

Farm Management Optimization Under Uncertainty with Impacts on Water Quality and Economic Risk

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January, 2022

Abstract

Farm management decisions under uncertainty are important not only for farmers trying to maximize their net income but also for policymakers responsible for incentives and regulations to achieve environmental goals. We focus on corn production as a significant contributor to the US Midwest economy. Nitrogen is one of the key nutrients needed to increase production efficiency, but its leaching and loss as nitrate through subsurface flow and agricultural drainage systems poses a threat to water quality. We build a novel two-stage stochastic mixed-integer program to find the annual farm management decisions that maximize the expected farm profit. A decomposition-based solution strategy is suggested to reduce the computational complexity resulting from the predominance of binary variables and complicated constraints. Case study results indicate that farmers may compensate for the additional risks associated with nutrient reduction strategies by increasing the planned nitrogen application rate. Significant financial incentives would be required for farmers to achieve substantial reductions in nitrate loss by fertilizer management alone. The complicated interactions between fertilizer management and crop insurance decisions observed in the numerical study suggest that crop insurance programs can affect water quality by influencing the adoption of environmentally beneficial practices.

Keywords: farm management optimization; stochastic programming; nitrate loss; crop insurance; fertilizer application.

1. Introduction

Farm management is a complex process that is exposed to a wide range of risks and uncertainties. Each year, farmers make several management decisions with the goal to maximize net income, but their profits are also subject to weather and market conditions beyond their

control. Planting time and fertilizer management are critical decisions that influence the farm yield. However, uncertain growing season precipitation and temperature can also significantly affect the yield, so that farmers cannot know the yield outcome of their decisions with certainty in advance. Furthermore, both planting and fertilizer application require suitable field moisture conditions, the lack of which may prevent execution of management decisions as planned. To mitigate economic risk, farmers may purchase crop insurance with benefits that depend on the uncertain yield and price at harvest time, as well as the specific terms of the plan purchased.

The interaction of farm management decisions and weather uncertainty also poses environmental risks. In the US Midwest, agriculture’s impact on water quality is a major concern. Nitrogen (N) is one of the key nutrients needed in agricultural production. Ideally, crops can be fed with enough nutrients at the right time to ensure healthy plant growth. The soil naturally holds many nutrients, but if it lacks enough nutrients to match the required plant uptake, the farm’s yield will suffer. Farmers commonly apply fertilizer to the soil to compensate for its nutrient deficiencies. Because N is water-soluble, it is easily washed away by water moving through agricultural drainage systems due to precipitation or irrigation. Nitrate-N loss from farmland causes nutrient loads in waterways and depletes the oxygen level in surface waters, a phenomenon known as hypoxia. Nitrate within the Mississippi River basin moves downstream and creates the Gulf of Mexico dead zone, one of the largest in the world at nearly 9,000 square miles (EPA, 2017). Although estimates differ, several studies agree that Iowa contributes a considerable amount (20-40%) of the nutrients in the Gulf compared to the eleven other states along the Mississippi River (Goolsby et al., 2000; Jones et al., 2018; Turner and Rabalais, 2004). In a major statewide study updated annually since 2012, Iowa State University et al. (2017) summarize ways to decrease N concentration in surface water and reveal that a 45% reduction in N loss statewide is required to achieve environmental goals set by the Mississippi River/Gulf of Mexico Watershed Nutrient Task Force (2008). However, practices to reduce nutrient loss remain voluntary, with effects are subject to the same weather uncertainty that affects profit.

Uncertainty is therefore not only a serious concern for farmers, but is also an important consideration for policymakers and social planners with environmental concerns. In the US, several agricultural conservation policies and state regulations concern nutrient man-

agement. Those preventive measures aim to promote and incentivize nutrient reduction practices to improve water quality. According to the most recent (2018) U.S. Farm Bill, the budget allocated to the popular conservation programs will continue to increase gradually until 2023 (McMinimy, 2019). Farmer concerns about uncertainty and the resulting risk are also acknowledged in the legislation, as some of the income support direct payments are converted to insurance subsidies.

Policy makers commonly measure the potential effectiveness of a conservation program according to the types and amounts of payments required for farmers to adopt nutrient reduction practices that they otherwise would not (Claassen et al., 2014). However, determining additionality (i.e., whether a subsidy causes adoption of a practice) is not a simple task, as various risk perceptions and attitudes lead to different decisions under uncertainty. Income risk is one of the primary reasons for farmers' neglect of environmental practices (Bosch and Pease, 2000; Minnesota Pollution Control Agency, 2014; Marra et al., 2003; Greiner et al., 2009). In the agronomy literature, farmers' behavior has even prompted debate about farmer rationality (Arbuckle Jr et al., 2015; Howley et al., 2015). Farmers who do not adopt nutrient reduction practices argue that incentives do not cover the additional costs and effort required to follow those practices. Environmentalists commonly cite experimental tests to validate additional economic benefits. Those experiments are usually observed under specific weather and soil conditions and do not reflect how a small change in any component would impact the outcome. Even the studies that explicitly aim to investigate the uncertainties interpret the final results in terms of expected conditions only. Regardless, those studies fall short of representing underlying risks from the farmer's perspective. To develop effective policies and promote nutrient reduction practices, social planners must first understand optimal farm management decisions under uncertainty. Based on that understanding, it may be possible to judge the effectiveness of existing policies, such as whether the current incentive rates are enough to accomplish the social goals, or how policies can be improved. Although farmers may not necessarily follow the management decisions found to be optimal in a model, optimization results can provide some motivation for policies as well insights into farmer responses to those policies.

To explore the nitrate water pollution impacts of farm management under a profit maximization goal, this study focuses on corn production in Iowa. The production of corn,

also known as maize, in the US has trended upward since the 1930s (USDA and NASS, 2020). Improved farm management strategies and technological advances that have boosted yield per acre (Shahhosseini et al., 2020) are the primary factors behind the long-term growth to meet the increasing global demand. Today, corn enjoys the highest demand of any grain product and represents more than 40% of all grain production worldwide (USDA, 2020). The US, as the world leader in corn production, meets more than 30% of the global corn demand, while the state of Iowa is the biggest corn supplier in the US. Although farmers have faced some struggles in recent years, agriculture is still a major contributor to Iowa’s economy by accounting for around 20% of jobs.

We explore the uncertainty in corn production from a farmer’s viewpoint. We investigate the major annual farm-level decisions, including planting time, fertilizer application rate and timing, and crop insurance purchase, to maximize the expected farm profit. The agricultural economics literature includes many investigations of the economic consequences of individual management decisions and their interactions with some uncertain elements, as described in Section 3.1. However, those studies are neither comprehensive nor concerned with optimization. Of the numerous articles on farming decisions from the operations research perspective (Moghaddam and DePuy, 2011; Capitanescu et al., 2017), none investigate the annual management decisions of a Midwest farmer growing a grain product under real-world uncertainties. To fill this gap, we propose a novel two-stage stochastic program for optimal annual farm management. The case study and numerical instances represent the state of Iowa but the model can be parameterized for any state in the US Midwest. Numerical solutions reveal useful information about a farmer’s management behavior under uncertainty and provide valuable input to social planners concerned with environmental issues. We consider five major questions: (i) What are the optimal annual farming decisions under uncertainty? (ii) What financial incentives would be needed to achieve N reduction targets by fertilizer management alone? (iii) What are the expected profit tradeoffs for meeting various water quality goals through fertilizer management alone? (iv) What is the combined effect of fertilizer management and crop insurance decisions on farm profitability and water quality? (v) What types of information are needed to improve research on how to achieve environmental goals via management practices?

Our numerical results suggest that current N reduction targets for Iowa cannot be

achieved by focusing only on N management practices, as Iowa has naturally high organic matter levels, which means that the potential for N losses is high even without any fertilizer application. We demonstrate that crop rotation improves the farmer’s profit and reduces the necessary incentive rates to improve water quality. We are aware of only one recent study in the literature that considers insurance programs as a means to achieve environmental goals (Thorburn et al., 2020). However, uncertainty is one of the biggest concerns in agriculture, and insurance programs are the primary economic tools available to farmers to mitigate the resulting risk. Therefore, this paper explicitly explores the interaction between N management and crop insurance. We demonstrate that fertilizer management and insurance policy selection decisions are highly interrelated. Specifically, we observe that crop insurance has a complementary role in reducing the N application rate, with positive environmental impact. On the other hand, for very low N application rates, the availability of crop insurance reduces the motivation to adopt environmentally beneficial N application timing practices. The complicated and contradictory interactions display the need for more extensive investigations of insurance programs and their impact on environmental practices. Finally, our results indicate that N is a risk-reducing factor, in that the additional risk associated with a nutrient reduction practice may be mitigated by applying more fertilizer to the soil. However, the existing agronomy data representing the farm yield and N loss generated through field trials are not adequate to inform policy-making. Agronomic research currently emphasizes the investigation of individual elements (decisions, uncertainties, and other known factors) as independent entities while overlooking more complex interactions. A more extensive investigation into farmer decision-making under uncertainty requires more comprehensive information about interactions among these elements. The rest of the paper is organized as follows. In Section 2, we review the related studies in the literature. Section 3 contains a detailed problem description and a two-stage stochastic programming formulation. In Section 4, we specify the parameters used in the computational study and in Section 5, we present the numerical results. Finally, concluding remarks are provided in Section 6.

2. Literature Review

Optimization models of agricultural management have been formulated frequently. Singh (2012) provides a detailed survey. Those applications broadly include resource management, cropping pattern optimization, groundwater and irrigation management, and increasing production efficiency. Although some of them concern farm management, each study’s content and methods vary widely due to the investigation of different crops, objectives, and assumptions. To the best of our knowledge, the existing literature does not investigate a US Midwest farmer’s annual management decisions for growing a grain product under real-world uncertainties to the extent discussed in this paper. In this section, we describe the existing literature most similar to this paper.

Bloemhof-Ruwaard and Hendrix (1996) is one of the first papers to investigate the relationship between land management and fertilizer application to maximize farming profits. A bilinear model is formulated to make land management and fertilizer application decisions considering manure application limits imposed by the government to reduce negative environmental impacts. Li et al. (2017) build an integer program to investigate irrigation water allocation and seed selection decisions to maximize annual farm profit. Liu et al. (2008) optimize crop insurance decisions of a cotton and peanut farm in Florida under weather uncertainty to minimize the expected loss using a CVaR constraint. The study also includes crop allocation and binary planting decisions. Moghaddam and DePuy (2011) explore the stochastic nature of farming yield due to weather uncertainty on a hay farm. The study also includes environmental policies to improve water quality in the form of chance constraints. Hyytiäinen et al. (2011) include nitrogen balance equations in the soil in a stochastic dynamic program to compare split and spring fertilizer application under a pollution tax. The available N amount in the soil and crops is introduced as a state variable, while transition probabilities are obtained through simulation using weather data and fertilizer related decisions as input. The study suggests that split application performs better under the pollution tax while spring application is better without any taxation. Peña-Haro et al. (2011) investigate fertilizer application and irrigation rate decisions to maximize the agricultural profits without exceeding nitrate leaching standards. Functions for crop yield and nitrate leached are imported from an agro-simulation tool. Most recently, Capitanescu et al. (2017) investigate the crop allocation and rotation decisions over a multi-year plan-

ning horizon to maximize the farm profit based on environmental constraints generated according to the water-food-energy nexus.

In the agronomy literature, numerous studies look for optimal N application rates (Rware et al., 2016; Sexton et al., 1996; Yong et al., 2018). However, those studies rely on previous empirical tests to identify the best alternative among the limited number of experiments and do not seek optimality in the mathematical programming sense. Researchers commonly use popular crop simulation tools to estimate several outputs, including yield and N loss, and couple those simulation models with genetic algorithms to select management practices that increase profit and improve water quality simultaneously (Kaini et al., 2012; Srivastava et al., 2002; Geng et al., 2019). In recent years researchers have applied machine learning models to predict yield and N loss, acknowledging the limitation of experimental tests and simulation-based estimations (Chlingaryan et al., 2018; Puntel et al., 2016; Shahhosseini et al., 2019; Archontoulis et al., 2020). However, those models have yet to be integrated with agricultural decision making in optimization models.

3. Model Definition

In this section, we formulate a stochastic mixed-integer mathematical program for the farmer’s annual decision problem. Full nomenclature of the model is presented in Table 1.

3.1 Major Farming Decisions

We first introduce the major farming decisions investigated in this study. We discuss the importance of each decision and present a decision timeline illustrating the annual corn production calendar involving those decisions. The decisions involve fertilizer application rate, fertilizer timing, planting time, and finally, insurance plan selection.

3.1.1 Fertilizer Decisions

Nutrients are essential for agricultural production. In this study, we specifically focus on N and its underground movement as nitrate-N. Because crops cannot take in N directly from the air, having enough N in the soil is a necessity for healthy crop growth. In a soil network, some portion of N supply occurs through natural processes (mineralization and

Table 1: Nomenclature for the model

<i>Sets</i>	
\mathcal{I}	Set of nitrate timing alternatives $\{1(\text{fall}), 2(\text{spring}), 3(\text{split}), 4(\text{sidedress})\}$ – indexed by i
\mathcal{J}	Set of planting time windows $\{1(\text{optimal}), 2(\text{delayed})\}$ – indexed by j
\mathcal{S}	Set of all future scenarios $\{1, \dots, S\}$ – indexed by s
\mathcal{S}'	Scenario group where soil conditions are not suitable for fieldwork in early spring which will delay planting time with spring and split application – indexed by s
\mathcal{L}	Number of piecewise functions generated based on yield and N Rate relation illustrated in Figure 3 ($\{1, \dots, L\}$) – indexed by l
\mathcal{V}	Set of insurance coverage alternatives $\{1, \dots, V\}$ – indexed by v
\mathcal{B}	Set of unfavorable outcomes of τ_1 that define \mathcal{S}'
<i>Decision Variables</i>	
x_i	Binary decision for nitrate timing alternative i Equal to 1 if timing alternative i is selected, otherwise equal to 0
z_j^s	Binary decision for a specific planting time j under scenario s Equal to 1 if planting window j is selected, otherwise equal to 0
t	N application rate (lbs/acre)
u_1	Impact of N application rate to yield (percent of maximum yield)
u_2^s	Impact of N application rate to yield for split applications (percent of maximum yield)
α_{ij}^s	Binary decision representing combination of fertilizer and planting timing decisions Equal to 1 if both x_i and z_j^s are also equal to 1 at the same time
y_{v1} and y_{v2}	Binary variable representing coverage level selection for insurance plans Equal to 1 if a coverage level v is chosen, otherwise equal to 0
σ_1^s and σ_2^s	Indemnity paid by insurance protection plans
w_{ij}^s	A continuous variable introduced for linearization purposes Equal to u_1 if α_{ij}^s is 1, otherwise equal to 0
$q_1^s, q_2^s, q_3^s, q_4^s$	Disjunctive variables used for big-M reformulation while formulating insurance options
π	Expected farm profit
<i>Parameters</i>	
g	Cost of N application (\$/lbs)
p^s	Probability of scenario s
a_l, b_l	Constants of piecewise linear function l generated according with respect to Figure 3
c_{v1}	Insurance premium cost for yield protection plan for coverage option v (\$/acre)
c_{v2}	Insurance premium cost for revenue protection plan for coverage option v (\$/acre)
r_0	Projected corn price (\$/bu)
H	Maximum achievable yield of the farm (bu/acre)
μ	Historical average yield of the farm (bu/acre)
f_v	Coverage rate for coverage option v
M	A sufficiently large number
β_{ij}^s	Fraction of maximum yield realized on scenario s based on combinations of decisions i, j (%)
k_i^s	Portion of N being able to applied to soil during growing season in scenario s (%)
I^s	Scenario dependent binary parameter Equal to 1 if $\tau_1 \in B$ in scenario s , otherwise equal to 0
<i>Random Variables</i>	
ω	Uncertain precipitation level during crop growing season (inches)
γ	Uncertain temperature level during crop growing season ($^{\circ}\text{F}$)
r^s	Uncertain crop price at the harvest time (\$/bu)
τ_1	Days suitable for fieldwork during early spring for N application
τ_2	Days suitable for fieldwork during summer sidedressing

nitrification of soil organic matter) as nitrate-N. The remaining N supply can be provided through alternative sources, including synthetic fertilizers and manures, in which all forms of N will be transformed into nitrate-N as a result of nitrification (Randall and Mulla, 2001). Because nitrate-N easily moves with water, it is susceptible to leaching. The resulting loss causes N loads in waterways and negatively impacts the water quality by contributing to eutrophication (Iowa State University et al., 2017). Nitrate-N loss through drainage systems is highly dependent on precipitation rates and available nitrate-N amount in the soil (Lawlor et al., 2008).

Farmers apply fertilizer to the soil to replenish the missing N and prevent a potential yield loss. Each year, farmers face two critical fertilizer application decisions that will impact the harvested crop yield and also have environmental consequences: (i) rate; i.e., the amount applied per unit of land area, and (ii) timing. The amount of N taken up by crops during the growing season varies according to the growth stage of the plant. Ideally, one needs to match the required N uptake at each stage by ensuring the N availability in the soil during the uptake timings to achieve maximum yield potential. Corn growth stages are defined as vegetative (V) stages and reproductive stages. The V stages are denoted by Vn , where n represents the number of visible leaf collars.

Most farmers traditionally prefer applying N to the soil either during the fall or in the spring before planting. Cao et al. (2018) surveys historical fertilizer application timing in US. The most recent data on N application timing for corn in US were collected in 2010. Of the Iowa respondents to this survey, 31% applied N in the fall, while more than 50% favored spring pre-plant application. Sidedressing strategies generally were not preferred by the farmers. Similarly, according to Bierman et al. (2012), the occurrence of fall, spring and sidedressing N application in Minnesota was 32.5%, 58.8%, and 8.7%, respectively, in 2009. The main concern with fall application is the unavailability of N in the soil in the spring and throughout the growing season. Some N loss is expected during the winter, with rate of this N loss depending on the winter precipitation. Spring application lowers the expected N loss because the duration of time between N application and uptake by the plant is significantly shorter. However, spring application poses another risk. If the soil is not suitable for fieldwork in the early spring pre-plant time due to high soil moisture, then it will not be possible to apply the fertilizer without avoiding planting delay. Such delay could

reduce the maximum yield potential. A third alternative, sidedressing, became popular in the last couple of decades as a result of nutrient reduction efforts. This alternative involves the application of some portion of N during the pre-planting window and applying the rest by sidedressing after planting during the summer, commonly within the V6-V8 growth stages of corn, with the idea of feeding N at the right time to reduce nitrogen loss to the environment and achieve a higher yield outcome (Nleya et al., 2016). The V6-V8 growth stages are expected to occur around June, depending on planting time, and each stage lasts two to three weeks. In this study, we investigate two sidedressing strategies: (i) split (40% pre-plant and 60% summer sidedress), and (ii) full summer sidedress. Sidedressing risks, however, may be even higher than those of spring application. First, a split application still could cause a planting delay due to the spring feed of the first portion of N. Second, if the soil is not suitable for fieldwork during the summer feed of the second portion of N by sidedress, there will be no choice but to apply less fertilizer to the soil than what was intended. Therefore, nitrate management is a complex process in which fertilizer rate and application timing decisions not only play a crucial role in the farmers' profitability but also have a notable impact on nitrate loss.

In the remainder of this paper, we denote the fertilizer application rate decision by a continuous decision variable, t , and fertilizer timing decisions by binary variables x_i where $i \in \mathcal{I} = \{1(\text{fall}), 2(\text{spring}), 3(\text{split}), 4(\text{full summer sidedress})\}$.

3.1.2 Planting Time

Planting can start when the soil is warm enough, not too wet, and not too dry. Those conditions are necessary for planting and other field operations, but there are also other considerations. The main goal when selecting planting time is to ensure that the time between planting and the end of the growing season is long enough so that crops can mature enough before harvesting (Elmore, 2013). For example, in colder climates, corn is expected to mature more slowly and harvest must occur earlier. Previous research investigates the impact of planting time on yield at different locations (Baum et al., 2020; Abendroth et al., 2017). For most locations, optimal planting windows (the period between first and last date to plant to obtain maximum yield) have been identified. In Iowa they can range between early April and mid-May, depending on the region. If farmers cannot plant during their

specific location’s planting window due to some delay, the crop will not reach its maximum yield potential. Fertilizer applications other than sidedressing can cause such delays in planting because they require similar soil conditions as planting and other field operations, and must be completed before planting operations start.

To represent planting time, we denote decisions by binary variables, z_j , where $j = 1$ represents the optimal planting window recommended by agronomists while $j = 2$ corresponds to a planting delay.

3.1.3 Crop Insurance Plan Selection

Uncertainties, including weather and thereby yield, market prices, and policies, significantly affect the farm income. Crop insurance is popular among farmers because it potentially reduces their risk exposure (Antón et al., 2013; Moschini and Hennessy, 2001). Farmers can purchase the insurance policies subsidized by the federal government for protection against a potential crop loss due to unexpected weather conditions and/or revenue loss due to unexpected price changes. Producers can pay the premium for their selected policy to receive an indemnity payment for covered loss.

We consider two common crop insurance plans, yield protection and revenue protection, and denote those choices by binary variables, y_{v1} and y_{v2} , respectively, where $v \in \mathcal{V}$ indicates the selected coverage level. Detailed explanation about the insurance plans is provided in Section 3.4.1.

3.1.4 Timeline

Figure 1 shows the timeline of the farmer’s decisions investigated in this study. Commonly, after the harvesting in the fall, a farmer must finalize fertilizer rate and application timing decisions without full information on random weather events and crop harvest prices. It is logical to expect that, if the farmer prefers a fall application, they can revisit the fertilizer decisions made during fall and modify them in springtime based on observed fall and winter precipitation. That is, the farmer can opt to apply fertilizer again in spring and/or in summer considering the N loss between fall and spring. To the best of our knowledge, however, there are no empirical studies in the literature that quantify the unique impact of

fall precipitation on either N loss or yield. Therefore, in this study, we ignore the potential alteration of fertilizer decisions made in fall because of this lack of data.

Just before spring begins, March 15 is the deadline for all insurance plan purchases for corn in the US. The length of optimal planting windows is expected to be no less than three weeks (Elmore, 2012). This length can be longer depending on the unique climate and weather conditions. If fall fertilizer application is selected ($x_1 = 1$), planting operations can start as soon as the soil becomes suitable for fieldwork. As with most field operations, the length of planting time depends on several factors such as total acres to be covered, implement width and speed, or daily working hours (Edwards, 2015). However, planting time traditionally is not considered as a time-consuming process that could force a planting delay on its own (Irwin and Hubbs, 2018). Therefore, because in this model we only consider planting and fertilizing farming operations, we assume fall fertilizer application will not cause any potential delay in planting.

Because spring and split applications occur just before planting, those management decisions can delay planting depending on weather conditions (Scharf et al., 2002). For this reason, the soil’s suitability for fieldwork in the first two weeks of April is important. We denote the total number of days suitable for fieldwork in early spring by τ_1 and represent its set of values unfavorable for timely planting by B .

Since farmers cannot know the weather conditions before making the fertilizer timing decisions, they take a risk of planting delay and resulting loss of yield by choosing spring or split applications in exchange for a potentially lower N loss, which can help to increase the yield and reduce the N cost (Randall et al., 2008; Gramig et al., 2017; Sawyer et al., 2016). Likewise, if split application is chosen, the remaining fertilizer application is planned to be completed in summer. This implies a second time window in which the soil is required to be dry enough for fieldwork. Unlike with early spring applications, if summer fertilizer sidedressing cannot be completed during the V6-V8 stages window, the uncompleted portion of the fertilizer will be missing (Gramig et al., 2017). Finally, uncertain precipitation and temperature during the growing season, from planting time in early spring until harvesting time in fall, as well as crop price at harvest, affect the farmer’s harvest time revenue.

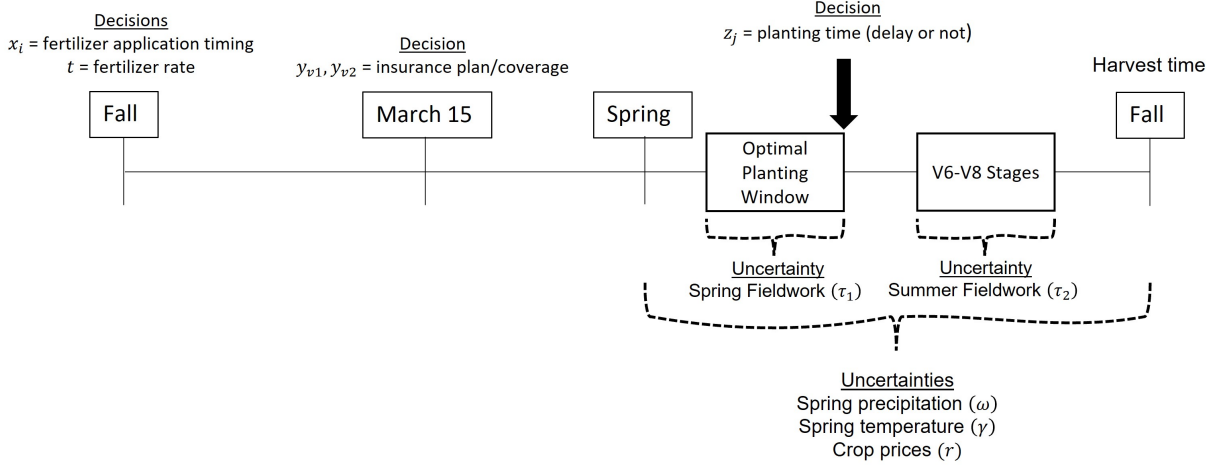


Figure 1: Farmer's decision model timeline

3.2 Available Information and Assumptions

Crop yield depends on several factors, including farm management decisions, weather conditions, and soil properties. Agronomists investigate their impact on crop yield through exhaustive analysis and empirical tests over various sites and conditions (Iowa State University, 2020; Randall and Mulla, 2001; Randall et al., 2008; Sawyer et al., 2016). However, it is highly challenging to observe all those conditions simultaneously and investigate complicated interactions. Thus, the literature largely consists of empirical studies investigating the impact of those factors disjointly by analyzing only one or two selected factors at a time. Accordingly, despite the interactions in the effects of farmer decisions and weather uncertainties on yield, we collect our data from distinct studies and treat their impact on yield as mutually independent. An alternative approach would be to estimate yield and N loss simultaneously through either numerical crop simulation tools (Archontoulis et al., 2020; Stockle et al., 1994) or machine learning approaches (Chlingaryan et al., 2018; Shahhosseini et al., 2019). A comprehensive explanation of the information used and assumptions considered is provided in the supplementary Sections S1.1-S1.3.

3.3 Deterministic Model

If the weather during the growing season and crop price at harvest time were known, a farmer could optimize management decisions according to the model below. Because there

is no risk exposure, insurance is unnecessary.

We denote the crop yield by $A(x, z, t)$, where x and z are binary vectors while t is a continuous variable. Denoting a maximum achievable crop yield of a single farm by H , the yield can be calculated as follows:

$$A(x, z, t) = H \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}(\omega, \gamma) \alpha_{ij} u(t) \quad (1)$$

$$x_i + z_j \geq 2\alpha_{ij} \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \quad (2)$$

where $u(t)$ is the percent of maximum yield given fertilizer application rate t (Section ??), and α_{ij} is another binary variable that equals 1 if and only if $x_i = z_j = 1$. Note that Equation (1) is a bilinear expression where α_{ij} is binary and t is continuous. Because the objective is to maximize the yield and revenue, we can linearize this expression by replacing $\alpha_{ij}u(t)$ with a continuous decision variable w_{ij} and appending Constraints (3) and (4):

$$u(t) - (1 - \alpha_{ij}) \leq w_{ij} \leq \alpha_{ij} \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \quad (3)$$

$$w_{ij} \leq u(t) \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \quad (4)$$

The right hand inequality of Constraint (3) ensures w_{ij} will equal 0 if α_{ij} is 0. Equation (4) and left hand inequality of Equation (3) together force w_{ij} to equal $u(t)$ if α_{ij} equals 1.

Recall that a split application or a full summer N sidedress may prevent the farmer from applying all of the intended fertilizer, depending on suitability of soil conditions for fieldwork. For that reason, we define decision variables u_1 and u_{2i} to replace $u(t)$, where u_1 denotes the percent of maximum yield obtained for fall and spring N applications, and u_{2i} indicates the percent of maximum yield achieved with split and full summer sidedress applications.

The farmer's deterministic mixed-integer program solved in fall, assuming full knowledge of growing season precipitation and temperature, corn harvest price and fieldwork suitability both in early spring and summer, is:

$$\text{Max } (\$/\text{acre}) \quad -gt + rH \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}(\omega, \gamma) w_{ij} \quad (5a)$$

s.t.

$$\sum_{i \in \mathcal{I}} x_i = 1 \quad (5b)$$

$$\sum_{j \in \mathcal{J}} z_j = 1 \quad (5c)$$

$$x_i + z_1 \leq 2 - I\{\tau_1 \in B\} \quad \forall i \in \{2, 3\} \quad (5d)$$

$$x_i + z_j \geq 2\alpha_{ij} \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \quad (5e)$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \alpha_{ij} = 1 \quad (5f)$$

$$u_1 \leq a_l + b_l t \quad \forall l \in \mathcal{L} \quad (5g)$$

$$u_{2i} \leq a_l + b_l k_i(\tau_2) t \quad \forall i \in \{3, 4\}, l \in \mathcal{L} \quad (5h)$$

$$u_1 - (1 - \alpha_{ij}) \leq w_{ij} \leq \alpha_{ij} \quad \forall i \in \{1, 2\}, j \in \mathcal{J} \quad (5i)$$

$$w_{ij} \leq u_1 \quad \forall i \in \{1, 2\}, j \in \mathcal{J} \quad (5j)$$

$$u_{2i} - (1 - \alpha_{ij}) \leq w_{ij} \leq \alpha_{ij} \quad \forall i \in \{3, 4\}, j \in \mathcal{J} \quad (5k)$$

$$w_{ij} \leq u_{2i} \quad \forall i \in \{3, 4\}, j \in \mathcal{J} \quad (5l)$$

$$0 \leq t \leq t_{max} \quad (5m)$$

$$x_i, z_j, \alpha_{ij} \in \{0, 1\} \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \quad (5n)$$

$$u_1, w_{ij} \geq 0 \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \quad (5o)$$

$$u_{2i} \geq 0 \quad \forall i \in \{3, 4\} \quad (5p)$$

The first term in the objective function (5a) represents the cost of using fertilizer rate t . The second term is the revenue obtained from selling harvested crop. Note that other costs of farming operations are excluded, under the assumption that they will not be affected by these management decisions. Constraints (5b)-(5f) involve fertilizer application timing, planting timing and their interactions. Equations (5b) and (5c), respectively, ensure that only one of the fertilizer timing and planting windows is selected. Recall that if spring or split application is selected and the soil is not suitable for fieldwork in early spring, the farmer must delay the planting operation. Constraint (5d) enforces this logic. The set of unfavorable τ_1 values which will delay the planting operation is denoted by B . The binary

parameter $I\{\tau_1 \in B\}$ equals 1 if $\tau_1 \in B$, and 0 otherwise. Equation (5e) guarantees α_{ij} equals 1 if both fertilizer application time $x_i = 1$ and planting time $z_j = 1$. Equation (5f) ensures only a single $\alpha_{ij} = 1$. To approximate the percent of maximum yield given a fertilizer rate t , we substitute piecewise linear functions for the data points shown in Figure 3. Equation (5g) defines the piecewise linear functions used to estimate the concave relationship between N rate and percent of maximum yield while Equation (5h) additionally takes into account the possibility of not being able apply all of the planned N with sidedress applications. The parameter $k_i(\tau_2)$ is the portion of N applied to soil calculated based on total workdays available for fieldwork during V6-V8 stages. This portion may be different for split and full summer sidedress applications. Therefore, the calculation of u_{2i} involves how much N is actually able to be applied to the soil for a given weather condition. Constraints (5i) to (5l) are used to linearize the bilinear terms. Equation (5m) defines the bounds for N application rate and the remaining constraints are the sign and binary restrictions on the decision variables.

Note that before introducing w_{ij} and constraints (5i) - (5l), the objective function would have bilinear terms $\alpha_{ij}u_1$, and $\alpha_{ij}u_{2i}$, while all constraints are linear expressions. A branch-and-cut solution procedure would create subproblems by fixing discrete variables to binary values. With all α_{ij} fixed to 0 or 1, the objective function would be linear. Then, if the fertilizer timing decision is $x_1 = 1$ or $x_2 = 1$, an optimal solution exists at one of the breakpoints for u_1 as a function of t . However, if $x_3 = 1$ or $x_4 = 1$ and the corresponding value of $k_i(\tau_1) < 1$, then a breakpoint combination of t and respective u_{2i} may not be optimal. However, restricting attention to breakpoint values of t proves to be a useful heuristic, as illustrated in Section 5.2.

3.4 Stochastic Program

The farm management decisions and growing season weather together determine the crop yield. Because farmers make fertilizer management decisions without full information on random weather events, the crop yield is the major uncertain element in this study. To summarize the connections between management decisions and uncertainties:

- Growing season average precipitation and temperature directly impact yield.

- The lack of enough days suitable for fieldwork during early spring causes planting delay if the spring or split fertilizer application decision was made during fall.
- The farmer will not be able to apply some portion of planned summer sidedress if there are not enough days suitable for fieldwork during summer.
- Crop yield uncertainty (depending on growing season weather), and harvest-time crop price uncertainty significantly affect the farmer's profit (generated by the combination of crop sale revenue and crop insurance).

Therefore, the average growing season precipitation, ω , the average growing season temperature, γ , the corn harvest price, r , the number of suitable workdays in early spring, τ_1 , and the number of suitable workdays in summer during the V6-V8 stages of the crop growth, τ_2 , are the uncertain elements in our model.

In the deterministic model, the crop yield is calculated using Equation (1). Note that $\beta_{ij}(\omega, \gamma)$ is the only parameter in that equation, and the first uncertain parameter of the stochastic program. The second uncertain parameter is the corn price, r , at harvest time. The third uncertain parameter used in model (5) is $I\{\tau_1 \in B\}$, an indicator takes the value of 1 if $\tau_1 \in B$, causing a planting delay. Finally, the fourth and fifth uncertain parameters are $k_2(\tau_2)$ and $k_3(\tau_2)$. Those parameters represent the portion of N that can be applied to soil the during the growing season, and depend on uncertain element τ_2 .

We structure a two-stage stochastic program by splitting the farmer's timeline into two periods, (i) from fall until spring, and (ii) from spring until harvest time in fall. Figure 2 depicts the decisions and recourse actions at each stage. The first stage involves fertilizer application timing, fertilizer rate and insurance planning decisions. Because the optimal planting windows are already identified, the planting time is a simple recourse action in the second stage after the realization of whether or not a planting delay occurs. After all uncertainties are realized, the resulting yield and revenue are observed.

Assuming we have a finite number of realizations for each of the random variables $(\omega, \gamma, r, \tau_1, \tau_2)$, we can define the scenario set $\mathcal{S} = \{1, \dots, S\}$ that consists of scenarios s , each of which represents a particular combination of realizations. As a result, we rewrite the parameters $\beta_{ij}(\omega, \gamma, \tau_1)$, r , $I\{\tau_1 \in B\}$, and $k_i(\tau_2)$ as β_{ij}^s , r^s , I^s , and k_i^s respectively.

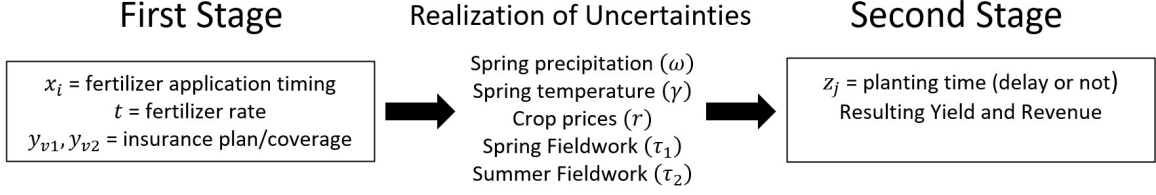


Figure 2: Staged Representation of Farmer's Problem

3.4.1 Modeling Insurance

We consider the two most common crop insurance plans: (i) yield protection, and (ii) revenue protection. Each alternative has options in the set $\mathcal{V} = \{1, 2, \dots, 8\}$ corresponding to coverage levels $\{50\%, 55\%, \dots, 85\%\}$, respectively. The premium rates for each plan and coverage level depend on several factors including the producer's county, their historical 10-year average yield, the yield trend, and the size of the farm (acres).

The yield protection plan offers a production based guarantee. The indemnity payment of this option, denoted by σ_1 , is calculated as $\max(\mu f_v r_0 - r_0 A, 0)$, where r_0 is the projected corn price, f_v is the coverage percentage, μ is the actual production history (average yield) for the farm, and A is the actual yield realized at harvest.

The revenue protection plan offers a revenue guarantee, and also takes harvest price uncertainty into account. The indemnity payment of this plan, denoted by σ_2 , is calculated as $\max(\mu f_v r_0 - r A, \mu f_v r - r A, 0)$, where r is the uncertain actual harvest price.

We define the binary decision variables, y_{v1} and y_{v2} , for the yield protection and revenue protection plan, respectively, to indicate which coverage level, $v \in \mathcal{V}$, is selected by the farmer. A two-stage insurance benefit model is formulated as follows:

$$\begin{aligned} \text{Max } (\$/\text{acre}) \quad & - \sum_{v \in \mathcal{V}} (c_{v1} y_{v1} + c_{v2} y_{v2}) + \sum_{s \in \mathcal{S}} p^s (\sigma_1^s + \sigma_2^s) \\ \text{s.t.} \end{aligned} \tag{6a}$$

$$\sigma_1^s \geq \sum_v \mu f_v r_0 y_{v1} - r_0 A^s \quad \forall s \in \mathcal{S} \tag{6b}$$

$$\sigma_1^s \leq \sum_v \mu f_v r_0 y_{v1} - r_0 A^s + M q_1^s \quad \forall s \in \mathcal{S} \tag{6c}$$

$$\sigma_1^s \leq M(1 - q_1^s) \quad \forall s \in \mathcal{S} \tag{6d}$$

$$\sigma_2^s \geq \sum_v \mu f_v r_0 y_{v2} - r^s A^s \quad \forall s \in \mathcal{S} \tag{6e}$$

$$\sigma_2^s \leq \sum_v \mu f_v r_0 y_{v2} - r^s A^s + M q_2^s \quad \forall s \in \mathcal{S} \quad (6f)$$

$$\sigma_2^s \geq \sum_v \mu f_v r^s y_{v2} - r^s A^s \quad \forall s \in \mathcal{S} \quad (6g)$$

$$\sigma_2^s \leq \sum_v \mu f_v r^s y_{v2} - r^s A^s + M q_3^s \quad \forall s \in \mathcal{S} \quad (6h)$$

$$\sigma_2^s \leq M q_4^s \quad \forall s \in \mathcal{S} \quad (6i)$$

$$q_2^s + q_3^s + q_4^s = 2 \quad \forall s \in \mathcal{S} \quad (6j)$$

$$\sigma_1^s, \sigma_2^s \geq 0 \quad \forall s \in \mathcal{S} \quad (6k)$$

$$\sum_v (y_{v1} + y_{v2}) \leq 1 \quad (6l)$$

$$y_{v1}, y_{v2} \in \{0, 1\} \quad \forall v \in \mathcal{V} \quad (6m)$$

$$q_1^s, q_2^s, q_3^s, q_4^s \in \{0, 1\} \quad \forall s \in \mathcal{S} \quad (6n)$$

The parameter c_{v1} is the insurance premium for the yield protection plan, while c_{v2} denotes the insurance premium of the revenue protection plan, with coverage level v . Accordingly, the first two terms in the objective function represent the cost of the insurance alternative selected. The third and fourth terms of the objective are the expected indemnity payments for the yield and revenue protection plan, respectively. The random crop yield harvested at the end of growing season is denoted by A^s while r^s is the random crop selling price. To calculate the yield protection plan indemnity, σ_1 , we introduce a new binary disjunctive variable q_1^s , and disjunctive constraints (6b)-(6d) by using a big-M reformulation. Similarly disjunctive variables, q_2^s , q_3^s , and q_4^s , and constraints (6e)-(6j) are introduced to calculate the revenue protection plan indemnity. A detailed explanation of the role of the disjunctive variables, and constraints (6b)-(6n) are provided in the supplementary material.

3.4.2 Two-Stage Stochastic Program for the Full Problem

The insurance model described in the previous section does not include the impact of fertilizer management or planting time on the actual yield realized at harvest. In this section, we combine all decisions presented in Figure 2, and build a two-stage stochastic programming model of the farmer's annual overall decision problem. Model (7) combines all constraints presented in sections 3.3 and 3.4.1. When calculating indemnities for both insurance plans, we replace the observed actual yield A^s mentioned in section 3.4.1 with

$$A^s(x, z, t) = H \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}^s w_{ij}^s.$$

$$\text{Max } (\$/\text{acre}) \quad \pi = -gt - \sum_{v \in \mathcal{V}} (c_{v1} y_{v1} + c_{v2} y_{v2}) \quad (7a)$$

$$+ \sum_{s \in \mathcal{S}} p^s \left[Hr^s \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}^s w_{ij}^s + \sigma_1^s + \sigma_2^s \right]$$

s.t.

$$t \leq t_{max} \quad (7b)$$

$$\sum_{i \in \mathcal{I}} x_i = 1 \quad (7c)$$

$$\sum_v (y_{v1} + y_{v2}) \leq 1 \quad (7d)$$

$$u_1 - b_l t \leq a_l \quad \forall l \in \mathcal{L} \quad (7e)$$

$$u_{2i}^s - b_l k_i^s t \leq a_l \quad \forall i \in \{3, 4\}, l \in \mathcal{L}, s \in \mathcal{S} \quad (7f)$$

$$\sum_{j \in \mathcal{J}} z_j^s = 1 \quad \forall s \in \mathcal{S} \quad (7g)$$

$$x_i + z_1^s \leq 2 - I^s \quad \forall s \in \mathcal{S}', i \in \{2, 3\} \quad (7h)$$

$$x_i + z_j^s - 2\alpha_{ij}^s \geq 0 \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, s \in \mathcal{S} \quad (7i)$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \alpha_{ij}^s = 1 \quad \forall s \in \mathcal{S} \quad (7j)$$

$$u_1 - (1 - \alpha_{ij}^s) \leq w_{ij}^s \leq \alpha_{ij}^s \quad \forall i \in \{1, 2\}, j \in \mathcal{J}, s \in \mathcal{S} \quad (7k)$$

$$w_{ij}^s - u_1 \leq 0 \quad \forall i \in \{1, 2\}, j \in \mathcal{J}, s \in \mathcal{S} \quad (7l)$$

$$u_{2i}^s - (1 - \alpha_{ij}^s) \leq w_{ij}^s \leq \alpha_{ij}^s \quad \forall i \in \{3, 4\}, j \in \mathcal{J}, s \in \mathcal{S} \quad (7m)$$

$$w_{ij}^s - u_{2i}^s \leq 0 \quad \forall i \in \{3, 4\}, j \in \mathcal{J}, s \in \mathcal{S} \quad (7n)$$

$$\sigma_1^s - \mu r_0 \sum_v f_v y_{v1} + Hr_0 \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}^s w_{ij}^s \geq 0 \quad \forall s \in \mathcal{S} \quad (7o)$$

$$\sigma_1^s - \mu r_0 \sum_v f_v y_{v1} + Hr_0 \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}^s w_{ij}^s - M q_1^s \leq 0 \quad \forall s \in \mathcal{S} \quad (7p)$$

$$\sigma_1^s - M(1 - q_1^s) \leq 0 \quad \forall s \in \mathcal{S} \quad (7q)$$

$$\sigma_2^s - \mu r_0 \sum_v f_v y_{v2} + Hr^s \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}^s w_{ij}^s \geq 0 \quad \forall s \in \mathcal{S} \quad (7r)$$

$$\sigma_2^s - \mu r_0 \sum_v f_v y_{v2} + Hr^s \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}^s w_{ij}^s - M q_2^s \leq 0 \quad \forall s \in \mathcal{S} \quad (7s)$$

$$\sigma_2^s - \mu r^s \sum_v f_v y_{v2} + Hr^s \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}^s w_{ij}^s \geq 0 \quad \forall s \in \mathcal{S} \quad (7t)$$

$$\sigma_2^s - \mu r^s \sum_v f_v y_{v2} + Hr^s \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \beta_{ij}^s w_{ij}^s - M q_3^s \leq 0 \quad \forall s \in \mathcal{S} \quad (7u)$$

$$\sigma_2^s - M q_4^s \leq 0 \quad \forall s \in \mathcal{S} \quad (7v)$$

$$q_2^s + q_3^s + q_4^s = 2 \quad \forall s \in \mathcal{S} \quad (7w)$$

$$t \geq 0, u_1 \geq 0 \quad (7x)$$

$$x_i, y_{v1}, y_{v2} \in \{0, 1\} \quad \forall i \in \mathcal{I}, v \in \mathcal{V} \quad (7y)$$

$$u_{2i}^s \geq 0 \quad \forall i \in \{3, 4\}, s \in \mathcal{S} \quad (7z)$$

$$w_{ij}^s, \sigma_1^s, \sigma_2^s \geq 0 \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, s \in \mathcal{S} \quad (7aa)$$

$$z_j^s, \alpha_{ij}^s, q_1^s, q_2^s, q_3^s, q_4^s \in \{0, 1\} \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, s \in \mathcal{S} \quad (7ab)$$

3.5 Fertilizer Rate Decomposition

Our computational experiments demonstrate that the two-stage stochastic mixed-integer program (7) is computationally expensive due to its disjunctive and linearization constraints, and the predominance of binary variables. In the literature, different formulation and solution strategies are suggested to overcome the difficulty of dealing with linearization (Adams and Sherali, 1990; Gupte et al., 2013) and disjunctive constraints (Sherali and Shetty, 2012). Our preliminary results show that the optimality gap of model (7) exceeds 80% after 12 hours of solution effort by CPLEX. In this section, we provide an alternative solution strategy using the unique structure that results from the assumptions made.

Among all the continuous variables (t , u_1 , u_{2i}^s , and w_{ij}^s) in the two-stage stochastic model (7), t is the only actual decision made by the farmer. The auxiliary variables, u_1 and u_{2i}^s , simply represent the impact of t on yield according to the piecewise linear approximation, and w_{ij}^s is a variable introduced for the purpose of linearization. Therefore, if t is fixed, all the remaining non-auxiliary decision variables are binary.

For a given fixed N application rate t' , let the parameters $\zeta_{ijv1}^s(t')$ and $\zeta_{ijv2}^s(t')$ denote the recourse indemnities for yield and revenue protection plans respectively:

$$\zeta_{ijv1}^s(t') = \max \left(\mu f_v r_0 - A_{ij}^s(t') r_0, \quad 0 \right) \quad \forall i, j, v, s \quad (8)$$

$$\zeta_{ijv2}^s(t') = \max \left(\mu f_v r_0 - A_{ij}^s(t') r^s, \quad \mu f_v r^s - A_{ij}^s(t') r^s, \quad 0 \right) \quad \forall i, j, v, s \quad (9)$$

where $A_{ij}^s(t')$ is a parameter representing the actual yield at harvest for N application time

i and planting time j in scenario s . Recall that the insurance indemnities are calculated using decision variables σ_1^s and σ_2^s in models (6) and (7). By fixing t to a value t' , we simply convert the decision variables σ_1^s and σ_2^s into parameters $\zeta_{ijv1}^s(t')$ and $\zeta_{ijv2}^s(t')$.

Similarly, we introduce binary decision variables η_{ijve}^s , where $e = 1$ represents the yield protection plan and $e = 2$ corresponds to the revenue protection plan. Decision variable η_{ijve}^s equals 1 if the protection plan e is selected with N application time i , planting time j and coverage level v , and 0 otherwise.

Then an alternative formulation, assuming the N rate decision t has been made, is:

$$\begin{aligned}
& \text{Max } (\$/\text{acre}) & \rho(t') = - \sum_{v \in \mathcal{V}} (c_{v1}y_{v1} + c_{v2}y_{v2}) + \\
& \sum_{s \in \mathcal{S}} p^s \left[\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \left(A_{ij}^s(t') r^s \alpha_{ij}^s + \sum_{v \in \mathcal{V}} (\zeta_{ijv1}^s(t') \eta_{ijv1}^s + \zeta_{ijv2}^s(t') \eta_{ijv2}^s) \right) \right] \\
& \text{s.t.} & \\
& & (7c), (7d), (7g), (7h), (7i), (7j), (7y), (7ab) \\
& & x_i + z_j^s + y_{v1} - 3\eta_{ijv1}^s \geq 0 & \forall i \in \mathcal{I}, j \in \mathcal{J}, v \in \mathcal{V}, s \in \mathcal{S} \\
& & x_i + z_j^s + y_{v2} - 3\eta_{ijv2}^s \geq 0 & \forall i \in \mathcal{I}, j \in \mathcal{J}, v \in \mathcal{V}, s \in \mathcal{S} \\
& & \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{v \in \mathcal{V}} (\eta_{ijv1}^s + \eta_{ijv2}^s) \leq 1 & \forall s \in \mathcal{S} \\
& & \eta_{ijv1}^s, \eta_{ijv2}^s \in \{0, 1\} & \forall i \in \mathcal{I}, j \in \mathcal{J}, v \in \mathcal{V}, s \in \mathcal{S}
\end{aligned}$$

The first term in the objective function (10a) represents the first stage costs, and corresponds to insurance premiums paid. The remaining terms are revenues from harvested yield and insurance. Note that, since the yield impact of t' and insurance indemnities of insurance decisions are now calculated as parameters in the form of ζ_{ijv1}^s and ζ_{ijv2}^s , no linearization or disjunctive constraints are required in this model. As a result, constraints (7c), (7d), (7g), (7h), (7i), (7j), (7y), and (7ab) are retained in model (10) while the remaining constraints from model (7) are replaced by constraints (10c)-(10f).

Using the simplified formulation, we decompose the problem by separating the fertilizer rate decision t from all the other decisions, and solve it using Algorithm 1:

Because the values for percent of maximum yield are identified for only a finite number of integer-valued t , one alternative to use Algorithm 1 is to enumerate over all t from 0 to

Algorithm 1 Fertilizer rate decomposition

```
1: Initiate BestResult = 0
2: for  $t'=0, t' \leq t_{max}, t' = t'_{next}, t'_{next} \in T'_{cand}$  do
3:   Solve model (10) using  $t'$ 
4:   NewResult =  $\rho(t') - gt'$ 
5:   if NewResult > BestResult then
6:     BestResult = NewResult and  $t^* = t'$ 
7:   end if
8: end for
```

t_{max} . Alternatively, we can use the L linear segments as described in section 3.4.2 and model (7) to approximately solve the model (10). In this heuristic approach we consider a set T'_{cand} that includes only the $L + 1$ breakpoints of the piecewise linear function. Instead of solving model (10) in step 3 of Algorithm (1), we solve the model (7) after fixing $t = t' \in T'_{cand}$. As discussed in section 3.3, equation (7f) is the only constraint that may prevent one of the breakpoint t values of the concave piecewise linear function from being optimal in (7). In section 5.1, we show that using this heuristic approach significantly improves the computation time with a small optimality gap when L is a small number. Furthermore, by increasing L , it is possible to come arbitrarily close to optimality without significantly increasing the computational time. In the remainder of the paper, this heuristic approach used to solve model (7) is referred to as the piecewise linear (PL) approximation heuristic.

4. Computational Study

The study is designed to represent a typical corn farm in Iowa, where typically corn is grown in rotation with soybeans but sometimes is repeated year after year.

The impact of the nitrogen application rate on yield is reflected in our model based on data points illustrated in Figure 3. We assume that this relationship of yield to N rate holds for fall N application. We use this information in two different ways. First, we generate piecewise linear (PL) functions representing this data to preserve the linearity of the main model as illustrated in Figure 3. We also explore the results by trying all potential fertilizer application rate points using Algorithm 1. To generate PL functions, we use the formulation of Jekel and Venter (2019) to identify the locations of a specified number of breakpoints that minimize the overall sum of squared differences between original data points and the

PL approximation. For illustration, we generate three linear pieces as shown in Figure 3. However, we also explore how increasing the number of linear segments affects the quality of the results of the heuristic approach discussed in Section 3.5. Note that we do not allow the percent of maximum yield to exceed 100%. That is, if the PL approximation exceeds 100% at any point, we replace the approximated function value with 100%.

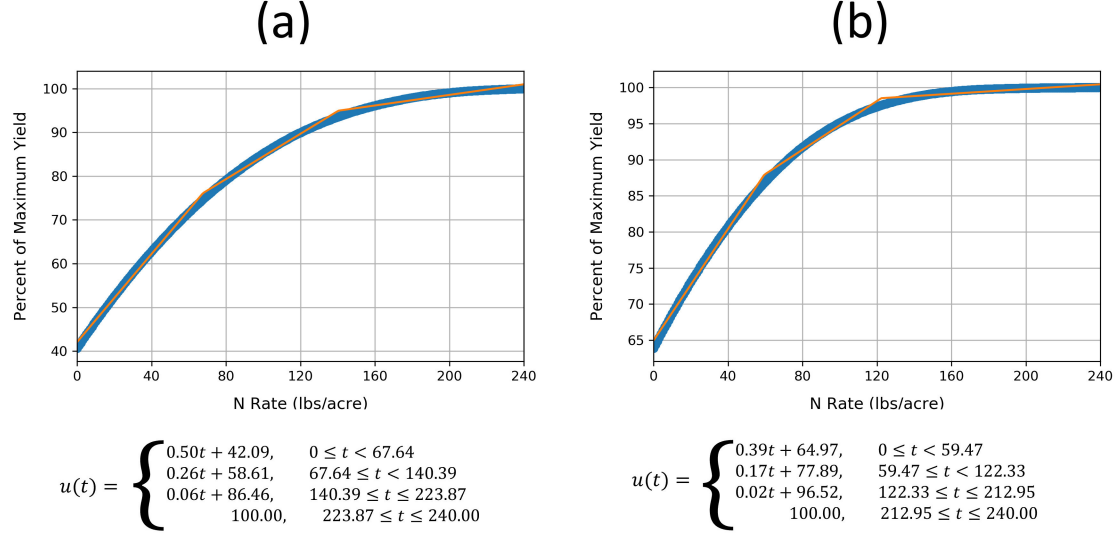


Figure 3: Impact of N rate on yield as discrete points and piecewise linear approximation with $L = 3$. (a) Corn-corn rotation (b) Soybean-corn rotation (Sawyer et al., 2020)

The rest of the information explaining the calculation of parameters and random variables is provided in the supplementary material. Since we collected information related to random variables, ω , γ , r^s , τ_1 and τ_2 , independently, we assume that they are mutually independent. Accordingly, we generate 768 combinations ($4 \times 4 \times 4 \times 2 \times 6$) as scenarios by multiplying the marginal probabilities. Similarly, realized yield β_{ij}^s is computed based on the respective yield impacts of each random component of scenario s , along with decisions x_i and z_j . Assuming the impacts of all decisions and uncertainties constituting a scenario path s , are independent of each other and multiplicative due to limited available information to reflect interactions among them, we calculate β_{ij}^s by multiplying the yield factors of i , j , γ and ω with a baseline $u(t)$ rate of 1.

5. Results and Discussion

In this section, we summarize the results of our computational runs by describing: (i) the computational performance and solution quality of the suggested models and the heuristic, and a suitable granularity for the PL approximation; (ii) optimal results for the baseline case and how different N application rates affect the profit and other management decisions; (iii) how higher crop insurance premiums affect the results; (iv) the water quality implications; and (v) the interactions between N management and crop insurance; specifically, how crop insurance programs affect environmentally beneficial N management practices.

We implemented the proposed models in Java and use IBM ILOG CPLEX as the optimization engine. We performed the computational experiments on a machine with Intel Core i7-7700HQ @ 2.80 GHz processor and 16 GB RAM.

5.1 Piecewise Linear (PL) Approximation Heuristic

Section 3.5 describes two alternative solution approaches using Algorithm 1: (i) enumerating over integer-valued fertilizer amounts using the data points provided in Figure 3 and optimizing the discrete decisions, and (ii) using the PL approximation to optimize all decisions simultaneously. In this section, we compare those two approaches in terms of computational performance and solution quality. We investigate how increasing L , the number of linear segments, affects the solution quality of the heuristic approach.

Table 2 summarizes the computational performance of the alternative solution approaches for corn following corn. The middle columns contain the solutions obtained using different numbers, L , of linear segments. The row labeled “N Rate” indicates the optimal values of t , which is the only management decision variable whose value differs according to the solution approach and value of L . Recall that the PL approximation uses $u(t)$ to generate percent of maximum yield. The enumeration strategy, on the other hand, uses the actual data points instead of $u(t)$ and enumerates over all integer-valued t from 0 to t_{max} , as illustrated in Figure 3. Therefore, the same decisions may yield slightly different expected profits when those two strategies are compared. To make a fair profit comparison between those two strategies, after having applied the PL approximation heuristic, we calculate $\rho(t')$ by fixing all the decisions generated from the heuristic in equation (10a). Thus,

we use the real percent of maximum yield data instead of $u(t)$ to report the profit values for the heuristic in Table 2. The piecewise linear approximation heuristic finds a solution within 25 minutes but enumeration over all integer N rates takes more than 13 hours. The profit achieved by implementing the PL approximation heuristic solution is nearly optimal if L is sufficiently large.

Table 2: Changing L and its relationship with optimality for corn-corn rotation

	PL Approximation Heuristic								Optimal Enum.
	$L = 3$	$L = 4$	$L = 5$	$L = 6$	$L = 7$	$L = 8$	$L = 9$	$L = 10$	
N Rate (lbs/acre)	223.87	228.75	231.75	185.25	189.47	191.64	204.09	205.05	205.00
Profit (\$/acre)	615.87	615.56	615.04	615.44	616.44	616.43	617.04	617.41	617.41
Optimality Gap (%)	0.25	0.30	0.38	0.32	0.16	0.16	0.06	0.00	-
Comp. (s)	803.16	910.25	964.92	1077.27	1145.43	1287.05	1454.43	1500.30	49539.25

The accuracy of the piecewise linear approximation with L sufficiently large indicates that the stochastic mixed-integer program (7) could be solved to find near-optimal solutions for the true nonlinear relationship between yield and N rate. In the remainder of the paper we enumerate over t using Algorithm 1 to explore the relationship between the N rate and the binary decisions.

5.2 Baseline Results

Figure 4 presents the optimal solutions and profits for corn following corn (C-C) and corn following soybean (S-C), respectively. For both crop rotations, full summer sidedress is the optimal fertilizer application timing decision, while the yield protection plan with maximum coverage rate at 85% is the best insurance decision when the N rate is set to its optimal value. For the C-C case, Figure 5 shows the components of expected profit to explain the nonconvex shape of the profit curve. While increasing t also increases the expected harvest income with diminishing returns, it reduces the expected insurance indemnity at a decreasing rate. However, the indemnity payment flattens out faster than harvest income. As a result, the expected profit initially shows a decreasing trend, after which it continues to increase until the optimal solution is reached.

The impact of the fertilizer application rate decision on other farming decisions is also investigated. Fertilizer application rate is a critical farming decision, not only affecting the farmer’s profitability but also causing environmental consequences. Environmentalists

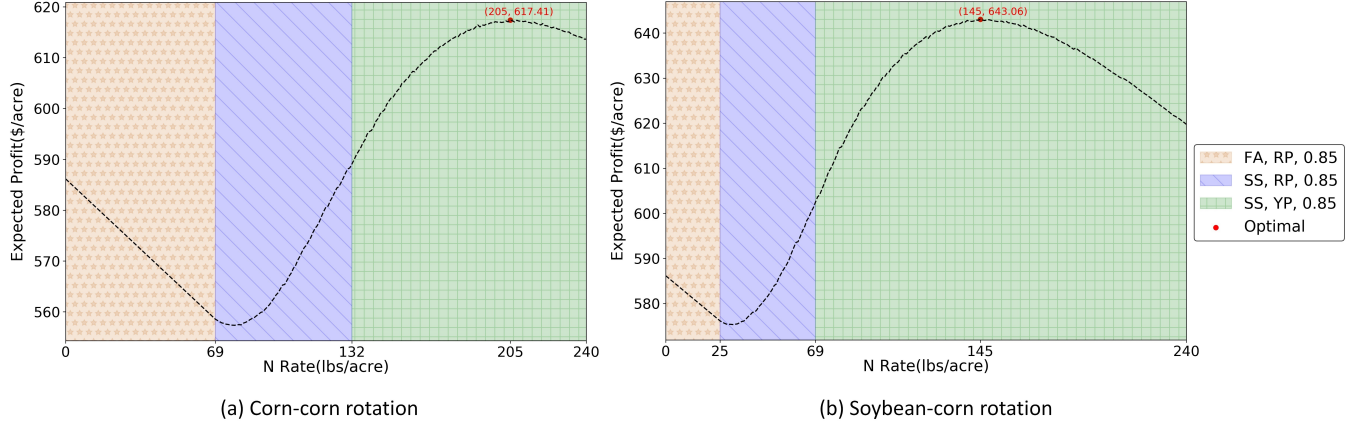


Figure 4: Baseline case results. Shaded regions are labeled by N timing decisions (FA = fall application, SS = summer sidedress), type of insurance (RP = revenue protection, YP = yield protection), and insurance coverage rate, f_v

and social planners ideally would prefer to reduce N application rate as much as possible to lower nitrate-N loss through leaching. Although we investigate the problem from a farmer's point of view, understanding how different N application rates affect the profit and other management decisions is just as important as knowing farmers' optimal solutions. Based on the applied fertilizer rate, we observe three combinations of optimal fertilizer timing and insurance decisions. Recall that fall N application is expected to produce the lowest yield, but it also imparts less risk than the other timing alternatives because the random variables τ_1 and τ_2 have no impact on subsequent decisions or yield. On the other hand, summer sidedress application is expected to result in the highest yield according to previous research, yet is also risky. For very low values of t (below 70 lbs/acre for C-C and or 26 lbs/acre for S-C), fall application is optimal. For any higher N application rate, summer sidedress is the best N timing decision. As illustrated in Figure 3, increasing the N application rate also increases the yield. It might be expected that, to overcome the lower yield resulting from low N rates, one would select a higher-yielding timing alternative. However, close examination reveals why low fertilizer rates and fall N application are selected together. When the N rate is low, the model relies on minimizing the harvest yield to maximize the crop insurance indemnity payment. Therefore, the insurance alternative providing the most protection, the revenue protection plan with the highest coverage, is selected. The perverse incentives that exist with low N rates are illustrated in Figure 4 by the decrease of profit as t increases for low values of t . Also note that, even if minimizing the harvest

yield with low N rate to maximize the insurance indemnity payments were optimal in one year, it would not be viable in the long term because indemnity payments depend on the actual production history of the farm.

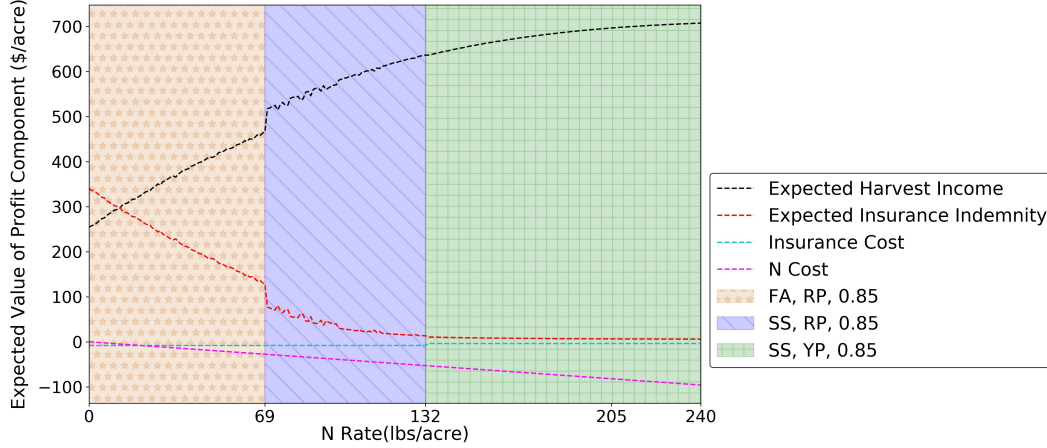


Figure 5: Expected value of profit components for corn-corn rotation. Shaded regions are labeled by N timing decisions (FA = fall application, SS = summer sidedress), type of insurance (RP = revenue protection, YP = yield protection), and insurance coverage rate, f_v

When the fertilizer application rate reaches 70 lbs/acre for C-C or 26 lbs/acre for S-C, maximizing the harvest yield and maximizing the profit align. As a result, summer sidedress becomes the best N timing decision. In this intermediate interval (70-132 lbs/acre for C-C or 26-69 lbs/acre for S-C), the revenue protection plan with the highest coverage rate at 85% is still the best crop insurance decision because the applied fertilizer amount is still not high enough to achieve good crop yield. Finally, when the fertilizer application rate exceeds 132 lbs/acre for C-C or 70 lbs/acre for S-C, the yield protection plan with the highest coverage becomes the best insurance decision as yield risk is reduced.

Due to the higher efficiency (greater percentage of maximum yield for a given N rate) of corn following soybean, as illustrated in Figure 3, the optimal N application rate is lower for S-C, while the expected profit per acre is higher than for C-C. Likewise, the N rates at which the timing and insurance decisions change are different for S-C and C-C.

The optimal N rate is 205 lbs/acre for C-C and 145 lbs/acre for S-C. However, if we calculate the expected application rates using discrete probability outcomes of τ_2 presented in supplementary Table S4, we find that the expected N rate actually applied is approximately 180 lbs/acre for C-C and 127 lbs/acre for S-C. That means the optimal solution

includes a higher N rate to benefit from the higher yield potential of summer sidedressing decision by compensating for the risk of random variable τ_2 . As a further note, when low risk, low yield fall application is forced to be selected, the optimal N rate is 184 lbs/acre for C-C and 130 lbs/acre for S-C.

5.3 Alternative Crop Insurance Premiums

Crop premiums can be higher than our baseline rates, depending on the yield trend of the farm and its surrounding county. In this section, we investigate the impact of the alternative, higher insurance premiums shown in supplementary Table S1. The results are illustrated in Figure 6.

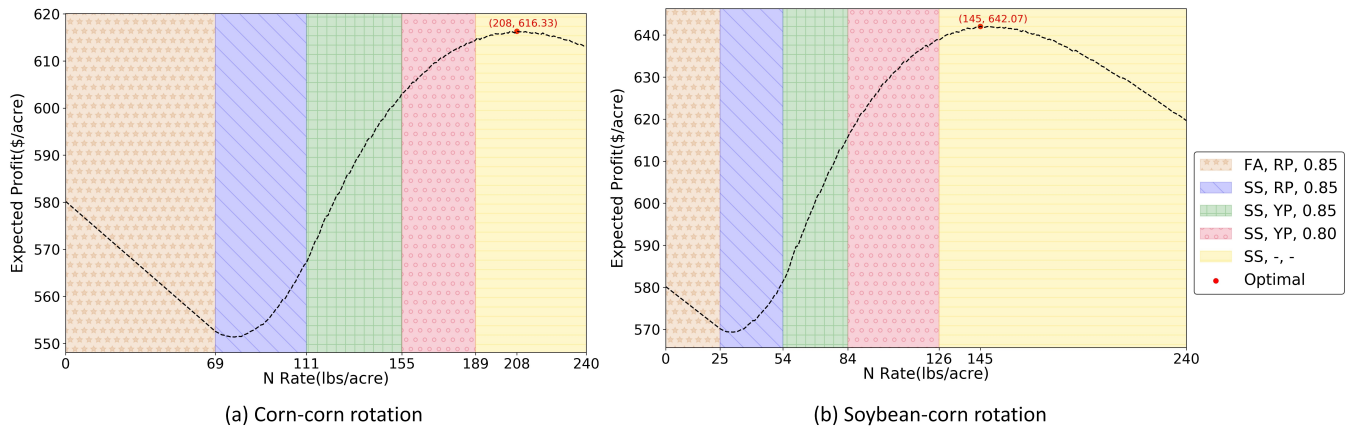


Figure 6: Results with higher crop insurance premiums. Shaded regions are labeled by N timing decisions (FA = fall application, SS = summer sidedress), type of insurance (RP = revenue protection, YP = yield protection), and insurance coverage rate, f_v

Increasing the insurance premiums does not cause any significant change in fertilizer rate or N timing decisions. For S-C, the optimal fertilizer rate and N timing decision with alternative insurance premiums are exactly the same as for the lower baseline insurance premium rates. Similarly, with C-C, we observe only a slight increase in the optimal N application rate compared to baseline premiums. The only significant change occurs in the crop insurance choices. With higher premium rates, the optimal solution foregoes insurance. Even with those high premium rates, the maximum or next highest coverage rate is selected for every fertilizer application rate. If the applied N rate is low, the revenue protection plan is selected with the highest coverage rate. The yield protection plan is selected for higher N application rates with coverage rates at either 80% or 85%. If the N application rate is

higher than 189 lb/acre for C-C and 126 for S-C, buying an insurance plan is not part of the optimal solution.

5.4 Water Quality Implications

Lawlor et al. (2008) estimate the nitrate-N concentration in subsurface drainage based on tests performed in Iowa. According to their study, a N rate application of 205 lbs/acre (the optimal result from C-C in the baseline case) results in a nitrate-N concentration of 20.23 mg/L, while an application of 145 lbs/acre (the optimal result from S-C) corresponds to 12.93 mg/L. According to Lawlor et al. (2008), applying no fertilizer will result in a N concentration of 7 mg/L.

Considering the current Iowa nitrate-N concentration target of 5-6 mg/L based on the 41% reduction goal (Iowa State University et al., 2017), it is highly unlikely to achieve this goal by simply focusing on fertilizer management strategies (i.e., additional nitrogen management, land use and edge-of-field nutrient practices are needed to achieve target reduction goals). In this section, we investigate how much water quality improvement can be achieved by simply focusing on fertilizer management practices. By exploring various N concentration targets achievable as illustrated in Table 3, we show the limitations of fertilizer management in improving water quality, and also indicate the incentives needed to achieve those concentration targets when only fertilizer management is considered. Table 3 displays the expected profit foregone by the farmer to achieve various N concentration targets. To generate the table, we extracted the fertilizer application rate corresponding to each nitrate-N concentration target, based on the information provided by Lawlor et al. (2008), and solved the optimization model repeatedly with fixed t equal to each fertilizer rate in turn. For example, when corn follows corn the farmer's profit from applying 100 lbs/acre to meet the 10 mg/L target is \$52.14 per acre lower (a 8.4% reduction) than the optimal profit achieved by applying 205 lbs/acre. This represents an opportunity cost that, alternatively, drops to \$12.93/acre for corn following soybean (a 2% reduction). These results also demonstrate the combined financial and environmental advantages of crop rotation. It is important to underline that those incentive rates are generated under two assumptions: (i) farmers are rational and have the single objective of maximizing their short-term profit and (ii) other nutrient reduction practices are not considered. Therefore,

the realistic fertilizer-based incentive rates are expected to be lower than what is reported in Table 3. Still, we believe the incentive rates reported for alternative N concentration targets provide a valuable insight to policymakers as those values represent the upper envelope of fertilizer-based incentives. In other words, those rates would ensure the cooperation of rational farmers under the current assumptions but true rates may be lower than what are reported.

Another environmental takeaway concerns the use of sidedressing strategies. The common consensus in agronomy suggests that summer sidedress application increases the farm yield and also reduces the N loss, compared to other N timing decisions such as fall or spring applications. Our results also indicate that this fertilizer timing option optimizes the farmer’s profit. However, this decision is highly susceptible to weather uncertainty. If the soil moisture is high during the summer, there is a high chance that the farmer will not be able to apply all of the intended fertilizer. This economic risk can be mitigated by increasing the planned N application rate which, if carried out, will increase the N loss. As a result, summer sidedress may not be the best decision from an environmental perspective when all uncertainties are considered.

Table 3: Farmer’s opportunity cost of achieving N concentration targets

Target(mg/L)	7	8	9	10	11	12	13	20
N Rate to achieve target (lbs/acre)	0	46	78	100	118	133	145	205
Foregone profit for C-C (\$/acre)	31.26	49.66	60.06	52.14	38.58	27.47	18.42	0
Foregone profit for S-C (\$/acre)	56.91	62.04	31.12	12.93	4.77	1.25	0	-

Note that trying to achieve a 7 and 8 mg/L N concentration appears less costly than 9 mg/L for C-C, and likewise 7 mg/L looks less costly than 8 mg/L for S-C. As discussed in section 5.2, this nonintuitive result occurs because, for low N application rates, it is optimal to minimize the yield in order to maximize the insurance payout. As a result, we observe a decreasing expected profit curve for low values of t .

5.5 Mitigation of N Management Risk by Insurance

Figure 7 illustrates how different N application timing decisions affect the expected farm profit. Expected reduction in profit (\$/acre) represents the cost of selecting a different fertilizer timing decision compared to the optimal baseline results provided in Figure 4.

To generate the plots, we fix x_i to a specific nitrate timing alternative i and enforce its selection in model (7). Then, we obtain the expected profit reduction by calculating the difference between newly obtained results and optimal results from Figure 4.

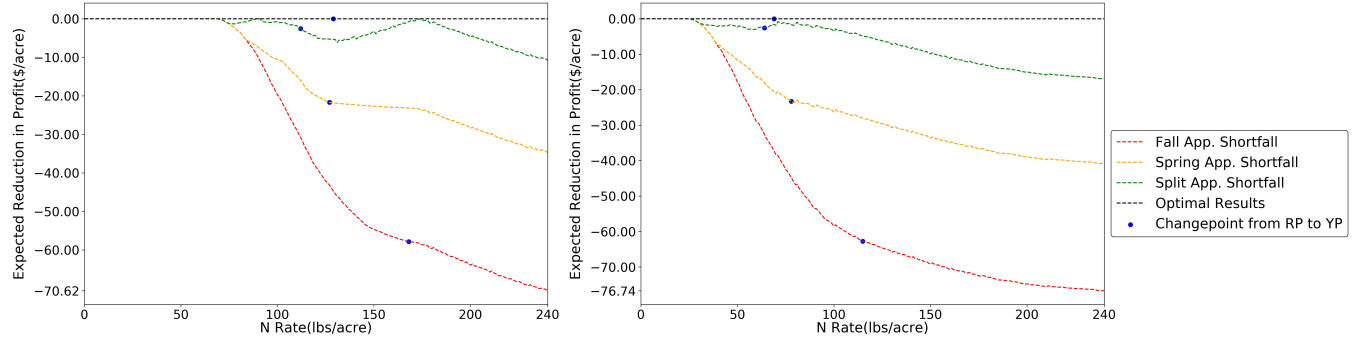


Figure 7: Comparison of N application timing decisions for the baseline case

The sidedressing strategies, split and complete summer sidedress, are considered as part of precision agriculture. Those strategies aim to apply the N during a period of growth and when it is needed most. The idea is to increase the crop uptake efficiency by timely synchronizing the nutrient availability in the soil, considering crop demand based on its growth stages. Therefore, sidedressing strategies are expected to improve water quality and farm yield compared to other fertilizer application timing decisions as less N leaching due to early fertilizer application is expected. For that reason, split fertilizer application is a risk reducing strategy since it reduces the risk of N loss. The results in Figure 7 align with the scientific expectations where both split and summer sidedress applications result in smaller profit reductions in the case study. Interestingly, the expected per-acre profit gap between different N time decisions increases with the N application rate. When the expected yield is very low, resulting from low N application rates, the insurance programs cover the economic deficits. Therefore, the reduction in profit is indistinguishable for different N application time decisions when the N application rate is very low (< 100 bu/acre). In the literature, reducing the N application rate and sidedressing N application timing are considered as two valuable nutrient reduction practices related to N management. However, our results demonstrate that when the N application rates are reduced, timing-related N reduction practices can be redundant for producers concerned only with maximizing their profits because insurance programs also act as a risk-reducing strategy. In other words, risk-reducing conservation practices such as split N application may be redundant

when combined with crop insurance policies. This insight demonstrates the importance of including insurance programs in environmental investigations and designing insurance programs so as to not undermine water quality efforts.

Similarly, Figure 8 highlights the expected reduction in profit when the purchase of insurance policies is not allowed (i.e., solutions to model (7) are forced to not select any insurance plan). N is a limiting nutrient in agriculture because plants cannot utilize atmospheric N directly in its gaseous form. By applying N, agricultural producers ensure the N availability in the soil to maximize yield potential. However, N is susceptible to leaching. Therefore, agricultural producers may tend to apply more N to the soil than necessary to cover the required N uptake by the crops. Figure 8 demonstrates that increasing the N application rate acts as a risk-reducing strategy for agricultural producers when crop insurance is taken out of the picture. As the N application rate increases, we observe that the expected benefit of insurance programs is diminishes to negligibility. This finding is important as it suggests that federal crop insurance programs significantly decrease the economic loss arising from the N application reduction. Specifically, in this case study, the expected C-C rotation profit range (\$/acre) is [557.3, 617.4] with insurance programs and [322.9, 616.9] without insurance. For S-C rotation, the corresponding ranges are [575.3, 643.1] and [514.4, 642.9], with and without insurance, respectively. It also means that the opportunity cost of achieving N concentration targets shared in Table 3 is expected to be higher when the insurance programs are not considered. That is, insurance programs can potentially complement nutrient reduction programs (i.e., they are effective instruments to mitigate the risk of yield loss from reduced N applications).

Uncertainty and the resulting risk are primary agricultural concerns, and our numerical results indicate that they significantly impact fertilizer rate and timing decisions. Because the purpose of insurance programs is to reduce risk exposure, the insurance purchase options that exist should be considered when studying N management from an environmental perspective. The environmental impact of insurance programs may be inconsistent and circumstantial. Specifically, we observe that crop insurance has a complementary role in reducing the N application rate with a positive environmental impact. However, if the N application rate drops below a certain level, the crop insurance reduces the motivation to use environmentally beneficial N timing strategies. Those inconsistent results demonstrate

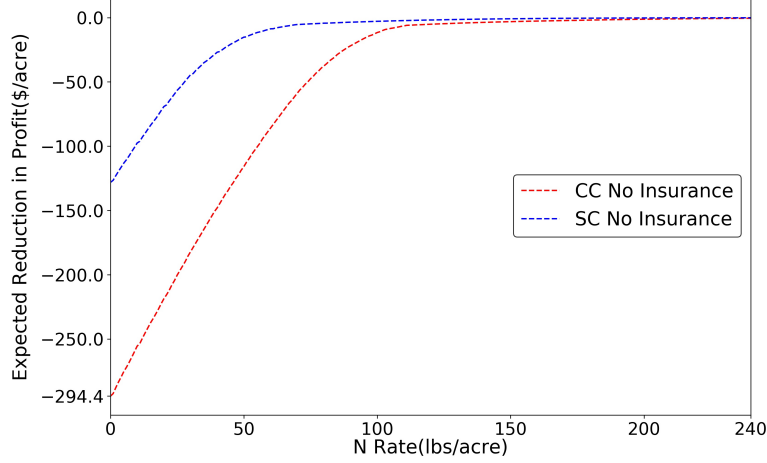


Figure 8: Impact of crop insurance programs on farm profitability

the complicated interactions between N management and crop insurance programs. The incentive rate estimates in Table 3 are generated based on the existing federal insurance program structure and parameters while considering N management decisions only. Updating the structure and parameters of existing crop insurance programs or integrating additional parametric insurance options could reduce the need for financial incentives for adopting environmental best practices. Appropriately designed insurance plans could be vehicles for aligning economic and environmental incentives.

6. Conclusion

This paper explores some major annual farming decisions of a corn producer under uncertain growing season precipitation and temperature, harvest price, and soil moisture during critical time windows. We built a two-stage stochastic mixed-integer program for annual farm management decisions to maximize the expected farm profit. Because the two-stage stochastic program is computationally expensive due to its disjunctive and linearization constraints and the predominance of binary variables, we suggested a heuristic solution approach that produces near-optimal solutions.

By examining the farmer's optimal behavior under uncertainty, the case study derives valuable input to policymakers concerned with developing effective policies and promoting nutrient reduction practices to reduce N loss. Previous field experiments in agronomy demonstrate the advantages of spring and sidedress N application compared to fall N ap-

plication. Sidedressing strategies specifically are expected to lower N loss and increase crop yields and, thus, appear advantageous for both farmers and the environment. Our results, however, indicate that other decisions taken to mitigate farming risks can negate the environmental benefits. Farmers maximizing expected profit would compensate for the additional risks resulting from weather uncertainties if sidedressing is chosen by increasing the planned N application rate. Spring and sidedressing strategies, especially, are more susceptible to the risk of insufficient days suitable for fieldwork, and could paradoxically increase N leaching if the farmer carries out the plan of applying more N to compensate for the yield risk.

To explore financial incentives that policymakers could offer to alter farmers' major annual decisions, we estimate the cost to the farmer, in terms of foregone profit, of achieving potential N reduction targets by fertilizer management alone. The results show that significant incentives are needed under corn-corn rotation for substantial changes in N loss while up to 20% N reduction is achievable under soybean-corn rotation with little impact on profit.

This research explores how crop insurance programs can influence the adoption of environmentally beneficial N management practices. How insurance interacts with other environmental practices constitutes a gap in the literature. If carefully designed, insurance programs have the potential to align economic and environmental incentives. Therefore, expanding the consideration to all available insurance tools and modifying them accordingly to incentivize environmental programs is a promising research direction. Future work could address (i) how insurance programs relate to other best management practices (i.e., use of inhibitors, cover crops, land use changes, etc.) and (ii) how available insurance tools can be used or modified to further incentivize environmentally beneficial practices.

This model has several limitations due to the “reductionist” character of traditional agronomy research (Drinkwater et al., 2016), which informed both the model structure and the case study inputs. The case study performed currently relies on empirical field tests to obtain information about critical outputs, including yield and N loss. These experiments are carefully designed to isolate the impact of one variable, such as fertilizer application rate, on yield. Unfortunately, they are inadequate to investigate all components of an agricultural system and their interactions simultaneously. Simplifying assumptions in our model, such

as independence of the effects of management decisions and uncertain factors on yield, are based on the empirical information available but could distort the optimization results. For a decision model to properly reflect the interactions among management decisions and uncertain elements as they unfold over time, more accurate multivariate functional relationships are needed. Numerical agronomic simulation models may help fill this gap and allow for better model fidelity to actual decision processes.

Data Availability Statement

Data can be made available on request.

Acknowledgement

This material is based upon work supported by the National Science Foundation under Grant No. DGE-1828942.

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