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# THE MAGNETIC CIRCULAR DICHROISM OF 

 BIOLOGICALLY INTERESTITMG MOLECULES byMichael Eäward McCarville

A Dissertation Submitted to the Graduate Faculty in Partial Fulfillment of The Requirements for the Degree of DOCTOR OF PHILOSOPEY<br>Major Subject: Biophysics

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## History

In the course of searching for a relationship between light and magnetism during the nineteenth century, a number of magneto-optical effects were discovered. In 1845 Faraday found that plane polarized light is rotated as it passes through a piece of lead glass in parallel with a magnetic field (1). Experimentation later indicated that any substance observed in this manner, whether diamagnetic or paramagnetic, exhibits the rotation of light to some degree. The discovery by Zeeman in 1896 that certain emission (or absorption) lines are split by a magnetic field provided an explanation for much of the Faraday effect observed in atoms (2).

During the early 1930's a practical quantum mechanical theory was developed for the magnetic optical activity of molecules (3). Also at about this time one of the first measurements of magnetic circular dichroism (MCD) was made through the absorption band of a molecular substance in solution (4). However, most of the applications of the Faraday effect were made on atomic or very simple linear molecular systems. An extensive bibliography of magneto-optics has recently been publishea which makes reference to much of this early work (5). The experimental apparatus necessary for sensitive and accurate measurements of circular dickroism was not available commercially until about 1960 (6). Modification of these machines for magnetic work has enabled a number of workers to explore the usefulness of the technique. An extension of the theory to cover the broad band systems of molecules was made by Stephens (7).

Several workers are active in the MCD field today. The laboratory of Dr. Paul Schatz at the University of Virginia is studying various problems, but the emphasis of their work is on inorganic molecules. The techniques of MCD are often quite quantitative for these molecular systems and the results are very helpful for the assignment of energy levels to electronic transitions (8).

Dr. Carl Djerassi at Stanford has had workers making measurements of MCD, however in the results published so far little or no attempt has been made to interpret the spectra (9).

Dr. A. Dratz recently published his Ph.D. thesis from the University of California at Berkeley where his research included the study of the electronic structure of the metal porphyrins by the method of MCD (10).

In this laboratory the initial attempts to measure MCD were made for two reasons: first, because the necessary apparatus was available and second, because it seemed MCD might provide an optical method of obtaining the type of information usually requiring measurements of magnetic susceptibility or electron spin resonance. In particular it was hoped that the magnetic state of the respiratory protein, hemocyanin, might be determined by this new technique. However, we soon found both experimentally and from a study of the theory of the Faraday effect that MCD would not in general be sensitive to effects due to electron spin. In the process of making these first measurements a number of interesting but sometimes puzzling spectra turned up. The MCD spectra of hemoglobin and cytochrome $c$ were particularly interesting, the porphyrin ring
prosthetic group being the cause of the absorption spectra. Investigation of this ring system by MCD and correlation of the results with molecular orbital theory has been advanced by the graduate work of Dratz mentioned above (10). It was decided here to design experiments which would help to interpret the MCD spectra of biologically important compounds. As a result of both experimental and theoretical limitations, the information which can be drawn from the MCD of these organic molecules is qualitative or semi-quantitative. Despite these short-comings our research has yielded a number of interesting developments. A portion of these results have been published in a series of papers (11,12,13,14). The underlying motive in this investigation is to determine the usefulness of the MCD technique for problems in molecular biology.

## II. EXPERIMENTAL

## Instrumentation

The MCD spectra were obtained using a Roussel-Jouan dichrograph which had undergone the following modifications:

1. Replacement of the original monochromator with a Cary Model 15 double prism monochromator.
2. Replacement of the original light sources with a 450 watt xenon arc.
3. Replacement of the original modulating crystal with a BairdAtomic crystal of potassium dideuterium phosphate, Model No. JV-1D(UV).
4. Installation of an Alpha Scientific Lab. electromagnet.
5. Replacement of the original signal detector and $A C$ amplifier With a Princeton Applied Research Model JB-4 Lock-in Amplifier and change of the modulating frequency from 60 cps to 82 cps .
6. Arrangement of the optical components on a five foot lathe bed manufactured by Logan Eng. Co.. The experimental arrangement is outlined in Fig. 1.

The light from the xenon lamp is focused on the monochromator entrance slit by means of a mirror. Light emerging from the monochromator is focused on the photomultiplier tube. The ammonium dihydrogen phosphate crystal was replaced with a thicker crystal of potassium dideuterium phosphate with gold electrodes, solving problems of high voltage breakdown experienced with the original crystal. The new modulating crystal requires only half the voltage required of the original to produce the


Fig. I. Experimental arrangement for the dichrograph
same retardation. The modulating frequency was altered to avoid the 60 cps line voltage and was set at 82 cps when this proved to be the frequency with the lowest noticeable noise.

The power supply for the xenon lamp (see Fig. 2) is similar to one built earlier in this laboratory (15). This supply source has proven to be very satisfactory. The 360 cps ripple is less than $0.1 \%$ of the total voltage as measured by an oscilloscope across the lamp terminals with the lamp operating.

The power supply for the modulator is shown in Fig. 3. The source is taken from the reference output of the Princeton amplifier, a signal which is synchronized with the phase sensitive detector of the amplifier.

The magnet is an Alpha Scientific Laboratories Model AL7500M equipped with the AL7500 power supply and 7500R regulator. The light beam passes through a 1 cm bore. The magnet is operated at a regulated 9.5 amperes. For measurements of 1 cm path length a 22 mm gap is used yielding a 10.4 kilogauss field. With a 1 mm path length a 3 mm gap is used giving 13.1 kilogauss. The magnetic field measurements were obtained with a Bell gaussmeter using an axial probe.

The dichrograph was calibrated with a $1 \mathrm{mg} / \mathrm{ml}$ solution of $5-\alpha-$ cholestan-3-one in methanol. For a 1 cm path length $\Delta A=0.00369$ at $290 \mathrm{~m} \mathrm{\mu}$, a value determined and supplied by Dr. Paul Schatz.

A diagram of the circuit used to operate the dichrograph automatically for multiple scans is shown in Fig. 4. The "gear microswitch" is coupled directly to the dichrograph wavelength drive where contacts are set to provide the desired frequency range. The interface needed


T - Superior Electric Eo. 3 phase autotransformer Type \#l:36-3
Rec - Westinghouse 6-3-1B size K set of selenium rectifiers
L - Chicago Standard Trans. Corp. . 005 Hys. C-2689 Filter Choke
$C_{1}$ - Mellory 1000 mfd 150 vdc HC 15010 A V.V. 6 in parallel
$\mathrm{C}_{2}^{1}$ - Mallory 1000 mfd 150 vdc HC 15010 A V.V. 2 in parallel
$\mathrm{B}^{2}$ - Ballast $85 \Omega 200-225 \mathrm{w}$ resistors IRC 6448 Type $10 \mathrm{l} / 2 \mathrm{H}$ in series
F - Fuse 30 amp .
M - Haydon hour meter
A - Electro-Mechanical Inst. Co. 30 amp DC ammeter
RE - Potter-Brumfield PR3AY relay 230v, 60 cycle coil rated at 25 amp . at 110 v

Fig. 2. Lamp Power Supply


Fig. 3. Modulator power supply


Fgi. 4. Automatic Scanner
between the final output of the dichrograph and the computer of average transients (CAT) input is shown in Fig. 5. The CAT used was a Japan Electron Optics Lab Co. Model JNM-RA-I.

The absorption spectra were determined on a Cary Model 15 spectrophotometer.

Measurements at other than room temperature were made with the sample held in a brass holder. Temperature of the brass holder was controlled by conduction from an attached reservoir. Depending on the temperatures desired, the reservoir was filled with liquid nitrogen, acetone-dry ice mixture, salt-ice mixture or heated oil. Temperature measurements were taken with a chromel-alumel thermocouple using a Keithley Instruments Model 149 milli-microvoltmeter.

## Materials

Most of the chemicals were used directly from the bottle unless their absorption spectra indicated impurities when compared to literature values. Special precautions had to be taken in this regard for samples of naphthalene, anthracene and benzimidazole. No further purification was deemed to be necessary for this work if the compound proved to be spectroscopically pure.

Other Experimental Arrangements
The dichrograph in its present form is truly a versatile instrument. With the addition of only a few optical components it is capable of measuring linear dichroism and birefringence. It is also theoretically capable of measuring magnetic linear dichroism and birefringence (the


Fig. 5. Dichrograph - CAT hookup


Fig. 6. Optics for use of dichrograph to measure linear dichroism


Fig. 7. Optics for use of dichrograph to measure linear birefringence

Cotton-Mouton effect), although the sensitivity is not presently available. The optical arrangements needed for these measurements are shown in Figs. 6 and 7.

In designing these arrangements it is necessary to know the state of polarization of the light beam at each point along the path, and to make certain that the signal emerging from the photomultiplier tube actually contains the information which one wishes to measure. Two particularly helpful aids which greatly simplify the optical analyses are the Poincare sphere and the Jones calculus. This will be illustrated for the operation of the dichrograph in its normal mode for the measurement of circular dichroism.

Points on the Poincare sphere represent polarized light of any degree of ellipticity (16, p. 15)(17). In Fig. 8 the poles represent right and left circular light, the equator represents the various azimuthal angles oi plane polarized light as shown and all other points represent some other degree of ellipticity. If we begin with plane polarized light of azimuthal angle $0^{\circ}$ (Fig. 9) from the dichrograph polarizer ( $P$ ) and next introduce a variable retardance ( $R$ ) at an angle of $45^{\circ}$, the effect of the retarder will be to rotate the sphere about the axis $C R$ to a degree depending on the retardation (i.e., depending on the modulation voltage). If the retardation is $1 / 4 \lambda$, circularly polarized light will be obtainea. Actually, because of the detection system used, the dichrograph modulator is made to retard somewhat more than $1 / 4 \lambda$ at maximum voltage, therefore at this point the light beam is elliptically polarized (18). The Poincare sphere makes it simple to visualize the state of the light entering the


Fig. 8. The Poincare sphere


Fig. 9. Polarization of light during modulation cycle of dichrograph
sample at any point in the modulation voltace cycle.
The voltage as seen at the photomultiplier output is shown in Fig. 10 for various types of samples. This may also be seen easily by using the Jones calculus (16, p. 118)(19). The initial state of polarization of the light beam is represented by a column matrix and each optical component by a square matrix. The matrices are then lined up from right to left in the same order as the components are seen by the light beam. Then the matrices are multiplied from right to left and the resulting matrix yields the state of the emerging light. This is illustrated below for the case of a circularly dichroic sample.

$$
\text { State of polarization }=[S][\mathrm{M}][\mathrm{P}]
$$

where $[P]=\left[\begin{array}{l}7 \\ 0\end{array}\right]$, the column matrix for plane polarized light at an azimuthal angle of $90^{\circ}$.

$$
[M]=\left[\begin{array}{ll}
\cos ^{2} \theta e^{i \frac{\delta}{2}+\sin ^{2} \theta e^{-i \frac{\delta}{2}}} \quad \cos \theta \sin \theta 2 i \sin \frac{\delta}{2} \\
\cos \theta \sin \theta 2 i \sin \frac{\delta}{2} & \cos ^{2} \theta e^{-i \frac{\delta}{2}+\sin ^{2} \theta e^{i \frac{\delta}{2}}}
\end{array}\right]
$$

the matrix for the modulator with a retardation $\delta=1.98 \sin \omega t$ and azimuthal angle of $\theta=45^{\circ}$.

$$
[s]=\left[\begin{array}{ll}
\alpha_{1}+\alpha_{2} & -i\left(\alpha_{1}-a_{2}\right) \\
i\left(\alpha_{1}-\alpha_{2}\right) & a_{1}+\alpha_{2}
\end{array}\right]
$$

the matrix for an optically active sample with $\alpha_{1}=10^{-2 A_{1}}$ and $\alpha_{2}=10^{-2 A_{r}}$. $A_{l}$ and $A_{r}$ are the respective absorbances for left and right circular light. Then we have
$[S][M][P]=\frac{1}{2}\left[\begin{array}{l}\left(\alpha_{1}+\alpha_{2}\right) \cos \frac{\delta}{2}+\left(\alpha_{1}-\alpha_{2}\right) \sin \frac{\delta}{2} \\ i\left(d_{1}-\alpha_{2}\right) \cos \frac{\delta}{2}+i\left(\alpha_{1}+\alpha_{2}\right) \sin \frac{\delta}{2}\end{array}\right] A A^{-}$


Fig. 10. Photomultiplier output for different types of circularly dichroic samples
where $A x$ and Ay are the amplitudes in the $x$ and $y$ directions. Now since the intensity I is equal to the square of the amplitude, or $I=A x A x^{*}+A y A y^{*}$,

$$
I=I o \frac{1}{2}\left[\left(a_{1}^{2}+d_{2}^{2}\right)+\left(d_{1}^{2}-d_{2}^{2}\right) \sin \delta\right]
$$

where $\sin \delta=\sin \left(\delta_{0} \omega t\right)$. The first term in the bracket represents the time independent part of the light.intensity due to the ordinary absorption and the second term is the time dependent part resulting from the circular dichroism of the optically active sample. So it is seen that the method of the Jones calculus gives quickly an easily visualized expression typifying the intensity of the light which the phototube will observe. Further analysis of the signal as it is processed by the machine follows exactly that given by Velluz et al. (18, p:60).
III. A SIMPLIFIED THEORY OF MCD

The classical and quantum mechanical theories of the molecular Faraday effect have been known for several decades. However it is only within the last few years that the theory has been developed in a practical form for molecular systems. Unfortunately little of the quantum mechanical theory of MCD has been stated explicitly in a language which those relatively unfamiliar with the methods and notation can readily grasp. (The recent thesis of Dratz (IO) handes certain aspects of the theory in a very lucid manner.) Therefore the purpose of this section will be to present the theory as clearly as possible, to indicate reierences which have been particularly helpful, and to use this simple theory to predict the type of information MCD might provide, especially for molecules of biological interest. Appenaix I contains a second-order perturbation theory derivation for MCD, also presented with considerable explanation of the techniques and mathematical methods used.

A considerable amount of the MCD effect can be understood in terms of the inverse Zeeman effect. Here "inverse" pertains of the absorption of circularly polarizea light rather than emission as is usually used to study the Zeeman effect. The inverse Zeeman effect is a change in the frequencies at which right and left circular light is absoribed, caused by the action of an external magnetic field upon moving electrons in a substance. Intuitively one would expect that for two charges orbiting in opposite directions an external magnetic field should favor one direction and retard the other. This will change the kinetic energies of the charges and therefore chainge the resonant frequencies at which they will
absorib light. Essentially this is what actually occurs for molecular electrons. The effect is observed experimentally by measuring the absorption of circular light passing through the sample in a direction parallel to the applied magnetic field. Fig. $\operatorname{ll}(\mathrm{a})$ illustrates this for sharp absorption lines and Fig. II(b) for broad bands. Since MCD is defined as the result of subtracting right circular absorption from left circular absorption, a spectrum such as Fig. ll(c) results fior broad bands.

The Zeeman effect is treated nicely in a number of textbooks. (20,21) However, to account for the MCD of all molecules it will be necessary to discuss some quantum mechanical aspects of the circular absorption process.

## Photons and Transitions

At the berinning it must be realized that single photons always contain one unit ( $\bar{n}$ ) of angular momentum (spin) (22). This is true of all photons, without regard to their frequency or origin. This angular momentum may be of either sign, and by convention we will say that a +1 angular momentum means the photon is left-circularly polarized and -l that it is a right-circularly polarized photon (23). Appendix II contains an explanation of the sign convention used in this thesis and includes the reasons for the choice of signs. Fig. 12 shows how the handedness is derineā. The electric (or magnetic) vector moves clockwise for right-hana light, counterclockwise for left-hand light as viewed looking toward the light source.

If a molecule in its ground state absorbs a circularly polarizeā


(c)

Fig. Il. Inverse Zeeman effect and MCD


Fig. 12. Handedness of circularly polarized light


Fig. 13. Absorption of circularly polarized light

Fig. 14. Emission of circularly polarized light
photon, it will gain a unit of angular momentum as shown in Fig. 13 (24). The quantum number $m$ is used to designate the state of orbital angular momentum of the electron involved in the transition. If the molecule emits the energy of the photon, it will release the angular momentum it absorbed, emitting a photon of the same handedness as it absorbed. This is requireā to satisfy the conservation of momentum. Fig. 13 illustrates the "selection rules" for circularly polarized transitions, for example, the requirement for right-hand absorption is that $\Delta m=-1$.

The Quantum Mechanical Language of MCD
One of the primary requisites for understanding quantum mechanical methods, and indeed for understanding any unfamiliar subject, is to know the language associated with the subject. Although quantum mechanics is quite an involved method, individual topics within it can often be visualized quite easily, a fact not at all obvious from reading most of the literature. Very often an interpretation or analogy can be iound which will give a "physical feeling" for some concept which has been written in a compact mathematical manner. Once these "physical feelings" have been gained the compact language used in quantum mechanical theory becomes much easier to read.

A number of the terms used in the literature associated with magnetic circular dichroism, such as "matrix element" and "vector potential", will be discussed below in an attempt to show why their use is quite natural.

The circular photons which strike our sample may be thought of as perturibations capable of producing changes in the system. In order to represent these properties mathematically an expression must be found for

the electromagnetic fields of the photons, for it is these fields which provide the perturbation. This could be done by using the electric field $\overline{\mathrm{E}}$ and magnetic field $\overline{\mathrm{H}}$ oit the photon. However, for reasons which will not be discussed, it is mathematically convenient to use a vector function related to both $\overline{\mathrm{E}}$ and $\overline{\mathrm{H}}$ called the vector potential $(\overline{\mathrm{A}})$. The relation of $\overline{\mathrm{A}}$ to $\overline{\mathrm{E}}$ and $\overline{\mathrm{H}}$ is shown in Eqs. I (25, p. 108).

$$
\begin{align*}
& \bar{E}=-\frac{l}{c} \frac{\partial \bar{A}}{\partial t}-\bar{\nabla} \phi \quad \text { where } \quad \bar{\nabla}=\left(\bar{i} \frac{\partial}{\partial x}+\bar{j} \frac{\partial}{\partial y}+\bar{k} \frac{\partial}{\partial z}\right) \\
& \bar{H}=\bar{\nabla} \times \bar{A} \tag{I}
\end{align*}
$$

The scalar potential $\phi$ is associated with an electrostatic fiela and will be ignored here since it is assumed that the photon is far from its electrostatic source. The electrostatic field is proportional to $\frac{l}{r^{2}}$, radiation proportional to $\frac{I}{r}$ and a dipole field is proportional to ${ }^{r} \frac{I}{r^{3}}$. The vector potential $\bar{A}$ (also called vector magnetic potential) has no real physical significance. It cannot be measured experimentally as can $\bar{E}$ and $\bar{H}$, but is a vector function which yields the magnetic field when the cross product with $\bar{\nabla}$ is taken. However we can obtain some feeling for the function if we examine its definition in Eq. 2. In this expression

$$
\begin{equation*}
\bar{A}=k \int_{\infty} \frac{\bar{J}}{r} d \tau \tag{2}
\end{equation*}
$$

$\bar{A}$ at some point in space at a distance $r$ from the electrostatic source is proportional to the integral over all space of the current density $\bar{J}$ at the source. Fig. 15 shows an oscillating dipole $\bar{P}$ (a changing current) as the source producing an oscillating vector function $\vec{A}$ at some point in space. ( 26 ) The current traveling back and forth in the dipole is $\bar{J}$.


Fig. 15. Vector potential ( $\bar{A}$ ) produced by a dipole ( $\bar{P}$ )

From the vector potential $\overline{\mathrm{A}}$ the fields $\overline{\mathrm{E}}$ an $\overline{\mathrm{H}}$ may be founa using Eqs. 1 .
The vector potential for a circular photon traveling in the $z$ direction can be consiôered a combination of the vector potentials from dipoles escillating in the $x$ and $y$ directions as in Eq. 3 .

$$
\begin{align*}
& \bar{A}_{r}=\bar{i} A_{x}+i \cdot \bar{j} A_{y} \\
& \bar{A}_{\ell}=\bar{i} A_{x}-i \bar{j} A_{y} \tag{3}
\end{align*}
$$

The imaginary quantity i in Eq. 3 represents a $90^{\circ}$ phase factor. The +i in $A_{r}$ implies that $A_{y}$ is $90^{\circ}$ ahead of $A_{x}$, the -i in $A_{1}$ means $A_{y}$ is behind $A_{X}$ by $90^{\circ}$.

Without going into the derivation of the perturbation operator, it is obvious that the vector potential $\bar{A}$ does not fully describe the perturibing force. A force $\left(F=m a=e\left(\overline{\mathrm{E}}+\frac{l}{c}[\overline{\mathrm{~V}} \times \overline{\mathrm{H}}]\right)\right)$ should be related to the mass, charge and velocity of the particles in the substance being illuminated. When the derivation has been accomplished $(25,27)$ the perturbation is found to be

$$
H=\frac{e}{m c} \bar{A} \cdot \vec{p}
$$

where $\bar{p}$ is the momentum operator $-i \quad h\left(\bar{i} \frac{\partial}{\partial x}+\bar{j} \frac{\partial}{\partial y}+\bar{k} \frac{\partial}{\partial z}\right)_{0}$. Now H contains the field, mass charge and particle velocity. ( $\frac{\partial}{\partial x}$ etc). The expression is considerably simplified by making two approximations. First, the size of the wavelength of light is considered to be much greater than the size of the molecule being perturbed, and therefore $A$ is constant over the molecule. This approximation cannot be made for the perturbation operator concerned with natural circular dichroism. Essentially this first approximation will mean that only the first term in an expansion of the space part of the vector potential $\left(e^{i \bar{k} \cdot \bar{Z}}\right.$ ) is needed, and the first term is unity $\left(e^{i \bar{k} \cdot Z} \simeq 1+i \bar{k} \cdot \bar{Z}+\cdots \cdot\right)$. Second, the so-called dipole approximation will be made in which we assume that the molecule absorbs radiation as a dipole. This allows $\bar{p}$ to be written in the form $\bar{p} \alpha i(\overline{i x}+\bar{j} y+\bar{k} z)$ (25,28). This is equivalent to saying that the velocity of the perturbed charge and the distance which it moves from its equilibrium point are directly related.

Now if for simplicity we exclude the constants involved, we see that the form of the right-hand perturbation operator will be

$$
\mathrm{Hr}=\overline{\mathrm{A} r} \cdot \overline{\mathrm{p}}=(\overline{\mathrm{i}} A x+i \bar{j} A y) \cdot i(\overline{i x}+\bar{j} y)=A i(x+i y)
$$

since $A x=A y$ for circular light. Doing likewise for the left-hand case gives $H_{l}=A i(x-i y)$ (29).

It will be convenient to transform the operators from cartesian to spherical coordinates, since integration will be simpler in this coordinate system. One finds that $x \pm i y$ transforms as $r \sin \theta e^{+i \phi}$. Again for simplicity only the essential part of this expression will be
used so that for $H_{r}$ we will use $e^{+i \phi}$ and for $H_{l}$ use $e^{-i \phi}$.
We are now interested in learning what effect the perturbation of a circular photon has upon a particular molecule. Since the electrons in a molecule exhibit a wave-like behavior, the state of a molecule is represented by wave functions $\psi$. We will represent the perturbation of the ground state of a molecule by a right circular photon as $H_{r} \psi_{1}$. This may have some affect upon the wave function and thereby change it slightly.

We would like to know whether the perturbation is capable of causing a transition of the molecule from state $\psi_{1}$ to a higher state $\psi_{2}$. In order for this to occur there must be some similarity in spatial configuration between the perturbed state $H_{r} \psi_{1}$ and $\psi_{2}$. One can think of this similarity as an overlap of the regions where an electron in state $H_{r} \psi_{1}$ resides with the region it would occupy in state $\psi_{2}$. This can be determined by calculating the integral $\int \psi_{2} \mathrm{E}_{\mathrm{r}} \psi_{工}^{*}$ dr, where $\mathrm{d} \tau$ means to integrate over all space. This integral gives the average or "expectation" value for the operation $H_{r}$ when the states $\psi_{1}$ and $\psi_{2}$ are involved. The form of the average value is dictated by a law of quantum mechanics (28). Since the perturbation operator in this case contains the dipole moment and may result in a transition, the integral is commonly called a "transition moment". If the result of the calculation is non-zero the transition is said to be "allowed", if zero it is "forbidden".

As an example of evaluating a transition moment we can ask the question, "what is the possibility of changing the state of a molecule from $m=0$ to $m=1$ using right-hand or left-hand photons?" We will use only
the parts of the wave functions describing the orbital angular momentum for simplicity. These parts can be written in the form $e^{i m \phi}$ to make the integrals for transition moment easy to evaluate.
right-hand $\int_{0}^{2 \pi} e^{+i \phi} e^{+i \phi} e^{0} d \phi=\int_{0}^{2 \pi} e^{+2 i \phi} d \phi=$ zero
leftt-hand $\int_{0}^{2 \pi} e^{+i \phi} e^{-i \phi} e^{0} d \phi=\int_{0}^{2 \pi} l d \phi=$ non zero
Therefore only left-hand photons can cause the transition.
The transition moment itself is not proportional to the intensity of absorption, but rather the product of the transition moment with its complex conjugate must be taken,

$$
\left.\left|\left\langle\psi_{2}\right| H_{r}\right| \psi_{1}\right\rangle\left.\right|^{2}=\left(\int \psi_{2} H_{r} \psi_{1}^{*} d \tau\right)\left(\int \psi_{2}^{*} H_{r}^{*} \psi_{1} d \tau\right)
$$

This has its analog classically in that the rate of energy absorption is proportional to the square of the dipole moment (28, p. 554). Quantum mechanically the need for the square results from the fact that the states are set up in such a way that the probability of a system being in the state $\bar{\psi}$ is $\bar{\psi} \dot{\psi}$. This topic is explained clearly by Barrow (30, p. 68 ). It should also be noted at this point that the transition moment $\left\langle\psi_{2}\right| H_{r}\left|\psi_{1}\right\rangle$ is not observable experimentally. This is apparent mathematically by the presence of the complex operator containing $i e^{i \phi}$ which will always result in imaginary answers. Imaginary quantities are not oidservable, a corollary of quantum mechanics (28, p. 169). The product oi the transition, moment with its complex conjugate will always result in a real quantity and will therefore be observable.

Integrals with a form such as $\left\langle\psi_{2}\right| \mathrm{H}\left|\psi_{1}\right\rangle$ are commonly calieà "matrix
elements" (31, p. 87). This results from the fact that it is convenient to use matrix methods to solve equations involving transition moments for a molecular system. The equations to be solved (32) are of the type $d_{j}=\underset{i}{\sum}\left\langle\psi_{j}\right| O p\left|\psi_{i}\right\rangle C_{i}$, which for the operator $O p$ and the system in Fig. 16 is

$$
\begin{aligned}
& \psi_{3} \\
& \psi_{2}
\end{aligned}
$$

## $\psi_{1}$

Fig. 16. System of levels

$$
\left(\begin{array}{c}
a_{1}  \tag{4}\\
a_{2} \\
a_{3}
\end{array}\right)=\left(\begin{array}{ccc}
\left\langle\psi_{1}\right| 0 p\left|\psi_{1}\right\rangle & \left\langle\psi_{1}\right| 0 p\left|\psi_{2}\right\rangle & \left\langle\psi_{1}\right| 0 p\left|\psi_{3}\right\rangle \\
\left\langle\psi_{2}\right| 0 p\left|\psi_{1}\right\rangle & \left\langle\psi_{2}\right| 0 p\left|\psi_{2}\right\rangle & \left\langle\psi_{2}\right| 0 p\left|\psi_{3}\right\rangle \\
\left\langle\psi_{3}\right| 0 p\left|\psi_{1}\right\rangle & \left\langle\psi_{3}\right| 0 p\left|\psi_{3}\right\rangle & \left\langle\psi_{3}\right| 0 p\left|\psi_{3}\right\rangle
\end{array}\right)\left(\begin{array}{c}
c_{1} \\
c_{2} \\
c_{3}
\end{array}\right)
$$

where $O p$ is the operator which will give the average (expectation) values for a particular property. The individual terms $\left\langle\psi_{j}\right| O p_{p}\left|\psi_{i}\right\rangle$ are called matrix elements. For the case of electric dipole absorption oniy those off-aiagonal elements in the lower left are of interest. In MCD matrix elements representing magnetic dipole transitions (using $O p=\mu$ ) and electric dipole transitions (using $O p=m$ ) are needed.

In order to simplify the writing of matrix elements the Dirac notation is oiten used. In this notation reversal of the order of the wave functions inäicates a complex conjugate,

$$
\langle a| A|b\rangle=\langle b| A|a\rangle^{*} \quad(33, p \cdot 307)
$$

We may write the intensity for a right hand transition as
$k\langle a| x+i y|b\rangle\langle b| x-i y|a\rangle$
following Dirac. This indicates that the matrix element is being muitiplied "-by its complex conjugate.

It is often stated that only Hermetian operators may be used in quantum mechanics (28, p. 156). However, the operator $x \pm i y$ or $e^{+i \phi}$ is not Hermetian since the requirement

$$
\left\langle\psi^{*}\right| A|\phi\rangle=\langle\phi| A^{*}\left|\psi^{*}\right\rangle
$$

is not met. This also means the expectation value is not necessarily real, a requirement for an observable. This does not matter to us since it is the product of the expectation value with its complex conjugate that we use, it will be real and therefore will be observable.

## The Atomic Case

To illustrate the theory of MCD we will begin with some simple cases, the most elementary of which are transitions between $s$ and $p$ atomic orbitals as shown in Fig. 17. The orbitally degenerate $p$ levels have

..Fig. 17. Circularly polarized transitions for $s \rightarrow p$
different energies when a magnetic field is applied, in this case in the $z$ direction. This change of energies is called Zeeman splitting. The splitting will be of a magnitude proportional to the magnetic moment of the electron involved and the external magnetic field $\Delta E=\mu \bar{H}$. The magnetic moment results from the magnetic field developed by the orbiting electron, and therefore depends upon the angular momentum of the orbital. The diagonal terms of Eq. 4 (such as $\left\langle\psi_{I}\right| L_{Z}\left|\psi_{I}\right\rangle$ ) are used to evaluate the magnetic moment of a state.

The $p_{x}$ and $p_{y}$ orbitals may now be thought of as containing electrons orbiting around the $z$ axis, but circulating in opposite directions for $p_{x}$ and $p_{y}$. Before the magnetic field was applied it was not possible to distinguish between the orbitais $p_{x}, p_{y}$ and $p_{z}(34, p$. 113 ). After the field is established the energies of these three orbitals attain a value depending upon their orbital angular momentum (m). This means it is now necessary for the $p_{x}$ and $p_{y}$ wave functions to be written including the complex quantity $e^{i m \phi},(35, p$. 8) which designates the value of m. or conversely, a wave function which must be written with such a complex quantity possesses orbital angular momentum. The angular parts of these wave functions are shown in Eq. 5. It can be seen that these are of the form $e^{+i \phi}$ and therefore we can calculate which circularly polarized transitions are permitted to each of the states by evaluating the transition moments. Since circularly polarized transitions are forbidden for $\Delta m=0$, we will not be interested in that case.

$$
\begin{equation*}
\psi_{p_{x}}=\frac{x+i y}{2} \quad \psi_{p_{y}}=\frac{x-i y}{2} \tag{5}
\end{equation*}
$$

If the transitions are now represented by absorption bands with Gaussian shape as in Fig. 18, the MCD resulting will be that of Fig. 19. This particular type of MCD is called "A term" MCD, and is the result of Zeeman splitting.


Fig. 18. Excited state splitting


Fig. 19. A term MCD

Both situations a and $b$ in $F i g .17$ will show $A$ term MCD, but in addition b will have another effect since the ground state is degenerate. This effect occurs because the Zeeman splitting will change the energies of the orbital components of the ground state (36, p. 204). They will no longer be populated thermally to the same extent, but rather their populations will depend on the Boltzmann relation $N=N_{o} e^{-\frac{E}{K T}}$. This leads to the results shown in Figs. 20 and 21. The effect due to the populations of


Fig. 20. Ground state splitting


Fig. 21. A plus C term MCD
the levels is called "C term" MCD.
So far nothing has been said about electron spin. However, in the event that there is no interaction between electron spin and orbit (approximately true for light atoms), it can be shown that spin has no effect on MCD. The splitting aue to spin will be of the same magnitude in both ground and excited states as shown in Fig. 22. The result is


Fig. 22. Effect or spin on MCD
that the frequencies of the transitions are unaffected. Transitions in which the spin quantum number changes (such as $m=0, s=-\frac{1}{2} \rightarrow m=+1$, $s=+\frac{I}{2}$ ) are highly forbidaen, so they do not contribute in Fig. 22.

Other examples of simple atoms could be show, such as transitions between $p$ and $d$ levels, or between $\mathbb{d}$ and $I$ levels. However, the results for these are almost identical to those for the $s \rightarrow p$ case descriod (36, p. 104).

The Molecular Case
Unfortunately the MCD of the compounds we will consider is not so simple as that of atoms. We take as an example a case which approaches more closely the situation in biologically interesting molecules.

Suppose an atom with $p$ orbitals has arranged around it charges (e.g., a crystal field or ligands) such that the degeneracy of the $p$ levels is now lifted electrostatically ( $35, \mathrm{pp} .65-78$ ). The moving electrons in the p orbitals interact strongly with the perturbing charges and may be


Fig. 23. Electrostatic splitting of $p$ levels
considered to exchange angular momentum. with the particles which bear the charges (28, p. 259). This exchange erases any trace oî an exact orbital angular momentum ( $m$ ) for the orbitals. Mathematically this is stated by writing the angular parts of the states $\psi_{p x}$ and $\psi_{p y}$ as Iinear combinations of the $m=+1$ and re-l states. The $z$ state is not of interest to us. It can be seen that these states

$$
\begin{aligned}
& \psi_{p x}=\frac{1}{\sqrt{2}}\left[\frac{x+i y}{\sqrt{2}}+\frac{x-i y}{\sqrt{2}}\right] \\
& \psi_{p y}=\frac{i}{\sqrt{2}}\left[\frac{x-i y}{\sqrt{2}}-\frac{x+i y}{\sqrt{2}}\right]
\end{aligned}
$$

can now be described by real functions, that is $\psi_{p x}=x$ and $\psi_{p y}=y$. Since these states are not complex, they no longer may be considered to possess orbital angular momentum. It is said that the orbital angular momentum
is "quenched". There is as much electron circulation in one direction around the $z$ axis as the other. There will be just as many right hand transitions as left hand transitions to each state and therefore no circular dichroism.

Now when a magnetic field is applied, there are no orbital degeneracies to split, so there is no possibility of $A$ or $C$ terms. But the magnetic field does have an effect in this case which tends to "unquench" the angular momentum to an extent linear with the external magnetic fiela strength.

If there is a possibility of a magnetic transition between two states, in this case between the $\psi_{p x}$ and $\psi_{p y}$ wave functions, there can be mixing of some of the $\psi_{p y}$ into the $\psi_{p x}$ and some of the $\psi_{p x}$ into the $\psi_{p y}$, unquenching the orbital angular momentum somewhat. This leads to a probability of more right hand transitions to one state, more left hand transitions to the other. MCD resulting from this effect is called "B term" MCD, and will be most important for the compounds which will be discussed.

The sign of the mixing term will depend upon the direction of the magnetic dipole transition, that is for the levels in Fig. 24 it depends upon whether the transition is $x \rightarrow y$ or $y \rightarrow x$. The signs will be opposite, $M_{x y}=-M_{J x}(37)$. This shows mathematically that the $B$ terms associated with the two interacting levels will be of opposite sign.

If the $x$ and $y$ transitions are now represented by Gaussian bands, the resulting MCD is shown in Fig. 24 .

From the perturbation derivation (See Appendix I) it is seen that the form of the $B$ term is


Fig. 24. B term MCD from two interacting levels

$$
\begin{align*}
\frac{\bar{M} \cdot(\overline{\mathrm{R}} \times \overline{\mathrm{R}})}{\Delta \mathrm{E}} \text { where } \overline{\mathrm{M}} & =k\left\langle\psi_{\mathrm{x}}\right| \mathrm{L}_{\mathrm{Z}}\left|\psi_{y}\right\rangle  \tag{6}\\
L_{\mathrm{Z}} & =\text { ang. mom. operator }\left(\mathrm{x} \frac{\partial}{\partial y}-\mathrm{y} \frac{\partial}{\partial \mathrm{x}}\right)
\end{align*}
$$

where $\overline{\mathrm{M}}$ is a magnetic dipole transition. between two of the levels and $\overline{\mathrm{R}}$ and $\overline{\mathrm{R}}$ ' are two electric dipole transition moments separated in energy by $\Delta E$. Note that $\bar{M}$ is an off-diagonal matrix element of the type in $E q .4$. From Eq. 6 we can see that certain features are required in order to have a B term:

1. Two electric transitions with some perpendicular component.
2. An allowed magnetic transition between states common to $\bar{R}$ .and $\bar{R}$ ' and possessing a component perpenaicular to the plane containing $\bar{R}$ and $\overline{R^{\prime}}$.

It shoulà be noted that the constant coefficients in $\bar{M}$ and $\bar{R}$ contain the charge of the particle, therefore the sign of the $B$ term will depen $\bar{a}$ on the sign of the charge.

As the electrostatic splitting of the two bands becomes smaller, the unquenching of the orbital angular momentum by the magnetic fiela becomes more efficient. In the limit that the two bands are orbitally degenerate we have returned to the A or C term case. Fig. 25 shows what the $B$ term Por two interacting bands looks like as the energy difference between the transitions is reauced. At a certain point it becomes impossible to tell $B$ term Irom $A$ term MCD.

We will now show by a simple derivation that when two states ( $j$ ana $k$ ) are magnetically mixed $B$ term MCD will result. The initial states (see Fig. 26) are assumed real and therefore possess no orbital angular



$$
\begin{array}{ll}
A & \Delta E=11 \\
B & \Delta E=4 \\
C & \Delta E=0
\end{array}
$$

Fig. 25. B term for two identical interacting banas with various energy differences
momentum. The transition monents for rieht hand and left hard cases ure written for the $g \rightarrow j$ transition, squared to obtain intensities, and then the rigint hand absorption is substracted from left hand absorption to yield MCD. The result is a real quantity as all observables must be. The magnetic mixing factor is written $\bar{M}=i a$ in order to make it easier to follow manipulation of the imaginary number $i$. Terms containing $a^{2}$ are aropped because they are very small and because we are interested only in the terms linear in a magnetic field.
$\longrightarrow k \rightarrow i a j$
$\longrightarrow j+i a k$

## $\longrightarrow \sigma$

Fig. 26. Levels for $B$ term derivation
right hand transition moment

$$
\begin{aligned}
& \langle 5| A i(x+i y)|j-i a k\rangle\langle j+i a k|-A i(x-i y)|g\rangle \\
& =\left(A i\left[g x_{j}+i g y j-i a g x k+a g y k\right]\right) \\
& x(A i[-j x g+i j y g-i a k x g-a k y g])= \\
& {\left[\begin{array}{l}
-g x j j x g-i g y j j x g+i a g x k j x g-a g y k j x g \\
+i g x j j y g-g y j j y g+a g x k j y g+i a g y k j y g \\
-i a g x j k x g+\text { agyjkxg - a }{ }^{2} g x k k x g-i a^{2} g y i k x g g \\
- \text { agxjkyg - iagyjkyg + ia }{ }^{2} g x k k y g-a^{2} g y k k y g
\end{array}\right]} \\
& x\left[-A^{2}\right]
\end{aligned}
$$

Here the term gxjjxg is shorthand for $\langle f| x|j\rangle\langle j| x|g\rangle$. left hand transition moment

$$
\begin{aligned}
& \langle g| A i(x-i y|j-i a k\rangle\langle j+i a k|-A i(x+i y)|g\rangle \\
& =(A i[g x j-i g y j-i a g x k-a g y k]) \\
& x(A i[-j x g-i j y g-i a k x g+a k y g]) \\
& {\left[\begin{array}{l}
-g x j j x g+i g y j j x g+i a g x k j x g+a g y k j k g \\
-i g x j j y g-g y j j y g-a g x k j y g+i a g y k j y g \\
-i a g x j k x g-a g y j k x g-a^{2} g x k k x g+i a^{2} g y k k x g \\
+ \text { +agxjkyg - iagyjkyg - ia }{ }^{2} g x k k y g-a^{2} g y k k y g
\end{array}\right] \quad x\left[-A^{2}\right]}
\end{aligned}
$$

Left hand - Right hand intensities $=-A^{2}$ [4agxjkyg - 4agyjkxg]
$=-4 A^{2} a[g x j k y g-g x k j y g]$
$=-(-i) 4 A^{2}(i a)[g x j k y g-g x k j y g]$
$=i 4 A^{2} \bar{M}[g x j k y g-g x k j y g]$
$M C D=i 4 A^{2} \bar{M}[\langle j| x|j\rangle\langle k| y|g\rangle-\langle g| x|k\rangle\langle j| y|g\rangle]$
d-ã Transitions
Another somewhat more complicated molecular arrangement which gives additional insight into the origin of MCD is provided by transition metal complexes. The spectra of these compounds are characterized by transitions between the d orbitals of the metal ions, transitions which are normally forbidden but slightly allowed by vibrations which mix in otner states with the d states.

The d orbitals of the free metal ion are degenerate, this has already been mentioned in the atomic case. But the liganäs of a metal-ligand complex will affect the energies of the various d orbitais to different degrees, partially or perhaps completely destroying the degeneracy. We
will illustrate using complexes of octahedral symmetry which split the $\dot{d}$ orbitals as shown in Fig. 27 (38, p. 53).

.Fig. 27. $\dot{a}$ shell less than half filled

The angular parts of the five d orbitals are listed below as they may be written before electrostatic splitting occurs (39, p. 63).

$$
\begin{aligned}
& d_{+2}=\sqrt{\frac{3}{8}}(x+i y)^{2} \\
& a_{+1}=-\sqrt{\frac{3}{2}}(x+i y) z \\
& a_{0}=\frac{1}{2}\left(3 z^{2}-r^{2}\right) \\
& a_{-2}=\sqrt{\frac{3}{2}}(x-i y) z \\
& d_{-I}=\sqrt{\frac{3}{8}}(x-i y)^{2}
\end{aligned}
$$

Aitter the splitting they must be written as below. The reasons for writing them in this way are similar to the case described above for the electrostatic splitting of $p$ orbitals.
$t_{2 g}{ }^{\circ}=\frac{1}{\sqrt{2}}\left(d_{2}-d_{-2}\right)=\frac{1}{\sqrt{2}}\left[\sqrt{\frac{3}{8}}(x+i y)^{2}-\sqrt{\frac{3}{8}}(x-i y)^{2}\right]$
$\begin{array}{ll}t_{2 g}=a_{-1} & =\sqrt{\frac{3}{2}}(x-i y) z \\ t_{2 g}{ }^{+}=d_{+i} & =-\sqrt{\frac{3}{2}}(x+i y) z \\ e_{g}^{a}=d_{0} & =\frac{1}{2}\left(3 z^{2}-r^{2}\right) \\ e_{g}{ }^{b}=\sqrt{\frac{1}{2}}\left(d_{2}+d_{-2}\right) & =\frac{1}{\sqrt{2}}\left[\sqrt{\frac{3}{8}}(x+i y)^{2}+\sqrt{\frac{3}{8}}(x-i y)^{2}\right]\end{array}$
If now the matrix elements for transitions between $e_{g}$ and $t_{2 g}$ levels are evaluated, we find that although both right-hand and left-hand transitions are allowed for $t_{2 g}{ }^{ \pm} \rightarrow e_{g}$, right-hand transitions are predominant for $t_{2 g}{ }^{-} \rightarrow e_{g}$ and left-hand transitions are predominant for $t_{2 g}+e_{g}$. So we are left with the situation as shown in Fig. 28.

Fig. 29 shows the same case for the d levels split by an octahedral field, but with the $\alpha$ shell more than half-filled, that is $\alpha^{n},(n=6,7,8,9)$. The levels are inverted (24, p.146) and it can be seen now that the sign (handedness) of the longest wavelength band should change sign. The MCD expected from this is shown in Fig. 30.

The reason for the inversion of levels for a more- than- half-filled shell is that the system now acts as though positive holes occupy the $\bar{\alpha}$ shell, changing the sign of the charge and therefore of the magnetic moment as in Eq. 7 (39, p. 69).

$$
\begin{equation*}
\mu=\frac{e p}{2 m} e^{p-\text { ang. mom. } \bar{n} \text { of particle of particle }} \tag{7}
\end{equation*}
$$

## Cylindrical Symmetry

In the case of diatomic molecules, or more complex molecules with cylindrical symmetry (such as benzene or metallo-porphyrin) there is the possibility that a component of orbital angular momentum may still remain


Fig. 28. Circularly polarized transitions, shells less than half file $\bar{\alpha}$


Fig. 29. Circularly polarized transitions, shells more than half filled


Fig. 30. MCD from d level.s
(a) less than half filled
(b) more than half filled
unquenched (40). The angular momentum may be defined with respect to the axis of the cylinder. In Fig. 3I this is illustrated by the $\mathrm{Cl}_{2}{ }^{-}$ion (35, p. 199).

From the figure it can be seen that the $x$ and $y$ orbitals remain essentially undisturbed and there is no reason for them to lose their exact quantum number $m$. The $z$ orbitals point in such a direction that there will be maximum interaction between the electrons and nuclei, quenching any discrete angular momentum they may have possessed before the molecule was formed.

Sigma bonding and antibonding orbitals are formed by the $z_{1}$ and $z_{2}$ atomic orbitals and pi bonding and antibonding orbitais come from the $y_{1}, y_{2}, x_{1}$, and $x_{2}$ orbitals. The $\pi_{u}$ and $\pi_{g}$ states are both orbitally degenerate, but since they are also occupied no $C$ term MCD would be expected, only A term.

## Polyatomic Molecules

For most of the molecules of biological interest the orbital angular momentum. is quenched ( $30, \mathrm{p} .267$ ). The orbits of the electrons cannot be determined exactly, although they can be known well enough to be characterized by their symmetry. It is in terms of these symmetries that we can best discuss the electronic structures of polyatomic molecules.

Exceptions to the quenching of orbital angular momentum in nonlinear molecules have been mentioned above. This may be expecteả to occur with molecules of sufficiently high symmetry such as the cylindrical symmetry of the metallo-porphyrins.

It should aiso be emphasized that the microsymmetry of the area

surrounding the chromophore is of the most importance in determining whether guenching occurs. For instance, an $-R$ substitution destroying the cylindrical symmetry of the complete metallo-porphyrin molecule has minor effects on both the absorption and MCD spectra (38, p. 90).

Since there is no orbital angular momentum associated with most polyatomic molecules, there is no spin-orbit coupling, A-term or $C$ term MCD to consider. The electrons are usually in orbits which may be classifiea as belonging to $\sigma$, or $\pi$ bonds or $n$ non-bonding orbitals. There is the possibility of magnetic mixing between all energy levels as discussed above, leading to $B$ term effects. This could lead to MCD spectra too complex to be useîul. Some knowledge about which levels are mixing may be inferred by studying several types of experimental and theoretical information. These are:

1. The observed sign and shape of the MCD.
2. The energy differences between the levels.
3. The symmetry permitted magnetic transitions between the levels.
4. The area under the MCD band.
5. Known directions of transition moments.

## Summary

From the theory above we can make several observations and predictions:

1. The sign of the MCD should depend on the sign of the charge involved in the transition.
2. MCD should be useful for the study of orbital angular momentum quenching effects and spin-orbit interactions.
3. Degeneracies in the ground state can be observed by studying the temperature dependence of MCD.
4. Some information about the relative directions of transitions moments is available from MCD.
IV. RESUUTS AND DISCUSSION

The Rare Earths
The temperature dependence of $M C D$ for a rare earth (didymium) giass reveal clear examples of $A$ and $C$ term $M C D$. The advantages and disadvantages of using didymium glass are given below.

## Advantages

1. The rare earths exhibit narrow band spectra of transitions between their $f$ electron levels. As seen from Fig. 18 , the narrower the bandwidth for any absorbance and splitting, the greater will be the MCD. Therefore the MCD of the rare earths is easy to obtain experimentally.
2. Didymium glass contains a number of rare earths.
3. The glass is easy to handle for temperature measurements, no problems of freezing, boiling or bubble formation are encountered as when using solutions.
4. The large number of transitions increases the possibility that several different types of MCD effects may be observed with one sample.

## Disadvantages

1. The rare earths have so many transitions so close together that it has not been possible to assign the energy levels with any degree of certainty. Therefore it is not possible to know which levels possess orbital degeneracy.
2. With absorption bands very close together or on top of one another it is difficult to distinguish two $C$ term effects


Fig. 32. Absorbance anả high temperature MCD of aiaymium giass


Fig. 33. Room and low tomperature MCI of didymium glass
of opposite sign from a single A term.
3. Since the rare earth atoms are in a condensed state, random crystal field effects will tend to broaden the absorption bands. However, this is not serious since the $f$ electrons are shielded by two other outer electron shells.

Interpretation of results
I. The bands all appear to be temperature dependent, the magnitudes vary roughiy as $1 / T$. From this we can conclude:
a) $B$ terms are relatively small or non-existent
b) The transitions originate in degenerate ground states.
2. As the temperature increases, the populations of the ground state levels tends to become equalized. Therefore the $C$ terms should become smaller while the $A$.term remains the same. This should result in spectra at higher temperatures with more of an A term appearance. This appears to be happening for the absorption bands at $419 \mathrm{~m} \mathrm{\mu}$ and $537 \mathrm{~m} \mathrm{\mu}$.
3. There is an increase in MCD with increasing temperature ior the banas appearing at $540-550 \mathrm{~m} \mathrm{\mu}$. This can be account $\epsilon \overline{\mathrm{a}}$ for by the population of a low-lying electronic level. If this level exists, and transitions take place to an excited state common with the 540 mp absorption, the level would be expected to lie about $30 \mathrm{~cm}^{-1}$ above the ground state. This state would be approximately $90 \%$ populated at the temperatures usea.

## Inorganic Systems

## Temperature dependence

The absorbance and temperature dependence of the $\operatorname{MCD}$ of $\mathrm{K}_{3} \mathrm{Fe}(\mathrm{CN})_{6}$ are shown in Fig. 34. The graph in Fig. 35 indicates that the MCD of ferricyanide ion follows very closely a I/T dependence. This may be interpreted as meaning $B$ terms are quite small for these bands. Only the $425 \mathrm{~m} \mu$ band indicates some slight leveling off of the MCD as temperature is increased, evidence of some B term contribution. It seems therefore that the spectra are almost exclusively $C$ terms. This is more likely to occur with wide band widths, as for any given Zeeman splitting the A terms will decrease as the band is made broader.

Further temperature dependence data is given in the table below.

Table 1. Temperature dependence data for the MCD of cobalt and nickel ions

| Ion | $\lambda m \mu$ | Temp. Range | MCD change | $1 / T$ change |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Co}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}^{+2}$ | 520 | $3^{\circ} \mathrm{C}--63^{\circ} \mathrm{C}$ | $12 \%$ | $18 \%$ |
| $\mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}^{+2}$ | 385 | $5^{\circ} \mathrm{C}--60^{\circ} \mathrm{C}$ | $15 \%$ | $17 \%$ |

It appears that the cobalt ion may exhibit a significant amount of $B$ term MCD for the $520 \mathrm{~m} \mu$ band, whereas the nickel probably has little or no B term effect. Tetrachloroplatinate (II) ion

Another inorganic system was investigated in an attempt to assign one of the excited state energy levels associated with the band at $330 \mathrm{~m} \mathrm{\mu}$. The absorption and MCD of tetrachloroplatinate are shown in Fig. 36.


Fig. 34. Absorbance $(\cdots \cdots \cdots)$ and $\operatorname{MCD}(\square)$ for $\mathrm{K}_{3} \mathrm{Fe}(\mathrm{CN}) 6$


Fig. 35. Temperature dependence oin MCD for $\mathrm{K}_{3} \mathrm{Fe}(\mathrm{CN})_{6}$ at trree wavelengths


Fice. 36. Absorption and MCD for tetrachloro platinate ion

The band at $330 \mathrm{~m} \mu$ presents an $A$ term appearance in the MCD winch inãicates excited state degeneracy. İ̃ this degeneracy is broken by a magnetic field each Zeeman component will be changed in energy by an amount $\delta$ for a total splitting of $2 \delta$. The splitting may take place in eitner the ground or excited state, giving $\delta g$ and $\delta m$ respectively. A comouter program was written using a least squares method to fit the experimental MCD to separate values of $\delta g$ and $\delta m$. The results gave $\delta g=0.00 \mathrm{~cm}^{-1}$ and $\delta \mathrm{m}=0.45 \mathrm{~cm}^{-1}$.

The lack of a Zeeman splitting of the ground state agrees with the assignment of the state to ${ }^{l^{\prime}} \mathrm{Ag}_{\mathrm{g}}$. The presence of a splitting of the excited state supports the assignment as ${ }^{1}$ Eg. (12) Also, the magnetic moment of ${ }^{I_{\text {Eg }}}$ level woula be expected to be one Bohr magneton. The experimental value of $\delta m$ corresponds to a magnetic moment of 0.8 Bonr magnetons, the correct order of magnitude for the expected moment.

$$
\Delta E=H \cap \beta
$$

$$
\begin{aligned}
0.45 \mathrm{~cm}^{-1} & =\left(12 \times 10^{3} \mathrm{~g}\right)(\mathrm{n})\left(4.67 \times 10^{-5} \mathrm{~cm}^{-1} \mathrm{~g}^{-1} \mathrm{dipole}\right. \\
\mathrm{n} & =\frac{0.45}{0.56}=0.8 \text { Bohr magnetons }
\end{aligned}
$$

## Sign oi MCD

In the theoretical section the effects of changing from an electron to a nole $\hat{\text { Iormalism were discussed. Essentially this would be expectea }}$ to change the sign of $\operatorname{MCD}$ in holes rather than electrons are involved in tine transitions. There is some experimental evidence from the inorganic systems that this does indeed occur.

Fig. 37 illustrates again the arrangement of levels and MCD to be expected if the states are inverted. This is, of course, a much simpliき゚ied


Fig. 37. MCD resulting from inverted Jevels
view of the true situation. However, the inversion of levels will still have essentially these effects.

Examples of case (a) in Fig. 37 are not available. The results of Stephens, Schatz, et al. (4I) on the $\mathrm{Cr}(\mathrm{III})$ complexes with $\mathrm{NH}_{3}$, $\mathrm{H}_{2} \mathrm{O}$ ana CN give examples of the (c) case. Since $\operatorname{Cr}\left(\right.$ III ) is a $d^{3}$ system, the electron formalism holds. Case (b) is represented by the $\mathrm{K}_{2} \mathrm{PtCl}{ }_{4}$ MCD discussed above. The tetrachloroplatinate ion has the $a^{8}$ configuration corresponding to two holes. Case (d) includes $\operatorname{Co}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}^{+2}$ which is a $a^{7}$ system, and the $395 \mathrm{~m}_{\mu}$ band of $\mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}^{+2}$, a $\mathrm{a}^{8}$ system. The MCD spectrum of the latter ion is shown in Fig. 38.

The ideal system in which to look for the sign change would be $d^{1}$ vs. $d^{9}$ configurations. These are represented by $\mathrm{Ti}^{1}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}^{+3}$ and $\mathrm{Cu}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}^{+2}$, respectively. Unfortunately Ti(III) exhibits no MCD for the $495 \mathrm{~m} \mu$ band representing the $t_{2 g} \rightarrow e_{g}$ excitation of the single d electron. This may be due to a Jahn-Teller aistortion which always tends to remove the degeneracy needed to have orbital angular momentum. Likewise, the $e_{g}$ ground state of $\mathrm{Cu}(I I)$ is distorted by the Jahn-Teller effect. Possibly some $d^{1}$ and $d^{9}$ systems exist which exhibit MCD, but this will be left for experts in inorganic chemistry to determine.


Fig. 38. $\operatorname{MCD}$ OA $\operatorname{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right) 6^{+2}$

The Supstituted Benzenes
A very important part of the spectroscopy of biological compounds deals with the abscrption of light by aromatic structures. Many biologically interesting molecules contain such groups and the spectra are in an easily accessible region.

The spectral effects observed in the more complex aromatic structures are often correlated with those found in simple benzene derivaties, therefore the substituted benzenes can serve as models. However, there still exist some uncertainties in understanding the spectra of even these simple molecules, notably in assigning transitions to certain symmetry changes and in direction of transition moments. These difficulties hamper the use of aromatic spectral properties to help solve biological problems. Therefore the first step should be to find what MCD can tell us about the model compounds.

From the theory discussed above, it was seen that if two bands mix and MCD occurs, then certain conditions must have been met. From this we can infer certain properties for the electronic structure of the molecule. In view of this possibility, it was decided to study the MCD of the substituted benzenes in an attempt to shed some new light on their electronic properties. This seemed particularly feasible after several randomiy picked benzene derivatives showed easily measurable and interesting MCD spectra. A portion of the results in this section has been reported previously (14).

## The ultraviolet absorption

The UV absorption of benzene is characterized by three bands, the
$A_{l g} \rightarrow B_{2 u}\left(L_{b}\right)$ band at $256 \mathrm{~m} \mu$, the $A_{l_{g}} \rightarrow B_{l u}\left(L_{a}\right)$ band at $203 \mathrm{~m} \mu$, and the $A_{l g} \rightarrow E_{l u}$ band at $l 80 \mathrm{~m} \mathrm{\mu}$. The $L_{a}$ and $I_{b}$ bands are forbidden, the $A_{l g} \rightarrow E_{I u}$ band is allowed. The transitions correspond to changes in the electronic structure of the molecule as shown in Fig. 40. The signs correspond to the signs of the electronic wavefunctions as seen on one side of the molecular plane.

The $A_{l g} \rightarrow B_{\text {lu }}$ transition involves changes in the wave functions at the positions of the atoms, so is called $I_{\underline{a}}$. The signs of the wave functions change in the bond region for $A_{1 g} \rightarrow B_{2 u}$ and are termed $I_{L_{0}}$ (42). The direction of the transition moment for the $L_{a}$ transition is considered to be through the atoms, the transition moment for the $I_{b}$ band is through the bonas (see Fig. 4I).

## Origin of MCD

There are a number of reasons for believing the MCD of the $I_{b}$ band is due primarily to mixing between the $B_{1 u}$ and $B_{2 u}$ levels. None of these reasons alone proves conclusively that such is the case, but taken together they suggest strongly that this is the correct interpretaition.

The levels closest to the $B_{1 u}$ and $B_{2 u}$ levels are those for the ground state ( $A_{I_{g}}$ ) and the next higher excited state ( $E_{I u}$ ). Therefore the $B_{\text {lu }}$ and $B_{2 u}$ levels could be mixing with either or both of these, or with another level even higher in energy than $I_{l u}$. However, when two energy levels are mixing with another level which is higher or lower in energy than either of them, they should both have the same MCD sign while the level they are mixing with should have the opposite sign (see the Simple Theory). This is not the case for the $I_{a}$ and $I_{b}$ bands, they always have opposite signs.


Fig. 39. The benzene electronic energy levels





$E_{1 u}$



Fig. 40. Symmetry changes for benzene transitions


La


Lb

Fig. Hi. Direction of transition moments for $L_{a}$ and $L_{b}$ bands

The MCD is inversely proportional to the energy difference between the mixing levels. These differences are
$E_{01}\left(B_{2 u}-A_{1 g}\right) \cong 38000 \mathrm{~cm}^{-1} \quad E_{23}\left(E_{1 u}-B_{1 u}\right) \cong 7500 \mathrm{~cm}^{-1}$ $E_{02}\left(B_{1 u}-A_{1 g}\right) \cong 48000 \mathrm{~cm}^{-1} \quad E_{13}\left(E_{1 u}-B_{2 u}\right) \cong 17500 \mathrm{~cm}^{-1}$ $E_{12}\left(B_{1 u}-B_{2 u}\right) \cong 10000 \mathrm{~cm}^{-1}$
The MCD is also proportional to the vector product of the electronic transition moments. We use the square roots of the $\varepsilon$ values for the transition moments and these are

$$
\begin{array}{lll}
R_{01} & \left(A_{l g}\right. & \left.B_{2 u}\right) \alpha \\
R_{02} & \left(A_{l g}\right. & \left.B_{I u}\right) \alpha \\
R_{1 G} & 85 \\
R_{03}\left(A_{l g}\right. & \left.E_{l u}\right) \alpha & 200
\end{array}
$$

The three mixing possibilities are shown in Fig. 42. The dotted lines represent magnetic transitions, the solid lines are electric transitions.

Assuming for the moment that the magnetic transition moments are the same for the substituted benzenes (some evidence that this may be true comes later), we can calculate how the $M C D$ is proportional to the $R$ and E values using
$\operatorname{MCD} \propto \frac{R_{i} \times R_{j}}{E_{i j}}$
For the case of Fig. 42(a) we cannot calculate the MCD expected since the order of magnitude of the electric transition moment between states 1 and 2 is not known. However since the $R_{01}$ an $R_{02}$ values are relatively small and $E_{01}$ and $E_{02}$ are relatively large, this contribution mignt be expected to be smaller than for the cases in Fig. 42 ( $b$ and $c$ ).

For mixing between states 1 and 2


Fig. $12 . \quad$ Magnetic (---) and electronic ( $\quad$ ) transition possibilities for benzene levels

$$
\operatorname{MCD} \alpha \frac{\left(R_{01}\right)\left(R_{02}\right)}{E_{12}}=0.13 .
$$

For the case of Fig. 42(c)

$$
\begin{aligned}
\text { MCD } & \alpha\left(\frac{I}{E_{13}}+\frac{I}{E_{23}}\right)\left(R_{03}\right)\left(R_{01}+R_{02}\right) \\
& =R_{03}\left(\frac{R_{01}}{E_{13}}+\frac{R_{01}}{E_{23}}\right)+R_{03}\left(\frac{R_{02}}{E_{13}}+\frac{R_{02}}{E_{23}}\right) \\
& =0.57+3.23=3.90
\end{aligned}
$$

Now we can draw hypothetical MCD spectra to illustrate what we would expect to see in each case. These.are shown in Fig. 43.

The MCD spectra observed look most like Fig. $43(\mathrm{~b})$ both in regard to signs and magnitudes.

A further argument against the likelihood of Fig. 42 (c) being correct is the fact that the magnetic transitions are symmetry forbidden for the planar substituted benzenes.

The MCD for both $L_{a}$ and $L_{b}$ bands of aniline is shown in Table 2 (see below). The larger value for the $I_{a}$ band suggest that the $B_{l u}$ state may be interacting with more than one energy level, but the general appearance of the MCD spectrum (Fig. 48) is very like that of Fig. 43(b).

As the $I_{a}$ and $I_{b}$ bands approach each other in enersy, as happens in certain compounds, the MCD values tend to be equal in magnituade and opposite in sign, just what would be expected for mixing of $B_{I u}$ and $B_{2 u}$ bands.

MCD cata
Data related to the $\mathbb{N G D}$ of the substituted benzenes is listed in


Fig. 43. MCD expected from hypothetical mixing cases

Table 2.
The values for $\sigma_{p}$ in Tabie 2 are from Ferguson (43, p. 415). These were determined from chemical data. The signs for the $\sigma_{p}$ of the halogens were changed when it was found that this greatly improved the fit of a plot of MCD vs $\sigma_{p}$ for a number of compounds (see below). It is realized that there is no strong justification for this sign change. However, there are some reasons for thinking the spectral and chemical properties might be of opposite sign for the halogens. For instance, the halogens are considered to be electron releasing by resonance and electron withdrawing by inductive effects. The $\sigma_{p}$ may be considered to consist of two contributions of opposite sign, related to resonance ( $\sigma_{R}$ ) and induction $\left(\sigma_{I}\right)$. The $\sigma_{R}$ effect is negative for the halogens (43, $p .394$ ). The $\sigma_{p}$ for the benzenes with more than two substituents are calculated by a simple addition of the $\sigma_{p}$ of the groups, again considering the halogens as having a negative sign.

Several references were used to obtain extinction coefficients ( $\varepsilon$ ) (44, p. 257)(45,46). Since there is some disagreement between sources for $\varepsilon$ values, they should be considered no more accurate than $10 \%$. The square roots of the $\varepsilon$ 's are proportional to the magnitudes of the transition moments.

The energies at the peak maxima for the $I_{a}$ and $I_{b}$ bands are needed for calculations, but are also listed since there is sometimes disagreement on the assignment of the $I_{b}$ band. When no other guidance was available, the most reasonable bands were chosen as $L_{a}$ and $L_{b}$. In cases where the $L_{a}$ and $L_{b}$ bands seemed to be almost degenerate, no attempt was made to

Table 2. Experimental data for the mono-substituted benzenes

| Group | CN | COOH | CHO | F |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{\text {p }}$ | +0.66 | $+0.45$ | +0.23 | -0.06 |
| $\varepsilon L_{b}$ | 1000 | 970 | 1000 | 1280 |
| $\sqrt{\varepsilon L_{b}},\left(R_{I}\right)$ | 31.5 | 31.1 | 31.5 | 35.7 |
| $\varepsilon L_{a}$ | 13000 | 11600 | 11400 | 7950 |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 113 | 108 | 106.5 | 88.8 |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 36900 | 36500 | 35400 | 38500 |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{a}}$ | 44700 | 43500 | 40800 | 4900 |
| $\mathrm{E}_{2}-\mathrm{E}_{1},\left(\mathrm{E}_{12}\right)$ | .7800 | 7000 | 5400 | 10500 |
| $\left(\mathrm{R}_{1}\right)\left(\mathrm{R}_{2}\right) / \mathrm{E}_{12}$ | 0.456 | 0.48 | 0.62 | 0.30 |
| [MCD] | +0.377 | +0.276 | +0.305 | 0 |
| solvent | EtOH | IN HCl-EtOH | 95\% EtOH | EtOH |
| symmetry | $\mathrm{c}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{~V}}$ | $\mathrm{C}_{2 \mathrm{~V}}$ |
| Fig. 44 no. | 1 | 2 | 3 | 4 |

Table 2. (Continued)

| Group | $\mathrm{C}_{2} \mathrm{H}_{5}$ | $\mathrm{CH}_{3}$ | I | Cl | Br |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma_{p}$ | -0.15 | -0.17 | -0.18 | -0.22 | -0.23 |
| $\varepsilon L_{b}$ | 160 | 225 | 700 | 190 | 192 |
| $\sqrt{\varepsilon L_{b}}$, $\left(\mathrm{R}_{\mathrm{l}}\right)$ | 2.16 | 15 | 26.4 | 13.7 | 13.8 |
| $\varepsilon L_{a}$ | 32000 | 7000 | 7000 | 7400 | 7900 |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 178 | 83.2 | 83.2 | 85.5 | 88.5 |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 38600 | 38500 | 38900 | 38000 | 38500 |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{e}}$ | , 48500 | 48300 | 47500 | 47500 |  |
| $\mathrm{E}_{2}-\mathrm{E}_{1},\left(\mathrm{E}_{12}\right)$. | 9900 | 10000 | 9400 | 9500 | 9000 |
| $\left(R_{1}\right)\left(R_{2}\right) / E_{12}$ | 0.39 | 0.124 | 0.233 | 0.125 | 0.136 |
| [MCD] | -0.0276 | -0.0283 | -0.044 | -0.065 | -0.071 |
| solvent | EtOH | 95\% EtOH | EtOH | EtOH | 95\% EtOH |
| symmetry | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{~V}}$ | $\mathrm{C}_{2 \mathrm{~V}}$ | $\mathrm{C}_{2 \mathrm{v}}$ |
| Fig. 44 no . | 5 | 6 | 7 | 8 | 9 |

Table 2. (Continued)

| Group | OH | $\mathrm{NH}_{2}$ | $\underset{\text { (benzene) }}{\mathrm{H}}$ | $\mathrm{OCH}_{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{\sigma}$ | -0.37 | -0.66 | 0 | -0.27 |
| $\varepsilon L_{b}$ | 1450 | 1430 | 200 | 1480 |
| $\sqrt{\varepsilon L_{b}},\left(R_{1}\right)$ | 38 | 37.6 | 14.1 | 38.3 |
| $\varepsilon L_{\text {a }}$ | 6200 | 8600 | 7400 | 6400 |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 78.3 | 92.3 | 85.6 | 79.7 |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 37100 | 35800 | 39000 | 37300 |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{a}}$ | 47500 | 43500 | 49000 | . 46000 |
| $E_{2}-E_{1},\left(E_{12}\right)$ | ' 10400 | 7700 | 10000 | 8700 |
| $\left(R_{1}\right)\left(R_{2}\right) / E_{12}$ | 0.286 | 0.45 | 0.12 | 0.35 |
| [MCD] | -0.224 | -0.417 | $\pm 0.0096$ | -0.266 |
| solvent | 0.1 NHCl | 95\% EtOH | cyclohexane | EtOH |
| symmetry | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | D6h | $\mathrm{C}_{2 \mathrm{v}}$ |
| Fig. 44 no. | 10 | 11 | 12 | 13 |

Iable 2. (Continued)

| Group | $\begin{aligned} & \mathbb{N}, \mathbb{N} \text { di.-Me } \\ & \text { aniline } \end{aligned}$ | $\mathrm{COO}^{-}$ | $\mathrm{SO}_{3} \mathrm{H}$ | $\mathrm{SO}_{2} \mathrm{NH}_{2}$ | $\mathrm{SO}_{2} \mathrm{Cl}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{\circ} \mathrm{p}$ | -0.83 | -- | --i | -- | -- |
| $\varepsilon L_{b}$ | 15500 | 560 | 290 | 740 | 1260 |
| $\sqrt{\varepsilon L_{b}},\left(R_{l}\right)$ | 124 | 23.6 | 17 | 27.1 | 35.5 |
| $\varepsilon L_{\text {a }}$ | 30000 | 8700 | 7800 | 9700 | 12000 |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 173 | 93 | 88 | 98 | 109 |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 40000 | 37400 | 38000 | 37800 | 37000 |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{a}}$ | , 50000 | 44700 | 47000 | 45800 | 47700 |
| $E_{2}-E_{1},\left(E_{12}\right)$ | 10000 | 7300 | 9000 | 8000 | 10700 |
| $\left(R_{1}\right)\left(R_{2}\right) / E_{12}$ | 2.15 | 0.30 | 0.166 | 0.332 | 0.360 |
| [MCD] | +2.28 | +0.224 | +0.0432 | +0.178 | +0.346 |
| solvent | EtOH | 0.1N NaOH | $\mathrm{Cl}_{2}$ water | $\mathrm{NH}_{4} \mathrm{OH}$ | EtOH |
| symmetry | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{~V}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{v}}$ |
| Fig. 44 no. | 14 | -- | -- | - | -- |

Table 2. (Continued)

| Group | $\mathrm{NH}_{3}^{+}$ | $\mathrm{NO}_{2}$ | $\begin{aligned} & 1-\mathrm{NH}_{2} \\ & 2-\mathrm{OH} \end{aligned}$ | $\begin{aligned} & \mathrm{I}-\mathrm{NH}_{2} \\ & 3-\mathrm{Cl} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{NH}_{2} \\ & 3-\mathrm{Br} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{CH}_{3} \\ & 3-\mathrm{OH} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma_{p}$ | -- | +0.78 | -1.03 | -0.88 | -0.89 | -0.54 |
| $\varepsilon L_{b}$ | 160 | -- | 3300 | 2100 | 2400 | 1600 |
| $\sqrt{\varepsilon I_{b}}$, $\left(\mathrm{R}_{1}\right)$ | 12.6 | -- | 57.3 | 45.9 | 48.8 | 40 |
| $\varepsilon L_{a}$ | 7500 | 10000 | 7100 | 8500 | 7800 | -- |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 86 | -- | 84 | 91.7 | 88 | -- |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{I}_{\mathrm{b}}$ | 39400 | -- | 34800 | 34700 | 34500 | 36300 |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{a}}$ | . 49400 | 39700 | 42600 | 42100 | 42100 | -- |
| $E_{2}-E_{1},\left(E_{12}\right)$ | 10000 | -- | 7800 | 7400 | 7600 | -- |
| $\left(\mathrm{R}_{1}\right)\left(\mathrm{R}_{2}\right) / \mathrm{E}_{12}$ | 0.109 | -- | 0.62 | $0.569$ | 0.567 | -- |
| [MCD] | -0.0057 | +0.586 | -0.411 | $-0.522$ | -0.491 | -0.19 |
| solvent | IN HCl | cyclohexane | EtOH | EtOH | EtOH | 95\% EtOH |
| symmetry | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{5}$ | $\mathrm{C}_{5}$ | $\mathrm{C}_{5}$ | $\mathrm{C}_{5}$ |
| Fig. 4 l no. | -- | 15 | 16 | 17 | 18 | 19 |

Table 2. (Continued)

| Group | $\begin{aligned} & 1-\mathrm{CH}_{3} \\ & 2-\mathrm{CH}_{3} \end{aligned}$ | $\begin{aligned} & \mathrm{I}-\mathrm{CH}_{3} \\ & 3-\mathrm{CH}_{3} \end{aligned}$ | $\begin{aligned} & \mathrm{l}-\mathrm{CH}_{3} \\ & 4-\mathrm{CH}_{3} \end{aligned}$ | $\begin{aligned} & \mathrm{l}-\mathrm{NO}_{2} \\ & 4-\mathrm{CHO} \end{aligned}$ | $\begin{aligned} & \mathrm{I}-\mathrm{CH}_{3} \\ & 2-\mathrm{I} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{Br} \\ & 3-\mathrm{F} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma_{p}$ | -0.34 | -0.34 | -0.34 | +1. 01 | -0.35 | -0.29 |
| $\varepsilon L_{b}$ | 260 | 280 | 520 | 3020 | -- | 1000 |
| $\sqrt{\varepsilon L_{l b}},\left(R_{l}\right)$ | 16.1 | 1.67 | 22.7 | 54.8 | -- | 31.5 |
| $\varepsilon L_{a}$ | 9000 | 8000 | 7500 | 14400 | 12000 | 8000 |
| $\sqrt{\varepsilon I_{a}},\left(R_{2}\right)$ | 94.4 | 89 | 86.2 | 119 | 109 | 89 |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 38000 | 37800 | 36600 | 33100 | -- | 37500 |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{a}}$ | - 47900 | 47000 | 47100 | 37600 | 43000 | 47000 |
| $E_{2}-E_{1},\left(E_{12}\right)$ | 9900 | 9200 | 10500 | 4500 | -- | 9500 |
| $\left(\mathrm{R}_{1}\right)\left(\mathrm{R}_{2}\right) / \mathrm{E}_{12}$ | 0.154 | 0.162 | 0.187 | 1.45 | -- | 0.295 |
| [MCD] | -0.046 | -0.038 | -0.090 | +0.757 | 0 | 0 |
| solvent | EtOH | EtOH | 95\% EtOH | $\mathrm{EtOH}-\mathrm{H}_{2} \mathrm{O}$ | EtOH | EtOH |
| symmetry | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{c}_{2 \mathrm{v}}$ | D2h | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{5}$ | $\mathrm{C}_{5}$ |
| Fig. 44 no. | 20 | 21 | 22 | 23 | 24 | 25 |

Table 2. (Continued)

| Group | $\begin{aligned} & 1-\mathrm{OH} \\ & 2-\mathrm{COOH} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{NH}_{2} \\ & 4-\mathrm{COOH} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{Cl} \\ & 4-\mathrm{OH} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{Br} \\ & 4-\mathrm{NO}_{2} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{\circ} \mathrm{p}$ | -0.12 | -0.21 | -0.59 | +0.55 |
| $\varepsilon L_{b}$ | 4000 | 18600 | 1600 | 9000 |
| $\sqrt{\varepsilon L_{b}},\left(R_{1}\right)$ | $6: 3$ | 136 | 40 | -- |
| $\varepsilon L_{a}$ | 32000 | 8900 | 8900 | 13000 |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 178 | 94 | 94 | -- . |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 32700 | 34600 | 35700 | -- |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{a}}$ | . 48900 | 45500 | 44500 | -- |
| $\mathrm{E}_{2}-\mathrm{E}_{1},\left(\mathrm{E}_{12}\right)$ | 16200 | 10600 | 8800 | -- |
| $\left(\mathrm{R}_{1}\right)\left(\mathrm{R}_{2}\right) / \mathrm{E}_{12}$ | 0.692 | 1.20 | 0.428 | -- |
| [MCD] | 0 | -2.24 | -0.357 | +0.648 |
| solvent | 95\% EtOH | EtOH | $\mathrm{H}_{2} \mathrm{O}$ | EtOH-H20 |
| symmetry | $\mathrm{C}_{5}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{~V}}$ |
| Fig. 44 no. | 26 | 27 | 28 | 29 |

Table 2. (Continued)

| Group | $\begin{aligned} & 1-B r \\ & 4-B r \end{aligned}$ | $\begin{aligned} & 1-\mathrm{Br} \\ & 4-\mathrm{Cl} \end{aligned}$ | penta- <br> methyl | hexachloro | $\begin{aligned} & \text { 1,3,5 tri- } \\ & \text { methyl } \end{aligned}$ | hexaethyl |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{\circ} \mathrm{p}$ | -0.46 | -0.45 | -0.85 | -1.32 | -0.51 | -0.90 |
| $\varepsilon L_{b}$ | 315 | 415 | 20 | 210 | 200 | 470 |
| $\sqrt{\varepsilon I_{b}},\left(R_{1}\right)$ | 17.7 | 20.4 | 4.45 | 14.5 | 14.1 | 21.6 |
| $\varepsilon L_{a}$ | 1.5000 | 13500 | -- | -- | -- | 8450 |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 122 | 116 | -- | -- | -- | 92 |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 38500 | 36500 | 37000 | 34400 | 37700 | 36700 |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{a}}$ | 43700 | 44700 | -- | -- | -- | 45500 |
| $E_{2}-E_{1},\left(E_{12}\right)$ | . 5200 | 8200 | -- | -- | -- | 8800 |
| $\left(\mathrm{R}_{1}\right)\left(\mathrm{R}_{2}\right) / \mathrm{E}_{12}$ | 0.416 | 0.29 | -- | -- | -- | 0.226 |
| [MCD] | -0.203 | -0.218 | -0.00247 | +0.0338 | 0 | +0.029 |
| solvent | EtOH | EtOH | EtOH | EtOH | 95\% EtOH | EtOH |
| symmetry | D2h | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{c}_{2 \mathrm{v}}$ | D3d | D3h | D6h |
| Fig. 44 no. | 30 | 31 | 32 | 33 | 34 | 35 |

Table 2. (Continued)

| Group | $\mathrm{CH}_{2} \mathrm{OH}$ | $0^{-}$ | $\begin{aligned} & 1-\mathrm{NH}_{2}^{+} \\ & 4-\mathrm{COOH} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{Br} \\ & 2-\mathrm{CH}_{3} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{OCH}_{3} \\ & 2-\mathrm{COOH} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{\circ} \mathrm{p}$ | -- | -- | -- | -0.40 | +0.18 |
| $\varepsilon L_{b}$ | 480 | 2600 | 1000 | 460 | 2950 |
| $\sqrt{\varepsilon L_{b}},\left(R_{l}\right)$ | 21.9 | 51 | 31.5 | 21.4 | 54 |
| $\varepsilon L_{\text {a }}$ | 8000 | 9400 | 11500 | 10000 | 7950 |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 89 | 96.6 | 107 | 100 | 89 |
| $E_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 38500 | 34900 | 37000 | 37400 | 35500 |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{a}}$ | 47600 | 42500 | 44200 | 48000 | 43700 |
| $\mathrm{E}_{2}-\mathrm{E}_{1},\left(\mathrm{E}_{12}\right)$ | 9100 | 7600 | 7200 | 10600 | 8200 |
| $\left(\mathrm{R}_{1}\right)\left(\mathrm{R}_{2}\right) / \mathrm{E}_{12}$ | 0.214 | 0.647 | 0.467 | 0.201 | 0.586 |
| [MCD] | -0.00529 | -0.717 | +0.273 | -0.0743 | 0 |
| solvent | $\mathrm{H}_{2} \mathrm{O}$ | 0.1. N NaOH | 0.1 NHCl | EtOH | $\mathrm{H}_{2} \mathrm{O}$ |
| symmetry | $\mathrm{C}_{2 \mathrm{~V}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{~V}}$ | $\mathrm{C}_{5}$ | $\mathrm{C}_{5}$ |
| Fig. 44 no. | -- | -- | -- | 36 | 37 |

Table 2. (Continued)

| Group | $\begin{aligned} & 1-\mathrm{F} \\ & 3-\mathrm{COOH} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{OH} \\ & 4-\mathrm{COCH}_{3} \end{aligned}$ | $\begin{aligned} & \mathrm{I}-\mathrm{NO}_{2} \\ & 4-\mathrm{NH}_{2} \end{aligned}$ | $\begin{aligned} & 1-\mathrm{Br} \\ & 3-\mathrm{NO}_{2} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{\circ} \mathrm{p}$ | +0.39 | +0.33 | +0.12 | +0.55 |
| $\varepsilon L_{b}$ | 1700 | -- | -- |  |
| $\sqrt{\varepsilon L_{b}}$, $\left(R_{1}\right)$ | 41 | -- | -- |  |
| $\varepsilon L_{a}$ | 11300 | 15000 | -- |  |
| $\sqrt{\varepsilon L_{a}},\left(R_{2}\right)$ | 106 | -- | -- |  |
| $\mathrm{E}_{1} \mathrm{~cm}^{-1}, \mathrm{~L}_{\mathrm{b}}$ | 34900 | -- | -- |  |
| $\mathrm{E}_{2} \mathrm{~cm}^{-1}, L_{a}$ | . 43500 | 40500 | -- |  |
| $\mathrm{E}_{2}-\mathrm{E}_{1},\left(\mathrm{E}_{12}\right)$ | 8600 | -- | -- |  |
| $\left(\mathrm{R}_{1}\right)\left(\mathrm{R}_{2}\right) / \mathrm{E}_{12}$ | 0.507 | -- | -- |  |
| [MCD] | +0.304 | + | 0 | + |
| solvent | $\mathrm{H}_{2} \mathrm{O}$ | EtOH | EtOH | EtOH |
| symmetry | $\mathrm{C}_{5}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{2 \mathrm{v}}$ | $\mathrm{C}_{5}$ |
| Fig. 44 no. | 38 | -- | -- | -- |

estimate their relative energies or extinction coefficients.
The quantity [MCD] is the molar magnetic circular dichroism (the $\Delta A$ which would be observed for al molar solution with al cm path length) normalized to a 10 kgauss field. It is calculated using the peak height for the MCD band. According to the theory, the area of an MCD bana rather than its maxima should be used in the calculation. However, most of the $B_{2 u}$ bands for the substituted benzenes have band widths within a few hundred wave numbers of $3300 \mathrm{~cm}^{-1}$, and therefore peak heights are very nearly proportional to areas. In those cases where the band width is considerably larger than $3300 \mathrm{~cm}^{-1}$ it is usually caused by a contribution from some absorption not related to the $\mathcal{B}_{2 u}$ absorption.

The symmetries given in the table are the local symmetries as seen by the benzene ring. For example, any monosubstituted benzene, regardless of the symmetry of the substituent, is called $C_{2 v}$.

Table 3 shows a correlation of the MCD of a number of monosubstituted benzenes with their directing properties. The order of the substituents was found by dividing them into ortho-para and meta directing groups and then arranging them in order of their effect on the wavelength of the $B_{\text {Iu }}$ band relative to benzene ( $44, \mathrm{p} .257$ ). For all of the compounds except $\mathrm{NO}_{2}$ the energy difference between the $\mathrm{B}_{1 \mathrm{u}}$ and $\mathrm{B}_{2 \mathrm{u}}$ bands is roughly of the same orcier.

The correlation in Table 3 is very good semiquantitatively with the exception of benzonitrile. However, even in this case the sign is correct. It appears that if a substituent is meta-directing it will have a positive sign for the MCD of the $L_{b}$ band, if it is ortho-para directing the sign

Table 3. MCD and wavelength change of $L_{a}$ band for mono-substituted benzenes

|  | Group | $\Delta \lambda \mathrm{m} \mu$ | [MCD] |
| :---: | :---: | :---: | :---: |
| meta | NO, | 60 | +0.586 |
|  | CHO | 46.5 | +0.305 |
|  | COOH | 26.5 | +0.276 |
|  | COO- | 20.5 | +0.224 |
|  | CN | 14.5 | +0.377 |
| Orthopara | $\mathrm{SO}_{2} \mathrm{NH}_{2}$ | 14 | +0.178 |
|  | $\mathrm{NH}_{3}^{2}+2$ | 0 | -0.0057 |
|  | $\mathrm{CH}_{3}^{3}$ | 3 | -0.0283 |
|  | Cl ${ }^{3}$ | 6 | -0.065 |
|  | Br | 6.5 | -0.071 |
|  | OH | 7 | -0.224 |
|  |  | 14 | -0.266 |
|  | $\mathrm{NH}_{2}{ }^{3}$ | 26.5 | -0.417 |
|  | $0-2$ | 31.5 | -0.717 |
|  | $\mathrm{N}\left(\mathrm{CH}_{3}\right)_{2}$ | ? | +2.28 |

Table 4. MCD and percent meta substitution for some mono-substituted benzenes

| Group | \% Meta | $[\mathrm{MCD}]$ |
| :--- | :--- | :--- |
| $\mathrm{NO}_{2}$ | 93.3 | +0.586 |
| $\mathrm{CN}^{2}$ | 81 | +0.377 |
| COOH | 80.2 | +0.276 |
| CHO | 79 | +0.305 |
| H | $(40)$ | $\pm 0.0096$ |
| $\mathrm{CH}_{3}$ | 4.4 | -0.0283 |
| $\mathrm{I}_{3}$ | 2.1 | -0.044 |
| $\mathrm{OCH}_{3}$ | 1.2 | -0.266 |
| $\mathrm{Br}^{\mathrm{Cl}}$ | 0.9 | -0.071 |
| F | 0.5 | -0.065 |
| $\mathrm{OH}_{3}$ | 0 | 0 |
| $\mathrm{NH}_{2}$ | 0.417 |  |

will be negative. Compared to unsubstituted benzene, the ortho-para directors result in a net increase of electron density in the ring, the meta directors produce a net decrease. Since there are fewer electrons than spaces for electrons in the latter case, one may speak of positive holes as circulating in the ring. The experimental evidence suggests that the sign of the charge carrier determines the sign of the MCD.

Table 4 is a correlation of MCD with the respective percent of meta substitution (43, p. 431). The quantitative agreement is poor but again the signs are positive for m-directors and negative for o-p directors. Although the sign of the MCD seems to have a reasonable explanation, the magnitude of the MCD is not so easily resolved. The form of the $B$ term equation suggests that the magnitude of the charge carrier (holes or excess electrons) should affect the size of the MCD effect. However, there is no independent measurement for the magnitude of the charge carrier to relate to the MCD. Tables 3 and 4 are attempts in this direction but the amount of charge carrier is not necessarily directly related to either correlation.

Fig. 44 shows a plot of MCD versus $\sigma_{p}$. The heavy rings inaicate points for monosubstituted benzenes. 31 out of the 38 compounas plotted follow an approximate straight line relationship which appears to be quite good considering experimental error.

It is somewhat surprising that nitrobenzene (15) fits in the straightline region since it is not an example of a compound with well separated $B_{I u}$ and $B_{2 u}$ bands. The MCD and absorption spectra of $\mathrm{NO}_{2}$-benzene are shown in Fig. 45. In view of the circumstances, the fit of nitrobenzene


Fig. 44. MCD vs. Hammett's sigma para


Fig. 45. Absorption (- - ) and MCD (-) of nitrobenzene
in Fig. 44 may be fortuitous. The shape of the absorption spectrum indicates that a single band is present with a maximum at $252 \mathrm{~m} \mathrm{\mu}$. However, the drastic electron-withdrawal of the nitro group is expected to move the $A_{l g} \rightarrow B_{l u}$ band to approximately the same position as the $A_{l g} \rightarrow B_{2 u}$ band. One would expect a wider bandwidth in this case, and this occurs, the bandwidth is about $6600 \mathrm{~cm}^{-1}$, or about twice the normal value. However it is not really obvious that the two bands are almost degenerate until one sees the MCD spectrum. The shape of the MCD makes it clear that two bands very close in energy are interacting. It is still not possible to determine the relative energies of the interacting bands, for the one with lowest energy will always give the same sign. But the usefulness of MCD for determining when an accidental degeneracy has taken place is quite evident.

N, N dimethyl aniline (see Fig. 46) seems to have some mixing with an $n \rightarrow \pi^{*}$ band at lower energy ( $300 \mathrm{~m} \mathrm{\mu}$ ) than the $\mathrm{B}_{2 \mathrm{u}}$ band. This may be the reason that neither the sign nor magnitude of the MCD fit the plot of Fig. 44. Para-amino benzoic acid is another example where the $L_{a}$ and $I_{b}$ bands may be almost. degenerate (see Fig. 47). The spectra for $p-\mathrm{NH}_{3}{ }^{+}$ benzoic acid are included in Fig. 47 to indicate the change which is brought about simply by putting a proton on the amino group.

The spectra of aniline are given in Fig. 48 as an example showing the MCD of both the $I_{a}$ and $I_{b}$ transitions. In most cases the $I_{a}$ band was too low in wavelength to give dependable experimental results, aithough the sign could usually be determined.

The points 32-35 are poor-fitting, the reason seems to be related


Fig. 46. Absorption (---) and MCD ( - ) of $N, N$ dimethyl aniline



Fig. 47. Absorption ( $-\cdots$ ) and MCD ( - ) of p-amino benzoic acid in (left) ethanol and (right) acidic solution


Fig. 48. Absorption (...-) and MCD ( - ) of aniline
to the symmetry of these compounds and will be discussed below.
In general, Fig. 44 indicates a good relationship for compounds in which the $L_{a}$ and $I_{b}$ bands are well-separated and in which no interfering bands mix with them. Although it is not possible to make any unambiguous interpretation of the relationship revealed by Fig. 44, there seems to be a clear indication that the sign of the MCD is related to the sign of the charge carrier in the aromatic ring.

## Symmetry effects

As mentioned above, symmetry seems to play a part in determining the MCD of some of the substituted benzenes. Four of the points which did not fit well on the graph in Fig. 44 are highly symmetrical compounds.

Table 5 indicates the symmetry conditions for transitions of the substituted benzenes. The groups correspond to molecular types as follows: D6h for benzene and hexa-substituted benzenes, C2v for monosubstituted benzenes, D3h for $1,3,5$ tri-substituted benzenes, Cs for any planar benzene derivative not of higher symmetry, D3d for the "puckered" molecule hexachlorobenzene, and D2h for para disubstituted benzenes.

The symmetry groups which correspond to $A_{l g}, B_{1 u}$ and $B_{2 u}$ were taken from correlation tables (47). Then the selection rules were evaluated from character tables (48). An 0 means the transition is forbidden by symmetry, + means it is allowed. The coordinate convention used is shown in Fig. 49.

From the form of the B term one can see that the relative directions are important since vectors are involved, $B{ }_{\alpha} \bar{M}_{12} \cdot \bar{R}_{1} \times \bar{R}_{2}$. In order to have an MCD there must be some perpendicuiar component between the two

Table 5. Symmetry relations for substituted benzenes

| Group | D6h Correlations |  |  | $\underline{B_{1 u} \leftrightarrow B_{2 u} \text { Magnetic }}$ |  |  | $\mathrm{L}_{\mathrm{a}}$ Electronic |  |  | $\underline{L}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | X | y | 2 | x | y | Z | x | y | z |
| D6h | ${ }^{\text {A }}$ lg | $\mathrm{B}_{\mathrm{lu}}$ | $\mathrm{B}_{2 \mathrm{u}}$ | 0 | 0 | + | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathrm{C}_{2 \mathrm{~V}}$ | $\mathrm{A}_{1}$ | $\mathrm{A}_{1}$ | $\mathrm{B}_{1}$ | 0 | $+$ | 0 | 0 | 0 | $+$ | + | 0 | 0 |
| D3h | $A_{1}{ }^{\prime}$ | $\mathrm{A}_{1}$ | $\mathrm{A}_{2}{ }^{\prime}$ | 0 | 0 | $+$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathrm{C}_{5}$ | $A^{\prime}$ | $A^{\prime}$ | $A^{\prime \prime}$ | + | + | 0 | + | + | 0 | 0 | 0 | + |
| D3d | ${ }^{A_{1 g}}$ | ${ }^{\text {A }}$ u | $A_{2 u}$ | 0 | 0 | $+$ | 0 | 0 | 0 | 0 | 0 | $+$ |
| D2h | $A_{g}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{B}_{3 \mathrm{u}}$ | 0 | 0 | $+$ | 0 | + | 0 | + | 0 | 0 |



$c_{2 v}$

Fig. 49. Symmetry coordinate conventions
electronic transition moments $\overline{\mathrm{R} I}$ and $\overline{\mathrm{R} 2}$, and there must be a parallel component between the magnetic transition moment $\vec{M}_{12}$ and the vector product of the electronic moments $\overline{\mathrm{RI}} \mathrm{x} \overline{\mathrm{R} 2}$. The ideal situation would be to have the three transitions allowed in mutually orthogonal directions. This is possible for the C2v, Cs and D2h symmetries. These are all represented by molecules with large MCD's.

Benzene has no allowed electronic transitions for the $B_{1 u}$ or $B_{2 u}$ symmetries. That the $B_{1 u}$ and $B_{2 u}$ bands are seen at all is due to a molecular vibration with $\mathrm{E}_{\mathrm{g}}$ symmetry which mixes in small amounts of allowed symmetries. The vibration-permitted transitions are, however, not necessarily in the directions needed for MCD. Benzene and the other D6h compound studied, hexaethylbenzene, have small MCD values which may be caused by their symmetry.

Hexachlorobenzene has a puckered shape (49), and as a result one direction for the $A_{l g} \rightarrow B_{2 u}$ transition is allowed. It does have a larger MCD than benzene or hexaethylbenzene but still fits poorly in Fig. 44.

Mesitylene (1,3,5 trimethyl benzene) has the same selection rules as compounds of D6h symmetry. No MCD was detected for this compound and
symmetry seems to be strongly implicated in this result.
A non-benzenoid molecule with $D 3 h$ symmetry was examined, it also has a small MCD. The compound is symmetrical-triazine, the spectrum is shown in Fig. 50. The band peaking at $272 \mathrm{~m} \mu$ is assigned as an $n \rightarrow \pi^{*}$ transition, with an obvious vibrational progression stating at about $320 \mathrm{~m} \mathrm{\mu}$ (50). The tropylium ion

An obvious test of the hypothesis that the sign of the charge carrier is determining the sign of the MCD is to examine molecules which by their structure should possess either an extra electron or a hole. In 1966 in a private communication, Professor W. Simpson of the University of Washington suggested that good examples to investigate would be the ions tropylium $\left(\mathrm{C}_{7} \mathrm{H}_{7}{ }^{+}\right)$and cyclopentadienide $\left(\mathrm{C}_{5} \mathrm{H}_{5}{ }^{-}\right.$). Unfortunately $\mathrm{C}_{5} \mathrm{H}_{5}{ }^{-}$ seems to have no spectrum in the accessible region.

The MCD and absorption spectra of $\mathrm{C}_{7} \mathrm{H}_{7} \mathrm{~B} \mathrm{~F}_{4}$ in conc. $\mathrm{H}_{2} \mathrm{SO}_{4}$ are shown in Fig. 52. One would expect the long wavelength absorption to be positive if holes circulate in the ring. Some data on the molecule is given in Table 6. The absorption coefficients are given by Ginsberg (5I).

Table 6. Optical data for the tropylium ion

```
Extinction coefficient ( }\varepsilon\mathrm{ )
    A1
    A1
Transition energy
    A_
    A_'
Molar MCD ([MCD])
    at 280 m\mu ...........+l. }6
```



Fig. 50. Absorption (---) and MCD (——) of symmetrical-triazine

Experimental and theoretical work on the tropylium ion have been done by J. N. Murrell and H. C. Longuet-Higgins, and W. von Doering and L. H. Knox (52,53). The energy level diagram is shown in Fig. 51 and the symmetry correlation between $D 7 h$ and $D 6 h$ is given by Table. 7 .


Fig. 51. Tropylium ion energy levels

Table 7. Symmetry correlations for the tropylium ion
D7h D6h
$A_{1} \quad A_{1 g}$ $E_{1} \quad E_{l u}$ $E_{3}{ }^{I} \quad B_{I u}, B_{2 u}$
$\qquad$

The MCD spectrum agrees with that expected from an $A \rightarrow E$ type (doubly degenerate) transition. If the $E_{3}^{\prime}$ state is truly degenerate the MCD would be the result of an A term interaction. Another possibility is that the two levels of the $E_{3}^{\prime}$ state are slightly different in energy as the result of some perturbation, although still appearing as essentially a single peak at $37,000 \mathrm{~cm}^{-1}$. The MCD in this event would be due to $B$ term interactions. In either case the MCD will have the same appearance. The sign . of the splitting for tropylium agrees with the assumption that a hole is the charge carrier. The presence of a small negative MCD at the long wavelength end of the absorption region is somewhat disturbing, for the lowest energy MCD should be positive. From the position of this small MCD relative to the absorption band, it may be related to the 0-0


00w \&甘า0w

Fig. 52. Absorption (---) and MCD ( $\quad$ ) of tropylium ion
vibrational transition. This transition is not normally seen in the electronic spectrum since it is forbidden. However, its MCD may be permitted by some mechanism exclusive of that allowing the rest of the band. This sort of situation occurs in natural circular dichroism. More analysis would be necessary to clarify this case.

## Correlations

The form of the $B$ term
MCD $\alpha \frac{\overline{\mathrm{M}}^{\mathrm{R}_{1}} \times \overline{\mathrm{R}}_{2}}{\mathrm{E}_{12}}$
suggests that we plot the data in Table 2 as
[MCD] vs. $\frac{\bar{R}_{1} \times \vec{R}_{2}}{\mathrm{E}_{12}}$
in order to determine whether the magnetic transition moment is appreciably different for the substituted benzenes. Since the relative directions for the transitions are not known, the vector products cannot be evaluated. Therefore we will approximate by performing the ordinary multiplication of $R_{1}$ and $R_{2}$. A plot of $[M C D]$ vs. $\frac{\left(R_{1}\right)\left(R_{2}\right)}{E_{12}}$ is shown in Fig. 53.

Considering the approximations made, the graph indicates a definite trend, suggesting that changes in the magnitude of the magnetic transition moment are minor between different substituted benzenes. Since the directions of the transition moments are not known (although they are probably the same for all monosubstituted benzenes), nothing more definite can be said.

Another correlation is suggested by the fact that circulating


0
electrons.in the benzene ring produce a diamagnetism. If the diamagnetism due to the atoms of the substituents could be determined separately, this could be subtracted from the total diamagnetic susceptibility to yield that due only to the atoms in the ring plus the ring current. The data from these calculations is shown in Table 8. The experimental values are from the literature (54,55,56). Pascal's values were used to evaluate the substituent susceptibilities (43, p. 53). For the disubstituted benzenes the value of the hydrogen atom susceptibility had to be added to obtain the $\mathrm{C}_{6} \mathrm{H}_{5}$ susceptibility.

As the graph of $[M C D]$ vs. $X$ shows (see Fig. 54), there are two trends which seem to take place, one for molecules with small MCD (no correlation) and another for those with relatively large MCD (positive correlation). The interpretation of this result is not obvious. Summary

A number of conclusions resulting from the study of the substituted benzenes are Iisted below.
I. The $B_{l u}$ and $B_{2 u}$ levels mix magnetically in the substituted benzenes, resulting in MCD. This implies that the directions of the two electronic transitions must have some perpendicular component relative to one another.
2. There is a good deal of evidence that the charge carrier in the benzene ring determines the sign of the MCD. This sign is negative if the charge carrier is an electron and is positive if the charge carrier is a hole.
3. Highly symmetrical molecules often exhibit no MCD or very

Table 8. MCD and diamagnetic susceptibility of substitute benzenes

| Group | diamag. susc. | Substit. susc. | C6H5 susc. | MCD |
| :---: | :---: | :---: | :---: | :---: |
| H | $-54.86 \times 10^{-6}$ | $-2.93 \times 10^{-6}$ | $-51.93 \times 10^{-6}$ | $\pm 0.01$ |
| CH3 | -65.54 | -14.79 | -50.75 | -0.028 |
| Et | -77.26 | -26.65 | -50.61 | -0.0276 |
| $\mathrm{NH}_{2}$ | -62.95 | -11.43 | -51.52 | -0.417 |
| $\mathrm{CN}^{2}$ | -65.19 | -6.74 | -58.45 | +0.377 |
| Cl | -69.97 | -17.00 | -52.97 | -0.065 |
| OH | -60.21 | -7.54 | -52.67 | -0.224 |
| $\mathrm{CH}_{2} \mathrm{OH}$ | -71.83 | -19.40 | -52.43 | -0.0053 |
| COOH | -70.28 | -16.90 | -53.38 | +0.276 |
| CHO | -60.78 | -7.21 | -53.57 | +0.305 |
| Br | 078.1 | -26.5 | -51.6 | -0.071 |
| I | -92.00 | -40.50 | -51.50 | -0.044 |
| $\mathrm{OCH}_{3}$ | -72.10 | -19.40 | -52.70 | -0.266 |
| $\mathrm{p}-\mathrm{Cl}, \mathrm{OH}$ | -77.6 | -24.54 | -55.99 | -0.357 |
| m-cresol | -71.90 | -22.33 | -52.50 | -0.19 |
| o-xylene | -77.78 | -29.58 | -51.13 | -0.046 |
| m-xylene | -76.56 | -29.58 | -49.91 | -0.038 |
| p -xylene | -76.81 | -29.58 | -50.16 | -0.090 |
| $\mathrm{m}-\mathrm{Br}, \mathrm{NH}_{2}$ | -84.89 | -33.90 | -51.00 | -0.491 |


small MCD. This is probably related to the symmetry-forbidden character of the electric or magnetic transitionsinvolved.4. The shape of the MCD spectrum for several of the compoundsstudied indicated the presence of near- or accidental degeneracyof absorption bands. This method of detecting accidentaldegeneracy could be an important supplement to other methods:
5. There is some evidence that the magnetic transition moment isapproximately the same for the substituted benzenes.

## Benzene and the Catacondensed Aromatics

## Benzene

The ultra-violet spectrum of the benzene molecule has undergone a great deal of study both theoretically and experimentally. There is general agreement that the sharp bands near $260 \mathrm{~m} \mathrm{\mu}$ constitute the electrically forbidden $A_{l g} \rightarrow B_{2 u}\left(I_{b}\right)$ transition, and that this is partially allowed by an $\mathbb{E}_{g}$ vibration. If there were simply mixing between the $B_{2 u}$ and $B_{l u}$ levels the MCD should have the same sign for the entire $L_{b}$ band, as occurs for almost all of the substituted benzenes studied. This is not the case, as can be seen from Fig. 55. The MCD spectrum suggests that the situation may be quite complex, e.g., possibly more than one type of vibration is involved in permitting the $L_{b}$ transition. The individual components of each vibronic structure could then partially overlap and interact to give the splittings observed.

The MCD of benzene is very weak and therefore the signal-to-noise ratio is quite small. One method of improving the results is to make many scans of the spectrum and then average them together. This is possible with a Computer of Average Transients (CAT). The spectrum in Fig. 55 represents 50 scans which should give a theoretical overall improvement of signal-to-noise ratio of $\sqrt{50} \cong 7$ times. The catacondensed aromatics

Klevens and Platt (57) assigned $L_{a}$ and $L_{b}$ bands for the series of catacondensed aromatics: benzene, naphthalene, antheacene, tetracene, pentacene. If their assignments were correct, this offers the interesting possibility of observing the MCD of the $I_{a}$ and $I_{b}$ transitions as


Fig. 55. Absorption and MCD of benzene


Fig. 56. Proposed energies for the $L_{a}$ and $L_{b}$ bands of the catacondensed aromatics
they change position with respect to their energies (see Fig. 56). The $I_{a}$ band is expected to be affected in energy to a greater degree by changes in the skeletal structure of the molecule. This results because the sign of the wave function changes at the positions of the atoms, so that addition of a substituent or another ring has a greater perturbation on the $I_{a}$ transition than the $I_{b}$ transition which has nodes at the atoms. The polarization of the $I_{a}$ and $I_{b}$ bands has been determined by effects of suostituents and is illustrated in Fig. 57.


Fig. 57. Polarization of $I_{a}$ and $I_{b}$ bands

One might predict that the MCD for the series in Fig. 56 would become larger in magnitude as the bands approach one another and then decrease again as they draw apart in tetracene and pentacene. The long wavelength band would be expected to retain the same sign but an interesting problem comes up if a narrow band occurs within a broader one. According to theory, the result should be as shown in Fig. 58(a). Another possibility which does not agree with theory but might be intuitively expected is in Fig. 58(b). The situation might well occur for the cases being studied, for Plat (57) has characterized the $I_{b}$ band as one with sharp vibrational
structure and a bandwidth of $300 \mathrm{~cm}^{-1}$, while the $L_{a}$ band has a width of approximately $500 \mathrm{~cm}^{-1}$.

The experimental results for the MCD of the series of compounds in Fig. 56 are shown in Fig. 59. Unfortunately, problems with oxidation of the pentacene in solution precluded the possibility of measuring the MCD for this compound. In order to make the results of the complicated spectra more easily visible, the molar magnetic circular dichroism of the peaks have been calculated and replotted as lines. The proposed assignments of Klevens and Platt are indicated in the figure and the positions of the main absorption peaks are marked along the abscissa.

As expected, the MCD becomes larger in magnitude as the $I_{a}$ and $I_{b}$ bands approach one another in naphthalene and anthracene. The MCD does not decrease again with tetracene in disagreement with the preaiction. Also, if the tetracene $I_{a}$ and $I_{b}$ bands are mixing, equal and opposite values would be expected for their respective MCD's, whereas the MCD for the $I_{a}$ bands is of the same sign as the $I_{b}$ MCD and barely detectabie experimentally. These difficulties make the assignment of the tetracene bands questionable. For the naphthalene and anthracene opposite signs are found generally for $I_{a}$ and $I_{b} M C D$ although they are not of equal magnitude (nor are the areas under their curves of equal size) suggesting that complex interactions are taking place. Since the vibrational bands are so close together for anthracene it is difficult to compare with the expected MCD of Fig. 58, but there is substantial agreement with Fig. 58 (0), that is, not as expected from theory. In any case, it seems improbable that, merely by coincidence, the only MCD band with a positive sign would be the

(a)

(b)

Fig. 58. Possible MCD (—) resulting from overlapping narrow and broad absorption bands (---)


Fig. 59. MCD results for the catacondensed aromatics
preaicted $I_{0}$ band at $28,000 \mathrm{~cm}^{-1}$.

## Substituted naphthalenes

The MCD spectra of several substituted naphthalenes were onserved in order to compare with the results found with the substituted benzenes. Since the $L_{a}$ and $I_{b}$ bands are both accessible experimentally for these compounds, spectra similar to those in Fig. 60 are expected.


Fig. 60. Expected MCD for the naphthalenes

Several of the substituted naphthalenes gave good examples or the spectra in Fig. 60. The only case of Fig. 60 (a) was a-nitro naphthalene. Examples of Fig. 60 (b) were the molecules with -OH or $-\mathrm{NH}_{2}$ in the $\alpha$ position, 1,5 naphthalene diol and $5-\mathrm{NH}_{2}$-l-naphthol. Once again the classification of suiostituents as electron-withdarawing and electrondonating correlates with the sign of the longest wavelength MCD.

Naphthalenes with $\beta$ substituents gave weak, complex MCD spectra for which no interpretation will be offered.

## The uitraviolet spectra

The electronic absorption spectrum of quinoline is very similar to that of naphthalene ( $44, \mathrm{p} .379$ ). From Fig. 61 it may be seen that several sharp bands appear at the longer wavelengths from 300-320 mu, and a broad band system peaks at about 270 mp . By noting the effects of substituents the former bands have been assigned as the $L_{b}$ transition and the latter as the $L_{a}$ transition. The intensity of the $I_{b}$ band is higher than for naphthalene, probably a result of the reduced symmetry. As with naphthalene, the polarization of the $I_{b}$ transition moment is along the long axis of the molecule, the $L_{a}$ moment is along the short axis. There are no recognizable $n \rightarrow \pi^{*}$ transitions for the quinolines, a situation oiten encountered for hetero-aromatics with only one ring nitrogen (58, p. 301).

The ring nitrogen ( $=\mathbb{N}-$ ) is electron-attracting for the $\pi$ electrons (58, p. 34). Hammett $\sigma$-values have been calculated for the quinoline molecule and are positive for all but the 8 position (59).

The substituted quinolines also have absorption spectra very similar to the corresponding substituted naphthalene. The most common result of substitution is a bathochromic shift of the $L_{a}$ band while the $L_{b}$ band moves very little. This effect is represented by the compounds in Figs. 63 and 64.

Adaition of a proton to quinoline seems to lower the intensity of the $L_{a}$ transition and shift it to the red (44, $p .369$ ). The $L_{a}$ band may be partially covered by the $L_{b}$ absorption. The absorption spectrum of


Fig. 61. Absorption (---) and MCD ( $\longrightarrow$ ) of quinoline
quinolinium ion is shown in Fig. 62.
The MCD spectra
The MCD spectrum of quinoline is show in Fig. 6I. The $L_{a}$ and $I_{b}$ bands appear to be mixing magnetically, giving opposite and almost equal MCD. The positive sign for the long wavelength transition is in agreement With the fact that the ring nitrogen causes this to be a $\pi$-electrondeficient system. Therefore the charges involved in the transitions can be considered to be holes rather than electrons. From the results of Fig. 6I one would assign those bands above $290 \mathrm{~m} \mu$ as $\mathrm{I}_{\mathrm{b}}$ and those from 250-290 mu as $I_{a}$.

The MCD of quinolinium ion (Fig. 62) retains the same sign as that for quinoline, but the crossover point has moved to the red approximately $20 \mathrm{~m} \mathrm{\mu}$. This indicates that the interacting bands have moved much closer together as suspected from the absorption spectrum. One might also expect the MCD to increase considerably with the bands more nearly equal in energy, but some cancellation of the positive and negative MCD contributions from the overlapping absorptions will reduce this.

A number of substituted quinolines were studied in an attempt to correlate the sign of the MCD with the type and position of the substituent. The position of the suibstituted group was found to be of litile effect, what was much more important was the relative electron-anating or -withdrawing power. MCD spectra very similar to that obiained for quinoline were recorded for the following substitutions: $5-\mathrm{NO}_{2}, 8-\mathrm{SO}_{3} \mathrm{H}^{\mathrm{I}}$, $2-\mathrm{CH}_{3}, 5-\mathrm{NH}_{3}^{+}, 8-\mathrm{CH}_{3}, 4-\mathrm{COO}^{-}, 8-\mathrm{NO}_{2}$. An attempt was made to find a suidstituent with strong enough electron-donating ability to overcome tine


Fig. 62. Absorption (..-) and MCD ( $\longrightarrow$ ) of quinolinium ion
effect oi the ring nitrogen and therefore change the sign of the MCD. Only two such compounds were found among those available, the $4-0 H$ and 5-OH quinolines in strongly basic solution. The spectra for the $4-0 \mathrm{~F}$ case are shown in Fig. 63. The hyaroxy group does not donate $\pi$ electrons so readily as $-0^{-}$and in, acid solution even more electrons must be witharawn to bind a proton to the ring nitrogen. It is only in basic solution where the suiostitutentis in anionic form that an excess of $\pi$ electrons is found in the ring.

Fig. 64 represents another case where a narrow band appears to be interacting with, and overlapped by, a broader band. The 5-0H and 8-0H quinolines in acidic solutions also are of this type. An understanding of these results may have to await the development of a theory for accidentally degenerate systems for the results do not agree with the assumption that the $B$ term equations we have been using are valid ior overlapping banas. This situation occurred above in the discussion or antinracene.


Fig. 63. Absorpiion ( - - ) and MCD ( - ) of 4-0E quinoline in (top) acidic and (bottom) basic solution


Fig. 64. Absorption (---) and MCD ( - ) of $5-\mathrm{NH}_{2}$ quinoline

## The Purines and Pyrimidines

The purines and pyrimidines are often involved in important biological systems. Knowledge of their optical properties is particularly necessary in view of the fact that optical spectroscopy is one of the most promising means of studying the structure and function of these biological systems. In particular, the incorporation of purines and pyrimidines into RNA and DNA leads to interactions between the bases which can be understood satisfactorily only if the spectroscopy of the independent bases has been mastered. An example of these interactions is round in the hypochromism of polynucleotides as they make the transition from a random to a helical structure. Information is needed on the types of electronic transitions, their energies and directions, factors which MCD studies may help determine.

## The purines

The UV spectra A considerable amount of work has been done on the electronic spectroscopy of the purines. Mason (60). collected absorption data on the purines, Stewart and Davidson (61) examined the polarized absorption spectra of crystals of $9-\mathrm{Me}$ adenine, Callis, Rosa and Simpson (62), Clark and Tinoco (63) and Cohen and Goodman (64) attempted to demonstrate by solvent effects and polarization of emission that an accidental degeneracy occurs in certain purine derivaties. Studies by Drobnik, Kleinwachter and Augenstein $(65,66,67,68)$ have been reported most recently, also using various solvents and polarized emission.

In the solvents used, ring nitrogens of both types, $(=\mathbb{N}-)$ and $\left(-\mathbb{N}^{\prime}-R\right)$, are found. Transitions of the $n \rightarrow \pi^{*}$ type are not always visible, but one is seen as a long wavelength tail in the purine spectrum in Fig. 65. The


Fig. 65. Absorption and MCD of purine
two $\pi \rightarrow \pi^{*}$ transitions of lowest energy are generally consiciered to be related with the $I_{a}$ and $I_{0}$ bands of benzene. Experimental determinations of the polarizations of these transitions are still quite uncertain. There is some theoretical speculation that the $I_{a}$ and $I_{b}$ bands are parallel in purine:(68). Clark and Tinoco observed a series of substituted purines which seem to show the gradual merging of the $I_{a}$ and $I_{b}$ bands as the substituent perturbation is increased (63). Considerable effort has been expended in an attempt to show that the $260 \mathrm{~m} \mathrm{\mu}$ band of adenine actually consists of two unresolvea transitions (63-68).

The MCD spectra The technique of $M C D$ is, at least in theory, well suited to the determination of accidental degeneracy. If two overlapping bands interact magnetically the result will in general be an MCD which changes sign at some energy within the absorption curve. This is a clear indication of an accidental- or near-degeneracy.

While the method of MCD can be a poweriful one for the situation of interest here, several limitations must be kept in mind to avoid making unwarranted conclusions. These precautions are listed below.

1. There is the possibility that the different vibrational components of a single absorption band might interact, giving the appearance Of an accidental degeneracy. This could occur if the different vibrational peaks were polarized with some perpendicuiar componerit relative to one another. This situation probably occurs very rarely, but is not unknown (44, p. 133). For this reason, supplementary evidence for acciadental degeneracy is still essential. Such evidence is difficult to find since the


#### Abstract

techniques of polarization studies are all affected by this problem.


2. Although this technique can reveal an accidental degeneracy, it can ofier no information concerning the relative energies of the interacting bands. The sign of the MCD will not depend on which band is at the longest wavelength.
3. Due to overlapping of the two opposite MCD contributions, the MCD maxima will usually not coincide with the maxima of the individual absorption. Information on the values for the absorption peak energies is available only after resolution of the bands into their separate components, a technique involving considerable uncertainty.

Clark and Tinoco studied the absorption of the series; purine, 6-chloro purine, 6-methoxy purine, adenine for evidence of a graciual merging of the $L_{a}$ and $I_{b}$ bands (63). The MCD for tinree of these are shown in Figs. 65, 66, 67: For purine the MCD peaks almost coincide with the absorption bands at $240 \mathrm{~m} \mathrm{\mu}\left(\mathrm{I}_{\mathrm{a}}\right)$ and $263 \mathrm{~m} \mathrm{\mu}\left(I_{b}\right)$, indicating that these are the interacting bands. The situation is almost the same ior 6-chloro purine although the $I_{a}$ band is now somewhat closer to the $I_{b}$ band. In adenine there is no evidence from the absorption spectrum for the presence of two bands, but the MCD changes sign at approximately tine wavelengtin oi the absorption peak. This crossover is at a wavelengih jetween the $I_{a}$ and $I_{b}$ peaks. The different sign for the $\operatorname{MCD}$ of tine long-wavelength band of aäenine can be attributed to the increased electrondonating ability of the amino group at the 6 position as comparea to


30 Mv日 8080 O
F.g 66. Absorption and MCD of 6-Cl purine


Fig. 67. Absorption and MCD of adenine
chlorine.
The MCD of 6-methoxy purine is in Fig. 68. This result is somewhat puzzling since it indicates that the interaction involves the very small tail at $270 \mathrm{~m} \mathrm{\mu}$. Comparison of this result with the other three members oi the series suggests that the peak at $251 \mathrm{~m} \mathrm{\mu}$ is $\mathrm{L}_{\mathrm{a}}$ and that at $270 \mathrm{~m} \mathrm{\mu}$ is $L_{b}$. The molar MCD may be small compared to the other compounds because the methoxy group donates just enough electrons so that only a small excess of holes is involved in the electronic transitions.

The MCD of guanine in Fig. 69 suggests that the band at 247 mp belongs to the $\mathrm{I}_{\mathrm{a}}$ transition, that at $274 \mathrm{~m} \mu$ is the $\mathrm{L}_{\mathrm{b}}$ band. When guanine is bonded at the 9 position, which is the case for purines in nucleotides, the relative intensities for the $L_{a}$ and $L_{b}$ transitions are changed. As seen from Fig. 70, these two bands are well separated and from the $M C D$ may be assigned as the $248 \mathrm{~m} \mu$ and $275 \mathrm{~m} \mu$ peaks respectively.

The accidental degeneracy of the absorption bands for two other purines was also detected. They were the $255^{\circ} \mathrm{mu}$ band or 9-methyl hypoxanthine (MCD peaks at $247 \mathrm{~m} \mathrm{\mu}$ and $264 \mathrm{~m} \mathrm{\mu}$ ), and the $260 \mathrm{~m} \mu$ band or 9-methyl adenine (MCD peaks at $252 \mathrm{~m} \mu$ and $270 \mathrm{~m} \mathrm{\mu}$ ).

The sign of the low energy MCD band was positive for only three of the purines studied, purine, 6-chloro purine and 6-methoxy purine. This may be interpreted as meaning that the substituents on the other purines were capaile of donating enough electrons to produce a $\pi$-electron excess in the ring.

The MCD found for the purines reveals some information concerning the relative directions for the $L_{a}$ and $I_{b}$ bands. The most obvious


Fig. 68. Absorption and MCD of 6-methoxy purine


Fig. 69. Absorption and MCD of guanine


Fig. 70. Absorption and MCD of 9-ethyl guanine
conclusion is that since MCD is observed the two transition monents cannot be parallel. Secondy, the magnitude of the MCD gives a rough idea of the size oit the angle between the transition moments. The molar MCD for purine is approximately +0.68 at $267 \mathrm{~m} \mu$, of the same order of magnitude as the MCD for naphthalene and anthracene where the angle is known to be nearly $90^{\circ}(44, p .305)$. Although other variables are involved (e.g., the size and direction of the magnetic transition moment) which cannot be specified with the information available, this comparison suggest that the angle is a large one.

The MCD of a relative of purine, benzimidazole, is snown in Fig. 71. The bands related to benzene may again be picked out and on the basis of the MCD are assigned as $I_{a}$ from $240-260 \mathrm{~m} \mu$ and $I_{b}$ above $260 \mathrm{~m} \mu$. The sign of the MCD indicates that the imidazole ring is causing a depletion of $\pi$ electrons from the benzene ring. Indole, which has only one heteronitrogen but is otherwise identical with benzimidazole, has an MCD much like Fig. 71 in appearance.

## The pyrimidines

The UV spectra Many of the properties of electronic absorption for the purines are also present for the pyrimidines. Bands related to the $I_{a}$ and $I_{b}$ transition of benzene are observed, and with some pyrimiaines $n \rightarrow \pi^{*}$ bands may be seen. Pyrimidine itself, Fig. 72, has an $n \rightarrow \pi^{*}$ band beginning at $320 \mathrm{~m} \mathrm{\mu}$ anc peaking at 293 mp . The $\mathrm{I}_{\mathrm{a}}$ transition is uncertain, but may be the shoulder at $210 \mathrm{~m} \mu(50)$. The substituted pyrimidines often retain the $I_{b}$ band $a t$ approximately 260 $280 \mathrm{~m} \mathrm{\mu}$.


Fig. 71. Absorption and MCD of benzimid azole


Fig. 72. Absorption and MCD of pyrimidine

The MCD spectra The MCD for pyrimidine is shown in Fig. 72. The $n \rightarrow \pi^{*}$ band is probably interacting with the $I_{b}$ band. Most $n \rightarrow \pi^{*}$ transitions are polarized perpendicular to the plane of the ring while the $I_{0}$ transition moment is expected to lie in the plane. The sign of the Long wavelength MCD indicates a $\pi$-deficient ring system due to the electron attraction of the two ring nitrogens. Unfortunately the $I_{a}$ bana for the pyrimidines seems to lie out of range of the instrument, therefore no help for assigning that band is forthcoming from these results. A compound related to the pyrimidines, 6-aza uracil, also has a long-wavelengtin $n \rightarrow \pi^{*}$ absorption and an MCD similar to that in Fig. 72 .

The MCD of uracil is pictured in Fig. 73. The negative sign of the Iowest energy band now indicates that the substituents have donated enough electrons to make this a $\pi$-excess ring system. Similar MCD results Were obtained for uracil at pH 13, cytosine in TMP, 5-iluoro uracil, dimethyl uracil and 6-methyl uracil.


Fig. 73. Absorption and NCD of uracil

## Miscellaneous Results

A number of results which did not fit into the categories already discussed should be reported, for they illustrate what may be expected in specifiic cases.

## A free radical - 2,2-diphenyl-l-picrylhydrazyl

This molecule contains a stable unpaired electron at room temperature and therefore is paramagnetic and is sometimes used as a standard for electron spin resonance. There are two easily accessible electronic absorption bands at $327 \mathrm{~m} \mu$ and $516 \mathrm{~m} \mu$. One might at first expect large MCD effects due to the unpaired electron, however, presumably because of the reasons discussed in the theoretical section, only a small MCD is observed. This is shown in Fig. 74. The pyridine, quinoline, acridine series

The spectra of these three compounds are compared in Fig. 75. The $I_{a}$ and $I_{b}$ transitions of acridine seem to be combined under the band system peaking at $356 \mathrm{~m} \mathrm{\mu}$. There is disagreement on the question or whether it is the $I_{a}$ or $I_{b}$ band which lies at the longest wavelength (69, p. 178). However, the two banas are considerably closer together in energy for acridine than for quinoline, which is probably at least part of the reason for the molar NCD of acridine to be the larger. Both quinoline and acridine have a sign of MCD which corresponds to a ti-deficient ring system, which is expected on account of the electronwithärawing character of the ring nitrogens.

The $I_{a}$ band of pyridine is probably at $205 \mathrm{~m} \mathrm{\mu}$, about $9900 \mathrm{~cm}^{-1}$ from the $I_{b}$ band, compared to $3900 \mathrm{~cm}^{-1}$ for quinoline. This difference alone


Fig. 74. Absorption (---) and MCD ( - ) or 2,2-aiphenyl-lpicrylhydrazyl


Fig. 75. Absorption (- - ) anà MCD ( $\quad$ ) of (top) pyriaine, (middle) quinoline anā (boțom) acriāine
will therefore not account for the order of magnitude decrease in molar MCD in goinc from quinoline to pyridine. Here again, as with the substituted benzenes, the comparatively high symmetry of pyridine ( $\mathrm{C}_{2 \mathrm{v}}$ ) may be a major infiuence.

The sign for the MCD of pyridine is anomalous compared to quinoline and acridine. This situation has occurred before, for the case of $N, N$ dimethyl aniline. In that case also, there was an $n \rightarrow \pi^{*}$ bana apparent. The MCD of pyridine appears only in nonpolar solvents, in aqueous solution neither the MCD nor the tail of the $n \rightarrow \pi$ * band from 275-290 mu could be measured. Therefore it seems that the presence of an $n \rightarrow \pi^{*}$ transition may be involved in the anomalous sign, and tinat the MCD of pyridine results from the interaction of the $n \rightarrow \pi^{*}$ transition with one of higher energy. This suggests that the MCD reveals the true position of the $n \rightarrow \pi^{*}$ absorption band.

The aromatic amino acids, proteins
Tyrosine, tryptophan and phenylalanine were examined for NCD with results very similar to the corresponding simple aromatic systems. Tyrosine, in acidic or basic solution, has an MCD Iike that oi phenol in the same solutions. Tryptophan's MCD is similar to that of indoie. No MCD was detected for phenylalanine, a situation corresponding to the very smail MCD found for toluene or ethyl benzene. The MCD of proteins containing tyrosine and tryptophan is just what would be expectea if tieir individual spectra were multiplied by the number of respective amino aciá residues in the protein and the results added. Due to limitation of the apparatus measurements were not made in the region of $225 \mathrm{~m} \mathrm{\mu}$, where the protein bands related to helical structure occur.
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VII. APPENDIX I

Derivation of an Expression for MCD using Perturbation Theory
In this section we will derive an expression for the MCD using perturbation theory. This derivation and a number of others similar to it have appeared in the literature, however, they are almost always in very compact form and therefore difficult to follow. In particular, the manipulation of frequency terms needs clarification. The following is an expansion of one of the clearest derivations in the literature and has been included for the benefit of those who might find it useful. The procedure will follow that of I'Haya (70) for the development of the Vevdet constant of molecules with narrow absorption bands and then continue using Stephen's method (3) to find the MCD of molecules with broad bands.

The calculations using time-dependent perturbation theory will follow the usual steps:

1. Write the Hamiltonian operator of the system.
2. Write the time dependent Schroedinger equation for the perturbed system.
3. Expand the perturbed wave functions in terms of unperturbed wave functions.
4. Substitute the expansions from step 3 into the Schroedinger equations.
5. Reduce the expression in step 4 by deleting the equal terms from each side of the equation.
6. Multiply both sides of the equation by the complex conjugate
wave functions and integrate over all space.
7. The integration in step 6 gives the transition rates, these may be integrated with respect to time to give the coefficients for the expansions in step 3.

With the completion of the perturbation calculation, the following steps can be taken.
8. Using the perturbed wave functions from step 3 with their coefficients from step 7, the desired expectation values may be calculated.
9. Calculate the expectation value at a given temperature.
10. Average the expectation value over all molecule orientations.
11. With the expectation values and using Maxwell's equations, the magnetic optical rotation may be calculated.
12. Expression for MCD developed according to Stephens.

## 1. The Hamiltonian operator of the system

Assuming that there is no spin-orbital coupling, the total Hamiltonian for the case of a molecule subjected to perturbations from both light ana a static magnetic field is (25, p. 109)

$$
X \dot{X}=\sum_{j}\left[\frac{1}{2 m}\left(\bar{p}_{j}-\frac{e}{c} \bar{A}_{j}\right)^{2}+e \phi_{j}\right]+U
$$

Eq. 1
where:
$j$ is the $j^{\text {th }}$ electron
m is the electronic mass
$\bar{p}$ is the linear momentum
$e$ is the electronic charge
c is the speed of light
$\bar{A}_{j}$ is the vector potential
$\phi$ is the scalar potential
$U$ is the interval potential
The vector potential $\bar{A}_{j}$ is the sum of two parts, that due to the light wave $\bar{a}_{j}$ and that due to the magnetic field $\bar{a}_{j} f^{f}$

$$
\begin{equation*}
\bar{A}_{j}=\bar{a}_{j}+\bar{a}_{j} f^{f} \tag{Eq. 2}
\end{equation*}
$$

The time dependence of $\bar{a}_{j}$ is $(71, p .35)(70, p .3)$,

$$
\bar{a}_{j}=\frac{1}{2}\left\{a_{0}^{0} i x p\left[v\left(t-\frac{n}{c}(\vec{r} \cdot \vec{k})\right)\right]+\text { complex conjugate }\right\} \quad \text { Eq. } 3
$$

where:
$a_{0}^{0}$ is the complex amplitude, $a e^{i \theta}$
ixpf is $e^{2 \pi i f}$
$v$ is the frequency of the light wave
$t$ is time
$n$ is the refractive index of the medium
$\bar{r}$ is the position vector
$\bar{k}$ is the unit vector in the $z$ direction
The expression for $a_{j}{ }^{f}$ is:

$$
a_{j}^{f}=\frac{I}{2}\left[H^{f} \cdot \bar{r}_{j}\right]
$$

Eq. 4
where

$$
\mathrm{H}^{\hat{\mathrm{I}}} \text { is the intensity of the magnetic field }
$$

The square in Eq. I may be expanded to. give;

$$
\left(p-\frac{e}{c} a-\frac{e}{c} a^{f}\right)^{2}=p^{2}-2 p \frac{e}{c} a^{f}+2 \frac{e^{2}}{c^{2}} a a^{f}
$$

$$
+\frac{e^{2}}{c^{2}} a^{f^{2}}+\frac{e^{2}}{c^{2}} a^{2}-2 p \frac{e}{c} a
$$

The fourth and fifth terms are dropped because we will neglect all nonlinear terms. This leaves:

$$
\begin{array}{ll}
p_{j}^{2} & \text { zeroth order perturbation term } \\
-2 \frac{e}{c} o_{j}\left(a_{j}+a_{j}^{f}\right) & \text { first order perturbation term } \\
2 \frac{e^{2}}{c^{2}} a_{j} a_{j}^{f} & \text { second order perturbation term }
\end{array}
$$

It is because of the cross term $a_{j} a_{j} f$ in the Hamiltonian as we apply simultaneously fields from the light wave and the static magnetic field that we call this derivation a second order perturbation. The first and second order Hamiltonian will now be:

$$
\begin{align*}
& \mathcal{X ( 1 )}=-\frac{e}{m c} \sum_{j}\left(p_{j} \cdot\left(a_{j}+a_{j}^{f}\right)\right)  \tag{Eq. 6}\\
& \mathcal{H}(2)=\frac{e^{2}}{m c^{2}} \sum_{j}\left(a_{j} \cdot a_{j}^{f}\right)
\end{align*}
$$

Eq. 7

The vector potential $a_{j}$ can be expanded in a Taylor's series:

$$
a_{j}=\bar{a}_{0}+\bar{r}_{j}[\nabla \cdot a]_{\partial}+\text { neglected terms }
$$

where $\quad \bar{a}_{0}=\frac{1}{2}\left\{a_{0}^{0} \operatorname{ixp}(\nu t)+a_{0}^{0} i \operatorname{ixp}(-\nu t)\right\}$

$$
\begin{aligned}
{[\nabla \cdot a]_{0} } & =\frac{1}{2}\left\{[\nabla \cdot a]_{0}^{0} i_{x p}(\nu t)+[\nabla \cdot a]_{0}^{0^{*}} i x_{p}(-\nu t)\right\} \\
\nabla & =\frac{d}{d x}+\frac{d}{d y}+\frac{d}{d z}
\end{aligned}
$$

The first order perturbation will now be

$$
\begin{align*}
g^{(1)}= & -\frac{e}{m c} \sum_{j} \bar{P}_{j}\left(\bar{a}_{0}+\bar{r}_{j} \cdot[\nabla \cdot a]_{0}+a_{j}^{f}\right) \\
= & -\frac{e}{m c} \sum_{j} \bar{P}_{j} \cdot \bar{a}_{0}-\frac{e}{m c} \sum_{j}\left(\overline{P_{j}} \times \overline{r_{j}}\right)[\nabla \cdot a]_{0} \\
& -\frac{e}{m c} \sum_{j}\left(\bar{P}_{j} \times \bar{r}_{j}\right) \frac{H^{f}}{2} \tag{Eq. 9}
\end{align*}
$$

now since the magnetic dipole moment is $M=\frac{e}{m c} \sum_{j}\left(\bar{r}_{j} \times \bar{P}_{j}\right)$,

$$
g^{(1)}=-\frac{e}{m c} \sum_{j} \bar{p}_{j} \cdot \bar{a}_{0}-2 \sum_{j} M[\nabla \cdot a]_{0}-\sum_{j} M H^{f}
$$

Eq. 10

The second order perturbation will be:

$$
q_{f}(2)=\frac{e^{2}}{m c^{2}} \sum_{j}\left[a_{j}^{f} \cdot \bar{a}_{0}+a_{j}^{f} \cdot[\nabla \cdot a]_{0} \cdot \bar{r}_{j}\right]
$$

$$
=\frac{e^{2}}{m c^{2}} \sum_{j}\left[\frac{H^{f}}{2} \bar{r}_{j} \cdot \bar{a}_{0}+\frac{H^{f}}{2} \bar{r}_{j} \cdot \bar{r}_{j}[\nabla \cdot a]_{0}\right]
$$

Eq. 12

The second term in Eq. 11 represents a quadrupole moment and will be neglected. Since the electric dipole moment is $R=\underset{j}{e \sum r_{j}}$, Eq. Il can be written as: $\quad H^{(2)}=\frac{e}{m_{c}}=R \frac{H^{f}}{2} \bar{a}_{0}$

Now we will rewrite the Hamiltonian operators in a form useful for making the actual calculations.

$$
\begin{aligned}
& g f(1) a=-\frac{e}{m c} \sum_{j} \bar{P}_{j} \cdot \bar{a}_{0}=-\frac{e}{2 m c} \sum_{j} \frac{\partial}{\partial F_{j}}\left\{a_{0}^{0} i x_{\rho}(\nu t)+\right. \\
&\left.a_{0}^{0}{ }^{*} i \times p(-\nu t)\right\}
\end{aligned}
$$

Eq. 13 is. written in a form using the linear momentum operator. However it may be shown that this is equivalent to the following since $p_{j}=m \frac{d r_{j}}{d t}$,

$$
\mathbb{H}^{(1) a}=-\frac{2 \pi i \nu\left(u^{\prime} n\right)}{c} a_{0} R=-\frac{\pi i \nu\left(n^{\prime} n\right) R}{c}\left\{a_{0}^{0} i x_{p}(\nu t)+\right.
$$

$$
\left.a_{0}^{\circ} \text { ixp }(-\nu t)\right\}
$$

Eq. 14
where

$$
\nu\left(n^{\prime} n\right)=\frac{E_{n^{\prime}}^{0}-E_{n}^{0}}{\hbar}
$$

$$
\forall(1) b=-2 \sum_{j} M[\nabla \cdot a]_{0}=-M\left\{[\nabla \cdot a]_{0}^{0} i x_{p}(\nu t)+\right.
$$

$$
\begin{align*}
& +[\nabla \cdot a]_{0}^{0 *} i x p(-\nu t) \\
& \mathcal{H}^{(1) c}=-M H^{f} \\
& \mathcal{H}^{\prime}(2)=\frac{e}{2 m c^{2}} R H^{f} \bar{a}_{0}=\frac{e}{4 m c^{2}} R H^{f}\left\{a_{0}^{0} i \times p(\nu t)+\right. \\
& \left.a_{0}^{0} * i \times p(-\nu t)\right\} \tag{Eq. 17}
\end{align*}
$$

$$
\mathrm{Eq} \cdot 15
$$

$$
\text { Eq. } 16
$$

## 2. The time dependent Schroedinger equation

We can now write the time-dependent Schroedinger equation,

$$
\begin{equation*}
i \hbar \dot{\Psi}_{n}=\left(H_{0}+\mathcal{H}^{(1)}+\mathcal{H}^{(2)}\right) \Psi_{n} \tag{Eq. 18}
\end{equation*}
$$

where the dot denotes the time derivative.
3. Expanding the perturbed wave functions

The $\Psi_{n}$ may be expanded in terms of the unperturbed wave functions $\mathbb{I}_{\mathrm{n}}^{\circ}$ as follows:

$$
\begin{equation*}
\Psi_{n}=\Psi_{n}^{\circ}+\sum_{n^{\prime} \neq n} c_{n}^{\prime}(t) \Psi_{n} \circ \tag{Eq. 19}
\end{equation*}
$$

The unperturbed wave functions satisfy

$$
i \hbar \dot{\Psi}_{n}^{\circ}=\mathscr{H}_{0} \Psi_{n}^{\circ}
$$

and have the form

$$
\begin{equation*}
\Psi_{n}^{0}=\Psi_{n}^{0} \exp \left(-i \frac{E_{n}^{0} t}{\hbar}\right)=\Psi_{n}^{0} i x_{p}(-\nu(n) t) \tag{Eq. 21}
\end{equation*}
$$

where $\Psi_{n}{ }^{\circ}$ is a time independent wave function and $E_{n}{ }^{\circ}$ is its energy.
4. Substitution of the wave functions into Schroedinger's equation Substituting Eq. 21 into Eq. 19, and then Eq. 19 into Eq. 18 we obtain:

$$
\begin{aligned}
& i \hbar\left[\psi_{n}^{0} \frac{d \exp \left(-i \frac{E_{n} t}{\hbar}\right)}{d t}+\sum_{n^{\prime} \neq n} c_{n}^{\prime}(t) \psi_{n^{\prime}}^{0} \frac{d \exp \left(-i \frac{E_{n^{\prime}}^{0} t}{\hbar}\right)}{d t}\right. \\
& +\sum_{n^{\prime} \neq n} \Psi_{n^{\prime}}^{0} \exp \left(-\frac{i E_{n^{\prime}}^{0} t}{\hbar}\right) \frac{d n_{n^{\prime}}(t)}{d t}=\left[\eta d_{0}+H^{(1)}+\eta t(2)\right] \\
& {\left[\psi_{n}^{0} \exp \left(-i \frac{E_{n}^{0} t}{\hbar}\right)+\sum_{n^{\prime} \neq n} c_{n}^{\prime}(t) \psi_{n^{\prime}}^{0} \exp \left(-i \frac{E_{n^{\prime}}^{0} t}{\hbar}\right) \quad \text { Eq. } 22\right.}
\end{aligned}
$$

## 5. Reducing Eq. 22

Since we know by definition Eq. 20, these terms in Eq. 22 are equal and can be canceled out. Multiplying out what remains we find:

$$
\begin{aligned}
& i \hbar \sum_{n^{\prime} \neq n} \Psi_{n^{\prime}}^{0} \exp \left(-i \frac{E_{n^{\prime}}^{0} t}{\hbar}\right) \frac{d c_{n^{\prime}}(t)}{d t}=\notin(1) \Psi_{n}^{0} \exp \left(-i \frac{E_{n^{0} t}}{\hbar}\right) \\
& +\notin(1) \sum_{n^{\prime} \neq n} c_{n^{\prime}}(t) \Psi_{n^{\prime}}^{0} \exp \left(-i \frac{E_{n^{\prime}}^{0}}{\hbar}\right)+\mathcal{H}^{(2)} \Psi_{n}^{0} \exp \left(-i \frac{E_{n}^{0} t}{\hbar}\right)
\end{aligned}
$$

$$
\begin{equation*}
+H^{(2)} \sum_{n^{\prime} \neq n} c_{n^{\prime}}(t) \Psi_{n^{\prime}}^{0} \exp \left(-i \frac{E_{n^{0}} 0 t}{\hbar}\right) \tag{Eq. 23}
\end{equation*}
$$

## 6. Multiplying by the complex conjugate wave functions and integrating

We will now multiply term by term Eq. 23 times the complex conjugate wave function $\Psi_{n} O^{\prime *}$, and integrate over all space.

$$
\begin{equation*}
i \hbar \int \sum_{n^{\prime}+n} \Psi_{n^{\circ}}^{0} \Psi_{n^{\prime}}^{0} d \tau \frac{d c_{n^{\prime}}(t)}{d t}=i \hbar \dot{C}_{n^{\prime}} \tag{Eq. 24}
\end{equation*}
$$

$$
\int \Psi_{n^{\prime}}^{0 \pi} \mathcal{H}^{(1)} \Psi_{n}^{0} d \tau=\int \psi_{n^{\prime}}^{0 *} d(1) \psi_{n}^{0} d T e^{2 \pi i}\left(\frac{E_{n^{\prime}}^{0}-E_{n}^{0}}{h}\right) T
$$

where $n '$ and $n$ stand for stationary states

$$
\begin{equation*}
=\left\langle n^{\prime}\right| \mathcal{X}^{\prime \prime}|n\rangle \operatorname{ixp}\left\{\nu\left(n^{\prime} n\right) t\right\} \tag{Eq. 25}
\end{equation*}
$$

Note the use of brackets to simplify notation for integration.

$$
\begin{gather*}
\int \Psi_{n^{\prime}}^{0 *} x^{(1)} \sum_{n^{\prime} \neq n} c_{n^{\prime}}^{\prime}(t) \Psi_{n^{\prime}}^{0} d \tau=\sum_{n^{\prime \prime} \neq n} c_{n^{\prime \prime}}\left\langle n^{\prime}\right| \mathcal{N}\left|(1) n^{\prime \prime}\right\rangle \\
\quad i \times p\left\{\nu\left(n^{\prime} n^{\prime \prime}\right)+\right\} \tag{Eq. 26}
\end{gather*}
$$

where we allow one of the states $n$ ' to called $n^{\prime \prime}$.

$$
\int \Psi_{n^{\prime}}^{\cdot *} f^{(2)} \Psi_{n}^{0} d T=\left\langle n^{\prime}\right| g^{(2)}|n\rangle i x p\left\{\nu\left(n^{\prime} n\right) t\right\}
$$

Other higher order terms could be developed but these will be neglected. Writing Eqis. 24-27 together again:

$$
\begin{aligned}
& i \hbar \dot{c}_{n^{\prime}}=\left\langle u^{\prime}\right| \not g^{(1)}|n\rangle \text { ix p }\left\{\nu\left(n^{\prime} n\right) t\right\}+\left\langle n^{\prime}\right| g^{\prime}(2)|n\rangle \\
& \quad i \lambda_{j}\left\{\nu\left(n^{\prime} n\right) t\right\}+\sum_{n^{\prime}+n} c_{n^{\prime \prime}}\left\langle n^{\prime}\right| \partial^{\prime}(1)\left|n^{\prime \prime}\right\rangle i x p\left\{\nu\left(n^{\prime} n^{\prime \prime}\right) t\right\}
\end{aligned}
$$

Eq. 28

This equation is I'Haya's.Eq. 18 and is the time derivative of the coefficient appearing in our Eq. 19.. Eq. 25. may be considered a first order perturbation correction to the wave functions and Eqs. 26 and 27 are second order corrections.

## 7. Calculation of the correction coefficients

We will now substitute the Hamiltonian (Eqs. 14-17) into Eq. 28 and integrate with respect to time to obtain the correction coefficients. The constant of integration is chosen so that $c_{n^{\prime}}=0$ when $t=0$.

First, substituting Eq. 14 into Eq. 25:

$$
\begin{aligned}
& i \hbar \int_{\hat{*}} \dot{c}_{n^{\prime}} d t=\left\langle n^{\prime}\right| R|n\rangle\left[-\frac{i \pi \nu\left(n^{\prime} n\right)}{c}\right] \int\left\{a_{0}^{0} i x_{p}(\nu t)+\right. \\
& \left.\quad a_{0}^{0}{ }^{*} i x_{p}(-\nu t)\right\} i x_{p}\left\{\nu\left(n^{\prime} n\right) t\right\} d t \\
& c_{n^{\prime}}=\left\langle n^{\prime}\right| R|n\rangle\left[-\frac{i \pi \nu\left(n^{\prime} n\right)}{c}\right]\left[\frac{1}{2 \pi i \cdot i \hbar}\right]\left[a_{0}^{0} F_{+}\left(n^{\prime} n\right)+a_{0}^{0^{*}} F_{-}\left(n^{\prime} n\right)\right]
\end{aligned}
$$

where

$$
\begin{equation*}
F_{+}\left(n^{\prime} n\right)=\frac{i x_{p}\left\{\nu\left(n^{\prime} n\right)+\nu\right\}+}{\nu\left(n^{\prime} n\right)+\nu} \quad F_{-}\left(n^{\prime} n\right)=\frac{i \times p\left\{\nu\left(n^{\prime} n\right)-\nu\right\} t}{\nu\left(n^{\prime} n\right)-\nu} \tag{Eq. 29}
\end{equation*}
$$

$$
\begin{equation*}
C_{n^{\prime}}=\frac{i \nu\left(n^{\prime} n\right)}{2 \hbar c}\left\langle n^{\prime}\right| R|n\rangle\left[a_{0}^{0} F_{+}\left(n^{\prime} n\right)+a_{0}^{0} F_{-}\left(n^{\prime} n\right)\right] \tag{Eq. 30}
\end{equation*}
$$

This is the first term in I'Haya's Eq. 21 and corresponds to a correction in the unperturbed wave function caused by the electric vector of the incident light.

Next we substitute Eq. 17 into Eq. 27:

$$
i \hbar \int \dot{c}_{n} d t=
$$

$$
\begin{align*}
& \left\langle n^{\prime}\right| R|n\rangle\left[\frac{e H^{f}}{4 m c^{2}}\right] \int\left\{a_{0}^{0} i x_{p}(\nu t)+a_{0}^{0^{*}} i x_{p}(-\nu t) i x p\left\{v\left(n^{\prime} n\right) t\right\} d t\right. \\
& c_{n^{\prime}}=-\frac{e H^{f}}{8 \pi m c^{2} \hbar}\left\langle n^{\prime}\right| R|n\rangle\left[a_{0}^{0} F_{t}\left(n^{\prime} n\right\rangle+a_{0}^{0} F_{-}\left(n^{\prime} n\right)\right] . \tag{Eq. 31}
\end{align*}
$$

This is the fourth term in I'Haya's Eq. Ll, a second order correction caused by both the electric vector of the light and the static magnetic field. We have neglected the second and third terms of I'Haya's Eq. 21 as they contain only the small magnetic dipole corrections. We will calculate only one more term from I'Haya's Eq. 21. It is by far the mast important term and is the only one we will need to account for the MCD of all but a few molecules. This expression is I'Haya's fifth term and results when we substitute Eq. 16 into Eq. 26. However, since the coefficient $c_{n}$ " is present in Eq. 26, we must find it first. The term $c_{n}$ " may be found by substituting Eq. 14 into Eq. 25 as this is the only part of $c_{n}$ " which will not give non-linear terms at the end of the calculotion for $c_{n}$.-

So finding $c_{n "}$ :

$$
\begin{aligned}
& i \hbar \int \dot{C}_{n \prime \prime} d t=\left\langle n^{\prime \prime}\right| R|n\rangle\left[-\frac{\pi i \nu\left(n^{\prime \prime} n\right)}{c}\right] \int\left\{a_{0}^{0} i x_{p}(\nu t)+\right. \\
& \left.\quad a_{0}^{0}{ }^{*} \operatorname{ixp}(-\nu t)\right\} \operatorname{ixp}\left\{\nu\left(n^{\prime \prime} n\right) t\right\} d t
\end{aligned}
$$

$$
c_{n}^{\prime \prime}=\frac{i \pi v\left(n^{\prime \prime} n\right)}{c h}\left\langle n^{\prime \prime}\right| R|n\rangle\left[a_{0}^{0} F_{f}\left(n^{\prime \prime}\right)+a_{0}^{0} F_{-}\left(\ldots_{0}^{\prime}\right)\right]
$$

Now putting this value for $c_{n}$ " into $E q .26$ and finding $c_{n}$,

$$
\begin{aligned}
& i \hbar \int \dot{C}_{n^{\prime}} d t=\sum_{n^{\prime \prime} \neq n} \sum_{n^{\prime} \neq n} \frac{i \pi \nu\left(n^{\prime \prime} n\right)}{c h}\left\langle n^{\prime \prime}\right| R|n\rangle\left\langle n^{\prime}\right| M\left|n^{\prime \prime}\right\rangle\left[-H^{f}\right] \\
& \int\left[a_{0}^{0} F+\left(n^{\prime \prime} n\right\rangle+a_{0}^{0} F_{-}\left(n^{\prime \prime} n\right) i x p\left\{\nu\left(n^{\prime} n^{\prime \prime}\right) t\right\} d t\right. \\
& C_{n}^{\prime}=\sum_{n^{\prime \prime} \neq n} \sum_{n^{\prime} \neq n}\left\langle n^{\prime \prime}\right| R|n\rangle\left\langle n^{\prime}\right| M\left|n^{\prime \prime}\right\rangle\left[\frac{i \nu\left(n^{\prime \prime} n\right) H^{f}}{4 \pi c \hbar^{2}}\right]\left[\frac{a_{0}^{0} F_{+}\left(n^{\prime} n\right)}{\nu\left(n^{\prime \prime} n\right)+\nu}+\frac{a_{0}^{0^{*}} F_{-}\left(n^{\prime} n\right)}{\nu\left(n^{\prime \prime} n\right)-\nu}\right]
\end{aligned}
$$

Eq. 32

Now we have found the important terms needed to write the perturbed wave functions. This is done by substituting Eqs. 30, 31 and 32 into Eq. 19.
8. Expectation value for the induced electric moment

The expectation value of the induced electric moment in the perturbed state $n$ is $\quad \int \Psi_{n}^{*} R \Psi_{n} d T=\langle R\rangle n$

Using our values for the perturbed wave functions:

$$
\langle R\rangle_{n}=\int\left[\Psi_{n}^{0 *}+\sum_{n^{\prime} \neq n} c_{n} \Psi_{n^{\prime}}^{0}\right] R\left[\Psi_{n}^{0}+\sum_{n^{\prime} \neq n} c_{n^{\prime}} \Psi_{n^{\prime}}^{0}\right] d \tau
$$

Expanding this

$$
\begin{aligned}
& =\int \Psi_{n}^{0 *} R \Psi_{n}^{0} d \tau+\int \Psi_{n}^{0} 0{ }_{n}^{*} \sum_{n^{\prime} \neq n} C_{n^{\prime}} \Psi_{n^{\prime}}^{0} d \tau \\
& +\int \sum_{n^{\prime} \neq n} c_{n^{\prime}} \Psi_{n^{\prime}}^{0 *} R \Psi_{n}^{0} d \tau+\int\left(\sum_{n^{\prime} \neq n} c_{n^{\prime}}{ }^{2}\right) \Psi_{n^{\prime}}^{0 *} R \Psi_{n^{\prime}}^{0} d \tau
\end{aligned}
$$

Eq. 33

In Eq. 33 the first term is the permanent electric dipole moment of the ground state

$$
\int \psi_{n^{0}}^{*} R \psi_{n}^{0} d \gamma=\langle n| R|n\rangle .
$$

The second and third terms of Eq. 33 are equal giving

$$
2 \sum_{n^{\prime} \neq n} c_{n^{\prime}}\left\langle n^{\prime}\right| R|n\rangle
$$

The fourth term contains squares and is neglected.
The resulting expression is:

$$
\langle R\rangle_{n}=\langle n| R|n\rangle+2 \sum_{n^{\prime} \neq n} c_{n^{\prime}}\left\langle n^{\prime}\right| R\left|n^{\prime}\right\rangle i x_{p}\left(\nu\left(n^{\prime} n\right) \mid\right)
$$

Eq. 34

Substituting our expression for $c_{n}$, into Eq. 34 , and using only the parts of $c_{n}$, which give linear terms for MCD, that is Eqs. 30 and 32, we find:

$$
\begin{aligned}
& \langle R\rangle_{n}=\langle n| R|n\rangle+2 \sum_{n^{\prime} \neq n} \sum_{n^{\prime} \neq n}\left[\frac{i \nu\left(n^{\prime} n\right)}{2 \hbar c}\langle n| R\left|n^{\prime}\right\rangle\left\langle n^{\prime}\right| R|n\rangle\right] \\
& {\left[a_{0}^{0} F_{+}\left(n^{\prime} n\right)+a_{0}^{0^{*}} F_{-}\left(n^{\prime} n\right)\right] i \times p\left[-\nu\left(n^{\prime} n\right) t\right]+}
\end{aligned}
$$

$$
\begin{aligned}
& +2 \sum_{n^{\prime \prime} \neq n} \sum_{n^{\prime} \neq n} \frac{i \nu\left(n^{\prime \prime} n\right)}{4 \pi c \hbar^{2}} H^{f}\left\langle n^{\prime}\right| R|n\rangle\left\langle n^{\prime \prime}\right| R|n\rangle\left\langle n^{\prime}\right| M\left|n^{\prime \prime}\right\rangle \\
& \left\{\frac{a_{0}^{0} F_{+}\left(n^{\prime} n\right)}{\nu\left(n^{\prime \prime} n\right)+\nu}+\frac{\left.a_{0}^{0 *} F_{-\left(n^{\prime} n\right)}^{\nu\left(n^{\prime \prime} n\right)-\nu}\right\} i x p\left[-\nu\left(n^{\prime} n\right)+\right]}{}\right.
\end{aligned}
$$

Eq. 35

We will find it convenient to express the vector potential in terms of the field strength, E. The time derivative of $a_{0}$ is:

$$
\dot{a}_{0}=i \pi \nu\left\{a_{0}^{0} \operatorname{ixp}(\nu t)-a_{0}^{0} \text { ixp }(-\nu t)\right\}=-c E
$$

Also:

$$
a_{0}=\frac{1}{2}\left\{a_{0}^{0} \operatorname{ixp}(\nu t)+a_{0}^{0} \text { ixp }(-\nu t)\right\}=\frac{c \dot{E}}{4 \pi^{2} \nu^{2}} \quad \text { (25, sec. 17.2) }
$$

Rewriting the frequency terms in brackets from the second term of Eq. 35
we get:

$$
\begin{aligned}
& \frac{a_{0}^{*} \operatorname{ixp}_{p}\left\{\nu\left(n^{\prime} n\right)+\nu\right\}+}{\nu\left(n^{\prime} n\right)+\nu}+\frac{a_{0}^{0} \operatorname{ixp}^{*}\left\{\nu\left(u^{\prime} n\right)-\nu\right\}+}{\nu\left(n^{\prime} n\right)-\nu} \\
& =\frac{a_{0}^{0} i x p\left\{\nu\left(u^{\prime} u\right)+\nu\right\}+\left[\nu\left(n^{\prime} n\right)-\nu\right]+a_{0}^{d}{ }^{\prime} i x p\left\{\nu\left(n^{\prime} n\right)-\nu\right\}+\left[\nu\left(n^{\prime} u\right)+\nu\right]}{\nu^{2}\left(n^{\prime} n\right)-\nu^{2}}
\end{aligned}
$$

$$
=T\left(n^{\prime} n\right)\left[\frac{\nu(n \cdot n) c \dot{E}}{2 \pi^{2} \nu^{2}}+\frac{c E}{i \pi}\right] i x_{p} \nu\left(n^{\prime} n\right) t
$$

Inserting this back into Eq. 35 :

$$
\begin{aligned}
& 2 \sum_{n^{\prime} \neq n}\left[\frac{i \nu\left(n^{\prime} n\right)}{2 \hbar c}\langle n| R\left|n^{\prime}\right\rangle\left\langle n^{\prime}\right| R|n\rangle\right] \tau\left(n^{\prime} n\right)\left[\frac{\nu\left(n^{\prime} n\right) c \dot{E}}{2 \pi^{2} \nu^{2}}+\frac{c E}{i \pi}\right] \frac{2 i 2 \pi c \pi^{2} \nu^{2}}{2 n c} \\
& \quad=\operatorname{Re} \sum_{n^{\prime} \neq n}\langle n| R\left|n^{\prime}\right\rangle\left\langle n^{\prime}\right| R|n\rangle \frac{\tau\left(n^{\prime} n\right)}{h \pi \nu^{2}}\left[2 \nu\left(n^{\prime} n\right) \pi \nu^{2} E+i \nu^{2}\left(n^{\prime} n\right) \dot{E}\right]
\end{aligned}
$$

If we now modity only the frequency terms in curly brackets of Eq. 35, we will have

$$
\begin{aligned}
& \frac{a_{0}^{0} i x p\left[\left(\nu\left(n^{\prime} n\right)+\nu\right)+\right]}{\left[\nu\left(n^{\prime} n\right)+\nu\right]\left[\nu\left(n^{\prime \prime} n\right)+\nu\right]}+\frac{a_{0}^{0} i x_{p}\left[\left(\nu\left(n^{\prime} n\right)-\nu\right)+\right]}{\left[\nu\left(n^{\prime \prime} n\right)-\nu\right]\left[\nu\left(n^{\prime} n\right)-\nu\right]} \\
& =\frac{a_{0}^{0} \operatorname{ixp}_{p}\left[\left(\nu\left(n^{\prime} n\right)+\nu\right) \dagger\right]\left[\nu\left(n^{\prime \prime} n\right) \nu\left(n^{\prime} n\right)-\nu \nu\left(n^{\prime \prime} n\right)-\nu \nu\left(n^{\prime} n\right)+\nu^{2}\right]}{\left[\nu^{2}\left(n^{\prime \prime} n\right)-\nu^{2}\right]\left[\nu^{2}\left(n^{\prime} n\right)-\nu^{2}\right]} \\
& +\frac{a_{0}^{*} \operatorname{ixp}\left[\left(\nu\left(n^{\prime} n\right)-\nu\right) t\right]\left[\nu\left(k^{\prime} n\right) \nu\left(n^{\prime \prime} n\right)+\nu \nu\left(n^{\prime} n\right)+\nu \nu\left(n^{\prime \prime} n\right)+\nu^{2}\right]}{\left[\nu^{2}\left(n^{\prime \prime} n\right)-\nu^{2}\right]\left[\nu^{2}\left(n^{\prime} n\right)-\nu^{2}\right]} \\
& =\tau\left(n^{\prime \prime} n\right) \tau\left(u^{\prime} n\right) \operatorname{ixp}\left(\nu\left(n^{\prime} n\right) t\right)\left[[ a _ { 0 } ^ { 0 } \operatorname { i x p } ( \nu t ) + a _ { 0 } ^ { 0 * } \operatorname { i x p } ( - \nu t ) ] \left[\nu\left(u^{\prime} n\right) \nu\left(u^{\prime \prime} n\right)+\right.\right. \\
& \left.\left.\nu^{2}\right]-\left[a_{0}^{0} \operatorname{ixp}(\nu t)-a_{0}^{0} \operatorname{ixp}(-\nu t)\right]\left[\nu \nu\left(u^{\prime} n\right)+\nu \nu\left(u^{\prime \prime} n\right)\right]\right]
\end{aligned}
$$

where

$$
T\left(n^{\prime \prime} n\right)=\frac{1}{\nu^{2}\left(x^{\prime \prime} n\right)-\nu^{2}}
$$

$$
\begin{aligned}
= & c T\left(n^{\prime \prime} n\right) \tau\left(n^{\prime} n\right) \operatorname{ixp}\left[\nu\left(n^{\prime} n\right) t\right]\left[\frac{\left[\nu\left(n^{\prime} n\right) \nu\left(n^{\prime \prime} n\right)+\nu^{2}\right]}{2 \pi^{2} \nu^{2}} \dot{E}\right. \\
& \left.+\frac{\left[\nu\left(n^{\prime} n\right)+\nu\left(n^{\prime \prime} n\right)\right]}{i \pi} E\right] \\
= & \frac{c T\left(n^{\prime} n\right) T\left(n^{\prime \prime} n\right)}{12 \pi^{2} \nu^{2}}\left[i \nu\left(n^{\prime \prime} n\right)\left[\nu\left(n^{\prime} n\right) \nu\left(n^{\prime \prime} n\right)+\nu^{2}\right] \dot{E}\right. \\
& \left.\quad+2 \pi \nu^{2} \nu\left(n^{\prime \prime} n\right)\left[\nu\left(n^{\prime} n\right)+\nu\left(n^{\prime \prime} n\right)\right] E\right] i x p\left[\nu\left(n^{\prime} n\right) t\right]
\end{aligned}
$$

Now when we insert these back into Eq. 35,

$$
\begin{align*}
& \langle R\rangle_{n}=\langle n| R|n\rangle+\operatorname{Re} \sum_{n^{\prime} \neq n}\langle n| R\left|n^{\prime}\right\rangle\left\langle n^{\prime}\right| R|n\rangle \frac{T\left(n^{\prime} n\right)}{n^{\prime} \nu^{2}}[ \\
& \left.2 \nu\left(n^{\prime} n\right) \pi \nu^{2} E+i \nu^{2}\left(n^{\prime} n\right) E\right] \\
& +\sum_{n^{\prime \prime} \neq n} \sum_{n^{\prime} \neq n} \frac{H^{f}}{\pi n^{2}} R_{e}\left\langle n^{\prime}\right| R|n\rangle\left\langle n^{\prime \prime}\right| R|n\rangle\left\langle n^{\prime}\right| M\left|n^{\prime \prime}\right\rangle \\
& \frac{\Gamma\left(n^{\prime} n\right) T\left(n^{\prime \prime} n\right)}{\nu^{2}}\left[i \nu\left(n^{\prime \prime} n\right)\left[\nu\left(n^{\prime} n\right) \nu\left(n^{\prime \prime} n\right)+\nu^{2}\right] \dot{E}+\right. \\
& 2 \pi \nu^{2} \nu\left\langle n^{\prime \prime} n\right)\left[\nu\left(n^{\prime} n\right)+\nu\left(n^{\prime \prime} n\right)\right] E \tag{Eq. 36}
\end{align*}
$$

where the $\operatorname{ixp}\left[\nu\left(n^{\prime} n\right) t\right]$ disappears since it is multiplied by the exp $[-\nu(n ' n) t]$ term from the time dependent wave function, and the real part (Re) is taken since only real results are observable. Eq. 36 is the desired part of I'Haya's Eq. 27.

## 9. The average moment at a given temperature

Since the energy states of the molecule will be populated according to the Boltzmann factor, it is necessary to find the average moment per molecule at a given temperature. We multiply by Bn where Bn is the probability that the molecule will be found in state $n$ with energy En at this temperature, and sum over all possible $n$ ( $n$ also includes both vibrational and rotational quantum numbers). The Boltzmann factor Bn. under the influence of a magnetic field is:

$$
\begin{aligned}
B_{n}= & \frac{\exp \left(-\frac{E_{n}}{k T}\right)}{\sum_{n^{\prime}} \exp \left(-\frac{E_{n^{\prime}}}{k T}\right)} \\
& =\frac{\exp \left[-\frac{E_{n^{0}}+\langle n| M|n\rangle H^{f}}{K T}\right]}{\sum_{n^{\prime}} \exp \left(-\frac{E_{n^{\prime}}}{k T}\right\rangle}
\end{aligned}
$$

Expanding the numerator and taking only the term linear in $\mathrm{H}^{f}$, we have:

$$
B_{n}=B_{n} 0\left\{1+\frac{\langle n| M|n\rangle H^{f}}{k T}\right\}
$$

where

$$
\begin{equation*}
B_{n^{0}}=\frac{\exp \left(-\frac{E_{n}^{0}}{k T}\right)}{\sum_{n^{\prime}} \exp \left(-\frac{E_{n^{\prime}}}{k T}\right)} \tag{Eq. 37}
\end{equation*}
$$

Applying this now to our expectation value, Eq. 36 ,

$$
\begin{aligned}
& \langle R\rangle_{n T}=\sum_{n} B_{n^{0}}^{0}\left[\langle R\rangle_{n}+\frac{\langle n| M|n\rangle H^{f}}{k T}\langle n| R|n\rangle+\frac{\langle n| M|n\rangle}{k T}\right. \\
& \left.\operatorname{Re} \sum_{n^{\prime} \neq n}\langle n| R\left|n^{\prime}\right\rangle\left\langle n^{\prime}\right| R|n\rangle \frac{T\left(n^{\prime} n\right)}{n \pi \nu^{2}}\left[2 \nu\left(n^{\prime} n\right) \pi \nu^{2} E+i \nu^{2}\left(n^{\prime} n\right) \dot{E}\right]\right]
\end{aligned}
$$

Eq. 38

Eq. 38 introduces the temperature-dependent paramagnetic terms. Only the first and third terms on the right-hand side of Eq. 38 contribute to the MOD .
10. The average moment over all orientations

Up to this point we have considered the molecules to be held rigidly in one position. Actually, in solution they are tumbling and so their properties must be averaged over all positions. The process of taking this average is rather involved and is discussed by a number of authors ( $25, \mathrm{p} .121$ ) (28, p.707)(72). However, the result for this case is simple, multiplication by the factor 1/3.
11. Calculation of the magnetic optical rotation

Only the terms of Eq. 38 containing E are of interest for MCD. These are after averaging over all orientations
$\langle R\rangle_{n T}=\frac{H^{f}}{6 \pi h^{2}} \sum_{n} B_{n^{0}} \operatorname{Im}_{m} \sum_{n^{\prime} \neq n} \sum_{n^{\prime \prime} \neq n}\left[\frac{T\left(n^{\prime} n\right) T\left(n^{\prime \prime} n\right)}{\nu^{2}}\right.$

$$
\left[i \nu\left(n^{\prime \prime} n\right)\left(\nu\left(n^{\prime} n\right) \nu\left(n^{\prime \prime} n\right)+\nu^{2}\right) \dot{E}\right]\left\langle n^{\prime}\right| R|n\rangle\left\langle n^{\prime \prime}\right| R|n\rangle\left\langle n^{\prime}\right| M\left|n^{\prime \prime}\right\rangle
$$

$$
-\frac{1}{\pi T} \frac{i \nu^{2}\left(n^{\prime} n\right)}{\pi h v^{2}}\langle n| M|n\rangle\left\langle n^{\prime}\right| R|n\rangle\langle n| R\left|n^{\prime}\right\rangle
$$

Since only complex wave functions contain angular or linear momentum they must be used here. Therefore we can replace $\operatorname{Re}[i(f)]$ with $-\operatorname{Im}[f]$. Since $\quad \frac{T\left(n^{\prime} n\right) \nu^{2}\left(n^{\prime} n\right)}{\nu^{2}}=\frac{1}{\nu^{2}}+T\left(n^{\prime} n\right)$
the frequency part of the first term in Eq. 39 becomes
$\frac{\tau\left(n^{\prime} n\right) \tau\left(n^{\prime \prime} n\right) \nu\left(n^{\prime \prime} n\right)\left[\nu\left(n^{\prime} n\right) \nu\left(n^{n} n\right)+\nu^{2}\right]}{\nu^{2}}=\frac{\tau\left(n^{\prime} n\right) \nu\left(n^{\prime} n\right)}{\nu^{2}}+\frac{\tau\left(n^{\prime} n\right)-\tau\left(n^{\prime \prime} n\right)}{\nu\left(n^{\prime \prime} n^{\prime}\right)}$

Only the last term still contains all three states $n, n^{\prime}$ and $n^{\prime \prime}$ and only it will be used in the final evaluation. However, two cases must be distinguished. If $v\left(n^{\prime} n\right)$ and $v\left(n^{\prime \prime} n\right)$ are far from being the same then $\frac{\tau\left(n^{\prime} n\right)-\tau\left(n^{\prime \prime} n\right)}{v\left(n^{\prime \prime} n^{\prime}\right)}$ will be used. If $v\left(n^{\prime} n\right)$ and $v\left(n^{\prime \prime} n\right)$ are almost degenerate as in the case of Zeeman splitting, then the term can be expanded in powers of $v\left(n^{\prime \prime} n^{\prime}\right) \tau\left(n^{\prime} n\right)$ because this quantity is small.
$\frac{\tau\left(n^{\prime} n\right)-T\left(n^{\prime \prime} n\right)}{\nu\left(n^{\prime \prime} n^{\prime}\right)}=2 \nu\left(n^{\prime} n\right) T^{2}\left(n^{\prime} n\right)\left[1-2 \nu\left(n^{\prime} n\right) \nu\left(n^{\prime \prime} n^{\prime}\right) \tau\left(n^{\prime} n\right)+\ldots\right]$

Neglecting the terms after the first, Eq. 39 becomes,

$$
\begin{aligned}
& \overline{\langle R\rangle_{n T}}=\frac{H^{f}}{6 \pi h^{2}} \sum B_{n^{\prime}} \operatorname{Im} \sum_{n^{\prime} \neq n} \sum_{n^{\prime \prime} \neq n}\left[-\frac{T\left(n^{\prime} n \mid-T\left(n^{\prime \prime} n\right)\right.}{\nu\left(n^{\prime \prime} n^{\prime}\right)}\right. \\
& \dot{E}\left\langle n^{\prime}\right| R|n\rangle\left\langle n^{\prime \prime}\right| R|n\rangle\left\langle n^{\prime}\right| M\left|n^{\prime \prime}\right\rangle-2 \nu\left(n^{\prime} n\right) T^{2}\left(n^{\prime} n\right) \dot{E} \\
& \left\langle n^{\prime}\right| R|n\rangle\langle n| R\left|n^{\prime}\right\rangle\left\langle n^{\prime}\right| M\left|n^{\prime}\right\rangle \\
& -\frac{\nu^{2}\left\langle n^{\prime} n\right) T\left(n^{\prime} n\right)}{k T \pi h \nu^{2}}\langle n| M|n\rangle\left\langle n^{\prime}\right| R\left|n^{\prime}\right\rangle\langle n| R\left|n^{\prime}\right\rangle .
\end{aligned}
$$

$$
\text { Eq. } 40
$$

From Maxwell's equations it may be shown that the indices of refraction for right and left handed light are:

$$
\begin{array}{ll}
n_{r}=\varepsilon^{1 / 2}+2 \pi v g & g=4 \pi \mathbb{N} \frac{-R_{c}}{c} \cdot \mathbb{N}=\begin{array}{c}
\text { no. molecules per } \\
\text { unit volume }
\end{array} \\
n_{\ell}=\frac{1 / 2}{\varepsilon}-2 \pi v g &
\end{array}
$$

where $\varepsilon$ is the dielectric instant of the medium (we will use $\varepsilon=1$ ). The rotation per unit length is. then $\frac{\pi v}{c}\left(n_{\ell}-n_{r}\right)=\phi \quad$.

$$
\begin{equation*}
\phi=\left(\frac{4 \pi^{2} \nu^{2}}{c}\right)\left(4 \pi N-\frac{R_{c}}{c}\right)=\frac{16 \pi^{3} \nu^{2} N}{c} \overline{\langle R\rangle n T} \tag{Eq. 41}
\end{equation*}
$$

Substituting Eq. 40 into Eq. 41 gives the value for the Ver.de.t constant. This may be compared to I'Haya's Eq. 67 and Stephen's Eq. 27 except that Stephens has not yet averaged over all orientations.
12. Development of the expression for MCD according to Stephens

Beginning with Stephens Eq. 27 which we see is identical with our Eq. 41 before averaging, we use his results of treating for wide absorption bands.

We are interested in the case where electronic transitions occur in a molecule in a liquid or solution and the absorption band consists of many unresolved vibrational bands. If the vibrational bands act similarly, (they will for an electronically allowed transition or one which is electronically forbidden but vibrationally allowed by a single vibration and where the absorption band consists of a totally symmetric vibrational progression super-imposed on a single quantum in this.vibration) then the various vibronic transitions may be added. These may be treated as sharp, well-resolved lines but with $\nu\left(n^{\prime \prime} n^{\prime}\right) \ll \Gamma\left(n^{\prime} n\right)$, that is, the Zeeman splitting is much smaller than the banawiath. In this event the shape of the rotation (or circular dichroism) band remains essentially the same although its magnitude changes as the Zeeman splitting changes. The rotation in this case will be

$$
\begin{aligned}
& \phi\left(n^{\prime} n\right)=-\frac{4 \pi}{\hbar c} N\left\{\frac{4 \pi \nu\left(n^{\prime} n\right) \nu^{2}\left[4 \pi^{2}\left(\nu^{2}\left(n^{\prime} \cdot n\right)-\nu^{2}\right)^{2}-\nu^{2} T^{\prime 2}\left(n^{\prime} n\right)\right] A\left(n^{\prime} n\right)}{\hbar\left[4 \pi^{2}\left(\nu^{2}\left(n^{\prime} n\right)-\nu^{2}\right)^{2}+\nu^{2} T^{2}\left(n^{\prime} n\right)\right]^{2}}\right. \\
& \left.+\frac{4 \pi^{2} \nu^{2}\left(\nu^{2}\left(n^{\prime} n\right)-\nu^{2}\right)}{4 \pi^{2}\left(\nu^{2}\left(n^{\prime} n\right)-\nu^{2}\right)^{2}+\nu^{2} T^{2}\left(n^{\prime} n\right)}\left[B\left(n^{\prime} n\right)+\frac{C\left(n^{\prime} n\right)}{k T}\right]\right\} H_{z}
\end{aligned}
$$

Now since the MORD and MCD are related by the Kramers-Kronig relation

$$
\theta\left(n^{\prime} n\right)(\nu)=-\frac{1}{\pi^{2}} P . V . \int_{0}^{\infty} \frac{\oint\left(n^{\prime} n\right)\left(\nu^{\prime}\right)}{\left(\nu^{\prime}\right)^{2}\left[\left(\nu^{\prime}\right)^{2}-\nu^{2}\right]} d \nu^{\prime}
$$

where P.V. is the Canchy principal value we find the MCD has the form:

$$
\begin{aligned}
& \theta\left(n^{\prime} n\right)=-\frac{4 \pi}{\hbar c} \cdot N\left\{\frac{16 \pi^{2} \nu\left(n^{\prime} n\right) \nu^{3}\left(\nu^{2}\left(n^{\prime}(n)-\nu^{2}\right) T\left(n^{\prime} n\right) A\left(n^{\prime} n\right)\right.}{\hbar\left[4 \pi^{2}\left(\nu^{2}\left(n^{\prime} n\right)-\nu^{2}\right)+\nu^{2} \Gamma^{2}\left(n^{\prime} n\right)\right]^{2}}\right. \\
& \left.+\frac{2 \pi \Gamma\left(n^{\prime} n\right)}{4 \pi^{2}\left(\nu^{2}\left(n^{\prime} n\right)-\nu^{2}\right)+\nu^{2} T^{2}\left(n^{\prime} n\right)}\left[B\left(n^{\prime} n\right)+\frac{C\left(n^{\prime} n\right)}{\hbar T}\right]\right\} H_{z}
\end{aligned}
$$

where the freq. term for the A term looks like

and the freq. term for the B and C terms looks like


There has been some confusion concerning the sign convention for magnetic circular dichroism. In the past the convention used in this laboratory has been the conventional one for Faraday rotation. This leads to a positive sign for the $520 \mathrm{~m} \mu$ band of cobaltous salts as obtained by Scherer (4). However, the other workers have chosen the opposite convention $(8,9,10)$.

Since the sign of MCD is of particular importance in this research, and since the other workers will eventually be providing the majority of the research papers, it was decided to adopt their convention throughout this thesis. In addition, it is seen (see the Simplified Theory) when using the circular dichroism convention of Condon, Altar and Eyring (29), the orbital angular momentum convention of Condon and Shortley (24, p. 146) and the photon angular momentum convention of Beth (23), that the sign for the cobalt band should be negative. These are the most commonly used conventions but caution should be used since the opposite conventions are often found in the literature, particularly in the field of high energy physics.

