TWO DIMENSIONAL DISLOCATION SOURCES IN FIBER COMPOSITE LAMINATES

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INTRODUCTION

The method which is frequently adopted in order to determine the response of an elastic body to internal impulsive dislocations is to make use of the representation theorem due to Burridge and Knopoff (see for example Vasudevan and Mal [1]). In this approach, the problem of evaluating the transient response at a specified location due to a point dislocation source is replaced by the problem of evaluating the stress state at the source due to an impulsive point load placed at the measuring location. This technique is particularly useful in the acoustic emission problem where the interest lies in the response measured at the surface of the body, arising from some internal source. In particular, if attention is restricted to evaluating the normal component of displacement at the surface, the response to dislocation sources at any internal point is determined from the solution of the transient response of the body to an impulsive normal load acting at the surface. The representation theorem approach has been applied by Vasudevan and Mal [1] and by Suzuki *et.al.* [2] in order to evaluate the response of isotropic plates to dislocation sources.

An alternative approach is based on the moment tensor representation due to Ohtsu and Ono [3], whereby the solution is expressed in terms of the derivatives of the Green's tensor components associated with impulsive point loads acting at the source. These can be interpreted in terms of point couples and point dipoles (double forces) acting at the source location and the response of an isotropic elastic plate to a variety of these sources has been calculated by Ceranoglu and Pao [4]. In the case of anisotropic elastic bodies, Green has evaluated the response of a cross-ply fiber composite laminate to internal line loads [5], line double forces [6], line couples [7], and to some line dislocations [8]. For point sources, Guo *et.al.* [9, 10] have employed a laminate theory with shear correction in order to model the elastic wave propagation in a cross-ply laminate and they have applied the moment tensor approach with this model in order to simulate the acoustic emission associated with a variety of dislocation sources. As the authors point out, the use of the modified laminate theory leads to a considerable simplification in solving the point source problem but at the expense of limiting the frequency range of the solution.

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For the fiber composite laminate, a convenient formulation of the full equations of dynamic elasticity in each layer is in terms of six coupled first order partial differential equations for the three components of displacement and for the three traction components across any plane parallel to the layering. By taking Fourier transforms with respect to the two spatial components in the plane of the layering and the Laplace transform with respect to the time, these equations reduce to a system of six coupled ordinary differential equations in the spatial coordinate normal to the layering. It is a straightforward matter to solve these equations in the presence of a dislocation (displacement discontinuity) acting across any plane parallel to the layering, and the response is then obtained by inverting the transform solutions. Whilst the direct solution obviates the need to evaluate the derivatives of the Green's functions for these dislocation sources, there remains the problem of determining the response to dislocations acting across planes whose normals lie in the plane of the layering, for which the direct solution is not possible. In general it is then necessary to have recourse to the equivalent force system.

The laminate problem is simplified in the case where the displacements and stresses are functions of the time, the coordinate normal to the plate and one of the two spatial coordinates in the plane of the plate. This corresponds to the disturbance generated by line loads and/or line dislocations, which give rise to straight crested waves travelling in the plate at right angles to the load line. The upper surface normal displacement resulting from an internal line dislocation is then obtained by solving the problem of a normal line load acting on the surface of the laminate and employing the representation theorem. In this work, each layer of the fiber composite material is modelled as a transversely isotropic elastic continuum with the axis of transverse isotropy parallel to the fiber direction. It turns out that, because of this particular form of anisotropy, it is possible to make use of the moment tensor representation in order to express the solution for any dislocation line source in terms of the Green's functions for line loads and the solutions for line dislocations acting across a surface element parallel to the layer plane, thus eliminating the necessity to evaluate the derivatives of the Green's tensor components.

The representation theorem approach is employed here in order to determine the response to crack opening line dislocations acting across planes normal to the layering. Results are obtained for dislocations located at the mid-plane and at each of the interfaces between the core and the outer layer. As a check on the calculations, the response is also determined using the moment tensor method. This employs a completely independent set of numerical results and it leads to transient normal surface displacements which are indistinguishable from the representation theorem solutions.

THEORY

The representation theorem which is applied to determine the response of an elastic body to an impulsive dislocation distributed over an internal area element S has the form

$$u_r(x_k,t) = \int_0^t d\tau \int_S [t_{ij}^{(r)}(\xi_k,\tau;x_k)n_j d_i(\xi_k,t-\tau)]dS(\xi_k). \qquad (1)$$

Here $t_{ij}^{(r)}(\xi_k,\tau;x_k)$ are the stress components at the point ξ_k at time τ arising from an impulsive point force acting in the direction Ox_r at the point x_k at time $t = 0$. The term $d_i(\xi_k,t)$, is the discontinuity at time t in the component of displacement u_i across the surface element dS with unit normal $(n_1,\ n_2,\ n_3)$ located at ξ_k. These stress components are given in terms of the Green's tensor $G_{pr}(\xi_k,t;x_k)$ $(p,q,r = 1,2,3)$, by the expressions $t_{ij}^{(r)} = C_{jipq}Gpr,q$ where $G_{pr,q}$ denotes the derivative of G_{pr} with respect to ξ_q. The constants C_{ijpq}, $(i,j,p,q = 1,2,3)$ are the elements of the elasticity tensor for the elastic material. The element G_{pr} of the Green's tensor denotes the component in the direction Ox_p of the displacement at the point ξ_k at time t due to a point force in the form of a delta function in time acting in the direction Ox_r, at the point x_k at time $t = 0$. The representation theorem in equation (1) is particularly useful in the acoustic emission problem, where it is required to determine the normal displacement at a surface location due to a variety of internal sources, since it allows this to be done by solving the single problem of a normal point load acting at that point. This approach has the drawback that it provides no information about the effect of the source at any interior point of the body and if this information is required an alternative formulation must be adopted.

This alternative formulation may be obtained by means of the reciprocal theorem of elasticity which in the present context yields the result that

$$G_{pr}(\xi_k, t; x_k) = G_{rp}(x_k, t; \xi_k) \quad (p, r, k = 1, 2, 3). \tag{2}$$

This allows the solution at any point x_k to be expressed in terms of the Green's tensor due to a load acting at the location of the displacement discontinuity, in the form

$$\begin{aligned} u_r(x_k, t) &= \int_0^t d\tau \int_S [C_{ijpq} G_{rp,q}(x_k, \tau; \xi_k) n_j d_i(\xi_k, t-\tau)] dS(\xi_k) \\ &= \int_0^t d\tau \int_S G_{rp,q}(x_k, \tau; \xi_k) m_{pq}(\xi_k, t-\tau) dS(\xi_k). \end{aligned} \tag{3}$$

Here, the alternative formulation involves the components m_{pq} of the moment tensor M defined by Ohtsu and Ono [3], which are given by the expressions $m_{pq} = C_{ijpq} n_j d_i$. The terms $G_{rp,q}$ are interpreted as the components along Ox_r of the displacement arising from the limit as $\epsilon - > 0$ of two equal and opposite forces of magnitude ϵ^{-1} acting parallel to Ox_p and separated by a distance ϵ in the direction Ox_q. In the case where the subscripts p and q are the same, this limiting force system corresponds to a double force or dipole, otherwise it corresponds to a couple.

Attention here is restricted to line loads and line dislocations acting in a cross-ply fiber composite laminate for which the coordinate axes are chosen so that Ox_1 is normal to the plane of the laminate and Ox_2 and Ox_3 are along the fiber directions in one or other of the plies. Choosing the load line to be $x_1 = \hat{x}_1,\ x = x_2 \sin\gamma + x_3 \cos\gamma = 0$, gives rise to disturbances which are functions of the spatial coordinates x_1 and x and of the time t. Taking Laplace transforms with respect to t and Fourier transforms with respect to x, equations (1) and (3) become

$$U_r(x_1, k, s) = T_{ij}^{(r)}(\hat{x}_1, k, s; x_1) n_j D_i(k, s), \tag{4}$$

and

$$U_r(x_1, k, s) = M_{pq} \tilde{G}_{rp,q}(x_1, k, s; \hat{x}_1). \tag{5}$$

Here U_r, $T_{ij}^{(r)}$, D_i, M_{pq} and $\tilde{G}_{rp,q}$ are the double transform of u_r, $t_{ij}^{(r)}$, d_i, m_{pq} and G_{rp} respectively, k is the Fourier transform parameter and s is the Laplace transform parameter. For the fiber composite laminate it is a straightforward matter to determine the solutions corresponding to line loads acting at the load location $x_1 = \hat{x}_1,\ x = 0$, in each of the three orthogonal directions, Ox_1, Ox and Oy, where $y = -x_2 \cos\gamma + x_3 \sin\gamma$, (Green [5]). It is equally straightforward to determine directly the solution arising from a dislocation with Burgers vector along each of these three directions, in the case when the normal vector n is along Ox_1, Green [6]. These latter solutions can be shown to yield the derivatives of the Green's functions with respect to the coordinate x_1 and the derivatives with respect to the two in-plane directions are obtained directly from the Fourier transform. It is then possible to obtain the response at any location within or on the boundary of the laminate, due to dislocation sources for which the normal vector n lies in the plane of the laminate, by using equation (5). Since the interest here lies in simulating the normal component of displacement at the surface, it is more convenient to adopt the formulation (4) and to determine the components $T_{ij}^{(r)}$ from the solution for a line load acting in the normal direction on the laminate surface. Results are presented for crack opening (mode I), line dislocations acting at the mid-plane and at the interfaces between the core and the outer layers for the two cases when the line of action is along Ox_2 and along Ox_3. As a check on the numerical accuracy, some of these results are also obtained by employing the formulation given in equation (5) which makes use of the alternative Green's function solutions.

RESULTS

The results presented here relate to a fiber composite material having the same material constants as that reported in [5]. The graphs in figures 1-6 show the variation with time of the normal displacement at the upper surface of the laminate due to sources located at varying depths within the laminate. The laminate is of overall depth $4h$, where h is the depth of a single ply. Each figure consists of 3 curves and in order to display these clearly, the origins of two of the curves have been displaced downwards by differing amounts. In each figure, the upper curve, (a) is the surface signal arising from a source located at the upper interface (depth h below the surface), the middle curve, (b) shows the response to a source at the mid-plane (depth $2h$ below the surface) and the lowest curve, (c) relates to a lower interface source (depth $3h$ below the surface). In each case, the time history of the source consists of a unit step function acting at time $t = 0$. The graphs show the scaled normal displacement plotted against the non-dimensional time $T = c_1 t/h$, the time taken for the wave with speed c_1 to travel through the depth of one layer of the laminate. Here, c_1 is the speed of the longitudinal wave which travels in any direction at right angles to the fibers.

In applying equation (4) to evaluate the response to sources located at the interfaces, curves (a) and (c), it is possible to make use of the stress transforms $T_{ij}^{(r)}$ calculated either in the core material or in the outer material. The results presented here in each case use the stress components in the core, so that the sources may be regarded as lying in the core material at each of the three locations. Two different orientations of a crack opening line dislocation are considered. In the first case, the line of action is along Ox_2 with $\gamma = 0^0$. This may be taken to represent fiber breakages in the core at each of the three locations. The line of action in the second case is along Ox_3 with $\gamma = 90^0$, which may be interpreted as matrix cracking in the core at each location.

Figures 1 and 2 show the signals arriving at a point on the upper surface located a distance of $x = 4h$ (one laminate depth) from the plane normal to the laminate through the line of action of the source. Figure 1 corresponds to a fiber break and Fig. 2 to a matrix crack. Note that there is a significant difference in the vertical scales of the two graphs. There are two striking features apparent in Fig. 1, namely the symmetrical relationship of curves (a) and (c), and the almost non-existence of the response to a mid-plane source in comparison with the other two locations. Both these phenomena are due to the fact that an off-center fiber break in a symmetric laminate will induce bending of the plate. A fiber break a distance d above the mid-plane will produce an equal and opposite flexure to that produced by a fiber break a distance d below the mid-plane. However, a fiber break at the mid-plane will not give rise to any plate bending. On comparing Fig. 2 with Fig. 1, it is clear that a matrix crack does not have the same dramatic effect on a laminated plate as does a fiber break. In order to show the detail of the response, the vertical scale has been enlarged by a factor of 7.5. It is evident that the arrival of the disturbance at the receiver depends on the depth of the source.

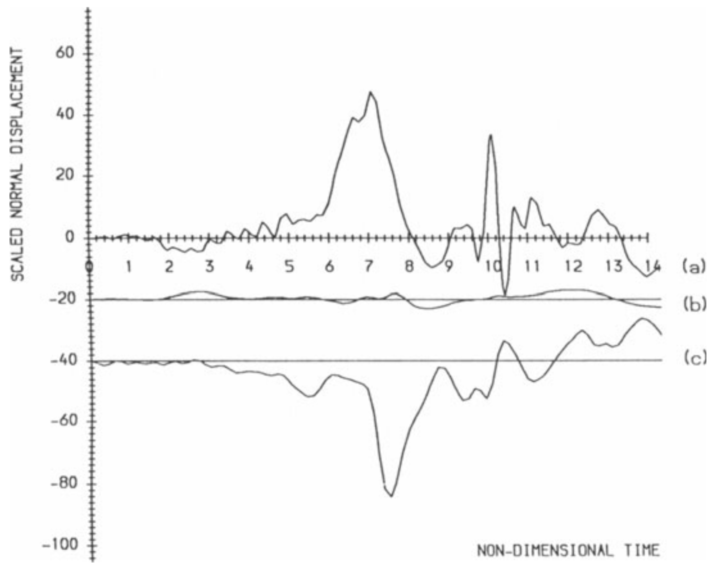

Figure 1. Upper surface normal displacement at $x = 4h$ (for $\gamma = 0^0$) due to fiber breakage dislocation sources at: (a) upper interface, (b) mid-plane, (c) lower interface.

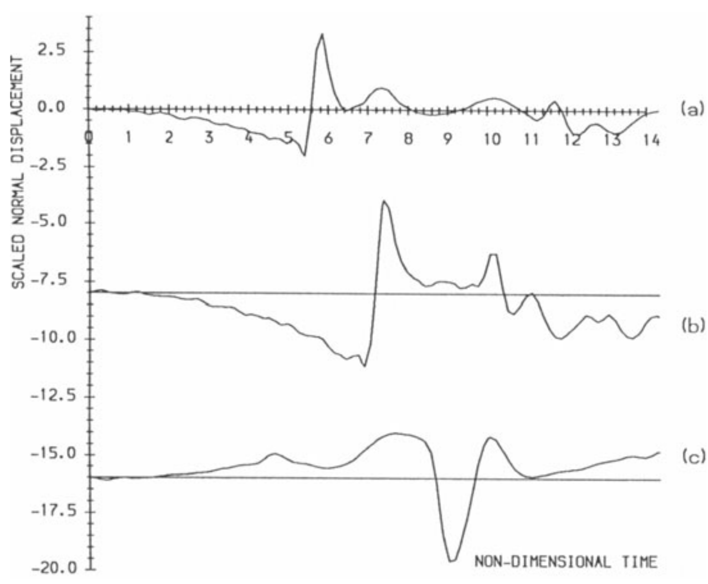

Figure 2. Upper surface normal displacement at $x = 4h$ (for $\gamma = 90^0$) due to matrix cracking dislocation sources at: (a) upper interface, (b) mid-plane, (c) lower interface.

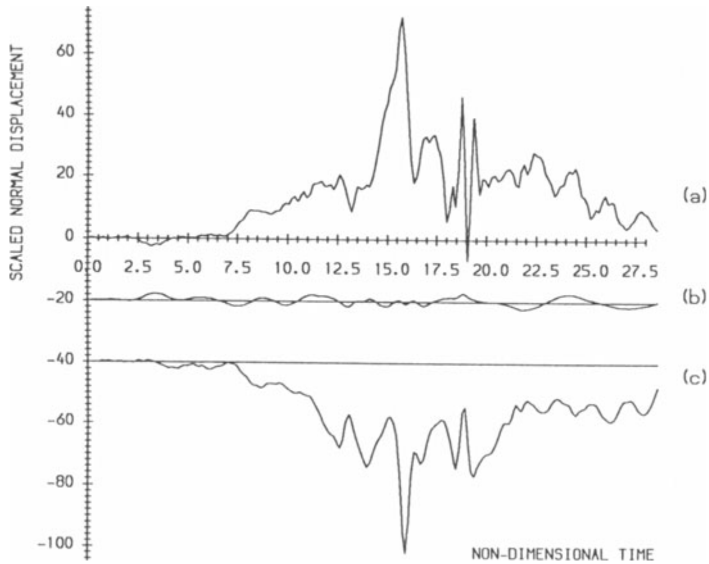

Figure 3. Upper surface normal displacement at $x = 8h$ (for $\gamma = 0^0$) due to fiber breakage dislocation sources at: (a) upper interface, (b) mid-plane, (c) lower interface.

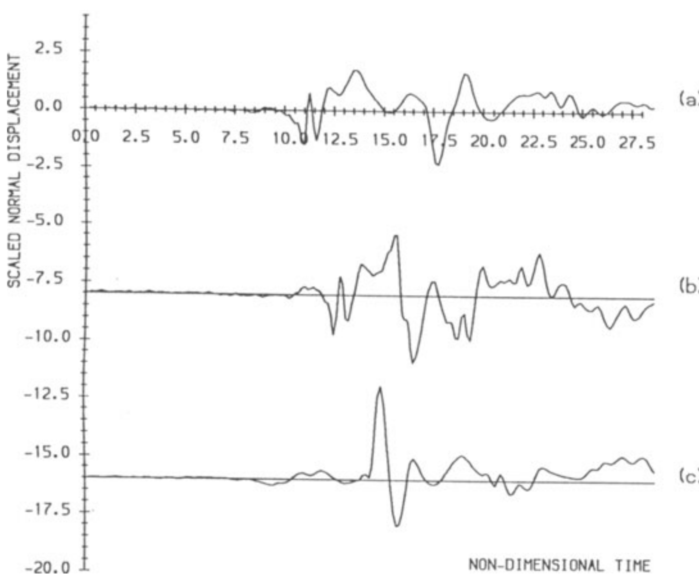

Figure 4. Upper surface normal displacement at $x = 8h$ (for $\gamma = 90^0$) due to matrix cracking dislocation sources at: (a) upper interface, (b) mid-plane, (c) lower interface.

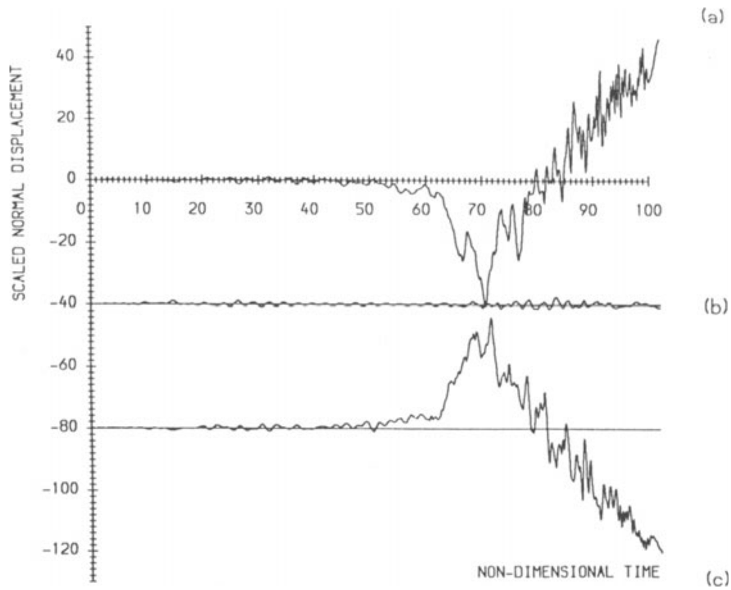

Figure 5. Upper surface normal displacement at $x = 40h$ (for $\gamma = 0^0$) due to fiber breakage dislocation sources at: (a) upper interface, (b) mid-plane, (c) lower interface.

The receiver has been moved to a location $x = 8h$, two plate thicknesses, for Figs. 3 and 4. The sources depicted in Fig. 3 are all fiber breaks whilst those for Fig. 4 are matrix cracks. There is a significant increase in the number of peaks and troughs present in each curve from when $x = 4h$. This is due to the increase in the number of internal reflections that occur at the interfaces and the free surfaces of the plate over the longer distance. Curve 1 in Fig. 3 has a pronounced spike occurring at time T = 19.0 which is not evident in either of curves 2 and 3, nor in any of the curves in Fig. 4. This spike is associated with the arrival of the surface wave which is generated by the upper level disturbance at $\gamma = 0^0$ but not by the sources at the other two levels. For the case of $\gamma = 90^0$, the surface wave arrival would be at time T=12.6 and there is clearly no trace of this in Fig. 4, a result which is in accord with theoretical predictions.

Figure 5 is included to show the effect of moving the receiver yet further from the source to a distance of 10 plate thicknesses. All the features previously remarked upon are also present here. In particular, the symmetry between the responses to the upper and lower interface sources is very clear. Also evident is the underlying plate bending induced by off-center sources which is superimposed by the high frequency oscillations caused by the multiple internal reflections.

As stated earlier, some of these results were also obtained by employing the alternative formulation given in equation (5). Figure 6 is an example of such a result for a fiber break at the three levels in the core layer, with the receiver located at $x = 4h$. These are precisely the same conditions as for Fig. 1. Though the two figures are derived from completely different numerical calculations, it is evident that they are indistinguishable. The results presented here are for waves propagating in perfectly elastic materials and arising from line sources in the form of a step function in time. In practice, most materials will exhibit some damping effects and real acoustic emission sources will be distributed over some finite area, with a finite rise time and having a finite duration. These results are not therefore intended as predictions of the observed signals arising from acoustic emission but

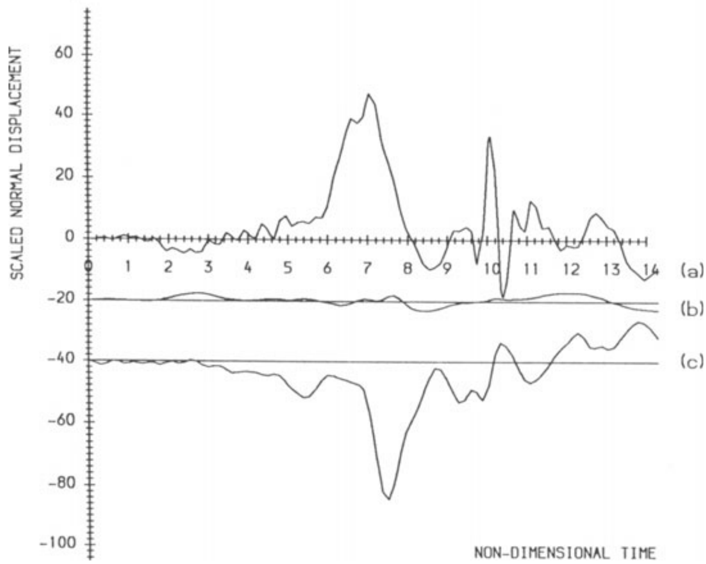

Figure 6. Upper surface normal displacement at $x = 4h$ (for $\gamma = 0^0$) due to fiber breakage dislocation sources at: (a) upper interface, (b) mid-plane, (c) lower interface.

rather they serve to enhance the understanding of the effects of specimen geometry and of material parameters on the mode of propagation of the signal. Nevertheless, the technique adopted here can be readily modified to include material damping and to take account of differing time histories with little effect on the computational effort required. It is also straightforward to evaluate the far-field response.

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