Three essays on trade policies in developing countries

by

Yoonho Choi

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
Eun Kwan Choi, Co-major Professor
Rajesh Singh, Co-major Professor
Peter F. Orazem
Juan C. Cordoba
Paul W. Gallagher

Iowa State University

Ames, Iowa

2017

TABLE OF CONTENTS

LIST OF FIGURES iv

LIST OF TABLES v

ACKNOWLEDGMENTS vi

ABSTRACT.. viii

CHAPTER 1 INTRODUCTION 1

CHAPTER 2: TARIFF EQUIVALENTS OF CHINA'S UNERVALUED YUAN 3
- 2.1 Abstract 3
- 2.2 Introduction 3
- 2.3 Basic Model 6
- 2.4 Equivalence of Devaluations and Tariffs 9
- 2.5 Graphical Illustration 14
- 2.6 Numerical Example 15
- 2.7 Tariff Equivalents of China's Undervalued Yuan 17
- 2.8 Concluding Remarks 25
- 2.9 References 27

CHAPTER 3: UNEMPLOYMENT AND OPTIMAL EXCHANGE RATE IN AN OPEN ECONOMY 36
- 3.1 Abstract 36
- 3.2 Introduction 36
- 3.3 The Two-Sector Keynesian Model with Unemployment 39
- 3.4 Devaluation and Exchange Rate Pass-Through 48
- 3.5 Yuan Devaluation, Income and Welfare 51
- 3.6 Yuan Devaluation and Unemployment 55
- 3.7 Numerical Example 56
- 3.8 Concluding Remarks 58
- 3.9 References 60

CHAPTER 4: PUBLIC CAPITAL AND OPTIMAL TARIFF 67
- 4.1 Abstract 67
- 4.2 Introduction 67
- 4.3 Basic Economic Model 70
- 4.4 Ramsey Optimal Tariff 75
- 4.5 An Algorithm for Computing Optimal Import Tariff Sequence 79
- 4.6 Numerical Example 82

4.7 Concluding Remarks.......... 85
4.8 References.......... 86

CHAPTER 5: CONCLUDING REMARKS.......... 89

LIST OF FIGURES

Figure 2.1. The Effect of Devaluation on Import Demand 30

Figure 2.2. The Effect of Devaluation on Export Supply 30

Figure 3.1 Yuan Devaluation and Terms of Trade 63

Figure 3.2 Yuan Devaluation and Indirect Utility 63

Figure 3.3 Yuan Devaluation and Output Effects 64

Figure 3.4 Effects on Yuan Devaluation 65

Figure 4.1 Ratio of Public Capital Stock to GDP 87

Figure 4.2 Ratio of Private Capital Stock to GDP 87

Figure 4.3 Share of Labor in Export Sector 87

Figure 4.4 Optimal Import Tariff 88

Figure 4.5 Consumer's Welfare 88

LIST OF TABLES

Table 2.1. Summary of the equivalence of tariffs and currency undervaluation 31

Table 2.2 Unit root test results 32

Table 2.3. Johansen test for Cointegration (Export Supply) 32

Table 2.4 Johansen test for Cointegration (Import Demand) 32

Table 2.5. Parameter Estimates of Import and Export Functions 32

Table 2.6 Tariff Equivalents of Devalued Yuan 33

Table 3.1. Variables in China and the Unites States 66

Table 4.1 The values of parameters 88

Table 4.2. The values of the steady state 88

ACKNOWLEDGMENTS

Undertaking this PhD has been a truly life-changing experience for me and it would not have been possible to do without the support and guidance that I received from many people.

First, I would like to say a very big thank you to my advisor Professor E. Kwan Choi for patiently guiding me in finishing this dissertation. Professor Choi provided me with every bit of guidance, assistance, and expertise. When I felt ready to venture into research on my own, he gave me valuable feedback, advice, and encouragement. In addition to our academic collaboration, I greatly value the close personal rapport that Dr. Choi and I have forged over the years. I quite simply cannot imagine a better adviser.

Second, I would like to thank Professor Rajesh Singh for his valuable instruction, advice and supports during my PhD study at Iowa State University. Professor Singh intrigued my interest in international economics. I secured my strict training in the usage of dynamic methods. Moreover, during my struggling period, he also provided me sincere suggestions and guidance to get over the frustrating obstacles. Meanwhile, my thanks also go to Professor Peter F. Orazem, Professor Juan C. Cordoba, and Professor Paul W. Gallagher for agreeing to be my committee members and supervising my research.

Third, I am indebted to Mam and Dad for always believing in me and encouraging me to follow my dreams. And my sister and brother in law for helping in whatever way they could during this challenging period.

Last but not least I am deeply thankful to my wife for her love, support, and sacrifices. She has been my side throughout this Ph.D., living every single minute of it, and without whom, I would not have had the courage to embark on this journey in the first place. To our two daughters Alexis and Genesis, their trust and supports are the source of happiness, strength and wisdom in my life. I will achieve nothing without my wife and two daughters.

ABSTRACT

During the past decade, foreign exchange reserves of China and Japan have increased dramatically. For instance, China's foreign exchange reserve rose from $954.6 billion in January 2007 to $3.5 trillion in April 2016. China and Japan seem to hold large foreign reserves, much more than are necessary to facilitate their imports. The WTO regulates only tariff and various non-tariff barriers but has made little effort to regulate the bilateral exchange rates because exchange rate practices are within the purview of the IMF. At present, the World Trade Organization (WTO) does not treat currency devaluation as a protective trade policy. In my dissertation, I have chosen three topics in the area of international economics. The first chapter argues that currency devaluation is equivalent to an import tariff, and hence currency devaluation should be treated as a trade policy instrument. The second chapter considers the employment effects of currency devaluations in a Keynesian open economy. Currency devaluation may decrease domestic employment and increase the social welfare. Under plausible conditions, the optimal policy is to get rid of domestic unemployment in input sectors. The third chapter investigates the effects of public capital investment in the export sector for the labor movement and capital formation and identifies the contribution of public capital and other economic factors to the productivity growth rate in the firm sector. We show that the optimal tariffs are positive but decrease to the steady state level.

CHAPTER 1. INTRODUCTION

This dissertation addresses two main issues in international economics, currency devaluation and import tariffs. John Maynard Keynes (1931, p.199) partly recognized the effect of devaluation, when he argued that "Precisely the same effects as those produced by a devaluation of sterling by a given percentage could be brought about by a tariff of the same percentage on all imports *together* [italics added] with an equal subsidy on all exports…" The second chapter shows whether yuan devaluations are equivalent to import tariffs in a two-good, two-currency model. Even if no tariffs or quotas are employed, our analysis shows that a country can restrict imports by undervaluing its currency. Contrary to Keynes's statement, a 50 percent devaluation from the benchmark exchange rate is shown to yield the same import price and volume as a 100 percent tariff on imports *without* an export subsidy. Using the available yuan-dollar exchange rates and bilateral U.S.-China trade data, it is shown that during the period 1994 - 2015, China's yuan has been grossly undervalued. On average, the yuan was devalued by 45 percent, and the average of tariff equivalents of the undervalued yuan was 87.5 percent for the period 1994-2015. The finding that tariffs and the undervalued yuan have the same effects on domestic prices and import volumes.

During the past two decades, many Asian Counties have accumulated foreign exchange reserves. For instance, China, Japan, and South Korea have accumulated $3.5 trillion, $1.32 trillion and $0.37 trillion as of April 2016, respectively. In developing countries, maintaining low unemployment rate is far more important than other economic problems. Thus, the Chinese government may adopt low yuan policy and accumulate foreign exchange reserves, not to take advantage of its trading partners, but to reduce unemployment. Since the exchange rate pass-through on the relative price of import good in the U.S. is not complete, The third chapter investigates optimal exchange rate for a Keynesian open economy. We consider an open economy that produces two tradable goods. In the benchmark equilibrium, all firms are price takers and the yuan price of the dollar is determined by the demand and supply of traded goods. Under a plausible scenario, the optimal yuan value of the dollar is less than that which ensures full employment. China may pursue the low yuan policy to reduce domestic unemployment and to maximize gross domestic product.

In many developing counties, lack of public infrastructures such as highways, transportation, power and water system is slowing economic growth because the infrastructure systems are crucial input factors for domestic production. Public investment plays an important role and a major policy issue in developing countries. A number of studies support that public capital has a powerful impact on the productivity of private capital. The fourth chapter addresses the relationship between the government expenditure in public capital stock and economic growth and examine an optimal import tariff for increase in public capital stock in a small open economy. This practice explains rationale for government provision of public goods based on the market failure, internalizes externalities in the private production function. Moreover, the paper is alternatively answering "Why does the government devalue its own currency and change the terms of trade in the short run? " The effect of change in exchange rate is the similar to that of import tariffs which changes the relative price in domestic economy. This distortion of government policy discourages current consumptions for import goods and encourage consumptions of home goods and capital investments.

CHAPTER 2. TARIFF EQUIVALENTS OF CHINA'S UNERVALUED YUAN

2.1 Abstract

This paper considers a two-good, two-currency model to demonstrate that undervalued currency is equivalent to an import tariff. Contrary to Keynes's statement, a 50 percent devaluation from the benchmark exchange rate is shown to yield the same import price and volume as a 100 percent tariff on imports *without* an export subsidy. A numerical example based on a Cobb-Douglas utility function illustrates the main proposition. Using the Chinese trade data and U.S. consumer price index, we show that China's yuan was undervalued by 45 percent on average for the period 1994-2015, and the average of equivalent tariffs was 87.5 percent on China's imports.

2.2 Introduction

The People's Bank of China (PBC), the central bank of China, began to regulate renminbi on January 1, 1994 by moving the official rate to the then-prevailing swap market rates, which contributed to the steady increase in China's foreign exchange reserve (Goldstein and Lardy 2009). During the past two decades, China's foreign exchange reserve rose significantly, surpassing $3.3 trillion as of December 2015.

Those who investigated China's yuan policy during the past ten years generally agreed that the Chinese yuan has been undervalued, but their estimates of yuan undervaluation vary widely. Funke and Rahn (2005) noted that in the aftermath of the Asian financial crisis, the yuan was stabilized but undervalued by 15 percent in 1999. Gan et al. (2013) estimates he RMB was undervalued by an average of 6.7 percent. In a multinational comparison of currency undervaluation, Chang (2007) noted that the yuan was undervalued by 22 percent in 2001. Wren-Lewis (2004) suggested a 20 percent devaluation of the yuan against the dollar in 2002, whereas Coudert and Couharde (2007) reported that the bilateral renminbi-dollar exchange rate was undervalued by 60 percent during the same year. Chang and Shao (2004) noted that RMB was undervalued 22.5 percent in 2003. Claud Meyer (2008, p. 7) stated that "over the period 2002-2007, the undervaluation of the RMB would be on this basis in the range of 10-15 percent against the US$." Garroway et al's (2012) estimate of undervaluation in 2007 for renminbi was 15 percent. Referring to the current value of the yuan, Bergsten (2010) observed that "The Chinese renminbi is undervalued by about 25 percent on a trade-weighted average basis and by about 40 percent against the dollar."

The dramatic rise in China's cumulative trade surplus has spurred a debate concerning China's currency valuation and misalignment. The common view is that China has intentionally depressed the value of its currency, the renminbi (RMB), to gain unfair advantages in the global market.[i] (Cheung et al., 2009, Cheung, 2012).

These estimates of yuan undervaluation often are used to justify policy recommendations to exert pressure on China to modify its currency policy. For example, Bergsten (2010) of the Peterson Institute for International Economics recently suggested that the RMB must appreciate by approximately 40 percent against the dollar to correct current "global imbalances" and urged the United States to take multilateral, and if necessary unilateral action, to pressure China to change its ways.

John Maynard Keynes (1931, p.199) partly recognized the effect of devaluation, when he argued that "Precisely the same effects as those produced by a devaluation of sterling by a given percentage could be brought about by a tariff of the same percentage on all imports *together* [italics added] with an equal subsidy on all exports…" Also, Kong (2012) recognized that long-term devaluation of a major currency such as the Chinese yuan may endanger the entire world trading system.

By design the International Monetary Fund (IMF) is concerned with exchange rate practices, while GATT /WTO is interested in regulating trade practices. Currency practices fall within the purview of the IMF, and are generally considered to be outside the jurisdiction of the WTO. Although currency devaluation affects international trade, the WTO has made little effort to regulate the exchange rate practices of member countries because any such attempt may be viewed as "intruding into the domain of the IMF." (Kong, 2012, p. 112) While the two institutions complement each other, neither institution has been willing to take actions on currency practices that have spillover effects on world trade. Hence, the hesitation of the IMF to take action against countries that manipulate exchange rates to gain "unfair advantage" over their trading partners.

Section 2.3 presents the basic model of two goods and two currencies, and Section 2.4 investigates the equivalence of devaluations and tariffs. Section 2.5 provides graphical illustrations of the effects of devaluation, while Section 2.6 uses a numerical example with a Cobb-Douglas utility function. Section 2.7 provides empirical estimates of China's import demand and export supply function, which yield estimates of yuan devaluations for the period 1994-2015, and Section 2.8 offers concluding remarks.

2.3 The Basic Model

In order to compare the effects China's tariffs and currency devaluation on trade, we first construct a basic model with two goods and two currencies. There are two countries, the home country, China, and the foreign country, the United States. We employ the following assumptions to describe the production side of China's economy:

(i) Supplies of the two tradable goods are subject to the production possibility function (PPF), $C = F(Z)$, where C and Z are the outputs of the exportable and the importable.

(ii) Each country fixes the price of its exportable good, i.e., the yuan price b of good C is fixed in China, and the dollar price P^* of good Z is fixed in the United States.

(iii) Perfect competition prevails in the domestic product and resources are fully employed.

Let ε denote the dollar price of Chinese yuan, and P be the yuan price of the importable good Z. China fixes the yuan price of its exportable good C, and the United States also fixes the dollar price of its exportable (good Z). That is, the yuan price b of good C and the dollar price $P^* = P\varepsilon$ of good Z are fixed and remain unaltered whether a tariff is imposed or the yuan is devalued. If no tariff is imposed, the relative price of the importable in China is $P / b = P^* / b\varepsilon$. Let c and z denote the domestic consumption of the exportable and importable, and let x and q denote the physical volumes of exports and imports. Then $x = C - c$ and $q = z - Z$. The dollar value of imports is qP^*.

Domestic producers are assumed to maximize yuan revenue $R = bF(Z) + PZ,$ where $b = 1,$ since the exportable is the numéraire. The first order condition for revenue maximization requires

$$bF' + P = 0, \qquad (1)$$

i.e., domestic price of the importable is equal to its marginal cost. The second order condition is $F'' < 0,$ which is satisfied if the PPF is concave to the origin.

Domestic supplies of the tradable goods depend on the domestic prices of the tradable goods, i.e., $C(b,P)$ and $Z(b,P)$. Supplies of the tradable goods are homogenous of degree zero in all prices, i.e., doubling all prices has no effect on the supplies. Differentiating (1) with respect to P yields $bF''(\partial Z/\partial P)+1=0,\ \ \partial Z/\partial P = -1/bF'' > 0$. That is, the domestic supply of the importable good is positively sloped. Accordingly, $\partial C/\partial P < 0,$ i.e., an increase in the price of the importable good shifts the supply of the exportable good to the left.

Let ε^o denote the equilibrium exchange rate under free trade,[ii] i.e., when $t=0$ and $P = \frac{P^*}{\varepsilon^o} = P^*.$ China's supply functions are written as $C(b,P)$ and $Z(b,P).$ Assume that the government imposes a tariff on its imports and devalues the yuan below the equilibrium level, ε^o. Then the domestic price of the importable is

$$P = P^* \frac{1+t}{\varepsilon}. \qquad (2)$$

The supply functions are written as: $C = C(b,P) = C\left(b, \frac{P^*(1+t)}{\varepsilon}\right),\ \ C_P(b,P) < 0,$ and $Z = Z(b,P) = Z\left(b, \frac{P^*(1+t)}{\varepsilon}\right), Z_P(b,P) > 0.$ Producer revenue in yuan is

$$R = bC\left(b, P^*(1+t)/\varepsilon\right) + PZ\left(b, P^*(1+t)/\varepsilon\right).$$

In order to compare tariffs and devaluation on an equal footing, we assume that consumers receive income only from production. Any tariff revenue is retained by the government as a budget surplus, and not rebated to consumers. Thus, consumer expenditure, *I* is the same, whether a tariff is used or the yuan is devalued from the benchmark equilibrium.

Consumer demands for the tradable goods can be rewritten as

$$c(b,P,I) = c\left(b, P^*(1+t)/\varepsilon, I\right), \quad z(b,P,I) = z\left(b, P^*(1+t)/\varepsilon, I\right). \tag{3}$$

where consumer expenditure $I = R.$ Recall that ε^o is the equilibrium exchange rate under free trade, i.e. $\varepsilon^o = 1$, when $t = 0$ and $P = \frac{P^*}{\varepsilon^o} = P^*.$

Let the dollar price of the exportable be denoted by $b^* = b\varepsilon$. Since the dollar price of the importable P^* is fixed in the United States, a yuan appreciation raises the domestic price of the importable, $P = P^*/\varepsilon.$ The first order conditions are

$$\frac{u_c}{u_z} = \frac{b}{P}. \tag{4}$$

Demand functions for the tradables can be written as: $c = c(b,P,I)$ and $z = z(b,P,I)$. Indirect utility is given by $V(b,P,I) = u\left(c(b,P,I), z(b,P,I)\right).$

For given consumer expenditure I, domestic price P also completely determines the import demand function and the export supply function,

$$q = z(b,P,I) - Z(b,P), \quad x = C(b,P) - c(b,P,I). \tag{5}$$

The yuan amount China's consumers pay for foreign imports is $qP,$ and the amount of money China receives in dollars for its exports is $\varepsilon x = \varepsilon(C - c)$.

2.4 Equivalence of Devaluations and Tariffs

In his *Committee on Finance & Industry Report* (1931, p. 199), apparently Keynes had thought that a sterling devaluation does not affect the relative prices of traded goods, because he said the same effect could be achieved by a tariff of the same percentage on all imports together with an equal subsidy on all exports, which does not affect the relative prices of importable or the exportable. It is apparent that despite the change in the price of gold, Kenyes thought that a sterling devaluation can keep the ratio of the price of the importable to that of the exportable unchanged. However, "by diminishing by 10 per cent. the gold parity of sterling" Keynes was necessarily changing the relative price of gold, whether gold is an importable, an exportable or a nontradable good.

We first consider the benchmark case where the tariff rate is zero and the exchange rate is at equilibrium, i.e., $\varepsilon^o = 1$, so that trade surplus in dollars is zero, $s = \varepsilon^o x - P^* q = 0$. Assume now that the yuan is devalued below the equilibrium exchange rate, $\varepsilon = 1 - \delta < 1$, where δ is the devaluation rate from the benchmark equilibrium. For example, $\delta = .1$ represents a 10 percent devaluation from the equilibrium rate. The associated yuan price of the importable is

$$P^\delta = \frac{P^*}{\varepsilon} = \frac{P^*}{1-\delta}, \tag{6}$$

which implies a yuan devaluation (a decrease in ε) raises the domestic price P^{δ} of the importable. Moreover, the relative price of good *C* in the United States is:

$$\frac{b*(\varepsilon)}{P*} = \frac{b/\varepsilon}{P(\varepsilon)/\varepsilon} = \frac{b}{P(\varepsilon)}.$$

That is, the relative price of good *C* is the same in both countries, but it can be manipulated by a currency devaluation.

Tariff Equivalent of a Yuan Devaluation

In a seminal paper Bhagwati (1965, p. 53) defined the tariff equivalent of an import quota "in the sense that a tariff rate will produce an import level which, if alternatively set as a quota, will produce an identical discrepancy between foreign and domestic prices." Thus, the tariff equivalent of an import quota yields the same domestic price and import volume. Likewise, the tariff equivalent of a devaluation is defined as the tariff which yields the same domestic price and import volume.

We now show that tariffs and devaluations are equivalent in the sense of Bhagwati. Let P^{t} denote the yuan price of the importable associated with a tariff. Given the benchmark exchange rate ($\varepsilon^{o} = 1$), domestic price of the importable is

$$P^{t} = P*(1+t). \qquad (7)$$

The yuan-dollar exchange rate when the yuan is devalued from the benchmark rate $\varepsilon^o = 1$ is $\varepsilon = 1 - \delta$. A tariff rate *t* is equivalent to a $100 \times \delta$ percent devaluation, if the same domestic price is achieved through the tariff, i.e.,

$$P^t = P^\delta = \frac{P*}{\varepsilon}. \qquad (8)$$

Thus, the tariff equivalent of a given exchange rate ε is:

$$t = \frac{1-\varepsilon}{\varepsilon}. \qquad (9)$$

For instance, a 10 percent devaluation is equivalent to an 11 percent tariff, i.e., $t = .1/.9 = 11.11\%$.

Effects on Production and Consumer Income

When a tariff is imposed and $\varepsilon = 1$, the supply of the importable is: $Z(b, P^t) = Z\left(b, P*(1+t)\right)$, where P^t is the yuan price of good Z when an import tariff t is imposed. When a devaluation occurs, $Z(b, P^\delta) = Z(b, P*/\varepsilon)$, where P^δ is the yuan price of Z when the rate of yuan devaluation is δ. Recall that the supply of the importable, $Z(b, P)$, is monotone increasing in *P*. Thus, $Z\left(b, P*(1+t)\right) = Z\left(b, P*/\varepsilon\right)$ if, and only if $P*(1+t) = P*/\varepsilon$, or $\varepsilon = \frac{1}{1+t}$. That is, the import volume of *Z* remains the same, whether a devaluation, $\delta = 1 - \varepsilon = \frac{t}{1+t}$, occurs, or its equivalent tariff is imposed. Likewise, under both regimes domestic production of the exportable is the same, $C(b,P^t)=C(b,P^\delta)$, as is the yuan value of China's income,

$$R^t \equiv bC(b,P^t)+P^tZ(b,P^t)=bC(b,P^\delta)+P^\delta Z(b,P^\delta)\equiv R^\delta.$$

Thus, when $\delta=\frac{t}{1+t}$, consumers have the same income, $I^t=I^\delta$, equal to producer revenue, $R^t=R^\delta$ under both regimes.

Effects on Import Volume

Recall from (5), the import demand function is $q(b,P,I)=z(b,P,I)-Z(b,P)$. Since all prices and income are the same under both regimes, consumer demands for the exportable and the importable are the same, $c(b,P^t,I^t)=c(b,P^\delta,I^\delta)$ and $\mathrm{z}(P^t,I^t)=\mathrm{z}(P^\delta,I^\delta)$. The import demand function with a tariff is given by $q^t=z(b,P^t,I^t)-Z(b,P^t)$. Likewise, the import demand function with the devalued yuan is $q^\delta=z(b,P^\delta,I^\delta)-Z(b,P^\delta)$. Thus, the import volumes are the same under both regimes, i.e., $q(b,P^t,I^t)=q(b,P^\delta,I^\delta)$. Since both t and δ yield the same domestic prices and the same volume of imports, there exists a tariff that is equivalent to any yuan devaluation, and vice versa.

Proposition 1: Suppose the government devalues the yuan below the benchmark equilibrium exchange rate, $\varepsilon^o=1$. Then the domestic price rises above the import price to $P^\delta=\frac{P*}{\varepsilon}$, and import volume falls to q^{δ}. The tariff equivalent, $t = \frac{1-\varepsilon}{\varepsilon}$, yields the same domestic price, $P^{t} = P^{\delta}$ and the same import volume, $q^{t} = q^{\delta}$.

The relationship between an exchange rate and its equivalent tariff in (9) can be written as:

$$t = \frac{\delta}{1-\delta}. \tag{10}$$

This shows that a yuan devaluation is more potent than an equal tariff. That is, a 10 percent yuan devaluation from the benchmark rate is more restrictive than an equal tariff rate in reducing the import volume. Alternatively, if an import tariff of 10 percent is used to restrict imports, the same volume of import can be achieved by less than a 10 percent devaluation of the yuan, since from (10), we have

$$\delta = \frac{t}{1+t} < t. \tag{11}$$

For instance, a 50 percent devaluation ($\delta = 1-\varepsilon = 0.5$) from the equilibrium exchange rate $\varepsilon^{o} = 1$ is equivalent to a 100 percent tariff, and domestic price rises 100 percent with the same volume of import in both regimes. Likewise, a 20 percent yuan devaluation is equivalent to a 25 percent tariff.

Tariff cum Devaluation

Next, consider the case where the yuan is devalued and a tariff is imposed simultaneously. In this case, domestic price rises to: $P = P^{*}\frac{1+t}{\varepsilon}$. We now show that a pairing (t, ε) of a tariff and an exchange rate is equivalent to a single tariff τ. Since both instruments yield the same domestic prices, the tariff equivalent of a joint tariff cum devaluation is defined by

$P^*(1+\tau) = P^*\frac{1+t}{\varepsilon}$, or $1+\tau = \frac{1+t}{\varepsilon}$. That is, an import tariff cum devaluation is equivalent to a single tariff τ. Notice that the single tariff exceeds the sum of the tariff rate and the rate of yuan devaluation,

$$\tau = \frac{t+\delta}{1-\delta} > t+\delta. \qquad (12)$$

Moreover, when both are used, the total effect on the import price is *superadditive*, i.e., a 10 percent devaluation plus a 10 percent tariff is equivalent to a single tariff of 22.22 percent.

2.5 Graphical Illustration

The effect of currency devaluation on imports is shown in Figure 2.1. Note that demand for the importable *z* depends on *P* and revenue *R*, which in turn depends on the domestic price *P*. Assume both the exportable and importable are normal goods. For the purpose of graphical illustration, we employ reduced form demand functions which incorporate the income effects. The reduced form demand functions for the importable and the exportable are: $z(b,P) \equiv z\left(b,P,R(b,P)\right)$ and $c(b,P) \equiv c\left(b,P,R(b,P)\right)$. An increase in *P* not only decreases the quantity of the importable demanded, but also raises producer revenue or income *R*, which partially offsets the direct effect. We assume that the direct effect is dominant, i.e., $\partial z(b,P)/\partial P < 0$.

The demand curve $z(b,P)$ and the supply curve $Z(b,P)$ are shown in Figure 2.1. Recall that the reduced form import demand $z(b,P)$ already includes the income effect of a rise in the domestic price on producer revenue. In the benchmark equilibrium, there is no tariff and the exchange rate is $\varepsilon^o = 1$ and the domestic price is $P = P^*$. Import volume is the gap *gd*. When the yuan is devalued, domestic price rises to P^*/ε and the import volume shrinks to the gap $a'h'$.

Figure 2.2 shows the demand and supply of the exportable. Recall that $\partial C / \partial P < 0$ and $\partial c / \partial P > 0$. An increase in *P* raises the quantity of the exportable demanded, and the consequent rise in consumer income further reinforces the direct effect. Thus, $\partial c(P) / \partial P > 0$, and $c(P)$ is positively sloped when plotting against the price of the importable. Also, $\partial C / \partial P < 0$ implies that the supply of the exportable falls with *P*.

In the benchmark equilibrium, there is no tariff, and the exchange rate is $\varepsilon^o = 1$, and the domestic price is $P = P^*$. Export volume is the gap between $C(P)$ and $c(P)$ at *P**, equal to *gd*. When the yuan is devalued, domestic price rises to P^*/ε, and the export volume shrinks to the gap $a'h'$.

2.6 A Numerical Example: The Case of a Cobb-Douglas Utility Function

Consumer preferences are represented by a Cobb-Douglas utility function

$$U(c,z) = c^{.25} z^{.25}, \tag{13}$$

and the production possibility frontier (PPF) is given by

$$a^2C^2 + \theta^2 Z^2 = K^2, \qquad (14)$$

where a, θ and K are parameters. Producers are assumed to maximize revenue $R = bC + PZ$, subject to the PPF. The Lagrangian function is: $\phi = bC + PZ + \lambda[K^2 - a^2C^2 - \theta^2 Z^2]$. The first order conditions yield the equilibrium condition, $\frac{b}{P} = \frac{a^2C}{\theta^2 Z}$, or

$$Z = \frac{a^2 P}{\theta^2 b} C. \qquad (15)$$

From equations (14) and (15), supplies of the exportable and importable are $C = \frac{K}{a}\frac{\theta b}{\sqrt{\theta^2 b^2 + a^2 P^2}}$ and $Z = \frac{K}{\theta}\frac{aP}{\sqrt{\theta^2 b^2 + a^2 P^2}}$. Since the PPF is concave, the supply of the importable good is positively sloped, $Z'(P) > 0$, and hence $C'(P) < 0$. Maximized producer revenue is:

$$R = \frac{K\sqrt{\theta^2 b^2 + a^2 P^2}}{a\theta}. \qquad (16)$$

The Case of Devaluation

When the yuan is devalued, the supplies of the tradables are $C^\delta = \frac{K}{a}\frac{\theta b}{\sqrt{\theta^2 b^2 + a^2 (P^\delta)^2}}$ and $Z^\delta = \frac{K}{\theta}\frac{aP^\delta}{\sqrt{\theta^2 b^2 + a^2 (P^\delta)^2}}$. Producer revenue is

$$I^\delta = R^\delta = \frac{K\sqrt{\theta^2 b^2 + a^2 (P^\delta)^2}}{a\theta}. \qquad (17)$$

Consumer demands are: $c^{\delta} = \frac{I^{\delta}}{2} = \frac{K\sqrt{\theta^2 b^2 + a^2 (P^{\delta})^2}}{2a\theta}$, and $z^{\delta} = \frac{I^{\delta}}{2P^{\delta}} = \frac{K\sqrt{\theta^2 b^2 + a^2 (P^{\delta})^2}}{2P^{\delta} a\theta}$.

The Case of Tariff

For any given tariff *t*, the supplies of the exportable and the importable are:

$C^t = \frac{K}{a}\frac{\theta b}{\sqrt{\theta^2 b^2 + a^2 (P^t)^2}}$, and $Z^t = \frac{K}{\theta}\frac{aP^t}{\sqrt{\theta^2 b^2 + a^2 (P^t)^2}}$. Producer revenue under a tariff is

$$I^t = R^t = \frac{K\sqrt{\theta^2 b^2 + a^2 (P^t)^2}}{a\theta}. \quad (18)$$

Domestic demands for the tradables are:

$c^t = \frac{I^t}{2} = \frac{K\sqrt{\theta^2 b^2 + a^2 (P^t)^2}}{2a\theta}$ and $z^t = \frac{I^t}{2P^t} = \frac{K\sqrt{\theta^2 b^2 + a^2 (P^t)^2}}{2P^t a\theta}$.

Equivalence of Yuan Devaluation and a Tariff

When an equivalent tariff is imposed, $P^t = (1+t)P^* = P^{\delta}$, and hence $C^t = C^{\delta}$, $Z^t = Z^{\delta}$ and consumer income is the same under both regimes, i.e., $I^t = I^{\delta}$. Clearly, if $P^t = P^{\delta}$ and $I^t = I^{\delta}$, then $c^t = c^{\delta}$, and $z^t = z^{\delta}$. Thus, the trade volumes are the same under both regimes, i.e., $q^t = q^{\delta}$ and $x^t = x^{\delta}$, as shown in Table 1.

2.7 Tariff Equivalents of China's Undervalued Yuan

In this section, we estimate the extent of undervaluation of China's yuan, using the trade and exchange rate data for the period 1994 -2015. While the yuan-dollar exchange rates are available in monthly data, the import and export statistics are quarterly data. Thus, the exchange rate of the last month of each quarter is used as the exchange rate of each quarter. China GDP data is obtained from the National Bureau of Statistics of China, while U.S. GDP data is from the Bureau of Labor Statistics. We also utilized the monthly data for Sino-US bilateral trade in goods from the US census (https://www.census.gov/foreign-trade/balance/c5700.html). China's monthly nominal exchange rates are obtained from the State Administration of Foreign Exchange (SAFE) of the People's Republic of China, while Consumer Price Index (CPI) for the United States and China are from the OECD (https://data.oecd.org/price/inflation-cpi.htm).

VAR (Vector Autoregressive) and VECM (Vector Error Correction Model) are most often used to find relationships with two or more endogenous variables. Even though variables are individually non-stationary, they may be cointegrated. For instance, export, income and exchange rate may have one cointegrating relationship. We employ Johansen's (1998) Vector Error Correction Model (VECM) to explore the long-term relationships and short-term dynamics among endogenous variables. The model shows that in the long run, endogenous variables converge to their cointegrated relations. The long run relationships are investigated in three steps. First, the order of integration of variables is determined by Augmented Dickey Fuller test (ADF). Second, if each variable is integrated in the same order, i.e., I(1), a sequence test (Johansen 1995) is performed to determine the cointegrating rank *r* which indicates the maximum possible number of cointegrated equations. We test the null hypothesis of $r = 0$ against $r \geq 1$ to determine whether there is one cointegrating relationship. If the hypothesis of $r = 0$ is rejected, there is no relationship among the variables and there is no need for VECM, in which case, a VAR model can be employed. Finally, if there exist cointegrated relationships, then Johansen's cointegration test is used to detect long term relationships between exports (imports) and exchange rate.

Consider China's import expenditure and export revenue functions, $X_t = g(GDP_t^*, P_t^*, \varepsilon_t)$ and $Q_t = f(GDP_t, P_t, \varepsilon_t)$, where GDP_t is China's gross domestic income (GDP), GDP_t^* is the United States' GDP, P* is the US CPI, P is Chinese CPI and ε_t is the dollar price of the yuan. This model is similar to Bahmani-Oskooee and Ardalani (2006) model which employed real effective exchange rate rather than CPI and nominal exchange rate.

The dollar values of exports (X) and imports (Q) are written as:

$$\ln X_t = \alpha_o + \alpha_1 \ln GDP_t^* + \alpha_2 \ln P_t^* + \alpha_3 \varepsilon_t + \omega_{Xt}, \qquad (19)$$

$$\ln Q_t = \beta_o + \beta_1 \ln GDP_t + \beta_2 \ln P_t + \beta_3 \varepsilon_t + \omega_{Qt}, \qquad (20)$$

where α's and β's are unknown parameters and ω_{Xt} and ω_{Qt} are error terms with zero mean and variances σ_X^2 and σ_Q^2, respectively. Our analysis suggests that α_2, and β_1 are positive while β_2 and β_3 are negative. However, α_1 may be negative or positive, depending on the characteristics of the exportable goods of China. If China's export is a normal (inferior) good, then α_1 is positive (negative).

We estimate the long-run relations between US-China bilateral trade and the exchange rate, using cointegration and error-correction modeling.

Unit Root Test

We examine the stochastic properties of the variables using Dickey and Fuller (1981) tests of unit roots. Unit root tests are based on estimating the following univariate models:

$$\Delta y_t = \alpha_0 + \alpha_1 y_{(t-1)} + \sum_{i=2}^{p} \beta_i y_{t-i+1} + \varepsilon_t \tag{21}$$

$$\Delta y_t = \alpha_0 + \alpha_1 y_{(t-1)} + \alpha_2 t + \sum_{i=2}^{p} \beta_i y_{t-i+1} + \varepsilon_t \tag{22}$$

where *y* is the variable to be tested for unit roots and *t* is the time trend. Equation (21) tests for unit roots around a constant, while equation (22) tests for unit roots around a constant and deterministic trend. The lag lengths, *p*, are chosen using Schwarz's information criterion. Under the null hypothesis, $H_o : \alpha_1 = 0$ implies that the variable has a unit root.

Cointegration Tests

Tests of cointegration under symmetric adjustment are based on the methodology developed by Johansen (1991), and Johansen and Juselius (1993). Johansen's method is to test the restrictions imposed by cointegration on the unrestricted VAR, involving the series. The mathematical form of a VAR is

$$y_t = A_o + A_1 y_{t-1} + \ldots + A_p y_{t-p} + Bx_t + \omega_t \tag{23}$$

where y_t is an *n*-vector of non-stationary I(1) variables, x_t is a *d*-vector of deterministic variables, $A_1, \ldots, A_p$ and *B* are matrices of coefficients to be estimated, and ε_t is a vector of innovations that may be contemporaneously correlated with each other. However, they are uncorrelated with their own lagged values and other right-hand side variables.

The corresponding vector error correction model can be written as,

$$\Delta y_t = B_o + B_1 \Delta y_{t-1} + \ldots + B_p \Delta y_{t-p} + B\Delta x_t + \pi y_{t-1} + v_t . \tag{24}$$

Granger's representation theorem asserts that if the coefficient matrix π has a reduced rank $r < n$, then there exist $n \times r$ matrices, α, and β, each with rank *r* such that $\pi = \alpha\beta'$ and $\beta' y_t$ are stationary. Here, *r* is the number of cointegrating relations, and each column of β is a cointegrating vector. For *n* endogenous non-stationary variables, there can be from (0) to (*n*-1) linearly independent, cointegrating relations. Cointegration implies the existence of stable relations among the variables in the model, and causality in at least one direction. The Error Correction Model (ECM) is used to test for the direction of causality in both short- and long-run (Engle and Granger, 1987). Thus, we examine causality between exports (or imports) and the nominal exchange rate using their corresponding ECM models. In particular, we are interested in the following two error-correction models associated with import demand and export supply function:

The export supply is given by:

$$\Delta \ln X_t = \alpha_1 + \lambda_1 ECT_{Xt} + \sum_{i=1}^{n} \theta_{1i}^{1} \Delta \ln X_{t-i} + \sum_{i=1}^{n} \theta_{1i}^{1} \Delta \ln GDP_{t-i}^{*} + \sum_{i=1}^{n} \theta_{2i}^{1} \Delta \ln P_{t-i}^{*} + \sum_{i=1}^{n} \theta_{3i}^{1} \Delta \ln \varepsilon_{t-i}, \tag{25}$$

and the import demand is written as:

$$\Delta \ln Q_t = \beta_1 + \lambda_2 ECT_{Xt} + \sum_{i=1}^{n} \theta_{1i}^{2} \Delta \ln Q_{t-i} + \sum_{i=1}^{n} \theta_{1i}^{2} \Delta \ln GDP_{t-1} + \sum_{i=1}^{n} \theta_{2i}^{2} \Delta \ln P_{t-1} + \sum_{i=1}^{n} \theta_{3i}^{2} \Delta \ln \varepsilon_{t-i}, \tag{26}$$

where *ECT* is the error correction term, derived from the long-run cointegration relationship and measures the magnitude of the past disequilibrium. In each equation, the change in the left-hand side

(LHS) variable is caused by past changes in the variable itself, past changes in other variables, as well as a fraction λ_2 of the previous equilibrium error. Given these specifications, the presence of short-run and long-run causality could be tested.

Empirical Results

The results of the unit root tests for series of Exports (*X*), Imports (*Q*), Chinese GDP (*GDP*), U.S. GDP (*GDP**), Chinese CPI (*P*), U.S. CPI (*P**) and the yuan value per dollar (ε) using ADF are reported in Table 2.2. The null hypothesis that the variables are non-stationary in level is not rejected across all variables. However, the null hypothesis of non-stationarity in first-difference of variables is rejected for all variables. Thus, all variables appear to be non-stationary in level and stationary after first differencing. Using the method of Paulsen (1984), the optimal lags of VEC model are 4 lags in the export supply model and 2 lags in the import supply model.

Table 2.3 and Table 2.4 show that the results of the Johansen Cointegration tests for the export supply and import demand. In both models, the trace and maximum eigenvalue tests show that the null hypothesis of the absence of cointegrating relation ($r = 0$) can be rejected at 5% level of significance. Similarly, the null hypothesis of the existence of at most one cointegrating relation ($r \leq 1$) cannot be rejected at 5% level of significance. In short, both tests suggest the existence of one cointegrating vectors driving the series with four common stochastic trends in the data. Thus, we can conclude that the variables in both models are cointegrated. That is, there is a long-run relationship among the export supply (or import demand), exchange rate, price or income. In Table 2.5, the long term cointegrating vector suggests that Chinese exports have causal relationships with U.S. income, U.S. domestic price and exchange rate. However, the VEC model does not support the long run relationships between Chinese imports and the exchange rate or China's domestic prices. These results show that the exchange rate has no effect on imports in the long run. Thus, there is no need to estimate the relationship between China's import demand and the exchange rate. The leading Chinese imports are electronic equipment, oil, and machinery, and demand for these industrial products tend to be price inelastic. Thus, only the coefficient of GDP is positive and significant at 1% confidence level.

The estimated long-run relationship in the export supply function is

$$\ln \hat{X} = -39.071 - 2.995 \ln \widehat{GDP}^* + 13.733 \ln \hat{P}^* + 4.136 \ln \hat{\varepsilon}. \qquad (27)$$

The coefficients of U.S. domestic price and the exchange rate are positive. This implies that as the prices of China's domestic goods rise and the yuan depreciates, China's export supply increases. However, the coefficient of U.S. GDP is negative. This indicates that China's exportable goods are inferior goods. That is, as U.S. GDP rises, U.S. imports of China's export goods fall. Moreover, the results show that the elasticities of Chinese exports with respect to U.S GDP, U.S. domestic price, and the exchange rate are all greater than unity. For instance, a 1-percent increase in the exchange rate and the domestic price level increases China's exports by 4.136% and 13.733%, respectively. Similarly, a 1-percent increase in U.S. GDP reduces China's exports by 2.995%.

All parameters are statistically significant at the 1-percent level. The estimated value of exports is given by $\hat{X}_t = e^{\hat{\alpha}_0} GDP_t^{*\hat{\alpha}_1} P_t^{*\hat{\alpha}_2} \varepsilon_t^{\hat{\alpha}_3}$, where $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, and $\hat{\alpha}_3$ are estimated parameters in equation (19), and $\mathrm{e} = 2.71828$ is the base of the natural logarithm. Since Chinese imports are mostly derived demands for industrial inputs, rather than consumption demands, we use China's actual import data Q, instead of estimating the import demand function. In other words, China's import demand is not responsive to changes in the exchange rates. Thus, the estimated equilibrium exchange rate $\hat{\varepsilon}_t^*$ at time t that yields bilateral trade balance is derived from the condition,[iii] $\hat{X}_t = \mathrm{e}^{\hat{\alpha}_0} GDP_t^{*\hat{\alpha}_1} P_t^{*\hat{\alpha}_2} \varepsilon_t^{\hat{\alpha}_3} = Q_t$, and is written as

$$\hat{\varepsilon}_t^* = \left(\frac{Q}{\mathrm{e}^{\hat{\alpha}_0} GDP_t^{*\hat{\alpha}_1} P_t^{*\hat{\alpha}_2}} \right)^{1/\hat{\alpha}_3}, \tag{28}$$

From the estimated equilibrium exchange rates in (28) and actual exchange rates, the rate of devaluation in period *t* is given by

$$\hat{\delta}_t = \left(\frac{\varepsilon_t - \hat{\varepsilon}_t}{\hat{\varepsilon}_t} \right), \tag{29}$$

and the *equivalent tariff rate* in period *t* is given by

$$\tau_t = \left(\frac{\hat{\delta}_t}{1 - \hat{\delta}_t} \right). \tag{30}$$

The quarterly equilibrium and actual exchange rates are shown in Table 2.6. For instance, in the fourth quarter of 1994, the actual exchange rate was 8.5, which indicates a 47.9 percent devaluation of the yuan below the equilibrium rate of 5.7. This undervaluation of the yuan is equivalent to an import tariff of 92.1 percent. The rate of devaluation ranged from 21.7 percent in the fourth quarter of 2013 to 63.9 percent in the first quarter of 2000. The corresponding tariff equivalent ranged from 27.8 percent to 176.8 percent during the same period. The average rate of devaluation from the equilibrium exchange rates was 45.1 percent, and the average of equivalent tariffs was 87.5 percent.

2.8 Concluding Remarks

While tariffs and non-tariff barriers are bound by various agreements in the WTO, there are no such agreements among countries on the bilateral exchange rates. The IMF tends to monitor only the exchange rates of developing countries that are beset by large balance of payments deficits. The WTO regulates only tariff and various non-tariff barriers, but has made little attempt to regulate the bilateral exchange rates because exchange rate practices are within the purview of the IMF.

This paper has shown that yuan devaluations are equivalent to import tariffs in a two-good, two-currency model of international trade and exchange rates. Even if no tariffs or quotas are employed, our analysis shows that a country can restrict imports by undervaluing its currency. Using the available yuan-dollar exchange rates and bilateral U.S.-China trade data, it is shown that during the period 1994 - 2015, China's yuan has been grossly undervalued. On average, the yuan was devalued by 45 percent, and the average of tariff equivalents of the undervalued yuan was 87.5 percent for the period 1994-2015.

The finding that tariffs and the undervalued yuan have the same effects on domestic prices and import volumes suggests that currency pegging can be viewed as a trade policy. Either the WTO or IMF could begin to consider regulation of currency practices of developing countries in the face of large trade imbalances between major trading countries.

2.9 References

Bahmani-Oskooee, M., and Z. Ardalani, 2006, “Exchange Rate Sensitivity of U.S. Trade Flows: Evidence from Industry Data,” *Southern Economic Journal* 72, 542–559.

Bergsten, C. F., 2007, “The Dollar and the Renminbi,” Statement before the Hearing on US Economic Relations with China: Strategies and Options on Exchange Rates and Market Access, Subcommittee on Security and International Trade and Finance, Committee on Banking, Housing and Urban Affairs, United States Senate, 23 May 2007.

Bergsten, C. F., 2010, “Correcting the Chinese exchange rate: An action plan,” Testimony before the Committee on Ways and Means, U.S. House of Representatives, Peterson Institute for International Economics, http://www.iie.com/publications/testimony/testimony.cfm?ResearchID=1523 (24.3.10), accessed November 25, 2014.

Bhagwati, Jagdish, 1965, “On the Equivalence of Tariffs and Quotas,” in R. E. Baldwin and others (eds.), *Trade, Growth, and the Balance of Payments- Essays in Honor of Gottfried Haberler*, Rand McNally and Company, Chicago.

Bhagwati, J., 1968, “More on the Equivalence of Tariffs and Quotas,” *American Economic Review* 58, 142-46.

Chang, G. H., and Shao, Q., 2004, “How much is the Chinese currency undervalued? A quantitative estimation,” *China Economic Review* 15, 366–71.

Chang, G. H., 2007, “Is the Chinese currency undervalued? Empirical evidence and policy implication,” *International Journal of Public Administration* 30, 137–148.

Cheung, Yin-Wong, 2012, “Exchange Rate Misalignment –The Case of the Chinese Renminbi,” *Handbook of Exchange Rates*, Jessica James, Ian W. Marsh and Lucio Sarno (eds.).

Choi, E. Kwan, and Hailong Jin, 2013, “Currency Intervention and Consumer Welfare in an Open Economy." *International Review of Economics and Finance 29, 47-56*

Coudert, Virginie and Cécile Couharde, 2007, "Real Equilibrium Exchange Rate in China: Is the Renminbi Undervalued?," *Journal of Asian Economics* 18, 568-594.

Engle, R.F. and Granger, C.W.J., 1987, "Co-integration and Error Correction: Representation, Estimation, and Testing", *Econometrica*, 55, 251-276.

Fernalda, John, Edison, Hali, and Lounganib, Prakash, 1999, "Was China the first domino? Assessing links between China and other Asian economies," *Journal of International Money and Finance*, 18, 515–535.

Funke, M., and Rahn, J., 2005, "Just how undervalued is the Chinese Renminbi?" *World Economy 28*, 465–489.

Garroway, Christopher, Burcu Hacibedel, Helmut Reisen, and Edouard Turkisch, 2012, "The Renminbi and Poor Country Growth," *World Economy* 35, 273-94

Johansen, S., 1988, "Statistical analysis of cointegration vectors" *Journal of Economic Dynamics and Control* 12, 231–254.

Johansen, S., 1995, "Likelihood-Based Inference in Cointegrated Vector Autoregressive Models," Oxford University Press.

Jin, Hailong, and E. Kwan Choi, 2013, "Profits and Losses from Currency Intervention," International Review of Economics and Finance 27, 14-20.

Jin, Hailong, and Choi, E. Kwan, 2014, "China's Profits and Losses from Currency Intervention, 1994-2011," Pacific Economic Review 19, 170-183.

Keynes, John Maynard, 1931, *Committee on Finance & Industry Report*, June 1931Addendum, Macmillan Report.

Kong, Qingjiang, 2012, "China's Currency Devaluation and WTO Issues," *China: An International Journal* 10, 110-118.

McKinnon, Ronald, and Gunther Schnabl, 2009, "China's Financial Conundrum and Global Imbalances," *China Economist* 21, 65-77.

Meyer, Claude, 2008, "Should China revalue its currency? Lessons from the Japanese experience," Policy Brief (April 2008), Groupe d'Économie Mondiale.

Paulsen, J., 1984, "Order determination of multivariate autoregressive time series with unit roots," *Journal of Time Series Analysis* 5, 115–127.

Wren-Lewis, S., 2004, The needed changes in bilateral equilibrium exchange rates. In C. F. Bergsten & J. Williamson (Eds.), *Dollar Adjustment: How Far? Against What?*. Institute of International Economics, Washington, DC.

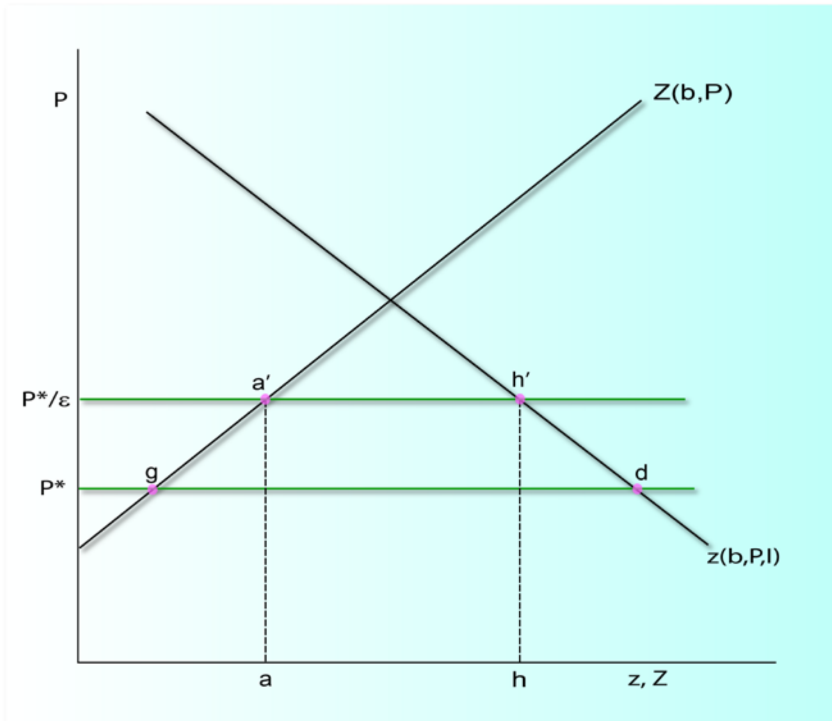

Figure 2.1. The Effect of Devaluation on Import Demand

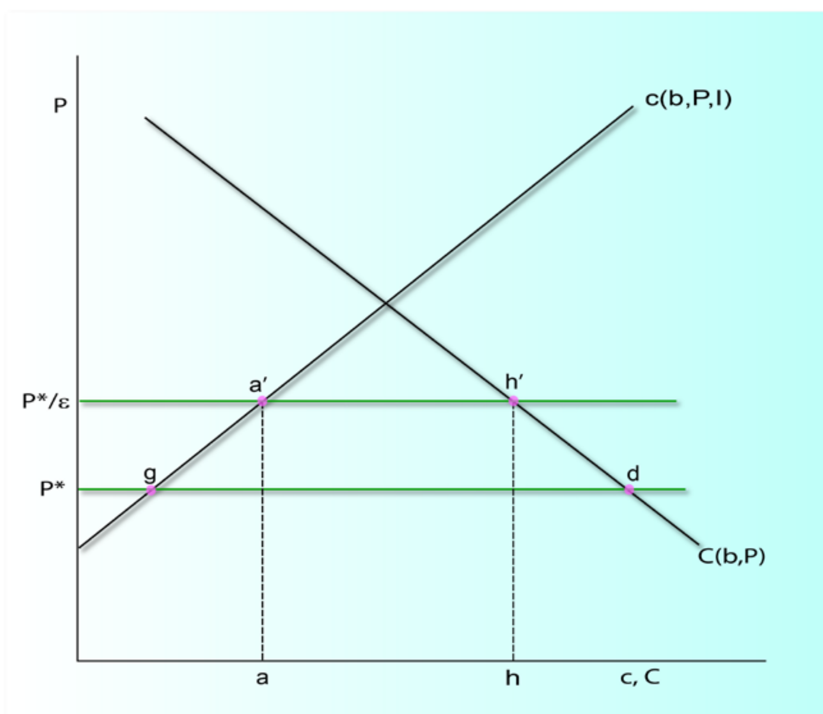

Figure 2.2. The Effect of Devaluation on Export Supply

Table 2.1. Summary of the equivalence of tariffs and currency undervaluation

$U(c,z)=c^{.25}z^{.25}$, $a^2C^2+\theta^2Z^2=K^2$, $b=1$, $P^*=.25$, $a=2$, $\theta=1$, $K=100$.

	Currency Undervaluation $\varepsilon=.8$ $(\delta=.2)$		Equivalent Tariff $t=(1-\varepsilon)/\varepsilon=.25$	
	General Case	Special Case	General Case	Special Case
Export Price	b	$b=1$	b	$b=1$
Import Price	$P^\delta=P^*/\varepsilon$	$P^\delta=5/16=.313$	$P^t=(1+t)P^*$	$P^t=5/16=.313$
Output C	$C^\delta=\frac{K}{a}\frac{\theta b}{\sqrt{\theta^2b^2+a^2(P^\delta)^2}}$	$C^\delta=\frac{50}{\sqrt{1+25/64}}$ $=\frac{50}{\sqrt{89/64}}=42.40$	$C^t=\frac{K}{a}\frac{\theta b}{\sqrt{\theta^2b^2+a^2(P^t)^2}}$	$C^t=\frac{50}{\sqrt{1+25/64}}$ $=\frac{50}{\sqrt{89/64}}=42.40$
Output Z	$Z^\delta=\frac{K}{\delta}\frac{aP^\delta}{\sqrt{\theta^2b^2+a^2(P^\delta)^2}}$	$Z^\delta=\frac{100}{1}\frac{5/8}{\sqrt{1+25/64}}$ $=\frac{125/2}{\sqrt{89/64}}=53$	$Z^t=\frac{K}{\theta}\frac{aP^t}{\sqrt{\theta^2b^2+a^2(P^t)^2}}$	$Z^t=\frac{100}{1}\frac{5/8}{\sqrt{1+25/64}}$ $=\frac{125/2}{\sqrt{89/64}}=53$
GDP R	$R^\delta=bC^\delta+P^\delta Z^\delta$	$R^\delta=42.40+(5/16)53=58.96$	$R^t=bC^t+P^tZ^t$	$R^t=42.40+(5/16)53=58.96$
Expenditure I	$I^\delta=R^\delta$	$I^\delta=58.96$	$I^t=R^t$	$I^t=58.96$
Consumption c	$c^\delta=\frac{I^\delta}{2b}$	$c^\delta=29.48$	$c^t=\frac{I^t}{2b}$	$c^t=29.48$
Consumption z	$z^\delta=\frac{I^\delta}{2P^\delta}$	$z^\delta=94.34$	$z^t=\frac{I^t}{2P^t}$	$z^t=94.34$

Table 2.2. Unit root test results

Variable	*X*	*Q*	*GDP*	*GDP**	*P*	*P**	ε
No time trend, No lag First-Difference	-9.24 *	-13.17*	-12.47*	-5.88*	-6.28*	-7.88*	-4.51*

Note: *indicates significance at 1%

Table 2.3. Johansen test for Cointegration (Export Supply)

Series: ln*X* ln*GDP** ln*P** lnε

Maximum Rank	LL	eigenvalue	Trace Statistic	5% critical Value
0	1068.35	.	52.427	47.21
1	1083.97	0.314	21.194*	29.68
2	1088.91	0.112	11.301	15.41
3	1093.18	0.098	2.769	3.76
4	1094.56	0.033		

Table 2.4. Johansen test for Cointegration (Import Demand)

Series: ln*Q* ln*GDP* ln*P* lnε

Maximum Rank	LL	eigenvalue	Trace Statistic	5% critical Value
0	634.37	.	50.27	47.21
1	649.59	0.30	19.8249*	29.68
2	654.97	0.12	9.08	15.41
3	659.34	0.10	0.32	3.76
4	659.50	0.00		

Table 2.5. Parameter Estimates of Import and Export Functions

	Constant	lnGDP*	ln GDP	lnP*	lnP	ln ε
ln(Export)	-39.071	-2.995 (0.990) *	-	13.733 (2.179)*	-	4.136 (0.595) *
ln(Import)	-5.059	-	0.820 (0.245)*	-	-	0.360 (1.939)

Note: *indicates significance at 1%

Table 2.6. Tariff Equivalents of Devalued Yuan

Time	ε	$\hat{\varepsilon}_t^*$	$\hat{\delta}_t = \left(\frac{\varepsilon_t - \hat{\varepsilon}_t^*}{\hat{\varepsilon}_t^*} \right)$	$\hat{\tau}_t = \left(\frac{\hat{\delta}_t}{1 - \hat{\delta}_t} \right)$
1994 Q1	8.724	5.771	0.512	1.048
1994 Q2	8.684	6.095	0.425	0.738
1994 Q3	8.558	6.431	0.331	0.494
1994 Q4	8.503	5.748	0.479	0.921
1995 Q1	8.448	5.889	0.435	0.768
1995 Q2	8.321	6.016	0.383	0.621
1995 Q3	8.337	6.125	0.361	0.565
1995 Q4	8.335	6.241	0.335	0.505
1996 Q1	8.350	5.868	0.423	0.733
1996 Q2	8.342	5.630	0.482	0.930
1996 Q3	8.334	5.815	0.433	0.764
1996 Q4	8.329	5.642	0.476	0.910
1997 Q1	8.326	5.588	0.490	0.960
1997 Q2	8.322	5.624	0.480	0.922
1997 Q3	8.317	5.794	0.436	0.772
1997 Q4	8.310	6.155	0.350	0.539
1998 Q1	8.308	5.970	0.392	0.644
1998 Q2	8.310	5.561	0.494	0.977
1998 Q3	8.306	5.708	0.455	0.835
1998 Q4	8.278	6.493	0.275	0.379
1999 Q1	8.279	5.250	0.577	1.365
1999 Q2	8.278	5.487	0.509	1.035
1999 Q3	8.277	5.487	0.508	1.034
1999 Q4	8.279	5.456	0.518	1.073
2000 Q1	8.279	5.052	0.639	1.768
2000 Q2	8.277	5.406	0.531	1.132
2000 Q3	8.279	5.682	0.457	0.842
2000 Q4	8.277	5.492	0.507	1.028
2001 Q1	8.278	5.052	0.638	1.766
2001 Q2	8.277	5.124	0.615	1.601
2001 Q3	8.277	5.177	0.599	1.491
2001 Q4	8.276	5.380	0.538	1.166
2002 Q1	8.277	5.302	0.561	1.279
2002 Q2	8.277	5.129	0.614	1.588
2002 Q3	8.277	5.317	0.557	1.256
2002 Q4	8.278	5.363	0.544	1.191
2003 Q1	8.277	5.297	0.563	1.287

Table 2.6 continued

2003 Q2	8.277	5.312	0.558	1.263
2003 Q3	8.277	5.278	0.568	1.317
2003 Q4	8.277	5.733	0.444	0.798
2004 Q1	8.277	5.544	0.493	0.973
2004 Q2	8.277	5.405	0.531	1.134
2004 Q3	8.277	5.345	0.548	1.215
2004 Q4	8.277	5.453	0.518	1.074
2005 Q1	8.277	5.261	0.573	1.343
2005 Q2	8.277	5.362	0.544	1.191
2005 Q3	8.092	5.328	0.519	1.078
2005 Q4	8.076	5.407	0.493	0.974
2006 Q1	8.035	5.212	0.542	1.182
2006 Q2	8.004	5.240	0.528	1.117
2006 Q3	7.933	5.334	0.487	0.950
2006 Q4	7.822	5.555	0.408	0.689
2007 Q1	7.737	5.279	0.466	0.871
2007 Q2	7.633	5.088	0.500	1.001
2007 Q3	7.521	5.069	0.484	0.937
2007 Q4	7.368	5.189	0.420	0.724
2008 Q1	7.072	4.999	0.415	0.709
2008 Q2	6.899	4.704	0.467	0.876
2008 Q3	6.831	4.642	0.472	0.892
2008 Q4	6.854	5.003	0.370	0.587
2009 Q1	6.836	4.601	0.486	0.944
2009 Q2	6.833	4.671	0.463	0.862
2009 Q3	6.828	4.600	0.484	0.939
2009 Q4	6.828	4.918	0.388	0.634
2010 Q1	6.826	4.884	0.398	0.660
2010 Q2	6.818	4.806	0.419	0.721
2010 Q3	6.740	4.956	0.360	0.562
2010 Q4	6.650	5.255	0.265	0.361
2011 Q1	6.565	4.850	0.353	0.547
2011 Q2	6.475	4.626	0.400	0.666
2011 Q3	6.389	4.612	0.385	0.626
2011 Q4	6.348	4.884	0.300	0.428
2012 Q1	6.313	4.623	0.365	0.576
2012 Q2	6.363	4.543	0.401	0.669
2012 Q3	6.320	4.537	0.393	0.647
2012 Q4	6.233	4.817	0.294	0.416
2013 Q1	6.215	4.593	0.353	0.546

Table 2.6 continued

2013 Q2	6.134	4.477	0.370	0.588
2013 Q3	6.120	4.432	0.381	0.615
2013 Q4	6.074	4.989	0.217	0.278
2014 Q1	6.0509	4.588	0.319	0.468
2014 Q2	6.2246	4.329	0.438	0.779
2014 Q3	6.1984	4.389	0.412	0.702
2014 Q4	6.1251	4.874	0.257	0.345
2015 Q1	6.2181	4.645	0.339	0.512
2015 Q2	6.2010	4.485	0.382	0.619
2015 Q3	6.2085	4.494	0.381	0.617

CHAPTER 3. UNEMPLOYMENT AND OPTIMAL EXCHANGE RATE IN AN OPEN ECONOMY

3.1 Abstract

China has been criticized for adopting a low yuan policy to take unfair advantage of its trading partners. This paper considers the optimal exchange rate policy of a Keynesian open economy with unemployed resources. In the case of Cobb-Douglas utility and production functions, indirect utility is monotone-increasing and concave in the exchange rate. Yuan devaluation is shown to reduce unemployment. Moreover, the optimal exchange rate is one which guarantees full employment. The United States may want to choose a different rate which ensures full employment. The two countries could negotiate an intermediate exchange rate for which some unemployment exists in both countries.

3.2. Introduction

Due to its mounting currency reserves since the 1990s, China's exchange rate policy has been under intense scrutiny. According to the State Administration of Foreign Exchange of People's Bank of China (PBC), China's foreign exchange reserve was $22 billion in 1993. China's foreign exchange reserve has since increased steadily, to $3.1 trillion in October 2016. Such a dramatic rise in China's cumulative trade surplus has provoked much debate concerning China's currency valuation.

Most major currencies except the renminbi are freely floating vis-à-vis other currencies, except the renminbi. It is argued that China may be deliberately depressing the yuan in the hope of stimulating domestic production. In the celebrated Mundell (1963)-Fleming(1964) model, currency devaluation influences a country's balance of payments, thereby affecting production and unemployment. In a study of ten countries, Gylfason and Schmid (1983) show that devaluation has positive output effects.

In an open economy, the government may be more interested in the output effects of currency devaluation. Helpman (1976) considered a single-period framework with a nontraded good and showed that devaluation increases employment, while Cuddington (1981) investigated the contemporaneous effect of devaluation. More recently, Batra and Beladi (2013) suggest that both China and Japan kept their currency values low relative to those of other nations such as the United States and Europe in order to maintain unemployment below a target rate.[iv] Jin and Choi (2013) noted that while some profits might be generated in the short run by slightly deviating from the equilibrium exchange rates, excessive hoarding of reserve assets in the long run can only result in losses to PBC's balance of payment account. Jin et al (2016) showed that in a two-period model nonintervention is the optimal exchange rate policy. However, the prevailing view is that China has intentionally depressed the value of the yuan to gain unfair advantages in the global market. (Cheung et al., 2009; Cheung, 2012)

In a developing country like China, the goal of keeping the unemployment rate low might take precedence over other economic issues. Reducing unemployment may be the principal motive for adopting the low yuan policy. For instance, Overholt (2010) argues that yuan appreciation would increase China's unemployment. Developing countries often encourage trade surpluses to prepare for future contingencies. Goldstein and Lardy (2006) suggest that China's undervaluation of the renminbi contributed to growing trade surpluses. Also, China wants the renminbi to be an international reserve currency, but the Chinese government is reluctant to make the yuan fully convertible.

This paper investigates the optimal exchange rate for a Keynesian open economy.[v] Open-economy macroeconomic models are predominantly based on an economy producing a single homogeneous good.[vi] Frenkel and Ros (2006) developed an unemployment model in which countries produce a nontraded good and a traded good to analyze the effect of exchanges on unemployment. Vasylenko and Vasylenko (2005) first considered the conditions for trade balance stabilization with two traded sectors. The present paper's main contribution is to analyze the effect of currency devaluation on unemployment and welfare in an open economy which produces two tradable goods. Currency devaluation changes the relative price of the exportable. Trade is balanced and hence no currency misalignment occurs.[vii] Using the Cobb-Douglas utility and production functions, we show that under certain conditions, the exchange rate which guarantees full employment is the optimal policy.

Section 3.2 introduces the basic two-sector, two-country model with unemployment. Section 3.3 investigates the effect of yuan devaluation on exchange rate pass-through into the yuan price of China's exportable good. Section 3.4 considers the effect of yuan devaluation on income and welfare, while Section 3.5 explores the effect on unemployment. Section 3.6 offers a numerical example to illustrate the main propositions. Section 3.7 provides concluding remarks.

3.3. The Two-Sector Keynesian Model with Unemployment

In this section we consider a Keynesian open economy model with two goods to consider China's optimal exchange rate policy. Let China's importable good *Z* be the numéraire, i.e., its dollar price $P^* = 1$, and let δ denote the yuan price of the dollar. Exchange rate pass-through into the import price is perfect, [viii] and the yuan price of the importable is P. An increase in δ represents an increase in the yuan price of the dollar, and hence a yuan depreciation. We assume that the dollar price of the importable good *P** is fixed in the importing country, and its yuan price is $P = P^*\delta$, where δ is the yuan price of the dollar.

Each country is assumed to fix the price of its exportable in terms of its own currency. That is, the yuan price of good *C*, which China exports, is *b*, while its dollar price is denoted by b^*. Likewise, the dollar price *P** of good *Z* is fixed, equal to unity. The yuan price of good *Z* is denoted by *P*. The relative price of good *Z* in China can be written as: $\frac{P}{b} = \frac{P^*\delta}{b} = \frac{P^*}{b/\delta} = \frac{P^*}{b^*}$. That is, the relative price of good *Z* is the same in both countries, regardless of the exchange rate. However, a change in the exchange rate may affect the relative price of good *Z*.

Assumptions

We now consider a two-sector Keynesian model of two countries producing two goods, *C* and *Z*. Unemployment exists in both the capital and labor markets. The wage rate *w* and capital rental *r* are assumed to be fixed in the short run. As a basis for analyzing the effects of yuan devaluation, we employ the following assumptions:

(i) Two factors, capital *K* and labor *L*, are used to produce two goods, *C* and *Z*. China is assumed to export *C* and import *Z*.[ix]

(ii) The dollar price of good *Z* is normalized, i.e., $P^* = 1$.

(iii) The Chinese government pegs the yuan to the dollar, and the yuan price of the importable is $P = P^*\delta = \delta$.

(iv) Cobb-Douglas production functions are used in both industries.

(v) Unemployment exists in both capital and labor markets.

(vi) Consumer preferences are represented by a Cobb-Douglas utility function in both countries.

China

China produces two goods, using two factors: capital (*K*) and labor (*L*) inputs. Domestic outputs of the traded goods are given by $C = F(L_C, K_C) = A_C L_C^{\alpha_1} K_C^{\beta_1}$, and $Z = G(L_Z, K_Z) = A_Z L_Z^{\alpha_2} K_Z^{\beta_2}$, where L_j and K_j denote the amounts of labor and capital inputs employed in sector j = *C*, *Z*. Both production functions are assumed to exhibit decreasing returns to scale (DRS), i.e., $\alpha_1 + \beta_1 < 1$, and $\alpha_2 + \beta_2 < 1$. DRS implies that the production functions $F(.)$ and $G(.)$ are monotone-increasing and concave.[x]

China's Supplies of Tradable Goods

Since labor and capital inputs are not fully employed and are immobile internationally, $w \neq w^*$ and $r \neq r^*$. Let Π_C and Π_Z denote the profits of industries, *C* and *Z*, respectively. Total profit of the Chinese economy in yuan is

$$\Pi = \Pi_C + \Pi_Z = bA_C L_C^{\alpha_1} K_C^{\beta_1} + PA_Z L_Z^{\alpha_2} K_Z^{\beta_2} - wL_C - rK_C - wL_Z - rK_Z, \qquad (31)$$

where *b* and *P* are the yuan prices of goods *C* and *Z*, L_i, and K_i are input demands of labor and capital in sector *i* = *C*, *Z*. Let $\bar{L}$ and $\bar{K}$ denote China's labor and capital endowments. The central planner's problem is to choose L_C, L_Z, K_C, and K_Z subject to $L_C + L_Z < \bar{L}$, $K_C + K_Z < \bar{K}$. Due to unemployment, a production mix of *C* and *Z* does not occur along a production possibility frontier (PPF). The first order conditions are

$$\begin{aligned} &b\alpha_1 A_C L_C^{\alpha_1-1} K_C^{\beta_1} = w,\ \ b\beta_1 A_C L_C^{\alpha_1} K_C^{\beta_1-1} = r, \\ &P\alpha_2 A_Z L_Z^{\alpha_2-1} K_Z^{\beta_2} = w,\ \ P\beta_2 A_Z L_Z^{\alpha_2} K_Z^{\beta_2-1} = r. \end{aligned} \qquad (32)$$

The input demands for labor and capital in the production of the two goods are as follows:

$$\begin{aligned} &L_C = \left(\frac{b\alpha_1^{\,1-\beta_1} \beta_1^{\,\beta_1} A_C}{w^{1-\beta_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}, K_C = \left(\frac{b\alpha_1^{\,\alpha_1} \beta_1^{\,1-\alpha_1} A_C}{w^{\alpha_1} r^{1-\alpha_1}} \right)^{1/(1-\alpha_1-\beta_1)}, \\ &L_Z = \left(\frac{P\alpha_2^{\,1-\beta_2} \beta_2^{\,\beta_2} A_Z}{w^{1-\beta_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}, K_Z = \left(\frac{P\alpha_2^{\,\alpha_2} \beta_2^{\,1-\alpha_2} A_Z}{w^{\alpha_2} r^{1-\alpha_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \qquad (33)$$

The optimal supplies of goods, *C* and *Z*, are functions of factor prices, *w* and *r*:

$$\begin{aligned} &C(b,P) = A_C L_C^{\alpha_1} K_C^{\beta_1} = \left(\frac{b^{\alpha_1+\beta_1} \alpha_1^{\,\alpha_1} \beta_1^{\,\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}, \\ &Z(b,P) = A_Z L_Z^{\alpha_2} K_Z^{\beta_2} = \left(\frac{P^{\alpha_2+\beta_2} \alpha_2^{\,\alpha_2} \beta_2^{\,\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \qquad (34)$$

China's Demands for Tradable Goods

The preferences of Chinese consumers are represented by a Cobb-Douglas utility function,

$$U(c,z) = c^{\gamma} z^{1-\gamma}, \tag{35}$$

where c and z are China's consumption of the exportable and importable, respectively.[xi] The equilibrium condition for optimal consumption is:

$$\frac{U_c}{U_z} = \frac{b}{P}, \tag{36}$$

where b and P are the yuan prices of exportable good C and importable good Z, respectively.

Thus, consumer demands for the two goods are written as: $c = \frac{\gamma I}{b}$ and $z = \frac{(1-\gamma)I}{P}$,

where I is China's income in yuan. Since both factors are unemployed and the total profit is distributed to consumers, the total income of China is

$I = w(L_C + L_Z) + r(K_C + K_Z) + \Pi = bC + PZ.$ The budget constraint in yuan is given by:

$$bc + Pz = I. \tag{37}$$

China's national income is

$$I = bC + PZ = \left(\frac{b\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{P\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}. \tag{38}$$

Suppose China lends S dollars to the United States. Then China's expenditure in yuan is

$$I = \left(\frac{b\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{P\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \qquad (39)$$

China's consumer demands for two tradable goods are written as:

$$c = \gamma \frac{\delta}{b} \left(\frac{b^{\frac{1}{1-\alpha_1-\beta_1}}}{\delta} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} \right),$$

$$z = (1-\gamma) \left(\frac{b^{\frac{1}{1-\alpha_1-\beta_1}}}{\delta} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} \right). \qquad (40)$$

United States

The United States also is assumed to produce the two goods, using two factors, capital (*K*) and labor (*L*) inputs. Since both inputs are unemployed, production does not occur on the PPF. Recall that the United States and China use the same technologies in the production of two goods, *C* and *Z*. U.S. outputs of the tradable goods are given by $C^* = F(L_C^*, K_C^*) = A_C L_C^{*\alpha_1} K_C^{*\beta_1}$, and $Z^* = G(L_Z^*, K_Z^*) = A_Z L_Z^{*\alpha_2} K_Z^{*\beta_2}$, where L_j^* and K_j^* denote the labor and capital inputs employed in sector $j = C^*, Z^*$.

U.S. Supplies of Tradable Goods

Let $\Pi_{C^*}^*$ and $\Pi_{Z^*}^*$ denote U.S. profits of industries *C** and *Z**. The total U.S. profit in dollars is:

$$\Pi^* = \Pi_{C^*}^* + \Pi_{Z^*}^* = b^* A_C L_C^{*\alpha_1} K_C^{*\beta_1} + P^* A_Z L_Z^{*\alpha_2} K_Z^{*\beta_2} - w^*(L_{C^*}^* + L_{Z^*}^*) - r^*(K_{C^*}^* + K_{Z^*}^*), \quad (41)$$

where b^* and $P^* = 1$ are dollar prices of good C and Z, and L_i^* and K_i^* are input demands of labor and capital in sector $i = C^*, Z^*$. Let $\bar{L}^*$ and $\bar{K}^*$ denote the U.S. endowments of labor and capital inputs. The central planner's problem is to choose L_C^*, L_Z^*, K_C^*, and K_Z^* to maximize the total profit in (41) subject to $L_C^* + L_Z^* < \bar{L}^*$, $K_C^* + K_Z^* < \bar{K}^*$.

The first order conditions are

$$\begin{aligned} &b^*\alpha_1 A_C L_C^{*\alpha_1 - 1} K_C^{*\beta_1} = w^*,\ b^*\beta_1 A_C L_C^{*\alpha_1} K_C^{*\beta_1 - 1} = r^*, \\ &P^*\alpha_2 A_Z L_Z^{*\alpha_2 - 1} K_Z^{*\beta_2} = w^*,\ P^*\beta_2 A_Z L_Z^{*\alpha_2} K_Z^{*\beta_2 - 1} = r^*. \end{aligned} \tag{42}$$

U.S. input demands are given by

$$\begin{aligned} L_C^* &= \left(\frac{b^*\alpha_1^{1-\beta_1}\beta_1^{\beta_1} A_C}{w^{*1-\beta_1}\ r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}, K_C^* = \left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{1-\alpha_1} A_C}{w^{*\alpha_1}\ r^{*1-\alpha_1}} \right)^{1/(1-\alpha_1-\beta_1)}, \\ L_Z^* &= \left(\frac{P^*\alpha_2^{1-\beta_2}\beta_2^{\beta_2} A_Z}{w^{*1-\beta_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}, K_Z^* = \left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{1-\alpha_2} A_Z}{w^{*\alpha_2}\ r^{*1-\alpha_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \tag{43}$$

The optimal supplies are:

$$\begin{aligned} C^*(b^*, P^*) &= A_C L_C^{*\alpha_1} K_C^{*\beta_1} = \left(\frac{b^{*\alpha_1+\beta_1}\ \alpha_1^{\alpha_1}\beta_1^{\beta_1} A_C}{w^{*\alpha_1}\ r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}, \\ Z^*(b^*, P^*) &= A_Z L_Z^{*\alpha_2} K_Z^{*\beta_2} = \left(\frac{P^{*\alpha_2+\beta_2}\ \alpha_2^{\alpha_2}\beta_2^{\beta_2} A_Z}{w^{*\alpha_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \tag{44}$$

U.S. Demands for Tradable Goods

Preferences of American consumers are represented by a utility function, $U(c^*, z^*) = (c^*)^{\gamma}(z^*)^{1-\gamma}$, where c^* and z^* are the U.S. demands for C and Z, respectively. The national income of the United States is

$I^* = w^*(L_C^* + L_Z^*) + r^*(K_C^* + K_Z^*) + \Pi^* = b^*C^* + P^*Z^*$. The U.S. expenditure in dollars is given by:

$$b^*c^* + P^*z^* = I^* = b^*C^* + P^*Z^*, \tag{45}$$

where b^* and P^* are the dollar prices of C^* and Z^*, respectively.

Revenue from production is

$$I^* = b^*C^* + P^*Z^* = \left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\, r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\, r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}. \tag{46}$$

The total U.S. expenditure of the United States is

$$I^* = \left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\, r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\, r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}. \tag{47}$$

The equilibrium condition for optimal consumption is:

$$\frac{U_{c^*}}{U_{z^*}} = \frac{b^*}{P^*}. \tag{48}$$

U.S. consumer demands satisfying the equilibrium condition in (48) and the budget constraint in (46) are written as:

$$\begin{aligned} c^* &= \frac{\gamma}{b^*}\left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\, r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \frac{\gamma}{b^*}\left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\, r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}, \\ z^* &= \frac{1-\gamma}{P^*}\left(\frac{b^*\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\, r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \frac{1-\gamma}{P^*}\left(\frac{P^*\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\, r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}, \end{aligned} \tag{49}$$

where w^* and r^* are the wage rate and capital rent, respectively.

World Market Equilibrium and Trade Balance

Recall that the relative price of the exportable is $b/P = b/\delta.$ A competitive equilibrium is attained when producers and consumers in both markets behave as price takers. Recall that when unemployment exists in the labor market, any increases in $\bar{K}$ or $\bar{L}$ have no effect on China's domestic outputs. Note that due to unemployment the supply of each good depends on its own price, but not on the price of the other good. Thus, the supply functions are written as: $C = C(b)$ and $Z = Z(P).$

The market clearing condition for good *C* is written as:

$$c(b,\delta,I) + c^*(b,\delta,I^*) = C(b,\delta) + C^*(b,\delta). \tag{50}$$

Despite unemployment in the factor markets, a Keynesian equilibrium exists when aggregate demand equals aggregate supply in each sector. The labor market imperfection only causes labor unemployment, but does not preclude the working of Walras' Law in the output markets. Thus, all outputs produced are sold at the equilibrium prices. Walras' Law suggests that if the market for good *C* is in equilibrium, the market for the other output, *Z*, also is in equilibrium, i.e.,

$$z(b,\delta,I) + z^*(b,\delta,I^*) = Z(b,\delta) + Z^*(b,\delta). \tag{51}$$

Thus, there exists a unique value of δ which clears the market for good *C* in (50). Note that when capital is not fully utilized, $\partial Z/\partial K = 0$. Likewise, when labor unemployment exists, $\partial Z/\partial L = 0.$ The same conditions hold if capital and labor are not fully employed in the United States, i.e., $\partial Z^*/\partial K^* = \partial Z^*/\partial L^* = 0.$

Exchange Rate Pass-Through

Let (b, δ) be a pair of yuan prices at the competitive benchmark equilibrium satisfying (50).[xii] Then by Walras' law, the price pair (b, δ) also satisfies (51). That is, if the market for good *C* clears for given prices (b, δ), the other product market (i.e., for good *Z*) also clears. Substituting (34), (40), (44) and (49) into (50), we obtain the world market clearing condition for good *C*:

$$b^{1/(1-\alpha_1-\beta_1)}\left(\frac{b\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + b^{1/(1-\alpha_1-\beta_1)}\left(\frac{(1/\delta)^{\alpha_1+\beta_1}\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\ r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}$$
$$= \delta\frac{\gamma}{(1-\gamma)}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\ r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} + \delta^{\frac{1}{1-\alpha_2-\beta_2}}\frac{\gamma}{(1-\gamma)}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}. \quad (52)$$

Since good *Z* is the numéraire, $P^* = 1$, we have $P = \delta$, and $b = b^*\delta$. Since the dollar price of the numéraire is fixed, its yuan price bears the full burden of adjustment and exchange rate pass-through into import price is complete. Recall that China and the United States have identical technologies, and in the absence of trade barriers, output prices are equalized. The equilibrium yuan price of good *C* is:[xiii]

$$b(\delta) = \frac{\left(\dfrac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}\left(\delta^{\frac{1}{1-\alpha_2-\beta_2}}\left(\dfrac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} + \delta\left(\dfrac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\ r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}\right)^{(1-\alpha_1-\beta_1)}}{\left(\left(\dfrac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} + \left(\dfrac{1}{\delta}\right)^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}\left(\dfrac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\ r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}\right)^{(1-\alpha_1-\beta_1)}}. \quad (53)$$

The equilibrium dollar price of good *C* is:[xiv]

$$b^*(\delta) = \frac{\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)} \left(\delta^{\frac{1}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} + \delta \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{*\alpha_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} \right)^{(1-\alpha_1-\beta_1)}}{\left(\delta^{\frac{1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{*\alpha_1}\ r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} \right)^{(1-\alpha_1-\beta_1)}}.$$

(54)

Thus, both the yuan and dollar prices of good *C* depend on the exchange rate, δ.

3.4. Devaluation and Exchange Rate Pass-Through

We now consider the effect of yuan devaluation on the yuan and dollar prices of the exportable good *C*. Yuan devaluation is represented by an increase in δ, which raises the yuan price *P* of good *Z*. Differentiating (53) with respect to δ, we have

$$\frac{db(\delta)}{d\delta} = \frac{(1-\alpha_1-\beta_1)(A_3)\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}}{(A_1)^{(1-\alpha_1-\beta_1)}(A_2)^{(\alpha_1+\beta_1)}} + \frac{(\alpha_1+\beta_1)\left(\frac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}\left(\frac{1}{\delta}\right)^{\frac{1}{1-\alpha_1-\beta_1}}(A_2)^{(1-\alpha_1-\beta_1)}}{\left(\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\ r^{*\beta_1}}\right)^{1/(\alpha_1+\beta_1-1)}(A_1)^{(2-\alpha_1-\beta_1)}}, \quad (55)$$

where

$$A_1 = \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{1}{\delta}\right)^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{*\alpha_1}\ r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)},$$

$$A_2 = \delta^{\frac{1}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} + \delta \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{*\alpha_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)},$$

$$A_3 = \frac{\delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}}}{1-\alpha_2-\beta_2} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} + \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{*\alpha_2}\ r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}.$$

If both industries exhibit DRS ($\alpha_1 + \beta_1 < 1$ and $\alpha_2 + \beta_2 < 1$), then $db / d\delta > 0$. If *b* rises proportionately, then $b / \delta = b^*$ is constant; i.e., yuan devaluation has no effect on the dollar price of good *C*.

Jacob and Uusküla (2016) observed that exchange rate pass-through is not only incomplete but also diminishing, and “the firm will resist transmitting exchange rate fluctuation to the sales price, and instead partially absorb the currency fluctuation into the price markup.” Gopinath et al. (2010) observed that the life-long pass-through rate is almost complete (0.98), and higher than the short-run pass-through rate for non-dollar priced goods (0.95). This observation suggests that in the short run the yuan export price decreasingly reflects the exchange rate changes (i.e., exchange rate pass-through becomes less and less complete) as the yuan appreciates.

Thus, we assume that *b* rises with *δ*, but less than proportionately. Differentiating (54) with respect to δ yields

$$\frac{\partial\left(b^*(\delta)\right)}{\partial\delta} = \frac{\partial\left(b^*(\delta)\right)}{\partial\delta} = \frac{(1-\alpha_1-\beta_1)\left(\dfrac{\gamma}{1-\gamma}\right)^{(1-\alpha_1-\beta_1)}}{\left(A_0\right)^{(2-\alpha_1-\beta_1)}\left(A_2\right)^{(\alpha_1+\beta_1)}}\left(A_0A_3 - A_2A_4\right),$$

where A_i's are functions of *δ* as follows:

$$A_0 = \delta^{\frac{1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{*\alpha_1} \; r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)},$$

$$A_2 = \delta^{\frac{1}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} + \delta \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{*\alpha_2} \; r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)},$$

$$A_3 = \frac{\delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}}}{1-\alpha_2-\beta_2} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} + \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{*\alpha_2} \; r^{*\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)},$$

$$A_4 = \frac{\delta^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}}{1-\alpha_1-\beta_1} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{*\alpha_1} \; r^{*\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)}.$$

If both industries exhibit the same DRS ($\alpha_2 + \beta_2 = \alpha_1 + \beta_1$), then devaluation of the yuan has no effect on the relative price of the good C, $\frac{d(b/\delta)}{d\delta} = 0.$ If industry C exhibits higher returns to scale than industry Z ($\alpha_1 + \beta_1 > \alpha_2 + \beta_2$), and the U.S. wage and capital rent are higher than those in China ($\delta w^* \geq w,$ and $\delta r^* \geq r$), then $A_o A_3 - A_2 A_4 < 0$,[xv] and $\frac{d(b/\delta)}{d\delta} < 0$. This implies that the dollar price of good C decreases with δ, i.e., $db^*/d\delta < 0.$ Figure 1 illustrates the case where the dollar price of good C decreases with δ until the full employment rate δ^f is reached, and remains constant because exchange rate pass-through into the export price is perfect.

Let δ^o denote the benchmark equilibrium value of δ, at which the market for good C clears when producers and consumers in both countries behave as price takers. In the absence of government intervention, the benchmark equilibrium also means trade is balanced between the two countries. If China devalues the yuan from the benchmark equilibrium rate, then the yuan price of good C increases. We assume that the yuan price of good C increases at a decreasing rate, i.e., $b'(\delta) > 0,$ and $b''(\delta) < 0.$ [xvi]

Once full employment is reached, it can be shown that the dollar price of good *C* is constant,

$$b^* = \left(\left(\frac{1-\alpha_1}{\alpha_1} \right) \left(\frac{\bar{L}+\bar{L}^*}{\bar{K}+\bar{K}^*} \right) \frac{(1-\gamma)}{\gamma} \frac{(1-\alpha_2)^{1+\alpha_2}}{(1-\alpha_1)^{1+\alpha_1}} \right)^{(\alpha_2-\alpha_1)} \left(1 - \left(\frac{\alpha_2}{1-\alpha_2} \right) \frac{(\bar{K}+\bar{K}^*)}{(\bar{L}+\bar{L}^*)} \right)^{(\alpha_2-\alpha_1)}$$
$$+ \left(\left(\frac{1-\alpha_1}{\alpha_1} \right) \left(\frac{\bar{L}+\bar{L}^*}{\bar{K}+\bar{K}^*} \right) \right)^{(\alpha_2-\alpha_1)} \left(\frac{A_Z (\alpha_2)^{\alpha_2} (1-\alpha_2)}{A_C (\alpha_1)^{\alpha_1} (1-\alpha_1)} \right),$$

(56)

where $\bar{K}$ and $\bar{L}$ are fixed endowments of China's capital and labor inputs, and $\bar{K}^*$ and $\bar{L}^*$ are U.S. endowments of the same inputs. Thus, once full employment is reached, a further increase in δ has no effect on the dollar price of good *C*, i.e., $db^*/d\delta = d(b/\delta)/d\delta = 0.$

If $\alpha_1 + \beta_1 > \alpha_2 + \beta_2,$ yuan devaluation lowers the dollar price of good *C*, which increases China's exports. If $\alpha_1 + \beta_1 = \alpha_2 + \beta_2,$ yuan devaluation has no effect on the terms of trade. On the other hand, if $\alpha_1 + \beta_1 < \alpha_2 + \beta_2,$ yuan devaluation raises the relative price of good *C* and may defeat the purpose of reducing unemployment.

3.5. Yuan Devaluation, Income and Welfare

Devaluation and GDP

We now consider the effect of yuan devaluation on national income. Let *I* denote the yuan value of China's outputs, i.e.,

$$\begin{aligned} I &= bC(b,P) + PZ(b,P) \\ &= b^{\frac{1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \delta^{\frac{1}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \qquad (57)$$

Differentiating (57) with respect to δ yields

$$\frac{dI}{d\delta} = b^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}} \left(\frac{\alpha_1^{\alpha_1} \beta_1^{\beta_1} A_C}{w^{\alpha_1} r^{\beta_1}} \right)^{1/(1-\alpha_1-\beta_1)} + \frac{1}{1-\alpha_2-\beta_2} \delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}} \left(\frac{\alpha_2^{\alpha_2} \beta_2^{\beta_2} A_Z}{w^{\alpha_2} r^{\beta_2}} \right)^{1/(1-\alpha_2-\beta_2)} > 0. \quad (58)$$

Thus, the yuan value of China's national income rises with δ.

Devaluation and Welfare

Next, consider the effect of a yuan devaluation on consumer welfare, using (38) and (53), and the Cobb-Douglas utility function in (35). Given the demand functions in (40), the indirect utility of the Chinese consumers is given by

$$V(b,\delta,I(\delta)) \equiv U\left(c(b,\delta,I(\delta)), z(b,\delta,I(\delta))\right). \quad (59)$$

We now explore whether China gains from yuan devaluation. Differentiating (59) with respect to δ, and using Roy's identities, $V_b = -V_I c$ and $V_P = -V_I z$, we obtain

$$\begin{aligned} \frac{dV}{d\delta} &= V_b(db/d\delta) + V_P + V_I\left(I'(\delta)\right) \\ &= V_b(db/d\delta) + V_P + V_I\left(b'(\delta)C + Z + bb'C'(b) + \delta Z'(\delta)\right) \\ &= V_I\left((C-c)b'(\delta) + (Z-z) + bb'C'(b) + \delta Z'(\delta)\right), \end{aligned} \quad (60)$$

where $\theta \equiv bb'C'(b) + \delta Z'(\delta)$ represents the change in national income resulting from a reduction in unemployment through a change in the exchange rate. Once full employment is reached, $\theta = 0.$ Thus, as δ approaches δ^f, $\theta(\delta)$ must converge to 0. Thus, we assume that $\theta'(\delta) < 0.$ Therefore, (60) is written as

$$\frac{dV}{d\delta}=V_I\left((C-c)b'(\delta)-\frac{b(\delta)}{\delta}(C-c)+\theta\right)=V_I\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right). \quad (61)$$

Note that θ is the income effect on welfare resulting from using idle resources, which is always positive, while $X(b'-b/\delta)$ is the price effect of yuan devaluation on welfare. A rise in the price of imports, $P=\delta$, reduces utility, and the price effect is negative. If the income effect dominates the negative price effect, then $\left.\frac{dV}{d\delta}\right|_{\delta^o}>0$ when evaluated at δ^o, i.e., yuan devaluation initially increases consumer welfare. If unemployment exists in China, then $db(\delta)/d\delta=b'<\frac{b}{\delta}=b^*$. Assume that there is a unique solution δ^* in the interval $(\delta^o,\delta^f]$ to the utility maximization problem. From (61), δ^* satisfies the first order condition,

$$\frac{dV}{d\delta}=V_I\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right)=0. \quad (62)$$

Recall that once full employment is reached, $\theta=0,$ and from (56), b^* is constant and $b'(\delta)-\frac{b(\delta)}{\delta}=0.$ Thus, δ^f satisfies (62). Differentiating (62) with respect to $\delta,$ we obtain

$$\begin{aligned}\frac{d^2V}{d\delta^2}&=V_{IP}\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right)+V_{Ib}\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right)b'(\delta)\\&+V_I\left(X'(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+X(\delta)b''(\delta)-\frac{X(\delta)}{\delta}\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta'(\delta)\right)\\&+V_{II}\left(X(\delta)\left(b'(\delta)-\frac{b(\delta)}{\delta}\right)+\theta\right)I'\end{aligned}$$

When evaluated at δ^f, $\frac{d^2V}{d\delta^2}$ reduces to

$$\frac{d^2V}{d\delta^2} = V_I\left(X(\delta)b''(\delta) + \theta'(\delta)\right) < 0,$$

which implies that the indirect utility is concave in δ. Thus, δ^f is the optimal solution.

Proposition 1: Assume that $b'' < 0$. Then China's optimal policy is yuan devaluation until full employment is reached.

An important policy implication of this proposition is that yuan devaluation initially proves consumer welfare. When unemployment is high, the positive income effect of currency devaluation probably will dominate any negative price effect. Thus, yuan devaluation is likely to improve consumer welfare. Moreover, there is no optimal exchange rate that maintains a moderate rate of unemployment.

Figure 3.2 illustrates that China's optimal policy is yuan devaluation, raising δ above δ^o. Since the indirect utility is monotone-increasing, δ can be raised to its upper limit, δ^f, thereby reaching a maximum utility. A further increase in δ does not affect the dollar price of good *C*, because the exchange rate pass-through into the export price is perfect.

Chen (2014) showed that in the case of China, from 1995 to 2007, exchange rate pass-through into the (long-term) export price was 42.6 percent. This finding suggests that China has not pursued the optimal exchange rate policy, and has devalued the yuan below the full employment rate, δ^f.

3.6. Yuan Devaluation and Unemployment

We now consider the effect of yuan devaluation on unemployment. The number of unemployed workers is

$$\begin{aligned} N &= L - L_C(\delta) - L_Z(\delta) \\ &= L - b^{1/(1-\alpha_1-\beta_1)}\left(\frac{\alpha_1^{1-\beta_1}\beta_1^{\beta_1}A_C}{w^{1-\beta_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} - \delta^{1/(1-\alpha_2-\beta_2)}\left(\frac{\alpha_2^{1-\beta_2}\beta_2^{\beta_2}A_Z}{w^{1-\beta_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}. \end{aligned} \quad (63)$$

Once the full employment exchange rate δ^f is reached, the labor constraint is binding, i.e.,

$$L - (b(\delta^f))^{1/(1-\alpha_1-\beta_1)}\left(\frac{\alpha_1^{1-\beta_1}\beta_1^{\beta_1}A_C}{w^{1-\beta_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} - (\delta^f)^{1/(1-\alpha_2-\beta_2)}\left(\frac{\alpha_2^{1-\beta_2}\beta_2^{\beta_2}A_Z}{w^{1-\beta_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} = 0.$$

Differentiating (63) with respect to δ yields

$$\frac{\partial N}{\partial \delta} = -b'\frac{b^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}}{(1-\alpha_1-\beta_1)}\left(\frac{\alpha_1^{1-\beta_1}\beta_1^{\beta_1}A_C}{w^{1-\beta_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)} - \frac{\delta^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}}}{(1-\alpha_2-\beta_2)}\left(\frac{\alpha_2^{1-\beta_2}\beta_2^{\beta_2}A_Z}{w^{1-\beta_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)} < 0.$$

This implies that a yuan devaluation reduces labor unemployment. Thus, there is some merit in the argument that China is undervaluing renminbi to reduce its domestic unemployment.

Proposition 2: Assume the Cobb-Douglas production in Section 2. Then a yuan devaluation reduces China's domestic unemployment.

Figure 3.3 illustrates the effect of yuan devaluation in the presence of unemployment in China. The initial benchmark equilibrium occurs at point 0, where both *K* and *L* are unemployed and which is inside the PPF, labeled BB'. Note that yuan devaluation affects the relative yuan price of good *C*. In this case, the supplies of *C* and *Z* rise, which causes movement from point 0 to 1 inside the PPF. Yuan devaluation always raises domestic production of both goods in so far as unemployment exists. Thus, China's optimal policy is to choose δ to eliminate unemployment until production occurs at point 2 along the PPF, i.e., $\frac{\delta}{b} = \frac{\delta^f}{b(\delta^f)}$.

Consider an alternative scenario in which capital is fully utilized at point 0, but unemployment exists in the labor market. Note that the supply of good *Z* increases with *δ*. An increase in *δ* not only raises the yuan price of good *C*, but also raises the shadow price of capital. The latter effect partly offsets the former effect. Thus, the supply response of good *C* is likely to be less pronounced than when unemployment exists in both factor markets.[xvii]

3.7. Numerical Example

Since there is no tariff, the yuan price of the importable is $P = P^*\delta = \delta.$ The price of the exportable good *C* is *b* and its dollar price of good *C* is b^*.

China and the United States

The United States and China are assumed to have identical production functions for the traded goods, *C* and *Z*: $C = F(L,K) = 1.3L^{.45}K^{.15}$ and $Z = G(L,K) = L^{.3}K^{.1}$. Assume that factor prices in China are: $w = .7$ and $r = 1$, and in the Unites States: $w^* = 1$ and $r^* = 0.7$. Labor and capital endowments are $\bar{L} = 4$ and $\bar{K} = 1$ in China and $\bar{L}^* = 1$ and $\bar{K}^* = 3$ in the United States. Direct utility functions of the Chinese and U.S. consumers are represented by $u(c,z) = (cz)^{.4}$ and $u(c^*, z^*) = (c^* z^*)^{.4}$.

From the world market clearing condition in equation (54), we obtain the optimal yuan and dollar prices of good *C*:

$$b(\delta) = \left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44\left(1/\delta\right)^{1.5}} \right)^{0.4},$$

$$b^*(\delta) = \frac{b(\delta)}{P} = \frac{1}{\delta}\left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44\left(1/\delta\right)^{1.5}} \right)^{0.4}.$$

Thus, as δ increases, the dollar price of good C, $b^*(\delta)$, decreases.

China's consumer demands for the two tradable goods are written as:

$$c = \frac{0.4}{b}\left(0.446\delta^{1.67} + 0.575\left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44\left(1/\delta\right)^{1.5}} \right)^{0.4} \right),$$

$$z = \frac{b}{\delta}c = \frac{0.4}{\delta}\left(0.446\delta^{1.67} + 0.575\left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44\left(1/\delta\right)^{1.5}} \right)^{0.4} \right).$$

China's consumer welfare is measured by the indirect utility,

$$\begin{aligned} V(b,\delta,I(\delta)) &= (cz)^{.4} \\ &= \frac{0.48}{b^{0.4}\delta^{0.4}}\left(0.446\delta^{1.67} + 0.575\left(\frac{0.396\delta + 0.446\delta^{1.67}}{0.575 + 0.44\left(1/\delta\right)^{1.5}} \right)^{0.4} \right)^{0.8}. \end{aligned} \quad (64)$$

We now determine whether China gains from a yuan devaluation. *Mathematica* was used to generate Figure 4, which shows the impacts of yuan devaluation on the yuan and dollar prices of good *C*, national income, indirect utility, and labor and capital unemployment. When $\delta \approx 2.44$, both capital and labor inputs are fully utilized and maximum utility is attained. Until δ^f is reached, the exchange rate pass-through into the yuan price is imperfect, i.e., $b' < b/\delta$. Once δ^f is reached, a further increase in δ does not affect the dollar price of good C, b^*.

3.8. Concluding Remarks

This paper investigates the optimal exchange rate policy for a two-sector, Keynesian open economy. A Cobb-Douglas utility function is used to investigate China's optimal exchange rate policy. In the plausible scenario where capital rents are equalized $(r = \delta r^*)$, China's wage is lower than the U.S. wage $(w < \delta w^*)$, and industry *C* exhibits higher returns to scale than industry *Z* $(\alpha_1 + \beta_1 > \alpha_2 + \beta_2)$, then the indirect utility function is monotone-increasing and concave in δ, which guarantees the existence of an optimal exchange rate.

China has been widely criticized for keeping the yuan low in order to take advantage of its trading partners. However, this paper suggests that instead China may have pursued the low yuan policy to reduce domestic unemployment. In the case of the Cobb-Douglas utility function, China's optimal exchange rate is that rate which guarantees full employment. Such a policy cannot be criticized as deliberately devaluing the yuan to take unfair advantage of China's trading partners. China may be preoccupied with reducing domestic unemployment.

This paper demonstrates that yuan devaluation reduces unemployment. Given the diminishing exchange rate pass-through into export price ($b''(\delta) < 0$), China's optimal exchange rate is that rate which yields full employment. The model is limited in scope as it assumes a balance of trade between two trading countries, and does not explicitly consider trade imbalance.

Given the plausible assumption of a concave indirect utility function in the exchange rate, the paper also shows that there is no optimal exchange rate that allows some unemployment.

From the U.S. perspective, there exists another exchange rate δ^* that guarantees full employment in the United States. This rate is lower than the rate δ^f that guarantees full employment in China. Instead of permitting the trading partner to set its own optimal exchange rate, the two countries could negotiate an intermediate exchange rate, which lies somewhere between the two optimal exchange rates, $\delta*$ and δ^f. To date, the United States has permitted China to choose its optimal rate, δ^f.

3.9. References

Al-Abri, Almukhtar Saif, “Optimal exchange rate policy for a small oil-exporting country: A dynamic general equilibrium perspective,” *Economic Modelling* 36 (2014), 88-98.

Batra, Ravi and Hamid Beladi, “The US Trade Deficit and the Rate of Interest,” *Review of International Economics* 21 (2013), 614-626.

Beladi, Hamid and Reza Oladi, “On Offshoring and Trade Deficit,” *Review of Development Economics* 18 (2014), 517-23.

Bruno, Michael, “The Two-Sector Open Economy and the Real Exchange Rate,” *American Economic Review* 66 (1976), 566-577.

Chen, Jiatao, “Empirical research on effect of exchange rate pass-through into China’s export price,” *Bio Technology An Indian Journal* 10 (2014), 1714-1720.

Cheung, Yin-Wong, “Exchange Rate Misalignment –The Case of the Chinese Renminbi,” *Handbook of Exchange Rates*, Jessica James, Ian W. Marsh and Lucio Sarno (eds.), 2012.

Cheung, Yin-Wong, Menzie D. Chinn, and Eiji Fujii, “China's Current Account and Exchange Rate,” *NBER Working Paper No. 14673*, 2009.

Cuddington, John, “Import Substitution Policies: A Two-Sector, Fix-Price Model,” *Review of Economic Studies* 48 (1981), 327-342.

Devereux, M. B., “How does a devaluation affect the current account?,” *Journal of International Money and Finance* 19 (2000), 833-851.

Fleming, J. Marcus, "Domestic financial policies under fixed and floating exchange Rates." *IMF Staff Papers* 9 (1962), 369–379.

Frenkel, Roberto and Jaime Ros, “Unemployment and the Real Exchange Rate in Latin America,” *World Development* 34 (2006), 631-646.

Goldstein, Morris and Nicholas Lardy, “China’s Exchange Rate Policy Dilemma,” *American*

Economic Review (96) 2006, 422-426.

Gopinath, Gita, Oleg Itskholi and Roberto Rigobon, “Currency Choice and Exchange Rate Pass-through,” *American Economic Review* 100 (2010), 304-336.

Gylfason, Thorvaldur and Michael Schmid, “Does Devaluation Cause Stagflation?,” *Canadian Journal of Economics* 16 (1983), 641-654.

Helpman, Elhanan, “Macroeconomic Policy in a Model of International Trade with a Wage Restriction,” *International Economic Review* 17 (1976), 262-277.

Holtemöller, Oliver and Sushanta Mallick, “Exchange rate regime, real misalignment and currency crises,” *Economic Modelling* 34 (2013), 5-14.

Jacob, Punnoose and Lenno Uusküla, “Deep Habits and Exchange Rate Pass-through,” Working Paper Series, Eesti Pank, 2016.

Jiang, Jiadan and David Kim, “Exchange rate pass-through to inflation in China,” *Economic Modelling* 33 (2013), 900-912.

Jin, Hailong and E. Kwan Choi, “Profits and Losses from Currency Intervention,” *International Review of Economics and Finance* 27 (2013), 14-20.

Jin, Hailong, Yoonho Choi and E. Kwan Choi, “Unemployment and Optimal Currency Intervention in an Open Economy,” *International Review of Economics and Finance* 41 (2016), 253-261.

Mundell, Robert A, “Capital Mobility and Stabilization Policy Under Fixed and Flexible Exchange Rates,” *Canadian Journal of Economic and Political Science* 29 (1963), 475-85.

Overholt, William, “China in the Global Financial Crisis: Rising Influence, Rising Challenges,” *The Washington Quarterly*, 33 (2010), 21-34.

Vasylenko, Yuriy and Dmytro Vasylenko, “Necessary and Sufficient Conditions for Foreign

Trade Balance Stabilization by Means of Devaluation in Ukraine," *Journal of European Economy* 4 (2005), 55-77.

Yue, Kan and Kevin Honglin Zhang, "How much does China's exchange rate affect the U.S. trade deficit?" *The Chinese Economy* 46 (2013), 80-93.

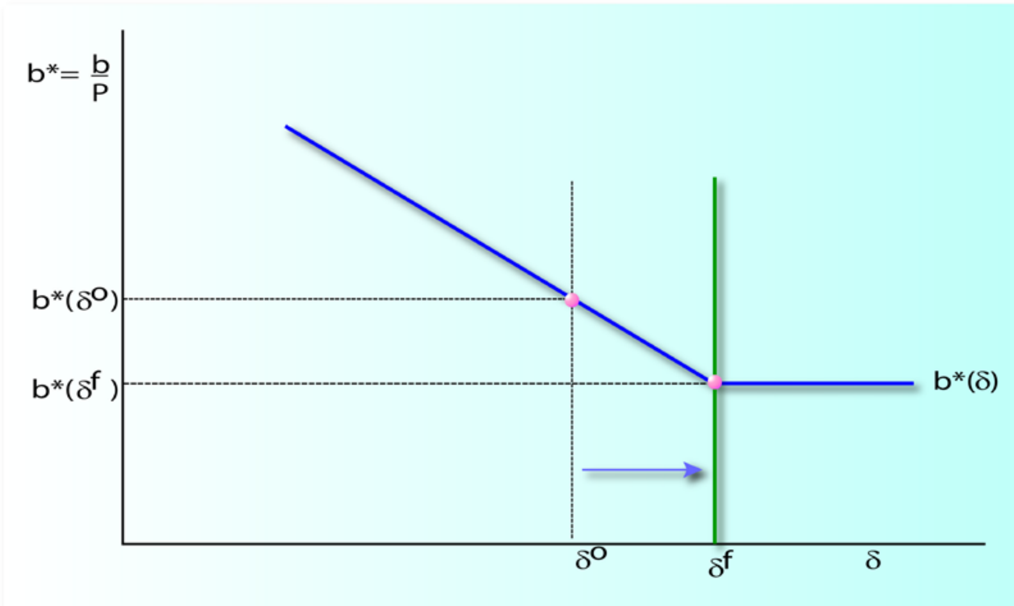

Figure 3.1. Yuan Devaluation and Terms of Trade

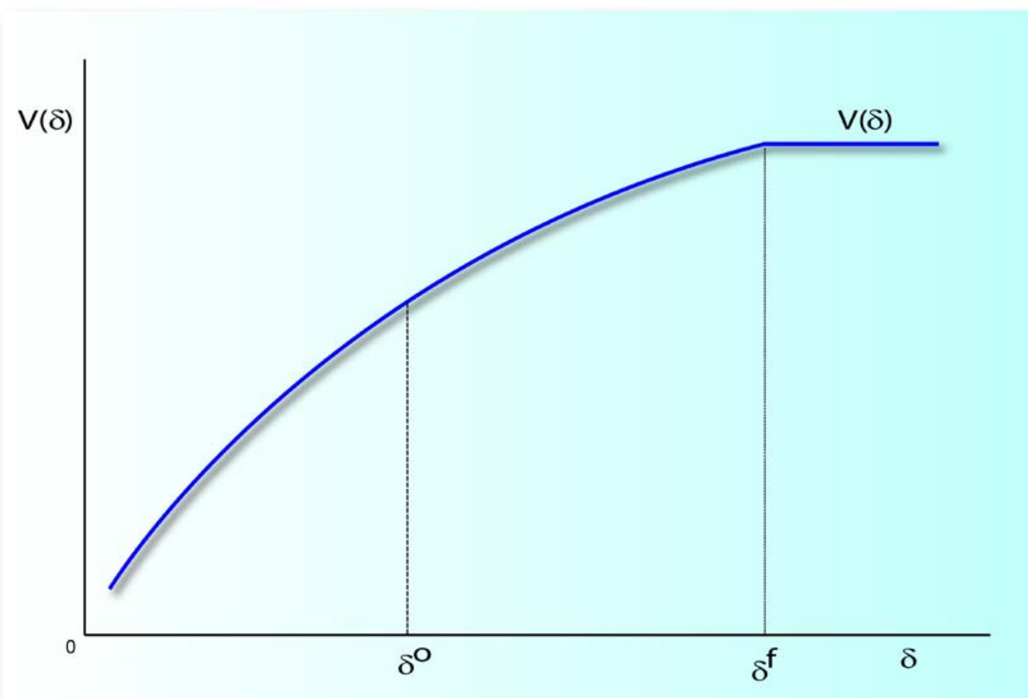

Figure 3.2. Yuan Devaluation and Indirect Utility

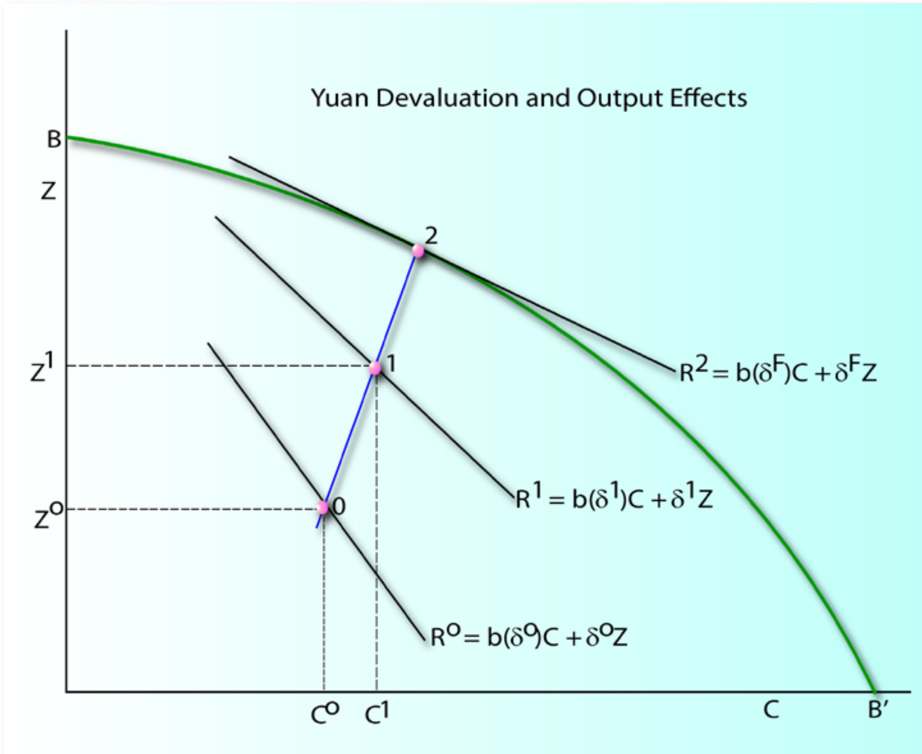

Figure 3.3. Yuan Devaluation and Output Effects

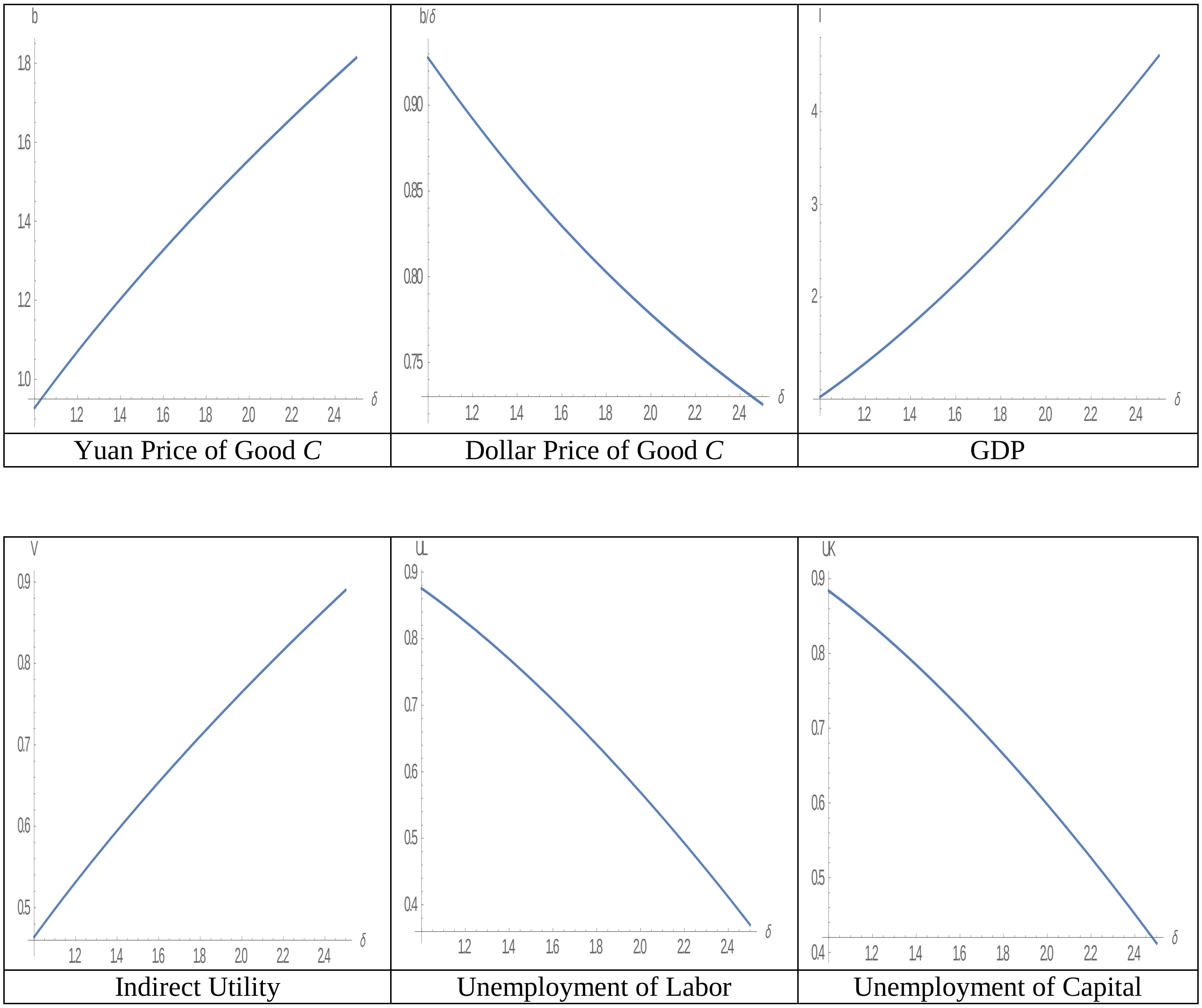

Figure 3.4. Effects on Yuan Devaluation

Table 3.1. Variables in China and the United States

Variables of China					
C	Producer supply of exportable good	Z	Producer supply of importable good	K_C	Capital demand for Good *C*
b	Yuan price of good *C*	P	Yuan price of good *Z*	K_Z	Capital demand for Good *Z*
c	Consumer demand for exportable good	z	Consumer demand for importable good	$\overline{K}$	Capital endowment
w	Wage rate	r	Capital rent	L_C	Labor demand for good *C*
Π_C	Profit of industry *C*	Π_Z	Profit of industry *Z*	L_Z	Labor demand for good *Z*
I	Gross income	N	Unemployment	$\overline{L}$	Labor endowment
U	Utility	V	Indirect Utility	R	Gross domestic product
Variables of the United States					
$C*$	Producer supply of importable good	$Z*$	Producer supply of exportable good	K_C^*	Capital demand for Good *C*
$b*$	Dollar price of good *C*	$P*$	Dollar price of good *Z*	K_Z^*	Capital demand for Good *Z*
$c*$	Consumer demand for importable good	$z*$	Consumer demand for exportable good	$\overline{K}*$	Capital endowment
$w*$	Wage rate	$r*$	Capital rent	L_C^*	Labor demand for good *C*
Π_C^*	Profit of industry *C*	Π_Z^*	Profit of industry *Z*	L_Z^*	Labor demand for good *Z*
$I*$	Gross income	$R*$	Gross domestic product	$\overline{L}*$	Labor endowment

CHAPTER 4. PUBLIC CAPITAL AND OPTIMAL TARIFF

4.1 Abstract

The objective of this paper is to address the relationship between the government expenditure in public capital stock and economic growth and examine an optimal import tariff for increase in public capital stock in a small open economy. The study provides theoretical model of the positive impacts of public capital on production sector. It also evaluates the effects of private capital investment on the export sector for labor and capital formation and identifies the contribution of public capital and other economic factors to the productivity growth rate in the firm sector. We show that the optimal tariffs are positive but decreases to the steady state level. Since the initial public and private capital stocks are lower than the levels of the steady state, the representative household spend more time to produce the exportable goods to obtain higher income compare to the level of the steady state. Increases in the public and private capital stock drops the share of labor in the export sector and the consumer spend his/her time on the home good sector.

4.2. Introduction

In many developing counties, lack of public infrastructures such as highways, transportation, power and water system is slowing economic growth because the infrastructure systems are crucial input factors for domestic production. Public investment plays an important role and a major policy issue in developing countries. A number of studies support that public capital has a powerful impact on the productivity of private capital. Aschauer (1989) insists the public capital is more important in determining productivity.

Barro (1990) also finds a positive relationship between public investment and output growth. Recently, they found that the public capital stock and economic growth is significantly connected especially in small income countries.

In the theoretical literature, government expenditure is introduced as an argument in the production function as an externality in production. This approach of including public capital in a private production function was proposed in Arrow and Kurz (1970) and all government investment was used for productions in the private sector. To invest in public capital, government uses two types of policies: domestic policy (income taxes or consumption tax) or international policy (import tariff) to finance government expenditure. Barro (1990) develops a simple model of endogenous growth in which the government uses tax revenue and found that all government spending was productive. Abe (1990, 1992) examined the theory of tariff reform in a small open economy. He examined that if the public production is complementary with the private goods, then the reduce tariffs increases welfare. However, if public production uses public inputs, then the welfare effect of lowering tariff is not clear.

The objective of this paper is to address the relationship between the government expenditure in public capital stock and economic growth and examine an optimal import tariff for increase in public capital stock in a small open economy. This practice explains rationale for government provision of public goods based on the market failure, internalizes externalities in the private production function. Moreover, the paper is alternatively answering "Why does the government devalue its own currency and change the terms of trade in the short run? " The effect of change in exchange rate is the similar to that of import tariffs which changes the relative price in domestic economy. This distortion of government policy discourages current consumptions for import goods and encourage consumptions of home goods and capital investments.

We consider a small open economy with three types of agents: household, firm and government. The household saves in the form of capital and lends to the firm, and produces a nontraded good or home good with only labor. The firm borrows labor and capital and produces an export good given public capital. The government imposes an import tariff to finance public service or public capital which is an input in the tradable output sector. The paper begins by deriving the equilibrium in a social planning problem, characterizing its steady-state and dynamic properties. The effects of import tariff shocks in such an economy are analyzed. An important aspect of our analysis concerns the design of an optimal tariff policy in an economy with gradually accumulating public capital.

We assume discrete time, infinite time-horizons and perfect foresight. The government chooses the path of the tax rate once and for all on date 0 by taking into account the competitive equilibrium. First, we present the competitive equilibrium and then solve the primal problem. Our framework builds on the approach to optimal taxation. This approach characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two simple conditions: a resource constraint and an implementability constraint. The implementability constraint is the household budget constraint in which the first-order conditions are used to substitute out for tariff policies. Thus both constraints depend only on allocations.

The study provides theoretical model of the positive impacts of public capital on production sector. It also evaluates the effects of private capital investment on the export sector for labor and capital formation and identifies the contribution of public capital and other economic factors to the productivity growth rate in the firm sector.

Section 2 introduces the basic economic model with public capital stock and import tariffs. Section 3 investigates the optimal tariff rates through the primal approach. Section 4 suggests the algorithm to computing the optimal import tariff sequence. Section 5 offers a numerical analysis with log-linear Cobb-Douglas utility function to illustrate the case of positive optimal tariffs and increasing consumer's welfare. Section 6 provides concluding remarks.

4.3. Basic Economic Model

We assume discrete time, infinite time-horizons and perfect foresight. The government chooses the path of the tax rate once and for all on date 0 by taking into account the competitive equilibrium. First, we present the competitive equilibrium and then solve the primal problem. Our framework builds on the approach to optimal taxation. This approach characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two simple conditions: a resource constraint and an implementability constraint. The implementability constraint is the household budget constraint in which the first-order conditions are used to substitute out for tariff policies.

We now consider a two-sector small open economic model producing the non-tradable good and the tradable good, h and x. The domestic consumers import foreign goods, m.

The Household

The representative household's preferences are given by

$$U = \sum_{0}^{\infty} u(c_{ht}, c_{mt}), \quad \text{subject to}$$
$$c_h(l_t) = B(1-l_t)^{\gamma}, \tag{65}$$
$$(1+\tau_t)c_{mt} + k_{t+1} = w_t l_t + (r_t + 1 - \delta_k)k_t.$$

where $u(c_{ht}, c_{mt})$ is the one-period utility function and $0 < \beta < 1$ is the discount rate. c_{ht} and c_{mt} are consumptions in home goods and imported goods (*h* and *m*) at date *t*. The representative household has a unit of time endowment. In period *t*, a fraction l_t of the time endowment is allocated to work in the exportable sector and the remaining fraction $1-l_t$ is allocated to the domestic sector. The utility $u(.)$ is strictly increasing in consumptions, c_h and c_m, and strictly concave.

The household owns technology for producing the home good c_h on date *t* described as

$$c_h(l_t) = B(1-l_t)^{\gamma}, \tag{66}$$

where the fraction $1-l_t$ is allocated to the home good sector and γ is a productivity parameter. The home good c_h is non-tradable and the consumption of the nontraded home good is the same as the production of the home good.

The household earn incomes from the labor (l_t) of the export sector and capital (k_t) at time *t*. Households own capital (k_t) and rent it to firms. The capital depreciates at a rate of δ_k. Denote that *w* and *r* are the market-determined the wage rate for labor and the rental rate of capital from producers. The representative household budget constraint is

$$(1+\tau_t)c_{mt} + k_{t+1} = w_t l_t + (r_t + 1 - \delta_k)k_t, \tag{67}$$

where τ_t is the rate of tariff levied by the government, $q_t \equiv 1+\tau_t$ is the price of the importable, and δ_k is a depreciation rate of private capital stock.

The household maximizes its utility (65), subject to the technology of home good (66) and the budget constraint (67) and the problem is written as:

$$\max_{\{c_{mt}, l_t, k_{t+1}\}} U = \sum_0^{\infty} u(c_{ht}, c_{mt}), \qquad \text{subject to}$$
$$c_h(l_t) = B(1-l_t)^{\gamma}, \tag{68}$$
$$(1+\tau_t)c_{mt} + k_{t+1} = w_t l_t + (r_t + 1 - \delta_k)k_t \quad \text{given } k_0.$$

China produces two goods, using two factors: capital (*K*) and labor (*L*) inputs.

The first order conditions with respect to c_{mt}, l_t and k_{t+1} are given by

$$c_{mt}: \; u_m(c_{ht}, c_{mt}) = \lambda_t(1+\tau_t), \tag{69}$$

$$l_t: \; u_l(c_{ht}, c_{mt}) = -\lambda_t w_t, \tag{70}$$

$$\lambda_t = \beta(r_{t+1} + 1 - \delta_k)\lambda_{t+1}. \tag{71}$$

where $u_l(.) < 0$, and $u_m(.) > 0$ are marginal utilities of labor allotted in the home good sector and the import good.

To simplify these conditions, (69) and (70) together yield, the consumption-labor trade off condition, the intratemporal condition,

$$-u_l(c_{ht}, c_{mt}) = \frac{w_t}{(1+\tau_t)} u_m(c_{ht}, c_{mt}). \tag{72}$$

Next, combining (69) and (71) yield the consumption Euler equation, the intertemporal condition as follows:

$$u_m(c_{ht}, c_{mt}) = \frac{\beta(1+\tau_t)(r_{t+1}+1-\delta_k)}{(1+\tau_{t+1})} u_m(c_{ht+1}, c_{mt+1}). \tag{73}$$

With a No-Ponzi condition $\left(\lim_{T\to\infty}\sum_{t=0}^{T}\frac{k_{T+1}}{\Pi_{s=1}^{t}(r_t+1-\delta_k)}=0\right)$, the budget condition reduces to

$$\sum_{t=0}^{\infty}\frac{1}{\Pi_{s=1}^{t}(r_t+1-\delta_k)}\left[(1+\tau_{t+1})c_{mt}-w_t l_t\right]=(r_0+1-\delta_k)k_0. \tag{74}$$

The Firm

The firm produces an exportable good at the world at price p_t which is normalize, i.e., $p_t = 1$ for all $t = 0,1,\cdots,\infty$. The production function of the exportable good on date t is

$$x = f(G_t, k_t, l_t) = AG_t^{\varphi} k_t^{\theta} l_t^{1-\theta}, \tag{75}$$

where G and k are public and private capital stocks, and φ and θ are productivity parameters of the public and private capital stocks. Firm takes the public capital stock G as given.

The production function is a constant returns to scale with private inputs, capital (k) and labor (l). Producer hires labor and rents capital from the household and pays the wage and the capital rent to household.

Firm's profit at time t is

$$\pi_t = f(G_t, k_t, l_t) - w_t l_t - r_t k_t. \tag{76}$$

where w_t and r_t are the market-determined wage rate for labor and the rental rate of capital. The firm's optimal conditions for private capital and labor are as follows:

$$r_t = f_k\left(G_t, k_t, l_t\right) = \theta AG_t^{\varphi} k_t^{\theta-1} l_t^{1-\theta}, \tag{77}$$

$$w_t = f_l\left(G_t, k_t, l_t\right) = \left(1-\theta\right) A G_t^{\varphi} k_t^{\theta-1} l_t^{-\theta}. \tag{78}$$

Since the production function is constant returns to scale and competitive pricing ensures that these returns are equal their marginal products, the equilibrium profits are zero.

The Government

Denote by G_t the public capital stock with δ_G its depreciation rate. The government finances its stream of purchases public capital $\{G_t\}_{t=0}^{\infty}$ by levying time varying tariffs on import goods at rate τ_t. Since the government has a borrowing constraint at each period, the government budget constraint in each period is:

$$G_{t+1} - (1-\delta_G)G_t = \tau_t c_{mt} \tag{79}$$

where δ_G is the depreciate rate for public capital stock and $0 < \delta_G < 1$, and c_{mt} is the consumption of the importable good.

Comparative Equilibrium

We assume that all markets are perfectly comparative. The central planer faces an aggregate goods market constraint and using (67), (77), (78), and (79), the resource constrain is obtained by

$$c_{mt} + G_{t+1} + k_{t+1} = f(G_t, k_t, l_t) + (1-\delta_G)G_t + (1-\delta_k)k_t. \tag{80}$$

Given the paths of the government policy instruments $\{\tau_t, G_t\}_{t=0}^{\infty}$ and initial conditions for G_0 and k_0, a comparative equilibrium is household's choices $\{c_{mt}, l_t, k_{t+1}\}_{t=0}^{\infty}$, firm's choices $\{l_t, k_t\}_{t=0}^{\infty}$, and prices $\{w_t, r_t\}_{t=0}^{\infty}$, such that: (i) the household optimizes, (72) and

(73); (ii) the firm optimizes, (77) and (78); (iii) the government budget constraint, (79); (iv) the goods market clears, (80).

In equilibrium, by using the firm's optimal condition of capital in (78), the intratemporal condition in (72) becomes

$$-u_l(c_{ht},c_{mt}) = \frac{f_l\left(G_t,k_t,l_t\right)}{(1+\tau_t)}u_m(c_{ht},c_{mt}). \tag{81}$$

Moreover, by using the firm's optimal condition of capital in (77), the intertemporal condition in (73) becomes

$$u_m(c_t,c_{mt}) = \frac{\beta(1+\tau_t)\left(f_k\left(G_t,k_t,l_t\right)+1-\delta_k\right)}{(1+\tau_{t+1})}u_m(c_{ht+1},c_{mt+1}). \tag{82}$$

Thus, we obtain new equilibrium conditions, (80), (81), and (82) in terms of $c_{mt}, G_t, k_t,$ and l_t.

4.4. Ramsey Optimal Tariff

To solve the Ramsey planer's problem where the government chooses allocations, $c_{mt}, l_t, G_{t+1},$ and k_{t+1}, rather than tariff rates, τ_t, first, by the process of recursively using successive household budget constraints (67) to eliminate successive k_{t+1} terms for t=0,⋯,∞, we have

$$\lim_{T\to\infty}\frac{k_{t+1}}{\Pi_{s=1}^{t}(r_t+1-\delta_k)}+\sum_{t=0}^{\infty}\frac{\left[(1+\tau_t)c_{mt}-w_tl_t\right]}{\Pi_{s=1}^{t}(r_t+1-\delta_k)}=\left(f_k\left(G_0,k_0,l_0\right)+1-\delta_k\right)k_0.$$

By imposing the transversality condition (No-Ponzi Condition) for capital stocks, i.e., $\left(\lim_{T \to \infty} \sum_{t=0}^{T} \frac{k_{T+1}}{\Pi_{s=1}^{t}(r_t + 1 - \delta_k)} = 0 \right)$, we obtain the household's present budget constraint as follows:

$$\sum_{t=0}^{\infty} \frac{\left[(1+\tau_t)c_{mt} - w_t l_t\right]}{\Pi_{s=1}^{t}(r_t + 1 - \delta_k)} = \left(f_k\left(G_0, k_0, l_0\right) + 1 - \delta_k \right) k_0. \tag{83}$$

If we use new household's intratemporal constraint (81) and intertemporal constrain (82), we have

$$w_t l_t = -\frac{u_l(c_{ht}, c_{mt})}{u_m(c_{ht}, c_{mt})}(1+\tau_t),$$
$$(1+\tau_t) = \Pi_{s=1}^{t}(r_s + 1 - \delta_k)\beta^t (1+\tau_0)\frac{u_m(c_{ht}, c_{mt})}{u_m(c_{h0}, c_{m0})}.$$

Substitute these equations into the household's present budget constraint (83), we obtain the implement constraint:

$$\sum_{t=0}^{\infty} \beta^t \left[u_m(c_{ht}, c_{mt})c_{mt} - u_l(c_{ht}, c_{mt})l_t\right] = \frac{\left(f_k\left(G_0, k_0, l_0\right) + 1 - \delta_k \right) k_0 u_m(c_{h0}, c_{m0})}{(1+\tau_0)}. \tag{84}$$

The Ramsey problem is as follows. The central planner chooses $c_{mt}, l_t, G_{t+1},$ and k_{t+1}, in order to maximize the representative household's utility in (65) subject to the implementability constraint (84) and the resource constraint (80).

Optimal Tariff

In the primal approach, the central planner chooses $\{c_{mt}, l_t, G_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ in order to maximize the representative household's utility in (65) subject to the implementability constraint (84) and the resource constraint (80). Let μ be the Lagrange multiplier on the impelmentability constraint (84) and define new preference function *W* to include the implementability constraint as follow:

$$W(c_{ht},c_{mt}) = u(c_{ht},c_{mt}) + \mu\left[u_m(c_{ht},c_{mt})c_{mt} - u_l(c_{ht},c_{mt})l_t\right] \tag{85}$$

The Ramsey problem is

$$\begin{aligned}&\max_{\{c_{mt},l_t,k_{t+1}\}_{t=0}^{\infty}} \sum_{0}^{\infty}\beta^t W(c_{ht},c_{mt}) - \frac{\mu\left(f_k\left(G_0,k_0,l_0\right)+1-\delta_k\right)k_0 u_m(c_{h0},c_{m0})}{(1+\tau_0)}\\ &+\sum_{t=0}^{\infty}\beta^t\phi_t\left[f(G_t,k_t,l_t)+(1-\delta_G)G_t - G_{t+1} + (1-\delta_k)k_t - k_{t+1} - c_{mt}\right]\\ &k_0 \text{ and } G_0 \text{ given}.\end{aligned} \tag{86}$$

where ϕ_t is the Lagrange multiplier on the resource constraint at time *t*.

The first order conditions for this problem are for $t \geq 1$,

$$c_{mt}: \;\; W_m(c_{ht},c_{mt}) - \phi_t = 0, \tag{87}$$

$$l_t: \;\; W_l(c_{ht},c_{mt}) - \phi_t f_l(G_t,k_t,l_t) = 0, \tag{88}$$

$$G_{t+1}: \;\; -\phi_t + \beta\phi_{t+1}\left[f_G(G_t,k_t,l_t) + 1 - \delta_G\right] = 0, \tag{89}$$

$$k_{t+1}: \;\; -\phi_t + \beta\phi_{t+1}\left[f_k(G_t,k_t,l_t) + 1 - \delta_k\right] = 0, \tag{90}$$

$$f(G_t,k_t,l_t) + (1-\delta_G)G_t - G_{t+1} + (1-\delta_k)k_t - k_{t+1} - c_{mt} = 0. \tag{91}$$

The Ramsey Allocations

Combining the optimal conditions, (87) and (88), we obtain an intratemporal condition:

$$-\frac{W_l(c_{ht},c_{mt})}{W_m(c_{ht},c_{mt})} = f_l(G_t,k_t,l_t). \tag{92}$$

Moreover, using (87) and (90), we obtain an intertemporal condition:

$$\frac{W_m(c_{ht}, c_{mt})}{W_m(c_{ht+1}, c_{mt+1})} = \beta\left[f_k(G_t, k_t, l_t) + 1 - \delta_k \right], \tag{93}$$

Equation (92) shows that the government's marginal rate of substitution between consumption of import goods and labor in the export sector equals to the marginal product of labor in the exportable good at date *t*. Similarly, equation (93) equates the government's marginal rate of substitution between consumption today and consumption tomorrow to the gross capital return (net of depreciation).

The optimal tariff can be determined by substituting the Ramsey allocation into the equilibrium conditions. Combining the household’s intratemporal constraint (72), the firm’s first order condition for labor (78) and the planer’s intratemporal constraint (92), we solve out for the optimal tariff for $t \geq 1$ as follows:

$$\tau_t^* = \frac{u_m(c_{ht}, c_{mt})}{W_m(c_{ht}, c_{mt})} \frac{W_l(c_{ht}, c_{mt})}{u_l(c_{ht}, c_{mt})} - 1. \tag{94}$$

Now we consider a separable utility function:

$$u(c_{ht}, c_{mt}) = \ln(c_{ht}^{\alpha} c_{mt}^{1-\alpha}) = \ln B^{\alpha} + \alpha\gamma \ln(1-l_t)^{\gamma} + (1-\alpha)\ln c_{mt}, \tag{95}$$

where α and $(1-\alpha)$ are the shares of expenditure on the home good and the importable good. Then $W(c_{ht}, c_{mt})$ can be written as

$$W(c_{ht}, c_{mt}) = \ln B^{\alpha} + \alpha\gamma \ln(1-l_t)^{\gamma} + (1-\alpha)\ln c_{mt} + \mu\left[(1-\alpha) + \alpha\gamma - \alpha\gamma\left(\frac{l_t}{1-l_t} \right) \right]. \tag{96}$$

Differentiating (96) with respect to c_{mt} and l_t, we obtain

$$W_m(c_{ht}, c_{mt}) = \frac{1-\alpha}{c_{mt}}, \tag{97}$$

$$W_l(c_{ht}, c_{mt}) = -\frac{\alpha\gamma(1+\mu) - \alpha\gamma l_t}{(1-l_t)^2}. \tag{98}$$

Substituting (97) and (98) into the optimal tariff, (94), for $t \geq 1,$ then the optimal import tariff is

$$\tau_t^* = \frac{\mu}{1-l_t^*}. \tag{99}$$

This implies that under the linear log utility function given by (95), we find that the optimal tariffs are positive.

4.5. An Algorithm for Computing Optimal Import Tariff Sequence

We compute the sequence of optimal tariffs and evaluate welfare gains through the following five steps:

First, from the Ramsey problem, we find the system of equations at the steady state given by four first order conditions and the planner's resource constraint as follows:

$$\frac{(1-\alpha)}{c_{mss}} - \phi_{ss} = 0, \tag{100}$$

$$\frac{\alpha\gamma(1+\mu) - \alpha\gamma l_{ss}}{(1-l_{ss})^2} - \phi_{ss}\left(1-\theta\right) AG_{ss}^{\varphi} k_{ss}^{\theta-1} l_{ss}^{-\theta} = 0, \tag{101}$$

$$-1 + \beta\left[\varphi AG_{ss}^{\varphi-1} k_{ss}^{\theta} l_{ss}^{1-\theta} + 1 - \delta_G\right] = 0, \tag{102}$$

$$-1 + \beta\left[\ \theta AG_{ss}^{\varphi} k_{ss}^{\theta-1} l_{ss}^{1-\theta} + 1 - \delta_k\right] = 0, \tag{103}$$

$$AG_{ss}^{\varphi}k_{ss}^{\theta}l_{ss}^{1-\theta}-\delta_G G_{ss}-\delta_k k_{ss}-c_{mss}=0. \quad (104)$$

In this system, there are six variables, $\phi_{ss}, G_{ss}, k_{ss}, l_{ss}, c_{mss}$, and μ. However we have only five equations. We assume that in the steady state, $l_{ss}=1/3$ and we solve the systems for the five unknown variables. The optimal values at the steady state as follows:

$$G_{ss}=\left(\frac{\beta\varphi^{1-\theta}\theta^{\theta}Al_{ss}^{1-\theta}}{(1-\beta(1-\delta_k))^{\theta}(1-\beta(1-\delta_G))^{1-\theta}}\right)^{\frac{1}{1-\theta-\varphi}}, \quad (105)$$

$$k_{ss}=\left(\frac{\beta\varphi^{\varphi}\theta^{1-\varphi}Al_{ss}^{1-\theta}}{(1-\beta(1-\delta_k))^{1-\varphi}(1-\beta(1-\delta_G))^{\varphi}}\right)^{\frac{1}{1-\theta-\varphi}}, \quad (106)$$

$$c_{mss}=AG_{ss}^{\varphi}k_{ss}^{\theta}l_{ss}^{1-\theta}-\delta_G G_{ss}-\delta_k k_{ss}, \quad (107)$$

$$\phi_{ss}=\frac{(1-\alpha)}{c_{mss}}, \quad (108)$$

$$\mu=\frac{\phi_{ss}\left(1-\theta\right)AG_{ss}^{\varphi}k_{ss}^{\theta-1}l_{ss}^{-\theta}(1-l_{ss})^2}{\alpha\gamma}+l_{ss}-1. \quad (109)$$

From the steady state, we obtain the value of the Lagrange multiplier μ which depends on l_{ss}.

Second, having computed the steady –state allocations, the Lagrange multiplier μ, we look at the first order conditions and the resource constraint for $t\geq 1$, which are given by

$$\frac{(1-\alpha)}{c_{mt}}-\phi_t=0, \quad (110)$$

$$\frac{\alpha\gamma(1+\mu)-\alpha\gamma l_t}{(1-l_t)^2}-\phi_t\left(1-\theta\right)AG_t^{\varphi}k_t^{\theta-1}l_t^{-\theta}=0, \quad (111)$$

$$-\phi_t+\beta\phi_{t+1}\left[\varphi AG_t^{\varphi-1}k_t^{\theta-1}l_t^{1-\theta}+1-\delta_G\right]=0, \quad (112)$$

$$-\phi_t + \phi_{t+1}\beta\left[\ \theta A G_{t+1}^{\varphi} k_{t+1}^{\theta-1} l_{t+1}^{1-\theta} + 1 - \delta_k\ \right] = 0, \tag{113}$$

$$A G_t^{\varphi} k_t^{\theta} l_t^{1-\theta} - G_{t+1} + (1-\delta_G)G_t - k_{t+1} + (1-\delta_k)k_{ss} - c_{mt} = 0. \tag{114}$$

Since Lagrange multiplier μ is obtain from the steady state, we have seven unknown variables, $G_{t+1}, k_{t+1}, l_{t+1}, l_t, c_{mt}, \phi_t$, and ϕ_{t+1}, and five optimal equations.

Using the optimal equations (110)-(114), we obtain two dynamic equations k_{t+1} and G_{t+1} as follows:

$$G_{t+1} + k_{t+1} = \frac{(\delta_k - \delta_G)G_t k_t}{\theta G_t - \varphi k_t} + (1-\delta_G)G_t + (1-\delta_k)k_{ss} - c_{mt}, \tag{115}$$

$$\begin{aligned} &\frac{\left((1+\mu)A^{\frac{1}{1-\theta}}(\theta G_t - \varphi k_t)^{\frac{1}{1-\theta}} - (\delta_k - \delta_G)^{\frac{1}{1-\theta}} G_t^{\frac{1-\varphi}{1-\theta}} k_t\right) G_t^{\frac{\theta-\varphi}{1-\theta}} (\theta G_t - \varphi k_t)}{\left(A^{\frac{1}{1-\theta}}(\theta G_t - \varphi k_t)^{\frac{1}{1-\theta}} - (\delta_k - \delta_G)^{\frac{1}{1-\theta}} G_t^{\frac{1-\varphi}{1-\theta}} k_t\right)^2 \beta\left[\varphi(\delta_k - \delta_G)k_{t+1} + (1-\delta_G)(\theta G_{t+1} - \varphi k_{t+1})\right]} \\ &= \frac{\left((1+\mu)A^{\frac{1}{1-\theta}}(\theta G_{t+1} - \varphi k_{t+1})^{\frac{1}{1-\theta}} - (\delta_k - \delta_G)^{\frac{1}{1-\theta}} G_{t+1}^{\frac{1-\varphi}{1-\theta}} k_{t+1}\right) G_{t+1}^{\frac{\theta-\varphi}{1-\theta}}}{\left(A^{\frac{1}{1-\theta}}(\theta G_{t+1} - \varphi k_{t+1})^{\frac{1}{1-\theta}} - (\delta_k - \delta_G)^{\frac{1}{1-\theta}} G_{t+1}^{\frac{1-\varphi}{1-\theta}} k_{t+1}\right)^2}. \end{aligned} \tag{116}$$

Third, since the steady-state levels of public and private capitals, G_{ss} and k_{ss}, and labor in the exportable sector, l_{ss} depends on the initial values of public and private capital stocks G_0 and k_0, we guess the initial values of public and private capital stocks G_0 and k_0. After the iterations converge, the public and private capital stocks are matched with the steady-state values obtained in the first step. If they don't, the initial guess for G_0 and k_0 is corrected. This is repeated, until the iterations converge to the steady-state allocations.

Fourth, we use the sequence of allocations $\{G_t, k_t\}_{t=1}^{T}$, to compute the entire sequence $\{l_t\}_{t=1}^{T}$ by using the optimal condition

$$l_t = \frac{(\delta_k - \delta_G) G_t^{\frac{1-\varphi}{1-\theta}} k_t}{A^{\frac{1}{1-\theta}} (\theta G_t - \varphi k_t)^{\frac{1}{1-\theta}}}. \tag{117}$$

Given initial public and private capital stocks G_0 and k_0, if the value for l_{ss} obtained in step 4 matches with the one given, we stop. Otherwise, the guess for l_{ss} is updated, and steps 1-4 is repeated, until convergence is achieved.

Finally, after steps 1-4 converge, the optimal sequence of import tariffs $\{\tau_t\}_{t=1}^{T}$ is obtained from

$$\tau_t = \frac{\mu}{1 - l_t}. \tag{118}$$

4.6. Numerical Analysis

We use a separable utility function $u(c_{ht}, c_{mt}) = \ln B^{\alpha} + \alpha\gamma \ln(1 - l_t)^{\gamma} + (1-\alpha)\ln c_{mt}$. We choose parameters $\alpha = 0.4$ and $\gamma = 3$, and nomalize *B*=1. We use the Cobb-Douglas production function $x_t = AG_t^{\varphi} k_t^{\theta-1} l_t^{1-\theta}$. We choose the shares of public and private capitals at $\varphi = 0.5$ and $\theta = 0.3$, and *A*=2. The annual time preference rate is $\beta = 0.9$, and the depreciation rates of public and private capital stocks are $\delta_G = 0.1$ and $\delta_k = 0.3$. The parameters are summarized in Table 4.1.

To solve the levels of the steady state, we assume that the level of labor in the exportable sector is $l_{ss} = 1/3$. Using the solutions for the steady-state, (105) - (109), we obtain the steady state levels of the variables, $\phi_{ss}, G_{ss}, k_{ss}, c_{mss}$, and μ, where μ is the Lagrange multiplier on the implement constraint as following Table 4.2.

From the steady state solutions, we find that the optimal tariff is $\tau_{ss} = 0.7041$ which is positive. We assume that the initial values of public and private capital stocks are lower than the levels of the steady states and set and the initial values of public and private capital stock as G_0=0.95885* G_{ss} =1.1583 and k_0=0.97* k_{ss} =0,6017. Using the value of the Lagrange multiplier on the implement constraint $\mu = 0.4694,$ and the initial values, we find the sequence of allocations of public and private capital stocks and a share of labor in the export sector $\{G_t, k_t, l_t\}_{t=1}^{T}$, converge to the steady state levels and obtain the optimal sequence of import tariffs $\{\tau_t\}_{t=1}^{T}$.

Trends in the Public and Private Capital Stock

In assessing the public and private capital stocks and its relationship to output, it is important to take into account both the volume of capital stocks (the physical amount of capital goods) and the ratios of capital stocks to gross domestic product (GDP) each period. Figure 4.1. and Figure 4.2. below describe that the ratios of public and private capital stocks to GDP.

The initial ratios of public and private capital to GDP are 1.408 and 0.731 and reach to the highest levels 1.457 and 0.73818. However, after reaches to the highest level, the ratios slightly drop and move to the steady state levels.

The Figure 4.3. illustrates the pattern of the share of labor in the export sector. Labor income from the export sector is the main source for household’s income. Using the income, the household consumes import goods and increases his/her utility. In this model, we assume that the initial levels of public and private capital stock are lower than those of the levels of steady state. The lower levels of capital stocks cause lower wage and capital rent from the private capital stock.

In order to increase household income, the consumer initially works more than the level of the steady state. From the government investments in the public capital financed through the import tariffs, the household increases private capital stock and reduces the share of labor in the export sector shown in Figure 4.3. This implies that the consumers enjoy more home goods (non-traded goods) such as gardening, home cooked meal or leisure. As the public and private capital stock reach to the levels of the steady state, the share of labor in the export sector also arrives the level of the steady state.

The countries endowed lower levels of public and private capital stocks initially levies higher tariff rates for import goods. As the both capital stocks have accumulated, the government reduces the import tariffs. Figure 4.4. provides the optimal tariff rates which are positive even in the level of steady state. The public capital is an important input to produce exportable goods and the higher level of a public capital stock increases the marginal product of labor and capital that are the same as the wage and capital rent. In other words, the increase in a public capital stock increases household's income. Thus, the income effect of the public capital stock is larger than the substitute effect of between shares of labor in the exportable sector and home good sector. The consumer reduces the share in the export sector.

Finally, Figure 4.5. depicts the trends of consumer's welfare. Increases both home goods and import goods consumptions improve the consumers' welfare. The level of the welfare gradually increases and reached the level of the steady state when the allocations of the public and private capital stocks and the share of labor in export sector stay in the levels of the steady state. This result shows that the use of import tariffs is the optimal trade policy under this special economy.

4.7. Concluding Remarks

Many developing countries face tighter budget. Although they have vast poor population, it is hard to increase public investments because of budget constraints. Public capital is the main component of private productions, for instance, primarily transportation systems, such as highways, mass transit, water supply, and power facilities. Public capital stocks plays a very import role for the firms' productivities. Therefore, allocating public investment becomes critically important for the public policy. This paper examines the contributions of public capital to economic growth and productivity growth of the economy as well as consumer's welfare. The study provides positive impacts of public capital on production sector. Government intervention in the international market through import tariffs improves domestic production and social welfare. We show that the optimal tariffs are positive but decreases to the steady state level. Since the initial public and private capital stocks are lower than the levels of the steady state, the representative household spend more time to produce the exportable goods to obtain higher income compare to the level of the steady state. Increases in the public and private capital stock drops the share of labor home in the export sector and the consumer spend his/her time on the home goods, for instance, leisure or gardening. When the capital stocks reach to the levels of steady state, the share of labor in the export sector also arrive the level of the steady state. Thus, in the special economy, the optimal trade policy is the imposition of tariffs.

4.8. References

Abe, K., “Tariff reform in a small open economy with public production,” *International Economic Review* 33 (1992), 209-22.

Abe, K., “A Public Input as a Determinant of Trade,” *Canadian Journal of Economics* 23 (1990), 400-07.

Arrow, K.J. and M. Kurz, Public investment, the rate of return and optimal fiscal policy (Johns Hopkins University, Baltimore, MD), 1970.

Aschauer, D. A., “Is public expenditure productive?” *Journal of Monetary Economics* 23 (1989), 177-200.

Barro, R. J., “Government spending in a simple model of endogenous growth” *Journal of Political Economy* 98 (1990), 103-25.

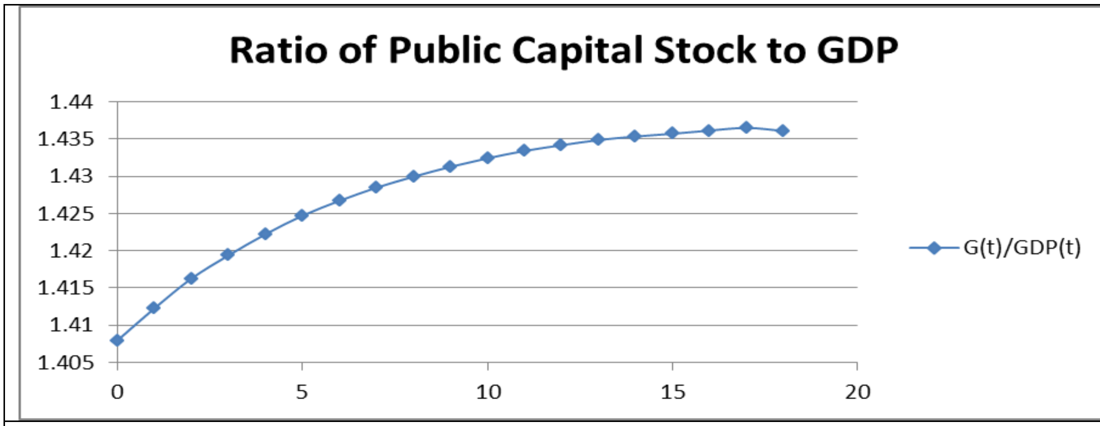

Figure 4.1. Ratio of Public Capital Stock to GDP

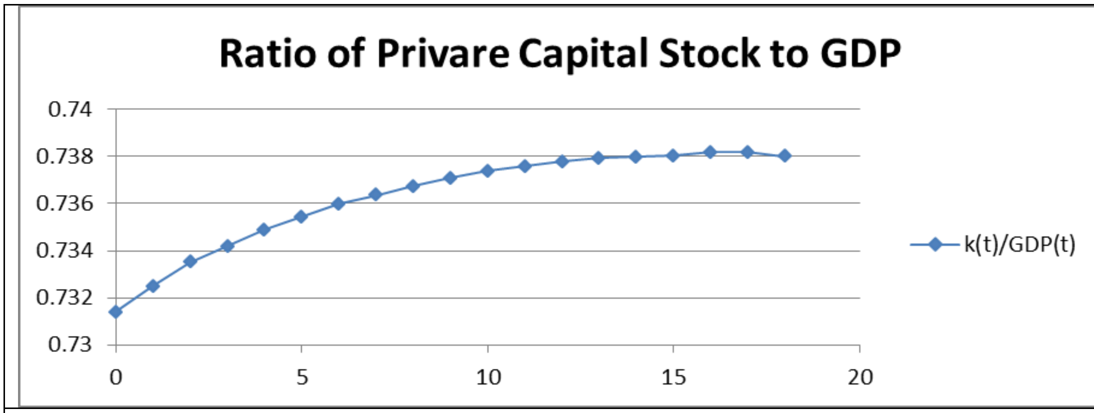

Figure 4.2. Ratio of Private Capital Stock to GDP

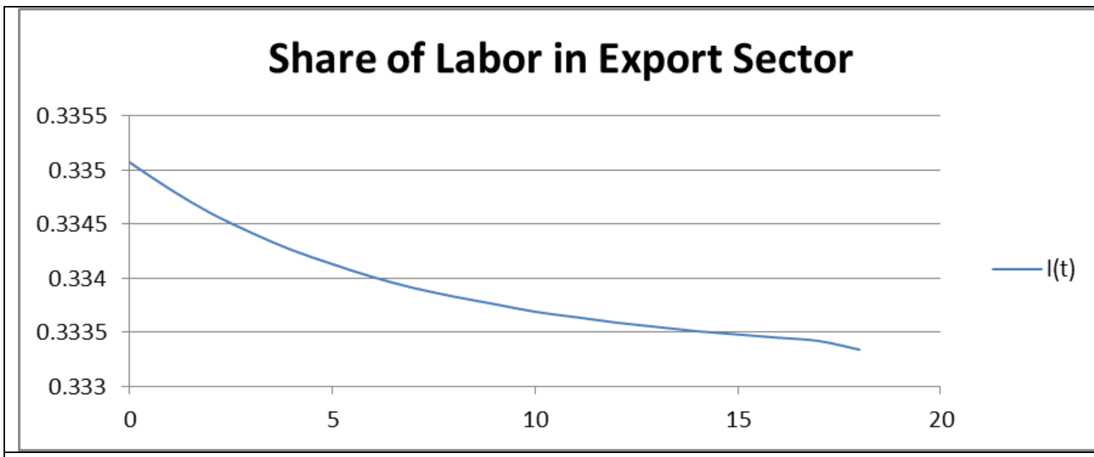

Figure 4.3. Share of Labor in Export Sector

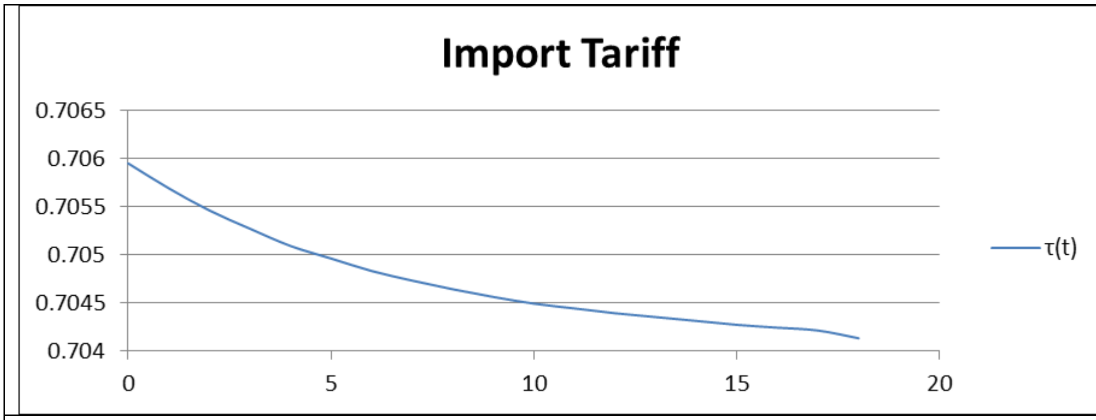

Figure 4.4. Optimal Import Tariff

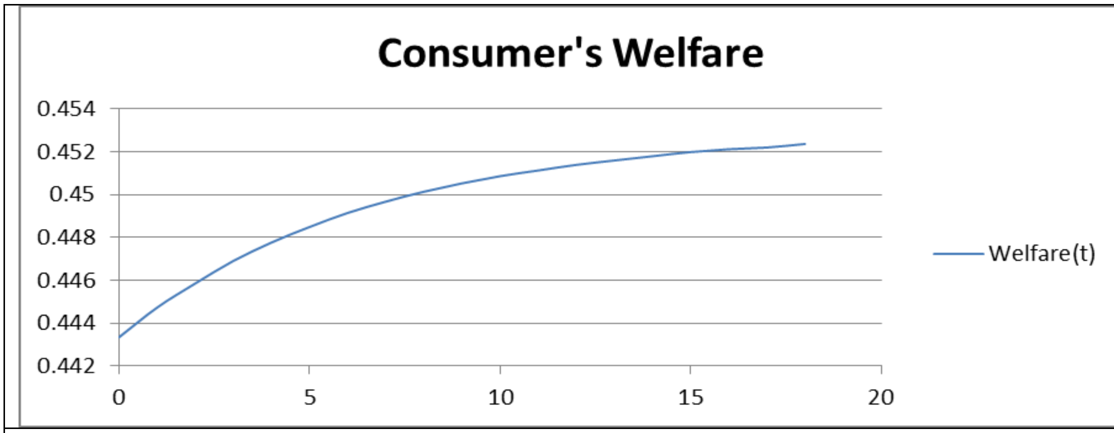

Figure 4.5. Consumer’s Welfare

Table 4.1. The values of parameters

L	A	B	θ	φ	β	γ	α	δ_k	δ_G
1	2	1	0.3	0.3	0.9	3	0.3	0.3	0.1

Table 4.2. The values of the steady state

G_{ss}	k_{ss}	l_{ss}	c_{mss}	c_{hss}	ϕ_{ss}	τ_{ss}	μ
1.208	0.6203	0.333	0.5431	0.2963	1.288	0.7041	0.4694

CHAPTER 5. CONCLUDING REMARKS

China has been accused of intentionally depressing the value of its currency, the renminbi, to gain unfair advantages in the global market. If producers and consumers in both countries act as price takers, the equilibrium prices of the traded goods are determined by the supplies and demands in the world market, and the benchmark yuan value of the dollar is determined in a competitive market. An important policy question is whether China will benefit from devaluing the yuan from the benchmark equilibrium rate. An optimal exchange rate may depend on the exchange rate pass-through to the yuan price of China's exports or the dollar price of U.S. imports. As the yuan peg is raised, the exchange rate pass-through may not be perfect. If the yuan price of China's export rises more than proportionately, then China's optimal policy is a yuan devaluation. China's elasticity of the export price index with respect to exchange rate has been less than unity since 2003. This implies that China's yuan devaluation policy has been consistent with the prediction of the main proposition.

While tariffs and non-tariff barriers are bound by various agreements in the WTO, there are no such agreements among countries on the bilateral exchange rates. The IMF monitors only the exchange rates of developing countries that are beset by large balance of payments deficits. The WTO regulates only tariff and various non-tariff barriers, but has made little effort to regulate the bilateral exchange rates because exchange rate practices are within the purview of the IMF. The finding that tariffs and the undervalued yuan have the same effects on domestic prices and import volumes suggests that currency pegging can be viewed as a trade policy. Either the WTO or IMF could begin to consider regulation of currency practices of developing countries in the face of large trade imbalances between major trading countries.

---

[i] Jin and Choi (2013) noted that while in the short run some profits might be generated by slightly deviating from the equilibrium exchange rates, in the long run excessive hoarding of reserve assets can only result in huge losses to PBC's balance of payment account.

[ii] If the equilibrium exchange rate is 6 (6 yuan = $1), then new renminbi can be issued so that 6 old yuan = 1 new yuan. With this currency conversion, the equilibrium dollar price of the new renminbi is 1. In the empirical work, we relax this assumption to derive the equilibrium exchange rates in each period.

[iii] In empirical analyses the value of ε^o cannot be fixed arbitrarily, but must be derived from the balanced trade condition.

[iv] The Chinese currency devaluation may not be the only source of U.S. trade deficits. For instance, Beladi and Oladi (2014) suggest that outsourcing may widen U.S. trade deficits. Also, Yue and Zhang (2013) emphasize that the U.S. trade deficit would not be reduced very much by a change in the Chinese exchange rate.

[v] Of course, the first-best policy is to remove wage and rent rigidity in the factor markets. Given this rigidity, Chinese government may be using yuan devaluation as a second-best policy.

[vi] In the same vein, Bruno (1976) considered a two-sector model, but defined the exchange rate as the ratio of the price of the tradable good to that of the nontradable good.

[vii] See Holtemöller and Mallick (2013) for a model of currency misalignment. They show that the higher the flexibility of the currency regime, the lower is the misalignment.

[viii] Devereux (2000) analyzed the impact of devaluation on the trade balance when exchange rate pass-through is imperfect.

[ix] For instance, if China is abundant in labor and production of *C* is labor-intensive, China is expected to export good *C* and import good *Z*.

[x] For instance, $F_{LL} = \alpha_1(\alpha_1 - 1)A_C L_C^{\alpha_1 - 2} K_C^{\beta_1} < 0$, $F_{KK} = \beta_1(\beta_1 - 1)A_C L_C^{\alpha_1} K_C^{\beta_1 - 2} < 0$, $F_{LK} = \alpha_1 \beta_1 A_C L_C^{\alpha_1 - 1} K_C^{\beta_1 - 1} > 0$, and $F_{LL}F_{KK} - \left(F_{LK}\right)^2 = \alpha_1\beta_1(1 - \alpha_1 - \beta_1)\left(A_C L_C^{\alpha_1 - 1} K_C^{\beta_1 - 1}\right)^2 > 0$.

[xi] Money as a financial instrument does not enter the utility function directly, but indirectly through a change in income, which ultimately affects consumption. See Al-Abri (2014) for a model in which utility depends on money balances.

[xii] If the benchmark equilibrium value of $1 is 6 RMB, new RMB notes can be issued at the rate of 6 RMB for 1 new RMB.

[xiii] $$\gamma\left(\frac{\delta\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}=(1-\gamma)\left(\frac{b\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}$$

[xiv] $$\gamma\left(\frac{\delta\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}=(1-\gamma)\left(\frac{b\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}$$

[xv] It can be shown that

$$\begin{aligned}
&A_0A_3-A_2A_4\\
&=\frac{(\alpha_2+\beta_2)\delta^{\frac{1}{1-\alpha_2-\beta_2}}}{(1-\alpha_2-\beta_2)}\left[1-\frac{(\alpha_1+\beta_1-\alpha_2-\beta_2)}{(1-\alpha_1-\beta_1)(\alpha_2+\beta_2)}\left(\frac{(\delta w^*)^{\alpha_1}(\delta r)^{*\beta_1}}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}\right]\\
&\times\left(\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{*\alpha_1}\,r^{*\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}\\
&-\delta^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}\delta^{\frac{1}{1-\alpha_2-\beta_2}}\frac{(\alpha_1+\beta_1)-(\alpha_2+\beta_2)}{(1-\alpha_1-\beta_1)(1-\alpha_2-\beta_2)}\left(\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}\\
&-\frac{\delta^{\frac{1}{1-\alpha_1-\beta_1}}(\alpha_1+\beta_1)}{1-\alpha_1-\beta_1}\left(\frac{\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{1/(1-\alpha_1-\beta_1)}\left(\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{*\alpha_2}\,r^{*\beta_2}}\right)^{1/(1-\alpha_2-\beta_2)}.
\end{aligned}$$

[xvi] Jiang and Kim (2013) show that exchange rate pass-through to the producer price index and the retail price index are both incomplete. Without concavity of $b(\delta)$, multiple optimal exchange rates may exist, including the exchange rate that yields some unemployment.

[xvii] Under assumption $\partial r/\partial\delta>0$, the effects of the exchange rate on output C and Z are

$$\begin{aligned}
\frac{\partial C(b,P)}{\partial\delta}&=\frac{1}{1-\alpha_1-\beta_1}\left(\frac{b^{\alpha_1+\beta_1}\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}\left(\frac{(\alpha_1+\beta_1)\delta}{b}\frac{\partial b}{\partial\delta}-\frac{\beta_1\delta}{r}\frac{\partial r}{\partial\delta}\right)\frac{1}{\delta}\\
&=\frac{1}{1-\alpha_1-\beta_1}\left(\frac{b^{\alpha_1+\beta_1}\alpha_1^{\alpha_1}\beta_1^{\beta_1}A_C}{w^{\alpha_1}r^{\beta_1}}\right)^{\frac{\alpha_1+\beta_1}{1-\alpha_1-\beta_1}}\left(-(\alpha_1+\beta_1)\varepsilon_{b\delta}-\beta_1\varepsilon_{r\delta}\right)\frac{1}{\delta},
\end{aligned}$$

$$\frac{\partial Z(b,P)}{\partial\delta}=\frac{1}{1-\alpha_2-\beta_2}\left(\frac{P^{\alpha_2+\beta_2}\alpha_2^{\alpha_2}\beta_2^{\beta_2}A_Z}{w^{\alpha_2}r^{\beta_2}}\right)^{\frac{\alpha_2+\beta_2}{1-\alpha_2-\beta_2}}\left((\alpha_2+\beta_2)\varepsilon_{P\delta}-\beta_2\varepsilon_{r\delta}\right)\frac{1}{\delta}>0.$$