SEPARATION OF SPATIAL AND TEMPORAL EFFECTS IN AN ULTRASONIC TRANSDUCER

Chung Chang and Wolfgang Sachse

Department of Theoretical and Applied Mechanics
Cornell University
Ithaca, New York 14850

ABSTRACT

This paper focuses on the separation of the smoothing effect of a signal caused by a finite-size transducer and that by an emission source of finite bandwidth. The total output response of a transducer as a receiver is a superposition of these two effects, resulting in a waveform whose frequency response is determined by the product of the spectrum of the excitation, the structure Green's function and the equivalent spectrum of the transducer's aperture. The true time function of a source can only be recovered by separating the aperture effect from the detected signal. Results are demonstrated with experimental data determined with capacitive transducers of varying apertures detecting signals on plates of various thicknesses.

INTRODUCTION

The smoothing of the received waveform, causing by the finite receiving area of a transducer, is known as the aperture effect. In general, a wave produced by a point source and detected by a point receiver is desired, because most theoretical calculations predict waves at a point such that a direct comparison between theoretical calculations and experimental results can be made. Most of the ultrasonic transducers have finite receiving areas. For a point receiver little energy can pass through it and no significant response can be detected. So it is very important to understand the aperture effect of a finite-size transducer on the measured waveform and know when a receiver can be approximated as a point receiver. Even when it can not be treated as a point receiver, one would like to remove or deconvolve the aperture effect through a proper processing scheme.

FORWARD FORMULATION

The vertical displacement excited by a point source and detected at the epicenter can be written as:

$$u_z(\underline{x},t) = \int_{-\infty}^{+\infty} s(\tau)d\tau \iint_A \rho_z(\underline{x}')\ G_{zz}(\underline{x}|\underline{x}',t-\tau)\ dA \tag{1}$$

where ρ_z is the spatial sensitivity function (which is a function of position inside the active region of the transducer) and s(t) is the source time function. Because the receiver is circularly symmetric with respect to an epicenter point source, it can be further reduced to a line receiver. This line receiver in general will not have a uniform sensitivity across its radius and it is this variation which is sought. The line receiver can be further discretized into several concentrated point receivers with appropriate weighting constants.

In order to see the smoothing effect which may result to the signals detected by this degenerated line receiver more clearly, the spatial sampling intervals are not equally spaced. The P-wave travel time from the source to each discrete receiver is consecutively delayed by one unit of the sampling time. The position of each point receiver from the epicenter can be calculated according to the following equation.

$$X_n = (n^2 \Delta t^2 C_p^{\,2}/H^2 + 2n\Delta t C_p/H)^{1/2}\ H \tag{2}$$

Here, Δt is the sampling time, H is the plate thickness, and C_p is the P-wave speed in the elastic plate. The weighting functions of the line receiver after discretization are:

$$W_i = A_i \cdot \bar{\rho}_i \tag{3}$$

where A_i is the area of each discretized ring and $\bar{\rho}_i$ is the average sensitivity over the ring.

The vertical displacement signal after discretizing both in time and space can be written in the matrix form as follows:

$$\begin{bmatrix} G_{x_0}(1) & & & & \\ G_{x_0}(2) & G_{x_1}(1) & & & \\ G_{x_0}(3) & G_{x_1}(2) & G_{x_2}(1) & & \\ G_{x_0}(4) & G_{x_1}(3) & G_{x_2}(2) & G_{x_3}(1) & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} W_0 \\ W_1 \\ W_2 \\ W_3 \\ \cdot \\ \cdot \end{bmatrix} * S(t) = \begin{bmatrix} U_z(1) \\ U_z(2) \\ U_z(3) \\ U_z(4) \\ \cdot \\ \cdot \end{bmatrix} \tag{4}$$

in which S(T) is the discretized source-time function for the point source and W(X) is the discretized weighting function. In the following, a capital index refers to a discretized variable. The subscript of G is the index of position and the index inside the parentheses indicates the discrete time. From this equation one can see that if the Green's functions are the same but only shifted in time, the multiplication between the G matrix and the spatial weighting function becomes a convolution operation. For a linear system, the operation of a shift and a sum corresponds to a convolution. The weighting function W(X) is now equivalent to a time function W(T). We note that W(X) is spatially not equi-spaced, though W(T) is equi-spaced in time. The equivalent spectrum of the transducer aperture is the Fourier transform of W(T). This function depends on the source-receiver arrangement. The case treated here is for a source above the center of the transducer. For off-center transducer-source arrangements, W(T) will be different, but the approach adopted here will still be valid.

RECOVERY OF SOURCE TIME FUNCTION

From Eq. 4 one can see that, if the aperture is not too big, only a few Green's functions are needed in the equation. We can further assume that all the Green's functions involved are position independent, they only shift in time. After deconvolution, the recovered time dependence is the convolution result of W(T)*S(T). For a known aperture, S(T) can be obtained by deconvolving the function W(T). For large aperture transducers, many Green's functions must be included in Eq. 4. If both W(T) and S(T) are unknown, an iterative deconvolution algorithm is used to obtain the unknowns [1,2].

EXPERIMENTAL RESULTS

Figure 1 is the experimental signal generated by breaking a glass capillary (0.05 mm ID and 0.08 mm OD) on top of a 1/4" thick glass plate and detected by a 1/4" diameter capacitive transducer. Previous work has shown that a capacitive transducer can respond as a displacement sensor [3] and it is useful for the quantitative characterization of AE sources [4,5,6]. To the authors' knowledge, in none of the published work has removal of the aperture effect been considered. That is, the previous experimental deconvolution results reported have implicitly assumed that the signals were generated by a point source and detected by a point receiver.

The deconvolution result for W(X) was obtained with an iterative method. The results are shown in Table I. The value of W(X) drops

Table I

Distance to Origin	Recovered W_i
$x_0 = 0.0$	0.01201
$x_1 = 0.26154$ h	0.01486
$x_2 = 0.37292$ h	0.01460
$x_3 = 0.46043$ h	0.01113
$x_4 = 0.53590$ h	0.00534
$x_5 = 0.60386$ h	-0.00043

almost to zero at 0.536 h. This apparent radius of the transducer is greater than the actual size of the transducer probably indicating the extent of the fringing effect of the capacitive transducer. The recovered source-time function is compared in Fig. 2 with the recovered source-time function which was obtained if a point source and a point receiver are assumed. It is seen that the transducer aperture effect leads to inaccuracies in the recovered source-time function.

CONCLUSION

A weighting function is defined for specifying the spatial sensitivity of a transducer. In obtaining a solution to the inverse source problem, the true time characteristics of the source are not recovered, unless the receiver's aperture is accounted for. If both the weighting function and

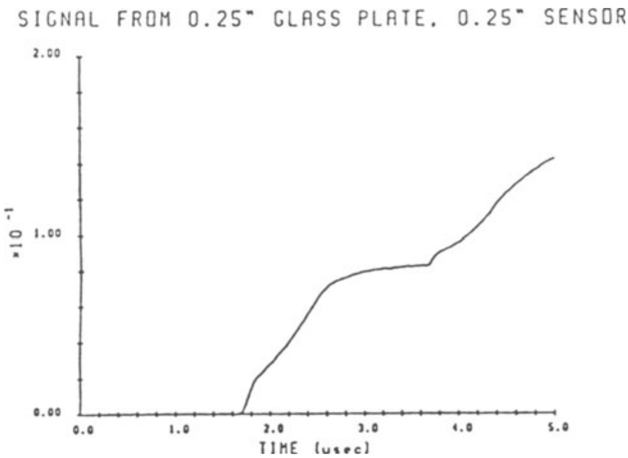

Fig. 1. Experimental signal generated by breaking glass capillary.

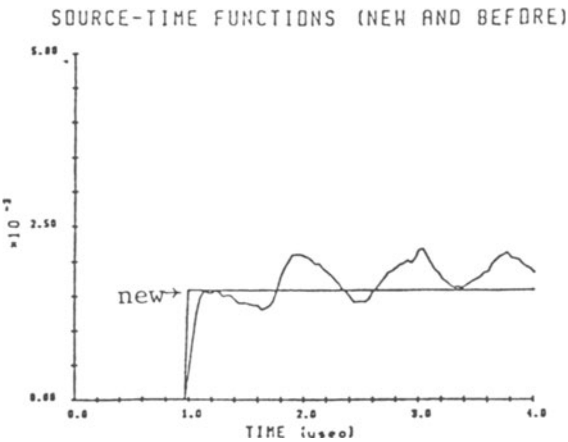

Fig. 2. Source-time functions (new and before).

source time function are not known, an iterative deconvolution algorithm must be used to recover the unknowns.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation through grants to the Materials Science Center and the College of Engineering at Cornell University. It is a pleasure to also acknowledge the special fellowship support provided to C. Chang by the Schlumberger-Doll Research Center.

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