 7 + 5 + 2 + \ldots + 6 = 112,$$ the sum of all measurements for all students.

Problems

Find the indicated sums using the following data and check your answers with the key.

i	1	2	3	4	5	6
B	7	4	5	3	6	2

1. $\sum_{i=1}^{6} B_i =$

2. $\sum_{i=1}^{3} B_i =$

3. $\sum_{i=1}^{4} B_i^2 =$

4. $(\sum_{i=1}^{3} B_i)^2 =$

5. $\sum_{i=3}^{6} B_i =$

6. $\sum_{i=1}^{4} 3B_i =$

7. $\sum_{i=1}^{3} (B_i - 4) =$

8. $\sum_{i=1}^{5} 6 =$

9. $\sum_{i=1}^{3} (2B_i - 3) =$

Find the indicated using the following data if H_{ij} represents the values in the table

j \ i	1	2	3	4
1	3	2	1	4
2	5	0	3	1
3	4	5	2	2

10. $H_{32} =$

11. $H_{21} =$

12. $\sum_{i=1}^{4} H_{i3} =$

13. $\sum_{j=1}^{3} H_{2j} =$

14. $\sum_{i=1}^{4} \sum_{j=1}^{3} H_{ij} =$

15. $\sum_{i=3}^{4} \sum_{j=2}^{3} H_{ij} =$

ABSOLUTE VALUE

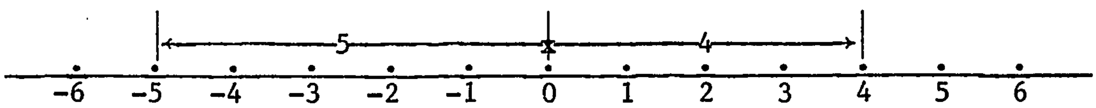
To denote the absolute value of a number, we enclose the number within vertical bars. That is, the symbol $|2|$ is read "the absolute value of 2."

Definition of absolute value

For any real number X, $|X| = X$ if $X \geq 0$, and $|X| = -X$ if $X \leq 0$. From this definition we see that $|2| = 2$, since $2 > 0$, while $|-3| = -(-3) = 3$ since $-3 < 0$. These examples illustrate the fact that the absolute value of a number is never negative.

Another way to look at it, is to say that the absolute value of a number is the value of the number without regard to its sign.

Still another way to look at it, is to say that the absolute value of a real number is the distance between that real number and 0 on the real number line, regardless of whether the number lies to the left or to the right of 0.



$|4| = 4$ ---- the distance from 4 to 0 is 4

$|-5| = 5$ ---- the distance from -5 to 0 is 5

Other examples

1. if $a = -4$, then $|a| = |-4| = -(-4) = 4$, here $a < 0$, hence $|a| = -a$
2. $|5 - 3| = |2| = 2$
3. $|4 - 7| = |-3| = -(-3) = 3$
4. $|-4 - 3| = |-7| = -(-7) = 7$
5. if $X = -5$, and $Y = 3$, then $|X - Y| = |-5 - (3)| = |-8| = -(-8) = 8$

6. $|5| + |2| = 5 + 2 = 7$

7. $|-3| + |-2| = 3 + 2 = 5$

8. $|-4| - |3| = 4 - 3 = 1$

9. $|3| - |-2| = 3 - 2 = 1$

Problems

Solve and compare your answers with the key.

1. $|3 - 5| =$

2. $|-2(8 - 3)| =$

3. $|-5| + |5| =$

4. $|3 - 2| - |5 - 6| =$

5. $|-5 - 3| + |5 - 3| =$

6. if $a = -4$ and $b = 3$ then $|a - b| =$

7. if $a = -4$ and $b = 3$ then $|b - a| =$

8. if $X = 5$ and $Y = -3$ then $3|X - Y| - 2|Y - X| =$

EQUATIONS WITH ONE UNKNOWN

In solving equations with one unknown, we will use several principles of equality or identities: the equation, $X = X$, is an identity. It is true for any number. Starting with, $X = X$, we have the following principles:

1. $X + a = X + a$

   The truth value of an equation is unchanged if the same number is added to both sides of an equation.

2. $X - a = X - a$

   The truth value of an equation is unchanged if the same number is subtracted from both sides of an equation.

3. $aX = aX$

   The truth value of an equation is unchanged if both sides are multipled by the same number.

4. $\frac{X}{a} = \frac{X}{a}$, $a \neq 0$

   The truth value of an equation is unchanged if both sides are divided by the same number.

Several examples should serve to clarify the different possibilities in solving equations.

1. $X + 3 = 10$ -------- subtract 3 from each side

   $X + 3 - 3 = 10 - 3$

   $X = 7$

2. $X - 5 = 30$ -------- add 5 to each side

   $X - 5 + 5 = 30 + 5$

   $X = 35$

3. $X + a = 5$ ---------- (find the solution for X) subtract a from each side

$X + a - a = 5 - a$

$X = 5 - a$

4. $5Z = 10$ ------------ divide both sides by 5

$\frac{\cancel{5}Z}{\cancel{5}} = \frac{\cancel{10}^{2}}{\cancel{5}}$

$Z = 2$

5. $\frac{Z}{4} = 3$ -------------- multiply both sides by 4

$4(\frac{Z}{4}) = 4(3)$

$Z = 12$

6. $\frac{3Z}{4} = 6$ ------------- multiply both sides by $\frac{4}{3}$

$\frac{4}{3}(\frac{3Z}{4}) = \frac{4}{3}(6)$

$Z = 8$

OR ----------------- multiply each side by 4

$4(\frac{3Z}{4}) = 4(6)$

$3Z = 24$ ------------ then divide each side by 3

$\frac{3Z}{3} = \frac{24}{3}$

$Z = 8$

7. $2X + 4 = 6$ --------- subtract 4 from each side

$2X = 2$ ------------- divide each side by 2

$X = 1$

8. $\frac{2X}{3} + \frac{3}{4} = \frac{7}{2}$ --------- multiply each term by the L.C.D. 12, OR clearing of fractions

$\overset{4}{\cancel{12}}(\frac{2X}{\cancel{3}}) + \overset{3}{\cancel{12}}(\frac{3}{\cancel{4}}) = \overset{6}{\cancel{12}}(\frac{7}{\cancel{2}})$

$8X + 9 = 42$ -------- subtract 9 from each side

$8X = 33$ ------------ divide each side by 8

$X = \frac{33}{8}$

9. $5X - 7 = 3X + 5$ ---- add 7 to each side

$5X = 3X + 12$ ------- subtract 3X from each side

$2X = 12$ ------------ divide each side by 2

$X = 6$

10. $3X + 4aX + 2bX = c$ --solve for X, factor X from each term on the left

$X(3 + 4a + 2b) = c$ --divide each side by $(3 + 4a + 2b)$

$$X = \frac{c}{3 + 4a + 2b}$$

In each case, if we multiply, divide, add, or subtract on one side of an equation, the same thing must be done to the other side of the equation.

Subsstitution

In many instances, we are asked to find the value of a certain variable in an equation, when the values of the other variables in the equation are given. For example:

1. If $P = \frac{S}{1 + rt}$, what is the value of P when $S = 450$, $r = 0.05$, $t = 12$.

Substituting: $P = \frac{450}{1 + 0.05(12)} = \frac{450}{1 + .60} = \frac{450}{1.60} = 281.25$

2. If $f = \frac{a(b + c)}{d}$, what is the value of b when $a = \frac{1}{2}$, $c = \frac{1}{6}$, $d = \frac{3}{4}$, $f = \frac{2}{3}$. In this case, it is probably easiest to solve for b first, and then substitute.

Solving for b: $f = \frac{a(b + c)}{d}$

$df = ab + ac$

$ab = df - ac$

$b = \frac{df - ac}{a}$

Substituting: $b = \frac{\frac{3}{4}(\frac{2}{3}) - \frac{1}{2}(\frac{1}{6})}{\frac{1}{2}} = \frac{\frac{1}{2} - \frac{1}{12}}{\frac{1}{2}} = \frac{\frac{5}{12}}{\frac{1}{2}} = \frac{5}{6}$

Problems

Solve and check you answers with the key.

1. $X + 8 = 13$
2. $X - 7 = 5$
3. $10 = 15 - X$
4. $2X - a = 7$ (solve for X)
5. $\frac{X}{3} + a = 5$ (solve for X)
6. $\frac{2}{7}X = 6$
7. $\frac{2}{3}X = \frac{3}{4}$
8. $3X - \frac{3}{4} = 7$
9. $\frac{2X}{3} - \frac{a}{5} = \frac{1}{6}$ (solve for X)
10. $4X + 3 = X + 6$
11. $3X - 8 = 6X - 2$
12. $3(2X - 1) = 2(2 - X)$
13. $5aX + 10cX + 2aX + 6cX = 10$ (solve for X)
14. $5X + aX + 7X + 4aX = a + 1$ (solve for X)
15. $7X - 2aX + 4 = 5X - aX + 11$ (solve for X)
16. If $X = \frac{3(a + b)}{2 - cd}$, what is the value of X when $a = 4$, $b = 7$, $c = -2$, $d = 3$
17. If $f - 3 = \frac{a(X - c)}{d}$, what is the value of X when $a = \frac{2}{5}$, $c = \frac{4}{3}$, $d = \frac{2}{3}$, $f = \frac{3}{4}$

INEQUALITIES

Inequalities are generally solved the same way that equations are solved. However, some operations do present a special problem in regard to the direction of the inequality sign. Several examples should help to clarify the possibilities.

1. $X + 4 \leq 8$ ----- subtract 4 from both sides

   $X \leq 4$

2. $4X - 3 > 9$ ---- add 3 to both sides

   $4X > 12$ ------- divide both sides by 4

   $X > 3$

3. $6 - X < 4$ ----- add X to both sides

   $6 < 4 + X$ ----- subtract 4 from both sides

   $2 < X$ (or, equivalently, $X > 2$)

   OR, The problem can by approached in another way in which confusion can arise.

   $6 - X < 4$ ----- subtract 6 from each side

   $-X < -2$ ------- multiply each side by -1

   The result appears to be "$X < 2$." But an example using $X = 3$ shows that "$-X < -2$" and "$X < 2$" are not equivalent. They are "$-3 < -2$" which is true and "$3 < 2$" which is false. This illustrates the principles that <u>when multiplying or dividing an inequality by a negative number, we must reverse the direction of the inequality sign</u>. Therefore in our example:

   $-X < -2$ ------- multiply each side by -1

   $X > 2$ --------- (reversing the direction of the inequality)

4. $7 - 3X \leq 16$ --- subtract 7 from each side

   $-3X \leq 9$ -------- divide each side by -3

   $X \geq -3$ --------- (reversing the direction of the inequality)

5. $9 - 4X \leq -2X + 14$ -- add 2X to each side

   $9 - 2X \leq 14$ --- subtract 9 from each side

   $-2X \leq 5$ -------- divide each side by -2

   $X \geq -\frac{5}{2}$ -------- (reversing the direction of the inequality)

Problems

Simplify and check your answers with the key.

1. $K - 9 \leq 7$
2. $K + 5 \geq 4$
3. $5 - 2X < 9$
4. $3X + 7 \geq X - 5$
5. $2X - 5 \leq 3X + 4$
6. $6 - 3X \geq 5 + X$
7. $\frac{2}{3}X - \frac{1}{4} < \frac{1}{5}$
8. $\frac{3}{8} - \frac{1}{4}X \geq \frac{2}{3}X - \frac{1}{6}$

EXPONENTS

When we write an expression like b^n (read "b to the nth" or "the nth power of b"), we are using n as an <u>exponent</u> and b is called the <u>base</u>. The <u>exponent</u> tells us how many times the <u>base</u> will be used as a factor. For example:

$5^4 = 5\cdot5\cdot5\cdot5$	The base is 5 and the exponent 4 tells us that 5 is used as a factor 4 times.
$a^3 = a\cdot a\cdot a$	The base is a and the exponent 3 tells us that a is used as a factor 3 times.
$2^n = \underbrace{(2)(2)(2)\cdots(2)}_{n \text{ factors}}$	The base is 2 and the exponent n tells us that 2 is used as a factor n times.
$m^k = \underbrace{m \quad m \quad m \cdots m}_{k \text{ factors}}$	The base is m and the exponent k tells us that m is used as a factor k times.

You must be careful not to confuse the meaning of an exponent with that of a numerical coefficient, such as the "4" in 4X, or in $4ab^2c^3$. For example:

5a means $a + a + a + a + a$

a^5 means $a\cdot a\cdot a\cdot a\cdot a$

$5X^3$ means $X^3 + X^3 + X^3 + X^3 + X^3$

$(5X)^3$ means $(5X)(5X)(5X)$

It should be noted that a numeral without an exponent can be thought of as having the exponent 1 "understood;" that is, it is to be taken only once as a factor. Thus "X^1" or X to the first power, means the same as "X." We do much the same thing when we regard a letter without a number before it as having a numerical coefficient of 1 "understood." Thus "X" is the same as 1X; we say $X + 7X = 1X + 7X = 8X$.

Observe also that an exponent applies only to the numeral next to which it is written. For example:

a^2b^3 means $a\cdot a\cdot b\cdot b\cdot b$

ab^4 means $a\cdot b\cdot b\cdot b\cdot b$

3^3b^2 means $3\cdot 3\cdot 3\cdot b\cdot b$

$4a^3$ means $4\cdot a\cdot a\cdot a$

If a and b are real numbers, then some of the laws of exponents include:

1. $b^m\cdot b^n = b^{m+n}$

Examples:

$$a^3\cdot a^2 = (a\ a\ a)(a\ a) = a^{3+2} = a^5$$

$$7^3\cdot 7^4 = (7\cdot 7\cdot 7)(7\cdot 7\cdot 7\cdot 7) = 7^{3+4} = 7^7$$

$$m\cdot m^3 = (m)(m\cdot m\cdot m) = m^{1+3} = m^4$$

$$X^2\cdot X^3\cdot X = X^{2+3+1} = X^6$$

$$(a^2b^5c^3)(a^3b\ c) = a^{2+3}b^{5+1}c^{3+1} = a^5b^6c^4$$

Notice that the base must be the same, thus, $a^m\cdot b^n \neq (ab)^{m+n}$ Observe also, that when there are numerical coefficients among the factors, the coefficients are multiplied, but the exponents of similar bases are added; thus:

$$(3a)(4a) = 3(4)(a)(a) = 12a^{1+1} = 12a^2$$

$$(2X^2Y)(3XY^4) = 2(3)(X^2Y)(XY^4) = 6X^{2+1}Y^{1+4} = 6X^3Y^5$$

$$(5a^5)(5a^5) = 5(5)(a^5)(a^5) = 25a^{5+5} = 25a^{10}$$

$$(3X^2)(-2X^3) = 3(-2)(X^2)(X^3) = -6X^{2+3} = -6X^5$$

2. $(b^m)^n = b^{mn}$

Examples:

$$(5^2)^3 = (5^2)(5^2)(5^2) = 5^{2+2+2} = 5^{2(3)} = 5^6$$

$$(a^3)^2 = (a^3)(a^3) = a^{3+3} = a^{3(2)} = a^6$$

$$(a^3)^4 = (a^3)(a^3)(a^3)(a^3) = a^{3+3+3+3} = a^{3(4)} = a^{12}$$

3. $(ab)^m = a^m b^m$

Examples:

$$(2a)^4 = (2a)(2a)(2a)(2a) = (2\cdot2\cdot2\cdot2)(a\cdot a\cdot a\cdot a) = 2^4a^4$$

$$(a^2b^3)^6 = (a^2)^6(b^3)^6 = a^{12}b^{18}$$

$$(2a^2b^3)^3 = 2^3(a^2)^3(b^3)^3 = 2^3a^6b^9$$

4. $a^m \div a^n = a^{m-n}$

Examples:

$$\frac{5^6}{5^3} = \frac{(5)(5)(5)(5)(5)(5)}{(5)(5)(5)} = 5^{6-3} = 5^3$$

$$\frac{a^4}{a^2} = \frac{a\cdot a\cdot a\cdot a}{a\cdot a} = a^{4-2} = a^2$$

$$\frac{a^5}{a^{-3}} = a^{5-(-3)} = a^8$$

$$\frac{a^2}{a^3} = \frac{a\cdot a}{a\cdot a\cdot a} = a^{2-3} = a^{-1} = \frac{1}{a}$$

The above example illustrates that $a^{-1} = \frac{1}{a}$

$$\frac{a^3}{a^6} = a^{3-6} = a^{-3} = \frac{1}{a^3}$$

$$\frac{a^4}{a^4} = \frac{a\cdot a\cdot a\cdot a}{a\cdot a\cdot a\cdot a} = 1 = a^{4-4} = a^0$$

The above example illustrates that $a^0 = 1$. Hence, any nonzero quantity to the zero power is equal to one.

$$\frac{(3a)^4}{(3a)^4} = (3a)^{4-4} = (3a)^0 = 1$$

$$\frac{(2b+3)^3}{(2b+3)^3} = (2b+3)^{3-3} = (2b+3)^0 = 1$$

5. $(\frac{a}{b})^m = \frac{a^m}{b^m}$

Examples:

$$(\frac{3}{4})^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

$$(\frac{2a}{3b})^3 = \frac{(2a)^3}{(3b)^3} = \frac{2^3a^3}{3^3b^3}$$

$$(\frac{2}{3})^{-3} = \frac{2^{-3}}{3^{-3}} = \frac{\frac{1}{2^3}}{\frac{1}{3^3}} = \frac{1}{2^3}\cdot\frac{3^3}{1} = \frac{3^3}{2^3} \equiv (\frac{3}{2})^3$$

6. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Examples:

$$\sqrt[3]{5^6} = 5^{\frac{6}{3}} = 5^2 = 25$$

$$\sqrt[4]{a^2} = a^{\frac{2}{4}} = a^{\frac{1}{2}} = a \quad \text{if } a \geq 0$$

$$\sqrt{3^3} = 3^{\frac{3}{2}}$$

$$\sqrt[4]{9^2} = 9^{\frac{2}{4}} = 9^{\frac{1}{2}} = \sqrt{9} = 3$$

These laws of exponents apply to quantities as well, however, caution must be exercised in many cases. For example:

$$(X + Y)^2 = (X + Y)(X + Y) = X(X + Y) + Y(X + Y) = X^2 + XY + XY + Y^2$$
$$= X^2 + 2XY + Y^2 \quad \underline{\text{and}}\ \underline{\text{not}} \quad X^2 + Y^2$$

$(a + b)^m(a + c)^n$ cannot be simplified because the quantities do not contain the same terms.

$$(X + Y)^{-1} = \frac{1}{X + Y} \quad \underline{\text{and}}\ \underline{\text{not}} \quad X^{-1} + Y^{-1}$$

$$X^{-1} + Y^{-1} = \frac{1}{X} + \frac{1}{Y} = \frac{Y + X}{XY}$$

Problems -- Simplify and check your answers with the key.

1. $X^6 \cdot X^3 =$
2. $a^5 \cdot a^{-2} =$
3. $-3X^2 \cdot 4X^4 =$
4. $(4a^2)(-7a) =$
5. $\frac{a^7}{a^2} =$
6. $X^6 \div X^4 =$
7. $\frac{a^5}{a^{-2}} =$
8. $\frac{-24a^3}{6a} =$
9. $(2a)^3 =$
10. $\frac{7^7}{7^4} =$
11. $\frac{24X^2Y^5}{-6X^2Y^3} =$
12. $(\frac{2z}{c^3})^3 =$
13. $(-2a)(-3a^3)(-4a^4) =$
14. $(3a^2bc^3)^3(-2ab^3c^2)^2 =$
15. $5^3 \div 5^3 =$
16. $(3X^2)^0 =$
17. $(b^{-2})^5 =$
18. $(5 - 2)^0 =$

RADICALS

In statistical work, squares and square roots are used often. In the equation $\sqrt{49} = 7$, the number under the radical sign is called the "<u>radicand</u>," which in this equation is 49.

Although $(-12)^2 = 144$ as well as $(+12)^2 = 144$, in statistical work <u>it is always assumed that the positive square root is indicated</u>.

To multiply two radicals, write the product of their radicands under the radical sign. In many cases this allows for simplification. For example:

$\sqrt{8}\ \sqrt{2} = \sqrt{8 \cdot 2} = \sqrt{16} = 4$

$\sqrt{5}\ \sqrt{5} = \sqrt{5 \cdot 5} = \sqrt{25} = 5$

$\sqrt{XY}\ \sqrt{XZ} = \sqrt{(XY)(XZ)} = \sqrt{X^2YZ} = \sqrt{X^2}\ \sqrt{YZ} = X\sqrt{YZ}$

To divide two radicals, write the quotient of their radicands under the radical sign. For example:

$\frac{\sqrt{63Y^2}}{\sqrt{9Y}} = \sqrt{\frac{63Y^2}{9Y}} = \sqrt{7Y}$

$\frac{\sqrt{75}}{\sqrt{3}} = \sqrt{\frac{75}{3}} = \sqrt{25} = 5$

$\frac{\sqrt{135}}{\sqrt{15}} = \sqrt{9} = 3$

It is sometimes useful to move a factor under the square root radical. To do this, square the factor, multiply it by the radicand, and then write the product under the radical. For example:

$2\sqrt{7} = \sqrt{4(7)} = \sqrt{28}$

$\frac{1}{X}\sqrt{35 + 4X} = \sqrt{\frac{1}{X^2}(35 + 4X)} = \sqrt{\frac{35}{X^2} + \frac{4}{X}}$

$2Y\sqrt{7Y - 8} = \sqrt{(4Y^2)(7Y - 8)} = \sqrt{28Y^3 - 32Y^2}$

The square root of a number between zero and one is always greater than the number itself; for example, in the equation $\sqrt{.25} = .5$, the square root .5 exceeds .25 which is the number itself. Likewise:

$\sqrt{.01} = .1$, $\sqrt{.09} = .3$, $\sqrt{0.0144} = .12$

Computation with radicals is often greatly facilitated by factoring the radicand into two factors, one of which is a perfect square. The root of this perfect square may then be extracted and the result multiplied by the indicated root of the remaining factor. For example:

$$\sqrt{640} = \sqrt{64}\ \sqrt{10} = 8\sqrt{10}$$

$$\sqrt{\frac{27}{16}} = \sqrt{\frac{9}{16}}\ \sqrt{3} = \frac{3}{4}\sqrt{3}$$

$$\sqrt{32a^3} = \sqrt{16a^2}\ \sqrt{2a} = 4a\sqrt{2a}$$

<u>Rationalizing a Denominator</u>

It is obviously easier to find the approximate value of an irrational number which calls for multiplication by a radical, as $7\sqrt{5}$ or $\frac{2}{3}\sqrt{106}$, than one which calls for division by a radical, as $\frac{5}{\sqrt{7}}$ or $\frac{2}{3\sqrt{106}}$. However, when both terms of the fraction $\frac{5}{\sqrt{7}}$ are multiplied by $\sqrt{7}$, we have $\frac{5}{\sqrt{7}}\ \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{\sqrt{49}} = \frac{5\sqrt{7}}{7}$ with the radical in the numerator where it is easier to work with arithmetically. This process is called <u>rationalizing the denominator</u>. Examples:

$$\sqrt{\frac{4}{27}} = \sqrt{\frac{4}{27}\cdot\frac{3}{3}} = \sqrt{\frac{12}{81}}$$

$$= \sqrt{\frac{4}{81}}\sqrt{3} = \frac{2}{9}\sqrt{3}$$

Multiplying both numerator and denominator by the smallest number which will make the denominator a perfect square.

$$\sqrt{\frac{35}{120} - \left(\frac{7}{60}\right)^2} = \sqrt{\frac{35}{120} - \frac{49}{3600}} = \sqrt{\frac{35}{120}\cdot\frac{30}{30} - \frac{49}{3600}}$$

$$= \sqrt{\frac{1050}{3600} - \frac{49}{3600}} = \sqrt{\frac{1001}{3600}} = \frac{1}{60}\sqrt{1001}$$

$$\sqrt{\frac{a^3}{b}} = \sqrt{\frac{a^3}{b}\cdot\frac{b}{b}} = \sqrt{\frac{a^2}{b^2}\cdot\frac{ab}{1}} = \frac{a}{b}\sqrt{ab}$$

$$7\sqrt{\frac{2}{7}} = 7\sqrt{\frac{2}{7}\cdot\frac{7}{7}} = 7\sqrt{\frac{14}{49}} = 7(\frac{1}{7})\sqrt{14} = \sqrt{14}$$

Problems

Simplify and check your answers with the key.

1. $\sqrt{2}\ \sqrt{18} =$
2- $\sqrt{3}\ \sqrt{15} =$
3. $\frac{6}{\sqrt{2}} =$
4. $\sqrt{40} =$
5. $\frac{\sqrt{32}}{\sqrt{8}} =$
6. $\sqrt{\frac{2}{3}} =$
7. $\sqrt{98} =$
8. Reduce $\sqrt{\frac{107}{24} - (\frac{5}{12})^2}$ to a form in which the radicand is a single integer.
9. $4\sqrt{3}\ \sqrt{3} =$
10. $3\sqrt{2} \cdot 5\sqrt{8} =$
11. $\frac{\sqrt{125X^3}}{\sqrt{5X}} =$
12. $6\sqrt{\frac{3}{2}} =$
13. $\frac{\sqrt{75AB^2}}{\sqrt{3B}} =$

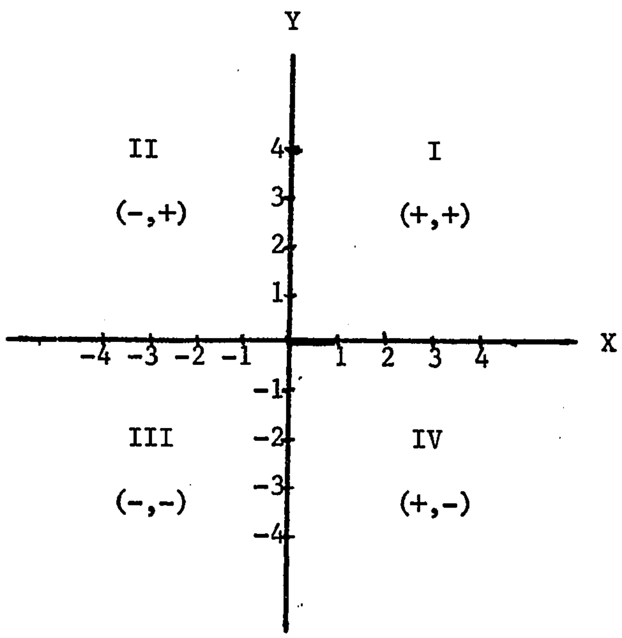
Transform the next three problems so that each has its entire expression simplified under a single radical.

14. $3X\sqrt{6 - 5X}$
15. $6\sqrt{\frac{7C}{9} + \frac{5}{12}}$
16. $\frac{1}{N}\sqrt{NS - T^2}$

GRAPHING

Graphing is a statistical tool that is used in many statistical procedures from simple data description to complex statistical analyses.

The point where the axes intersect at right angles is called the origin. The horizontal axis is usually designated the X-axis, while the vertical axis is usually designated the Y-axis, although other letters would serve just as well.



The axes divide the plane into four quadrants, numbered as indicated in the figure. Each point in the plane is represented by an ordered pair (X,Y). The first number is the X-coordinate, or abscissa, while the second number is the Y-coordinate, or ordinate. Together they are called the coordinates. The quadrants have coordinates with signs as indicated in the figure. We call X the independent variable; and since the value of Y depends upon the value of X, we call Y the dependent variable.

In graphing a straight line, one method is to find ordered pairs which satisfy the equation of the line. For example, for the line $2X - Y = 4$, some ordered pairs which satisfy the equation include:

$2X - Y = 4$

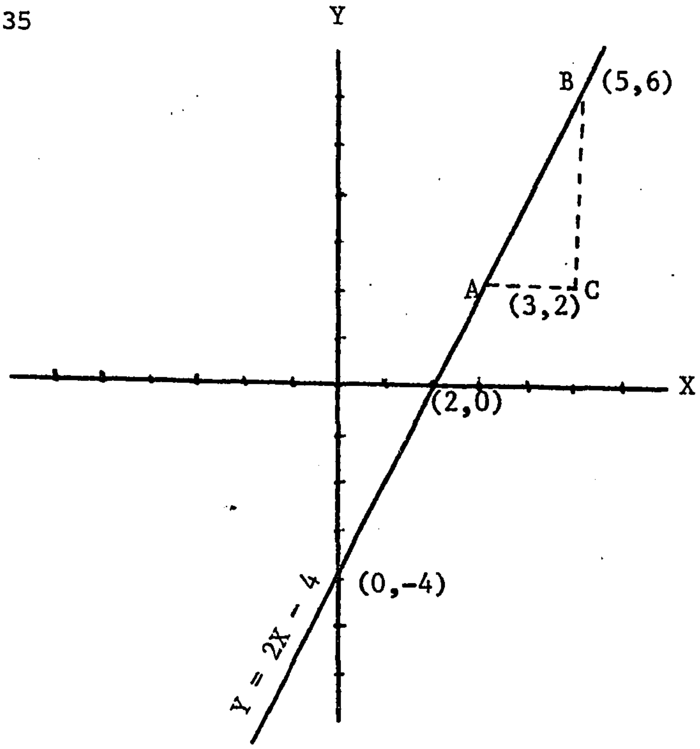
$Y = 2X - 4$

if $X = 0$, $Y = 2(0) - 4 = -4$

if $X = 3$, $Y = 2(3) - 4 = 2$

if $X = 2$, $Y = 2(2) - 4 = 0$

X	0	2	3
Y	-4	0	2



The equation $Y = 2X - 4$ is of the form $Y = mX + b$, where 2 is a special value of m and -4 is a special value of b. If we choose two points A and B on the graph where the coordinates of A are (3,2) and those of B are (5,6), we see that when a point moves on the graph from A to B, X increases from 3 to 5 and Y increases from 2 to 6. In other words, Y has an increase of 4 when X has an increase of 2. This allows us to find the <u>slope</u> which is defined as follows: The <u>slope</u> of a line equals the change in Y divided by the change in X.

From this definition the slope of the line is:

$$\text{slope} = \frac{\text{change in Y}}{\text{change in X}} = \frac{CB}{AC} = \frac{4}{2} = 2$$

In general, the slope is the ratio of the change of the dependent variable to the change of the independent variable.

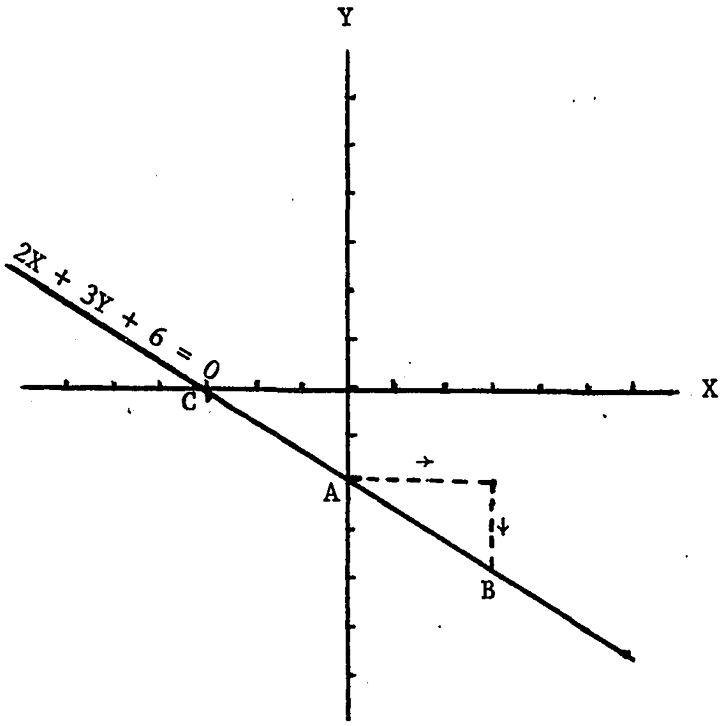
The above graph intersects the X-axis where X = 2 and the Y-axis where Y = -4. We say that the <u>X intercept</u> is 2 with coordinates (2,0) and the Y intercept is -4 with coordinates (0,-4). For the equation $Y = 2X - 4$, we have found the slope to be 2 and the Y intercept to be -4.

This shows that for an equation of the form $Y = mX + b$, the slope is m and the Y-intercept is b. Therefore, when a linear equation is written in the form $\underline{Y = mX + b}$, we know immediately the slope and Y-intercept of its graph, and can easily draw the graph.

Example: graph $2X + 3Y + 6 = 0$

solving for Y we have, $\underline{Y = -\frac{2}{3}X - 2}$

The slope is $-\frac{2}{3}$ and the Y intercept is -2. We plot the point A(0,-2). Since the slope is negative, Y decreases as X increases. As X increases 3 units, Y decreases 2 units. Then we find the point B on the graph by starting at A, going 3 units to the right and then 2 units down. The abscissa of B is 3 more than that of A and the ordinate of B is 2 less than that of A. We draw the straight line through A and B and check the solution with the point C (-3,0).



Problems

Graph the following and check with the key.

1. $Y = 2X + 4$

2. $Y = \frac{3}{4}X - 1$

3. $Y = -\frac{1}{2}X + 3$

4. $Y = \frac{2}{5}X$

5. $3X + 4Y = 16$

6. $2(X - 2Y) - 2(2X - 1) = 5$

SOLVING PAIRS OF EQUATIONS BY ADDITION OR SUBTRACTION

A pair of linear equations can be solved if they can be combined to form a third equation which has only one unknown.

Example 1. solve
$$\begin{aligned} 3X + Y &= 7 \\ X - Y &= 1 \end{aligned}$$

Solution. In order to find the common solution for these two equations, it is necessary to derive a third equation containing only X or Y. We can eliminate Y by adding the two equations.

$$\begin{array}{r} 3X + Y = 7 \\ \underline{X - Y = 1} \\ 4X \quad\;\; = 8 \end{array}$$

We then divide each side by 4 to obtain X = 2. We can solve for Y by substituting X = 2 in either of the two equations. Substituting X = 2 in the first equation, we have 3(2) + Y = 7. Solving this equation we get Y = 1. The solution of the set of equations is X = 2, Y = 1.

We could solve for Y by eliminating X. This can be accomplished by multiplying the second equation by three and then subtracting.

$$\text{Multiply by 3} \left|\begin{array}{r} 3X + Y = 7 \\ \underline{X - Y = 1} \end{array}\right. \qquad \begin{array}{r} 3X + \;\; Y = 7 \\ \underline{3X - 3Y = 3} \\ 4Y = 4 \\ Y = 1 \end{array} \quad \text{subtract}$$

Example 2. solve
$$\begin{aligned} 5X + 2Y &= -5 \\ -3X + 4Y &= 29 \end{aligned}$$

Solution.

To solve for Y we must eliminate X. The smallest number that can be exactly divided by 5 and 3 is 15. We must therefore have 15X in each equation.

$$\begin{array}{ll} \text{Multiply by 3} & \left| \; 5X + 2Y = -5 \right. \\ \text{Multiply by 5} & \left| \; \underline{-3X + 4Y = 29} \right. \\ & \quad 15X + 6Y = -15 \\ & \quad \underline{-15X + 20Y = 145} \\ \text{adding} & \qquad\quad 26Y = 130 \\ & \qquad\qquad Y = 5 \end{array}$$

To solve for X we must eliminate Y. The smallest number that can be exactly divided by 2 and 4 is 4. We must therefore have 4Y in each equation.

$$\begin{array}{ll} \text{Multiply by 2} & \left| \; 5X + 2Y = -5 \right. \\ \text{Multiply by 1} & \left| \; \underline{-3X + 4Y = 29} \right. \\ & \quad 10X + 4Y = -10 \\ & \quad \underline{-3X + 4Y = \;\; 29} \\ \text{subtracting} & \quad 13X \qquad\; = -39 \\ & \quad\;\; X \qquad\;\; = -3 \end{array}$$

After the value of Y has been found, we can find the value of X by substituting $Y = 5$ in the equation $5X + 2Y = -5$. We then have $5X + 2(5) = -5$. Solving, we get $X = -3$, as we found in the right hand column above.

You can check your values for X and Y by showing that they satisfy both equations. For example 2, we would have:

$$\begin{array}{rl} 5X + 2Y = -5 & 5(-3) + 2(5) = -5 \rightarrow -5 = -5 \\ -3X + 4Y = 29 & -3(-3) + 4(5) = 29 \rightarrow 29 = 29 \end{array}$$

Problems

Solve and check your answers with the key.

1. $$\begin{array}{r} 5X - 2Y = 39 \\ \underline{3X - 2Y = 25} \end{array}$$

2. $$\begin{array}{r} a + 2b = 8 \\ \underline{2a - b = 6} \end{array}$$

3. $$\begin{array}{r} 5X - 2Y = 0 \\ \underline{X - Y = -3} \end{array}$$

4. $$\begin{array}{r} 3X + 2Y = -9 \\ \underline{5X + 3Y = -19} \end{array}$$

5. $$\begin{array}{r} 4(X + Y) + 5(X + 1) = -16 \\ \underline{3(3X - 2Y) - (2X - 3Y) = 2} \end{array}$$

6. $$\frac{7a}{4} + \frac{5b}{3} + 3 = 0$$
$$\frac{a}{2} - \frac{b}{3} = 4$$

LINEAR INTERPOLATION

Linear interpolation is an important concept in the use of many of the tables required in statistics. Some examples should clarify the concept.

Example 1. Assuming we have a table with the following entries.

X	Y
0.76	0.2764
0.77	0.2794
0.78	0.2823
0.79	0.2852

Suppose we have a value of 0.774 for X and want to know what value of Y corresponds to this value of X.

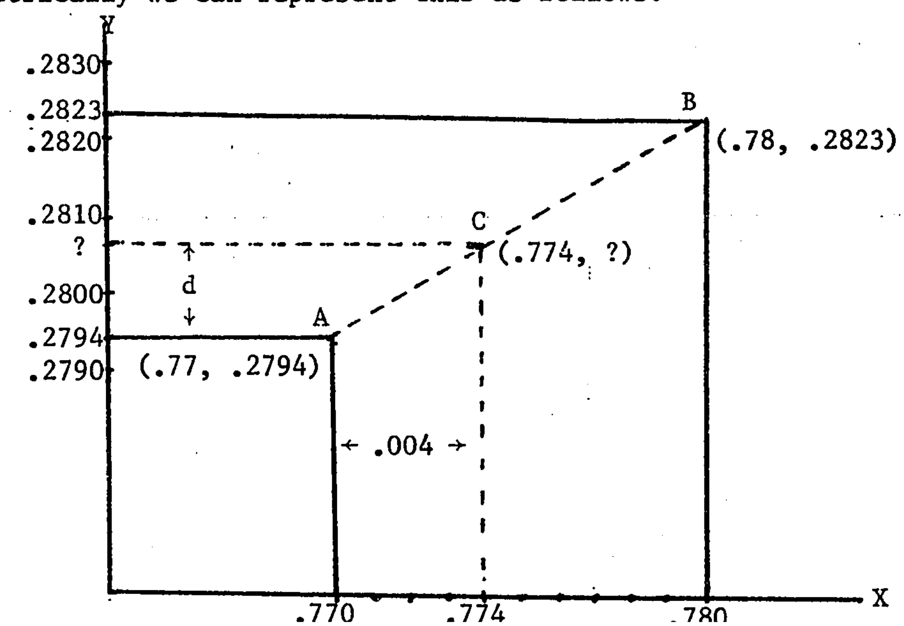
		X	Y		
.010	.004	0.770	0.2794	d	.0029
		0.774	?		
		0.780	0.2823		

Using 0.774, we see that we are $\frac{0.004}{0.010} = \frac{4}{10}$ of the way from 0.770 to 0.780. We can then take $\frac{4}{10}$ of the difference in the Y values to obtain d, and then add d to the smaller Y value to get our desired result.

$d = \frac{4}{10}(0.0029) = \frac{0.0116}{10} = 0.00116 = 0.0012$ if we round to 4 places past the decimal as the other values of Y are. Hence our desired result for Y is:

$Y = 0.2794 + d = 0.2794 + 0.0012 = 0.2806$

Geometrically we can represent this as follows:



The problem is to find the Y coordinate when we know that X = 0.774. This can be accomplished by drawing a vertical line at X = 0.774 to locate point C on the line segment AB, and then drawing a horizontal line from C to the Y-axis to determine the Y coordinate. This is too time consuming to be convenient, therefore, it is usually done as illustrated previously.

We can also interpolate in reverse. Suppose we have a value of 0.2841 for Y, and want to know what value of X corresponds to this Y value. By checking the table, we see that the X value will be between 0.780 and 0.790.

		X	Y		
0.010	c	0.780	0.2823	0.0018	0.0029
			0.2841		
		0.790	0.2852		

Using 0.2841, we see that we are $\frac{0.0018}{0.0029} = \frac{18}{29}$ of the way from 0.780 to 0.790. We can then take $\frac{18}{29}$ of the difference in the X values to obtain c, and then add c to the smaller X value to get our desired result.

$c = \frac{18}{29}(0.010) = 0.0062 = 0.006$ if we round to 3 places past the decimal as the other values of X are. Hence our desired result for X is:

$$X = 0.780 + c = 0.780 + 0.006 = 0.786$$

Problems

Solve and check your answers with the key. Given the table with the following entries.

X	Y
0.53	0.2019
0.54	0.2054
0.55	0.2088
0.56	0.2123
0.57	0.2157
0.58	0.2190

1. Find Y when X = 0.563
2. Find X when Y = 0.2110
3. Find Y when X = 0.548
4. Find X when Y = 0.2173

FACTORIALS

The factorial sign "!" indicates a special repeated multiplication which is used frequently in statistical applications. Here are some examples:

$3! = 3 \cdot 2 \cdot 1 = 6$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

In general, when n is an integer,

$n! = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$

<u>0! is defined to be 1</u>

Here are some examples that show "canceling."

$$\frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 5 \cdot 4 = 20$$

$$\frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 6 \cdot 5 \cdot 4 = 120$$

$$\frac{5!}{4!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 5$$

$$\frac{5!}{4!} = \frac{5 \cdot 4!}{4!} = 5$$

$$\frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3!}{3!} = 5 \cdot 4 = 20$$

The above examples show that $\frac{n!}{r!} = n(n-1)(n-2) \cdots (r + 1)$ where r is some integer less than n. In the example $\frac{5!}{3!}$, let $n = 5$ and $r = 3$. Then $r + 1 = 4$, and the answer is $5 \cdot 4 = 20$. Also, $\frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4$ because $n = 7$, $r + 1 = 4$ and $n(n-1) \cdots (r + 1) = 7 \cdot 6 \cdot 5 \cdot 4$. In general, $n! = n(n-1)! = n(n-1)(n-2)! = \cdots = n(n-1)(n-2) \cdots 2 \cdot 1$.

For example: $7! = 7 \cdot 6! = 7 \cdot 6 \cdot 5! = 7 \cdot 6 \cdot 5 \cdot 4! = \cdots = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$.

Problems

Solve and check your answer with the key.

1. $6! =$

2. $(4!)(2!) =$

3. $\frac{9!}{7!} =$

4. $4! + 3! =$

5. $4! - 3! =$

6. $\frac{8!\ 3!}{7!} =$

7. $\frac{5!}{6!\ 2!} =$

CALCULATING SQUARE ROOT

First, break the number under the radical sign into groups of two digits starting at the decimal point.

```
1.  √5'90!49
```

Second, estimate the square root of the first group of digits. This is the first digit in the answer. In the example, this is 2, so we write it above the 5.

```
2.   2
    √5'90!49
```

Third, square the estimate ($2^2 = 4$) and subtract it as you would in long division. Bring down the next two digits.

```
3.   2
    √5'90!49
     4          ← 2 x 2
     1 90
```

Fourth, double the 2 to make it 4 and multiply this by 10 to make it 40. Write the 40 to the left of 190.

```
4.   2
    √5'90!49
     4          ← 2 x 2
 40  1 90
```

Fifth, the 40 is to be divided into 1 90 after an adjustment. Since 190 ÷ 4 is 4, or more, let 4 be the next digit in the answer. Add the 4 to the 40 to give 44. Subtract 4 x 44 from the 190. Bring down the next two digits.

```
5.   2  4
    √5'90!49
     4          ← 2 x 2
 40  1 90
+ 4  1 76       ← 44 x 4
 44    14 49
```

Sixth, steps 4 and 5 are repeated. Double 24 to make it 48 and multiply this by 10 to make it 480. Then 1449 ÷ 480 is about 3, so the next digit in the answer is 3. Add the 3 to the 480 and divide the 1449 by the result. Subtract 3 x 483 from 1449 to show no remainder. Insert the decimal point. The solution (24.3) is complete, and there is no rounding error in this example.

```
6.          2  4  3
          √5'90!49
            4        ← 2 x 2
   40       1 90
   +4       1 76     ← 44 x 4
   44  480  14 49
       + 3  14 49  ← 483 x 3
       483       0
```

You can continue the process for as much accuracy as you like.

```
                        7. 5  8  0
                      √57!46'00'00                   √57.460000 = 7.580
                        49         ← 7 x 7
140                      8 46
+ 5                      7 25      ← 145 x 5
145  1500                1 21 00
      + 8                1 20 64   ← 1508 x 8
     1508  15160            1 36 00
```

Here are two additional worked examples:

```
                       . 2  4  1
                     √0!05'82'25
                         4        ← 2 x 2
40                       1 82
+4                       1 76     ← 44 x 4
44   480                    6 25
     + 1                    4 81 ← 481 x 1
     481   4820             1 44 00

                     2. 7  1  1
                     √7!35'00'00
                      4
40                    3 35
+7                    3 29
47   540                 6 00
     + 1                 5 41
     541   5420            59 00
            + 1            54 21
           5421             4 79
```

Problems

Calculate the following square roots and check your answers with the key.

1. $\sqrt{2513.0169}$ to the nearest .01

2. $\sqrt{137789.44}$ to the nearest .1

3. $\sqrt{65.98}$ to the nearest .001

4. $\sqrt{0.0342}$ to the nearest .001

5. $\sqrt{1.0346}$ to the nearest .001

KEY

Summation

1. 27
2. 16
3. $(7^2 + 4^2 + 5^2 + 3^2) = 99$
4. $(7 + 4 + 5)^2 = 256$
5. $(5 + 3 + 6 + 2) = 16$
6. $3(7 + 4 + 5 + 3) = 57$
7. $(7 + 4 + 5) - 3(4) = 4$
8. $5(6) = 30$
9. $2(7 + 4 + 5) - 3(3) = 23$
10. 3
11. 2
12. $(4 + 5 + 2 + 2) = 13$
13. $(2 + 0 + 5) = 7$
14. $(3 + 5 + \cdots + 2) = 32$
15. $(3 + 1 + 2 + 2) = 8$

Absolute Value

1. 2
2. 10
3. 10
4. 0
5. 10
6. 7
7. 7
8. 8

Equations with One Unknown

1. $X = 5$
2. $X = 12$
3. $X = 5$
4. $X = \frac{a + 7}{2}$
5. $X + 3a = 15$
   $X = 15 - 3a$
6. $X = 21$
7. $X = \frac{9}{8}$
8. $12X - 3 = 28$
   $12X = 31$
   $X = \frac{31}{12}$
9. $20X - 6a = 5$
   $20X = 6a + 5$
   $X = \frac{6a + 5}{20}$
10. $X = 1$
11. $X = -2$
12. $6X - 3 = 4 - 2X$
    $8X = 7$
    $X = \frac{7}{8}$

13. $$\begin{aligned} 7aX + 16cX &= 10 \\ X(7a + 16c) &= 10 \\ X &= \frac{10}{7a + 16c} \end{aligned}$$

14. $X = \dfrac{a + 1}{5a + 12}$

15. $$\begin{aligned} 2X - aX &= 7 \\ X(2 - a) &= 7 \\ X &= \frac{7}{2 - a} \end{aligned}$$

16. $X = \dfrac{3(4 + 7)}{2 - (-2)(3)} = \dfrac{3(11)}{2 + 6} = \dfrac{33}{8}$

17. $$\begin{aligned} d(f - 3) &= aX - ac \\ df - 3d + ac &= aX \\ X &= \frac{df - 3d + ac}{a} \\ X &= \frac{\frac{2}{3}(\frac{3}{4}) - 3(\frac{2}{3}) + \frac{2}{5}(\frac{4}{3})}{\frac{2}{5}} \end{aligned}$$

$$= \frac{\frac{1}{2} - 2 + \frac{8}{15}}{\frac{2}{5}} = \frac{\frac{15}{30} - \frac{60}{30} + \frac{16}{30}}{\frac{2}{5}}$$

$$= \frac{-\frac{29}{30}}{\frac{2}{5}} = \frac{29}{30}\cdot\frac{5}{2} = -\frac{29}{12}$$

Inequalities

1. $K \le 16$

2. $K \ge -1$

3. $$\begin{aligned} -2X &< 4 \\ X &> -2 \end{aligned}$$

4. $X \ge -6$

5. $-9 \le X$ or $X \ge -9$

6. $$\begin{aligned} -4X &\ge -1 \\ X &\le \frac{1}{4} \end{aligned}$$

7. $$\begin{aligned} 40X - 15 &< 12 \\ 40X &< 27 \\ X &< \frac{27}{40} \end{aligned}$$

8. $$\begin{aligned} 8 &= (2)(2)(2) \\ 4 &= (2)(2) \\ 3 &= (1)(3) \\ 6 &= (2)(3) \\ \text{LCD} &= (2)(2)(2)(3) = 24 \end{aligned}$$

$$24(\tfrac{3}{8}) - 24(\tfrac{1}{4}X) \ge 24(\tfrac{2}{3}X) - 24(\tfrac{1}{6})$$

$$9 - 6X \ge 16X - 4$$

$$13 \ge 22X$$

$$\frac{13}{22} \ge X \text{ or } X \le \frac{13}{22}$$

Exponents

1. X^9

2. a^3

3. $-12X^6$

4. $-28a^3$

5. a^5

6. X^2

7. $a^{5-(-2)} = a^7$

8. $-4a^2$

9. $8a^3$

10. $7^{7-4} = 7^3$

11. $-4y^2$

12. $\frac{8z^3}{c^9}$

13. $-24a^{1+3+4} = -24a^8$

14. $(27a^6b^3c^9)(4a^2b^6c^4) = 108a^8b^9c^{13}$

15. $\frac{5^3}{5^3} = 5^{3-3} = 5^0 = 1$

16. 1

17. b^{-10}

18. 1

Radicals

1. $\sqrt{36} = 6$

2. $\sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5}$

3. $3\sqrt{2}$

4. $2\sqrt{10}$

5. 2

6. $\frac{1}{3}\sqrt{6}$

7. $\sqrt{49(2)} = 7\sqrt{2}$

8. $\sqrt{\frac{107}{24}(\frac{6}{6}) - \frac{25}{144}} = \sqrt{\frac{642}{144} - \frac{25}{144}} = \sqrt{\frac{617}{144}} = \frac{1}{12}\sqrt{617}$

9. $4(3) = 12$

10. $15\sqrt{16} = 15(4) = 60$

11. $\sqrt{25X^2} = 5X$ if $X \geq 0$

12. $6\sqrt{\frac{6}{4}} = \frac{6}{2}\sqrt{6} = 3\sqrt{6}$

13. $\sqrt{25AB} = 5\sqrt{AB}$

14. $\sqrt{9X^2(6 - 5X)} = \sqrt{54X^2 - 45X^3}$

15. $\sqrt{36(\frac{7c}{9} + \frac{5}{12})} = \sqrt{36(\frac{7c}{9}) + 36(\frac{5}{12})} = \sqrt{28c + 15}$

16. $\sqrt{\frac{1}{N^2}(NS - T^2)} = \sqrt{\frac{S}{N} - \frac{T^2}{N^2}}$

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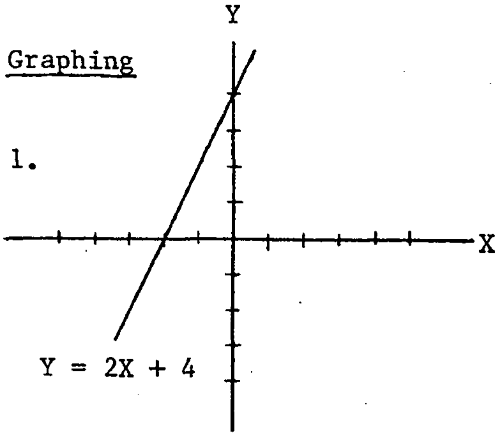
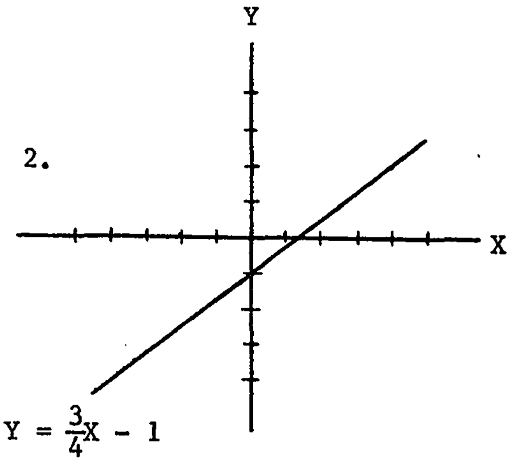
Graphing

1.



2.



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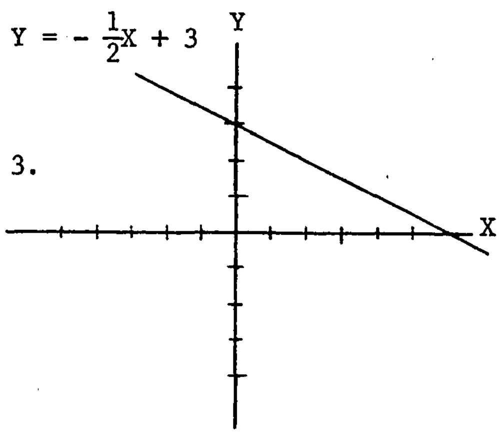

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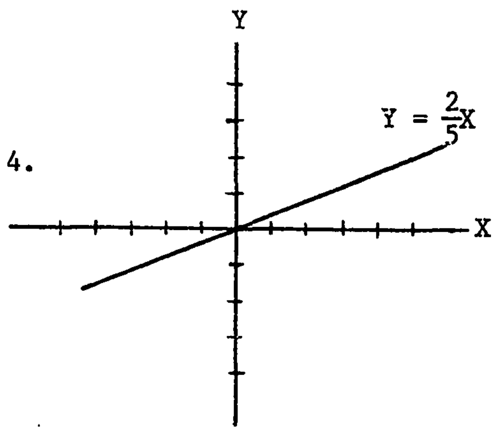

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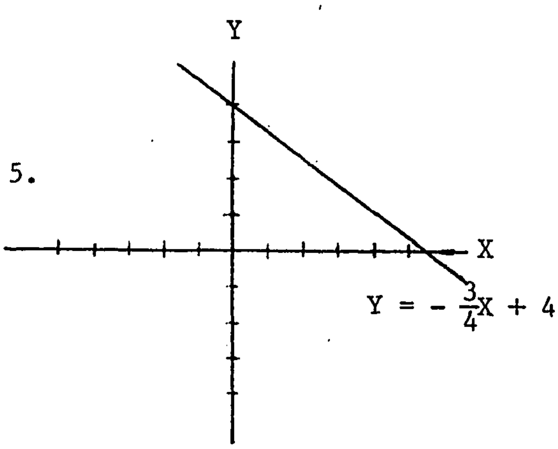
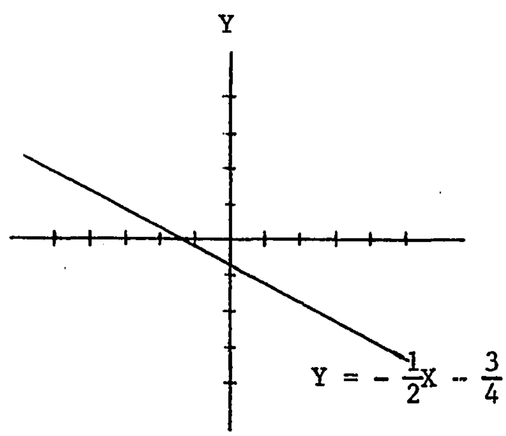


6. $2X - 4Y - 4X + 2 = 5$

$$-2X - 4Y = 3$$
$$2X + 4Y = -3$$
$$4Y = -2X - 3$$
$$Y = -\frac{1}{2}X - \frac{3}{4}$$



Solving Pairs of Equations by Addition or Subtraction

1. X = 7, Y = -2

2. a = 4, b = 2

3. X = 2, Y = 5

4. X = -11, Y = 12

5. $4X + 4Y + 5X + 5 = -16$
   $9X - 6Y - 2X + 3Y = 2$

   $$\begin{array}{rcl} 9X + 4Y &=& -21 \\ \underline{7X - 3Y} &=& \underline{2} \end{array} \qquad \begin{array}{rcl} 27X + 12Y &=& -63 \\ 28X - 12Y &=& 8 \\ \hline 55X &=& -55 \end{array}$$

   X = -1, Y = -3

6. $$\begin{array}{rcl} 21a + 20b &=& -36 \\ 3a - 2b &=& 24 \end{array} \qquad \begin{array}{rcl} 21a + 20b &=& -36 \\ 30a - 20b &=& 240 \\ \hline 51a &=& 204 \end{array}$$

   a = 4, b = -6

Linear Interpo ation

1.

		X	Y		
.010	.003	0.560	0.2123	d	.0034
		0.563			
		0.570	0.2157		

We want $\frac{0.003}{0.010} = \frac{3}{10}$ of 0.0034. $d = \frac{3}{10}(0.0034) = \frac{0.0102}{10} = 0.0010$

(rounding to 4 places past decimal)

Y = 0.2123 + 0.0010 = 0.2133

2.

		X	Y		
.010	c	0.550	0.2088	.0022	.0035
			0.2110		
		0.560	0.2123		

We want $\frac{0.0022}{0.0035} = \frac{22}{35}$ of 0.010. $c = \frac{22}{35}(0.010) = 0.0063 = 0.006$

(rounding to 3 places past decimal)

X = 0.550 + 0.006 = 0.556

3. Y = 0.2081

4. X = 0.575

Factorials

1. 720
2. 48
3. 72
4. 24 + 6 = 30
5. 24 - 6 = 18
6. $\frac{8 \cdot 7!3!}{7!} = 8 \cdot 3! = 8(6) = 48$
7. $\frac{5!}{6 \cdot 5!2!} = \frac{1}{6 \cdot 2!} = \frac{1}{12}$

Square Root

1. 50.13
2. 371.2
3. 8.123
4. 0.185
5. 1.017

APPENDIX C: ATTITUDINAL QUESTIONNAIRES

NAME________________________

ATTITUDINAL QUESTIONNAIRE

This is not an examination; it is part of a project to study the attitudes of students toward statistics. Please write your name in the upper right hand corner so that different groups of students can be compared; however, no results will be used in any way that will affect your grade in this or in any other course. We are interested in your feelings or opinion about each statement.

After you have read each statement, please circle the "A" (agree) if you agree with the statement or the "D" (disagree) if you disagree with the statement. Once you have made this decision, please indicate how strongly you agree or disagree with the statements by circling one of the numbers which appears to the right of each statement.

For example, consider the statement:

		slight strong
All men are created equal.	A D	1 2 3 4 5

Do you agree or disagree with this statement? Circle "A"("D"). How strongly do you agree(disagree) with this statement? Circle the appropriate number.

Please be sure to circle both a number and a letter after each statement, unless you are completely undecided whether you agree or disagree with the statement. In that case, circle both "A" and "D", but do not circle any of the numbers. This response indicates that you neither agree nor disagree with the statement.

There are no right or wrong answers to the statements. The answers which will be most helpful to this project are the ones which best reflect your own feelings about each of the statements. Thank you for your cooperation.

		slight strong
1. It scares me to have to take statistics.	A D	1 2 3 4 5
2. The feeling I have toward statistics is a good feeling.	A D	1 2 3 4 5
3. Statistics can be made understandable to almost every college student.	A D	1 2 3 4 5
4. I can't see where statistics will ever help me.	A D	1 2 3 4 5
5. I don't think I can ever do well in statistics.	A D	1 2 3 4 5
6. Only people with a very special talent can learn statistics.	A D	1 2 3 4 5
7. Statistics is fascinating and fun.	A D	1 2 3 4 5
8. I feel a sense of insecurity when attempting statistics.	A D	1 2 3 4 5
9. I feel at ease in statistics.	A D	1 2 3 4 5
10. Statistics is something which I enjoy a great deal.	A D	1 2 3 4 5
11. When I hear the word statistics, I have a feeling of dislike.	A D	1 2 3 4 5
12. I do not like statistics.	A D	1 2 3 4 5
13. Statistics makes me feel secure.	A D	1 2 3 4 5

Item		slight strong
14. Statistics is stimulating.	A D	1 2 3 4 5
15. Very few people can learn statistics.	A D	1 2 3 4 5
16. It makes me nervous to even think about having to do a statistics problem.	A D	1 2 3 4 5
17. Statistics is enjoyable.	A D	1 2 3 4 5
18. I approach statistics with a feeling of hesitation—hesitation from a fear of not being able to do statistics.	A D	1 2 3 4 5
19. I really like statistics.	A D	1 2 3 4 5
20. I wish I were not required to study any statistics.	A D	1 2 3 4 5
21. Statistics makes me feel uncomfortable, restless, irritable, and impatient.	A D	1 2 3 4 5
22. Any person of average intelligence can learn to understand a good deal of statistics.	A D	1 2 3 4 5
23. I would like to study more statistics whether or not it is required for my program.	A D	1 2 3 4 5
24. Statistics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.	A D	1 2 3 4 5
25. Almost anyone can learn statistics if he is willing to study.	A D	1 2 3 4 5
26. Statistics is very interesting to me.	A D	1 2 3 4 5

NAME________________________

ATTITUDINAL QUESTIONNAIRE

The use of self-instructional materials is the method by which you have just learned. We are interested in your feelings or opinion about each statement.

After you have read each statement, please circle the "A" (agree) if you agree with the statement or the "D" (disagree) if you disagree with the statement. Once you have made this decision, please indicate how strongly you agree or disagree with the statements by circling one of the numbers which appears to the right of each statement.

For example, consider the statement:

		slight strong
All men are created equal.	A D	1 2 3 4 5

Do you agree or disagree with this statement? Circle "A"("D"). How strongly do you agree(disagree) with this statement? Circle the appropriate number.

<u>Please be sure to circle both a letter and a number after each statement</u>, <u>unless</u> you are <u>completely undecided</u> whether you agree or disagree with the statement. In that case, circle <u>both</u> "A" and "D", but do not circle any of the numbers. This response indicates that you neither agree nor disagree with the statement.

	slight strong
1. The use of the self-instructional mathematics materials helped me to learn the mathematics I need for statistics.	A / D 1 2 3 4 5
2. Probably the best way to learn the mathematics I need for statistics is with a teacher.	A / D 1 2 3 4 5
3. With the self-instructional mathematics materials, I know exactly how I am doing all the time.	A / D 1 2 3 4 5
4. There is no thinking involved in learning with the self-instructional mathematics materials.	A / D 1 2 3 4 5
5. The self-instructional mathematics materials are a boring method of learning.	A / D 1 2 3 4 5
6. There is no pressure on me when I use the self-instructional mathematics materials.	A / D 1 2 3 4 5
7. The self-instructional mathematics materials offered no challenge to me.	A / D 1 2 3 4 5
8. The use of the self-instructional mathematics materials has helped to reduce my anxiety with regard to statistics.	A / D 1 2 3 4 5
9. The self-instructional mathematics materials did not provide a clear explanation of the topics presented.	A / D 1 2 3 4 5
10. The mathematical topics presented in the self-instructional mathematics materials were not sufficient to enable me to do the work in statistics.	A / D 1 2 3 4 5
11. With the use of the self-instructional mathematics materials, I never get left behind the class.	A / D 1 2 3 4 5
12. The level of difficulty of the self-instructional mathematics materials was appropriate.	A / D 1 2 3 4 5
13. The self-instructional mathematics materials are more trouble than they are worth.	A / D 1 2 3 4 5

	slight	strong

14. The use of the self-instructional mathematics materials helped to reduce the amount of time required to do my statistics assignments.

A/D	1 2 3 4 5

15. With the self-instructional mathematics materials, good students are not held back by the class.

A/D	1 2 3 4 5

16. The time I spent using the self-instructional mathematics materials would have been better spent studying statistics.

A/D	1 2 3 4 5

17. The use of the self-instructional mathematics materials is a good way to learn.

A/D	1 2 3 4 5

18. The self-instructional mathematics materials do not contain things that I can use in statistics.

A/D	1 2 3 4 5

19. The self-instructional mathematics materials would have been more useful if I could have used them before I started statistics.

A/D	1 2 3 4 5

20. The self-instructional mathematics materials would have reduced my anxiety with regard to statistics if I could have used them before I started statistics.

A/D	1 2 3 4 5