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A shuttle system model based on two interdependent queues

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NOMENCLATURE

τ	=	fixed ferry transit time
C	=	capacity of the ferry
s	=	side A or B
i	=	actual docking number
j	=	docking number of a particular ferry
f	=	denotes a ferry
D_{fj}	=	delay of a particular ferry f due to loading and unloading at its j th docking ($j=1,2,\ldots,m$)
t_{nk}	=	clock time of docking of a ferry at one of the sides; n,k are dummy variables such that $i=2n-3+k$, $i>1$
X_{si}	=	total number of arrivals waiting on side s at i th docking
a	=	mean interarrival time on side A
b	=	mean interarrival time on side B
N_{fi}	=	number of cars boarding ferry f at i th docking
F_{fi}	=	number of cars on ferry f at i th docking
F_{si}	=	number of cars on the ferry docking at side s on the i th time
P	=	denotes the Poisson distribution of the number of arrivals per unit time
h_s	=	a constant time ferry spends on side s of the channel
y_s	=	a function of F at side s
g_s	=	a function of N at side s
Δ^f_{nk}	=	time at t_{nk} for ferry f to reach destination
N_{si}	=	number of cars taken aboard from side s by the ferry at i th docking
D_{si}	=	delay of ferry f at side s, due to loading and unloading at i th docking

W_{fi} = waiting time of ferry f at i th docking

t_i^s = cumulative time at the end of i th unloading of ferry at side s

$X_{s,i(s)}$ = total number of arrivals waiting on side s at the end of i th unloading at side (s)

γ_s = a per car unloading constant for side s

β_s = a per car loading constant for side s

L_i^s = time taken for loading at side s during i th docking

U_i^s = time taken for unloading at side s during i th docking

$N_{A,i(A)}$ = number of cars arriving at side A during the time interval $(t_i^A - t_i^B)$

$N(t)$ = number of car arrivals at time t

$I(t)$ = interarrival time at time t

$E(x)$ = expected number of variable x

d_i^s = cumulative distribution function for the queue sizes on shore at the 1st, 2nd,..., i th unloadings at side s, i=1,2,...,m

$\delta_{i'}^s$ = cumulative distribution function for the queue sizes on shore at the m th, (m-1)st,...,(m-i'+1)st unloadings at side s, i'=1,2,...,m

m = total number of dockings for a particular ferry

R = multiple correlation coefficient

λ = Weibull scale parameter

α = Weibull shape parameter

μ = Weibull location parameter

$F(t)$ = cumulative distribution function

$f(t)$ = density function

k = a constant

I. INTRODUCTION

The population of the earth has been persistently increasing throughout recorded history, but only within the twentieth century has its size become an important obstacle to orderly civilization. One of the problems created by this growth, which has proved to be of some mathematical interest, is that of congestion. On land and in the air, in vehicles and on foot, people now get in each others' way to an extent far surpassing that of any previous age. Congestion is seen not only in transportation, but in virtually every aspect of modern life: communication, urban development, commercial organization, mass production, and perhaps even agriculture.

The scientific study of congestion, whether intended to describe or to ameliorate, has been a natural consequence of man's enforced interest in his increasingly overcrowded world. The most fully developed mathematical theory of congestion is "queueing theory" which deals with accumulation at a fixed point caused by the need for "service". The subject is more than sixty years old and is now being extended very vigorously, both in depth of formulation and in breadth of application.

As a source of congestion, the motor vehicle occupies a unique position, both from the practical and from the mathematical point of view and in recent years has therefore been spotlighted by engineers. Estimates of the importance of transportation by car are difficult to make, but one can be sure that in an industrialized society its effect is enormous, whether measured economically, politically, in terms of public health, psychologically, industrially, or purely as a fraction of transportation in general. Some of the central concepts of this dissertation, such as traffic streams and traffic delays, are popular concepts and quite justifiably so. Few areas of applied mathematics have such widespread and directly intuitive importance in our lives.

On the mathematical side many genuinely interesting aspects of traffic flow are found. The development in the past two decades of a substantial theory of vehicular movement has come not only from the need to understand more exactly the empirical results of the traffic engineering profession, but also as a natural extension of the theory of queues. Although the problems are difficult to formulate and still more difficult to solve, there is by now a considerable literature in traffic flow theory.

A. Literature Review

As a simple consequence of its maturity, traffic flow theory has been developed by research workers of widely varying interests: mathematicians, statisticians, physicists, traffic engineers, economists and more recently practitioners of operations research. The field is sprawling, diffuse and in many ways rather baffling. There is no general agreement on notation or terminology, much of which has been inherited from the traffic engineer. There is little agreement on methodology, or on which quantities are significant, or on how these quantities should be measured. Only a portion of the literature relevant to this dissertation topic will be systematically exposited.

1. Theories of traffic flow

In an extensive literature search, it became apparent that, in most cases, the descriptive theories concerning vehicular traffic were inadequate or restricted to a very limited situation. There are basically three types of theories. The first, an analytic and deterministic model, considers the characteristics of the vehicle and assumes driver behavior. A second class of selections involves queue theory treatments of a stochastic model. Queue theory necessitates that all vehicles enter at one point, a major simplification of the problem. Reasonable results may be obtained when traffic is actually queued, velocity is uniform, and the driver has few decisions. In a third approach, which describes traffic flow in a continuum, the individual vehicles are treated analogously to molecules of a semi-compressible fluid; traffic flow must then obey appropriate differential equations of fluid flow.

Pearce (34) considered a single server queueing system with a service mechanism that operated regardless of whether or not customers were present, such as a bus or ferry service that operates even when there are no passengers available. A customer arriving at an empty queue would thus not, in general, be able to commence service immediately. Pearce considered the equilibrium behavior of such time dependent systems in the case of negative exponential services and a general class of stationary but not necessarily recurrent inputs.

Vaughan (43) investigated the distribution of hourly traffic volumes. This author divided the distributions into hourly distribution of regular trips and chance trips. These divisions led to an exponential-normal model where the exponential component represents the journeys of a chance nature, such as social trips, farmers' and suppliers' trips etc., and the normal component represents the regular trips such as work trips. Based on this model and assuming that the distribution of volumes for each hour of the week has the same form for all weeks of the year, but with a different scale parameter, Vaughan attempted to explain traffic behavior in rural, suburban, recreational and urban road sites.

Miller (29) proposed that on roads which are uninterrupted by traffic signals, intersections etc., vehicles could be considered as travelling in random queues unless the concentration of traffic is so high that there are no gaps in the stream of vehicles. This author used a crude model to study the formation of these queues in an attempt to derive the distribution of queue lengths. The independent random queue model was then used to study waiting times for pedestrians or vehicles wishing to cross one lane of traffic. The problem with this model, acknowledged by Miller, was that it was only realistic on roads with fairly uniform characteristics, that is, for roads which are of uniform width and either continuously straight or uniformly winding.

Dawson and Chimini (9) were concerned with the development of the hyperlang probability distribution as a generalized time headway model for single-lane traffic flows on two-lane, two-way roadways. The authors assumed that a traffic stream will always contain both free and constrained vehicles, where constrained vehicles were those under the influence of other vehicles in the traffic stream. Their proposed hyperlang headway model was a linear combination of a translated exponential function and a translated Erlang function. The exponential component of the distribution described the free (unconstrained) headways in the traffic stream, and the Erlang component described the constrained headways.

Buckley (5) postulated a generalized semi-Poisson model of traffic flow. The basis of his model was the simple conjecture that in a single traffic lane the only inhibition to the underlying Poisson traffic process is the existence of a zone of emptiness in front of the rear of each vehicle. This author concluded that the headway distribution associated with his semi-Poisson model, which was a generalization of the displaced delta-exponential, delta-exponential, displaced exponential, and exponential distributions, predicted headways fairly well and could yield some insight into the nature of road traffic.

Serfling (39) sought a suitable non-Poisson model for a traffic flow with a moderately high density or restricted overtaking. Using second-order linear-difference equations this author developed a heuristic solution, leading to a counting distribution whose expression involved a "clustering tendency" function. Serfling concluded that one may incorporate into the model the phenomenon of bunching of vehicles and the parameters associated with this phenomenon.

Potts et al. (36) developed a discrete Markov model to describe the time series of events of vehicles passing a point on a roadway. Their Markov model contained two fundamental properties. First, the times between arrivals were independently and identically distributed and second, the model implied the existence of correlations between the counts of vehicles in successive time intervals, an assumption not necessary for the random arrivals model. Thus, in their model the authors were able to account for the bunching tendency of traffic by assuming correlations between successive vehicles. The authors tested the adequacy of the model for describing the arrivals of vehicles at a point on a roadway when passing was hindered and the traffic flow was medium to heavy and obtained satisfactory results.

Oliver (31) derived a traffic counting distribution in which a minimum spacing or headway between units of traffic was taken into account such as airplanes separated by a minimum space or time interval for reasons of safety. This researcher also derived explicit expressions for the mean and variance of count as well as the probability that the interval of interest was completely filled by vehicles.

2. Traffic simulation models

Simulation has experienced widespread application in various fields of science and engineering. Until recent years, however, traffic and transportation engineering applications were limited to simulation which utilized physical models. With the rapid development of electronic digital computers it has now become feasible to consider simulation of vehicular traffic flow, such as simulation of street intersections, on ramp areas and highway interchanges, by mathematical or symbolic models.

Digital simulation may be defined as the technique of setting up a stochastic model of a real system which neither oversimplifies the system to the point where the model becomes trivial nor incorporates so many features of the real system that the model becomes untractable or prohibitively clumsy. Two decades have now passed since the first use of digital simulation in the study of traffic phenomena. In these years considerable work has been done in the simulation of traffic flow which has also broadened the body of knowledge in theory of traffic flow. This traffic flow theory and the rapid improvement of the electronic digital computer as well as the techniques of digital simulation have mutually been responsible for the development of simulation as a design tool in traffic and transportation engineering.

Perchonok and Levy (35) devised a simulation model for use by highway design engineers to determine ramp and acceleration area configurations for given traffic conditions. The basis for their simulation was the statistical analysis of data from a number of interchange locations which describe flow and driver behavior in the merging process. Through the use of Monte Carlo techniques and a general purpose digital computer, each vehicle in the portion of roadway under study was allowed to maneuver through the model access area with the same freedom of decision as do their real-life counterparts. The authors' investigation showed that simulation methods can aid the design engineer by supplying information on added service to the driver by length of on-ramp, etc., and thereby allow him to weigh these factors in determining the most favorable design for given traffic conditions.

Kell (23) developed a simulation model for the intersection of two 2-lane, two-directional streets, with one street being controlled by stop signs. This author determined the total vehicular delay experienced at intersections with respect to approach volumes and turning movements. The effect of installing a signal at an intersection on vehicular delay, which provided a basis for examining and refining existing traffic signal warrants, was evaluated. Kell also determined the effect of turning movement restrictions on intersection operation.

Evans et al. (13) conducted a simulation study of queueing at a stop sign for a single main stream of traffic. They assumed that the headways on the main highway were distributed exponentially with arbitrary mean headway and that the side road arrivals were Poisson with arbitrary mean arrival rate. The gap acceptance functions employed were either a step function or trapezoidal function with arbitrary parameters. The authors compared their results to those predicted by an analytically tractable theory and found them to be in good agreement.

Levy et al. (25) developed a simulation model of a general purpose, limited access highway system which has been designated to help to determine how complex models need be to reproduce reality faithfully. Their model assigned to each vehicle characteristics such as: 1) desired velocity, 2) minimum acceptable gap, 3) desired following distance, 4) type of vehicle, chosen in a random manner from prescribed distributions. Using the model, Levy and his associates carried out a series of experiments with the objective of gaining quantitative knowledge of the effect of slight changes in the input data on the output. The results of the authors' analysis showed that the mean gap acceptance did not significantly affect anything except number of weaves and number of velocity changes. The variance of gap acceptance and the variance of following distance were shown not to be important.

Stark (40) constructed a computer model which simulated the volume and movement of traffic on a nine-block section of a city street. The simulated cars were reviewed every quarter-second and were moved according to rules for movement which have been built into the computer program. The simulation run on the computer produced two outputs. In the first output, the quarter-second car positions were plotted on an oscilloscope and photographed, resulting in a moving picture which could be shown in real time. The other output was a series of tables that cataloged all vehicles as they entered and left the model. These tables furnished an abundance of quantitative data for measuring and evaluating the performance of the model.

Rhee (38) simulated the movement of traffic on a network of streets controlled by traffic signals. This author applied his program to an actual traffic bottleneck consisting of several streets and four traffic signals. Two types of traffic signal control mechanisms were considered, a real-time adaptive method and a fixed-time method. Rhee found that the adaptive mechanism, which made use of the current data on traffic conditions, reduced queues on some arms considerably compared with the fixed-time system.

Francis and Lott (14) investigated by simulation the behavior of traffic in a road network controlled by fixed-time traffic signals. In their program vehicles were considered to be all of the same type and were indistinguishable, and therefore the system did not allow the path of any particular vehicle to be followed. The network was considered to consist of road junctions connected together by links which bore single streams of vehicles travelling from one junction arm to another. Vehicles were fed into and left the network at certain peripheral arms.

The authors determined the flows and delays in all links, the average delays and average queue lengths at all junction arms, plus the average total number of vehicles queueing throughout the network at any moment.

Blum (3) developed a GPSS model describing a traffic network as a series of interconnected intersection modules. His model offered a large amount of flexibility in specifying the network geometrical characteristics and vehicle input information unique to a particular problem. For example, vehicles varying in size may change lanes, turn, change speed and merge. Blum's vehicle traffic simulator depicts the traffic network as a series of intersection or junction modules connected by traffic lanes. Within the simulator program, the intersection module was reproduced by a single subprogram which processed vehicles for the entire network. Each vehicle entering the network was assigned a speed from an empirical or hypothetical distribution which was retained until the vehicle's free flow was inhibited by a preceding vehicle or signal light. With his model, Blum was able to test various alternative arrangements for signal settings in order to reduce travel and queue delay times.

Carrol and Bronzini (6) programmed a model to simulate the movement of shallow draft barge tows through a waterway with an interconnected network of ports. Within the model, tows having preassigned characteristics and itineraries arrived at the system end-points at a specified average arrival rate by means of a Poisson process. Similarly, as the tows encountered the various locks listed in their itineraries, the actual values of locking times were chosen from the appropriate input distributions, using Monte Carlo techniques. The model output included statistics concerning system operations, such as the number of tows and barges processed at each lock, service and delay times and average queue lengths. Using the model, the authors investigated traffic flows, delays, and congestion costs arising from designated alternative system designs.

Nanda et al. (30) developed a passenger arrival simulation model to evaluate facility utilization and operating alternatives at airports. Processing of passengers included deplanning passengers and baggage, federal inspection, baggage handling, passenger luggage matching and incidentals. Observations were taken over a lengthy period to identify processes for arrivals and the influencing parameters as well as cumulative distributions and influencing parameters. Utility of their simulation model included establishing the reduction in waiting time for increasing federal inspectors and various rules for baggage assignment.

Brant and McAward (4) developed a simulation model and used it to evaluate the performance of the proposed Dallas-Forth Worth Regional Airport layout plan. The time oriented simulation model of aircraft ground operations was used to evaluate the functioning of the proposed airfield layout under anticipated loading. The results of their simulation led to modifications to the initial development plan, providing substantial saving in initial airport construction costs.

3. Shuttle transportation systems

Urban transportation planning and with it the role of transportation engineer is becoming more complex. No longer relegated to mere data manipulations-a process which has not led to rational decisions- the transportation professional is being asked to give policy makers more objective, extensive and intensive information than ever before. The reasons are simple: Vitality of a city depends upon the freedom with which people can move into, out of, and around a metropolitan area and transportation has a crucial effect upon people and their environment. In an urban traffic environment, an inefficient method of controlling traffic results in costly aggregate delays to the motoring public. Solution of this problem is one of the primary tasks confronting urban planners in cities in many parts of the world. The healthy growth of the city and its metropolitan area can not be achieved until people can travel conveniently and economically to work, to school, to shop and to play.

Renewed interest in urban transportation systems as possible solutions for the increasingly unmanageable traffic snarls in large metropolitan centers has focused attention on the variables that affect such systems. These variables are many, and unfortunately, they have been poorly understood. One of the more specific problems that the urban transportation planner is faced with is that of the problems arising from shuttle transportation systems. Unfortunately, there have been only a few studies done in this area.

Reynolds (37) considered the problem of assigning shuttle cars to sections of a mine with the objective of maximizing expected output. In his model every continuous miner has assigned to it two shuttle cars that make periodic trips from the continuous miner to the conveyor belt. A shuttle car having transferred its load to the conveyor belt, waits in a byway while the other shuttle car is still being loaded. Thus, when one car becomes inoperative the remaining car absorbs the delay normally incurred in going from the conveyor belt to the continuous miner. Developing a mathematical model, Reynolds found a solution to this problem that could be used readily by any mine foreman.

Panico (33) looked at an optimization problem with ferries operating on the Ohio River. This author assumed that originally ferry boats were the only way to cross the river, but at present either the free bridge or the ferry could be used. Since ferry boats operated almost on the doorstep of the large plants, considerable time could be saved if this service were used, but the demand was frequently so great that drivers would forego the ferry for the bridge and drive the additional miles. This avoided the cost of the ferry but resulted in the additional per-mile costs and a possible loss of time if the choice was ill-conceived. Assuming that cars arrived in a Poisson fashion, Panico developed formulas to find the optimum service rate with respect to minimized costs and investigated whether it is economically justified to expand the ferry service facilities by adding an extra ferry.

Kosten (24) considered an unscheduled ferry problem where a ferry transported cars between a port A and a port B. In both ports cars arrived according to Poisson processes. The ferry needed one unit of time for a trip from A to B or B to A. Loading and unloading were supposed to take no time. The ferry did not sail according to a time table. It started whenever the number of cars awaiting transport in the sailing direction was at least a given constant. The capacity of the vessel was so large that it could take along all cars waiting in the port of departure. Given these assumptions, Kosten analytically determined the average waiting-time per car.

B. Thrust of the Present Research

A conclusion from the research of the literatures which was sampled in the foregoing sections is that most of the modeling studies have concentrated mainly on the traffic flow theory and simulation of traffic networks and only a few investigations were made in the area of modeling shuttle systems and related traffic streams. Thus, a definite need exists for the development of a methodological framework for the improvement of shuttle transportation systems.

It was one of the objectives of this research to demonstrate alternative ways of modeling traffic streams approaching a shuttle system such as various ferry boats operating across a channel, or shuttle trains, or flights operating between two cities, or a monorail operating between two sections of a city, or even a ski lift operating in a winter resort area. All of these systems have one thing in common in that they are what may be called "interdependent" in nature which is demonstrated mathematically in chapter II, section B. A second objective was to develop mathematical and simulation models which can be used to describe the behavior of shuttle systems. The final objective of this research was to conduct sensitivity studies to observe how such systems respond to changes in model parameters and to deduce certain conclusions as to the efficiency of the shuttle system under various inputs and constraints.

The overall keynote of the present research would be the infusion into the modeling of shuttle systems of a higher degree of realism and flexibility than seems heretofore to have been attained.

II. MODEL DEVELOPMENT

A. Simulation Model

As seen in section B of this chapter mathematical analysis of multi-shuttle systems is very cumbersome because of the many random variables involved; thus, simulation offers a good alternative to help analyze the system.

Using exponential and Weibull interarrival distributions, two GPSS simulation models were developed, capable of simulating real-life conditions as well as simplistic cases, to determine the effect of changes in the input on the output measurables. Results of the various simulation runs were compared with each other and with actual data and were used to make certain predictions of system behavior. This section explains the development of the simulation model, research methodology followed, and the cases investigated as part of the simulation sensitivity studies.

1. Data acquisition

As an example of the multi-shuttle system that was considered in this research, the ferry system in operation at the Istanbul Bosphorus, Turkey, was chosen. A schematic of the system is shown in Figure 1. Up-to - four ferries with varying capacities carry cars back and forth across the Bosphorus strait that is about one and a quarter miles wide. Traffic flow to the ferry docks is interrupted by various traffic lights and one traffic policeman. Only two docks exist on the Asian side whereas there are three on the European side. Although parking lots have finite capacity, when they are full cars sometime form a queue line

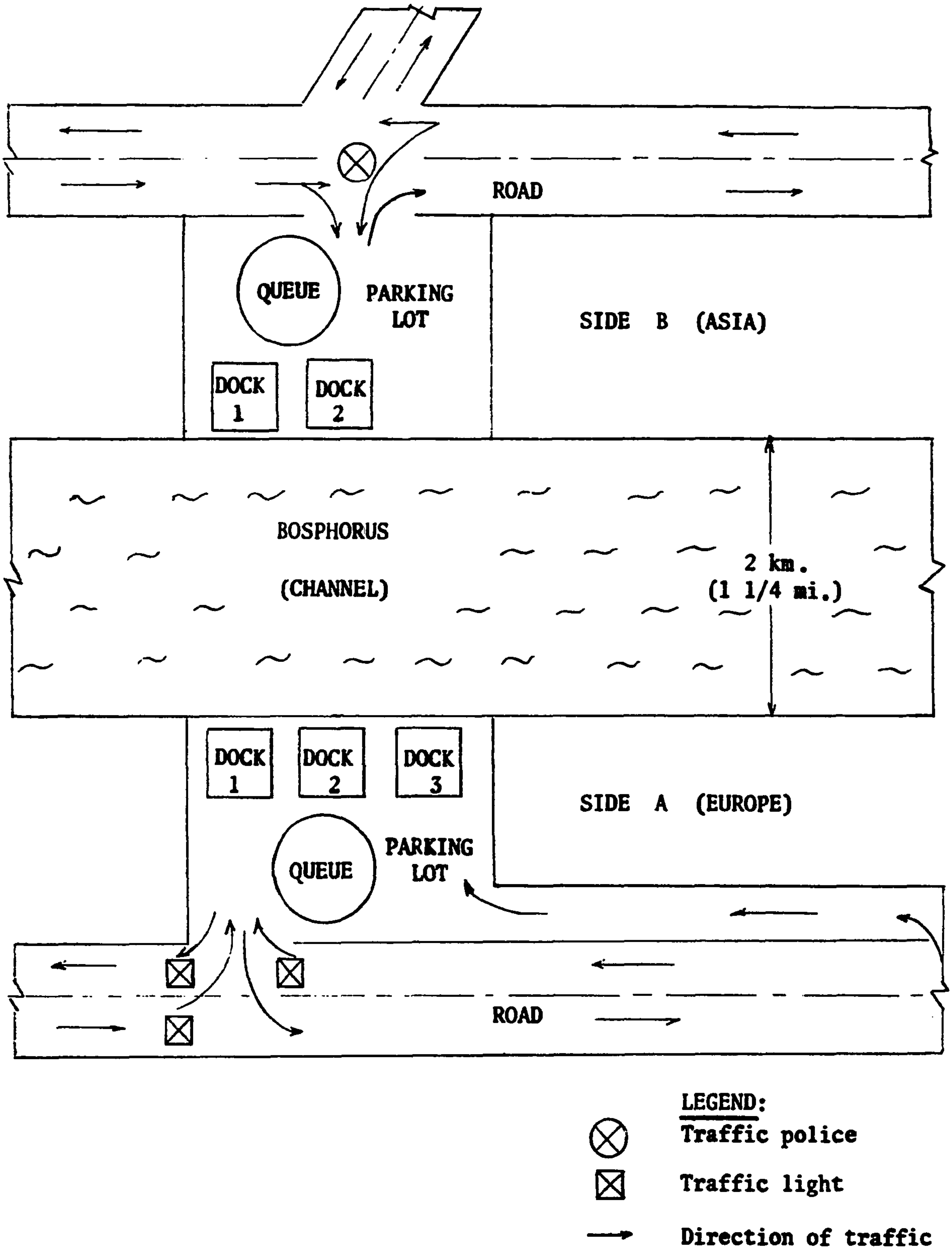


Figure 1. Schematic of Istanbul Bosphorus ferry system

on the street. On the average it takes the ferries about twelve minutes to go across from one side to the other, but travel time is a random variable depending on the efficiency of the ferry and current conditions of the channel. Ferries observe a fixed time schedule after midnight, but they have no such schedule during the day. Normally, but not always, they leave as soon as they are full.

On Sunday, April 3, 1973, a twelve-hour study was conducted by four observers. They recorded minute-by-minute car arrivals to the queue lines on both sides of the Bosphorus, number of cars embarking and disembarking the ferries, their loading and unloading times, total time each ferry spent at the dock and the time each spent crossing the channel. These data are given in Appendix A.

In general, counting interval can not be so long as to neglect gross variations in the traffic, nor must it be so short as to over-emphasize the random variation of traffic over short periods. The minute used in recording arrivals fulfilled both of these requirements and was a convenient unit of time with which to work.

2. Analysis of data

Arrival data totaled for each half-hour period for sides A and B are shown in Figures 2 and 3 respectively. Several observations were made here. First, there was a general upward trend in the amount of traffic at both sides except for the dips occurring right before lunch and dinner hours. Secondly, the average arrival rate was higher for side A. Finally, assuming that cars at the end of their trip all return to the side from

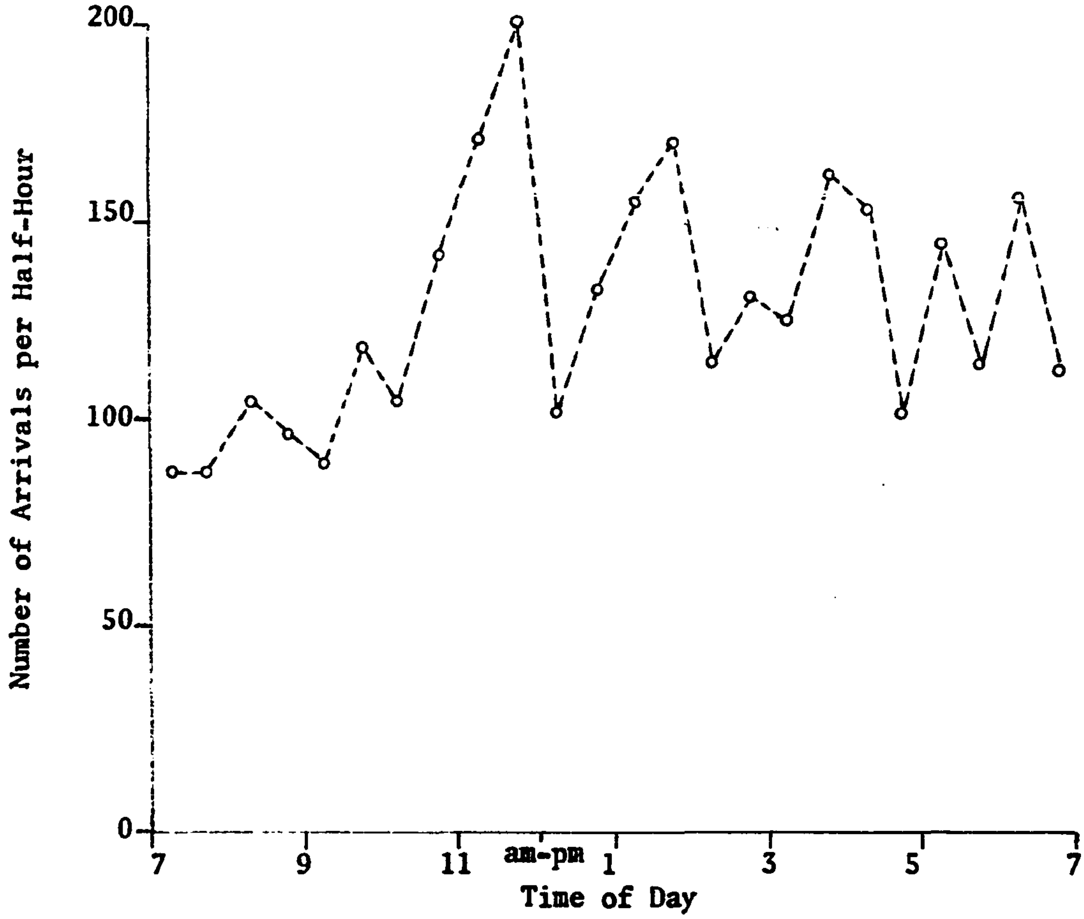


Figure 2. Number of arrivals per half-hour for side A, real-life data

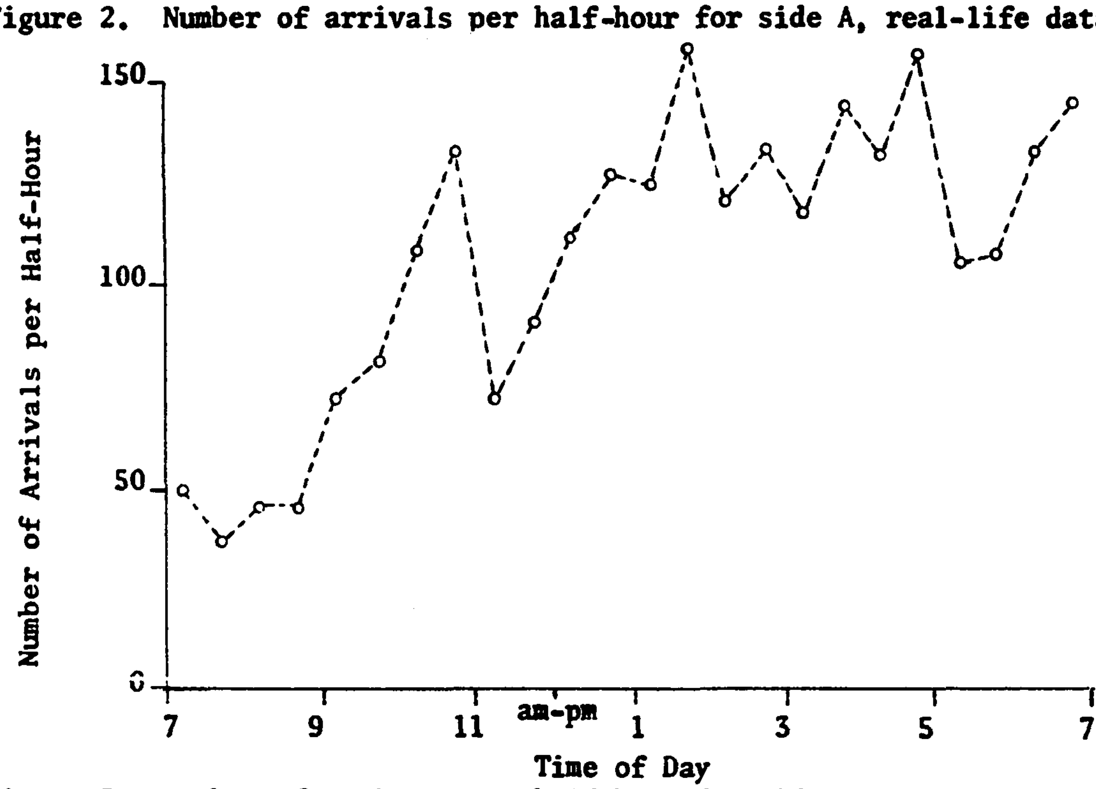


Figure 3. Number of arrivals per half-hour for side B, real-life data

which they originated, it was observed that not all cars had returned to side A by the end of the study period.

a. Non-stationary "half-hour" Weibull input Because of its flexibility, generality and ease of interpretation it was decided to fit Weibull distributions to the car arrival data. Density and cumulative distribution functions of Weibull are given in Appendix C. Among its three parameters, α shows to what extent the distribution is skewed or symmetrical, λ shows the scale of the distribution, and μ is the absolute minimum value observed between occurrences of events. It is noted that when the shape parameter α is equal to one, the Weibull becomes an exponential distribution and thus Weibulls include the exponential.

Using one-hour overlapping intervals of arrival data, average interarrival times and the three Weibull parameters were calculated for successive "half-hour" periods assuming independence among various time points. Appendices B and C illustrate the details of these calculations. The resulting parameters were plotted with respect to time in Figures 4 and 5 for sides A and B respectively. A general observation was made that, at least for side B, the shape parameter α is relatively constant. The "half-hour" Weibull distributions have basically the same shape, and the other parameters are variable with respect to time. A smoothed version of these parameter functions was used in simulation runs while investigating the transient behavior of the model for one day. Smoothed equations of the parameters are listed in the Fortran subroutine named Weibull, given in Appendix E, section A.

Calculation of average interarrival times is illustrated in Appendix C,

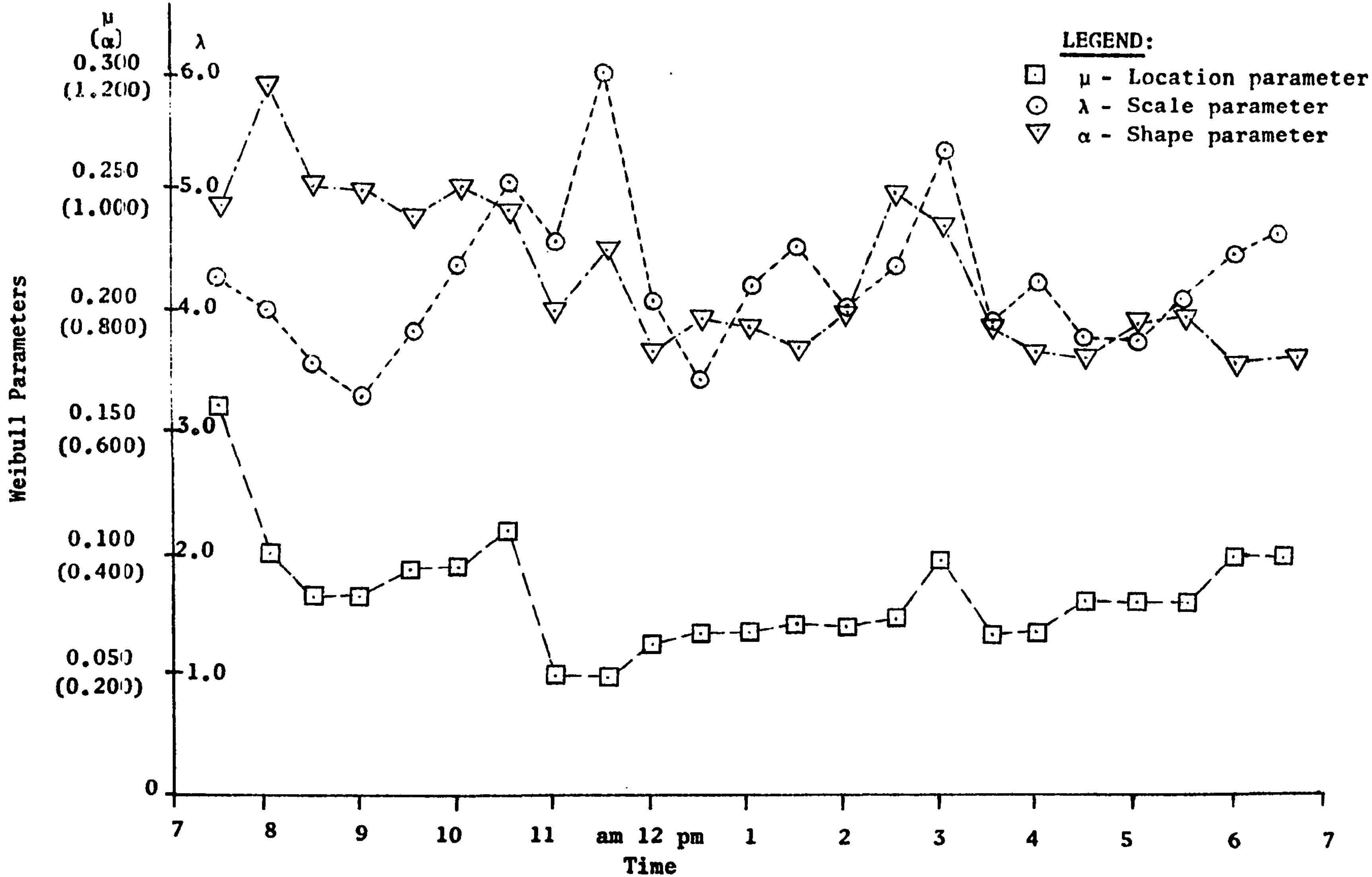


Figure 4. Parameters of Weibull distribution, side A

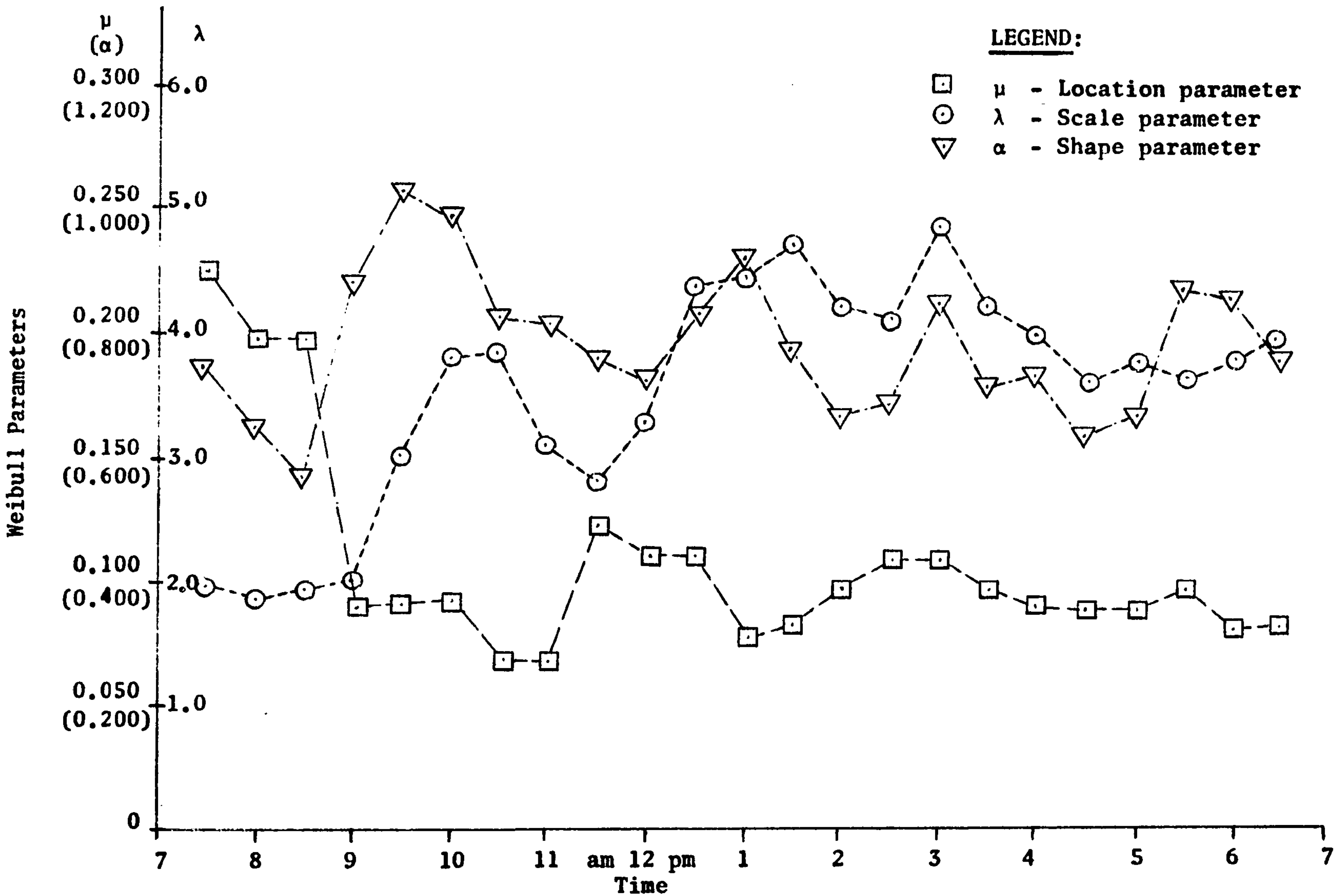


Figure 5. Paremeters of Weibull distribution, side B

section B. Resulting mean functions for sides A and B are shown in Figures 6 and 7 respectively. The same spikes in slope were observed, corresponding to the dips indicated in Figures 2 and 3.

Histograms of transit times for ferries travelling from side A to B and from side B to A are shown in Figure 8. These actual frequencies were incorporated into the simulation model as variable functions in determining ferry transit times.

Loading and unloading times of the ferries were regressed against the number of cars loaded and unloaded, for each side and ferry individually. A sample of loading and unloading times with respect to cars embarked and disembarked for ferry number 4 is shown in Figures 9 and 10 for sides A and B respectively. Resulting regression coefficients (slopes) which are listed in the main GPSS simulation program, Appendix E, were used in simulation studies for both Weibull and exponential models in determining loading and unloading times for a particular ferry and side.

A Fortran subroutine was written in connection with the GPSS simulation program which calculated an independent Weibull cumulative distribution function at any point in time using the three known Weibull parameters. Then a random number determined the next interarrival time from the cumulative distribution function and transferred this information to the main simulation program. Derivation of the Weibull inverse function used in this subroutine to calculate interarrival times is given in Appendix D, section A. A list of the subroutine and the main GPSS simulation program, which is flexible enough to incorporate most parameter changes, is given in Appendix E.

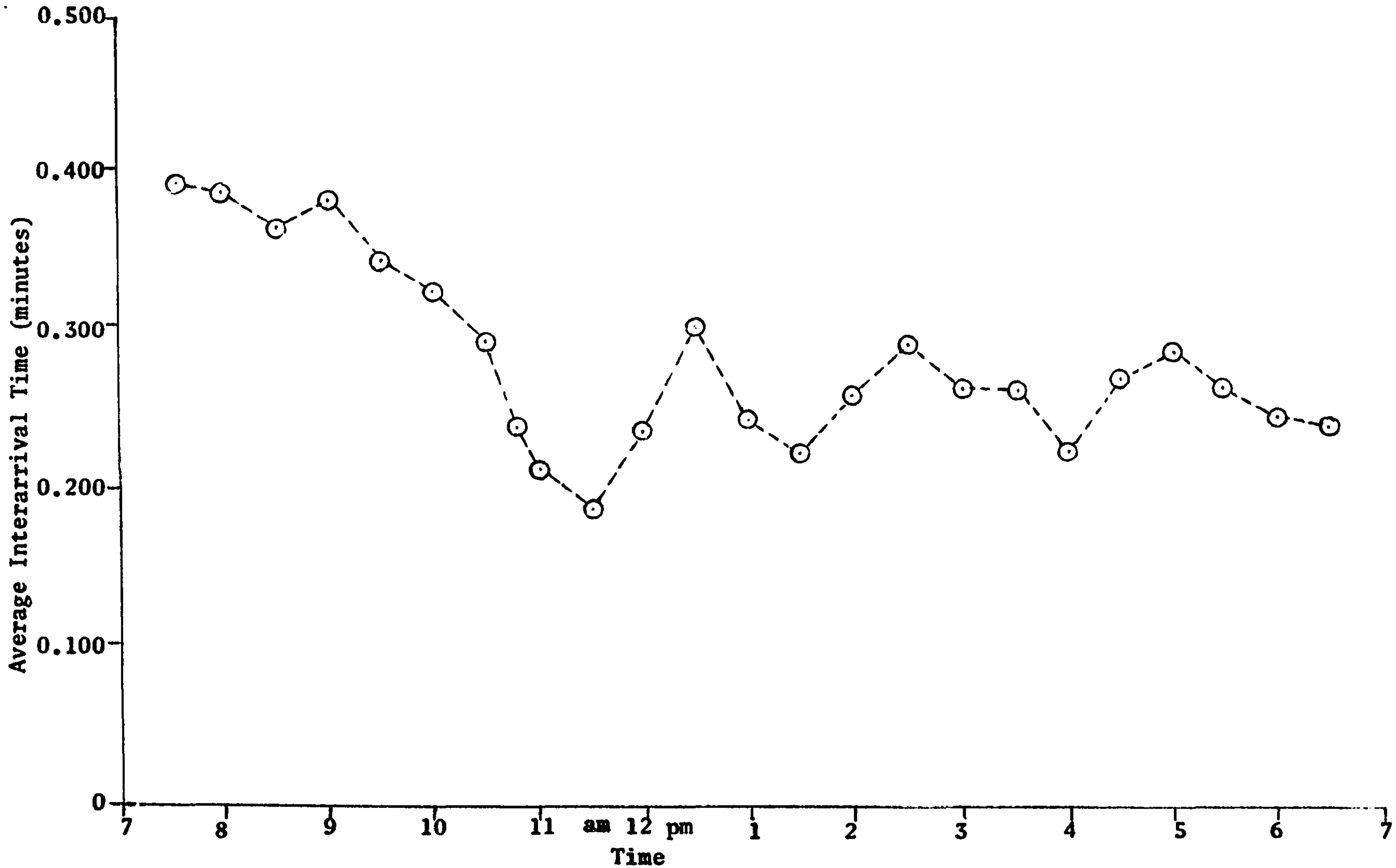


Figure 6. Average interarrival times in minutes, side A

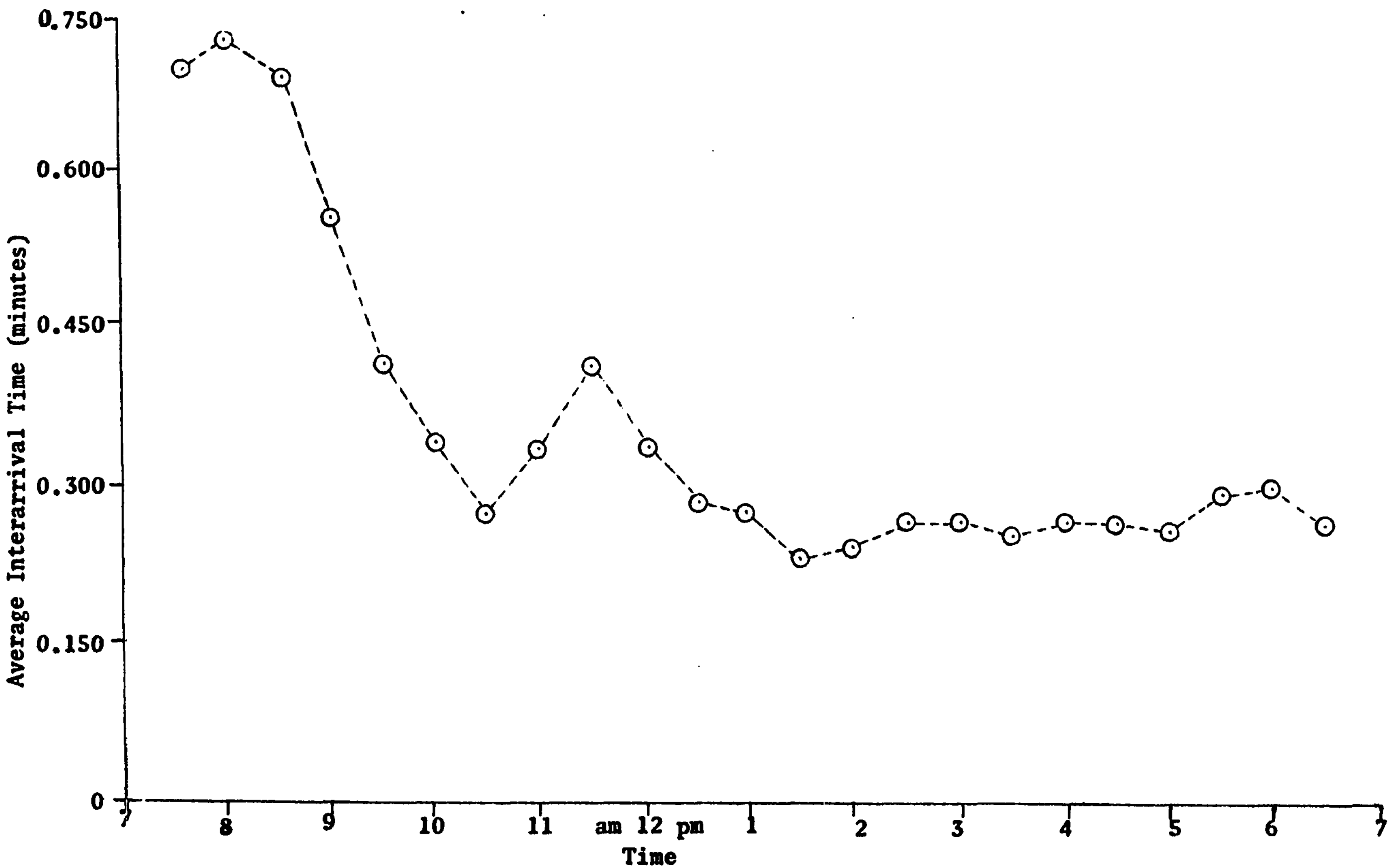


Figure 7. Average interarrival times in minutes, side B

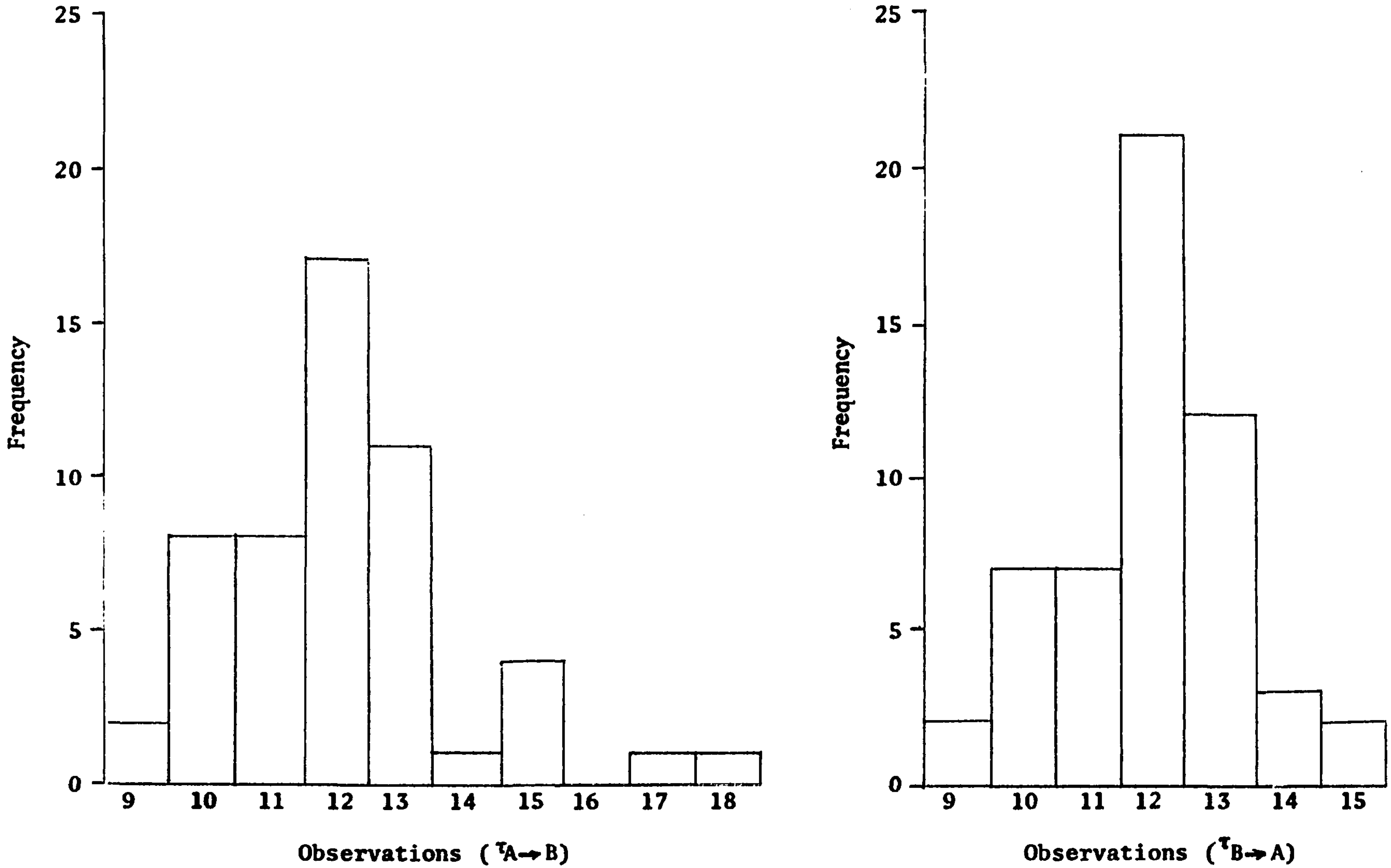


Figure 8. Histograms of ferry transit times where τ is in minutes

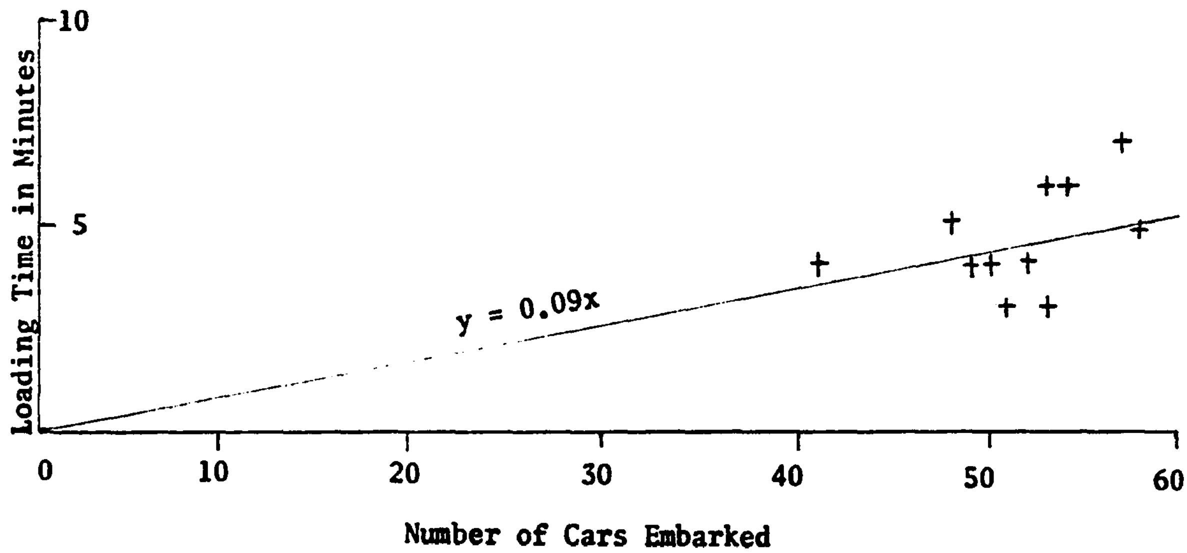


Figure 9. Loading function of ferry number 4, side A

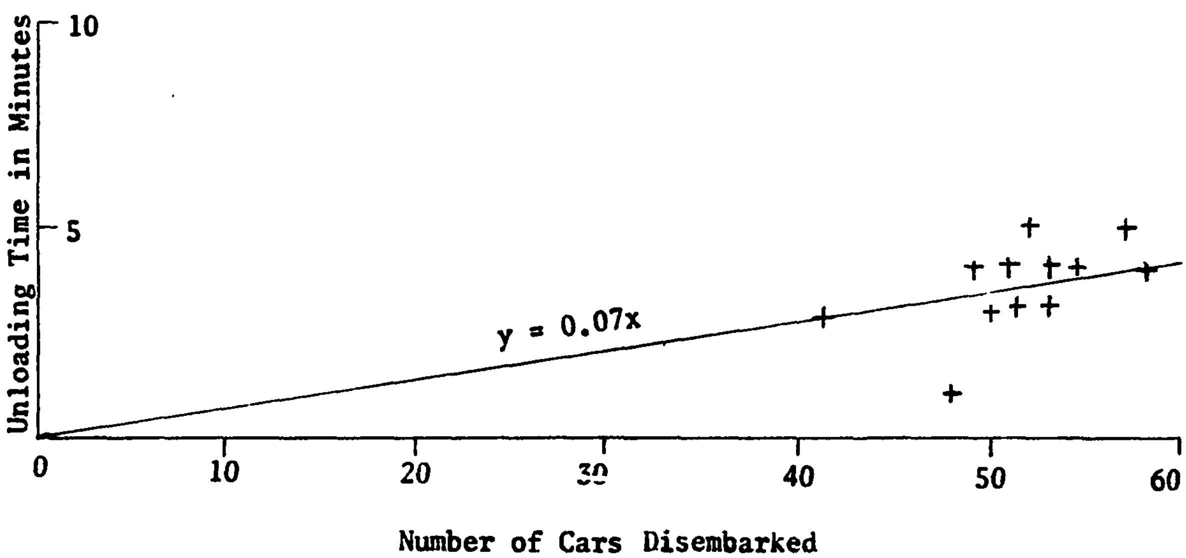


Figure 10. Unloading function of ferry number 4, side B

Assuming non-stationary conditions, independent smoothed cumulative Weibull distribution functions were created as simulation proceeds. Transient and stationary behavior of the system was investigated during a twelve-hour period using real-life conditions. Validity of the simulation model was verified against the actual data and some of the system parameters were varied to observe system responses and performances.

b. Non-stationary "continuous" Weibull input It was observed in the previous section that the Weibull shape parameter was relatively constant, particularly for side B. Taking the shape parameter as fixed, it is possible to further refine the "half-hour" approach by calculating the Weibull distributions in a "continuous" way. This can be accomplished by using a similar Fortran subroutine in connection with the main GPSS simulation model which calculates the Weibull parameters at any point in time, given Weibull mean and variance functions plus a fixed Weibull shape parameter deduced from the previous studies. The rationale behind this technique can be explained by assuming Weibull interarrival times are given by the model:

$$I_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 \cos \omega t + \varepsilon_t = E(I_t) + \varepsilon_t$$

where

t = time in minutes

$\beta_0, \beta_1, \beta_2$ = constants calculated from ordinary regression

$\beta_3 \cos \omega t$ = a periodic term which takes into account the effect of traffic light on one side and traffic policeman on other

$E(I_t)$ = expected interarrival times

ε_t = error term

but, since

$$\hat{\varepsilon}_t = I_t - \hat{I}_t$$

or

$$\hat{\varepsilon}_t^2 = (I_t - \hat{I}_t)^2$$

then,

$$E(\hat{\varepsilon}^2) = E(I_t - \hat{I}_t)^2 = V(I_t)$$

or

$$\hat{\varepsilon}^2 = \gamma_0 + \gamma_1 t + \gamma_2 t^2 + r_t$$

where

γ_0, γ_1 and γ_2 = regression constants

r_t = residual

$V(I_t)$ = variance of interarrival times .

Thus, using the following models:

$$E(I_t) = f_E(t) \equiv \beta_0 + \beta_1 t + \beta_2 t + \beta_3 \cos\omega t$$

and

$$V(I_t) = f_V(t) \equiv \gamma_0 + \gamma_1 t + \gamma_2 t^2$$

$\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\gamma}_1, \hat{\gamma}_2$ values can be determined by computer regression analysis. Therefore, since

$$E(I_t) = \lambda_t^{\frac{-1}{\alpha}} \Gamma\left(\frac{1}{\alpha}+1\right) + \mu_t$$

and

$$V(I_t) = \{\Gamma(\tfrac{2}{\alpha} + 1) - \Gamma^2(\tfrac{1}{\alpha} + 1)\}/\lambda_t^{\frac{2}{\alpha}}$$

where

$$\Gamma(x) = (x-1)!$$

then,

μ_t and λ_t can be expressed as functions of

$$\lambda_t = f_1[E(I_t), V(I_t)]$$

and

$$\mu_t = f_2[E(I_t), V(I_t)]$$

which, in turn can be solved as a function of $t(\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\gamma}_1, \hat{\gamma}_2)$. Thus, these $\hat{\lambda}_t, \hat{\mu}_t$ and $\alpha = 0.85$ values can be used in a simulation model to define Weibull distributions at each point in time.

c. Stationary exponential input This is the special case of the Weibull input when shape parameter is equal to one. It was used mainly for mathematical stationary analysis. The same loading and unloading equations were used as in the Weibull model. A fixed ferry transit time of twelve minutes was determined by taking the mode of the actual data. Constant times, which may be defined as total time a ferry spends at dock -

(unloading time + loading time of ferry), were determined as $h_A = 1.8$ and $h_B = 1.6$ minutes for sides A and B respectively by taking the average of the actual data. These data are listed in Tables 12 and 13 in Appendix A. Derivation of exponential inverse function which was used in determining interarrival times in the main GPSS simulation program is given in Appendix D, section B. A list of the main program, flow charts, GPSS definitions used in the program and a sample of output using exponential input are given in Appendix E.

3. System parameters

The following are the parameters of the multi-ferry transportation system that could be varied in the course of a simulation sensitivity study:

1. Model for incoming traffic streams on both sides:
    i. Stationary exponential interarrival times with rate parameters a, b for sides A and B respectively
    ii. Non-stationary "continuous" Weibull interarrival times
    iii. Non-stationary "half-hour" Weibull interarrival times
    iv. Time series input
    v. Other inputs
2. Number of ferries:
    i. Only one ferry operating across the channel
    ii. Two ferries operating across the channel
    iii. Three ferries operating across the channel
    iv. Four ferries operating across the channel
    v. Five or more ferries operating across the channel

3. Ferry capacities:

   i. Capacity of two cars

   ii. Capacity of forty-two cars

   iii. Actual real-life capacities of forty-two, sixty-four, forty-two and fifty-one for ferries 1, 2, 3 and 4 respectively

   iv. Capacity of eighty-four cars

   v. An infinitely large capacity

   vi. Any other capacity

4. Number of ferry docks:

   i. Only one ferry dock available on each side

   ii. Only two ferry docks available on each side

   iii. Only three ferry docks available on side A and two on side B

   iv. Infinitely large number of docks available on each side

   vi. Any other combination of docks available on each side

5. Ferry transit times:

   i. A fixed transit time of twelve minutes for both ways

   ii. Random transit times based on actual data for both ways

   iii. Any other time

6. Ferry dock parking lot capacity:

   i. A lot with infinitely large capacity

   ii. A lot with a finite car capacity

7. Ferry discipline (operating rule):

   i. Ferry loads only those cars which are waiting at the end of its unloading and leaves immediately

   ii. Ferry is not allowed to leave until there is a minimum number of cars aboard

iii. Ferry operates according to a fixed time schedule

iv. Other operating rules

4. <u>Sensitivity case studies</u>

A series of experiments were conducted with the objective of gaining a quantitative knowledge of the relationship between input and output. The following cases were specifically investigated (small Roman numerals for each case indicate a specific parameter used from subdivision 3):

a. <u>i,i,i,i,i,i,i</u> This is the "null" case which matches the mathematical one-ferry model developed. Under this case, steady-state conditions may be reached faster than other cases investigated. First, intensity parameters of a = 25 minutes/car and b = 25 minutes/car were used and the results were compared with the mathematical model. Transient and stationary behavior of the simulation model were investigated using half-hour snaps and different random number sequences at each run.

Again, under this case, the effect of imbalance on the incoming traffic streams of both sides of the channel was investigated using various combinations of intensity parameters. Contour lines of the overall service and median car-waiting times were derived to determine the efficiency of the system.

b. <u>i,i,iv,i,i,i,i</u> Various intensity parameters likely not to explode the system were used. Mean interarrival times a and b were taken equal, possibly accelerating the tendency to stability. This case was compared with case c by finding the difference in average waiting times per car and plotting them against different intensity parameters.

c. i,ii,ii,i,i,i,i This case featured the parameters of the mathematical two ferry model. The same intensity parameters were used as in case b, and the results of average waiting times were compared with the previous case.

d. iii,iv,iii,iii,ii,i,i This case simulated the actual situation under non-stationary "half-hour" Weibull input. System attributes and incoming traffic stream characteristics were compared with real-life data and with case e to determine which simulated the actual situation better in both the short and the long-run.

e. i,iv,iii,iii,ii,i,i Exponential input was used to compare various attributes of the system with the actual data and with simulation run under Weibull input.

B. Mathematical Model

1. Multi-shuttle system model

Taking ferries operating across a channel as an illustration of a multi-shuttle system, Poisson-exponential mathematical models for single and two shuttle systems were formulated as interdependent queueing systems. The aim of these models was to derive the probability distributions for the number of cars waiting on shore at successive ends of unloading times using Markovian equations of transition. These probabilities were then compared against the results of simulation runs.

a. Single shuttle model The simplest case of a multi-shuttle system based on two interdependent queues is that a single shuttle system or one-ferry system taken as an example. In order to derive the relevant equations for this system, the following simplifying assumptions are made:

1. Ferry transit time τ is constant.

2. At the beginning ($t = 0$) there are no cars waiting on either side. Ferry is assumed to be in the middle of the channel going toward side A carrying k number of cars.

3. Ferry leaves the dock as soon as it is either loaded to capacity or there are no more cars waiting at the dock.

4. The number of arrivals per unit time has a Poisson (P) distribution.

5. Car parking lots at ferry docks are infinitely large.

Defining:

t = clock time of docking of a ferry at one of the sides of the channel

X_{si} = total number of arrivals waiting on either side s at i th docking

y_s, g_s = some functions of F_{si} and N_{si} respectively

C = capacity of the ferry

D_{si} = delay of ferry at side s, due to loading and unloading at i th docking

a = mean interarrival time on side A

b = mean interarrival time on side B

s = side A or B

N_{si} = number of cars taken aboard from side s by the ferry at i th docking

F_{si} = number cars on the ferry docking at side s at i th time

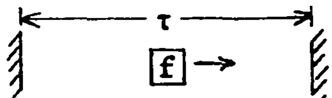
P = denotes the Poisson distribution of the number of arrivals per unit time

h_s = a constant on either side s .

Then one can derive a set of equations for the following clock times:

<u>Clock time</u>

$t = 0$



$t = \frac{\tau}{2}$

ferry has just docked at side A

$X_{A1} = P(\frac{\tau}{2} \times \frac{1}{a})$; $F_{A1} = k$ where $0 \leq k \leq C$

$D_{A1} = y_A(F_{A1}) + g_A(N_{A1}) + h_A$ that is, delay is a function of unloading and loading of ferry plus some constant h where

$N_{A1} = \text{minimum}\ (X_{A1}, C)$

$t = \frac{3\tau}{2} + D_{A1}$

ferry has just docked at side B

$X_{B1} = P[(\frac{3\tau}{2} + D_{A1})(1/b)]$; $F_{B1} = N_{A1}$

$D_{B1} = y_B(F_{B1}) + g_B(N_{B1}) + h_B$ where

$N_{B1} = \text{minimum}\ (X_{B1}, C)$. In a similar fashion at

$t = \frac{5\tau}{2} + D_{A1} + D_{B1}$

$X_{A2} = P[(2\tau + D_{A1} + D_{B1})(1/a)] + \text{maximum}\ (0, X_{A1} - C)$

$F_{A2} = N_{B1}$; $D_{A2} = y_A(F_{A2}) + g_A(N_{A2}) + h_A$ where

$N_{A2} = \text{minimum}\ (X_{A2}, C)$ and thus at

$t = \frac{7\tau}{2} + D_{A1} + D_{B1} + D_{A2}$

$X_{B2} = P[(2\tau + D_{B1} + D_{A2})(1/b)] + \text{maximum}\ (0, X_{B1} - C)$

$F_{B2} = N_{A2}$; $D_{B2} = y_B\,(F_{B2}) + g_B(N_{B2}) + h_B$ where

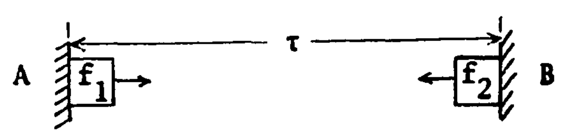
$N_{B2} = \text{minimum}\,(X_{B2}, C)$ and so on.

One can see the interdependent nature of the queueing system by noting that $F_{B1} = N_{A1}$ and $F_{A2} = N_{B1}$ etc. That is, each ferry's delay time due to loading or unloading on each side is dependent upon the number of cars taken aboard from the other side. Thus, whatever one ferry does on one side affects the service time of the cars on the other, and this is the interactive nature of the queueing system.

b. <u>Two shuttle model</u> In order to analyze the two-ferry system, one should start with the most simplistic case by assuming a constant loading and unloading time D on both sides of the channel. Assuming ferries are docked at side A and B initially, it is possible to express the state of each side, that is the number of cars waiting at dock A or B, by the following equations:

<u>Clock time</u>

$t = 0$



Clock time	State A	State B
$t = \tau$	$X_{A1} = P[(\tau)(1/a)]$	$X_{B1} = P[(\tau)(1/b)]$
$t = 2\tau + D$	$X_{A2} = P[(\tau + D)(1/a)]$ $+ \text{maximum}\,(0, X_{A1} - C)$	$X_{B2} = P[(\tau + D)(1/b)]$ $+ \text{maximum}\,(0, X_{B1} - C)$
$t = 3\tau + 2D$	$X_{A3} = P[(\tau + D)(1/a)]$ $+ \text{maximum}\,(0, X_{A2} - C)$	$X_{B3} = P[(\tau + D)(1/b)]$ $+ \text{maximum}\,(0, X_{B2} - C)$ etc.

Assumptions made here are not very realistic. Just as it is in the one-ferry case, delay times of the ferries are not constants but a function of the loading and unloading times plus waiting time for the other ferry, if any. Thus, a more realistic two-ferry model needs to be formulated. The following assumptions are made for this second model:

1. Travel time τ from one side to another is constant.

2. At time $t = 0$ there are no cars waiting on either side. Ferries are assumed to be in the middle of the channel going in opposite directions.

3. Ferries leave the dock as soon as they are either loaded to capacity or there are no more cars waiting at the dock.

4. There is one dock on each side.

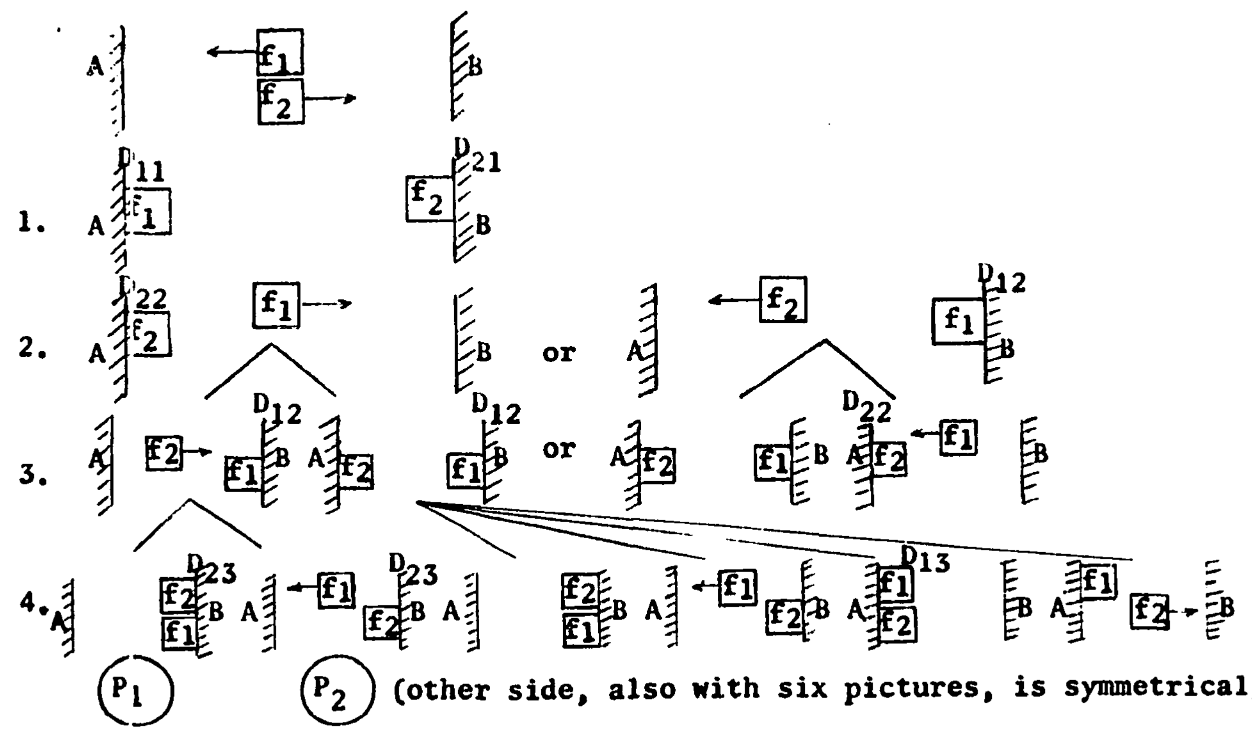
5. The number of arrivals per unit time has a Poisson (P) distribution.

6. If the second ferry boat arrives at a side at which the first ferry is still docked, all cars arriving after the arrival of the second ferry are not boarded on the first ferry.

7. Car parking lots are infinitely large.

1). Clock times Defining D_{fj} as the delay time due to loading and unloading for ferry f ($f = 1,2$) on the j th docking ($j = 1,2,\ldots,m$), the clock times (t_{nk}), times of arrivals of ferries at one of the sides, in pairs of (n,k) can be derived as shown in Figure 11; n and k are dummy variables such that $i = 2n-3+k, i>1$. Proceeding in the same manner as demonstrated in Figure 11, docking for the fifth time occurs when:

Docking no. (i)	Clock time at docking (t_{nk})



$t_0 = 0$

$t_1 = \frac{\tau}{2}$

(1st docking)

$t_{21} = \frac{3\tau}{2} + \min.(D_{11}, D_{21})$

(2nd docking)

$t_{22} = \frac{3\tau}{2} + \max.(D_{11}, D_{21})$

(3rd docking)

$t_{31} = \frac{5\tau}{2} + \min.(D_{11}+D_{12}, D_{21}+D_{22})$

(4th docking)

Figure 11. Derivation of clock times for two-ferry system

$$t_{32} = \frac{5\tau}{2} + \max.(D_{11}+D_{12}, D_{21}+D_{22})$$

and docking for the sixth time occurs when

$$t_{41} = \frac{7\tau}{2} + \min.(D_{11}+D_{12}+D_{13}, D_{21}+D_{22}+D_{23}) \quad \text{etc.},$$

and in general

$$t_{nk} = \frac{(2n-1)\tau}{2} + U(1,k)\min.\left(\sum_{j=1}^{n-1} D_{1j}, \sum_{j=1}^{n-1} D_{2j}\right) + U(2,k)\max.\left(\sum_{j=1}^{n-1} D_{1j}, \sum_{j=1}^{n-1} D_{2j}\right)$$

where

$$U(q,k) = \begin{cases} 1 \text{ if } q = k \\ 0 \text{ if } q \neq k \end{cases} \quad \text{and} \quad n \geq 2 .$$

Note that

$$D_{fj} = WT + DT$$

which represents the waiting time for the other ferry to leave plus the delay time due to loading and unloading. Thus, for example, considering pictures p_1 and p_2 in Figure 11, one has to test to see if f_2 arrives at side B before f_1 leaves B. Then if $t_{31}-t_{22}>D_{12}$ holds true, f_1 has left and picture p_2 represents the situation. In this case, $D_{23} = DT$ is a random variable consisting only of loading and unloading times; but if $t_{31}-t_{22}<D_{12}$, then f_1 has not left, and picture p_1 represents the situation. In this case, the delay of the ferry D_{23} consists of waiting time until f_1 leaves plus the loading and unloading time, that is

$$D_{23} = WT + DT$$

or

$$D_{23} = D_{12}-(t_{31}-t_{22}) + DT .$$

2). <u>States of sides A and B</u> In a manner similar to the one-ferry case, the number of cars waiting at docks A and B can be derived as follows:

Docking no. (i)	Clock time	State A	State B
1	t_1	$X_{A1} = P[(\frac{\tau}{2})(1/a)]$	$X_{B1} = P[(\frac{\tau}{2})(1/b)]$
2	t_{21}	$X_{A2} = P[(t_{21}-t_1)(1/a)] + \max.(0,X_{A1}-C)$	$X_{B2} = P[t_{21}-t_1)(1/b)] + \max.(0,X_{B1}-C)$
3	t_{22}	$X_{A3} = P[(t_{22}-t_{21})(1/a)] + \begin{cases} X_{A2} \text{ (if docking at } t_{21} \text{ was at dock B)} \\ \text{or} \\ \max.(0,X_{A2}-C) \text{ (if docking at } t_{21} \text{ was at dock A)} \end{cases}$	$X_{B3} = P[(t_{22}-t_{21})(1/b)] + \begin{cases} X_{B2} \text{ (if docking at } t_{21} \text{ was at dock A)} \\ \text{or} \\ \max.(0,X_{B2}-C) \text{ (if docking at } t_{21} \text{ was at dock B)} \end{cases}$
4	t_{31}	$X_{A4} = P[(t_{31}-t_{22})(1/a)] + \begin{cases} X_{A3} \\ \text{or} \\ \max.(0,X_{A3}-C) \end{cases}$	$X_{B4} = P[(t_{31}-t_{22})(1/b)] + \begin{cases} X_{B3} \\ \text{or} \\ \max.(0,X_{B3}-C) \end{cases}$ etc.

3). <u>Times for ferries to reach their destinations at the appropriate clock times</u> Derivation of formulas is illustrated by third and fourth dockings. Defining Δf_{nk} = time at t_{nk} for ferry f to reach destination and letting $D_{11}<D_{21}$ and $D_{11}+D_{12}<D_{21}+D_{22}$, then third docking time is given by the equation:

$$t_{22} = \frac{3\tau}{2} + D_{21} .$$

The next docking time for f_1 is given by:

$$t_{31} = \frac{5\tau}{2} + D_{11} + D_{12}$$

thus at t_{22}, the time for f_1 to reach the next destination is:

$$\Delta_{22}^{1} = t_{31} - t_{22}$$

or

$$\Delta_{22}^{1} = \frac{5\tau}{2} + D_{11} + D_{12} - \frac{3\tau}{2} - D_{21}$$

or

$$\Delta_{22}^{1} = \tau + D_{12} - (D_{21} - D_{11}) \cdot$$

But now suppose $D_{11} > D_{21}$, then third docking time can be expressed as

$$t_{22} = \frac{3\tau}{2} + D_{11} ,$$

and thus at t_{22} the time for f_1 to reach next destination is

$$\Delta_{22}^{1} = t_{31} - t_{22}$$

or

$$\Delta_{22}^{1} = \frac{5\tau}{2} + D_{11} + D_{12} - \frac{3\tau}{2} - D_{11}$$

or

$$\Delta_{22}^{1} = \tau + D_{12} \cdot$$

Therefore,

$$\Delta_{22}^{1} = \tau + D_{12} + \begin{cases} 0 \\ -(D_{21} - D_{11}) \end{cases}$$

and by symmetry for f_2

$$\Delta_{22}^{2} = \tau + D_{22} + \begin{cases} 0 \\ -(D_{11} - D_{21}) \end{cases} \cdot$$

To find the times for ferries f_1 and f_2 to reach their destinations at fourth docking, let

$$D_{11} + D_{12} > D_{21} + D_{22} \cdot$$

Then the fourth docking time is given by the equation

$$t_{31} = \frac{5\tau}{2} + D_{21} + D_{22}$$

and the next docking time for f_1 can be expressed as

$$t_{32} = \frac{5\tau}{2} + D_{11} + D_{12} \cdot$$

Thus, at t_{31}, the time for f_1 to reach its next destination is

$$\Delta_{31}^{1} = t_{32} - t_{31}$$

$$\Delta^1_{31} = \frac{5\tau}{2} + D_{11} + D_{12} - \frac{5\tau}{2} - (D_{21}+D_{22})$$

$$\Delta^1_{31} = \sum_{j=1}^{2} D_{1j} - \sum_{j=1}^{2} D_{2j} \; .$$

But if $D_{11} + D_{12} < D_{21} + D_{22}$, then f_1 is the ferry which has just docked at time $t_{31} = \frac{5\tau}{2} + D_{11} + D_{12}$. Thus, the time for f_1 to reach its next destination is simply:

$$\Delta^1_{31} = \tau + D_{13} \; .$$

Therefore,

$$\Delta^1_{31} = \begin{cases} \tau + D_{13} \\ \sum_{j=1}^{2} D_{1j} - \sum_{j=1}^{2} D_{2j} \end{cases}$$

and, similarly, by symmetry for f_2

$$\Delta^2_{31} = \begin{cases} \tau + D_{23} \\ \sum_{j=1}^{2} D_{2j} - \sum_{j=1}^{2} D_{1j} \end{cases} \qquad \text{etc.}$$

Times for both ferries to reach their destinations, derived in a similar fashion, are given in Table 1.

Table 1. Times for ferries to reach their destinations at appropriate clock times

Docking no. (i)	Clock time (t_{nk})	Time for f_1	Time for f_2
1	t_1	$\Delta^1_1 = \tau + D_{11}$	$\Delta^2_1 = \tau + D_{12}$
2	t_{21}	$\Delta^1_{21} = \begin{cases} \tau + D_{12} \\ D_{11} - D_{21} \end{cases}$	$\Delta^2_{21} = \begin{cases} \tau + D_{22} \\ D_{21} - D_{11} \end{cases}$

Table 1. (Continued)

Docking no. (i)	Clock time (t_{nk})	Time for f_1	Time for f_2
3	t_{22}	$\Delta^1_{22} = \tau + D_{12} + \begin{cases} 0 \\ -(D_{21}-D_{11}) \end{cases}$	$\Delta^2_{22} = \tau + D_{22} + \begin{cases} 0 \\ -(D_{11}-D_{21}) \end{cases}$
4	t_{31}	$\Delta^1_{31} = \begin{cases} \tau + D_{13} \\ \sum_{j=1}^{2} D_{1j} - \sum_{j=1}^{2} D_{2j} \end{cases}$	$\Delta^2_{31} = \begin{cases} \tau + D_{23} \\ \sum_{j=1}^{2} D_{2j} - \sum_{j=1}^{2} D_{1j} \end{cases}$
5	t_{32}	$\Delta^1_{32} = \tau + D_{13} + \begin{cases} 0 \\ -(\sum_{j=1}^{2} D_{2j} - \sum_{j=1}^{2} D_{1j}) \end{cases}$	$\Delta^2_{32} = \tau + D_{23} + \begin{cases} 0 \\ -(\sum_{j=1}^{2} D_{1j} - \sum_{j=1}^{2} D_{2j}) \end{cases}$
6	t_{41}	$\Delta^1_{41} = \begin{cases} \tau + D_{14} \\ \sum_{j=1}^{3} D_{1j} - \sum_{j=1}^{3} D_{2j} \end{cases}$	$\Delta^2_{41} = \begin{cases} \tau + D_{24} \\ \sum_{j=1}^{3} D_{2j} - \sum_{j=1}^{3} D_{1j} \end{cases}$
7	t_{42}	$\Delta^1_{42} = \tau + D_{14} + \begin{cases} D \\ -(\sum_{j=1}^{3} D_{2j} - \sum_{j=1}^{3} D_{1j}) \end{cases}$	$\Delta^2_{42} = \tau + D_{24} + \begin{cases} 0 \\ -(\sum_{j=1}^{3} D_{1j} - \sum_{j=1}^{3} D_{2j}) \end{cases}$
and in general:			
		$\Delta^1_{n1} = \begin{cases} \tau + D_{1n} \\ \sum_{j=1}^{n-1} D_{1j} - \sum_{j=1}^{n-1} D_{2j} \end{cases}$	$\Delta^2_{n1} = \begin{cases} \tau + D_{2n} \\ \sum_{j=1}^{n-1} D_{2j} - \sum_{j=1}^{n-1} D_{1j} \end{cases}$
		$\Delta^1_{n2} = \tau + D_{1n} + \begin{cases} 0 \\ -(\sum_{j=1}^{n-1} D_{2j} - \sum_{j=1}^{n-1} D_{1j}) \end{cases}$	$\Delta^2_{n2} = \tau + D_{2n} + \begin{cases} 0 \\ -(\sum_{j=1}^{n-1} D_{1j} - \sum_{j=1}^{n-1} D_{2j}) \end{cases}$

4). <u>Number of cars on and boarding ferry</u> Defining:

N_{fi} = number of cars boarding ferry f at i th docking

F_{fi} = number of cars on ferry f at i th docking

and assuming ferries carry k number of cars at time t = 0, the following relationships for the number of cars on and boarding ferry are derived:

Docking no. (i)	Number on f_1	Number on f_2	Number boarding f_1	Number boarding f_2
1	$F_{11} = k$	$F_{21} = k \ (0 \leq k \leq C)$	$N_{11} = \min.(X_{A1}, C)$	$N_{21} = \min.(X_{B1}, C)$
2	$F_{12} = N_{11}$	$F_{22} = N_{21}$	$N_{12} = \begin{cases} \min.(X_{B2}, C) \text{ (if } f_1 \text{ docks first)} \\ 0 \text{ (if } f_2 \text{ docks first)} \end{cases}$	$N_{22} = \begin{cases} \min.(X_{A2}, C) \text{ (if } f_2 \text{ docks first)} \\ 0 \text{ (if } f_1 \text{ docks first)} \end{cases}$
3	$F_{13} = N_{12}$	$F_{23} = N_{22}$	$N_{13} = \begin{cases} \min.(X_{B3}, C) \\ 0 \end{cases}$	$N_{23} = \begin{cases} \min.(X_{A3}, C) \\ 0 \end{cases}$
4	$F_{14} = N_{13}$	$F_{24} = N_{23}$	$N_{14} = \begin{cases} \min.(X_{A4}, C) \\ 0 \end{cases}$	$N_{24} = \begin{cases} \min.(X_{B4}, C) \\ 0 \end{cases}$
5	$F_{15} = N_{14}$	$F_{25} = N_{24}$	$N_{15} = \begin{cases} \min.(X_{A5}, C) \\ 0 \end{cases}$	$N_{25} = \begin{cases} \min.(X_{B5}, C) \\ 0 \end{cases}$ etc.

5). <u>Waiting times of the ferries</u> Defining:

W_{fi} = waiting time of ferry f at i th docking

then the waiting times of the ferries would be as follows:

Docking no. (i)	Waiting time of f_1	Waiting time of f_2
1	$W_{11} = 0$	$W_{21} = 0$
2	$W_{12} = \Delta(D_{21},D_{11}+\tau)$ $[D_{21}-D_{11}-\tau]$	$W_{22} = \Delta(D_{11},D_{21}+\tau)$ $[D_{11}-D_{21}-\tau]$
3	$W_{13} = 0$	$W_{23} = 0$
4	$W_{14} = \Delta(D_{21}+D_{22},D_{11}+D_{12}+\tau)$ $[D_{21}+D_{22}-D_{11}-D_{12}-\tau]$	$W_{24} = \Delta(D_{11}+D_{12},D_{21}+D_{22}+\tau)$ $[D_{11}+D_{12}-D_{21}-D_{22}-\tau]$
5	$W_{15} = 0$	$W_{25} = 0$
6	$W_{16} = \Delta(D_{21}+D_{22}+D_{23}, D_{11}+D_{12}+D_{13}+\tau)$ $[D_{21}+D_{22}-D_{23}-D_{11} -D_{12}-D_{13}-\tau]$	$W_{26} = \Delta(D_{11}+D_{12}+D_{13}, D_{21}+D_{22}+D_{23}+\tau)$ $[D_{11}+D_{12}+D_{13}-D_{21} -D_{22}-D_{23}-\tau]$

etc.

where

$$\Delta(q,k) = \begin{cases} 1 & \text{if } q>k \\ 0 & \text{otherwise .} \end{cases}$$

Table 2 gives a summary of the parameters of the two-ferry system.

Table 2. Summary of the parameters of two-ferry system

Docking no. (i)	Clock time (t_{nk})	State A (number waiting at dock A)
1	$t_1 = \frac{\tau}{2}$	$X_{A1} = P[(\tau/2)(1/a)]$
2	$t_{21} = \frac{3\tau}{2} + \min.(D_{11},D_{21})$	$X_{A2} = P[(t_{21}-t_1)(1/a)] + \max.(0,X_{A1}-C)$
3	$t_{22} = \frac{3\tau}{2} + \max.(D_{11},D_{21})$	$X_{A3} = P[(t_{22}-t_{21})(1/a)] + \begin{cases} X_{A2} \\ \max.(0,X_{A2}-C) \end{cases}$
4	$t_{31} = \frac{5\tau}{2} + \min.(D_{11}+D_{12},D_{21}+D_{22})$	$X_{A4} = P[(t_{31}-t_{22})(1/a)] + \begin{cases} X_{A3} \\ \max.(0,X_{A3}-C) \end{cases}$
5	$t_{32} = \frac{5\tau}{2} + \max.(D_{11}+D_{12},D_{21}+D_{22})$	$X_{A5} = P[(t_{32}-t_{31})(1/a)] + \begin{cases} X_{A4} \\ \max.(0,X_{A4}-C) \end{cases}$

State B (number waiting at dock B)	Number of cars on f_1	Number of cars on f_2	Number boarding f_1
$X_{B1} = P[(\tau/2)(1/b\ ]$	$F_{11} = k$	$F_{21} = k$	$N_{11} = \min.(X_{A1}, C)$
$X_{B2} = \begin{matrix} P[(t_{21}-t_1)(1/b)] \\ +\max.(0, X_{B1}-C) \end{matrix}$	$F_{12} = N_{11}$	$F_{22} = N_{21}$	$N_{12} = \begin{cases} \min.(X_{B2}, C) \\ 0 \end{cases}$
$X_{B3} = \begin{matrix} P[(t_{22}-t_{21})(1/b)] \\ + \begin{cases} X_{B2} \\ \max.(0, X_{B2}-C) \end{cases} \end{matrix}$	$F_{13} = N_{12}$	$F_{23} = N_{22}$	$N_{13} = \begin{cases} \min.(X_{B3}, C) \\ 0 \end{cases}$
$X_{B4} = \begin{matrix} P[(t_{31}-t_{22})(1/b)] \\ + \begin{cases} X_{B3} \\ \max.(0, X_{B3}-C) \end{cases} \end{matrix}$	$F_{14} = N_{13}$	$F_{24} = N_{23}$	$N_{14} = \begin{cases} \min.(X_{A4}, C) \\ 0 \end{cases}$
$X_{B5} = \begin{matrix} P[(t_{32}-t_{31})(1/b)] \\ + \begin{cases} X_{B4} \\ \max.(0, X_{B4}-C) \end{cases} \end{matrix}$	$F_{15} = N_{14}$	$F_{25} = N_{24}$	$N_{15} = \begin{cases} \min.(X_{A5}, C) \\ 0 \end{cases}$

Table 2. (Continued)

Number boarding f_2	Delay time for f_1	Delay time for f_2
$N_{21} = \min(X_{B1},C)$	$D_{11} = \begin{matrix} y(F_{11})+g(N_{11}) \\ +h_A+W_{11} \end{matrix}$	$D_{21} = \begin{matrix} y(F_{21})+g(N_{21}) \\ +h_B+W_{21} \end{matrix}$
$N_{22} = \begin{cases} \min.(X_{A2},C) \\ 0 \end{cases}$	$D_{12} = \begin{cases} y(F_{12})+g(N_{12}) \\ +h_B+W_{12} \\ 0 \end{cases}$	$D_{22} = \begin{cases} y(F_{22})+g(N_{22}) \\ +h_A+W_{22} \\ 0 \end{cases}$
$N_{23} = \begin{cases} \min.(X_{A3},C) \\ 0 \end{cases}$	$D_{13} = \begin{cases} y(F_{13})+g(N_{13}) \\ +h_B+W_{13} \\ 0 \end{cases}$	$D_{13} = \begin{cases} y(F_{23})+g(N_{23}) \\ +h_A+W_{23} \\ 0 \end{cases}$
$N_{24} = \begin{cases} \min.(X_{B4},C) \\ 0 \end{cases}$	$D_{14} = \begin{cases} y(F_{14})+g(N_{14}) \\ +h_A+W_{14} \\ 0 \end{cases}$	$D_{24} = \begin{cases} y(F_{24})+g(N_{24}) \\ +h_B+W_{24} \\ 0 \end{cases}$
$N_{25} = \begin{cases} \min.(X_{B5},C) \\ 0 \end{cases}$	$D_{15} = \begin{cases} y(F_{15})+g(N_{15}) \\ +h_A+W_{15} \\ 0 \end{cases}$	$D_{25} = \begin{cases} y(F_{25})+g(N_{25}) \\ +h_B+W_{25} \\ 0 \end{cases}$ etc.

2. Markov single shuttle model

Using an approach similar to the one-ferry model, probability distributions for the number of cars waiting on shore at successive end of unloading times are derived using Markovian equations of transition and assuming infinitely large parking lots on both sides of the channel.

The following notation is used in developing the general expressions for the probability distributions. Let:

t_i^s	=	cumulative time at the end of i th unloading of ferry at side s
$X_{s,i(s)}$	=	total number of arrivals waiting at side s at the end of i th unloading at side s
L_i^s	=	time taken for loading at side s during i th docking
U_i^s	=	time taken for unloading at side s during i th docking
β_s	=	a per car loading constant for side s
γ_s	=	a per car unloading constant for side s

a. Development of Markovian equations of transition   Assuming that at time t = 0 the ferry is at side A, B $\|\quad \boxed{f}\|$ A, initially there are no cars waiting on either side, and first docking starts with side B, then at

$$t_1^B = \tau + h_A, \quad B \|\boxed{f} \quad \| A,$$

there are $X_{B,1(B)}$ and $X_{A,1(B)}$ arrivals waiting at side B and A respectively and the ferry loads min. $[X_{B,1(B)}, C]$ number of cars. The time taken for loading at side B would be

$$L_1^B = (\beta_B)[\min.(X_{B,1(B)}, C)] + h_B \quad .$$

The ferry returns to side A at time

$$t = t_1^B + L_1^B + \tau = t_1^B + \tau + (\beta_B)[\min.(X_{B,1(B)}, C)] + h_B, \quad B \|\quad \boxed{f}\| A,$$

The time to unload the ferry at side A is given by

$$U_1^A = (\gamma_A)[\min.(X_{B,1(B)},C)]$$

then

$$t_1^A = t_1^B+L_1^B+\tau+U_1^A = t_1^B+\tau+(\gamma_A+\beta_B)[\min.(X_{B,1(B)},C)]+h_B \ .$$

At this point the number of cars waiting at side A is given by the equation

$$X_{A,1(A)} = X_{A,1(B)}+P\{(1/a)[L_1^B+\tau+U_1^A]\}$$

or

$$X_{A,1(A)} = X_{A,1(B)}+P\left\{(1/a)[(\gamma_A+\beta_B)\{\min.(X_{B,1(B)},C]\}+h_B+\tau]\right\} .$$

The ferry loads a $\{\min.(C,X_{A,1(A)})\}$ number of cars, which requires a loading time of

$$L_1^A = (\beta_A)[\min.(C,X_{A,1(A)})]+h_A \ .$$

When the ferry returns to side B, time at the end of unloading would be

$$t_2^B = t_1^A+L_1^A+\tau+ (\gamma_B)[\min.(C,X_{A,1(A)})],$$

or

$$t_2^B = t_1^B+\tau+(\gamma_A+\beta_B)[\min.(X_{B,1(B)},C)]+h_B+(\beta_A)[\min.(C,X_{A,1(A)})]$$
$$+h_A+\tau+(\gamma_B)[\min.(C,X_{A,1(A)})] \ .$$

Time elapsed between t_1^B and t_2^B is, therefore,

$$t_2^B-t_1^B = 2\tau+h_A+h_B+(\beta_B+\gamma_A)[\min.(X_{B,1(B)},C)]$$
$$+(\gamma_B+\beta_A)\left[\min.\left(C,X_{A,1(B)}+P\left\{(1/a)\left((\gamma_A+\beta_B)\{\min.(X_{B,1(B)},C)\}+h_B+\tau\right)\right\}\right)\right]$$
$$= \rho_B(X_{B,1(B)},X_{A,1(B)}) \ .$$

Thus, the number of cars waiting at side B at the end of 2 nd unloading at side B is

$$X_{B,2(B)} = X_{B,1(B)}-\min.[X_{B,1(B)},C]+P\left\{(1/b)[\rho_B(X_{B,1(B)},X_{A,1(B)})]\right. \ .$$

In general,

$$t_{i+1}^B-t_i^B = 2\tau+h_A+h_B+(\beta_B+\gamma_A)[\min.(X_{B,i(B)},C)]$$
$$+(\gamma_B+\beta_A)\left[\min.\left(C,X_{A,i(B)}+P\left\{(1/a)\left((\gamma_A+\beta_B)\{\min.(X_{B,i(B)}, C)\}+h_B+\tau\right)\right\}\right)\right]$$

and

$$X_{B,(i+1)(B)} = \max.[0,X_{B,i(B)}-C]+P\{(1/b)[\rho_B(X_{B,i(B)},X_{A,i(B)})]\}.$$

Similarly, the time taken by ferry from t_1^B to t_1^A is

$$t_1^A-t_1^B = \tau+(\gamma_A+\beta_B)[\min.(X_{B,1(B)},C)]+h_B,$$

and the time elapsed from t_1^A to t_2^B is

$$\begin{aligned} t_2^B-t_1^A &= t_1^B+\tau+(\gamma_A+\beta_B)[\min.(X_{B,1(B)},C)]+h_B+(\beta_A)[\min.(C,X_{A,1(A)})] \\ &\quad +h_A+\tau+(\gamma_B)[\min.(C,X_{A,1(A)})]-t_1^B-\tau-(\gamma_A+\beta_B) \\ &\quad [\min.(X_{B,1(B)},C)]-h_B \\ &= (\beta_A+\gamma_B)[\min.(C,X_{A,1(A)})]+h_A+\tau \end{aligned}$$

or

$$\begin{aligned} t_2^B-t_1^A &= h_A+\tau+(\gamma_B+\beta_A)\Big[\min.\Big(C,X_{A,1(B)}+P\{(1/a)[(\gamma_A+\beta_B) \\ &\quad (\min.(X_{B,1(B)},C))+h_B+\tau]\}\Big)\Big] \\ &= \rho_A(X_{B,1(B)}+X_{A,1(B)}). \end{aligned}$$

This equation represents the ferry service time from side A to side B.

Thus,

$$\begin{aligned} X_{A,2(B)} &= \max.\Big[0,X_{A,1(B)}+P\{(1/a)(t_1^A-t_1^B)\}-C\Big] \\ &\quad +P\{(1/a)[\rho_A(X_{B,1(B)},X_{A,1(B)})]\}. \end{aligned}$$

In general,

$$\begin{aligned} t_{i+1}^B-t_i^A &= h_A+\tau+(\gamma_B+\beta_A)\Big[\min.\Big(C,X_{A,i(B)}+P\{(1/a)[(\gamma_A+\beta_B) \\ &\quad (\min.(X_{B,i(B)},C))+h_B+\tau]\}\Big)\Big] \end{aligned}$$

and

$$\begin{aligned} X_{A,(i+1)(B)} &= \max.\Big[0,X_{A,i(B)}+P\{(1/a)\big(\tau+(\gamma_A+\beta_B)[\min.(X_{B,i(B)},C)] \\ &\quad +h_B\big]\}-C+P\{(1/a)[\rho_A(X_{B,i(B)},X_{A,i(B)})]\}. \end{aligned}$$

In summary, conditionally on $H = \{(X_{A,0(B)},X_{B,0(B)})\ldots(X_{A,(i-1)(B)},X_{B,(i-1)(B)})\}$, $X_{B,(i+1)(B)}$ and $X_{A,(i+1)(B)}$ can be expressed as:

$$X_{B,(i+1)(B)} = \upsilon+P(\lambda_1) \qquad (1)$$

$$X_{A,(i+1)(B)} = \max.[0,\eta+P(\lambda_2)]+P(\lambda_3) \qquad (2)$$

where

$$\upsilon = \max.[0, X_{B,i(B)} - C] \tag{3}$$

$$\eta = X_{A,i(B)} - C \tag{4}$$

and the three Poisson variables are independent, with parameters

$$\lambda_1 = (1/b)[\rho_B(X_{B,i(B)}, X_{A,i(B)})] \tag{5}$$

$$\lambda_2 = (1/a)\{\tau + (\gamma_A + \beta_B)[\min.(X_{B,i(B)}, C)] + h_B\} \tag{6}$$

and

$$\lambda_3 = (1/a)[\rho_A(X_{B,i(B)}, X_{A,i(B)})] . \tag{7}$$

Under the assumption that the number of arrivals at either side form a Poisson process, it is now easily seen that the random process $(X_{A,(i+1)(B)}, X_{B,(i+1)(B)})$ is a bivariate Markov chain.

A random process $[X_{i,} \; i = 0,1,2,\ldots,\infty]$ is called a Markov chain if X_i is a discrete random variable for each i and for any sequence of states $k_0, k_1, \ldots, k_{i+1}$ the following holds:

$$\text{Prob.}[X_{i+1} = k_{i+1}/X_i = k_i, X_{i-1} = k_{i-1}, \ldots, X_0 = k_0]$$
$$= \text{Prob.}[X_{i+1} = k_{i+1}/X_i = k_i] . \tag{8}$$

This conditional probability is called the Markovian transition probability.

Thus, to show that $(X_{A,(i+1)(B)}, X_{B,(i+1)(B)})$ is a bivariate Markov chain, it is needed only to verify that the probability identity in equation 8 holds. But it is easily seen from equations 1 through 7 that the probability distribution of $(X_{A,(i+1)(B)}, X_{B,(i+1)(B)})$, given H, depends only on $(X_{A,i(B)}, X_{B,i(B)})$ and not on H. Therefore, the probability identity in equation 8 holds for $(X_{A,(i+1)(B)}, X_{B,(i+1)(B)})$. Note also that $X_{A,(i+1)(B)}, X_{B(i+1)(B)}$ are conditionally independent, that is, for any i, the pair of the random variables are independent because of the cars arriving at each side independently. Next subsection gives an illustration of the calculation of a transition probability.

Since the vector $(X_{A,(i+1)(B)}, X_{B,(i+1)(B)})$ is a classical bivariate Markov chain, one can evaluate the long-run probability distributions for the number of cars waiting at either side at the end of an unloading at side B.

A comment has to be made here about the car parking lots at the ferry docks. In the development of the simulation and mathematical models, an infinitely large parking lot capacity is assumed. However, in an actual situation this may or may not be true. If the cars start forming a queue line on the street when the parking lot is full, then this may still be considered an infinitely large parking lot. However, there is still the possibility that the driver may decide not to get into the queue line if he sees that the parking lot is full. Thus, if one assumes independent Poisson arrivals with a rate of λ_1 when parking lot is not full, then the arrival rate may change to λ_2 when the parking lot is full. That is, the arrival rate may be related to the number of cars in the parking lot and the probabilities become conditional.

b. Illustration of a transition probability computation   As a numerical example, let

$$\tau = 12 \text{ minutes}; \; h_A = 1.8 \text{ minutes}; \; h_B = 1.6 \text{ minutes}$$

$$C = 2 \text{ cars}; \; \gamma_A = 0.08; \; \gamma_B = 0.08$$

$$\beta_B = 0.14; \; \beta_A = 0.13; \; a = 25 \text{ minutes/car}; \; b = 25 \text{ minutes/car}.$$

For illustration purposes, the following transition probability is calculated:

$$\text{Prob.}(X_{A,2(B)} = 1, X_{B,2(B)} = 1 / X_{A,1(B)} = 1, X_{B,1(B)} = 1).$$

Earlier it was found that

$$X_{A,1(A)} = X_{A,1(B)}+P\left\{(1/a)[L_1^B+\tau+U_1^A]\right\}.$$

This equation can be expressed as

$$X_{A,1(A)} = X_{A,1(B)}+N_{A,1(A)}$$

where $N_{A,1(A)}$ is the number of cars arriving at side A during time interval $(t_1^A-t_1^B)$, or

$$N_{A,1(A)} = P\left\{(1/a)\left[(\gamma_A+\beta_B)[\min.(X_{B,1(B)},C)]+h_B+\tau\right]\right\}$$

which is a function of $X_{B,1(B)}$. Figure 12 shows the parameters involved in the calculations. Assuming that the random variables $X_{B,1(B)},X_{A,1(B)}$ and $N_{A,1(A)}$ and the other constants of the system are given, then $X_{A,1(A)}$ and $(t_1^A-t_1^B)$ can be calculated deterministically from the equations already developed. Time difference $(t_2^B-t_1^A)$ is also deterministic once $X_{A,1(A)}$ is known since loading, transit and unloading times are known. Defining $N_{A,2(B)}$ as the number of cars arriving at side A during time interval $(t_2^B-t_1^A)$, then $N_{A,2(B)}$ is a Poisson random variable with parameter $(t_2^B-t_1^A)/a$. In a similar fashion, $X_{B,2(B)}$ is a function of $(t_2^B-t_1^B)$ and the other previously calculated or given variables. Proceeding in this manner one can calculate the times $(t_{i+1}^B-t_i^A)$ and $(t_{i+1}^B-t_i^B)$ and the transition probability "matrices" by independent Poisson probabilities.

Thus, since $(X_{A,2(B)},X_{B,2(B)}/X_{A,1(B)},X_{B,1(B)})$ is a function of the random variable $N_{A,1(A)}$, then the joint probabilities of $X_{A,2(B)}$ and $X_{B,2(B)}$ are derived by averaging over the distribution of $N_{A,1(A)}$ for all possible values. That is,

$$\text{Prob.}(\ddot{X}_{A,2(B)} = \dot{k}_1,\ddot{X}_{B,2(B)} = \dot{k}_2/\ddot{X}_{A,1(B)},\ddot{X}_{B,1(B)}) =$$

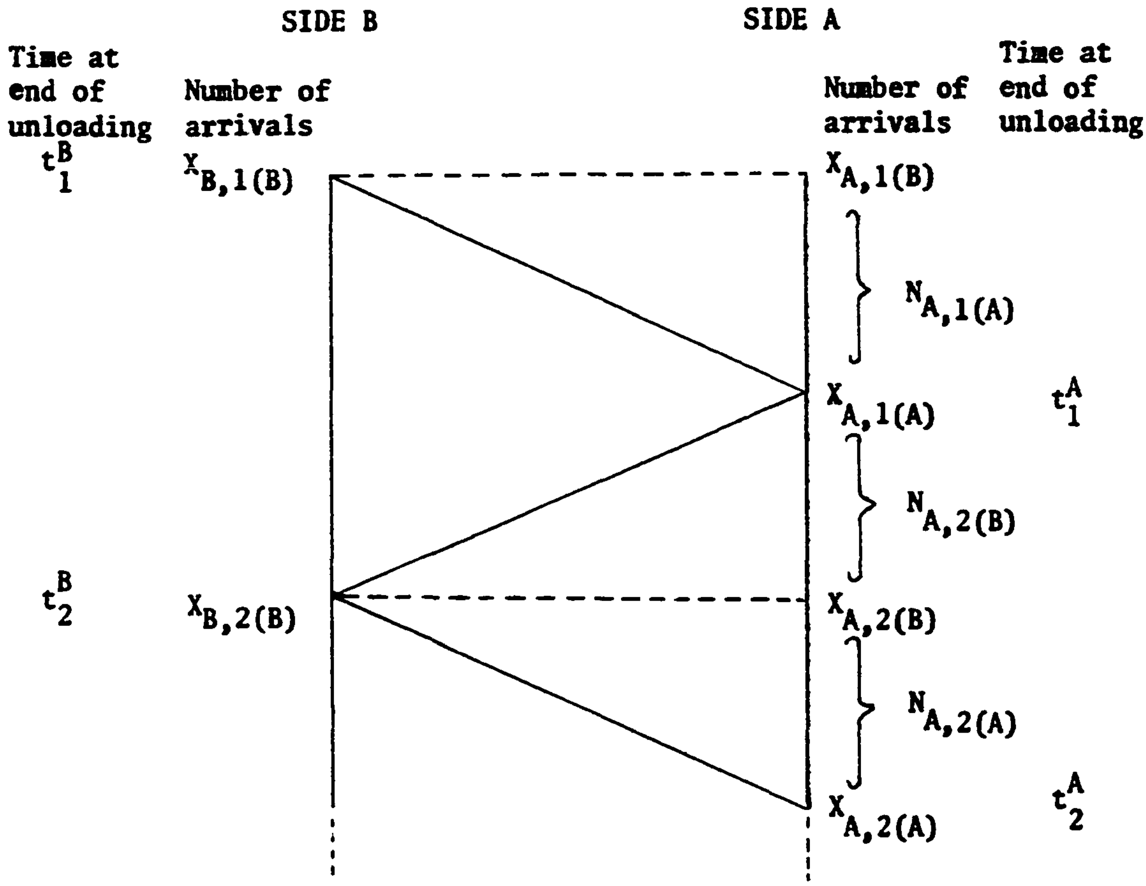


Figure 12. Relevant parameters involved in calculation of transition probabilities

$$\sum_{k=0}^{\infty}\text{Prob.}(N_{A,1(A)} = k/X_{A,1(B)},X_{B,1(B)})\text{Prob.}(X_{A,2(B)} = k_1/X_{A,1(B)},X_{B,1(B)},$$
$$N_{A,1(A)} = k)\text{Prob.}(X_{B,2(B)} = k_2/X_{A,1(B)},X_{B,1(B)},N_{A,1(A)} = k) \qquad (9)$$

where

$$X_{B,2(B)} = \text{max.}[0,X_{B,1(B)}-C]+P\Big\{(1/b)[2\tau+h_A+h_B+(\gamma_A+\beta_B)\text{min.}(X_{B,1(B)},C)+(\gamma_B+\beta_A)\text{min.}(C,X_{A,1(B)}+N_{A,1(A)})]\Big\}$$

$$X_{A,2(B)} = \text{max.}[0,X_{A,1(B)}+N_{A,1(A)}-C] +P\Big\{(1/a)[h_A+\tau+(\gamma_B+\beta_A)\text{min.}(C,X_{A,1(B)}+N_{A,1(A)})]\Big\}.$$

Similary,

$$\text{Prob.}(X_{A,2(B)} = k_1/X_{A,1(B)},X_{B,1(B)}) = \sum_{k=0}^{\infty}\text{Prob.}(N_{A,1(A)} = k/X_{A,1(B)},X_{B,1(B)})\text{Prob.}(X_{A,2(B)} = k_1/X_{A,1(B)},X_{B,1(B)},N_{A,1(A)} = k) \qquad (10)$$

and

$$\text{Prob.}(X_{B,2(B)} = k_2/X_{A,1(B)}, X_{B,1(B)}) = \sum_{k=0}^{\infty} \text{Prob.}(N_{A,1(A)} = k/ X_{A,1(B)}, X_{B,1(B)})\text{Prob.}(X_{B,2(B)} = k_2/ X_{A,1(B)}, X_{B,1(B)}, N_{A,1(A)} = k) \ . \tag{11}$$

For the above example,

$$\text{Prob.}(X_{A,2(B)} = 1, X_{B,2(B)} = 1/X_{A,1(B)} = 1, X_{B,1(B)} = 1)$$
$$= \sum_{k=0}^{\infty} \text{Prob.}(N_{A,1(A)} = k/X_{A,1(B)} = 1, X_{B,1(B)} = 1)\text{Prob.}(X_{A,2(B)} = 1/ 1,1,N_{A,1(A)} = k)\text{Prob.}(X_{B,2(B)} = 1/1,1,N_{A,1(A)} = k)$$

but

$$\text{Prob.}(N_{A,1(A)} = k/X_{A,1(B)} = 1, X_{B,1(B)} = 1) = \text{Prob.}(R = k)$$

where R is a Poisson random variable with parameter

$$\lambda = (1/a)[(\gamma_A+\beta_B)\{\min.(X_{B,1(B)}, C)\}+h_B+\tau]$$

$$\lambda = (1/25)[(0.08+0.14)+1.6+12]$$

$$\lambda = 0.554 \ .$$

Poisson density function is defined as $f(k) = e^{-\lambda}\lambda^k/k!$. Thus, for $\lambda = 0.554$, Prob.(R = k) for various values of k is as follows:

k	Prob.(R = k)
0	0.575
1	0.318
2	0.088
3	0.016
4	0.002 etc.

When $N_{A,1(A)} = 0$, $X_{A,2(B)} = 1$ if $\max.[0,1+0-2]+R_1 = 1$ or if $R_1 = 1$ where R_1 is a Poisson random variable with parameter

$$\lambda_1 = (1/a)[h_A+\tau+(\gamma_B+\beta_A)\min.(C, X_{A,1(B)}+N_{A,1(A)})]$$

$$\lambda_1 = (1/25)[1.8+12+(0.08+0.13)\min.(2,1+0)]$$

$$\lambda_1 = (1/25)(14.01) = 0.5604 \ .$$

Therefore,

$$\text{Prob.}(X_{A,2(B)} = 1/1,1,N_{A,1(A)} = 0) = \text{Prob}(R_1=1)$$
$$= (e^{-0.5604})(0.5604) = 0.319 \ .$$

Similarly, $X_{B,2(B)} = 1$ if $\max.[0,1-2]+R_2 = 1$ or if $R_2 = 1$ where R_2 is a Poisson random variable with parameter

$$\lambda_2 = (1/b)[2\tau+h_A+h_B+(\gamma_A+\beta_B)\min.(X_{B,1(B)},C)$$
$$+(\gamma_B+\beta_A)\min.(C,X_{A,1(B)}+N_{A,1(A)})]$$
$$\lambda_2 = (1/25)[24+1.8+1.6+(0.08+0.14)\min.(1,2)$$
$$+(0.08+0.13)\min.(2,1+0)]$$
$$\lambda_2 = (1/25)[27.83] = 1.112 \ .$$

Thus,

$$\text{Prob.}(X_{B,2(B)} = 1/1,1,N_{A,1(A)} = 0) = \text{Prob.}(R_2=1)$$
$$= (e^{-1.112})(1.112) = 0.367 \ .$$

Proceeding in a similar manner, when $N_{A,1(A)} = 1$, $X_{A,2(B)} = 1$ if $\max.[0,1+1-2]+R_1 = 1$ or if $R_1 = 1$ where

$$\lambda_1 = 14.22/25 = 0.569 \ .$$

Therefore,

$$\text{Prob.}(X_{A,2(B)} = 1/1,1,N_{A,1(A)} = 1) = \text{Prob.}(R_1=1)$$
$$= (e^{-0.569})(0.569) = 0.322 \ .$$

Also

$$X_{B,2(B)} = 1 \text{ if } R_2 = 1 \text{ where } \lambda_2 = (28.04/25) = 1.123 \ .$$

Thus,

$$\text{Prob.}(X_{B,2(B)} = 1/1,1,N_{A,1(A)} = 1) = \text{Prob.}(R_2=1)$$
$$= (e^{-1.123})(1.123) = 0.366 \ .$$

When $N_{A,1(A)} = 2$, $X_{A,2(B)} = 1$ if $\max.[0,1+2-2]+R_1 = 1$ or if $R_1 = 0$ where

$$\lambda_1 = 0.569 \text{ and Prob.}(X_{A,2(B)} = 1/1,1,N_{A,1(A)} = 2) = \text{Prob.}(R_1=0)$$
$$= e^{-0.569} = 0.566 .$$

Similarly,

$$X_{B,2(B)} = 1 \text{ if } R_2 = 1 \text{ where } \lambda_2 = 1.123$$

or

$$\text{Prob.}(X_{B,2(B)} = 1/1,1,N_{A,1(A)} = 2) = \text{Prob.}(R_2=1) = 0.366 .$$

When $N_{A,1(A)} = 3$, $X_{A,2(B)} = 1$ if $\max.[0,1+3-2]+R_1 = 1$ or if $R_1 = -1$

but

$$\text{Prob.}(X_{A,2(B)} = 1/1,1,N_{A,1(A)} = 3) = \text{Prob.}(R_1=-1) = 0 .$$

It is seen that the rest of the terms in equation 9 are zero. From equation 9

$$P(X_{A,2(B)} = 1, X_{B,2(B)} = 1/X_{A,1(B)} = 1, X_{B,1(B)} = 1)$$
$$= (0.575)(0.319)(0.367) + (0.318)(0.322)(0.366)$$
$$+ (0.088)(0.566)(0.366) = 0.123 .$$

Using equations 10 and 11

$$\text{Prob.}(X_{A,2(B)} = 1/X_{A,1(B)} = 1, X_{B,1(B)} = 1)$$
$$= (0.575)(0.319)+(0.318)(0.322)+(0.088)(0.566) = 0.336$$

$$\text{Prob.}(X_{B,2(B)} = 1/X_{A,1(B)} = 1, X_{B,1(B)} = 1)$$
$$= (0.575)(0.367)+(0.318)(0.366)+(0.088)(0.366) = 0.360 .$$

To confirm the above result, the joint probability of $X_{A,2(B)}$ and $X_{B,2(B)}$ can be calculated, using independence, as

$$P(X_{A,2(B)} \cap X_{B,2(B)}) = P(X_{A,2(B)})P(X_{B,2(B)})$$
$$= (0.336)(0.360) = 0.121 .$$

c. Illustration of the computation of expected ferry travel times

Using the same numerical values as in previous subsection, expected travel times $(t_2^B - t_1^A)$ and $(t_2^B - t_1^B)$ of ferry are calculated to compare, in chapter III, against the values obtained from simulation runs. From subsection a.

$$t_2^B - t_1^A = \rho_A(X_{B,1(B)}, X_{A,1(B)}) = h_A + \tau + (\gamma_B + \beta_A)\Big[\min.\Big(C, X_{A,1(B)} + P\Big\{(1/a)[(\gamma_A + \beta_B)\{\min.(X_{B,1(B)}, C)\} + h_B + \tau]\Big\}\Big)\Big]$$

$$\rho_A(X_{B,1(B)}=1, X_{A,1(B)}=1) = 1.8+12+(0.08+0.13)\Big[\min.\Big(2, 1 + P\Big\{(1/25)[(0.08+0.14)\{\min.(1,2)\}+1.6+12]\Big\}\Big)\Big]$$

$$= 13.8+0.21[\min.\{2, 1+P(0.554)\}] .$$

Taking expectations of both sides:

$$E\{\rho_A(1,1)\} = 13.8+0.21E[\min.\{2,1+P(0.554)\}]$$

but

$$\min.\{2,1+P(0.554)\} = 1 \text{ if } P(0.554) = 0$$

$$= 2 \text{ if } P(0.554) \geq 1$$

since $E(x) = \sum_{\text{all } x} xf(x)$ and Poisson density function is given by $f(x) = e^{-\lambda}\lambda^x/x!$, it follows that

$$E[\min.\{2,1+P(0.554)\}] = (1)e^{-0.554}+(2)(1-e^{-0.554}) = 1.426 .$$

Therefore,

$$E\{t_2^B - t_1^A\} = E\{\rho_A(1,1)\} = 13.8+(0.21)(1.426) = 14.1 \text{ minutes} .$$

This is the expected ferry service time from side A to side B. Similarly,

$$t_2^B - t_1^B = \rho_B(X_{B,1(B)}, X_{A,1(B)}) = 2\tau + h_A + h_B + (\beta_B + \gamma_A)[\min.(X_{B,1(B)}, C)] + (\gamma_B + \beta_A)\Big[\min.\Big(C, X_{A,1(B)} + P\Big\{(1/a)[(\gamma_A + \beta_B)\{\min.(X_{B,1(B)}, C)\} + h_B + \tau]\Big\}\Big)\Big]$$

$$\rho_B(X_{B,1(B)}=1, X_{A,1(B)}=1) = 24+1.8+1.6+(0.14+0.08)[\min.(1,2)] + (0.08+0.13)[\min.\{2,1+P(0.554)\}]$$

$$= 27.62+0.21[\min.\{2,1+P(0.554)\}] .$$

Taking expectations of both sides:

$$E\{\rho_B(1,1)\} = 27.62+0.21E[\min.\{2,1+P(0.554)\}]$$

$$= 27.62+(0.21)(1.426)$$

$$E\{\rho_B(1,1)\} = E(t_2^B - t_1^B) = 27.92 \text{ minutes} .$$

III. RESULTS AND DISCUSSION

A. Results of Simulation Case Studies

In this section the results are presented, using the notation of chapter II, section A, for each case in the same order given on pages 32-33.

1. Case a

This is the case with input (i,i,i,i,i,i,i). Using intensity parameters of a = 25 minutes/car and b = 25 minutes/car, two simulations were run, using different random number sequences. These intensity parameters were chosen such that it would be possible to obtain many ($X_{A,i(B)} = 1$, $X_{B,i(B)} = 1$) states. A total of nine such states was obtained from the two runs combined. $X_{A,(i+1)(B)} = 1$ and $X_{B,(i+1)(B)} = 1$ occurred three times each. Jointly they occurred only once. Thus, computed values of Prob.($X_{A,2(B)} = 1/1,1$) = 0.336 and Prob.($X_{B,2(B)} = 1/1,1$) = 0.360 from section B of chapter II are comparable with simulation result of 0.333. Their joint probability of 0.123 agrees closely with the simulation result of 1/9 = 0.111 .

In the previous chapter $E\{t^B_{i+1} - t^B_i\} = E\{\rho_B(X_{B,i(B)} = 1, X_{A,i(B)} = 1)\}$ was calculated to be 27.92 minutes. The two simulation runs resulted with an average of 27.90 minutes. Similarly, $E\{t^B_{i+1} - t^A_i\} = E\{\rho_A(X_{B,i(B)} = 1, X_{A,i(B)} = 1)\}$ was calculated to be 14.1 minutes. This value also agrees very closely with the 14.08 minute average from the two simulation runs. Table 3 gives a summary of the above comparisons.

Table 3. Comparison of results obtained from simulation and mathematical analysis for case a.

Comparison	Mathematical analysis	Simulation
Prob.$(X_{A,2(B)} = 1/1,1)$	0.336	0.333
Prob.$(X_{B,2(B)} = 1/1,1)$	0.360	0.333
Prob.$(X_{A,2(B)} = 1,\ X_{B,2(B)} = 1/1,1)$	0.123	0.111
$E\{\rho_A(1,1)\} = E\{t_2^B - t_1^A\}$	14.10 minutes	14.08 minutes
$E\{\rho_B(1,1)\} = E\{t_2^B - t_1^B\}$	27.92 minutes	27.90 minutes

A twelve-hour simulation period was used to investigate the behavior of the system and the stability of the $X_{A,i}$ and $X_{B,i}$ distributions. Defining:

d_i^s = cumulative distribution function for the queue sizes on shore at the 1 st, 2 nd,..., i th unloadings at side s, i = 1,2,...,m

$\delta_{i'}^s$ = cumulative distribution function for the queue sizes on shore at the m th , (m-1) st,...,(m-i'+1) st unloadings at side s, i' = 1,2,...,m.

Then max.$|d_i^B - d_{i+1}^B|$ and max.$|\delta_{i'}^B - \delta_{i'+1}^B|$ values were calculated. Results of the two simulation runs for side B were combined in a pairwise fashion starting with i = i' = 14. These calculations were computed for both docking directions because it was expected that the distributions were more and more "unlike" in the direction of i' which may be an indication of less stability. Table 4 shows an example of max.$|d_i^B - d_{i+1}^B|$ calculations. Values in Table 4 are obtained from Table 5 which shows the number of occurrences for each queue size at side B. Figure 13 gives a plot of the absolute

Table 4. Sample computation of $\max.|d_i^B - d_{i+1}^B|$

$X_{B,i(B)}$	d_{14}	d_{15}	d_{16}	$\lvert d_{14}-d_{15}\rvert$	$\lvert d_{15}-d_{16}\rvert$
0	9/28	10/30	11/32	0.013	0.009
1	20/28	21/30	22/32	0.015	0.013
2	25/28	27/30	29/32	0.006	0.006
3	27/28	29/30	31/32	0.002	0.003
4	28/28	30/30	32/32	0.000	0.000
			$\max.\lvert d_i^B - d_{i+1}^B\rvert$	0.015	0.013

maximum differences of cumulative queue size distributions versus ferry docking numbers. The lower curve of Figure 13 indicates some stability in the direction it is expected and it is settling down slightly, but the upper curve shows an even stronger tendency to stabilize in the opposite direction (with decreasing time). All of this suggests that stability does indeed set in very early. This was further put to the test, for the data of side B, by the following statistical test of homogeneity. Dividing the data of Table 5 into three non-overlapping groups of equal size (skipping the first 2 dockings) and proceeding with a chi-square test, the observed and expected numbers are determined as follows:

Table 5. Number of occurrences for each queue size at side B - two simulation runs combined

Docking no. (i) / $X_{B,i(B)}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
0	1	1	0	0	0	0	1	1	1	1	1	0	1	1	1	1	0	0	0	1	1	1	1	0	0	0
1	1	0	1	1	2	1	0	1	1	1	0	1	0	1	0	0	1	1	1	0	0	0	0	0	0	0
2	0	0	0	1	0	1	1	0	0	0	0	1	1	0	1	1	0	0	0	1	0	0	1	1	2	2
3	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0
4	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0

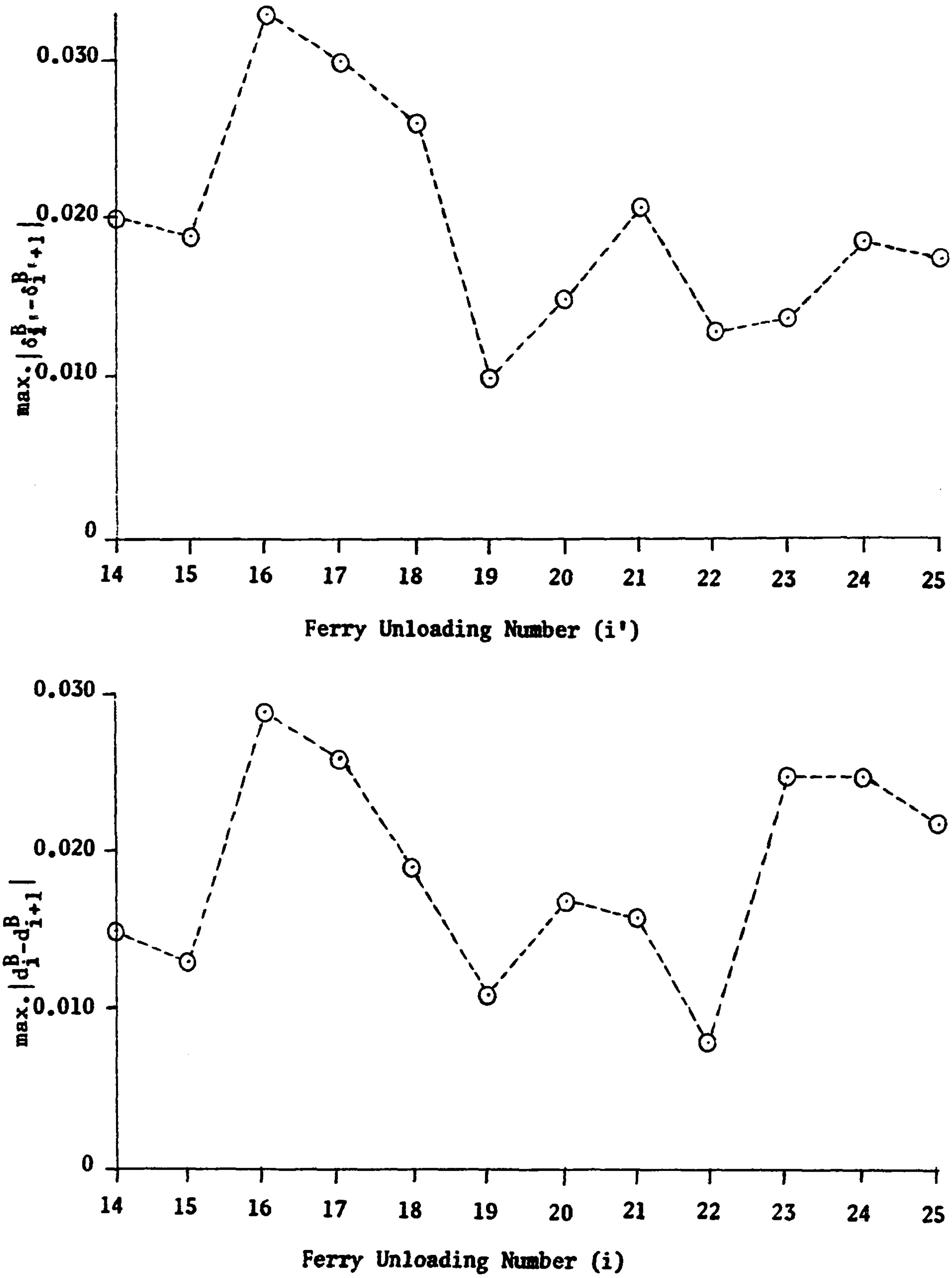


Figure 13. Transient behavior of simulation model under case a.

Observed

Group / $X_{B,i(B)}$	I	II	III	Total
0	4	5	4	13
1	8	4	1	13
2	3	4	7	14
3to6	1	3	4	8
Total	16	16	16	48

Expected

Group / $X_{B,i(B)}$	I	II	III	Total
0	(13)(16)/48 = 4.33	(13)(16)/48 = 4.33	(13)(16)/48 = 4.33	13
1	(13)(16)/48 = 4.33	(13)(16)/48 = 4.33	(13)(16)/48 = 4.33	13
2	(14)(16)/48 = 4.67	(14)(16)/48 = 4.67	(14)(16)/48 = 4.67	14
3to6	(8)(16)/48 = 2.67	(8)(16)/48 = 2.67	(8)(16)/48 = 2.67	8
Total	16	16	16	

$$\chi^2 = \sum_{i=1}^{12} (O_i - E_i)^2/E_i = (4-4.33)^2/4.33 + (8-4.33)^2/4.33$$

$$+ (3-4.67)^2/4.67 + (5-4.33)^2/4.33 + (4-4.33)^2/4.33$$

$$+ (4-4.67)^2/4.67 + (4-4.33)^2/4.33 + (1-4.33)^2/4.33$$

$$+ (7-4.67)^2/4.67 + (1-2.67)^2/2.67 + (3-2.67)^2/2.67$$

$$+ (4-2.67)^2/2.67$$

$$= 9.452 .$$

But $\chi^2_{1-\alpha,(1-r)(1-c)} = \chi^2_{0.95,6} = 12.59 > 9.452$.

Therefore, there is no reason to reject the null hypothesis that the three distributions are coincident, and they come from the same parent population. Observed values were what one expects them to be when the queue size distribution at side B is generated from a single distribution. A similar procedure may also be used for side A.

The reader should note that strictly speaking, an "ergodic" assumption was made here about the $X_{B,i(B)}$ process, namely, that, near stability, observations taken at successive times have approximately the same probabilistic structure as repeated independent observations at a fixed time.

For the same "null" case, the effect of imbalance on the incoming traffic streams of both sides was investigated and contours of overall service time and median waiting time of cars were derived using various combinations of intensity parameters, as shown in Figure 14. Because of

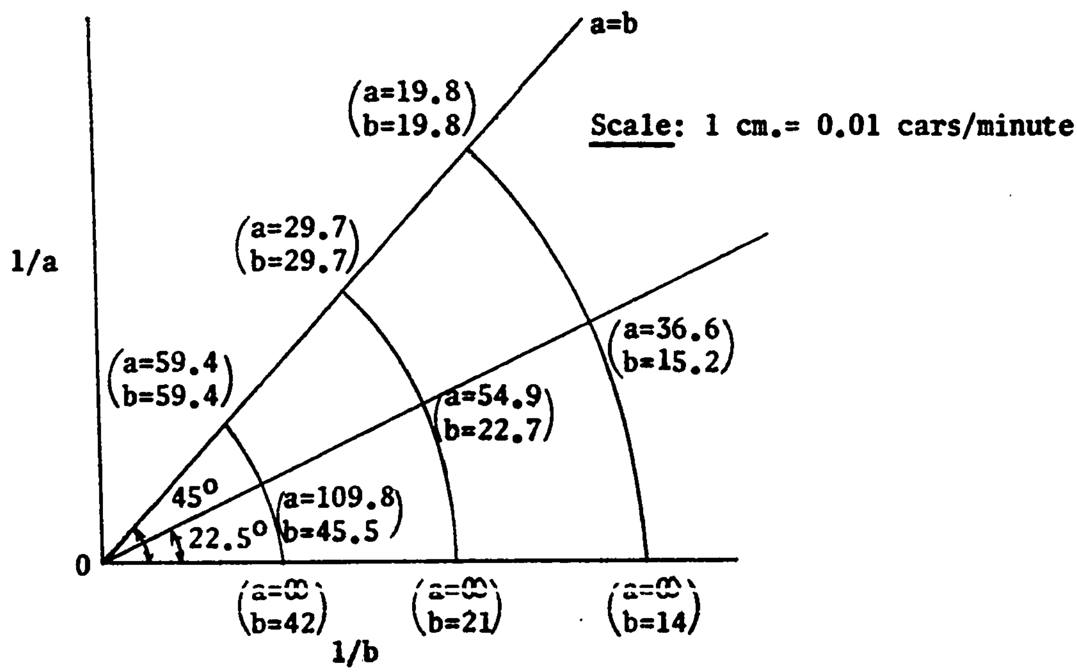


Figure 14. Combinations of intensity parameters, in minutes per car, used in deriving service contour lines for 2-car capacity ferry

its symmetrical nature, only the lower half of the 45^{o} line is used. Figures 15 and 16 show the simulation results of the overall service and median-car waiting times, in minutes, corresponding to the intensities shown in Figure 14. The overall service times of the cars in the system were calculated by finding the weighted average of the car-waiting times on both shores and the car-service times which includes the loading, transit and unloading times of the cars by the ferry for both crossings. Median-car waiting times were calculated combining both sides of the channel.

The resulting contour lines in minutes, as shown in Figures 17 and 18, were found by interpolation of the values obtained from Figures 15 and 16. These contours can be thought of as performance indices of the system. They indicated that ferries were more efficient if demand was the same on both sides. To see this one could consider line AB in Figure 17. This is the line for which $\left(\frac{1}{a} + \frac{1}{b}\right)$ is constant, that is, the total traffic processed by the system on the average is fixed. The overall residence time of the cars in the system is 30 minutes when inter-arrival times are equal, a = b = 59.4 minutes/car, on both sides. It takes approximately 33 minutes if no cars arrive on shore A, that is, when $a = \infty$, b = 29.7 minutes/car, determined by interpolation.

The same study was repeated for a ferry with a 42-car capacity. Various intensities used in this investigation are given in Figure 19. Figures 20 and 21 show the resulting overall service and the median-car waiting times, in minutes, corresponding to these intensities used.

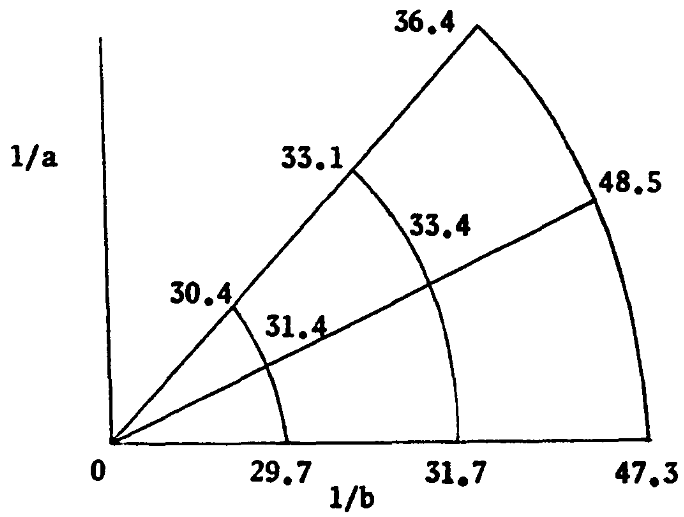


Figure 15. Overall service times in minutes corresponding to intensities shown in Figure 14.

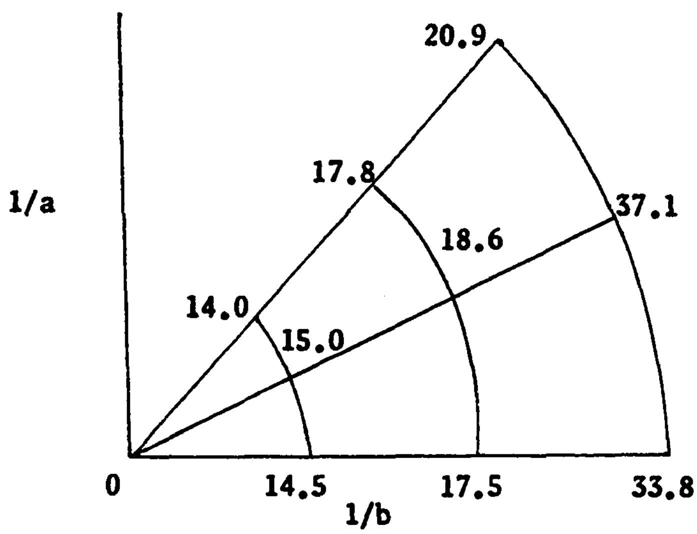


Figure 16. Median-car waiting times in minutes corresponding to intensities shown in Figure 14.

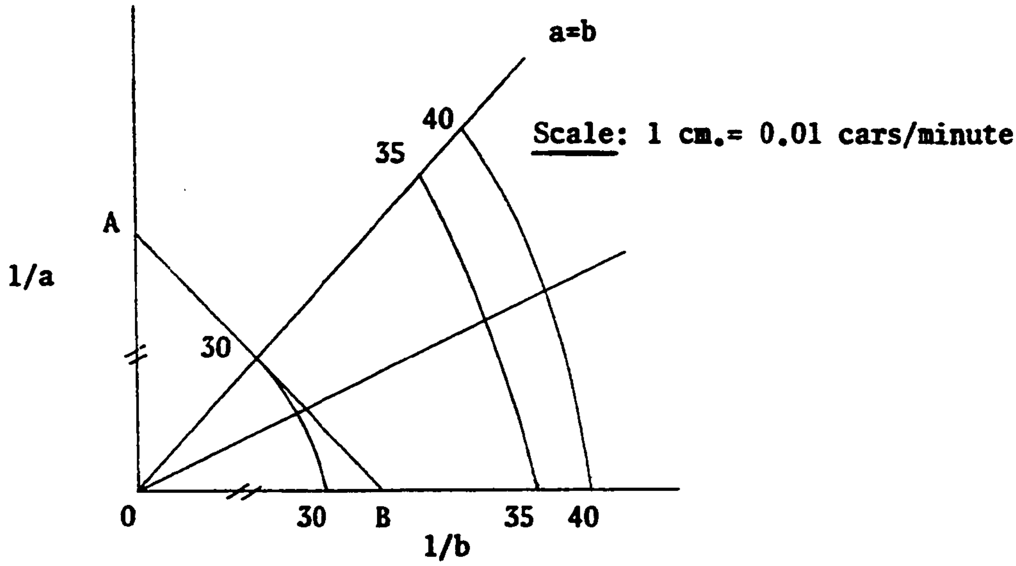


Figure 17. Overall service time contours of cars in the system, in minutes. Ferry with 2-car capacity

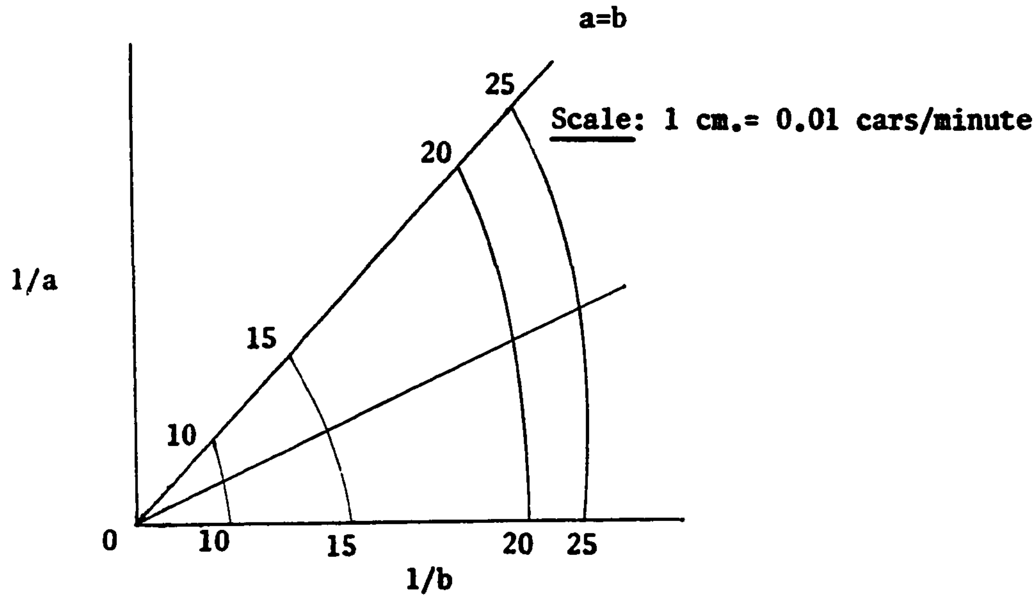


Figure 18. Median-car waiting time contours, in minutes, both sides combined. Ferry with 2-car capacity

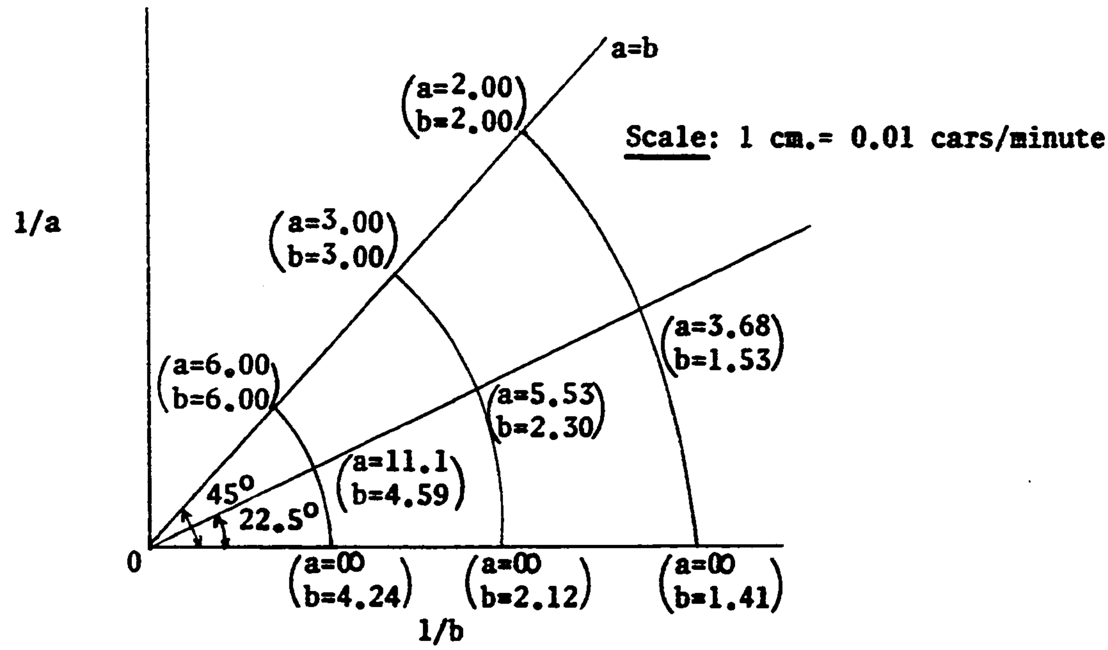


Figure 19. Combinations of intensity parameters, in minutes per car, used in driving service contour lines for 42-car capacity ferry

Contours obtained for overall service times of cars in the system and median-car waiting times for both shores combined are shown in Figures 22 and 23 respectively.

By changing intensity parameters a and b, it is also possible to establish an explosion region bounded by an explosion curve which may be defined as that line after which the average waiting time per car on either side continually increases as time proceeds. Actual a and b parameters on the explosion line were not determined, however, due to the excessive computer costs involved.

2. Cases b and c

These are the cases corresponding to inputs (i,i,iv,i,i,i,i,) and (i,ii,ii,i,i,i,i) respectively. Equal intensity parameters for both sides

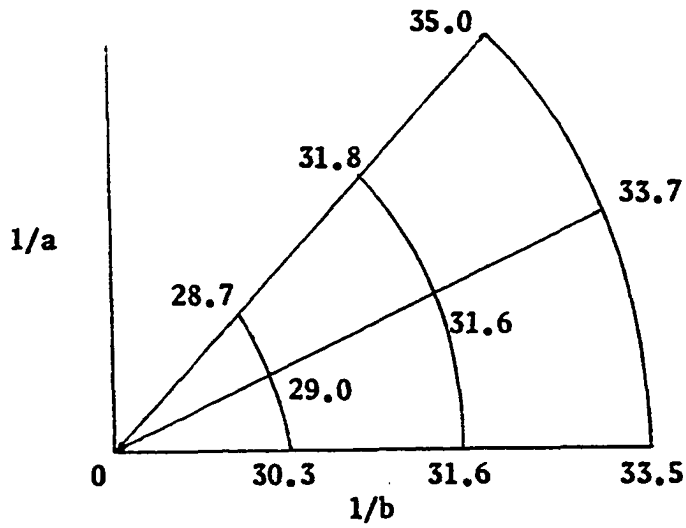


Figure 20. Overall service times, in minutes, corresponding to intensities shown in Figure 19.

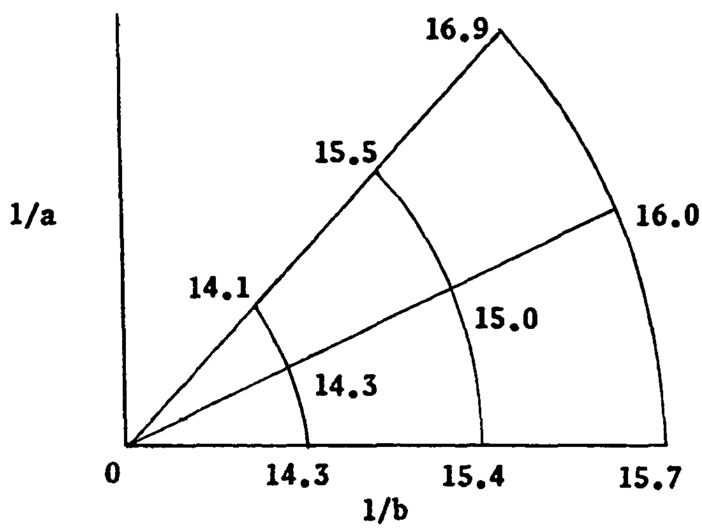


Figure 21. Median-car waiting times, in minutes, corresponding to intensities shown in Figure 19.

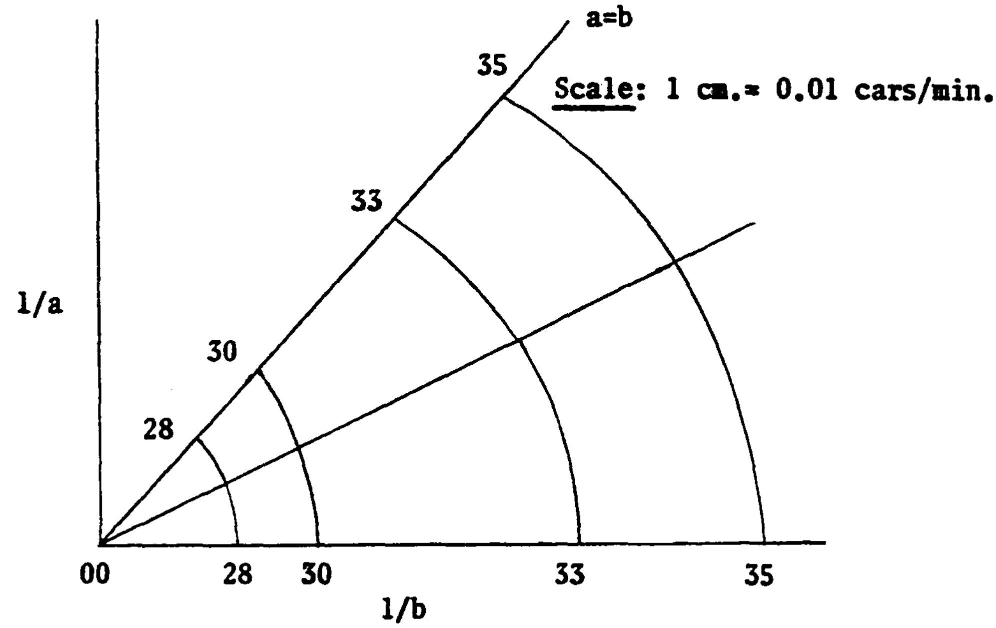


Figure 22. Overall service time contours of cars in the system, in minutes. Ferry with 42-car capacity

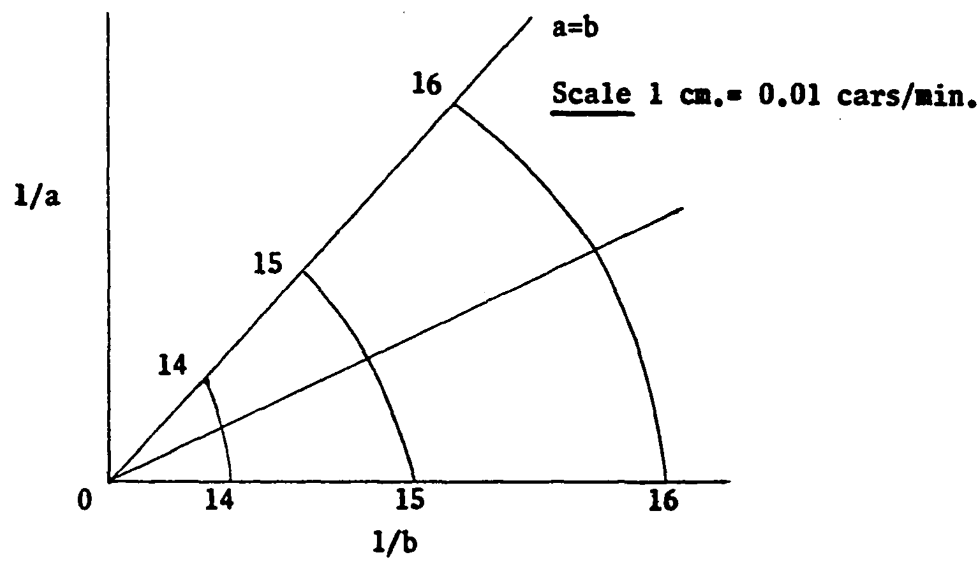


Figure 23. Median-car waiting time contours, in minutes, both sides combined. Ferry with 42-car capacity

in the non-explosive range were used in each case to measure the average waiting times per car on both shores combined. Results shown in Figures 24 and 25 indicate that, one 84-capacity ferry is most competitive for large interarrival times. In the parameter range used, the two 42-size ferry case always results in shorter average waiting time per car and thus is more advantageous than the one 84-size ferry. It may also be reasoned that two 42-size ferries are more flexible. They can always act as one 84-size ferry, once one has caught up with the other, by docking at the same time, and thus they have a distinct advantage over an 84-size ferry. Therefore, in the light of above argument, the results were not too surprising.

3. Cases d and e

These are the cases corresponding to inputs (iii,iv,iii,iii,ii,i,i) and (i,iv,iii,iii,ii,i,i) respectively. Using non-stationary "half-hour" Weibull and stationary exponential inputs, the real-life situation was simulated over a twelve-hour period. The results obtained were compared against the actual data by considering incoming traffic streams and system attributes separately.

a. Comparison of incoming traffic streams   Incoming traffic stream statistics of the twelve-hour study period for the real-life situation, Weibull and exponential models are given in Tables 6, 7 and 8 respectively. Means and standard deviations of these statistics are also shown at the bottom of each table. From these tables it is seen that the mean of the arrival rates and the mean of standard deviations about mean

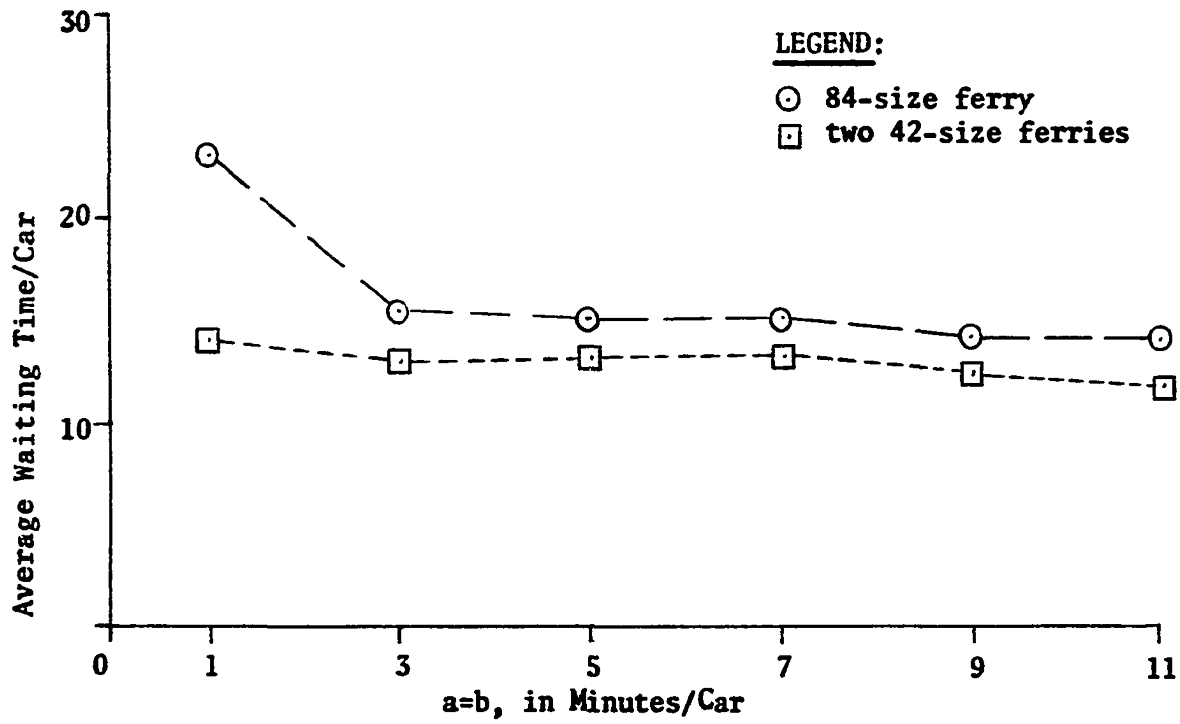


Figure 24. Average waiting time per car. 84-car size ferry versus two 42-size ferries. Both sides combined

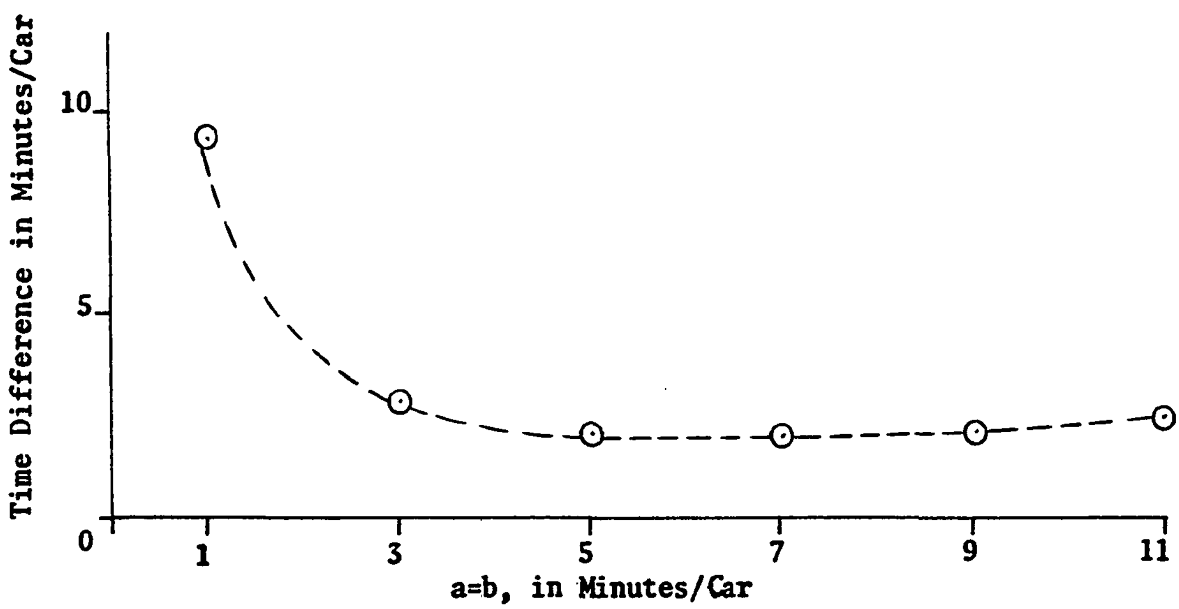


Figure 25. Difference in average waiting times per car. 84-car size ferry versus two 42-size ferries. Both sides combined

Table 6. Incoming traffic stream statistics, real-life situation

Time	SIDE A Number of arrivals	SIDE A Mean (arrivals per minute)	SIDE A Standard deviation about mean	SIDE B Number of arrivals	SIDE B Mean (arrivals per minute)	SIDE B Standard deviation about mean
7-7:30 am	89	2.967	1.416	49	1.667	1.184
7:30-8	89	2.933	1.257	35	1.167	1.234
8-8:30	105	3.500	2.300	45	1.500	1.570
8:30-9	97	3.200	2.469	44	1.467	1.547
9-9:30	90	3.000	2.243	72	2.467	2.161
9:30-10	120	4.000	2.406	81	2.933	2.049
10-10:30	106	3.500	2.029	109	3.833	2.102
10:30-11	141	4.700	2.451	133	4.667	4.212
11-11:30	172	5.767	4.174	72	2.433	2.028
11:30-12	200	6.700	3.975	90	3.067	2.625
12-12:30 pm	101	3.367	3.232	111	3.667	2.630
12:30-1	132	4.433	3.962	127	4.300	1.896
1-1:30	156	5.200	3.880	125	4.200	3.076
1:30-2	171	5.700	3.860	157	5.200	3.209
2-2:30	114	3.867	3.070	120	4.033	2.684
2:30-3	131	4.367	2.008	133	4.433	2.762
3-3:30	124	4.167	2.574	117	3.900	1.881
3:30-4	163	5.367	4.172	144	4.767	3.420
4-4:30	153	5.167	4.026	131	4.367	2.999
4:30-5	101	3.400	2.513	158	5.200	3.377
5-5:30	144	4.833	3.404	105	3.500	2.529
5:30-6	114	3.800	2.696	108	3.600	2.357
6-6:30	158	5.267	3.016	132	4.400	3.222
6:30-7	113	3.967	2.525	145	4.800	2.917
Mean	129	4.300	2.902	107	3.565	2.486
Standard deviation		1.023	0.869		1.218	0.752

were lower for the Weibull than the exponential model and the real-life data for both sides; exponential model appears to give a closer approximation to reality.

Figures 26 and 27 show the number of arrivals per half-hour against

Table 7. Incoming traffic stream statistics, Weibull input

Time	SIDE A Number of arrivals	SIDE A Mean (arrivals per minute)	SIDE A Standard deviation about mean	SIDE B Number of arrivals	SIDE B Mean (arrivals per minute)	SIDE B Standard deviation about mean
7-7:30 am	87	2.900	0.959	23	0.767	0.971
7:30-8	90	3.000	0.982	50	1.667	0.994
8-8:30	91	3.033	1.217	49	1.633	1.217
8:30-9	96	3.200	1.270	56	1.867	1.224
9-9:30	98	3.267	1.337	68	2.267	1.284
9:30-10	89	2.967	1.519	79	2.633	1.376
10-10:30	97	3.233	1.277	70	2.333	1.604
10:30-11	98	3.267	1.412	88	2.933	1.284
11-11:30	98	3.267	1.142	99	3.300	1.488
11:30-12	93	3.100	1.604	92	3.067	1.638
12-12:30 pm	116	3.867	1.634	104	3.467	1.166
12:30-1	116	3.867	1.525	94	3.133	1.591
1-1:30	115	3.833	1.743	111	3.700	1.511
1:30-2	127	4.233	1.755	109	3.633	1.449
2-2:30	122	4.067	2.116	126	4.200	1.517
2:30-3	131	4.367	1.351	89	2.967	1.629
3-3:30	120	4.000	1.438	122	4.067	1.552
3:30-4	117	3.900	1.668	110	3.667	1.787
4-4:30	109	3.633	1.938	98	3.267	1.799
4:30-5	118	3.933	2.211	120	4.000	1.339
5-5:30	151	5.033	1.670	106	3.533	1.676
5:30-6	131	4.367	2.108	115	3.833	1.662
6-6:30	141	4.700	1.878	108	3.600	2.077
6:30-7	131	4.367	1.771	89	2.967	2.157
		3.716	1.563	Mean	3.020	1.499
		0.611	0.343	Standard deviation	0.878	0.294

the time for Weibull and exponential models for sides A and B respectively. Comparing these figures with Figures 2 and 3 for the real-life data it may be observed that the number of "up" and "down" runs, eight for side A of the real-life data, compared favorably with seven for both the Weibull and

Table 8. Incoming traffic stream statistics, exponential input

Time	SIDE A Number of arrivals	SIDE A Mean (arrivals per minute)	SIDE A Standard deviation about mean	SIDE B Number of arrivals	SIDE B Mean (arrivals per minute)	SIDE B Standard deviation about mean
7-7:30 am	131	4.367	1.810	116	3.867	2.240
7:30-8	149	4.967	2.553	99	3.300	1.601
8-8:30	127	4.233	2.528	112	3.733	2.033
8:30-9	123	4.100	2.249	108	3.600	1.694
9-9:30	122	4.067	1.799	102	3.400	2.078
9:30-10	130	4.333	2.155	90	3.000	1.145
10-10:30	159	5.300	1.932	98	3.267	1.507
10:30-11	138	4.600	2.358	110	3.667	1.688
11-11:30	121	4.033	2.008	106	3.533	1.961
11:30-12	125	4.167	2.001	128	4.267	1.874
12-12:30 pm	135	4.500	1.978	111	3.700	2.261
12:30-1	148	4.933	2.243	107	3.567	1.736
1-1:30	136	4.533	1.814	96	3.200	2.041
1:30-2	156	5.200	2.709	108	3.600	2.061
2-2:30	125	4.167	1.895	104	3.467	1.613
2:30-3	120	4.000	1.597	110	3.667	1.845
3-3:30	140	4.667	2.758	117	3.900	1.788
3:30-4	148	4.933	2.599	108	3.600	1.589
4-4:30	124	4.133	2.097	108	3.600	2.044
4:30-5	143	4.767	2.459	121	4.033	1.921
5-5:30	134	4.467	1.814	114	3.800	1.846
5:30-6	116	3.867	2.113	114	3.800	2.188
6-6:30	125	4.167	2.260	117	3.900	2.524
6:30-7	124	4.133	2.030	97	3.233	1.654
		4.443	2.156	Mean	3.612	1.872
		0.400	0.319	Standard deviation	0.291	0.294

exponential simulation results. Similarly, the number of "up" and "down" runs, again eight for side B of the real-life data, were comparable with nine of the Weibull and six of the exponential input simulators.

Runs above and below the mean may also be compared. The number of

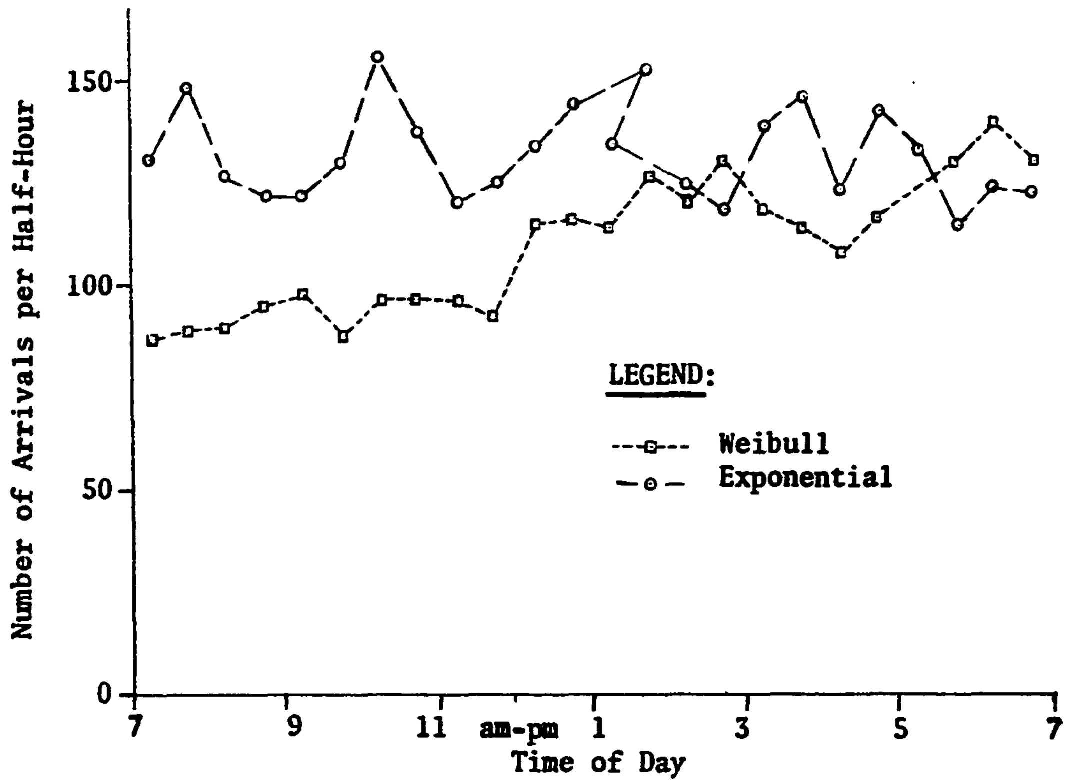


Figure 26. Number of arrivals per half-hour for side A, simulation with Weibull and exponential inputs

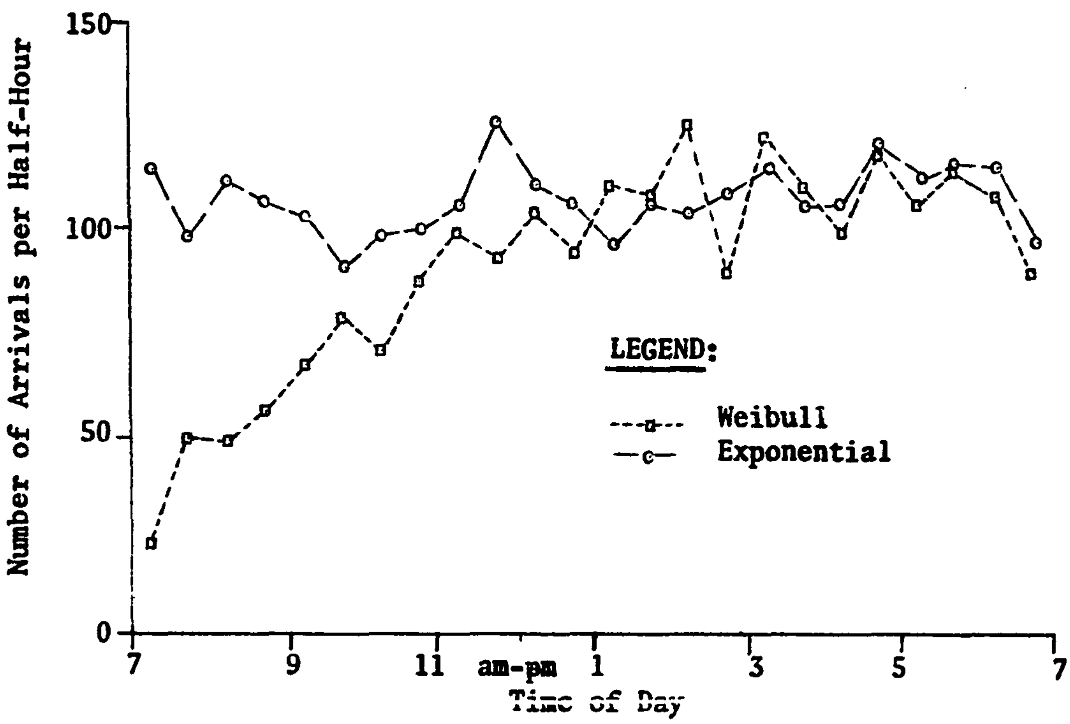


Figure 27. Number of arrivals per half-hour for side B, simulation with Weibull and exponential inputs

"low and "high" runs for side A of the real-life data were eight and seven respectively. For Weibull model, these runs were both two, whereas for exponential they were five. Similarly, the number of "low" and "high" runs for side B of the real-life data was three each. For Weibull these runs were three and two respectively, whereas for exponential they were both six. It may be concluded that, in general, Weibull simulated the trend of the actual data better than the exponential model.

Figures 28 and 29 compare the cumulative number of arrivals for sides A and B respectively. A maximum difference of 444 arrivals between the real-life data and Weibull input occurred at time period 4-4:30 pm for side A. For side B the maximum difference of 368 arrivals occurred at time period 6:30-7 pm. Similarly, a maximum difference of 245 arrivals occurred at 10-10:30 am between real-life data and exponential input for side A and maximum difference of 339 arrivals occurred at 11:30-12 am and 12-12:30 pm for side B. On these cumulative statistics the exponential model performed the best.

The sum of absolute differences between mean arrival rates for each half-hour of real-life data and Weibull model was 19.24 and 19.07 arrivals per minute for sides A and B respectively. Similarly, absolute mean arrival rate differences between real-life data and exponential model were 21.14 and 23.93 arrivals per minute for sides A and B respectively. This non-cumulative comparison definitely was in favor of the Weibull model as the better of the two input simulators.

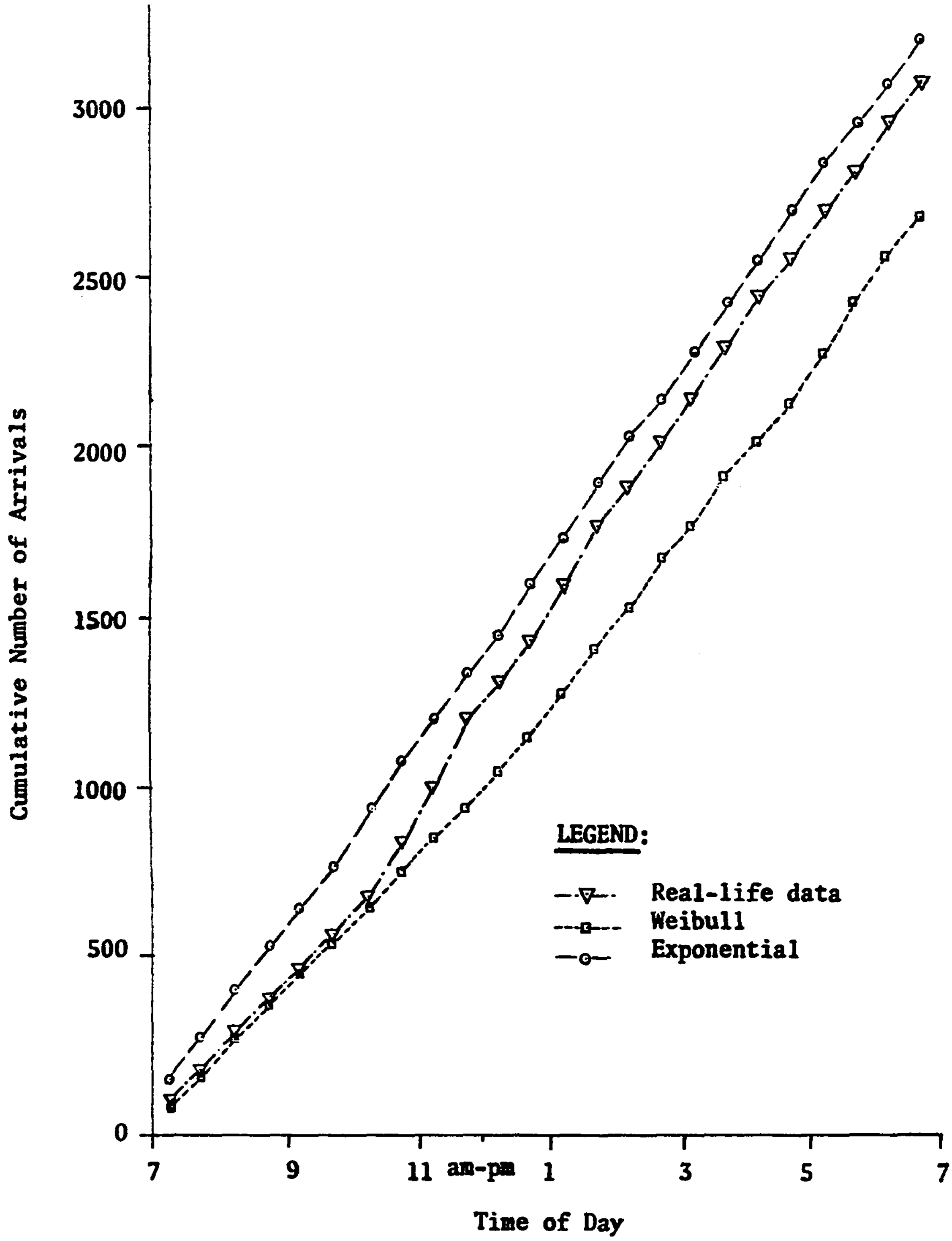


Figure 28. Cumulative number of arrivals per half-hour at side A

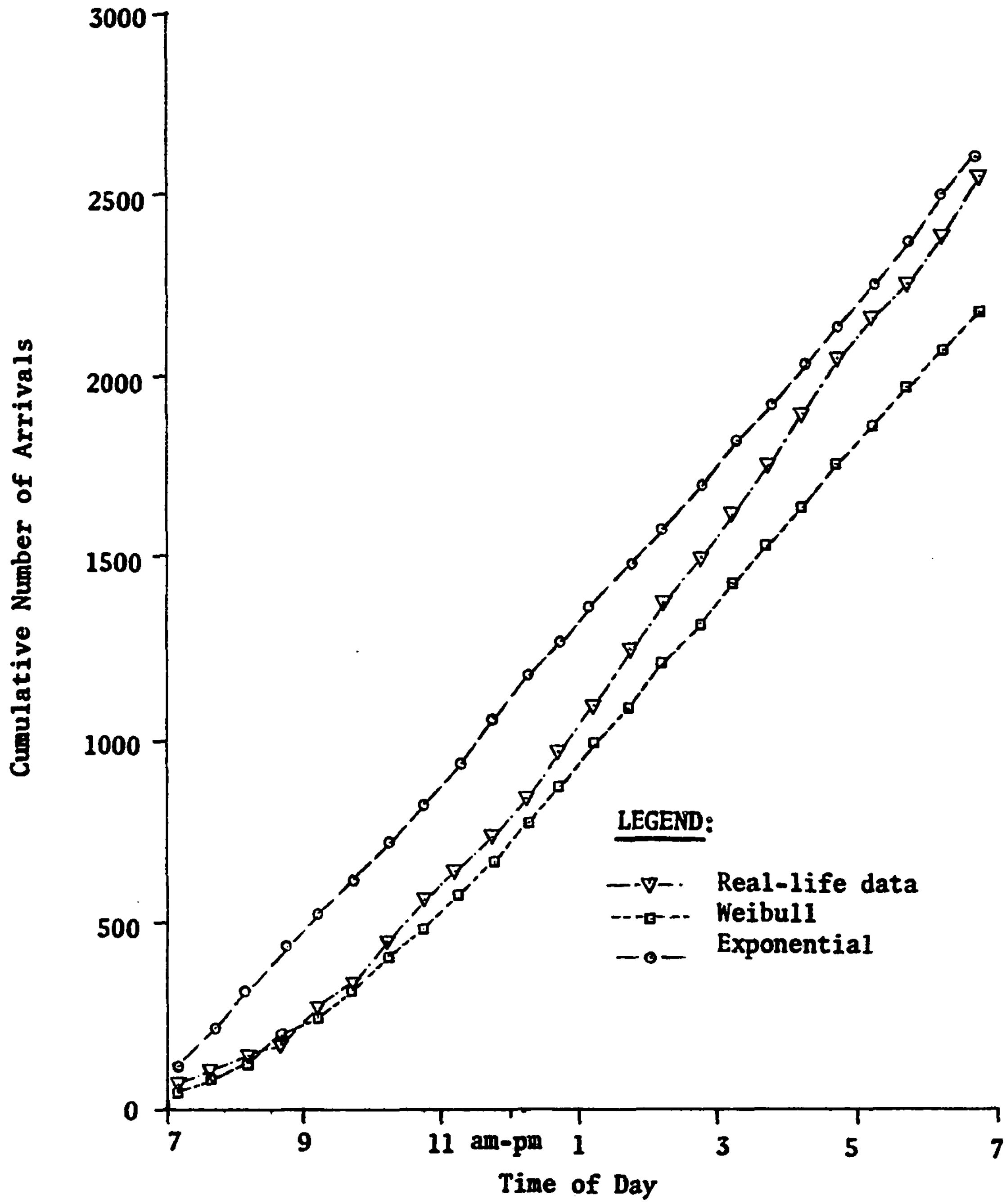


Figure 29. Cumulative number of arrivals per half-hour at side B

b. System comparison Table 9 shows comparison of various system attributes combining both sides A and B whenever possible.

Table 9. Comparison of system attributes

Input	Average time between dockings (minutes)	Standard deviation of interdocking times (minutes)	Average ferry service time	% Time ferries carried cars less than half capacity	Total number of arrivals during twelve-hour period side A	side B
Real-life	12.56	6.36	23.19	0	3,084	2,543
Weibull	10.06	11.17	20.56	28	2,682	2,175
Exponential	10.59	9.16	21.23	9	3,199	2,601

Cumulative number of arrivals carried by all ferries from side A to B and vice versa are shown in Figures 30 and 31 respectively. Maximum differences of 481 and 767 cars occurred between real-life data and Weibull model. Similarly, the maximum differences between real-life data and exponential model were 83 and 535 cars. As Table 9 indicates, the exponential model seems to be performing better in the long-run.

This portion of the research fell short of meeting all of the multiple objectives originally set because of the complexities involved in the fitting process. A further discussion seems to be appropriate at this point to account for the reasons why the non-stationary Weibull model did not perform better than the stationary exponential model as an input to the simulation of incoming traffic streams.

Looking at Figure 28 as an example, it is seen that the Weibull

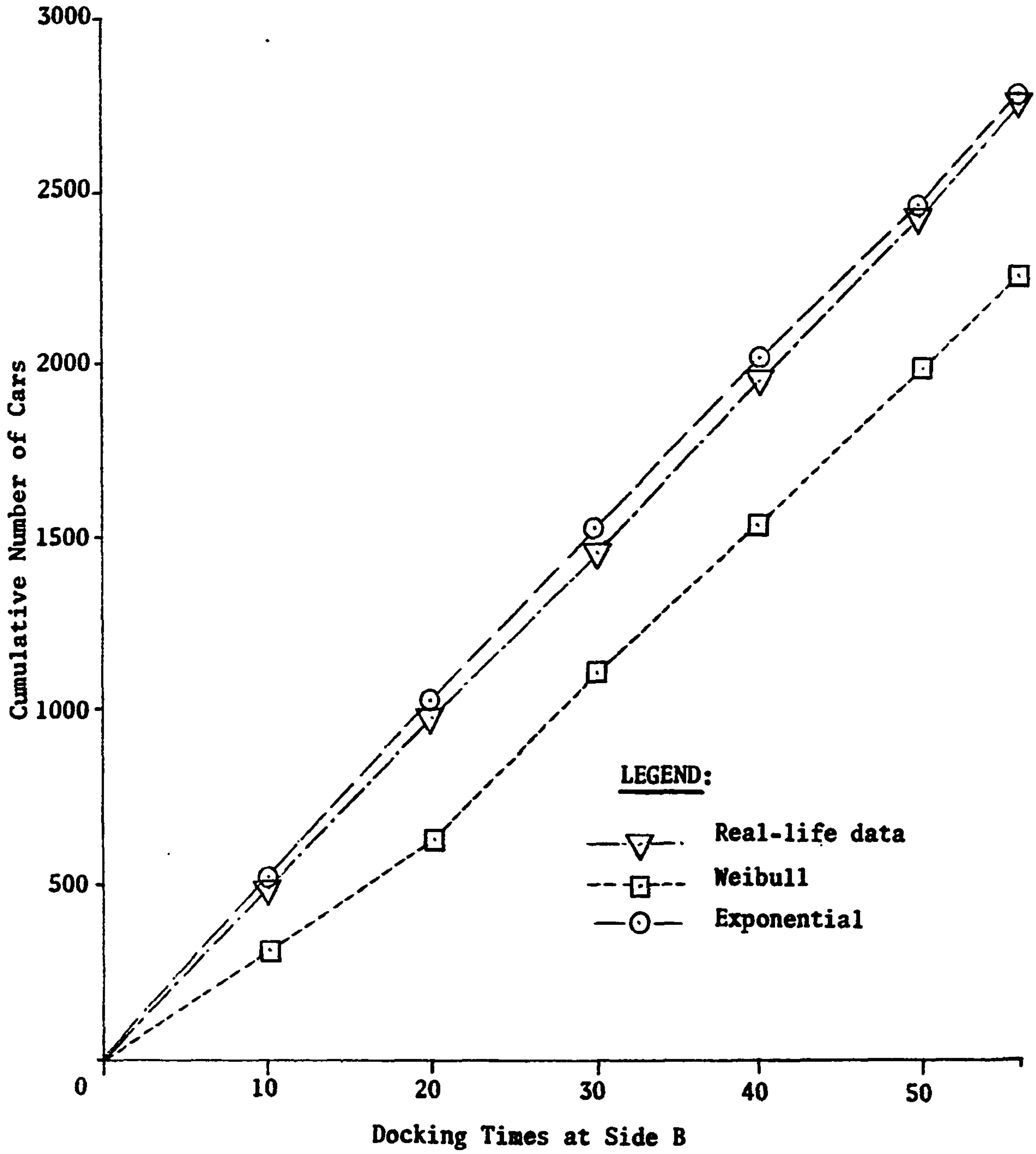


Figure 30. Cumulative number of cars carried by all ferries from side A to side B

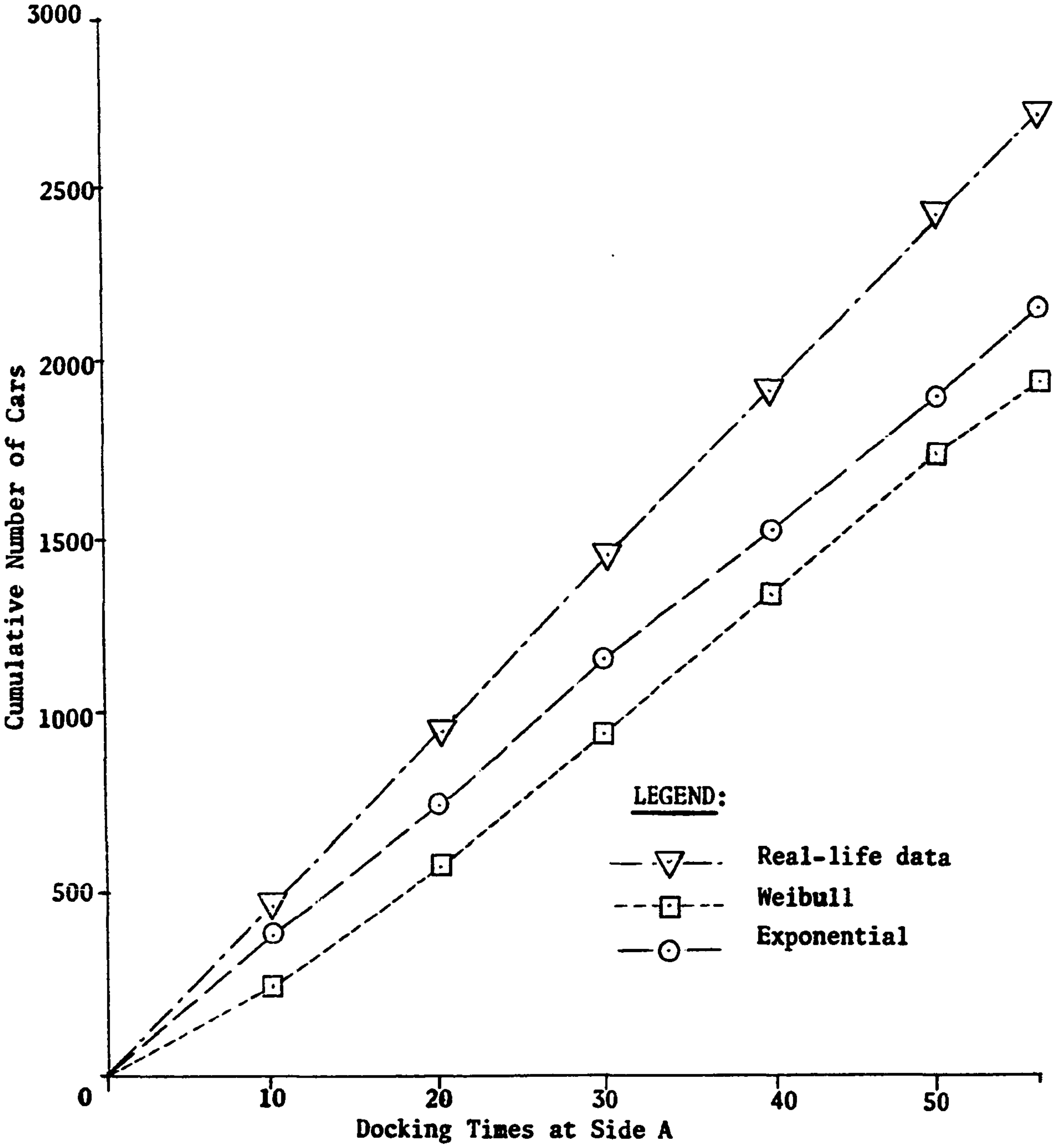


Figure 31. Cumulative number of cars carried by all ferries from side B to side A

simulation starts out well, and follows the real-life data rather closely until 10:45 am. Somehow the gap widens further between 10:45-11:45 am and stays basically the same until the end of the simulation period. Without the discrepancy that occurred between 10:45 and 11:45 am, the cumulative number of arrivals from the Weibull model would have kept up with the real-life data since the slopes of both curves after 11:45 am stay about the same.

The gap between Weibull output and the actual data may be attributed to the following reasons:

1. Original data was taken in terms of number of arrivals per minute. In converting these data to the interarrival times certain assumptions were made, as seen in Appendix B; this may have caused a loss of information. Taking the data in terms of interarrival times would have been more appropriate and should have been done originally.
2. In smoothing Weibull parameters, attempts were made to find a function for each parameter that gave the best fit. In spite of this, low coefficients of determinations were obtained (e.g. $R^2 = 0.18$) for location and scale parameters for side A and shape parameter for side B.
3. Due to smoothing, the "hump" in Figure 2, in case of side A, was completely missed, which may have also accounted for the gap occurring just before noon that caused the Weibull to lag behind the actual data.
4. Emphasis was placed in this research on simulating the "shape" of

Weibull distributions for each half-hour period determined by the three parameters, whereas no attention was paid to the half-hour Weibull means. However, means seem to have made the difference in fitting the data as far as can be deduced from the performance of the exponential model in Figures 28 and 29.

In the light of previous discussion, it is concluded that simulation of means at certain time intervals play an important role. In that respect, the exponential distribution, which doesn't keep track of the details of the arrival data but duplicates the overall mean, gives a better approximation of actual conditions. Use of an exponential distribution based on half-hour means rather than the overall mean interarrival times may give even better results and should be explored as a possible future research area.

In the short-run or locally the Weibull distribution, because of its non-stationary nature, may give the better fit for simulating the real-life data. The Weibull inputs might be further improved by paying more attention to spikes and dips occurring in arrival data and attempting to preserve half-hour mean arrival rates; this should be investigated further.

IV. SUMMARY AND CONCLUSIONS

This study considered the problem of modeling a shuttle transportation system. The thrust of the research was threefold. The first objective was to consider alternative ways of modeling traffic streams approaching the shuttle system. A second objective was to develop mathematical and simulation models encompassing various parameters to help explain the behavior of shuttle systems. The final objective of this research was to conduct sensitivity studies to observe how such a system responds to changes in model parameters.

Review of the literature, although limited in view of the vast amount of published material in the field of traffic flow theory and simulation of traffic and transportation networks, indicated that only a few prior investigations had considered the area of multi-shuttle systems, but, as a rule, without substantial attention to realistic detail.

As an example of the multi-shuttle system, the ferry system in operation at the Istanbul Bosphorus, Turkey was chosen. Analysis of the arrival data indicated non-stationarity, and a certain non-stationary Weibull interarrival process was fitted initially. Factors such as flexibility and ease of interpretation also added to the decision to proceed with a Weibull analysis. Using one-hour overlapping intervals, average interarrival times and the three Weibull parameters were calculated for successive half-hour periods. A stationary exponential model also was fitted to the incoming traffic streams to provide a bench mark for the non-stationary Weibull model.

Poisson-exponential mathematical models for single and two shuttle systems were formulated as interdependent queueing systems. In addition, the vector (X_A, X_B) of cars waiting at shore A and B respectively at the end of an unloading at one of the shores was shown to form a bivariate Markov chain, leading to the possibility of computing a long-run probability distribution for (X_A, X_B). A transition probability was calculated and expected ferry travel times for a specific case were approximated as an illustration.

Loading and unloading times of the ferries were regressed against the number of cars loaded and unloaded, for each side and ferry individually. Resulting regression coefficients, plus the ferry transit and constant times which were determined from the actual data, were used in the simulation and mathematical models.

A simulation model was developed using GPSS language which is flexible enough to incorporate most parameter changes. Using the simulation model, the transient and stationary behavior of the system was examined under various inputs and contraints. Effect of imbalance of the incoming traffic streams of both sides of the channel was investigated using various combinations of intensity parameters. Contour lines of the overall service and median-car waiting times were derived to determine the efficiency of the system.

Using Weibull and exponential inputs, the real-life situation was simulated over a twelve-hour period. Incoming traffic streams and system attributes were compared against actual data. It was found that, in general, cumulative comparisons were in favor of the exponential model which duplicated the overall mean arrival rates on both shores. However, in the short-run or locally, the Weibull distribution, due to its non-stationary nature, gave a better fit for simulating the real-life data. The Weibull inputs might be further improved by preserving the local mean arrival rates; this should be investigated further. Thus, the results indicated that the modeling approach should be modified according to the length of the simulation period under consideration, or, more generally, according to the specific objectives of a study.

From a broader perspective, the simulation and mathematical models developed herein are but a modest beginning in the application of systems analysis to multi-shuttle system improvement projects.

V. BIBLIOGRAPHY

1. Andrews, F. C. A statistical theory of traffic flow on highways.-I. Steady-state flow in low-density limit. Transportation Research 4, No. 4:359-366. 1970.

2. Andrews, F. C. A statistical theory of traffic flow on highways.-II. Three-car interactions and the onset of queueing. Transportation Research 4, No. 4:367-377. 1970.

3. Blum, A. M. A general purpose digital traffic simulator. Simulation 14, No. 1:9-25. 1970.

4. Brant, A. E., Jr., and P. J. McAward, Jr. Evaluation of airfield performance by simulation. Preprint No. 1486. Joint ASCE-ASME Transportation Engineering Meeting, Seattle, July 26-30. 1971.

5. Buckley, D. J. A Semi-Poisson model of traffic flow. Transportation Science 2, No. 2:107-133. 1968.

6. Carrol, J. L., and M. S. Bronzini. Simulation of waterway transport systems. American Society of Civil Engineers Proceedings 97, TE 3:527-539. August 1971.

7. Cassel, A., and M. S. Janoff. A simulation model of a two-lane rural road. Highway Research Record, No. 257:1-13. 1968.

8. Clenahan M. C., and H. J. Simkowitz. The effect of short cars on flow and speed in downtown traffic:A simulation model and some results. Transportation Science 3, No. 2:126-139. May 1969.

9. Dawson, R. F., and L. A. Chimini. The hyperlang probability distribution-A generalized traffic headway model. National Research Council, Highway Research Board, No. 230:1-4. 1968.

10. Dickey, J. W., and S. P. Hunter. Grouping of travel time distributions. Transportation Research 4, No. 1:93-102. 1970.

11. Dietrich, G., and H. Wagner. Traffic simulation and its application in telephoney. Electrical Communication 38, No. 4:524-533. 1963.

12. Dixon, W. J., and F. J. Massey, Jr. Introduction to statistical analysis. McGraw-Hill Book Company, New York. 1969.

13. Evans, D., R. Herman, and G. H. Weiss. Queueing at a stop sign. International Symposium on the Theory of Traffic Flow Proceedings 2:182-189. 1963.

14. Francis, J. G. F., and R. S. Lott. A simulation programme for linked traffic signals. International Symposium on the Theory of Traffic Flow Proceedings 2:257-259. 1963.

15. Garwood, F. The sampling and use of traffic flow statistics. Applied Statistics, A Journal of the Royal Statistical Society, Series C, 11, No. 1:1-15. 1962.

16. General Purpose Simulation System/360 OS and DOS, Version 2, User's Manual, No. SH20-0694-1. International Business Machines Corporation, White Plains, New York. September 1971.

17. Gerlough, D. L. Simulation of freeway traffic by an electronic computer. Highway Research Board Proceedings 35:543-547. 1956.

18. Gerlough, D. L., and F. C. Barnes. Poisson and other distributions in traffic. Eno Foundation for Transportation, Saugatuck, Connecticut. 1971.

19. Gerlough, D. L., and F. A. Wagner, Jr. Simulation of traffic in a large network of signalized intersections. International Symposium on the Theory of Traffic Flow Proceedings 2:249-252. 1963.

20. Goode, H. H., C. H. Pollmar, and J. B. Wright. The use of digital computer to model signalized intersection. Highway Research Board Proceedings 35:548-557. 1956.

21. Haight, F. A. Mathematical theories of traffic flow. Academic Press, Inc., New York. 1963.

22. Hildrebrand, F. B. Advanced calculus for engineers. Prentice-Hall, Inc., Englewood Cliffs, N.J. 1958.

23. Kell, J. H. Analyzing vehicular delay at intersections through simulation. Highway Research Board Bulletin 356:28-39. 1962.

24. Kosten, L. Stochastic theory of service systems. International series of monographs in pure and applied mathematics, Volume 103. Pergamon Press, Ltd., London. 1973.

25. Levy, S. L., M. Carter, and A. Glickstein. Traffic and simulation. International Symposium on the Theory of Traffic Flow Proceedings 2: 253-256. 1963.

26. Lewis, R. M., and H. L. Michaels. The simulation of traffic flow to obtain volume warrants for intersection control. Highway Research Record 15:1-43. 1963.

27. Longley, D. A simulation study of a traffic network control scheme. Transportation Research 5, No. 1:1-14. April 1971.

28. Mason, R. M., and W. B. Stewart. Computer simulation of transit operation and costs. American Society of Civil Engineers Proceedings 95, TE 1:57-66. February 1969.

29. Miller, A. J. A queueing model for road traffic flow. Journal of the Royal Statistical Society, Series B, 23, No. 1:64-75. 1961.

30. Nanda, R., J. J. Browne, and B. R. Lui. Simulating passenger arrivals at airports. Industrial Engineering 4, No. 3:12-19. March 1972.

31. Oliver, R. M. A traffic counting distribution. Operations Research 9, No. 6:802-809. 1961.

32. Oliver, R. M. Distribution of gaps and blocks in traffic stream. Operations Research 10, No. 2:197-217. 1962.

33. Panico, J. A. Queueing theory. Prentice-Hall Inc., Englewood Cliffs, N.J. 1969.

34. Pearce, C. E. Queueing Systems with transport service processes. Transportation Science 1, No. 3:218-223. 1967.

35. Perchonok, P. A., and S. L. Levy. Application of digital simulation techniques to freeway on ramp traffic operations. Highway Research Board Proceedings 39:506-523. 1960.

36. Potts, R. B., M. C. Dunne, and R. W. Rothery. A discrete Markov model of vehicular traffic. Transportation Science 2, No. 3:233-251. 1968.

37. Reynolds, G. H. A shuttle car assignment problem in the mining industry. Management Science 17, No. 9:652-655. May 1971.

38. Rhee, S. Y. An urban traffic control simulator. International Symposium on the Theory of Traffic Flow Proceedings 2:260-263. 1963.

39. Serfling, R. J. Non-Poisson models for traffic flow. Transportation Research 3, No. 3:299-306. 1969.

40. Stark, M. C. Computer simulation of traffic on nine blocks of a city street. Highway Research Board Bulletin 356:40-47. 1962.

41. Tindall, J. I. Simple time series and seasonal factors for road traffic sampling. Third Conference- Australian Road Research Board Proceedings 3, Part 1:711-728. 1966.

42. Tomlin, S. G. Time-dependent traffic distributions. Transportation Research 4, No. 1:77-86. 1970.

43. Vaughan, R. J. The distribution of traffic volumes. Transportation Science 4, No. 1:97-110. 1970.

44. Welding, P. I. Time series analysis as applied to traffic flow. International Symposium on the Theory of Traffic Flow Proceedings 2:60-72, 1963.

45. Wohl, M. Simulation- its application to traffic engineering, Part 2. Traffic Engineering 31:19-25,56. October 1960.

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VII. APPENDIX A:

ACTUAL FERRY BOAT DATA

TAKEN AT ISTANBUL BOSPHORUS, TURKEY[1]

Table 10. Sample car arrivals at side A

Time	Number of arrivals	Time	Number of arrivals	Time	Number of arrivals	Time	Number of arrivals
7:01 am	5	7:26	3	7:51	3	8:16	4
7:02	3	7:27	3	7:52	4	8:17	0
7:03	6	7:28	3	7:53	3	8:18	9
7:04	3	7:29	0	7:54	3	8:19	0
7:05	2	7:30	3	7:55	4	8:20	4
7:06	1	7:31	1	7:56	4	8:21	4
7:07	3	7:32	3	7:57	4	8:22	0
7:08	4	7:33	3	7:58	4	8:23	5
7:09	0	7:34	3	7:59	4	8:24	5
7:10	3	7:35	4	8:00	4	8:25	5
7:11	4	7:36	3	8:01	4	8:26	7
7:12	4	7:37	2	8:02	3	8:27	0
7:13	3	7:38	5	8:03	4	8:28	3
7:14	3	7:39	2	8:04	4	8:29	0
7:15	1	7:40	2	8:05	0	8:30	3
7:16	6	7:41	3	8:06	4	8:31	1
7:17	2	7:42	2	8:07	5	8:32	4
7:18	3	7:43	4	8:08	4	8:33	4
7:19	4	7:44	0	8:09	0	8:34	3
7:20	5	7:45	4	8:10	4	:	:
7:21	4	7:46	0	8:11	4	:	:
7:22	3	7:47	3	8:12	4	6:57 pm	5
7:23	2	7:48	1	8:13	4	6:58	3
7:24	1	7:49	4	8:14	6	6:59	4
7:25	2	7:50	2	8:15	6	7:00	2

[1]Data taken on Sunday, April 3, 1973 .

Table 11. Sample car arrivals at side B

Time	Number of arrivals	Time	Number of arrivals	Time	Number of arrivals	Time	Number of arrivals
7:01 am	1	7:26	0	7:51	0	8:16	5
7:02	1	7:27	3	7:52	2	8:17	1
7:03	1	7:28	0	7:53	1	8:18	4
7:04	4	7:29	1	7:54	2	8:19	3
7:05	1	7:30	1	7:55	2	8:20	0
7:06	2	7:31	1	7:56	0	8:21	0
7:07	3	7:32	3	7:57	0	8:22	2
7:08	1	7:33	0	7:58	1	8:23	0
7:09	1	7:34	2	7:59	2	8:24	3
7:10	2	7:35	2	8:00	0	8:25	3
7:11	1	7:36	0	8:01	2	8:26	1
7:12	2	7:37	2	8:02	0	8:27	1
7:13	4	7:38	1	8:03	4	8:28	0
7:14	2	7:39	2	8:04	3	8:29	2
7:15	4	7:40	0	8:05	0	8:30	1
7:16	1	7:41	0	8:06	3	8:31	0
7:17	2	7:42	0	8:07	0	8:32	1
7:18	2	7:43	2	8:08	0	8:33	3
7:19	1	7:44	0	8:09	4	8:34	0
7:20	4	7:45	4	8:10	0	:	:
7:21	1	7:46	0	8:11	0	:	:
7:22	1	7:47	0	8:12	0	6:57 pm	4
7:23	2	7:48	2	8:13	0	6:58	4
7:24	1	7:49	4	8:14	1	6:59	4
7:25	0	7:50	0	8:15	2	7:00	11

Table 12. Ferry docking data at side A

Ferry number	Number of cars dis-embarked	Unloading time (min.)	Number of cars embarked	Loading time (min.)	Total time spent at dock (min.)	Constant time[a] (min.)	Transit time to side B (min.)
1	40	5	42	6	14	3	13
2	55	5	62	9	17	3	13
3	40	4	42	22	29	3	12
1	41	4	43	17	24	3	12
2	60	4	64	8	15	3	13
3	44	1	40	8	13	4	12
1	42	4	42	9	15	3	12
2	62	4	61	5	10	1	13
3	43	3	41	4	8	1	11
4	50	5	51	21	28	2	12
1	43	3	43	3	8	2	12
2	61	1	64	4	9	4	12
3	41	3	41	4	7	0	12
4	49	3	48	5	10	2	12
1	44	3	45	3	8	2	11
2	61	4	62	6	10	0	13
4	54	3	49	4	9	2	10
1	43	3	42	6	11	2	10
3	40	4	40	5	11	2	12
2	63	4	64	7	13	2	11
4	51	3	53	6	9	0	10
1	42	3	40	3	9	3	10
3	42	3	39	5	9	1	10
2	64	4	66	7	12	1	9
4	49	5	54	6	11	0	10
1	41	3	39	7	11	1	13
3	40	3	40	4	9	2	10
2	63	3	59	5	9	1	10
4	48	4	53	3	9	2	9
1	42	3	41	4	9	2	12
3	41	3	41	7	12	2	13
2	62	5	64	7	14	2	10
4	49	2	52	4	9	3	14
1	41	4	38	5	10	1	15

[a]Constant time = total time spent at dock
- (loading time + unloading time) .

Table 12. (Continued)

Ferry number	Number of cars dis-embarked	Unloading time (min.)	Number of cars embarked	Loading time (min.)	Total time spent at dock (min.)	Constant time (min.)	Transit time to side B (min.)
3	43	3	42	5	10	2	13
4	48	5	50	4	10	1	17
2	61	6	65	8	17	3	12
1	42	3	39	3	7	1	18
3	44	5	45	3	9	1	11
4	48	1	51	3	11	7	12
2	64	5	62	4	10	1	12
1	43	3	40	6	12	3	13
3	42	3	46	6	9	0	15
2	63	5	64	5	11	1	11
4	49	3	57	7	13	3	13
1	42	3	39	5	9	1	13
3	41	3	39	4	9	2	12
2	62	4	59	4	9	1	11
4	50	3	58	5	9	1	11
1	40	4	42	7	11	0	12
3	41	3	42	5	9	1	15
2	65	3	67	7	11	1	12
4	48	5	41	4	10	1	15
1	43	3	-	5	9	1	-
3	41	3	-	6	11	2	-

Table 13. Ferry docking data at side B

Ferry number	Number of cars dis-embarked	Unloading time (min.)	Number of cars embarked	Loading time (min.)	Total time spent at dock (min.)	Constant time (min.)	Transit time to side A (min.)
1	40	3	40	10	16	3	12
2	60	4	55	6	11	1	13
3	41	3	40	6	11	2	12
1	42	3	41	15	21	3	12
2	62	4	60	5	10	1	12
3	42	3	44	11	17	3	12
1	43	4	42	6	13	3	12
2	64	4	62	6	12	2	12
4	49	4	50	4	9	1	13
3	40	3	43	3	7	1	10
1	42	3	43	4	8	1	11
2	61	4	61	6	10	0	12
3	41	4	41	6	11	1	12
4	51	3	49	5	11	3	13
1	43	3	44	7	12	2	14
2	64	5	61	11	20	4	12
4	48	1	54	5	9	3	15
1	45	3	45	8	12	1	12
3	41	3	40	5	10	2	12
2	62	4	62	5	10	1	13
1	42	4	42	6	11	1	13
3	40	3	42	5	9	1	13
2	64	4	64	6	11	1	15
4	53	4	49	6	11	1	14
1	40	3	41	6	11	2	12
3	39	4	40	7	13	2	13
2	66	4	63	6	11	1	12
4	54	4	48	4	10	2	10
1	39	3	42	4	7	0	13
3	40	3	41	6	11	2	12
2	59	3	62	5	9	1	10
4	53	3	49	4	7	0	9
1	41	3	41	6	10	1	12
3	41	3	43	4	11	4	11

Table 13. (Continued)

Ferry number	Number of cars dis-embarked	Unloading time (min.)	Number of cars embarked	Loading time (min.)	Total time spent at dock (min.)	Constant time (min.)	Transit time to side A (min.)
4	52	5	48	2	11	4	11
2	64	3	61	5	10	2	11
1	38	5	42	3	10	2	10
3	42	3	44	4	9	2	13
4	50	3	48	7	11	1	9
2	65	5	64	4	10	1	11
1	39	3	43	3	7	1	12
3	45	3	42	7	11	1	10
4	51	4	49	6	11	1	13
2	62	3	63	6	9	0	12
1	40	3	42	5	10	2	14
3	46	4	41	6	12	2	13
2	64	5	62	5	12	2	12
4	57	5	50	5	11	1	10
1	39	3	40	4	7	0	10
3	39	4	41	5	11	2	13
2	59	5	65	4	11	2	11
4	58	4	48	5	12	3	12
1	42	4	43	9	14	1	12
3	42	4	41	7	12	1	11
2	67	4	-	6	11	1	-
4	41	3	-	7	12	2	-

VIII. APPENDIX B: CALCULATION AND TABULATION OF INTERARRIVAL TIMES

A. Calculation of Interarrival Times

Using arrival data shown on Tables 10 and 11, the interarrival times are calculated the following way.

Defining:

t = time

N(t) = number of arrivals at time t

I(t) = interarrival time at time t

1. If $N(t)>0$

then, $$I(t) = \frac{1}{N(t)} .$$

2. If $N(t) = 0$ and $N(t-1)>0$ and $N(t+1)>0$

then, $$I(t) = 1.0 + \frac{I(t-1)}{2} + \frac{I(t+1)}{2} .$$

3. If $N(t) = N(t+1) = 0$ and $N(t-1)>0$ and $N(t+2)>0$

then, $$I(t) = 2.0 + \frac{I(t-1)}{2} + \frac{I(t+2)}{2} .$$

4. If $N(t) = N(t-1) = 0$

then, I(t) is disregarded.

5. If $N(t) = N(t+1) = N(t+2) = 0$ and $N(t-1)>0$ and $N(t+3)>0$

then, $$I(t) = 3.0 + \frac{I(t-1)}{2} + \frac{I(t+3)}{2} .$$

6. If $N(t) = N(t-1) = N(t-2) = 0$

then, I(t) is disregarded etc.

Samples of interarrival times calculated in a similar manner, using the Fortran program listed in section B of this Appendix, are tabulated in Tables 14 and 15 for sides A and B respectively.

Table 14. Sample interarrival times for side A

Time	Interarrival time	Time	Interarrival time	Time	Interarrival time	Time	Interarrival time
7:01 am	0.200	7:26	0.500	7:51	0.333	8:16	0.250
7:02	0.333	7:27	0.333	7:52	0.250	8:17	1.181
7:03	0.167	7:28	0.333	7:53	0.333	8:18	0.111
7:04	0.333	7:29	1.333	7:54	0.333	8:19	1.181
7:05	0.500	7:30	0.333	7:55	0.250	8:20	0.250
7:06	1.000	7:31	1.000	7:56	0.250	8:21	0.250
7:07	0.333	7:32	0.333	7:57	0.250	8:22	1.225
7:08	0.250	7:33	0.333	7:58	0.250	8:23	0.200
7:09	1.292	7:34	0.333	7:59	0.250	8:24	0.200
7:10	0.333	7:35	0.250	8:00	0.250	8:25	0.200
7:11	0.250	7:36	0.333	8:01	0.250	8:26	0.143
7:12	0.250	7:37	0.500	8:02	0.333	8:27	1.238
7:13	0.333	7:38	0.200	8:03	0.250	8:28	0.333
7:14	0.333	7:39	0.500	8:04	0.250	8:29	1.333
7:15	1.000	7:40	0.500	8:05	1.250	8:30	0.333
7:16	0.167	7:41	0.333	8:06	0.250	8:31	1.000
7:17	0.500	7:42	0.500	8:07	0.200	8:32	0.250
7:18	0.333	7:43	0.250	8:08	0.250	8:33	0.250
7:19	0.250	7:44	1.250	8:09	1.250	8:34	0.333
7:20	0.200	7:45	0.250	8:10	0.250	:	:
7:21	0.200	7:46	1.292	8:11	0.250	:	:
7:22	0.250	7:47	0.333	8:12	0.250	6:57 pm	0.200
7:23	0.333	7:48	1.000	8:13	0.250	6:58	0.333
7:24	0.500	7:49	0.250	8:14	0.167	6:59	0.250
7:25	1.000	7:50	0.500	8:15	0.167	7:00	0.500

Table 15. Sample interarrival times for side B

Time	Interarrival time	Time	Interrarrival time	Time	Interarrival time	Time	Interarrival time
7:01 am	1.000	7:26	-	7:51	-	8:16	0.200
7:02	1.000	7:27	0.333	7:52	0.500	8:17	1.000
7:03	1.000	7:28	1.667	7:53	1.000	8:18	0.250
7:04	0.250	7:29	1.000	7:54	0.500	8:19	0.333
7:05	1.000	7:30	1.000	7:55	2.750	8:20	2.417
7:06	0.500	7:31	1.000	7:56	-	8:21	-
7:07	0.333	7:32	0.333	7:57	1.000	8:22	0.500
7:08	1.000	7:33	1.417	7:58	0.500	8:23	1.417
7:09	1.000	7:34	0.500	7:59	1.500	8:24	0.333
7:10	0.500	7:35	0.500	8:00	0.500	8:25	0.333
7:11	1.000	7:36	1.500	8:01	0.500	8:26	1.000
7:12	0.500	7:37	0.500	8:02	1.375	8:27	1.000
7:13	0.250	7:38	1.000	8:03	0.250	8:28	1.750
7:14	0.500	7:39	0.500	8:04	0.333	8:29	0.500
7:15	0.250	7:40	3.500	8:05	1.333	8:30	1.000
7:16	1.000	7:41	-	8:06	0.333	8:31	2.000
7:17	0.500	7:42	-	8:07	2.292	8:32	1.000
7:18	0.500	7:43	0.500	8:08	-	8:33	0.333
7:19	1.000	7:44	1.375	8:09	0.250	8:34	2.417
7:20	0.250	7:45	0.250	8:10	4.625	:	:
7:21	1.000	7:46	2.375	8:11	-	:	:
7:22	1.000	7:47	-	8:12	-	6:57 pm	0.250
7:23	1.500	7:48	0.500	8:13	-	6:58	0.250
7:24	1.000	7:49	0.250	8:14	1.000	6:59	0.250
7:25	2.667	7:50	2.375	8:15	0.500	7:00	0.090

B. Fortran Program Listing for

Calculation of Interarrival Times

```
$JOB            E5556,TIME=60,PAGES=30
      CHARACTER*80 IMAGE
      CHARACTER*1 IWORK(80),KZERO,KBLNK
      EQUIVALENCE (IMAGE,IWORK(1))
      DATA KZERO/1HO/,KBLNK/1H /
      IZSW=0
5     READ(5,8001,END=99) IMAGE
8001  FORMAT(A80)
      IF(IWORK(6).EQ.KZERO) READ(IMAGE,8002)ITIME,NOA
8002  FORMAT(I3,1X,I2)
      IF(IWORK(6).EQ.KBLNK) READ(IMAGE,8003)ITIME,NOA
8003  FORMAT(I3,1X,I1)
      IF(NOA.NE.0) GO TO 10
      IZSW=1
      GO TO 5
10    CONTINUE
      IF(IZSW.NE.0) GO TO 20
15    FIAT=1.0/FLOAT(NOA)
      WRITE(6,7001) ITIME,NOA,FIAT
7001  FORMAT(' ',I3,1X,I2,1X,F5.3)
      WRITE(7,7001)ITIME,NOA,FIAT
      ITIMES=ITIME
      FIATS=FIAT
      GO TO 5
20    CONTINUE
      IDIF=ITIME-ITIMES-1
      FW1=IDIF+0.5*FIATS+0.5*(1/FLOAT(NOA))
      FI=0.0
      I=0
      J=ITIMES+1
      WRITE(6,7001) J,I,FW1
      WRITE(7,7001) J,I,FW1
      IF(IDIF.EQ.1) GO TO 2002
      L1=J+1
      L2=ITIME-1
      DO 2001 J=L1,L2
      WRITE(6,7001) J,I,FI
      WRITE(7,7001) J,I,FI
2001  CONTINUE
2002  CONTINUE
      IZSW=0
      GO TO 15
99    WRITE(6,7009)
7009  FORMAT('0','E.0.J.')
      STOP
      END
```

IX. APPENDIX C:

CALCULATION AND TABULATION OF WEIBULL PARAMETERS AND AVERAGE INTERARRIVAL TIMES

A. Calculation and Tabulation of Weibull Parameters for Each Half-Hour Period

Weibull density function is given by

$$f(t) = \alpha\lambda^{\alpha}(t-\mu)^{\alpha-1}e^{-\lambda(t-\mu)^{\alpha}}$$

and the cumulative distribution function is

$$F(t) = 1-\exp[-\lambda(t-\mu)^{\alpha}]$$ where

λ = scale parameter

α = shape parameter (slope)

μ = location parameter.

Taking natural logarithms of both sides twice

$$\ln[(1-F(t))^{-1}] = \lambda(t-\mu)^{\alpha}$$

and

$$\ln\{\ln[(1-F(t))^{-1}]\} = \ln\lambda + \alpha\ln(t-\mu) \quad . \tag{12}$$

Let

$$Y = \ln\{\ln[(1-F(t))^{-1}]\}$$

$$a = \ln\lambda \quad ; \; b = \alpha$$

$$x = \ln(t-\mu)$$

then equation 12 reduces to a simple linear equation in the form of $Y = a+bx$, and it is possible to plot interarrival time (t) versus F(t). Statistical Analysis System (SAS) at Iowa State University is used to calculate the three Weibull parameters for one-hour overlapping periods for

Table 16. Weibull parameters, side A

Time period	R^2	$\ln\lambda$	Scale parameter λ	Shape parameter α	Location parameter μ	Average interarrival time T_0 (minutes)
7:00-8:00	0.972	1.441	4.23	0.964	0.160	0.395
7:30-8:30	0.952	1.379	3.97	1.165	0.100	0.390
8:00-9:00	0.950	1.264	3.54	1.009	0.083	0.368
8:30-9:30	0.969	1.187	3.28	0.990	0.083	0.386
9:00-10:00	0.963	1.333	3.79	0.951	0.095	0.348
9:30-10:30	0.928	1.462	4.32	0.997	0.095	0.327
10:00-11:00	0.932	1.613	5.02	0.960	0.108	0.297
10:30-11:30	0.760	1.507	4.51	0.795	0.048	0.219
11:00-12:00	0.970	1.781	5.94	0.891	0.048	0.192
11:30-12:30	0.972	1.396	4.04	0.729	0.061	0.240
12:00-1:00	0.935	1.210	3.39	0.772	0.065	0.305
12:30-1:30	0.969	1.421	4.14	0.765	0.065	0.249
1:00-2:00	0.974	1.499	4.48	0.737	0.069	0.227
1:30-2:30	0.986	1.391	4.02	0.794	0.069	0.266
2:00-3:00	0.978	1.469	4.35	0.985	0.072	0.298
2:30-3:30	0.977	1.664	5.28	0.931	0.097	0.271
3:00-4:00	9.964	1.351	3.86	0.776	0.065	0.269
3:30-4:30	0.953	1.443	4.23	0.722	0.066	0.233
4:00-5:00	0.967	1.327	3.77	0.718	0.082	0.277
4:30-5:30	0.976	1.318	3.74	0.763	0.080	0.289
5:00-6:00	0.951	1.406	4.08	0.783	0.080	0.271
5:30-6:30	0.973	1.496	4.46	0.702	0.099	0.249
6:00-7:00	0.973	1.534	4.64	0.710	0.099	0.244

each side of the channel. Location parameter μ is estimated such that R^2 = coefficient of determination is maximized or equivalently the error sum of squares is minimized.

Tabulation of Weibull parameters thus calculated (Tables 16 and 17), a sample calculation of cumulative distribution function and interarrival times for one-hour period (Table 18 and Figure 32), and calculation of average Weibull interarrival times (section B) are given in the following pages.

Table 17. Weibull parameters, side B

Time period	R^2	$\ln\lambda$	Scale parameter λ	Shape parameter α	Location parameter μ	Average interarrival time T_0 (minutes)
7:00-8:00	0.997	0.682	1.98	0.748	0.224	0.703
7:30-8:30	0.989	0.611	1.84	0.650	0.199	0.735
8:00-9:00	0.980	0.673	1.96	0.563	0.199	0.696
8:30-9:30	0.958	0.716	2.05	0.877	0.091	0.563
9:00-10:00	0.972	1.110	3.04	1.024	0.091	0.426
9:30-10:30	0.937	1.335	3.80	0.979	0.095	0.353
10:00-11:00	0.970	1.350	3.86	0.824	0.069	0.282
10:30-11:30	0.978	1.133	3.11	0.818	0.067	0.346
11:00-12:00	0.970	1.029	2.80	0.755	0.121	0.424
11:30-12:30	0.976	1.192	3.30	0.727	0.110	0.347
12:00-1:00	0.967	1.477	4.38	0.832	0.110	0.298
12:30-1:30	0.950	1.487	4.43	0.919	0.077	0.283
1:00-2:00	0.972	1.548	4.71	0.778	0.082	0.240
1:30-2:30	0.944	1.441	4.23	0.663	0.097	0.250
2:00-3:00	0.970	1.420	4.14	0.689	0.110	0.272
2:30-3:30	0.977	1.576	4.84	0.851	0.107	0.278
3:00-4:00	0.976	1.440	4.22	0.719	0.097	0.264
3:30-4:30	0.958	1.384	3.99	0.725	0.090	0.273
4:00-5:00	0.963	1.288	3.63	0.631	0.090	0.274
4:30-5:30	0.976	1.336	3.80	0.669	0.090	0.270
5:00-6:00	0.975	1.293	3.65	0.874	0.097	0.303
5:30-6:30	0.978	1.330	3.78	0.855	0.080	0.309
6:00-7:00	0.978	1.384	3.99	0.763	0.082	0.273

B. Calculation of Average Weibull Interarrival Times

Average Weibull interarrival times are calculated as follows:

$$T_0 = \mu + \lambda^{-1/\alpha}\Gamma(1+1/\alpha)$$

where

$$\Gamma(x) = (x-1)! = (x-1)(x-2)\ldots(x_0+1)x_0\Gamma(x_0) \text{ for } x>2 \; ; \; 1\leq x_0\leq 2 \; .$$

As an illustration, using parameter values from Table 16 (for side A), the average interarrival time for 7-8 am period is calculated as follows:

Table 18. Sample calculation of cumulation distribution function and interarrival times for 7-8 am, side A

Number of arrivals per minute		Number of occurrences		Total	Inter-arrival times	Adjusted inter-arrival times[a]	Cumulative distribution function	Smoothed cumulative distribution function
6	*	2	=	12	0.166	0.023	0.067	0.033
5	*	3	=	15	0.200	0.057	0.150	0.111
4	*	16	=	64	0.250	0.107	0.506	0.328
3	*	21	=	63	0.333	0.190	0.856	0.678
2	*	8	=	16	0.500	0.357	0.944	0.901
1	*	10	=	10	1.000	0.857	0.978	0.960
					1.250	1.107	0.983	0.977
		Total	=	180	1.333	1.190	1.000	0.989

[a]In the above example, μ is estimated as 1/max. no. of arrivals+1 = 1/6+1 = 0.143 . Various other estimates of μ were tried for each time interval and the ones which maximized R^2 were chosen.

$$T_{0_{7-8}} = 0.160+(4.225)^{-1/0.964}\Gamma(1+1/0.964)$$

but

$$\Gamma(2.104) = 1.104\Gamma(1.104) = (1.104)(0.9514) = 1.047$$

therefore,

$$T_{0_{7-8}} = 0.160+(1.047)(0.224) = 0.395 \text{ minutes.}$$

Other average interarrival times are calculated in a similar manner.

Results are tabulated in Tables 16 and 17.

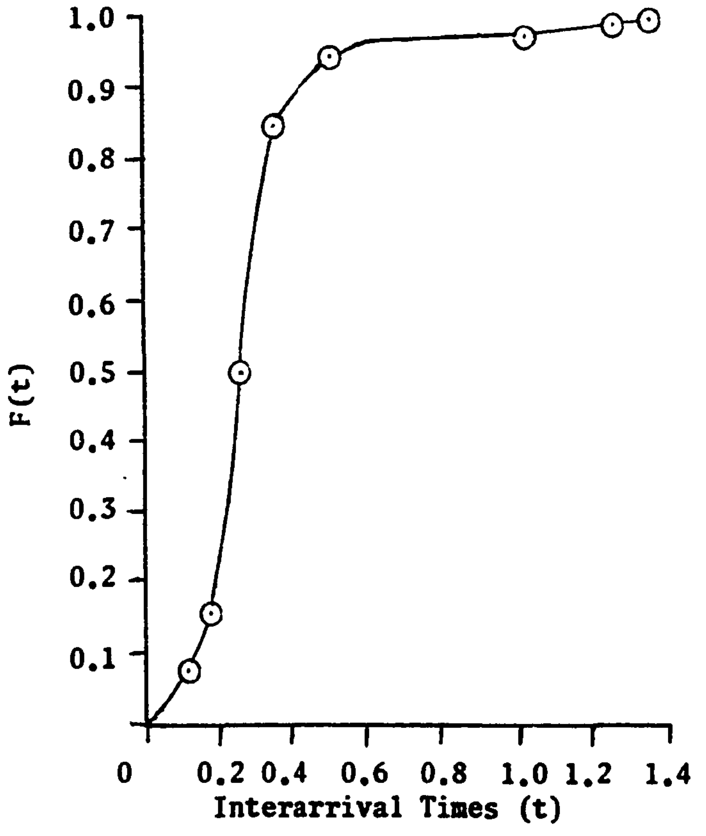


Figure 32. Sample Weibull cumulative distribution function for 7-8 am period, side A.

X. APPENDIX D:

DERIVATION OF INVERSE FUNCTIONS

A. Derivation of Weibull Inverse Function

Cumulative distribution function for Weibull distribution is given by the equation:

$$F(t) = 1-\exp[-\lambda(t-\mu)^{\alpha}] \quad \text{where}$$

λ = scale parameter

α = shape parameter

μ = location parameter .

Rearranging above expression

$$(1-F(t)) = \exp[-\lambda(t-\mu)^{\alpha}]$$

and taking logarithms of both sides

$$\ln(1-F(t)) = -\lambda(t-\mu)^{\alpha}$$

or

$$(t-\mu) = [-\frac{1}{\lambda}\ln(1-F(t))]^{1/\alpha} .$$

Taking logarithms again

$$\ln(t-\mu) = \frac{1}{\alpha}\ln[-\frac{1}{\lambda}\ln(1-F(t))]$$

or

$$(t-\mu) = \exp\{\frac{1}{\alpha}\ln[-\frac{1}{\lambda}\ln(1-F(t))]\}$$

or

$$t = \exp\{\frac{1}{\alpha}\Big(\ln[-\ln(1-F(t))]-\ln\lambda\Big)\}+\mu \quad .$$

This is the formula used in calculating Weibull interarrival times corresponding to uniform values of F(t).

B. Derivation of Exponential Inverse Function

Density function of exponential distribution is given by the expression

$$f(t) = \frac{1}{a}\exp(-t/a) \qquad t>0 \qquad \text{where}$$

a = mean interarrival time.

Integrating f(t), the cumulative distribution becomes

$$F(t) = 1-\exp(-t/a)$$

or

$$\exp(-t/a) = 1-F(t) \ .$$

Taking logarithms of both sides

$$-t/a = \ln[1-F(t)]$$

or

$$t = -a\ln[1-F(t)] \ .$$

The tabulation follows:

F(t)	-ln[1-F(t)] = (FN$EXPO)
0.0	0.0
0.1	0.104
0.2	0.222
:	:
:	:
0.999	7.0
0.9998	8.0

In forming above tabulation, the factor "a" has not been used in the second column because in GPSS simulation language when a Generate Block B Operand is FNj, the Function value is used multiplicatively, without integerizing, to modify the A Operand (a). Then the integerized product is used as interarrival time.

Mean interarrival times for sides A and B of a = 0.23 minutes/car and

$b = 0.28$ minutes/car respectively are determined simply by dividing the total observation period of 720 minutes by the total number of cars which arrived on each side of the channel during this elapsed time period.

XI. APPENDIX E:

DOCUMENTATION OF THE COMPUTER SIMULATION PROGRAM

This Appendix is divided into four major sections; program listing, flow charts of the main GPSS program, a list and brief description of various entities used in GPSS program, and a sample output for case (i,iv,iii,i,ii,i,i).

The computer program for the Weibull input consists of one main GPSS program and one Fortran subroutine which calculates the Weibull interarrival times at any given point in time and returns the information back to the main simulation program.

A. Program Listing

```
      SUBROUTINE WBULL(IS1,IS2,IS3,IS4,IS5,IS6)
C     ROUTINE TO DETERMINE AN INTERARRIVAL  TIME
C     IS1:  CURRENT CLOCK TIME (ABSOLUTE) FROM GPSS.
C     IS2:  DEBUG/DUMP SWITCH  = 0 DO NOT DUMP,
C                              = 1  DUMP.
C     IS3:  1 =  SIDE 1
C           2 =  SIDE 2
C           RETURN TO GPSS THE INTERARRIVAL  TIME
C     IS4-IS6:  DUMMY PARAMETERS (REQD. FOR PROPER LINKAGE).
C     SEED TO RANDON U(0,1) GENERATOR.
      DATA M /218341/ ,  IALPHA /65539/
C     NOTE - SET AT LOAD TIME. THIS METHOD IS VALID ONLY WHEN
C     THIS ROUTINE IS CORE-RESIDENT THROUGHOUT THE SIMULATION.
C     (DYNAMIC LOADING WILL GENERATE A CONSTANT INSTEAD
C     OF A SERIES OF RANDOM DEVIATES.)
      FS1=FLOAT(IS1)/100.0
C        DETERMINE APPROPRIATE SIDE OF CHANNEL
      IF(IS3.EQ.2) GO TO 5
C        SIDE 1(A) COMPUTATIONS
C     DETERMINE A:(SHAPE PARAMETER)
       A=1.0251-0.00048*FS1
C     DETERMINE B:(SCALE PARAMETER)
       B=4.136+0.0002*FS1+0.354*COS(3.1416*(FS1-7.5)/105)
C     DETERMINE U:(LOCATION PARAMETER)
       U=0.1779*FS1**(-0.1433)
       GO TO 10
    5 CONTINUE
C         SIDE 2(B) COMPUTATIONS
C     DETERMINE A:(SHAPE PARAMETER)
       A=0.7998-0.000056*FS1
C     DETERMINE B:(SCALE PARAMETER)
       B=1.2357+0.0128*FS1-0.0000.4*(FS1**2)
C     DETERMINE U:(LOCATION PARAMETER)
       U=0.458*FS1**(-0.2663)
   10 CONTINUE
C     DETERMINE A UNIFORM (0,1) RANDOM NUMBER.
C     POWER RESIDU METHOD
       M=M*IALPHA
       R01 = 0.5 + FLOAT(M) * 0.2328306E-9
      IF(IS2.EQ.1) WRITE(6,79) R01
   79 FORMAT(' ','R01: ',F6.4)
C     ENSURE A USABLE RESULT
      IF(R01.LE.0.0001.OR.R01.GE.0.9999) GO TO 10
C     COMPUTE INTERARRIVAL TIME
      T1=EXP((1.0/A)*ALOG(-1.0*ALOG(1-R01))-(1.0/A)*ALOG(B))+U
```

```
C     SCALE ACCORDINGLY AND TRUNCATE
      IS3=IFIX(T1*100.0)
C     CHECK FOR DUMP OF RESULTS
      IF(IS2.EQ.0) GO TO 99
      WRITE(6,80) IS1,A.B.U,IS3
   80 FORMAT(' ',5X,'CLOCK: ',I6,' A: ',F10.5,' B: ',F10.5,' U: ',
      IF10.5,' I.A. TIME: ',I6)
C     RETURN CONTROL BACK TO GPSS. NOTE-BACK TO HELP BLOCK.
   99 RETURN
      END


*LOC    OPERATION  A,B,C,D,E,F,G              COMMENTS
        LOAD       WBULL                  ENSURES THAT WBULL REMAINS CORE-R
        SIMULATE
*       MULTI-SHUTTLE (FERRY) SYSTEM
*
*       SET MULTIPLE UNIQUE RANDOM NUMBER SEQUENCE
        RMULT      30,31,32,33,34
*
*       DEFINE FUNCTIONS
*
 EXPO   FUNCTION   RN2,C24      INTERARRIVAL TIME DISTRIBUTION
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,1.2/.75,1.38/
.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/.94,2.81/.95,2.99/.96,3.2/
.97,3.5/.98,3.9/.99,4.6/.995,5.3/.998,6.2/.999,7/.9998,8
 TAU1   FUNCTION   RN1,C10      TRAVEL TIME DISTR.FROM EUROPE TO ASIA
0,0/.038,900/.189,1000/.340,1100/.661,1200/.869,1300/.888,1400/
.963,1600/.982,1700/1,1800
 TAU2   FUNCTION   RN3,C8       TRAVEL TIME DISTR. FROM ASIA TO EUROPE
0,0/.037,900/.167,1000/.297,1100/.686,1200/.908,1300/.964,1400/1,1500
 CONS1 FUNCTION    RN4,C6       CONSTANT TIME DISTR. EUROPEAN SIDE
0,0/.445,100/.732,200/.945,300/.982,400/1,700
 CONS2 FUNCTION    RN5,C5       CONSTANT TIME DISTR. ASIAN SIDE
0,0/.52,100/.80,200/.945,300/1,400
*
*       INITIALIZE SAVE VALUES
*
        INITIAL    XH1,0/XH2,0/XH3,0
        INITIAL    X6,23/X7,28
*
*       DEFINE TABLES
*
 1      TABLE      MP3,1100,100,60 SERVICE TIME OF FERRY(EUROPE TO ASIA)
 2      TABLE      MP4,1100,100,60 SERVICE TIME OF FERRY(ASIA TO EUROPE)
*
*       DEFINE MATRICES
*
 1      MATRIX     X,115,7      NO.OF CARS,Q SIZE,DOCKING TIME,FERY ID.EUR
 2      MATRIX     X,115,7      NO.OF CARS,Q SIZE,DOCKING TIME,FERY ID.ASI
```

```
*
*       DEFINE ARITHMETIC VARIABLES
*
 LOD11 FVARIABLE  13*CH1       LOADING FUNCTION FOR FERRY 1,EUROPEAN SIDE
 LOD12 FVARIABLE  10*CH2       LOADING FUNCTION FOR FERRY 2,EUROPEAN SIDE
 LOD13 FVARIABLE  12*CH3       LOADING FUNCTION FOR FERRY 3,EUROPEAN SIDE
 LOD14 FVARIABLE  9*CH4        LOADING FUNCTION FOR FERRY 4,EUROPEAN SIDE
*
 LOD21 FVARIABLE  14*CH11      LOADING FUNCTION FOR FERRY 1,ASIAN SIDE
 LOD22 FVARIABLE  9*CH12       LOADING FUNCTION FOR FERRY 2,ASIAN SIDE
 LOD23 FVARIABLE  14*CH13      LOADING FUNCTION FOR FERRY 3,ASIAN SIDE
 LOD24 FVARIABLE  10*CH14      LOADING FUNCTION FOR FERRY 4,ASIAN SIDE
*
 ULD11 FVARIABLE  8*CH11       UNLOADING FUNCTION FOR FERRY 1,EUROPE
 ULD12 FVARIABLE  7*CH12       UNLOADING FUNCTION FOR FERRY 2,EUROPE
 ULD13 FVARIABLE  8*CH13       UNLOADING FUNCTION FOR FERRY 3,EUROPE
 ULD14 FVARIABLE  7*CH14       UNLOADING FUNCTION FOR FERRY 4,EUROPE
*
 ULD21 FVARIABLE  8*CH1        UNLOADING FUNCTION FOR FERRY 1,ASIAN SIDE
 ULD22 FVARIABLE  6*CH2        UNLOADING FUNCTION FOR FERRY 2,ASIAN SIDE
 ULD23 FVARIABLE  8*CH3        UNLOADING FUNCTION FOR FERRY 3,ASIAN SIDE
 ULD24 FVARIABLE  7*CH4        UNLOADING FUNCTION FOR FERRY 4,ASIAN SIDE
*
*       DEFINE BOOLEAN VARIABLES
*
 42     BVARIABLE  CH5'GE'XH1  CONDITION TO TAKE FERRY
 51     BVARIABLE  CH5'GE'XH2  CONDITION TO TAKE FERRY
 64     BVARIABLE  CH5'GE'XH3  CONDITION TO TAKE FERRY
 43     BVARIABLE  CH6'GE'XH1  CONDITION TO TAKE FERRY
 52     BVARIABLE  CH6'GE'XH2  CONDITION TO TAKE FERRY
 65     BVARIABLE  CH6'GE'XH3  CONDITION TO TAKE FERRY
*
*       STORAGE 1 INITIALIZATION SEGMENT
*
        GENERATE   ,,,1        CREATE ONE DUMMY FERRY,EUROPEAN SIDE
        ENTER      1           ENTER DOCK AT EUROPE
        TERMINATE  0           FERRY LEAVES THE SYSTEM
*
*       FERRY SEGMENT
*
        GENERATE   ,,,1,,,F    CREATE ONE FERRY,EUROPEAN SIDE
        ASSIGN     1,K42       PARAMETER 1 BECOMES CAPACITY OF FERRY(BVR)
        ASSIGN     2,K43       PARAMETER 2 BECOMES NO. OF BVARIABLE
        ASSIGN     7,K1        PARAMETER 7 BECOMES I.D.NO OF FERRY
        ASSIGN     8,K11       PARAMETER 8 BECOMES I.D.NO OF FERRY
        TRANSFER   ,TEST       TRANSFER TO TEST
        GENERATE   ,,,1,,,F    CREATE ONE FERRY,EUROPEAN SIDE
        ASSIGN     1,K64       PARAMETER 1 BECOMES CAPACITY OF FERRY(BVR)
```

```
         ASSIGN     2,K65        PARAMETER 2 BECOMES NO. OF BVARIABLE
         ASSIGN     7,K2         PARAMETER 7 BECOMES I.D.NO OF FERRY
         ASSIGN     8,K12        PARAMETER 8 BECOMES I.D.NO OF FERRY
         TRANSFER   ,TEST        TRANSFER TO TEST
         GENERATE   ,,,1,,,F     CREATE ONE FERRY,EUROPEAN SIDE
         ASSIGN     1,K42        PARAMETER 1 BECOMES CAPACITY OF FERRY(BVR)
         ASSIGN     2,K43        PARAMETER 2 BECOMES NO. OF BVARIABLE
         ASSIGN     7,K3         PARAMETER 7 BECOMES I.D.NO OF FERRY
         ASSIGN     8,K13        PARAMETER 8 BECOMES I.D.NO OF FERRY
         TRANSFER   ,TEST        TRANSFER TO TEST
         GENERATE   14700,,,1,,,F CREATE ONE FERRY, EUROPEAN SIDE
         ASSIGN     1,K51        PARAMETER 1 BECOMES CAPACITY OF FERRY(BVR)
         ASSIGN     2,K51        PARAMETER 2 BECOMES NO. OF BVARIABLE
         ASSIGN     7,K4         PARAMETER 7 BECOMES I.D.NO OF FERRY
         ASSIGN     8,K14        PARAMETER 8 BECOMES I.D.NO OF FERRY
         TRANSFER   ,SAVE        TRANSFER TO SAVE
TEST     TEST E     BV*1,K1      HAVE MINIMUM REQMNTS FOR CROSSING SATISFIED?
         SAVEVALUE  10,P7,H      PUT FERRY ID.NO.IN SAVEVALUE NO.10
         UNLINK     5,FERY1,P1   PUT CARS(CAPACITY)ON ACTIVE STATUS,EUROPE
         MARK       3            START OF LOADING TIME(EUROPE)BECOMES P3
         PRIORITY   0,BUFFER     PUT FERRY AT END OF CURRENT EVENTS CHAIN
         TEST E     P7,K1,TES12  IS THIS FERRY 1?
         ADVANCE    V$LOD11      LOADING TIME ELAPSES FOR FERRY 1,EUROPE
         TRANSFER   ,CONS1       TRANSFER TO CONSTANT TIME
TES12    TEST E     P7,K2,TES13  IS THIS FERRY 2?
         ADVANCE    V$LOD12      LOADING TIME ELAPSES FOR FERRY 2,EUROPE
         TRANSFER   ,CONS1       TRANSFER TO CONSTANT TIME
TES13    TEST E     P7,K3,TES14  IS THIS FERRY 3?
         ADVANCE    V$LOD13      LOADING TIME ELAPSES FOR FERRY 3,EUROPE
         TRANSFER   ,CONS1       TRANSFER TO CONSTANT TIME
TES14    ADVANCE    V$LOD14      LOADING TIME ELAPSES FOR FERRY 4,EUROPE
CONS1    ADVANCE    FN$CONS1     CONSTANT TIME ELAPSES
         SAVEVALUE  9+,K1,H      UPDATE COUNTER FOR FERRIES
         TEST G     XH9,K4,LEAV1 IS THIS THE 5TH FERRY?
         MSAVEVALUE 2,P12,6,C1   PUT DEPARTURE TIME FROM EUROPE IN ROW 1, COL.6
LEAV1    LEAVE      1            LEAVE DOCK AT EUROPE
         SAVEVALUE  4+,K1,H      UPDATE COUNTER FOR FERRIES(NO.LEFT FOR ASIA)
         TEST LE    XH4,K3,ADV1  IS THIS THE 4 TH FERRY?
         ENTER      1            SEIZE DOCK AT EUROPE
ADV1     ADVANCE    FN$TAU1      FERRY TRANSIT TIME ELAPSES(EUROPE TO ASIA)
         SAVEVALUE  5+,K1,H      UPDATE COUNTER FOR MATRIX ROW(NO.DOCKED ASIA)
         ASSIGN     11,XH5       PARAMETER 11 BECOMES NUMBER IN XH5
         MSAVEVALUE 1,P11,5,C1   PUT DOCKING TIME IN ROW 1,COLUMN 5
         QUEUE      DOCK2        GET INTO QUEUE LINE AT DOCK(ASIA)
         ENTER      2            SEIZE DOCK AT ASIA
         DEPART     DOCK2        LEAVE QUEUE LINE
         TEST E     P7,K1,TES22  IS THIS FERRY 1?
         ADVANCE    V$ULD21      UNLOADING TIME ELAPSES FOR FERRY 1,ASIA
         TRANSFER   ,ULNK1       TRANSFER TO(UNLINK)BLOCK
TES22    TEST E     P7,K2,TES23  IS THIS FERRY 2?
```

```
      ADVANCE    V$ULD22     UNLOADING TIME ELAPSES FOR FERRY 2,ASIA
      TRANSFER   ,ULNK1      TRANSFER TO(UNLINK)BLOCK
TES23 TEST E     P7,K3,TES24 IS THIS FERRY 3?
      ADVANCE    V$ULD23     UNLOADING TIME ELAPSES FOR FERRY 3,ASIA
      TRANSFER   ,ULNK1      TRANSFER TO(UNLINK)BLOCK
TES24 ADVANCE    V$ULD24     UNLOADING TIME ELAPSES FOR FERRY 4,ASIA
ULNK1 MSAVEVALUE 1,P11,1,CH*7 PUT NO.OF CARS IN ROW 1,COLUMN 1
      MSAVEFALUE 1,P11,2,Q$ASIA PUT Q SIZE AT ASIA IN ROW 1,COLUMN 2
      MSAVEVALUE 1.P11,3,C1  PUT END OF UNLOADING TIME IN ROW 1,COLUMN 3
      MSAVEVALUE 1,P11,4,*7  PUT ID NUMBER OF FERRY IN ROW 1,COLUMN 4
      MSAVEVALUE 1,P11,7,Q$EUROP PUT Q SIZE AT EUROPE IN ROW 1,COLUMN 7
      UNLINK     *7,DEPT1,P1 PUT CARS(CAPACITY)ON ACTIVE STATUS
      TABULATE   1           TABULATE SERVICE TIME OF FERRY(EURO.TO ASIA)
      TEST E     BV*2,K1     ARE MINIMUM REQMNTS FOR CROSSING SATISFIED?
      SAVEVALUE  20,P8,H     PUT FERRY ID.NO.IN SAVEVALUE NO.20
      UNLINK     6,FERY2,P1  PUT CARS(CAPACITY)ON ACTIVE STATUS,ASIA
      MARK       4           START OF LOADING TIME(ASIA)BECOMES P4
      PRIORITY   0.BUFFER    PUT FERRY AT END OF CURRENT EVENTS CHAIN
      TEST E     P7,K1,TET22 IS THIS FERRY 1?
      ADVANCE    V$LOD21     LOADING TIME ELAPSES FOR FERRY 1,ASIA
      TRANSFER   ,CONS2      TRANSFER TO CONSTANT TIME
TET22 TEST E     P7,K2,TET23 IS THIS FERRY 2?
      ADVANCE    V$LOD22     LOADING TIME ELAPSES FOR FERRY 2,ASIA
      TRANSFER   ,CONS2      TRANSFER TO CONSTANT TIME
TET23 TEST E     P7,K3,TET24 IS THIS FERRY 3?
      ADVANCE    V$LOD23     LOADING TIME ELAPSES FOR FERRY 3,ASIA
      TRANSFER   ,CONS2      TRANSFER TO CONSTANT TIME
TET24 ADVANCE    V$LOD24     LOADING TIME ELAPSES FOR FERRY 4,ASIA
CONS2 ADVANCE    FN$CONS2    CONSTANT TIME ELAPSES
      MSAVEVALUE 1,P11,6,C1  PUT DEPARTURE TIME FROM ASIA IN ROW 1,COL.6
      LEAVE      2           LEAVE DOCK AT ASIA
      ADVANCE    FN$TAU2     FERRY TRANSIT TIME ELAPSES(ASIA TO EUROPE)
SAVE  SAVEVALUE  6+,K1,H     UPDATE COUNTER FOR MATRIX ROW(NO.DOCKED EU)
      ASSIGN     12,XH6      PARAMETER 12 BECOMES NUMBER IN XH6
      MSAVEVALUE 2,P12,5,C1  PUT DOCKING TIME IN ROW 1,COLUMN 5
      QUEUE      DOCK1       GET INTO QUEUE LINE AT DOCK(EUROPE)
      ENTER      1           SEIZE DOCK AT EUROPE
      DEPART     DOCK1       LEAVE QUEUE LINE
      TEST E     P7,K1,TET12 IS THIS FERRY 1?
      ADVANCE    V$ULD11     UNLOADING TIME ELAPSES FOR FERRY 1,EUROPE
      TRANSFER   ,ULNK2      TRANSFER TO(UNLINK)BLOCK
TET12 TEST E     P7,K2,TET13 IS THIS FERRY 2?
      ADVANCE    V$ULD12     UNLOADING TIME ELAPSES FOR FERRY 2,EUROPE
      TRANSFER   ,ULNK2      TRANSFER TO(UNLINK)BLOCK
TET13 TEST E     P7,K3,TET14 IS THIS FERRY 3?
      ADVANCE    V$ULD13     UNLOADING TIME ELAPSES FOR FERRY 3,EUROPE
      TRANSFER   ,ULNK2      TRANSFER TO(UNLINK)BLOCK
TET14 ADVANCE    V$ULD14     UNLOADING TIME ELAPSES FOR FERRY 4,EUROPE
ULNK2 MSAVEVALUE 2,P12,1,CH*8 PUT NO.OF CARS IN ROW 1,COLUMN 1
```

```
       MSAVEVALUE 2,P12,2,Q$EUROP PUT Q SIZE AT EUROPE IN ROW 1,COLUMN 2
       MSAVEVALUE 2,P12,3,C1  PUT END OF UNLOADING TIME IN ROW 1,COLUMN 3
       MSAVEVALUE 2,P12,4,*7  PUT ID NUMBER OF FERRY IN ROW 1,COLUMN 4
       MSAVEVALUE 2,P12,7,Q$ASIA PUT Q SIZE AT ASIA IN ROW 1,COLUMN 7
       UNLINK     *8,DEPT2,P1 PUT CARS(CAPACITY)ON ACTIVE STATUS
       TABULATE   2           TABULATE SERVICE TIME OF FERRY(ASIA TO EUR.)
       TRANSFER   ,TEST       TRANSFER BACK TO TEST
*
*      EUROPEAN SEGMENT(SIDE A)
*
       GENERATE   X6,,,,1     CARS ARRIVE AT EUROPEAN SIDE
       GATE LR    3           GATE IS LOCKED AT END OF SIMULATION
       SAVEVALUE  1,AC1       PUT ABSOLUTE CLOCK TIME IN X1
       SAVEVALUE  6,K1        FOR SIDE 1
       HELPB      WBULL,1,2,6,3,4,5
       QUEUE      EUROP       JOIN THE LINE FOR FERRY
       LINK       5,FIFO      TO USER CHAIN UNCONDITIONALLY
 FERY1 DEPART     EUROP       LEAVE QUEUE LINE
 3     QTABLE     EUROP,0,100,60 CAR WAITING TIME STATISTICS,EUROPE
       ASSIGN     7,XH10      FERRY ID.NO.BECOMES VALUE OF(CAR)P7
 CARS1 QUEUE      1           SERVICE TIME OF FERRY,EUROPE TO ASIA
       LINK       P7,FIFO     TO USER CHAIN UNCONDITIONALLY(NO.IN P7)
 DEPT1 DEPART     1           LEAVE QUEUE LINE
       TERMINATE  0           CARS LEAVE THE SYSTEM
*
*      ASIAN SEGMENT(SIDE B)
*
       GENERATE   X7,,,,1     CARS ARRIVE AT ASIAN SIDE
       GATE LR    4           GATE IS LOCKED AT END OF SIMULATION
       SAVEVALUE  1,AC1       PUT ABSOLUTE CLOCK TIME IN X1
       SAVEVALUE  7,K2        FOR SIDE 2
       HELPB      WBULL,1,2,7,3,4,5
       QUEUE      ASIA        JOIN THE LINE FOR FERRY
       LINK       6,FIFO      TO USER CHAIN UNCONDITIONALLY
 FERY2 DEPART     ASIA        LEAVE QUEUE LINE
 4     QTABLE     ASIA,0,100,60 CAR WAITING TIME STATISTICS,ASIA
       ASSIGN     8,XH20      FERRY ID.NO.BECOMES VALUE OF (CAR)P8
 CARS2 QUEUE      2           SERVICE TIME OF FERRY,ASIA TO EUROPE
       LINK       P8,FIFO     TO USER CHAIN UNCONDITIONALLY(NO.IS IN P8)
 DEPT2 DEPART     2           LEAVE QUEUE LINE
       TERMINATE  0           CARS LEAVE THE SYSTEM
*
*      TIMER SEGMENT
*
       GENERATE   72000       CREATE A TIMER AFTER TWELVE-HOURS
       LOGIC S    3           CLOSE GATE,EUROPEAN SIDE
       LOGIC S    4           CLOSE GATE,ASIAN SIDE
       TEST E     N$CARS1,N$DEPT1 WAIT UNTIL LAST FERRY COMPLETES SERVICE
       TEST E     N$CARS2,N$DEPT2 WAIT UNTIL LAST FERRY COMPLETES SERVICE
```

```
 TERM   TERMINATE  1             TURN OFF THE SIMULATION
*
*       CONTROL CARDS
*
 1      STORAGE    3             NO.OF DOCKS IN EUROPEAN SIDE
 2      STORAGE    2             NO.OF DOCKS IN ASIAN SIDE
        START      1             FIRST RUN
*
        REPORT
        EJECT
 CHA    INCLUDE    CH1-CH14/1,2,3,4,5,6
 QUE    INCLUDE    Q1-Q6/1,2,3,4,5,6,7,8,10
 STO    INCLUDE    S1-S2/1,2,3,4,5,6,7,8
 HSAV   INCLUDE    ,XH1-XH20
 MSAV   INCLUDE    ,MX1-MX2
 TAB    TITLE      ,SERVICE TIME STATISTICS(EUROPE TO ASIA)
 TAB    INCLUDE    T1/1,2,3,4,5,10,11,12,13
 TAB    TITLE      ,SERVICE TIME STATISTICS(ASIA TO EUROPE)
 TAB    INCLUDE    T2/1,2,3,4,5,10,11,12,13
 TAB    TITLE      ,CAR WAITING TIME STATISTICS,EUROPE
 TAB    INCLUDE    T3/1,2,3,4,5,10,11,12,13
 TAB    TITLE      ,CAR WAITING TIME STATISTICS,ASIA
 TAB    INCLUDE    T4/1,2,3,4,5,10,11,12,13
        END
```

B. Flowcharts of Main GPSS Simulation Program

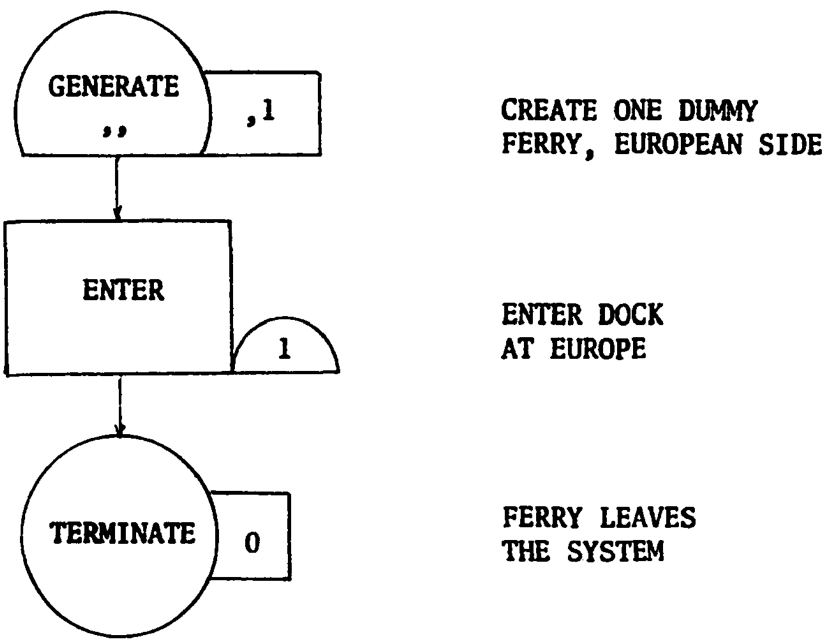


Figure 33. Storage 1 initialization segment

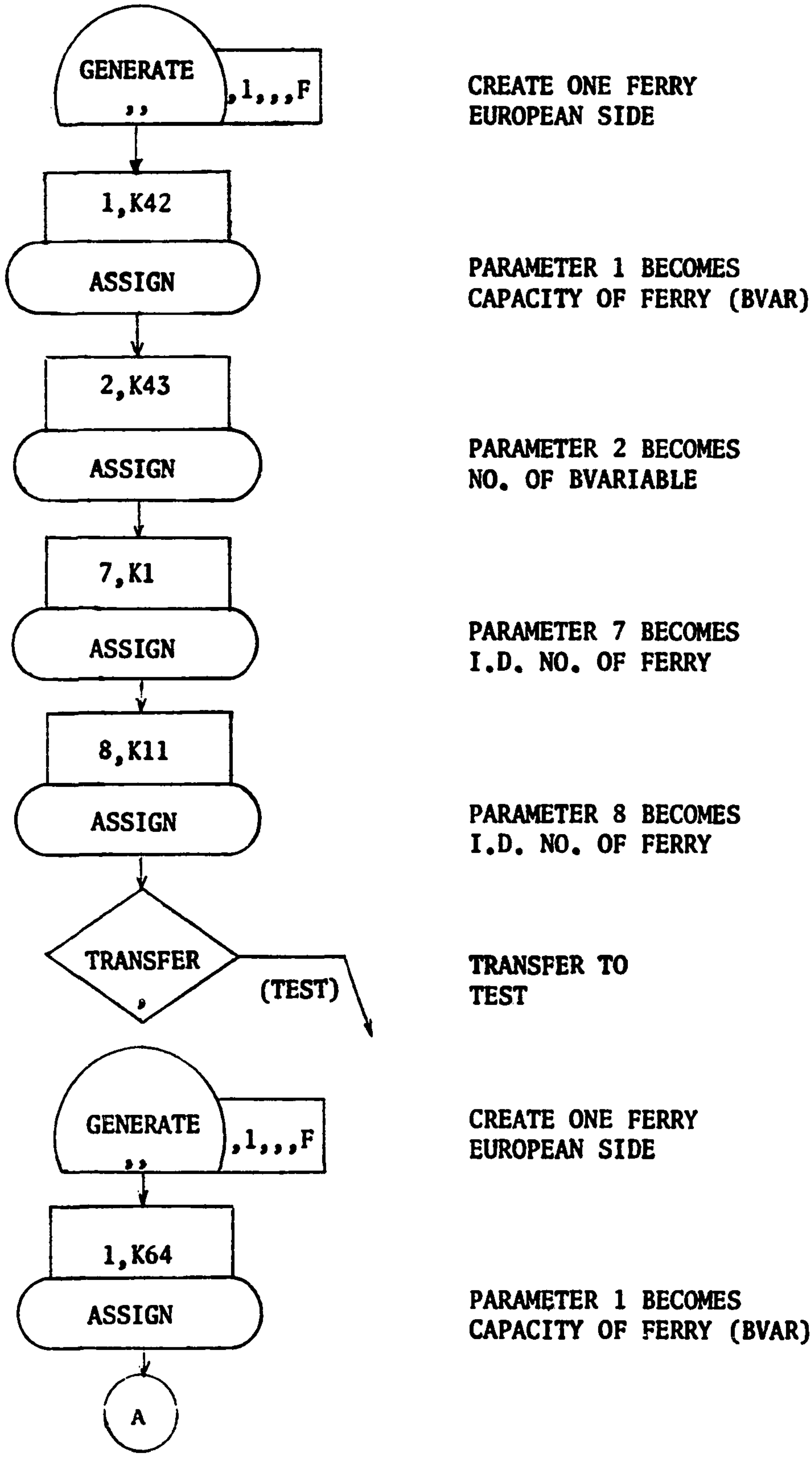


Figure 34. Ferry segment

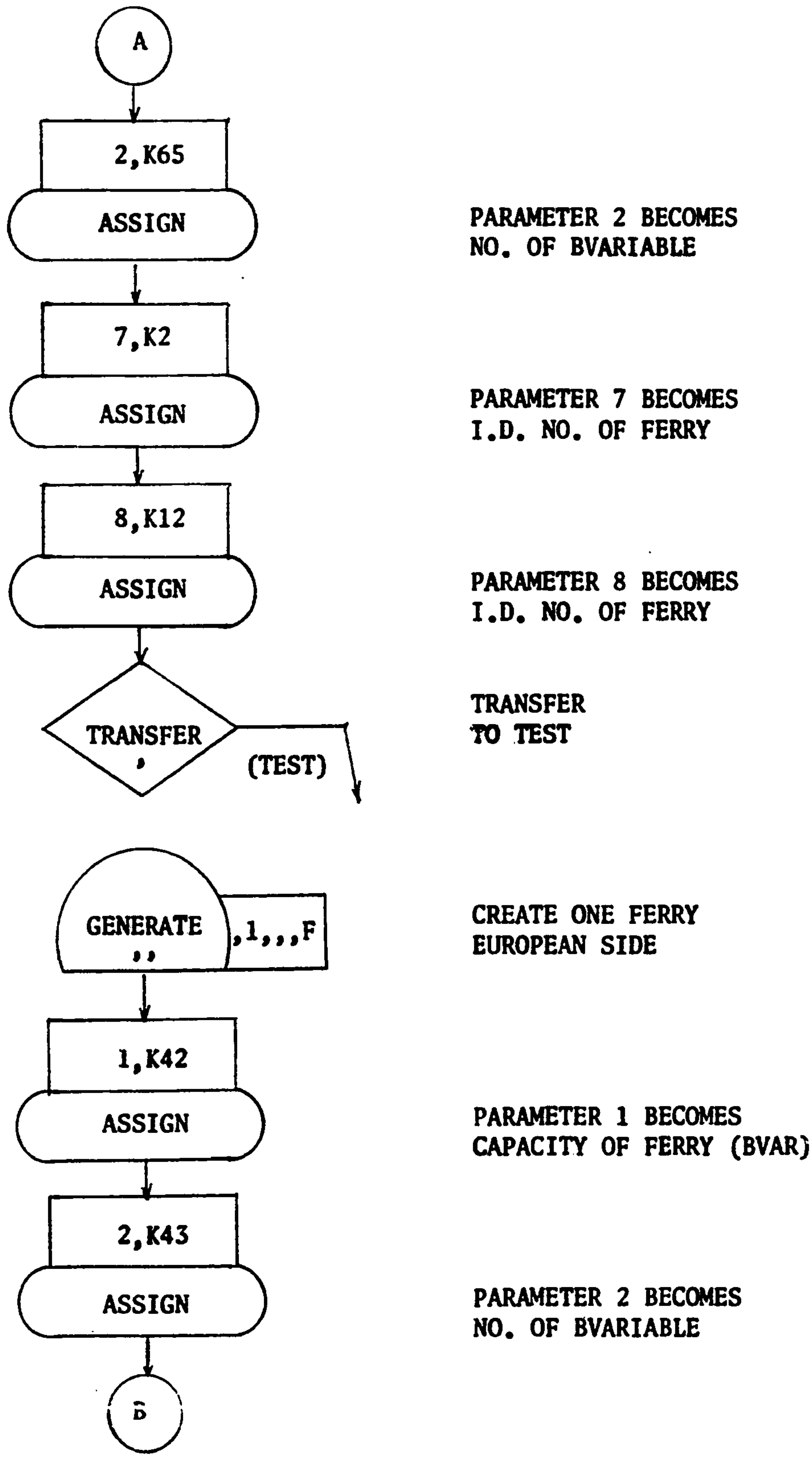


Figure. 34. (Continued)

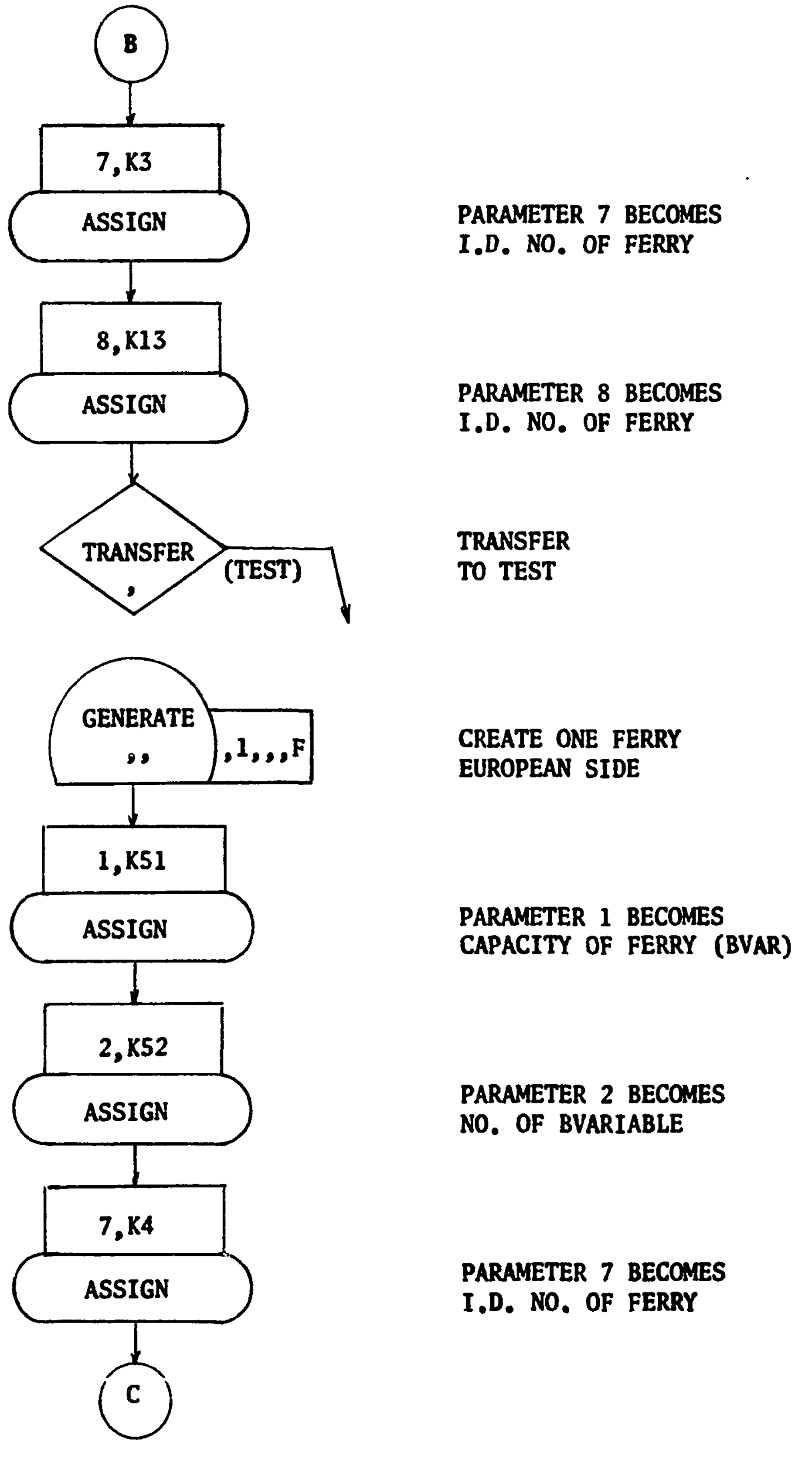


Figure 34. (Continued)

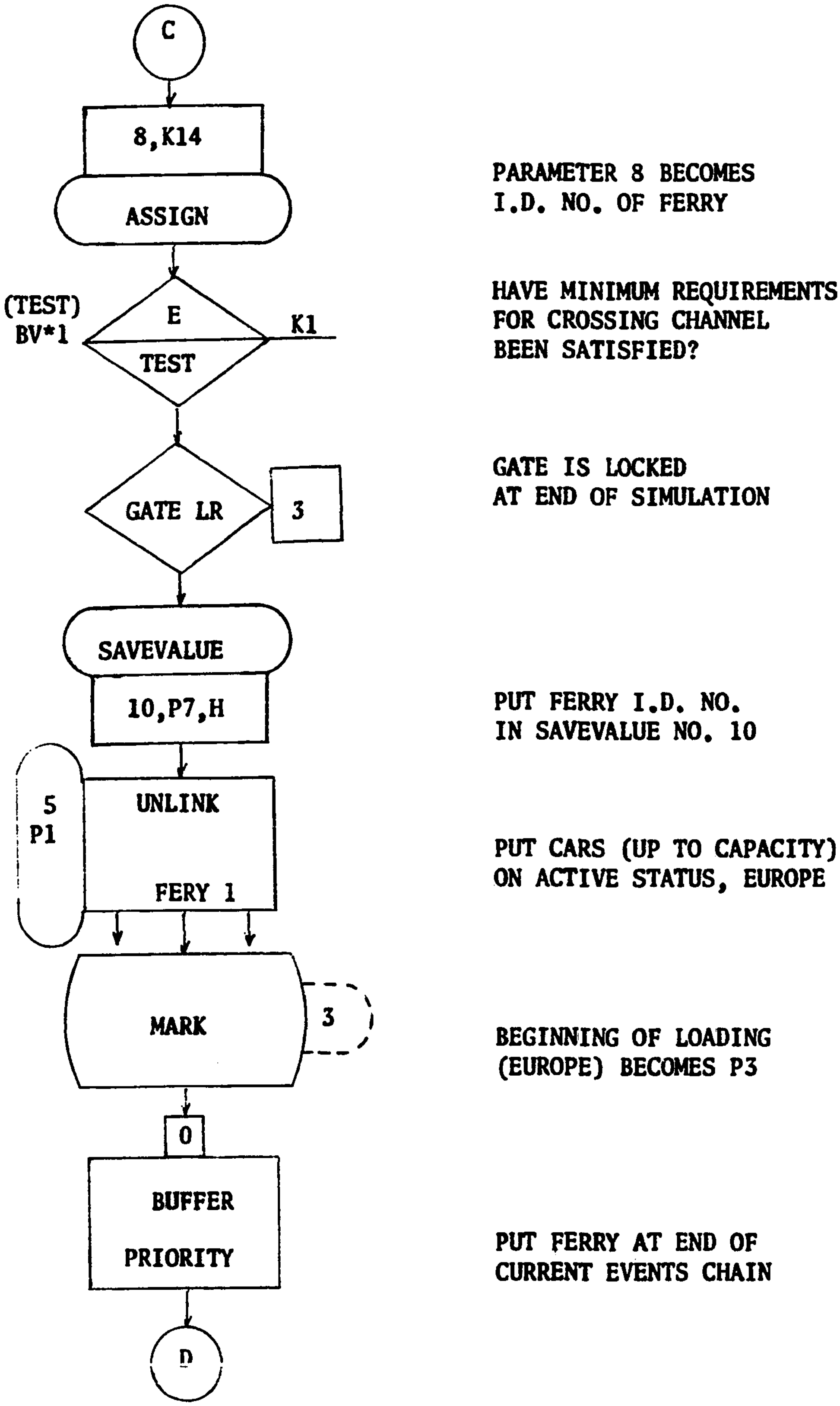


Figure 34. (Continued)

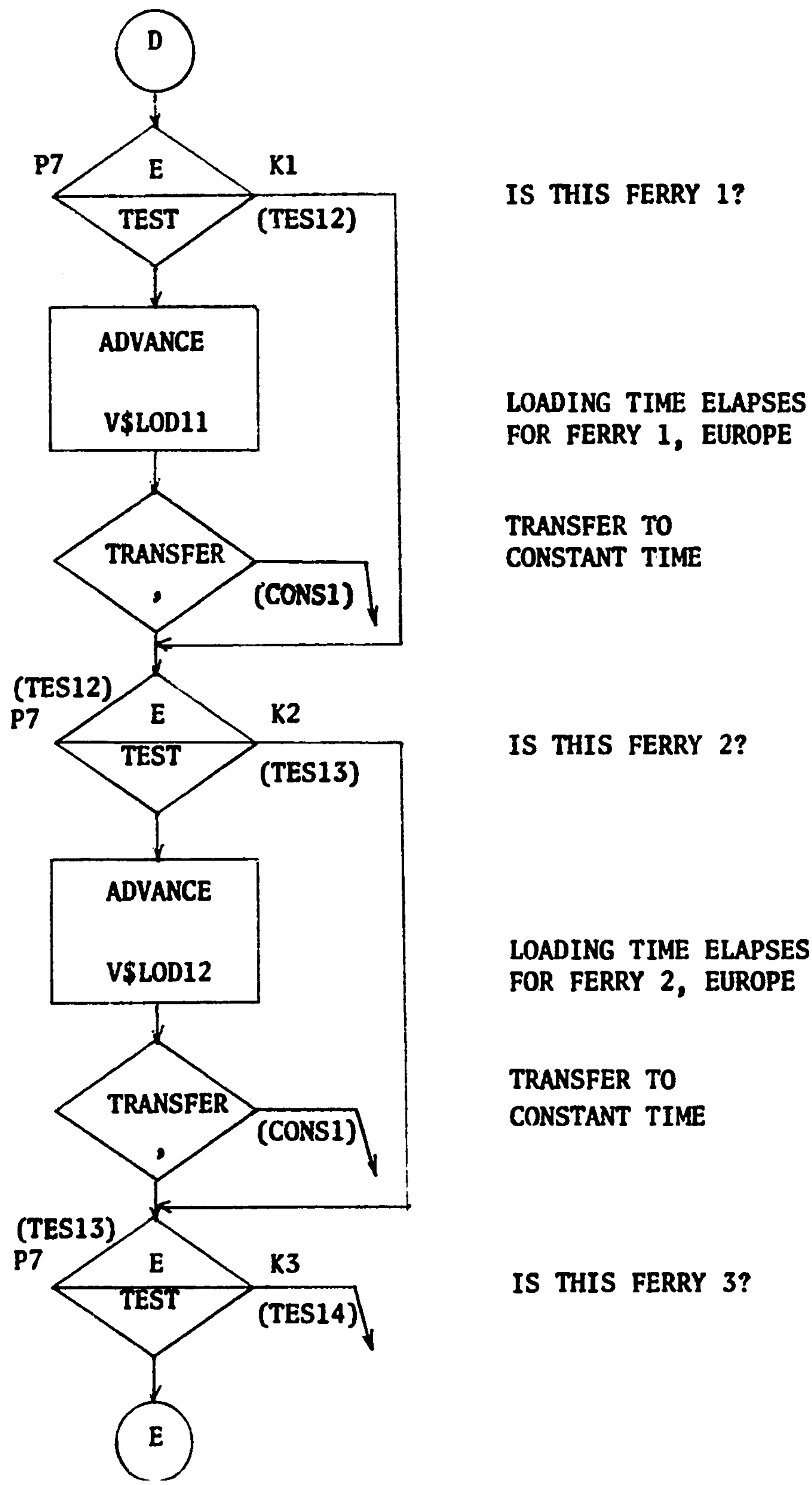
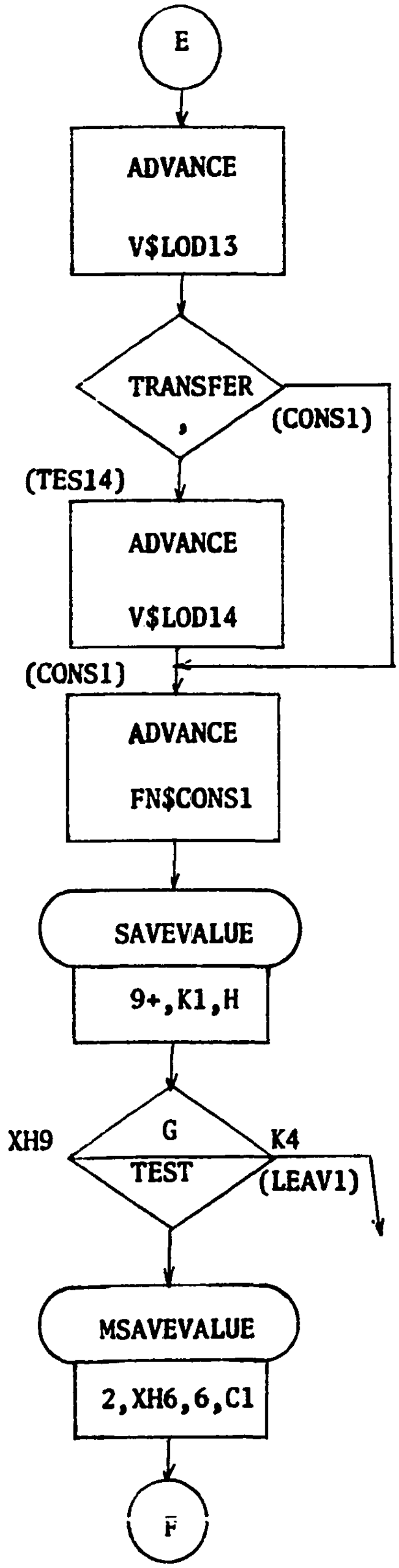


Figure 34. (Continued)



LOADING TIME ELAPSES
FOR FERRY 3, EUROPE

TRANSFER TO
CONSTANT TIME

LOADING TIME ELAPSES
FOR FERRY 4, EUROPE

CONSTANT TIME
ELAPSES

UPDATE COUNTER
FOR FERRIES

IS THIS THE
5 TH FERRY?

PUT DEPARTURE TIME
FROM EUROPE IN ROW 1,COL.6

Figure 34. (Continued)

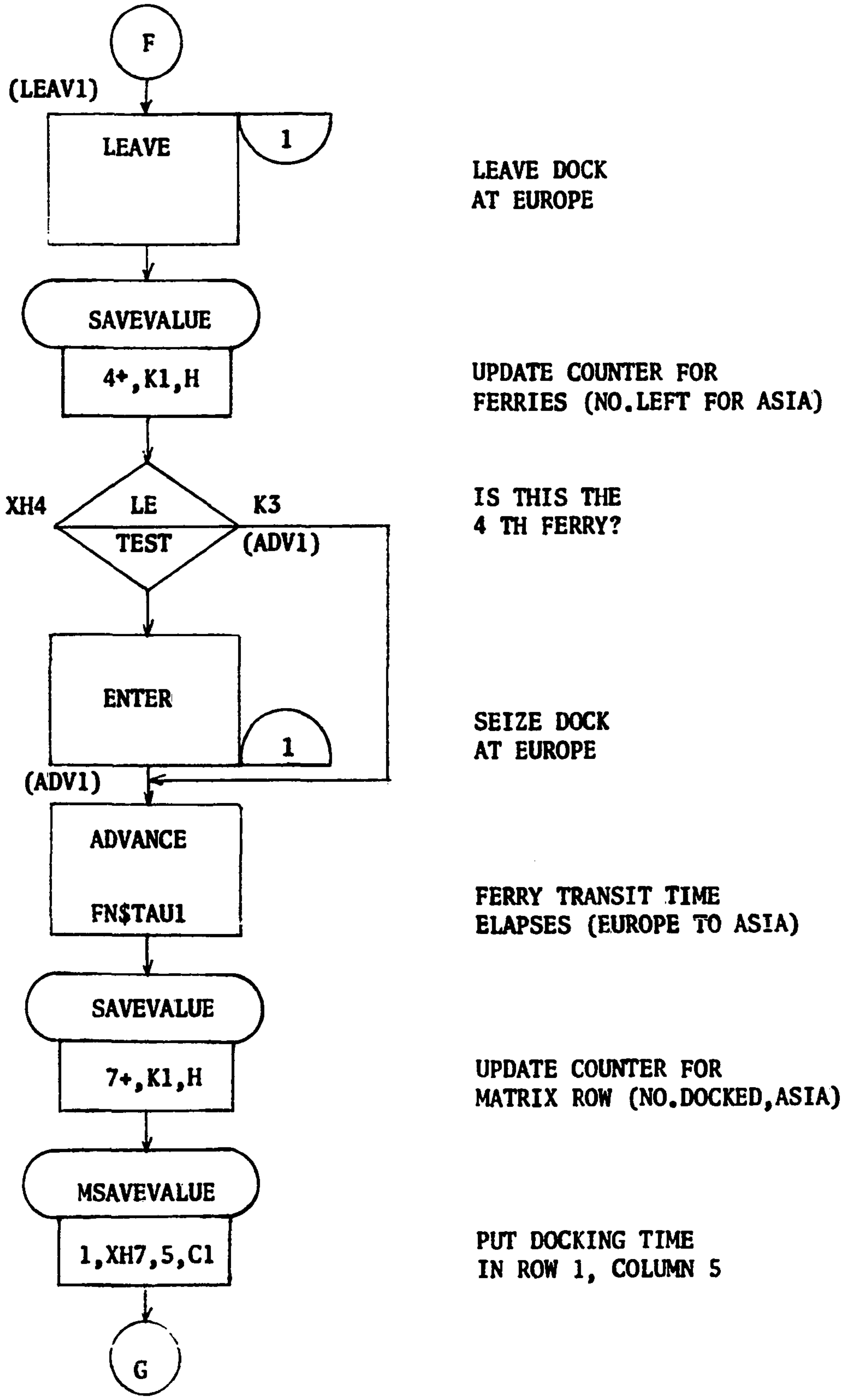


Figure 34. (Continued)

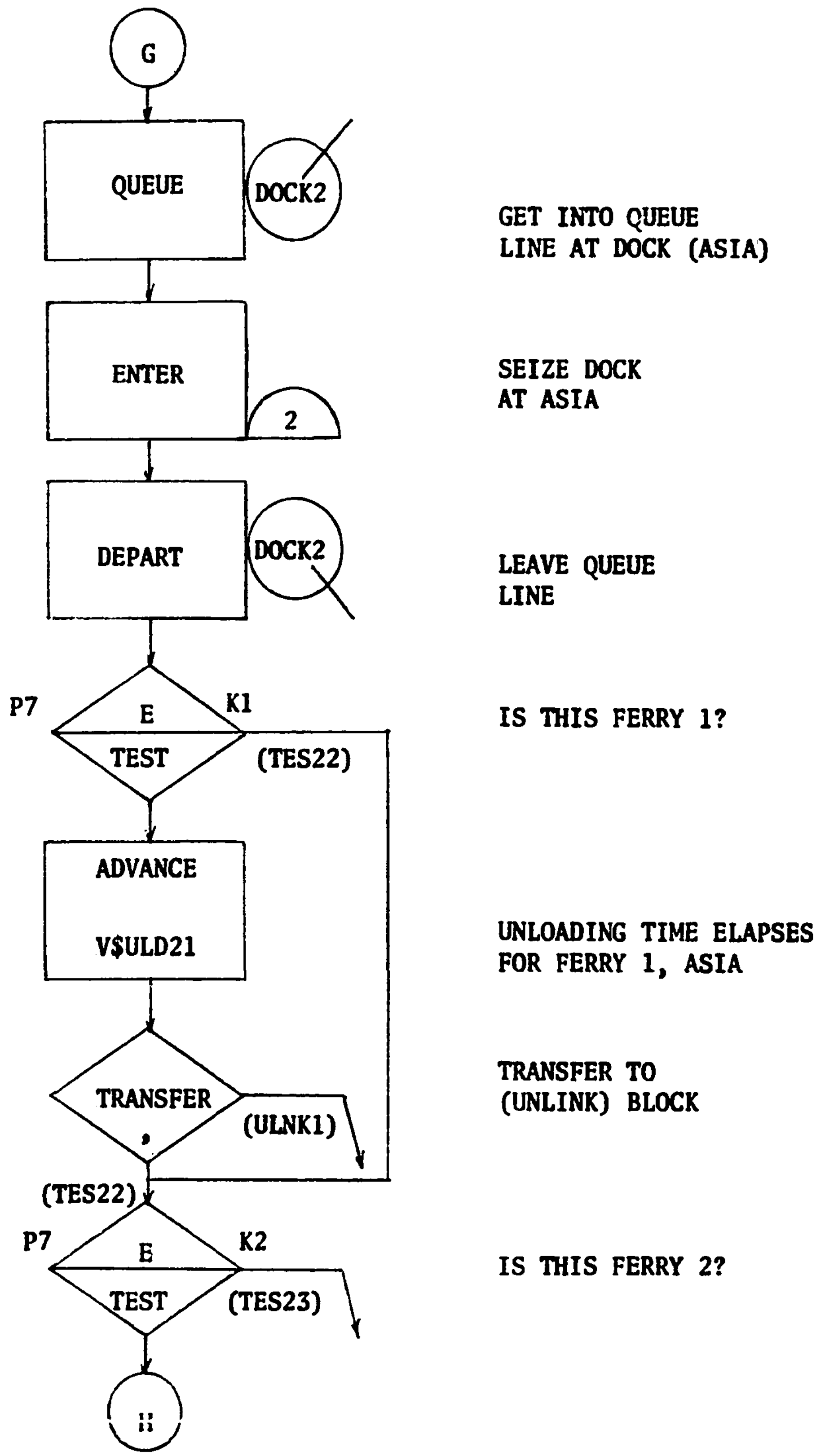
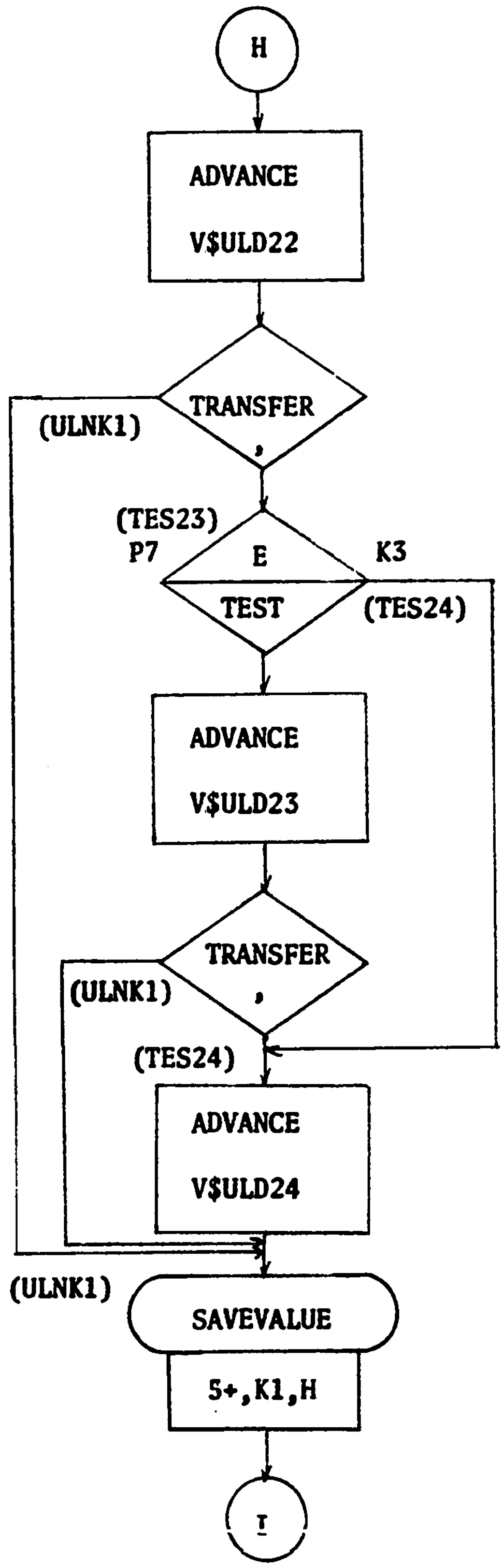


Figure 34. (Continued)



UNLOADING TIME ELAPSES
FOR FERRY 2, ASIA

TRANSFER TO
(UNLINK) BLOCK

IS THIS FERRY 3?

UNLOADING TIME ELAPSES
FOR FERRY 3, ASIA

TRANSFER TO
(UNLINK) BLOCK

UNLOADING TIME ELAPSES
FOR FERRY 4, ASIA

UPDATE COUNTER
FOR MATRIX ROW

Figure 34. (Continued)

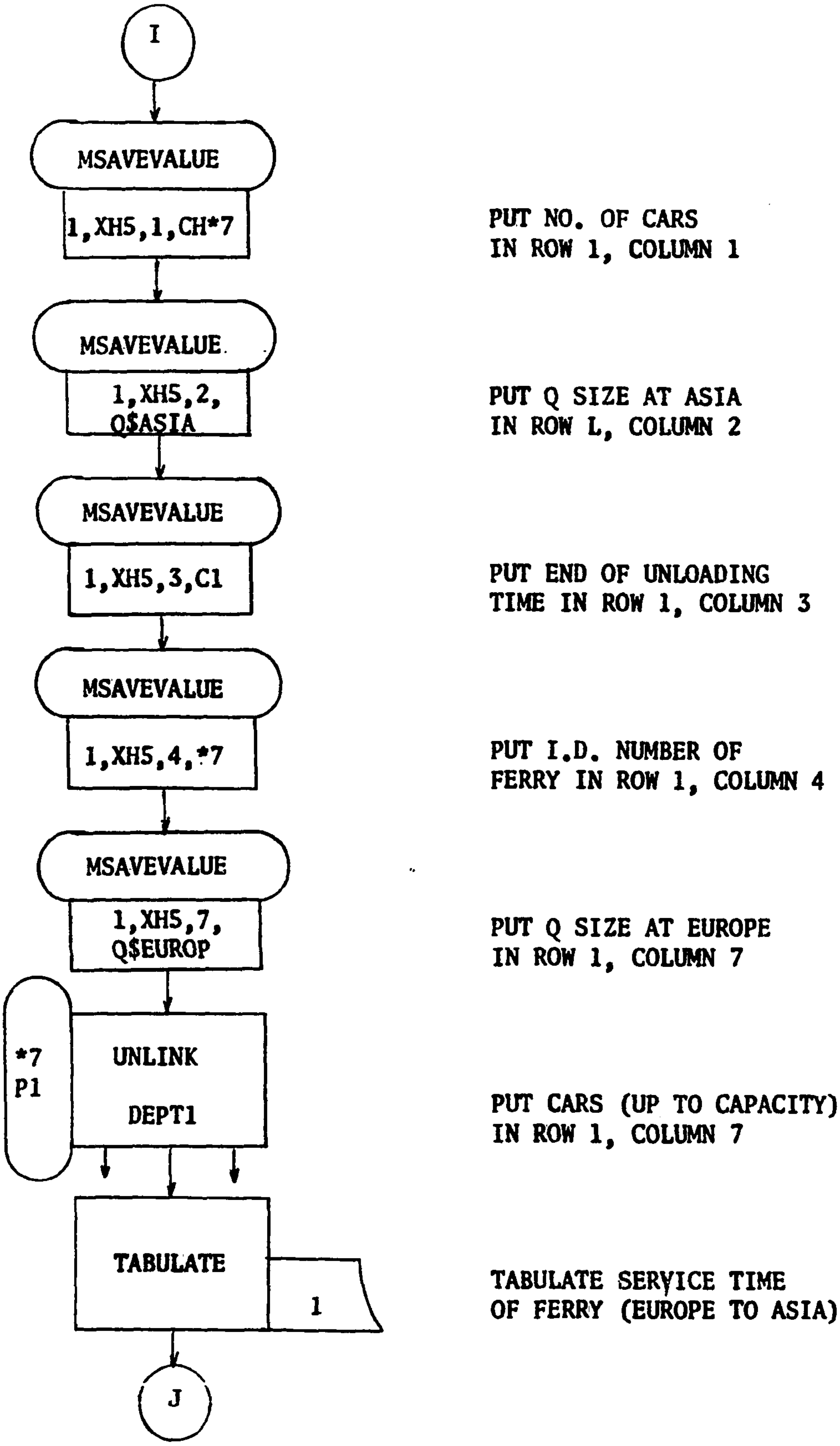
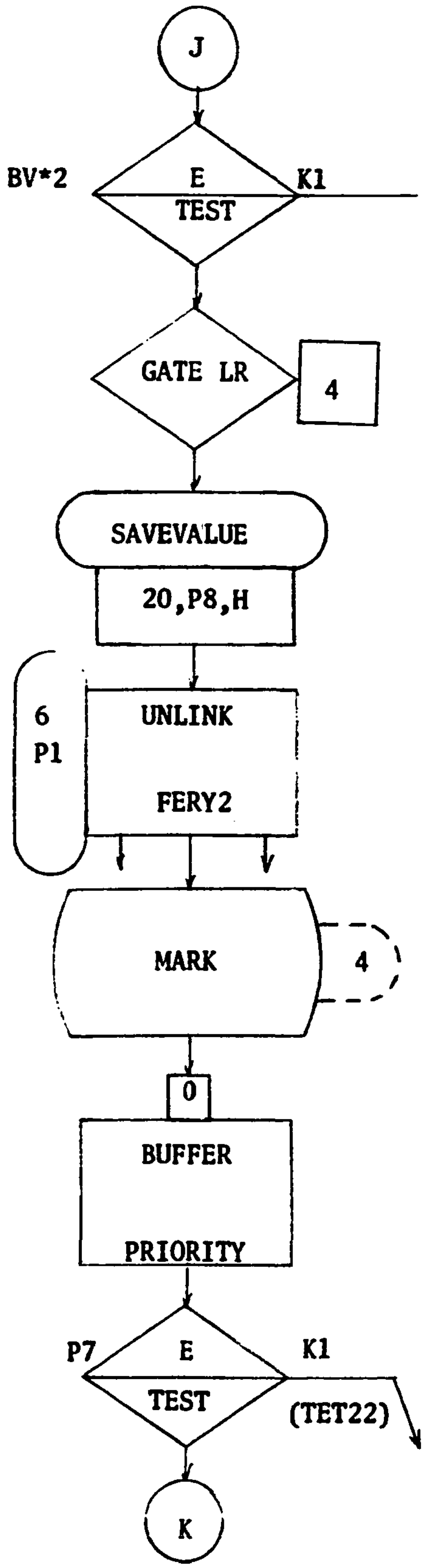


Figure 34. (Continued)



HAVE MINIMUM REQUIREMENTS FOR CROSSING CHANNEL BEEN SATISFIED?

GATE IS LOCKED AT END OF SIMULATION

PUT FERRY I.D. NO. IN SAVEVALUE NO. 20

PUT CARS (UP TO CAPACITY) ON ACTIVE STATUS, ASIA

START OF LOADING TIME (ASIA) BECOMES P4

PUT FERRY AT END OF CURRENT EVENTS CHAIN

IS THIS FERRY 1?

Figure 34. (Continued)

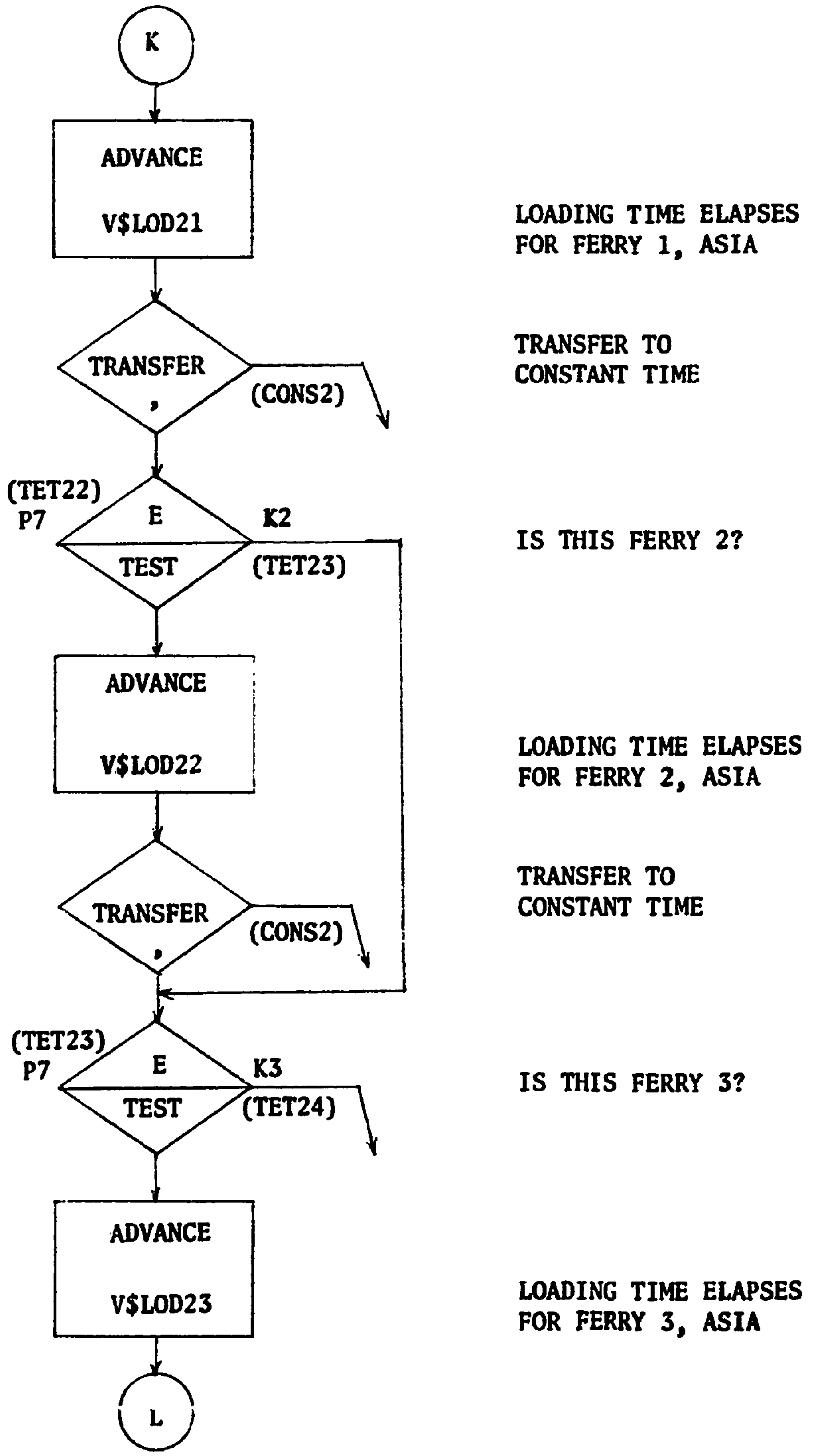


Figure 34. (Continued)

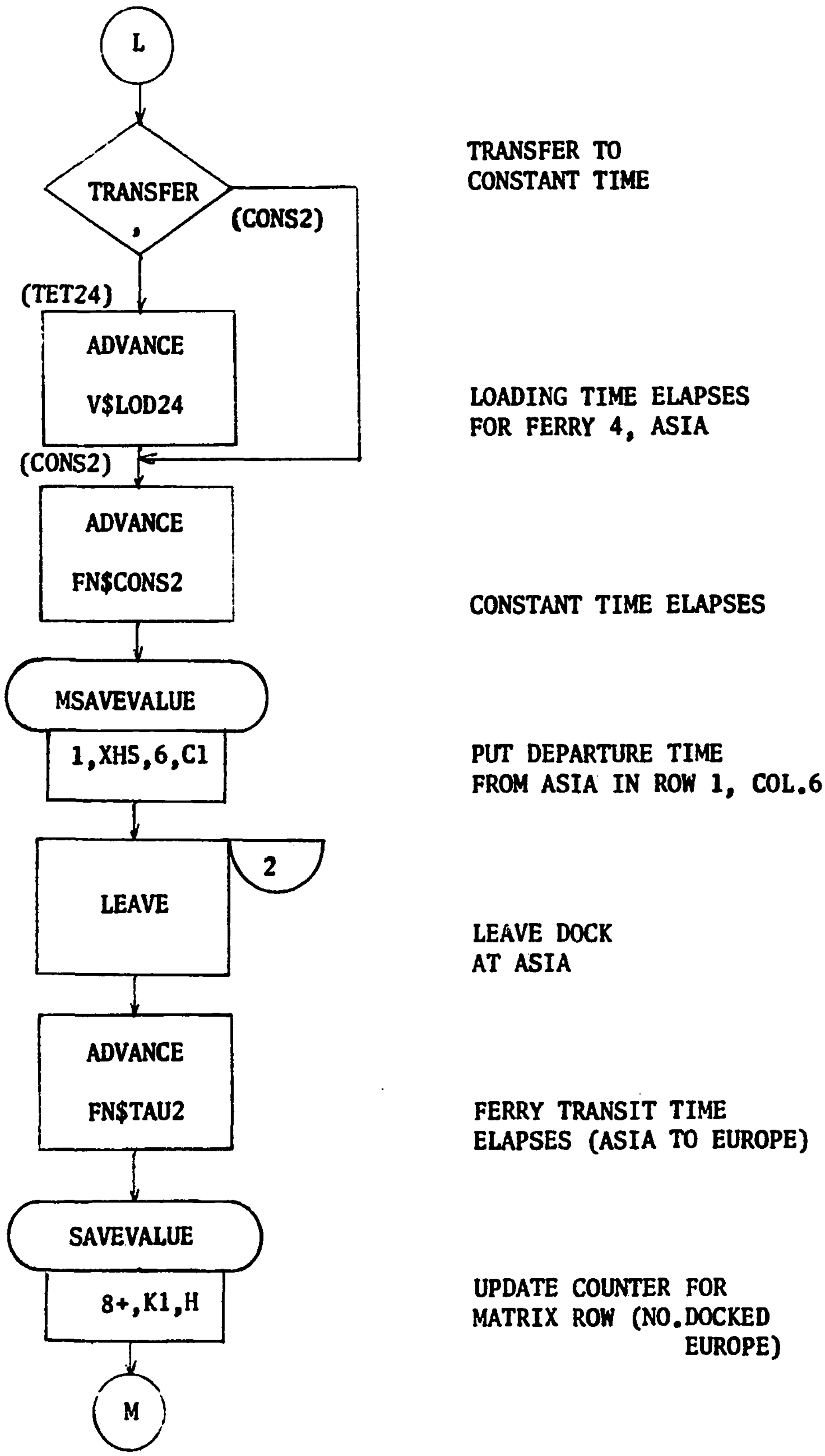


Figure 34. (Continued)

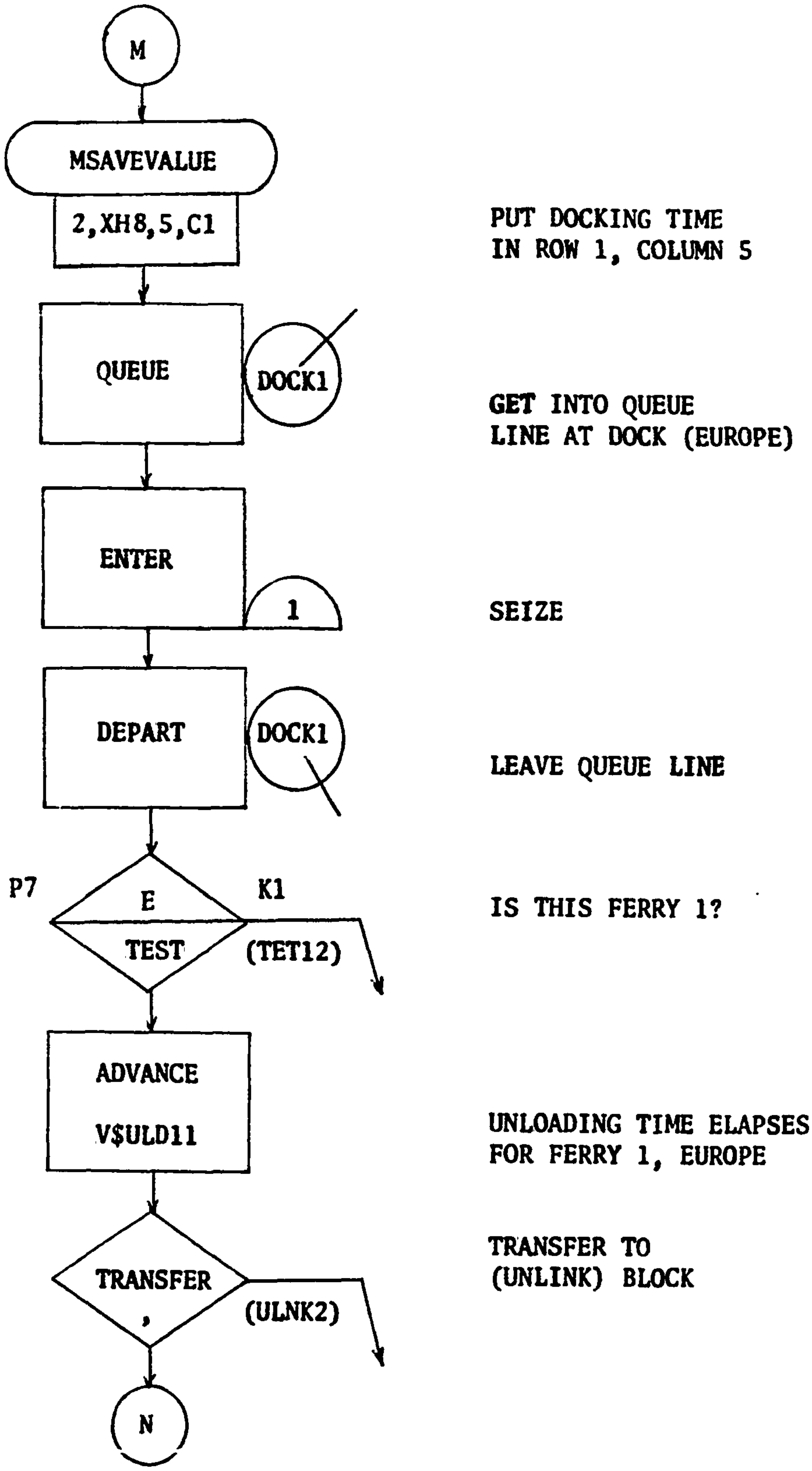


Figure 34. (Continued)

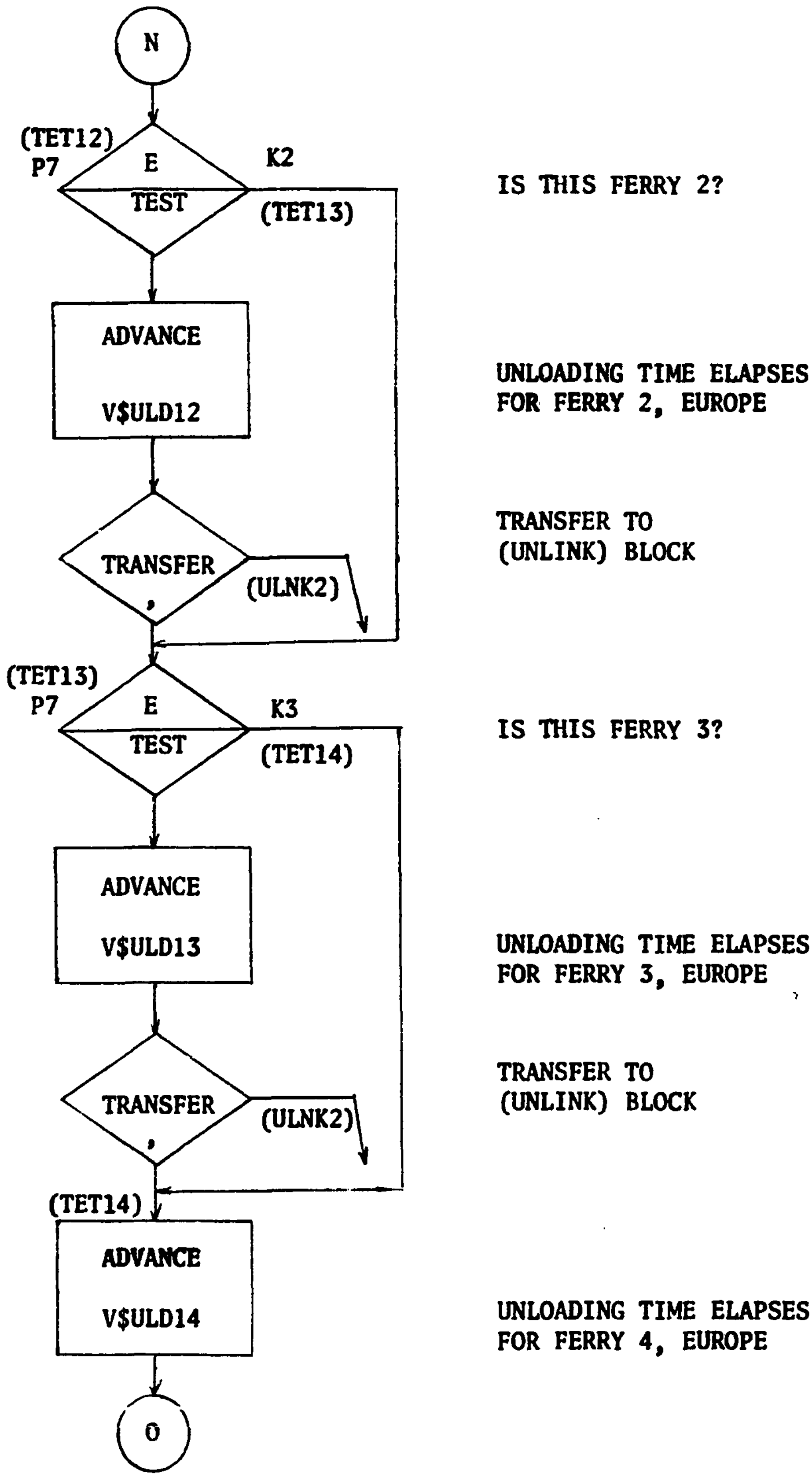


Figure 34. (Continued)

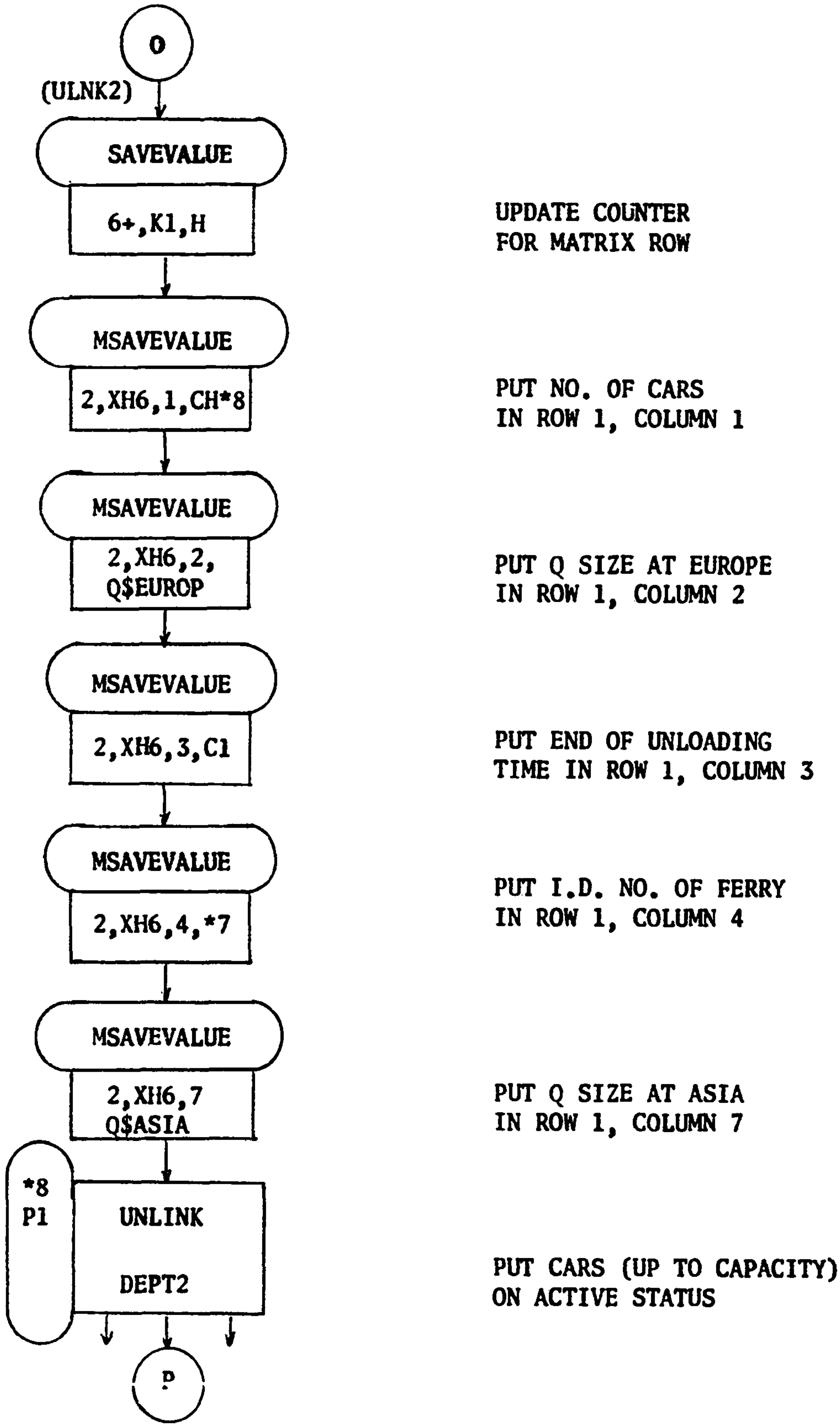
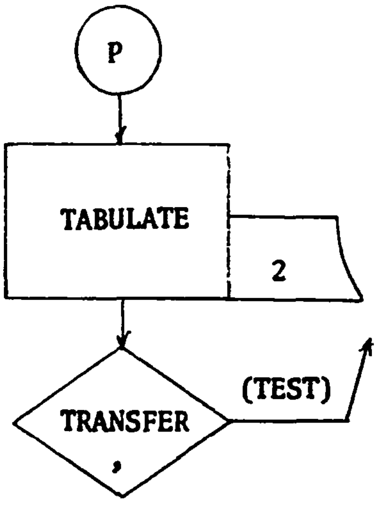


Figure 34. (Continued)



TABULATE SERVICE TIME
OF FERRY (ASIA TO EUROPE)

TRANSFER BACK
TO TEST

Figure 34. (Continued)

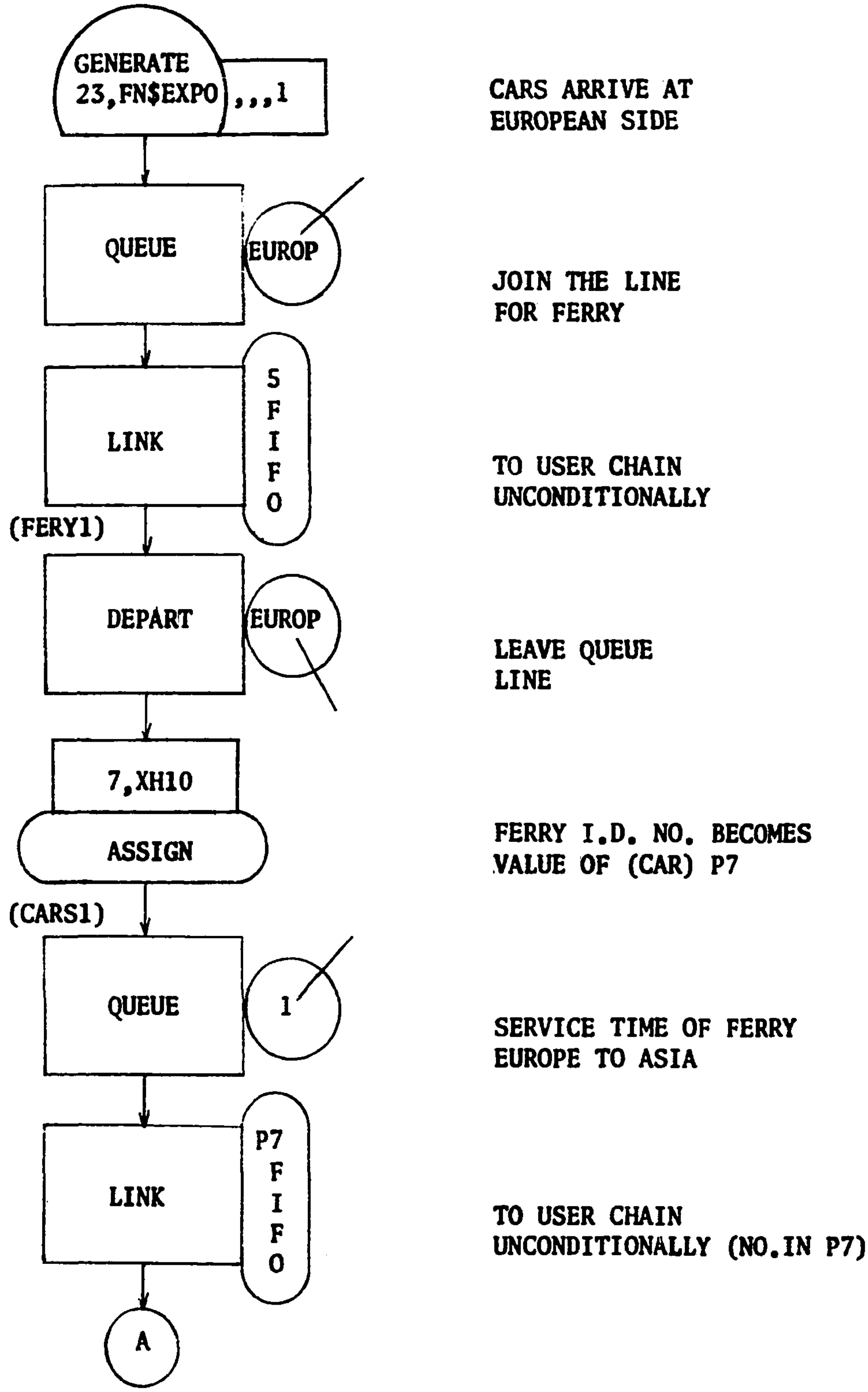


Figure 35. European segment (side A)

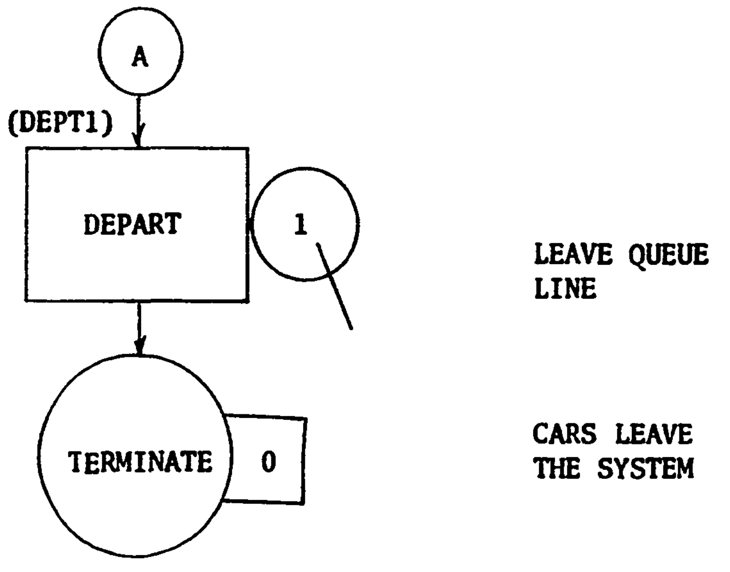


Figure 35. (Continued)

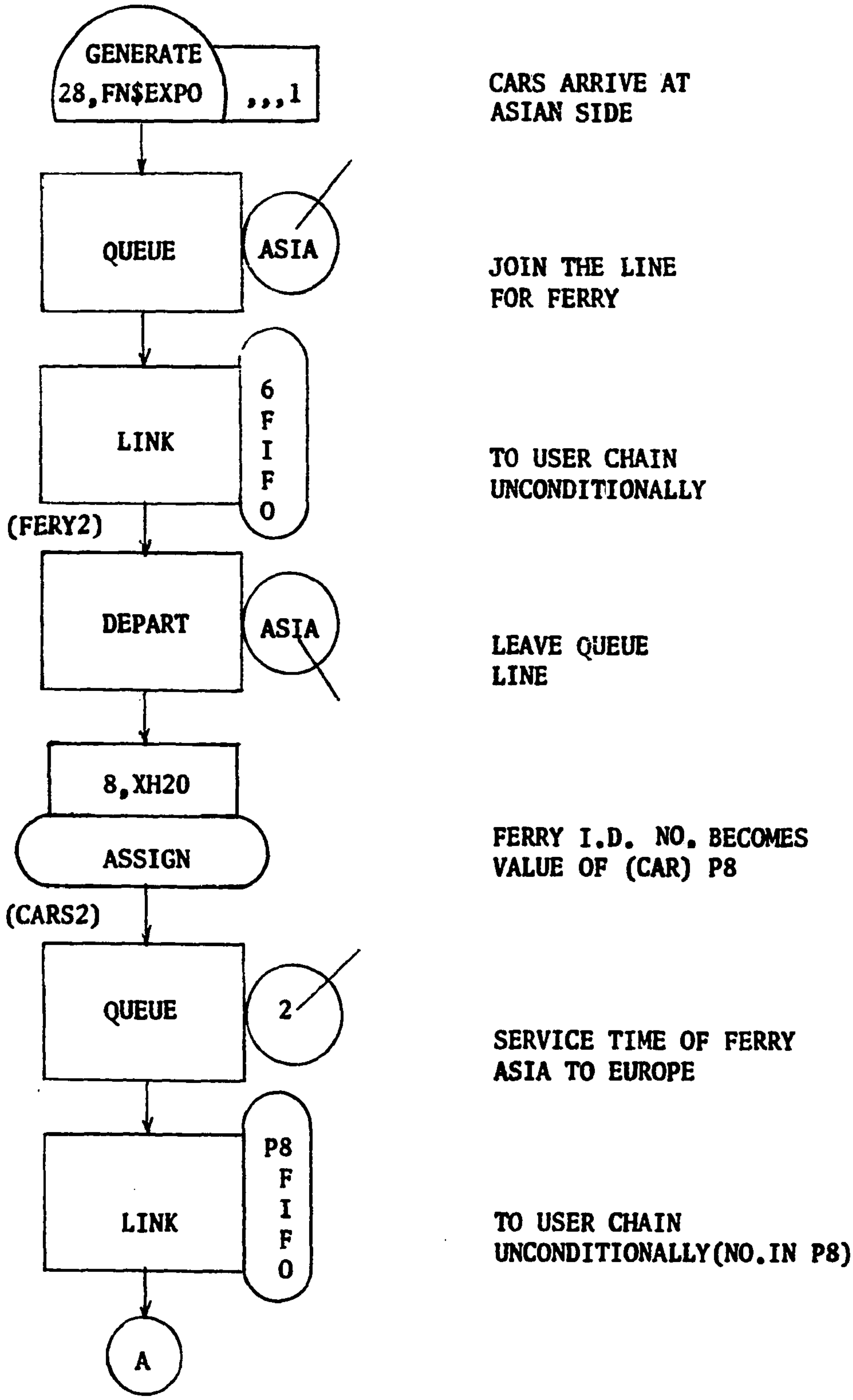


Figure 36. Asian segment (side B)

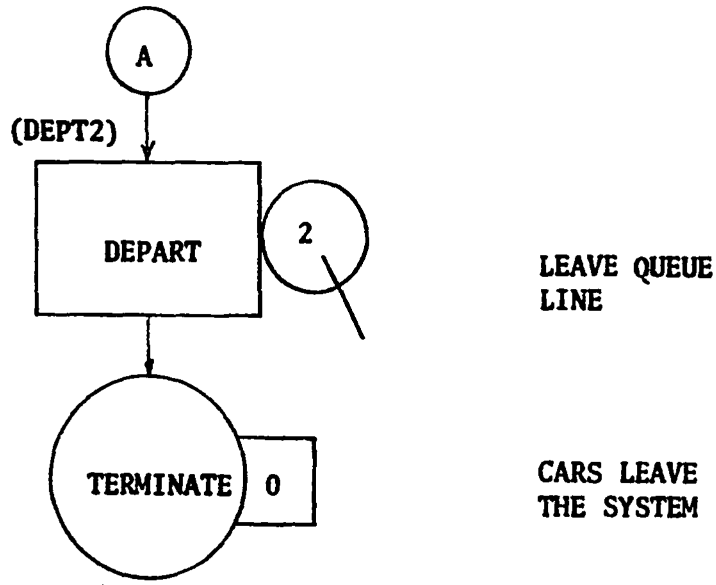


Figure 36. (Continued)

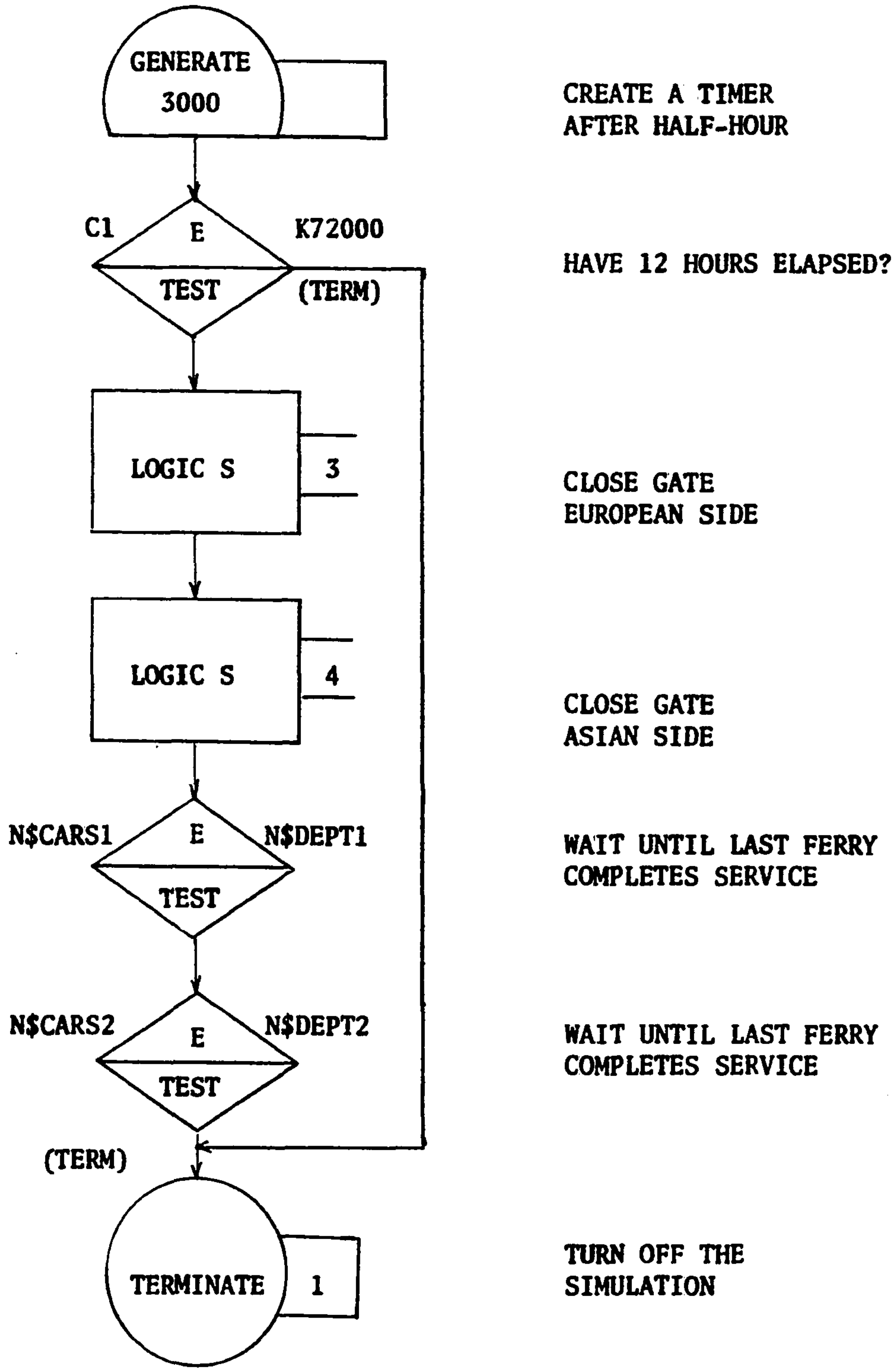


Figure 37. Timer segment

C. GPSS Definitions in the Program

GPSS ENTITY	INTERPRETATION
Transactions	
Storage 1 initialization segment	A dummy ferry
Ferry segment	One of the ferries
European segment	A car
Asian segment	A car
Timer segment	A timer
Parameters	
P1	Capacity of a ferry and Boolean Variable number for side A (Europe)
P2	Boolean Variable number for side B (Asia)
P7, P8	Identification number of the ferry
P11, P12	Counters for matrix row
Functions	
EXPO	Exponential interarrival time distribution of cars
TAU1	Transit time distribution of ferry from Europe to Asia
TAU2	Transit time distribution of ferry from Asia to Europe
CONS1	Time distribution not associated with loading and unloading function of ferry, European side
CONS2	Time distribution not associated with loading and unloading function of ferry, Asian side

GPSS ENTITY	INTERPRETATION
<u>Logic Switches</u>	
3,4	When set at the end of simulation program in the one ferry model, it allows the ferry to complete its service. For the multi-ferry case, last customers are served.
<u>Queues</u>	
EUROP, ASIA	Queue line of cars waiting to take the ferry from European and Asian sides respectively
1,2	Service time of ferry (loading time + transit time + unloading time) going from Europe to Asia and Asia to Europe respectively
DOCK1, DOCK2	Waiting line of ferry before it is allowed to unload at European and Asian sides respectively
<u>Storages</u>	
1,2	Storages simulating the number of ferry docks on European and Asian sides respectively
<u>Tables</u>	
1,2	Service time statistics of ferries traveling from Europe to Asia and Asia to Europe respectively
3,4	Car waiting time statistics at European and Asian sides respectively
<u>Variables</u> (Arithmetic)	
LOD11,LOD12,LOD13,LOD14	Loading function at European side for ferries 1,2,3 and 4 respectively
LOD21,LOD22,LOD23,LOD24	Loading function at Asian side for ferries 1,2,3 and 4 respectively

GPSS ENTITY	INTERPRETATION
ULD11,ULD12,ULD13,ULD14	Unloading function at European side for ferries 1,2,3 and 4 respectively
ULD21,ULD22,ULD23,ULD24	Unloading function at Asian side for ferries 1,2,3 and 4 respectively
Variables (Boolean)	
42,51,64	Variables which are true only when the conditions to travel from Europe to Asia are satisfied
43,52,65	Variables which are true only when the conditions to travel from Asia to Europe are satisfied
Savevalues (Halfword)	
1,2,3,	Minimum number cars waiting on shore necessary before each ferry is allowed to leave
4,9	Counters for number of ferries
5,6,7,8	Counters for matrix row
10,20	Ferry I.D. numbers
Savevalues (Fullword)	
1,2	Used to pass the absolute clock time to Weibull generators
3,4,5	Dummy arguments required for proper linking
6,7	Values associated with Weibull inter-arrival times return from generator
Msavevalues	
1,2	Matrices containing statistics on docking, end of unloading, and departure times, queue sizes on both sides, number of cars carried and I.D. numbers of ferries docking at Asian and European sides respectively

GPSS ENTITY	INTERPRETATION
User Chains	
1,2,3,4	Chains which contain number of cars serviced by ferries going from Europe to Asia
5,6	Chains which contain number of arrivals waiting at European and Asian sides respectively
11,12,13,14	Chains which contain number of cars serviced by ferries going from Asia to Europe

Time Unit 0.01 minute

Simulation Period

Simulation for each specific case is carried out over a twelve-hour period with snaps every half-hour, to investigate transient behavior of the system.

Output Editing

GPSS standard output is suppressed so that only meaningful portions are printed out.

Run Time

Total run time (including assembly) for a twelve-hour simulation period, using half-hour snaps, took about 1.5 minutes.

D. Sample Output

USER CHAIN	TOTAL ENTRIES	AVERAGE TIME/TRANS	AVERAGE CONTENTS	MAXIMUM CONTENTS
1	673	2348.898	20.506	42
2	1024	2459.562	32.671	64
3	672	2332.625	20.334	42
4	854	2308.744	25.576	51
5	3223	3136.019	131.114	208
6	2653	795.334	27.371	105
11	580	2206.112	16.598	42
12	809	2183.855	22.918	64
13	627	2276.505	18.516	42
14	637	2187.827	18.078	51

QUEUE	MAXIMUM CONTENTS	AVERAGE CONTENTS	TOTAL ENTRIES	ZERO ENTRIES	PERCENT ZEROS	AVERAGE TIME/TRANS
1	157	99.089	3223		.0	2370.025
2	157	76.111	2653		.0	2211.571
DOCK2	2	.197	69	37	53.6	220.811
ASIA	105	27.371	2653	2	.0	795.334
EUROP	208	131.114	3223	1	.0	3136.019
DOCK1	3	.164	67	32	47.7	188.731

STORAGE	CAPACITY	AVERAGE CONTENTS	AVERAGE UTILIZATION	ENTRIES	AVERAGE TIME/TRAN
1	1	.815	.815	71	885.408
2	1	.796	.796	69	890.217

CONTENTS OF HALFWORD SAVEVALUES (NON-ZERO)

SAVEVALUE	NR,	VALUE	NR,	VALUE	NR,	VALUE	NR,	VALUE	NR,	VALUE	NR,	VALUE
	4	71	5	69	6	67	7	69	8	67	9	71

MATRIX FULLWORD SAVEVALUE 1

	COLUMN 1	2	3	4	5	6	7
ROW 1	0	34	1171	4	1171	1518	65
2	1	11	1526	1	1285	1704	81
3	0	7	1704	3	1450	1849	93
4	0	3	1849	2	1562	1983	97
5	51	105	4800	4	4443	5367	98
6	42	97	6059	1	5723	6830	92
7	42	105	7166	3	5908	7850	90
8	64	103	8234	2	6627	8832	144
9	51	68	9189	4	8778	9742	134
10	42	68	10490	1	10154	11282	97
11	42	65	11618	3	10906	12286	100
12	64	66	12740	2	12356	13343	107
13	42	37	13679	1	13127	14232	151

14	51	28	14589	4	13307	14886	143
15	42	37	15788	3	15452	16674	133
16	64	50	17059	2	16675	17685	100
17	42	52	18171	1	17835	18788	103
18	51	45	19145	4	18427	19613	151
19	42	59	20714	3	20378	21584	112
20	64	64	21968	2	21355	22734	119
21	51	47	23091	4	21918	23578	121
22	42	31	23914	1	22213	24459	155
23	42	41	24979	3	24643	25730	150
24	64	57	26642	2	26258	27191	119
25	51	44	27595	4	27238	28236	119
26	42	41	28673	1	28337	29383	166
27	42	54	30182	3	29846	30962	129
28	64	56	31346	2	30870	31855	143
29	51	42	32371	4	32014	33049	187
30	42	29	33385	1	32472	33819	185
31	42	43	34606	3	34270	35235	181
32	64	52	35935	2	35551	36403	153
33	51	41	37145	4	36788	37560	170
34	42	36	37896	1	37421	38427	146
35	42	39	38865	3	38529	39614	187
36	64	37	39998	2	39612	40510	137
37	51	28	40942	4	40585	41232	181
38	42	57	43023	1	41687	42771	130
39	42	63	43302	3	42966	44042	127
40	64	58	44426	2	43636	45028	128
41	51	32	45385	4	43697	45705	163
42	42	37	46497	1	46161	47113	108
43	42	41	47608	3	47272	48436	113
44	64	42	48820	2	48048	49228	128
45	51	28	49585	4	49038	50067	154
46	42	37	50652	1	50316	51374	153
47	42	42	51779	3	51443	52512	100
48	64	48	52896	2	52485	53492	113
49	51	33	53849	4	53096	54358	160
50	42	53	55533	1	55197	56194	119
51	42	53	56530	3	55720	57489	113
52	64	70	57873	2	57057	58621	128
53	51	44	58978	4	57424	59427	128
54	42	34	59982	1	59646	60807	177
55	42	42	61143	3	60751	61965	161
56	64	48	62422	2	62038	63064	115
57	51	54	63871	4	63514	64680	144
58	42	44	65016	1	64072	65657	138
59	42	46	65993	3	64984	66711	187
60	64	38	67095	2	66025	67605	178
61	51	33	68168	4	67811	68568	185
62	42	35	69139	1	68803	69745	196
63	42	69	71060	3	70724	71770	153

MATRIX FULLWORD SAVEVALUE 2

	COLUMN	1	2	3	4	5	6	7
ROW	1	34	146	2843	4	2605	3357	33
	2	11	124	3445	1	2829	4156	54
	3	7	116	4212	3	3117	4788	90
	4	3	98	4809	2	3135	5524	54
	5	51	133	6953	4	6596	7594	95
	6	42	149	8373	1	8037	8927	44
	7	42	136	9263	3	8842	9784	21
	8	64	155	10421	2	9973	11122	66
	9	51	146	11479	4	10764	12138	62
	10	42	138	12474	1	12088	13110	58
	11	42	153	13471	3	13405	14268	2
	12	64	167	15004	2	14556	15726	8
	13	37	139	16022	1	15394	16655	7
	14	28	138	16851	4	15996	17495	39
	15	37	142	18081	3	17785	18811	50
	16	50	163	19407	2	19057	20061	11
	17	42	140	20397	1	19747	20994	45
	18	45	142	21309	4	20896	21909	41
	19	42	158	22977	3	22641	23492	46
	20	64	175	24253	2	23805	25065	11
	21	47	165	25394	4	24726	26038	13
	22	31	150	26286	1	25577	27112	49
	23	41	157	27440	3	26950	28584	39
	24	57	180	28983	2	28278	29640	12
	25	44	167	29948	4	29412	30472	47
	26	41	164	30956	1	30628	31503	41
	27	42	192	32556	3	32220	33094	5
	28	56	190	33486	2	33014	34372	5
	29	42	183	34666	4	34180	35510	3
	30	29	185	35742	1	35260	36288	45
	31	42	196	36745	3	36409	37323	24
	32	52	192	37687	2	36699	38381	21
	33	41	189	38913	4	38626	39510	2
	34	36	171	39876	1	39588	40492	32
	35	39	190	41095	3	40783	41667	4
	36	37	186	41926	2	41461	42579	53
	37	28	160	42775	4	42388	43359	41
	38	42	166	44289	1	43953	44867	56
	39	42	170	45541	3	45205	46083	8
	40	58	172	46489	2	46057	47330	37
	41	32	161	47554	4	46898	48032	37
	42	37	160	48519	1	48223	49068	31
	43	41	163	49757	3	49429	50302	8
	44	42	158	50721	2	50427	51415	1
	45	28	142	51611	4	51241	52143	33

46	37	153	52861	1	52565	53705	46
47	42	168	54041	3	53691	54605	7
48	48	171	55083	2	54747	55890	37
49	33	146	56121	4	55597	56669	37
50	42	168	57776	1	57440	58419	67
51	42	170	58975	3	58639	59594	44
52	64	187	60242	2	59794	61120	10
53	44	170	61428	4	60553	61927	9
54	34	146	62199	1	61926	62933	40
55	42	164	63482	3	63146	64070	37
56	48	173	64406	2	63227	65116	28
57	51	199	66219	4	65862	66686	12
58	42	179	67119	1	66783	67718	0
59	42	198	68493	3	68157	69258	10
60	38	208	69524	2	68840	70320	10
61	33	179	70551	4	69849	71183	44

SERVICE TIME STATISTICS(EUROPE TO ASIA)
TABLE 1

ENTRIES IN TABLE	MEAN ARGUMENT	STANDARD DEVIATION
69	2313.854	412.000

UPPER LIMIT	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE
1100	0	.00	.0
1800	5	7.24	7.2
2500	45	65.21	72.4
3200	17	24.63	97.1
3900	2	2.89	100.0

RAMAINING FREQUENCIES ARE ALL ZERO

SERVICE TIME STATISTICS(ASIA TO EUROPE)
TABLE 2

ENTRIES IN TABLE	MEAN ARGUMENT	STANDARD DEVIATION
67	2189.164	255.500

UPPER LIMIT	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE
1100	0	.00	.0
1800	5	7.46	7.4
2500	60	89.55	97.0
3200	2	2.98	100.0

REMAINING FREQUENCIES ARE ALL ZERO

CAR WAITING TIME STATISTICS,EUROPE
TABLE 3

ENTRIES IN TABLE	MEAN ARGUMENT	STANDARD DEVIATION
3223	3136.019	642.000

UPPER LIMIT	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE
0	1	.03	.0
100	0	.00	.0
200	0	.00	.0
300	0	.00	.0

400	0	.00	.0
500	0	.00	.0
600	0	.00	.0
700	0	.00	.0
800	6	.18	.2
900	4	.12	.3
1000	3	.09	.4
1100	2	.06	.4
1200	4	.12	.6
1300	2	.06	.6
1400	6	.18	.8
1500	4	.12	.9
1500	9	.27	1.2
1700	6	.18	1.4
1800	12	.37	1.8
1900	26	.80	2.6
2000	29	.89	3.5
2100	52	1.61	5.1
2200	68	2.10	7.2
2300	56	1.73	8.9
2400	101	3.13	12.1
2500	115	3.56	15.6
2600	127	3.94	19.6
2700	142	4.40	24.0
2800	183	5.67	29.7
2900	168	5.21	24.9
3000	200	6.20	41.1
3100	190	5.89	47.0
3200	211	6.54	53.5
3300	190	5.89	59.4
3400	186	5.77	65.2
3500	171	5.30	70.5
3600	177	5.49	76.0
3700	175	5.42	81.4
3800	133	4.12	85.6
3900	999	3.07	88.6
4000	99	3.07	91.7
4100	76	2.35	94.1
4200	56	1.73	95.8
4300	48	1.48	97.3
4400	30	.93	98.2
4500	12	.37	98.6
4600	8	.24	98.8
4700	14	.43	99.3
4800	5	.15	99.4
4900	4	.12	99.5
5000	4	.12	99.7
5100	2	.06	99.7
5200	2	.06	99.8
5300	5	.15	100.0

REMAINING FREQUENCIES ARE ALL ZERO

CAR WAITING TIME STATISTICS,ASIA

TABLE 4

ENTRIES IN TABLE	MEAN ARGUMENT	STANDARD DEVIATION
2653	795.334	554.000

UPPER LIMIT	OBSERVED FREQUENCY	PER CENT OF TOTAL	CUMULATIVE PERCENTAGE
0	2	.07	.0
100	157	5.91	5.9
200	171	6.44	12.4
300	171	6.44	18.8
400	200	7.53	26.4
500	192	7.23	33.6
600	213	8.02	41.6
700	190	7.16	48.8
800	215	8.10	56.9
900	200	7.53	64.4
1000	162	6.10	70.5
1100	149	5.61	76.2
1200	125	4.71	80.9
1300	85	3.20	84.1
1400	63	2.37	86.5
1500	75	2.82	89.3
1600	71	2.67	92.0
1700	41	1.54	93.5
1800	31	1.16	94.7
1900	17	.64	95.3
2000	17	.64	96.0
2100	12	.45	96.4
2200	12	.45	96.9
2300	11	.41	97.3
2400	18	.67	98.0
2500	17	.64	98.6
2600	10	.37	99.0
2700	11	.41	99.4
2800	8	.30	99.7
2900	4	.15	99.8
3000	3	.11	100.0