Application of Shuffled Complex Evolution Optimization Approach

to Concrete Pavement Backanalysis

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Abstract

This paper focuses on the development of a new backcalculation method for concrete pavements based on a hybrid evolutionary global optimization algorithm, namely Shuffled Complex Evolution (SCE). Evolutionary optimization algorithms are ideally suited for intrinsically multi-modal, non-convex, and discontinuous real-world problems such as pavement backcalculation because of their ability to explore very large and complex search spaces and locate the globally optimal solution using a parallel search mechanism as opposed to a point-by-point search mechanism employed by traditional optimization algorithms. Shuffled Complex Evolution (SCE), a type of evolutionary optimization algorithms based on the tradeoff of exploration and exploitation, has been proved to be an efficient method for many global optimization problems and in some cases it does not suffer the difficulties encountered by other evolutionary computation techniques. The SCE optimization approach is hybridized with a Neural Networks (NN) surrogate forward pavement response model to enable rapid computation of global or near-global pavement layer moduli solutions. It is shown that the developed approach is robust and produces consistent results.

Key Words: Evolutionary optimization, backcalculation, rigid pavements, neural networks.

Introduction

Transportation agencies across the world, in charge of maintaining the highway systems, frequently evaluate the structural condition of road pavements as part of their routine maintenance schedule. The Falling Weight Deflectometer (FWD) is the most commonly used non-destructive type pavement deflection testing equipment used for such purposes. The measured deflections from FWD can be correlated to in-situ material stiffness of each layer in the pavement structure through a procedure known as *backcalculation* or *inverse analysis*. Determination of in-situ material stiffness is essential in assessing the structural condition of exiting pavement for estimation of pavement remaining life and in determining the thickness of new overlay.

The backcalculation methodology is an inverse process to determine in-situ materials stiffness of pavement layer by matching the measured and the theoretical deflection with iteration or optimization schemes. The most common approach in current commercial backcalculation software requires inputting initial seed modulus which is an assumed layer modulus for an iterative process. Thus, the reliability of the final optimized solution is dependent upon the initial seed modulus. It is not uncommon that minor deviations between

measured and computed deflections usually result in significantly different moduli and the various combinations of modulus values essentially produce the same deflection basin (Mehta and Roque 2003).

Over the years, numerous pavement backcalculation approaches have been developed. Each approach has its own pros and cons and researchers continue to explore advanced hybrid approaches to pavement moduli backcalculation with the main aim of facilitating speed of convergence, robustness, and computational efficiency (Gopalakrishnan et al. 2010).

Evolutionary optimization algorithms are ideally suited for intrinsically multi-modal, non-convex, and discontinuous real-world problems because of their ability to explore very large and complex search spaces and locate the globally optimal solution using a parallel search mechanism as opposed to a point-by-point search mechanism employed by traditional optimization algorithms (Muttil and Liong 2004). Shuffled Complex Evolution (SCE), a type of evolutionary optimization algorithms based on the tradeoff of exploration and exploitation, has been proved to be an efficient method for many global optimization problems and in some cases it does not suffer the difficulties encountered by other evolutionary computation techniques (Muttil and Liong 2004).

This paper proposes a hybrid SCE-based heuristic optimization algorithm for analysis of rigid pavement non-destructive test data and backcalculation of concrete pavement layer moduli.

Shuffled Complex Evolution (SCE) Algorithm

The SCE algorithm developed at the University of Arizona is reported to be an efficient global optimization method that can be used to handle non-linear problems with highparameter dimensionality (Duan et al. 1992, Duan et al. 1993, Duan et al. 1994, Muttil and Liong 2004). It consists of all the four principles for global optimization: the controlled random search, the implicit clustering, the complex shuffling, and the competitive evolution. The search for the optimal solution begins with a randomly selected complex of points spanning the entire feasible space. The implicit clustering helps to concentrate the search in the most promising of the regions. The use of complex shuffling provides a freer and more extensive exploration of the search space in different directions, thereby reducing the chances of the search getting trapped in local optima. Three of these principles are coupled with the competitive complex evolution (CCE) algorithm, which is a statistical reproduction process employing the complex geometric shape to direct the search in the correct direction. The synthesis of these concepts makes the SCE algorithm not only effective and robust, but also flexible and efficient (Nunoo and Mrawira 2004).

Basic Algorithm

The SCE control parameters should be determined in advance to achieve the required exploration process. These parameters include the number of points in a complex (m), the number of points in a sub complex (q), the number of complexes (p), the number of consecutive offspring generated by each sub complex (α) , and the number of steps inevolution taken by each complex (β) . Duan et al (1994) provides guidelines for proper selection of these parameters.

The basic algorithm for SCE described by Duan et al (1993) is represented in Figure 1 and can be outlined as follows:

1. An initial population of points is sampled randomly from the feasible solution space (Ω) in the real space (\mathbb{R}^n) .

- 2. The selected population is partitioned into one or more complexes, each containing a fixed number of points.
- 3. Each complex evolves according to a competitive complex evolution (CCE) algorithm.
- 4. The entire population is periodically shuffled and points are reassigned to complexes to share the information from the individual complexes.
- 5. Evolution and shuffling are repeated so that the entire population is close to convergence criteria, and are stopped if the convergence criteria are satisfied.

The CCE algorithm is a sub-route in SCE algorithm. CCE algorithm employs the downhill simplex method (Nelder and Mead 1965) in generating offsprings. The simplex method facilitates evolution of each complex independently in an improvement direction. The CCE procedure is described by Duan et al (1993).



Figure 1: Shuffled Complex Evolution (SCE) Algorithm (after Duan et al. 1993)

SCE Applications: A Brief Review

The SCE algorithm developed at the University of Arizona (also referred to as SCE-UA) was originally to deal with the rainfall – runoff models (Duan et al. 1992). The SCE has been used extensively for the calibration of various rainfall-runoff models and other water-related fields (Muttil and Jayawardena, 2008) since then. In the field of civil engineering, Nunoo and Mrawira (2004) applied the SCE in decision-making process of infrastructure management. Since the decision-making process of infrastructure management involves a large number of sections and a large number of possible treatment alternatives, optimization of infrastructure preservation activities for available resources is a difficult task. The findings of this study showed that the SCE algorithm is very efficient and consistent in simultaneous consideration of the trade-off among various infrastructure preservation strategies. Sanson and Shibayama (2007) also demonstrated that SCE algorithm could be used to optimize the planning of maintenance and rehabilitation of a road network in Japan.

Barakat and Altoubat (2009) evaluated three optimization techniques to solve nonlinear constrained structural optimization problems. These methods are SCE, simulated annealing (SA) and genetic algorithm (GA). They concluded that the robust search capability of SCE algorithm technique is well suited for solving the structural problem in hand. Gopalakrishnan (2010) successfully demonstrated the use of global optimization techniques like Particle Swarm Optimization (PSO) and SCE in the backcalculation of conventional flexible pavement layer moduli. The current study focuses on developing SCE-based rigid pavement backcalculation models.

SCE-based Rigid Pavement Backcalculation

The elastic modulus of the slab, E, and modulus of subgrade reaction, k, are the two most important backcalculated concrete pavement properties. Over the years, researchers have developed many different methodologies for backcalculation of concrete pavement properties from FWD measurements, including the AREA method for rigid pavements (Ioannides et al., 1989; Ioannides, 1990; Barenberg and Petros, 1991), ILLI-BACK (Ioannides, 1994), graphical solution using IILI-SLAB (Foxworthy and Darter, 1989), use of regression analysis to solve AREA method for rigid pavements (Hall, 1992; Hall et al., 1996), use of best fit algorithm to find radius of relative stiffness (I) (Hall et al., 1996; Smith et al., 1996), among others.

Based on a backcalculation study of concrete pavement properties using 277 deflection basins obtained from the Denver International Airport (DIA), Rufino et al. (2002) studied the effect of slab modeling (number of layers, interface condition, and model type) as well as effect of different methodologies and sensor configurations on backcalculated pavement properties. It was found that backcalculated slab modulus of elasticity (E) is lower on average when the pavement layers on top of subgrade are bonded versus unbonded interface. Higher backcalculated k-values are obtained when the slab is modeled as plate compared to modeling the slab as elastic layer. Modeling of the slab and base as elastic layers seem to yield more reasonable backcalculated results since the interface bonding condition can be reflected both in the backcalculated slab elastic modulus and subgrade k-value.

Over the past decade, the use of computational intelligence techniques in pavement systems modeling, analysis, and design, has become increasingly common. Ceylan (2002) employed Artificial Neural Networks (ANNs) in the analysis of concrete pavement systems and developed ANN-based design tools that incorporated the ISLAB 2000 (Tabatabaie and

Barenberg, 1978; Khazanovich, 1994; Khazanovich et al, 2000) finite element solutions into routine practical design at several orders of magnitude faster than ISLAB 2000.

Khazanovich and Roesler (1997) developed a program called DIPLOBACK for backcalculation of moduli values of composite pavements based on ANNs. ANNs have also been applied along with dimensional analysis to backcalculate joint properties from FWD testing (Ioannides et al., 1996). The advantage of using ANN and dimensional analysis together is that they both reduce the database size necessary to accurately estimate pavement properties (Rufino et al., 2002). In the development of the new Mechanistic-Empirical Pavement Design Guide (MEPDG) for the American Association of State Highway and Transportation Officials (AASHTO), ANNs were recognized as nontraditional, yet very powerful computing techniques and ANN models were used in preparing the concrete pavement analysis package (Khazanovich et al, 2001). Ceylan et al. (2009) developed a suite of ANN-based flexible, rigid, and composite pavement backcalculation models from comprehensive synthetic databases.

This paper discusses the implementation of the SCE optimization approach for a slabon-grade rigid pavement structure although it can be used for a variety of pavement geometry and types owing to its flexible and integrated modular systems approach. The objective (fitness) function or the cost function for the proposed SCE optimization approach is the difference between measured FWD deflections and computed pavement surface deflections.

SCE Implementation

The proposed SCE global optimization backcalculation approach is presented in Figure 2. This approach treats backcalculation as a global optimization problem where the cost functions to be minimized is defined as the differences in measured and computed deflections. The optimal solution (elastic modulus of the slab, E, and modulus of subgrade reaction, k) is searched for in the multi-modal solution space by the SCE algorithm as described previously. Thus, for every update of the population of moduli solutions in the SCE search scheme, the forward pavement response model has to be invoked to compute the resulting surface deflections. In this paper, the SCE optimization technique is hybridized with a Neural Networks (NN) surrogate forward pavement response model for rapid prediction of surface deflections using elastic moduli and thicknesses of pavement layers as inputs. This reduces the computational time of SCE significantly considering the number of times the surface deflections need to be computed using different sets of pavement layer moduli during the optimization process. Thus, the resulting hybrid backcalculation model, NN-SCE combines the robustness of SCE with the computational efficiency of NNs.



Figure 2: Neural Networks (NN) - Shuffled Complex Evolution (SCE) Hybrid Global Optimization Backcalculation Approach

The SCE, in essence, finds the optimal values of the NN inputs (pavement layer moduli) iteratively such that the corresponding values of the network outputs (deflections) match the measured pavement surface deflections to minimize the differences between the measured and computer deflections. Although the error-minimization deflection-based objective function can be defined in a number of ways, a simple objective function representing sum of the squared differences between measured and computed deflections as shown in Equation 1 was selected for this study (where n = 6):

$$f = \sum_{i=1}^{n} (D_i - d_i)^2$$
(1)

The NN-SCE hybrid optimization toolbox for rigid pavement backcalculation was implemented in MATLAB. The input variables to the toolbox include six FWD measured surface deflections at 300-mm radial offsets starting from the center of the FWD loading plate (D0, D300, D600, D900, D1200, and D1500), PCC layer thickness, and the corresponding min-max ranges of pavement layer moduli. For ease of implementation, all values were normalized in the range of 0.1 to 0.9

A trained NN serves as a surrogate forward pavement response model that has learned the mapping between pavement layer moduli and resulting pavement surface deflections for a variety of case scenarios generated using the ISLAB 2000 pavement finite element program, as described in the next section.

The choice of the SCE algorithm's parameters is crucial in achieving convergence of solution for the problem under consideration. In this study, the following guidelines proposed by Duan et al. (1992) were used for determining the SCE parameters (m = number of points in each complexes, p = number of complexes, q = number of parent solutions, α and β are user specified parameters):

- m = 2n, where n = dimension or number of parameters being estimated
- q = n+1 and p = 5
- $\beta = 1$ and $\alpha = 1$

NN Surrogate Forward Pavement Response Model

A BackPropagation (BP) type NN model was trained in this study with the results from the ISLAB 2000 finite element program to develop the surrogate forward pavement response model. BP NNs are very powerful and versatile networks that can be taught a mapping from one data space to another using example of the mapping to be learned. The term "back-propagation network" actually refers to a multi-layered; feed-forward neural network trained using an error back-propagation algorithm. The learning process performed by this algorithm is called "back-propagation learning" which is mainly an "error minimization technique" (Haykin, 1999). Backpropagation networks excel at data modelling with their superior function approximation capabilities (Haykin, 1999).

A total of 41,106 data vectors generated by modeling slab-on-grade concrete pavement systems using ISLAB 2000 were used for NN training and testing. Concrete pavements analyzed in this study were represented by a six-slab assembly, each slab having dimensions of 6.1 m by 6.1 m (20 ft by 20 ft). The dense liquid model, proposed by Winkler (1867), was used to characterize the subgrade behavior. To maintain the same level of accuracy in the results from all analyses, a standard ISLAB 2000 finite element mesh was constructed for the slab. This mesh consisted of 10,004 elements with 10,209 nodes. The 40-kN (9,000-lb) FWD loading condition was simulated in ISLAB 2000.

The ISLAB 2000 solutions database was generated by varying E, k and thickness of PCC (h_{pcc}) over a range of values representative of realistic variations in the field. The E ranged from 6.9 to 103.4 GPa (1,000 to 15,000 ksi); k ranged from 13.6 to 217 MPa/m (50 to 800 psi/in); and h_{pcc} ranged from 152 to 635 mm (6 to 25 in) considering that most design thicknesses would be in this range. Note that the US design approach for concrete pavements limits k to 0.16-0.20 N/mm³. A Poisson's ratio (μ) of 0.15 was assumed for PCC. Thus a total of 41,106 ISLAB 2000 analyses (51 different values of E × 31 different values of k × 26 different values of h_{pcc}) were conducted to represent a complete factorial of all the input values.

Histograms and quantile box plots of inputs (E, k, and h_{pcc}) for generating the synthetic database are presented in Figure 3. Frequency histogram is shown on the left side and a box plot is shown on the right side. The rectangular box plot extends from the first to the third quartile with the median marked in the center.

A scatterplot for each pair of variables in the synthetic database is displayed in a matrix arrangement in Figure 4. A 95% bivariate normal density ellipse is imposed on each scatterplot with the correlation values shown. If the variables are bivariate normally distributed, this ellipse encloses approximately 95% of the points. The correlation of the variables is seen by the collapsing of the ellipse along the diagonal axis. If the ellipse is fairly round and is not diagonally oriented, the variables are uncorrelated. It is observed that the

modulus of subgrade reaction, k, is strongly correlated to deflections at the outer sensors (especially D1200 and D1500). Relatively, E is poorly correlated to all the deflections.



Figure 3: Histograms and quantile box plots of inputs (E, k, and h)

A network with two hidden layers was exclusively chosen for the NN models trained in this study. Satisfactory results were obtained in the previous studies with these types of networks due to their ability to better facilitate the nonlinear functional mapping (Ceylan, 2002; Ceylan et al, 2005).

In the BP NNs used in this study, the connection weights are initially selected at random. Inputs from the mapping examples are propagated forward through each layer of the network to emerge as outputs. The errors between those outputs and the correct answers are then propagated backwards through the network and the connection weights are individually adjusted to reduce the error. After many examples (training patterns) have been propagated through the network many times, the mapping function is learned with some specified error tolerance. This is called supervised learning because the network has to be shown the correct answers for it to learn (Haykin, 1999; Ceylan et al, 2005).

The 3-60-60-6 architecture (see Figure 5: 3 inputs [E, k, and PCC layer thickness], 60 nodes in the first and second hidden layers, and 6 outputs [D0 ~ D1500], respectively) was chosen as the best architecture based on its lowest training and testing Mean Squared Errors (MSEs). The NN training and testing were conducted with the MATLAB NN toolbox using the Levenberg-Marquardt training algorithm. In the BP learning algorithm, the error energy used for monitoring the progress toward convergence is the generalized value of all errors that is calculated by the least-squares formulation and represented by a Mean Squared Error (MSE) as follows (Haykin 1999):

$$MSE = \frac{1}{MP} \sum_{1}^{P} \sum_{k=1}^{M} (d_{k} - y_{k})^{2}$$
(2)

where y_k and d_k are actual and desired outputs, respectively, M is the number of neurons in the output layer and P represents the total number of training patterns. Other performance measures such as the Root Mean Squared Error (RMSE), Average Absolute Error (AAE), etc. are also used.

Figure 6 shows the training and testing MSE progress curves for the 3-60-60-6 network. Both the training and testing curves for the output are in the same order of magnitude thus depicting proper training. The almost constant MSEs obtained for the last few iterations also provide a good indication of adequate training of this network. Exceptional prediction performance of the NN surrogate response models were achieved validating their suitability for use in SCE hybrid optimization approach.





Figure 5: NN Surrogate Forward Pavement Response Model Architecture



Figure 6: NN Surrogate Forward Pavement Response Model training progress curve

Prediction Performance of NN-SCE Backcalculation Tool

Hypothetical data covering wide ranges of layer thicknesses and FWD deflections commonly encountered in the field were first used to evaluate the prediction accuracy of the developed NN-SCE hybrid concrete pavement backcalculation tool. A total of about 150 datasets were independently selected from the comprehensive synthetic FE solutions database to assess the prediction performance. The performance of NN-SCE optimization approach in backcalculating concrete pavement layer moduli is reported in Figure 7. As shown in the plots, all 150 NESCE backcalculation predictions fell on the line of equality for the two pavement layer moduli (concrete modulus, E and modulus of subgrade reaction, k) thus indicating a proper training and excellent performance of the proposed hybrid backcalculation model.

The *goodness-of-fit* statistics for the NN-SCE model predictions were performed using statistical parameters such as the correlation coefficient (R^2) and the average absolute error (AAE). Average Absolute Errors (AAEs) were calculated as sum of the individual absolute relative errors divided by the number of independent testing patterns:

Average Absolute Error (AAE), % =

$$\sum_{i=1}^{n} \frac{y_{actual} - y_{predicted}}{y_{actual}} \bigg|_{i} *100$$
(3)

Where i is the *i*th testing pattern among n testing patterns.

The R^2 is a measure of correlation between the predicted and the measured values and therefore, determines accuracy of the fitting model (higher R^2 equates to higher accuracy). The AAE indicates the relative improvement in accuracy and thus a smaller value is indicative of better accuracy.



(a)



Figure 7: Prediction performance of NN-SCE backcalculation tool with hypothetical data: (a) E; (b) k

The modulus of subgrade reaction (k) is the stress that will cause one unit of deflection in the underlying soil. Soils such as clay will have a lower k-value compared to cement treated or asphalt treated bases. Research has shown that the value of k depends on certain soil characteristics such as density, moisture, soil texture and other factors that influence the strength of the soils. The k-value of a particular soil will also vary with size of the loaded area and the amount of deflection. The modulus of subgrade reaction is directly proportional to the loaded area and inversely proportional to the deflection. Modulus of subgrade reaction is obtained by conventional plate bearing tests as described in AASHTO T222, correlation with soil properties or other soil tests and also by backcalculation from FWD testing on concrete pavements.

It is well known that the backcalculated pavement properties are strongly related to the backcalculation model (Rufino et al., 2002). For slab-on-grade systems, Ioannides (1990) proposed a closed-form procedure for backcalculating foundation properties based on principles of dimensional analysis by recognizing the existence of a unique relationship between AREA (Hoffman and Thompson, 1981) and radius of relative stiffness (l_k). Once the radius of relative stiffness is known, Westergaard's (1926) maximum deflection solution for interior loading can be used to backcalculate the subgrade k-value. Once the subgrade properties and radius of relative stiffness are known, the slab modulus of elasticity (E) can also be determined. This approach has been coded in a computer program called ILLI-BACK by Ioannides (1990). The following equations are used:

$$AREA = 6*\left[1 + 2\left(\frac{D_{12}}{D_0}\right) + 2\left(\frac{D_{24}}{D_0}\right) + \left(\frac{D_{36}}{D_0}\right)\right]$$
(4)

$$l_{k} = \left[\frac{\ln\left(\frac{36 - AREA}{1812.279133}\right)}{-2.559340}\right]^{4.387009}$$
(5)

$$k = \left(\frac{P}{8D_0 l_k^2}\right) \left\{ 1 + \left(\frac{1}{2\pi}\right) \left[\ln\left(\frac{a}{2l_k}\right) - 0.673 \right] \left(\frac{a}{l_k}\right)^2 \right\}$$
(6)

$$E = \left(\frac{12l_k^{\ 4}k(1-\mu^2)}{h^3}\right)$$
(7)

where the deflections are in inches; P is the FWD/HWD load in lbs; a is the radius of load plate (usually 6 inches); and h is the effective slab thickness in inches. Note that the k-value obtained from this backcalculation procedure is the dynamic-k value which is typically twice the k value obtained from plate bearing tests. Using this closed-form backcalculation procedure, the k-values were backcalculated using the same hypothetical data used for testing the prediction performance of NN-SCE approach.

For the sake of illustration, the comparison between the closed-form procedure k values and NN-SCE predicted k values are presented in Figure 8. There is a good agreement between the two especially at lower k values. The NN-SCE ANN predictions were also in agreement with the NN based backcalculation results reported by Ceylan et al. (2008).



Figure 8: NN-SCE k predictions compared with closed-form solutions

Summary and Conclusions

The backcalculation methodology is an inverse process to determine in-situ materials stiffness of pavement layer by matching the measured and the theoretical deflection with iteration or optimization schemes. Over the years, numerous pavement backcalculation approaches have been developed. Each approach has its own pros and cons. This paper focused on the development of a new backcalculation method for concrete pavements based on a hybrid evolutionary global optimization algorithm, namely Shuffled Complex Evolution (SCE). The SCE algorithm developed at the University of Arizona is reported to be an efficient global optimization method that can be used to handle non-linear problems with high-parameter dimensionality. This approach treats backcalculation as a global optimization problem where the cost function to be minimized is defined as the differences in measured and computed deflections. The optimal solution (elastic modulus of the slab, E, and modulus of subgrade reaction, k) is searched for in the multi-modal solution space by the SCE algorithm. Hypothetical data covering wide ranges of layer thicknesses and FWD deflections commonly encountered in the field were first used to evaluate the prediction accuracy of the developed NN-SCE hybrid concrete pavement backcalculation tool. The results demonstrated the excellent performance of the developed backcalculation tool. Future studies will focus on validating the developed approach with actual field data.

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