AGRONOMIC AND ECONOMIC EVALUATION OF DIRECT AND RESIDUAL CROP RESPONSES TO VARIOUS FERTILIZER NUTRIENTS

by

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INTRODUCTION

More and more emphasis has been placed on efficiency in agricultural production in recent years. Increased production costs coupled with lower crop prices have made it imperative that farmers have more exact information on costs and returns from different farm enterprises if they are to make better choices among various alternatives. It is the task of agricultural research to furnish this exact information.

In the area of soil fertility, the potential yield increases made possible through the addition of fertilizer nutrients are of considerable importance. Of the 9 practices that would contribute to a 35 bushel increase in the average corn yield in Iowa listed in the Iowa Production Capacity Study of 1952 (28), the practice of proper fertilizer and lime use was credited with over 34 percent of the increase.

Since fertilizers can play such an important part in crop production, and since efficiency in crop production is of paramount importance today, there is a great need to know, with greater exactness, the economic optimum for fertilizer applications on a given crop to be grown on a given soil. There is also a need to know the rate at which nutrients substitute for one another in producing a given yield so as to have a basis for determining least cost nutrient combinations.

Agronomic experiments in the past have not been specifically designed to provide the economic answers to the above but rather to answer agronomic questions. Single variable nutrient studies have been common, and where several nutrient combinations have been studied, too few rates of each nutrient have been included to provide an adequate basis for economic analysis which would have given rise to the answers needed.

The above points up the need to conduct some part of fertility research within a framework that lends itself to useful economic analysis. By joining forces, agronomists and economists can make faster progress toward the goal of more exact and economically sound fertilizer recommendations than each could make alone. This dissertation study is an example of a joint agronomic-economic investigation.

REVIEW OF LITERATURE

Economic

While the law of diminishing returns is an established phenomenon in most production fields, the mathematical form of the law as it applies to each field is less well established. As far as soils are concerned, Mitscherlich (39) states that this general law was known before Liebig's time.

Liebig's (33) law of the minimum states that the yield of the crop is limited by that nutrient which is found in smallest available quantity in the soil. Mitscherlich (39) states that Wollny went a step further and said that when nutrients are present in adequate available supply, crop yields would be limited by whatever physical property of the soil was in the "minimum".

Mitscherlich (39), at the turn of the century, was the first to propose a mathematical equation to express his ideas relative to the nature of yield responses to fertilizer nutrients. On the basis of his own experiments and other data, he formulated a yield response equation of the form $y = A(1-\underline{e}^{-cx})$ where y equals the yield increase per unit of x nutrient applied, A equals the maximum yield increase possible, c is a constant and e is the base of the natural logarithms. He maintained that "c" was constant for a given nutrient over different crops and growing conditions. While there was little controversy about the form of the equation, a great deal of disagreement occurred over the constancy of "c". Most workers have generally considered "c" a parameter whose value is to be estimated from the data.

Baule (1), a German mathematician, in looking at Mitscherlich's work said that the latter's mathematical concept of the yield response function was completely different from that of Liebig's as stated in the law of the minimum. According to Liebig, plants took up nutrients in a particular ratio which, mathematically, would result in a linear response function. Mitscherlich's concept, on the other hand, permitted an asymptotic approach to a maximum yield for a given nutrient up to the point where another nutrient or growth factor became limiting.

Baule (1) further proposed the incorporation of an effect quantity "h" into Mitscherlich's "c" value. This effect quantity was that amount of nutrient which resulted in one-half the maximum yield. He also expanded Mitscherlich's single variable equation so as to handle multivariable experiments.

Spillman (58), in 1933, independently proposed the equation $y = m - ar^x$, an algebraic equivalent of the Mitscherlich equation in which y is the predicted yield, m is the maximum yield that can be obtained with the fertilizer nutrient, a is the difference between m and the yield without fertilizer, r is the ratio by which one yield increment exceeds the previous increment (comparable to Mitscherlich's "c" value), and x is the rate of fertilizer nutrient applied. Spillman used the least squares method to estimate the parameters of his equations. Today, most workers (15, 20, 26, 27, 40, 44, 45, 47, 60) use the method of least squares to fit regression lines to the data regardless of the form of equation used to characterize the response function.

While the use of the least squares method improves the Mitscherlich equation in characterizing the response function, it does not solve its basic inadequacies. Heady (21) points out that the equation (a) assumes an elasticity of response of less than 1.0 over all ranges of fertilizer applications and hence cannot show ranges of increasing returns; (b) assumes that declining yields will not occur with heavy rates of fertilizer application; (c) assumes that the ratio of successive yield increments is equal over all fertilizer inputs and (d) assumes that when two nutrients are involved, the maximum yield can be obtained with a large number of nutrient combinations with the result that isoclines do not converge at a point of maximum yield.

Another general equation which has been used to characterize yield response is the Cobb-Douglas or power function $y = aF^b$ where y is the yield, a and b are constants and F is the level of the factor (35). While this equation allows for increasing, decreasing or constant yield increases, it does not allow for more than one type of yield increase to be expressed in any one response curve. Furthermore, the percentage increase in yield is constant which makes it comparable to the Mitscherlich-Spillman equations in this regard. For decreasing yield increases, the total yield also continues to increase without limit.

The quadratic polynomial avoids all of the above restrictions and, therefore, has greater validity for characterizing yield response functions. Heady (21) points out that this form of equation allows for (a) flexible marginal elasticity and flexible yield ratios for successive fertilizer inputs; (b) the reaching of maximum yields followed by declining yields; and (c), converging isoclines at the point of maximum yield when two or more nutrients are applied. Mason (36) outlines the advantages of the polynomial as (a) furnishing the flexibility needed for biological data; (b) being easily fitted to the data by the method of least squares and (c), allowing for the calculation of the standard errors for the parameters of the equation. Each of the previously mentioned equations, even after they have been modified as was recently the case for the Mitscherlich equation (35), lack some aspect of the points brought out by Heady or Mason and, therefore, are less desirable for general use in characterizing fertilizer response functions.

Numerous workers are using modified forms of the quadratic equation[1] to characterize their data. Heady *et al.* (22) used a square-root transformation of the quadratic to characterize the response surface for two nutrient variable experiments on corn and red clover hay. Brown (6) and Brown *et al.* (7) used the ordinary and mixed quadratic form while Jensen (30) used the ordinary quadratic form to characterize yield curves or surfaces.

Hurst and Mason (25) and Hader *et al.* (18) used the composite experimental designs proposed by Box and Wilson (5) and Box (4). These designs help solve the physical problems of the large areas required for factorial experiments mentioned by Pesek (46). The reduction in area size is achieved through the use of key, orthogonal comparisons which make for easy computation and at the same time, adequately character-

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[1]Hereafter, an equation of the form $Y = aX + bX^2 + cZ + dZ^2 + \cdots + k$ will be called the quadratic. If the variables are shown in the form $Y = aX^{1/2} + bX + cZ^{1/2} + dZ + \cdots + k$, the equation will be called the square-root transformation of the quadratic or, more simply, the root form. If an equation contains both of the above forms, it will be called the mixed form of the quadratic.

ize quadratic type responses. With reasonable replication, the precision inherent in the usual factorial experiment can be approached by the Box design.

As Heady (21) says, there is perhaps no one algebraic form of equation that will adequately characterize the response function for any one crop under all soil and climatic conditions. However, a generalized function such as shown by Jensen (30) and Hanway and Dumenil (19) is possible under a given climatic condition and on a given soil for a specific crop. In this dissertation, specific forms of the general quadratic polynomial equation are used to characterize the data.

Agronomic

Nutrient composition

Relevant questions in the area of nutrient composition revolve about (a) crop yield and nutrient supply in the soil; (b) nutrient composition of the plant and nutrient supply in the soil; and (c), nutrient composition of the plant and crop yield.

As far as crop yield and nutrient supply in the soil are concerned, Liebig (33) said that crop yield was dependent on the nutrient in minimum supply in the soil. He further stated that the addition of the limiting nutrient resulted in a linear increase in yield. Mitscherlich, (39) on the other hand, proposed the law of diminishing returns for yield with increasing applications of the limiting nutrient. The accumulated evidence from about 1900 to date supports Mitscherlich's general view, if not his specific equation for characterizing the yield curve.

As far as nutrient composition of the plant and nutrient supply in the soil are concerned, Goodall and Gregory (17) state that the internal concentration of a nutrient in a plant or its parts is not necessarily a good indication of the adequacy of the available nutrient supply in the soil. For example, a deficiency of nitrogen in the plant reduces growth but may not greatly reduce phosphorus or potassium absorption. Hence, these latter nutrients accumulate and show a relatively high concentration in the plant even though the available supply in the soil is far from adequate. The same would be true when lack of available phosphorus reduces growth. Nitrogen and potassium would tend to accumulate and plant analysis might indicate a high available supply of these nutrients in the soil. This high status may be far from the true state of affairs had phosphorus been present in amounts sufficient to promote good plant growth.

Goodall and Gregory (17) have further shown that the time of sampling during the development of the plant as well as the plant part sampled influence the concentration of nutrient found. These investigators, as well as Lundegårdh (34), who worked with small grains, point out that drouth may increase the concentration of each nutrient present in whole plant tissue. According to Lundegårdh, nitrogen percentage generally shows the greatest increase, with potassium intermediate and phosphorus the lowest increase.

Spies (57) found the critical nitrogen percentage in corn leaves at silking time to be 2.33 and 2.49 percent, respectively, for two experiments conducted under dry conditions. These results agree with the critical level of 2.5 percent nitrogen found for corn leaves taken at the same time under drouth conditions by Nicholson et al. (43) and are lower than those obtained by Tyner (61) and Bennett et al. (3) for corn leaves obtained from plants growing under more favorable moisture conditions. Thus, the corn leaf analyses did not follow Lundegarth's observations for small grain at harvest but this may be due to crop differences and the different times of sampling.

In light of the above, the use of plant composition as an indicator of the nutrient status of the soil would be relatively unsatisfactory unless correlated with a specific plant part sampled at a specific time during growth under specific environmental conditions. Even then, results could be very misleading in view of possible accumulations of certain other nutrients in the plant when a specific nutrient is in deficient supply in the soil.

In spite of the limitations of plant or leaf composition as a diagnostic tool in assessing fertility requirements of a soil, Lundegårdh (34) insists that the method has greater reliability than chemical soil tests. In 1942, he compared leaf analyses with soil analyses made for 59 field experiments with cereals and 53 field experiments with grasses and found that the leaf or plant analyses were considerably better estimators of nitrogen, phosphorus and potassium requirements than were the soil tests using Egner's method (13). However, he did not state whether the soil tests had been thoroughly calibrated against field experiments previously conducted on the various soils concerned.

The relation of yield to internal nutrient concentration has been studied by many workers (2, 23, 29, 51, 52, 65). Macy (35) distinguished three portions of the curve relating yield to internal concentration; (a) the minimum percentage region where yield rises but nutrient percentage remains relatively constant; (b) the poverty adjustment region where both yield and nutrient percentage increase but where yield increases at a decreasing rate; and (c), the luxury consumption region where nutrient percentage continues to increase but yields no longer increase. Macy called the point at which yields no longer increase as the "critical percentage".

Goodall and Gregory (17) point out that while Macy's ideas can be substantiated for yield increases due to application rates of a single nutrient with all others present in adequate quantities, they are less well substantiated when two or more nutrients are varied together. They state that under such conditions, it is unlikely that the concentration of one nutrient in the plant will be related to yield over the whole range of nutrient application. This point needs more investigation since much of the work reported in the literature deals with varying rates of one nutrient while others are held constant in relatively adequate amounts.

Tyner (61), working with the 6th corn leaf sampled at full silk, tentatively proposed 2.90 percent nitrogen, 0.295 percent P_2O_5, and 1.30 percent K_2O as the critical nutrient percentages for corn. In his study, each nutrient was varied independently with the other major nutrients present in adequate amounts in the soil.

Bennett _et al._ (3), in studying the response of corn to nitrogen side-dressing in 8 different experiments, found that the percent nitrogen and phosphorus in the 6th corn leaf increased with nitrogen applications. The correlation between increase in nitrogen percentage and increase in yield was 0.96 and was highly significant. The critical nitrogen percentage appeared to range from 2.8 to 3.0 percent.

Ellis et al. (14) found that leaf sampling date for corn was very important. For the 3rd leaf from the base of the plant, the nitrogen content for maximum yield varied from as much as 3.00 percent when corn was 3 feet high to only 1.90 percent at early ear stage. Phosphorus content also decreased with aging but calcium and magnesium content increased. An increase in nitrogen fertilization rate was almost always reflected in an increase in nitrogen percentage in the leaf. Phosphorus applications were reflected in increased phosphorus percentages in the leaf early in growth.

Viets et al. (62) and Krantz and Chandler (32) found that nitrogen additions increased nitrogen percentages in the corn leaf located below the ear when nitrogen, phosphorus and potassium were applied together in row fertilizers with or without a subsequent side-dressing of nitrogen. Both groups of workers also found that the nitrogen additions increased phosphorus uptake.

Smith et al. (55) found that rates of nitrogen up to 90 pounds per acre on wheat in an NPK factorial experiment increased the percent nitrogen in the forage clippings on an upland soil (Lufkin fine sandy loam) but decreased the nitrogen percentage on a bottomland soil (Norwood silt loam). The percent phosphorus in the clippings was decreased with increasing nitrogen applications on both soils. Yield response to nitrogen was high in both cases but response to phosphorus or potassium was low or non-existent. The decrease in nitrogen percentage on the bottomland soil was explained on the basis that yields were still increasing linearly up to the 90 pound nitrogen rate and hence the quantity of nitrogen applied was not sufficient to permit maximum yields.

All of the above emphasize that nutrient uptake by plants does not follow a simple, well defined or agreed upon pattern. Critical percentages are likely to vary with the soil concerned, moisture conditions, ion antagonism, magnitude of crop response, etc. Leaf analysis can be helpful in diagnosing deficiency symptoms but must be interpreted carefully in light of growing conditions. The degree of deficiency of any single element in the soil other than the one being checked must also be considered.

Nutrient carryover

Nutrient carryover depends on the soil and nutrient concerned, the rate of application, the length of time involved, crop utilization, microbial immobilization, etc. While each factor contributes to the reduction in carryover for a given nutrient, one or two factors may play a more dominant role than the others.

With nitrogen, for example, leaching may play a larger role in effective carryover than some of the other factors concerned. White (66), in two oat experiments in Iowa in 1953, calculated residual nitrogen from the previous year's corn to range from 0 to 49 percent for rates from 60 to 180 pounds per acre. Dumenil (12) estimated residual nitrogen from corn to oats to range from 10 to 15 percent following wet seasons and from 35 to 40 percent following dry seasons under Iowa conditions.

Cook and Scarseth (9) found that oats, wheat and corn showed a tremendous response to residual nitrogen from 42 pounds of nitrogen as cyanamid applied as a plow-down application to corn in 1939. Oat yields the following year were increased 31.6 bushels, wheat yields 3.4 bushels and corn yields 17.1 bushels. Scarseth et al. (53) further reported that 84 pounds of nitrogen as cyanamid applied for two consecutive years to corn resulted in a 59 bushel increase in oat yields the following year.

Hunter and Yungen (24) found that carryover from 100 pounds of nitrogen applied to irrigated corn in Oregon in 1952 resulted in a 25.6 bushel increase in yield of wheat in 1953 even though the increase in corn yield in 1952 was 35.5 bushels. Such carryover is unusual but may be the result of other factors becoming more favorable for growth and yield because of the presence of available nitrogen.

Phosphorus carryover may be influenced equally by more than one factor. Of these factors, phosphorus fixation or reversion can be of considerable importance. Excellent reviews of the literature on this subject have been presented by Wild (67) and Midgley (38).

Midgley (38), in reviewing representative papers on the subject, suggests that adsorption and chemical precipitation are the main mechanisms by which phosphorus is fixed in the soil. Of the two mechanisms, chemical fixation appears to be the most common under both acid and alkaline-calcareous soil conditions.

Even though fixation or reversion of applied phosphorus takes place in every soil, the degree to which this occurs may vary considerably. If periodic applications of phosphorus are made, the available soil phosphorus generally increases with time. For example, Prince (48) found that annual applications of P_2O_5 over a 36-year period on an Orangeburg fine sandy loam increased available residual soil phosphorus as measured by P^{32} uptake by crimson clover.

Caldwell et al. (8), using P^{32}, found that after 6 years, the available phosphorus in the soil as measured by Fried and Dean's "A" value had increased as much as 100 percent after annual applications of 40 pounds of P_2O_5 per acre in either citrate or water soluble form. However, only the hay yields in the corn-soybean-oats-hay rotation were increased significantly.

Webb and Pesek (64), in 1952, found that superphosphate applications which averaged 19 pounds of P_2O_5 per acre per year in a long-time rotation experiment at Ames, Iowa resulted in a buildup of available soil phosphorus as measured by soil tests and "A" values. Correlations between soil tests and yields and between "A" values for different times of sampling the oat plants and final yields were all highly significant. This buildup in available soil phosphorus indicates effective carryover of applied P_2O_5.

Smith (54) found that phosphorus carryover for applications made one and two years previously as measured by "A" values obtained in the greenhouse on three Iowa soils ranged from 55 to 75 percent. This high carryover is somewhat unusual when compared with the early work in the eastern and southeastern parts of the United States and in other parts of the world where fixation is apparently much greater.

Such fixation may have been a large part of the problem in the work reported by Cooke and Gasser (10) in England. They found that on the whole, periodic phosphorus applications over a 6-year period did not result in appreciable carryover in terms of newly applied superphosphate. Poor drainage and pH of 5.6, however, may also have contributed to the low availability of the residual phosphorus.

It has been found that on many soils, large initial applications of phosphorus may be effective for quite some time. Volk (63) found that yields of seed cotton decreased gradually when grown without phosphorus fertilization on a Hartsells fine sandy loam following a 5-year period of phosphorus fertilization at varying rates. The yield of cotton on the plot receiving 30 pounds of P_2O_5 per acre per year during the fertilized period went below that of the original check yield about 3 years after fertilization was discontinued. It took 7 years for the 60 pound rate to reach this point while all higher rates produced more cotton than was produced originally by the check plots.

McAuliffe _et al_. (37), using radio-tracer techniques, found that whole-plant oat yields and phosphorus content were increased in 1949 for P_2O_5 applications of 0, 100, 200, and 400 pounds per acre made in 1941.

Stelly and Morris (59) found that the yield of seed cotton as well as phosphorus content were increased over checks in 1951 for rates of phosphorus applied to cotton in 1949.

The above suggests that phosphorus carryover is somewhat variable. On some soils and under some conditions, carryover can be fairly large. On other soils under the same conditions carryover can be quite low. Since carryover of any nutrient affects the economics of following nutrient applications, it is important that quantitative data on carryover be obtained for various soils, rates of application, and crops grown.

OBJECTIVES

This dissertation was concerned with a rather intensive study of one experiment over a 4-year period for purposes of --

1. Delineating a methodological approach which results in equations that adequately characterize both direct and residual crop responses, nutrient percentages and nutrient yields.
2. Applying economic principles in the analysis of agronomic data for purposes of increasing the usefulness of these data in fertilizer recommendation work.
3. Evaluating total crop response over a rotation period to "once in the rotation" fertilization.
4. Studying the relationships between chemical composition and crop yield.

EXPERIMENTAL PROCEDURES

Experimental Conditions

This experiment was initiated in the spring of 1953 on a calcareous-variant Webster silty clay loam in Wright county. Prior cropping for 4 years consisted of corn, oats, corn and oats, with the last oat crop occurring in 1952. The latter crop was not seeded to a legume.

Soil test levels as obtained by the Iowa State College Soil Testing Laboratory for the surface 6 inches of soil were as follows: pH, 7.8; available nitrogen, 167 pounds per acre (high); available phosphorus, 3 pounds per acre (low); available potassium, 228 pounds per acre (medium). For comparison purposes, the classification ranges for each test are very low, low, medium and high.

Available nitrogen and phosphorus decreased from the surface to a depth of 3 feet while available potassium increased and then decreased with depth (see Table 48 in the Appendix).

A rainfall record was not kept at the experimental site until 1956. However, a weather station is located at Clarion about 9 miles northeast of the experiment. Precipitation by months for this station as well as the amounts recorded at the experimental site in 1956 are shown in Table 1.

Experimental Design and Harvesting Procedures

The fertilizer was broadcast by hand on fall-plowed ground on April 22 on 23 1/3 foot square plots arranged in a twice-replicated, randomized-block, 5 x 4 x 3 NPK[1] factorial experiment. Nutrient rates were 0, 40, 80, 160, and 240 pounds of N, 0, 40, 80 and 120 pounds of P_2O_5 and 0, 40 and 80 pounds of K_2O per acre. Pioneer 352 was planted by the cooperating farmer[2] who also performed all tillage operations.

Table 1. Monthly precipitation in inches during the growing season for individual years[a] and period of years.

Month	Mean precipitation[b]	1953	1954	1955	1956	Precipitation at experiment in 1956
April	2.57	4.38	4.65	2.18	1.45	----
May	4.10	2.35	4.06	2.86	3.38	3.36
June	5.27	4.30	9.67	1.42	1.99	2.48
July	3.40	3.31	2.17	2.28	3.08	2.70
Aug.	3.46	2.46	8.01	3.02	2.63	1.19
Sept.	2.98	0.51	2.20	4.00	1.63	----

[a]Clarion weather station for 1953 through 1956.

[b]Humboldt - 25 year normal.

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[1]Subsequent references to nitrogen, phosphorus and potassium when used in the sense of fertilizer application or response will be symbolized by N, P or P_2O_5 and K or K_2O, respectively. References to plant or soil nitrogen, phosphorus and potassium will be spelled out.

[2]Mr. Ovid Toresdahl, R. R. #1, Eagle Grove, Iowa.

Twenty hills of corn were picked from each plot in the fall, and the yield adjusted to 15.5 percent moisture shelled corn. Grain samples were saved for chemical analysis.

Clinton oats were sown in the spring of 1954 and seeded to Grimm alfalfa by the cooperating farmer. Since K_2O had no effect on corn yields the previous year, the check plots and those K_2O plots not receiving N and P_2O_5 originally were topdressed with rates of N or P_2O_5 after the oats emerged. One set of 3 plots were each split 4 ways and topdressed with 0, 15, 30 and 45 pounds per acre of N. The other 3 plots were handled in the same way but topdressed with 0, 30, 60 and 90 pounds of P_2O_5.

Two quadrats of oats, each 9 square feet in area, were harvested from the center of the residual and topdressed N and P_2O_5 plots. The residual K_2O plots were not harvested since response to this nutrient was not obtained on corn and response of oats was not expected. Grain yields and whole plant yields[1] were computed on the basis of 4 percent moisture. Whole plant samples were saved for chemical analysis.

No fertilizer applications were made in 1955 but on

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[1]Whole plant yields refer to the above ground portion of the plants.

April 10, 1956, each of the remaining N plots not receiving P_2O_5 originally was split 4 ways and topdressed with 0, 30, 60 and 90 pounds per acre of P_2O_5.

Hay yields represent three cuttings in both 1955 and 1956. Each harvest was obtained by cutting a 3-foot, 2-inch x 21-foot, 7-inch swath through the middle of each plot. Yields were computed on the basis of dry matter containing 4 percent moisture. Samples were taken for chemical analysis from every plot.

Chemical Analyses

Nitrogen, phosphorus and potassium analyses for both grain and plant material were made by the Iowa State College Soil Fertility Laboratory.[1] Nitrogen determinations were made by a modified Kjeldahl procedure. Phosphorus was determined by a modified Kitson and Mellon (31) procedure while potassium was determined separately by means of the Perkin-Elmer flame photometer using lithium nitrate as an internal standard.

Statistical Analyses

Analyses of variance and multiple regression equations were computed according to Snedecor (56) for crop yield,

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[1]All chemical analyses were made under the direction of Dr. J. J. Hanway of the Iowa State College Agronomy department.

nutrient percentage and nutrient yield. The regressions, together with the standard errors for each partial regression coefficient were computed by the Iowa State College Statistical Laboratory.[1]

[1] All statistical analyses were made under the direction of Dr. E. H. Jebe of the Iowa State College Statistics department.

RESULTS AND DISCUSSION

Corn - 1953

General observations

Highly significant N and P_2O_5 responses and significant NP interaction effects were obtained with the 1953 corn crop (Tables 2 and 3). The mean response to K_2O over all N and P_2O_5 treatments was positive but not significant.

It is of interest to note in Table 2 that P_2O_5 had essentially no effect on yields in the absence of N. In the presence of N, however, the P_2O_5 was very effective. N rates were effective at the zero level of P_2O_5 but to a much smaller degree than when associated with P_2O_5.

Statistical procedures

An analysis of variance of the data was the first step in the process of developing a procedure for determining the variables and their form which would best characterize the yield data. From Table 2, it is easy to see that, of the seven treatment combinations, the N, P and NP treatments account for most of the treatment variance. Thus, these variables should represent the nature of the data fairly well.

At this point, however, the analysis of variance does not indicate the form of the yield curve or surface being

Table 2. Mean corn yields in bushels per acre obtained in 1953 for various nutrient combinations. Each value is a mean of 2 observations.[a]

Pounds of P_2O_5/A	Pounds of K_2O/A	Pounds of N/A: 0	40	80	160	240
0	0	89.4	87.1	95.8	82.7	94.1
0	40	77.9	90.9	104.0	93.6	99.3
0	80	75.2	92.5	91.8	100.2	97.4
40	0	72.8	92.3	108.7	108.5	112.9
40	40	87.3	108.3	112.6	92.3	122.2
40	80	76.0	99.8	110.8	120.0	122.1
80	0	81.1	99.6	121.3	119.6	126.4
80	40	65.8	109.8	112.1	122.2	123.3
80	80	80.2	116.2	118.4	123.9	122.2
120	0	75.5	97.3	123.4	111.2	110.1
120	40	77.0	102.6	121.4	124.0	123.3
120	80	79.4	97.9	115.3	122.1	110.5

Mean yield for main effects

Pounds of N/A	Mean yield (24 obs.)	Pounds of P_2O_5/A	Mean yield (30 obs.)	Pounds of K_2O/A	Mean yield (40 obs.)
0	78.1	0	91.4	0	100.5
40	99.5	40	103.1	40	103.5
80	111.3	80	109.5	80	103.6
160	110.0	120	106.0		
240	113.6				

[a]The mean stand level was 14,553 stalks per acre.

analyzed. In order to ascertain this form, each significant main treatment effect sum of squares must be partitioned into its linear, quadratic, etc. components, the components tested for significance[1] and the results compared with the mean main effect response lines. If the main effect variables were significant to begin with, there should be reasonably good agreement between the general trend of the response lines and the general curvature which the significant components of the partitioned sum of squares bring into being.

By following the above procedure, it was found that the linear, quadratic and cubic components of the N sum of squares were significant while only the linear and quadratic components of the P sum of squares were significant (Table 3). Thus on the basis of statistics, all of these variables should be included in the regression equation if the observed yields are to be adequately characterized.

Agronomic logic, however, precludes a cubic effect for first year responses except under unusual conditions. If they do occur, the shape of the response curve is usually sigmoid. The mean yield response lines for N in Figure 1 do

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[1]The statistical method of testing the significance of the individual components of the treatment sum of squares for equally spaced, orthogonal comparisons given by Snedecor (56, pp. 409-11) was used to determine the significance of the linear, quadratic, etc. components. Since the N rates were not equally spaced in this experiment, some bias is introduced into these particular results through the use of this procedure. However, it is doubtful that this bias was large enough to have been a deciding factor in the acceptance or rejection of a specific component for use in the regression equation.

Table 3. Analysis of variance for 1953 corn yields.

Source of variation	Degrees of freedom		Sum of squares	Mean square	F
Total	119		41,876.92		
Replication	1		1,993.49	1,993.49	16.64**
Treatments	59		32,817.23	556.22	4.64**
Nitrogen (N)	4		20,664.39	5,166.09	43.13**
Linear		1		15,965.86	133.30**
Quadratic		1		4,035.89	33.70**
Cubic		1		506.05	4.23*
Quartic		1		156.59	1.31
Phosphorus (P)	3		5,502.04	1,834.01	15.31**
Linear		1		3,774.54	31.51**
Quadratic		1		1,697.26	14.17**
Cubic		1		30.24	0.25
Potassium (K)	2		246.20	123.10	1.03
NP	12		3,267.86	272.32	2.27*
NK	8		899.59	112.45	0.94
PK	6		420.76	70.13	0.58
NPK	24		1,816.39	75.68	0.63
Error	59		7,066.20	119.77	

**In all future tables, the double asterisk will indicate a probability level less than or equal to 0.01.

*In all future tables, the single asterisk will indicate a probability level less than or equal to 0.05.

not indicate a sigmoid effect but do indicate the reason for the significant cubic effect in the erratic response to N at the 0 and 40 pound levels of P_2O_5. Since such behavior is not logical nor explainable, it must be assumed to be capricious. The cubic term was, therefore, left out of the regression equation.

The path traced by the majority of the yield lines in Figure 2 for rates of P_2O_5 at various levels of N illustrates the validity of including the statistically significant linear and quadratic components of the phosphorus sum of squares shown in Table 3. The divergence of the response lines as they leave the Y axis in Figures 1 and 2 is caused by the NP interaction.

The yield lines of Figures 1 and 2 also point out the need for an equation which allows for some depression in yield at high nutrient levels. While this need is less apparent in Figure 1, it is quite apparent in Figure 2. The use of the Mitscherlich-Spillman or power functions, therefore, would not have characterized the observed response surface properly.

Regression analyses

The selection of the significant variables and the power to which they are to be raised were achieved through the procedures used thus far. The problem now is to choose a cer-

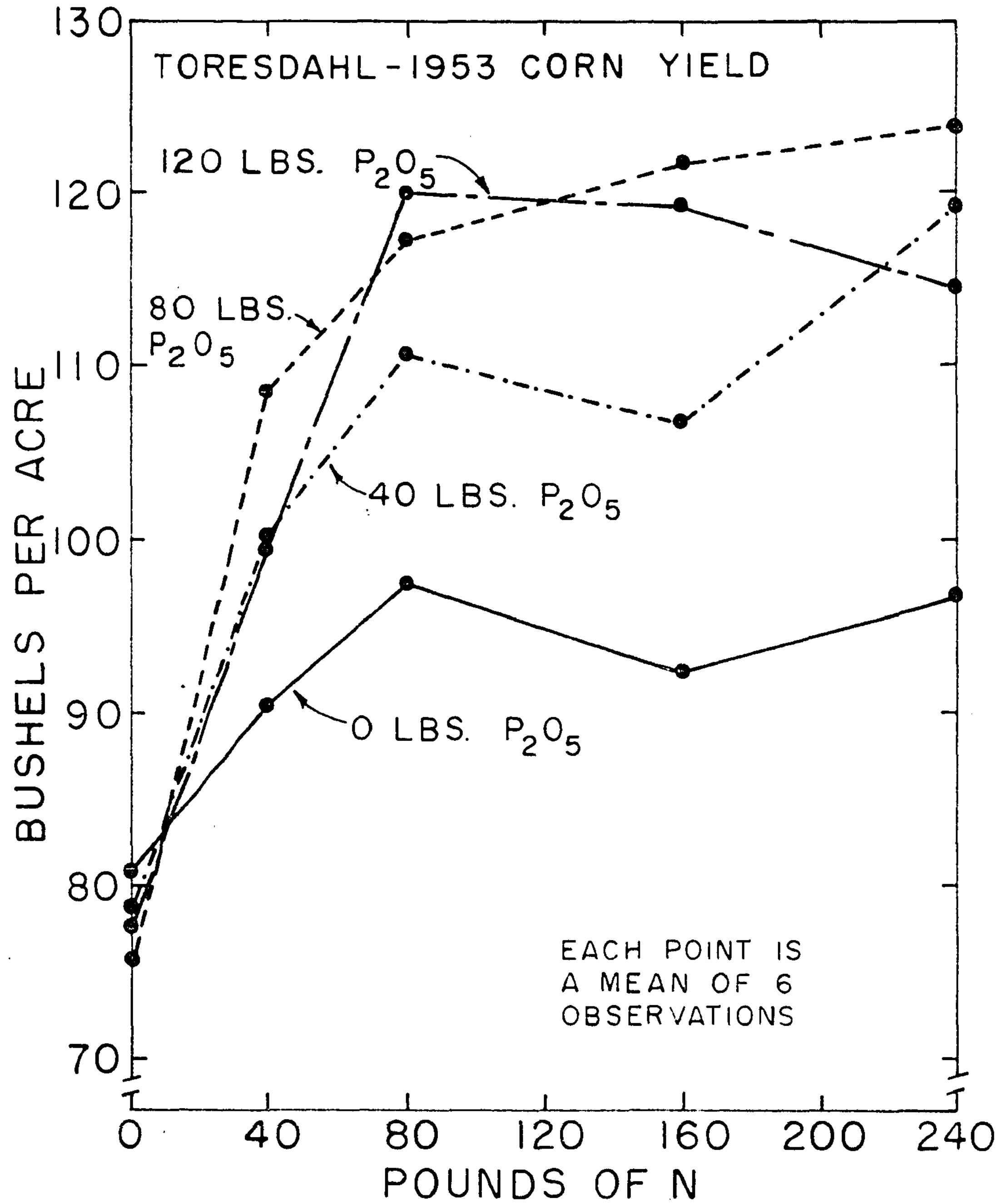


Figure 1. Average yield of corn as a function of N rates at different levels of P_2O_5.

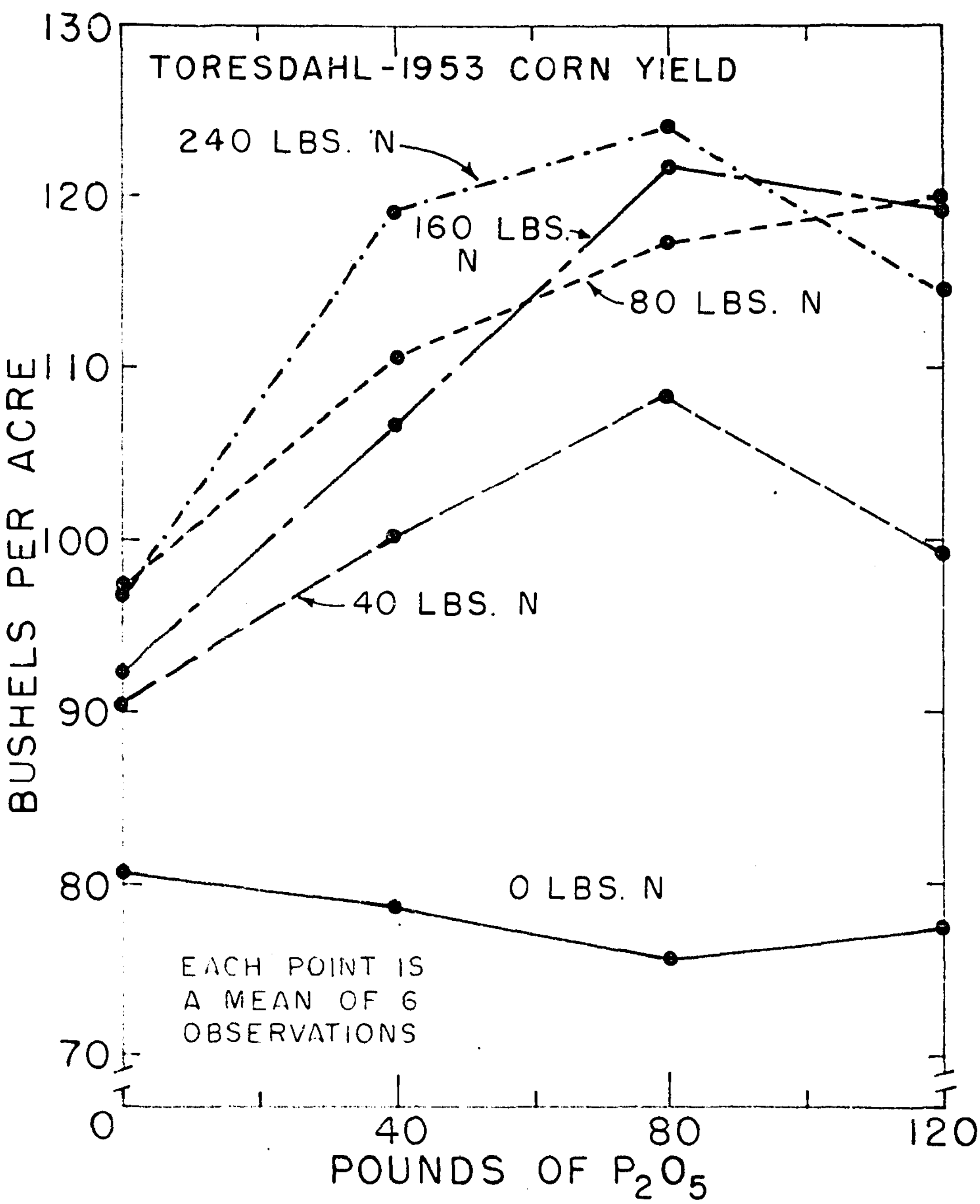


Figure 2. Average yield of corn as a function of P_2O_5 rates at different levels of N.

tain form for the main-effect variables which will result in a curve or surface which conforms most closely to the observed yields.

Heady et al. (22) found that expressing the variables in the quadratic form results in a good fit when responses occur rather uniformly throughout the range of nutrient application while the root form fits best when response is large for low application rates and then essentially levels off.

When the yield lines in Figures 1 and 2 are viewed in light of the above criteria, it appears that the root form of the N variable and the quadratic form of the P variable would result in the best fitting yield equation. To check this point, regressions were computed which contained only the quadratic or root forms for both the N and P variables as well as one which contained both forms. The resulting equations with standard errors for the partial regression coefficients are

$$\begin{aligned} Y^1 = {} & \underset{\pm 0.0584}{0.364992N} - \underset{\pm 0.0002}{0.001186N^2} + \underset{\pm 0.104}{0.333257P} - \underset{\pm 0.0008}{0.002351P^2} \\ & + \underset{\pm 0.0003}{0.000714NP} + 74.895792, \end{aligned} \tag{1}$$

$$\begin{aligned} Y = {} & \underset{\pm 0.72}{3.604763N^{\frac{1}{2}}} - \underset{\pm 0.04}{0.163192N} + \underset{\pm 1.02}{1.460571P^{\frac{1}{2}}} - \underset{\pm 0.086}{0.143028P} \\ & + \underset{\pm 0.048}{0.180316N^{\frac{1}{2}}P^{\frac{1}{2}}} + 76.926320, \end{aligned} \tag{2}$$

and

---

[1] In subsequent equations and in text, Y refers to predicted yields while P refers to P_2O_5.

$$Y = 3.646N^{\frac{1}{2}} - 0.163N + 0.1513P - 0.001332P^2 + 0.174N^{\frac{1}{2}}P^{\frac{1}{2}} + 76.304. \quad (3)$$
$$\pm 0.706 \quad \pm 0.04 \quad \pm 0.106 \quad \pm 0.0007 \quad \pm 0.046$$

The coefficients of determination (R^2) and t values for the partial regression coefficients are shown in Table 4.

Table 4. Values of R^2 and t for two variable regressions for 1953 corn.

Equation	R^2	Value of t for coefficients in the order shown in equations				
(1)	0.755	6.25**	5.43**	3.20**	2.99**	2.19*
(2)	0.828	6.43**	4.06**	1.43[d]	1.67[d]	3.78**
(3)	0.834	5.16**	4.13**	1.43[d]	1.90[c]	3.74**

[c]In all future tables, superscript c will indicate a probability level less than or equal to 0.10.

[d]In all future tables, superscript d will indicate a probability level less than or equal to 0.20.

As the form of each variable in an equation changes, the percent of the total variation explained by it may also change. This change in turn causes increases or decreases in the percent variation explained by other variables in the equation. If the net effect of the changes is in favor of the increases, the R^2 becomes greater. Thus, the higher the R^2, the better the equation fits the observed yields.

The higher R^2 for a given equation, however, may not result in higher t values for all the partial regression coefficients in that equation in comparison to those t values of another equation having different forms of the same variable. This effect should not be cause for concern as long as the t indicates a reasonable probability that the coefficient is greater than zero. As long as the variable has an agronomic basis and satisfies the equation model, the coefficient is the best estimate of the parameter available (18).

<u>Yield curves and surfaces</u>

Equation 3 would be used to characterize the corn yield data on the basis that it has the highest R^2. However, it is of interest to see how predicted yields vary for each equation. The predicted and mean observed yields are grouped for easy comparison in Table 5.

It is evident in Table 5 that the predicted yields vary some among themselves and from the mean observed yields. As would be expected, each equation fits better at some points than at others. The differences in the predicted yields for the separate equations show up in the curves of Figures 3, 4, 5, 6, 7 and 8.

For example, the N yield curves for equation 1 vary considerably from the original mean yield lines in Figure 1. The

Table 5. Comparison of 1953 corn yields in bushels per acre predicted by three equations among equations and between mean observed yields.[a]

Pounds P_2O_5/A	Pounds N/A						
	0	40	80	120	160	200	240
0	80.8	90.2	97.2	--	92.2	--	96.9
	74.9	87.6	96.5	101.6	102.9	100.5	94.2
	76.9	93.2	96.1	96.8	96.4	95.3	93.6
	76.3	92.8	95.9	96.7	96.3	95.3	93.7
40	78.7	100.1	110.7	--	106.9	--	119.1
	84.5	98.3	108.4	114.6	117.1	115.7	110.6
	80.4	103.9	109.8	112.8	114.4	114.9	114.8
	80.2	103.7	109.6	112.7	114.2	114.8	114.6
80	75.7	108.5	117.2	--	121.9	--	124.0
	86.5	101.5	112.7	120.1	123.7	123.5	119.5
	78.5	105.0	112.2	116.1	118.4	119.7	120.2
	79.9	106.3	113.0	117.3	119.6	120.9	121.4
120	77.3	99.2	120.0	--	119.1	--	114.6
	81.0	97.2	109.5	118.0	122.8	123.7	120.9
	75.8	104.5	112.6	117.3	120.2	122.0	123.0
	75.3	103.9	111.9	116.5	119.4	121.2	122.2

[a]Upper figure is the mean observed yield, second figure indicates predicted yields for the quadratic equation 1, third figure indicates predicted yields for the root form, equation 2, and fourth figure indicates predicted yields for the mixed form, equation 3.

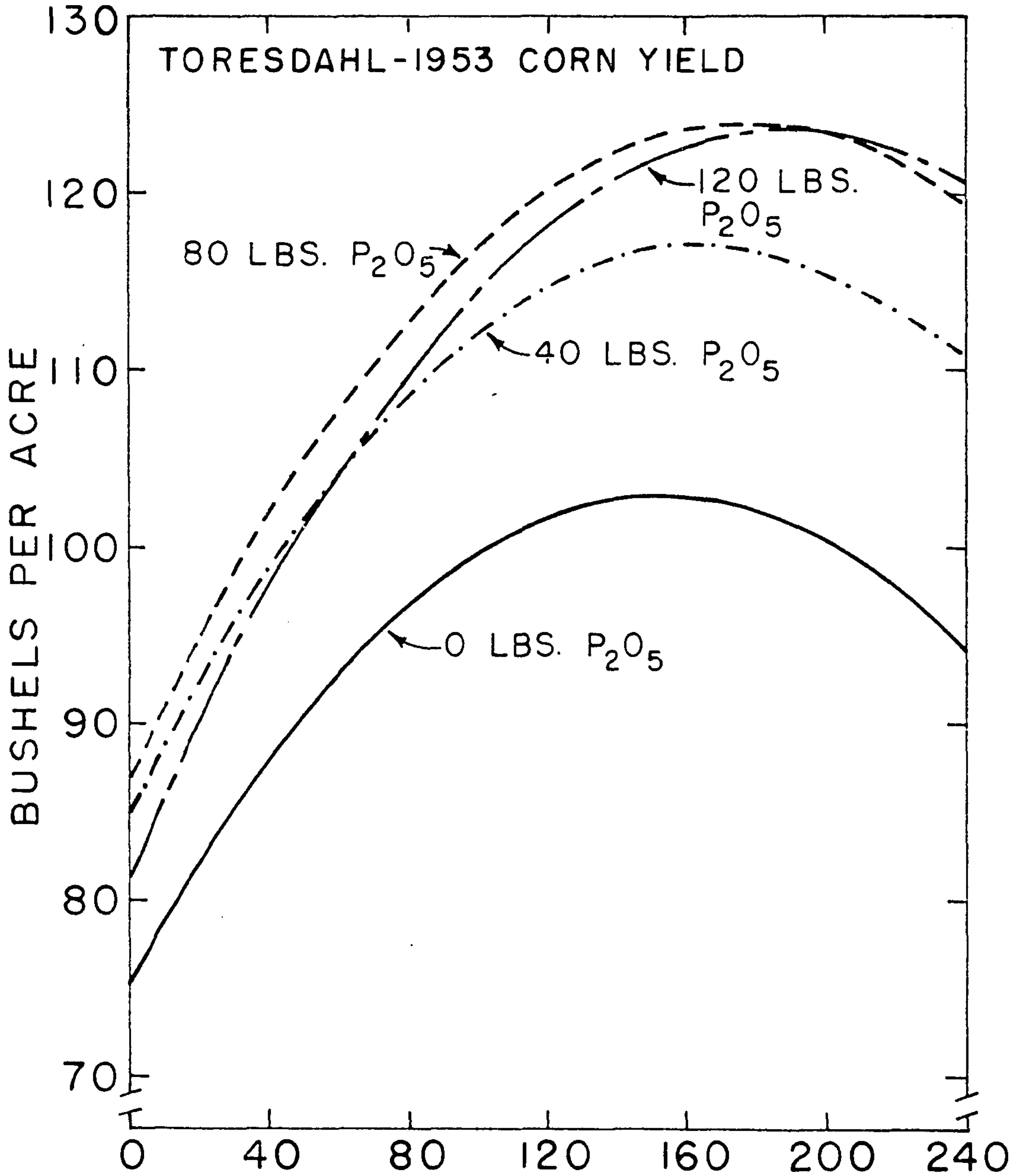


Figure 3. Yield of corn as a function of N rates at different levels of P_2O_5 predicted by the quadratic equation.

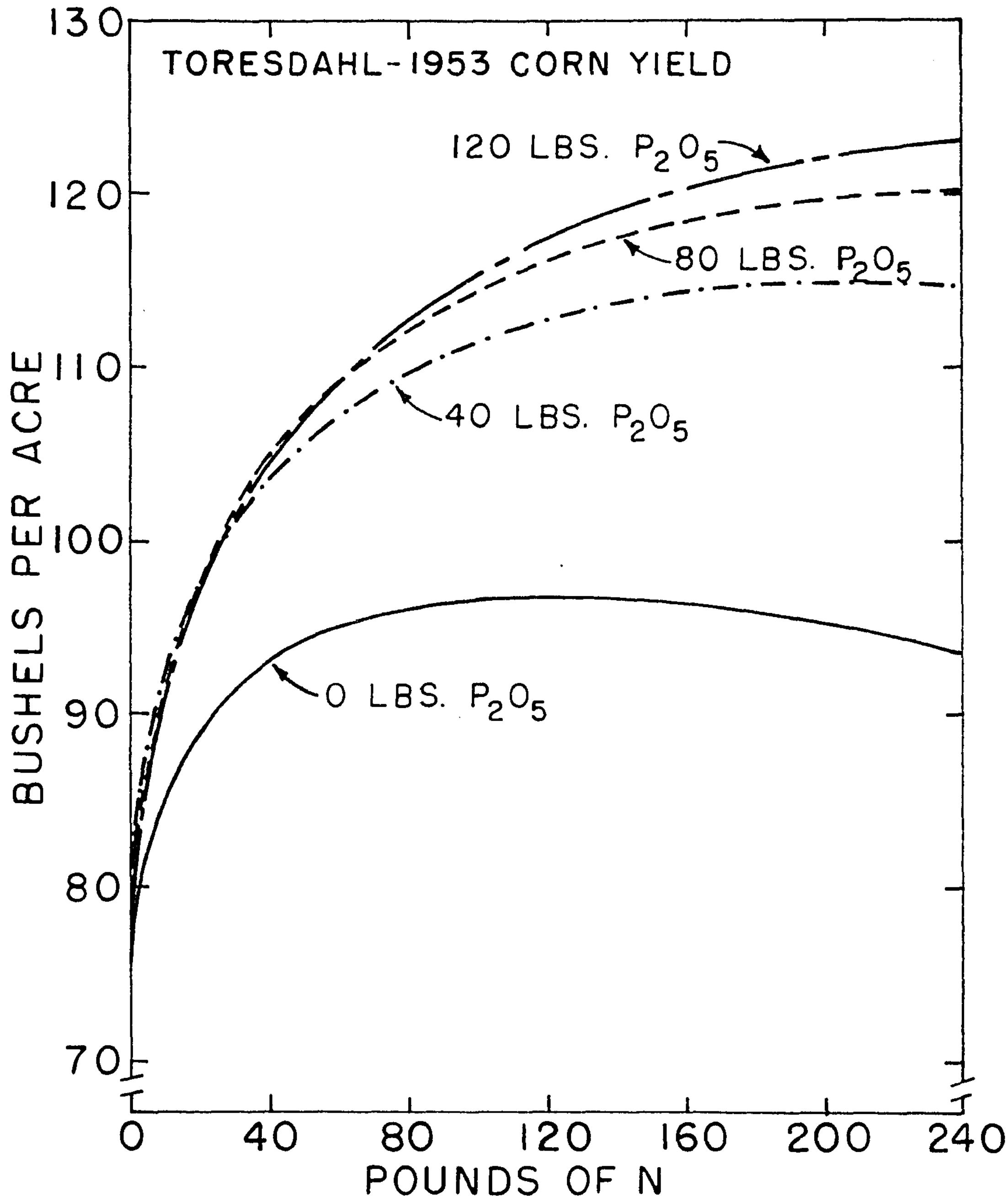


Figure 4. Yield of corn as a function of N rates at different levels of P_2O_5 predicted by the root form of the quadratic equation.

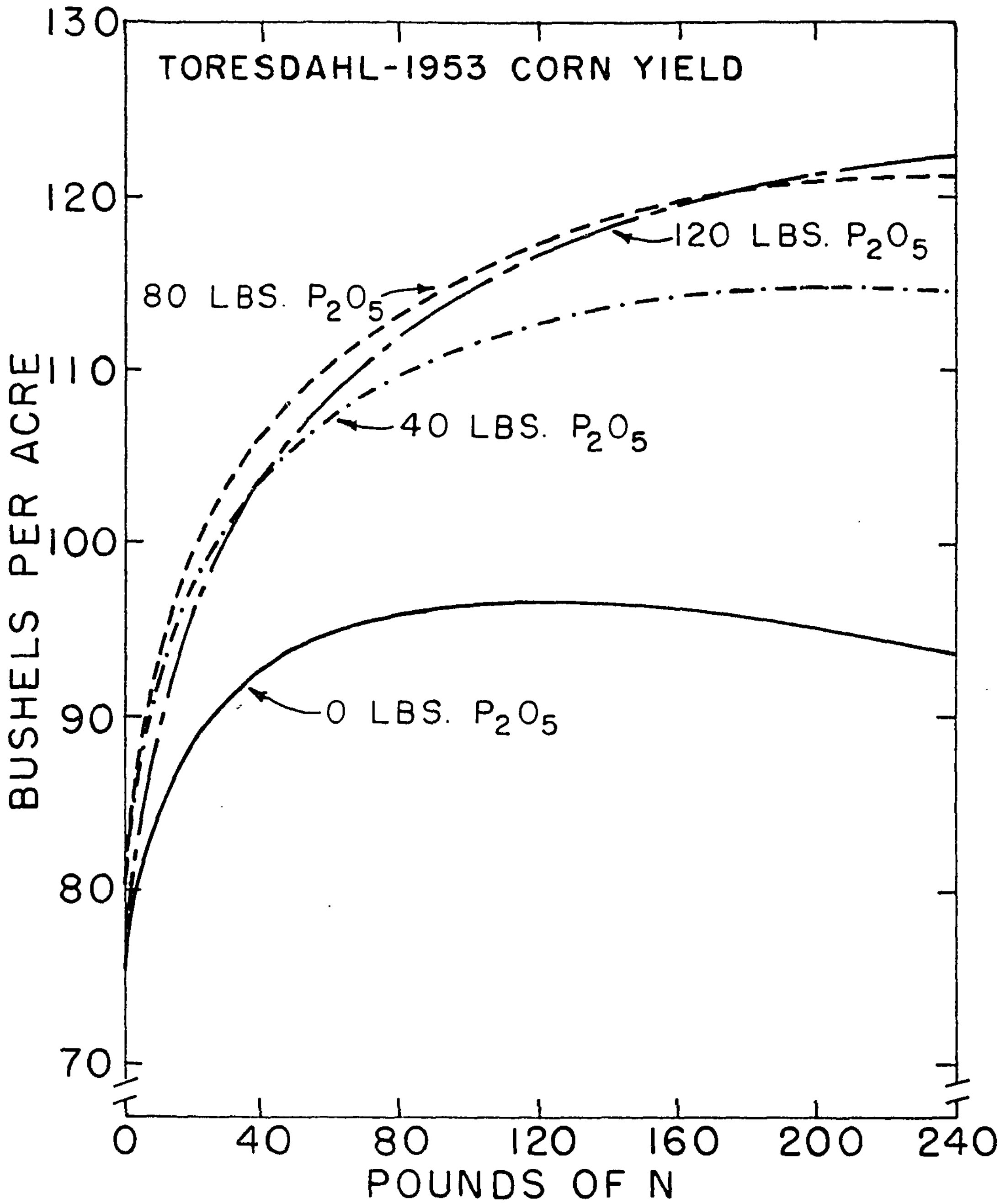


Figure 5. Yield of corn as a function of N rates at different levels of P_2O_5 predicted by the mixed form of the quadratic equation.

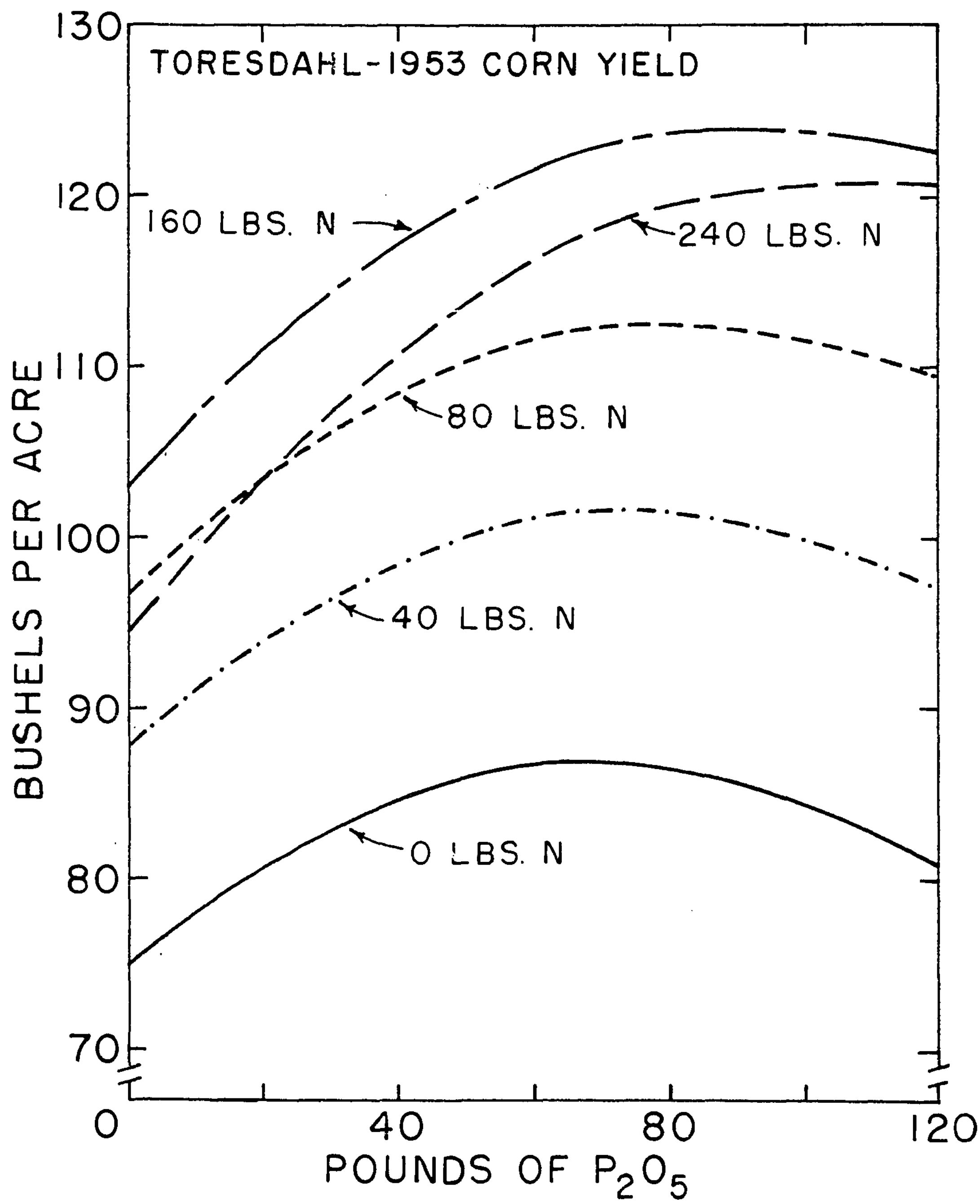


Figure 6. Yield of corn as a function of P_2O_5 rates at different levels of N predicted by the quadratic equation.

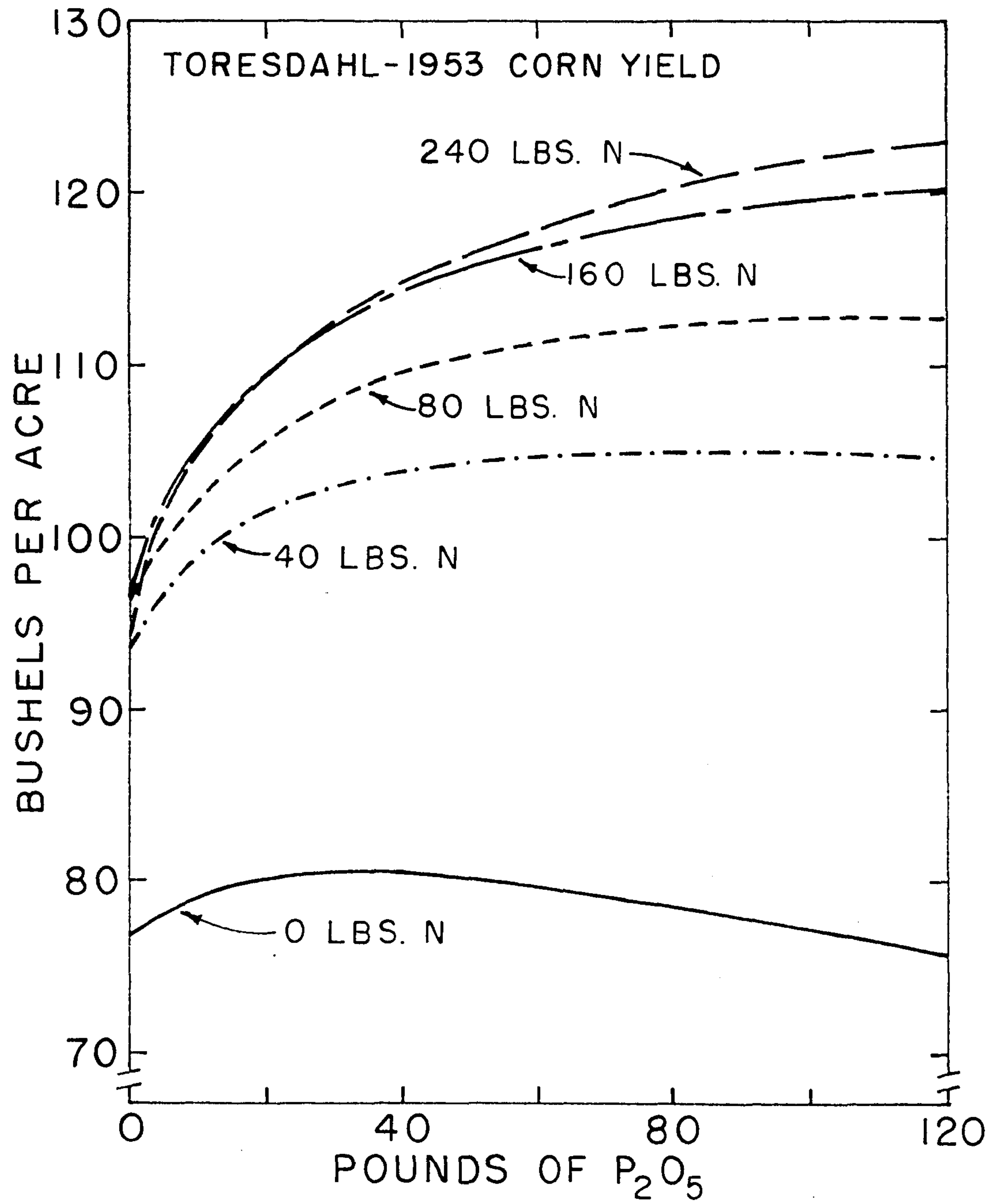


Figure 7. Yield of corn as a function of P_2O_5 rates at different levels of N predicted by the root form of the quadratic equation.

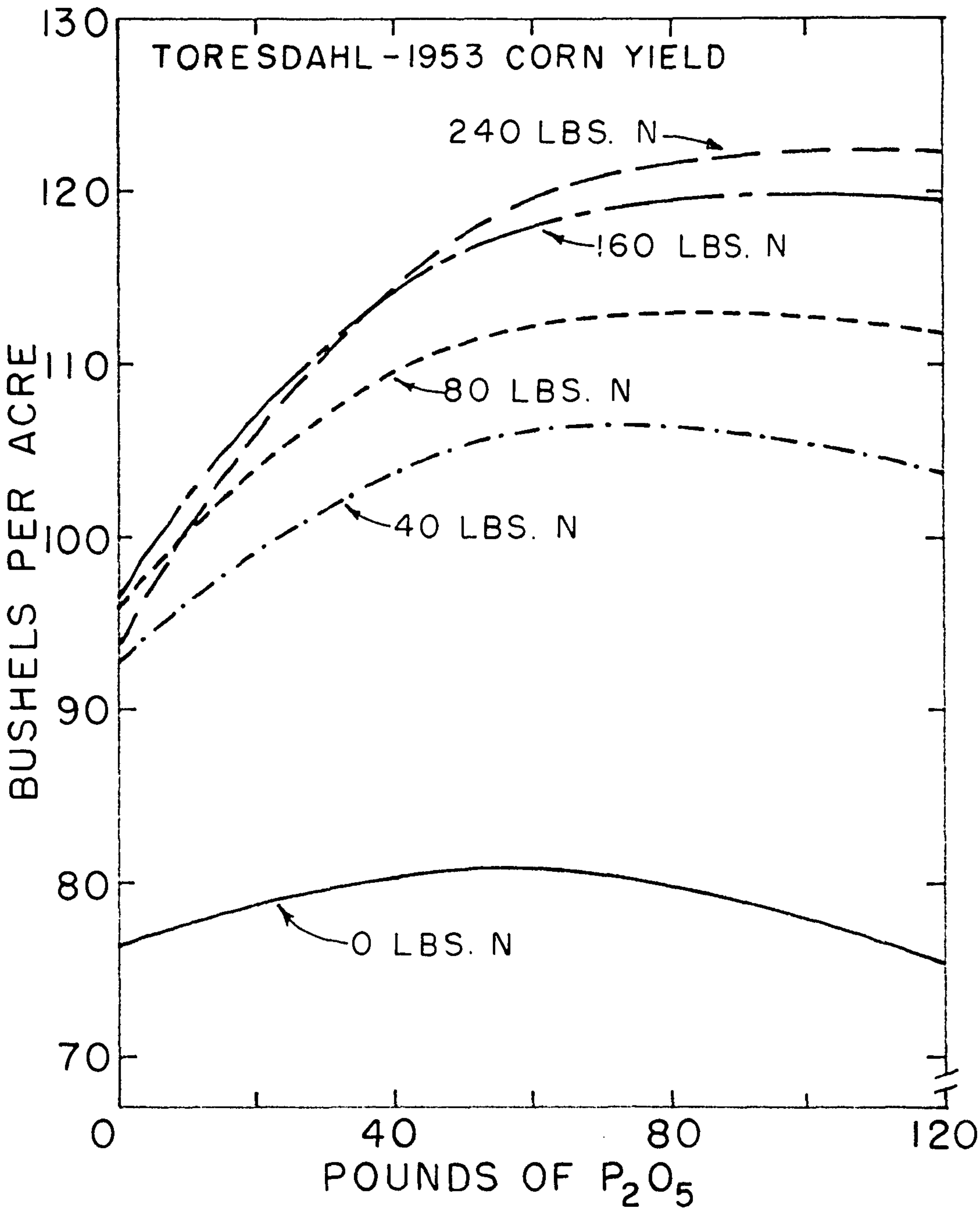


Figure 8. Yield of corn as a function of P_2O_5 rates at different levels of N predicted by the mixed form of the quadratic equation.

curves resulting from equation 2 conform much more closely to these mean yield lines. This graphic contrast shows very clearly why the root form for the N variable improves the fit obtained for equation 3.

It is also of interest to note that the choice of the quadratic or root form of N also affects the curvature of the P_2O_5 yield curves for equations 1 and 3 even though both equations contain the quadratic form for P_2O_5. These comparisons serve to emphasize that the selection of the proper forms of the variates for each variable is of considerable importance if the predicted yields for each nutrient are to correspond closely to observed yields.

The individual curves, plus others, have been combined to form the yield surfaces shown in Figures 9, 10 and 11. These surfaces show the NP interaction as well as the variation in curvature of the individual yield planes.

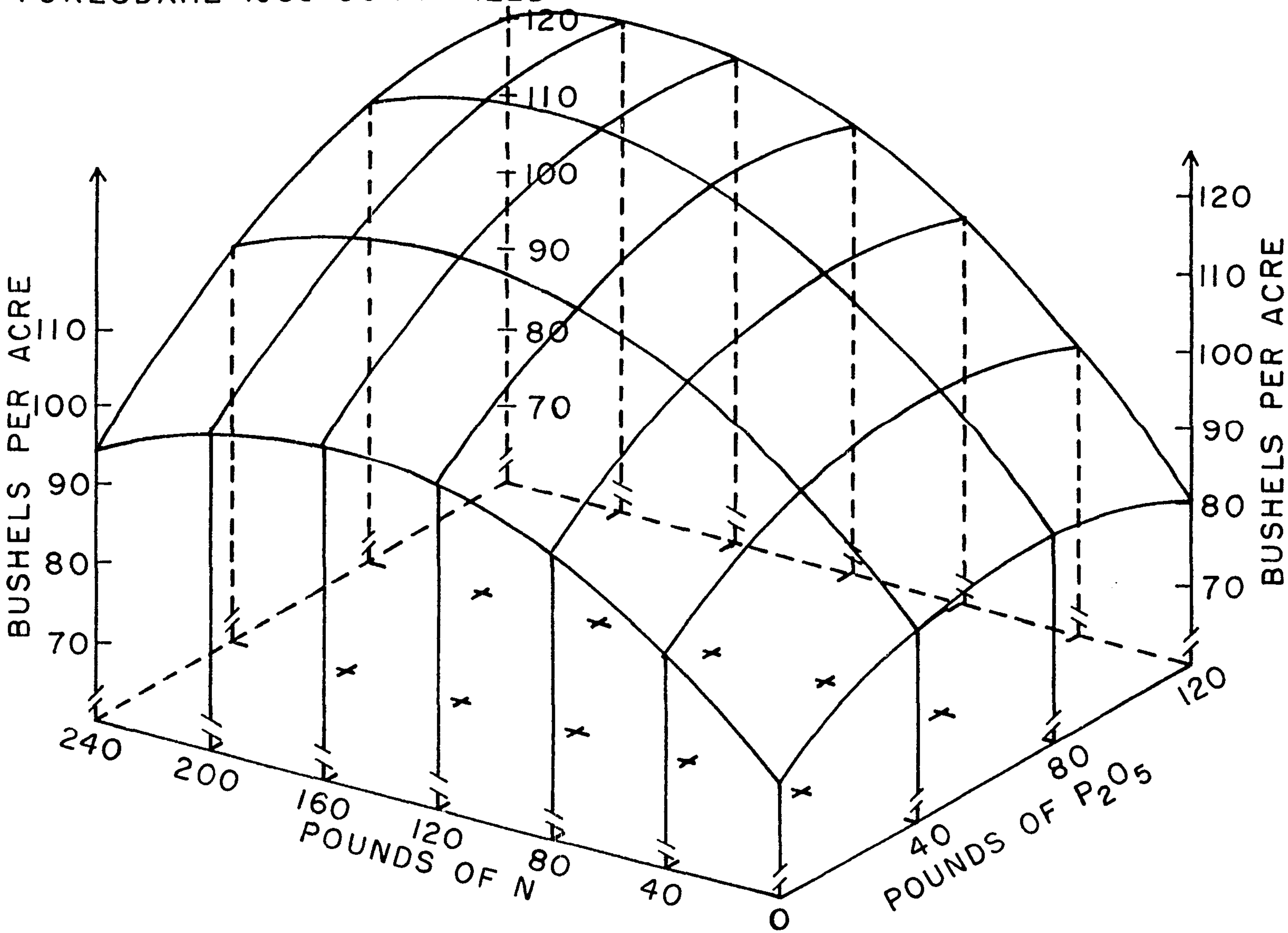
The expected depression in yield for high rates of a nutrient at the zero level of another is well illustrated by all of the surfaces. The quadratic shows this effect throughout the surface while the root and mixed forms of the quadratic show increasing yields as both N and P_2O_5 are increased together. The latter effect is more reasonable for the conditions involved in the experiment and for the yield levels attained.

Figure 9. Corn yield surface for rates of N and P_2O_5 predicted by the quadratic equation.

Figure 10. Corn yield surface for rates of N and P_2O_5 predicted by the root form of the quadratic equation.

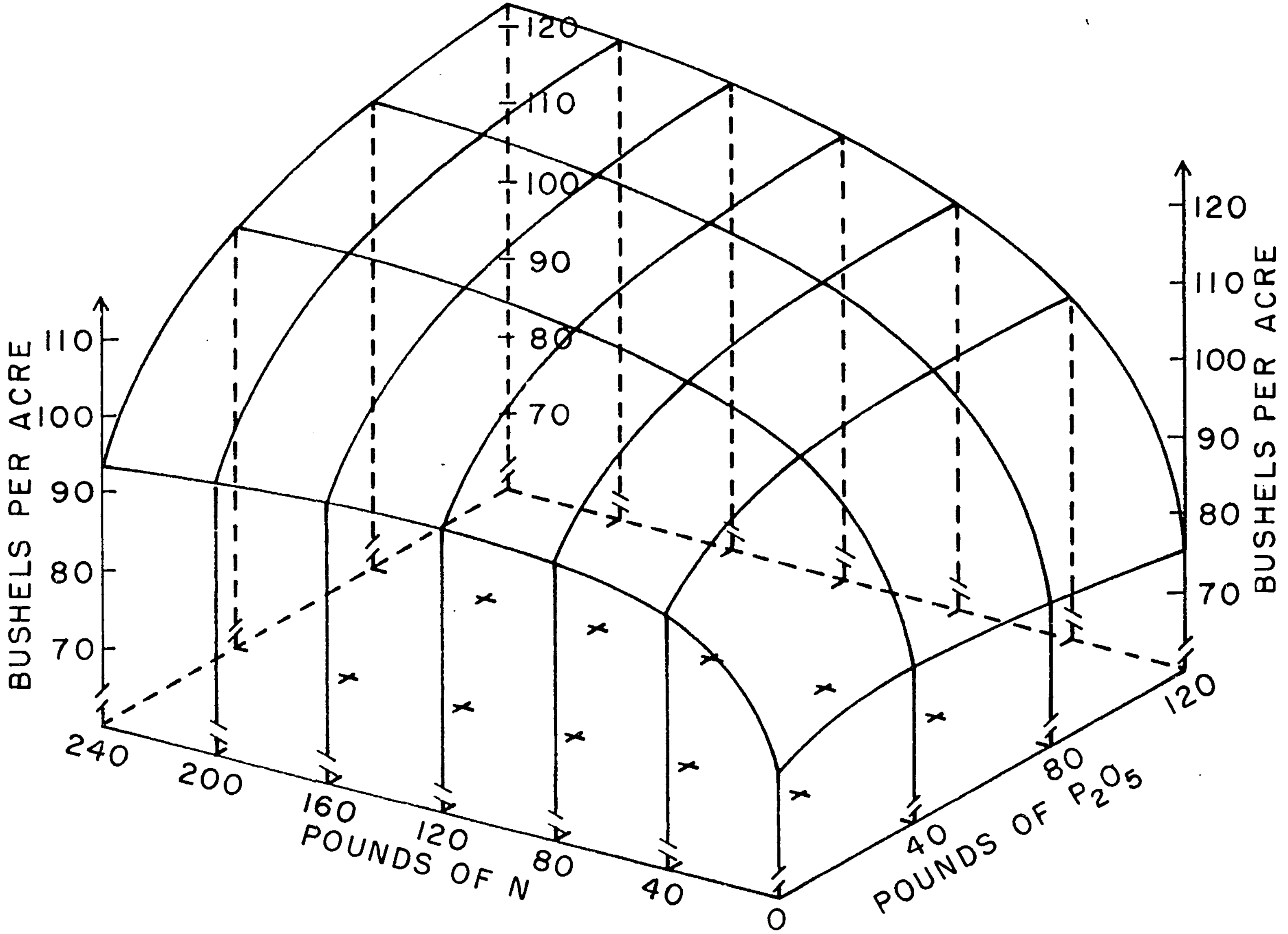
Figure 11. Corn yield surface for rates of N and P_2O_5 predicted by the mixed form of the quadratic equation.

TORESDAHL-1953 CORN YIELD

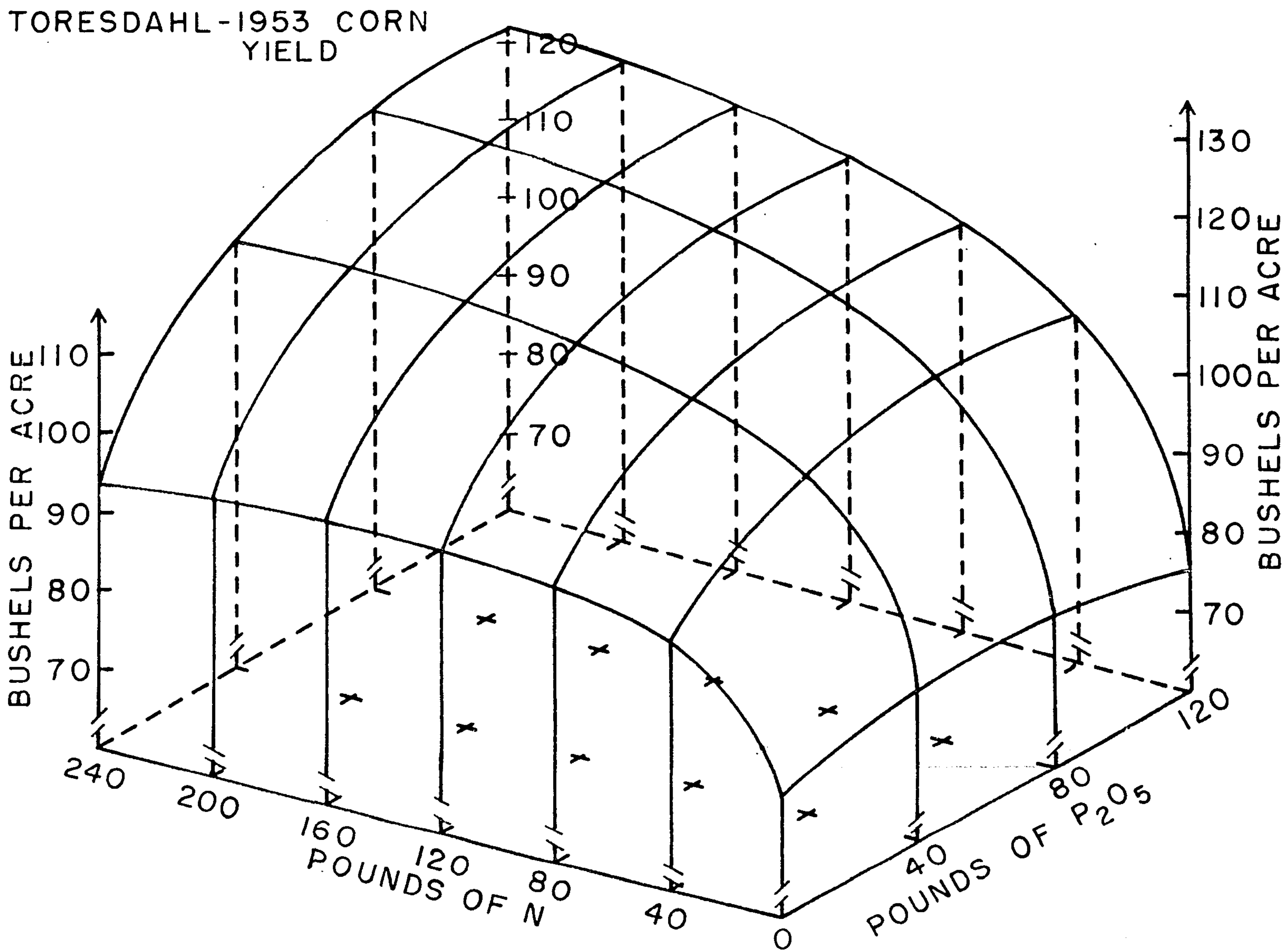


TORESDAHL-1953 CORN YIELD

TORESDAHL-1953 CORN YIELD



TORESDAHL-1953 CORN



TORESDAHL-1953 CORN
YIELD
BUSHELS PER ACRE
70
80
90
100
110
120
130
240
200
160
120
80
40
0
POUNDS OF N
POUNDS OF P_2O_5

Isoquants, isoolines and ridgelines

Another feature of the yield surface is the visualization of yield contours on the surface. These yield contours are called isoquants and are derived from the regression equation used for determining the yield surface. The procedure is to set Y equal to a certain yield and to solve for either N or P_2O_5 at given levels of the other nutrient. The respective isoquant equations for the quadratic root and mixed form of yield equations are

$$P = 70.876 + 0.15185N - \frac{\sqrt{0.003908N - 0.00001064N^2 + 0.815380 - 0.009404Y}}{-0.004702}, \qquad (4)$$

$$P = \left[5.106 + 0.630N^{\frac{1}{2}} + \frac{\sqrt{2.589N^{\frac{1}{2}} - 0.061N + 46.144 - 0.572Y}}{-0.286} \right]^2, \qquad (5)$$

and

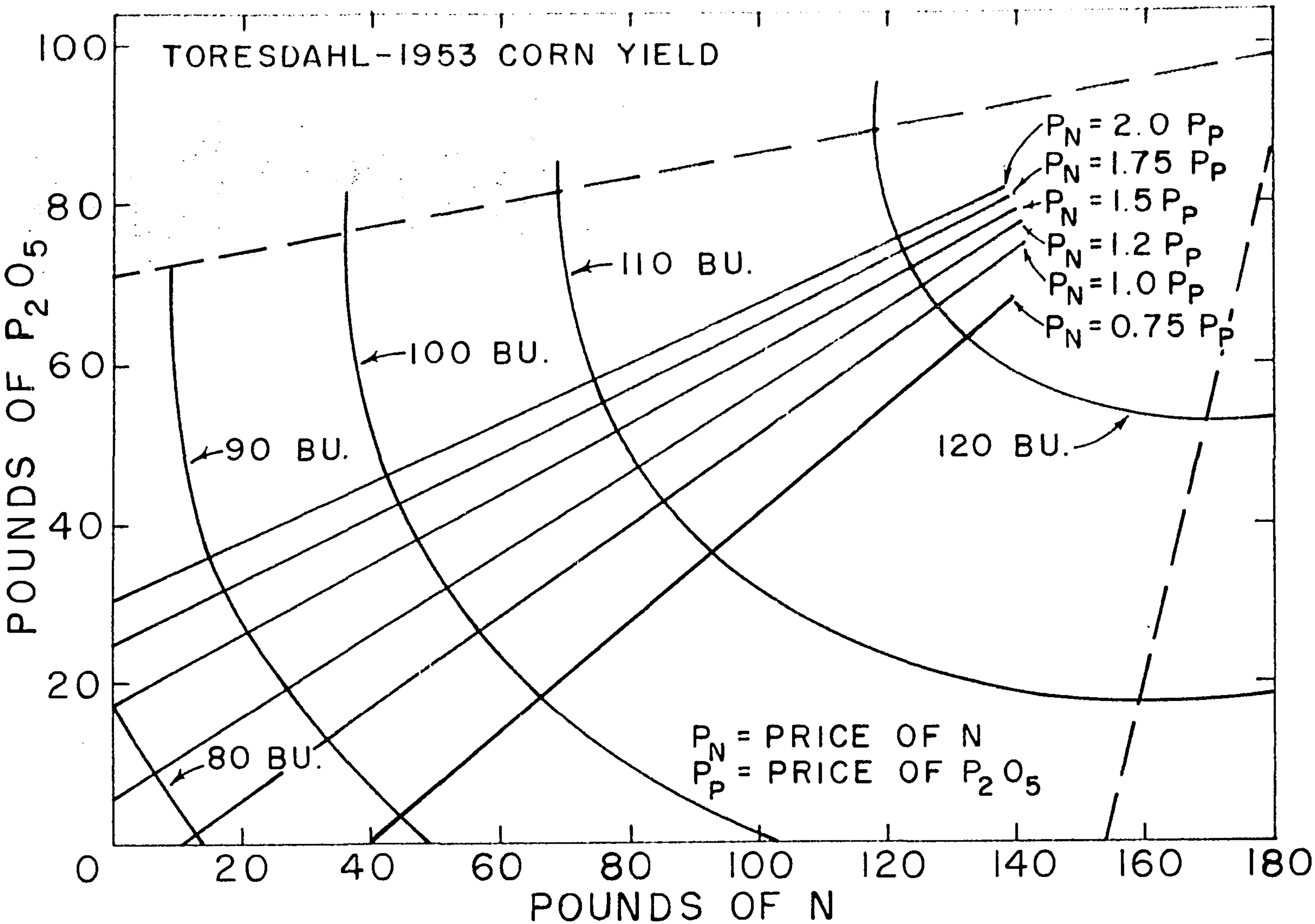
$$N = \left[11.184 + 0.534P^{\frac{1}{2}} + \frac{\sqrt{1.269P^{\frac{1}{2}} + 0.129P - 0.000868P^2 + 63.044 - 0.652Y}}{-0.326} \right]^2. \qquad (6)$$

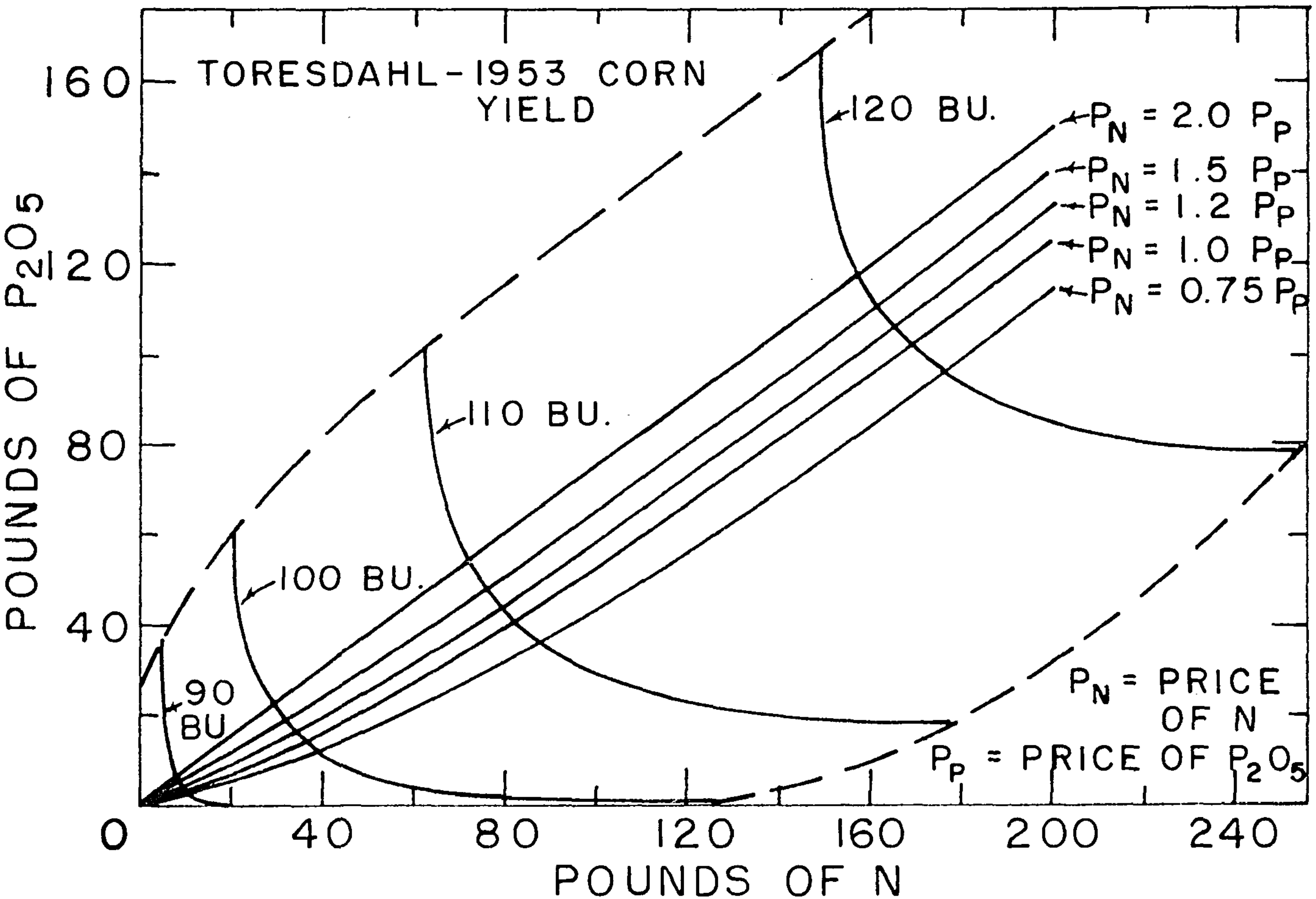
Isoquant maps for equations 4, 5 and 6, are shown in Figures 12, 13 and 14. Because of the nature of the yield surface for the quadratic equation, the 80 bushel yield can be drawn into its respective isoquant map and be seen. However, this yield level is so close to the check yield for the other equations that its visualization is of little value.

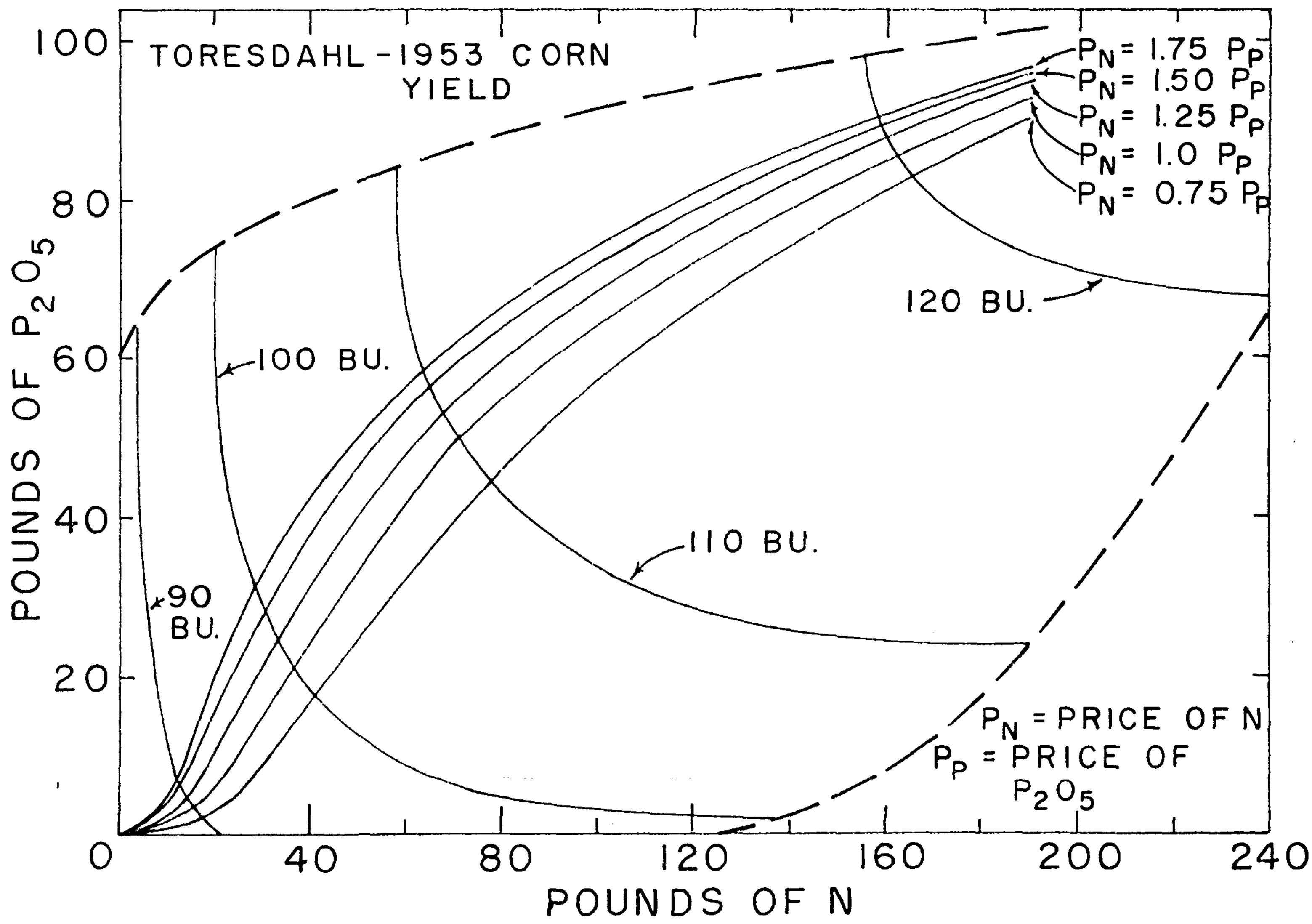
Figure 12. Corn yield isoquants, isoclines and dashed reidgelines for rates of N and P_2O_5 derived from the quadratic equation.

Figure 13. Corn yield isoquants, isoclines and dashed ridgelines for rates of N and P_2O_5 derived from the root form of the quadratic equation.

Figure 14. Corn yield isoquants, isoclines and dashed ridgelines for rates of N and P_2O_5 derived from the mixed form of the quadratic equation.


TORESDAHL-1953 CORN YIELD
POUNDS OF P_2O_5
POUNDS OF N
0
20
40
60
80
100
120
140
160
180
80 BU.
90 BU.
100 BU.
110 BU.
120 BU.
$P_N = 2.0\ P_P$
$P_N = 1.75\ P_P$
$P_N = 1.5\ P_P$
$P_N = 1.2\ P_P$
$P_N = 1.0\ P_P$
$P_N = 0.75\ P_P$
P_N = PRICE OF N
P_P = PRICE OF P_2O_5


TORESDAHL-1953 CORN YIELD
120 BU.
110 BU.
100 BU.
90 BU
$P_N = 2.0\ P_P$
$P_N = 1.5\ P_P$
$P_N = 1.2\ P_P$
$P_N = 1.0\ P_P$
$P_N = 0.75\ P_P$
P_N = PRICE OF N
P_P = PRICE OF P_2O_5
POUNDS OF P_2O_5
POUNDS OF N
0
40
80
120
160
200
240


TORESDAHL - 1953 CORN YIELD
POUNDS OF P_2O_5
POUNDS OF N
0
20
40
60
80
100
120
160
200
240
P_N = 1.75 P_P
P_N = 1.50 P_P
P_N = 1.25 P_P
P_N = 1.0 P_P
P_N = 0.75 P_P
120 BU.
100 BU.
110 BU.
90 BU.
P_N = PRICE OF N
P_P = PRICE OF P_2O_5

The 80 bushel isoquant in Figure 13 shows that this yield can be produced with either N or P_2O_5, alone. The production of 90 or 100 bushels of corn, however, can no longer be obtained with P_2O_5 alone but can be obtained with N alone. Such a result is to be expected for N when soil phosphorus is limiting because N applications increase root growth, thus increasing the total surface area for phosphorus absorption. If yields are to go still higher, however, they can be achieved only if P is also added. The latter effect is brought out clearly in all the isoquant maps.

Each map also brings out the wide range of economic substitution of N for P and vice versa that exists in the production of a given yield beyond 90 bushels. This effect is particularly evident in Figure 12.

Decreasing returns to scale are evident in all maps as indicated by the increasing distance between isoquants which represent successive 10 bushel yield increases.

The lines running diagonally across the isoquant map are called isoclines and serve to trace out points on successive isoquants which have the same slope. The isocline equation is derived from the basic yield equation by dividing the partial derivative of Y with respect to N or P by the partial derivative of Y with respect to P or N respectively, (i.e. the marginal rate of substitution of one nutrient for the other) and setting it equal to the respective N/P or P/N price ratio.

Performing these operations on equations 4, 5 and 6, leads to

$$P = \frac{0.002372N + \alpha 0.000714N + \alpha 0.333257 - 0.364992}{\alpha 0.004702 + 0.000714}, \qquad (7)$$

$$P = \left[\frac{0.163N^{\frac{1}{2}} - \alpha 0.143N^{\frac{1}{2}} - 1.802 + \sqrt{(-0.163N^{\frac{1}{2}} + \alpha 0.143N^{\frac{1}{2}} + 1.802)^2 - 0.360(-\alpha 0.730N^{\frac{1}{2}} - \alpha 0.090N)}}{0.180}\right]^2, \qquad (8)$$

and

$$N = \left[\frac{P^{\frac{1}{2}}(\alpha 0.002664P - \alpha 0.151 - 0.163) + \sqrt{P^{\frac{1}{2}}(\alpha 0.002664P - \alpha 0.151 - 0.163)^2 + \alpha(0.634404P^{\frac{1}{2}} + 0.030276P)}}{\alpha 0.174}\right]^2 \qquad (9)$$

where α is the N/P price ratio.

In addition to tracing out points of equal slope on the family of isoquant curves, the isoclines indicate the least cost combination of N and P_2O_5 needed for the production of any given yield for a given price ratio of nutrients. If, for example, a generalized prediction equation were available for a given soil type, a farmer with limited funds could choose some yield level below the optimum yield and by following the appropriate isocline for the current N/P price ratio, come up with the least cost amount of N and P_2O_5 to apply to obtain that yield.

Both the straight line and curved line isoclines show that the ratio of N to P_2O_5 needed to produce a given yield varies from one yield to another. In Figure 12, for example, approximately 20 pounds of N and 26 pounds of P_2O_5 per acre are needed to produce 90 bushels of corn when $P_N = 1.5\ P_P$.[1] To produce 120 bushels at this same price ratio, approximately 125 pounds of N and 72 pounds of P_2O_5 are needed. In Figure 14 for the same price ratio, approximately 30 pounds of N and 28 pounds of P_2O_5 are needed to produce 100 bushels of corn. To produce 120 bushels of corn at this least cost combination, approximately 160 pounds of N and 89 pounds of P_2O_5 are needed.

The significance of the above should not be overlooked. Too many times in the past the same nutrient ratios suggested for average yields have been recommended for higher yields. Although the higher yields may have been achieved, the above results would indicate that they probably were not obtained at least cost.

Isoclines also serve as expansion paths in the determination of optimum yields. With fixed N and P_2O_5 prices but varying crop prices, the optimum point will move up or down a given isocline. Changes in the price of either nutrient necessitates a change to a different isocline with a recalculation for the optimum point.

---

[1]The price of N and P_2O_5 will be designated as P_N and P_P respectively, in all future references to N/P price ratios.

The two outside, dashed isoclines in Figures 12, 13 and 14 are called ridgelines. These lines denote the economic limits of nutrient substitution and are determined by setting the partial derivative of Y with respect to N and Y with respect to P equal to zero and solving each derivative separately in terms of one nutrient at given levels of the other.

At the point of junction with the ridgelines, the isoquants have either zero or infinite slope, and no amount of N or P_2O_5, as the case may be, will substitute for the other nutrient. Ridgelines, since they are specific isoclines, converge at the point of maximum yield.

The ridgelines in Figures 12, 13 and 14, also serve to emphasize the importance of selecting the proper form of the variables to be used in the yield equation. In view of the 120-bushel yields observed in this experiment, it is reasonable to expect the fitted equation to predict a maximum yield in the neighborhood of 120 bushels, and that this maximum yield be obtained from rates of N and P_2O_5 close to those used in the experiment. Table 6 shows that the predicted maximum yields fall near the 120 bushel mark but that the pounds of N and P_2O_5 needed to produce this result vary considerably for the three different yield equations.

The exact maximum yield is predicted by setting the partial derivative of Y with respect to N and P equal to zero and solving both equations simultaneously.

Table 6. Predicted nutrient requirements in pounds per acre for maximum corn yields for three different yield equations.

Type of equation	Nutrient requirement N	Nutrient requirement P_2O_5	Maximum yield in bushels/A
Quadratic (1)	183.6	98.7	124.9
Root form (2)	452.0	341.9	128.8
Mixed form (3)	280.6	108.8	122.7

Economic optima

The optimum nutrient rates for this experiment were obtained with nutrient costs and the corn prices prevalent in 1953. Nitrogen was charged at $0.15 and P_2O_5 at $0.10 per pound applied to the land. Corn was valued at $1.50 per bushel.

To determine the optimum nutrient rates for equations 1, 2 and 3, the partial derivatives of Y with respect to N and P were set equal to the respective $\frac{\text{nutrient cost}}{\text{price of corn}}$ ratios and solved simultaneously. The optimum amounts of N and P_2O_5 for each equation were placed into the yield equation to determine the optimum yield. These yields, nutrient rates and net profit are shown in Table 7.

The results shown in Table 7 are of interest in that the separate equations show some divergence in optimum nutrient

Table 7. Predicted optimum nutrient rates, optimum corn yields, and net profit from yield increases for three different yield equations.

Type equation	Nutrient rates N	Nutrient rates P_2O_5	Optimum yield in bushels/A	Yield increase over check	Net return[a]
Quadratic (1)	135.0	77.2	121.7	46.8	42.23
Root form (2)	89.3	57.6	112.2	35.3	33.79
Mixed form (3)	93.9	69.6	114.5	38.2	36.25

[a]Corn at \$1.50 per bushel; N at \$0.15 and P_2O_5 at \$0.10 per pound.

rates for the production of only slightly different optimum yields. Furthermore, equation 3, which had the highest R^2, is only intermediate in net profit. This result is only an apparent discrepancy, however. If it is assumed that equation 3 models the true situation in the field, the nutrient rates for equation 1, placed into equation 3, would produce an increase of 41.9 bushels at a cost of \$27.97. With corn at \$1.50 per bushel, the net would be \$34.88 as contrasted to \$36.25 for the optimum nutrient rates computed for equation 3.

While the reduction of \$1.37 profit per acre is not large, another factor must be considered. The farmer who fertilized on the basis of equation 1 expects \$42.23 profit. However, since equation 1 does not model the true situation in the field, his net profit is only $34.88. In his eyes, therefore, his profit is reduced $7.35, not $1.37 per acre. Hence, the use of a poor model for prediction purposes in the field could result in a serious loss of confidence.

Methodological summary

The computations made in analyzing the 1953 corn yields point the way to a reasonable methodological approach in selecting a yield equation. Stepwise, the approach is as follows:

1. By statistical analysis of variance, select variables on the basis of a predetermined level of significance.
2. Subdivide the sums of squares for selected variables into their linear, quadratic, etc., components. Select those components whose sums of squares are significant at a predetermined level of significance and which have agronomic validity in terms of curvature. Interaction terms should not be raised to an order higher than the power to which the main effect has been raised.
3. Plot initial response lines for each significant main effect nutrient. If the linear and quadratic components of the main effects were found significant in step 2, and if yields rise rapidly and then level out, use the root form of the variable. If yields rise

gradually over the entire range of nutrient application, use the quadratic forms of the variable in the equation.

On the basis of the above criteria, the mixed form of the quadratic equation would have been selected to characterize the yield surface for the 1953 corn. The resulting goodness of fit of the data, based on both statistical and agronomic logic would appear to result in the most meaningful economic interpretations.

Percent nitrogen and nitrogen yield

The grain samples taken at harvest for moisture determination were saved for chemical analysis of total nitrogen and phosphorus. Unfortunately, some of the samples were damaged in storage so that they could not be analyzed. However, there were enough samples to represent all combinations of N and P. Since K had no effect in the experiment, each N, P or NP treatment could have been represented by 6 analyses. Actually, the number ranged from 1 to 4 analyses.

Since statistical analysis with odd numbers of replications would have been difficult, the selection of the significant variables and their form was based on the originally significant variables for yield and the characteristics of the observed percentage lines drawn from the mean of each treatment percentage shown in Table 49 of the Appendix. The mean nitrogen percentage lines for rates of N and P_2O_5 are shown in Figures 15 and 16. From the trends and slopes of these lines, it was felt that the quadratic equation would best characterize the N effect while a linear equation adequately characterized the P_2O_5 effect. An NP interaction effect was not evident and so this term was left out. The resulting equation with standard errors for the partial regression coefficient is

$$\underset{}{Y\%N} = \underset{\pm 0.000475}{0.004552N} - \underset{\pm 0.000002}{0.000012N^2} - \underset{\pm 0.000237}{0.000904P} + 1.255475 \, . \qquad (10)$$

The coefficient of determination for equation 10 is 80.2 percent and the t values for the partial regression coefficients in the order given are 9.58, 6.29 and 3.82 respectively. All of these t's are highly significant. The predicted nitrogen percentages for N and P_2O_5 rates are shown in Table 8. The corresponding response surface is shown in Figure 17.

Table 8. Predicted nitrogen percentages in corn grain for various nutrient combinations.

Pounds of P_2O_5/A	Pounds of N/A						
	0	40	80	120	160	200	240
0	1.26	1.42	1.54	1.63	1.68	1.69	1.66
40	1.22	1.38	1.51	1.59	1.64	1.65	1.62
80	1.18	1.35	1.47	1.56	1.60	1.61	1.58
120	1.15	1.31	1.43	1.52	1.57	1.58	1.55

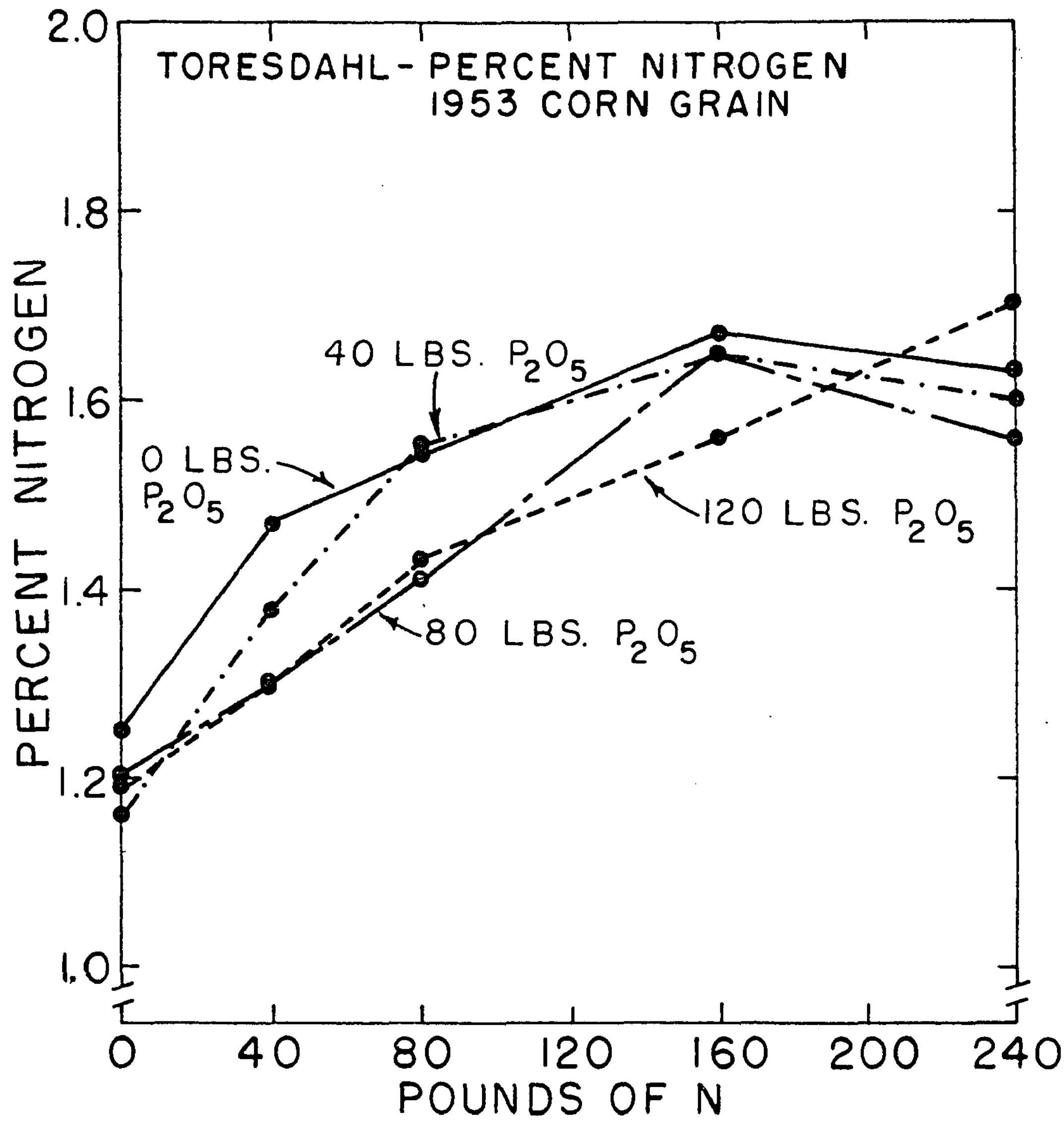


Figure 15. Average percent nitrogen in corn grain as a function of N rates at different levels of P_2O_5.

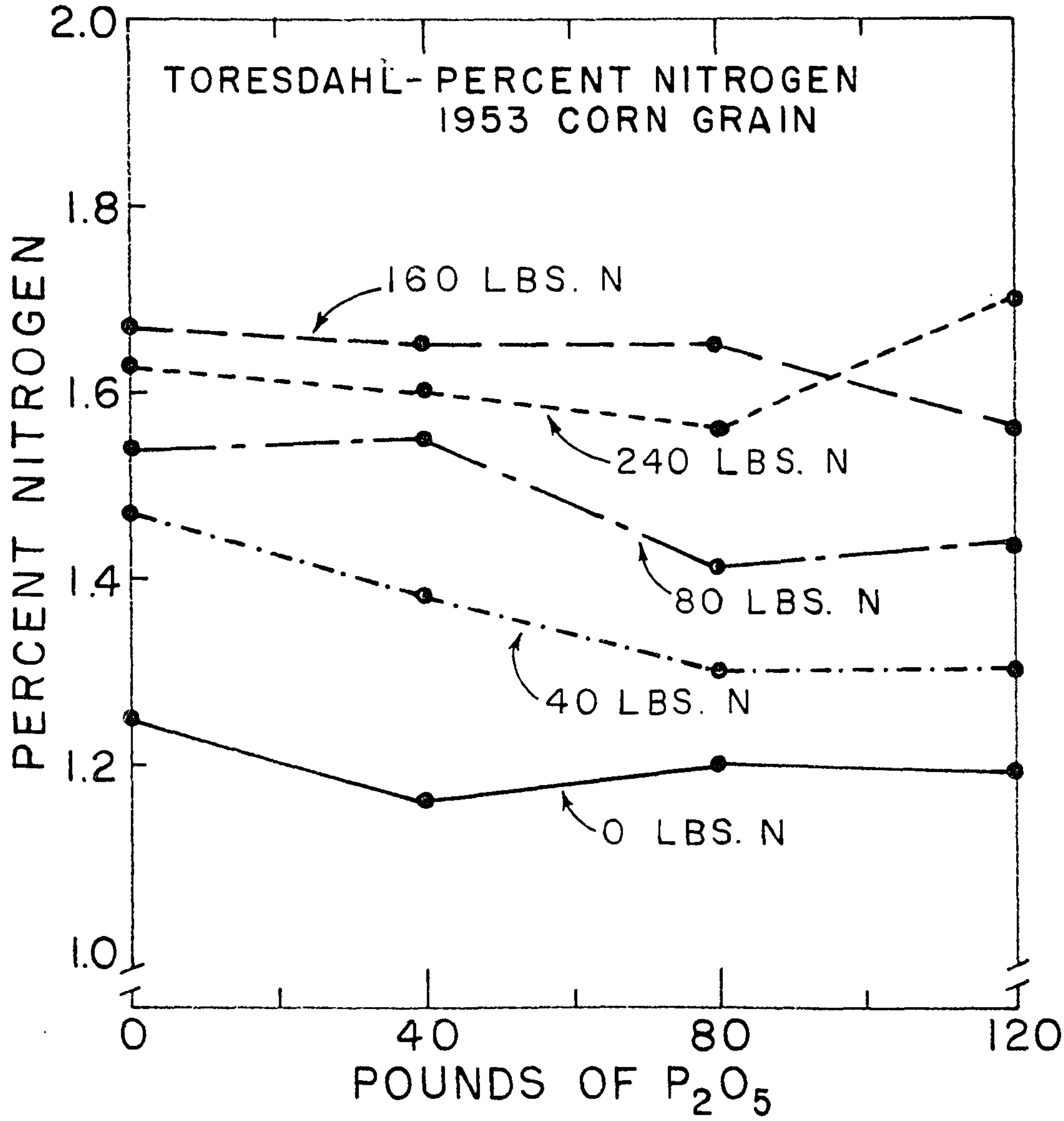


Figure 16. Average percent nitrogen in corn grain as a function of P_2O_5 rates at different levels of N.

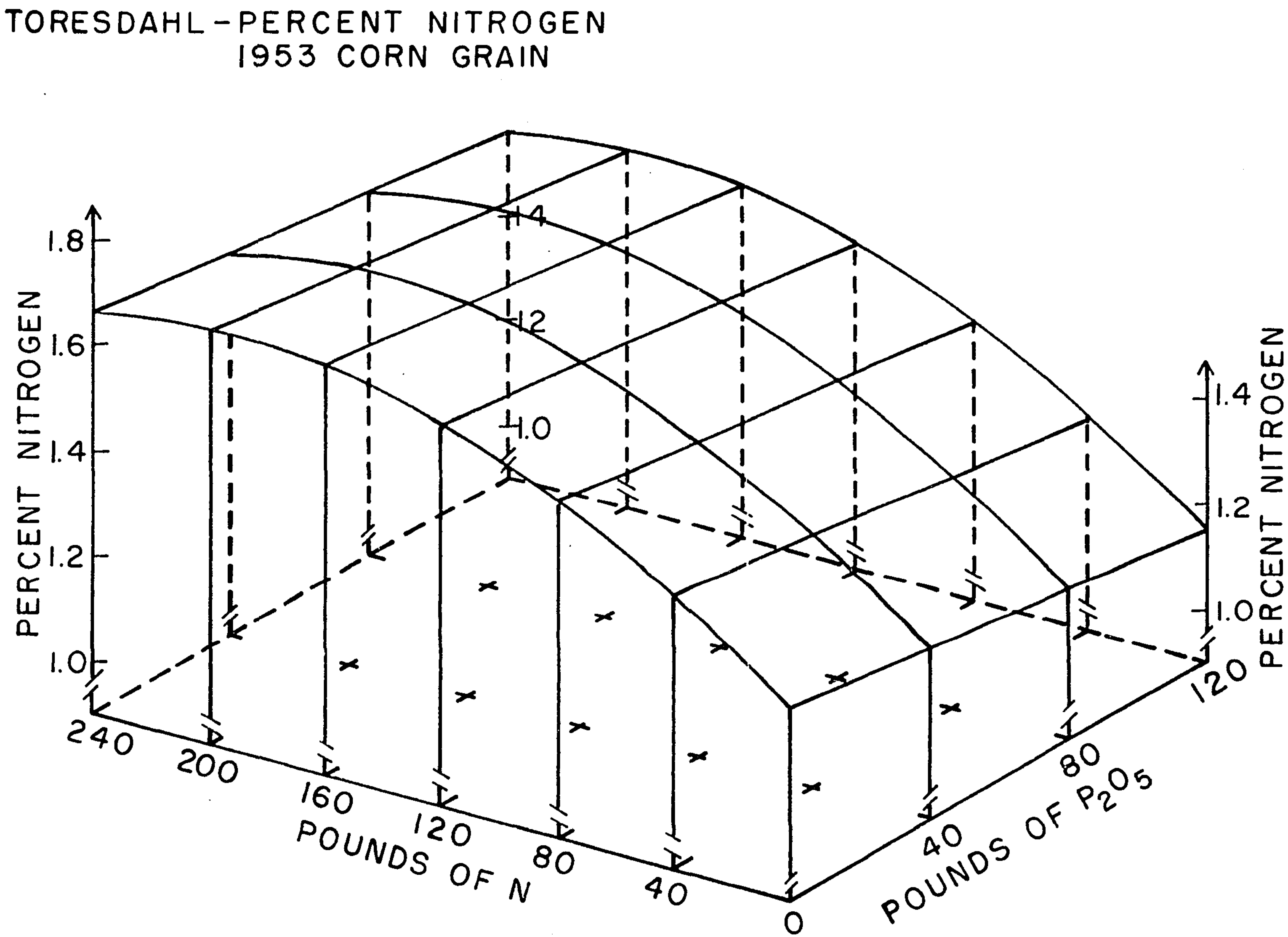


Figure 17. Predicted surface for percent nitrogen in corn grain as a function of N and P_2O_5 rates.

Percent nitrogen at the zero level of P_2O_5 in the surface of Figure 17 illustrates luxury consumption of N quite well. The yield of corn leveled off after 120 pounds of N (Figure 11) but the percent N in the grain (Figure 17) continued to rise up to 200 pounds of N. With the addition of P_2O_5 however, this effect does not occur. Percent nitrogen and yield increased together.

The negative effect of P_2O_5 rates on percent nitrogen may or may not be real since the effect of P_2O_5 on percent nitrogen was not determined by analysis of variance. If the effect is real, it is probably a dilution effect due to the increase in corn yield when P_2O_5 was applied.

When percent composition is translated into nitrogen yield, the observed mean nitrogen yield lines taken from the figures in Table 49 of the Appendix show the form exhibited in Figures 18 and 19. Nitrogen effects appear to take the quadratic form while the phosphorus effects more nearly approach the root form of the quadratic - particularly at high levels of N. A definite interaction is evident in both figures. The equation with standard errors for the partial regression coefficients is

$$Y_{N\ yield} = \underset{\pm 0.067}{0.447148N} - \underset{\pm 0.00025}{0.001697N^2} + \underset{\pm 1.34}{0.771196P^{\frac{1}{2}}} - \underset{\pm 0.1175}{0.154499P} + \underset{\pm 0.00466}{0.022720NP^{\frac{1}{2}}} + 57.345654. \qquad (11)$$

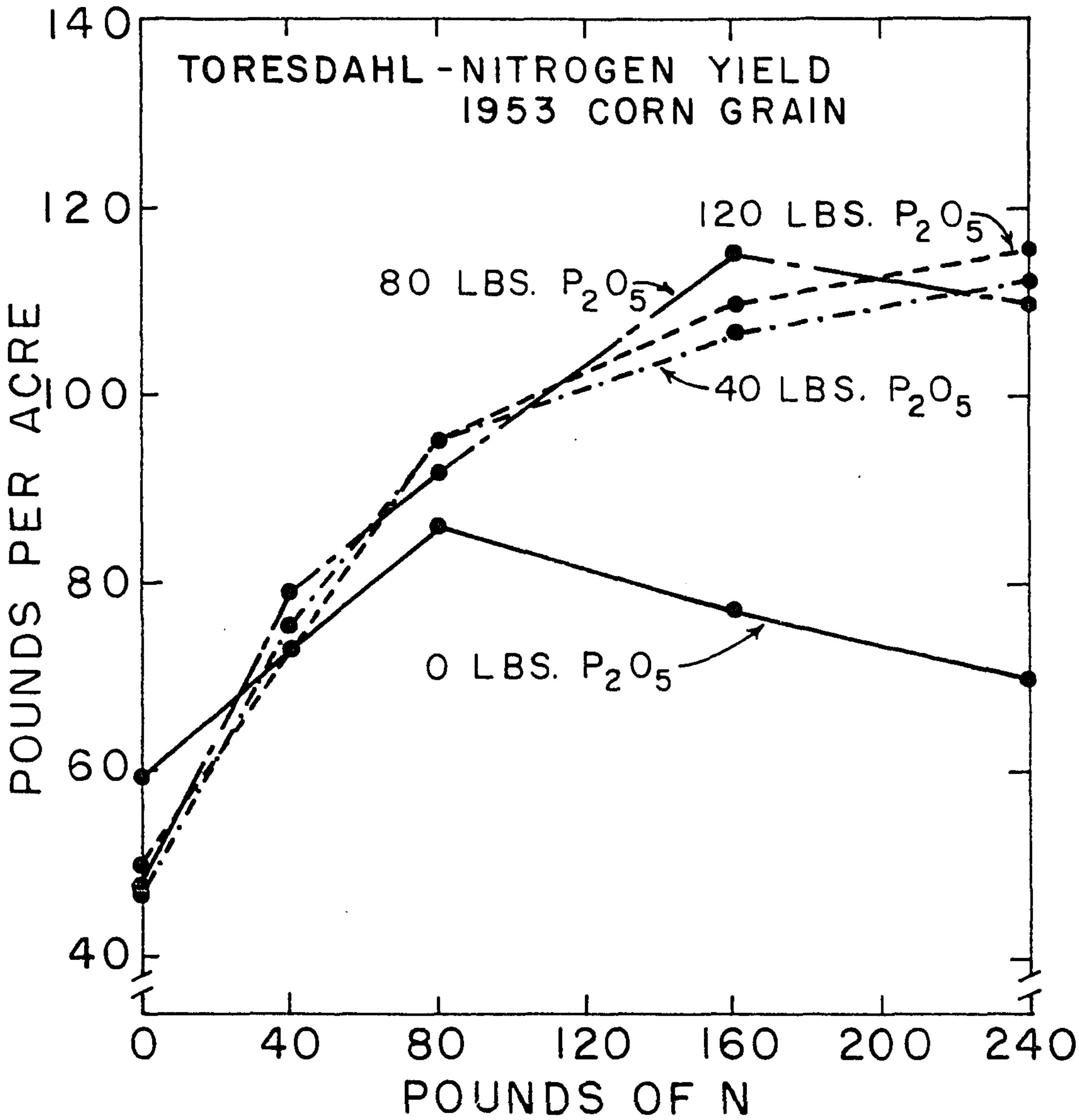


Figure 18. Average nitrogen yield of corn as a function of N rates at different levels of P_2O_5.

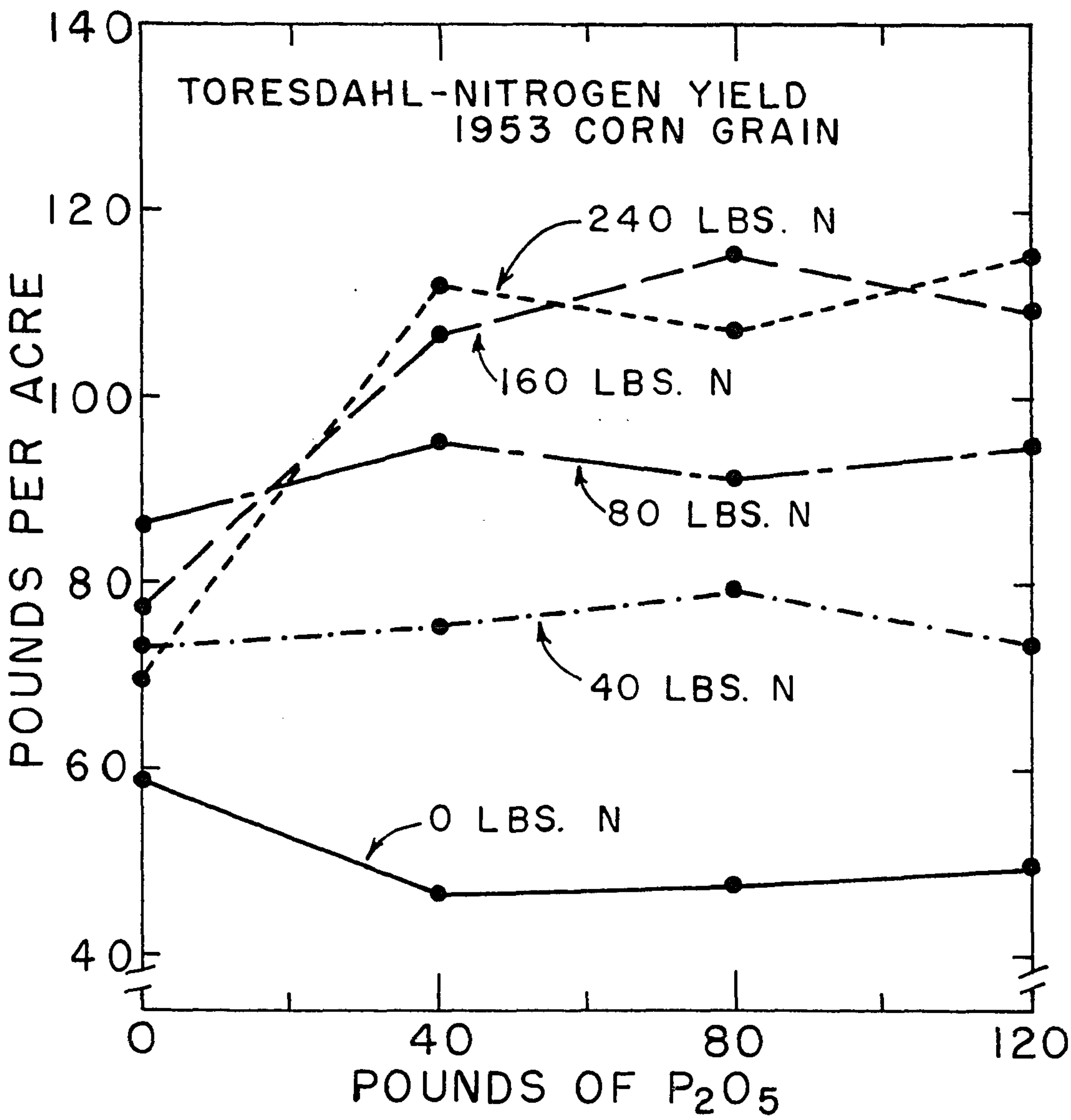


Figure 19. Average nitrogen yield of corn as a function of P_2O_5 rates at different levels of N.

The R^2 for equation 11 is 81.0 percent and the t values for the partial regression coefficients in the order shown are 6.64, 6.73, 0.58, 1.31 and 4.87. The first, second and fifth t's are highly significant, while the others are significant only at a low level of probability.

The predicted nitrogen yields obtained for rates of N and P_2O_5 are shown in Table 9. The corresponding response surface is shown in Figure 20.

Table 9. Predicted nitrogen yield in pounds per acre for corn grain obtained with various nutrient combinations.

Pounds of	Pounds of N/A						
P_2O_5/A	0	40	80	120	160	200	240
0	57.3	72.5	82.3	86.6	85.4	78.9	66.9
40	56.0	77.0	92.5	102.5	107.1	106.3	100.1
80	51.9	75.2	93.1	105.5	112.5	114.1	110.2
120	47.3	72.4	92.1	106.3	115.2	118.6	116.6

From the low t values for the P terms, it appears that the P_2O_5 effect should have been linear as it was in the case of percent nitrogen. This linear effect would probably have been evident if an analysis of variance could have been made for the data.

The quadratic effect, however, does not seriously affect the interpretation of the surface. There is a marked resem-

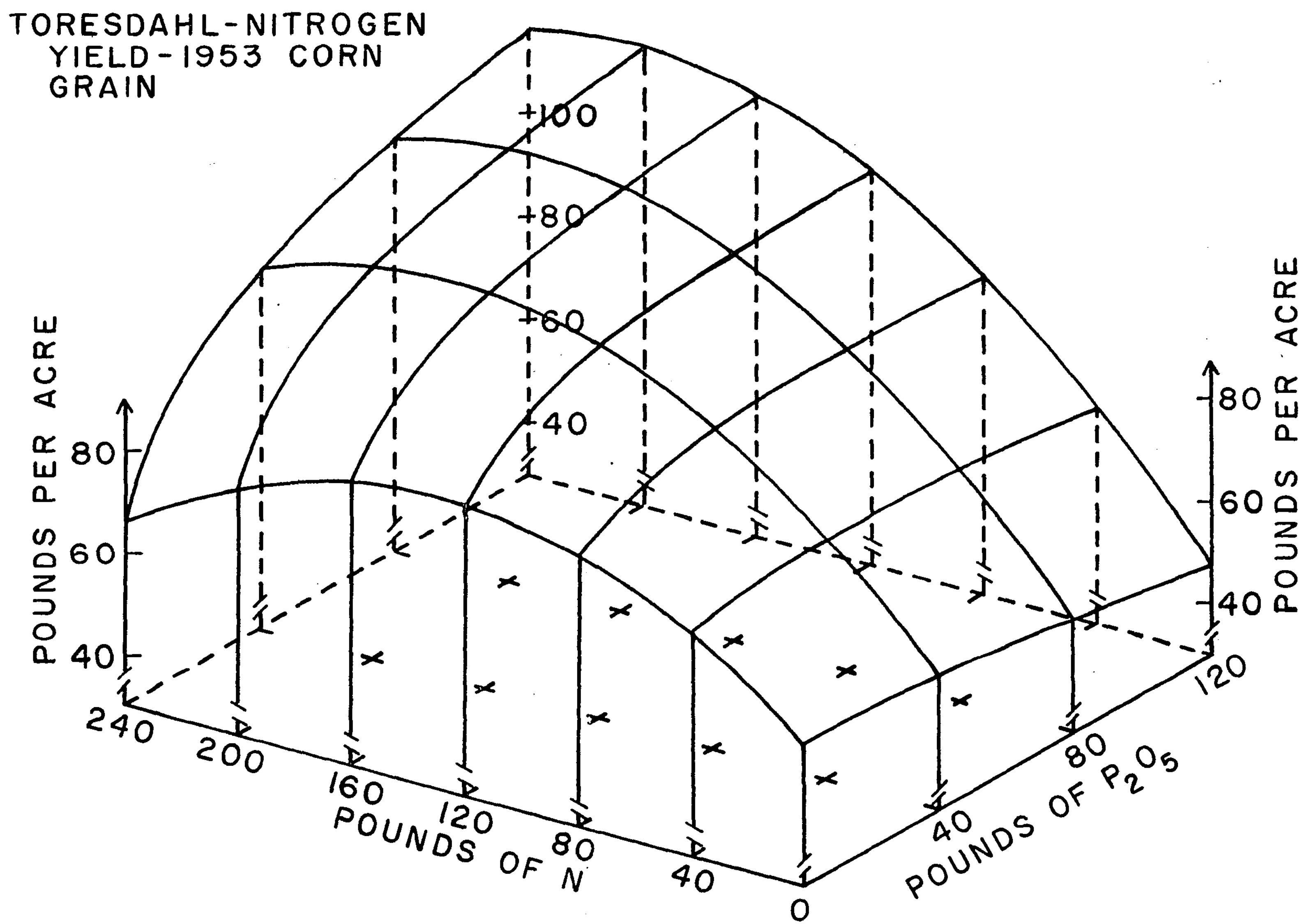


Figure 20. Predicted surface for nitrogen yield of corn as a function of N and P_2O_5 rates.

blance between the nitrogen yield surface and the percent nitrogen surface at P_2O_5 levels of 40 or more pounds per acre. The specific depression in nitrogen yield at the zero P_2O_5 level is due to some rather low yields for the particular samples used for analysis from the high N rates, (see Table 49 in Appendix). Aside from this fact, the major point of interest is the similarity of the yield, percent nitrogen and nitrogen yield surfaces.

<u>Percent phosphorus and phosphorus yield</u>

To determine the percent phosphorus surface, the mean observed percentage lines drawn from the means of the treatments shown in Table 50 of the Appendix were plotted as in Figures 21 and 22. From these drawings, it is evident that there is a linear N effect, quadratic P effect and an NP interaction. The resulting equation for these variables together with the standard errors for the partial regression coefficients is

$$Y_{\%P} = \underset{\pm 0.000067}{-0.000269N} + \underset{\pm 0.00024}{0.000741P} - \underset{\pm 0.00000187}{0.00000435P^2} + \underset{\pm 0.00000088}{0.00000275NP} + 0.197325. \qquad (12)$$

The R^2 for equation 12 was 56.5 percent and the t values for the partial regression coefficients in the order shown were 3.99, 3.05, 2.33 and 3.12 respectively. All of these t's are highly significant.

The predicted phosphorus percentages are shown in Table 10 and the corresponding surface in Figure 23.

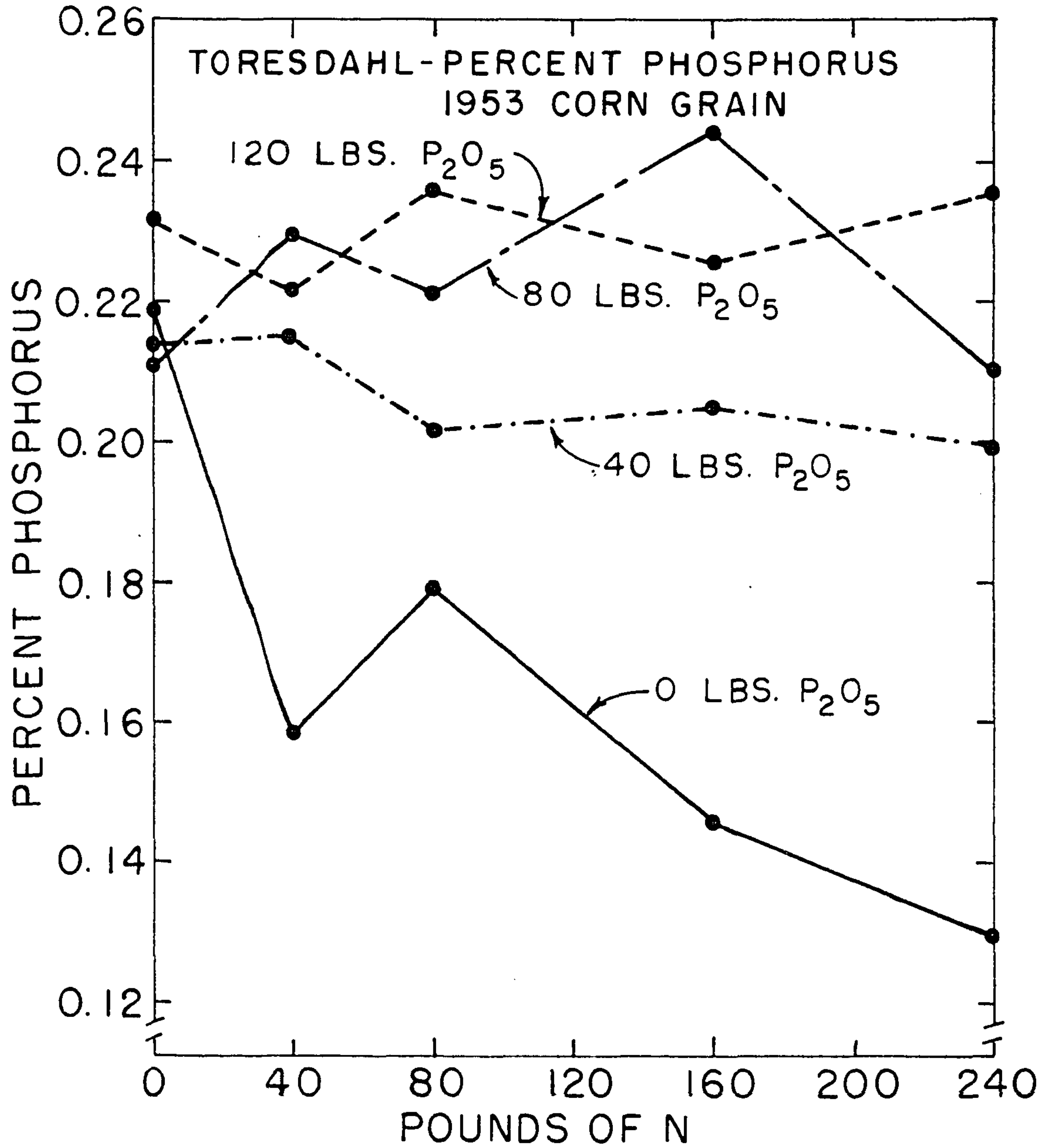


Figure 21. **Average percent phosphorus in corn grain as a function of N rates at different levels of P_2O_5.**

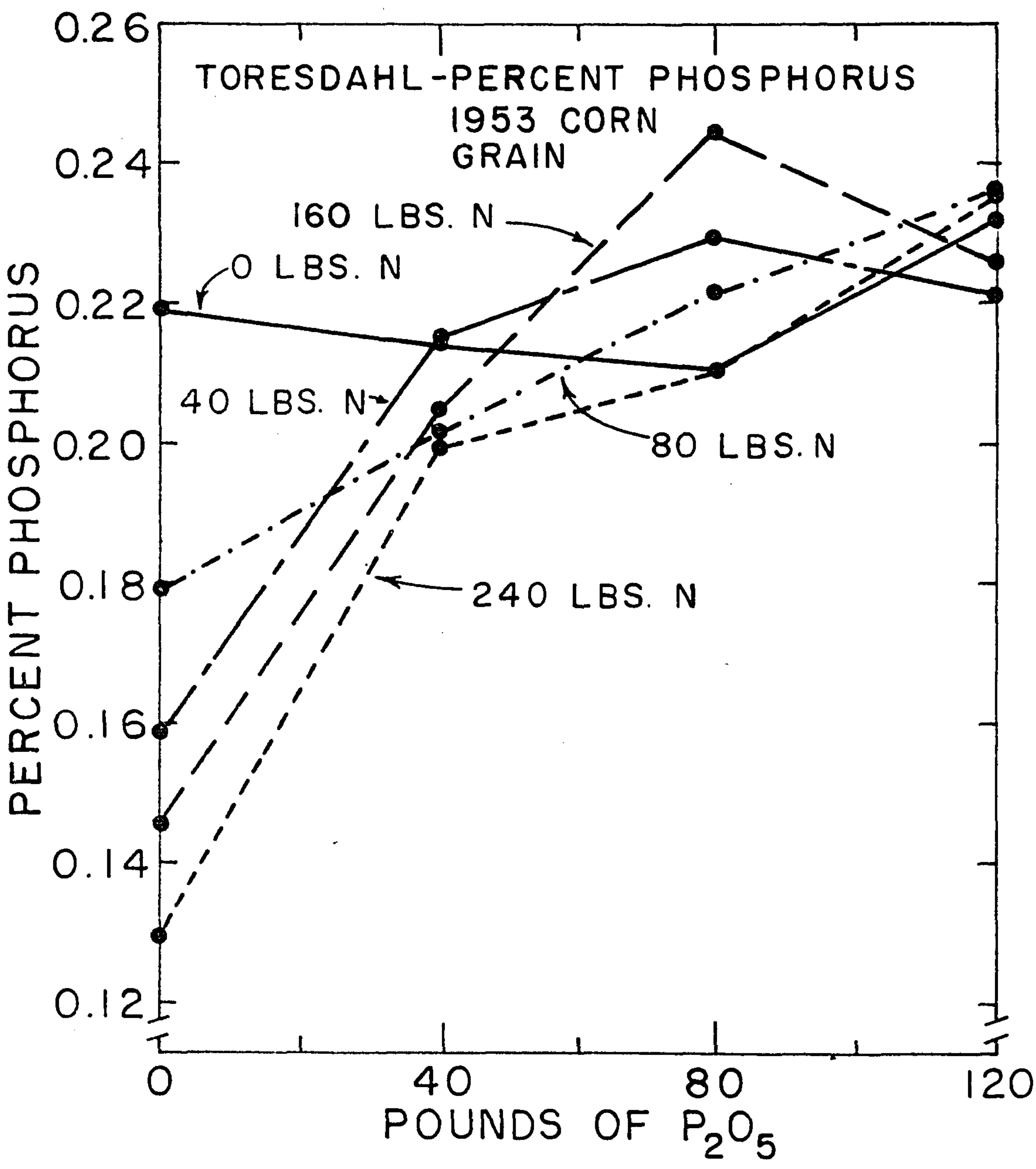


Figure 22. Average percent of phosphorus in corn grain as a function of P_2O_5 rates at different levels of N.

Table 10. Predicted percent phosphorus in corn grain for various nutrient combinations.

Pounds of P_2O_5/A	Pounds of N/A						
	0	40	80	120	160	200	240
0	0.197	0.187	0.176	0.165	0.154	0.144	0.133
40	0.220	0.214	0.207	0.201	0.195	0.188	0.182
80	0.229	0.227	0.225	0.223	0.221	0.219	0.217
120	0.224	0.226	0.228	0.231	0.233	0.236	0.238

Percent phosphorus decreases with increasing N rates at the zero level of P_2O_5 in Figure 23 just as percent nitrogen decreased with increasing P_2O_5 rates at the zero level of N in Figure 17. This result is probably a dilution effect in both cases due to yield increases accruing to the addition of one limiting nutrient to the soil when the other nutrient is also limiting but is not added to the soil.

While the above explanation is reasonable for additions of one nutrient at zero levels of the other, it does not explain continued depression of the one nutrient percentage if that nutrient is added to the soil along with the other as is the case in Figure 17. Neither does it explain why a positive interaction should occur between N and P as is shown in Figure 23. A possible explanation for the latter, however, might be the adequacy of both nutrients in the soil which increases root growth and permits absorption of both nutrients to keep

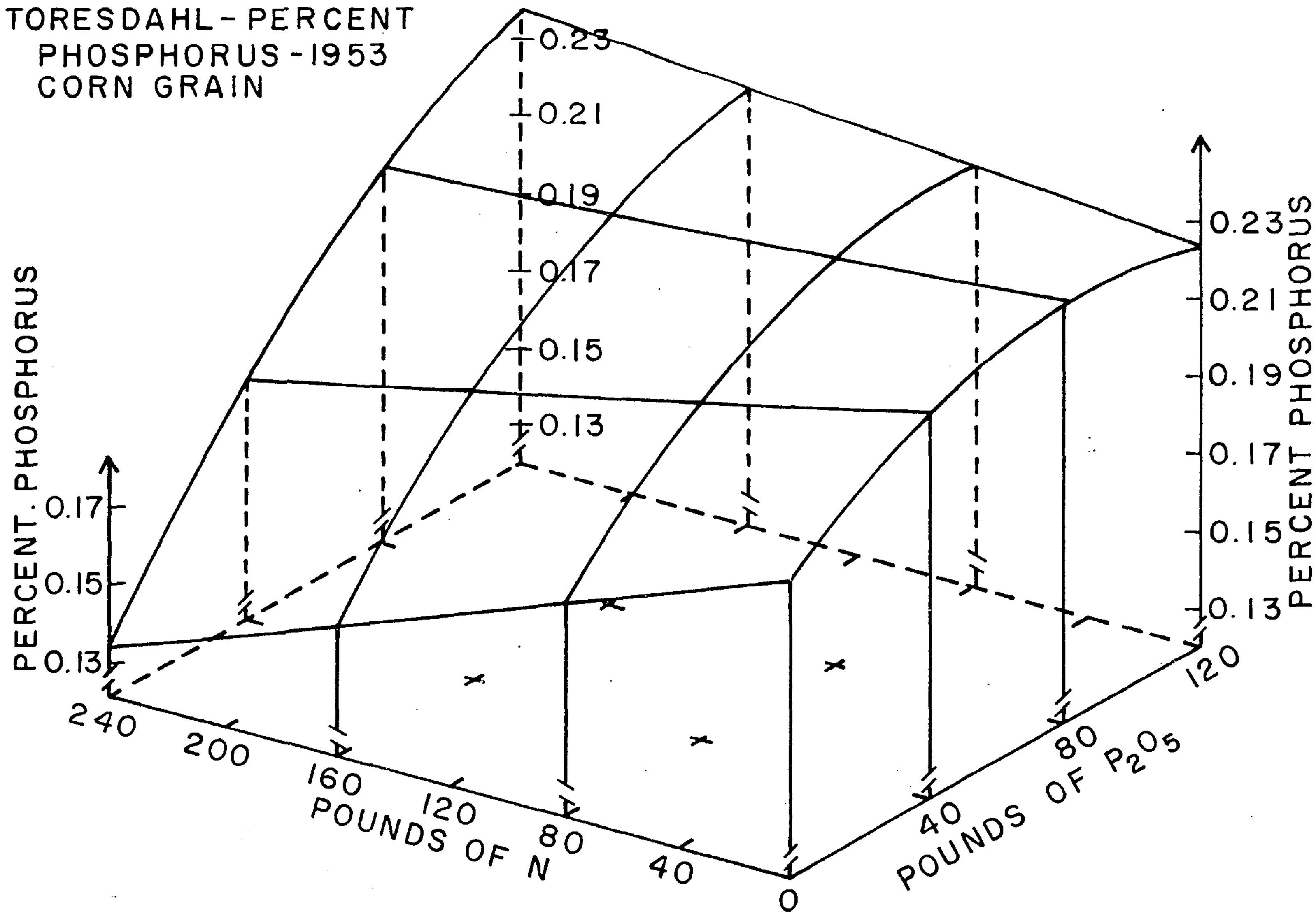


Figure 23. Predicted surface for percent phosphorus in corn grain as a function of N and P_2O_5.

pace with growth and dry matter accumulation. However, this is only a hypothesis and is not substantiated by any data in this dissertation.

The translation of phosphorus percentages into phosphorus yields results in the mean observed phosphorus yield lines shown in Figures 24 and 25. The data points are the means of the values shown in Table 50 of the Appendix. It is evident from these drawings that the N effect is basically of the root form of the quadratic and the P_2O_5 effect of the quadratic form. Subdivision of the NP sum of squares resulted in a significant linear x linear NP interaction which makes this term $N^{\frac{1}{2}}P$. The resulting equation for these variables with standard errors for the partial regression coefficients is

$$Y_{P\,yield} = \underset{\pm 0.19}{0.213868N^{\frac{1}{2}}} - \underset{\pm 0.0112}{0.021582N} + \underset{\pm 0.025}{0.048815P} - \underset{\pm 0.000185}{0.000424P^2} + \underset{\pm 0.00125}{0.005620N^{\frac{1}{2}}P} + 8.923547. \qquad (13)$$

The R^2 for equation 13 was 64.0 percent and the t values for the partial regression coefficients in the order shown were 1.13, 1.93, 1.98, 2.29 and 4.48. The first t is significant at a low level of probability. The next two t's are significant at the 10 percent level while the fourth and fifth t's are significant at the 5 and 1 percent levels, respectively.

Predicted phosphorus yields are shown in Table 11 and the

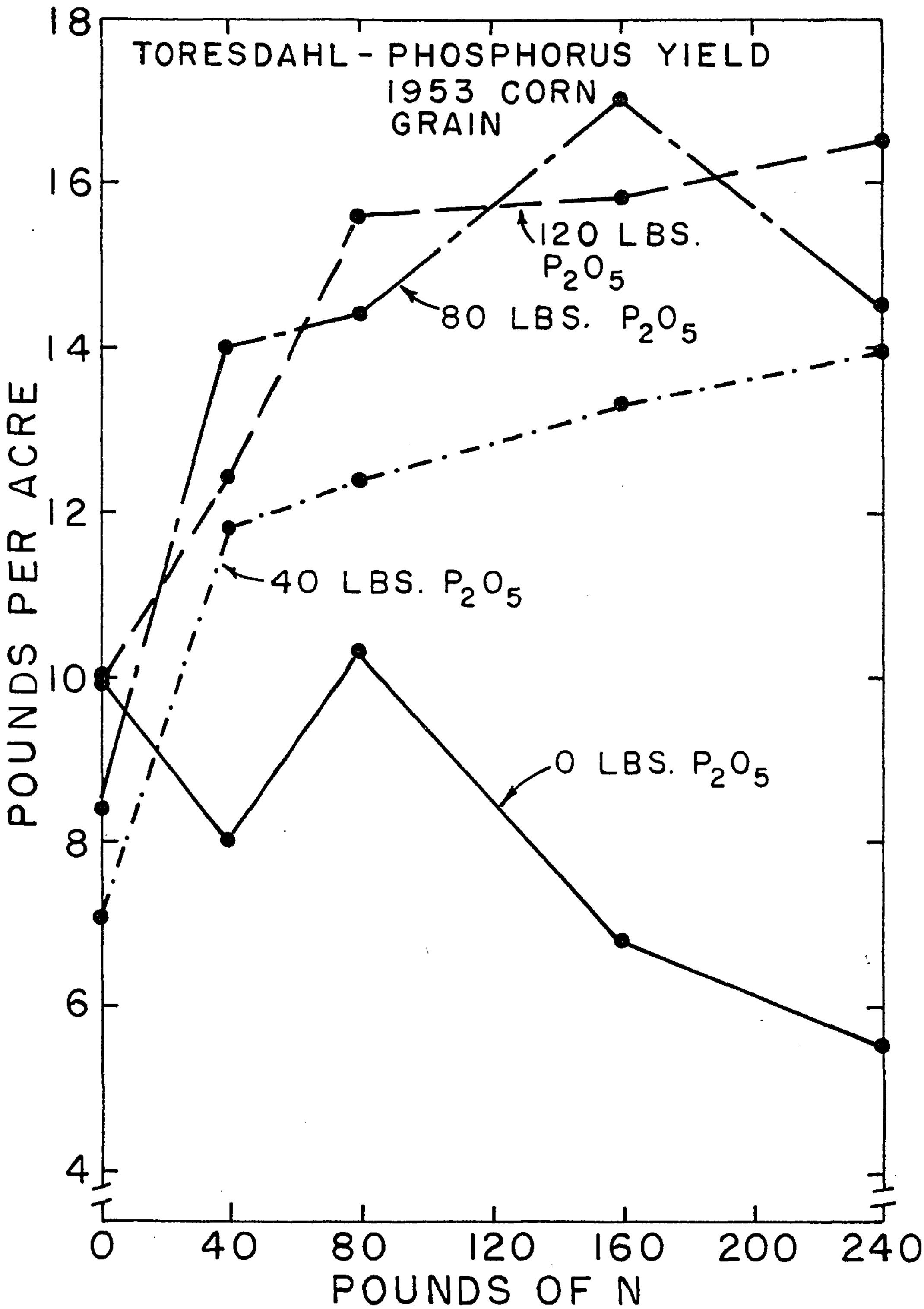


Figure 24. Average phosphorus yield of corn as a function of N rates at different levels of P_2O_5.

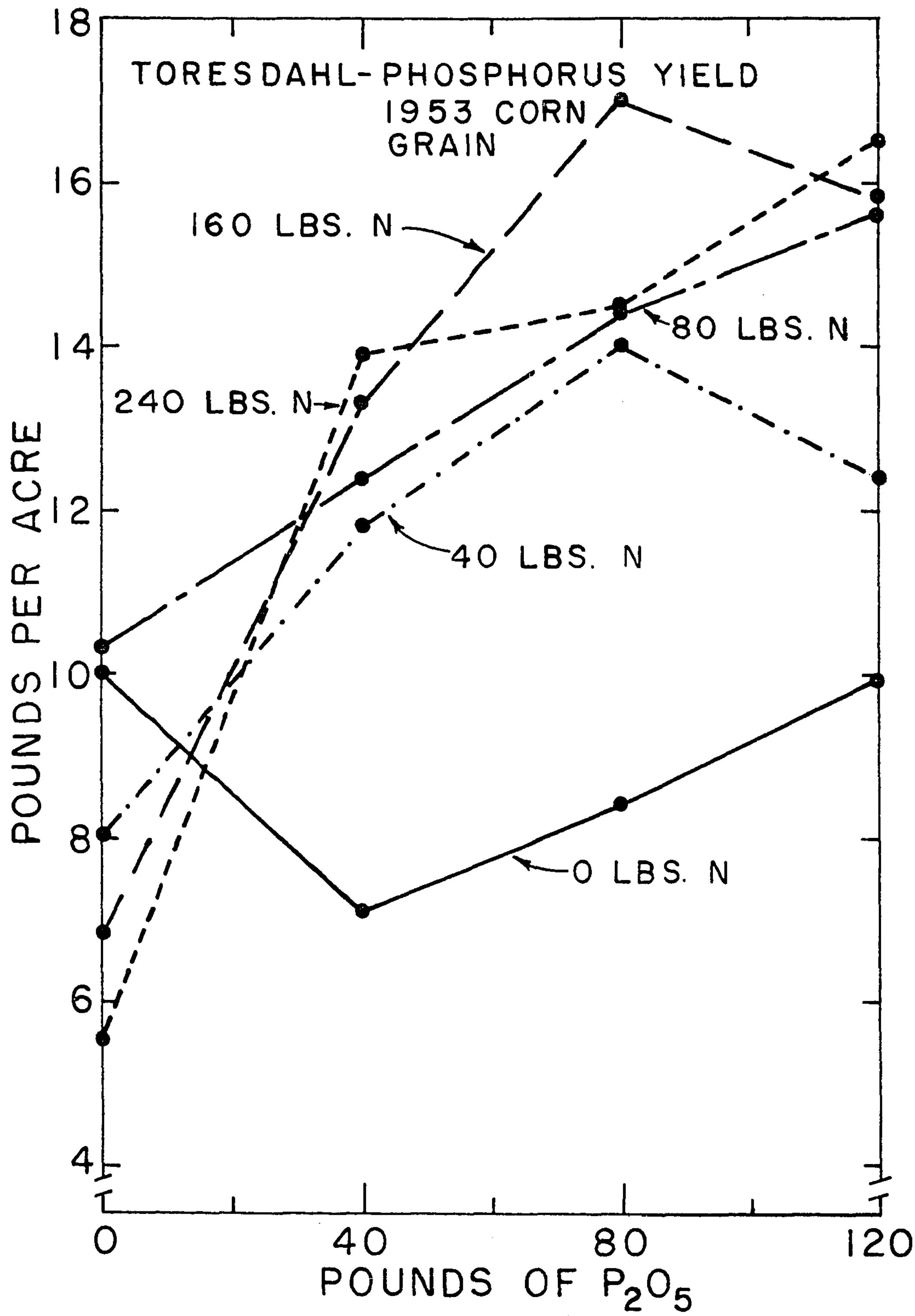


Figure 25. Average phosphorus yield of corn as a function of P_2O_5 rate at different levels of N.

corresponding surface in Figure 26.

Table 11. Predicted phosphorus yield of corn grain for nutrient combinations.

Pounds of P_2O_5/A	Pounds of N/A						
	0	40	80	120	160	200	240
0	8.9	9.4	9.1	8.7	8.2	7.6	7.1
40	10.2	12.1	12.4	12.4	12.3	12.1	11.8
80	10.1	13.4	14.3	14.8	15.1	15.2	15.2
120	8.7	13.4	14.9	15.8	16.5	16.9	17.3

Although there are differences in the respective surfaces for grain, nitrogen and phosphorus yields, (Figures 11, 20 and 26), there are some similiarities. The strong NP interaction is evident in all surfaces as is the general trend of lines. Phosphorus yield is low at high levels of N at zero levels of P_2O_5 due to a definite depression in percent uptake as shown in Figure 23 together with low yielding plots which were referred to earlier under the nitrogen yield discussions.

One difference in the phosphorus yield surface as compared to the surfaces in Figures 11 and 20 is the more rapid rise in yield of phosphorus at high levels of N than occurs for either grain or nitrogen yield. This result is an expression of the same trend which showed up in percent phosphorus in the grain. While the reason for this effect is obscure, it may be the

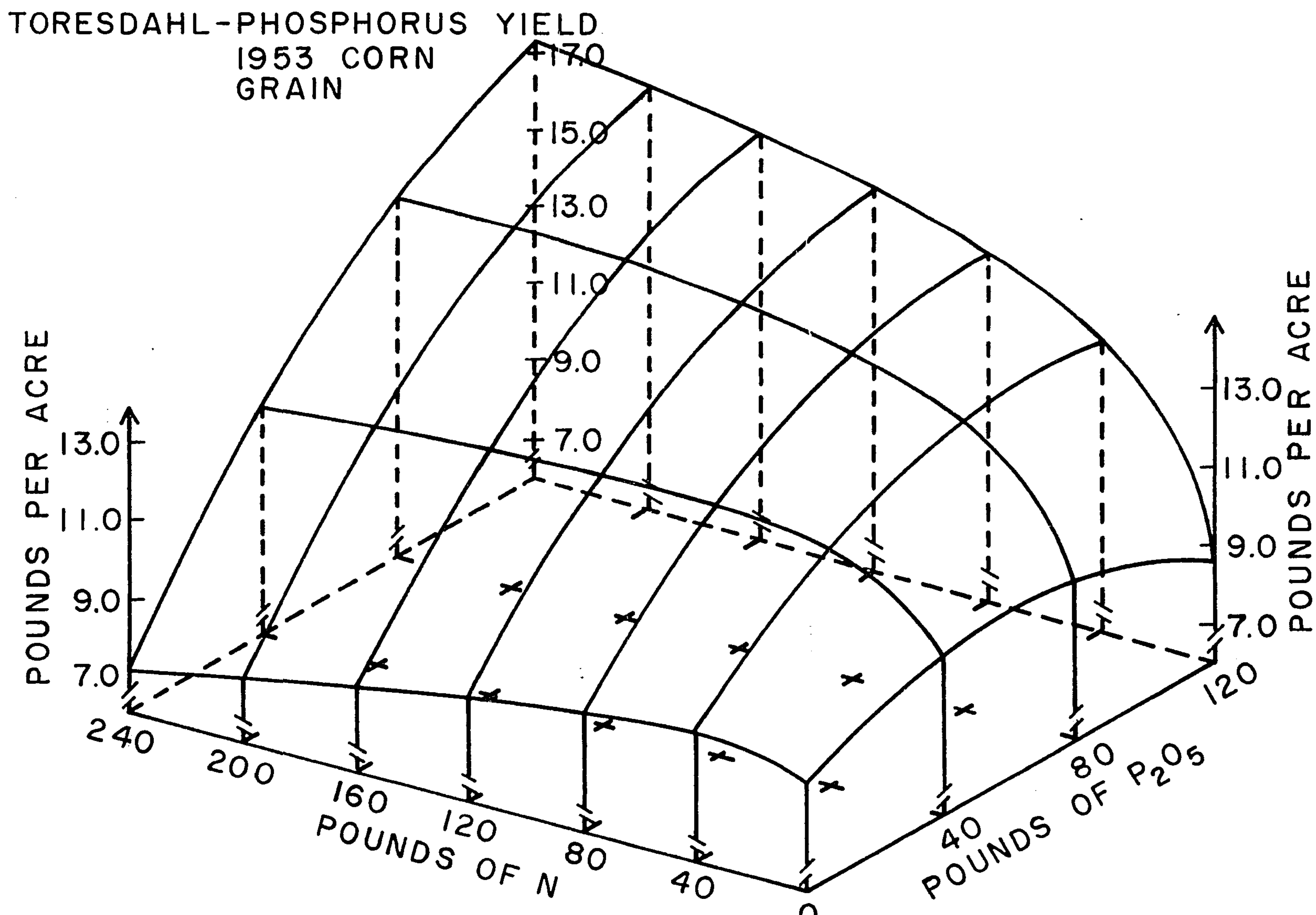


Figure 26. Predicted surface for phosphorus yield of corn as a function of N and P_2O_5 rates.

result of the increasing adequacy of both N and P as both rates increase together along with an enlarged root system which increases the soil volume from which P_2O_5 can be absorbed.

Both nitrogen and phosphorus yield surfaces exhibited curvature for both N and P effects. Within the framework proposed by Fried and Dean (16) for measuring available soil phosphorus, each independent phosphorus curve in each surface apparently falls into class 2 (curvilinear yield of phosphorus curves) mentioned by Dean (11). The reason for the divergence of both the nitrogen and phosphorus yield curves from a straight line is not known.

Summary - 1953 corn

1. The yield equation with the highest R^2 predicted yield increases up to 45.9 bushels per acre for combinations of N and P_2O_5.
2. A method of determining the best fitting yield equation with a minimum of labor was presented. See methodological summary on page 53.
3. Optimum and maximum nutrient rates are presented for 3 different equations. The greatest net profit accrues to the equation having the highest R^2 on the assumption that it represents the true situation in the field.

4. Luxury consumption of N or P_2O_5 occurred for the corn grain only at zero levels of the other nutrient. When both nutrients were added together, luxury consumption did not occur.
5. Percent phosphorus was depressed for increasing N rates up to about 100 pounds per acre of P_2O_5. At the 120 pound P_2O_5 rate, increasing N rates increased the percent phosphorus in the grain.
6. Nutrient yield surfaces conformed more closely to crop yield surfaces than did those for nutrient percentages.

Oats - 1954

As outlined under objectives, this study was also concerned with measuring residual fertilizer effects. In order to provide a standard by which to measure these effects, rates of N and P_2O_5 were topdressed on separate plots which had not received N or P_2O_5 originally. These plots were the original check plots and those receiving K_2O alone. Since oat yields were not increased by the topdressed P_2O_5 they are not reported in this thesis.

General observations

Residual oat yields for the various treatment combinations and for the main effects are shown in Table 12. Potassium treated plots were not harvested because of the apparent lack of response.

The yields in Table 12 indicate a very good response to residual fertility. Both N and P_2O_5 effects, as well as the NP interaction, were highly significant as is shown by the analysis of variance in Table 13.

The oats did not lodge on any of the residual fertility treatments. This result was somewhat unexpected, particularly at the 240 pound N rate. Leaching losses, previous utilization by the corn crop and microbial tieup of N in decomposing corn stalks probably reduced available N to the point

Table 12. Oat yield in bushels per acre obtained in 1954 from residual fertilizer treatments applied to corn in 1953. Each value is a mean of 2 observations.

Pounds of P_2O_5/A	Pounds of N/A				
	0	40	80	160	240
0	16.0	18.3	26.9	22.8	24.7
40	16.3	25.4	24.1	41.0	38.2
80	19.2	22.5	29.1	47.8	58.3
120	19.2	19.4	29.0	49.6	55.6

Mean yield for main effects

Pounds of N/A	Mean yield (4 obs.)	Pounds of P_2O_5/A	Mean yield (5 obs.)
0	17.7	0	21.7
40	21.4	40	29.0
80	27.3	80	35.4
160	40.3	120	34.6
240	44.2		

where it was no longer a problem in this regard. Subsoil samples taken in April of 1954 showed only 60 pounds more nitrate per acre in the top 3 feet of soil for the 240 + 80 + 80 plots than for the check plots. Heavy rains in May could have caused additional leaching with the result that

Table 13. Analysis of variance for 1954 residual oat grain yields.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	39	7,894,991		
Replication	1	332,296	332,296	11.14**
Treatment	19	6,996,033	368,212	12.35**
Nitrogen (N)	4	4,433,602	1,108,401	37.16**
Linear	1		4,243,001	142.27**
Quadratic	1		32,385	1.09
Cubic	1		104,170	3.49[c]
Quartic	1		54,046	1.81
Phosphorus (P)	3	1,215,299	405,100	13.58**
Linear	1		1,027,863	34.46**
Quadratic	1		167,288	5.61*
Cubic	1		20,148	0.68
NP	12	1,347,132	112,261	3.76**
Error	19	566,662	29,824	

[c]Significance level is explained in footnote of Table 4.

the effective residual nitrogen on these plots was probably less than 60 pounds per acre.

Regression analyses

Following the procedure established previously, the mean yield lines resulting from Table 12 were plotted as shown in Figures 27 and 28. A cubic effect is evident for the N re-

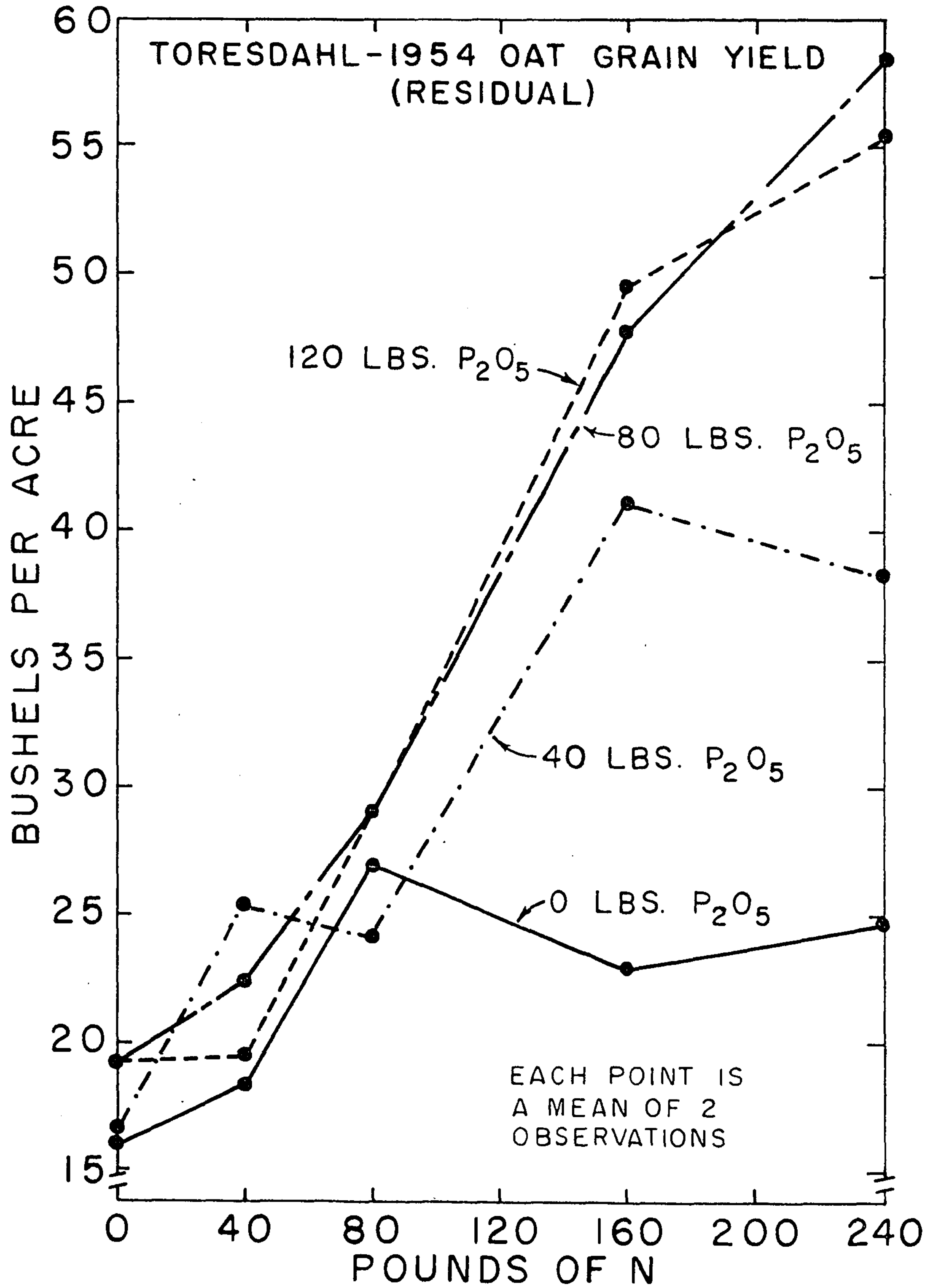


Figure 27. Average yield of oat grain as a function of residual rates at different levels of residual P_2O_5 in the second year after application.

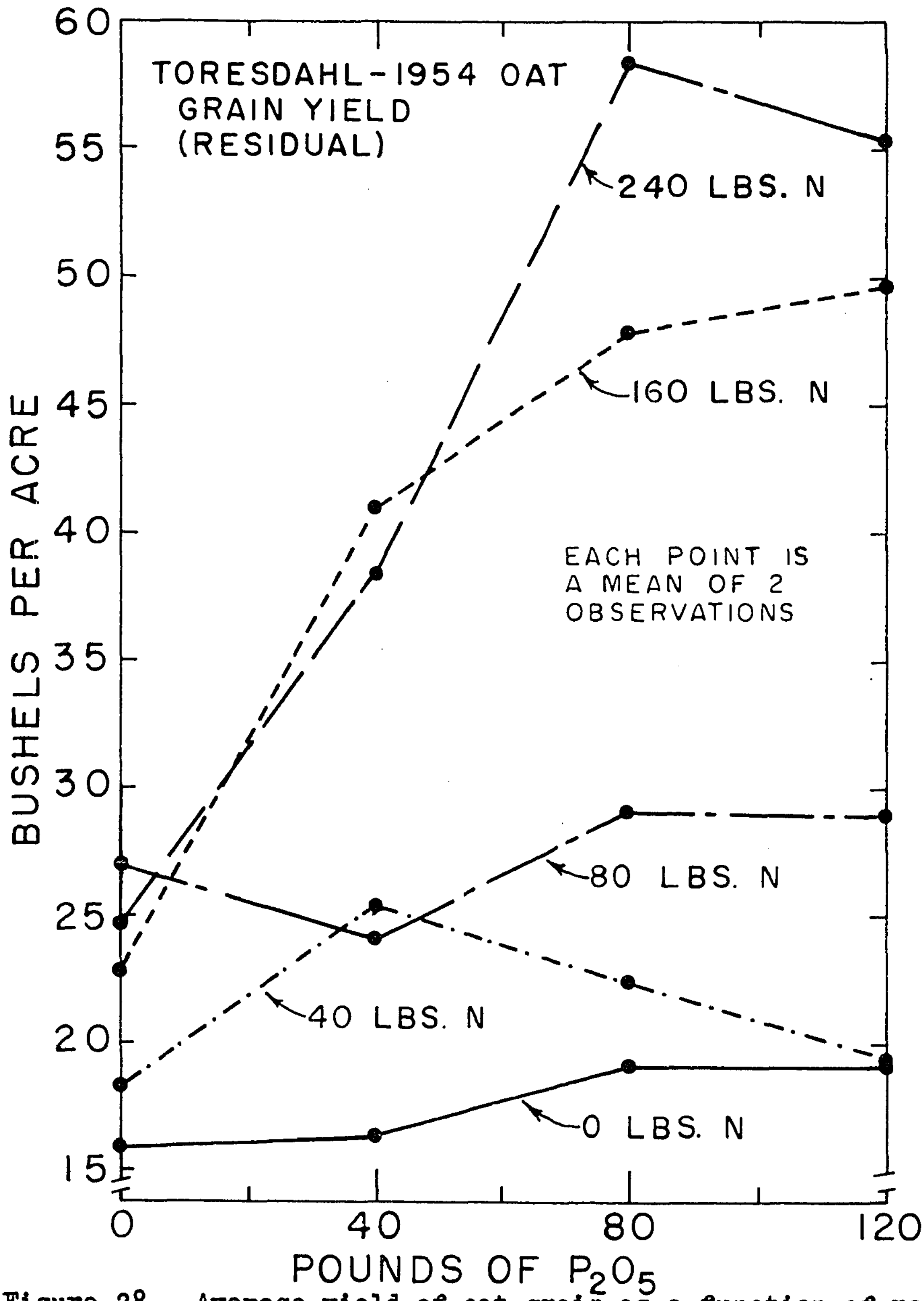


Figure 28. Average yield of oat grain as a function of residual P_2O_5 rates at different levels of residual N in the second year after application.

sponse in Figure 27 while the quadratic effect predominates for P_2O_5 effect in Figure 28. A large NP interaction is also evident.

At this point, and prior to the subdivision of the main effect sum of squares shown in Table 13, it was decided to include logical agronomic phenomena provided the level of significance was equal to or less than a probability of 0.20 and preferably, equal to or less than 0.10.

On the basis of the above statistical criteria and the agronomic plausibility of a sigmoid yield curve for residual N for the rates used in the experiment, a multiple regression equation containing a cubed N term was fitted to the data. The yield equation with standard errors for the partial regression coefficients is

$$
\begin{aligned}
Y = \underset{\pm 0.088}{-0.0267N} + \underset{\pm 0.00092}{0.00122N^2} - \underset{\pm 0.000003}{0.000004N^3} + \underset{\pm 0.077}{0.133P} - \underset{\pm 0.00058}{0.001263P^2} \\
+ \underset{\pm 0.00024}{0.001256NP} + 16.77. \quad (14)
\end{aligned}
$$

The correlation index (R^2) for equation 14 is 94.2 percent and the t values for the partial regression coefficients in the order shown are 0.29, 1.25, 1.49, 1.63, 2.05 and 4.92. All of these values have relatively low levels of significance except the last one for the NP interaction which is significant at the 1 percent level.

Yield curves

Predicted yields for various residual nutrient combinations are shown in Table 14. The corresponding yield curves are shown in Figures 29 and 30.

Table 14. Predicted 1954 oat grain yields in bushels per acre for various nutrient combinations applied to corn in 1953.

Pounds of P_2O_5/A	Pounds of N/A						
	0	40	80	120	160	200	240
0	16.8	17.4	20.4	24.2	27.2	27.9	24.8
40	20.1	22.7	27.8	33.5	38.5	41.2	40.1
80	19.3	24.0	30.9	38.7	45.7	50.5	51.4
120	14.5	21.2	30.1	39.9	49.0	55.7	58.6

The sigmoid yield curve for N in Figure 29 is reasonable in light of the relatively large response of corn to the 40 and 80 pound N rates in 1953 combined with some leaching and N tieup in decomposing residues. Factors more limiting than N cause the leveling off in yield at the higher N rates.

The quadratic effect for P_2O_5 is also reasonable in light of relatively smaller removal of P_2O_5 compared to N in the corn crop. Furthermore, there would be no leaching and relatively less tieup of P_2O_5 by microorganisms.

Figure 29. Predicted yield of oat grain as a function of residual N at different levels of residual P_2O_5 in the second year after application compared to N topdressed in 1954.

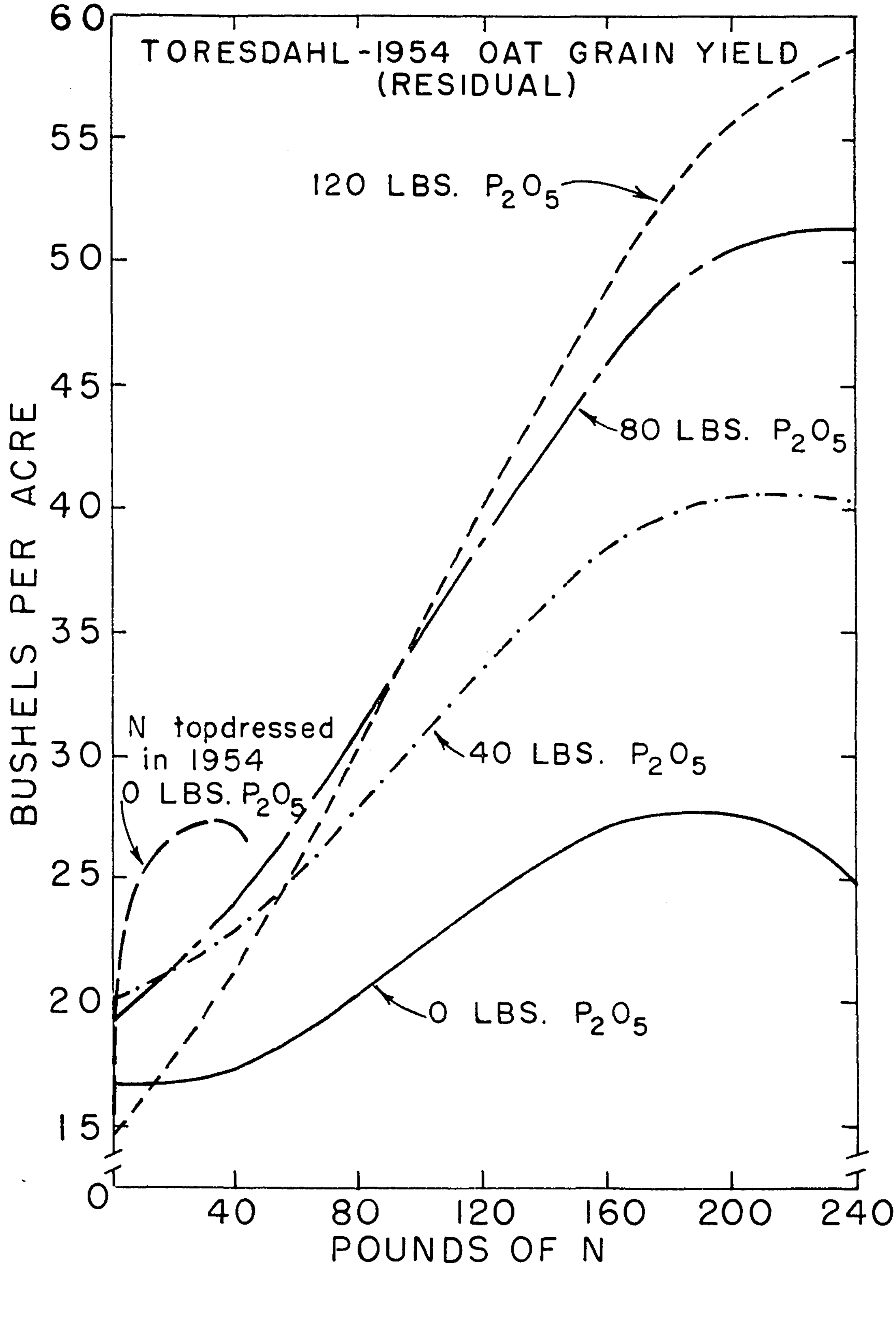
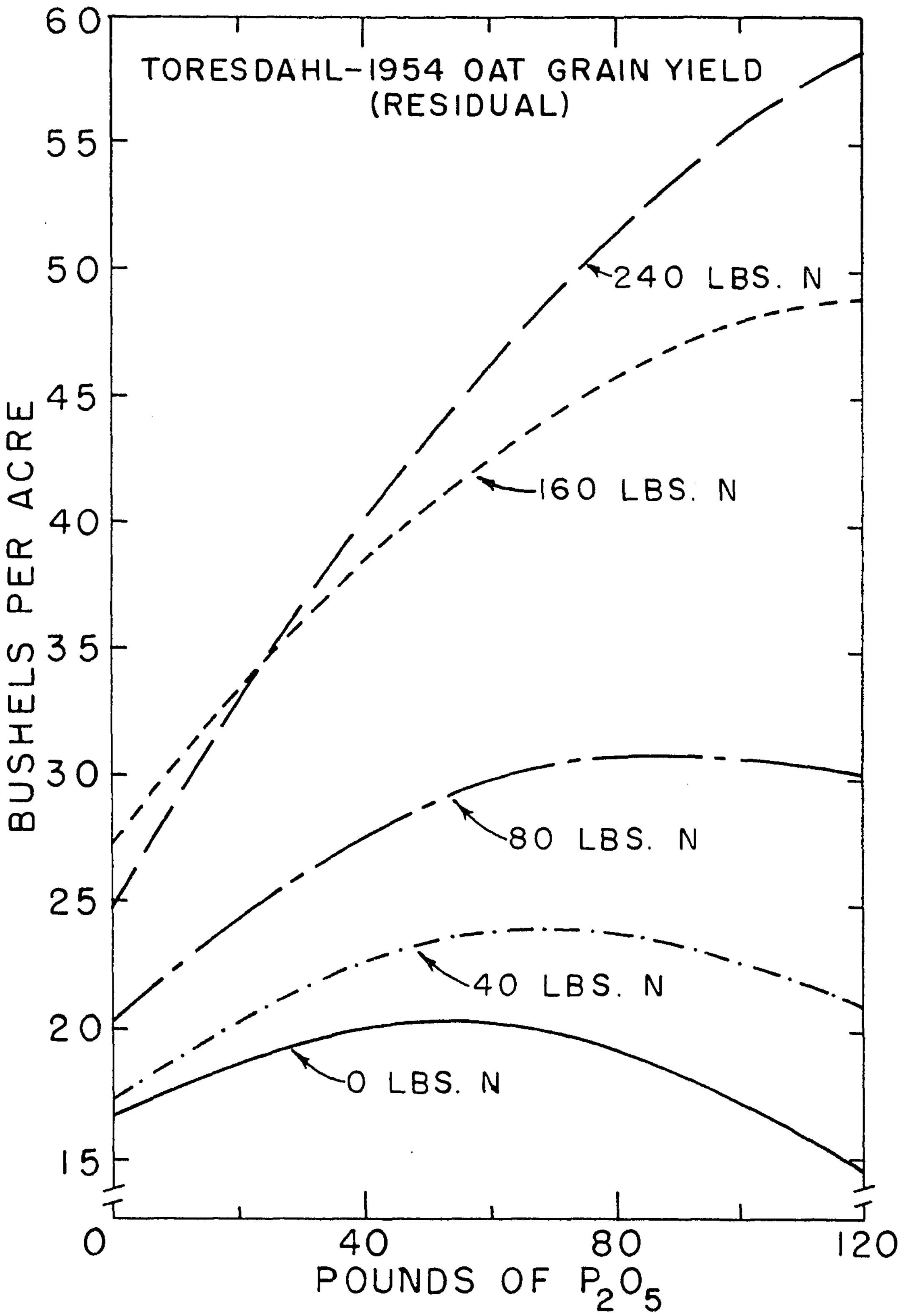

TORESDAHL-1954 OAT GRAIN YIELD
(RESIDUAL)
BUSHELS PER ACRE
POUNDS OF N
0
40
80
120
160
200
240
15
20
25
30
35
40
45
50
55
60
120 LBS. P_2O_5
80 LBS. P_2O_5
40 LBS. P_2O_5
0 LBS. P_2O_5
N topdressed
in 1954
0 LBS. P_2O_5

Figure 30. Predicted yield of oat grain as a function of residual P_2O_5 rates at different levels of residual N in the second year after application.


TORESDAHL-1954 OAT GRAIN YIELD
(RESIDUAL)
240 LBS. N
160 LBS. N
80 LBS. N
40 LBS. N
0 LBS. N
BUSHELS PER ACRE
60
55
50
45
40
35
30
25
20
15
0
40
80
120
POUNDS OF P_2O_5

Nitrogen carryover

In view of the large NP interaction, it was unfortunate that the topdressed fertilizer could not have been applied in an NP factorial design as was the original experiment. The small plot size as well as limited plot numbers precluded such an attempt for the split plot treatments.

However, a measure of N carryover is still possible by comparing the topdressed N with the residual N at the zero P_2O_5 level. Through the analysis of variance shown in Table 15, it was found that the linear and quadratic components of the sum of squares were significant at the 1 and 2 percent level respectively. The cubic term was rejected on agronomic grounds even though the level of significance was within the acceptable probability range. The shape of the response line was of the square root form.

The resulting multiple regression equation with standard errors for the partial regression coefficients is

$$Y = \underset{\pm 0.917}{4.34}N^{\frac{1}{2}} - \underset{\pm 0.138}{0.394}N + 15.39 \qquad (15)$$

The R^2 value is 98.7 percent. Predicted yields are plotted in Figure 29 and shown in Table 52 of the Appendix. The observed yields are shown in Table 51 of the Appendix.

Table 15. Analysis of variance of 1954 oat grain yield obtained from N topdressed in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	393,787		
Replication	2	40,948	20,474	2.5
Treatment	3	304,164	101,388	12.5**
Linear	1		189,113	23.3**
Quadratic	1		89,114	11.6*
Cubic	1		25,937	3.2[d]
Error	6	48,675	8,112	

[d]Significance level is explained in footnote of Table 4.

Carryover percentages will vary at various points along the curve as well as for comparisons made at various levels of P_2O_5. This variation is shown in Table 16 in which nutrient levels were taken directly from the curves in Figure 29.

The high carryover percentages for residual NP combinations are shown in Table 16 and are unrealistic since they include the contribution of the NP interaction. Had NP combinations been topdressed, it is likely that the percentage carryover would have come closer to those values given for N at zero P_2O_5.

Table 16. Variation in nitrogen carryover for various nutrient combinations producing equivalent yields.[a]

Topdressed		Residual		Percent carryover
N	P_2O_5	N	P_2O_5	
20	0	152	0	13.2
20	0	72	40	27.8
20	0	57	80	35.1
20	0	65	120	30.8
40	0	165	0	24.2
40	0	77	40	51.9
40	0	62	80	64.5
40	0	69	120	58.0

[a]Nutrient rates are in pounds per acre.

Yield surface and isoquants

The combined yield curves of Figures 29 and 30 form the yield surface shown in Figure 31. This figure shows the large NP interaction as well as the depressing effect of each nutrient on yield when applied at high rates at the zero level of the other nutrient.

Yield contours which can be visualized for the surface in Figure 31 are computed by

$$P = 52.543+0.494N+ \frac{\sqrt{0.203N+0.008N^2-0.000021N^3+105.040-0.162Y}}{-0.081} \quad (16)$$

Because of the nature of the yield surface, the isoquant lines shown in Figure 32 are somewhat different than usual. The lower yield isoquants are concave to the origin while those of higher yield are convex to the origin. Increasing, constant and decreasing returns to scale are all illustrated.

As with corn, a wide range of residual nutrient substitution is possible for the production of any given yield. As yields increase, however, the range of substitution narrows until at the highest yield, (predicted, not shown) the rate of substitution is zero.

Isoclines, other than ridgelines, are not shown for the reason that economic choice is not possible within a specific year in a residual study. The reason for this is that the nutrient rates involved do not really apply to the specific crop at face value. Furthermore, even if the rates were interpolated to the correct carryover amounts, (not possible in this experiment because only N was topdressed), they would be meaningless since they could not be applied. It is felt that a much better approach is a single final economic evaluation for the time period covered as will be done in this dissertation.

Economic optima and maxima

While it is not useful to compute economic optima for

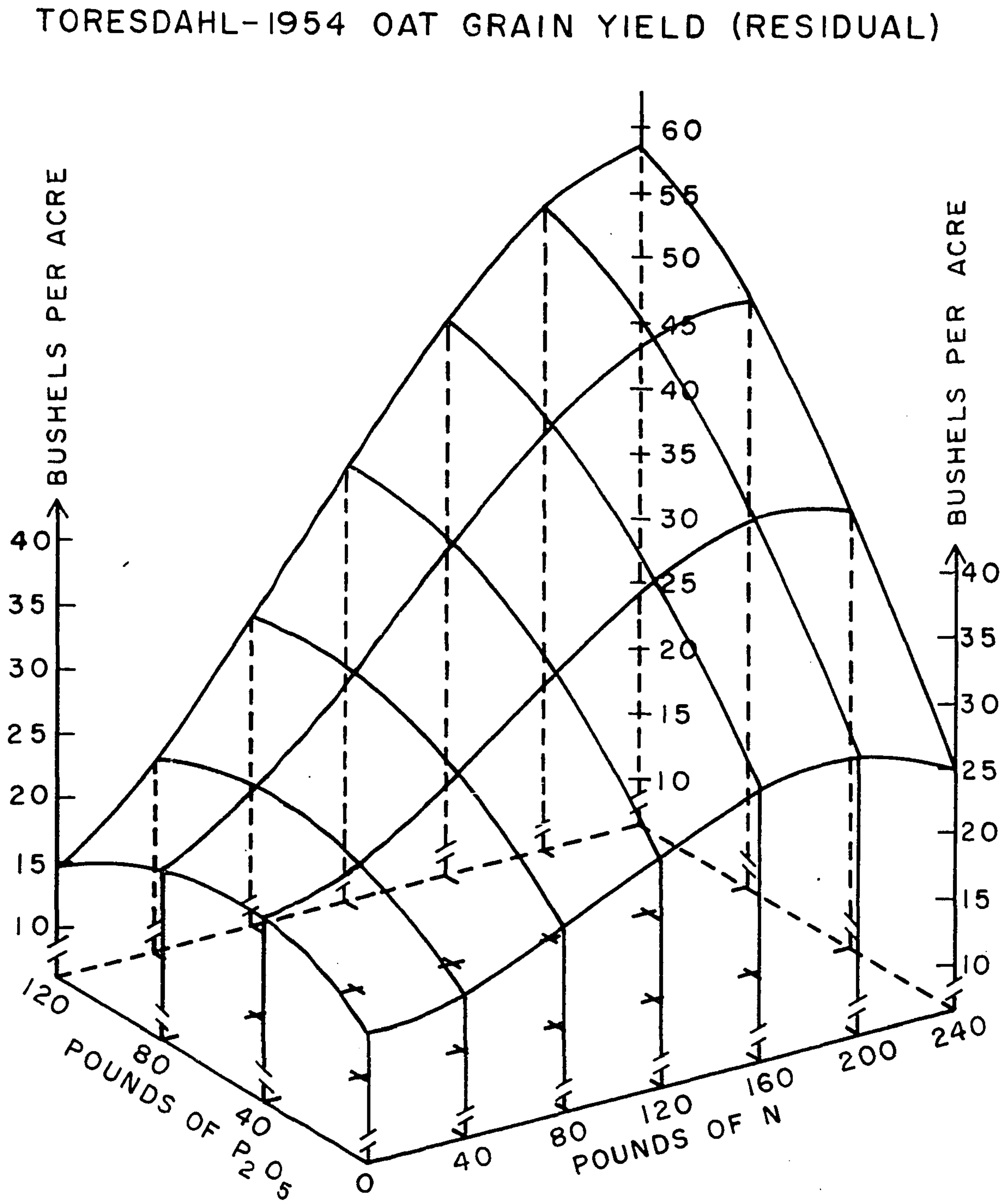


Figure 31. Predicted oat yield surface as a function of residual N and P_2O_5 rates in the second year after application.

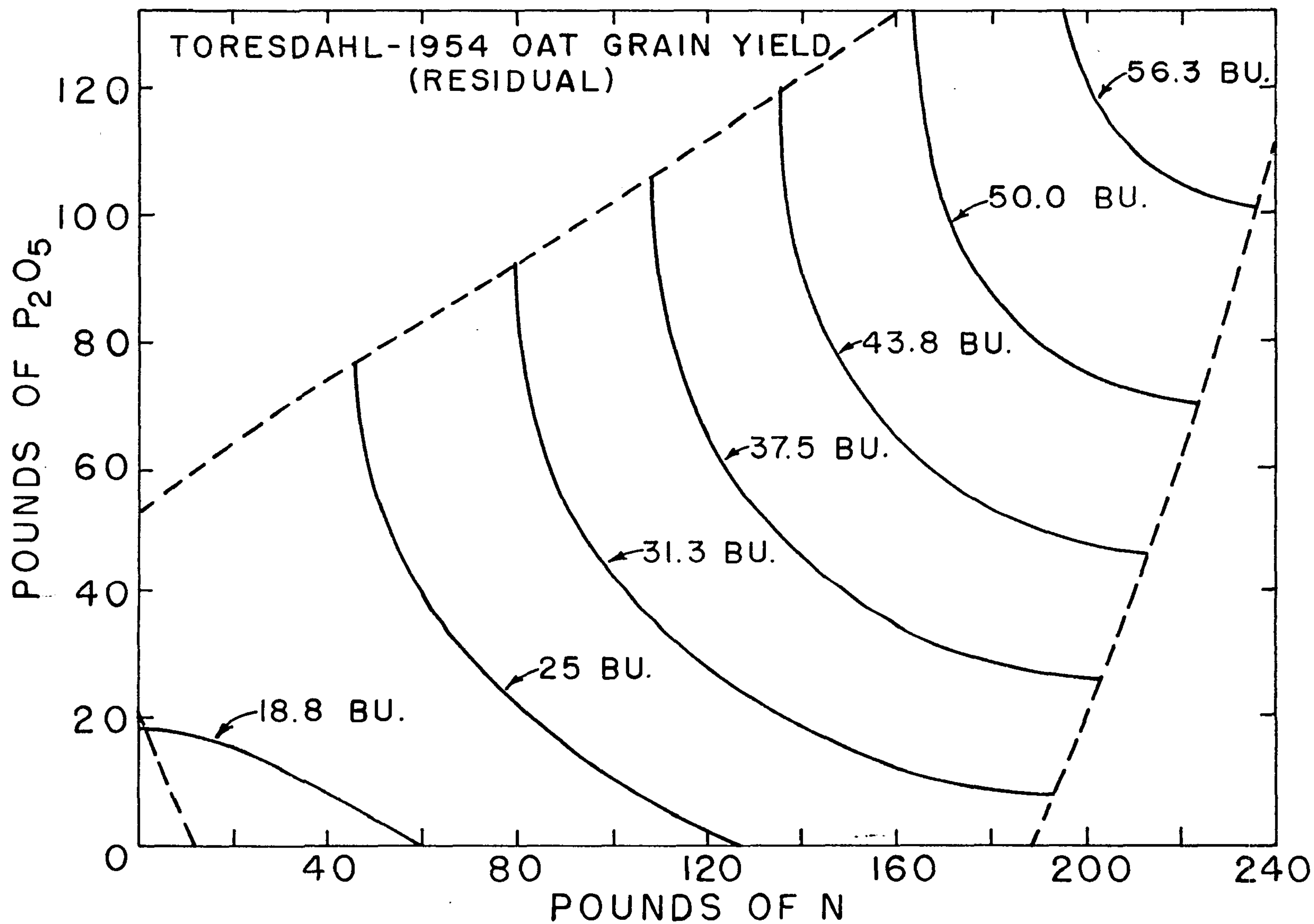


Figure 32. Oat yield isoquants and dashed ridgelines for residual N and P_2O_5 rates in the second year after application.

the residual nutrient rates for the reasons stated above, the optimum topdressed N rate is of interest. With N at \$0.15 per pound and the oats at \$0.60 per bushel, the optimum N rate is 11.4 pounds per acre which results in an optimum yield of 25.5 bushels. With the topdressed N check yield at 15.4 bushels, the yield increase of 10.1 bushels results in a net profit of \$6.06.

Maximum N rate and yield comparisons can be made only between topdressed and residual N at the zero rate of P_2O_5. The computations lead to 30.3 pounds per acre for the topdressed N and 189.9 pounds for residual N. The resulting maxima of 27.3 and 28.1 bushels, respectively, are in reasonably good agreement.

Maximum residual N and P rates and yield are also of interest. The computations lead to 276.1 pounds of N and 188.9 pounds of P_2O_5 which result in a maximum yield of 62.7 bushels. Only limited confidence can be placed in these results, however, since they fall beyond the range of the data.

Oats - total dry matter

In harvesting the oat crop, the entire aerial portions were harvested, dried and weighed before threshing in order to compute total dry matter yields. These yields are shown in Table 17 and illustrated in Figures 33 and 34.

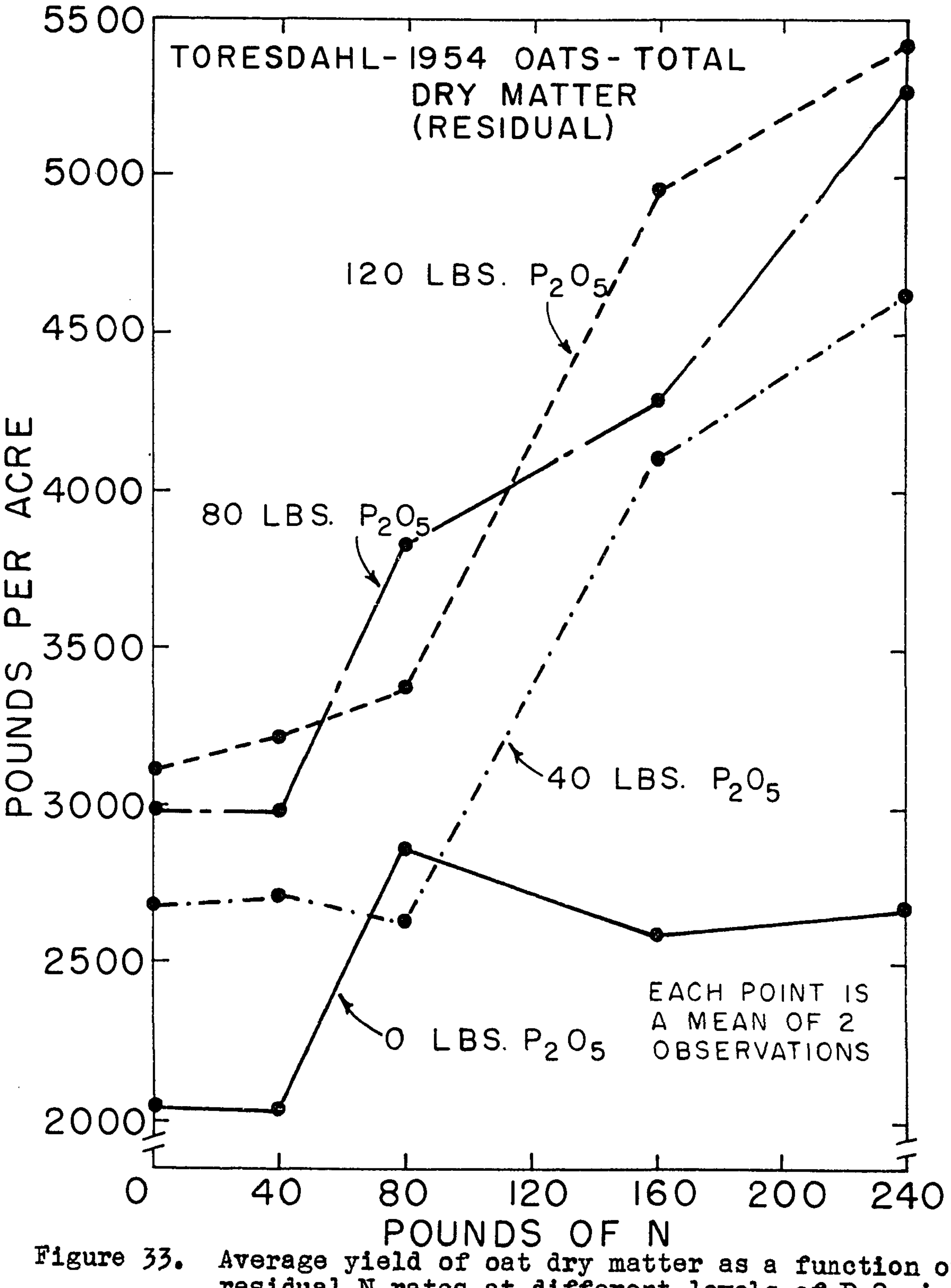


Figure 33. Average yield of oat dry matter as a function of residual N rates at different levels of P_2O_5 in the second year after application.

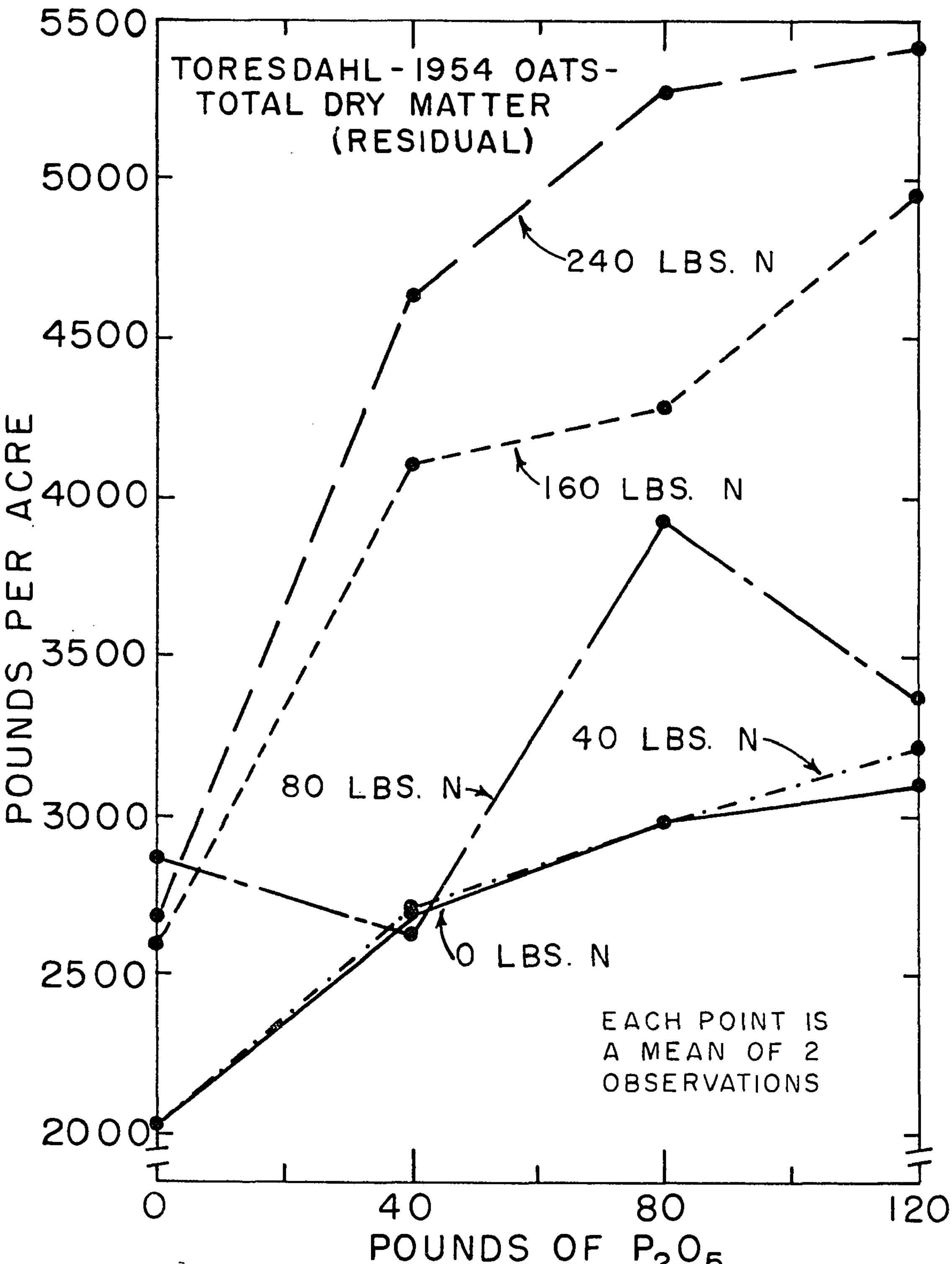


Figure 34. Average yield of oat dry matter as a function of P_2O_5 rates at different levels of N in the second year after application.

Table 17. Dry matter yields in pounds per acre obtained in 1954 from residual fertilizer treatments applied to corn in 1953. Each value is a mean of 2 observations.

Pounds of P_2O_5/A	Pounds of N/A				
	0	40	80	160	240
0	2048	2039	2865	2598	2678
40	2678	2705	2625	4104	4637
80	2985	2982	3838	4291	5277
120	3105	3211	3371	4957	5410

Mean yield for main effects

Pounds of N/A	Mean yield (4 obs.)	Pounds of P_2O_5/A	Mean yield (5 obs.)
0	2704	0	2446
40	2734	40	3350
80	3675	80	3874
160	3988	120	4011
240	4501		

As with the oat grain yields, N, P and NP interaction effects were large. The analysis of variance for the main and interaction effects together with subdivision of the main effects are shown in Table 18.

As in the oat grain analysis, the analysis of variance as shown in Table 18 confirms the cubic effect of N and the

Table 18. Analysis of variance for 1954 residual oat dry matter yields.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	39	44,025,549		
Replication	1	1,100,880	1,100,880	8.57**
Treatments	19	40,484,766	2,130,777	16.59**
Nitrogen(N)	4	20,402,294	5,100,574	39.72**
Linear	1		18,889,265	147.09**
Quadratic	1		1,041,814	8.11*
Cubic	1		432,224	3.37[c]
Quartic	1		38,991	0.30
Phosphorus(P)	3	15,008,515	5,002,838	38.96**
Linear	1		13,572,727	105.69**
Quadratic	1		1,435,539	11.18**
Cubic	1		249	0.00
NP	12	5,073,957	422,830	3.29**
Error	19	2,439,903	128,416	

[c]Significance level is explained in footnote of Table 4.

quadratic effect of P_2O_5. The yield equation resulting from the inclusion of the significant variables in Table 18 is

$$\begin{aligned} Y = &\underset{\pm 7.183}{-4.947N} + \underset{\pm 0.075}{0.104N^2} - \underset{\pm 0.002076}{0.000283N^3} + \underset{\pm 6.286}{20.641P} - \underset{\pm 0.047}{0.118P^2} \\ &+ \underset{\pm 0.0197}{0.063NP} + 2108.277. \quad (17) \end{aligned}$$

The R^2 for equation 17 is 93.2 percent and the t values for the partial regression coefficients in the order shown are

0.76, 1.52, 1.50, 3.61, 2.74 and 3.45. The first 3 values are significant at relatively low levels of probability. The third and fifth values are significant at the 1 percent level while the fourth value is significant at the 2 percent level of probability.

Dry matter curves and surfaces

Predicted yields are shown in Table 19 and the corresponding yield curves in Figures 35 and 36.

Table 19. Predicted 1954 oat dry matter yields in pounds per acre for various nutrient combinations applied to corn in 1953.

Pounds of	Pounds of N/A						
P_2O_5/A	0	40	80	120	160	200	240
0	2108	2058	2233	2523	2820	3015	2999
40	2745	2796	2072	3463	3860	4156	4241
80	3004	3156	3533	4024	4523	4919	5105
120	2886	3139	3616	4208	4807	5305	5591

The yield curves of Figures 35 and 36 and surfaces in Figure 37 are similar to the curves and surface for oat grain yield. Again, the nature of the surface is due primarily to the large nitrogen removals by the 1953 corn crop at the 40 and 80 pound N rates together with some leaching loss and tie-up of N by soil organisms in the decomposing cornstalk residue.

Figure 35. Predicted yield of oat dry matter as a function of residual N at different levels of residual P_2O_5 in the second year after application compared to N topdressed in 1954.

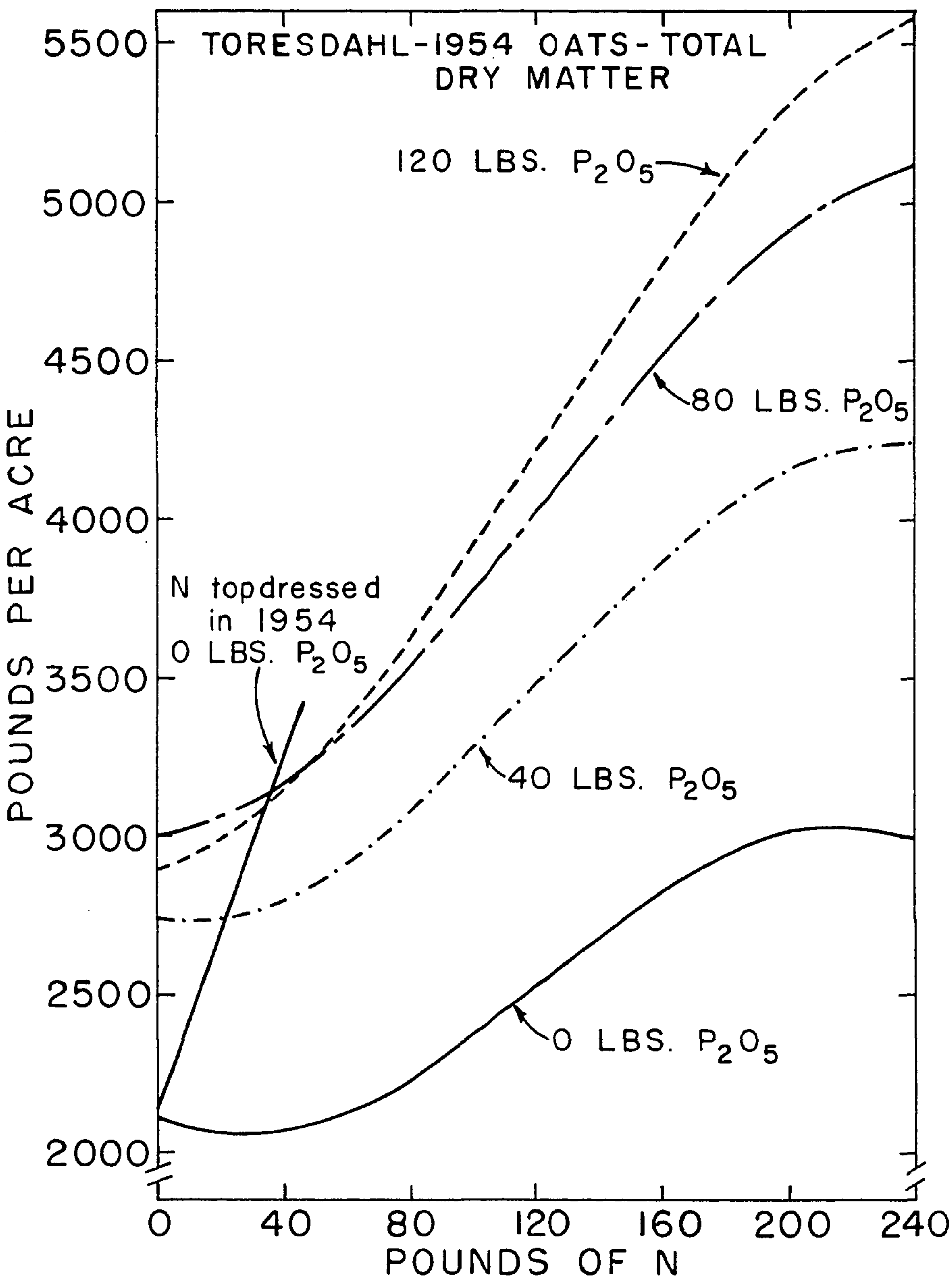
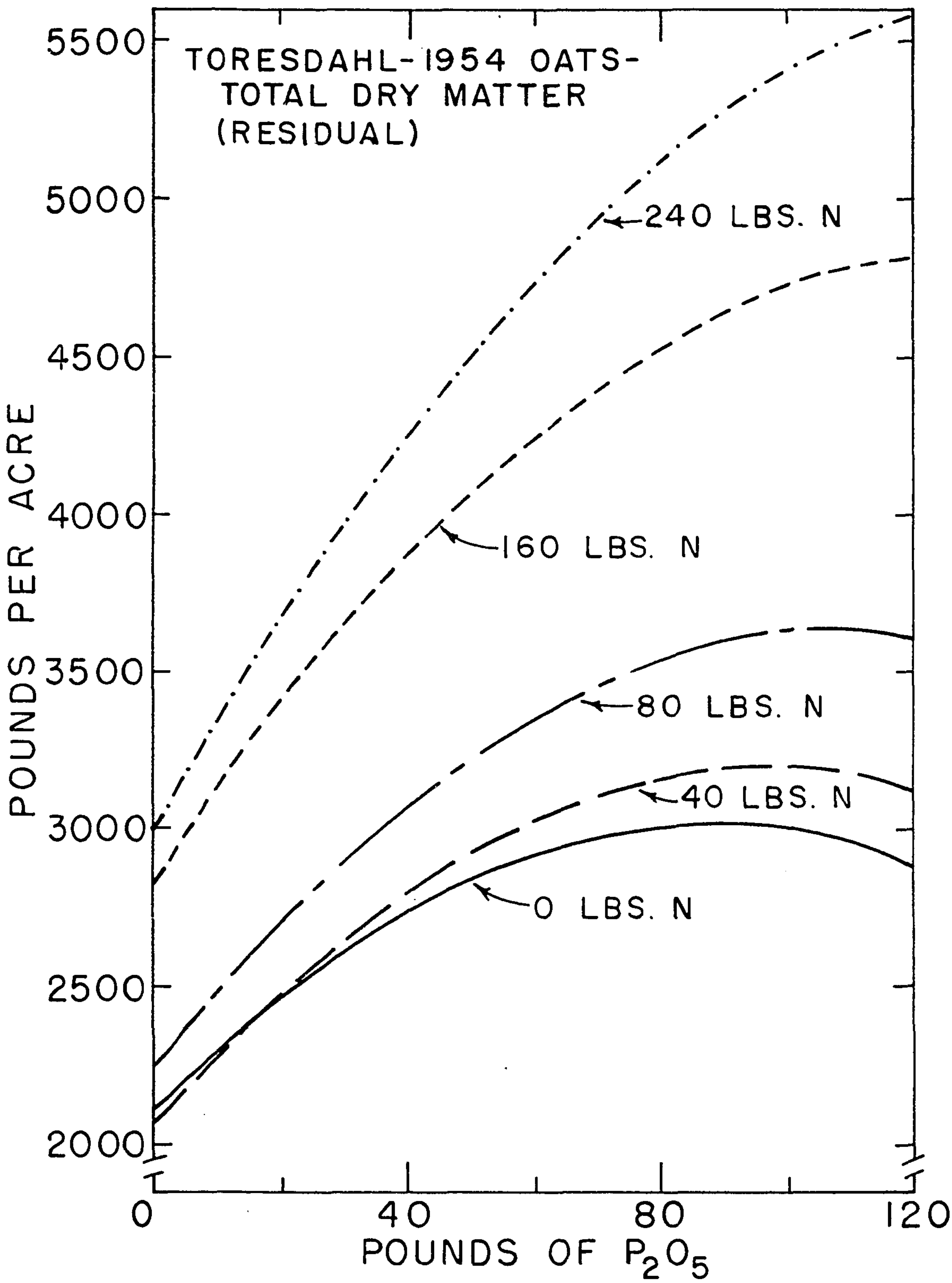

TORESDAHL-1954 OATS-TOTAL DRY MATTER
120 LBS. P2O5
80 LBS. P2O5
40 LBS. P2O5
0 LBS. P2O5
N topdressed in 1954
0 LBS. P2O5
POUNDS PER ACRE
POUNDS OF N
5500
5000
4500
4000
3500
3000
2500
2000
0
40
80
120
160
200
240

Figure 36. Predicted yield of oat dry matter as a func
of residual P_2O_5 rates at different levels
residual N in the second year after applica

Figure 36. Predicted yield of oat dry matter as a function of residual P_2O_5 rates at different levels of residual N in the second year after application.


TORESDAHL-1954 OATS-
TOTAL DRY MATTER
(RESIDUAL)
240 LBS. N
160 LBS. N
80 LBS. N
40 LBS. N
0 LBS. N
POUNDS PER ACRE
5500
5000
4500
4000
3500
3000
2500
2000
0
40
80
120
POUNDS OF P_2O_5

TORESDAHL-1954 OATS-TOTAL DRY MATTER
(RESIDUAL)

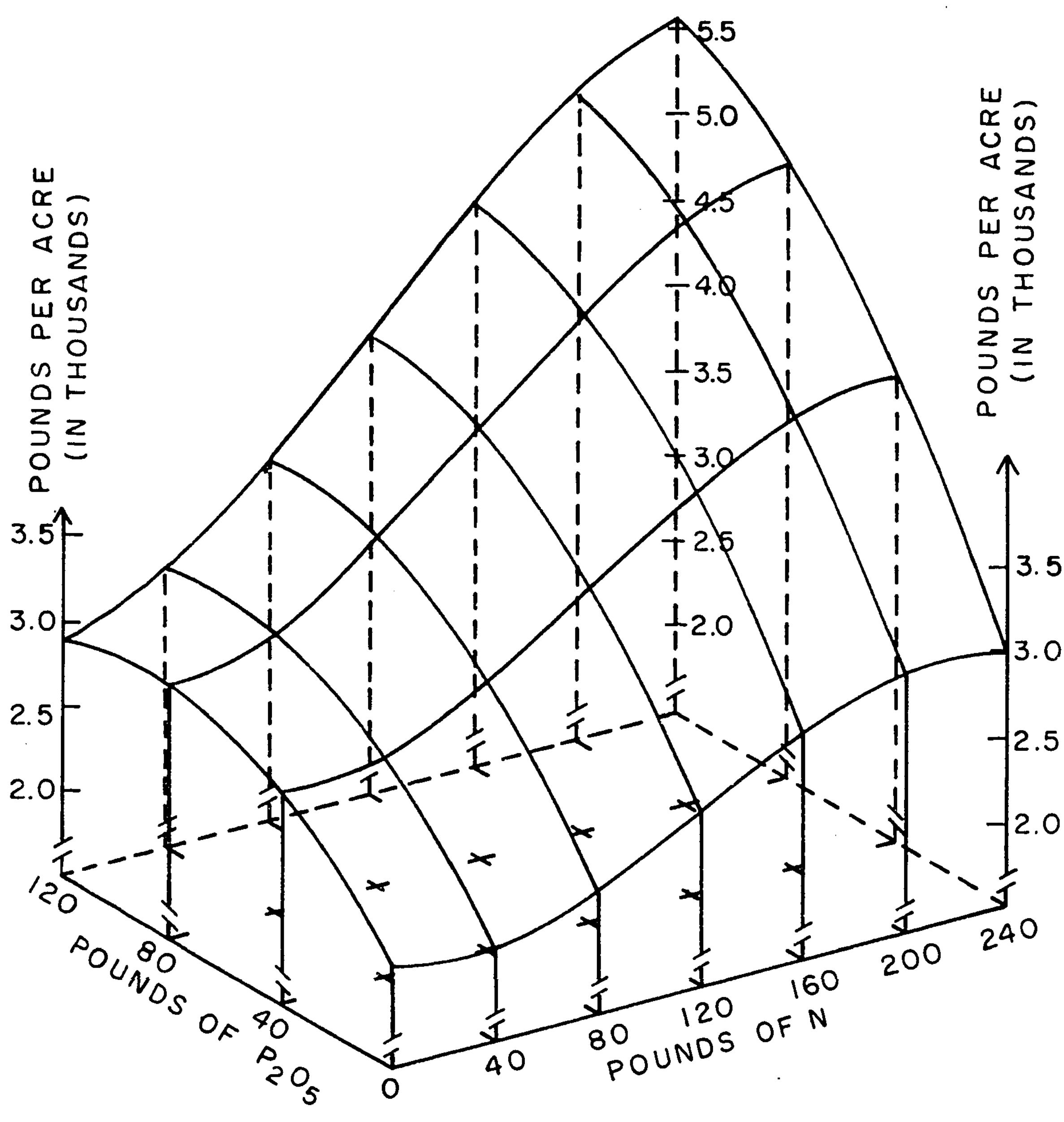


Figure 37. Predicted oat dry matter surface as a function of residual N and P_2O_5 rates in the second year after application.

The tendency to level off in grain yield as well as in grain plus straw yield may or may not be real since the N^3 term was not significant at a very high level of probability in either case. However, for the range and level of N rates used, together with the fact that the crop was grown only one year after application, it is reasonable to expect some leveling off in yields. Agronomic logic precludes continuously increasing returns to scale which would be the case if only the linear and quadratic terms were used.

<u>Nitrogen carryover for dry matter yield</u>

An estimate of carryover is also possible for oat dry matter yield at the zero P_2O_5 level. As is shown in Table 20, the response to topdressed N, however, is linear in contrast to the curvilinear response for topdressed N on oat grain.

Table 20. Analysis of variance of 1954 oat dry matter yields obtained from N topdressed in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F
Total	11	4,144,126		
Replication	2	877,307	438,654	10.6*
Treatment	3	3,018,544	1,006,181	24.3**
Linear	1		2,727,340	65.9**
Quadratic	1		147,985	3.6[d]
Cubic	1		143,219	3.5[d]
Error	6	248,275	41,379	

[d]Significance level is explained in footnote of Table 4.

Both the quadratic and cubic components of the sum of squares were significant at $P \leq 0.20$, but since the sums of squares were of the same magnitude, and since there was no agronomic justification for a cubed term because the mean observed yield line was opposite to that of a sigmoid function, both terms were rejected. The equation with its standard error for the regression coefficient is

$$\underset{\pm 6.7}{Y = 28.44N} + 2140.4, \qquad (18)$$

and is plotted in Figure 35. The coefficient of determination (r^2) was 90.0 percent. Predicted and observed yields are shown in Tables 56 and 55 respectively, of the Appendix.

If comparisons are made directly from Figure 35, 15 and 30 pounds of topdressed N are equal to 125 and 195 pounds, respectively, of residual N. The values convert to 12.0 and 15.4 percent carryover which is reasonably comparable to the 10.3 and 18.2 percent found for these rates with oat grain.

<u>Percent nitrogen and nitrogen yield</u>

Whole-plant samples were collected at harvest time for the purpose of determining percent nitrogen and phosphorus. Observed percentages and nutrient yields are shown in Table 51 of the Appendix. The significant variables for percent nitrogen and their subdivision into linear, quadratic, etc. components are shown in the analysis of variance in Table 21.

Table 21. Analysis of variance of percent nitrogen for 1954 residual oat dry matter yields.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	39	2.81		
Replication	1	0.00	---	
Treatments	19	2.18	0.115	3.48**
Nitrogen(N)	4	1.12	0.280	8.48**
Linear	1		0.822	24.92**
Quadratic	1		0.291	8.82**
Cubic	1		0.003	0.08
Quartic	1		0.000	--
Phosphorus(P)	3	0.43	0.143	4.33*
Linear	1		0.333	10.09**
Quadratic	1		0.083	2.51[d]
Cubic	1		0.014	0.43
NP	12	0.63	0.053	1.61
Error	19	0.63	0.033	

[d]Significance level is explained in footnote of Table 4.

A subdivision of the NP sum of squares showed the N linear x P linear interaction term to be highly significant. This term was, therefore, included in the multiple regression analysis. The equation with standard errors for the partial regression coefficients is

$$Y_{o/oN} = \underset{\pm 0.0012}{-0.004533N} + \underset{\pm 0.000005}{0.00001849N^2} + \underset{\pm 0.0022}{0.007889P} - \underset{\pm 0.000017}{0.00002843P^2} - \underset{\pm 0.000007}{0.00002343NP} + 1.828316 \quad (19)$$

The R^2 for equation 19 is 84.4 percent and the t values for the coefficients in the order shown are 3.68, 3.98, 3.57, 1.70 and 3.38. Only the fourth t value is significant at a low level of probability. All of the others are highly significant.

Predicted nitrogen percentages for equation 19 are shown in Table 52, of the Appendix and the corresponding surface in Figure 38.

The percent nitrogen surface in Figure 38 is in marked contrast to the yield surface in Figure 37. As yield goes up for rates of N up to 120 pounds, percent nitrogen goes down at every level of P_2O_5. Only after the flex point on the yield curve is reached in Figure 37, does the percent N begin to increase.

This behavior for N rates up to 140 pounds at the zero and 40 pound P_2O_5 level is probably due to a dilution effect resulting from the rapid accumulation of dry matter. Beyond the 140 pound N rate, dry matter accumulation begins to slow down with the result that nitrogen can again begin to accumulate. At the higher P_2O_5 levels, however, the rate of dry matter increase is such that nitrogen accumulates only after the 200 pound N rate is reached.

A possible explanation for the increase in percent

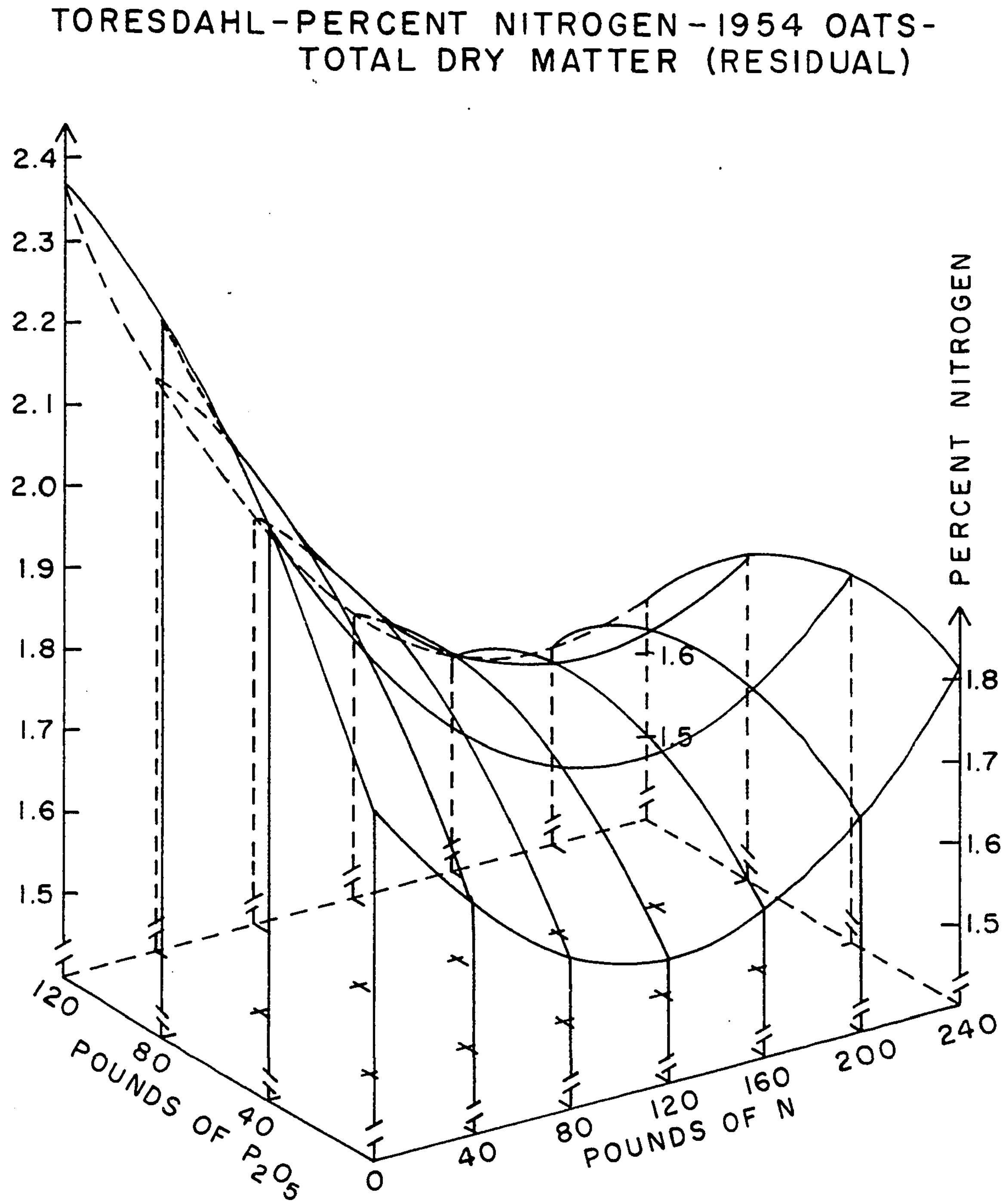


Figure 38. Predicted surface for percent nitrogen in oat dry matter as a function of N and P_2O_5 rates in the second year after application.

nitrogen for rates of P_2O_5 at low levels of N lies in the moderate accumulation of dry matter combined with an enlarged root system. If the latter occurred, greater absorption of N would have been possible and could have accumulated because of the slow rate of dry matter accumulation.

At higher N levels, the overall percent nitrogen is decreased considerably. While this overall trend was noted and discussed above, the specific increase and noticeable decrease in percent nitrogen for a given P_2O_5 plane beyond the 160 pound N rate has not been mentioned. Just why this effect should occur is obscure. It may be dry-matter dilution just as above or it may be an artifact of the regression equation.

While the percent nitrogen surface may be a little unusual, the nitrogen yield surface in Figure 39 appears to be more like the crop yield surface in Figure 37. The sigmoid effect is no longer present in the nitrogen yield surface but the depression in nitrogen percentage has not been completely erased. The nitrogen yield equation with standard errors for the regression coefficients is

$$\begin{aligned} Y_{N\,yield} = &\underset{\pm 0.063}{-0.073792N} + \underset{\pm 0.000234}{0.000603N^2} + \underset{\pm 0.112}{0.586301P} - \underset{\pm 0.0008}{0.002834P^2} \\ &+ \underset{\pm 0.00035}{0.000397NP} + 37.875077. \quad (20) \end{aligned}$$

The R^2 for equation 20 is 89.7 percent and the t values for the partial regression coefficients in the order shown are

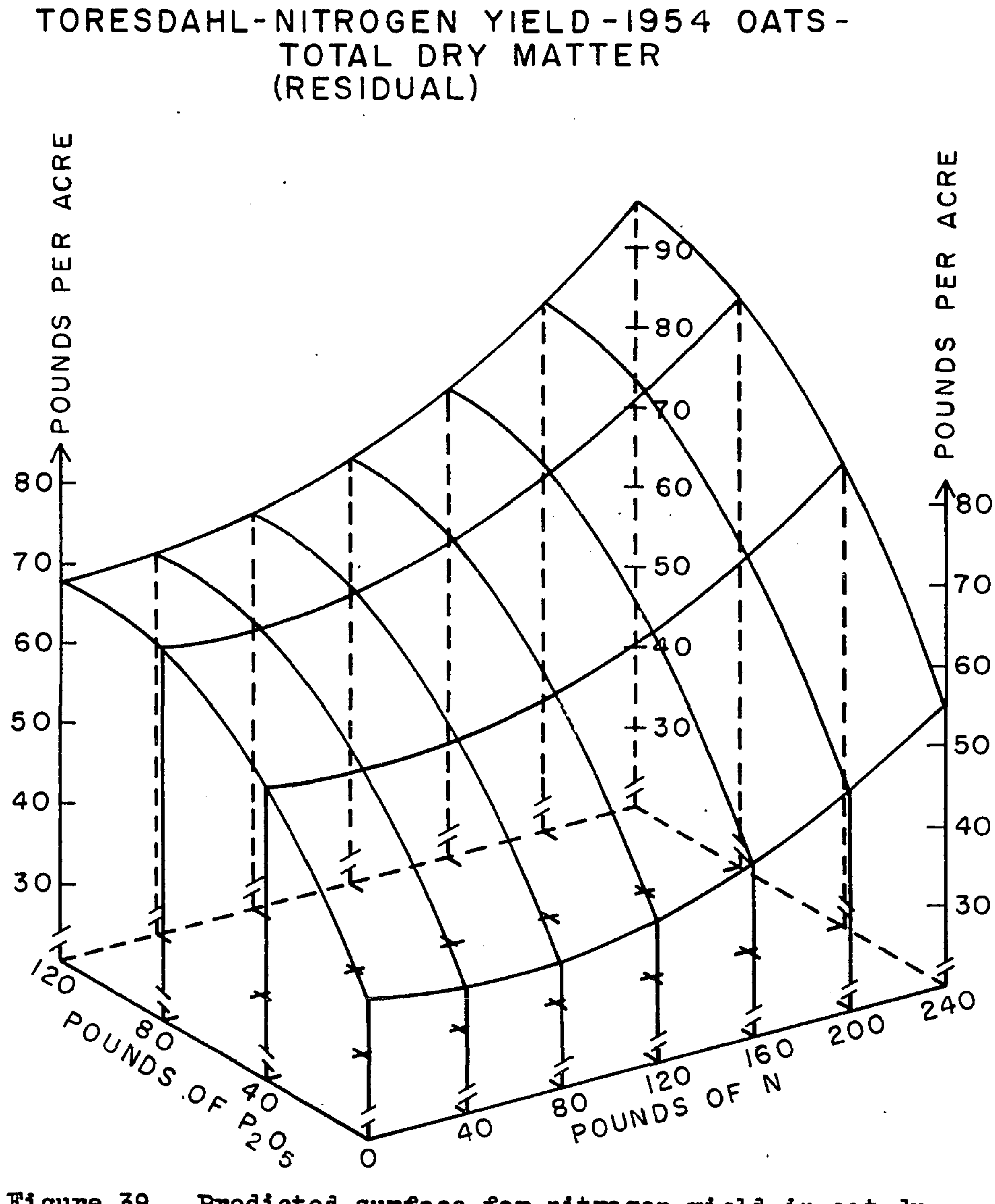


Figure 39. Predicted surface for nitrogen yield in oat dry matter as a function of residual N and P_2O_5 rates in the second year after application.

1.18, 2.57, 5.26, 3.37 and 1.14. The first and last values are significant at low probability levels. The third and fourth values are highly significant while the second is significant. Predicted and observed nitrogen yields and the analysis of variance are shown in Tables 51, 52 and 53 of the Appendix, respectively.

Percent phosphorus and phosphorus yield

Phosphorus analyses were also made on the same samples that were analyzed for percent nitrogen. The significant variables and their subdivision into linear, quadratic, etc. components are shown in Table 22.

Subdivision of the NP sum of squares showed the N linear x P linear interaction term to be significant at the 10 percent level. This term was, therefore, included in the regression equation which is

$$\begin{aligned} Y_{o/oP} = & -0.000185N - 0.00000071N^2 + 0.000964P - 0.0000018NP \\ & \pm 0.000153 \quad \pm 0.00000057 \quad \pm 0.0001 \quad \pm 0.0000009 \\ & + 0.192181, \qquad (21) \end{aligned}$$

and is shown in Figure 40. The R^2 for equation 21 is 95.7 percent and the t values for the coefficients in the order shown are 1.21, 0.12, 8.36 and 2.06. The first and second t values are significant at a low level of probability. The fourth is significant at the 10 percent level while the third

Table 22. Analysis of variance for percent phosphorus for 1954 residual oat dry matter yields.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	39	0.093946		
Replication	1	0.004040	0.004040	10.05**
Treatment	19	0.082271	0.004330	10.77**
Nitrogen(N)	4	0.028522	0.007131	17.74**
Linear	1		0.027085	67.38**
Quadratic	1		0.001157	2.88[d]
Cubic	1		0.000278	0.69
Quartic	1		0.000002	---
Phosphorus(P)	3	0.049103	0.016368	40.72**
Linear	1		0.048797	121.39**
Quadratic	1		0.000102	0.25
Cubic	1		0.000204	0.51
NP	12	0.004646	0.000387	0.96
Error	19	0.007635	0.000402	

[d]Significance level is explained in footnote of Table 4.

is highly significant.

With the exception of the increasing nitrogen percentages beyond the 140 pound N rate at the zero and 40 pound P_2O_5 levels, both nitrogen and phosphorus percentages decreased about 33 percent with additions of N. In Figure 40, this percentage decrease occurs at all rates of P_2O_5 over the entire N range while in Figure 38, this decrease occurs at the 80 and 120

TORESDAHL - PERCENT PHOSPHORUS - 1954 OATS - TOTAL DRY MATTER (RESIDUAL)

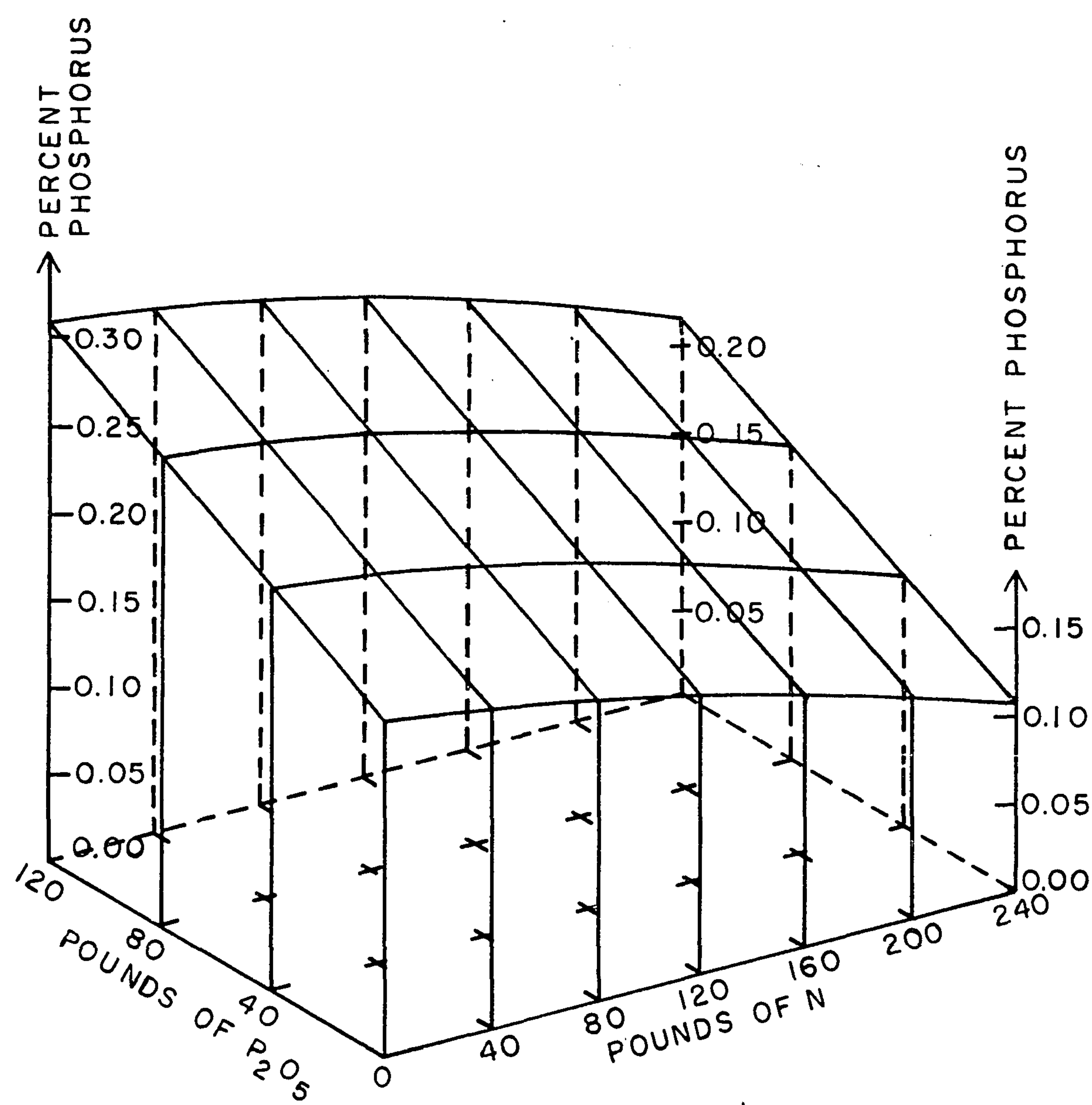


Figure 40. Predicted surface for percent phosphorus in oat dry matter as a function of residual N and P_2O_5 rates in the second year after application.

pounds P_2O_5 rate through the 160 pound N rate. Thus, the dilution effect for both N and P for rates of N appears to be similar for increasing amounts of dry matter.

On the other hand, phosphorus percentage increased continuously for additions of P_2O_5 at every level of N in Figure 40 as contrasted to a decrease and then an increase in percent nitrogen for additions at N at every level of P_2O_5 in Figure 38. These results seem to indicate a greater availability or adequacy of P_2O_5 relative to N for the rate and amount of dry matter accumulation thus eliminating the dilution effect.

The phosphorus yield equation with standard errors for the coefficients is

$$Y_{Pyield} = \underset{\pm .0036}{0.001841N} + \underset{\pm 0.0154}{0.063404P} - \underset{\pm 0.0001}{0.000170P^2} + \underset{\pm 0.000048}{0.000079NP} + 3.940069, \qquad (22)$$

and is plotted in Figure 41. The R^2 for equation 22 is 94.7 percent and the t values for the regression coefficients in the order shown are 0.51, 4.11, 1.46 and 1.64. The first, third and fourth values are significant at a low level of probability while the second t is highly significant. See Tables 51, 52 and 54 respectively for observed and predicted phosphorus yields and analyses of variance.

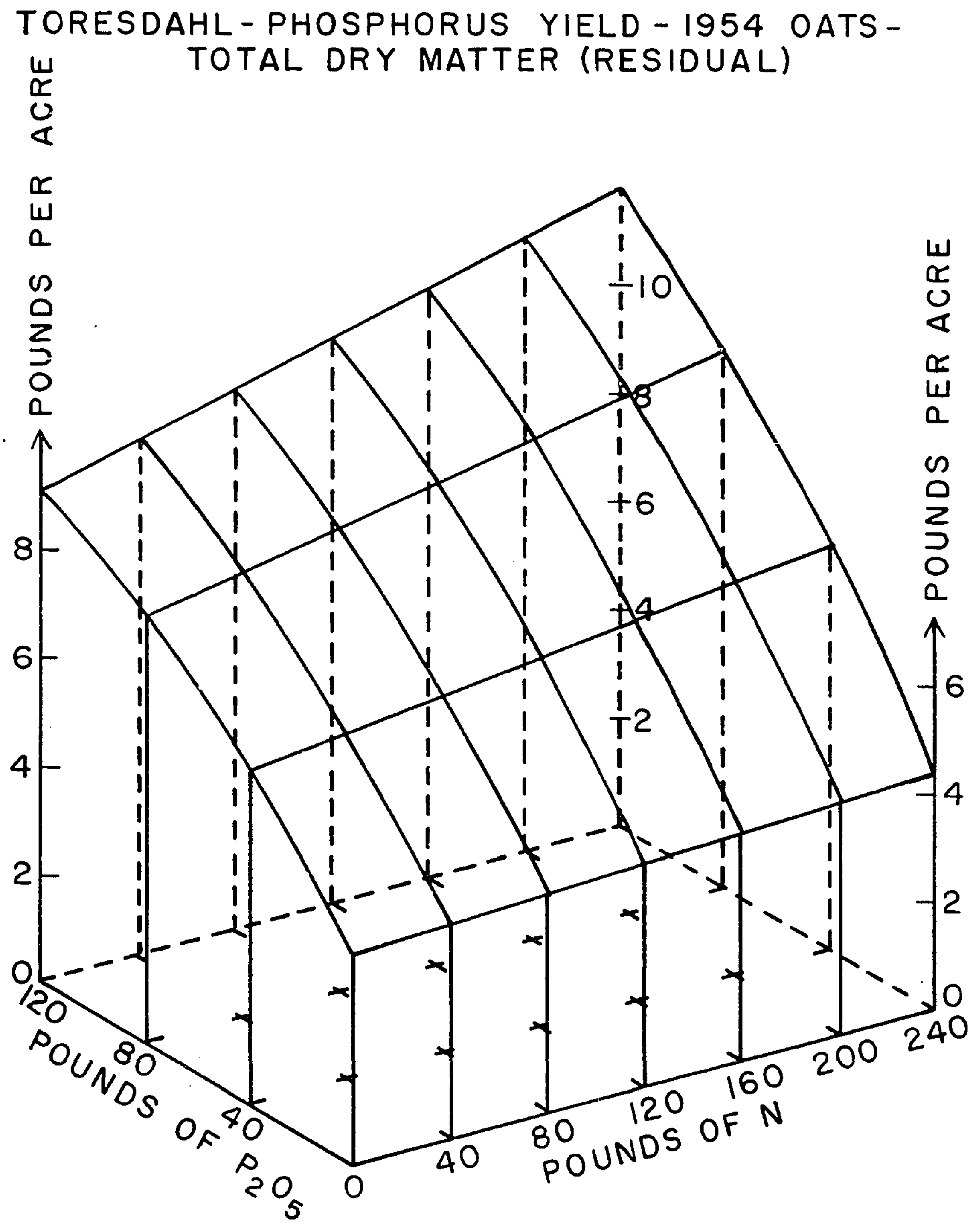


Figure 41. Predicted surface for phosphorus yield in oat dry matter as a function of residual N and P_2O_5 rates in the second year after application.

The phosphorus yield surface in Figure 41 is similar to the nitrogen yield surface in Figure 39, and with the exception of the cubic effect, is similar to the crop yield surface in Figure 37. This consistency is of interest especially since the surfaces for nutrient percentage were generally different from the yield surface.

Nutrient-yield comparisons for residual N

Figure 42 is a summary of specific individual curves already discussed separately but now grouped together for comparison purposes. All of the curves characterize the residual oat dry matter yield curve at zero level of P_2O_5 and K_2O.

The sigmoid crop yield curve illustrates previous removal of N by the corn crop, leaching losses and tieup of N by soil microorganisms in cornstalk residue decay. At low yields of dry matter, percent nitrogen and phosphorus are high. As N rates increase, dry matter yields increase but percent nitrogen and phosphorus decrease. Nitrogen percentage reaches a low point at the flex point of the yield curve which occurs at about the 120 pound N rate. From this point on, yield increases occur at a decreasing rate which allows nitrogen to accumulate. Percent phosphorus however, continues to decrease throughout the range of N rates. Unless this re-

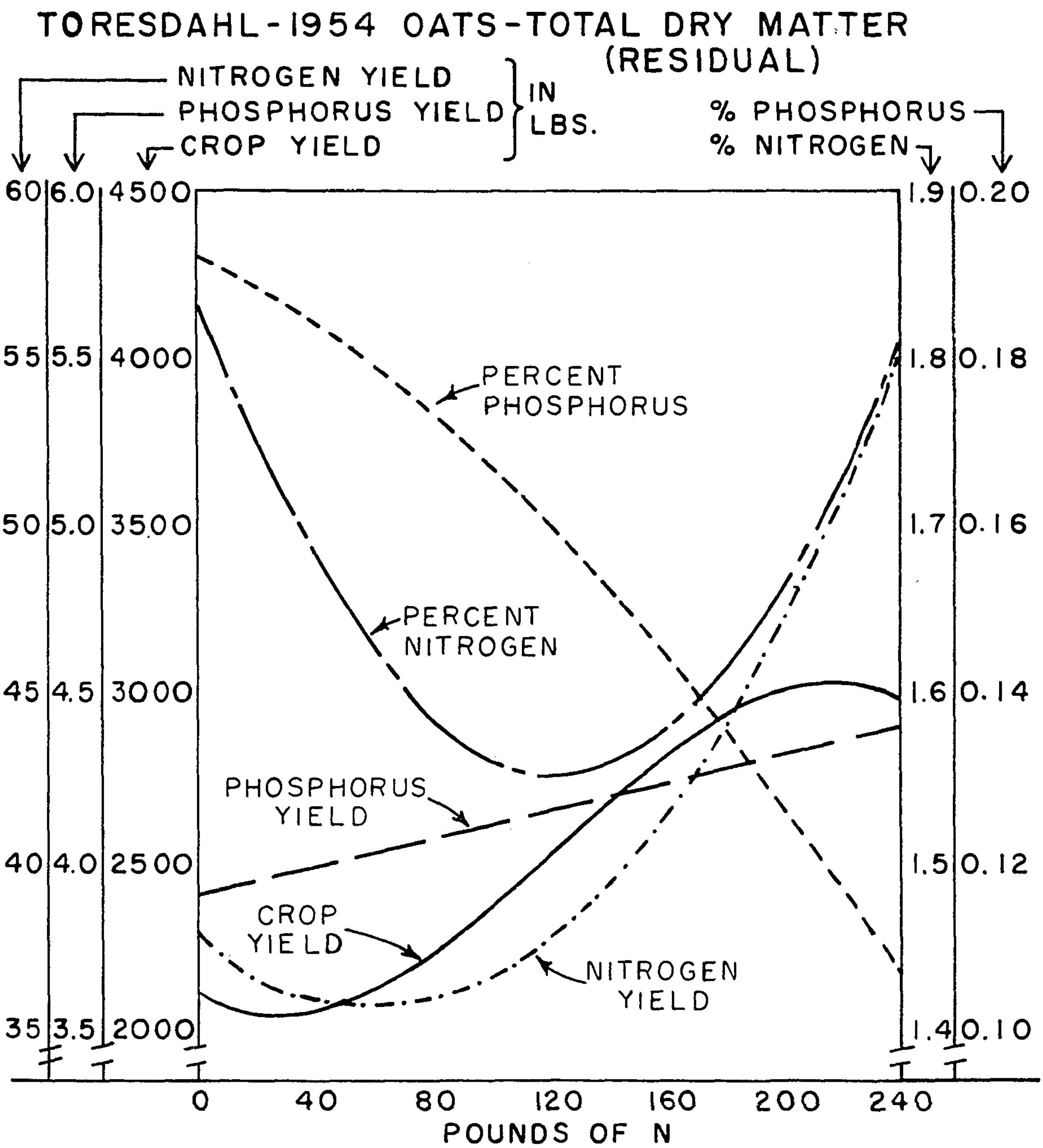


Figure 42. Yield and nutrient relationships for oat dry matter as a function of residual N rates at the zero P_2O_5 level in the second year after application.

sult is some phase of a dilution effect, the reason for the behavior is obscure and may be an isolated phenomenon.

When percent nitrogen and phosphorus are converted to nutrient yields, curves are obtained which resemble the yield curves somewhat more closely. The linear phosphorus yield curve reflects the canceling out of the high and low combinations of the respective crop yield and percent phosphorus curves. The nitrogen yield curve, however, continues to reflect the large variation in nitrogen percentage with the result that a curvilinear response is obtained.

In terms of Macy's (35) definition, luxury consumption of N occurs after the 220 pound N rate is reached. This effect, however, occurred only at the zero level of P_2O_5.

Nutrient-yield comparisons for topdressed N

Except for their linearity, the curves for topdressed N in Figure 43 show the same trend as those in Figure 42. As before, percent nitrogen and percent phosphorus decrease as crop yield increases. The rates involved are too low to exhibit curvilinearity and hence, it is not possible to note the change in yield of dry matter at the point where percent nitrogen is lowest. Had rates been high enough to produce a significant quadratic effect in the crop yield curve, it is

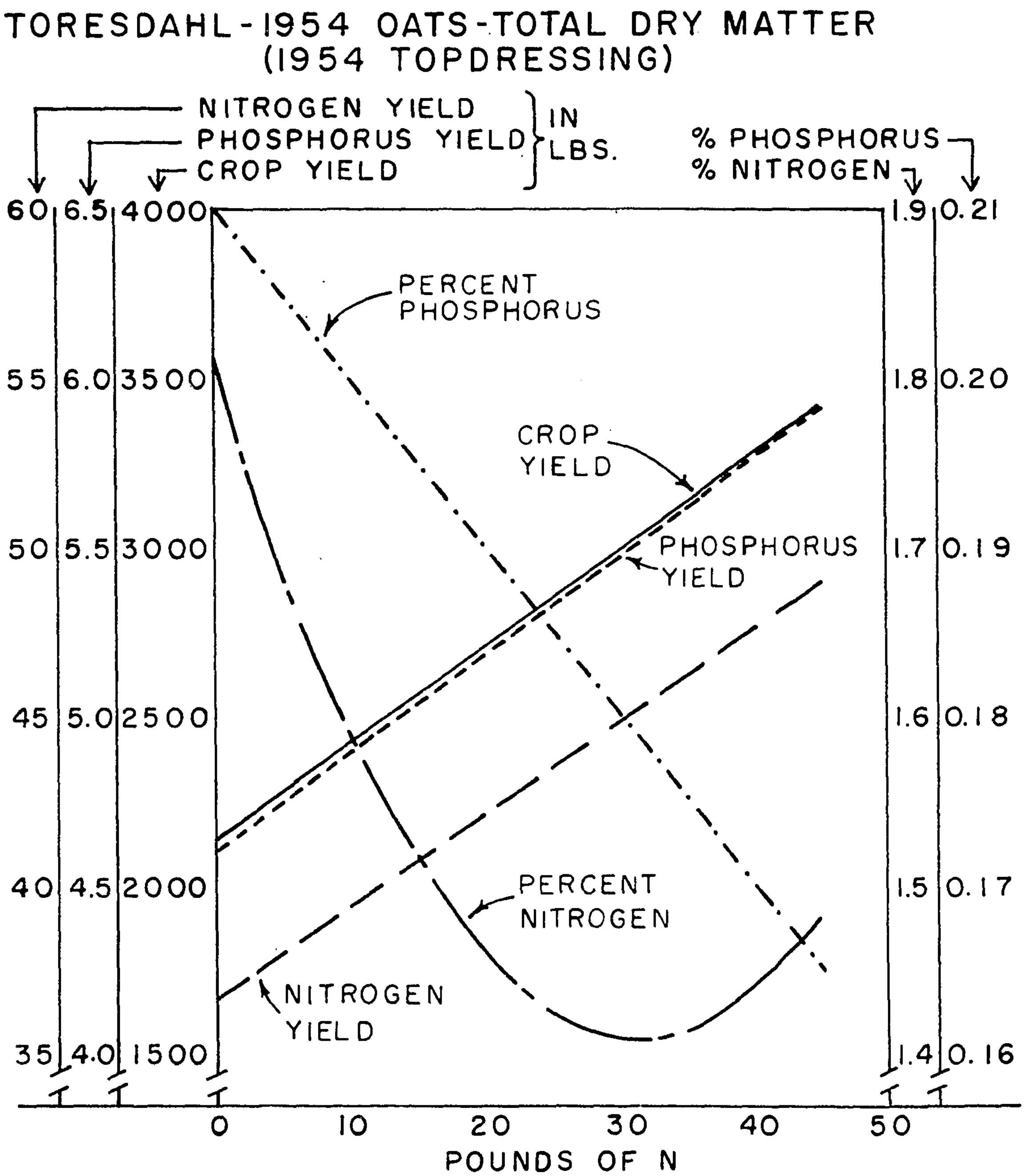


Figure 43. Yield and nutrient relationships for oat dry matter as a function of N rates topdressed in 1954, at a zero level of P_2O_5.

likely that yield increases would have begun to show decreasing returns to scale in the vicinity of the low point on the percent nitrogen curve.

The nitrogen and phosphorus yield curves behave in the same manner as in Figure 42 except that they are linear. For the individual regression equations, analyses of variance, initial and predicted plot values for topdressed N, see Tables 55 through 60 in the Appendix.

The trends observed in the nutrient analysis of the residual material were confirmed by those of the topdressed material. While the results may appear strange at first glance, they are not at variance with agronomic or plant physiology logic.

Summary - 1954 oats

1. Residual N and P_2O_5 combinations resulted in predicted yield increases up to 41.8 bushels per acre.
2. On the basis of the prediction equations, 20 and 40 pounds of N topdressed in April of 1954 were equal to 152 and 165 pounds of residual N. Residual N for these rates amounts to 13.2 and 24.2 percent, respectively.
3. The yield response to residual N was sigmoid in nature. It was postulated that this effect resulted from the large response of the previous corn crop to the low N rates, leaching and tieup by soil micro-

organisms. The residual P_2O_5 effect, however, was still quadratic and convex to the abscissa.

4. Whole-plant analyses showed nitrogen and phosphorus percentages, in general, to vary inversely with total dry matter yield. Nitrogen and phosphorus yield, however, conformed more closely to the crop yield curves and surfaces if the lower part of the sigmoid yield curve is excluded from the comparison.
5. Economic analyses were deferred to a final overall rotation evaluation since residual effects were not measured for comparable NP nutrient combinations.

Alfalfa Hay - 1955

To continue the study of residual fertilizer effects, 3 cuttings of hay were taken in 1955 to determine the total yield. Plant samples were also taken for chemical analysis. Plots which had been topdressed with 0, 30, 60 and 90 pounds of P_2O_5 in 1954 were also harvested.

Residual hay yield analysis

The mean hay yields for the various residual N, P_2O_5 and K_2O treatments and mean yields for main effects are shown in Table 23.

Table 23. Hay yields in tons per acre obtained in 1955 from residual nutrients applied to corn in 1953. Each value is a mean of 2 observations.

Pounds of P_2O_5/A	Pounds of K_2O/A	Pounds of N/A				
		0	40	80	160	240
0	0	2.43	0.96	1.96	1.27	0.99
0	40	1.48	1.26	1.02	2.13	1.35
0	80	1.20	1.55	1.43	2.55	1.03
40	0	1.91	1.38	1.55	1.11	0.72
40	40	1.60	1.44	1.23	1.74	1.29
40	80	2.13	1.74	1.81	2.70	0.89
80	0	3.00	2.69	4.08	1.71	3.70
80	40	2.93	2.30	2.09	2.96	2.46
80	80	3.21	3.39	2.72	2.07	2.48
120	0	3.39	4.28	4.33	3.32	2.74
120	40	4.32	4.37	3.34	2.56	3.19
120	80	4.60	4.23	3.46	3.24	3.12

Mean yield for main effects

Pounds of N/A	Mean yield (24 obs.)	Pounds of P_2O_5/A	Mean yield (30 obs.)	Pounds of K_2O/A	Mean yield (40 obs.)
0	2.68	0	1.51	0	2.37
40	2.46	40	1.55	40	2.25
80	2.42	80	2.78	80	2.48
160	2.28	120	3.63		
240	2.00				

Table 23 shows a distinct negative effect of N on yields. While this effect is significant at only the 10 percent level when averaged over all N treatments as shown by the analysis of variance in Table 24, the linear component is highly significant.

Table 24. Analysis of variance for 1955 residual hay yields.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	116	209.415		
Replication	1	37.811	37.811	55.93**
Treatments	59	133.769	2.267	3.35**
Nitrogen(N)	4	6.147	1.537	2.27[c]
Linear	1		5.809	8.59**
Quadratic	1		0.086	0.13
Cubic	1		0.239	0.35
Quartic	1		0.013	0.02
Phosphorus(P)	3	95.594	31.865	47.14**
Linear	1		86.974	128.66**
Quadratic	1		4.856	7.18**
Cubic	1		3.764	5.57*
Potassium(K)	2	1.014	0.507	0.75
NP	12	10.742	0.895	1.32
NK	8	7.203	0.900	1.33
PK	6	1.870	0.312	0.46
NPK	24	11.199	0.467	0.69
Error	56	37.835	0.676	

[c]Significance level is explained in footnote of Table 4.

The mean yields show a response to phosphorus. However, it is evident that a significant response is not obtained until the 80 pound P_2O_5 rate is reached. Potassium had no effect, as is born out in the analysis of variance.

The mean yield response lines in Figures 44 and 45 illustrate the data in Table 23. While the direction of the lines in Figure 45 show reasonable consistency, those of Figure 44 do not. In order to ascertain what effects are significant in these lines, the treatment sums of squares were partitioned into their various components as shown in Table 24. The analysis of variance shows only the linear effect of N to be significant, while the linear, quadratic and cubic effects are all significant for P. Since all of these effects fit agronomic logic, they were included in the regression analysis.

The NP interaction term was significant at a low level of probability. On a statistical basis, therefore, it might not have been included in the regression. However, since there was a significant interaction effect of N and P on the oat crop and since both N and P had a significant effect on hay yields, the term was left in the equation.

<u>Stand level analysis</u>

The negative effect of N on yields immediately leads one to suspect a difference in stand level due to treatment as the latter may have affected the growth of the companion crop of oats during the seeding year. Stand counts were taken, therefore, after the third cutting in 1955 to determine the effect of treatment on stand.

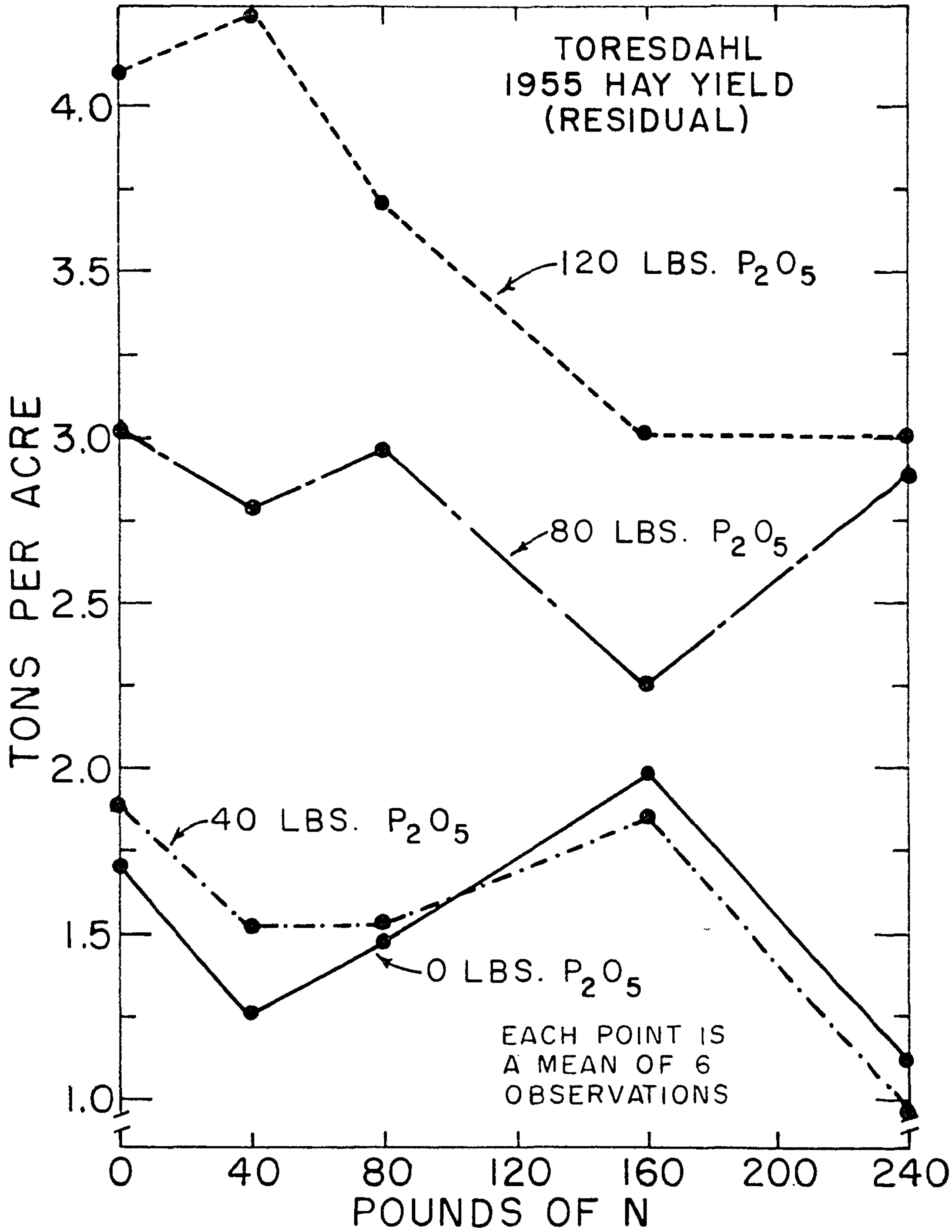


Figure 44. Average yield of alfalfa hay as a function of residual N rates at different levels of residual P_2O_5 in the third year after application.

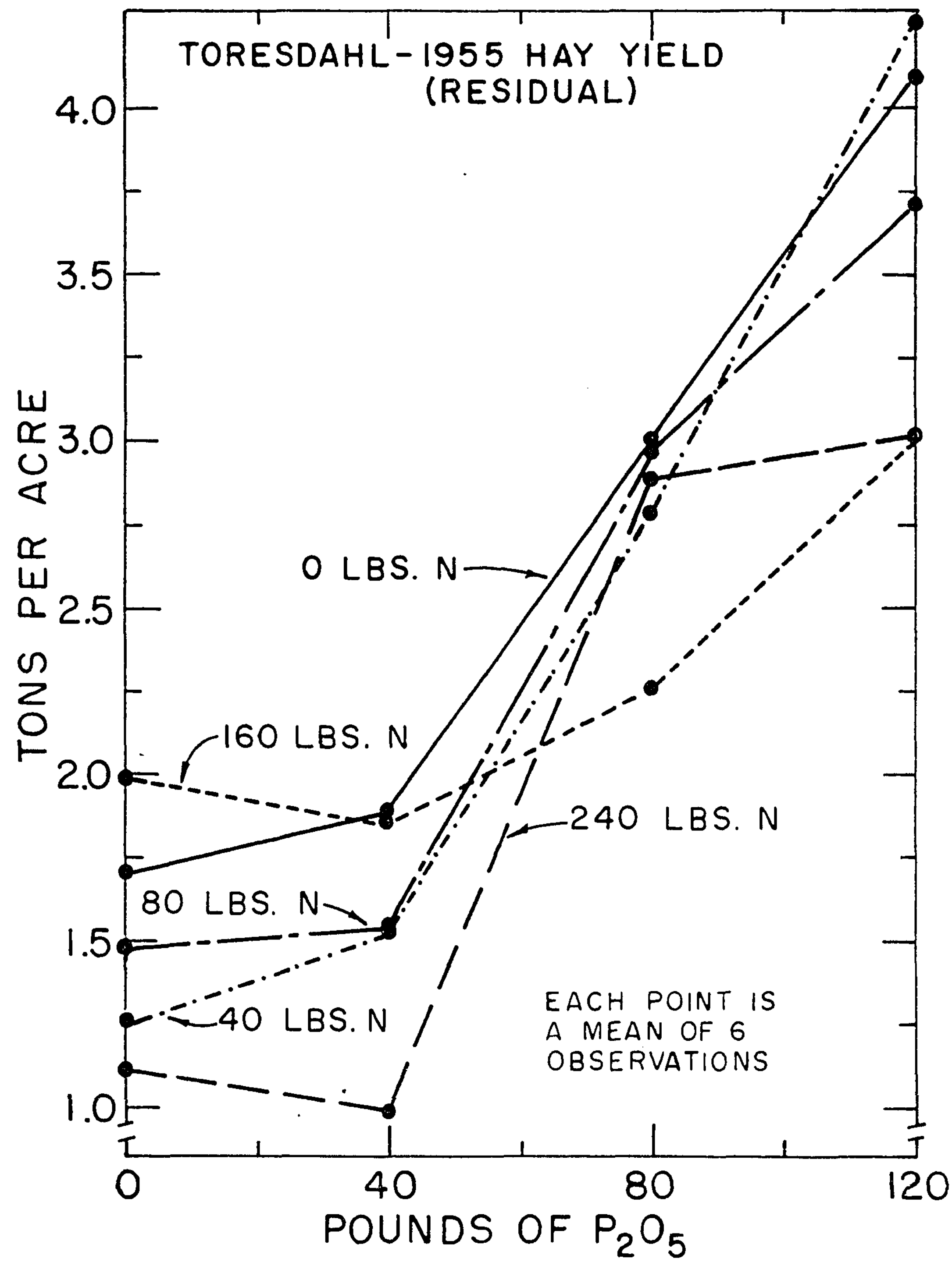


Figure 45. Average yield of alfalfa hay as a function of residual P_2O_5 rates at different levels of residual N in the third year after application.

The stand data are shown in Table 61 of the Appendix and the analysis of variance of the main effects in Table 25 below.

Table 25. Analysis of variance of alfalfa stand levels for residual N, P_2O_5 and K_2O treatments.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	109	458		
Replication	1	78	78.00	21.37**
Treatments	59	201	3.41	0.93
N	4	9	2.25	0.62
P	3	3	1.00	0.27
K	2	2	1.00	0.27
All other treatments	50	187	3.74	
Error	49	179	3.65	

As is evident in Table 25, treatment had no effect on stand level. However, a glance at Table 61 in the Appendix shows that stand levels for all plots are adequate for maximum yields on the basis that from 5 to 6 plants per square foot are sufficient for this purpose.[1] To verify this point, a correlation analysis was made between stand and yield. The resulting r = 0.128 for 107 d.f. was not significant at the 5 percent level. This result indicates that the variation in stand, while great enough to cause a large error term and thus reducing the precision with which differences could be measured, was beyond the minimum level where yields would be affected.

Chemical analyses showed that percent phosphorus in the plant was not influenced significantly by the N treatments. While the effect does not appear to be nutritional at this stage, this does not preclude the possibility of lasting detrimental nutritional effects which may have occurred during the seeding year.

Nelson *et al.* (42) also observed a depressing effect of N on hay yields. These workers found that 40 and 80 pound N applications on oats reduced hay yields the following year. Pritchett (49) in following up on this problem, found that

---

[1]Johnson, I. J. Iowa State College, Ames, Iowa. Effect of alfalfa stand levels on hay yields. Private communication. 1958.

decreased light intensities under both greenhouse and field conditions, with or without N, reduced top and root growth as well as nodulation of the legumes.

In the field, where oats respond vigorously to N, a shading of the seeding occurs which results in decreased sugar production and attendant reduction in root growth and nodule formation. This effect might not be serious if N was adequate in the soil. However, N is not likely to be adequate because of competition from the oat crop. Because of the reduced number of nodules, the shaded legume obtains even less N from the air with the result that a nitrogen deficiency occurs to some degree for a period of time. While the nitrogen deficiency may be only temporary, the plants may never recover sufficiently from the setback to produce yields comparable to those obtained from unshaded plants.

The above explanation could account for the decreased hay yields obtained in this experiment as a result of carry-over N. Perhaps more important than reduced nodule formation in this case was a reduced root system. This point was not checked but would be a reasonable assumption in view of the large oat response in 1954. If a reduced root system did exist for the N treated plots, the alfalfa in these plots would have been more dependent on surface moisture in 1955 than the alfalfa on the zero N plots. Since surface moisture was considerably below normal in 1955, the plants with the more shallow root systems would have been at a disadvantage. Yield reduction would be the logical consequence.

Regression analysis

While the cubic effect could have been questioned for the residual oat yield curves because of the relatively low level of significance for this term, it cannot be questioned for the 1955 residual hay yield curves since the term is significant. Thus the inclusion of the term is justified by agronomic logic as well as probability.

The equation expressing yield of first year hay is

$$Y = \underset{\pm 0.0017}{-0.000562N} - \underset{\pm 0.0166}{0.023565P} + \underset{\pm 0.00036}{0.000868P^2} - \underset{\pm 0.000002}{0.000004P^3} - \underset{\pm 0.000022}{0.000034NP} + 1.563. \qquad (23)$$

The R^2 for equation 23 is 77.4 percent and the t values for the partial regression coefficients in the order shown are 0.33, 1.38, 2.33, 2.01 and 1.47. The first value is not significant. The second and fifth values are significant at the 20 percent level, while the third and fourth are significant at the 5 percent level.

The predicted hay yields are shown in Table 25 and the corresponding yield curves in Figures 46 and 47.

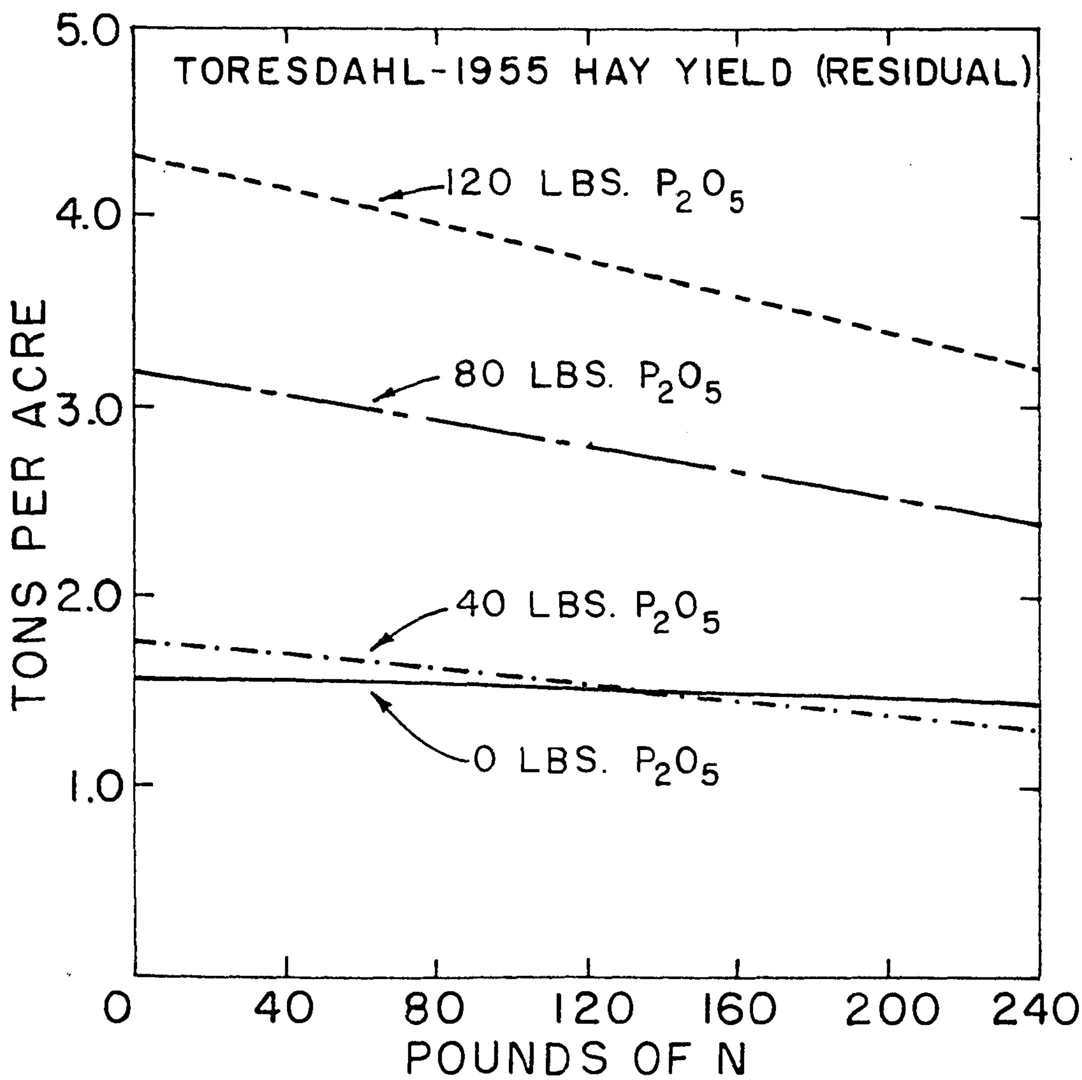


Figure 46. Predicted yield of alfalfa hay as a function of residual N rates at different levels of residual P_2O_5 in the third year after application.

Table 26. Predicted 1955 hay yields in tons per acre from residual nutrient combinations applied to corn in 1953.

Pounds of P_2O_5/A	Pounds of N/A						
	0	40	80	120	160	200	240
0	1.56	1.54	1.52	1.50	1.47	1.45	1.43
40	1.75	1.68	1.60	1.52	1.45	1.37	1.29
80	3.19	3.05	2.92	2.79	2.66	2.53	2.40
120	4.32	4.14	3.95	3.77	3.58	3.39	3.21

Residual yield curves

Figure 46 shows an interesting relationship. At the zero P_2O_5 level, hay yield is depressed only slightly. This effect is to be expected in light of the low response of oats to N alone and, hence, a minimum nurse crop effect. With N plus P_2O_5 additions, the response of oats was quite large, resulting in maximum nurse crop effect which could have reduced root growth or possibly nodulation of the legume much more so than in the plots receiving N alone. If the above represents the true situation in the field, the larger yield depressions for the NP combinations are a logical consequence.

The response of hay to residual P_2O_5, presented in Figure 47, shows the same trend as that of oats to the residual rates of N in 1954. In this case, the sigmoid effect may be

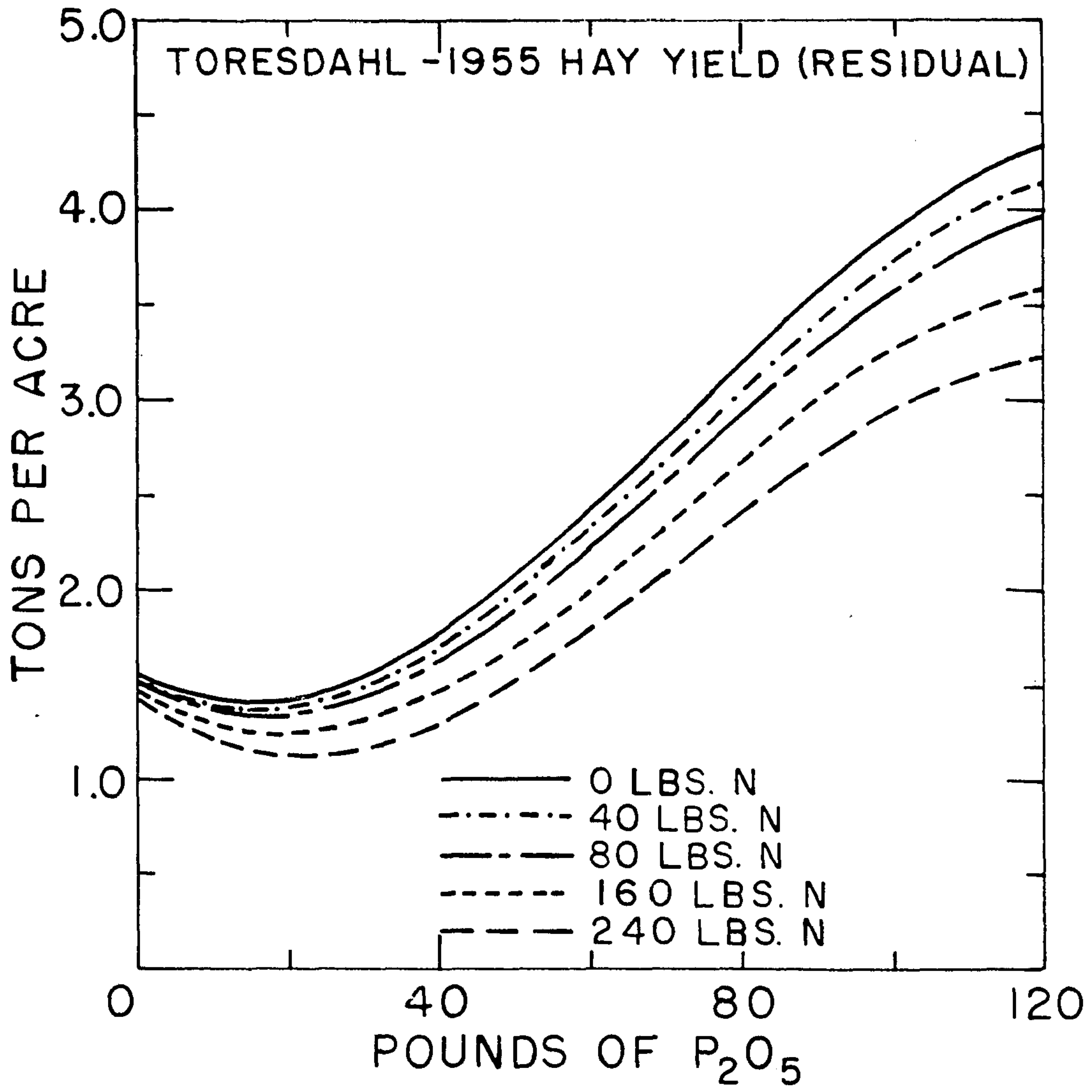


Figure 47. Predicted yield of alfalfa hay as a function of residual P_2O_5 at different levels of residual N in the third year after application.

due to factors suoh as fixation, previous utilization and dry weather, operating alone or together.

Fixation at the low application rates could be of considerable importance since the mean pH of 7.8 together with the "low P" soil test (3.0 pounds per acre) would make ideal conditions for fixation reactions.

Previous utilization would add to the above effect. Prediction equations for phosphorus yield for the previous 2 crops show a range of from 8 to 16 pounds more of P_2O_5 removed from the 40 pound P_2O_5 plots than from the check plots over the range of N rates.

Dry weather could have had some effect provided the residual 40 pound P_2O_5 rate was not sufficient to produce root systems comparable to the 80 and 120 pound P_2O_5 rates in the seeding year. If this was the case, the plants with the more shallow roots would have been at a disadvantage because of reduced soil volume from which P could be absorbed and because of greater dependency on surface moisture.

Residual and topdressed yield curves

The reason for the similar nature of the residual and topdressed yield curves in Figure 48 is not well understood. Fixation of residual P_2O_5 to the extent of a sigmoid effect

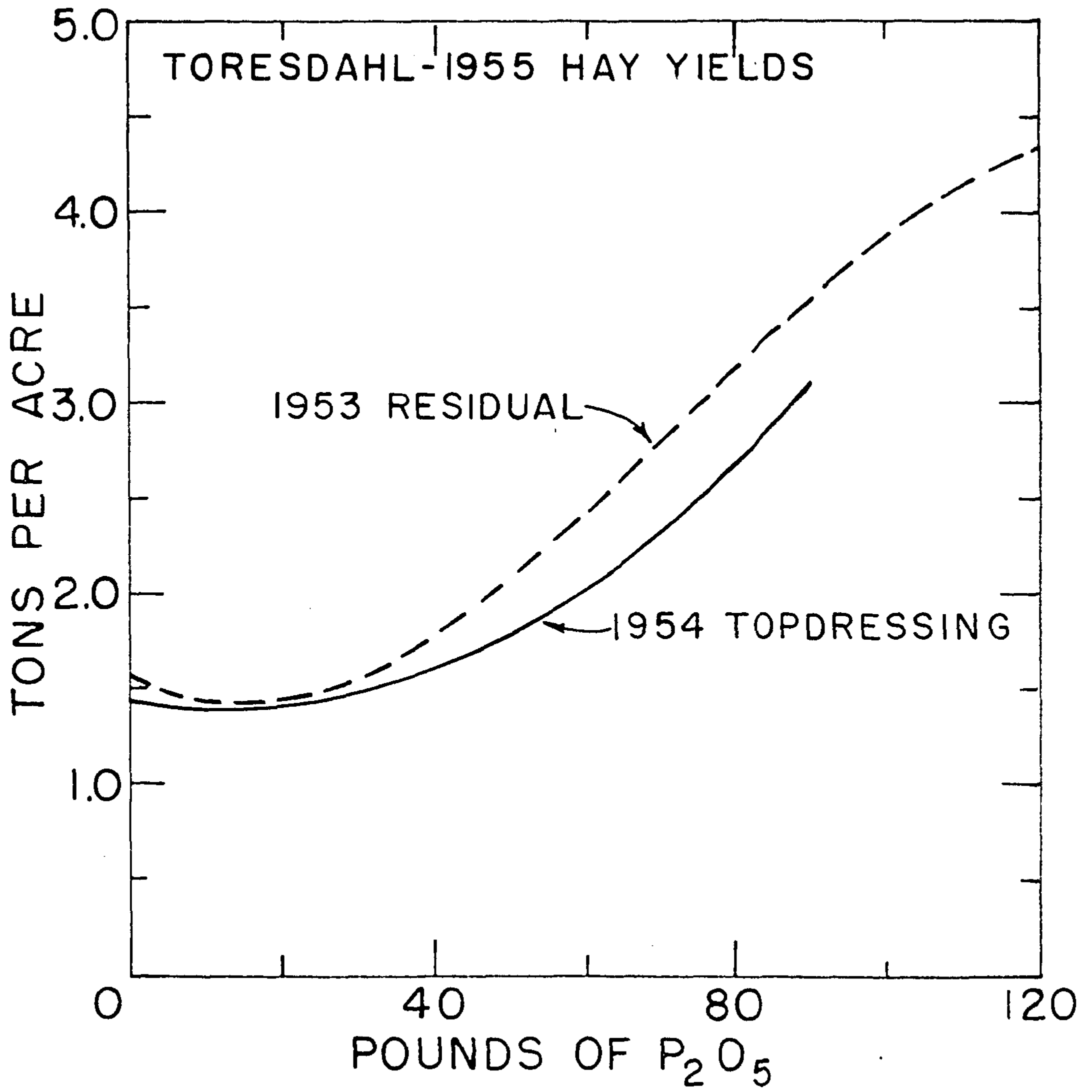


Figure 48. Predicted yields of alfalfa as a function of residual and topdressed P_2O_5 rates at zero level of N.

was not experienced for oats in 1954, yet for the same time period, the residual topdressing of P_2O_5 tends to be sigmoid in nature as well as producing lower yields.

One possible explanation is a difference in P_2O_5 availability due to positional placement. While May and early June were wet in 1954, it was dry and hot for the remainder of the summer until mid August. Because of the dry surface conditions, the topdressed P_2O_5, especially the lower rates, may have been less effective in promoting the kind of root growth which would normally be expected.

There is no way of knowing whether the above hypothesis is valid since root samples were not taken. On the assumption that root development was reduced, the reason for the sigmoid effect may not be one of response or lack of it to P_2O_5 rates during the hay year, but rather an effect of greater soil volume from which those plants, favored in the seeding year with adequate P_2O_5, were able to absorb soil phosphorus. Such an effect would also apply to the negative yield results obtained for N rates discussed earlier.

It is of interest to note that beyond the 40 pound residual P_2O_5 rate, carryover was high. Predicted increases for the 90 pound residual and topdressed rates were 1.9 and 1.7 tons respectively. The equation representing the topdressed curve is

$$Y = \underset{\pm\, 0.013}{-\, 0.006608P} + \underset{\pm\, 0.000142}{0.000283P^2} + 1.417 \,. \qquad (24)$$

The R^2 is 97.7 percent. Predicted and initial yields and the analysis of variance are shown in Tables 63, 64 and 65 of the Appendix.

Residual yield surface and isoquants

Combining the individual yield curves in Figures 46 and 47 resultsin the yield surface shown in Figure 49. The negative effect of N, the effect of the negative NP interaction and the sigmoid response to P are all illustrated in this surface.

As with the previous surfaces, yield contours can be visualized around this surface. The isoquant equation for these contours is

$$N = \frac{Y + 0.023565P - 0.000868P^2 + 0.000004P^3 - 1.563}{- 0.000562 \quad - 0.000034P} \,, \qquad (25)$$

and is illustrated in Figure 50. The two 1.5 ton yield isoquants look strange but are consistent with the surface. Higher yield isoquants follow the same pattern but are not shown because they fall in zones outside of the experimental observations.

Because of the negative effect of N, there is no range

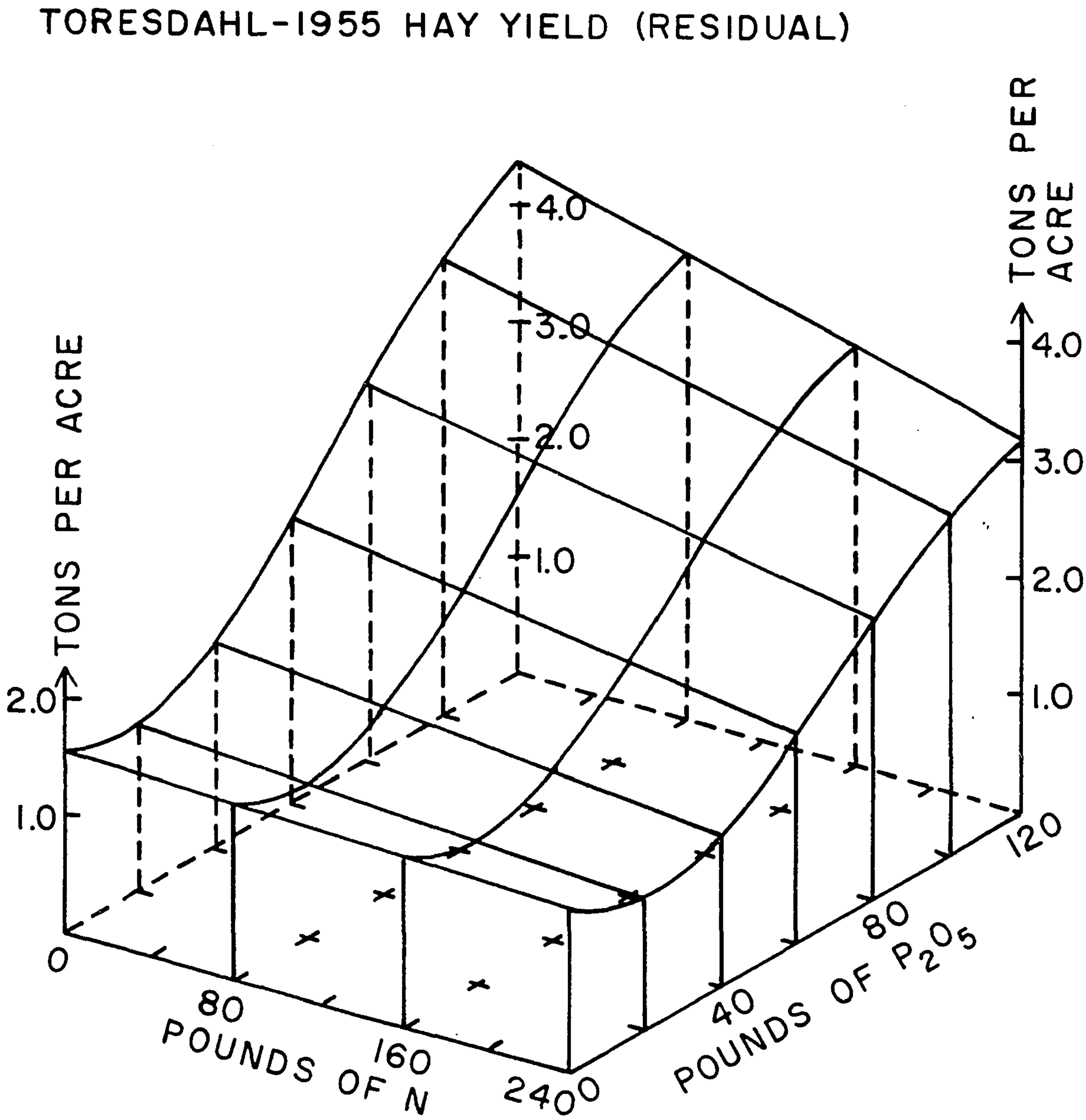


Figure 49. Predicted alfalfa yield surface as a function of residual N and P_2O_5 rates in the third year after application.

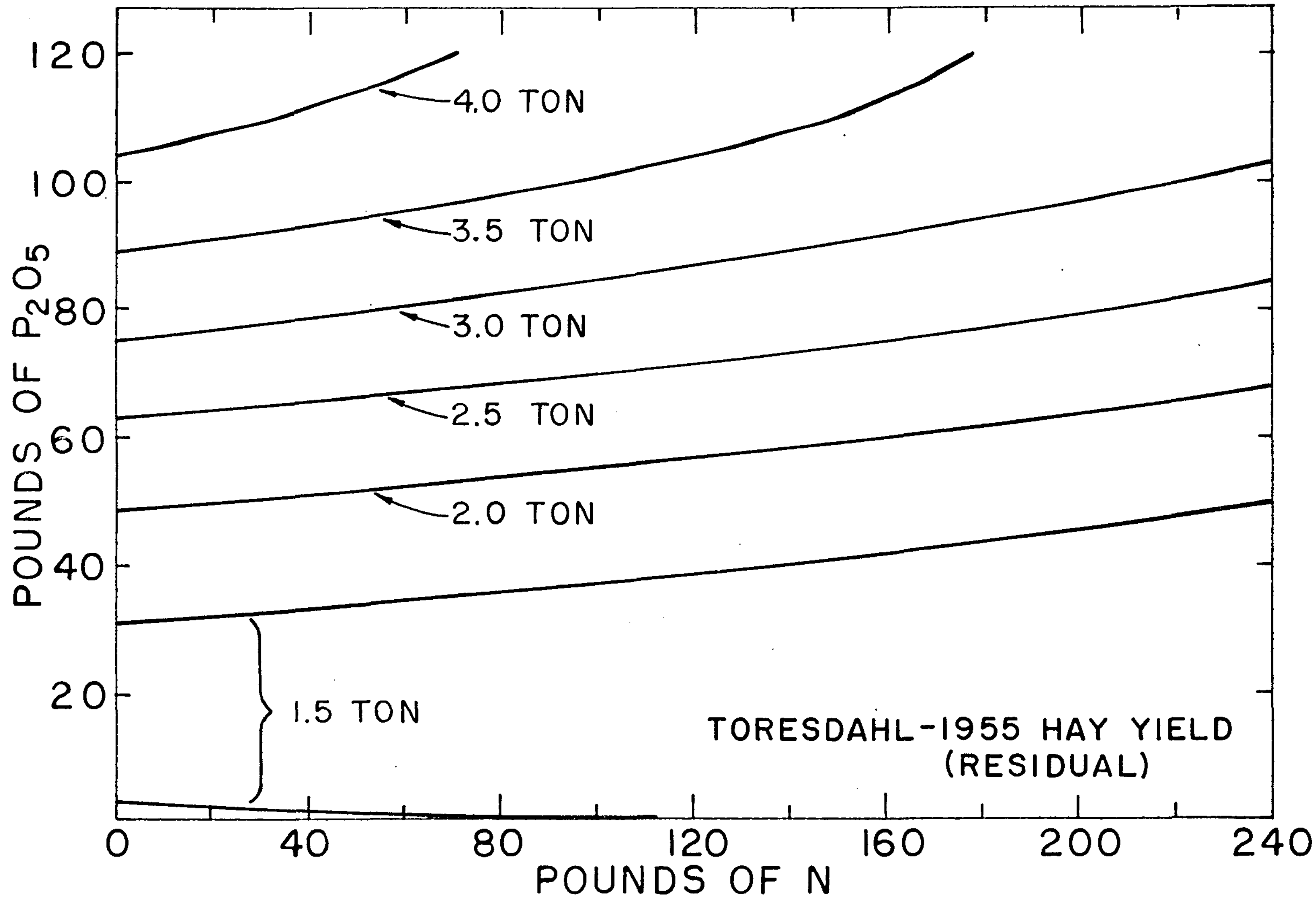


Figure 50. Alfalfa yield isoquants as a function of residual N and P_2O_5 rates in the third year after application.

of substitution of nutrients as is usually the case. In this situation, a farmer should use zero pounds of N since, if N is increased, a given yield can be obtained only by increasing P_2O_5.

Since there is no point where rates of nutrient substitution exist within the range of the data, isoclines and ridgelines are omitted from the drawing. This isoquant map is of interest primarily from the standpoint of showing the variation occurring in maps from year to year.

Economic optima and maxima

Economic optima are of academic interest only since residual P_2O_5 is being evaluated without regard for previous responses. However, since the residual P_2O_5 was superior to the topdressed material, it is of interest to see just what rates of N and P_2O_5 would be optimum.

Because N had a negative effect on hay yield, the optimum level for this nutrient is zero. With the price of P_2O_5 at $0.10 per pound and hay at $20.00 per ton, an optimum of 125.5 pounds of P_2O_5 is obtained. This optimum leads to an optimum yield of 4.37 tons.

The above calculations are interesting in view of the fact that the predicted yield for 120 pounds of P_2O_5 was 4.32

tons. On the basis of the 4.32 tons and the above costs and prices, the gross returns for the 120-pound residual increase is $43.20 without considering the response obtained on the two previous crops.

To obtain the maximum yield, N was set at zero for reasons given earlier and the partial derivative of the yield equation with respect to P was set at zero. Solving for the quantity of P_2O_5 to give a zero marginal product, the result was 129.3 pounds of P_2O_5 with an estimated hay yield of 4.38 tons.

Percent phosphorus

Plant samples were taken for each of the 3 cuttings and analyzed for phosphorus. The final percentages used for calculation purposes were a weighted average of the 3 cuttings based on yield. These computed percentages and phosphorus yields are shown in Table 62 of the Appendix. The analysis of variance for percent phosphorus is shown in Table 27.

Since only phosphorus had a significant effect on percent phosphorus, this term alone was included in the regression analysis. The equation expressing percent phosphorus is

$$Y_{\%P} = \underset{\pm 0.000475}{-0.00088930P} + \underset{\pm\, 0.00001}{0.00001987P^2} - \underset{\pm\, 0.00000006}{0.00000009P^3} + 0.180466. \quad (26)$$

The t values are 1.87, 1.89, and 1.51 respectively, with an R^2

Table 27. Analysis of variance of percent phosphorus in 1955 hay due to residual treatments.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	119	0.109285		
Replication	1	0.056768	0.056768	227.07**
Treatments	59	0.037753	0.000640	2.56**
Nitrogen(N)	4	0.000921	0.000230	0.92
Phosphorus(P)	3	0.023975	0.007992	31.97**
Linear	1		0.016981	67.92**
Quadratic	1		0.005307	21.23**
Cubic	1		0.001687	6.75*
Potassium(K)	2	0.000781	0.000391	1.56
Linear	1		0.000285	1.14
Quadratic	1		0.000496	1.98
PK	6	0.000964	0.000161	0.64
Aoo other treatments	44	0.011112	0.000253	
Error	59	0.014764	0.000250	

*Significance level is explained in footnote of Table 3.

of 63.5 percent. The first two values are significant at the 10 percent level while the last value is significant at the 20 percent level. Predicted phosphorus percentages for the 0, 40, 80 and 120 pound P_2O_5 rates are 0.180, 0.170, 0.190 and 0.204 respectively.

In view of the depressing effect of N on yields, it would have been reasonable to expect N to have an effect on phosphorus concentration in the plant. While there was some effect, it was not significant.

Percent phosphorus follows the trend of the yield curve for rates of P_2O_5. This result could be due to any one or all of the factors previously discussed in the residual yield curve and residual and topdressed yield curve sections.

Phosphorus yield

While N had no effect on percent phosphorus in the plant, it had an effect on phosphorus yield as shown in the analysis of variance in Table 28. This effect was negative just as it was for hay yield.

The P effect and the NP interaction were also significant. Hence, all of these terms were included in the regression equation which is

$$\begin{aligned} Y_{P\ yield} = &- \underset{\pm 0.009}{0.006667N} - \underset{\pm 0.089}{0.183577P} + \underset{\pm 0.0018}{0.004611P^2} - \underset{\pm 0.00001}{0.000019P^3} \\ &+ \underset{\pm 0.000378}{0.000271NP} - \underset{\pm 0.000003}{0.000004NP^2} + 6.983241. \quad (27) \end{aligned}$$

The correlation index was 78.1 percent while the t values were 0.71, 2.06, 2.57, 1.97, 0.72 and 1.26 respectively. Predicted phosphorus yields are shown in Table 29 and are illus-

Table 28. Analysis of variance of phosphorus yield in 1955 hay due to residual treatments.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	119	4,445.39		
Replication	1	1,129.15	1,129.15	105.53**
Treatment	59	2,685.09	45,51	4.25**
Nitrogen(N)	4	126.85	31.71	2.96*
Linear	1		121.70	11.37**
Remainder	3		5.15	--
Phosphorus(P)	3	1,878.86	626.29	58.53**
Linear	1		1,610.15	150.48**
Quadratic	1		188.25	17.59**
Cubic	1		80.45	7.52**
Potassium(K)	2	25.00	12.50	1.17
NP	12	258.68	21.56	2.01*
PK	6	35.06	5.84	0.55
All other treatments	32	360.64	11.27	--
Error	59	631.15	10.70	

Table 29. Predicted phosphorus yields in pounds per acre for 1955 hay obtained from residual nutrients applied to corn in 1953.

Pounds of	Pounds of N/A						
P_2O_5/A	0	40	80	120	160	200	240
0	7.0	6.7	6.4	6.2	5.9	5.6	5.4
40	5.8	5.7	5.6	5.5	5.4	5.3	5.3
80	12.1	11.7	11.2	10.8	10.4	10.0	9.5
120	18.5	17.3	16.0	14.7	13.4	12.2	10.9

trated in the surface shown in Figure 51.

Figure 51 resembles Figure 49 to a considerable degree. Such a resemblance is to be expected in view of the fact that the nutrient percentages follow the yield trend. The NP interaction effect, as well as the overall negative effect of N is much stronger in the phosphorus yield surface than in the hay yield surface. However, it is unlikely that these effects have agronomic significance beyond those already discussed.

Percent potassium

The concentration of this nutrient in the alfalfa showed a trend similar to that of phosphorus as is indicated in the analysis of variance of Table 30. Only P had a significant effect on the percent potassium in the hay. Since the yield response to P_2O_5 was cubic in nature and agronomically reasonable, and since the concentration of potassium could follow the characteristics of the yield curve, this term was included in the regression equation.

The resulting regression equation for percent potassium is

$$Y_{\%K} = \underset{\pm 0.003}{-\,0.002750P} + \underset{\pm 0.00006}{0.00012245P^2} - \underset{\pm 0.0000005}{0.00000082P^3} + 1.687328, \quad (28)$$

with an R^2 of only 15.3 percent. This result indicates that the variables chosen to represent the data explain very little

TORESDAHL-PHOSPHORUS YIELD-1955 HAY (RESIDUAL)

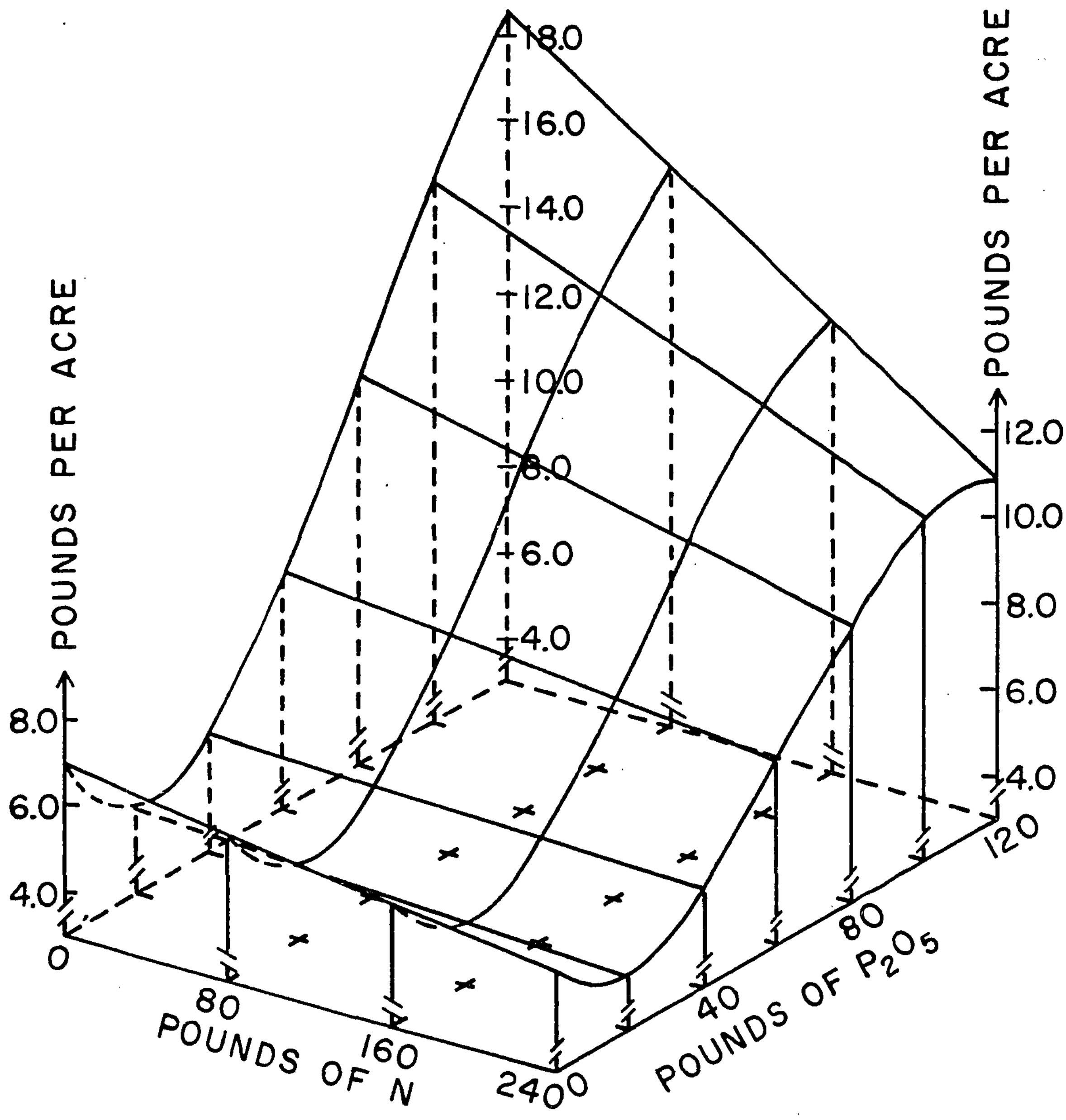


Figure 51. Predicted surface for phosphorus yield of alfalfa as a function of residual N and P_2O_5 rates in the third year after application.

Table 30. Analysis of variance of percent potassium in 1955 hay due to residual treatments.

Source of variation	Degrees of freedom		Sum of squares	Mean square	F
Total	119		5.5114		
Replication	1		0.2862	0.2862	6.17*
Treatments	59		2.4892	0.0422	0.91
Nitrogen(N)	4		0.0727	0.0182	0.39
Phosphorus(P)	3		0.3816	0.1272	2.74*
Linear		1		0.0356	0.77
Quadratic		1		0.1968	4.24*
Cubic		1		0.1492	3.22[c]
Potassium(K)	2		0.0064	0.0032	0.07
All other treatments	50		2.0285	0.0406	--
Error	59		2.7360	0.0464	

[c]Significance level is explained in footnote of Table 4.

of the total variation in percent potassium. In view of the analysis of variance results, it also means that no treatment had much effect on potassium concentration. The t values for the above coefficients are 0.88, 2.06 and 1.79 respectively. The predicted potassium percentages were 1.69, 1.72, 1.83 and 1.70 respectively for 0, 40, 80 and 120 pounds of P_2O_5.

Potassium yield

Since potassium concentration followed the yield trend for P_2O_5 rates while N had no effect, it is not surprising that the analysis of variance in Table 31 shows results similar to that for hay yield and phosphorus yield.

Table 31. Analysis of variance of potassium yield for 1955 residual hay yields.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	119	306,099.41		
Replication	1	32,095.05	32,095.05	23.90**
Treatments	59	194,774.76	3,301.27	2.46**
Nitrogen(N)	4	6,388.65	1,597.16	1.19
Linear	1		5,853.90	4.36*
Remainder	3		534.75	--
Phosphorus(P)	3	119,462.94	39,820.98	29.65**
Linear	1		107,942.46	80.38**
Quadratic	1		2,518.08	1.88[d]
Cubic	1		9,002.40	6.70*
Potassium(K)	2	1,398.01	699.01	0.52
NP	12	18,695.08	1,557.92	1.16
PK	6	3,903.72	650.62	0.48
All other treatments	32	44,926.36	1,403.95	--
Error	59	79,229.60	1,342.87	

[d]Significance level is explained in footnote of Table 4.

The significant terms in the analysis of variance, together with the linear by linear NP interaction which was significant at the 20 percent level of probability, were included in the regression analysis. The resulting equation for potassium yield is

$$
\begin{aligned}
Y_K \text{ yield} = &- \underset{\pm 0.07}{0.011685N} - \underset{\pm 0.7}{1.065681P} + \underset{\pm 0.015}{0.039166P^2} - \underset{\pm 0.000084}{0.000202P^3} \\
&- \underset{\pm 0.0009}{0.001191NP} + 52.563966, \qquad (29)
\end{aligned}
$$

with an R^2 of 65.8 percent. The t values for the individual coefficients are 0.17, 1.53, 2.56, 2.40 and 1.27 respectively. Predicted potassium yields are shown in Table 32 and are illustrated by the surface in Figure 52.

Table 32. Predicted potassium yields in pounds per acre for 1955 hay obtained from residual nutrients applied to corn in 1953.

Pounds of P_2O_5/A	Pounds of N/A						
	0	40	80	120	160	200	240
0	52.6	52.1	51.6	51.2	50.7	50.2	49.8
40	59.7	57.3	54.9	52.6	50.2	47.8	45.4
80	114.5	110.3	106.0	101.7	97.4	93.2	88.9
120	139.6	133.4	127.2	121.1	114.9	108.7	102.5

As would be expected from the results obtained for percent potassium, the potassium yield surface in Figure 52 is almost a replica of the hay yield and phosphorus yield sur-

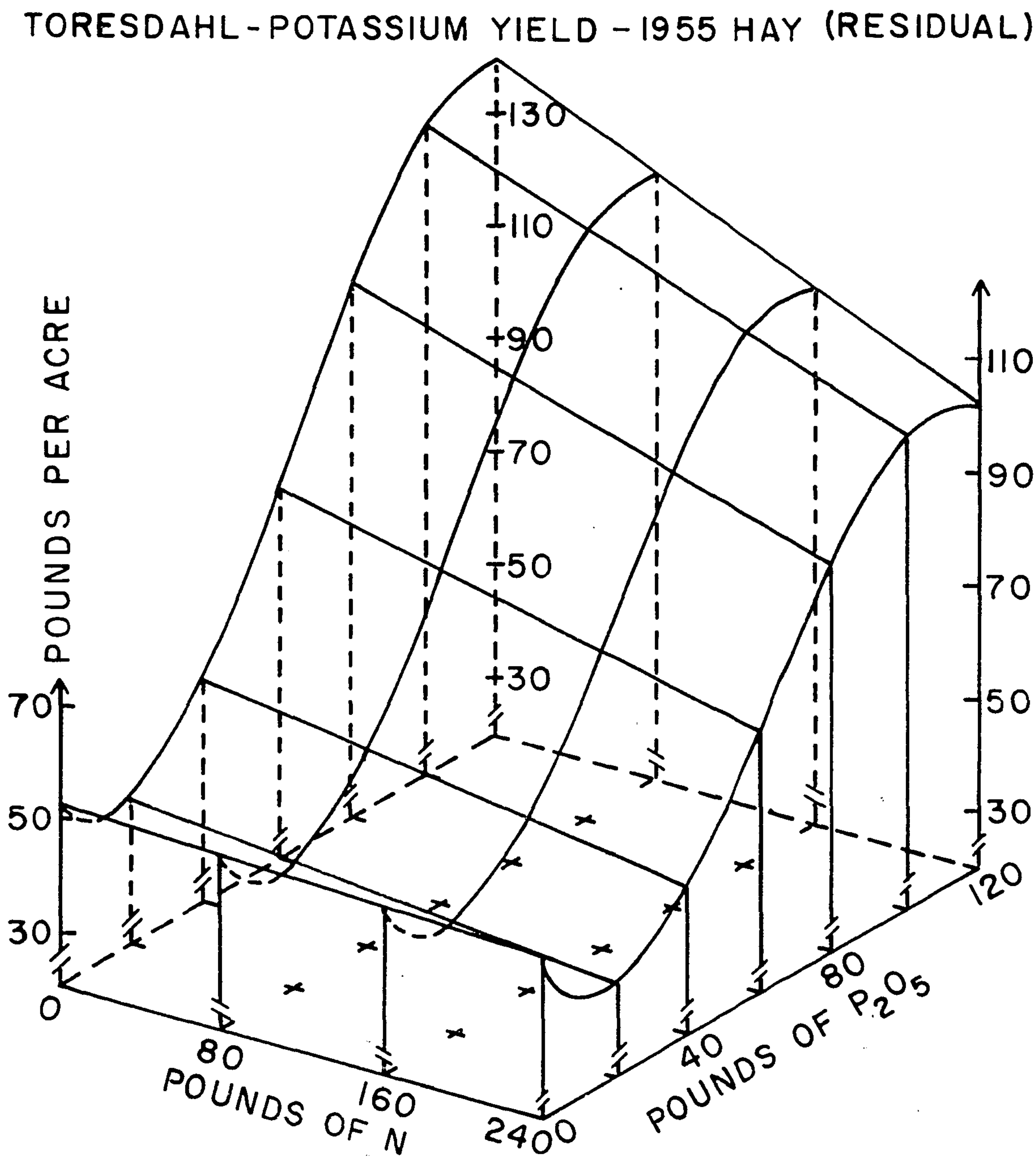


Figure 52. Predicted surface for potassium yield of alfalfa as a function of residual N and P_2O_5 rates in the third year after application.

faces. Apparently, potassium is in adequate supply in the soil and is being taken up in quantities corresponding to dry matter production. As the factors which limit hay yield, either at the low or high P_2O_5 rates, come into play, total absorption of K follows suit.

<u>Nutrient comparisons for residual treatments</u>

Figure 53 summarizes the relationships between hay yield, nutrient percentage and nutrient yield. Percent phosphorus and phosphorus yield follow the trend established by the crop yield curve. The potassium curves do likewise except that percent potassium fluctuates drastically. In view of the low R^2 for the latter, this large fluctuation has no agronomic significance.

All of the curves are dominated by phosphorus since it is the most limiting nutrient in the system. The absorption of both phosphorus and potassium reflects the effect of those factors, expressed in rates of P_2O_5, which influence residual hay yield.

<u>Nutrient comparisons for topdressed treatments</u>

Figure 54 summarizes the nutrient-yield relationships for the P_2O_5 topdressed in 1954. Nutrient yield curves conform to the crop yield curve just as they did in Figure 53 but the percent nutrient curves do not.

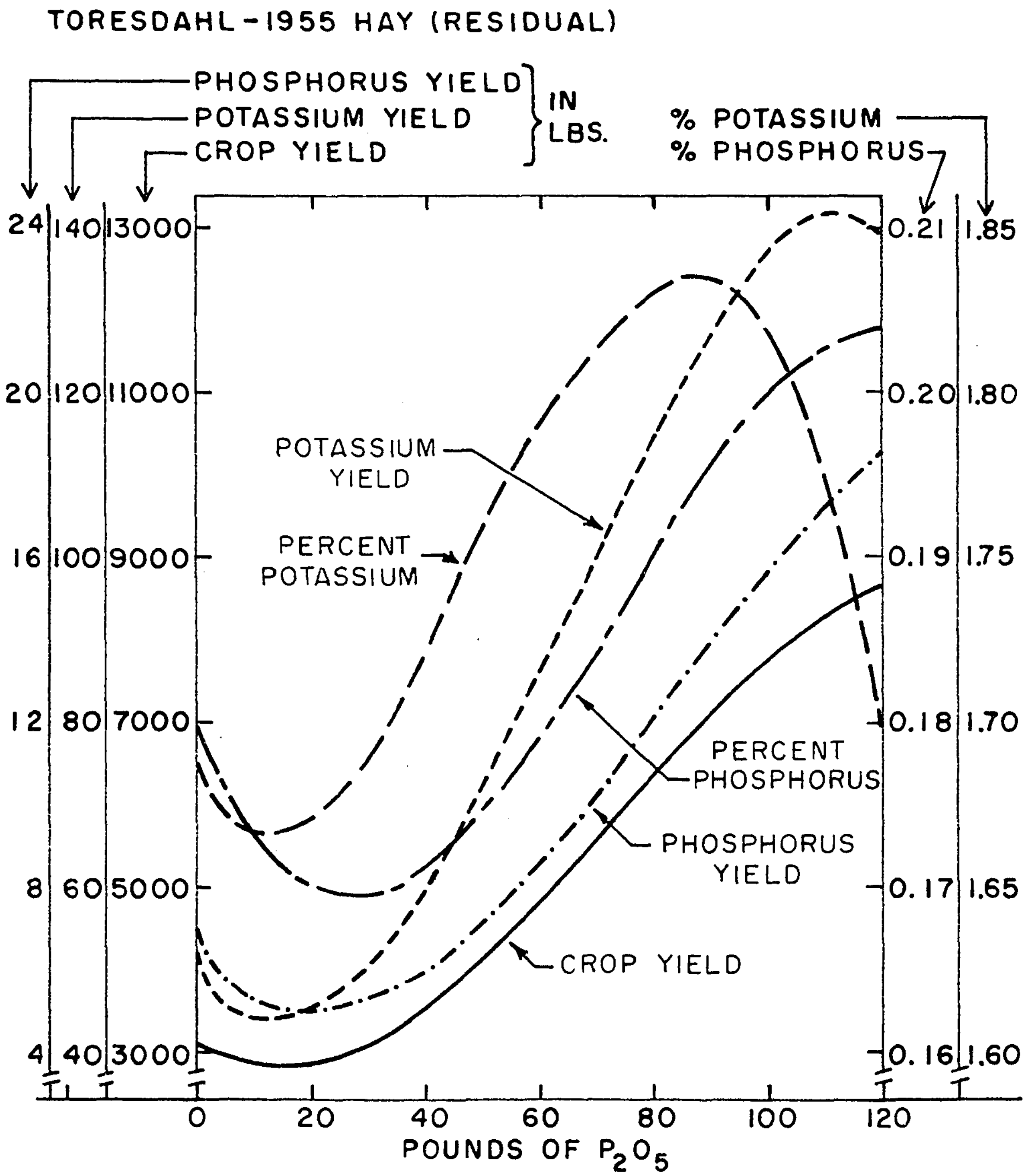


Figure 53. Yield and nutrient relationships for alfalfa hay as a function of residual P_2O_5 rates at the zero level of N in the third year after application.

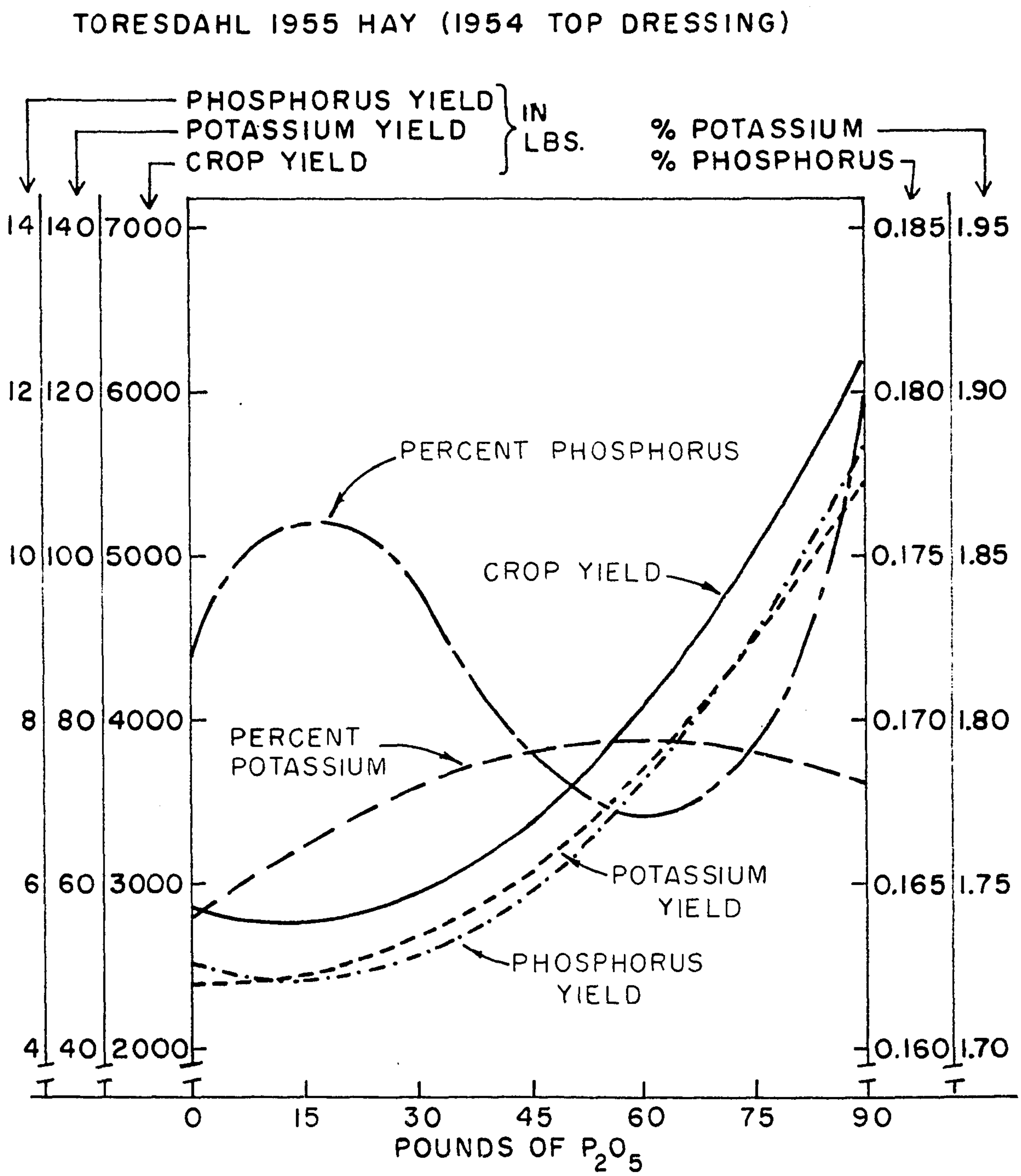


Figure 54. Yield and nutrient relationships for alfalfa hay as a function of topdressed P_2O_5 at a zero N level in the second year after application.

With the exception of the percent phosphorus curve for residual P in Figure 53, the percent phosphorus curve in Figure 54 and the percent potassium curves in both figures are meaningless. These curves are presented for the purpose of illustrating the importance of both the various significance levels in the analysis of variance and the plotting of the mean observed data.

Phosphorus and potassium effects were not significant in the original analysis of variance for the three curves showing the odd nutrient concentrations for rates of P_2O_5. However, the partitioned sum of squares showed that the various components were significant at the 20 percent level of probability or less and were, therefore, acceptable for inclusion into the regression equation on the basis of previously established criteria. If only these levels of significance are looked at, without regard for their agronomic validity in terms of the kind of curve that will be produced, as would be evident from plotting the mean observed data, it is likely that the results will be meaningless. The curves may fit the data well, but if the data did not show significant effects in the first place, not much will come from the mathematical manipulations.

The above emphasizes once again that the tool of statistical significance at low levels of probability must be used with caution. In most cases, it is desirable for the main effect to be significant at the 5 percent level of probability or less before it is included in the regression equation. For variables obtained through partitioning the sum of squares of a significant variable, the level should be 10 percent or less. If the main effect was not significant at the 5 percent level or less, the level of significance for variables obtained by partitioning sum of squares should be restricted to 5 percent or less. In all cases, the resulting curves must be reasonable from an agronomic standpoint.

The original plot data, predicted yields, nutrient composition data, analyses of variance and regression equations for all of the topdressed material are shown in Tables 63 through 69 of the Appendix.

<u>Summary - 1955 hay</u>

1. P_2O_5 effects from the 1953 application were highly significant and resulted in predicted yield increases up to 2.76 tons per acre.
2. N rates had a significant negative effect on hay yields. It was postulated that this effect was due to reduced root growth during the seeding year as a result of competition from the companion crop of oats which responded well to N and particularly to NP combinations.

3. A sigmoid response curve was obtained for P_2O_5 rates. This result is reasonable in terms of possible fixation of the low rates under the high-lime conditions together with the low phosphorus status of the soil, previous P_2O_5 utilization and possibly reduced root growth of the legume at low rates of application. The top end of the curve results from the fact that available P_2O_5 was less limiting than some other yield factor.
4. The 1953 residual P_2O_5 was superior to that topdressed in 1954 throughout the range of application rates. This result should not be interpreted as 100 percent carryover since the extremely dry season probably reduced the effectiveness of the topdressed material because of positional placement. Nevertheless, the magnitude of the residual response indicates that a very substantial carryover occurred.
5. With the exception of residual phosphorus percentage, P and K nutrient percentages were not affected significantly by treatment. Nutrient yields conformed to hay yields for both residual and topdressed P_2O_5. The dominating effect of P_2O_5, as modified by N, influenced the uptake of P and K in conformity with its effect on crop yield.
6. Economic analyses were deferred for a final rotation evaluation because residual effects were not measured for comparable nutrient combinations.

Alfalfa Hay - 1956

The measurement of residual fertilizer effects was continued during 1956 by taking 3 cuttings of hay from each of the residual 1953, residual 1954 and newly topdressed 1956 treatments. Samples were saved from each cutting for chemical analysis. The final nutrient percentages are a weighted average based on yields obtained from each plot at each cutting and are shown in Table 70 of the Appendix.

Residual hay yield analysis

The mean hay yields for the various 1953 residual treatments together with the mean yields for the main effects are shown in Table 33. The data (Table 33) indicate a possible negative trend for the N main effects and a quadratic effect for both P_2O_5 and K_2O main effects. The significance of these trends is shown in the analysis of variance in Table 34.

Instead of the N effect being strictly negative as in 1955, a significant linear and cubic effect showed up in the 1956 analysis of variance. The reason for the latter is evident from the large erratic effect shown in the mean N yield lines at the zero and 40 pound P_2O_5 levels in Figure 55. There is no good agronomic reason for such an effect at these

Table 33. Hay yields in tons obtained in 1956 from residual N, P_2O_5 and K_2O applied to corn in 1953. Each value is a mean of 2 observations.

Pounds of P_2O_5/A	Pounds of K_2O/A	Pounds of N/A 0	40	80	160	240
0	0	2.79	1.85	2.06	1.89	1.62
0	40	1.94	1.63	1.26	1.44	1.49
0	80	1.74	1.72	1.16	3.64	1.51
40	0	1.78	1.48	1.60	1.40	0.71
40	40	1.45	1.74	1.37	2.01	1.79
40	80	2.35	1.11	1.91	3.27	0.96
80	0	2.33	2.10	3.18	1.57	3.15
80	40	1.57	1.52	1.77	2.66	1.96
80	80	2.09	2.37	2.19	1.50	1.64
120	0	3.26	2.86	3.14	2.40	1.92
120	40	2.55	2.29	2.06	1.43	2.21
120	80	3.47	2.88	2.53	2.67	1.58

Mean yield for main effects

Pounds of N/A	Mean yield (24 obs.)	Pounds of P_2O_5/A	Mean yield (30 obs.)	Pounds of K_2O/A	Mean yield (40 obs.)
0	2.28	0	1.85	0	2.15
40	1.96	40	1.66	40	1.80
80	2.02	80	2.10	80	2.11
160	2.15	120	2.48		
240	1.71				

Table 34. Analysis of variance of 1956 hay obtained from residual 1953 fertilizer applications

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	119	112.7289		
Replication	1	32.0953	32.0953	64.37**
Treatments	59	51.2151	0.8681	1.74*
Nitrogen(N)	4	4.4026	1.1007	2.21[c]
Linear	1		2.1169	4.25*
Quadratic	1		0.0550	0.11
Cubic	1		2.1793	4.37*
Quartic	1		0.0514	0.10
Phosphorus(P)	3	11.3787	3.7929	7.61**
Linear	1		8.2791	16.60**
Quadratic	1		2.3688	4.75*
Cubic	1		0.7308	1.47
Potassium(K)	2	2.9088	1.4544	2.92[c]
Linear	1		0.0312	0.06
Quadratic	1		2.8776	5.77*
NP	12	8.8945	0.7412	1.49[d]
NK	8	8.1356	1.0170	2.04[c]
PK	6	3.9565	0.6594	1.32
All other treatments	24	11.5384	0.4808	---
Error	59	29.4185	0.4986	

[c,d] Significance levels are explained in footnotes of Table 4.

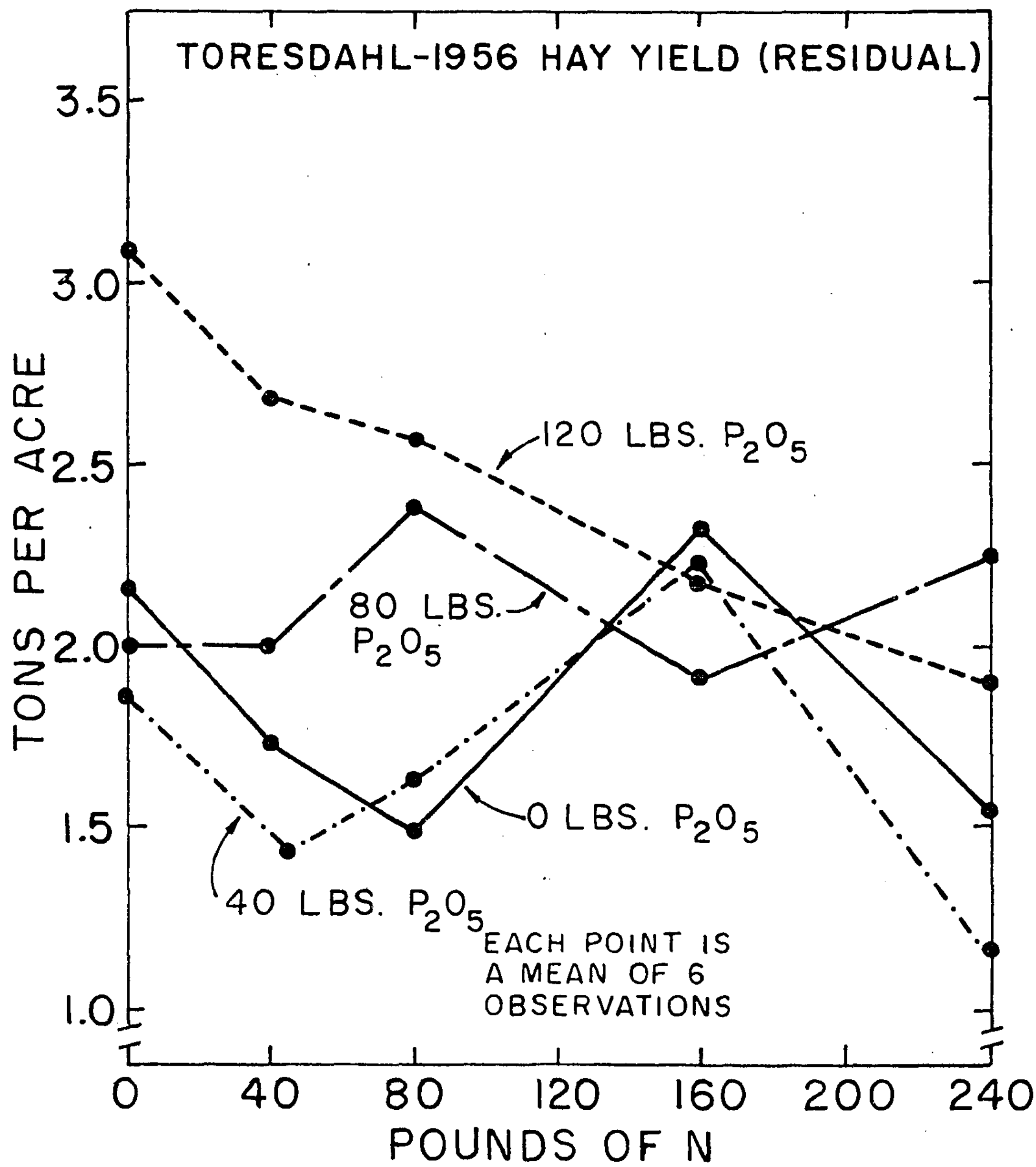


Figure 55. Average yield of alfalfa hay as a function of residual N rates at different levels of residual P_2O_5 in the fourth year after application.

rates and since the cubic effect was not significant in 1955, only the linear component was used in the 1956 regression analysis.

The phosphorus effect contained significant linear and quadratic components but no longer contained a significant cubic effect. Hence only the former were included in the regression. The mean response lines are shown in Figure 56.

The potassium analysis was the most surprising. While a quadratic effect had shown up in the mean yield figures in both 1955 and 1956, it had not been significant until now. The effect could have been rejected but since chemical analyses had not yet been made and because the criterion of significance of at least 20 percent was being adhered to, it was decided to include the linear and quadratic components in the regression.

The significance levels of the NP and NK interaction also prompted their inclusion in the regression analysis.

Regression analysis and yield curves

The yield equation is

$$\begin{aligned} Y = &\underset{\pm 0.0027}{-0.000409N} \underset{\pm 0.009}{- 0.002P} \underset{\pm 0.00007}{+ 0.000088P^2} \underset{\pm 0.00003}{- 0.000025NP} \underset{\pm 0.013}{- 0.018K} \\ &\underset{\pm 0.000146}{+ 0.000205K^2} \underset{\pm 0.00004}{+ 0.000008NK} + 1.983, \qquad (30) \end{aligned}$$

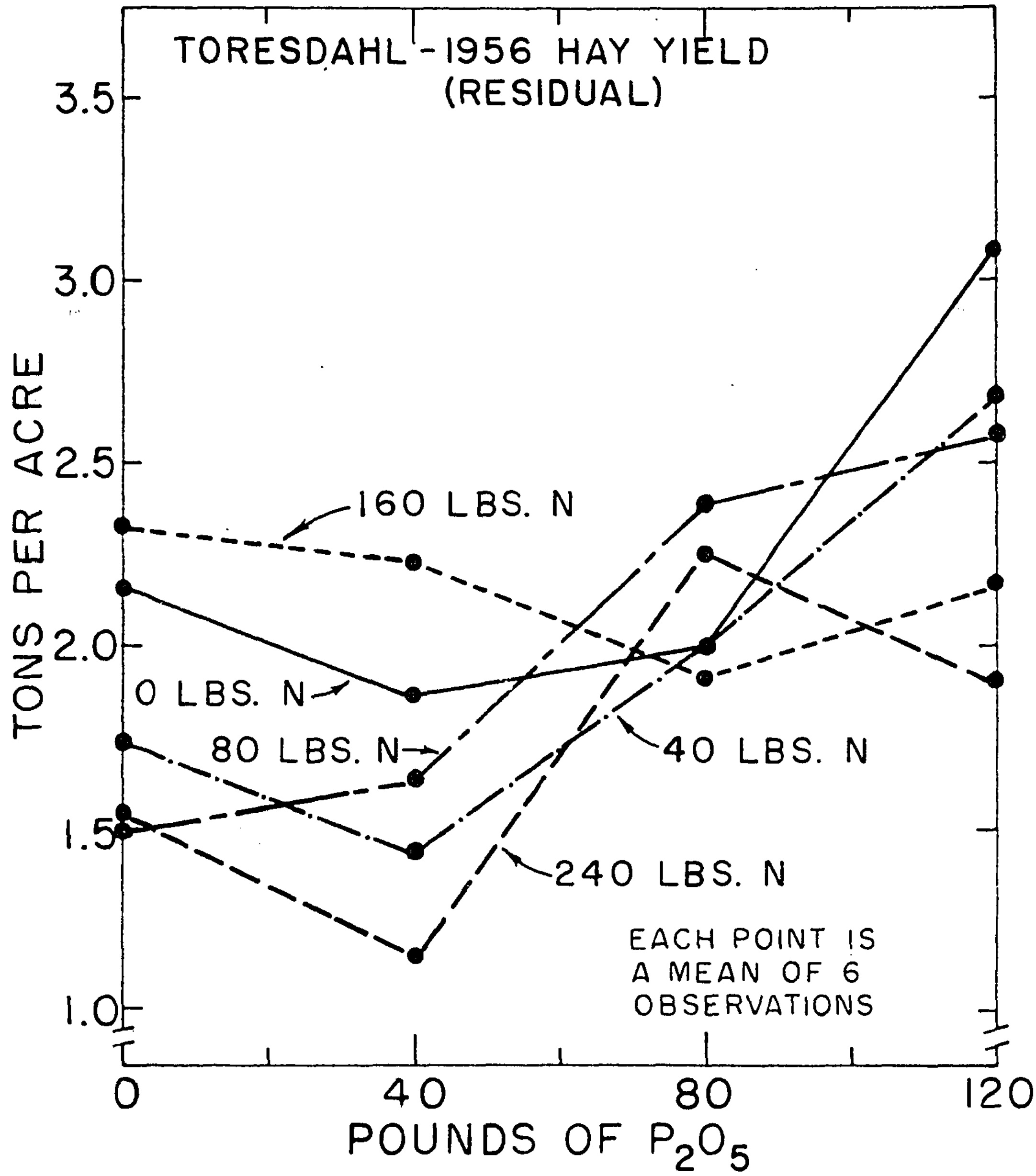


Figure 56. Average yield of alfalfa hay as a function of residual P_2O_5 at different levels of residual N in the fourth year after application.

with an R^2 of 32.9 percent. The t values are significant only at levels of probability greater than 10 percent. From these results, it is evident that the variability in hay yields was rather large and that, apparently, the variables selected did not explain much of the total variation in the experiment.

Actually, such results might be expected in fourth year residual results. Only the most limiting element is likely to have an appreciable carryover effect and even this effect will tend to be more variable than in previous years. That such is the case is illustrated by the difference in the coefficients of variation which were 11.0 and 34.9 percent, respectively, for 1955 and 1956 yields.

Predicted yields at the zero K_2O levels are shown in Table 35 and the resulting curves in Figures 57 and 58.

Table 35. Predicted 1956 hay yields in tons per acre from residual nutrient combination applied to corn in 1953.

Pounds of	Pounds of N/A						
P_2O_5/A	0	40	80	120	160	200	240
0	1.98	1.97	1.95	1.93	1.92	1.90	1.88
40	2.04	1.99	1.93	1.87	1.82	1.76	1.71
80	2.39	2.29	2.19	2.10	2.00	1.90	1.81
120	3.01	2.87	2.74	2.60	2.46	2.33	2.19

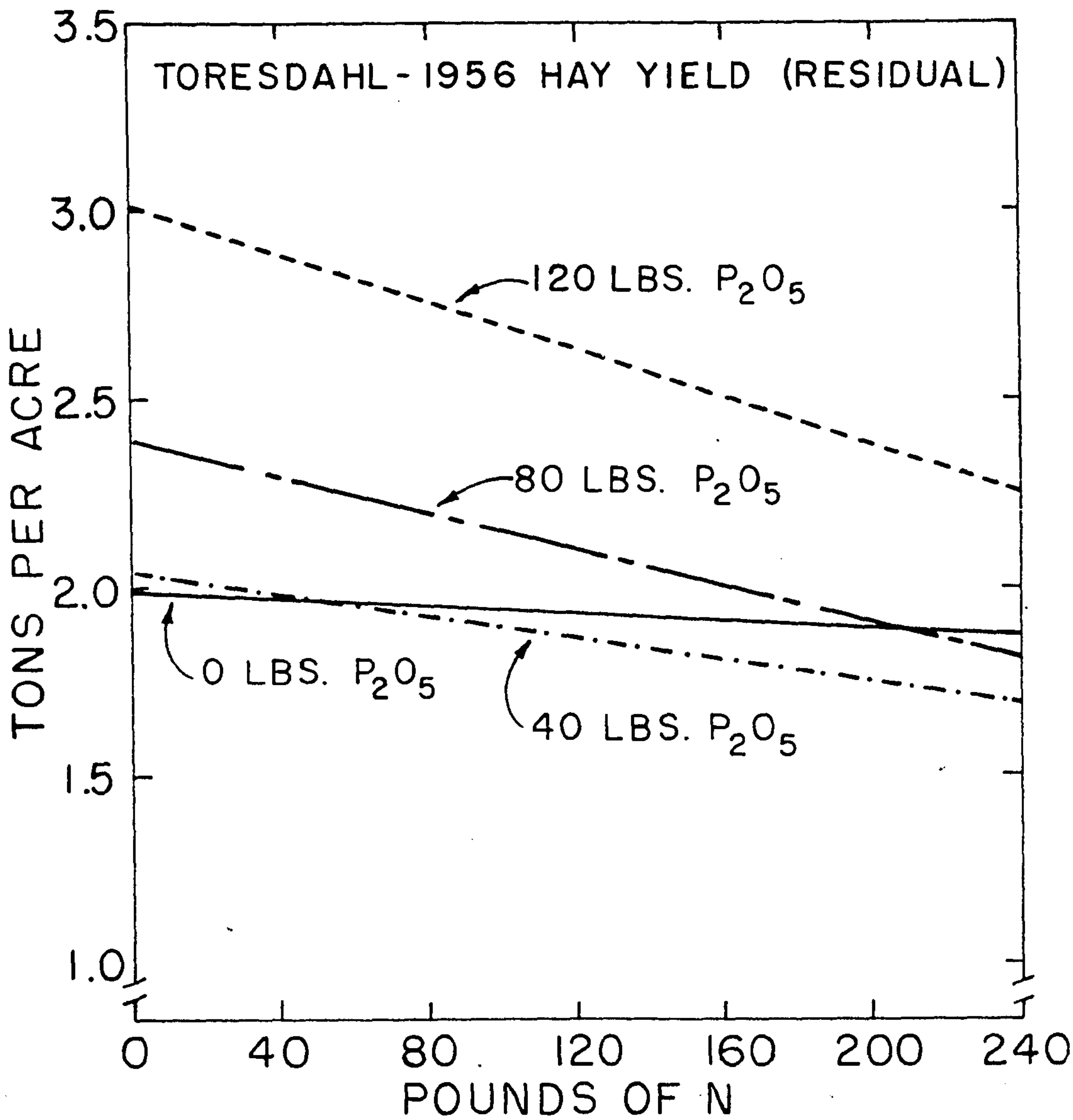


Figure 57. Predicted yield of alfalfa hay as a function of residual N rates at different levels of residual P_2O_5 in the fourth year after application.

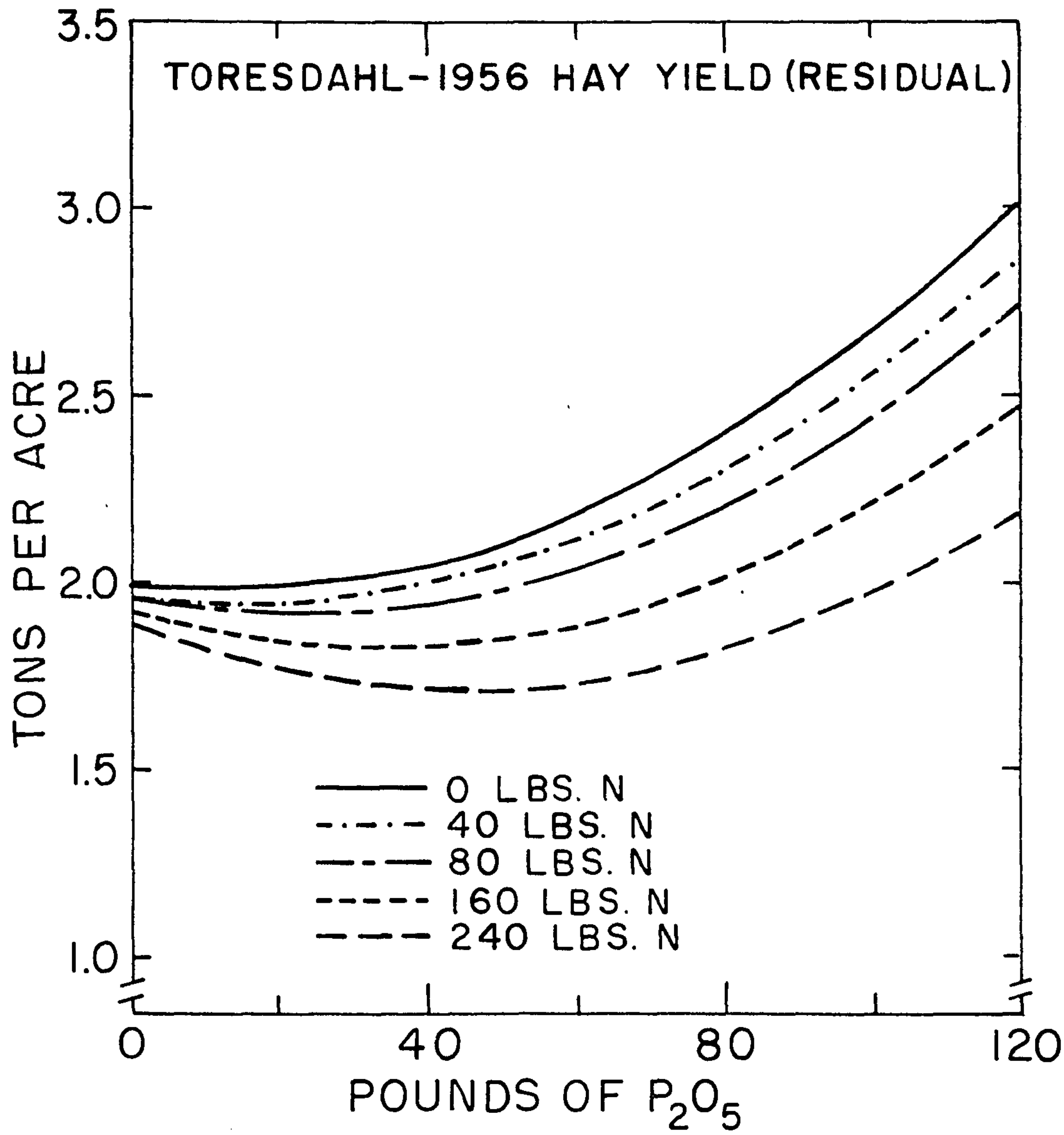


Figure 58. Predicted yield of alfalfa hay as a function of residual P_2O_5 at different levels of residual N in the fourth year after application.

The negative effect of nitrogen is the same as in 1955. Although stand counts were not made in 1956, a careful check did not indicate that stand level was any different than in 1955. Even if stand had been reduced some, the level should have been high enough in all plots to be adequate for high yields. Hence, it is felt that the negative effect is due to the initial effects occurring during the seeding year.

Although the NP interaction term appears to be small in the equation, it accounts for a half ton yield depression at the 120 pound P_2O_5 level as compared to the zero P_2O_5 level for the high rate of N. This effect is a continuation of 1955 trend and again is explained on the basis of effects occurring in the seeding year. With the onset of the dry 1955 season and the relatively dry 1956 season, the initial detrimental effect was never overcome.

The curves in Figure 58 reflect the effect of a diminishing supply of available P_2O_5. Had another higher rate or two been used, it is likely that a sigmoid effect would have again occurred.

Another contrast between the 1955 and 1956 hay crops is the difference in yields. In 1955, the maximum observed yield, averaged over all of the 120 pound P_2O_5 plots, was 4.1 tons while in 1956, the observed maximum was only 3.1 tons.

Predicted maximum yield increases were 2.76 and 1.03 tons, respectively, for 1955 and 1956. Reduced supplies of available phosphorus probably account for most of this difference.

The significant effect for K_2O was not expected in 1956 since a significant response to this nutrient had not occurred on any of the previous crops. Furthermore, the effect was negative which is difficult to explain. Chemical analyses showed that neither K_2O nor any other treatment had any effect on potassium concentration in the plant. With the exception of the significant effect of P_2O_5 on percent potassium in the 1955 hay, the 1956 results agree with those obtained in 1955. For these reasons, it is felt that the K_2O effect in 1956 is a chance occurrence.

Yield surface, isoquants, isoclines, and economic optima

The separate yield curves shown in Figures 57 and 58 combine to form the yield surface in Figure 59. Increasing returns to scale for P_2O_5 are plainly evident as is the depressing effect of N. The P_2O_5 effect, however, is evidence of appreciable carryover even into the fourth year after application.

Isoquants and isoclines are similar to those obtained in 1955 and are, therefore, not shown. A given yield can be

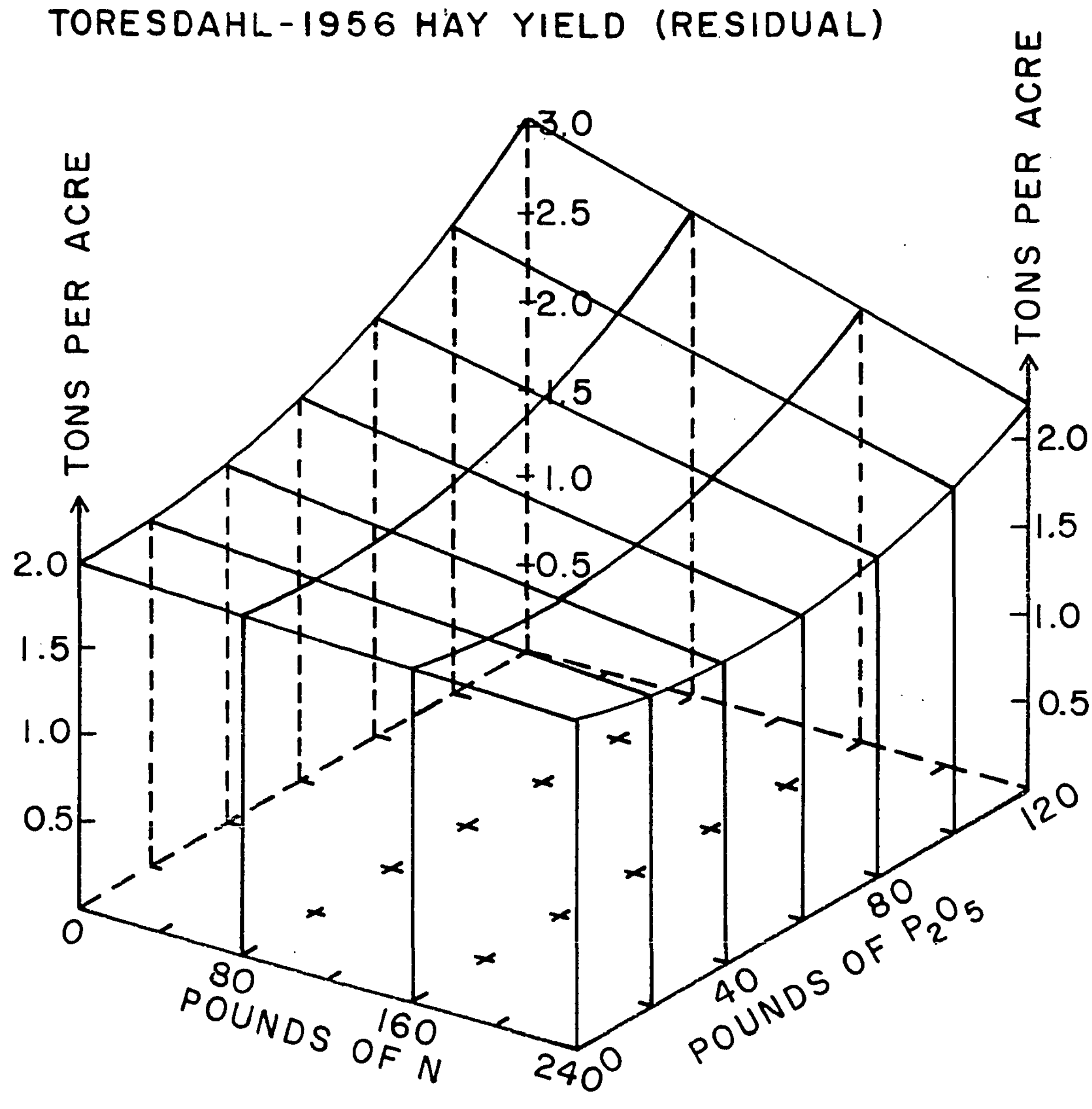


Figure 59. Predicted alfalfa yield surface as a function of residual N and P_2O_5 rates in the fourth year after application.

obtained only by increasing or decreasing N and P_2O_5 rates simultaneously. Hence, there is no substitution of nutrients.

Economic optima are not applicable for the same reasons given earlier. If the optimum was of value, it would occur at the zero N and 120 pound P_2O_5 level because of the negative N effect and increasing returns to scale for P_2O_5.

Percent phosphorus and phosphorus yield

An analysis of variance of percent phosphorus showed no significant effect of any nutrient on the concentration of this nutrient in the plants. Hence, it is to be expected that the yield of phosphorus curves or surface will be a replica of the crop yield curves or surfaces. For this reason, these drawings are not shown.

Carryover comparisons

The three curves in Figure 60 summarize 1953 and 1954 carryover effects in relation to effects from P_2O_5 topdressed in 1956. While rainfall was below normal in 1956, it was well distributed in small showers. This distribution was the major reason for the effectiveness of the topdressed material.

The response to 1956 topdressing was also measured with a great deal more precision than the 1954 topdressing because

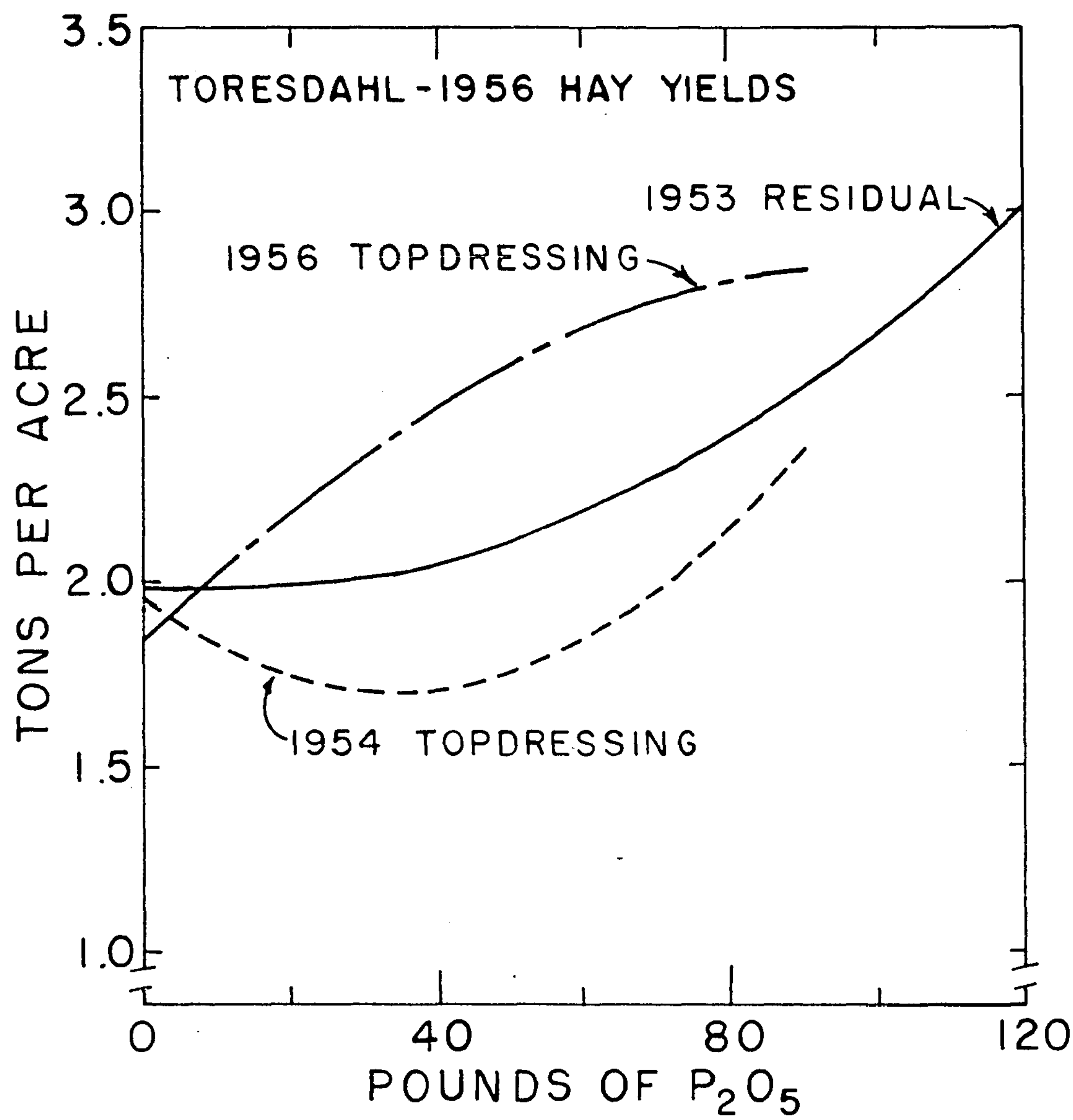


Figure 60. Effect of residual and topdressed P_2O_5 rates on the predicted yield of alfalfa hay.

it was possible to replicate the treatments 8 times as compared to 3 times for the 1954 applications. Those plots receiving N rates originally were split four ways each to achieve this goal. The analysis of variance is shown in Table 36.

Table 36. Analysis of variance of 1956 hay yields obtained from P_2O_5 topdressed in 1956.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	31	18.34		
Replication	7	12.18	1.74	29.0**
Treatment	3	4.82	1.61	26.8**
Linear	1		4.58	76.3**
Quadratic	1		0.22	3.7[c]
Cubic	1		0.02	0.3
Error	21	1.34	0.06	

[c]Significance level is explained in footnote of Table 4.

Although the quadratic effect was small, it was above that level of significance set previously as a criterion for inclusion in the regression. The regression equation is

$$\underset{\pm 0.0055}{Y = 0.019467P} - \underset{\pm\,0.000059}{0.000091P^2} + 1.839, \qquad (31)$$

with an R^2 of 99.8 percent. Observed and predicted yields are shown in Tables 73 and 74 of the Appendix.

The analysis of variance for the 1956 hay yields obtained from P_2O_5 topdressed in 1954 is shown in Table 37.

Table 37. Analysis of variance of 1956 hay obtained from P_2O_5 topdressed in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	8.1342		
Replication	2	6.5382	3.2691	23.7**
Treatment	3	0.7687	0.2562	1.9
Linear	1		0.2535	1.8
Quadratic	1		0.4485	3.3[d]
Cubic	1		0.0667	0.5
Error	6	0.8273	0.1379	

[d]Significance level is explained in footnote of Table 4.

From the analysis of variance it is evident that there is relatively no effect from the P_2O_5 topdressed in 1954. However, the quadratic effect is within the range of significance previously accepted, and since it was desirable to characterize whatever response was present, a multiple regression was fitted to the data. The yield equation is

$$Y = \underset{\pm 0.025}{-0.015P} + \underset{\pm\, 0.00027}{0.000215P^2} + 1.957, \qquad (32)$$

with an R^2 of 91.3 percent. Initial and predicted yields are shown in Tables 71 and 72 of the Appendix.

Comparisons of P_2O_5 rates topdressed in 1956 with residual treatments producing equal predicted yields are shown in Table 38. These comparisons were made directly from the yield curves.

Table 38. Residual P_2O_5 equivalents and percent carryover in terms of P_2O_5 topdressed in 1956.

Pounds of P_2O_5/A topdressed in 1956	1953 residual rate equal to 1956 topdressing	Percent carryover
30	76.0	39.5
60	101.5	59.1
90	111.5	80.7

Only the 30 pound 1956 P_2O_5 rate had an equivalent 1954 rate within the range of the predicted yields. This equivalent rate was 90 pounds, making the carryover percentage 33.3 percent.

The carryover percentages in Table 38 are large in view of the 3 year interval which had elapsed since the 1953 material was applied. The mean carryover percentage is 59.8 percent.

This large carryover must not be taken literally. For one thing, comparisons are being made between topdressed and disked-in material. One would generally expect a lower response to the former because of positional placement. While such an effect may not be true for wet years, it is likely to be true in normal or below normal rainfall years such as were experienced at this experimental site.

Another factor to be considered is that P_2O_5 absorption was limited during the corn and oat years on the zero N plots which are the basis for the 1953 residual curve. Only the 1955 residual hay crop removed appreciable amounts of P_2O_5 from these plots. Carryover, therefore, would tend to be larger than on those plots which had received both N and P_2O_5 applications initially. This effect is evident when one refers back to Figure 58 which shows lower residual response to P_2O_5 as the N rate increases. Since NP topdressings were not made, there is no way of actually determining whether carryover would be greatly different than on the zero N plots. Even if the computed carryover was reduced 15 to 20 percent for the NP plots, the relative carryover for the time period involved would still be quite large.

Nutrient composition comparisons

The various nutrient-yield relationships for the 1956 topdressed hay are shown in Figure 61 while Figure 62 shows the nutrient-yield relationships of the 1956 hay for P_2O_5 topdressed in 1954. In both cases, every nutrient measure-

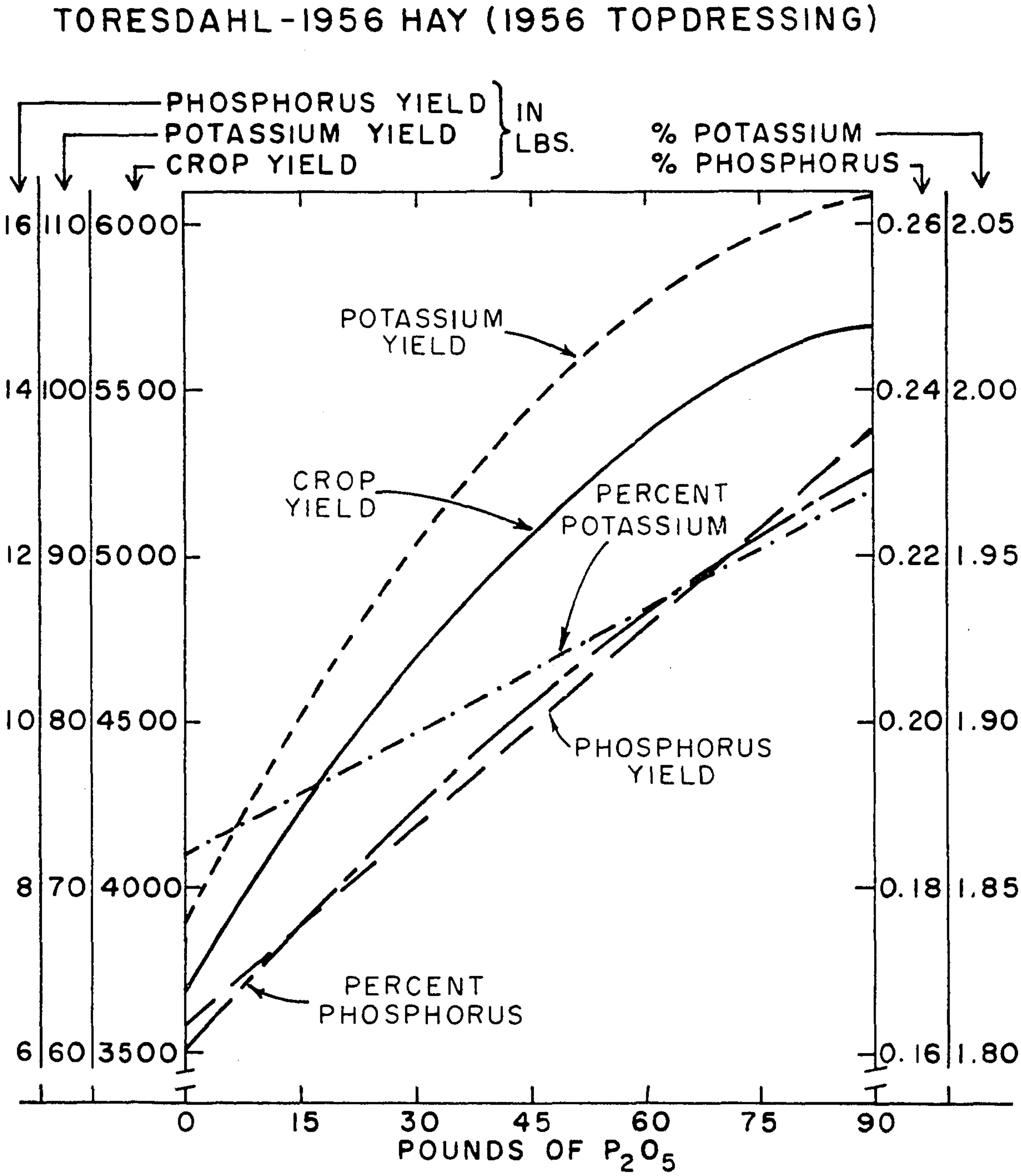


Figure 61. Yield and nutrient relationships for 1956 alfalfa hay as a function of P_2O_5 rates topdressed in 1956.

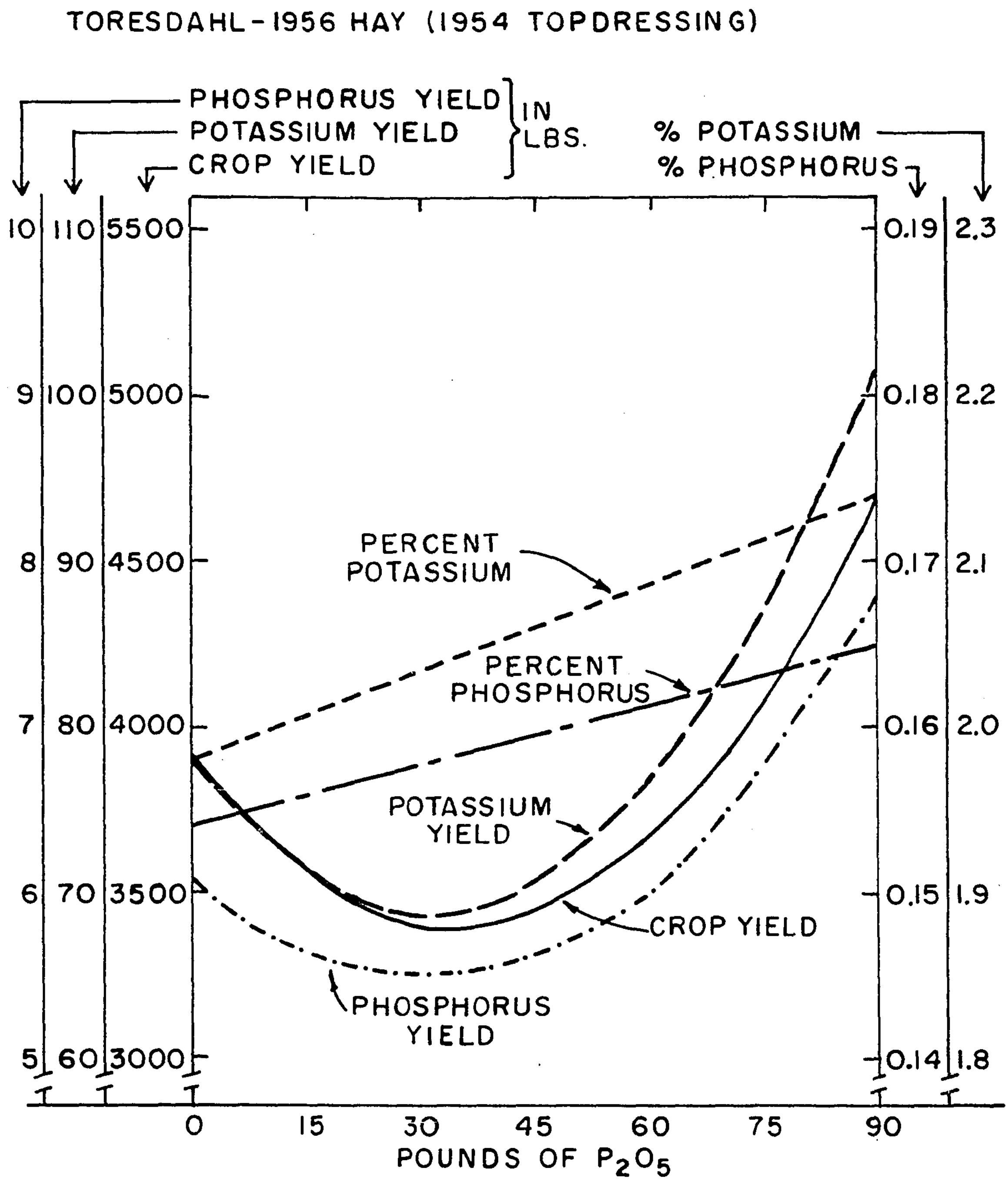


Figure 62. Yield and nutrient relationships for 1956 alfalfa hay as a function of residual P_2O_5 topdressed in 1954.

ment corresponds to its respective yield curve in either a linear or a quadratic fashion, and thus is in general agreement with the results obtained for the 1955 residual nutrient-yield relationships. The consistency of these three comparisons supports the view that the nutrient percentage results for the 1955 hay obtained from P_2O_5 topdressed in 1954 (Figure 54) are due to random variation and not to treatment.

The original plot data, predicted yields, nutrient composition data, analyses of variance and regression equations for all topdressed material are shown in Tables 71 through 82 of the Appendix.

Economic optima

An evaluation of the 1956 topdressed P_2O_5 is possible since no carryover effects are involved. The derivative of yield with respect to P for equation 31 is set equal to 0.005 which is the price ratio between the cost of P_2O_5 at $0.10 per pound and alfalfa at $20.00 per ton. These calculations lead to an optimum rate of 79.5 pounds of P_2O_5 per acre which results in an optimum yield of 2.81 tons of hay. At the above prices, the net profit (excluding additional harvesting costs, storage, etc.) is $11.45 per acre.

If the above derivative is set equal to zero, the predicted maximum yield is calculated to be 2.88 tons per acre and is attained with 107 pounds of P_2O_5 per acre. This yield and rate are reasonable but are somewhat outside of the experimental observations.

Summary - 1956 hay

1. P_2O_5 effects from the original application made in 1953 were still significant and resulted in predicted yield increases of up to 1 ton of hay per acre.
2. The response was quadratic in nature and concave to the abcissa indicating fixation, previous utilization and increasing returns to scale.
3. In terms of 1956 topdressed rates of 30 to 90 pounds of P_2O_5 per acre, 1953 residual carryover ranged from 39.5 to 80.7 percent.
4. The carryover from P_2O_5 topdressed in 1954 was no longer significant in 1956. Fixation and utilization by the 1955 crop are probably responsible for this result.
5. Nutrient composition and nutrient yield corresponded with crop yield for both the 1954 and 1956 topdressings of P_2O_5. Residual P_2O_5 resulted in significant increases in hay yield but had no effect on percent phosphorus or potassium in the crop. A possible reason for this behavior is that the yield increase, while significant, was relatively small. This fact, combined with considerable

soil variability, reduced the precision of statistical measurement of possible effects.

6. Economic analyses were deferred until a complete summary could be made covering the entire rotation.

Economic Evaluation

Fertilizer effects have usually been evaluated for a single crop on the basis of first year responses. When fertilizer effects have to be evaluated for a number of crops grown over a period of years, risk and uncertainty, as well as carryover effects, become important in the evaluation. The consideration of these items complicates the evaluation somewhat, but at the same time, increases the number of approaches that can be used for this purpose.

Two evaluation methods, of the many that might be used, are explored in this dissertation with the idea of focusing attention on the problems involved in economic evaluation of single fertilizer applications intended for a sequence of crops. Both approaches involve the use of a common denominator for the non-additive crops and some system of discounting to take care of risk, uncertainty and time *per se*. The first approach involves conversion of the crop yields to total digestible nutrients[1] and the second, to a total value

[1]TDN will be used in all subsequent references to total digestible nutrients.

product. A year by year economic analysis of the experimental results was not possible because comparable NP combinations were not applied in the successive years.

Total digestible nutrient approach

The percentage of TDN for corn, oats and alfalfa, listed by Morrison (41), was 80.1, 70.1 and 50.3, respectively. Using these percentages, each plot yield for each year was converted to pounds of TDN per acre.

The mean TDN yields for the various treatments together with the mean yields for the main effects are shown in Table 39 and illustrated in Figures 63 and 64. Individual plot yields are shown in Table 83 of the Appendix.

The mean TDN yields for the main effects in Table 39 indicate significant N and P_2O_5 effects but doubtful K_2O effects. The specific effect and the nature of this effect were determined from the analysis of variance shown in Table 40.

The NP interaction sum of squares was partitioned but the N linear x P linear component was not significant. Hence, only the significant variables shown in Table 40 were included in the regression equation. The quadratic form of the N variable was used in the equation because of the nature

Table 39. TDN yields in thousands of pounds per acre for a 4 year rotation of corn, oats, meadow and meadow.

Pounds of P_2O_5/A	Pounds of K_2O/A	Pounds of N/A				
		0	40	80	160	240
0	0	9.7	7.1	9.0	7.4	7.4
0	40	7.3	7.3	7.6	8.3	7.9
0	80	6.7	7.9	7.4	11.2	7.5
40	0	7.3	7.6	8.6	8.3	7.4
40	40	7.4	8.7	8.2	8.8	9.5
40	80	8.3	7.9	9.3	12.3	8.2
80	0	9.4	9.8	13.4	9.8	13.9
80	40	7.9	9.3	9.6	12.2	11.3
80	80	9.4	11.5	10.9	10.2	10.9
120	0	10.5	12.0	13.7	11.9	10.9
120	40	10.8	11.8	11.6	10.7	12.2
120	80	12.1	12.0	11.9	12.5	10.6

Mean yield for main effects

Pounds of N/A	Mean yield (24 obs.)	Pounds of P_2O_5/A	Mean yield (30 obs.)	Pounds of K_2O/A	Mean yield (40 obs.)
0	8.9	0	8.0	0	9.7
40	9.4	40	8.5	40	9.4
80	10.1	80	10.6	80	9.9
160	10.3	120	11.7		
240	9.8				

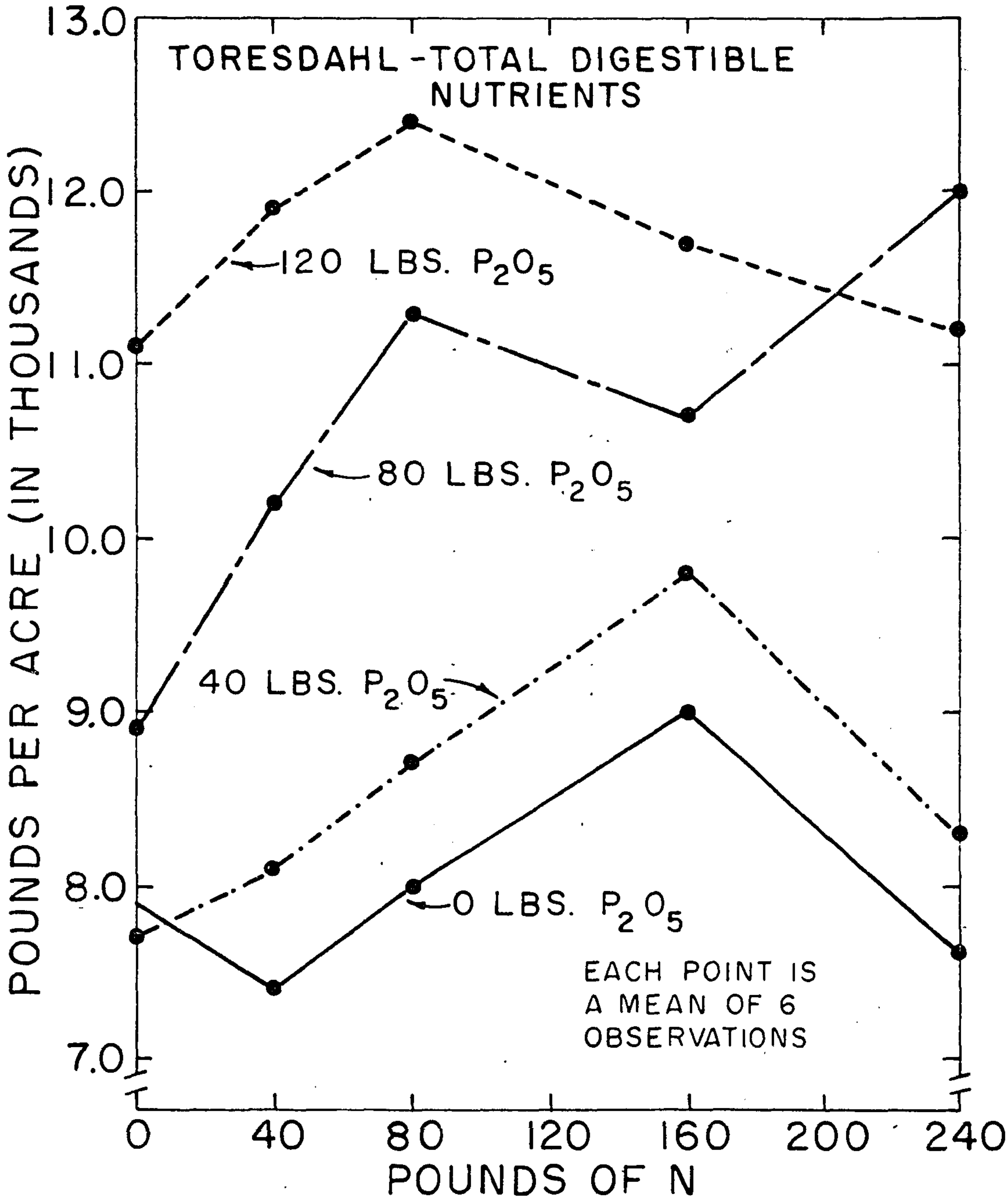


Figure 63. Average yield of TDN obtained from corn, oats, and alfalfa over a four year period as a function of N rates at different levels of P_2O_5 applied once in the rotation to corn.

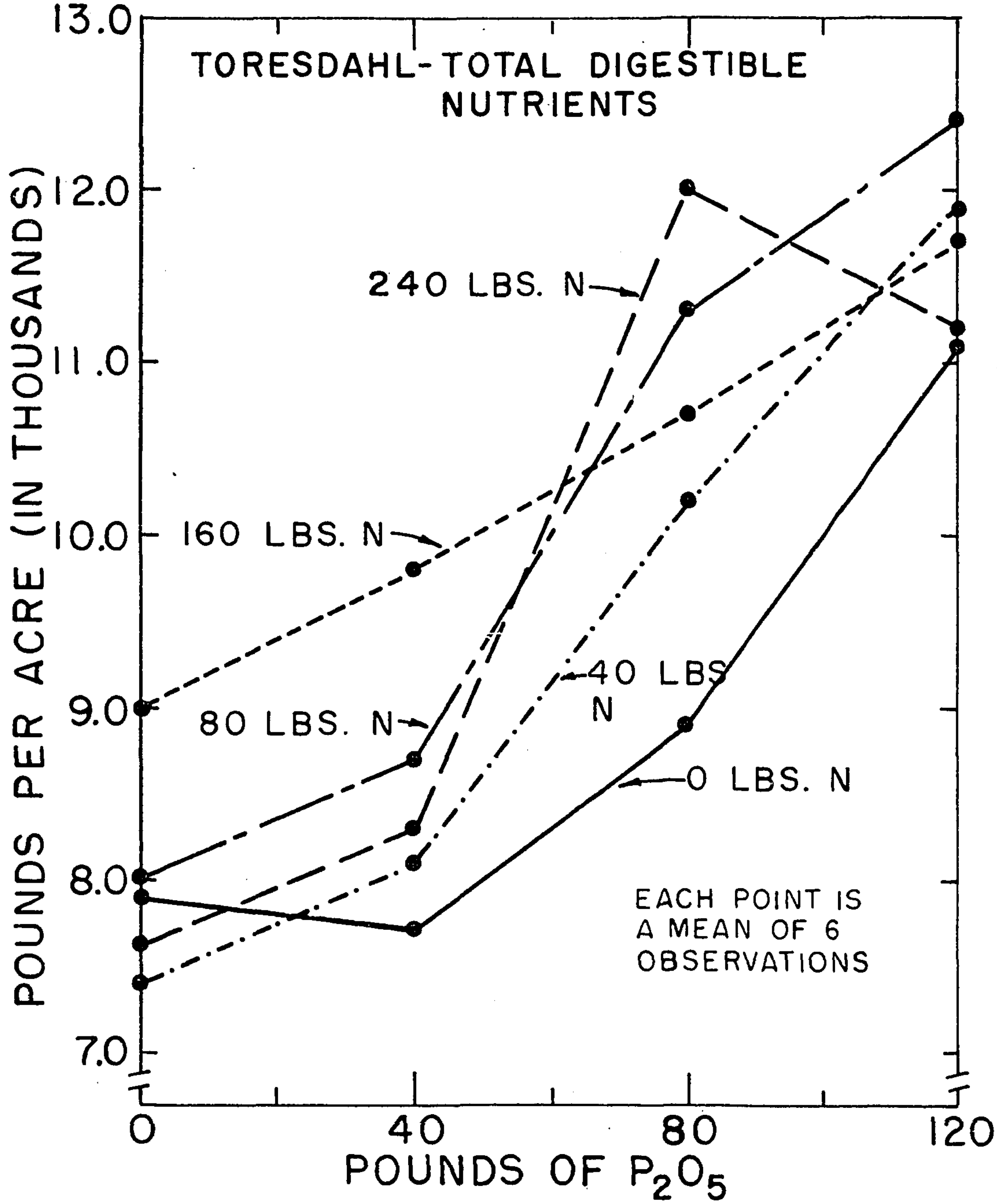


Figure 64. Average yield of TDN obtained from corn, oats and alfalfa over a four year period as a function of P_2O_5 rates at different levels of N applied once in the rotation to corn.

Table 40. Analysis of variance for TDN yields

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	119	811.21		
Replication	1	213.34	213.34	71.59**
Treatments	59	421.97	7.15	2.40**
Nitrogen (N)	4	30.27	7.57	2.54**
Linear	1		17.71	5.94*
Quadratic	1		10.50	3.52[c]
Cubic	1		2.01	0.67
Quartic	1		0.05	0.02
Phosphorus (P)	3	274.91	91.64	30.75**
Linear	1		262.68	88.15**
Quadratic	1		1.83	0.61
Cubic	1		10.40	3.49[c]
Potassium (K)	2	5.60	2.80	0.94
NP	12	34.32	2.86	0.96
NK	8	40.22	5.03	1.69
PK	6	13.43	2.24	0.75
NPK	24	23.22	0.97	0.33
Error	59	175.90	2.98	

[c]Significance level is explained in footnote of Table 4.

of the mean TDN lines in Figure 63. The resulting equation is

$$Y_{TDN} = \underset{\pm 0.008}{0.020N} - \underset{\pm 0.000032}{0.000065N^2} - \underset{\pm 0.036}{0.028P} + \underset{\pm 0.0008}{0.001311P^2} - \underset{\pm 0.000004}{0.000007P^3} + 7.103, \quad (33)$$

and has an R^2 of 72.2 percent. Predicted TDN yields are shown in Table 41 and illustrated in Figures 65 and 66.

Table 41. Predicted TDN yields in thousand of pounds per acre for crops grown in a 4 year rotation.

Pounds of P_2O_5/A	Pounds of N/A						
	0	40	80	120	160	200	240
0	7.1	7.8	8.3	8.6	8.6	8.5	8.2
40	7.6	8.3	8.8	9.1	9.2	9.0	8.7
80	9.7	10.4	10.9	11.1	11.2	11.1	10.7
120	10.5	11.2	11.7	12.0	12.1	11.9	11.6

The quadratic response to N is reasonable in light of the large quadratic response of corn to N in the first year. The sigmoid effect of N on oat yield and the negative effect of N on hay yields were not large enough to offset the quadratic N effect on corn, but the negative N effect on hay yields causes TDN yields to decline beyond the 160 pound N rate.

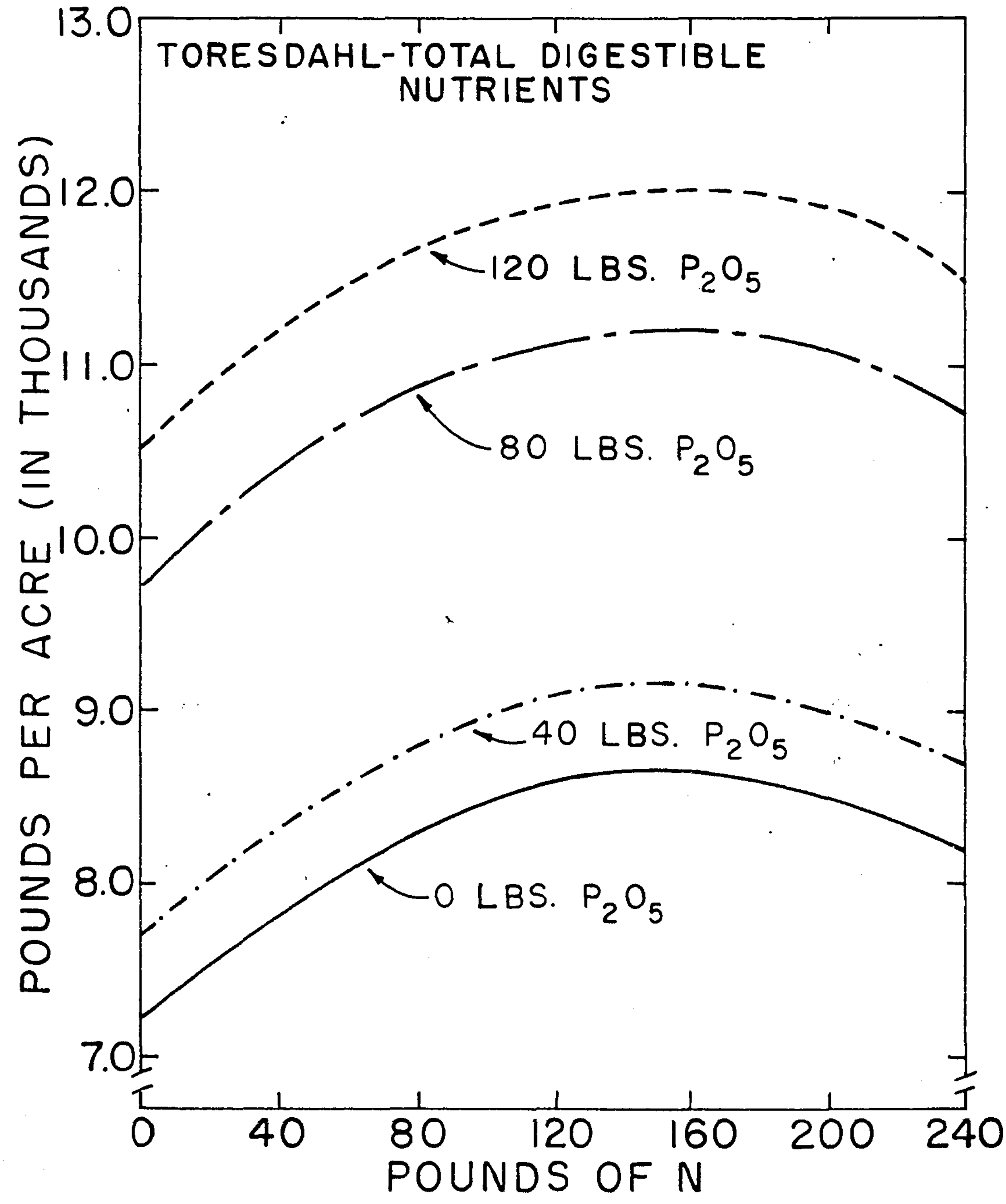


Figure 65. Yield of TDN predicted for corn, oats and alfalfa over a four year period as a function of N rates at different levels of P_2O_5 applied once in the rotation to corn.

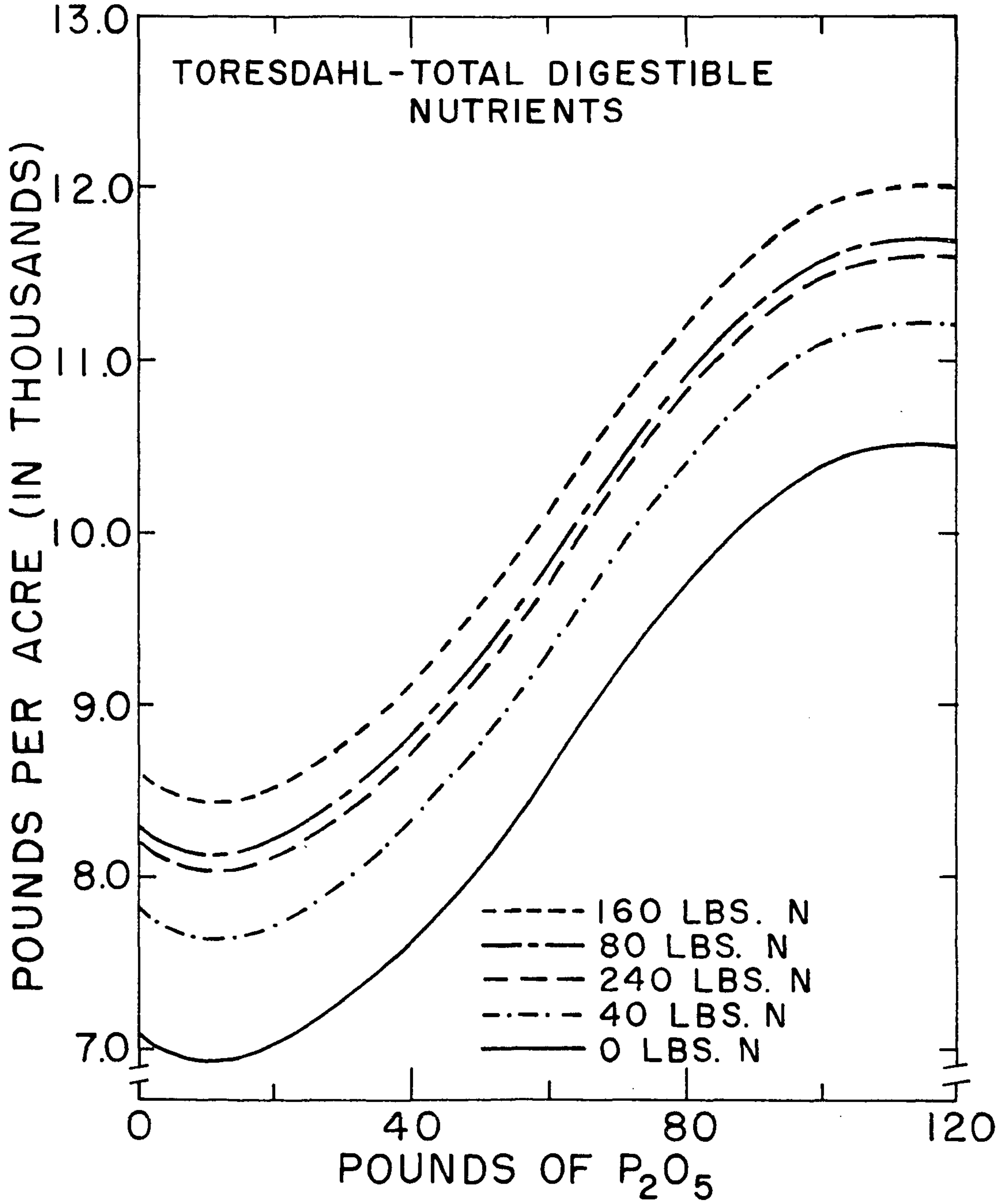


Figure 66. Yield of TDN predicted for corn, oats, and alfalfa over a four year period as a function of P_2O_5 rates at different levels of N applied once in the rotation to corn.

The cubic effect of P_2O_5 on TDN response shows the effect of the hay crops. However, it must be remembered that most of the sum of squares for P_2O_5 were linear which indicates that the substantial quadratic response of corn and oats to P_2O_5 almost counterbalanced the cubic effect of the hay crops.

It is of interest to note that the corn, oats, alfalfa and alfalfa crops contributed 47.5, 7.0, 24.5 and 21.0 percent respectively to the total TDN. These figures also help to explain why the TDN response curves are quadratic for N and cubic for P_2O_5.

Figure 67 shows the combined yield curves expressed as a surface. The curvature of the lines for N rates remains the same throughout the surface because there is no NP interaction.

The isoquant equation is

$$N = \frac{-0.020 + \sqrt{-0.00000728P + 0.00000034P^2 - 0.0000000018P^3 + 0.002247 - 0.000260Y}}{-0.000130}, \quad (34)$$

and is illustrated in the isoquant map shown in Figure 68.

An unusual feature about this map is that increasing, constant and decreasing returns to scale are all illustrated.

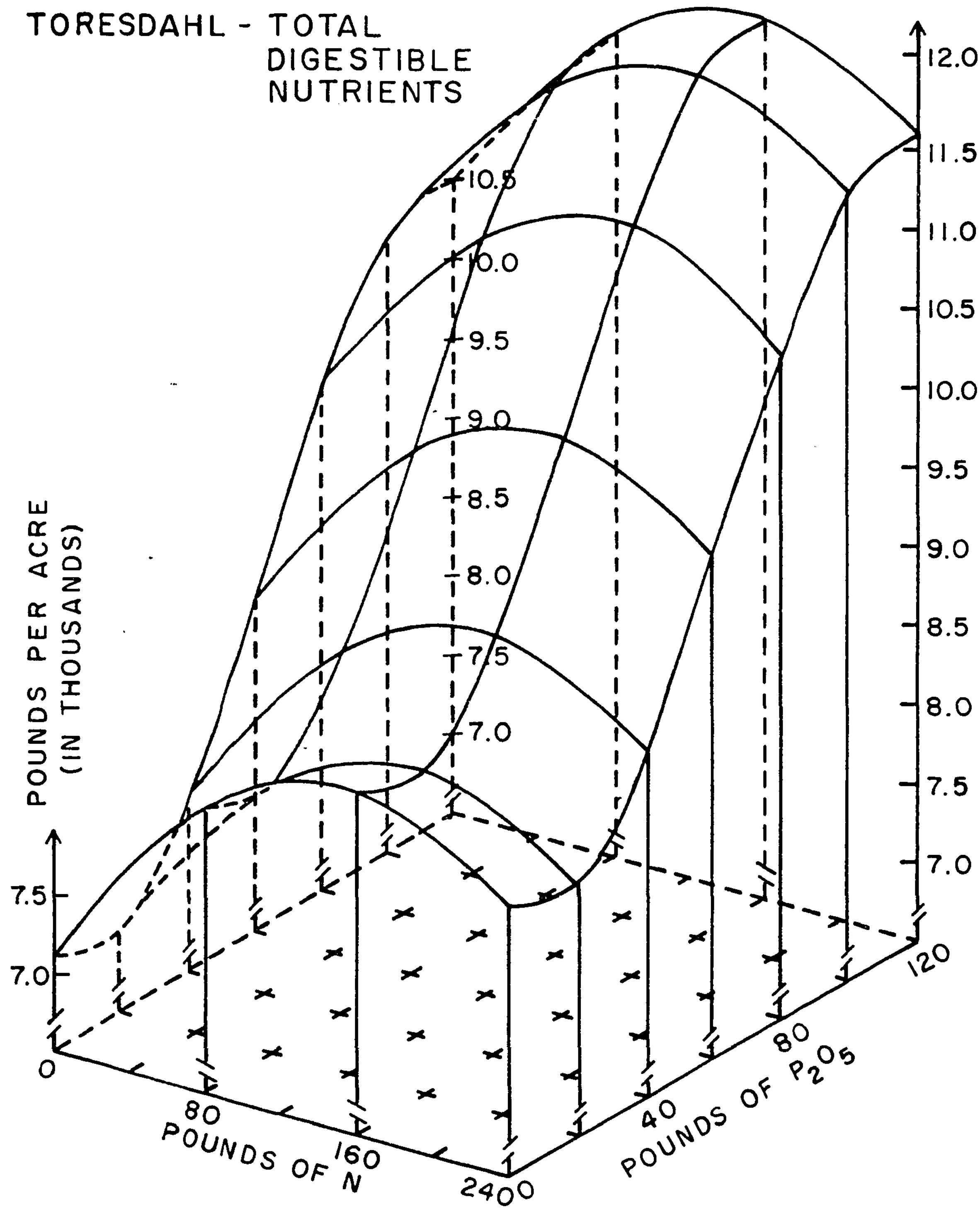


Figure 67. TDN yield surface predicted for the combined crops of corn, oats and alfalfa grown over a four year period as a function of N and P_2O_5 rates applied once in the rotation to corn.

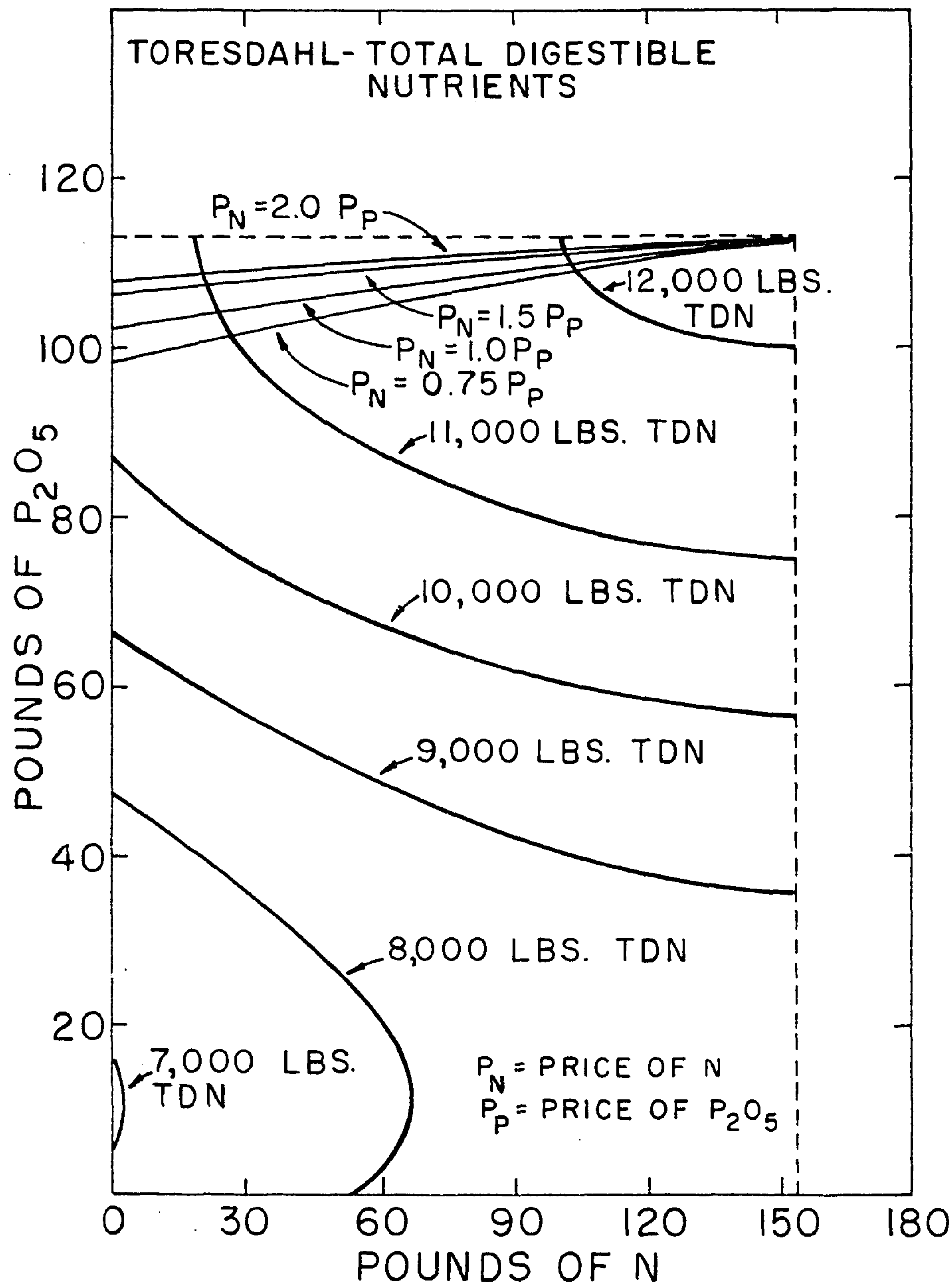


Figure 68. TDN yield isoquants, isoclines and dashed ridgelines obtained for corn, oats and alfalfa over a four year period as a function of N and P_2O_5 rates applied to corn.

Furthermore, both the optimum and maximum yield points fall within the nutrient application rates of the experiment.

The isocline equation is

$$N = \frac{-\alpha(0.002622P) + \alpha(0.000021P^2) + \alpha(0.028) + 0.020}{0.000130}, \quad (35)$$

where α designates the P_N/P_P ratio.

The closeness of the isoclines and the slopes of the isoquants at the points of intersection result in a very small range of nutrient substitution for the price ratios involved. For example, 11,000 pounds of TDN could be produced with a change of approximately 6 pounds of N and 9 pounds of P_2O_5 for the entire price range from $P_N = 0.75P_P$ through $P_N = 2.0\ P_P$.

The ridgelines are straight vertical and horizontal lines because there is no interaction term in the equation. If N were free, one would always operate at the maximum N rate with the minimum amount of P_2O_5 needed for a given yield. The same would be true if P_2O_5 were the free nutrient except that the nutrient combinations would be reversed. If an interaction term had been present, the N and P combinations at zero rates of nutrient substitution would change over the full range of isoquant curves.

Economic optima and maxima

The above operations have been relatively straightforward since they deal with mathematical manipulation of physical data. To properly evaluate these data is now the problem. In order to do this one basic assumption had to be made.

This basic assumption is that the ratio of the crops produced in the rotation is optimum for the livestock program of the farm. In substance, this means that the rotation supplies the optimum combination of TDN, from each feed produced, in the proportions fed, for the livestock involved. In other words, the marginal productivities of a pound of TDN in corn, oats and hay are equal for the proportions fed within each kind and among kinds of livestock for the prices existing for the feed and livestock.

On the assumption that the above exists, the prevalent 1953 corn price of $1.50 per bushel, the 1954 oat price of $0.60 per bushel and the 1955-56 alfalfa hay price of $20.00 per ton were converted to price per pound of crop and these prices in turn divided by the respective percentages of TDN in the crop to arrive at $0.03344, $0.02675 and $0.01988 per pound, respectively, of TDN in corn, oats and hay.

The earlier assumption of equal marginal productivities allows the use of a mean value of the above TDN prices in computing the optimum N and P_2O_5 rates. Setting the derivatives of the yield equation with respect to N and P equal to the respective price ratios of N/TDN and P/TDN leads to the predicted optimum rates of 110.6 and 111.2 pounds of N and P_2O_5, respectively. These rates in turn result in an optimum yield of 11,992 pounds of TDN per acre. The yield increase over the check is 4,892 pounds per acre which has a value of $130.57 at the mean TDN value of $0.02669 per pound. Subtracting the fertilizer cost of $27.71 from $130.57 results in a net[1] of $102.86 per acre.

The maximum rates predicted by the TDN equation when the above derivatives are set equal to zero are 153.8 and 113.0 pounds of N and P_2O_5, respectively, which result in a maximum yield of 12,117 pounds of TDN per acre.

The above optimum fails to take into account risk, uncertainty and time factors associated with "once in the rotation" fertilization as practiced in this experiment. Since time is involved in the practice, prices might change and weather could be adverse. Time is also involved from the standpoint of alternative uses of capital. Furthermore,

[1]The word net will mean the difference between gross value of the product and the fertilizer cost whenever it is used in the above manner in the remainder of this dissertation.

interest is due on that money invested in fertilizer which does not return a profit the year it is applied. In light of these considerations, a discounting procedure was used which adjusted the separate TDN prices in proportion to the discounted contribution made by each crop to the total TDN.

Discount rates of 0, 5, 10 and 20, and 0, 10, 20 and 40 percent were chosen for use in discounting the TDN produced in the successive years of the rotation. For example, the TDN from corn would not be discounted. That from oats would be discounted 5 percent while that from the successive hay years would be discounted 10 and 20 percent, respectively. The resulting values were then converted to percent contribution being made to the total TDN and these percentages in turn multiplied by the respective TDN price for corn, oats and hay mentioned earlier. The end result was a weighted TDN price based on a discount of future yields. Total and discounted TDN yields and percent contribution to the total are shown in Table 42.

The net effect of this particular discounting procedure was to raise the weighted price per pound of TDN with increasing discount rates because corn, which was the highest in price per pound of TDN, accounted for a greater and greater share of the total TDN. Thus, the final price per 1000

Table 42. Total and discounted TDN yields in pounds per acre for corn, oats and hay grown in a four year rotation.

Crop	Zero discount rate		0,5,10 and 20% discount rate		0,10,20 and 40% discount rate	
	Pounds of TDN/A	Percent contribution	Pounds of TDN/A	Percent contribution	Pounds of TDN/A	Percent contribution
Corn	275,883	47.48	275,883	51.06	275,883	55.22
Oats	40,592	6.99	38,562	7.14	36,533	7.31
Hay	142,462	24.52	128,216	23.73	113,970	22.81
Hay	122,108	21.01	97,686	18.07	73,265	14.66
Total	581,045	100.00	540,347	100.00	499,651	100.00

pounds of TDN for the undiscounted, low discount and high discount rates was \$26.80, \$27.29 and \$27.87, respectively.

Optimum N and P_2O_5 rates were obtained by setting the derivatives of the TDN yield equation with respect to N and P equal to the respective price ratios of N/TDN and P/TDN. Table 43 shows the results of these calculations.

Table 43. Economic optima for TDN yields

Criterion	Pounds of N/A	Pounds of P_2O_5/A	Pounds of TDN/A
Not discounted	107.9	111.3	11,977
0,5,10 and 20 percent discount	111.6	111.3	11,998
0,10,20 and 40 percent discount	112.4	111.4	12,003

The results of Table 43 are reasonable although contrary to what might have been expected for the discounting procedure to begin with. As prices go up, the optimum rates, as well as yield, go up.

These results indicate that the use of a discounting procedure which affects prices but not the yield equation can produce optimum rates which either increase or decrease depending on the direction in which the price is moving.

Total dollar value approach

In view of the above relationships, it appeared that discounting might best be done on the regression equations which characterized the yields for the successive years and then to add the successive discounted equations together for the final summary equation as suggested by Heady *et al.* (22). However, because of the kind of terms used in the separate equations, such a procedure would have resulted in final derivatives which would have been time consuming to solve.

In the interest of conserving time, it was decided to convert mean treatment yields into dollars and then to compute separate regressions on the discounted dollar yields. The discounting of yields was achieved by discounting the prices of the crops for the successive years with the same rates used previously in the TDN approach and multiplying each mean treatment yield by the discounted price. In this way, the non-additive crops were given a common denominator, and with each successively higher discount series, the overall experiment produced a lower value product.

A mean yield for multiplication purposes was obtained by averaging a given treatment over both replications at all levels of K_2O since the latter had no effect on yield except in the last year, in which case it was considered a chance occurrence.

An analysis of variance was made of the dollar values for each discount series to determine the magnitude of the separate effects. Because of the method used to arrive at total dollar values for each treatment, there was no replication and hence no error term. Levels of significance, therefore, could not be obtained. Table 44 illustrates the above for the undiscounted series.

From Table 44, it is evident that the N effect is both linear and quadratic while the P effect is primarily linear. The NP sum of squares accounted for about 8.5 percent of the total treatment variance, and so was also included in the regression equation. These variables were also included in the regression equations for the discounted series since the nature of the main effects and NP effects were similar to

Table 44. Relative magnitude of variance for undiscounted total dollar value of corn, oats and two hay crops in a rotation.

Source of variation	Degrees of freedom	Sum of squares	Mean square
Treatment	19	33,511	1,764
Nitrogen(N)	4	7,076	1,769
Linear	1		5,359
Quadratic	1		1,661
Cubic	1		42
Quartic	1		14
Phosphorus(P)	3	23,590	7,863
Linear	1		22,801
Quadratic	1		0
Cubic	1		789
NP	12	2,845	237

those of the undiscounted value product. The separate regression equations are

Undiscounted - R^2 = 89.2 percent

$$Y = \underset{\pm 0.161}{0.616484N} - \underset{\pm 0.0006}{0.002017N^2} + \underset{\pm 0.12}{0.681876P} + \underset{\pm 0.0009}{0.000703NP} + 187.176, \quad (36)$$

0, 5, 10 and 20 percent discount - R^2 = 89.7 percent

$$Y = \underset{\pm 0.158}{0.655648N} - \underset{\pm 0.00059}{0.002131N^2} + \underset{\pm 0.1189}{0.648274P} + \underset{\pm 0.00088}{0.000830NP} + 172.450, \text{ and} \quad (37)$$

0, 10, 20 and 40 percent discount - R^2 = 90.2 percent

$$Y = \underset{\pm 0.153}{0.691458N} - \underset{\pm 0.00057}{0.002246N^2} + \underset{\pm 0.116}{0.599830P} + \underset{\pm 0.00086}{0.001046NP} + 158.643. \qquad (38)$$

The surface for equation 36 is shown in Figure 69 for comparison with the TDN surface in Figure 67. Except for the cubic effect for P_2O_5, the surface for dollar value is similar to the TDN surface. The dollar yields from which the separate regressions were computed are shown in Table 84 of the Appendix.

Economic optima

Since the P effect is linear and has a slope greater than the value of P_2O_5, the optimum P_2O_5 is 120 pounds in both the undiscounted and discounted cases. The remaining problem is to determine the optimum N rate.

Since the yield is now in dollars, the value of the yield is considered 1. Hence, the derivative of the yield equations with respect to N are set equal to the ratio of $\frac{\text{price of N}}{\text{price of dollars}} = \frac{0.15}{1.00}$. The computational results are shown in Table 45.

Again, in Table 45, N rates increase with increasing discount rates just as they did in the TDN approach. In this

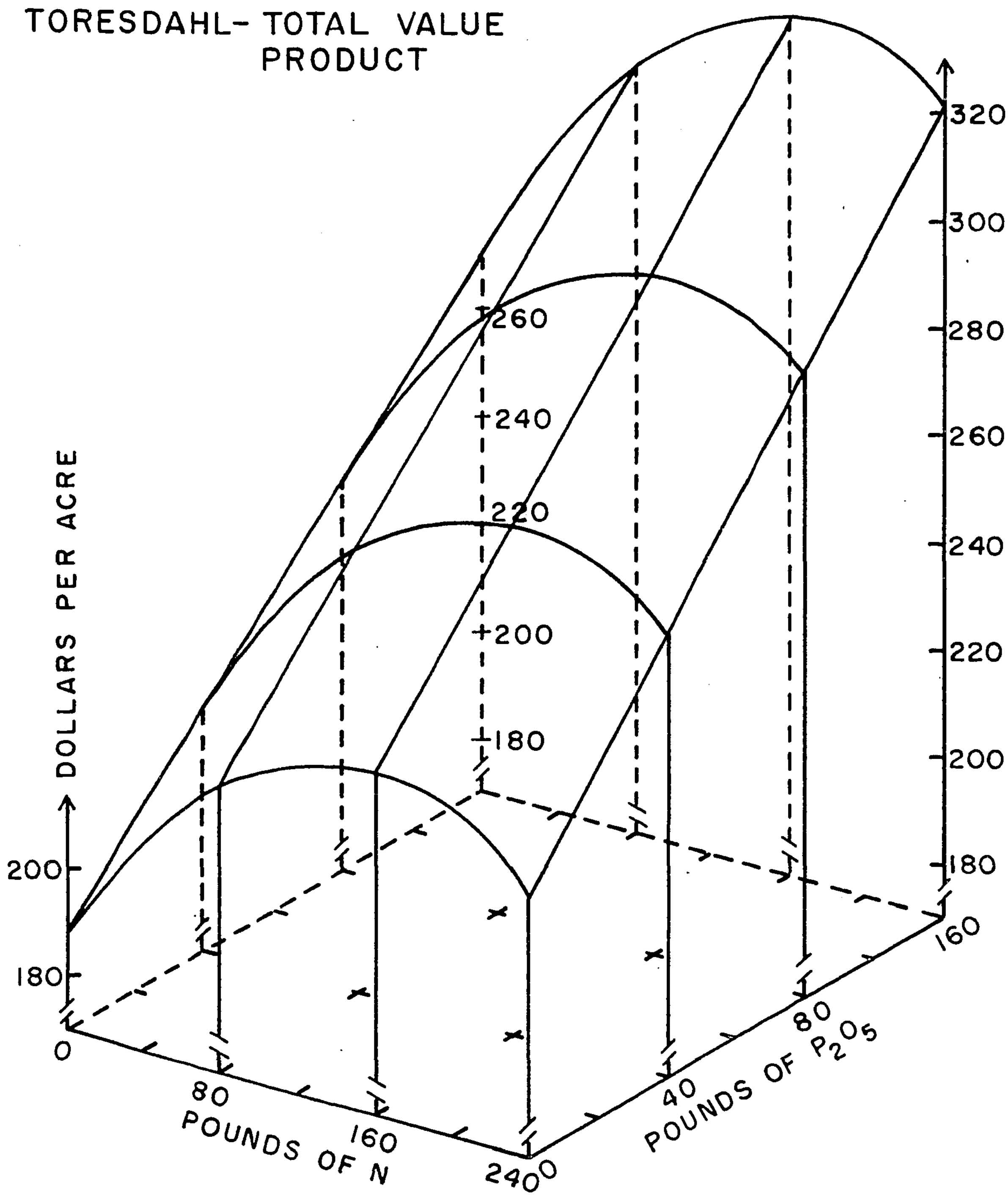


Figure 69. Undiscounted value surface predicted for the combined crops of corn, oats and alfalfa grown over a four year period as a function of N and P_2O_5 rates applied once in the rotation to corn.

Table 45. Economic optima for the total value product.

Criterion	Pounds N/A	Pounds P_2O_5/A	Dollar yield
Not discounted	136.6	120.0	327.10
0, 5, 10 and 20 percent discount	142.0	120.0	314.52
0, 10, 20 and 40 percent discount	148.5	120.0	302.41

case, however, the reason is different. The total value product in this approach decreased because of the discounted prices used to multiply the mean physical yields for the successive years. At the same time, the slopes of the curves resulting from the fitted equations decreased with the result that with constant fertilizer cost, the tangency point on the yield curve moved to higher and higher N rates for lower and lower dollar yields which occurred for the successively higher discount series.

Evaluation of methods

Neither of the above procedures resulted in decreasing optimum nutrient rates which would normally be expected for an increasing series of discount rates. The procedures presented, therefore, are primarily of academic interest and serve to point up some problems that may arise in trying to simplify discounting procedures for purposes of evaluating yield responses obtained over a rotation period.

As mentioned earlier, lack of time precluded further investigation into a discounting procedure which should have the desired effect of reducing optimum nutrient rates over time. This procedure would involve the multiplication of the coefficients of each yield equation by a factor which would be obtained by discounting a given crop price to a present value. For example, if a discount rate of 5 percent were used, the individual coefficients in the equation for corn would be multiplied by the ratio of the expected price/(expected price + 0.05),[1] those for oats with the ratio EP/ $(EP + 0.05)^2$, those for first year hay with the ratio EP/ $(EP + 0.05)^3$ and those for the second year hay with the ratio EP/$(EP + 0.05)^4$. Before or after the multiplication, the respective coefficients would have to be converted to a common denominator of pounds or tons and then added together to obtain the equation characterizing the cumulative discounted yields. Such a procedure should result in decreasing optimum rates and thus take into account time, risk and uncertainty factors for "once in the rotation" fertilization.

<u>Net returns</u>

Profits were reasonably large for the optimum nutrient rates used in this experiment as indicated by the results in Table 46. The undiscounted TDN yields or dollar values were used in the computations since they were not biased by any discounting techniques.

The results in Table 46 are very comparable as might have been expected since discounting is not involved and since

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[1]Expected price will be designated by EP in future mention of these discounting ratios.

Table 46. Costs and returns in dollars per acre at optimum nutrient rates for undiscounted crop yields obtained over a four year period.

Criterion	TDN approach	Dollar value approach
Dollar value of yield increases	130.61	139.92
Total fertilizer cost	27.32	32.49
Average return for dollars invested	4.78	4.31
Net return	103.29	107.43

the constants employed in the analysis were derived from the same base prices. Beyond this observation, the point of greatest interest is the overall net return of more than $100 for the four year period. While proper discounting would reduce this figure somewhat, the potential returns should still be high enough to merit serious consideration and further study on other soils of the practice of "once in the rotation" fertilization with the proper nutrients.

<u>Limitations and future work</u>

This dissertation should not be closed without some comment on the limitations of this study and suggestions for future work.

The first limitation is that the experiment was not designed to fully measure all residual effects. Only single nutrient carryover effects were measured. Thus the effect of the NP interactions as they affected carryover could not be measured nor could complete economic evaluation of the individual years be made.

Legume roots were not dug from the soil to determine whether decreased root growth was the cause of decreased hay yields on the N and NP treated plots.

Rainfall and temperature measurements should have been made each year at the experimental site in order to better evaluate the various results. The weather data from 9 miles away may not be too appropriate for the specific experimental site.

Whole corn plant samples could have been collected at specific times to determine nutrient status relative to corn grain. For the oat crop, separate analyses of the oat grain and straw may also have been helpful in evaluating nutrient-yield relationships.

From the experience gained in the conduction of this experiment, it is felt that the following items would be of importance at most experimental sites:

1. Determine available soil moisture to a depth of 5 feet at the time the experiment is set out.
2. Keep rainfall and temperature records.
3. Take one soil sample per replication for the soil testing laboratory for calibration purposes.

In view of the response to N on this soil which tested high in nitrogen, it is felt that a need exists to further calibrate the present nitrogen test used in the soil testing laboratory. Such calibration is of particular concern on major soil types which test alike in the surface but which vary considerably as far as depth of organic matter is concerned.

In light of the residual effects observed in this experiment, it seems desirable to explore more fully "once in the rotation" fertilization with N, P and K applied at the most logical time in the cropping sequence. More experiments should be initiated on various soil types to compare all of the P or K for the rotation applied to corn or to oats or split between the two. For continuous corn, basic plow-down P and/or K applications intended for a 4 to 6 year period could be compared to yearly applications. Such comparisons should be split for row fertilizer treatments.

SUMMARY

Large direct and residual responses were obtained over a four year period for corn, oats and alfalfa grown on a calcareous-variant Webster soil to which a single application of N and P_2O_5 combinations had been applied ahead of corn. The N carryover into the oat year, as measured by 15 and 30 pound N rates topdressed on the oats was 10.3 and 18.2 percent, respectively. However, the residual N had a negative linear effect on the following alfalfa yields. It is postulated that this effect may have been due to increased competition from the responding oat crop which resulted in reduced legume root systems.

On the corn and oat crops, the P_2O_5 applications were effective only in association with N. On the hay crops, however, significant yield responses were obtained for P_2O_5 alone. The P_2O_5 carryover, as measured by topdressed P_2O_5 in both 1954 and 1956, was high. In 1956, the carryover amounted to 39.5 and 59.1 percent, respectively, as measured by the 30 and 60 pound rates of topdressed P_2O_5.

Both residual N and P_2O_5 effects were characterized by sigmoid type yield curves. For N, the lower part of the curve is probably due to previous utilization, microbial tieup and leaching, while for P_2O_5, this effect could be due to possible fixation, previous utilization, and possible reduced root

systems.

Nitrogen and phosphorus percentage curves or surfaces for corn grain and oat dry matter generally showed dilution effects which resulted in decreasing nutrient percentages as dry matter or grain yields increased. The greatest dilution effect generally occurred when a response was obtained to one limiting element in the absence of another. When both limiting nutrients were added together in combinations leading to optimum yields, the dilution effects were reduced in varying degrees depending on the crop. On the hay crops, however, phosphorus and potassium percentages followed the crop yield trends without showing any dilution effects.

Nutrient yield curves and surfaces followed the trends of the crop yield curves and surfaces rather consistently in all cases.

A procedure for determining the best fitting equation for characterizing yield data without trial and error was investigated. The procedure involves the selection of significant variables by analysis of variance, subdividing significant treatment variable sum of squares into their linear, quadratic, etc., components and testing them for significance, accepting or rejecting significant components on the basis of agronomic logic, choosing the quadratic or root form of the variable on the basis of the shape of the plotted mean observed yield lines when significant quadratic effects occur and limiting the order of the interaction terms to the power to which the main variable was raised.

The use of the above procedure brought out the nature of response lines in the data which, at times, would have been difficult to determine from a plot of the mean observed yield lines alone. Significant linear N effects in the hay and TDN data and cubic, sigmoid effects for N on oats and P on hay were brought out very clearly by the use of this technique.

Returns over the four year period for "once in the rotation" fertilization were over $100 per acre or over $25 per acre per year on the average. Even though these results are discounted for time, risk and uncertainty, they are large enough to merit further consideration in more detailed experiments on this and other major soils in the state.

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APPENDIX

Table 47. Observed 1953 corn yields in bushels per acre. Upper figures, block I; lower figures, block II.

Pounds of P_2O_5/A	Pounds of K_2O/A	Pounds of N/A				
		0	40	80	160	240
0	0	98.4 80.4	90.3 83.9	115.2 76.4	109.0 56.3	106.9 81.4
0	40	73.8 82.0	99.0 32.8	117.3 90.6	116.7 70.5	107.1 91.5
0	80	74.2 76.1	97.3 87.6	94.1 89.5	118.1 82.3	118.6 76.1
40	0	85.5 60.1	98.5 86.0	107.8 109.6	109.0 107.9	117.4 108.4
40	40	95.5 79.1	110.2 106.3	114.3 110.9	73.8 110.7	125.4 119.0
40	80	82.8 69.2	104.6 95.0	110.6 110.9	120.5 119.5	124.4 119.7
80	0	78.8 83.3	102.1 97.1	125.0 117.5	120.6 118.6	121.5 131.3
80	40	70.1 61.5	115.7 103.9	110.0 114.1	121.8 122.5	129.0 117.6
80	80	85.0 75.4	126.4 105.9	118.6 118.1	127.3 120.4	125.1 119.3
120	0	76.9 74.1	99.2 95.4	128.7 118.1	106.0 116.4	115.4 104.9
120	40	81.2 72.7	100.1 105.0	121.9 120.8	126.0 122.0	125.2 121.4
120	80	78.2 80.5	109.8 85.9	104.5 126.0	128.8 115.4	99.8 121.1

Table 48. Soil test levels by depth for block I and II for various $N+P_2O_5+K_2O$ fertilizer treatments on a calcareous variant Webster soil in Wright Co.[a]

Sampling depth (inches)	0 + 0 + 0		40 + 80 + 80		80+ 80 + 80		160 + 80 + 80		240 + 80 + 80	
	I	II	I	II	I	II	I	II	I	II
					Initial Nitrate					
0-6	12	6	15	6	6	6	9	15	9	12
6-12	6	6	9	6	-	9	6	12	9	12
12-18	3	3	6	3	6	9	9	15	12	12
18-24	3	3	3	3	6	9	6	24	15	24
24-36	1	1	3	3	3	9	9	18	18	36
					Nitrate-Nitrified in 2 weeks					
0-6	183	150	177	174	162	183	138	144		
6-12	48	48	36	84	72	120	30	90		
12-18	12	6	15	6	9	6	12	27	b	
18-24	72	6	6	6	3	9	12	27		
24-36	3	3	6	3	3	3	3	3		
					Available P					
0-6	3.0	2.5	4.0	10.0	8.5	3.5	3.5	4.0	7.5	4.0
6-12	1.0	1.0	2.5	2.5	3.5	1.0	1.0	1.0	2.5	1.5
12-18	1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
18-24	2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.5
24-36	1.0	1.0	1.0	1.0	1.0	1.0	1.5	1.0	0.5	0.5

[a]Samples taken April 29, one year after fertilizer was applied (1953) and tested by the Iowa State College Soil Testing Laboratory.

[b]Samples misplaced in the soil testing laboratory. Found after one year, but after this time period, they would no longer nitrify.

Table 48. (continued)

Sampling depth (inches)	0 + 0 + 0		40 + 80 + 80		80 + 80 + 80		160 + 80 + 80		240 + 80 + 80	
	I	II	I	II	I	II	I	II	I	II
					Available K					
0-6	220	236	280	220	244	204	200	240	188	200
6-12	264	232	240	168	184	188	180	204	164	200
12-18	292	204	276	188	216	204	192	228	188	220
18-24	264	192	260	168	188	184	152	188	164	192
24-36	244	172	248	128	192	160	136	176	148	160
					pH					
0-6	7.8	7.9	7.5	7.8	7.3	7.8	7.7	7.8	7.5	7.9
6-12	7.6	7.9	7.4	7.7	7.1	7.7	7.9	7.9	7.3	7.8
12-18	7.6	8.1	7.4	7.5	7.2	8.0	8.1	8.1	7.3	8.0
18-24	7.7	8.2	7.5	7.8	7.7	8.2	8.2	8.1	7.6	8.1
24-36	7.8	8.3	7.9	8.2	8.1	8.3	8.3	8.4	8.2	8.3

Table 49. Yield, percent nitrogen, and nitrogen yield for 1953 corn grain.

Treatment[a]	Yield lbs./A	Percent N	N yield lbs./A
000	5510.4	1.23	67.8
000	4132.8	1.21	50.0
000	4502.4	1.30	58.5
010	3365.6	1.13	38.0
010	4636.8	1.19	55.2
020	4664.8	1.15	53.6
020	3925.6	1.20	47.1
020	3444.0	1.24	42.7
030	4306.4	1.26	54.3
030	4149.6	1.15	47.7
030	4071.2	1.17	47.6
100	5544.0	1.33	73.7
100	4636.8	1.51	70.0
100	4905.6	1.56	76.5
110	5516.0	1.28	70.6
110	4816.0	1.54	74.2
110	6171.2	1.33	82.1
120	5717.6	1.23	70.3
120	5437.6	1.28	69.6
120	7078.4	1.39	98.4
130	5605.6	1.21	67.8
130	6148.8	1.51	92.8
130	5880.0	1.23	72.3
130	4810.4	1.26	60.6
200	6451.2	1.63	105.2
200	4278.4	1.61	68.9
200	6568.8	1.42	93.3
200	5073.6	1.51	76.6

[a]Whenever treatment designations 000,010, etc. occur in the following tables, they refer to $N+P_2O_5+K_2O$. The actual pounds per acre are obtained by multiplying each figure by 40.

Table 49. (continued)

Treatment[c]	Yield lbs./A	Percent N	N yield lbs./A
210	6036.8	1.57	94.8
210	6193.6	1.51	93.5
210	6210.4	1.56	96.9
220	6160.0	1.36	83.8
220	6641.6	1.39	92.3
220	6613.6	1.48	97.9
230	7207.2	1.36	98.0
230	6826.4	1.58	107.9
230	5852.0	1.43	83.7
230	6613.6	1.35	89.3
400	6104.0	1.62	98.9
400	3152.8	1.69	53.3
400	4608.8	1.71	78.8
410	6199.2	1.67	103.5
410	6748.0	1.62	109.3
420	6860.0	1.65	113.2
420	7128.8	1.64	116.9
430	7056.0	1.55	109.4
430	7212.8	1.62	116.8
430	6832.0	1.51	103.2
600	4261.6	1.63	69.5
610	7022.4	1.59	111.7
610	6966.4	1.61	112.2
620	7224.0	1.65	119.2
620	6585.6	1.60	105.4
620	7005.6	1.57	110.0
620	6680.8	1.41	94.2
630	7011.2	1.65	115.7

Table 50. Yield, percent phosphorus and phosphorus yield for 1953 corn grain.

Treatment	Yield in lbs./A	Percent P	P yield in lbs./A
000	5510.4	0.238	13.1
000	4132.8	0.210	8.7
000	4502.4	0.208	9.4
000	4155.2	0.220	9.1
010	3365.6	0.213	7.1
020	4664.8	0.211	9.8
020	3925.6	0.220	8.6
020	3444.0	0.201	6.9
030	4306.4	0.228	9.8
030	4149.6	0.234	9.7
030	4071.2	0.227	9.2
030	4547.2	0.240	10.9
100	5544.0	0.164	9.1
100	4636.8	0.149	6.9
100	4905.6	0.162	7.9
110	5516.0	0.212	11.7
110	4816.0	0.229	11.0
110	6171.2	0.205	12.7
120	5717.6	0.241	13.8
120	5437.6	0.213	11.6
120	7078.4	0.233	16.5
130	5605.6	0.236	13.2
130	6148.8	0.226	13.9
130	5880.0	0.218	12.8
130	4810.4	0.204	9.8
200	6451.2	0.252	16.3
200	4278.4	0.153	6.5
200	6568.8	0.168	11.0
200	5073.6	0.144	7.3
210	6036.8	0.191	11.5
210	6193.6	0.219	13.6
210	6210.4	0.196	12.2

Table 50. (continued)

Treatment	Yield in lbs./A	Percent P	P yield in lbs./A
220	6160.0	0.219	13.5
220	6641.6	0.248	16.5
220	6613.6	0.201	13.3
230	7207.2	0.241	17.4
230	6826.4	0.230	15.7
230	5852.0	0.248	14.5
230	6613.6	0.224	14.8
400	6104.0	0.146	8.9
400	3152.8	0.140	4.4
400	4608.8	0.153	7.1
410	6199.2	0.182	11.3
410	6748.0	0.227	15.3
420	6860.0	0.262	18.0
420	7128.8	0.225	16.0
430	7056.0	0.237	16.7
430	7212.8	0.220	15.9
430	6832.0	0.215	14.7
600	4261.6	0.129	5.5
610	7022.4	0.211	14.8
610	6966.4	0.187	13.0
620	7224.0	0.253	18.3
620	6585.6	0.186	12.2
620	7005.6	0.189	13.2
620	6680.8	0.213	14.2
630	7011.2	0.235	16.5

Table 51. Observed plot yields of 1954 oat grain and nutrient composition of total dry matter obtained from fertilizer treatments applied to corn in 1953. Upper figure, block I; lower figure, block II.

Treatment	Oat grain yield in lbs/A	Total dry matter: Yield lbs/A	Percent N	N yield lbs/A	Percent P	P yield lbs/A
000	15.7	2140.9	1.91	40.9	0.219	4.7
000	16.2	1954.3	1.62	31.7	0.170	3.3
010	18.8	3144.7	2.34	73.6	0.255	8.0
010	13.8	2212.0	2.19	48.4	0.210	4.6
020	18.8	3118.1	2.44	76.1	0.270	8.4
020	19.5	2851.5	2.21	63.0	0.272	7.8
030	23.7	3278.0	2.01	65.9	0.300	9.8
030	14.7	2931.5	2.55	74.8	0.280	8.2
100	18.2	2025.4	1.55	31.4	0.179	3.6
100	18.5	2052.0	1.58	32.4	0.161	3.3
110	20.8	2451.9	1.95	47.8	0.214	5.2
110	30.0	2958.2	1.60	47.3	0.255	7.5
120	25.0	3011.5	2.09	62.9	0.280	8.4
120	20.0	2851.6	2.29	65.3	0.269	7.7
130	19.7	2984.8	2.19	65.4	0.290	8.7
130	19.2	3437.9	2.11	72.5	0.305	10.5
200	31.6	3304.6	1.57	51.9	0.198	6.5
200	22.2	2425.2	1.70	41.2	0.146	3.5
210	25.5	2904.9	2.03	59.0	0.204	5.9
210	22.7	2345.2	1.43	33.5	0.200	4.7
220	33.6	4077.5	1.81	73.8	0.270	11.0
220	24.5	3597.7	2.02	72.7	0.224	8.1
230	32.0	3464.5	1.77	61.3	0.305	10.6
230	26.0	3278.0	1.75	57.4	0.250	8.2
400	26.3	2851.5	1.69	48.2	0.205	5.8
400	19.3	2345.2	1.80	42.2	0.147	3.4

Table 51. (continued)

Treat-ment	Oat grain yield in lbs/A	Total dry matter Yield lbs/A	Percent N	N yield lbs/A	Percent P	P yield lbs/A
410	42.6	4237.4	1.66	70.3	0.187	7.9
410	39.3	3970.9	1.61	63.9	0.163	6.5
420	46.3	4184.0	1.81	75.7	0.214	9.0
420	49.3	4397.3	1.58	69.5	0.200	8.8
430	58.0	5276.7	1.58	83.4	0.243	12.8
430	41.3	4637.1	1.82	84.4	0.240	11.1
600	31.1	3198.0	1.67	53.4	0.161	5.1
600	18.2	2158.7	1.91	41.2	0.125	2.7
610	48.0	4850.3	1.83	88.8	0.171	8.3
610	28.5	4423.9	1.75	77.4	0.156	6.9
620	68.6	5969.6	1.64	97.9	0.179	10.7
620	48.0	4583.8	1.82	83.4	0.210	9.6
630	55.8	5196.8	1.76	91.5	0.228	11.8
630	55.3	5623.2	1.74	97.8	0.187	10.5

Table 52. Predicted nutrient composition of total dry matter of 1954 oats obtained from N and P_2O_5 applied to corn in 1953.

Treat-ment	Percent N	N yield lbs/A	Percent P	P yield lbs/A
000	1.83	37.9	0.192	3.9
010	2.10	56.8	0.231	6.2
020	2.28	66.6	0.269	7.9
030	2.37	67.4	0.308	9.1
100	1.68	35.9	0.184	4.0
110	1.91	55.4	0.222	6.4
120	2.05	65.9	0.260	8.3
130	2.10	67.3	0.298	9.6

Table 52. (continued)

Treat-ment	Percent N	N yield lbs/A	Percent P	P yield lbs/A
200	1.58	35.8	0.173	4.1
210	1.78	56.0	0.211	6.6
220	1.88	67.1	0.249	8.6
230	1.90	69.2	0.287	10.0
300	1.55	37.7	0.160	4.2
310	1.71	58.5	0.197	6.8
320	1.77	70.3	0.235	8.9
330	1.75	73.0	0.273	10.5
400	1.58	41.5	0.144	4.2
410	1.70	63.0	0.182	7.0
420	1.73	75.3	0.219	9.2
430	1.66	78.7	0.257	10.9
500	1.66	47.2	0.127	4.3
510	1.74	69.3	0.164	7.2
520	1.74	82.4	0.201	9.6
530	1.64	86.3	0.238	11.4
600	1.81	54.9	0.107	4.4
610	1.85	77.6	0.144	7.4
620	1.80	91.3	0.181	9.9
630	1.67	95.9	0.217	11.8

Table 53. Analysis of variance of nitrogen yield for total dry matter of 1954 oats obtained from N and P_2O_5 applied to corn in 1953.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	39	13,237.60		
Replication	1	355.21	355.21	7.74*
Treatments	19	12,010.49	632.13	13.77**
Nitrogen	4	3,413.37	853.34	18.60**
Linear	1		2,274.84	49.57**
Quadratic	1		1,077.56	23.48**
Cubic	1		58.14	1.27
Quartic	1		2.83	0.06
Phosphorus	3	7,448.15	2,482.72	54.10**
Linear	1		6,612.50	144.09**
Quadratic	1		822.65	17.93**
Cubic	1		130.00	2.83
NP	12	1,148.97	95.75	2.09
Error	19	871.90	45.89	

Table 54. Analysis of variance of phosphorus yield for total dry matter of 1954 oats obtained from N and P_2O_5 applied to corn in 1953.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	39	278.13		
Replication	1	16.00	16.00	15.38**
Treatments	19	242.42	12.76	12.27**
Nitrogen	4	14.32	3.58	3.44**

Table 54. (continued)

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Linear	1		12.80	12.31**
Quadratic	1		0.12	0.12
Cubic	1		1.25	1.20
Quartic	1		0.15	0.14
Phosphorus	3	213.58	71.19	68.45**
Linear	1		209.92	201.84**
Quadratic	1		2.97	2.86[d]
Cubic	1		0.69	0.66
NP	12	14.52	1.21	1.16
Error	19	19.71	1.04	

[d]Significance level is explained in footnote of Table 4.

Table 55. Observed plot yields of 1954 oat grain and nutrient composition of total dry matter obtained from N topdressed in 1954. Figures within each group of 3 refer to block I, II and III.

Pounds N/A	Oat grain yield in bu./A	Total dry matter				
		Yield lbs/A	Percent N	N yield lbs/A	Percent P	P yield lbs/A
0	17.8	2238.6	1.66	37.2	0.300	6.7
0	15.3	2078.7	2.19	45.5	0.156	3.2
0	12.8	1625.6	1.56	25.4	0.165	2.7
15	28.7	2931.5	1.51	44.3	0.223	6.5
15	27.0	2931.5	1.57	46.0	0.230	6.7
15	24.8	2611.7	1.50	39.2	0.155	4.0
30	28.8	3038.1	1.37	41.6	0.220	6.7
30	27.5	3224.7	1.46	47.1	0.180	5.8
30	23.0	2611.7	1.38	36.0	0.125	3.3
45	23.7	3917.6	1.58	61.9	0.223	8.7
45	32.8	3384.5	1.55	52.5	0.167	5.7
45	25.0	2771.6	1.32	36.6	0.108	3.0

Table 56. Predicted 1954 yields and nutrient composition for total dry matter of oats obtained from N topdressed in 1954.

Pounds N/A	Oat grain yield bu./A	Total dry matter				
		Yield lbs/A	Percent N	N yield lbs/A	Percent P	P yield lbs/A
0	15.4	2140.4	1.81	36.6	0.210	4.6
15	26.3	2567.0	1.52	40.7	0.195	5.0
30	27.3	2993.6	1.41	44.8	0.180	5.5
45	26.8	3420.2	1.48	49.0	0.165	5.9

Table 57. Analysis of variance of percent nitrogen for total dry matter of 1954 oats obtained from N topdressed in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	0.5493		
Replication	2	0.1310	0.0655	2.68
Treatment	3	0.2718	0.0906	3.71[c]
Linear	1		0.1760	7.21*
Quadratic	1		0.0954	3.91[c]
Cubic	1		0.0004	0.02
Error	6	0.1465	0.0244	

[c]Significance level is explained in footnote of Table 4.

$$Y_{\%N} = \underset{\pm 0.009}{-0.025056N} + \underset{\pm 0.0002}{0.000396N^2} + 1.805833 \qquad R^2 = 99.9 \text{ percent}$$

Table 58. Analysis of variance of nitrogen yield for total dry matter of 1954 oats obtained from N topdressed in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	930.36		
Replication	2	435.60	217.80	7.17*
Treatment	3	312.57	104.19	3.43[c]
Linear	1		255.85	8.42*
Quadratic	1		2.00	0.07
Cubic	1		54.72	1.80
Error	6	182.19	30.37	

[c]Significance level is explained in footnote of Table 4.

$$Y_{Nyield} = \underset{\pm 0.03}{0.275333}N + 36.579999 \qquad R^2 = 81.9\%.$$

Table 59. Analysis of variance of percent phosphorus for total dry matter of 1954 oats obtained from N topdressed in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	0.031297		
Replication	2	0.021439	0.010795	10.49*
Treatment	3	0.003686	0.001229	1.19
Linear	1		0.003405	3.31[d]
Quadratic	1		0.000016	0.01
Cubic	1		0.000265	0.26
Error	6	0.006172	0.001029	

[d]Significance level is explained in footnote of Table 4.

$$Y_{\%P} = \underset{\pm 0.00135}{-0.001004}N + 0.210267 \qquad R^2 = 92.4\%.$$

Table 60. Analysis of variance of phosphorus yield for total dry matter of 1954 oats obtained from N topdressed in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	41.41		
Replication	2	30.48	15.24	15.24**
Treatment	3	4.92	1.64	1.64
Linear	1		2.82	2.82[d]
Quadratic	1		0.75	0.75
Cubic	1		1.35	1.35
Error	6	6.01	1.00	

[d]Significance level is explained in footnote of Table 4.

$$Y_{Pyield} = \underset{\pm 0.017}{0.02889N} + 4.6 \qquad R^2 = 57.3\%.$$

Table 61. Observed 1955 alfalfa stand levels in plants per square foot. Each value is a mean of 4 throws of a one foot square quadrat per plot. Upper figure, block I; lower figure, block II.

Pounds P_2O_5/A	Pounds K_2O/A	Pounds of N/A 0	 40	 80	 160	 240
0	0	6 7	15 10	11 7	8 9	7 10
0	40	10 9	10 9	9 7	11 10	7 8
0	80	11 12	10 10	7 5	7 6	13 5
40	0	10 9	8 8	10 5	10 8	11 9
40	40	11 5	9 7	11 10	11 8	7 9
40	80	11 8	9 7	12 8	8 8	12 8
80	0	10 8	13 8	13 9	9 6	10 8
80	40	12 9	7 8	10 9	8 7	7 10
80	80	9 7	10 9	10 9	9 9	14 7
120	0	6 10	13 8	8 10	10 8	8 6
120	40	11 8	9 9	11 8	12 8	11 8
120	80	9 8	10 8	6 8	8 5	8 10

Table 62. Observed plot yields and nutrient composition of 1955 hay obtained from fertilizer treatments applied to corn in 1953. Upper figure, block I; lower figure, block II.

Treatment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
000	3.69	0.207	15.2	2.17	160.2
000	1.17	0.166	3.9	1.68	39.2
001	2.04	0.183	7.4	1.36	55.3
001	0.92	0.164	3.0	1.76	32.4
002	2.03	0.186	7.6	1.69	68.5
002	0.36	0.154	1.1	1.76	12.7
010	2.70	0.198	10.7	1.80	97.2
010	1.12	0.152	3.4	1.82	40.9
011	1.41	0.170	4.8	1.73	48.7
011	1.78	0.159	5.7	1.66	59.2
012	2.73	0.179	9.8	1.57	85.9
012	1.53	0.151	4.6	1.96	60.0
020	3.30	0.195	12.8	2.09	137.5
020	2.69	0.200	10.8	1.60	86.2
021	3.55	0.205	14.6	1.77	125.4
021	2.30	0.170	7.8	2.02	93.0
022	3.76	0.251	18.9	1.53	115.3
022	2.65	0.165	8.8	1.59	84.5
030	2.77	0.258	14.2	1.59	87.9
030	4.01	0.190	15.2	1.80	144.5
031	4.33	0.253	21.9	1.39	120.6
031	4.30	0.178	15.3	1.67	144.0
032	4.92	0.245	24.1	1.70	167.4
032	4.27	0.219	18.7	1.74	148.9
100	1.10	0.184	4.1	1.75	38.4
100	0.81	0.177	2.9	1.86	30.2

Table 62. (continued)

Treat-ment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
101	2.24	0.195	8.7	1.78	79.8
101	0.27	0.152	8.2	1.60	8.7
102	2.86	0.207	11.8	1.38	79.1
102	0.24	0.133	6.4	1.61	7.7
110	2.17	0.198	8.6	1.55	67.1
110	0.59	0.167	2.0	1.80	21.2
111	1.52	0.176	5.3	1.68	51.0
111	1.35	0.141	3.8	1.75	47.3
112	2.51	0.163	8.2	1.67	83.9
112	0.96	0.151	2.9	1.80	34.5
120	3.69	0.231	17.1	1.57	115.6
120	1.69	0.167	5.6	1.69	57.0
121	3.14	0.228	14.3	1.44	90.3
121	1.46	0.148	4.3	1.85	54.2
122	3.91	0.222	17.4	2.23	174.6
122	2.87	0.166	9.6	1.83	105.2
130	4.57	0.227	20.8	1.62	148.2
130	3.99	0.188	15.0	1.98	158.4
131	4.07	0.239	19.5	1.55	126.4
131	4.67	0.187	17.4	2.04	190.9
132	4.70	0.233	21.9	1.63	153.1
132	3.76	0.179	13.5	2.00	150.6
200	3.09	0.209	12.9	1.57	96.8
200	0.82	0.158	2.6	1.75	28.6
201	1.51	0.189	5.7	1.48	44.7
201	0.52	0.184	1.9	1.94	20.1
202	2.38	0.176	8.4	1.48	70.5
202	0.48	0.158	1.5	1.83	17.5

Table 62. (continued)

Treat-ment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
210	1.60	0.177	5.7	1.69	54.2
210	1.49	0.185	5.5	1.89	56.4
211	1.22	0.170	4.1	1.69	41.2
211	1.23	0.153	3.8	2.06	50.8
212	3.11	0.185	11.5	1.53	95.4
212	0.51	0.137	1.4	1.70	17.4
220	4.50	0.222	20.0	2.14	192.4
220	3.65	0.157	11.5	2.15	157.2
221	2.53	0.202	10.2	1.85	93.4
221	1.64	0.147	4.8	2.04	66.9
222	3.79	0.228	17.3	1.59	120.8
222	1.65	0.160	5.3	1.83	60.4
230	4.85	0.254	24.6	1.61	156.4
230	3.81	0.168	12.8	2.30	175.0
231	3.32	0.230	15.3	1.56	103.4
231	3.36	0.195	13.1	2.04	137.2
232	3.40	0.264	18.0	1.45	98.7
232	3.51	0.185	13.0	1.57	110.4
400	2.15	0.202	8.7	1.31	56.4
400	0.39	0.164	1.3	1.67	13.0
401	3.81	0.233	17.7	1.60	121.6
401	0.45	0.155	1.4	1.60	14.4
402	4.19	0.212	17.8	2.32	194.7
402	0.91	0.190	2.7	1.64	23.3
410	1.74	0.198	6.9	1.50	52.2
410	0.48	0.161	1.5	1.78	17.1
411	3.11	0.207	12.9	1.52	94.8
411	0.37	0.138	1.0	1.68	12.4
412	4.29	0.215	18.4	2.08	178.1
412	1.11	0.169	3.8	1.58	35.1

Table 62. (continued)

Treatment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
420	2.11	0.206	8.7	1.83	77.1
420	1.31	0.167	4.4	1.68	43.9
421	2.18	0.194	8.5	1.73	75.4
421	3.74	0.164	12.2	2.20	164.8
422	1.99	0.193	7.7	1.80	71.7
422	2.14	0.176	7.5	2.11	90.5
430	3.18	0.240	15.3	1.55	98.4
430	3.46	0.193	13.3	1.65	113.9
431	3.05	0.232	14.1	1.47	89.8
431	2.07	0.160	6.6	1.85	76.6
432	4.31	0.237	20.4	1.60	138.2
432	2.16	0.181	7.8	1.64	70.7
600	1.64	0.186	6.1	1.47	48.1
600	0.33	0.150	1.0	1.50	9.9
601	1.89	0.215	8.1	1.67	63.2
601	0.80	0.164	2.6	1.85	29.6
602	1.70	0.199	6.8	1.88	63.9
602	0.36	0.162	1.2	1.66	12.0
610	0.97	0.198	3.8	1.78	34.6
610	0.46	0.139	1.3	1.47	13.5
611	2.04	0.205	8.4	1.81	74.0
611	0.54	0.152	1.6	1.55	16.7
612	1.32	0.197	5.2	1.89	49.8
612	0.46	0.142	1.3	1.63	15.0
620	2.82	0.219	12.4	1.62	91.1
620	4.58	0.200	18.3	2.26	207.0
621	3.79	0.216	16.4	2.20	166.6
621	1.13	0.171	3.9	1.56	35.4
622	3.41	0.220	15.9	1.54	105.3
622	1.55	0.164	5.1	1.58	48.9
630	2.73	0.225	12.3	1.62	88.5
630	2.75	0.174	9.6	1.63	89.6

Table 62. (continued)

Treat-ment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
631	3.94	0.209	16.5	1.66	130.9
631	2.44	0.176	8.6	1.69	82.6
632	4.32	0.194	16.8	1.85	159.6
632	1.91	0.158	6.0	1.61	61.6

Table 63. Observed plot yields and nutrient composition of 1955 hay obtained from P_2O_5 topdressed on oats in 1954. Figures within each group of 3 refer to block I, II and III.

Pounds P_2O_5/A	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
0	2.03	0.186	7.6	1.69	68.5
0	0.92	0.164	3.0	1.76	32.4
0	1.16	0.166	3.9	1.68	38.9
30	2.46	0.185	9.1	1.61	79.4
30	1.40	0.172	4.8	1.98	55.4
30	0.98	0.164	3.2	1.79	35.0
60	2.54	0.190	9.7	1.81	92.1
60	1.51	0.152	4.6	2.01	60.7
60	1.65	0.160	5.3	1.75	57.8
90	3.45	0.187	12.9	1.71	118.2
90	3.38	0.183	12.4	1.90	128.2
90	2.66	0.178	9.5	1.59	84.6

Table 64. Predicted 1955 hay yields and nutrient composition for P_2O_5 topdressed on oats in 1954.

Pounds P_2O_5/A	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
0	1.42	0.172	5.0	1.74	47.7
30	1.47	0.174	5.1	1.78	53.2
60	2.04	0.167	7.2	1.79	73.6
90	3.11	0.180	11.4	1.78	109.2

Table 65. Analysis of variance of 1955 hay yields obtained from P_2O_5 topdressed on oats in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	8.5800		
Replication	2	2.2927	1.14635	12.3**
Treatment	3	5.7277	1.9092	20.5**
Linear	1		4.8167	51.6**
Quadratic	1		0.7803	8.4*
Cubic	1		0.1307	1.4
Error	6	0.5596	0.0933	

Table 66. Analysis of variance of percent phosphorus for 1955 hay obtained from P_2O_5 topdressed on oats in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean squares	F
Total	11	0.001735		
Replication	2	0.001028	0.000514	9.18*
Treatment	3	0.000371	0.000124	2.21
Linear	1		0.000099	1.76
Quadratic	1		0.000140	2.50[d]
Cubic	1		0.000132	2.36[d]
Error	6	0.000336	0.000056	

[d]Significance level is explained in footnote of Table 4.

$$Y_{\%P} = \underset{\pm 0.00096}{0.00051836P} - \underset{\pm 0.000028}{0.00002092P^2} + \underset{\pm 0.00000021}{0.00000018P^3} + 0.172001$$

$R^2 = 100.0\%$

Table 67. Analysis of variance of phosphorus yield for 1955 hay obtained from P_2O_5 topdressed on oats in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	135.49		
Replication	2	41.21	20.61	10.90**
Treatment	3	82.96	27.65	14.63**
Linear	1		66.99	35.44**
Quadratic	1		13.23	7.00*
Cubic	1		2.74	1.45
Error	6	11.32	1.89	

$$Y_{P yield} = \underset{\pm 0.0746}{-0.034557P} + \underset{\pm 0.00079}{0.001167P^2} + 5.046679 \quad R^2 = 96.7\%.$$

Table 68. Analysis of variance of percent potassium for 1955 hay obtained from P_2O_5 topdressed on oats in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	0.1971		
Replication	2	0.1163	0.0582	8.31*
Treatment	3	0.0389	0.0130	1.86
Linear	1		0.0027	0.39
Quadratic	1		0.0320	4.57[c]
Cubic	1		0.0042	0.60
Error	6	0.0419	0.0070	

[c]Significance level is explained in footnote of Table 4.

$$Y_{\%K} = \underset{\pm 0.003}{0.001961P} - \underset{\pm 0.000079}{0.000017P^2} + 1.738167 \qquad R^2 = 89.2\ \%$$

Table 69. Analysis of variance of potassium yield for 1955 hay obtained from P_2O_5 topdressed on oats in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	10,546.67		
Replication	2	2,535.51	1,267.76	7.92*
Treatment	3	7,051.36	2,350.45	14.69**
Linear	1		6,291.45	39.33**
Quadratic	1		681.01	4.26[c]
Cubic	1		78.90	0.49
Error	6	959.80	159.97	

[c]Significance level is explained in footnote of Table 4.

$$Y_{Kyield} = \underset{\pm 0.600}{-0.070677P} + \underset{\pm 0.0064}{0.008370P^2} + 47.746782 \qquad R^2 = 98.9\%$$

Table 70. Observed plot yields and nutrient composition of 1956 hay obtained from fertilizer treatments applied to corn in 1953. Upper figure, block I; lower figure, block II.

Treat-ment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
000	3.88	0.202	15.7	2.80	217.5
000	1.70	0.154	5.2	1.84	62.7
001	2.59	0.184	9.5	2.16	112.1
001	1.28	0.153	3.9	2.07	52.9
002	2.79	0.154	8.6	2.02	112.7
002	0.69	0.159	2.2	2.12	29.2
010	2.65	0.171	9.0	2.06	109.2
010	0.91	0.149	2.7	2.05	37.3
011	1.08	0.154	3.3	1.99	43.0
011	1.82	0.161	5.8	2.02	73.5
012	3.55	0.175	12.4	2.00	142.2
012	1.15	0.145	3.3	2.15	49.4
020	2.03	0.171	6.9	2.40	97.3
020	2.62	0.182	9.6	1.88	98.6
021	1.99	0.164	6.5	2.23	88.6
021	1.15	0.143	3.3	2.15	49.5
022	2.66	0.172	9.1	2.08	110.7
022	1.52	0.149	4.5	2.11	64.2
030	4.14	0.195	16.2	2.12	175.4
030	2.37	0.158	7.5	2.33	110.3
031	2.92	0.173	10.1	1.98	115.8
031	2.18	0.179	7.8	2.19	95.4
032	4.05	0.195	15.8	2.03	164.7
032	2.88	0.170	9.8	2.09	120.5
100	2.27	0.163	7.4	1.79	81.2
100	1.43	0.152	4.3	2.31	66.1

Table 70. (continued)

Treat-ment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
101	2.69	0.168	9.0	2.22	119.5
101	0.56	0.142	1.6	1.80	20.2
102	3.04	0.193	11.8	1.76	107.2
102	0.40	0.142	1.1	1.93	15.4
110	2.27	0.169	7.7	1.87	84.7
110	0.69	0.142	2.0	1.87	25.9
111	1.80	0.166	6.0	1.72	62.1
111	1.67	0.148	4.8	2.14	69.7
112	0.94	0.151	2.8	1.92	36.0
112	1.28	0.155	4.0	1.82	46.6
120	2.82	0.150	8.4	1.86	105.1
120	1.38	0.142	3.9	1.98	54.7
121	1.81	0.169	6.1	2.01	72.6
121	1.23	0.148	3.6	1.96	48.3
122	2.31	0.191	8.8	2.41	111.3
122	2.43	0.155	7.5	2.02	98.0
130	3.81	0.173	13.2	2.23	170.0
130	1.90	0.172	6.6	2.19	83.3
131	2.43	0.160	7.8	2.09	101.6
131	2.15	0.148	6.4	2.15	92.5
132	3.91	0.189	14.8	1.96	153.4
132	1.85	0.154	5.7	2.22	82.3
200	2.89	0.176	10.2	1.92	111.1
200	1.22	0.150	3.7	1.76	42.8
201	1.63	0.160	5.2	2.03	66.2
201	0.88	0.163	2.9	2.03	35.8
202	1.52	0.165	5.0	1.64	49.8
202	0.79	0.147	2.3	1.86	29.4

Table 70. (continued)

Treat-ment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
210	1.47	0.161	4.7	2.01	59.1
210	1.73	0.165	5.7	1.91	66.0
211	1.25	0.152	3.8	1.95	48.9
211	1.48	0.148	4.4	2.06	60.9
212	3.26	0.186	12.1	2.22	144.7
212	0.56	0.147	1.6	1.85	20.8
220	3.29	0.185	12.2	2.66	175.3
220	3.06	0.171	10.5	2.30	140.9
221	2.22	0.158	7.0	2.19	97.3
221	1.31	0.147	3.8	2.20	57.6
222	3.00	0.185	11.1	1.95	117.2
222	1.37	0.147	4.0	1.88	51.6
230	4.46	0.194	17.3	2.25	200.8
230	1.81	0.142	5.1	2.21	80.2
231	2.15	0.161	6.9	2.35	100.9
231	1.96	0.151	5.9	2.18	85.4
232	3.05	0.178	10.9	2.23	135.8
232	2.01	0.145	5.8	2.08	83.6
400	3.02	0.181	11.0	1.77	107.1
400	0.75	0.140	2.1	1.74	26.1
401	2.04	0.173	7.1	2.03	82.7
401	0.83	0.150	2.5	1.77	29.3
402	4.51	0.206	18.6	2.94	265.3
402	2.76	0.175	9.7	1.92	106.4
~~410~~	2.19	0.165	7.2	1.69	73.9
410	0.61	0.139	1.7	1.71	20.9
411	3.51	0.182	12.8	1.87	131.0
411	0.50	0.138	1.4	1.80	18.0

Table 70. (continued)

Treat-ment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
412	3.82	0.196	15.0	2.70	206.6
412	2.72	0.167	9.1	1.85	100.8
420	1.97	0.165	6.5	2.01	79.1
420	1.17	0.155	3.6	2.02	47.2
421	2.23	0.170	7.6	1.97	88.0
421	3.08	0.176	10.8	2.21	136.5
422	1.26	0.151	3.8	1.89	47.6
422	1.74	0.157	5.5	2.38	82.8
430	2.07	0.173	7.2	2.05	84.9
430	2.73	0.171	9.3	2.21	120.8
431	1.67	0.160	5.4	1.82	60.9
431	1.19	0.149	3.6	2.02	48.1
432	4.18	0.186	15.5	1.96	163.8
432	1.16	0.160	3.7	1.93	44.8
600	2.53	0.166	8.4	1.76	89.0
600	0.70	0.145	2.0	1.66	23.2
601	2.10	0.171	7.2	1.87	78.7
601	0.88	0.159	2.8	1.99	35.1
602	2.19	0.170	7.5	2.07	90.5
602	0.82	0.153	2.5	1.81	29.7
610	0.79	0.155	2.5	1.80	28.4
610	0.62	0.155	1.9	1.59	19.7
611	2.73	0.170	9.3	2.03	110.9
611	0.84	0.156	2.6	1.73	29.0
612	1.28	0.154	3.9	2.00	51.2
612	0.64	0.153	2.0	1.82	23.3
620	3.02	0.174	10.5	2.04	123.3
620	3.28	0.210	13.8	2.55	167.9

Table 70. (continued)

Treat-ment	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
621	2.48	0.176	8.7	2.25	111.5
621	1.44	0.169	4.9	2.04	58.7
622	1.94	0.186	7.2	1.75	67.8
622	1.33	0.152	4.1	1.98	52.8
630	1.88	0.161	6.0	2.08	78.0
630	1.96	0.155	6.1	2.05	80.2
631	2.73	0.172	9.4	2.07	112.8
631	1.68	0.150	5.0	2.09	70.1
632	1.65	0.169	5.6	2.04	67.4
632	1.50	0.171	5.1	1.95	58.6

Table 71. Observed plot yields and nutrient composition of 1956 hay obtained from P_2O_5 topdressed on oats in 1954. Figures within each group of 3 refer to block I, II and III.

Pounds P_2O_5/A	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
0	1.70	0.153	5.2	1.84	62.6
0	1.28	0.153	3.9	2.07	53.0
0	2.79	0.154	8.6	2.02	112.7
30	0.77	0.155	2.4	1.98	30.5
30	1.46	0.155	4.6	2.16	63.5
30	3.16	0.172	10.9	2.03	128.3
60	1.04	0.155	3.2	1.97	41.0
60	1.24	0.152	3.8	2.16	53.6
60	2.91	0.165	9.6	2.06	119.9
90	1.66	0.161	5.3	2.03	67.4
90	2.39	0.171	8.2	2.16	103.2
90	3.09	0.167	10.3	2.25	139.1

Table 72. Predicted 1956 hay yields and nutrient composition for P_2O_5 topdressed on oats in 1954.

Pounds P_2O_5/A	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
0	1.96	0.154	6.1	1.98	77.9
30	1.70	0.158	5.5	2.03	68.7
60	1.83	0.161	6.0	2.09	76.6
90	2.35	0.165	7.8	2.14	101.5

Table 73. Observed plot yields and nutrient composition of 1956 hay obtained from P_2O_5 topdressed in 1956. The successive values represent replications, I through VIII.

Pounds P_2O_5/A	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
0	2.27	0.163	7.4	1.79	81.2
	2.88	0.176	10.2	1.92	110.7
	3.02	0.181	11.0	1.77	107.1
	2.53	0.166	8.4	1.76	89.0
	1.43	0.152	4.3	2.31	66.1
	1.22	0.150	3.7	1.76	42.8
	0.75	0.140	2.1	1.74	26.1
	0.70	0.145	2.0	1.66	23.2
30	2.48	0.184	9.1	1.89	93.7
	2.93	0.201	11.8	2.13	124.2
	2.94	0.203	11.9	1.71	100.5
	2.87	0.185	10.6	1.89	108.3
	1.89	0.193	7.3	2.22	83.8
	2.49	0.183	9.1	1.90	94.6
	1.50	0.181	5.4	1.96	58.8
	1.34	0.197	5.3	1.61	43.2
60	2.79	0.226	12.6	2.06	114.9
	2.82	0.218	12.3	1.89	106.6
	3.47	0.209	14.5	1.88	130.4

Table 73. (continued)

Pounds P_2O_5	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
60 (contd.)					
	3.55	0.215	15.3	2.08	147.8
	2.44	0.208	10.1	2.35	114.7
	2.68	0.197	10.6	1.89	101.1
	2.07	0.209	8.6	1.97	81.4
	1.91	0.206	7.9	1.60	61.1
90	2.99	0.247	14.8	1.93	115.1
	3.00	0.239	14.4	2.06	123.7
	3.12	0.231	14.4	1.76	109.5
	3.85	0.233	17.9	2.09	160.7
	2.24	0.229	10.2	2.01	90.2
	2.77	0.224	12.4	2.03	112.3
	2.24	0.210	9.4	1.99	89.2
	2.53	0.233	11.8	1.67	84.4

Table 74. Predicted 1956 hay yields and nutrient composition for P_2O_5 topdressed in 1956.

Pounds P_2O_5	Yield tons/A	Percent P	P yield lbs/A	Percent K	K yield lbs/A
0	1.84	0.160	6.3	1.86	67.6
30	2.34	0.189	8.7	1.90	90.5
60	2.68	0.213	11.1	1.93	105.1
90	2.85	0.230	13.5	1.97	111.3

Table 75. Analysis of variance of percent phosphorus for 1956 hay obtained from P_2O_5 topdressed on oats in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	0.000609		
Replication	2	0.000145	0.000073	2.28
Treatment	3	0.000272	0.000091	2.84
Linear	1		0.000191	5.97*
Quadratic	1		0.000002	0.06
Cubic	1		0.000079	2.47
Error	6	0.000192	0.000032	

$$Y_{\%P} = \underset{\pm 0.00005}{0.000119P} + 0.154067 \qquad R^2 = 70.2\ \%.$$

Table 76. Analysis of variance of phosphorus yield for 1956 hay obtained from P_2O_5 topdressed on oats in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	98.87		
Replication	2	76.63	38.32	19.65**
Treatment	3	10.57	3.52	1.81
Linear	1		4.82	2.47[d]
Quadratic	1		4.08	2.09[d]
Cubic	1		1.67	0.86
Error	6	11.67	1.95	

[d]Significance level ~~is explained~~ in footnote of Table 4.

$$Y_{P\,yield} = \underset{\pm 0.095}{-0.039445P} + \underset{\pm 0.001}{0.000648P^2} + 6.066672 \qquad R^2 = 84.2\ \%.$$

Table 77. Analysis of variance of percent potassium for 1956 hay obtained from P_2O_5 topdressed on oats in 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	0.1325		
Replication	2	0.0717	0.0359	12.38**
Treatment	3	0.0434	0.0145	5.00*
Linear	1		0.0400	13.79**
Quadratic	1		0.0000	0.00
Cubic	1		0.0034	1.17
Error	6	0.0174	0.0029	

$$Y_{\%K} = \underset{\pm 0.00046}{0.001722P} + 1.983333 \qquad R^2 = 92.2\%.$$

Table 78. Analysis of variance of potassium yield for 1956 hay, obtained from P_2O_5 topdressed on oats 1954.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	11	15,104.41		
Replication	2	12,078.91	6,039.45	34.96**
Treatment	3	1,988.86	662.95	3.84[c]
Linear	1		935.36	5.41[c]
Quadratic	1		868.70	5.03[c]
Cubic	1		184.80	1.07
Error	6	1,036.64	172.64	

[c]Significance level is explained in footnote of Table 4.

$$Y_{K yield} = \underset{\pm 1.158}{-0.587609P} + \underset{\pm 0.01233}{0.009454P^2} + 77.854982 \qquad R^2 = 90.7\%.$$

Table 79. Analysis of variance of percent phosphorus for 1956 hay obtained from P_2O_5 topdressed in 1956.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	31	0.025715		
Replication	7	0.002008	0.000287	4.28**
Treatment	3	0.022297	0.007432	110.92**
Linear	1		0.021949	327.5**
Quadratic	1		0.000300	4.48*
Cubic	1		0.000048	0.72
Error	21	0.001410	0.000067	

$Y_{\%P} = 0.00108709P - 0.00000340P^2 + 0.159675$ $R^2 = 99.8\%$.
± 0.0002 ± 0.000002

Table 80. Analysis of variance of phosphorus yield for 1956 hay obtained from P_2O_5 topdressed in 1956.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	31	468.70		
Replication	7	210.05	30.01	20.55**
Treatment	3	228.02	76.01	52.06**
Linear	1		225.63	154.54**
Quadratic	1		2.00	1.37
Cubic	1		0.39	----
Error	21	30.63	1.46	

$Y_{Pyield} = 0.079167P + 6.337500$ $R^2 = 99.0\%$.
± 0.00637

Table 81. Analysis of variance of percent potassium for 1956 hay obtained from P_2O_5 topdressed in 1956.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	31	1.1202		
Replication	7	0.8017	0.1145	9.79**
Treatment	3	0.0726	0.0242	2.07
Linear	1		0.0526	4.50*
Quadratic	1		0.0190	1.62
Cubic	1		0.0010	0.09
Error	21	0.2459	0.0117	

$$Y_{\%K} = \underset{\pm 0.00057}{0.001208P} + 1.860625 \qquad R^2 = 72.5\%.$$

Table 82. Analysis of variance of potassium yield for 1956 hay obtained from P_2O_5 topdressed in 1956.

Source of variation	Degrees of freedom	Sum of squares	Mean square	F
Total	31	31,756.99		
Replication	7	18,317.21	2,616.74	12.84**
Treatment	3	9,160.95	3,053.65	14.99**
Linear	1		8,520.56	41.82**
Quadratic	1		559.45	2.75[d]
Cubic	1		80.94	0.40
Error	21	4,278.83	203.75	

[d]Significance level is explained in footnote of Table 4.

$$Y_{Kyield} = \underset{\pm 0.516}{0.904638P} - \underset{\pm 0.005}{0.004646P^2} + 67.563604 \qquad R^2 = 99.1\%.$$

Table 83. Total TDN yields in thousands of pounds per acre obtained from corn and oat grain and alfalfa hay in a corn, oat, alfalfa alfalfa rotation. Upper figure, block I; lower figure, block II.

Pounds P_2O_5/A	Pounds K_2O/A	Pounds of N/A				
		0	40	80	160	240
0	0	12.4	7.8	11.9	10.7	9.7
		6.9	6.4	6.0	4.1	5.1
0	40	8.3	9.6	9.1	11.7	9.5
		6.3	5.0	6.0	4.9	6.2
0	80	8.5	10.7	8.9	14.6	9.9
		4.8	5.0	5.8	7.8	5.0
40	0	9.6	9.4	8.5	9.8	8.1
		5.0	5.8	8.7	6.8	6.6
40	40	7.2	8.8	8.2	10.9	11.5
		7.5	8.5	8.2	6.7	7.4
40	80	10.5	8.6	11.9	14.5	9.3
		6.1	7.2	6.6	10.1	7.1
80	0	9.3	11.7	14.2	10.6	12.9
		9.5	7.9	12.6	8.9	14.9
80	40	9.1	10.7	10.5	10.9	13.6
		6.7	7.8	8.6	13.5	8.9
80	80	10.7	12.5	12.9	10.0	12.5
		8.0	10.5	8.9	10.4	9.3
120	0	10.9	13.3	15.9	11.3	11.1
		10.1	10.6	11.5	12.4	10.7
120	40	11.5	11.5	11.7	11.7	13.6
		10.1	12.0	11.4	9.7	10.8
120	80	13.1	14.0	11.9	15.6	11.7
		11.1	9.9	11.8	9.4	9.5

Table 84. Mean dollar values discounted at various rates for total crops produced during a four year period and from which regression equations were computed.

Treat-ment	Undis-counted values	0, 5, 10 and 20 percent dis-count rate	0, 10, 20 and 40 percent dis-count rate
000	208	195	183
010	203	191	179
020	226	211	196
030	271	250	229
100	206	196	186
110	225	215	205
120	272	258	244
130	300	280	260
200	221	211	202
210	244	233	223
220	300	284	267
230	323	304	286
400	238	224	210
410	266	253	239
420	295	281	267
430	313	296	280
600	213	204	195
610	244	236	228
620	324	307	290
630	304	288	273