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RANKING CAPITAL INVESTMENT ALTERNATIVES: A COMPUTER SIMULATION

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Ranking capital investment alternatives:

A computer simulation

by

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CHAPTER I. INTRODUCTION

There is an increasing interest throughout the world in the development of quantitative techniques and models to assist decision makers. When one considers the ever changing spectrum of problems decision makers face, the need for such tools is very apparent. The development of the modern, high speed computer has made possible several modeling techniques whose computational requirements would be nearly impossible to satisfy without use of a computer. An example of this is simulation. This technique has been known and practiced on a small scale for some time. However, the advent of the computer has made it possible to use simulation to solve larger scale problems that cannot be readily solved using classical solution methods.

Capital budgeting involves expending funds for long lived (more than one year) projects. One of the most important recurring tasks in capital budgeting is to allocate the financial resources of an enterprise, either public or private, in a manner that best achieves enterprise goals. Typically, capital budgeting decisions must be made in an environment characterized by uncertainty, incomplete information, and various other complex interactions. The impact of such decisions often continues far into the future.

Modern management principles emphasize the use of a systematic approach to improve upon intuitive analysis. This stimulus has encouraged the development of mathematical techniques for analyzing investment opportunities. These techniques have provided a theoretical basis for decision-making, and much of the research in capital budgeting has focused on developing and refining these quantitative solution procedures. This research describes a new approach for study of the classical capital budgeting dilemma of how to rank capital investment alternatives.

Ranking Capital Investment Alternatives

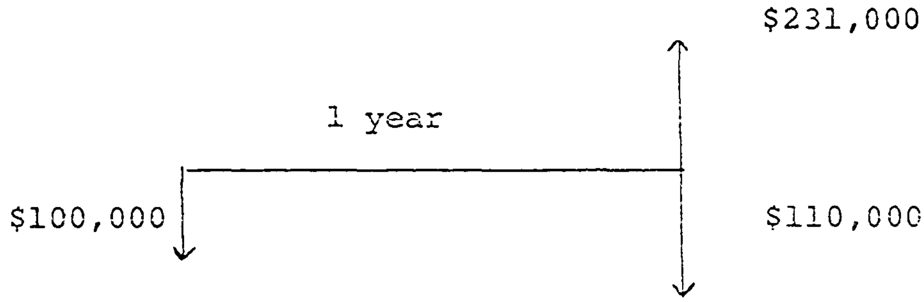
A number of methods for ranking capital investment alternatives have been advocated in the literature. Some authors argue that net present value is best, while others advocate rate of return, annual worth, payoff period, or other methods. From the perspective of the firm, the best method is the one that provides for the greatest net worth over some time horizon.

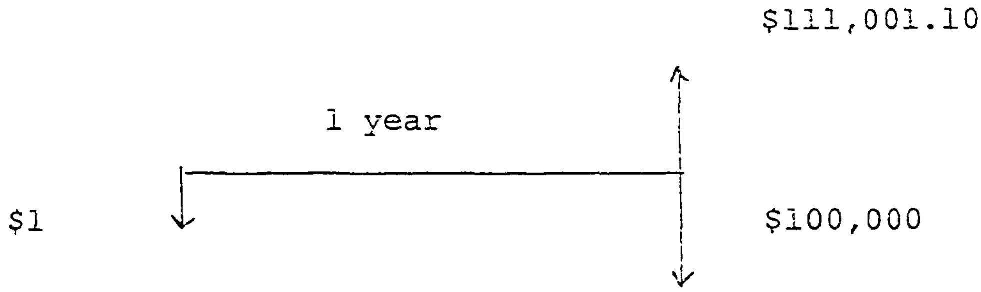
The difficulty that arises is that all of the methods currently in use may yield different results when applied to the same set of projects. At first this may appear surprising, but the various ranking criteria are measuring different things, and there is no reason to expect these different measures will yield the same ranking of projects. An analogy that sometimes proves useful in clarifying this inconsistency is to consider the high jump event at a track meet. There are different criteria that one can use to evaluate the participants. The usual method is to measure the greatest height an individual can jump over. However, there are other measures that could be used to determine a winner. Possibilities include: measuring the height and then dividing by the person's weight; measuring the height and then multiplying by the person's age; etc. The list of possible measures is practically endless, bounded only by one's imagination. In this example, determination of the winner is dependent upon the measuring method used.

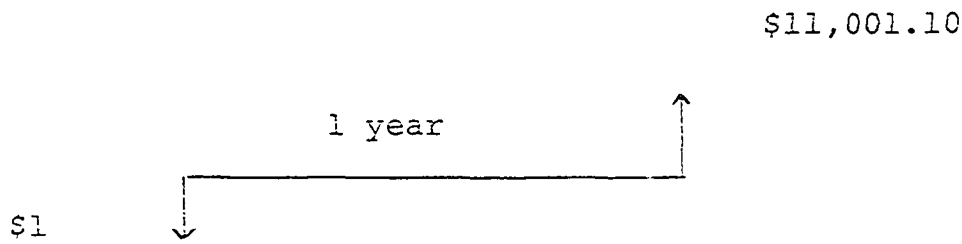
This same type of phenomenon has occurred in the ranking of capital investment alternatives. Many different methods have been proposed with each method having its proponents. Fortunately, the criterion for selecting the best ranking method is generally agreed upon as the one that provides the highest net worth of the firm.

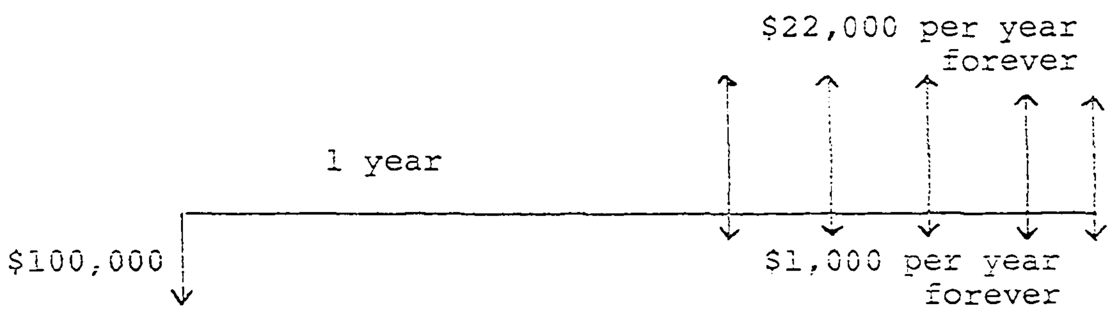
A short example will illustrate how six ranking methods, when applied to the same set of projects, can yield different rankings.

Consider the following projects with cash flows as given.









Assume that these cash flow diagrams represent four independent projects and that the projects must be ranked according to some criteria. For this example, the projects will be ranked using the following ranking methods.

ROR = rate of return

PEX = present equivalent excess of revenues over costs

PEX/B, where B is the initial investment at time zero

AEX = annual equivalent excess of revenues over costs

AEX/B, where B is the initial investment at time zero

PER/PEC, where PER is the present equivalent of revenue and PEC is the present equivalent cost. PER/PEC = (PEX+PEC)/PEC = (PEX/PEC)+1. It is analogous to the conventional benefit-cost ratio as in Smith (1979, p. 233).

PAYBACK = time required to recover initial investment

Using i = 10% and performing the necessary computations yields the following results:

	Project Number				Resulting
	1	2	3	4	Ranking
ROR	21%	1,100,010%	1,100,010%	21%	2-3,1-4
PEX	$10,000	$10,000	$10,000	$110,000	4,1-2-3
PEX/B	0.100	10,000	10,000	1.100	2-3,4,1
AEX	$11,000	$11,000	$11,000	$11,000	1-2-3-4
AEX/B	0.100	11,000	11,000	0.110	2-3,1-4
PER/PEC	0.05	0.11	10,000	1.00	3,4,2,1
PAYBACK	0.83	0.000009	0.000009	4.76	2-3,1,4

Even though the data are obviously contrived to illustrate the point, this simple example shows that contradictory rankings can result from proper application of various ranking methods. If available funding were either zero or infinite, the ranking procedure is of no consequence. Otherwise, the ranking method employed can affect the portfolio of projects selected.

Research Objectives

The purpose of this research is to compare various capital budgeting methods through further development and use of a previously developed computer simulation. The model is a basic multi-period horizon model, with extensions for interperiod borrowing and lending, budgetary constraints, and provisions for uncertainty. Particular attention is focused on the effects various parameters have on investment selection by the ranking process under capital rationing. The model is constructed to include both mutually exclusive and independent projects.

CHAPTER II. LITERATURE REVIEW

Literature pertaining to this research can be divided into three categories: 1) Capital Budgeting, 2) Mathematical Programming, and 3) Capital Budgeting Simulation.

Capital Budgeting

There is general agreement that the objective of capital budgeting is to allocate the capital resources of the firm so as to maximize the total wealth of the firm at some future date. The specific criteria employed to evaluate investment alternatives to achieve this goal has been an area of controversy, and has attracted significant attention. Criteria frequently analyzed include net present value, annual worth, benefit-cost ratio, internal rate of return, and payback.

Dean (1951) published a comprehensive monograph aimed at systematizing management's approach to capital budgeting. Dean advocates use of the rate of return index as a method that is easily understood in the business world, and is independent of an exogenously determined discount rate. McKean (1963) and Merrett and Sykes (1973) also assert that the internal rate of return criteria is both technically and practically superior. Osteryoung (1974) and Bierman and Smidt (1971) conclude that in a constrained environment (e.g., capital rationing), mathematical programming should be used in order to examine all combinations of projects.

Solomon (1959) attempted to resolve this conflict by suggesting explicit assumptions about reinvestment rates. Mao (1966) argues that the conflict between the net present value and internal rate of return criteria is traceable to the differing reinvestment rates implicitly assumed by the two criteria. Smith (1979) points out that the conflict over ranking is a result of "over specification" of the supply of funds, and that net present value and rate of return yield consistent results when properly applied. Jeynes (1968), Grant (1966), and Bedel and Mains (1973) all conclude that neither net present value nor rate of return makes any implicit assumptions about reinvestment rates.

Pegels (1968), and Leautaud and Swalm (1974) compare several ranking criteria and conclude that investment proposals should be evaluated on the basis of several decision criteria, rather than on the basis of one preselected criterion. Ross (1973) argues that when there is uncertainty, there is "no one best way" to make an investment decision.

Gitman and Forrester (1977) surveyed 110 United

States corporations and found that 53.6% use rate of return as the primary capital budgeting technique, with 44% using payback as a secondary technique. Net present value was used by 11% of the firms as the primary criteria, and by 25.8% of the firms as the secondary method.

Mathematical Programming

Lorie and Savage (1955) discussed some of the principal limitations of the rate of return approach for project selection, and presented a present value model designed to overcome some of these limitations. The objective of the model was to maximize net present value, subject to constraints on total expenditures in several periods. The model, which assumed that all cash flows are known with certainty, that projects are independent, and that fractional investments are allowed, used a form of the LaGrange multiplier technique to select a set of projects which explicitly considered the budgetary interactions of the projects.

Charnes, Cooper, and Miller (1959) demonstrated in general terms how linear programming could be used as a tool to help optimally allocate funds within an enterprise. Weingartner (1963), in his doctoral dissertation, showed that the Lorie and Savage problem could be expressed as a linear programming problem of the form:

$$\text{Maximize} \quad \sum_j b_j x_j$$

$$\text{Subject to} \quad \sum_j c_{tj} x_j \leq C_t \quad t = 1, \ldots, T$$

$$0 \leq x_j \leq 1$$

where

c_{tj} = cost of project j in period t

C_t = budget ceiling in period t

b_j = net present value of project j

x_j = fraction of project j accepted

Weingartner also presented the use of integer programming for indivisible projects and extended the basic linear programming model to include cases involving multiple budgets. In addition, Weingartner presented extensive analysis of the economic interpretations of the duality aspects of linear programming.

Baumol and Quandt (1965) criticized Weingartner for not addressing the problems of reinvesting funds and selecting the proper discount rate. They presented a model that makes use of a subjective utility index, and also provides an objective measure of the discount rate. They also argued that firms should maximize the utility of funds rather than the net present value.

Weingartner (1966) answered Baumol and Quandt by recognizing the difficulty of choosing a discount rate, and suggested a formulation that maximized dividend growth.

Meyers (1974) demonstrated that although the Baumol and Quandt model incorporated utility concepts, there is little difference between this model and Weingartner's.

Bernhard (1971) used mathematical programming to compare several capital budgeting ranking criteria. Bernhard concluded that in an unconstrained situation under certainty, with complete freedom to borrow or lend at one rate of interest, the present worth method is correct.

One of the major difficulties with the programming approach is the assumption of certainty. Chance constrained programming attempts to incorporate risk into the analysis by identifying those factors that when varied, significantly affect the solution. Naslund (1971) developed a chance constrained programming model that paralleled Weingartner's deterministic model. However, the result was a nonlinear programming problem with nonlinearities appearing in the constraints. Without simplifying assumptions, this model renders all but trivial problems too time consuming for solution.

Goal programming has been proposed as a technique that provides a more realistic model of the capital budgeting problem. Traditional programming formulations are restricted to the consideration of only a single objective function, whereas most real world problems involve several conflicting objectives. Lee and Lerro (1974) illustrated the advantages of incorporating multiple objectives into the selection of capital investments. Ignizio (1976) presented a multiple objective capital budgeting goal programming model that constrained variables to be zero or one. Taylor and Keown (1978) formulated a goal programming model where both profit and nonprofit projects are in competition for limited resources.

Capital Budgeting Simulation

Sundem (1975) constructed a manual simulation model to compare the performance of six capital budgeting models. The models included in the simulation were: 1) mean variance portfolio (MV), 2) MV with a diagonal simplification (MVD), 3) variability of returns (VR), 4) chance constrained programming (CCP), 5) net present value (NPV), and 6) payback. Sundem reported a high level of performance for the variability of returns model, a low level of performance for the net present value model in highly uncertain environments, and a decline in the performance level of the payback model between medium and high uncertainty environments. Sundem added that the results are completely dependent on the simulated environment and the specific parameters that were chosen.

Parra-Vasquez and Oakford (1976) described in general terms a model for using computer simulation as a technique for comparing decision procedures. Simulation was used to compare the effectiveness of: 1) sequential versus batch decision procedures, 2) logically exact versus approximate selection algorithms in the batch decision procedure, 3) three different decision procedures (maximum prospective value, net present value, and rank on prospective growth rate) when the marginal growth rate of the firm cannot be estimated accurately. The authors concluded: 1) firms should investigate the annual decision-making procedure as an alternative to sequential decision-making, or possibly consider a mix of the two procedures, 2) a relatively small improvement in average growth rate was achieved at relatively high computer cost by the exact mathematical programming models, 3) the three ranking procedures are almost equally effective if the marginal growth rate can be accurately estimated. If the marginal growth rate cannot be accurately estimated, the authors suggest the use of either maximum prospective value or rank on prospective growth rate.

Thomson (1976) used computer simulation to study six methods of ranking capital investment alternatives. The methods studied were: 1) internal rate of return,

2) modified rate of return, 3) annual worth, 4) net present value, 5) payback, and 6) random. Thomson reported that although the results were not conclusive regarding which ranking method is superior, the results indicated that heuristic modifications to known decision processes could improve the results of investment.

Salazar (1979) developed a computer simulation program to study the long-term consequences of consistently applying a variety of decision criteria under various conditions of uncertainty and incomplete information. Criteria studied were: 1) internal rate of return, 2) internal rate of return with cutoff, 3) net present value, 4) adjusted net present value, and 5) random. Salazar concluded that if an orderly decision procedure is used (any of the above except random), the choice of a procedure is not as important as maintaining the growth of the firm's investment opportunities, and of obtaining accurate estimates of the expected cash flows of investment proposals.

CHAPTER III. THE SIMULATION MODEL

Basis of the Model

The technique of simulation has long been an important tool in engineering. Applications include simulating airplane flight in a wind tunnel, plant layouts using scale models, and charts and graphs to simulate lines of communication. One of the major strengths of the simulation approach is that it abstracts the essence of the problem and thereby reveals its underlying structure. This enables one to gain insight into the cause and effect relationships within the system under consideration. One advantage of simulation is that it allows the system to be sub-divided into smaller component parts, combines these components into their natural order, and then allows the computer to determine the nature of their interaction with each other.

If it is possible to synthesize a mathematical model that closely represents the problem and is amenable to solution, the analytical approach is usually superior to simulation. However, many problems are so complex with so many interactive elements, that they cannot be solved analytically. In this case, simulation often provides the only practical way to solve the problem.

Given that a system and its associated measures of performance have been defined, Pritsker (1974) states that four basic steps should be performed in a simulation project.

1. Determine that the problem requires simulation;
2. build a model to solve the problem;
3. write a computer program to convert the model into an operating simulation program; and
4. use the computer simulation as an experimental device to resolve the problem.

The problem under study in this research involves investigating the performance of several investment opportunity selection criteria under various operating conditions. The criteria included in this study are:

AEX = Annual equivalent excess of revenues over costs

AEX/B, where B is the initial investment at zero

PEX = Present equivalent excess of revenues over costs

PEX/B, where B is the initial investment at time zero

PAYBACK = Time required to recover the initial investment

RANDOM = Randomly selecting projects for investment

Incr ROR = Rate of return on incremental investment

Incr AEX/B = AEX/B on incremental investment

Incr PEX/B = PEX/B on incremental investment

Incr PAYBACK = PAYBACK on incremental investment

One of the objectives of this research is to apply each of these criteria to the same set of data in a multiperiod context so that the relative desirability of the methods can be compared. The computational requirements of such a study are particularly suited to computer simulation.

In the next section a simulation model is presented. It was conceived and developed by Dr. Gerald W. Smith of Iowa State University and programmed in Fortran IV. Thomson (1976) used the simulation to compare the AEX, PEX, ROR, and PAYBACK ranking methods (although mutually exclusive alternatives were not considered, the procedure used by Thomson is consistent with incremental forms of ROR and PAYBACK). The model has since been expanded by Smith to include AEX/B, PEX/B, Incr AEX/B, Incr PEX/B, and Incr PAYBACK. This research uses and further develops this simulation model.

Development of the Model

One objective of studying ranking criteria is to establish the performance of the various criteria relative to each other. One way to compare this relative performance is to apply the various criteria to an identical data set, and then examine the results. This is readily accomplished in a simulation.

The model begins at time zero with a given set of independent and mutually exclusive projects. Initial financing is provided, and the first period investment activity is determined by ranking the projects in descending order of desirability according to a pre-established criteria. Projects are then accepted until the available funds are exhausted. If a project is rejected in one period, it cannot later be accepted in a subsequent period.

The next period begins with generation of a new set of projects. The available funds in this second period consist of cash flows generated by projects accepted in the first period, plus any cash carried over from the previous period. This links the capital budget in any period directly to the project selection matrix of previous periods. Newly generated projects are again ranked according to the preselected criteria, and projects are accepted until available funds are exhausted. This period by period selection of projects continues until the horizon date is reached. At this point, there will be some projects accepted in previous periods that have cash flows extending beyond the horizon date. These cash flows are discounted at some rate of interest to the horizon date. To those discounted cash flows the cash on hand at the horizon date is added. This sum represents the value of the firm at the horizon date.

Figure 3-1 illustrates how this model functions.

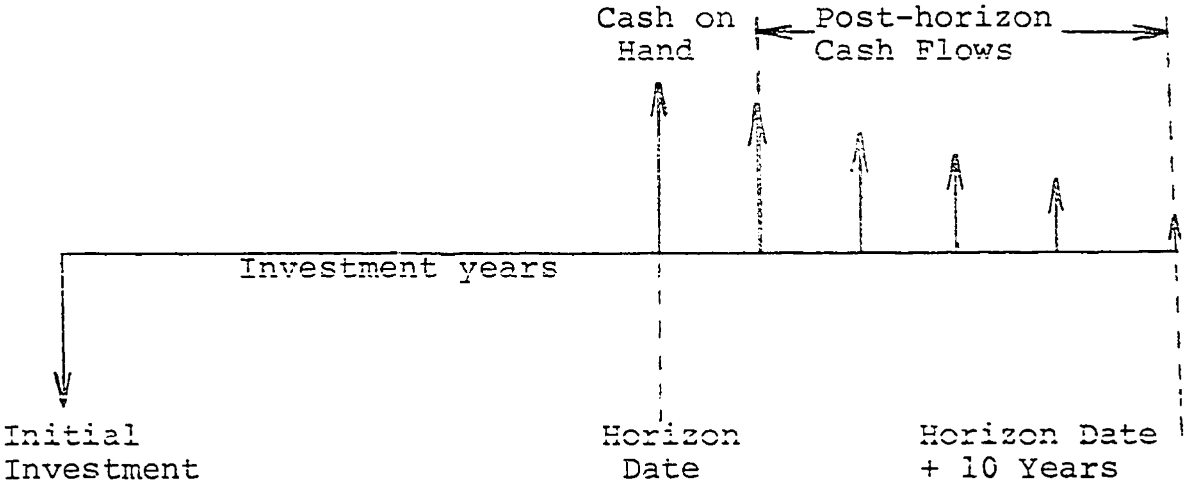


Figure 3-1. Model function

One objective is to adopt an inter-period project selection matrix that maximizes the net value of the firm at the horizon date. An alternative objective is to maximize the rate of return realized from: 1) the initial investment, 2) the cash on hand at the horizon date, and 3) the projected post-horizon date cash flows.

Quantifying, the objectives become:

Maximize net value at horizon, where

$$(\text{Net value}) = (\text{Cash on hand}) + \sum_{z=m}^{n} X_z (1+i)^{-(z-m)}$$

or

Maximize rate of return realized i, when

$$(\text{Initial investment}) = (\text{Cash on hand})(1+i)^{-m} + \sum_{z=m}^{n} X_z (1+i)^{-z}$$

where

m = horizon date

z = post horizon cash flow period ($m<z\leq n$)

X_z = cash flow at end of period z

n = horizon date plus life of longest-lived project

Figure 3-2 shows the logic and steps that are followed in the model just developed.

The model describes a basic framework for constructing a computer simulation program to dynamically study the relative performance of several ranking criteria. Such a program was conceived and developed by Dr. Gerald W. Smith of Iowa State University. This program, with various modifications and adaptations made by the author represent the basic tool used in this research.

The next section presents the program in sub-sections as an aid to the reader's understanding of the program.

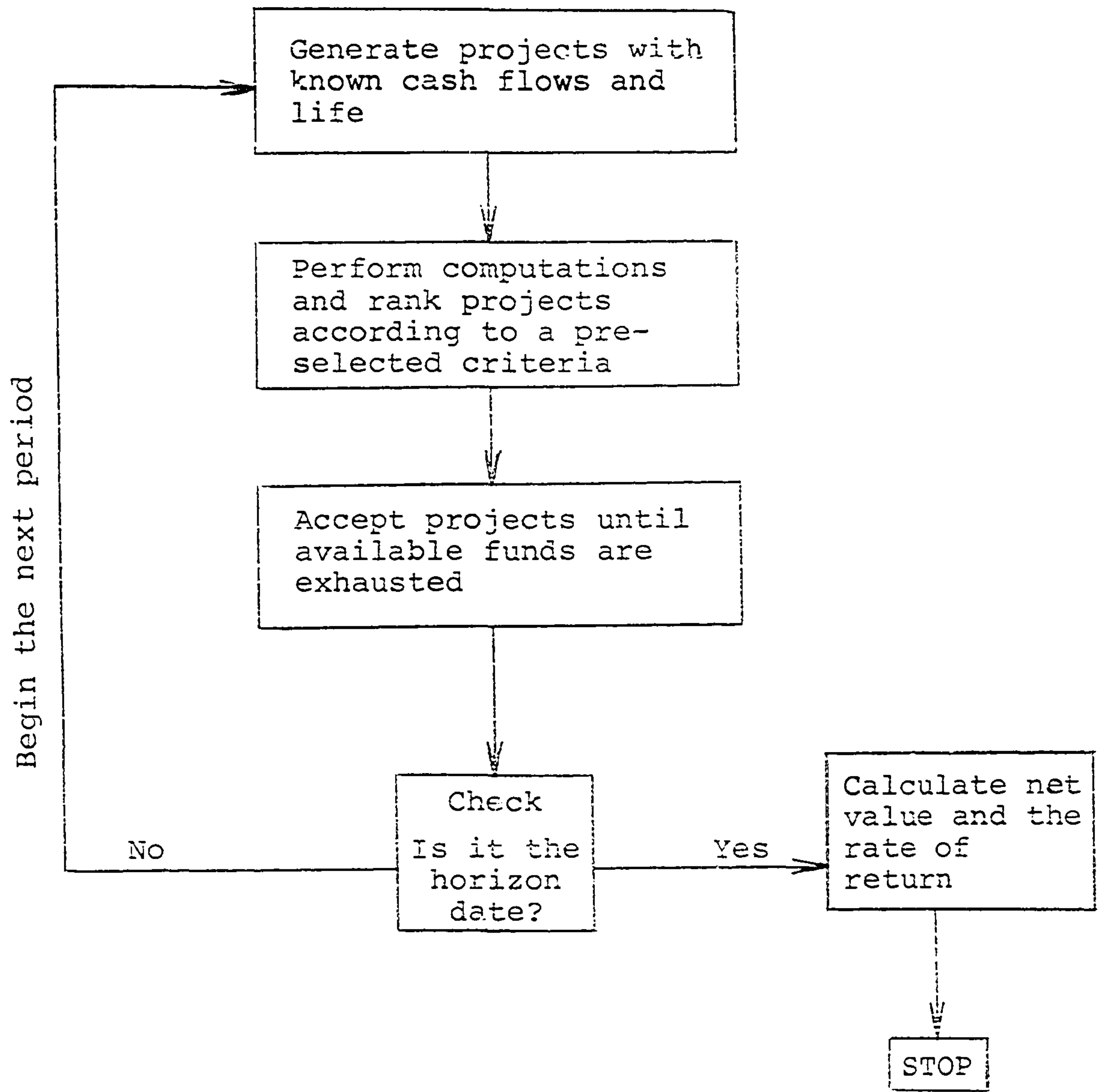


Figure 3-2. Simplified flow chart

The computer simulation program

The flow chart of Figure 3-3 represents an extension of the model presented in the previous section.

After input parameters have been initialized, the program generates one project with MX alternatives. For nonincremental decision criteria, the best of the alternatives is recorded. For incremental decision criteria, Smith's (1979, pp. 109-111) network diagram methodology is used to determine the relevant incremental choices, and the incremental decision path is recorded. The program continues in this manner until NP projects have been generated. Previously recorded projects are then rank ordered from most to least desirable. Projects are then accepted for investment until the available funds are spent. The next period then begins with generation of NP new projects with MX alternatives per project. This process continues on a period by period basis until the horizon date is reached. At this point the net value of the firm and the rate of return realized on initial funds supplied are calculated.

Although the program could be set to simulate any number of investment periods, with any number of projects available per period, the cost of running the simulation imposes some practical limitations on each.

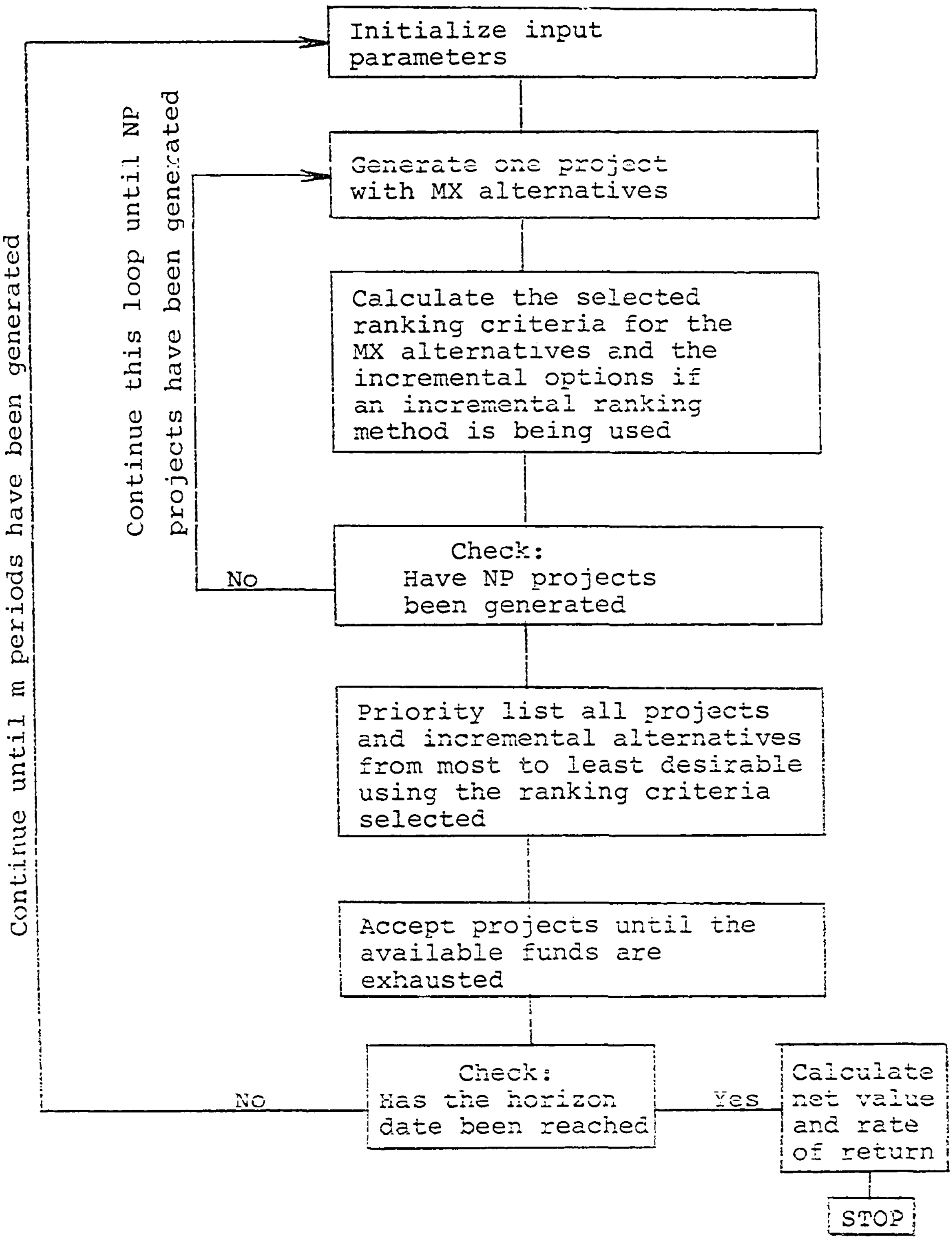


Figure 3-3. Simulation block diagram

Optional characteristics of the program include:

1. Internal rationing of funds
2. Stochastic cash flows
3. Inter period borrowing and lending

Setting the environment of the firm

To start the simulation, several beginning parameters need to be chosen:

1. The number of independent projects (NP) per period and the number of mutually exclusive (MX) alternatives within each project set is variable.
2. M, the number of periods of experience simulated (the horizon date).
3. The beginning capital budget.
4. Project indivisibility options.
5. Proportion of mandatory projects.
6. Relationship between forecast and actual cash flows.
7. Project characteristics such as life, rate of return, etc.

Items three through seven are detailed in the material that follows.

Beginning capital budget

The beginning capital input, which is user-variable, has a direct impact on the degree of capital rationing encountered in later periods. Program experience showed that the number of projects accepted in the first few periods was somewhat erratic. This start up transient effect, common in many simulation studies, was reduced by providing an additional capital input in year two. This supplemental input allows more projects to be accepted in year two, and stabilizes the number of projects accepted and the cutoff rate of return in later years. Capital inputs are specified by the statements:

ATCF1 = 700000.

ATCF2 = 600000.

ATCF1 and ATCF2 denote the after tax cash flows one and two. This represents the external funds supplied to the firm in years one and two.

Project indivisibility options

The program provides for three project indivisibility options. The first option is to accept only whole projects, so the allocation of funds ceases when the next-ranking project cannot be fully funded. The second option is to accept any portion of the next ranking project permitted by remaining funds. The third option is to search the list of remaining projects and accept the next project in line that has a first cost less than or equal to the funds remaining. Any funds not invested at the end of a period are carried over to the next period. In option two, when a fractional project is accepted, the portion of the project not accepted is not carried into the next period. Accepting fractional projects thus generally allows for complete spending of funds. The indivisibility options are controlled by the following:

```
      GO TO (76,72,73)INDIV
72    ACC(K)=(AVAIL-CASHMN)/BI(K)
      IF(ACC(K).LT.0.0)ACC(K)=0.0
      AVAIL=AVAIL-BI(K)*ACC(K)
      GO TO 76
73    KK=KK+1
      IF(KK.GT.NIC)GO TO 76
      K=ORDER(KK)
      KPLUS1=ORDER(KK+1)
      IF(BI(K).GT.BI(KPLUS1))GO TO 73
      AVAIL=AVAIL-BI(K)
      IF(AVAIL.LT.CASHMN)GO TO 75
      ACC(K)=1.
      GO TO 73
75    AVAIL=AVAIL+BI(K)
76    DO 77 K=1,NIC
      ACCEPT=ACCEPT+ACC(K)
77    CAPEXP=CAPEXP+BI(K)*ACC(K)
```

Proportion of mandatory projects

The proportion of the NP projects generated that require mandatory acceptance is a user controlled variable PM ($0 \leq PM \leq 1$). The status of a project, either mandatory or discretionary is determined by the value of a uniformly distributed, zero to one random number. While selecting a value for PM causes approximately this percentage of projects generated to be mandatory, it does not mean that PM percent of the available funds will be spent on mandatory projects. If a project is mandatory, only the least cost project alternative must be accepted. Investment in any of the remaining incremental alternatives in that set is discretionary. If a project is mandatory, investment in discretionary project increments is determined for incremental ranking methods by using Smith's (1979, pp. 109-111) network diagram methodology.

Relationship between forecast and actual cash flows

The program allows for two options in determining the funds available (AVAIL) for investment in each period; deterministic and stochastic. The after-tax cash flows forecast for the coming period are multiplied by a normally distributed random variable with a mean of one, and a user selected variance. If the variance is zero, the forecast cash flows are multiplied by one, and the result is a deterministic inter-period amount. If the variance is any positive number, funds available will be a normally distributed random variable. This is accomplished by the following:

```
      V=1.0
      IF(SIGMA.EQ.0.0)GO TO 45
  39  AT=0.0
      AM=1.0
      S=SIGMA
      DO 527 I=1,12
 527  AT=AT+RAN(1)
      V=(AT-6.0)*S+AM
  45  AVAIL=ATCF(1+NOW)*V+CARRYO
```

Project characteristics

Other parameters that need to be specified before investment activity can begin are the characteristics of the individual projects.

First cost

Recall that the model generates NP independent projects with MX mutually exclusive alternatives per project. The program is constructed so that the first cost of each MX alternative is an integer multiple of the lowest cost alternative. For example, if project set 3 has four mutually exclusive alternatives, 3A, 3B, 3C, and 3D, alternative 3B will have a first cost two times that of project 3A. Alternative 3C will have a first cost three times that of 3A, and 3D will have a first cost four times that of 3A.

In period one, the lowest cost alternative is assigned a first cost of \$50,000. This results in mutually exclusive alternatives with first costs of \$100,000, \$150,000 etc., respectively. As the firm's wealth increases through investment, the first cost of each alternative is increased by a fixed percentage in each succeeding period:

$$B_z = \$50,000(x)^{z-1} \quad z=1,\ldots,M$$

where

M = horizon date

\$50,000 = cost of project in year one

B_z = cost of project at beginning of year z

z = current investment year

x = one plus the rate of increase

This assumption of increasing first cost is used so that approximately the same number of projects will be selected each period. The growing wealth of a firm could otherwise permit acceptance of all projects generated, in which case the ranking criteria employed would be irrelevant.

<u>Life, cash flow pattern, and mandatory status</u>

The life of each project is either two or ten years, determined on a random basis, with an equal probability of either life. These lives were chosen to represent relatively short-lived and long-lived projects.

The cash flow pattern in each project is either positive uniform or positive gradient. The cash flow pattern is determined randomly, with 50% being uniform and 50% being gradient.

```
N(I)=2
SLOPE(I)=0
MAND(I)=1
IF(YFL(1).GT.0.5)N(I)=10
IF(YFL(1).GT.0.5)YFL(1)=YFL(1)-0.5
IF(YFL(1).GT.0.25)SLOPE(I)=1
IF(YFL)(1).GT.0.25)YFL(1)=YFL(1)-0.25
YFL(1)=YFL(1)*4
IF(YFL(1).GT.PM)MAND(I)=0
```

YFL(1) is a uniformly distributed, zero to one random number, that determines life, cash flow pattern, and mandatory status of the project as follows:

ORIGINAL YFL(1)	N	SLOPE	MANDATORY IF PM=0.250
0.0000-0.0625	2	0	1
0.0625-0.2500	2	0	1
0.2500-0.3125	2	1	0
0.3125-0.5000	2	1	0
0.5000-0.5625	10	0	0
0.5625-0.7500	10	0	0
0.7500-0.8125	10	1	0
0.8125-1.0000	10	1	0

Rate of return

If first cost, life, cash flow pattern, and rate of return are known, year-by-year cash flows can be calculated. Rate of return is randomly generated for each alternative by:

ROR(J,1)=2.00/(5.**(1+YFL(J+1))

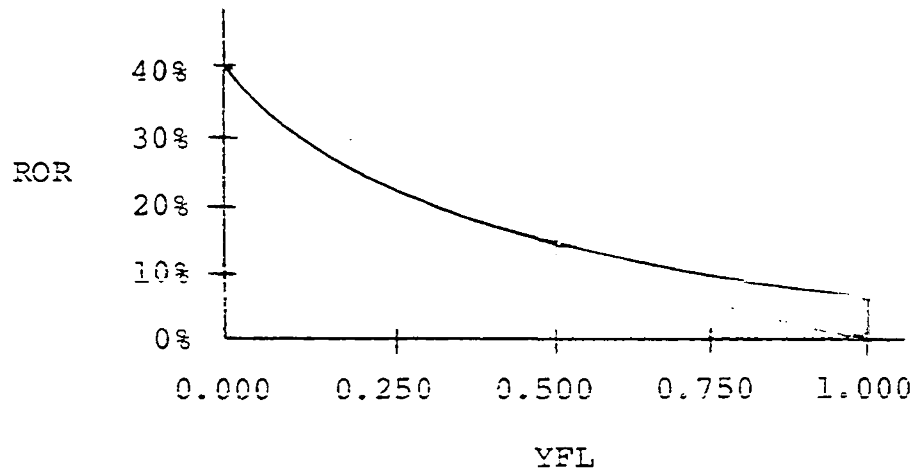
where

YFL( ) is a uniformly distributed, zero to one random number

The 2.00/5. establishes the limits of the rate of return, and is user variable. The table below lists various inputs and the resulting limits on project rates of return.

Input Variable	ROR Limits
2.00/5.	8% to 40%
1.28/4.	8% to 32%
0.90/3.	10% to 30%

Use of the equation above results in the following distribution of rates of return, with a median of 17.89% and a mean of 19.96%.



The rate of return statement is part of a loop that generates the cash flows for each mutually exclusive alternative:

```
      DO 1 J=1,MX
      ROR(J,1)=2.00/5.**(1.+YFL(J+1))
      B(J)=J*50000.*1.20**(NOW-1)
      A(J+1,1)=-B(J)
      NN=N(I)
      IA=ROR(J,1)
      AEP=IA/(1-(1+IA)**(-NN))
      PEG=((1-(1+IA)**(-NN))/IA-NN*(1+IA)**(-NN))/IA
      CFL(J)=B(1)*AEP*(1-SLOPE(I))
    1 G(J)=SLOPE(I)*B(1)/PEG
      DO 510 J=2,MX
      CFL(J)=CFL(J-1)+CFL(J)
  510 G(J)=G(J-1)+G(J)
```

If for example, project number one has four mutually exclusive alternatives, the alternatives would be labelled 1A, 1B, 1C, 1D.

The program generates the rates of return along the diagonal of the rate of return matrix:

Project	ROR on incremental investment compared to			
	0	A	B	C
1A	x			
1B		x		
1C			x	
1D				x

After generating the rates of return on the diagonal, the remaining rates of return can be calculated. An example of a complete rate of return matrix is:

Project	ROR on incremental investment compared to			
	0	A	B	C
1A	35.2			
1B	23.4	10.0		
1C	22.2	15.1	19.8	
1D	19.3	13.4	14.9	9.7

First cost, life, slope, rate of return, and the associated cash flows have now been determined for each of the MX alternatives of project number one. The program then repeats these steps for each of the NP projects to be generated.

Ranking criteria

The program calculates the following ranking criteria:

1. AEX
2. AEX/B
3. PEX
4. PEX/B
5. PAYBACK
6. RANDOM
7. ROR on incremental investment = Incr ROR
8. AEX/B on incremental investment = Incr AEX/B
9. PEX/B on incremental investment = Incr PEX/B
10. PAYBACK on incremental investment = Incr PAYBACK

RANDOM indicates that alternatives will be ranked randomly. Methods one through six are nonincremental decision rules. Method one, for example, requires us to select for further consideration the AEX-maximizing alternative in each project set, then rank those in descending order of AEX. Methods seven through ten are incremental ranking methods. These methods begin by selecting the best alternative from each independent project, and then use Smith's (1979, pp. 109-111) network diagram methodology to select additional project increments.

The ranking process

For the nonincremental ranking criteria (numbers one through six) the best alternative is selected for each independent project from its associated set of mutually exclusive alternatives. This is done for each independent project and results in a list of NP projects. If there are mandatory projects, the least cost alternative in the set is treated as mandatory, then the best increment from that least cost alternative is considered. If the increment has a positive value it is treated as discretionary; if negative, it is ignored. When mandatory projects are involved, the list of ranked projects can thus be greater than NP.

For incremental ranking methods (numbers seven through ten) the best alternative is selected within the set. Smith's (1979, pp. 109-111) network diagram methodology is then used to select remaining alternatives in the set. When mandatory projects are involved, the lowest cost alternative is selected first. Smith's (1979, pp. 109-111) network diagram methodology is then used to select remaining alternatives in the set.

The program then tabulates these investments and incremental investments in rank order, from most to least desirable. This is accomplished by the following program segment.

```
      ORDER(I)=I
      DO 31 I=1,NIM1
      IPLUS1=I+1
      DO 31 J=IPLUS1,NI
      IF(Y(I).GE.Y(J)) GO TO 31
      TEMP=Y(I)
      Y(I)=Y(J)
      Y(J)=TEMP
      TEMP=ORDER(I)
      ORDER(I)=ORDER(J)
      ORDER(J)=TEMP
   31 CONTINUE
```

Allocating the available funds

Investment opportunities have been generated and ranked according to the selected ranking criteria. The program now accepts investments and incremental investments until the available funds are exhausted. The hierarchy of project acceptance is:

1. Accept all mandatory projects
2. Accept all projects with a rate of return greater than the preset level RORGO
3. Accept additional projects as funds allow
4. Never accept projects below a preset minimum rate of return RORMIN, even if funds allow

RORGO and RORMIN are input parameters that are user variable. With the ROR generator limits of 0.08 and 0.40, the user can bypass lines 2 and 4 above by setting RORGO greater than 0.40 and RORMIN less than 0.08.

The program has provisions for inter-period borrowing and lending. If it is necessary to borrow, (for example, to accept all mandatory projects) extra funds are available at a user-specified rate of interest (30% in this simulation). If there are unspent funds in any period, they can be invested for the next period at a user-specified rate of interest (5% in this simulation).

Next period investment

The program now begins period two. NP new projects with MX mutually exclusive alternatives are generated. These projects are ranked, selected for investment, and the available funds are spent. This process continues on a period by period basis for M periods.

Computing the results of investment

After the horizon date has been reached, (1) the sum of current cash plus the present worth of future cash flows, and (2) the rate of return realized on initial funds supplied are calculated in the following statements.

```
      PEATCF=0.0
      DO 48 K=MPLUS1,MNY
   48 PEATCF=PEATCF+ATCF(K)/(1+IAR)**(K-M)
      FLAG=1
      AA=0.0
      DELTA=0.10
   80 AA=AA+DELTA
      PEX=0.0
      DO 81 K=MPLUS1,MNY
   81 PEX=PEX+ATCF(K)/(1+AA)**K
      PEX=-ATCF1+PEX+CARRY0/(1+AA)**M-ATCF2/(1+AA)
C
      IF(AA.LT.0.0)GO TO 85
      IF(PEX)82,85,83
   82 IF(FLAG.EQ.1)DELTA=-DELTA/2.
      FLAG=0
      GO TO 84
   83 IF(FLAG.EQ.0)DELTA=-DELTA/2.
      FLAG=1
   84 IF(ABS(DELTA).GT..0.0002)GO TO 80
   85 RR(ICYCLE)=AA
```

Additional cycles

The sequence of events just completed represents one cycle of the program. The program is structured so that one or more cycles are run using the same ranking criteria. After the desired number of cycles have been simulated, the program can be directed to start over again using another ranking criteria. If this option is selected, the program generates exactly the same set of projects that were generated for the previous ranking criteria. This is accomplished by restarting the random number generator at precisely the same point anytime a different ranking criteria is selected. This results in the inter-period project matrix being exactly the same for all the ranking criteria.

CHAPTER IV. RESULTS FROM SIMULATION

One of the objectives of this study is to investigate the relative desirability of capital budgeting ranking techniques under various operating conditions. In the simulation there are several input parameters that affect these conditions of the study, and many of these parameters have a large number of possible values. This results in an almost limitless number of feasible combinations.

Each program execution requires between 80,000 and 300,000 statement executions per cycle, depending on the ranking method selected. The constraint of a finite computer budget makes it impossible to study all of the parameter combinations. Therefore, this study focuses on investigating the effect that some of the major input parameters have on the relative performance of the ranking methods.

Unless otherwise specified, the parameters are set at:

1. ATCF1 = \$700,000
   ATCF2 = \$600,000
2. Length of simulation, M = 9 years
3. Number of projects, NP = 10
4. Number of mutually exclusive alternatives, MX = 4
5. Project lives, 2 or 10 years
6. Percent mandatory, PM = 0.0
7. Project indivisibility, INDIV = 2, accept fractional projects
8. Sigma = 0.0

Table 4-1 presents the results obtained by simulation performed with the above parameter values. Table 4-1 and all succeeding tables (unless otherwise noted) will present data that represent the average value obtained for five complete cycles. Both the net value in millions of dollars at the horizon date, and the rate of return realized on initial funds supplied the firm are given in this and most succeeding tables.

Table 4-1. Results achieved through five cycles of simulation

Rank by	Net value in millions	ROR realized
AEX	$10.31	25.14%
AEX/B	10.53	25.52
PEX	10.23	24.98
PEX/B	10.52	25.34
PAYBACK	7.81	22.48
RANDOM	5.99	19.64
Incr ROR	11.08	25.94
Incr AEX/B	11.10	25.92
Incr PEX/B	10.88	25.66
Incr PAYBACK	6.46	20.48

Table 4-1 shows that the ranking methods do yield different results for the net value of the firm at the horizon date and for the rate of return realized on initial funds supplied. Incr ROR, Incr AEX/B, and Incr PEX/B yield the highest net value and ROR. AEX/B and PEX/B are next, followed by AEX and PEX. PAYBACK and Incr PAYBACK do worse than any of the other methods except RANDOM. The ROR realized by the RANDOM ranking method, 19.64%, is close to the mean ROR of the random generator, 19.96%.

ATCF - The External Funds Invested

ATCF1 and ATCF2, hereafter referred to as ATCF, represent the total funds invested in the firm and directly control the number of projects accepted for investment in periods one and two. This in turn has a strong influence on determining the level of rationing encountered by the firm in later periods.

Several values of ATCF were tried in the simulation to determine the resulting effect on the relative performance of the ranking methods. Table 4-2 presents the data obtained by allowing ATCF to range from $3,700,000 to $25,000.

The data in Table 4-2 show that at all ATCF levels,

Table 4-2. Net value and ROR realized at various ATCF levels

ATCF	AEX	AEX/B	PEX	PEX/B	PAYBACK	RANDOM	Incr ROR	Incr AEX/B	Incr PEX/B	Incr PAYBACK
Net value										
$3,700,000	19.97	17.87	20.10	17.85	17.31	15.11	20.69	20.72	20.82	18.36
2,800,000	17.59	15.74	17.63	15.79	14.51	12.46	17.85	17.87	17.95	15.03
2,200,000	15.15	14.02	15.23	14.10	13.55	10.21	15.46	15.52	15.58	11.54
1,700,000	12.61	12.28	12.69	12.29	10.28	7.59	13.10	13.16	13.12	8.60
1,300,000	10.31	10.53	10.23	10.52	7.81	5.99	11.08	11.10	10.88	6.46
900,000	7.64	8.30	7.55	8.23	5.32	3.98	8.69	8.68	8.46	4.53
700,000	6.20	7.02	6.07	6.86	4.13	3.15	7.31	7.30	7.08	3.68
500,000	4.71	5.57	4.47	5.39	3.08	2.39	5.75	5.66	5.52	2.78
300,000	2.96	3.84	2.76	3.73	1.82	1.50	3.96	3.88	3.76	1.85
150,000	1.53	2.22	1.41	2.13	.981	.754	2.31	2.25	2.15	1.02
75,000	.793	1.16	.704	1.08	.556	.422	1.23	1.17	1.09	.560
25,000	.270	.412	.249	.397	.207	.110	.473	.412	.397	.207

Table 4-2 (Continued)

ATCF	AEX	AEX/B	PEX	PEX/B	PAYBACK	RANDOM	Incr ROR	Incr AEX/B	Incr PEX/B	Incr PAYBACK
ROR REALIZED										
$3,700,000	21.29	20.15	21.35	20.15	19.83	18.45	21.63	21.64	21.70	20.42
2,800,000	22.82	21.70	22.84	21.74	20.94	19.34	22.96	22.98	23.04	21.28
2,200,000	23.75	22.95	23.74	23.01	21.88	19.68	23.95	24.00	23.98	21.02
1,700,000	24.52	24.29	24.52	24.27	22.54	19.73	24.91	24.97	24.86	20.70
1,300,000	25.14	25.52	24.98	25.34	22.48	19.64	25.94	25.92	25.66	20.48
900,000	25.71	26.77	25.52	26.51	22.34	19.18	27.21	27.15	26.72	20.59
700,000	26.11	27.61	25.79	27.19	22.34	19.31	28.01	27.91	27.41	21.10
500,000	26.65	28.56	26.02	28.08	22.77	19.91	28.98	28.67	28.20	21.69
300,000	26.94	29.66	26.14	29.30	22.62	20.30	30.14	29.62	29.26	22.76
150,000	27.16	31.12	26.29	30.63	23.55	20.26	31.56	31.08	30.59	23.99
75,000	27.35	31.31	26.29	30.63	25.08	21.25	32.40	31.25	30.60	25.14
25,000	27.25	31.11	26.10	30.63	25.52	17.88	32.41	31.11	30.63	25.52

the incremental ranking methods, Incr ROR, Incr AEX/B, and Incr PEX/B, perform better than either the nonincremental ranking methods or Incr PAYBACK. The relative superiority of these methods increases as the level of ATCF decreases. The table also shows that PAYBACK and Incremental PAYBACK produced worse results than any of the other methods except RANDOM. At all levels of ATCF there is little difference in the results obtained by AEX and PEX. The same is true for the pair AEX/B and PEX/B, and for three incremental methods (Incr ROR, Incr AEX/B, and Incr PEX/B). There seem to be important differences however, in the values obtained among these three groups.

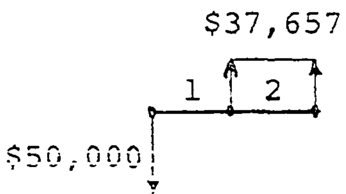
Bias of Ranking Methods

The ranking methods yield different results because each method selects a somewhat different set of projects each investment period. All of the ranking methods tend to favor certain project characteristics such as high or low first cost, long or short life, etc. The projects selected for investment by the various methods will have different life and first cost characteristics which reflect the tendencies of the respective methods to favor these project characteristics.

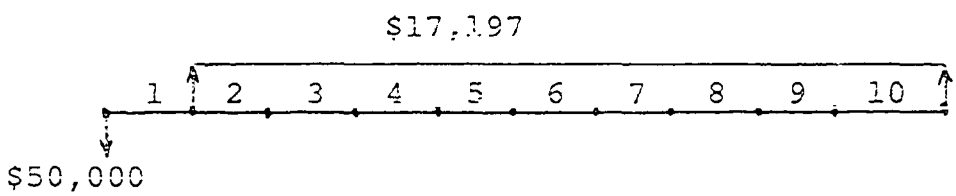
To illustrate this relative bias of the ranking methods, four examples will be presented showing how different ranking methods tend to favor different project characteristics. In the simulation the RANDOM criteria can be used as a benchmark for comparison. In these examples, ROR will be used as the benchmark, and the AEX, AEX/B, PEX, PEX/B, and PAYBACK methods will be compared relative to the ROR method. The choice of ROR as a benchmark is one of convenience only; any of the methods could be used.

Each example will consist of two independent alternatives with the same rate of return, but with different life and/or first cost characteristics. The discount rate used in all examples is 20%.

<u>Example 1: Equal first cost, unequal life</u>



ROR = 32.3%   PEX/B = .1506
PEX = $7532   AEX/B = .0986
AEX = $4930   PAYBACK = 1.328

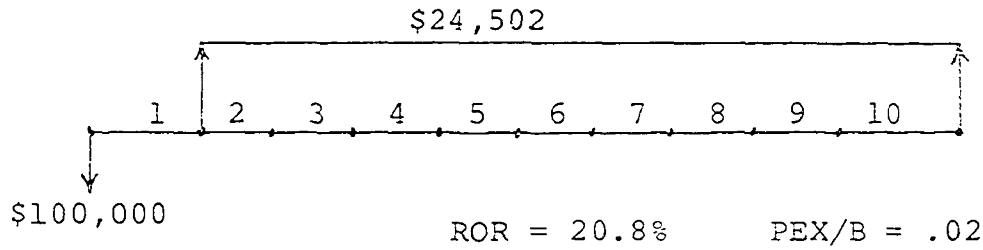


ROR = 32.3%   PEX/B = .4418
PEX = $22,090   AEX/B = .1054
AEX = $5269   PAYBACK = 2.907

This example illustrates that when two alternatives have the same first cost, unequal lives, and an ROR greater than the discount rate, PEX, AEX, PEX/B, and

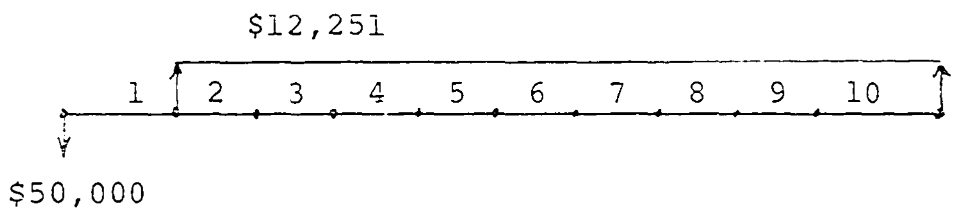
AEX/B criteria will favor the longer-lived project, while PAYBACK will favor the shorter-lived project.

Example 2: Equal life, unequal first cost



ROR = 20.8%
PEX = $2712
AEX = $647

PEX/B = .0271
AEX/B = .0065
PAYBACK = 4.08

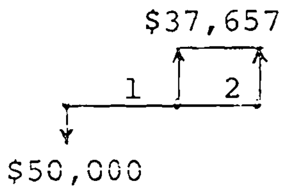


ROR = 20.8%
PEX = $1356
AEX = $324

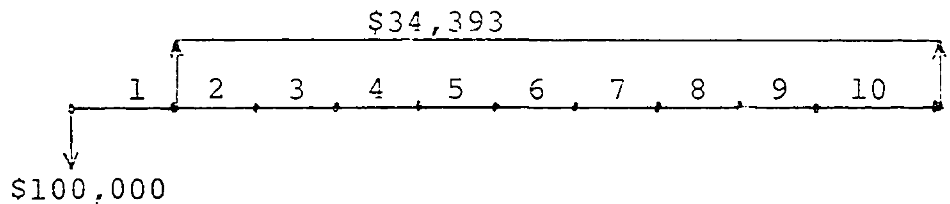
PEX/B = .0271
AEX/B = .0065
PAYBACK = 4.08

This example illustrates that when two alternatives have the same life, different first costs, and an ROR greater than the discount rate, PEX, and AEX criteria will favor the higher first cost alternative, while PEX/B, AEX/B, and PAYBACK indicate a standoff.

Example 3: Unequal life and first cost with longer-lived alternative having a higher first cost



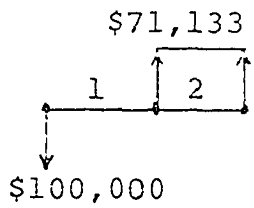
ROR = 32.3%   PEX/B = .1506
PEX = $7532   AEX/B = .0986
AEX = $4930   PAYBACK = 1.328



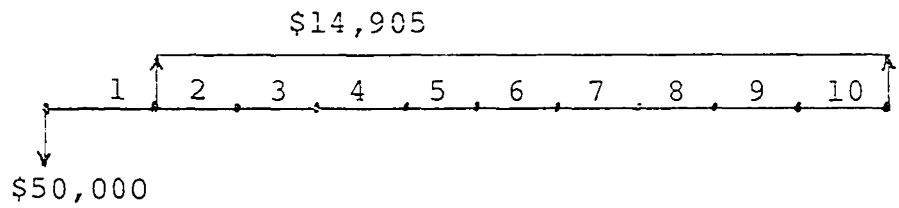
ROR = 32.3%   PEX/B = .4418
PEX = $44,178   AEX/B = .1054
AEX = $10,537   PAYBACK = 5.81

This example illustrates that when two alternatives have unequal lives and first costs, with the longer-lived alternative costing more and an ROR greater than the discount rate, PEX, AEX, PEX/B, and AEX/B criteria will favor the longer-lived, higher first cost project, while PAYBACK will favor the shorter-lived, lower first cost project.

Example 4: Unequal life and first cost with shorter-lived alternative having a higher first cost



ROR = 27.1%　PEX/B = .0869
PEX = $8691　AEX/B = .0569
AEX = $5689　PAYBACK = 1.406



ROR = 27.1%　PEX/B = .2496
PEX = $12,481　AEX/B = .0595
AEX = $2977　PAYBACK = 6.709

This example illustrates that when two alternatives have unequal lives and first costs, with the shorter-lived alternative costing more, and an ROR greater than the discount rate, PEX, PEX/B, and AEX/B will favor the longer-lived, lower first cost project, while AEX and PAYBACK will favor the shorter-lived, higher first cost project.

In general, when compared relative to the ROR ranking method, and when project rate of return is greater than the discount rate, AEX will favor high first cost, long-lived projects in that order. When choosing between a high first cost short-lived project or a low first cost long-lived project, AEX will favor the high first cost short-lived project. PEX will favor long-lived, high first cost projects in that order. This method tends to favor a long-lived low first cost project over a short-lived high first cost project. AEX/B and PEX/B have the same bias as their AEX and PEX counterparts. However, dividing by B, the first cost, reduces the extent of the bias. PAYBACK favors short-lived projects with relative indifference regarding first cost.

The bias of ROR relative to these methods is of course just the reverse of the biases discussed above. For example, ROR when compared relative to PEX tends to favor short-lived, low first cost projects.

Projects Accepted for Investment

The projects generated in this simulation are short-lived (2 years) or long-lived (10 years), with four levels of first cost providing a range of low to high first cost projects. These project characteristics were chosen to provide an opportunity for the relative bias of the various ranking methods to affect which projects would be selected for investment.

Table 4-3 shows the first cost and life characteristics of the projects accepted for investment by each of the

Table 4-3. Characteristics of projects accepted for investment

RANKBY	Number of Projects Accepted per Cycle	Avg. First Cost	Avg. Life
AEX	33.6	$327,572	6.5 yrs
AEX/B	52.8	216,508	6.4
PEX	30.4	313,675	7.7
PEX/B	50.6	211,301	6.8
PAYBACK	57.6	253,442	4.6
RANDOM	35.2	233,401	6.2
Incr ROR	58.0	210,703	6.5
Incr AEX/B	56.4	203,874	6.7
Incr PEX/B	50.0	198,481	7.5
Incr PAYBACK	73.8	230,279	2.9

ranking methods. The number of projects selected per cycle is also shown. The figures represent the average value obtained over five complete cycles of simulation activity.

The simulation data tabulated in Table 4-3 verify that the relative bias of the individual ranking methods does have an impact on the type of projects selected by each respective method.

Using RANDOM as a standard, it is seen that the incremental methods, Incr ROR, Incr AEX/B, and Incr PEX/B all tend to favor lower first cost projects. Incr PEX/B retains the bias of PEX for long-lived projects. AEX has a bias for high first cost projects, while PEX has a bias for long-lived, high first cost projects. Dividing by B reduces the bias of the AEX or PEX methods. PAYBACK and Incr PAYBACK favor short-lived projects.

Average Annual Capital Expenditures

Table 4-4 shows the annual expenditure (funds returned for reinvestment from previous projects) averaged over

Table 4-4. Average annual capital expenditure for the various ranking methods

RANKBY	Average Annual Capital Expenditure	Net Value At Horizon
AEX	$1,051,488	$10.31
AEX/B	1,111,436	10.53
PEX	893,832	10.23
PEX/B	1,040,019	10.52
PAYBACK	1,430,314	7.81
RANDOM	879,116	5.99
Incr ROR	1,173,524	11.08
Incr AEX/B	1,106,426	11.10
Incr PEX/B	974,741	10.88
Incr PAYBACK	1,716,925	6.46

five complete 9 year cycles.

PAYBACK and Incr PAYBACK provide for the highest average annual capital expenditures. However, these methods tend to select very short-lived projects, and at the horizon date cash flows do not extend very far into the future. This results in a relatively low net value of the firm.

Incr ROR and Incr AEX/B yield the highest results for the net value of the firm. These methods tend to select projects with similar life and first cost characteristics. Close behind is Incr PEX/B. This method tends to select relatively long-lived projects, and yields a smaller average annual capital expenditure.

The tendency of AEX to favor high first cost projects, results in this method selecting a smaller number of projects each period, which in turn results in a smaller amount of cash returned for reinvestment. The ultimate result is a relatively smaller net value of the firm at the horizon date.

The tendency of PEX to favor long-lived projects results in this method choosing longer-lived projects that return relatively smaller amounts to the firm in each future period. This results in a lower average annual capital expenditure, and a lower net value of the firm.

AEX/B and PEX/B behave similarly to their AEX and PEX counterparts. However, much of the bias for long-lived and high first cost projects is removed, and these methods yield higher figures for the net value of the firm than AEX or PEX.

Cutoff Rate of Return

For very large values of ATCF, there is little or no rationing. When the percent mandatory (PM) is zero, non-incremental ranking methods can accept a maximum of NP independent projects per period, so if ATCF is large, these methods tend to run out of projects before all available funds are spent. This results in a significant level of carryover cash from period to period. If ATCF is large, AEX/B and PEX/B do worse than AEX and PEX because these methods select lower first cost projects, thus producing an even higher level of carryover cash earning only 5% interest.

As the value of ATCF declines, the degree of capital rationing increases, and the cutoff rate of return, as determined by the Incr ROR method, increases.

Under severe rationing, AEX and PEX, because of their bias for long-lived and high first cost projects, do worse than all methods other than PAYBACK and RANDOM. However, AEX/B and PEX/B, which have some of that bias removed, perform nearly as well as the incremental methods.

The relationship between the cutoff rate of return and the discount rate is important as it affects the performance of the ranking methods, and warrants further examination.

Table 4-5 gives the average cutoff rate of return determined by the Incr ROR ranking method at the various ATCF levels. The table also gives the net value of the firm obtained by the PEX and Incr ROR methods.

Table 4-5. Average cutoff rate of return at various ATCF levels

ATCF	Avg. Cutoff ROR	Net value obtained by: PEX	Net value obtained by: Incr ROR	((Incr ROR-PEX)/PEX) x 100%
$3,700,000	16.2%	$20.10	$20.69	2.93%
2,800,000	19.8	17.63	17.85	1.24
2,200,000	21.5	15.23	15.46	1.51
1,700,000	23.2	12.69	13.10	3.23
1,300,000	24.1	10.23	11.08	8.31
900,000	25.4	7.55	8.69	15.1
700,000	26.9	6.07	7.31	20.4
500,000	28.3	4.47	5.75	28.6
300,000	30.3	2.76	3.96	43.5
150,000	32.5	1.41	2.31	63.8
75,000	35.4	0.704	1.23	74.7
25,000	36.0	0.249	0.473	90.0

The cutoff rate of return is closest to the discount rate (20%) when ATCF = $2,800,000, and the cutoff rate of return is 19.8%. If the discount rate and the cutoff rate are the same, the PEX and Incr ROR ranking techniques will not necessarily yield the same result due to period by period fluctuations. Table 4-5 shows that the smallest difference between PEX and Incr ROR, 1.24%, occurs when the cutoff rate of return is closest to the discount rate of 20%. As ATCF decreases, there is an increasing disparity between the cutoff rate of return and the discount rate.

Table 4-6 shows how the first cost and life characteristics of the projects selected by each method for investment change as the level of rationing increases.

The data suggest that as the level of rationing increases (ATCF decreases), the relative bias of the individual ranking methods becomes more important in determining the projects that are selected for investment.

Table 4-6. Characteristics of projects accepted for investment at three ATCF levels

RANKBY	ATCF Level		
	$2,800,000	$1,300,000	$300,000
AVERAGE FIRST COST OF PROJECTS ACCEPTED:			
AEX	$285,017	$327,527	$381,002
AEX/B	240,679	216,508	180,563
PEX	281,372	313,675	359,424
PEX/B	241,856	211,301	176,080
PAYBACK	249,871	253,442	258,167
RANDOM	272,198	233,401	291,165
Incr ROR	221,280	210,703	184,980
Incr AEX/B	220,347	203,874	184,895
Incr PEX/B	215,743	198,481	176,526
Incr PAYBACK	219,581	230,279	243,649
AVERAGE LIFE OF PROJECTS ACCEPTED:			
AEX	6.2 yrs	6.5 yrs	6.6 yrs
AEX/B	6.3	6.4	6.8
PEX	6.2	7.7	9.7
PEX/B	6.2	6.8	8.7
PAYBACK	5.7	4.6	2.3
RANDOM	6.3	6.2	6.0
Incr ROR	6.3	6.5	6.4
Incr AEX/B	6.3	6.7	7.0
Incr PEX/B	6.2	7.5	9.0
Incr PAYBACK	4.6	2.9	2.0

Relative Effectiveness of the Ranking Methods

RANDOM ranks projects on a random basis. The outcomes generated by this method can be used to establish a standard against which the other ranking methods can be compared.

Define the effectiveness of a ranking method as:

$$\frac{\text{Observed score - Random score}}{\text{Best score - Random score}}$$

This effectiveness index can be computed for both the net value and ROR realized at the ATCF values given in Table 4-2. Table 4-7 presents the results of these computations.

The ranking method with the highest average effectiveness index is Incr ROR. This is followed closely by Incr AEX/B and Incr PEX/B. The ranking methods, listed from most effective to least effective are:

	Average effectiveness index	
	Net value	ROR realized
1. Incr ROR	.994	.994
2. Incr AEX/B	.968	.969
3. Incr PEX/B	.934	.938
4. AEX/B	.828	.858
5. PEX/B	.801	.832
6. AEX	.724	.780
7. PEX	.686	.741
8. PAYBACK	.287	.401
9. Incr PAYBACK	.219	.316
10. RANDOM	.000	.000

Table 4-7. Relative effectiveness of the ranking methods at various ATCF levels

ATCF Net value	AEX	AEX/B	PEX	PEX/B	PAYBACK	RANDOM	Incr ROR	Incr AEX/B	Incr PEX/B	Incr PAYBACK
$3,700,000	.851	.483	.874	.480	.385	0.00	.977	.982	1.00	.569
2,800,000	.934	.597	.942	.607	.373	0.00	.982	.985	1.00	.468
2,200,000	.920	.709	.935	.724	.436	0.00	.978	.989	1.00	.248
1,700,000	.896	.833	.911	.835	.454	0.00	.989	1.00	.992	.135
1,300,000	.845	.888	.830	.886	.356	0.00	.996	1.00	.957	.090
900,000	.777	.917	.758	.902	.285	0.00	1.00	.998	.951	.117
700,000	.733	.930	.702	.892	.236	0.00	1.00	.998	.945	.127
500,000	.690	.946	.619	.893	.205	0.00	1.00	.973	.932	.116
300,000	.593	.951	.512	.907	.130	0.00	1.00	.967	.919	.142
150,000	.499	.942	.422	.884	.146	0.00	1.00	.961	.897	.171
75,000	.459	.913	.349	.814	.166	0.00	1.00	.926	.827	.171
25,000	.496	.832	.383	.791	.267	0.00	1.00	.832	.791	.267
Average	.724	.828	.686	.801	.287	0.00	.994	.968	.934	.219

Table 4-7 (Continued)

ATCF ROR Realized	AEX	AEX/B	PEX	PEX/B	PAYBACK	RANDOM	Incr ROR	Incr AEX/B	Incr PEX/B	Incr PAYBACK
$3,700,000	.874	.523	.892	.523	.425	0.00	.978	.982	1.00	.606
2,800,000	.941	.638	.946	.649	.432	0.00	.978	.984	1.00	.529
2,200,000	.942	.757	.940	.771	.509	0.00	.988	1.00	.995	.310
1,700,000	.914	.870	.914	.866	.536	0.00	.989	1.00	.979	.185
1,300,000	.873	.933	.848	.905	.451	0.00	1.00	.997	.956	.133
900,000	.813	.945	.790	.913	.394	0.00	1.00	.993	.939	.176
700,000	.782	.954	.745	.906	.348	0.00	1.00	.989	.931	.206
500,000	.743	.954	.674	.901	.315	0.00	1.00	.966	.914	.196
300,000	.675	.951	.593	.915	.236	0.00	1.00	.947	.911	.250
150,000	.611	.961	.534	.918	.291	0.00	1.00	.958	.914	.330
75,000	.547	.902	.452	.841	.343	0.00	1.00	.897	.839	.349
25,000	.645	.911	.566	.877	.526	0.00	1.00	.911	.877	.526
Average	.780	.858	.741	.832	.401	0.00	.994	.969	.938	.316

Statistical Significance

An important feature of the simulation model is the ability to generate an identical sequence of investment proposals that can then be operated on by the different ranking criteria. This procedure eliminates one potential source of random variation, and permits a direct comparison of the effectiveness of the various ranking criteria.

Each ranking criterion operates on an identical stream of investment proposals because each ranking method is passed through an initial seed for the random number generation. Thus, data generated by each of the ranking criteria for any individual cycle are based on application of the criteria to the exactly same set of investment opportunities that were made available to other ranking criteria.

By replicating this process for several cycles, a paired sample t test can be used to make statements about the statistical significance of the observed average differences among the various ranking criteria.

The measures of effectiveness are the net worth of the firm at the horizon date, and the rate of return on the initial funds supplied (ATCF). To use the paired sample t test, the difference between methods a and b for each cycle needs to be determined. The average difference $\bar{d}$, and the standard deviation of the difference, s_d, are then calculated.

Assuming that the differences are normally distributed, a t test can be used to test the hypothesis:

H_o: the mean of the difference, $\bar{d}_{b-a}$, is zero

H_A: the mean of the difference, $\bar{d}_{b-a}$, is not zero

Using the initial set of input parameters, the simulation was run for 50 cycles for each ranking method. Table 4-8 presents the results obtained.

Table 4-8. Results achieved through fifty cycles of simulation

RANKBY	Average Net Value (in millions)	Average ROR Realized
Incr ROR	$10.80	25.68%
Incr AEX/B	10.80	25.66
Incr PEX/B	10.64	25.43
AEX	10.20	25.05
AEX/B	10.18	25.08
PEX/B	10.16	25.00
PEX	10.09	24.87
PAYBACK	7.57	22.15
Incr PAYBACK	6.34	20.18
RANDOM	5.69	19.08

The number of possible paired combinations is:

$$\frac{10!}{(10-2)!} = 90$$

The number of unique t values is 90/2 = 45, since the t value for a-b is the negative of the t value for b-a. The paired sample t statistic was calculated for all 45 unique combinations for both the net value and rate of return. Table 4-9 gives the results.

For a two tailed t test, $H_A \neq 0$, the critical t values for 49 degrees of freedom and various levels of significance are:

Level of significance	.100	.050	.025	.010	.005	.001
Critical t value	1.667	2.009	2.312	2.680	2.940	3.501

For a one tailed t test, $H_A > 0$ or $H_A < 0$, the critical t values are:

Level of significance	.100	.050	.025	.010	.005	.001
Critical t value						
$H_A > 0$	1.299	1.667	2.009	2.312	2.680	3.291
$H_A < 0$	-1.299	-1.667	-2.009	-2.312	-2.680	-3.291

For the two tailed test, if the absolute value of the

Table 4-9. Paired sample t statistic claculated for fifty cycles of simulation

	Incr ROR	Incr AEX/B	Incr PEX/B	AEX	AEX/B	PEX/B	PEX	PAYBACK	Incr PAYBACK	RANDOM
NET VALUE										
Incr ROR	0	.49	-5.70	-14.19	-12.75	-12.66	-14.14	-30.34	-37.36	-41.39
Incr AEX/B	-.49	0	-6.83	-14.88	-12.74	-12.64	-14.69	-31.10	-38.26	-42.39
Incr PEX/B	5.70	6.83	0	-10.71	-9.09	-10.09	-14.65	-30.66	-39.09	-42.32
AEX	14.19	14.88	10.71	0	-.22	-.57	-3.08	-28.34	-37.34	-41.99
AEX/B	12.75	12.74	9.09	.22	0	-1.32	-1.48	-26.04	-32.79	-35.21
PEX/B	12.66	12.64	10.09	.57	1.32	0	-1.18	-26.38	-33.24	-35.64
PEX	14.14	14.69	14.65	3.08	1.48	1.18	0	-28.40	-38.18	-39.82
PAYBACK	30.34	31.10	30.66	28.34	26.04	26.38	28.40	0	-18.15	-26.38
Incr PAYBACK	37.36	38.26	39.09	37.34	32.79	33.24	38.18	18.15	0	-5.73
RANDOM	41.39	42.93	42.32	41.99	35.21	35.64	39.82	26.83	5.73	0

Table 4-9 (Continued)

	Incr ROR	Incr AEX/B	Incr PEX/B	AEX	AEX/B	PEX/B	PEX	PAYBACK	Incr PAYBACK	RANDOM
ROR REALIZED										
Incr ROR	0	-2.05	-8.78	-17.50	-15.12	-16.15	-12.10	-32.96	-41.19	-42.45
Incr AEX/B	2.05	0	-8.63	-16.74	-14.24	-15.30	-16.63	-33.55	-41.78	-43.12
Incr PEX/B	8.78	8.63	0	-10.18	-7.25	-9.82	-15.78	-31.36	-40.82	-40.41
AEX	17.50	16.74	10.18	0	-.41	-.85	-5.34	-28.25	-38.25	-38.93
AEX/B	15.12	14.24	7.25	.41	0	-3.78	-3.05	-26.58	-34.12	-35.26
PEX/B	16.15	15.30	9.82	.85	3.78	0	-2.15	-26.33	-33.75	-34.93
PEX	17.10	16.63	15.78	5.34	3.05	2.15	0	-26.89	-36.99	-35.41
PAYBACK	32.96	33.55	31.36	28.25	26.58	26.33	26.89	0	-17.51	-16.75
Incr PAYBACK	41.19	41.78	40.82	38.25	34.12	33.75	36.99	17.51	0	-5.64
RANDOM	42.45	43.12	40.41	38.93	35.26	34.93	35.41	16.75	5.64	0

observed t value is greater than the critical t value, the observed difference in the means is statistically significant.

For the one tailed test, if the observed t value is either greater than or less than the appropriate critical t value, the observed difference in the means is statistically significant.

For example, the observed t value for PEX/B-PEX is 1.182. This indicates that the null hypothesis, H_0: $\bar{d}=0$ would not be rejected at any of the significance levels given.

Table 4-10 shows those combinations where the null hypothesis would be rejected at a .01 significance level for the two tailed test, or at a .005 significance level for the one tailed test.

As an example, consider the AEX column. AEX is statistically worse than Incr ROR, Incr AEX/B, and Incr PEX/B; is not statistically different than AEX/B or PEX/B; and is statistically better than PEX, PAYBACK, Incr PAYBACK, and RANDOM.

Notice that the top two methods, Incr ROR and Incr AEX/B, are not statistically different. However, both methods are statistically better than any of the other

Table 4-10. Statistical significance of paired combinations

	Incr ROR	Incr AEX/B	Incr PEX/B	AEX	AEX/B	PEX/B	PEX	PAYBACK	Incr PAYBACK	RANDOM
NET VALUE										
Incr ROR	--	NO	YES	YES	YES	YES	YES	YES	YES	YES
Incr AEX/B	NO	--	YES	YES	YES	YES	YES	YES	YES	
Incr PEX/B	YES	YES	--	YES	YES	YES	YES	YES	YES	YES
AEX	YES	YES	YES	--	NO	NO	YES	YES	YES	YES
AEX/B	YES	YES	YES	NO	--	NO	NO	YES	YES	YES
PEX/B	YES	YES	YES	NO	NO	--	NO	YES	YES	YES
PEX	YES	YES	YES	YES	NO	NO	--	YES	YES	YES
PAYBACK	YES	YES	YES	YES	YES	YES	YES	--	YES	YES
Incr PAYBACK	YES	YES	YES	YES	YES	YES	YES	YES	--	YES
RANDOM	YES	YES	YES	YES	YES	YES	YES	YES	YES	--
ROR REALIZED										
Incr ROR	--	NO	YES	YES	YES	YES	YES	YES	YES	YES
Incr AEX/B	NO	--	YES	YES	YES	YES	YES	YES	YES	YES
Incr PEX/B	YES	YES	--	YES	YES	YES	YES	YES	YES	YES
AEX	YES	YES	YES	--	NO	NO	YES	YES	YES	YES
AEX/B	YES	YES	YES	NO	--	YES	YES	YES	YES	YES
PEX/B	YES	YES	YES	NO	YES	--	NO	YES	YES	YES
PEX	YES	YES	YES	YES	YES	NO	--	YES	YES	YES
PAYBACK	YES	YES	YES	YES	YES	YES	YES	--	YES	YES
Incr PAYBACK	YES	YES	YES	YES	YES	YES	YES	YES	--	YES
RANDOM	YES	YES	YES	YES	YES	YES	YES	YES	YES	--

methods.

Varying Program Parameters

As discussed at the beginning of this chapter, there are several program variables that set the conditions of the simulation. The remainder of this chapter will present results obtained by varying some of these program variables.

The constraint of a finite computer budget does not allow every parameter to be varied for every ranking method. Therefore, the parameters will be varied only for PEX and Incr ROR, two of the more widely advocated ranking methods.

Simulation length, M

The length of the simulation, M, sets the number of periods to be simulated, and thus determines the horizon date. M was allowed to vary from 2-19 years, with ATCF = $1,300,000. Table 4-11 gives the results.

At every value from 2 to 19, Incr ROR yields a higher net value and rate of return than PEX. When M = 2, the relative superiority of Incr ROR is fairly small, providing

Table 4-11. Results achieved by varying M, the number of periods in each cycle

M	Net Value (in millions)			ROR Realized		
	PEX	Incr ROR	(Incr ROR-PEX)/PEX x 100%	PEX	Incr ROR	(Incr ROR-PEX)/PEX x 100%
2	1.92	1.93	0.52	22.10	22.48	1.72
3	2.50	2.53	1.20	23.02	23.64	2.69
4	3.05	3.19	4.59	23.13	23.98	3.67
5	3.85	4.07	5.71	23.61	24.61	4.24
6	4.91	5.34	8.76	24.02	25.15	4.70
7	6.32	6.82	7.91	24.46	25.53	4.37
8	7.88	8.56	8.63	24.70	25.76	4.29
9	10.23	11.08	8.31	24.98	25.95	3.88
10	12.68	13.85	9.23	24.99	25.99	4.00
11	16.20	17.84	10.12	25.08	25.94	3.43
12	20.62	22.54	9.31	25.23	26.05	3.25
13	25.94	28.44	9.64	25.37	26.17	3.15
14	33.06	36.04	9.01	25.41	26.11	2.75
15	42.50	46.63	9.72	25.60	26.33	2.85
16	52.68	58.04	10.17	25.59	26.29	2.74
17	67.12	73.19	9.04	25.68	26.27	2.30
18	80.69	88.70	9.93	25.54	26.13	2.31
19	91.52	103.71	13.32	24.90	25.66	3.05

only a 0.52% higher net value and a 1.72% higher rate of return. As M increases, the relative superiority of Incr ROR over PEX increases, and then stabilizes roughly around a 10% advantage in net value, and a 3% advantage in rate of return.

Number of projects, NP

The next parameter to be varied is NP, the number of projects generated in each investment period. NP was allowed to vary from 2 to 20. Table 4-12 presents the results.

As the number of projects per period increases, the net value and rate of return provided by both PEX and Incr ROR increase. However, as NP gets larger, Incr ROR increases faster than PEX, and the relative difference between the two ranking methods grow larger.

Number of mutually exclusive alternatives, MX

MX determines the number of mutually exclusive alternatives per independent project. The program is constructed to handle from 2 to 7 alternatives. Table 4-13 presents the results of varying MX.

At all values of MX, Incr ROR resulted in a higher net value and rate of return than PEX.

Table 4-12. Results achieved by varying NP, the number of independent projects per period

NP	Net Value (in millions)			ROR Realized		
	PEX	Incr ROR	(Incr ROR-PEX) / PEX x 100%	PEX	Incr ROR	(Incr ROR-PEX) / PEX x 100%
2	5.08	5.35	5.31%	17.80	18.49	3.88
3	6.72	6.92	2.98	20.71	21.05	1.64
4	7.62	7.83	2.76	22.00	22.33	1.50
5	8.39	8.50	1.31	23.14	23.30	.69
6	8.61	8.92	3.60	23.34	23.70	1.54
7	9.44	9.92	5.08	24.19	24.72	2.19
8	9.84	10.42	5.89	24.63	25.30	2.72
9	9.65	10.43	8.08	24.53	25.48	3.87
10	10.23	11.08	8.31	24.98	25.95	3.88
11	10.33	11.33	9.68	25.05	26.27	4.87
12	9.95	11.50	15.58	24.78	26.38	6.46
13	10.37	12.06	16.30	25.05	26.63	6.31
14	10.47	12.20	16.52	25.14	26.97	7.28
15	10.43	12.29	17.83	25.22	27.23	7.97
16	10.86	12.85	18.32	25.60	27.49	7.38
17	11.21	13.18	17.57	25.87	27.77	7.34
18	11.16	14.79	32.53	25.77	29.02	12.61
19	11.57	17.93	54.97	26.18	31.20	19.17
20	11.55	21.75	88.31	26.19	33.37	27.42

Table 4-13. Results by varying MX, the number of mutually exclusive alternatives per independent project

MX	Net Value (in millions) PEX	Net Value (in millions) Incr ROR	Net Value (in millions) $\frac{\text{Incr ROR-PEX}}{\text{PEX}}$ x 100%	ROR Realized PEX	ROR Realized Incr ROR	ROR Realized $\frac{\text{Incr ROR-PEX}}{\text{PEX}}$ x 100%
2	8.84	8.98	1.58	23.60	23.79	.81
3	9.85	10.04	1.93	24.79	25.06	1.13
4	10.23	11.08	8.31	24.98	25.95	3.88
5	9.58	11.37	18.68	24.23	26.07	7.59
6	9.76	11.14	14.14	24.50	26.00	6.12
7	9.43	11.15	18.24	24.16	26.15	8.24

Project life

The program is constructed to generate projects of 2 and 10 year lives. By multiplying the zero to one uniform random number, YFL(J), by 8, adding 2, and then truncating the decimal portion, project lives of 2 to 9 years, uniformly distributed are generated.

Table 4-14 presents the results obtained by generating projects with lives of 2 and 10 years, and then relaxing this constraint to allow projects with lives of 2 to 9 years.

Table 4-14. Results achieved by varying project life

RANKBY	2 and 10	2 to 9	% Change
Net Value (in millions):			
PEX	$10.23	$10.44	+2.05%
Incr ROR	11.08	11.43	+3.16
ROR Realized:			
PEX	24.98%	25.22%	+0.96
Incr ROR	25.94	26.27	+1.27

Allowing project life to vary has a consistent effect on both the PEX and Incr ROR ranking criteria. It results in a small increase in the net value and the rate of return for both methods.

Percent mandatory, PM

The percentage of mandatory projects generated is user controlled and can be set from zero to 100%. Table 4-15 presents the values obtained for various levels of PM for two different levels of ATCF; $1,300,000 and $300,000. The results are presented only for PEX, because as the level of mandatory projects increases, significance of the ranking method decreases.

Table 4-15. Results achieved by varying PM, the percentage of mandatory projects (results presented only for PEX)

ATCF	Percent Mandatory 0.00	0.25	0.50	0.75	1.00
<u>Net Value (in millions):</u>					
$1,300,000	10.23	9.57	8.37	7.20	4.95
$300,000	2.76	1.02	-0.24	-2.05	-4.41
<u>ROR Realized:</u>					
$1,300,000	25.00	24.3	23.0	21.5	18.1
$300,000	26.1	17.0	11.5	8.0	5.8

As the level of PM increases, both the net value and the ROR realized decrease. When ATCF = $300,000, the net value turns negative as PM increases. This occurs because the firm is required to accept all mandatory projects. If funds are not available, the firm must borrow at 30%. Since the average rate of return of the generated projects is 19.96%, the firm loses money, and its net value turns negative.

<u>Project indivisibility, INDIV</u>

The program is constructed so that when INDIV = 1, only whole projects are accepted. When INDIV = 2, fractional projects are accepted.

Table 4-16 gives the results obtained for INDIV = 1 and INDIV = 2 at the two levels of ATCF used in the previous section.

Table 4-16. Results achieved by varying INDIV, the project indivisibility option

ATCF	INDIV	RANKBY PEX	RANKBY Incr ROR	RANKBY $\left(\frac{\text{Incr ROR-PEX}}{\text{PEX}}\right)$ x 100%
Net Value:				
1,300,000	2	10.23	11.08	8.31%
	1	9.54	10.55	10.59
% Change from INDIV = 2 to INDIV = 1		-6.74%	-4.78%	
300,000	2	2.76	3.96	43.48%
	1	1.75	3.18	81.71
% Change from INDIV =2 to INDIV = 1		-36.6%	-19.7%	
ROR Realized:				
1,300,000	2	25.0	25.9	3.6%
	1	24.3	25.5	4.9
% Change from INDIV = 2 to INDIV = 1		-2.8%	-1.5%	
300,000	2	26.1	30.1	15.33%
	1	21.6	28.5	31.94
% Change from INDIV = 2 to INDIV = 1		-17.2%	-5.32%	

Accepting only whole projects (INDIV = 1) rather than accepting fractional projects (INDIV = 2) decreases the net value and ROR realized of the firm. This occurs because accepting fractional projects allows the firm to invest all of its available capital in projects. Accepting only whole projects forces the firm to have some carryover cash, which earns only 5% interest.

CHAPTER V. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

There are several methods that can be used to rank capital investment alternatives. Unfortunately, there is no general consensus regarding the best method to use. The model presented in this dissertation uses computer simulation to investigate the relative performance of several ranking techniques. Specifically, the criteria studied are:

1. AEX
2. AEX/B
3. PEX
4. PEX/B
5. PAYBACK
6. RANDOM
7. Incr ROR
8. Incr AEX/B
9. Incr PEX/B
10. Incr PAYBACK

The model consists of a cash flow simulator that generates independent and mutually exclusive projects. These projects are then ranked according to one of the above criteria and accepted for investment until the available funds are exhausted. This continues for several periods, and results in the firm increasing its wealth through investment. The net value of the firm at the horizon date, and the rate of return realized on the initial funds supplied (ATCF) are the measures of effectiveness used to compare ranking criteria.

Conclusions

With regard to the study reported here, the following conclusions may be stated:

1. The method employed to rank capital investment alternatives does have an impact on the future net value of the firm.
2. The relationship between the cutoff rate of return and the discount rate is important as it affects the characteristics of the projects that are selected for investment by the discounted cash flow ranking methods.
3. The data indicate that for the assumptions and parameters incorporated in this model, Incr ROR and Incr AEX/B provide a net value of the firm, and rate of return realized on initial funds supplied, that is statistically significantly better than any of the other ranking criteria

tested (Incr PEX/B, AEX, AEX/B, PEX/B, PEX, PAYBACK, Incr PAYBACK, and RANDOM).

Recommendations for Future Study

With regard to this study, some suggestions for future study are:

1. Generate investment proposals with more diverse characteristics of first cost, and the duration and pattern of period by period cash flows. Patterns such as decreasing gradients, and projects with just a single future cash flow x years hence are examples.
2. Thomson (1976) found that heuristic modifications could improve the performance of Incr ROR as a ranking criteria if the period-to-period cutoff rate of return is time-variant. Additional heuristics might be sought in a future study.
3. Mandatory projects studied here had the same ROR distribution as did discretionary projects. A future study might investigate the effect of economically disadvantageous mandatory projects (as for pollution control, meeting OSHA requirements, and so forth).

4. Investigate the effects of generating project characteristics from distributions other than uniform.
5. The computer program should be tested for increased efficiency. Currently, each cycle requires between 80,000 and 300,000 statement executions, and between two and five seconds CPU time.
6. Expand the model to permit the inclusion of pre-requisite projects.

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BIBLIOGRAPHY

Ang, J. S. 1978. A Graphical Presentation of an Integrated Capital Budgeting Model. The Engineering Economist 23(2):101-116.

Baumol, W. J. and R. E. Quandt. 1965. Investment and Discount Rates Under Capital Rationing-A Programming Approach. The Economic Journal 75:317-329.

Baumol, W. J. 1972. Economic Theory and Operations Analysis. Prentice-Hall, Englewood Cliffs, New Jersey.

Bedel, D. P. and J. D. Mains. 1973. Three Problems in Using Ranking Techniques for Capital Budgeting. Cost and Management 47(4):21-23.

Beenhakker, H. L. 1973. Discounting Indices Proposed for Capital Investment Evaluation: A Further Examination. The Engineering Economist 18(3):149-168.

Beltz, M. H. 1973. Statistical Methods in the Process Industries. Halstead Press, New York.

Bernhard, R. H. 1971. A Comprehensive Comparison and Critique of Discounting Indices Proposed for Capital Investment Evaluation. The Engineering Economist 16(3):157-186.

Bernhard, R. H. 1979. "Modified" Rates of Return for Investment Project Evaluation-A Comparison and Critique. The Engineering Economist 24(3):161-167.

Bey, R. P. and R. B. Porter. 1977. An Evaluation of Capital Budgeting Portfolio Models Using Simulated Data. The Engineering Economist 23(1):41-65.

Bierman, H. and S. Smidt. 1971. The Capital Budgeting Decision. Macmillan, New York.

Bradley, S. P., Frank, R. S., and S. C. Frey Jr. 1978. Determining the Appropriate Discount Rates in Pure Capital Rationing. Decision Sciences 9(3):391-401.

Charnes, A., Cooper, W. W., and M. H. Miller. 1959. Application of Linear Programming to Financial Budgeting and the Costing of Funds. Pages 229-255 in E. Soloman, The Management of Corporate Capital, Macmillan, New York.

Dean, J. 1951. Capital Budgeting. Columbia University Press, New York.

Gitman, L. J. and J. R. Forrester Jr. 1977. A Survey of Capital Budgeting Techniques Used by Major U.S. Firms. Financial Management 6(3):661-71.

Grant, E. L. 1966. Reinvestment of Cash Flows Controversy-A Review. The Engineering Economist 11(3):23-29.

Harris, W. O. 1971. Discounted Cash Flow Economic Evaluation. IBM System Reference Library. IBM, Hawthorne, New York.

Hoskins, C. G. and G. A. Mumey. 1979. Payback: A Maligned Method of Asset Ranking? The Engineering Economist 25(1):53-65.

Ignizio, J. P. 1976. An Approach to the Capital Budgeting Problem with Multiple Objectives. The Engineering Economist 21(4):259-272.

Jeynes, P. 1968. Profitability and Economic Choice. Iowa State University Press, Ames, Iowa.

Keown, A. J. and J. D. Martin. 1977. A Chance Constrained Goal Programming Model for Working Capital. The Engineering Economist 22(3):153-174.

Kryzanowski, L., Lusztig, P., and B. Schwab. 1972. Monte Carlo Simulation and Capital Expenditure Decisions: A Case Study. The Engineering Economist 18(1):31-48.

Lorie, J. and L. J. Savage. 1955. Three Problems in Capital Rationing. Journal of Business 28:229-239.

Leautaud, J. L. and R. Swaim. 1974. On the Fundamentals of Economic Evaluations. The Engineering Economist 19(2):105-125.

Lee, S. M. and A. J. Lerro. 1974. Capital Budgeting for Multiple Objectives. Financial Management 3(1):58-66.

Mallik, A. K. 1979. Engineering Economy with Computer Applications. Engineering Technology, Mamomet, Illinois.

Mao, J. C. T. 1966. The Internal Rate of Return as a Ranking Criterion. The Engineering Economist 11(4):1-13.

McKean, R. N. 1963. Efficiency in Government Through Systems Analysis. Wiley, New York.

Merret, A. J. and A. Sykes. 1973. The Finance and Analysis of Capital Projects. Halstead Press, New York.

Meyers, S. C. 1974. Interaction of Corporate Financing and Investment Decisions-Implications for Capital Budgeting. The Journal of Finance 29(1):1-25.

Naslund, B. 1971. A Model of Capital Budgeting Under Risk. Pages 48-70 in R. F. Byrne et al., eds. Studies in Budgeting, North Holland, Amsterdam.

Newnan, D. G. 1980. Engineering Economic Analysis. Engineering Press, San Jose, California.

Oakford, R. V., Bhimjee, S. A., and J. V. Jucker. 1977. The Internal Rate of Return, the Pseudo Internal Rate of Return, and NPV and Their Use in Financial Decision Making. The Engineering Economist 22(3): 187-201.

Osteryoung, J. S. 1974. Capital Budgeting: Long Term Asset Selection. Grid, Columbus, Ohio.

Parra-Vasquez, A. S. and R. V. Oakford. 1976. Simulation as a Technique for Comparing Decision Procedures. The Engineering Economist 21(4):221-236.

Pegels, C. C. 1968. A Comparison of Decision Criteria for Capital Investment Decisions. The Engineering Economist 13(4):211-220.

Petry, G. H. 1975. Effective Use of Capital Budgeting Tools. Business Horizons 19(5):57-65.

Pritsker, A. A. 1974. The Gasp IV Simulation Language. Wiley and Sons, New York.

Quirin, G. D. 1967. The Capital Expenditure Decision. Irwin, Homewood, Illinois.

Rendleman, R. J. Jr. 1978. Ranking Errors in CAPM-Capital Budgeting Applications. Financial Management 7(4):40-44.

Ross, M. H. 1973. Probability, Games, Regret and Investment Criteria. The Engineering Economist 18(3):191-198.

Salazar, A. 1979. Simulation of Capital Budgeting Decisions. American Society for Engineering Education, 87th Annual Conference, Louisiana State University, June 25-28.

Snedecor, G. W. and W. G. Cochran. 1967. Statistical Methods. The Iowa State University Press, Ames, Iowa.

Soloman, E. 1959. The Management of Corporate Capital. Macmillan, New York.

Smith, G. W. 1962. Payoff Period: An Inverted Rate of Return? The Engineering Economist 7(4):26-29.

Smith, G. W. 1973. Engineering Economy: Analysis of Capital Expenditures. Iowa State University Press, Ames, Iowa.

Smith, G. W. 1979. Engineering Economy: Analysis of Capital Expenditures. Third Edition. Iowa State University Press, Ames, Iowa.

Sundem, G. L. 1975. Evaluating Capital Budgeting Models in Simulated Environments. The Journal of Finance 30(4):977-992.

Taylor, B. W. and A. J. Keown. 1978. A Goal Programming Approach of Capital Budgeting Selection in the Production Area. AIIE Transactions 10(1):52-57.

Theusen, H. G., Fabrycky, W. J. and G. J. Theusen. 1971. Engineering Economy. Prentice-Hall, Englewood Cliffs, New Jersey.

Thomson, J. S. 1976. Capital Budgeting Simulation: A Heuristic Approach. M.S. Thesis. Iowa State University, Ames, Iowa.

Weingartner, H. M. 1963. Mathematical Programming and the Analysis of Capital Budgeting Problems. Markham Publishing Company, Chicago.

Weingartner, H. M. 1966. Capital Budgeting of Interrelated Projects: Survey and Synthesis. Management Science 12:485-516.

Whitmore, G. A. and L. R. Amey. 1973. Capital Budgeting Under Rationing: Comments on the Lusztig and Schwab Procedure. Journal of Financial and Quantitative Analysis 9(1):127-135.