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UNIVERSAL SWING CURVES FOR TWO-MACHINE STABILITY

PROBLEM WITH MULTIPLE SWITCHING

by

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I. INTRODUCTION

Synchronism between synchronous machines must be maintained for successful operation of electrical power systems. This condition should be met not only under steady-load conditions, but also under transient disturbances. A transient stability study is aimed at determining whether a power system is stable under a particular disturbance or transient.

Solutions of transient stability problems involve a considerable amount of numerical calculations. To save much of the calculations needed for solving two-machine transient stability problems, several attempts have been made by others to develop generalized solutions.

The main limitations of the previously developed solutions are one or both of the following:

1. The solutions apply only for networks with no resistance.
2. They apply only for sustained transients, that is, for cases where the network between the two machines does not change after the occurrence of the transient.

In this dissertation, a universal set of solutions that overcome these limitations is presented.

The solutions are in the form of curves, that were calculated with a step-by-step process, using the Network Analyzer of Iowa State College.

II. REVIEW OF LITERATURE

A. Two-Machine Transient Stability Problem

When synchronous machines were first operated in parallel, it was realized that there is a limit to the power that can be transmitted between them. This was known as the stability limit. With the growth of electrical power systems, the complexity of their networks as well as the need for understanding their performance increased. One of the major problems that confronted the engineers was to determine the behaviour of the power system under transient disturbances. This was needed to design and operate power systems without frequent loss of synchronism. This led to what is known as transient stability studies.

The stability problem was first carefully investigated in 1924 by the Southern California Edison Company (6). Information on system operating characteristics under different layouts and loads, and at various disturbances, was obtained by means of transient recording apparatus.

In the following years many contributions were made. By the mid-thirties, the problem was well defined, and an accepted method of analysis was developed. A report by the A.I.E.E. Subcommittee on Interconnection and Stability Factors (10) in 1937, gives a good summary of the factors affecting stability and the method of analysis of the problem.

The first detailed treatment of the two-machine stability problem

was done by R. H. Park and E. H. Bancker (7) in 1929. In their paper they discussed the factors affecting stability, and the methods of calculating the angle-time curves. The torque-angle equation, which is usually known as the swing equation, was developed. The idea of reducing a two-machine system to an equivalent one machine against an infinite bus was introduced. The concept of calculating generalized swing curves for a two-machine system on non-dimensional time basis was discussed. Moreover the equal area criterion was mentioned, although not much discussed. All these ideas will be discussed in detail in the text of this dissertation.

Park and Bancker in their paper confined their discussion to lossless networks, and single instant of switching, i.e. sustained transients. However, the author believes that their brilliant work presented a satisfactory analysis of the problem. The later papers published on the same problem were mere refinements of their work.

In 1930, I. H. Summers, and J. B. McClure (8), published a paper, which they recognized as a continuation to the work of Park and Bancker. The treatment of the two-machine stability problem by the equal area criterion was extended to include the effect of resistance in the network. They also included this effect in the point-by-point solution of the problem. Moreover, they published a complete set of precalculated swing curves, using the Intergraph of the Massachusetts Institute of Technology. The curves were developed for a system without losses, and for one instant of switching. Although it was proved later that these curves could be used for a general system (with losses), the fact

that they were confined to sustained transients offered serious limitations to their use.

F. R. Longley (9), published a paper in 1930 in which he made a thorough discussion of the step-by-step method for calculating swing curves. Special attention was given to the effect of excitation systems on stability.

Griseom, Lewis, and Ellis (13), 1932, introduced some generalized stability solutions for metropolitan systems. Their curves are for lossless line, and limited to systems with such multiplicity of circuits that the reactance between a generator and a system does not change appreciably when the fault is cleared.

The last major attempt to develop generalized solutions of the two-machine stability problem, was done by Byrd and Pritchard (11), in 1933. Their curves are for determining the maximum switching time (to clear the fault) without losing synchronism. This switching time is plotted for values of initial load, network conditions before, during, and after the fault is cleared.

In 1940, Skilling and Yamakawa (12), developed a method for calculating swing curves for a two-machine system by means of graphical integration. It is based on knowing the power angle curve of the system, and is a continuation of the equal area criterion method. It is essentially a different step-by-step method for calculating swing curves. However it can be adapted easily to include the effect of excitation systems, governor action, and other factors that depend upon the speed of the generators.

The above are the major developments in the method of analysis of the two-machine stability problem by analytical means. It has been felt that, while the solution of the problem by the point-by-point method is quite satisfactory, there is a need for a set of generalized solutions that can offer quick answers to the engineer. This dissertation aims at fulfilling this need. Generalized solutions for a system with losses, and one or more switching operations, are developed. The assumptions made are universally accepted as reasonable, namely:

1. Constant flux linkage in the rotor circuit of the generators.
2. Angular momentum of generators remain constant.
3. Neglect of the effect of damper windings in the generators.
4. Negative sequence torques neglected.
5. The first swing is sufficient for determining stability.
6. Generator input torque is constant.
7. A synchronous machine can be represented by an e.m.f. behind transient reactance, and this e.m.f. is constant in magnitude.

In developing most of these generalized curves, the Iowa State College Network Analyzer was used. Some of the data needed to complement them were calculated.

III. THE SWING EQUATION

A. One Machine against an Infinite Bus*

1. Derivation of the swing equation

A synchronous machine, while in operation, obeys the laws that govern the motion of rotating bodies. At any instant, the net torque on the machine T is given by

$$T = I \frac{d^2\theta}{dt^2} \quad (1)$$

where

t = time

I = moment of inertia of rotating parts

θ = angular positions of the rotor with respect to
a reference axis in space.

By taking the reference as an axis rotating with a constant angular velocity ω_0 , equal to the synchronous speed, we define the angle as

$$\delta = \theta - \omega_0 t. \quad (2)$$

Hence, equation (1) can be written as

$$T = I \frac{d^2\delta}{dt^2}. \quad (3)$$

*Kimbark (1, 2)

The angle δ determines the position of the machine rotor with respect to the synchronous speed. Thus the object of a stability study is to solve equation (3) for δ as a function of t .

The torque T is the sum of several torque components. If their algebraic sum is equal to zero there is no net acceleration on the machine and the rotor will maintain a constant angular velocity. However, if the net torque is not zero the acceleration will not be zero, and the rotor may accelerate or decelerate depending on the nature of the net torque.

The components of the torque T can be divided into two main components

1. The input torque T_i , which is the mechanical shaft torque corrected for rotational losses. In a transient stability study this torque is considered constant, since the governor on the prime mover does not interfere to oppose the loss of synchronism during the first swing. It is assumed that if the machine is stable after the first swing, synchronism will be maintained and the system is stable*.

2. The electromagnetic torque T_u , which includes the synchronous electromagnetic torque, the damping torques in the rotor circuit and amortisseur windings, and the negative sequence torque that might develop due to unbalanced faults close to the generator.

*This point is discussed further on p. 10.

In general, the damping torques (induction and negative sequence torques) in large synchronous machines are of such a magnitude that they have little effect on the transient stability. However, their effect may be included in the analysis if it were felt necessary. This is seldom done since they represent rather complex functions that complicate the analysis considerably without increasing appreciably its accuracy. For a comprehensive discussion of this subject the author suggests chapter 6 of reference 4.

Thus equation (3) can be written as

$$I \frac{d^2 \delta}{dt^2} = T_1 - T_u \quad (4)$$

where T_u is the synchronous electromagnetic torque (all other electromagnetic torques neglected).

Equation (4) is recognized by many authors as the swing equation, e.g. Mr. Cray in references 3 and 4. In this dissertation, power will be dealt with instead of the torque. This simplifies the calculations, especially when a calculating board is used, and is the form used in the recent publications, e.g. Mr. Kimbark in references 1 and 2, and in reference 5.

Multiplying both sides of equation (4) by the angular velocity ω

$$I\omega \frac{d^2 \delta}{dt^2} = T_1\omega - T_u\omega \quad (5)$$

but, the angular momentum $M = I\omega$

and, the power $P = T\omega$.

Then equation (5) can be written as

$$M \frac{d^2 \delta}{dt^2} = P_1 - P_u \quad (6)$$

where P_1 = shaft input power corrected for rotational losses

P_u = electromagnetic synchronous power output.

Equation (6) is usually known as the swing equation, and will be referred to as such in this text.

The angular speed ω usually does not deviate considerably from the synchronous speed ω_0 until synchronism is lost. To illustrate this point, it is clear that ω_0 for a 60-cycle machine is 377 radians per second, or (377×57.3) electrical degrees per second. Assuming that the rotor of the machine would "slip" from synchronous speed at the rate of 180 degrees per second, the change in speed is 0.83 per cent.

Thus the angular momentum M , which is the product of the moment of inertia I (which is constant) and the angular speed (which is practically constant), can be considered constant. This assumption, while leading to a very slight error, leads to a considerable simplification in the solution of equation (6).

The electromagnetic synchronous power P_u depends upon the network between the synchronous machine terminals and the infinite bus, the voltage at the infinite bus, and the e.m.f. of the synchronous machine and its reactance.

The network is assumed to be at steady state at any one instant. This implies that the occurrence of a transient, or its removal consumes no time. This is true for all practical purposes since the time constant of the network is much smaller than the time constant of the

machine (1, p. 44).

The voltage of the infinite bus, by definition, is constant in magnitude and phase.

In transient stability studies, the synchronous machine is usually represented by an e.m.f. of constant magnitude behind a constant reactance. This representation assumes that the direct axis transient reactance is equal to the quadrature axis transient reactance and transient saliency neglected (4, chap. 2). It also assumes that the excitation system response is such that to keep the e.m.f. behind the transient reactance constant (4, chap. 6). This may not be a true representation of the actual system of excitation, but for most machines it is a good approximation. It is possible to include the effect of the excitation system response in the solution of the swing equation. A good discussion of this point is given in reference (9).

The above mentioned approximation will be used in this dissertation.

Examining equation (6), it is seen that the machine acceleration depends upon the network condition and the severity of the transient. If the power output differs from the power input, the machine will accelerate (or decelerate). But the power output changes with the change in the rotor position, and hence the acceleration is modified, and so on. The rotor of the machine may swing far enough for the machine to lose synchronism, or it may swing back and forth in damped oscillations until it comes to rest with its rotor at a new position with respect to the infinite bus. The nature of these swings is such that the first swing is the severest, and the effect of the damping torques, which is

neglected in the analysis, is in the direction of aiding stability. This is why the swing equation is solved for one oscillation.

Another significant aspect of the above assumptions is the fact that the angle δ in equation (6) can be used to represent the angle of e.m.f. behind transient reactance, instead of the angle of the rotor in space. This results from assuming that the transient reactance does not change with the position of the rotor.

2. The output power (1, chap. 3)

Consider a system of n nodes (other than ground). The voltages at these nodes are E_1, E_2, \dots, E_n , referred to node 0 which is ground.

Y_{0i} = admittance between node i and ground

y_{ij} = admittance between nodes i and j ,

therefore $Y_{ii}/\theta_{ii} = Y_{0i} + Y_{1i} + Y_{2i} + \dots + Y_{ni}$

and $Y_{ij}/\theta_{ij} = Y_{ji}/\theta_{ji}$
 $= -y_{ij}.$

The power at node i is given by

$$P_i = \sum_{j=1}^n E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) . \quad (7)$$

3. Application to one machine against an infinite bus (1, p. 126)

Referring to the finite machine as node 1, and the infinite bus as node 2, and taking the reference axis in phase with the voltage of the infinite bus, the output of machine 1 from an application of equation (7) is

$$P_{u1} = E_1^2 Y_{11} \cos \theta_{11} + E_1 E_2 Y_{12} \cos(\theta_{12} - \delta)$$

or

$$P_{u1} = P_0 + P_M \sin(\delta - \delta) \quad (8)$$

where

$$P_0 = E_1^2 Y_{11} \cos \theta_{11}$$

$$P_M = E_1 E_2 Y_{12}$$

$$\delta = \theta_{12} - 90^\circ .$$

Equation (8) represents a sinusoid with its zero displaced horizontally by an angle δ and vertically by a value of $P_u = P_0$.

The swing equation then becomes

$$M \frac{d^2 \delta}{dt^2} = (P_1 - P_0) + P_M \sin(\delta - \delta) \quad (9)$$

B. Two Finite Machines

There are two methods of approach for a two finite machines stability problem. One of them is to treat each machine separately and find the angular position of each machine δ_1 and δ_2 with respect to a common axis of reference at different intervals of time, then the relative angular position of the two machines at any instant is simply the difference between the angles δ_1 and δ_2 at that instant. The other method is to reduce the system of two finite machines to a system of one machine against an infinite bus, and then treat this equivalent system as mentioned above in equation (9).

The two methods are essentially the same. They are two ways that lead to the same thing, and both of them are used by different authors.

However, the second method is particularly suited for the use of pre-calculated solutions. Therefore, it will be used in this dissertation.

If P_{11} , and P_{12} are the initial power input to machines 1 and 2 corrected for rotational losses

P_{u1} , and P_{u2} are the electromagnetic output powers of machines 1 and 2

M_1 , and M_2 are the angular momentums of the two machines then the terms in equation (9) for the equivalent one machine against an infinite bus are given (1, p. 132-136) by

$$M = \frac{M_1 M_2}{M_1 + M_2} \quad (10)$$

$$P_i = \frac{M_2 P_{11} - M_1 P_{12}}{M_1 + M_2} \quad (11)$$

$$P_o = \frac{M_2 E_1^2 Y_{11} \cos \theta_{11} - M_1 E_2^2 \cos \theta_{22}}{M_1 + M_2} \quad (12)$$

$$P_M = \frac{E_1 E_2 \sqrt{M_1^2 + M_2^2 - 2 M_1 M_2 \cos 2\theta_{12}}}{M_1 + M_2} \quad (13)$$

$$\delta = -\tan^{-1} \left(\frac{M_1 + M_2}{M_1 - M_2} \tan \theta_{12} \right) - 90^\circ. \quad (14)$$

IV. SOLUTION OF THE SWING EQUATION

A. Analytical Solution

A formal solution of the swing equation is not possible. The universally accepted method of solution is the step-by-step method (1, p. 36-39), (7, Appen. VII), (3, p. 47-50). In this method a certain approximation is adopted that makes it possible to calculate the rotor position at the end of a small time interval, and then proceed with the next time interval and so on. To demonstrate this, let the swing equation be put in the form

$$Ma = P_a \quad (15)$$

where P_a is the accelerating power, which is a function of δ and $\frac{d\delta}{dt}$, and a = angular acceleration

$$= \frac{d^2\delta}{dt^2}.$$

Let the angular acceleration a be related to the time t by a curve similar to that in Figure I(a). The step-by-step method divides the time axis to several short and equal time intervals each of which is equal to Δt . The basic assumption is that the angular acceleration is constant over an interval of time starting from the middle of one interval to the middle of the next interval. The reason for this is the fact that the acceleration usually is known at the beginning of some interval. Thus, it is assumed that this acceleration is constant over the period

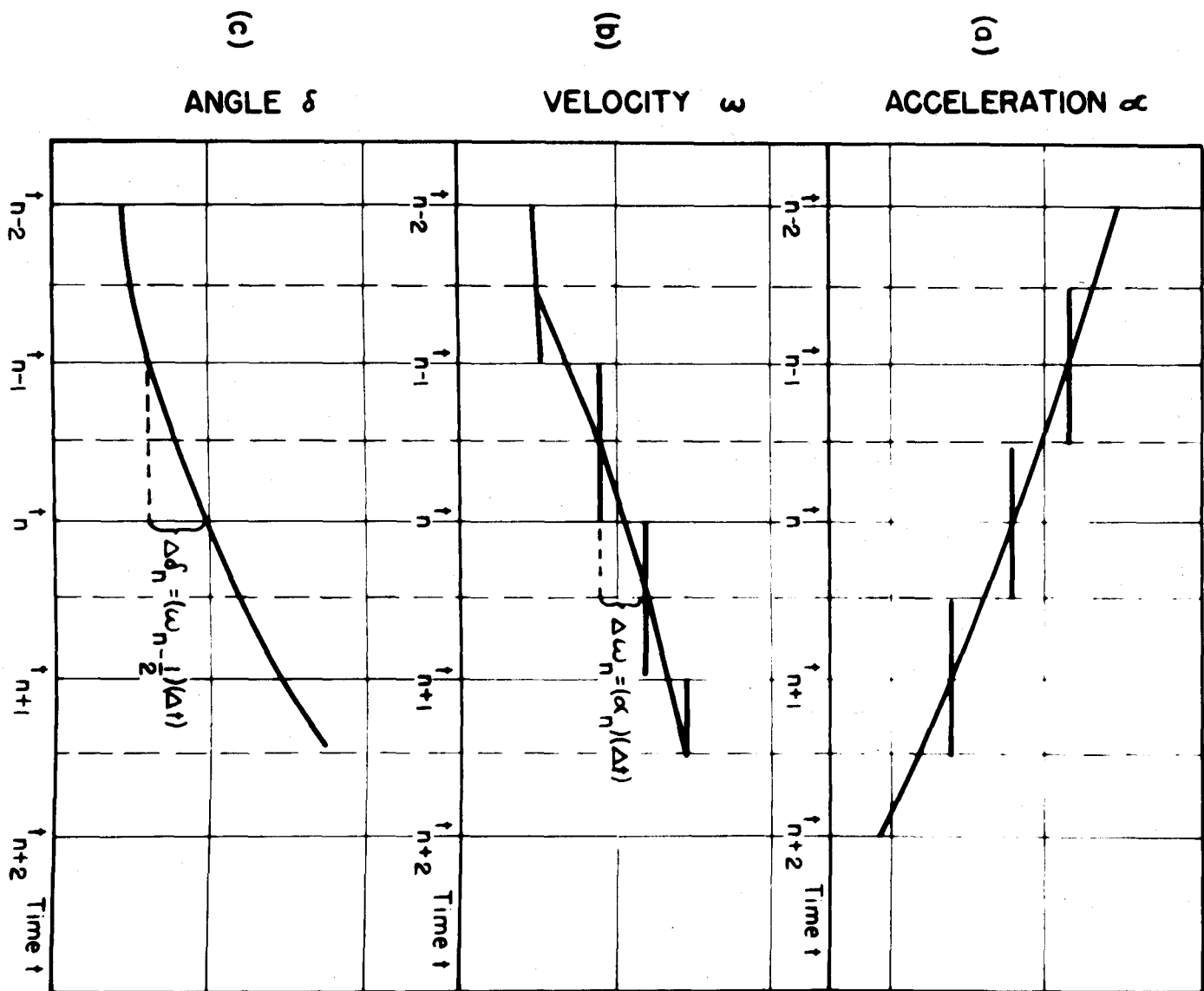


FIG. 1. STEP-BY-STEP METHOD

from the middle of the previous interval to the middle of the interval under consideration. It is evident from Figure I(a), that the integrated effect of α over the interval in which it is assumed to be constant, is almost the same. The shorter the interval Δt , the more justified this view will be.

If the interval under consideration is from t_n to t_{n+1} , let α_n , and also the angular velocity $\omega_{n-\frac{1}{2}}$, be known. From the above α_n is considered to be constant from $t_{n-\frac{1}{2}}$ to $t_{n+\frac{1}{2}}$. The velocity at $t_{n+\frac{1}{2}}$ is given by

$$\omega_{n+\frac{1}{2}} = \omega_{n-\frac{1}{2}} + \Delta t \times \alpha_n. \quad (16)$$

Then the next important assumption is that the angular velocity $\omega_{n+\frac{1}{2}}$ is constant over the time interval Δt starting from t_n to t_{n+1} .

From this assumption, the angle δ at t_{n+1} is given by

$$\delta_{n+1} = \delta_n + (\omega_{n+\frac{1}{2}})(\Delta t). \quad (17)$$

The new value of δ_{n+1} determines $P_{a_{n+1}}$, hence α_{n+1} , and with the above procedure δ_{n+2} is determined, and so on.

The angular velocity ω needs more explanation at this stage of the discussion. The angular velocity $\omega_{n+\frac{1}{2}}$ is taken as the average angular velocity in the interval under consideration (t_n to t_{n+1}). However, it should always be remembered that under the assumption of constant α 's, the values of ω 's obtained correspond to the instants of time at the middle of the intervals. Thus if a velocity-time curve is to be plotted, the points plotted to form the curve must be plotted at the instants

$$t_{n-\frac{1}{2}}, t_{n+\frac{1}{2}}, t_{n+\frac{3}{2}}, \text{etc.}^*$$

The angular velocity ω can be eliminated from equation (17) if it is so desired, as follows:

$$\begin{aligned}\omega_{n+\frac{1}{2}} &= \omega_{n-\frac{1}{2}} + \Delta \omega_{n+\frac{1}{2}} \\ &= \omega_{n-\frac{1}{2}} + (\Delta t)(a_n)\end{aligned}$$

but

$$\begin{aligned}\omega_{n-\frac{1}{2}} &= \frac{\Delta \delta_n}{\Delta t} \\ \omega_{n+\frac{1}{2}} &= \frac{\Delta \delta_n}{\Delta t} + (\Delta t)(a_n) \\ \delta_{n+1} &= \delta_n + \Delta \delta_n + (\Delta t)^2 a_n\end{aligned}$$

but

$$a_n = \frac{F_{an}}{M}.$$

$$\text{Therefore, } \delta_{n+1} = \delta_n + \Delta \delta_n + \frac{(\Delta t)^2}{M} P_a. \quad (18)$$

1. Discontinuities

The above discussion overlooked the discontinuities in the acceleration-time curve (or power-time curve). An example of such discontinuities is at the occurrence of a fault when the acceleration power instantly changes from zero to a large value, or the removal of a fault when the accelerating power changes to a lower value (often to a negative value).

*This point will be discussed further on p. 19.

If a discontinuity occurs at the beginning of a time interval, the velocity at the middle of the interval is the sum of the contributions of the accelerating powers before and after the discontinuity, each effective only for half an interval. This is equivalent to averaging the accelerating power before and after the discontinuity, and applying it over the total interval.

If there is a discontinuity at the middle of an interval, no special allowance for it is made since the acceleration is considered constant from the middle of one interval to the middle of the following interval.

For a discontinuity occurring at some other instant during an interval a weighted average of its effect over the interval should be considered.

2. Summary of the step-by-step method

Referring to equation (8), it is clear that the output power P_u of the synchronous machine is a function of one variable only, that is, the angle δ . At the beginning of every interval, δ is known, then P_u is calculated from equation (8). From this value of P_u the accelerating power P_a is calculated from the equation $P_a = P_i - P_u$.

The acceleration a is then calculated since it is equal to P_a/M , M being a constant. The value of a determines the increase in the velocity $\Delta \omega$, which, when added to the velocity at the middle of the previous interval, gives the velocity at the middle of the interval under consideration. This value of ω determines the change in δ , and

thus the value of δ at the end of the interval is determined, and the process is repeated.

The output power P_u can be calculated knowing P_o , P_M , δ in equation (8), or if the network is set on a network calculator, the output power P_u can be determined.

While the step-by-step method is widely used for solving transient stability problems, there are inherent errors in the method that should not be overlooked.

The assumption of constant accelerating power within an interval of time is the main source of error. Also the assumption that the velocity at the middle of an interval is the average velocity in the interval is not quite accurate. This is equivalent to assuming that the acceleration decreases (or increases) in equal steps in the intervals (i.e. linear acceleration curve), which is not the case.

The errors caused by the above assumptions can be minimized by choosing small time intervals (1, p. 40).

If the velocity ω is of interest -- as it is in the case in this dissertation -- the velocity-time curve will be determined by plotting the values of ω at the middle of the intervals and joining them by a smooth curve. The author believes that this leads to the least error since these points represent the only available information about the velocity ω .

B. Precalculated Solutions

It was explained in the discussion relative to the swing equation

that a two-machine power system can be represented by an equivalent one-machine against an infinite bus. This reduces the number of degrees of freedom to one, and makes a universal set of solutions possible.

The swing equation can be put in the form

$$M \frac{d^2 \delta'}{dt^2} = P_1' - P_M \sin \delta' \quad (19)$$

where

$$P_1' = P_1 - P_e$$

$$\delta' = \delta - \delta_0.$$

To put the swing equation into a dimension-less form, the modified time τ is introduced. By definition

$$\tau = t \sqrt{\frac{\pi}{180} \frac{P_M}{M}} \quad (20)$$

and equation (19), becomes

$$\frac{d^2 \delta'}{d\tau^2} = \frac{180}{\pi} \left[\frac{P_1'}{P_M} - \sin \delta' \right] \quad (21)$$

or

$$\frac{d^2 \delta'}{d\tau^2} = [p - \sin \delta'] \frac{180}{\pi} \quad (22)$$

where

$$p = \frac{P_1 - P_e}{P_M} = \text{constant}$$

and

$$\delta' = \delta - \delta_0.$$

Equation (22) is a general swing equation that is independent of the network and inertia constant of the machine. The constant p depends

on the ratio of the input power to amplitude of the power-angle curve.

There are three quantities that determine the solution of equation (22). The initial angle δ'_0 , initial angular velocity ω_0 , and the constant p are these quantities. If a set of solutions for equation (22) is given for different values of δ'_0 , ω_0 , and p , a two-machine stability problem can be solved by calculating these quantities for the system under consideration, then selecting the proper precalculated solutions, or interpolating between them if necessary. This idea was realized by Park and Hancher (7) as was mentioned in the Review of Literature. Messrs. Summers and McClure (8) developed a set of solutions for different values of $\sin \delta'_0$ and p , but only for $\omega_0 = 0$. This limits the use of these solutions since many transient stability studies involve changing the network during the transient, e.g. when breakers operate.

It is the object of this dissertation to develop a universal set of solutions covering a wide range of $\sin \delta'_0$, p , and ω_0 . Changing the network during the transient would only mean moving to a different solution with different values of ω_0 , p , and $\sin \delta'_0$.

V. UNIVERSAL SET OF SOLUTIONS

A. Method of Obtaining the Swing Curves

The swing curves were obtained by setting a system of two machines connected through an impedance network on the Network Analyzer. The voltage of one machine was kept constant in magnitude and phase to represent the infinite bus. The voltage of the other machine was kept constant in magnitude only. For simplifying the calculations the voltage of both machines was arbitrarily chosen at one per unit.

The network between the two generators was first set at a certain value to give the required initial power P_1 . Then, the network was changed to represent conditions after the occurrence of the transient. The swing curve was calculated by a step-by-step process as explained in part 2. This was repeated for different swing curves by using different networks.

The network used between the two machines was uniform impedance units with phase angle of 78.1 degrees. This gave a ratio between the resistance and the reactance in the network of about 1:5, which is similar to actual conditions in power systems. It should be noted here, however, that this particular angle was chosen as it was found to be convenient. The transient was mostly represented by a fault on the network between the two generators, that is, connecting some point on

the network to ground through an impedance. In some cases this was not possible, and the transient was represented by merely increasing the impedance between the two generators.

1. Example

To illustrate the above procedure, an example of the network used to obtain the swing curve for $\sin \delta'_0 = 0.5$, $\omega_0 = 0$, $p = 0.80^*$ is shown below.

$$\sin \delta'_0 = 0.5$$

$$\delta'_0 = 30 \text{ degrees}$$

$$\delta_0 = 41.9 \text{ degrees}$$

(a) Unfaulted network. The network is shown in Figure II. All values are in per unit.

$$E_1 = 1.0/\underline{\delta}^\circ$$

$$E_2 = 1.0/\underline{0}^\circ$$

$$Z = 1.5/\underline{78.1}^\circ$$

$$Y_{11} = 0.667/\underline{-78.1}^\circ, \quad \theta_{11} = -78.1^\circ$$

$$Y_{12} = 0.667/\underline{101.9}^\circ, \quad \theta_{12} = 101.9^\circ$$

$$P_1 = E_1^2 Y_{11} \cos \theta_{11} + E_1 E_2 Y_{12} \cos(\theta_{12} - \delta_0)$$

$$= 1 \times 0.667 \times \cos(-78.1^\circ) + 1 \times 1 \times 0.667 \times \cos(101.9^\circ - 41.9^\circ)$$

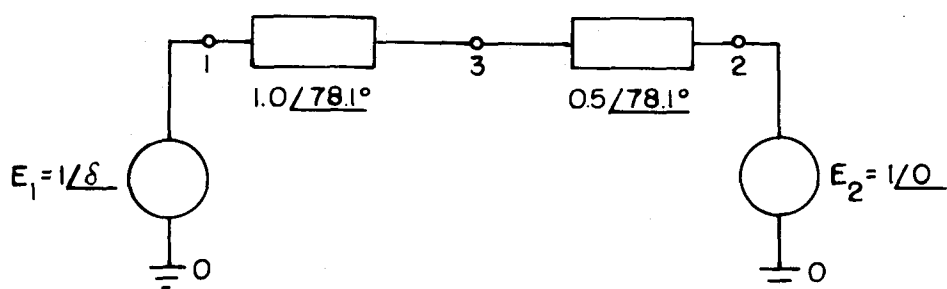
$$= 0.667(0.206 + 0.500)$$

$$= 0.667 \times 0.706$$

$$= 0.470$$

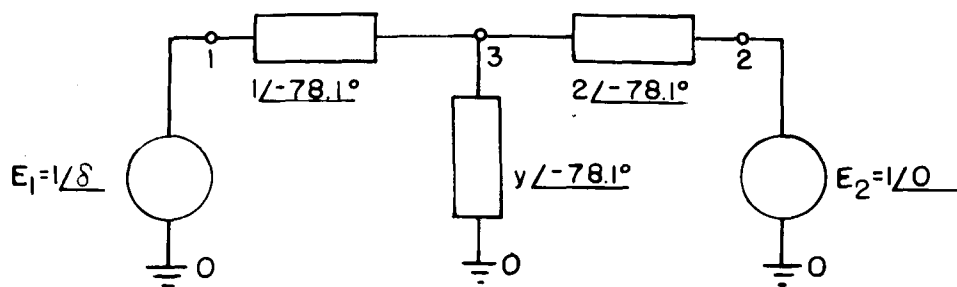
per unit.

*Fig. 11 of Appendix.



Z-DIAGRAM

FIG. II. NETWORK BEFORE THE FAULT



Y-DIAGRAM

FIG. III. NETWORK AFTER THE FAULT

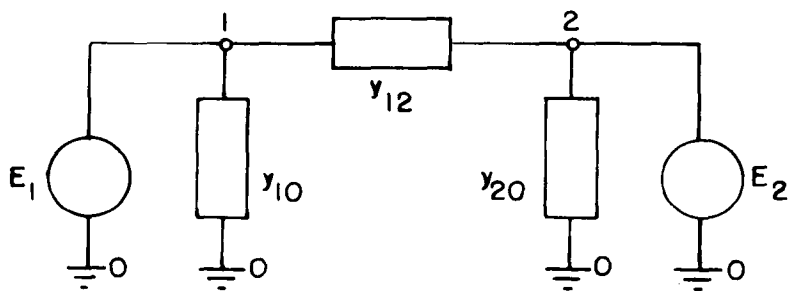


FIG. IV. EQUIVALENT FAULTED NETWORK

(b) Faulted network. The fault was represented by grounding point 3 in Figure II through an impedance. The new faulted network is shown in Figure III, with admittance used instead of impedance. To calculate the power, this network was reduced to that in Figure IV.

$$\Sigma Y = (3+y) \angle -78.1^\circ$$

$$Y_{12} = \frac{2}{3+y} \angle -78.1^\circ$$

$$Y_{10} = \frac{Y}{3+y} \angle -78.1^\circ$$

$$Y_{11} = \frac{2+y}{3+y} \angle -78.1^\circ$$

$$Y_{12} = \frac{2}{3+y} \angle 101.9^\circ .$$

From equation (8)

$$P_M = E_1 E_2 |Y_{12}|$$

$$= \frac{2}{3+y}$$

$$P_o = E_1^2 Y_{11} \cos \theta_{11}$$

$$= \frac{2+y}{3+y} \times 0.206 .$$

From equation (23)

$$P = \frac{P_1 - P_o}{P_M}$$

$$P = \frac{0.470 - 0.206 \frac{2+y}{3+y}}{\frac{2}{3+y}}$$

$$= \frac{1}{2} [0.470(3+y) - 0.206(2+y)]$$

$$= 0.132y + 0.500$$

or

$$y = \frac{P - 0.500}{0.132} .$$

(24)

Equation (24) gives the relation between the admittance to ground representing the fault with the parameter p . After a swing curve was obtained for some value of p and $\sin \delta_0$, the same network is used for other values of p except changing the impedance setting of the Network Analyzer unit representing the fault impedance. In this way, only one network was used for each value of $\sin \delta_0$, such that only ten different networks were used for the entire set of swing curves.

To illustrate the method of calculating the swing curves, the above example is carried further as follows:

Substituting $p = 0.8$ in equation (24), we get a value of $y = 2.27$ per unit. Therefore, the setting of the Network Analyzer unit representing the fault will be

$$Z = \frac{1}{2.27} = 0.44 \quad \text{per unit.}$$

The calculation of the swing curve is carried out as in Table 1 where

P_u = power output of finite machine.

P_a = accelerating power of finite machine.

ω = angular velocity at the middle of the interval,

which is considered the average angular velocity in the interval.

a = angular acceleration.

The angle δ is the actual angle of the finite machine that was set on the Network Analyzer. P_u is output power of the finite machine as indicated by the board meter. P_a is the difference between P_i and

P_u . The accelerating power P_a was then multiplied by $(\frac{180}{\pi})(\frac{1}{P_M})$ to obtain a non-dimensional angular acceleration. This is clear from equation (21) where the power was multiplied by $(\frac{180}{\pi})(\frac{1}{P_M})$ to obtain a non-dimensional form.

In this example

$$P_M = \frac{2}{3+y} = \frac{2}{5.27}$$

$$\frac{1}{P_M} = \frac{5.27}{2}$$

$$\alpha = (\frac{5.27}{2} \times \frac{180}{\pi}) P_a.$$

This number appears as a multiplier in the α column in Table 1. The change in angular velocity ($\Delta\omega$) was obtained by multiplying α by ΔT . This is added to the previous value of ω to give the average value of ω in the new interval. Then the change in the angle $\Delta\delta'$ was obtained by multiplying ω by ΔT . Then δ' at the end of the interval (or at the beginning of the next interval) was obtained by adding $\Delta\delta'$ to the value of δ' at the beginning of the interval. The angle δ equals the sum of δ' and δ'' , where δ'' equals 11.9 degrees.

It should be noted here that the first value of ω , which in this example is equal to zero, is the value of the initial angular velocity ω_0 . Also the value of P_a in the first interval was divided by two, to account for the discontinuity as explained in part 2.

For the curve of $\sin\delta'_0 = 0.5$, $p = 0.80$, and $\omega_0 = 10$ degrees per unit T , the same network on the Network Analyzer was used. Also the same procedure was followed to calculate the swing curve. The only

Table 1. Calculation of Swing Curve
 for
 $\sin \delta'_0 = .50$
 $p = .80$

Initial Vel. = 0			Degrees per unit τ			$P_1 = .470$		
τ	P_u	P_a	$a = P_a \times 151$	$\Delta\omega = \Delta\tau \times a$	Average ω	$\Delta\delta'$	δ'	δ
		.115			0			
0+	.355	.0575	8.7	4.35	4.35	2.2	30.0	41.9
0.5	.365	.105	15.8	7.9	12.25	6.1	32.2	44.1
1.0	.40	.07	10.5	5.25	17.5	8.7	38.3	50.2
1.5	.445	.025	3.8	1.9	19.4	9.7	47.0	58.9
2.0	.48	-.01	-1.5	-.75	18.65	9.3	56.7	68.6
2.5	.51	-.04	-6.0	-3.0	15.65	7.8	66.0	77.9
3.0	.53	-.06	-9.0	-4.5	11.15	5.6	73.8	85.7
3.5	.537	-.067	-10.1	-5.05	6.1	3.0	79.4	91.3
4.0	.540	-.07	-10.5	-5.25	+.85	.4	82.4	94.3
4.5	.540	-.07	-10.5	-5.25	-4.4	-2.2	82.8	94.7
5.0	.538	-.068	-10.2	-5.1	-9.5	-4.7	80.6	92.5
5.5	.535	-.065	-9.8	-4.9	-14.4	-7.2	75.9	87.8
6.0	.518	-.048	-7.2	-3.6	-18.0	-9.0	68.7	80.5
6.5	.492	-.022	-3.3	-1.65	-19.65	-9.8	59.7	71.6
7.0	.456	.014	2.1	1.05	-18.6	-9.3	49.9	61.8
7.5	.406	.064	9.6	4.8	-13.8	-6.9	40.6	52.5
8.0							33.7	

Table 2. Calculation of Swing Curve

for
 $\sin \delta'_0 = .50$
 $p = .80$

Initial Vel. = 10		Degrees per unit τ			$P_1 = .470$	
τ	P_u	P_a	$a=P \times 151$	$\Delta\omega = \Delta(xa)$	Average ω	$\Delta\delta'$ δ' δ
		.115			10	
0+	.355	.0575	8.7	4.35	14.35	7.2 30.0 41.9
0.5	.396	.074	11.1	5.55	19.9	9.9 37.2 49.1
1.0	.444	.026	3.9	1.95	21.85	10.9 47.1 59.0
1.5	.486	-.016	-2.4	-1.2	20.65	10.3 58.0 69.9
2.0	.518	-.048	-7.2	-3.6	17.05	8.5 68.3 80.2
2.5	.535	-.065	-9.8	-4.9	12.15	6.1 76.8 88.7
3.0	.538	-.068	-10.3	-5.15	7.0	3.5 82.9 94.8
3.5	.540	-.07	-10.5	-5.15	1.85	.9 86.4 98.3
4.0	.54	-.07	-10.5	-5.15	-3.3	-1.6 87.3 99.2
4.5	.539	-.069	-10.4	-5.2	-8.5	-4.2 85.7 97.6
5.0	.537	-.067	-10.1	-5.05	-13.55	-6.8 81.5 93.4
5.5	.532	-.062	-9.4	-4.7	-18.25	-9.1 74.7 86.6
6.0	.509	-.039	-5.9	-2.95	-21.2	-10.6 65.6 77.5
6.5	.473	-.003	-.4	-.2	-21.4	-10.7 55.0 66.9
7.0	.430	+.04	6.0	3.0	-18.4	-9.2 44.3 56.2
7.5						35.1
8.0						
8.5						
9.0						

difference was that a value of $\omega_0 = 10$ was used at the top of the velocity column, as shown in Table 2.

B. Accuracy Considerations

In using the Network Analyzer to calculate the swing curves, the following steps were taken to minimize the experimental errors.

1. Residuals

Every unit of the Network Analyzer contains a number of relays that connect the unit to the metering system when the unit is keyed to be metered. The contacts of these relays have a finite resistance that may vary slightly with time, and may vary from one unit to another. The resistance of the relay contacts and the impedance of the cables connecting the units to the center patching board is usually known as residual impedance.

To minimize the error due to the residual impedances, only four analyzer units were used for all measurements. Their residual R and L were measured carefully by an inductance bridge, and the relay contacts of these units were checked periodically.

2. Reading the wattmeter

The central desk wattmeter of the Analyzer has three scales, namely, 0.04 to 0.20 p.u., 0.20 to 1.00 p.u., and 1.00 to 5.00 p.u. scales. The scales are calibrated to give best accuracy at 70 per cent

of full scale. To avoid discrepancy between scales*, and to avoid possible poor accuracy at the lower end of the scale, the networks were chosen such that the power was read at the middle scale only (from 0 to 1.0 p.u.) and in the range of 0.3 to 0.8 p.u.

3. The VAR problem

As was explained before, three-phase faults in the network were used to represent the transients. This caused the generators to supply a high value of reactive volt-amperes compared to the value of the watts supplied by the generator. This would have forced the operator to read relatively small amount of power on a high scale with a poor accuracy. To avoid this, static capacitors were connected in parallel with the generator representing the finite machine. While the capacitors supplied most of the VARs, the generator power was kept in the range of the middle scale to be read with good accuracy.

4. Inherent errors in the board

The Network Analyzer of Iowa State College can be operated successfully within an accuracy of the order of two per cent. This can be obtained easily especially when precautions similar to the ones mentioned above, are taken. However, it is indeed very difficult to determine the exact cause of the errors. It can be caused by one or more of the

*This was done only as a precaution, although in the author's experience with the board there has not been any appreciable discrepancy between the wattmeter reading at different scales.

circuits of the board.

The electrical quantities metered were voltage, voltage phase, and power. In calibrating the metering system of the board, the calibration is done with respect to the desk voltmeter. Thus it was reasonable to assume that the error in the voltage reading is nil. Then, if any error in the power was noticed it had to be assigned to either the voltage phase meter or the wattmeter, or both. All the error was assumed to be in the wattmeter reading. This was arrived at after calculating some swing curves assuming all the error in the phase meter, then calculating the same swing curves assuming all the error in wattmeter, and comparing them to the curves predicted from long-hand calculations. This also was reasonable since the difference of two power readings were of importance and not the actual value of power.

C. The Range Covered by the Curves

The curves obtained covered initial angles in the range from 0 degree to 126.9 degrees. They were plotted in families of constant values of $\sin \delta'_0$ in steps of 0.10, and ω_0 in steps of ten degrees per unit T. As the difference between the angle of $\sin^{-1} = 0.9$ and the angle of $\sin^{-1} = 1.0$ is large (25.8 degrees), two more groups of families were plotted for $\sin \delta'_0 = 0.975$ ($\delta'_0 = 77.2$ degrees, and 102.8 degrees). Values of δ'_0 greater than 90 degrees are designated by placing a bar above the value of $\sin \delta'_0$. For example, $\sin \delta'_0 = \overline{0.9}$ means $\delta'_0 = 115.8$ degrees.

The curves chosen do not cover every possible circumstance, since this could have increased their number tremendously without justification. Instead, a range to cover almost all practical cases was chosen. For cases beyond that, extrapolation from the available curves can be useful.

Therefore, the families for $\sin \delta_0'$ from 0 to 0.5 were plotted only for $\omega_0 = 0$, assuming that a machine usually "swings" more than that before switching occurs. A maximum initial angular velocity of 60 degrees per unit T was found sufficient to cover most practical cases. Also a maximum initial angle of 126.9 degrees was thought to be a reasonable limit, although it is well recognized that a system can "swing" to larger angles and become stable after the network between the two machines is changed such that the constant p of the new network is small enough.

The curves are shown in the Appendix. Each initial condition is represented by two figures: one is a plot of the angle δ' against the modified time T , and the other is a plot of the angular velocity ω against the modified time T .

The angle-time curve was obtained by plotting the angles at the beginning of the time intervals and then joining between them with a smooth curve.

The velocity-time curves were obtained by plotting the values of ω (as in Tables 1 and 2) at the middle of the time intervals as explained in part 2, and then smooth curves joined between them.

The curves were first drawn on six inch by nine inch graph paper,

then reduced in size by a photostatic process.

D. How to Use the Curves

To determine the angle-time characteristic of a two-machine system for a particular transient with the use of the precalculated swing curves, the following steps should be followed:

1. The system must be reduced to one machine against an infinite bus. The equivalent constant M and the equivalent initial power P_1 are then to be calculated.

2. The quantities P_0 , P_M , and δ for the system during the duration of the transient and for every new network caused by the switching operations are to be calculated.

3. For condition of fault (or transient) on, the values of p , $\sin \delta'_0$, and T are calculated.

4. For the values of $\sin \delta'_0$, p as in step 3 and $\omega_0 = 0$, the proper precalculated swing curve is selected to fit these initial conditions. If no one curve is found to suit these conditions, interpolation among some of the curves is necessary.

5. If switching is known to occur at a certain time, this time is transformed to the modified time T by the relation determined in step 3. Then at that instant, the angle δ' is determined from the angle-time curve, and the velocity- ω is determined from the velocity-time curve. These two quantities will determine the initial conditions for the new precalculated swing curve to be followed after switching.

6. The new initial angular velocity ω_0 is given by

$$\omega_0 = \omega_s \frac{T_1}{T_2} \quad (25)$$

where $\omega_s = \omega$ at instant of switching as determined from first precalculated swing curve.

T_1, T_2 are the modified times for before and after switching respectively.

The reason for the use of equation (25) is the fact that ω in the precalculated curves is the rate of change of the angle with respect to the modified time and not the actual angular velocity. Thus, as the modified time changes, ω must be adjusted accordingly.

7. A new precalculated curve is selected for the new values of $\sin \delta'_0, \omega_0$, and p . Again interpolation may be necessary.

8. This process is to be continued for as many switching intervals as required.

VI. EXAMPLES AND DISCUSSION

A. Example 1

Consider a system of one machine connected to an infinite bus through a reactive network. A fault occurs in the network which is cleared after 0.15 second by isolating part of the transmission network. The following information applies to this system:

$$P_1 = 1.0 \quad \text{per unit}$$

$$\delta_0 = 17.5 \quad \text{degrees}$$

$$M = 4 \times 10^{-4} \quad \text{per unit}$$

$$P_M \text{ during fault} = 0.5 \quad \text{per unit}$$

$$P_M \text{ fault cleared} = 1.25 \quad \text{per unit.}$$

It is required to determine whether the system is stable.

1. Step-by-step solution

First, a solution by the usual step-by-step method is obtained and then a solution with the aid of the precalculated swing curves is shown.

For the analytical solution, a time interval of 0.05 second is chosen and the computations are given in Table 3. Looking at the last column of Table 3, it is evident that the angle δ is increasing with no limit, and the system is unstable.

$$\Delta t = 0.05$$

$$\frac{(\Delta t)^2}{M} = \frac{(0.05)^2}{4 \times 10^{-4}} = 6.25.$$

Table 3. Step-by-Step Solution of Example 1

$$P_1 = 1.0$$

t sec.	P_M	$\sin \delta$	$P_u =$ $P_M \sin \delta$	$P_a =$ $P_1 - P_u$	$6.25 P_a$ $= \frac{\Delta t^2}{M} P_a$	$\Delta \delta$	δ
0 ⁻				0.00			
0 ⁺	0.50	0.30	0.15	0.85			17.5
0 av.				0.425	2.7		
						2.7	
0.05	0.50	0.345	0.173	0.827	5.2		20.2
						7.9	
0.10	0.50	0.471	0.235	0.765	4.7		28.1
						12.6	
0.15 ⁻	0.50	0.652	0.326	0.674			40.7
0.15 ⁺	1.25	0.652	0.814	0.186			
0.15 av.				0.430	2.7		
						15.4	
0.20	1.25	0.830	1.040	-0.040	-0.2		56.1
						15.2	
0.25	1.25	0.950	1.18	-0.18	-1.2		71.3
						14.0	
0.30	1.25	0.997	1.245	-0.245	-1.5		85.3
						12.5	
0.35	1.25	0.990	1.24	-0.240	-1.5		97.8
						11.0	
0.40	1.25	0.947	1.18	-0.18	-1.1		108.8
						9.9	
0.45	1.25	0.877	1.09	-0.09	-0.9		118.7
						9.0	
0.50	1.25	0.791	0.986	+0.014	+0.1		127.7
						9.1	
0.55	1.25						136.8

2. Solution by means of precalculated curves

To solve the same problem with the available swing curves, the following steps are taken:

Conditions during the fault:

$$p = \frac{P_i}{P_M} = \frac{1.0}{0.50} = 2.0$$

$$\begin{aligned} T &= t \sqrt{\frac{1}{57.3} \frac{P_M}{M}} \\ &= t \sqrt{\frac{1}{57.3} \frac{0.50}{4 \times 10^{-4}}} = 4.68 t. \end{aligned}$$

The switching instant is at $t = 0.15$, or $T = 0.70$.

Since the network is reactive, $\delta' = \delta$, $\delta'_0 = 17.5$, $\sin \delta'_0 = 0.30$.

From Figure 7 and Figure 8 of Appendix, for $\sin \delta'_0 = 0.3$, and $\omega_0 = 0$, we obtain

$$\text{for } p = 2.85, \quad \delta' = 43, \quad \omega = 72$$

$$\text{for } p = 1.75, \quad \delta' = 37, \quad \omega = 50$$

therefore

$$\text{for } p = 2.0, \quad \delta' = 40, \quad \omega = 61.$$

Fault cleared:

$$\delta'_0 = 40^\circ, \quad \sin \delta'_0 = 0.643$$

$$p = \frac{1.0}{1.25} = 0.8$$

$$T = t \sqrt{\frac{1}{57.3} \frac{1.25}{4 \times 10^{-4}}} = 7.4 t$$

$$\omega_0 = 61 \times \frac{4.68}{7.4} = 38.6$$

degrees per unit T .

Information concerning the angle δ' is obtained by interpolation from the families of curves for $\sin \delta'_0 = 0.6$, $\sin \delta'_0 = 0.7$, $\omega_0 = 30$, and $\omega_0 = 40$, as shown in Table 4

$$t' = t - 0.15.$$

Table 4. Solution of Example 1 from Precalculated Swing Curves

t	t'	T	$\sin \delta'_0 = 0.6$			$\sin \delta'_0 = 0.7$			$\sin \delta'_0 = 0.643$
			$\omega_0=30$	$\omega_0=40$	$\omega_0=38.6$	$\omega_0=30$	$\omega_0=40$	$\omega_0=38.6$	
0.35	0.2	1.48	84	97	95	87	100	98	96.5
0.40	0.25	1.85	92	108	106	94	110	108	107
0.50	0.35	2.6	106	129	126	105	128	125	125.5
0.55	0.40	2.96	110	139	135	114	136	133	136

It is clear from the values of δ' in the last column of Table 4, that the system is unstable. Also these values of δ' compare favorably with the corresponding values of δ in Table 3.

B. Example 2

A system of one finite machine against an infinite bus. A fault occurs in the network between them. The fault is cleared in 0.25

seconds. The following data applies to this system:

$$\begin{aligned} M &= 2.5 \times 10^{-4} && \text{per unit} \\ P_i &= 0.500 && \text{per unit} \\ \delta'_0 &= 17.5 && \text{degrees.} \end{aligned}$$

Network with fault on:

$$\begin{aligned} P_c &= E_1^2 Y_{AA} \cos \theta_{AA} \\ &= -0.05 && \text{per unit} \\ P_M &= E_1 E_2 Y_{12} \\ &= 0.25 && \text{per unit} \\ \delta &= \theta_{12} - 90^\circ \\ &= 8 && \text{degrees.} \end{aligned}$$

Network with fault cleared:

$$\begin{aligned} P_c &= -0.4 && \text{per unit} \\ P_M &= 1.50 && \text{per unit} \\ \delta &= 12 && \text{degrees.} \end{aligned}$$

1. Step-by-step solution

Analytical solution with the step-by-step method is given in Table

5.

$$\Delta t = 0.05 \quad \text{second}$$

$$\begin{aligned} \frac{(\Delta t)^2}{M} &= \frac{25 \times 10^{-4}}{4 \times 10^{-4}} \\ &= 10 . \end{aligned}$$

From Table 5 it is seen that the system is stable when the fault is cleared at 0.25 seconds.

Table 5. Step-By-Step Solution of Example 2

t	P_M	$P_M \sin \delta'$	$P_u = P_c + P_M \sin \delta'$	$P_a = .50 - P_u$	$10 P_a$	$\Delta \delta$	δ	$\delta' = \delta - \delta$	$\sin \delta'$
0^-				0.000					
0^+	0.250	0.075	0.025	0.475			25.50	17.50	0.300
0 av.				0.238	2.38	2.38			
0.05	0.250	0.085	0.035	0.465	4.65	2.38	27.88	19.88	0.340
0.10	0.250	0.113	0.063	0.437	4.37	7.03	34.91	26.91	0.462
0.15	0.250	0.155	0.105	0.395	3.95	11.4	46.31	38.31	0.620
0.20	0.250	0.202	0.152	0.348	3.48	15.35	61.66	53.66	0.806
0.25 ⁻	0.250	0.238	0.188	0.312		18.85	69.49	72.49	0.954
0.25 ⁺	1.500	1.390	0.990	-0.490			80.49	68.49	0.930
0.25 av.				-0.089	-0.89				
0.30	1.500	1.496	1.096	-0.596	-5.96	17.94	98.43	86.43	0.998
0.35	1.500	1.482	1.082	-0.562	-5.82	11.98	110.41	98.41	0.989
0.40	1.500	1.450	1.050	-0.550	-5.50	6.16	116.57	104.57	0.967
0.45	1.500	1.440	1.040	-0.540	-5.40	1.16	117.75	105.75	0.962
0.50	1.500					-4.24	113.49	101.49	

2. Solution by means of precalculated curves

Fault on:

$$\sin \delta'_0 = 0.3$$

$$p = \frac{0.50 - (0.05)}{0.25} = 2.20$$

$$T = t \sqrt{\frac{1}{57.3} \frac{0.25}{2.5 \times 10^{-4}}} \\ = 4.18 t.$$

Condition at switching instance:

$$T = 0.25 \times 4.18 = 0.84$$

From Figures 7 and 8 of Appendix, for $p = 2.20$, $\delta' = 74.3^\circ$, and

$$\omega = 98 \quad \text{degrees per unit } T$$

$$\delta = \delta' + \delta = 74.3 + 8 = 82.3 \text{ degrees.}$$

Fault cleared:

$$\delta_0 = 82.3 \text{ degrees}$$

$$\delta'_0 = \delta_0 - \delta = 82.3 - 12 = 70.3 \text{ degrees}$$

$$\sin \delta'_0 = 0.941$$

$$p = \frac{0.50 - (-0.40)}{1.50} = 0.60$$

$$T = t \sqrt{\frac{1}{57.3} \frac{1.50}{2.5 \times 10^{-4}}} \\ = 10.25 t$$

$$\omega_0 = 98 \times \frac{4.18}{10.25} = 40 \quad \text{degrees per unit } T.$$

Table 6 contains the information about the angle δ' obtained from the precalculated curves in the Appendix. It also shows that the system is stable since the angle δ' (or the angle δ) decreases after $t = 0.45$.

Table 6. Solution of Example 2 from Precalculated Swing Curves

t	t'	τ	δ'		δ	
			$\sin \delta'_0 = 0.9 \quad \sin \delta'_0 = 0.975 \quad \sin \delta'_0 = 0.941$		$= \delta' + 12$	
0.45	0.2	2.05	101	114	108	120
0.50	0.25	2.55	97	111	104.5	116.5

C. Example 3

This is the same example given in the Westinghouse Transmission and Distribution Reference Book (5, p. 464-468).

Two power stations are connected through a double circuit transmission line and transformers. The capacity of the sending end station is 60,000 KVA, while the rating of the machines at the receiving end station is 125,000 KVA, with a shunt load of 115,000 KW at 0.85 power factor located at the receiving end bus. A double-line-to-ground fault occurs at the receiving end of one of the transmission lines. The fault is cleared in 0.15 second by isolating the faulty line.

The system is essentially a two-finite-machine system connected through an impedance network. Choosing a base power of 60,000 KVA, the rating of the sending-end machine is 1.0 per unit and the rating of the receiving-end machine is 2.08 per unit.

The following data concerning the machines and the initial conditions are taken from the example as solved in the Westinghouse Reference Book.

G = KVA rating of machine in per unit.

H = Inertia constant of machine in kilowatt-second per KVA.

Subscripts s and r refer to sending end and receiving end machines, respectively. f is the frequency.

$$H_s = 3 \quad \text{per unit}$$

$$H_r = 5 \quad \text{per unit}$$

$$M = \frac{GH}{180f}$$

$$M_s = \frac{3}{180 \times 60} \quad \text{per unit}$$

$$M_r = \frac{2.08 \times 5}{180 \times 60} \quad \text{per unit}$$

$$P_{is} = 0.917 \quad \text{per unit}$$

$$P_{ir} = 1.083 \quad \text{per unit}$$

$$\delta_{os} = 30 \quad \text{degrees}$$

$$\delta_{or} = 4.7 \quad \text{degrees .}$$

The power equations during the fault and after the fault is cleared are then given. It is to be noted here that these power equations contain information concerning the quantities P_e , P_M , and δ .

During the fault:

$$P_s = 0.1753 + 0.335 \sin (\delta_s - 0.7^\circ)$$

$$P_r = -1.797 + 0.335 \sin (\delta_r + 0.7^\circ).$$

After clearing the fault:

$$P_s = 0.276 + 1.092 \sin (\delta_s - 3.2^\circ)$$

$$P_r = -1.726 + 1.092 \sin (\delta_r + 3.2^\circ).$$

1. Constants of the system

First the system is to be reduced to an equivalent one machine against an infinite bus. The equivalent angular momentum and input power is given by

$$\begin{aligned} M &= \frac{M_s M_r}{M_s + M_r} \\ &= \frac{1}{180 \times 60} \left[\frac{3 \times 5 \times 2.08}{3 + 5 \times 2.08} \right] \\ &= 2.16 \times 10^{-4} \quad \text{per unit} \end{aligned}$$

$$\begin{aligned} P_i &= \frac{M_r P_{is} - M_s P_{ir}}{M_s + M_r} \\ &= \frac{5 \times 2.08 \times 0.917 - 3 \times 1.083}{13.4} \\ &= 0.461 \quad \text{per unit.} \end{aligned}$$

The equivalent values of P_e , P_M , and δ for conditions during the fault and after the fault is cleared are then to be determined.

Fault on:

$$E_s^2 Y_{ss} \cos \theta_{ss} = 0.1753$$

$$E_r^2 Y_{rr} \cos \theta_{rr} = -1.797$$

$$E_s E_r Y_{sr} = 0.335$$

$$\theta_{sr} = 89.3^\circ$$

$$P_c = \frac{M_r(E_s^2 Y_{ss} \cos \theta_{ss}) - M_s(E_r^2 Y_{rr} \cos \theta_{rr})}{M_r + M_s}$$

$$= \frac{10.4 \times 0.1753 - 3 \times (-1.797)}{13.4}$$

$$= -0.267$$

$$P_M = E_s E_r Y_{sr} \frac{\sqrt{M_s^2 + M_r^2 - 2M_s M_r \cos 2\theta_{sr}}}{M_r + M_s}$$

$$= \frac{0.335}{13.4} \sqrt{(10.4)^2 + (3)^2 - 2 \times 3 \times 10.4 (-0.9997)}$$

$$= 0.335$$

$$\delta = -\tan^{-1} \left(\frac{M_s + M_r}{M_s - M_r} \tan \theta_{sr} \right) - 90^\circ$$

$$= -\tan^{-1} \left(\frac{13.4}{-7.4} \times 81.8 \right) - 90^\circ$$

$$= 89.6^\circ - 90^\circ = -0.4^\circ$$

Fault cleared:

$$E_s^2 Y_{ss} \cos \theta_{ss} = 0.276$$

$$E_r^2 Y_{rr} \cos \theta_{rr} = -1.726$$

$$E_s E_r Y_{sr} = 1.092$$

$$\theta_{sr} = 86.8$$

$$P_c = \frac{10.4(0.276) - 3(-1.726)}{13.4}$$

$$= \frac{2.870 - 5.184}{13.4}$$

$$= -0.173$$

$$P_M = \frac{1.092}{13.4} \sqrt{117 - 2 \times 3 \times 10.4 (-0.994)}$$

$$= 1.092$$

$$\begin{aligned}
\delta &= -\tan^{-1}\left(\frac{13.4}{-7.4} \times 17.9\right) - 90^\circ \\
&= 88.2^\circ - 90^\circ \\
&= -1.8^\circ.
\end{aligned}$$

2. Solution by means of precalculated curves

To obtain the values of δ and ω at the switching instance, i.e., when the fault is cleared

$$\begin{aligned}
\delta_0 &= 30 - 4.7 = 25.3 && \text{degrees} \\
\delta'_0 &= 25.3 - \delta \\
&= 25.3 + 0.4 = 25.7 && \text{degrees} \\
\sin \delta'_0 &= 0.434 \\
p &= \frac{P_1 - P_0}{P_M} \\
&= \frac{0.461 - (-0.267)}{0.335} = 2.18 \\
T &= t \sqrt{\frac{1}{57.3} \frac{P_M}{M}} \\
&= t \sqrt{\frac{1}{57.3} \frac{0.335}{2.16 \times 10^{-4}}} \\
&= 5.22 t.
\end{aligned}$$

At $t = 0.15$ second, $T = 0.78$

for $\sin \delta'_0 = 0.4$, $\omega_0 = 0$, $p = 2.18$, $\delta' = 53.0^\circ$, $\omega = 75.0^\circ$ per T

for $\sin \delta'_0 = 0.5$, $\omega_0 = 0$, $p = 2.18$, $\delta' = 59^\circ$, $\omega = 68.5^\circ$ per T

therefore, for $\sin \delta'_0 = 0.434$, $\omega_0 = 0$, $p = 2.18$, $\delta' = 55.0^\circ$,

$$\omega = 72.5^\circ \text{ per } T$$

$$\delta = 55.0 - 0.4 = 54.6^\circ.$$

Fault cleared:

$$\delta_o = 54.6^\circ$$

$$\delta'_o = 54.6 + 1.8 = 56.4^\circ$$

$$\sin \delta'_o = 0.834$$

$$p = \frac{0.461 + 0.173}{1.092} = 0.59$$

$$T = t \sqrt{\frac{1}{57.3} \frac{1.092}{2.16 \times 10^{-4}}} = 9.4 t$$

$$\omega_o = 72.5 \times \frac{5.22}{9.4} = 40 \quad \text{degrees per unit } T.$$

Values of δ' obtained from the precalculated curves and the values of δ are tabulated in Table 7.

Table 7. Solution of Example 3 from Precalculated Curves

t	t'	T	δ'			δ = $\delta' - 1.8$
			$\sin \delta'_o = 0.8$	$\sin \delta'_o = 0.9$	$\sin \delta'_o = 0.834$	
0.35	0.2	1.88	94.5	85.0	98.0	96.2
0.45	0.3	2.82	84	88.0	85.4	83.6

From Table 7, it is seen that the angle δ decreases after reaching a maximum. Therefore the system is stable. The step-by-step solution given in p. 488 of the Westinghouse Reference Book arrives at the same conclusion. Also the values of δ at the two above instances are 97.5 and 80.4 degrees respectively. These values agree with the values

given in Table 7.

D. Discussion of Results

The three examples discussed above were selected to represent three different types of two-machine power systems, namely, a system of one machine against an infinite bus with reactive network, a system of one machine connected to an infinite bus through a general impedance network, and a general two-machine system.

As was seen from the solutions given, the amount of calculations involved in solving transient stability problems for these types of systems varies considerably. Therefore, comparison of solutions for each type of system may be useful. The results will be examined in the light of two factors, namely, the accuracy, and the amount of calculations involved.

The accuracy of the step-by-step method of solution was discussed before in the text of this dissertation. The accuracy of the solutions with the aid of the precalculated swing curves are to be determined by comparing them with the results obtained from the step-by-step method, since the precalculated curves were themselves obtained by a step-by-step process.

In the three examples, the same general results were arrived at with both methods of solutions, namely, that a particular system is stable or unstable under a particular transient. Moreover, values of the angle δ at corresponding instances agreed reasonably well for both

types of solutions and for the three systems. In most cases the angle δ agreed within one degree, and the greatest discrepancy was 3.2 degrees. Indeed with all the approximations involved in transient stability studies such results are considered excellent.

To determine whether the use of the precalculated solutions is justified as a time saver it is felt that each type of system has to be discussed separately.

For a system of one machine connected to an infinite bus through a reactive network, the calculations involved in the step-by-step process are relatively simple. The amount of calculations saved by using the precalculated curves may not be appreciable, especially if a considerable amount of interpolation between curves is needed.

When the network between the machines contains resistance, the calculations become more cumbersome as seen in Examples 2 and 3. This is where the use of the precalculated curves can be of great advantage. Not only is the amount of calculations reduced, but the possibility of error in results (due to some miscalculations) is also reduced.

Another point to be considered in this respect is the fact that the step-by-step method requires calculating a complete swing curve, with many points, to determine whether a system is stable, while in the other method of solution only two or three points are needed to reach the same result. As an illustration, only two points were needed in Example 3 to determine that the system is stable while many more points were needed in the step-by-step process.

In the author's opinion, the greatest advantage the precalculated

solutions offer, is when several stability studies are needed for one particular system. When using the precalculated solutions, often the same constants P_M , P_C , δ , etc....., for the different switching operations, are used for more than one study, while the step-by-step process requires a separate swing curve for each study. For this reason, the precalculated solutions can be particularly useful in determining critical switching times, transient stability limits, predicting better system layouts, and similar problems.

VII. CONCLUSION

A universal set of swing curves for two-machine transient stability problems has been developed. The curves can be used where the network between the machines is changed due to switching operations that may take place as a result of the occurrence of the transient.

The curves included in this dissertation have been selected to cover almost all practical problems. Their use yields results that agree favorably with the usual step-by-step method of solution.

With the aid of the curves, the amount of work involved in two-machine transient stability studies can be reduced considerably, especially when the network between the machines contains resistance, and when several studies of one particular system are needed.

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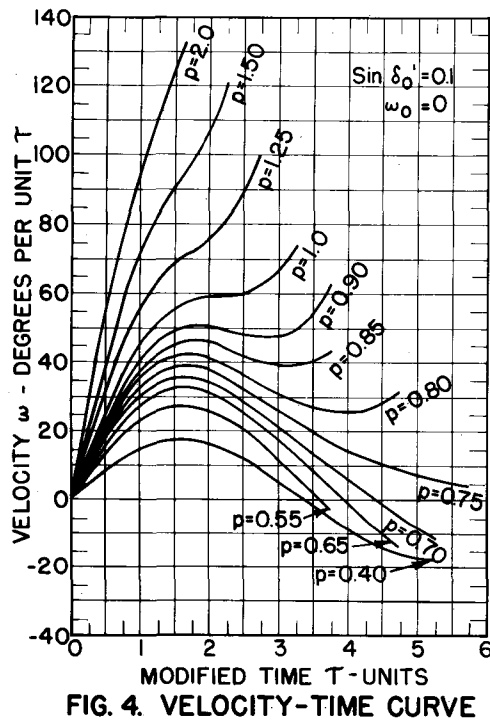
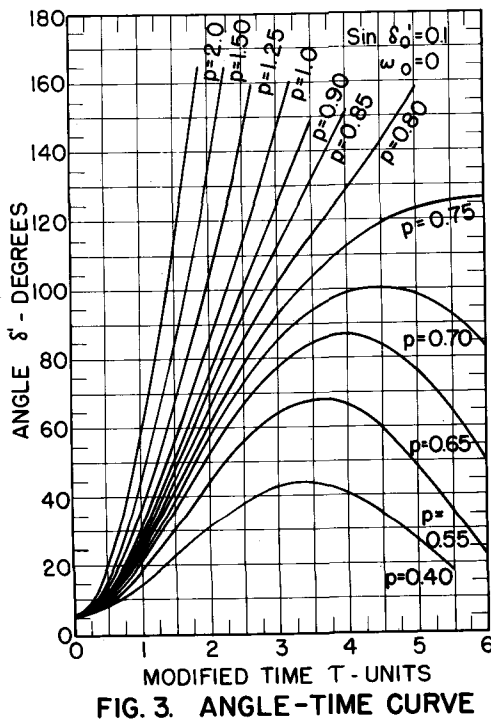
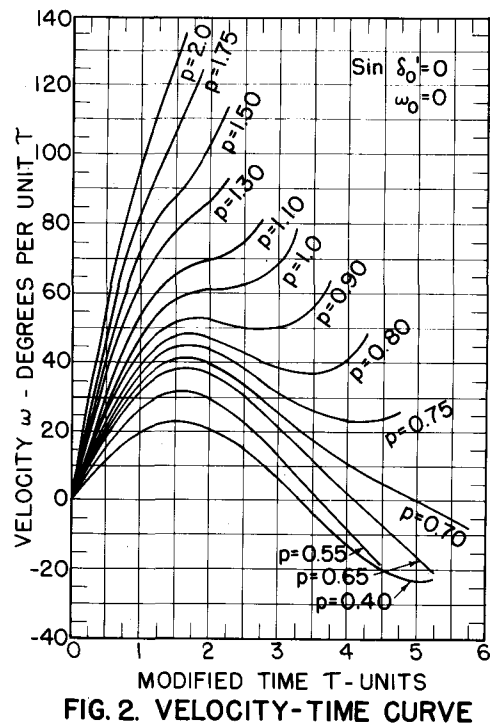
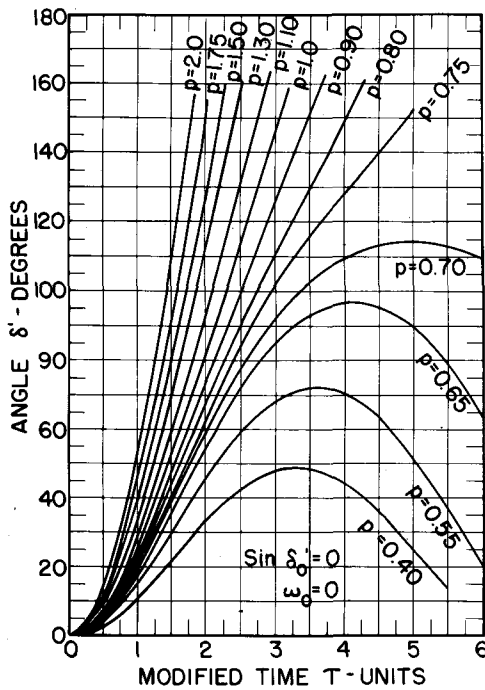
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X. APPENDIX



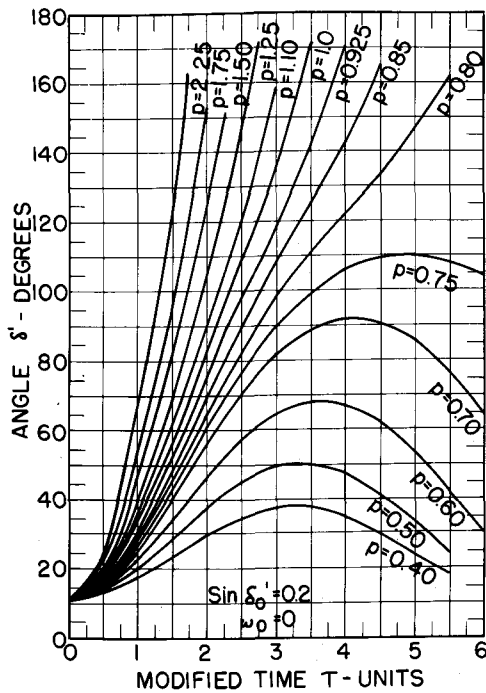


FIG. 5. ANGLE-TIME CURVE

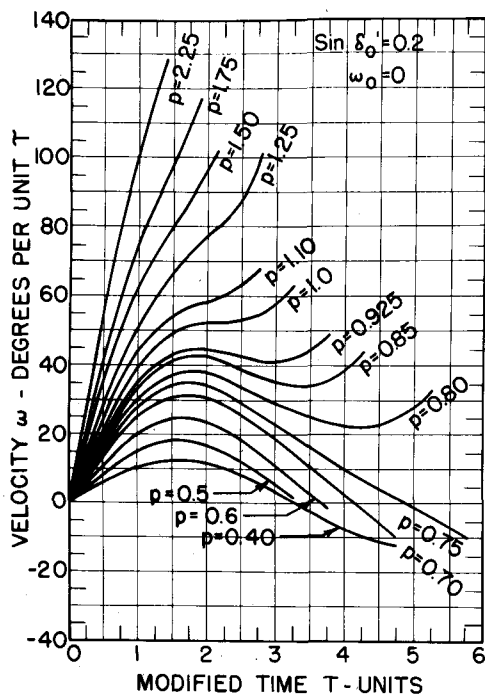


FIG. 6. VELOCITY-TIME CURVE

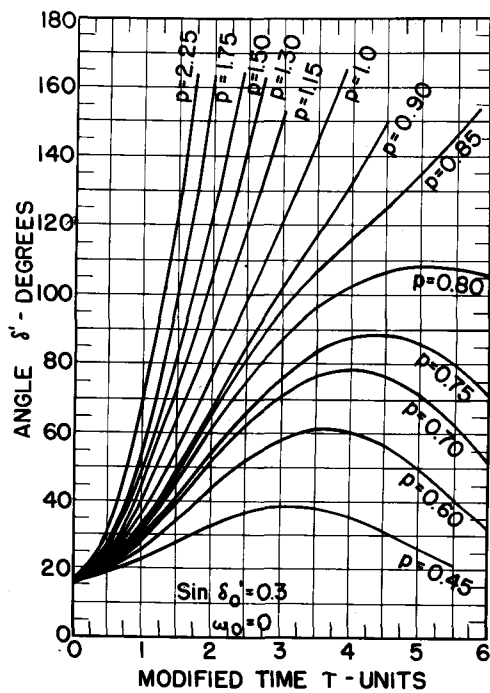


FIG. 7. ANGLE-TIME CURVE

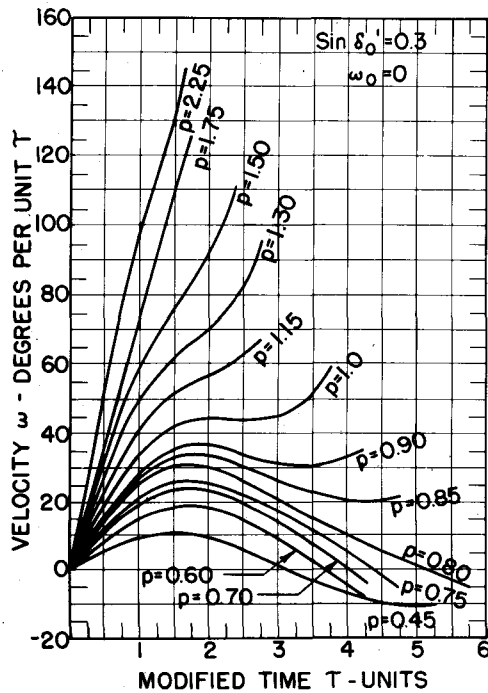
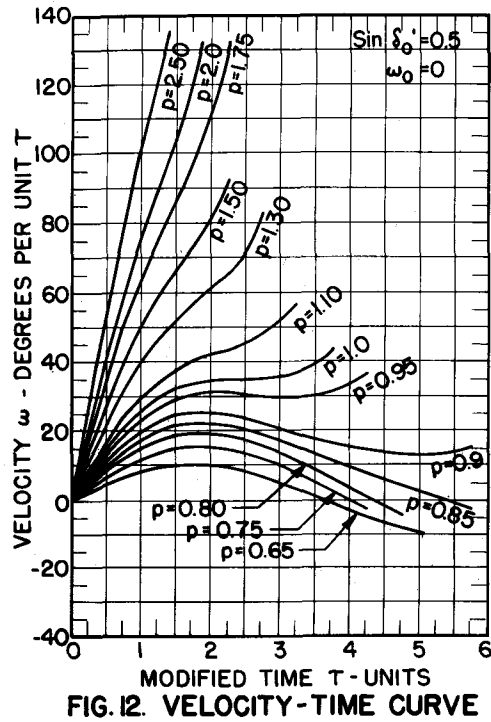
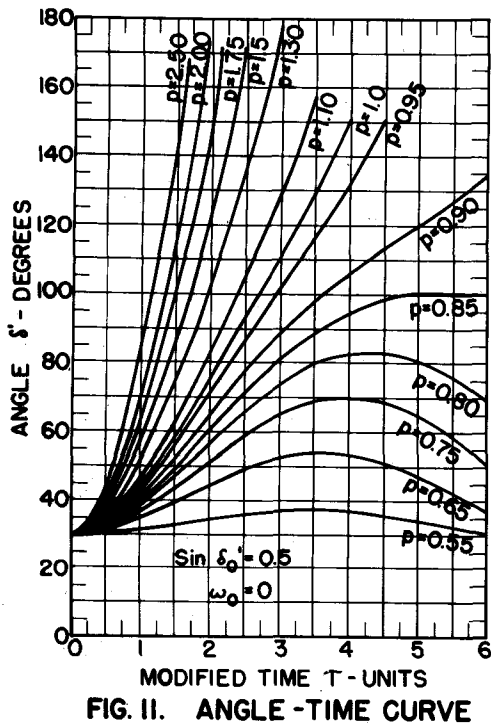
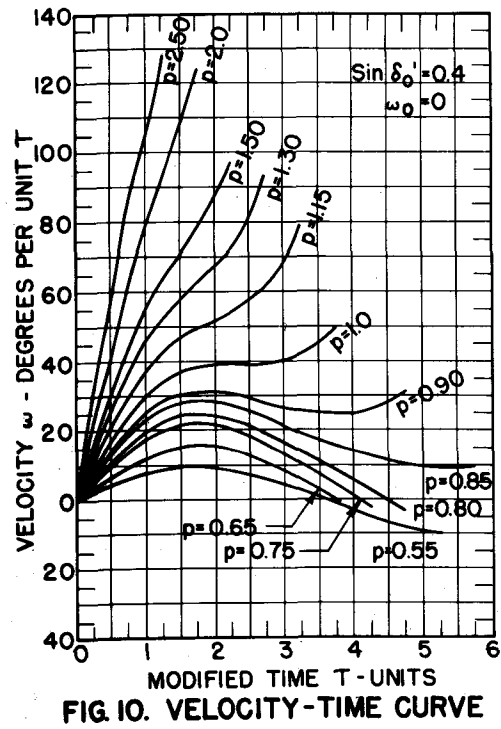
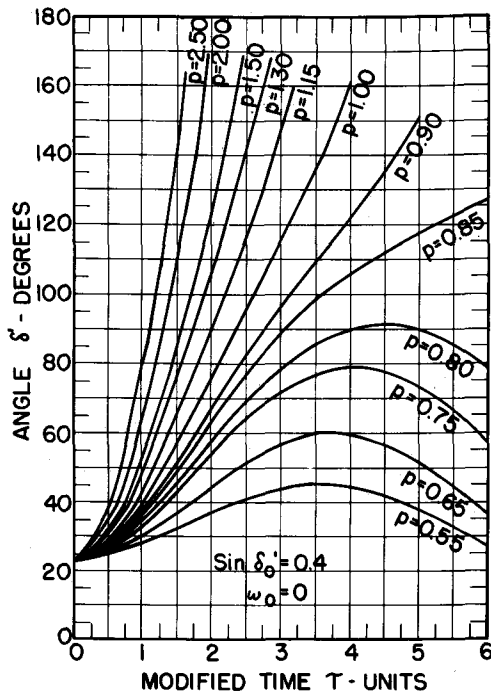


FIG. 8. VELOCITY-TIME CURVE



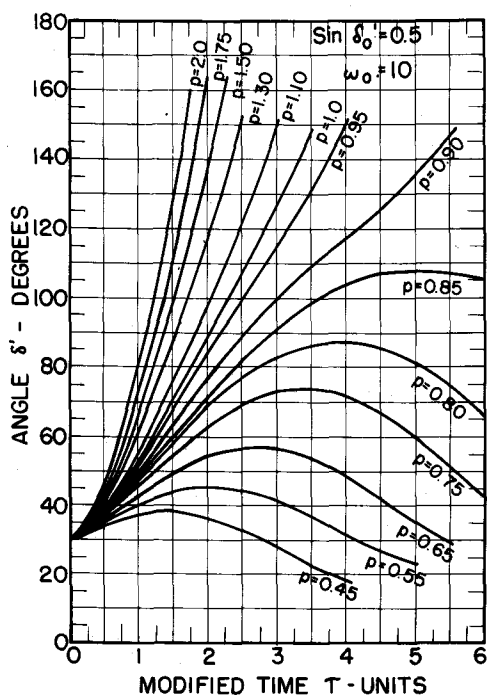


FIG. 13. ANGLE-TIME CURVE

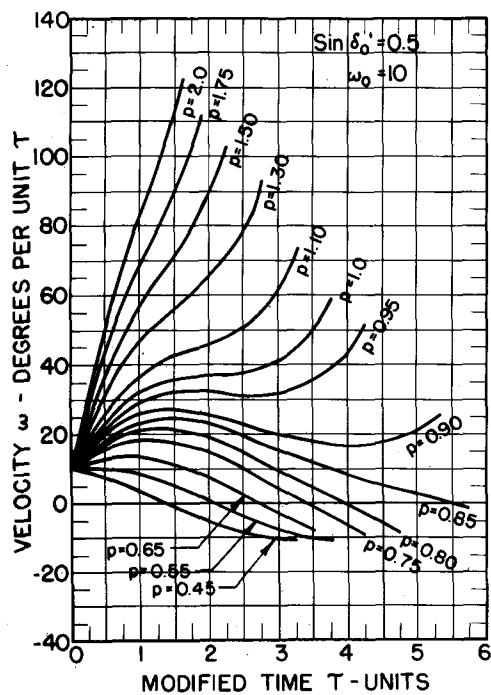


FIG. 14. VELOCITY-TIME CURVE

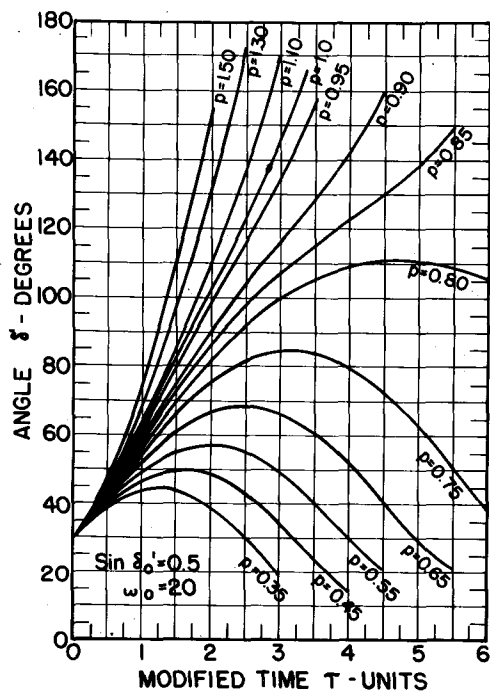


FIG. 15. ANGLE-TIME CURVE

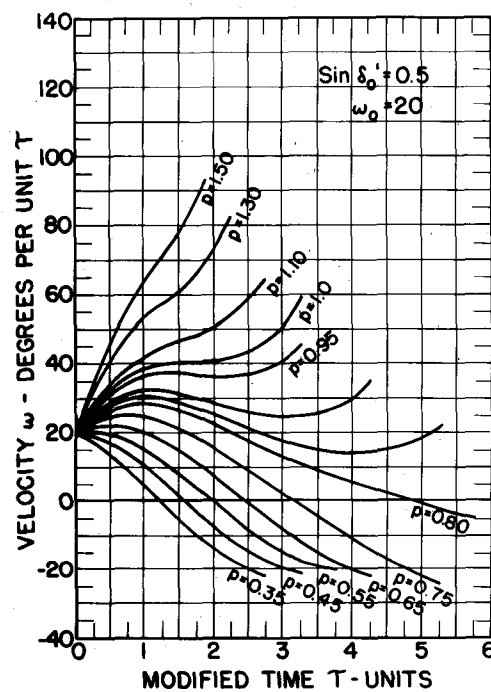
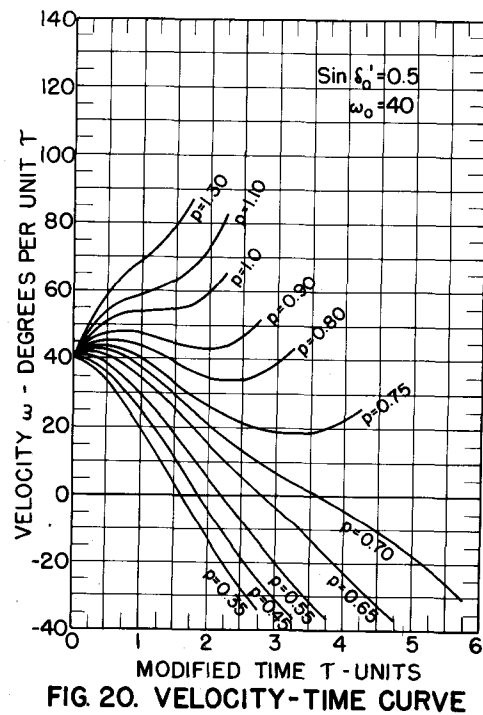
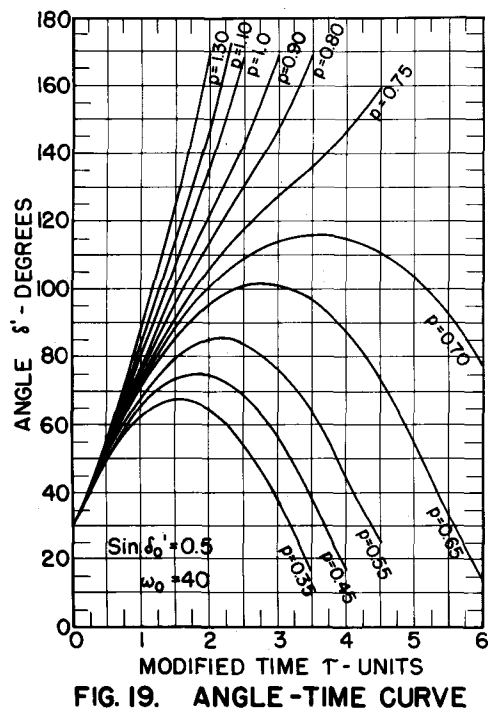
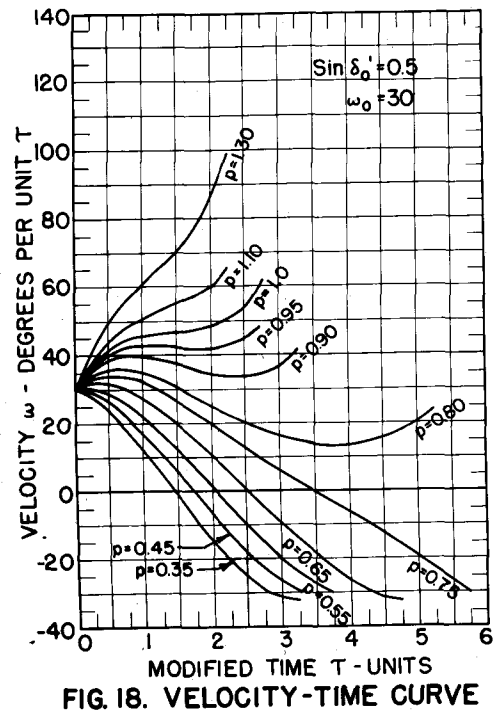
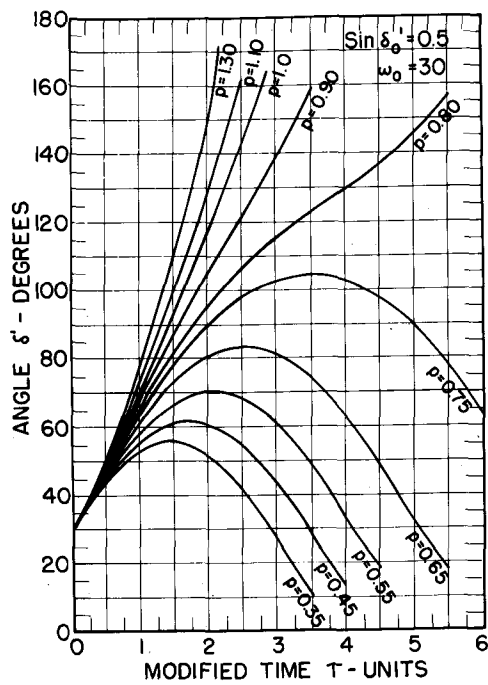
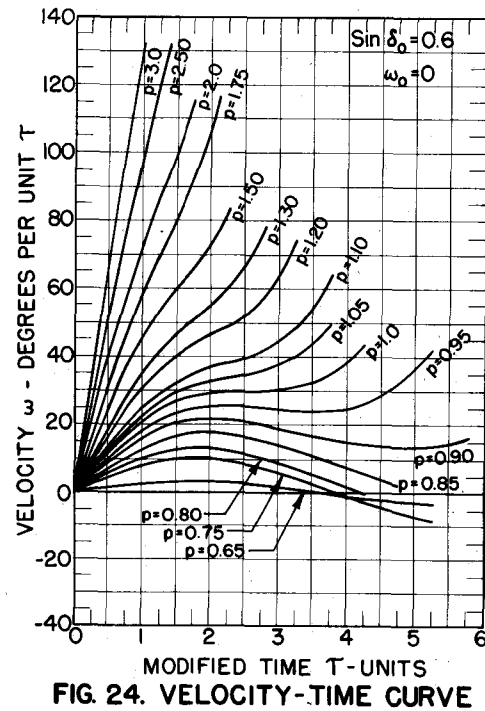
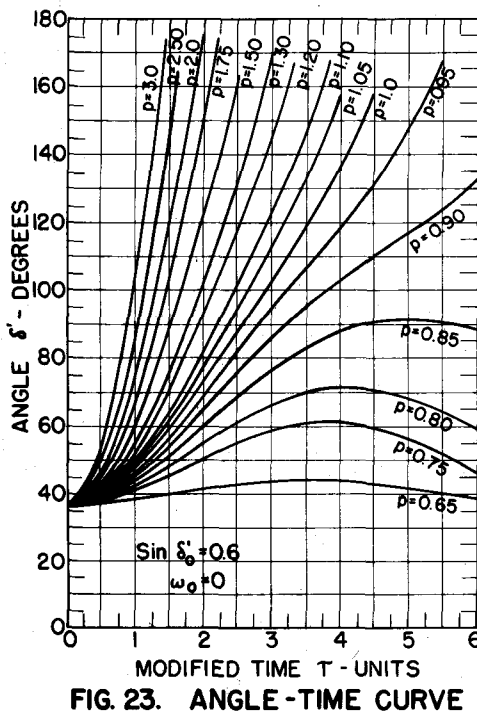
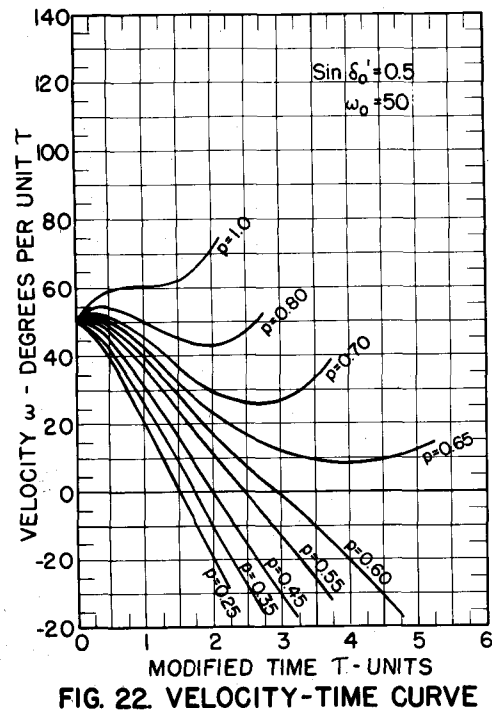
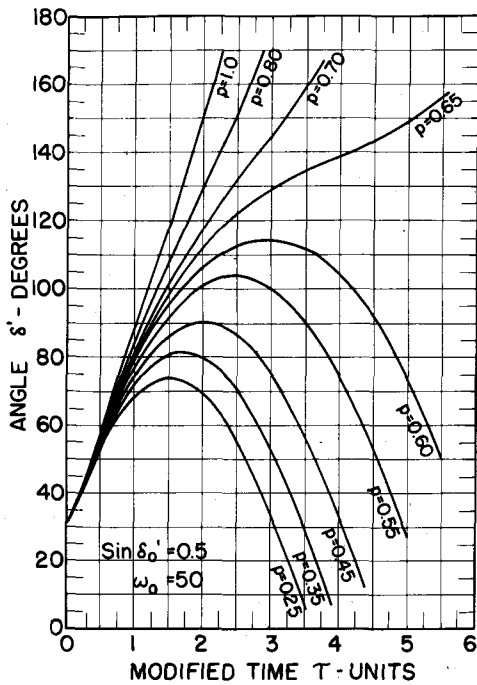


FIG. 16. VELOCITY-TIME CURVE





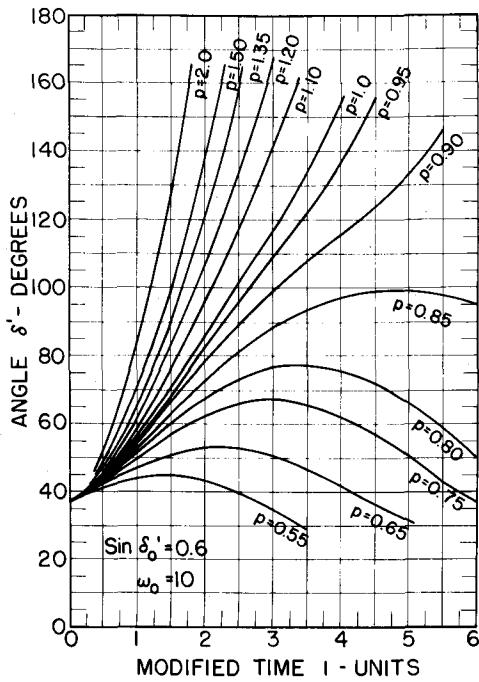


FIG. 25. ANGLE-TIME CURVE

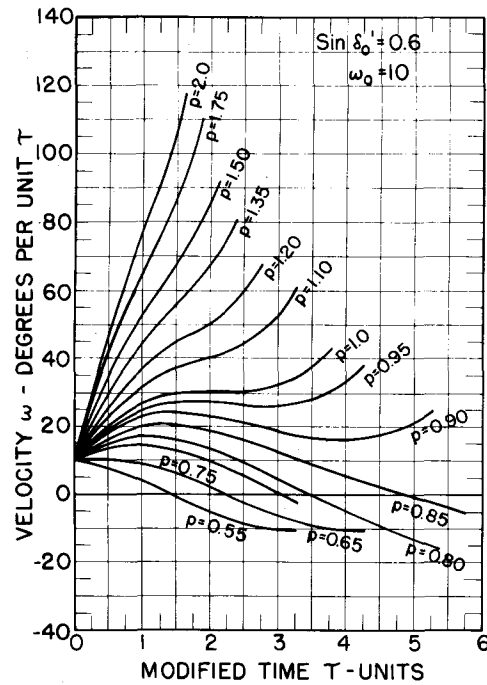


FIG. 26. VELOCITY-TIME CURVE

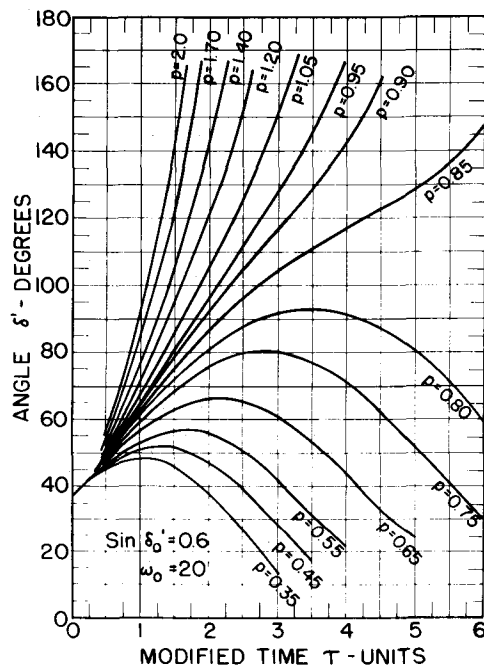


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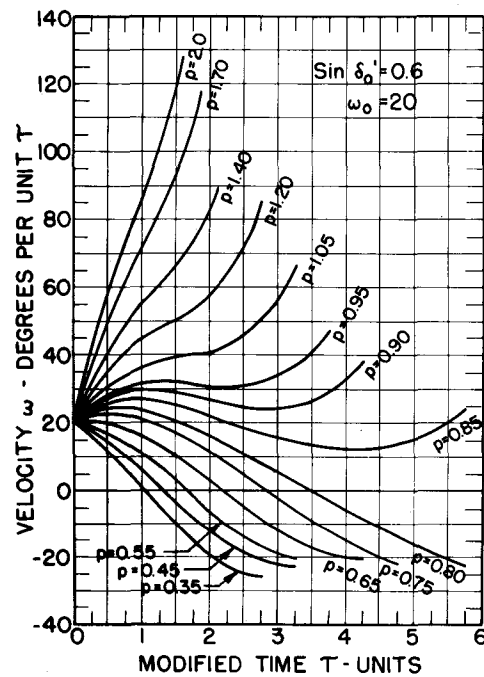
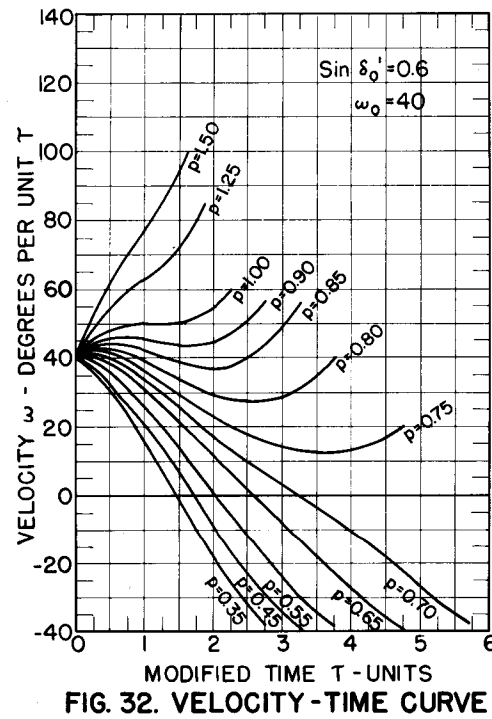
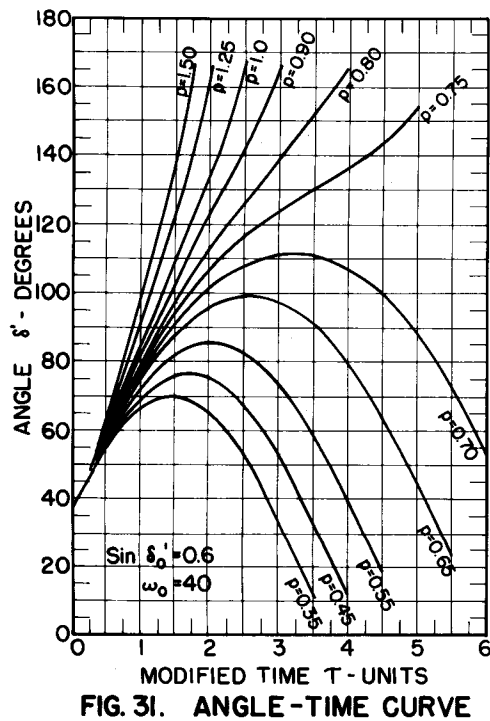
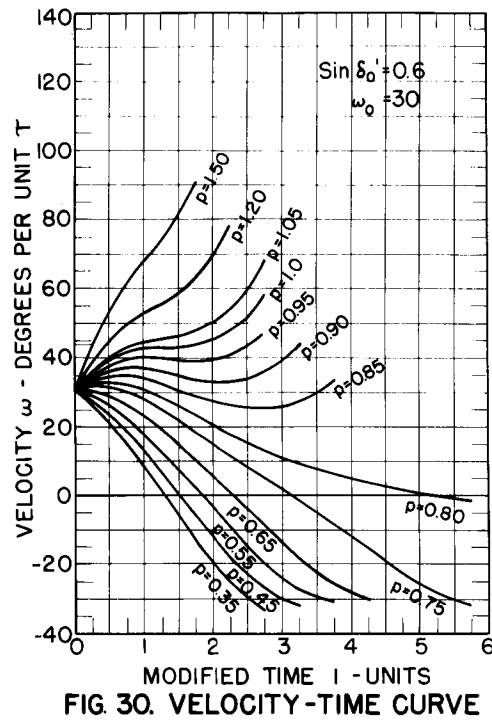
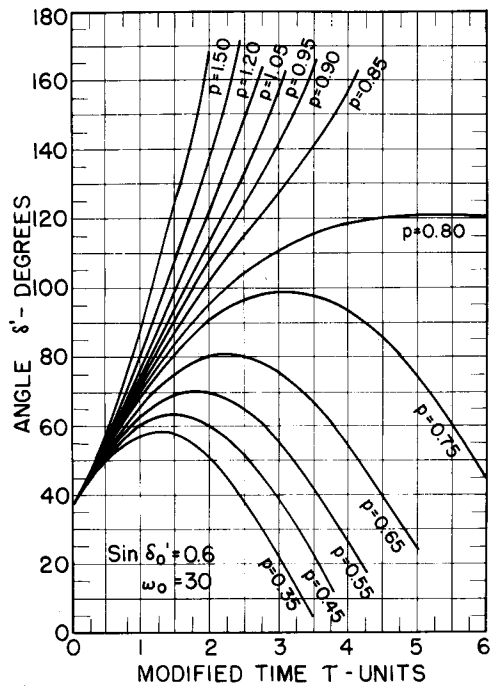


FIG. 28. VELOCITY-TIME CURVE



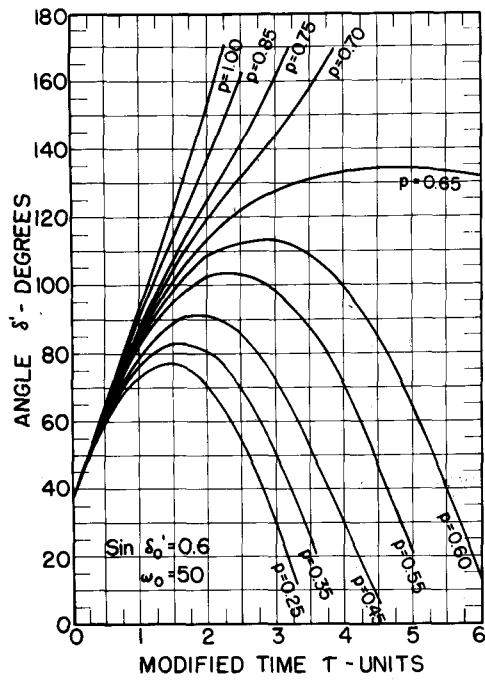


FIG. 33. ANGLE-TIME CURVE

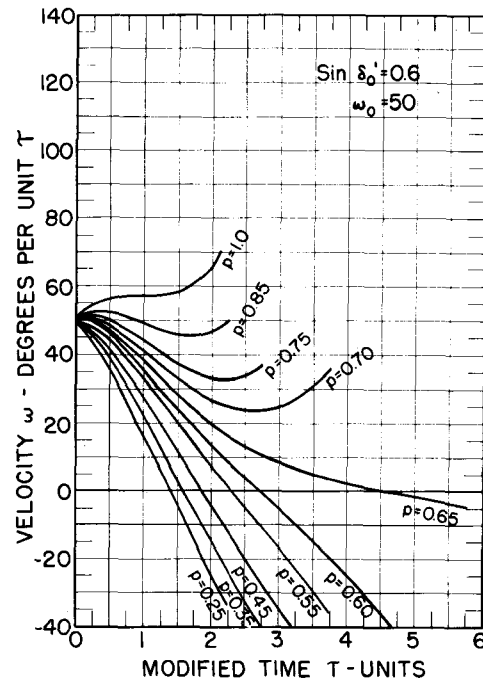


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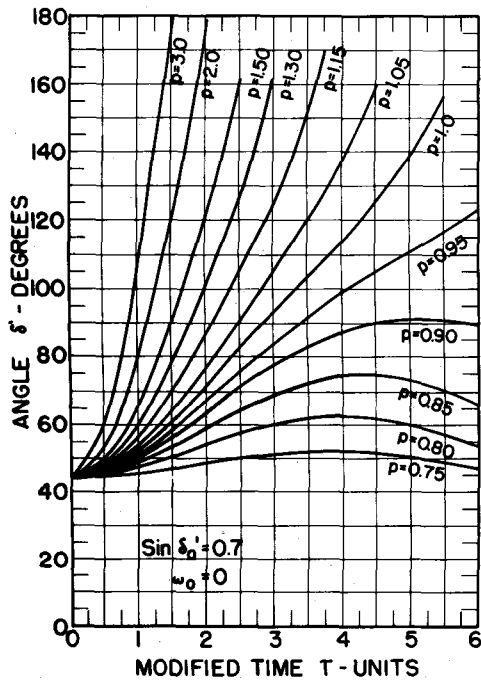


FIG. 35. ANGLE-TIME CURVE

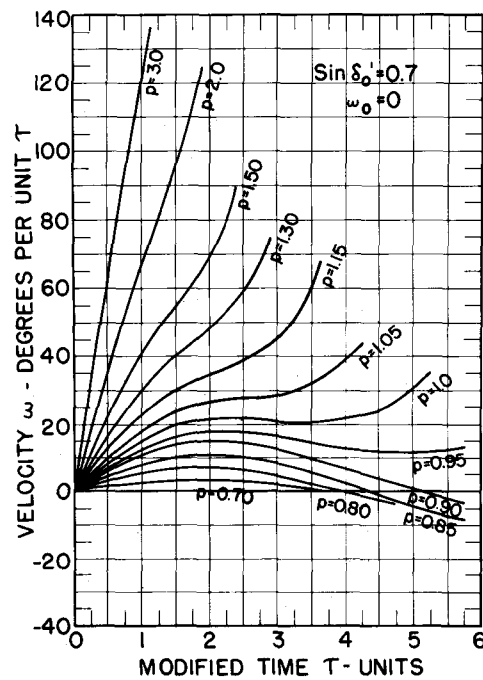


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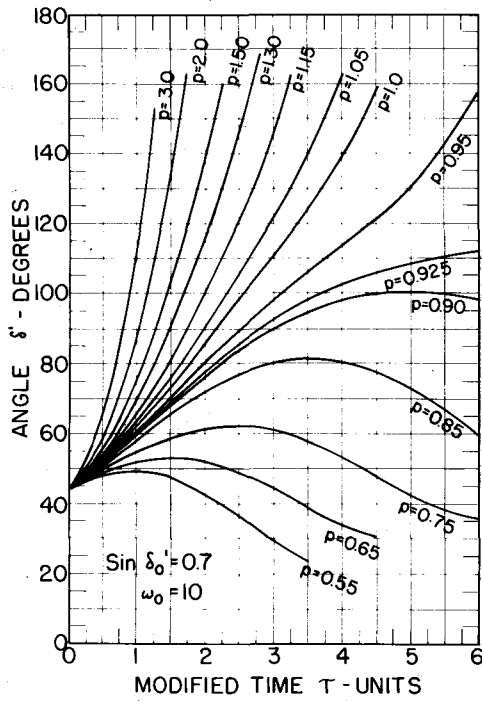


FIG. 37. ANGLE-TIME CURVE

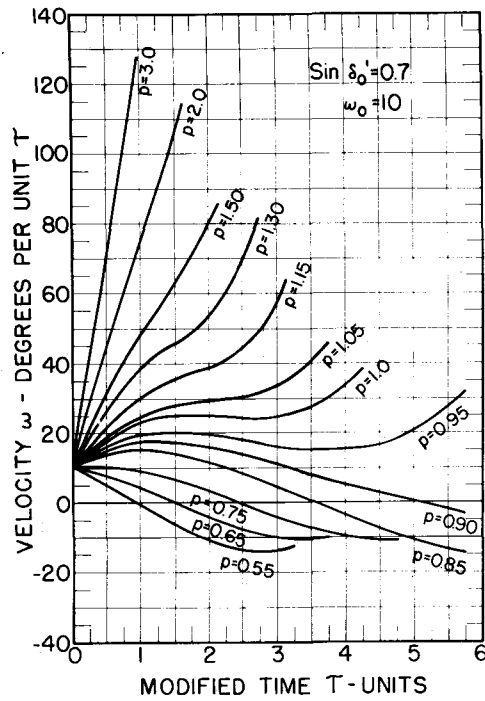


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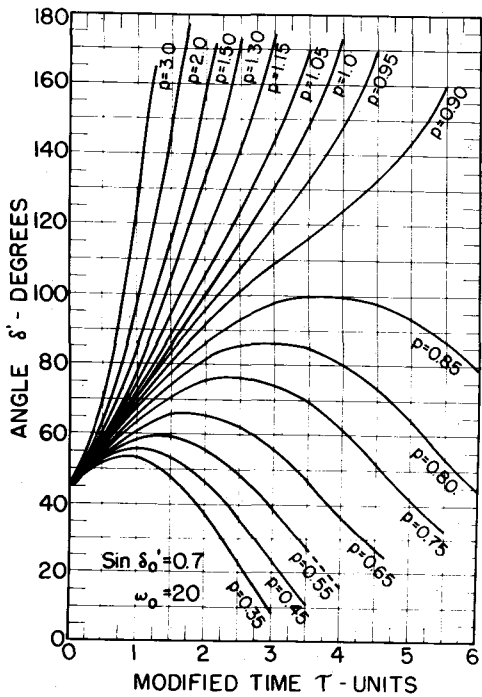


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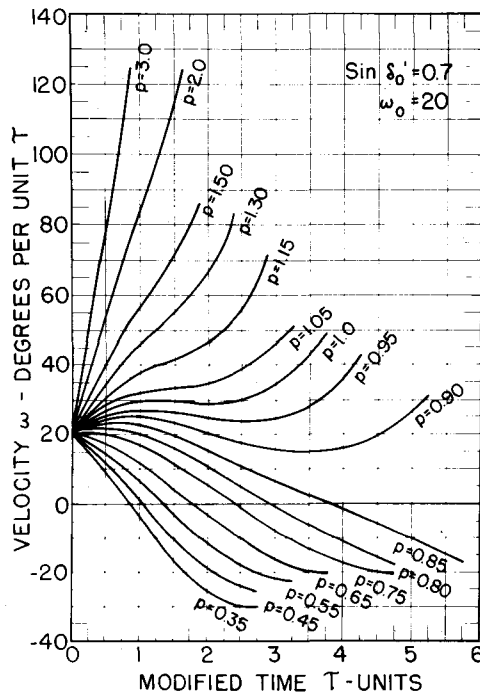
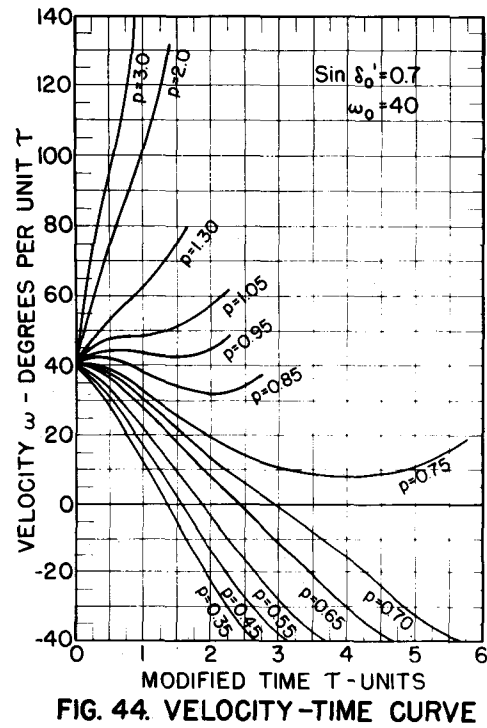
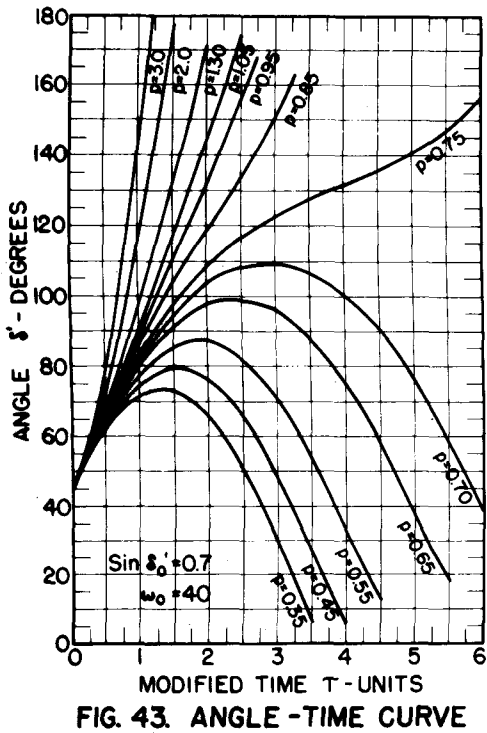
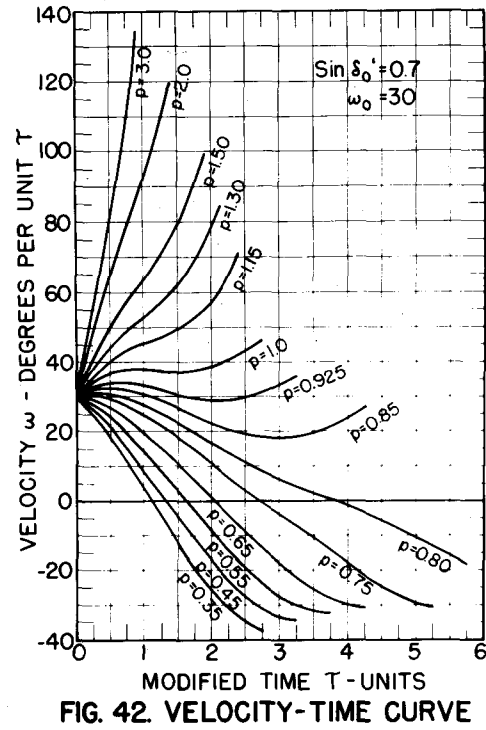
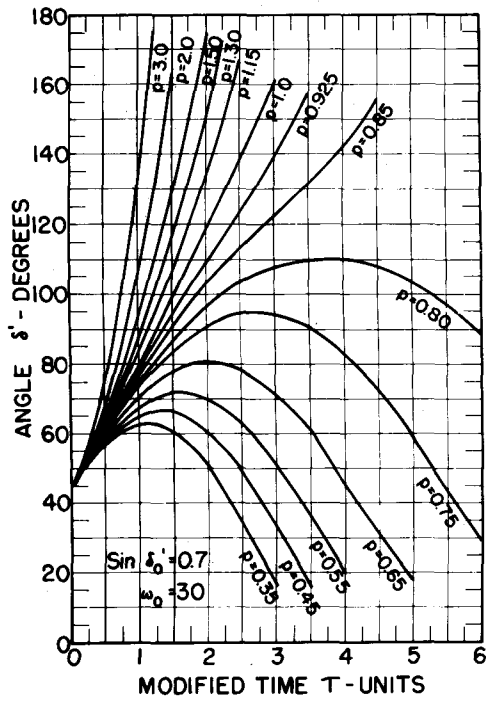
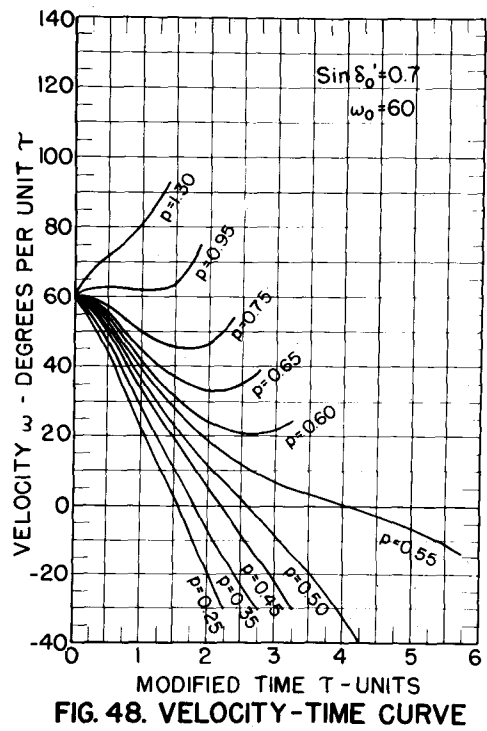
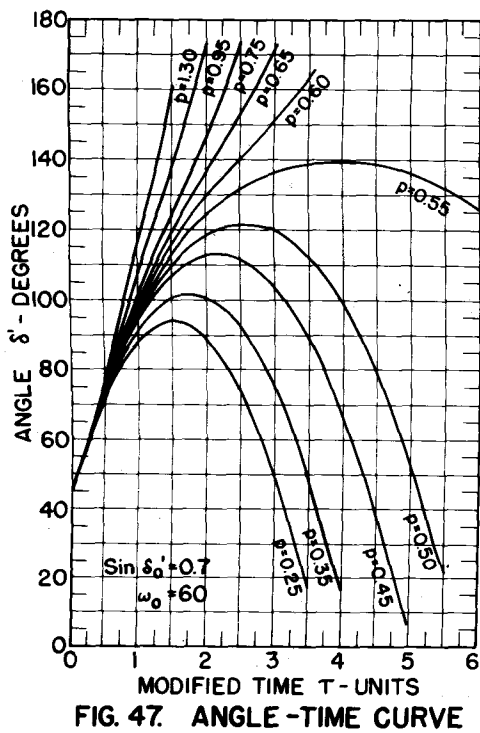
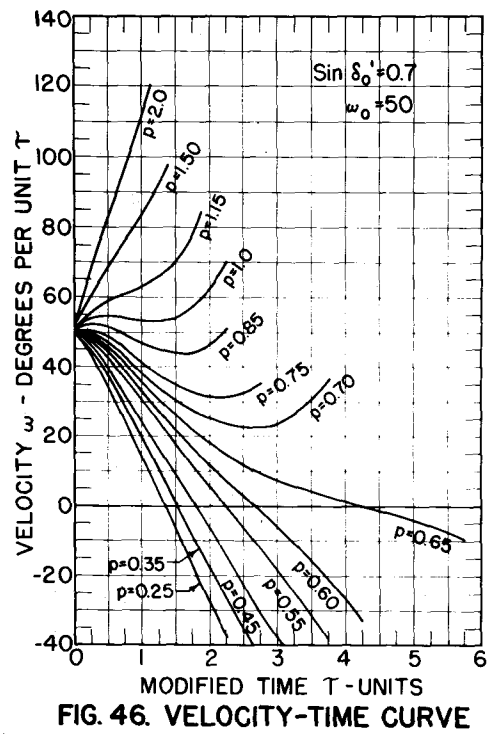
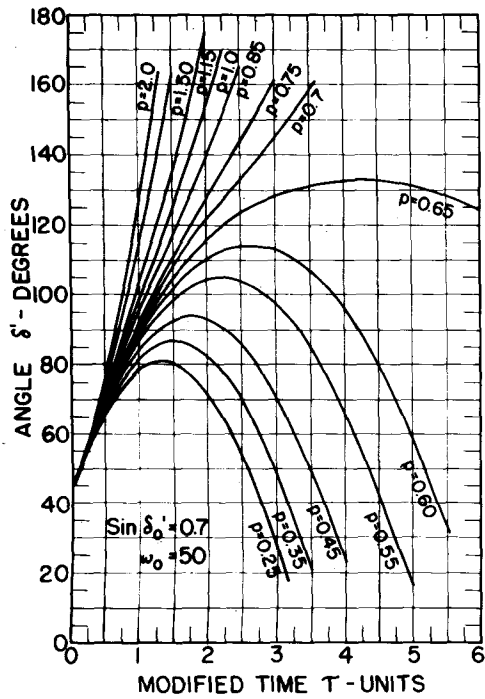


FIG. 40. VELOCITY-TIME CURVE





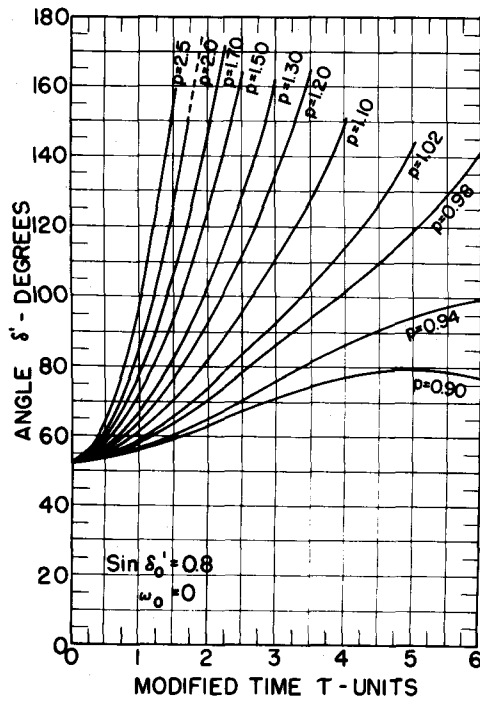


FIG. 49. ANGLE-TIME CURVE

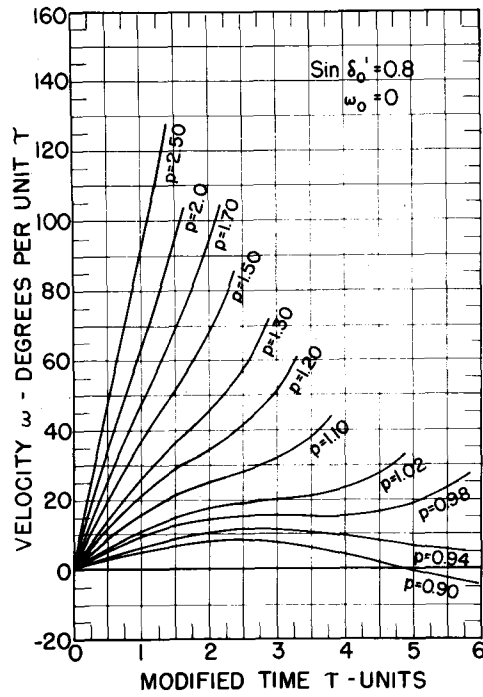


FIG. 50. VELOCITY-TIME CURVE

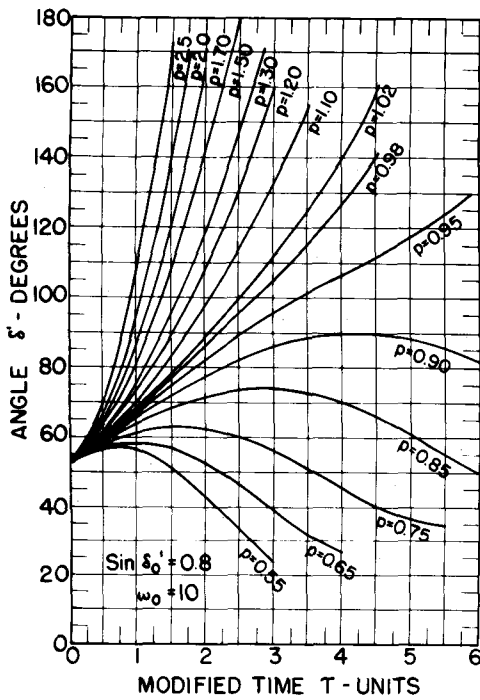


FIG. 51. ANGLE-TIME CURVE

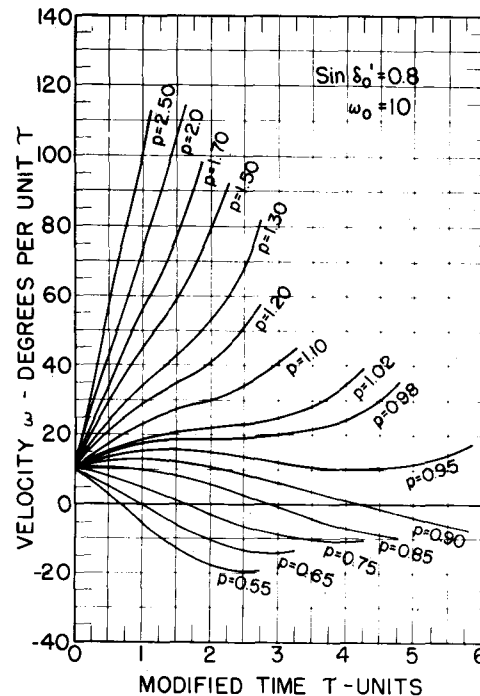


FIG. 52. VELOCITY-TIME CURVE

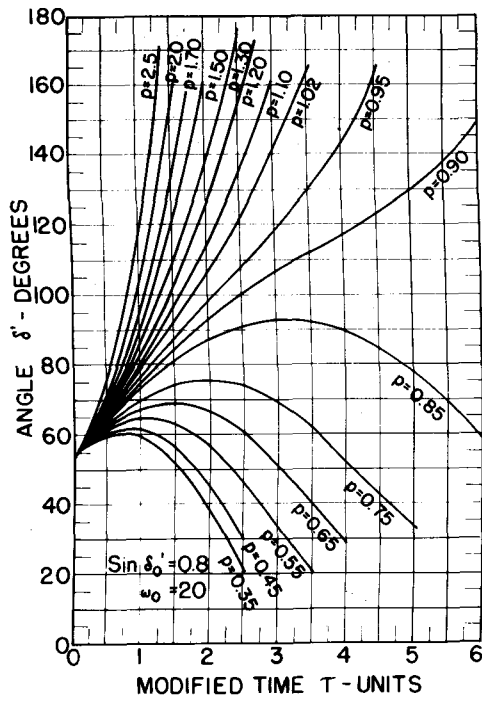


FIG. 53. ANGLE-TIME CURVE

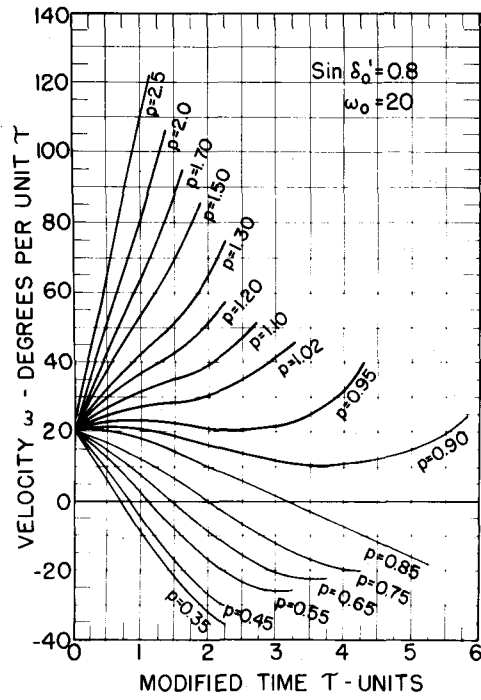


FIG. 54. VELOCITY-TIME CURVE

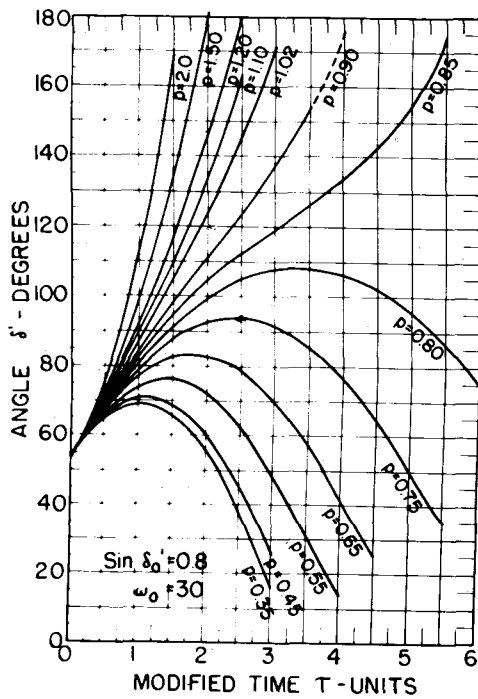


FIG. 55. ANGLE-TIME CURVE

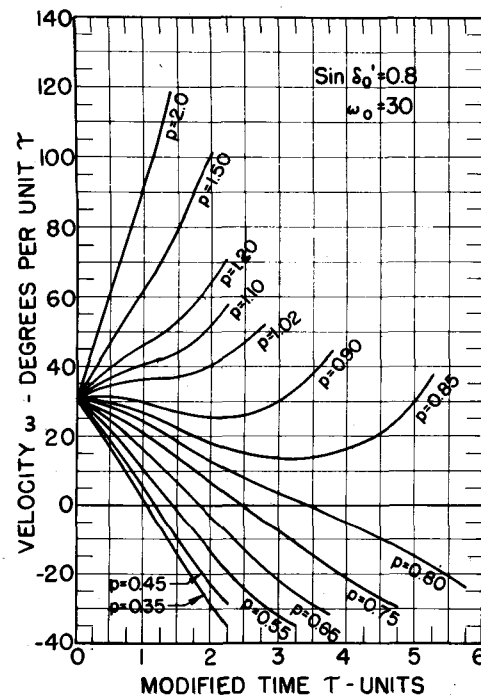
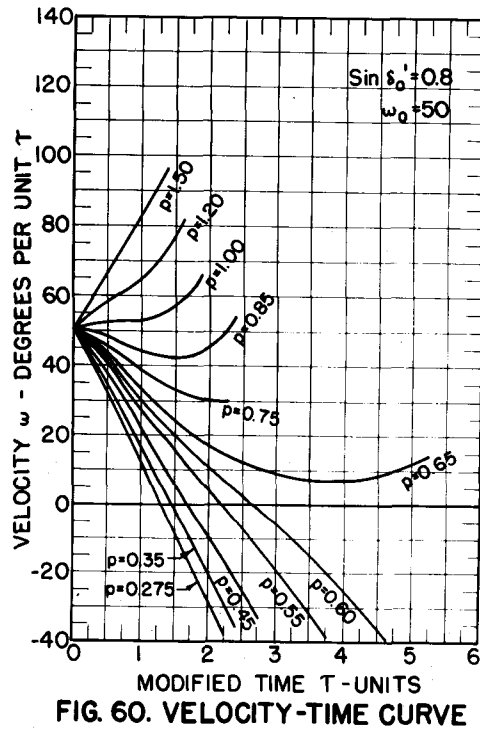
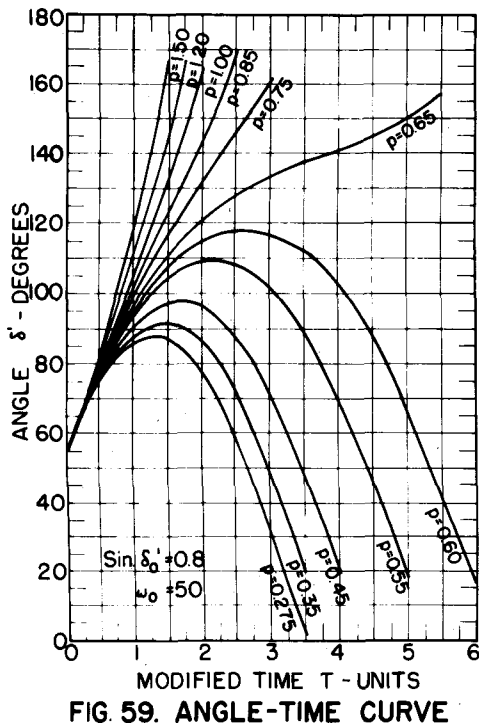
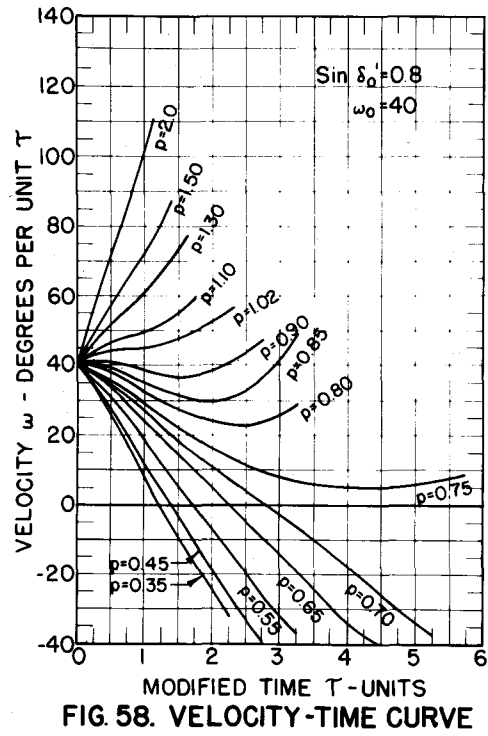
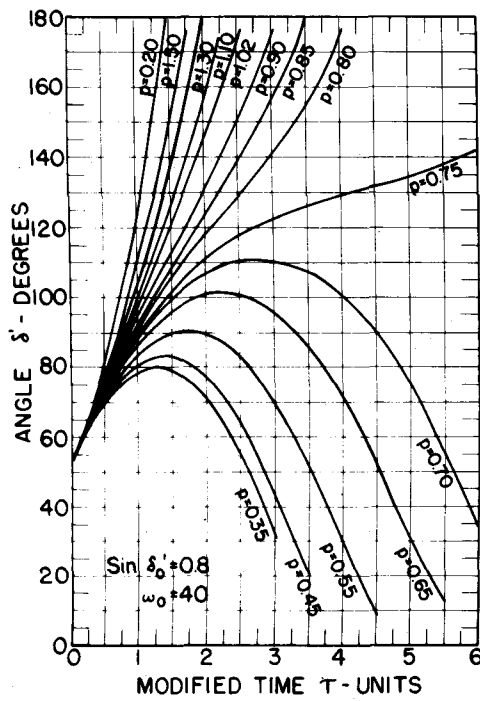
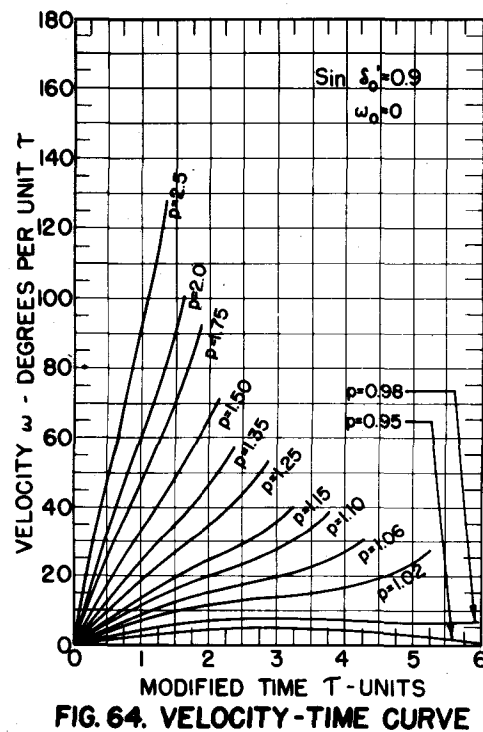
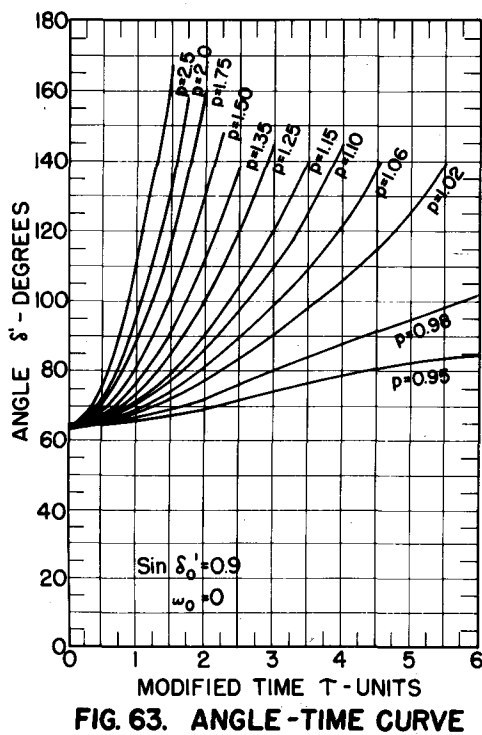
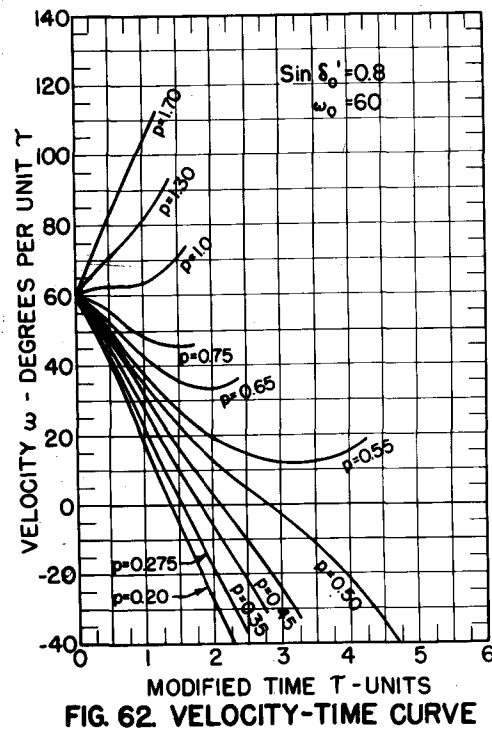
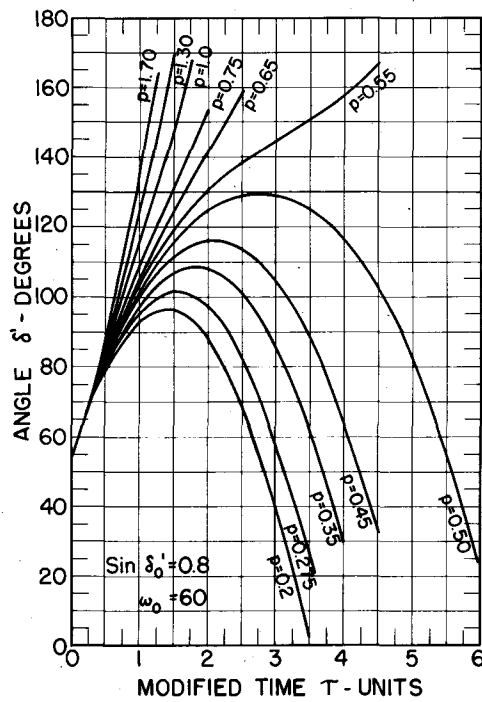
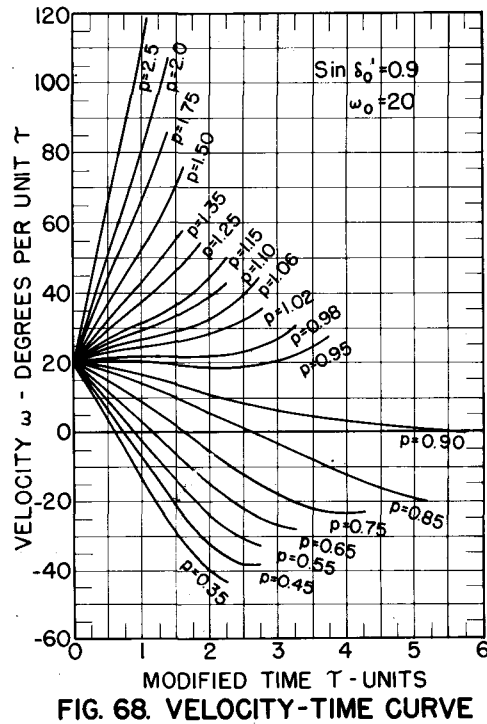
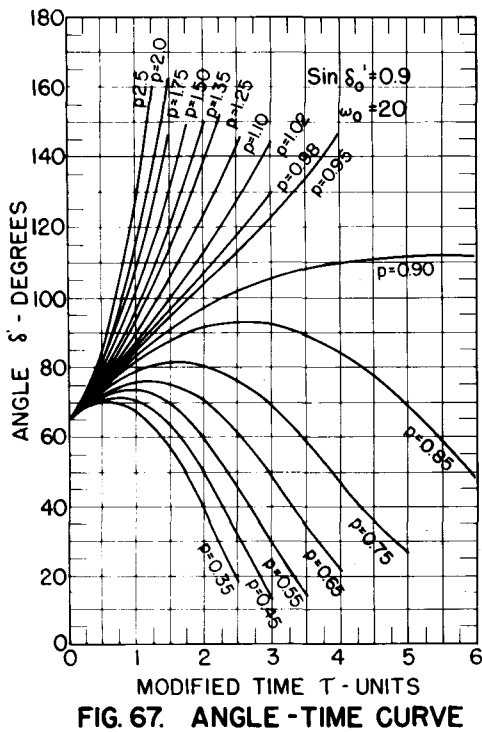
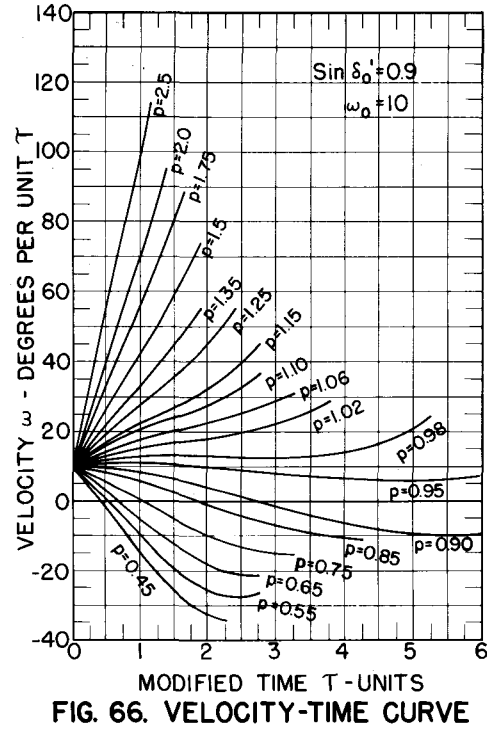
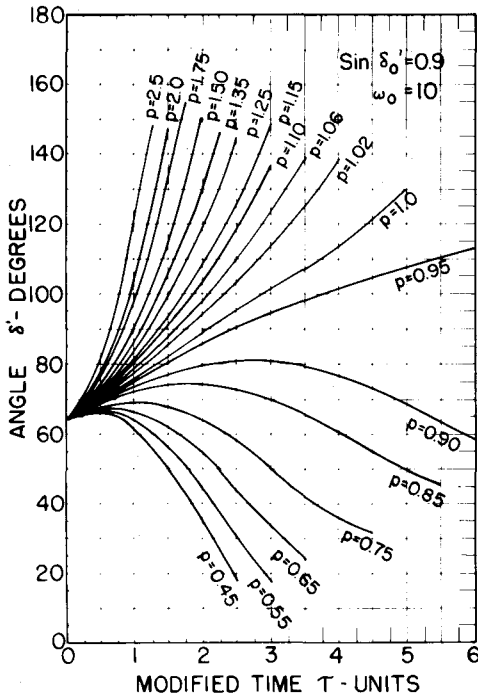
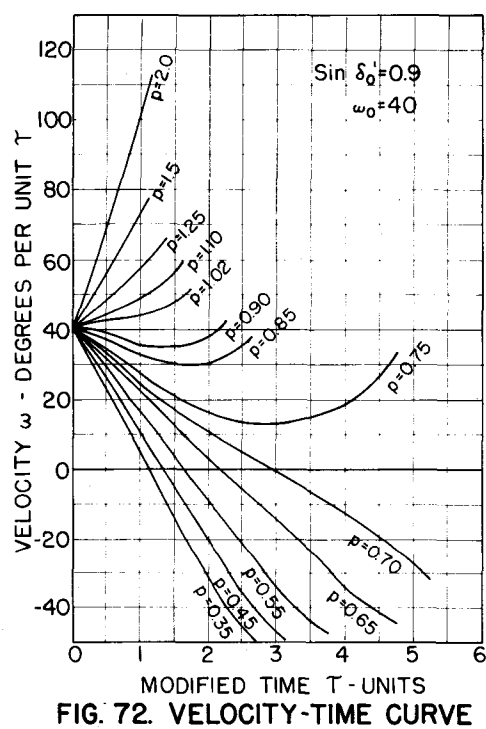
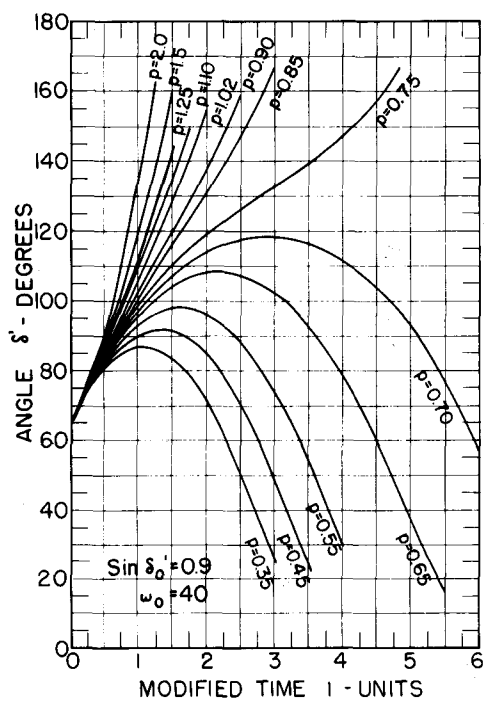
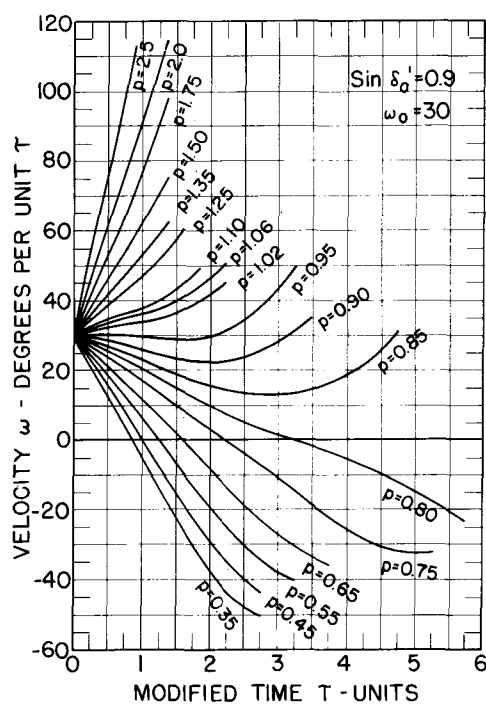
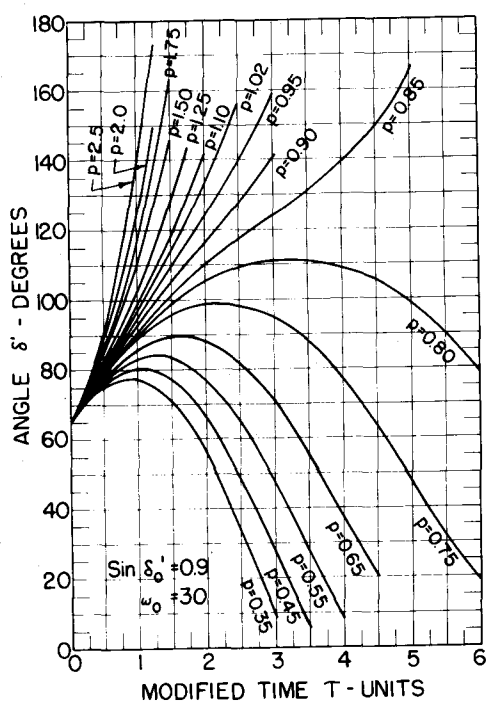


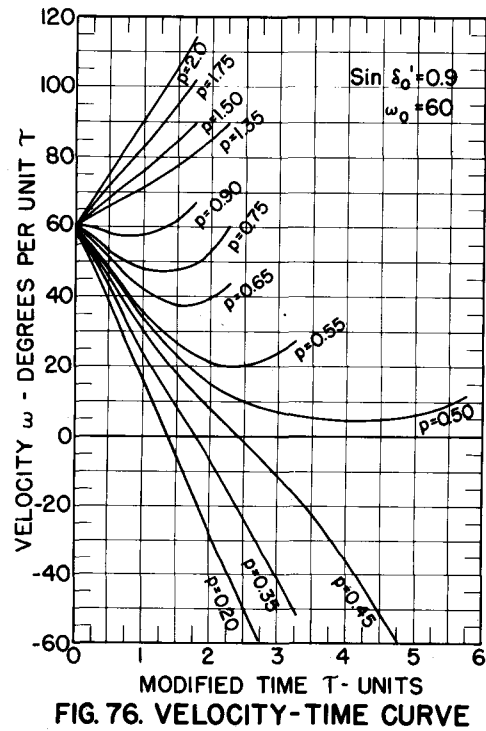
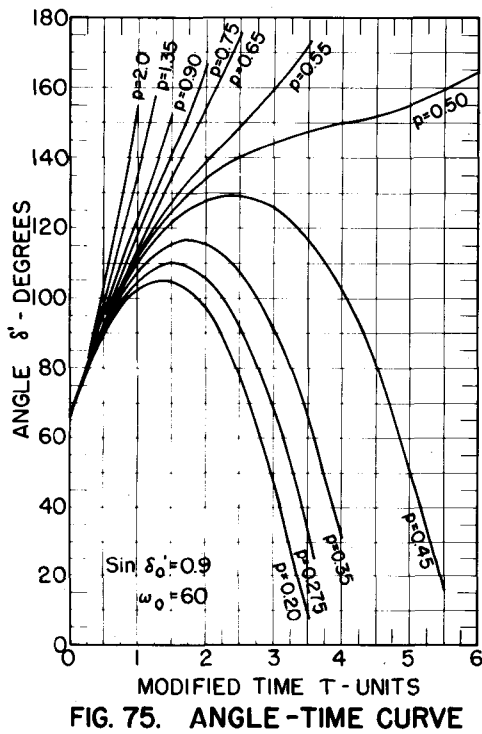
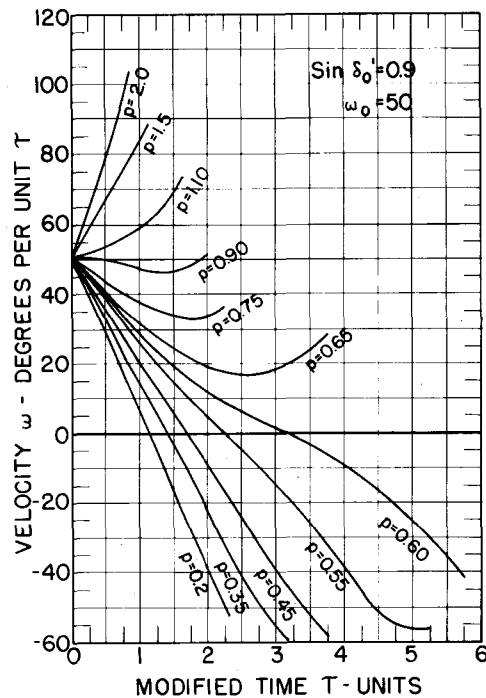
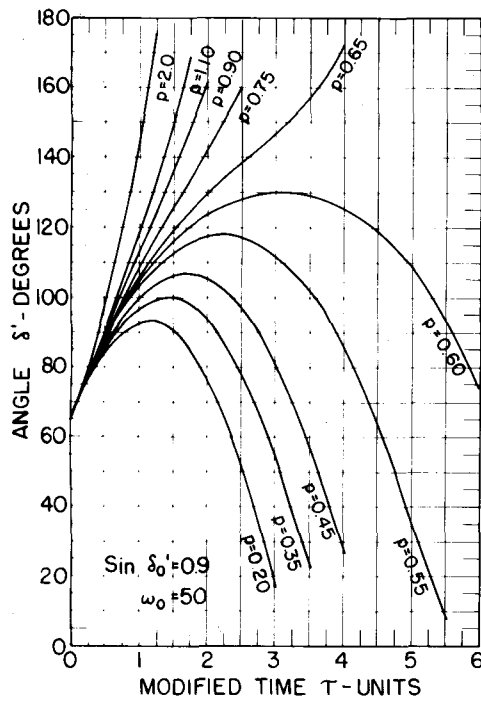
FIG. 56. VELOCITY-TIME CURVE











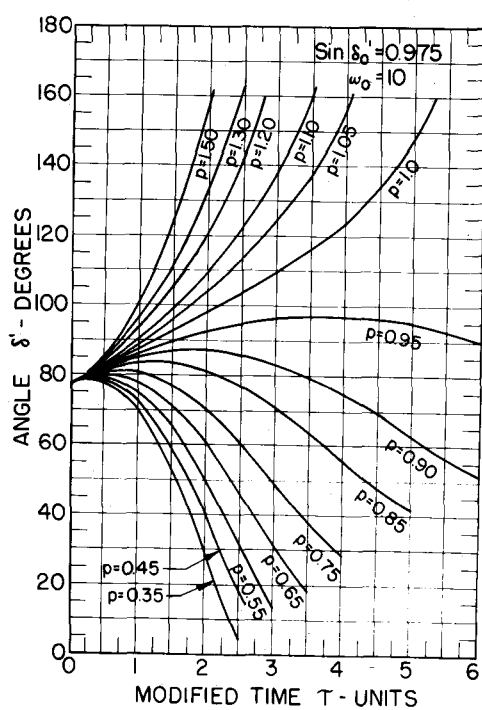


FIG. 77. ANGLE-TIME CURVE

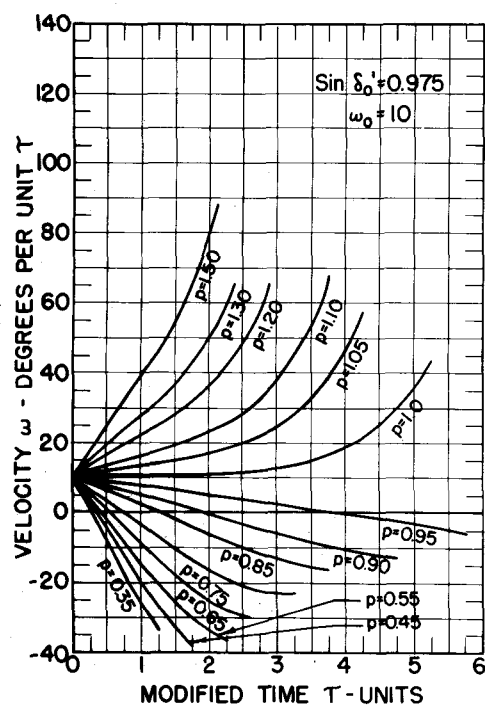


FIG. 78. VELOCITY-TIME CURVE

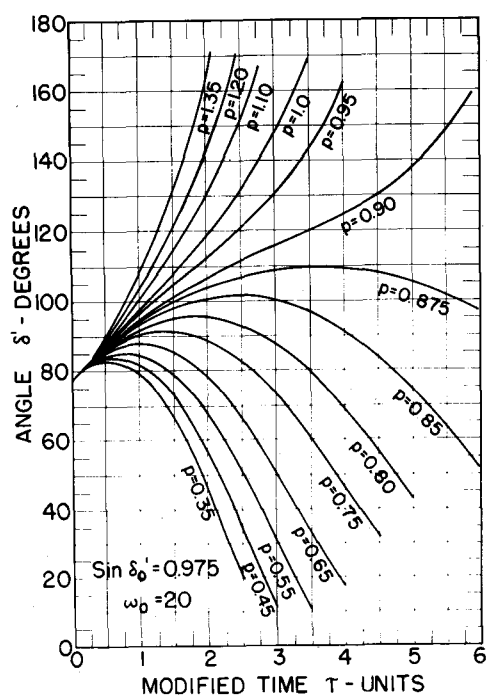


FIG. 79. ANGLE-TIME CURVE

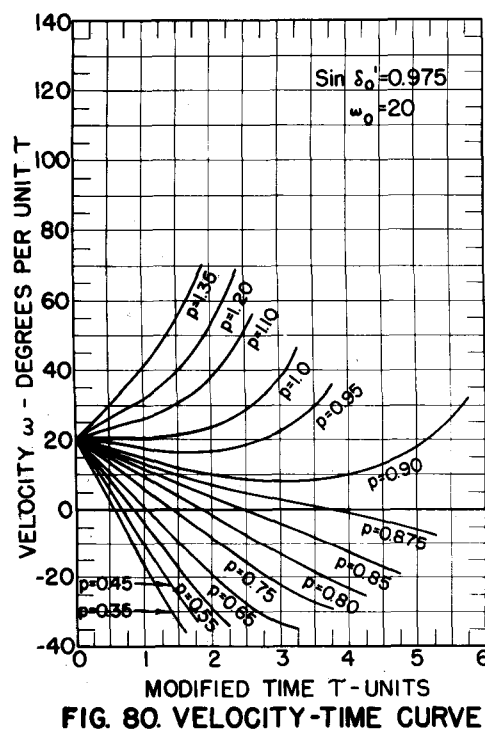
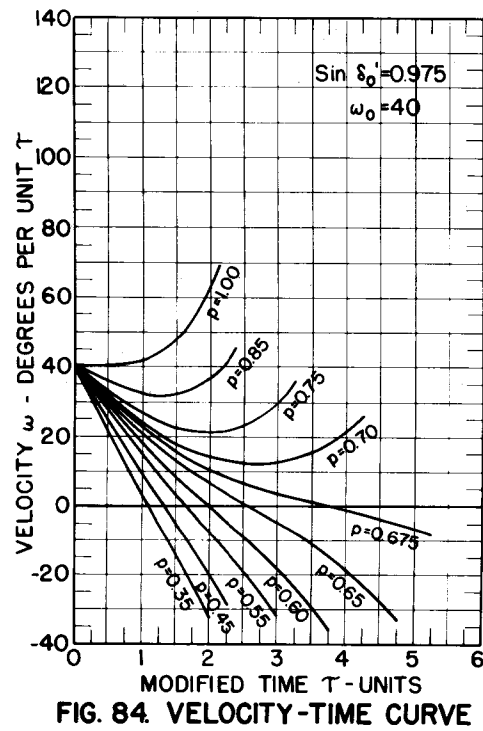
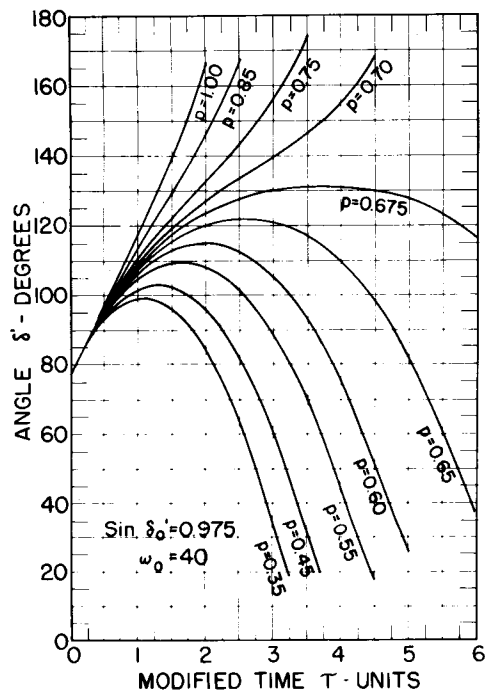
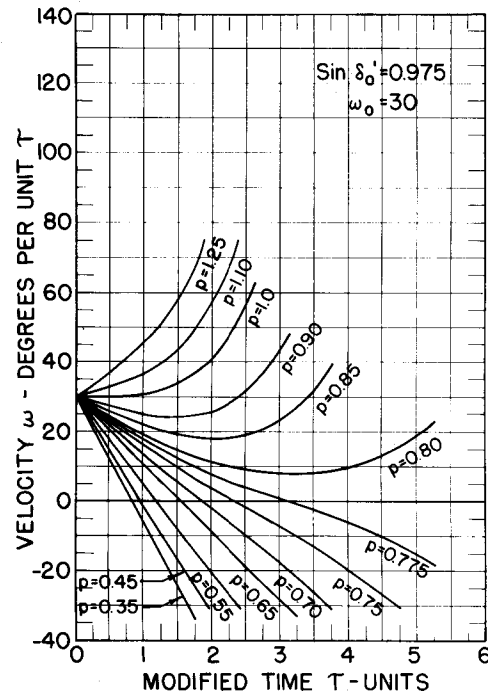
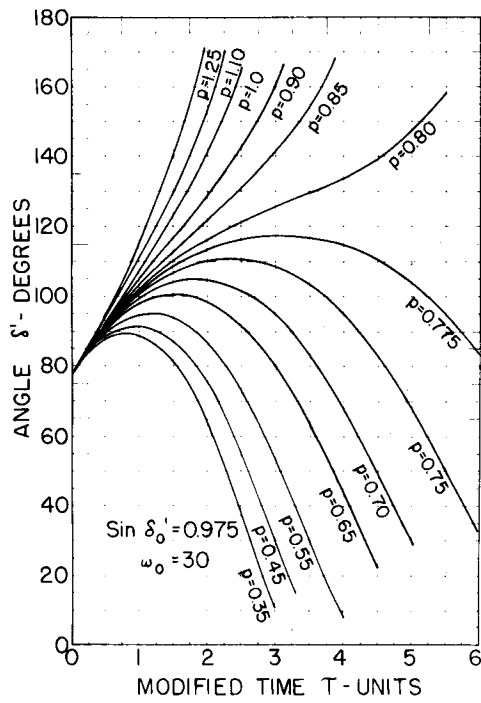


FIG. 80. VELOCITY-TIME CURVE



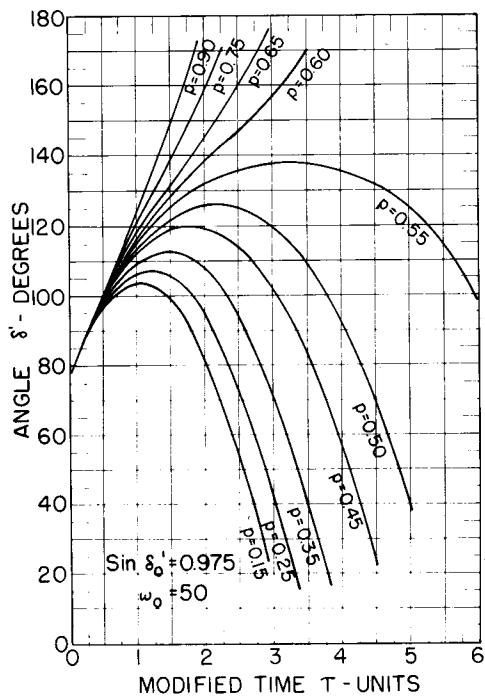


FIG. 85. ANGLE-TIME CURVE

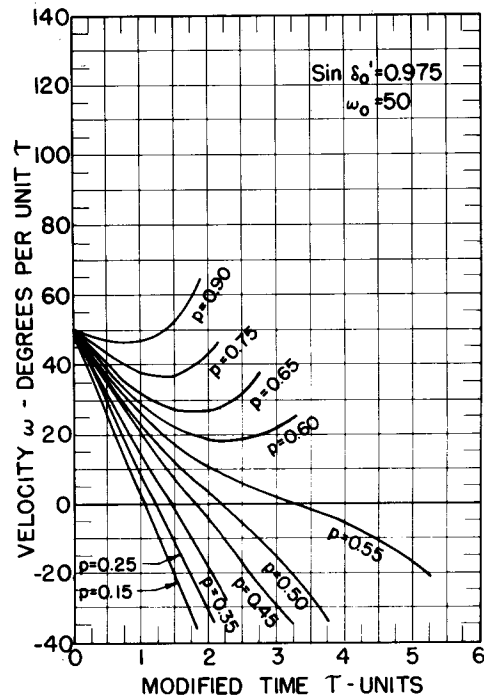


FIG. 86. VELOCITY-TIME CURVE

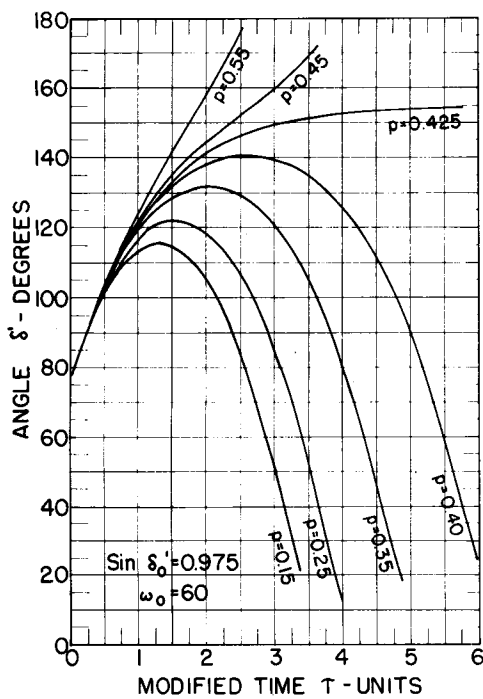


FIG. 87. ANGLE-TIME CURVE

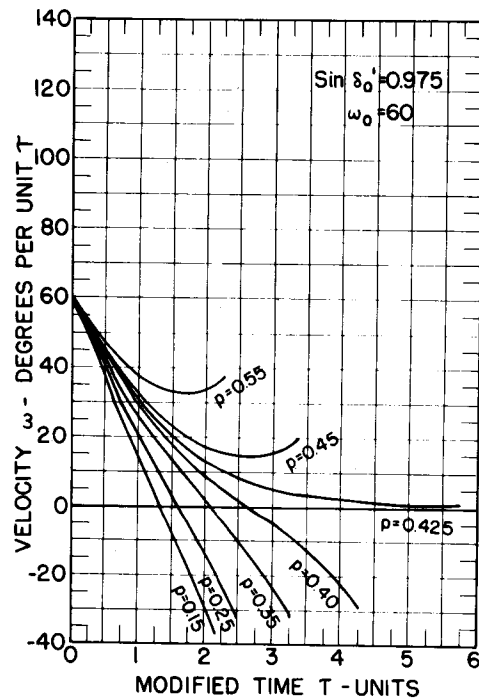
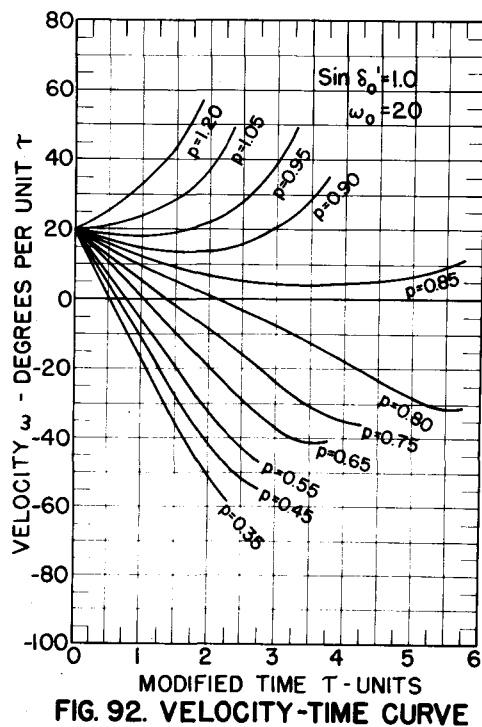
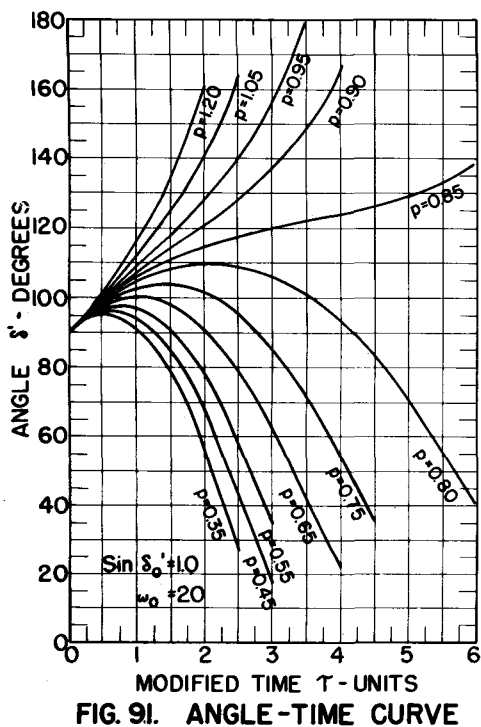
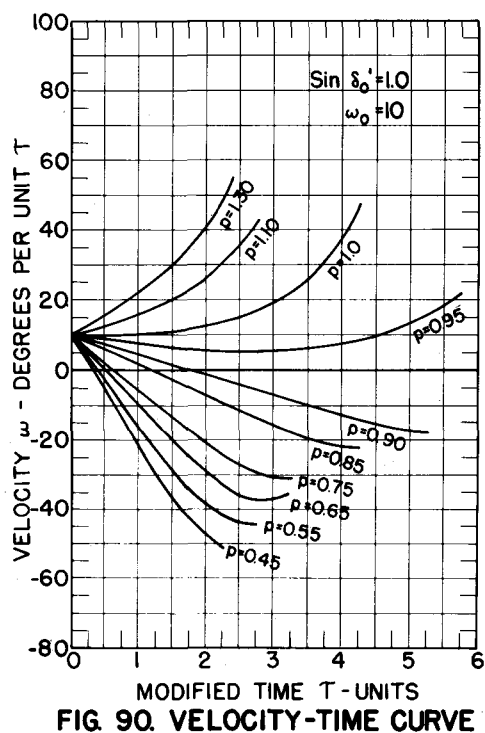
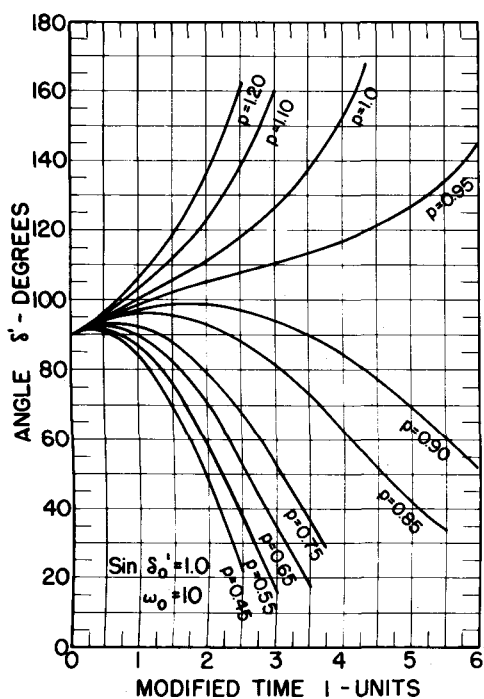


FIG. 88. VELOCITY-TIME CURVE



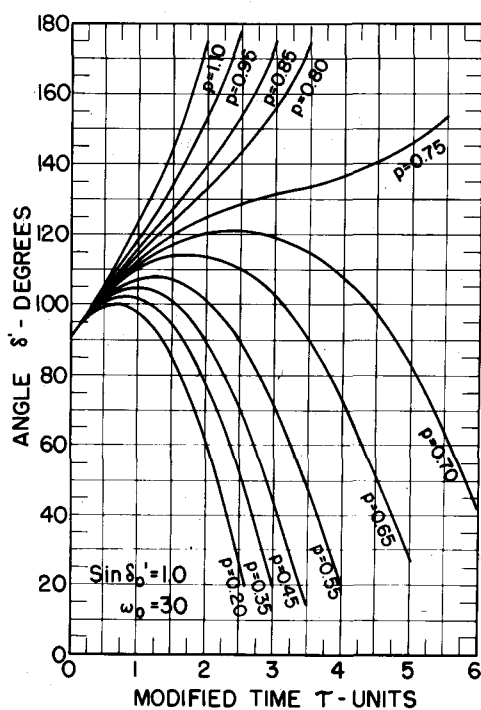


FIG. 93. ANGLE-TIME CURVE

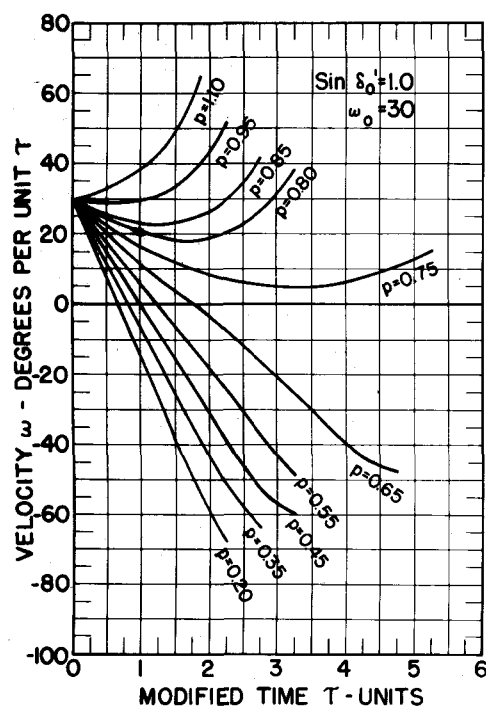


FIG. 94. VELOCITY-TIME CURVE

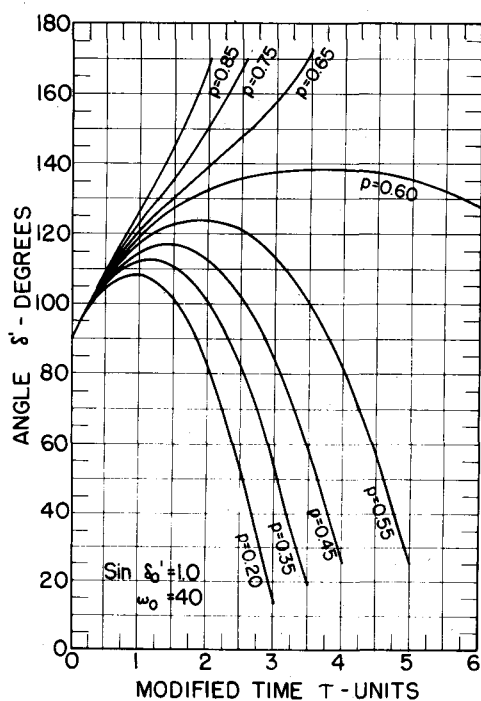


FIG. 95. ANGLE-TIME CURVE

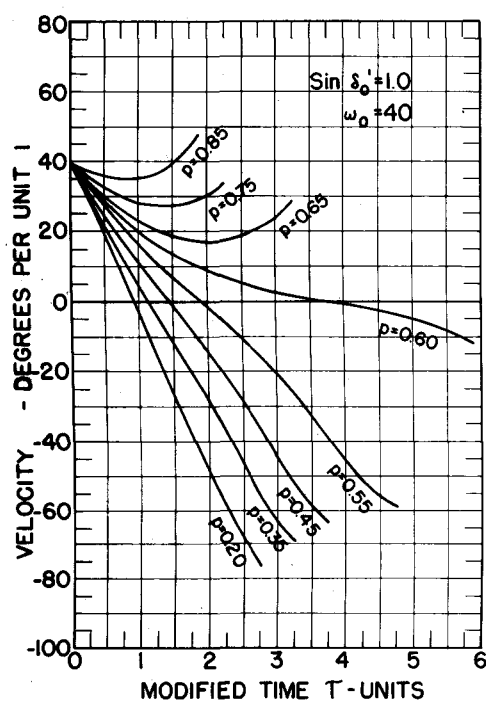


FIG. 96. VELOCITY-TIME CURVE

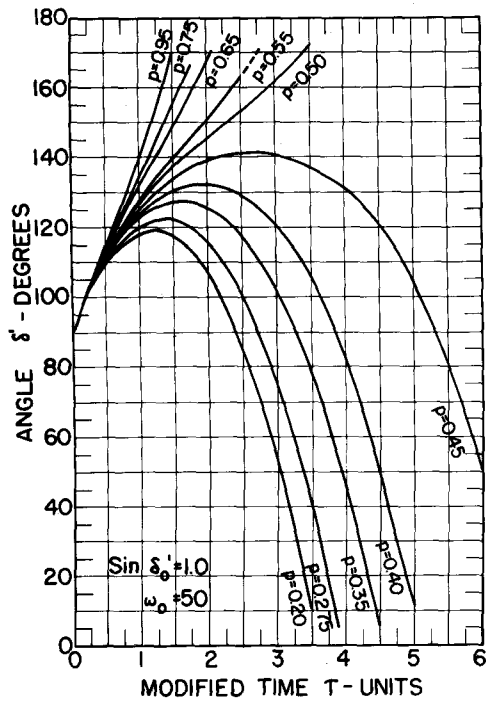


FIG. 97. ANGLE-TIME CURVE

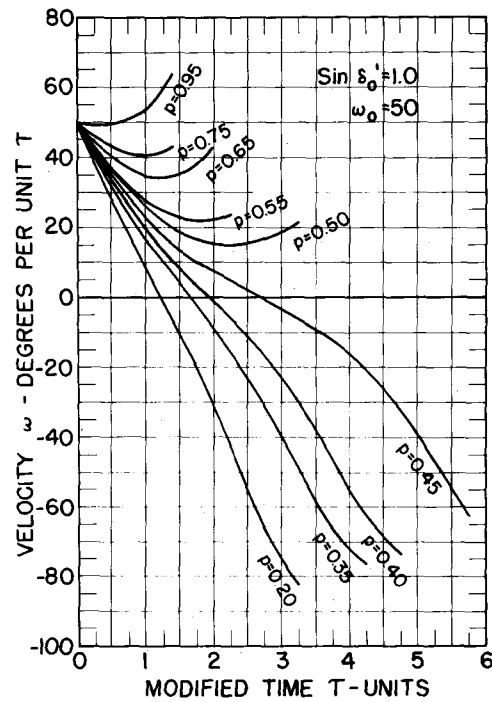


FIG. 98. VELOCITY-TIME CURVE

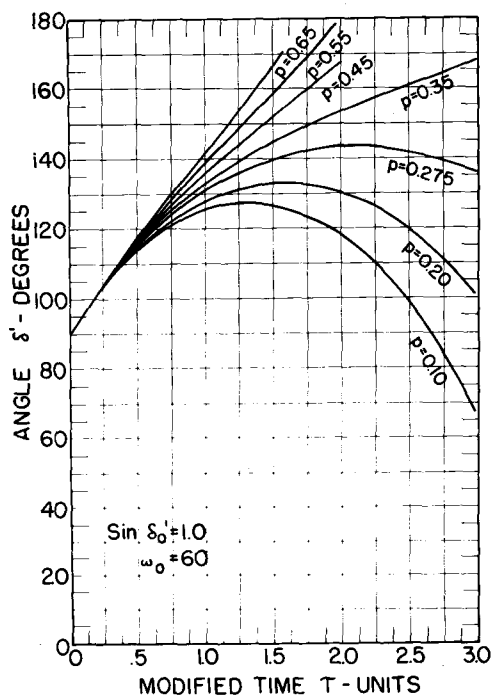


FIG. 99. ANGLE-TIME CURVE

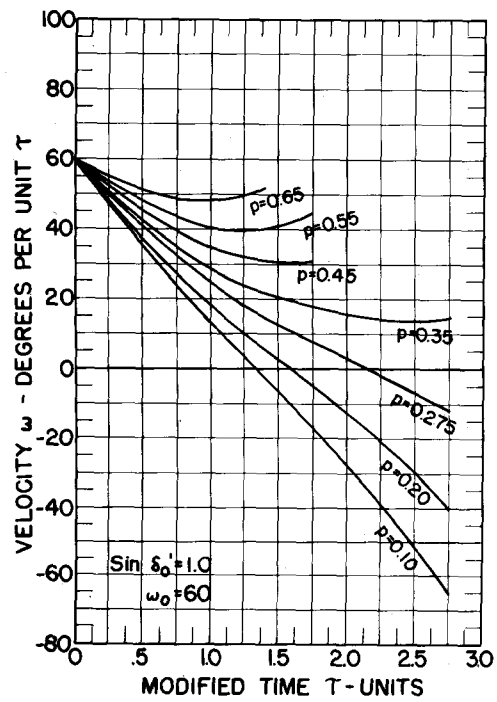
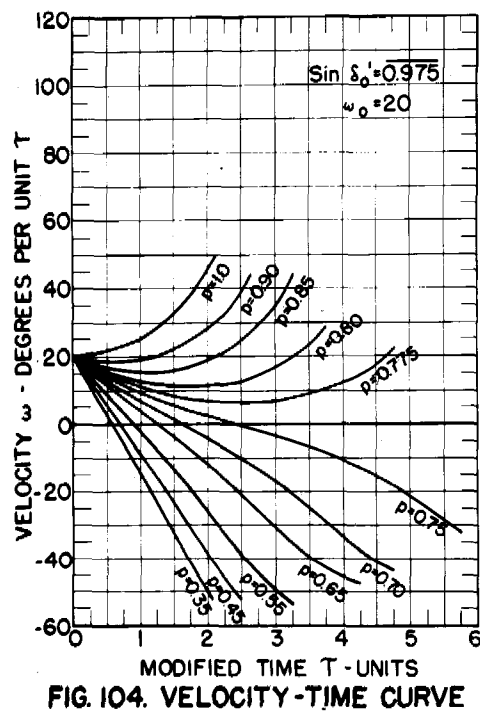
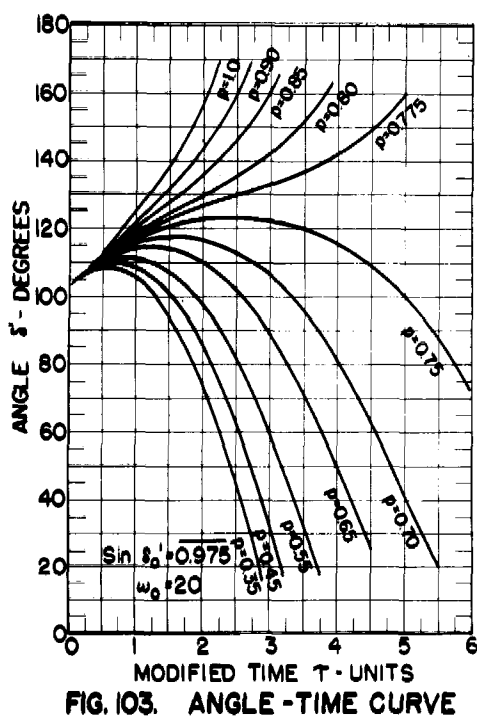
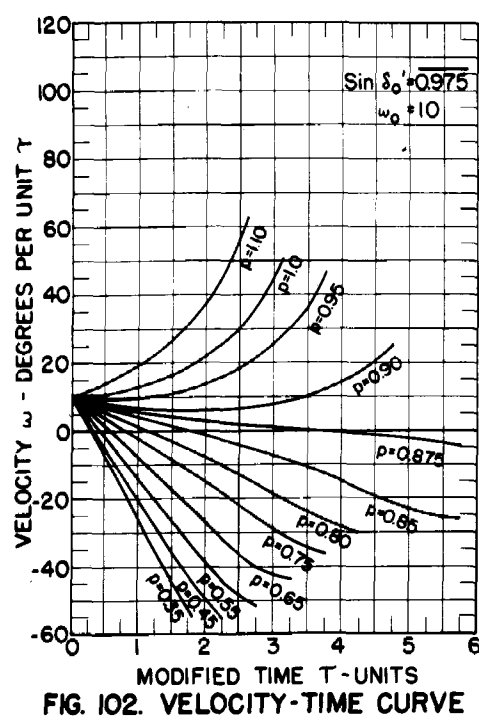
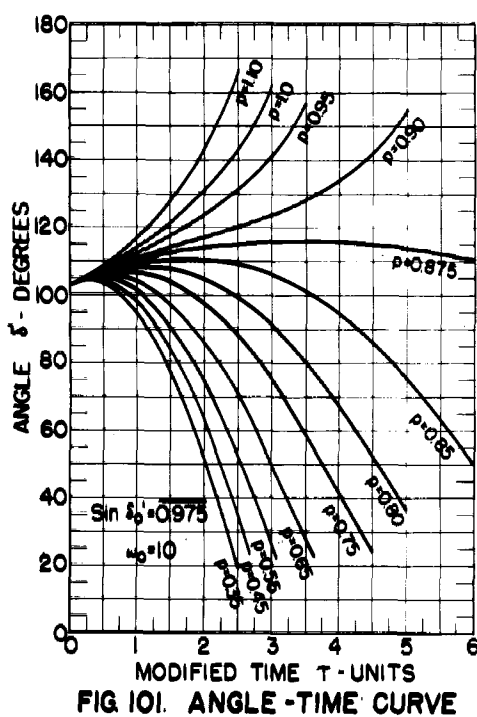
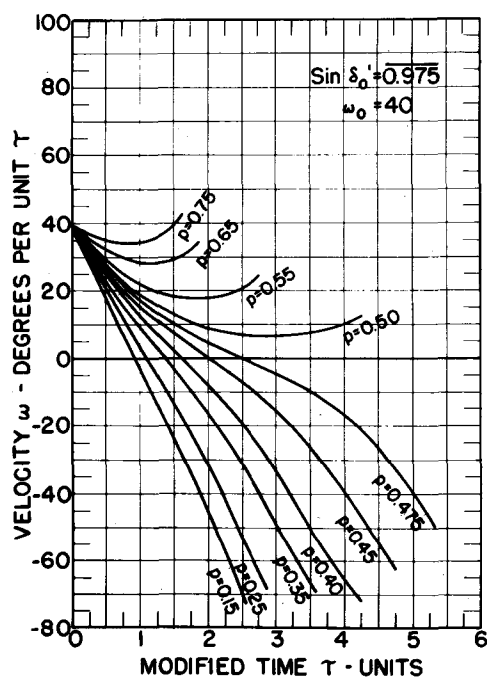
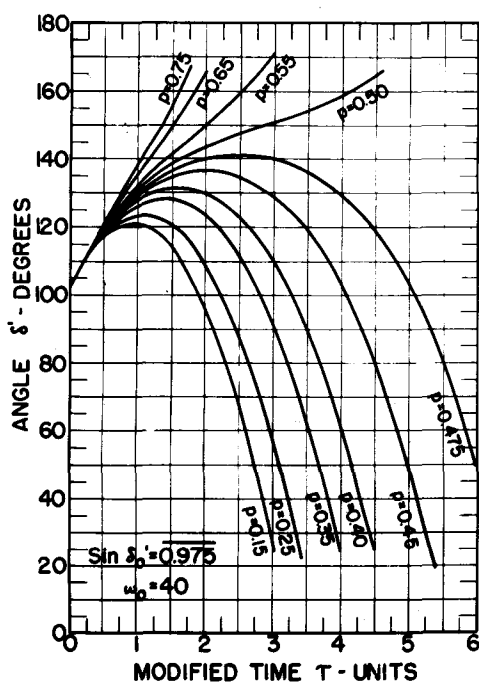
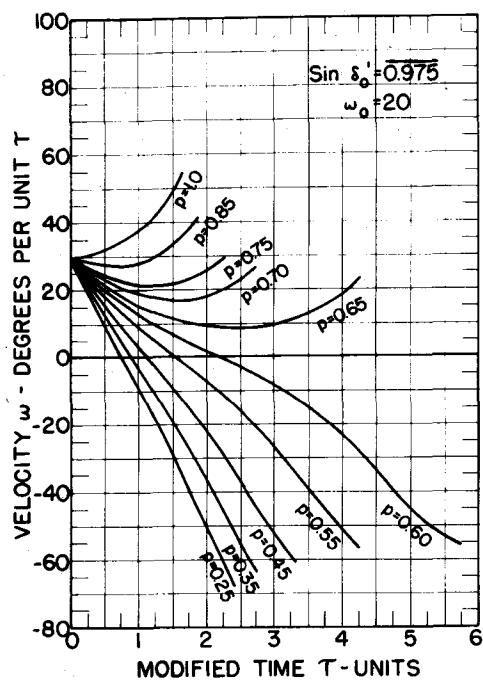
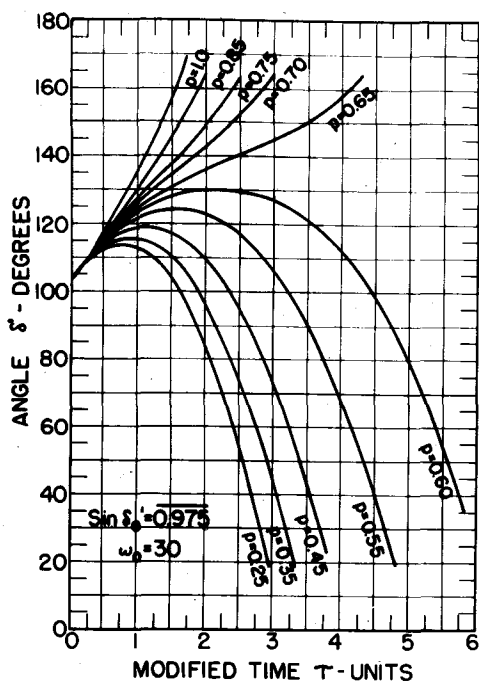


FIG. 100. VELOCITY-TIME CURVE





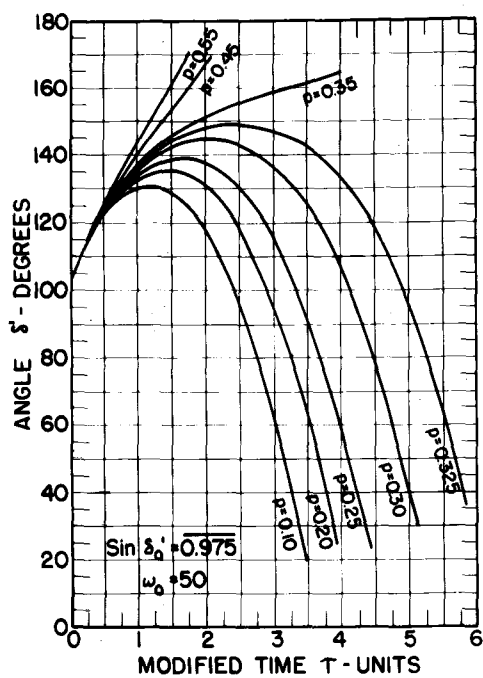


FIG. 109. ANGLE-TIME CURVE

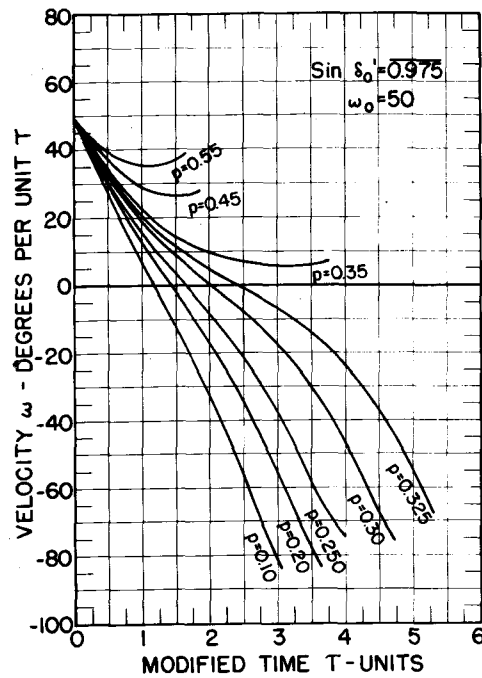


FIG. 110. VELOCITY-TIME CURVE

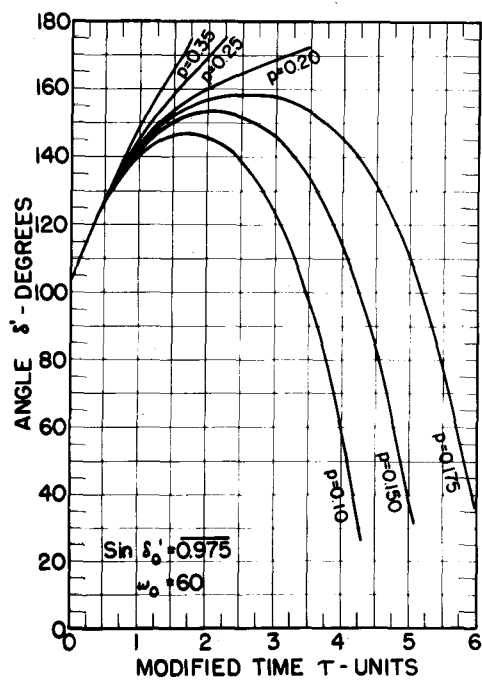


FIG. III. ANGLE-TIME CURVE

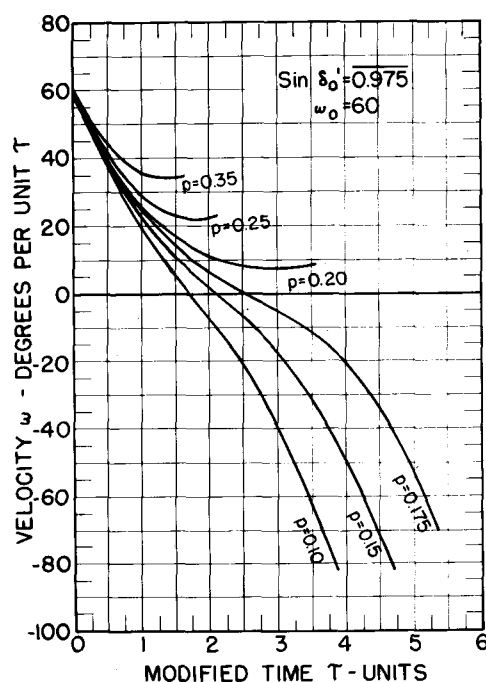


FIG. II2. VELOCITY-TIME CURVE

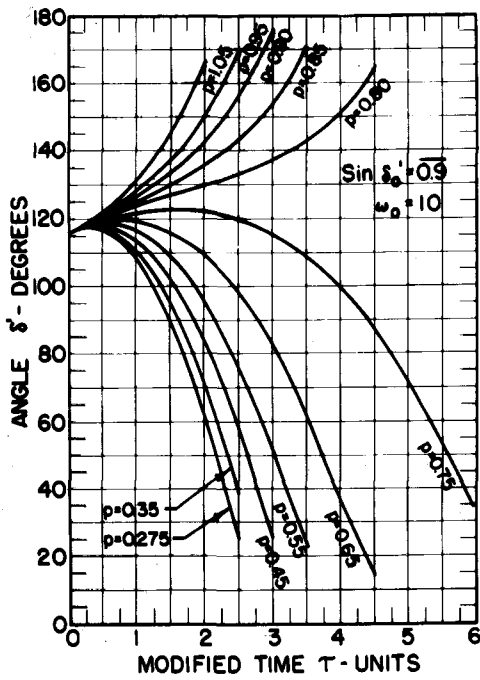


FIG. 113. ANGLE-TIME CURVE

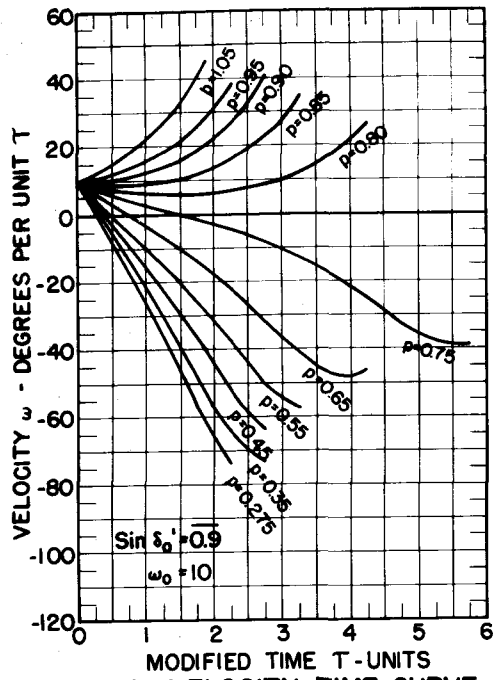


FIG. 114. VELOCITY-TIME CURVE

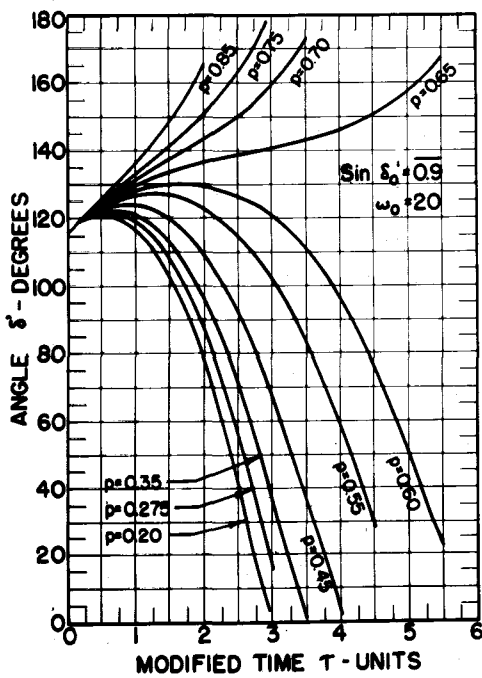


FIG. 115. ANGLE-TIME CURVE

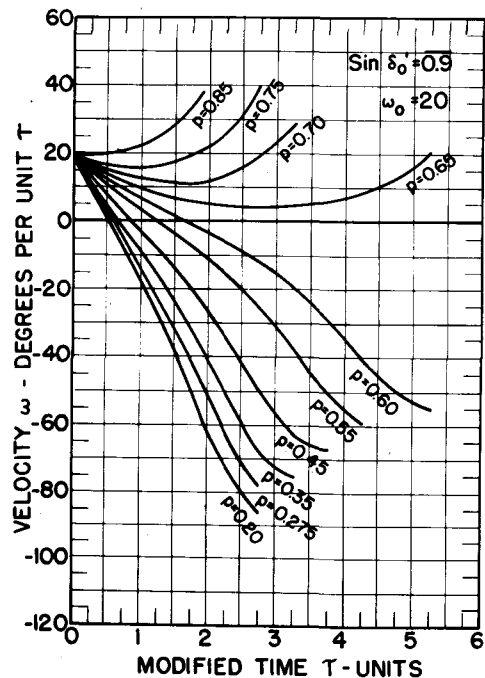
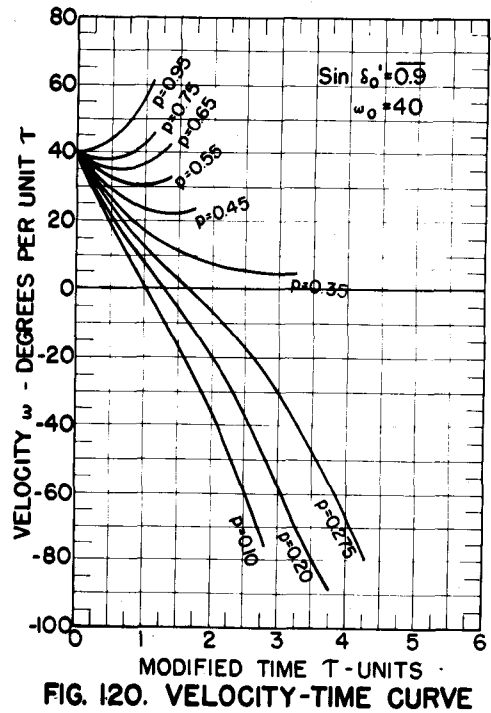
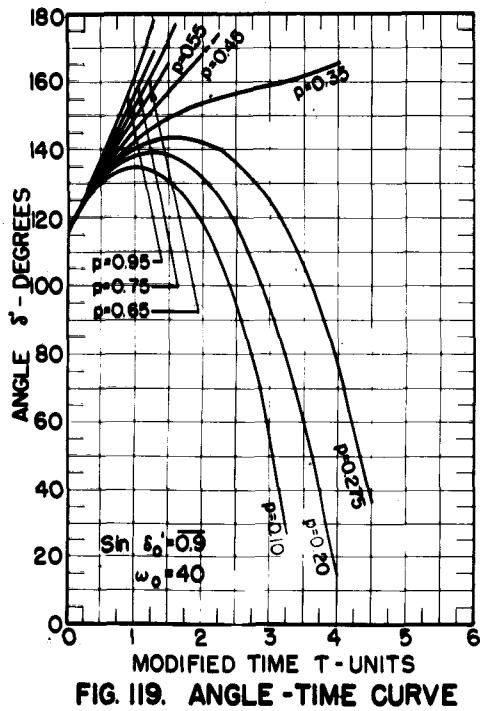
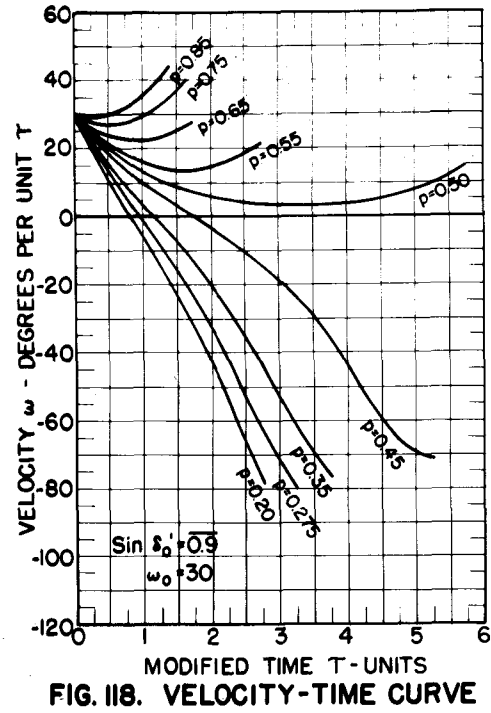
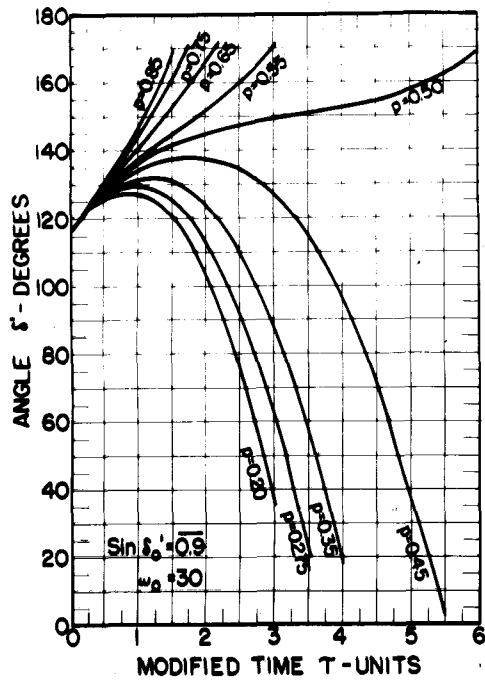


FIG. 116. VELOCITY-TIME CURVE



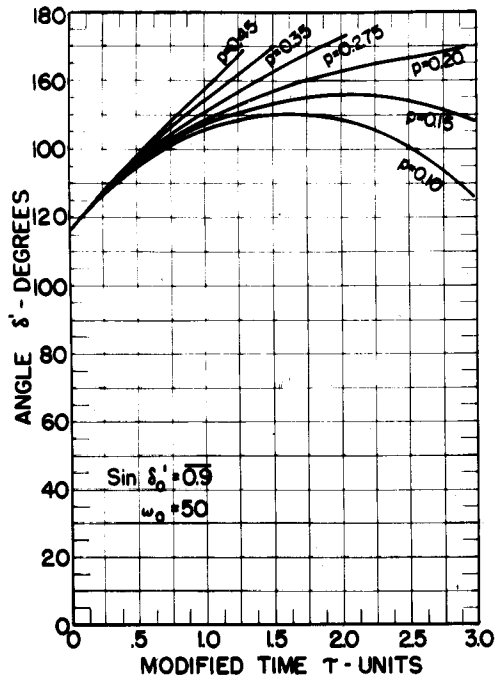


FIG. 121. ANGLE-TIME CURVE

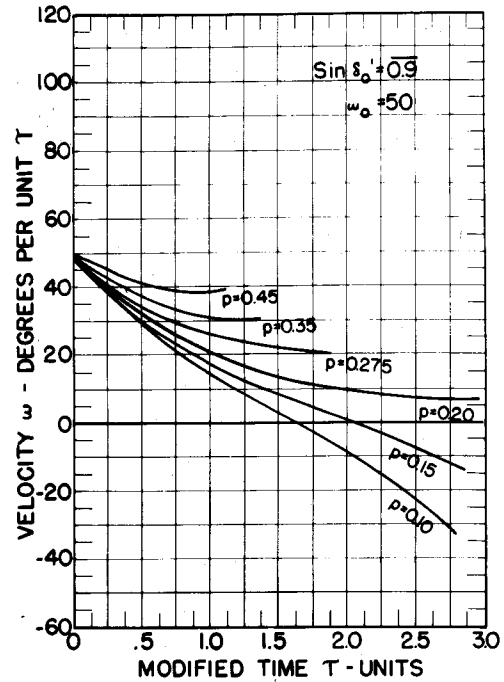


FIG. 122. VELOCITY-TIME CURVE

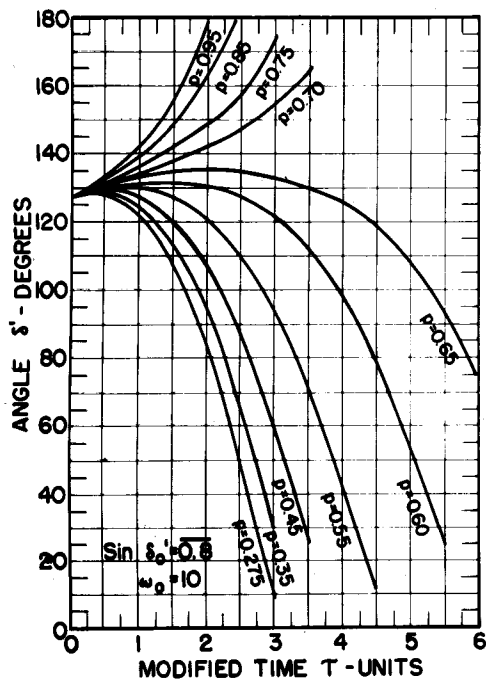


FIG. 123. ANGLE-TIME CURVE

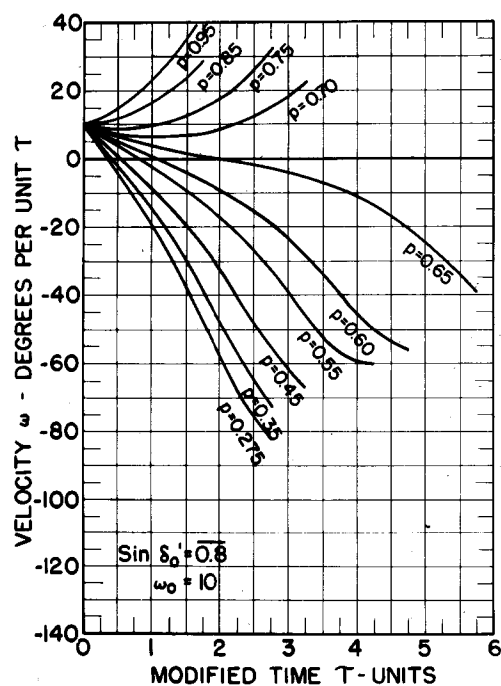
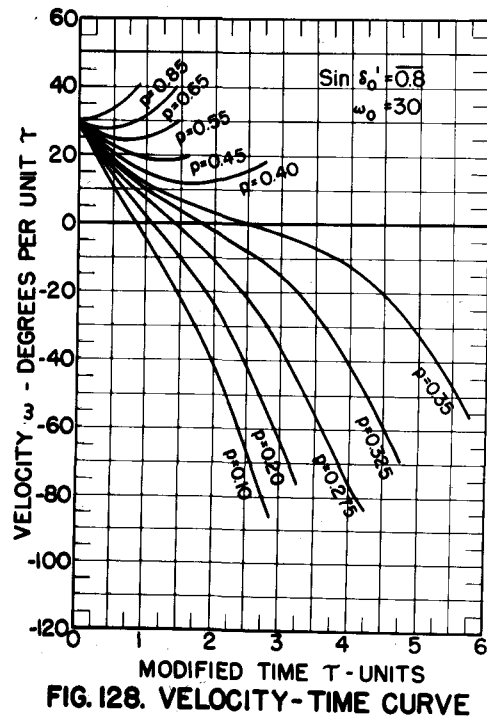
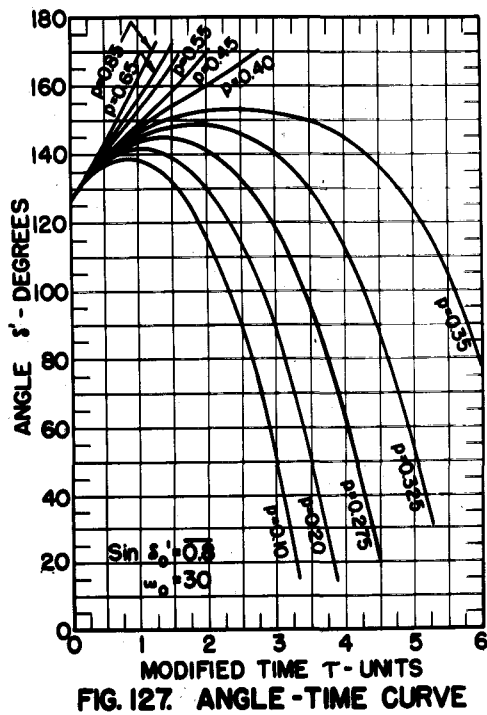
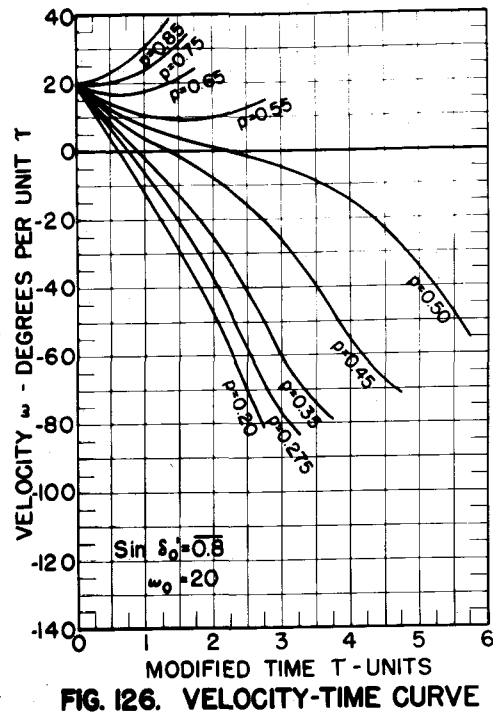
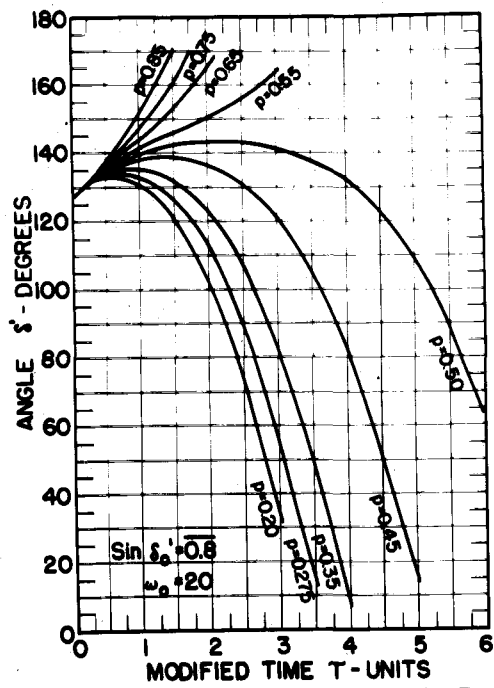


FIG. 124. VELOCITY-TIME CURVE



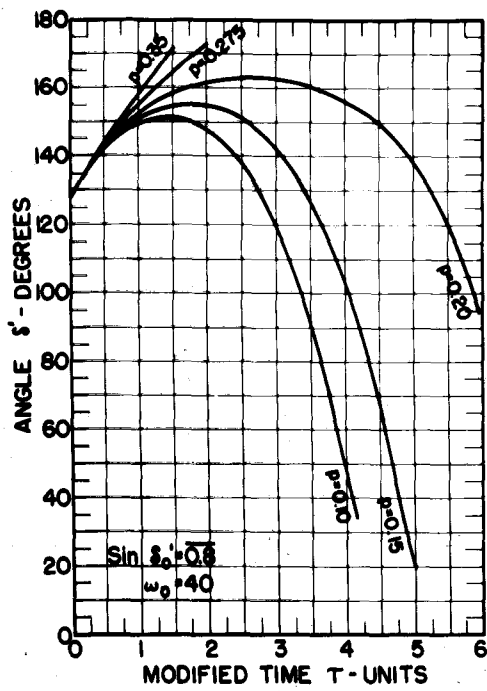


FIG. 129. ANGLE-TIME CURVE

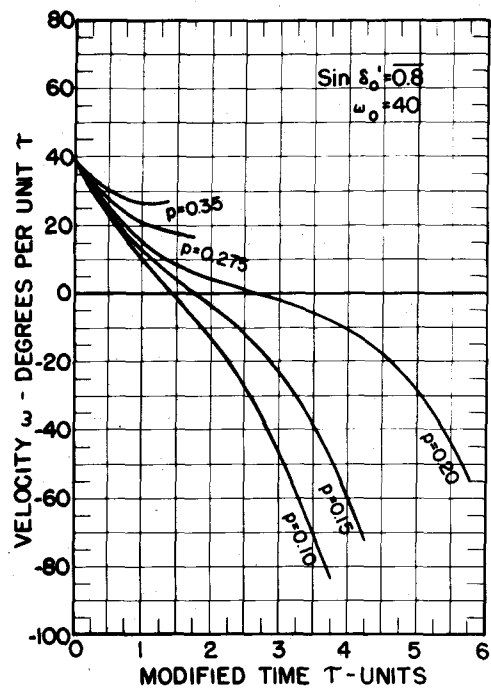


FIG. 130. VELOCITY-TIME CURVE