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THE DEVELOPMENT OF A COOPERATIVE MODEL TO ANALYZE THE EFFECTS OF DIFFERENTIAL MEMBER TREATMENT

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The development of a cooperative model to analyze the

effects of differential member treatment

by

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CHAPTER I. INTRODUCTION

Problem Statement

The environment in which cooperatives operate has
changed a great deal since 1844 when the Rochdale pioneers
established the first permanent cooperative on Toad Lane and
laid down a set of operating principles. Robert Owen,
Charles Fourier and others based their ideas of cooperative
philosophy on the premise that all members are homogeneous
and should be treated equally. Even though the founding
Rochdale pioneers included individuals of different
economic, social, and political classes, no member was
intentionally treated any better or worse than anyone else.
Today's interpretation of the cooperative ideology also
presumes that all members are equal and should be treated
similarly. All members have historically been treated
similarly because no single member was distinquished from
the others. If a difference among members existed, the
dissimilarity did not have any influence on how the members
were treated. The cooperative principle of operating at
cost by paying patronage refunds exhibits the pioneers'
intent that patrons should be accountable to the cooperative
for the cost associated with the provision of a product or
service. The cooperative business operates at cost by

returning any surplus to the members in the form of a patronage refund. Although cooperative philosophy was based on the idea of a homogeneous membership, the diversity of modern American agriculture has brought about a trend towards non-homogeneity of agricultural producers. The decline in farm numbers coupled with the increasingly bimodal distribution of farm size causes potential operating problems for cooperatives which treat all patrons exactly alike. For example, should larger producers with larger patronage volumes be treated any differently than smaller producers with correspondingly smaller patronage volumes? Strict adherence to the wording of the Rochdale Principle "equal treatment of members" would say no, however the actual intent may yield a different answer. There may be a problem of semantics here. If the original intent of the pioneers was to have individual members pay the costs of being provided the service, different member types could be expected to pay different prices. Of course, if a cooperative's cost of supplying a service is independent of the volume of an individual's patronage, all patrons will still pay the same amount. The problem of patrons not paying cost-justified prices could arise if the cooperative's costs are dependent on an individual's patronage level. Historically, patrons have paid the same price without any consideration given to the individual's

volume of patronage. There is an increasing trend for large

volume patrons to demand a more favorable price than the

average patron because other businesses will try to attract

these patrons by offering them a better price. Using a

simplistic cooperative scenario looking only at the pricing

policies of the decision-maker (omitting any financing

concerns) the issue becomes more apparent. Table 1.1

illustrates the situation where a cooperative supplies a

service to patrons with varying amounts of patronage but

charges only one price for the service. For simplicity,

assume that this price includes any value of future

patronage refunds.

Table 1.1. Example of a cooperative using a single price

Patron Group	% of Business	Average Cost of Provision	Competitor Price	Cooperative Price	Producer Benefit of Patronage
A	70	$5.00	$5.20	$5.40	$(.20)
B	20	6.00	6.24	5.40	.84
C	10	7.00	7.28	5.40	1.88
weighted average		5.40			

Assumptions:

1. The cooperative knows the exact average total cost of providing the service to each group.

2. The cooperative members can purchase the same service from a non-cooperative competitor which has a pricing policy of cost plus 4%.

3. The cooperative's average cost of supplying the service
   to patrons is negatively related to the volume of
   patronage of the patron.

This example shows the paradoxical situation of a

cooperative with a heterogeneous membership. Patrons in

group A who account for 70% of the total cooperative

business volume pay the same amount for the service ($5.40)

as patrons in groups B and C, even though the cooperative

incurs a lower average cost in providing the service to

group A patrons ($5.00). Looking solely at the monetary

benefits from the cooperative, patrons in group A would have

negative benefits since the service could be obtained

through alternative sources for 20 cents less. Is it

equitable that these large volume patrons subsidize the cost

of supplying lower volume patrons? In the case of a supply

cooperative, if business operations are to be done at cost,

an argument (as shown in Table 1.2) can be made that each

member group should pay for the cost of obtaining the

services that they demand. Table 1.2 illustrates the

implications for a supply cooperative which takes into

consideration the actual cost of supplying the service to

specific member groups in determining the prices charged to

those groups.

Table 1.2. Example of a cooperative using multiple prices

Patron Group	% of Business	Average Cost of Provision	Competitor Price	Cooperative Price	Producer Benefit of Patronage
A	70	$5.00	$5.20	$5.00	$.20
B	20	6.00	6.24	6.00	.24
C	10	7.00	7.28	7.00	.28

The assumptions for Table 1.2 are the same as those for Table 1.1 except that instead of one average price being charged for the service, each member group pays only the cost the cooperative incurs in providing the service to them. Using this type of pricing strategy, each member pays a price that is better than the price from alternative sources and still covers all costs associated with its provision.

By differentially treating members the cooperative may be able to retain the members of group A. Without the members in group A the weighted average cost of provision increases from $5.40 to $6.33. If the cooperative can keep the business of group A, the members in the other groups would benefit. It would be to the advantage of these other groups to allow the cooperative decision-maker to differentially treat those in group A if it was the only way to maintain their patronage. The additional business volume created by group A may enable to cooperative to gain economies of scale which could be passed back in varying

proportions to all groups. This illustrates why differential treatment could be beneficial to all groups of members.

The objective of this paper is to look at the effects on the cooperative when each member group can be treated in a different manner from the others. This research will look into the effects on cooperative membership, finance, and profit. It would be interesting to look at the fairness or equitability of differential member treatment, however defining what is equitable is difficult and goes beyond the scope and intent of this research. We will not say whether one situation is more "equitable" than another. No subjective judgements will be made. Only comparisons of the quantifiable effects of different cases will be attempted, leaving the interpretation of "fairness" to others.

Problem Formulation

Previous models (Royer 1978, VanSickle 1980, Fischer 1984) have analyzed the operational procedures of cooperatives. These theoretical models provided cooperative management with information that could help them in striving for optimal operating and financing strategies. Royer [52] devised an optimizing model for determining pricing and production policies. The work done by VanSickle [64] was

designed to examine the cooperative decision nexus of the relationship between the production and pricing decision and the financing decision. Fischer [18] assesses cooperative financial concerns by forming a model that contains both the pricing and production problem and the financial problem. None of these models allowed individual members or member groups to be treated differentially since it was assumed the cooperative membership was homogeneous. Prior research identifies only a "typical member" of a cooperative. The model presented in Chapter III maintains that a more pertinent view would be to look at a typical member of a categorical group within the cooperative. How these groups are devised can be based on many different criteria. For reasons mentioned later, this work considers member groups to be defined on the basis of varying levels of cooperative patronage. By looking at specific types of groups within the cooperative, the implications for optimal operating strategies for a cooperative can be determined. The model proposed in Chapter III will enable the cooperative decision-maker to look at the problems of pricing, production, and financing from this vantage point. The decision-maker can now determine how treating distinct groups of members dissimilarly will affect the cooperative pricing, production, and financial strategies.

Review of the Literature

There has been no previous research looking specifically at the differential treatment of cooperative members. In reviewing the literature of cooperative associations, there has been no discussion of heterogeneous members. From the earliest research of Nicholls [46] and Emelianoff [15] to recent models of Royer [52] and Fischer [18], none have mechanisms to study a specific member. Past research has looked at the cooperative and/or a typical member. A brief discussion of the previous research on cooperative theory will be presented.

There are two distinct approaches found in the literature. One considers the cooperative as an optimizing, decision-making unit while the other does not. Most theoretical models after Helmberger and Hoos's [27] 1962 article regard a cooperative as an enterprise which can make decisions as a distinct unit. The majority of the earlier literature saw the cooperative only as an extension of a group of individual members and, as Phillips [48, p. 249] put it, cooperatives do not have a "separate economic identity;...". This latter school of thought does not recognize a cooperative as a business enterprise separate from that of the members. Since there is little question now that the cooperative is an economic entity, only brief

attention will be given to the literature that does not view the cooperative as a decision-making unit. The emphasis of this literature review will be on the decision-maker cooperative models, with an indepth analysis of the Royer [52], VanSickle [64], and Fischer [18] work.

Emelianoff, one of the earlier cooperative theorists, was provoked by the question [15, p. 246] "what have we got to do to be entitled to be considered a cooperative?". In answering this question he concluded that a cooperative was an organization of independent economic units (patrons) which is coordinated, owned and controlled by these same economic entities. Another conclusion that Emelianoff [15] reached was that to remain a stable and viable cooperative, the membership must be homogeneous. He called this the "unwritten law" of cooperation. The belief was that a heterogeneous membership could cause social unrest in the cooperative. Many others followed Emelianoff's school of thought including, most notably, Robotka [50], Phillips [48], Aresvik [4], and Trifon [58]. All these authors echo the sentiment that a cooperative is an economically lifeless unit. Robotka asserts that members of the cooperative, while independent from each other, mutually operate the cooperative for the joint benefit of all members. The cooperative has an economic purpose only in the sense that producers can band together and benefit from its existence;

it has no economic life of its own. Phillips [48, pp. 74-75] affirms Robotka's convictions when he stated a "cooperative has no more economic life or purpose apart from that of the participating economic units than one of the individual plants of a large multi-plant firm...". An integral concept of Phillips work was that of proportionality. That is, all participating firms of the joint multi-plant will share on a proportional basis the economic endeavors of the cooperative. The economic use of the cooperative, all the costs, financial responsibility, and economic benefits, if any, would be shared among members on a proportional basis according to patronage. A cooperative was viewed as a vertically integrated · multi-plant firm with individual members representing separate plants which supply inputs to the multi-plant firm. Each individual member (plant) allocates its resources between its own production process and the jointly owned multi-plant. Phillips maintained that a cooperative was a multi-plant firm and that corresponding economic theory could be used to derive the optimal allocation of firm resources. For the individual firms (members) to maximize profits they must meet the following conditions: (a) marginal productivity of each resource used by the member and cooperative must be equal, and (b) individual members will equate the sum of their marginal costs to both the

marginal cost and marginal revenue of the cooperative. The individual members will attain these optimality conditions by varying their own patronage volumes.

Both Aresvik [4] and Ohm [47] expressed skepticism concerning Phillip's second optimality condition. They maintained that in practice producers will use the concept of average cost or average revenue rather than marginal cost or marginal revenue. Trifon also expounded on the criticism of Aresvik and Ohm by saying that this second condition was even incompatible with Phillip's concept of proportionality. It was pointed out that by using Phillip's intrepretation of proportionality, members should expect to pay average cost and receive average revenue, not the respective marginal concepts. Since members customarily do not know the marginal cost or marginal revenue associated with the cooperative, they use the associated average concepts. Another criticism of Phillip's work was the analogy of the cooperative as a multi-plant firm. Trifon argued that a cooperative can not be considered a vertically integrated multi-plant firm since the former serves a single economic interest whereas the latter serves many simultaneously.

Enke [16] proposed a model for consumer cooperatives where a decision-maker allocates resources such that the sum of members' consumer surplus and profits are maximized. A crucial assumption of Enke's model is that members determine

their level of patronage solely on the initial price charged with no consideration given to eventual refunds. This simplifies the cooperative decision-maker's selection of output level since quantity is now only a function of the initial price charged. Following this assumption the decision-maker should set the price charged equal to the associated marginal cost. Future researchers questioned why patronage refunds would not be involved with the members determination of patronage level and developed models which incorporated this idea. When members do anticipate some refund it generally disrupts Enke's condition that price should be set equal to the marginal cost [18, p. 44]. Assuming member patronage is a function of net price, the cooperative decision-maker should set the price charged equal to the average cost, not marginal cost.

Clark [10] presented another model in which the cooperative acted as a decision-maker. The cooperative had an objective function that minimized the cost of providing goods to members. The cooperative decision-maker minimizes the cost of providing goods by operating at the point where the average cost equals the marginal cost of providing the good (point of minimum average cost). Since Clark assumed that all members had a fixed level of "physical patronage", the decision-maker can achieve the optimal level of operations by regulating the level of cooperative

membership. Clark maintained that the cooperative principle of service at cost was very important and should be preserved even if it meant denying new memberships or terminating current ones. Aizsilnieks [2] and Gislason [22] both criticized Clark's model specification and assumptions. The model was inappropriate in the sense that cooperatives can not realistically manipulate their membership size and if they could, the level determined by the cooperative may not be the level desired by members.

Because of dissatisfaction with existing objective functions, Helmberger and Hoos [27] proposed a new one with a different approach in analyzing the cooperative enterprise. A theoretical model of a single product marketing cooperative that maximizes the price paid to members was developed. By using marginal analysis, two optimality conditions were derived: (a) whatever production level the cooperative chooses it must be done at minimum cost, and (b) the cooperative's surplus must be maximized. Helmberger [26] proceeded to use this model to study the effect cooperatives had on the performance of agricultural product markets.

Hardie [25a] extended the Helmberger and Hoos model into a multi-product marketing cooperative using linear programming. A separable programming "pooling constraint" was employed to overcome several restrictions that occurred

in simple linear programming models. Using this model, the
cooperative decision-maker could use the shadow prices as
the basis for determining the allocation of cooperative
"profits" among different products. A major emphasis of
Hardie's work was that both the cooperative and the
individual member are decision-makers who attempt to
optimize their respective objective functions. Members were
assumed to maximize their surplus income over costs while
the cooperative decision-maker maximized the aggregate rent
accruing to fixed resources of members and the cooperative.
By maximizing the rents accruing to fixed factors, the
cooperative decision-maker is essentially maximizing the
aggregate surplus of all member firms.

Ladd [32] also extended the analysis of the Helmberger
and Hoos model to a multi-product cooperative. In Ladd's
model a bargaining cooperative was assumed to perform three
services, (a) selling a production input to both members and
non-members, (b) providing an excludable public good to
members only, and (c) bargaining with processors for higher
raw product prices for its members. Two different objective
functions were suggested: maximization of the price received
by the members for their raw materials and maximization of
the quantity of raw material marketed through the
cooperative. First order conditions for the different
objective functions were derived using the price charged by

the cooperative for production inputs and the level of
excludable public goods provided as choice variables. It
was shown that these two sets of first order conditions
differed from each other and from those of a profit
maximizing proprietary firm.

Royer's work [52] in modelling cooperative associations
incorporated many of the ideas presented by earlier
researchers but filled in several gaps that caused
difficulty in the earlier models. A nonlinear programming
model of a multi-product cooperative was presented that
allowed the cooperative to do non-member business on a
profit basis, permitted members to patronize other firms,
and acknowledged that expected patronage refunds entered the
individual member's decision-making process. By assuming
that individual producers maximized expected profits, profit
functions for a typical member and non-member were written:

$$\pi = \sum_{i \in X} p_i q_i - \sum_{i \in Y} p_i q_i - fc + ds + pvpr \qquad (1.1)$$

where p_i = the price of the i-th product

q_i = the quantity of the i-th product

X = a set of outputs produced by members and non-member patrons

Y = a set of variable inputs purchased by member and non-member patrons

fc = the fixed costs of a typical patron

ds = the dividends on stock held by the typical member patron

pvpr = the present value of allocated patronage refunds of a typical member

$$= [s+(1-s)/(1+d)^\tau] \sum_{i \varepsilon C} r_i^* q_i \qquad (1.2)$$

where s = a constant proportion of patronage refunds paid in cash

τ = a constant length of the cooperative's revolving period

d = a typical producer's discount rate

C = a set of products sold to or variable inputs purchased from the cooperative

r_i^* = the cooperative member's expected per unit patronage refund

By means of adding production and fixed factor usage constraints, a Lagrangian function was formed that yielded Kuhn-Tucker conditions for individual producers. Solving these Kuhn-Tucker conditions yield output supply and input demand functions for typical member and non-member patrons. These supply and demand functions were represented as:

$$q_i = q_i(P_x, P_y, R_c^*, Q_G) \qquad i \varepsilon X,Y \qquad (1.3)$$

where P_x = a price vector for products in set X

P_y = a price vector for products in set Y

R_c^* = a vector of expected per-unit patronage refunds

Q_G = a vector of public goods provided by the cooperative

The cooperative Lagrangian function is analagous to the individual's except that there are several additional constraints to consider. The cooperative's profit function is the sum of the individual member's profit functions and is written as:

$$\pi = \sum_{i \varepsilon X} P_i q_{ic} - \sum_{i \varepsilon Y} P_i q_{ic} - FCM + DS + PVPR \qquad (1.4)$$

where q_{ic} = the quantity of the i-th product purchased or sold by the member patrons

FCM = the cooperative's fixed cost

DS = the total dividends on stock

PVPR = the present value of all allocated patronage refunds

Three constraints, production, fixed factor, and the allowable amount of non-member patronage, were placed on the cooperative profit function. The Kuhn-Tucker conditions for the typical cooperative decision-maker are of questionable use because as Royer [52] noted, not only are the optimality conditions complex, but there is a great amount of information which is necessary to evaluate them. Royer's framework was applied to single product marketing and single product supply cooperatives in order to compare the optimality conditions with those of previous models. By making different sets of assumptions, Royer's model gave results similar to both Enke's and Phillip's work. Eversull [17] employed Royer's model in an empirical study

to show other practical applications of the model.
Hypothetical cooperatives were simulated to analyze the
optimality conditions and provide insight into cooperative
management practices. By assuming that producer demand and
supply functions were linearly related to basis values
(prices), Eversull utilized quadratic programming to solve
for optimal basis values that the cooperative should use. A
cooperative enterprise selling two products and purchasing
two products was set up as a base model that was solved
under several scenarios. Modifications in the model were
formulated to test changes in cooperative storage capacity,
limits on basis values, and interrelated demand and supply.
VanSickle [64] also utilized Royer's work and estimated the
supply and demand equations represented in (1.3) for several
commodities using data from Iowa cooperatives. This
empirical analysis of the pricing and production nexus
showed the validity and applicability of Royer's theoretical
model.

VanSickle [64] is the first work which addresses the
cooperative decision nexus between production, pricing, and
financing. Whereas VanSickle integrated these decisions
into one model, previous work assumed the decisions to be
independent. A submodel using Royer's work is used as the
basis for the production and pricing decisions and a
submodel which maximizes total collective profits of all

members is used to arrive at financial decisions. The study separated the cooperative decision nexus into three interdependent steps, (a) short-run determination of pricing and production practices, (b) long-run investment portfolio determination, and (c) determining the long-run cooperative financial structure. The production and pricing decisions were solved for by using an enhanced Royer model whereas a cooperative financial model was developed to answer long-run financial questions. The financial sub-model provided a membership function of the cooperative which in turn provided implications for long-run cooperative financial structure. The optimality conditions were derived, but as in the case of Royer's work, their complexity diminished the practical usage by cooperative decision-makers. VanSickle and Ladd [67] used the theoretical model presented by VanSickle [64] and derived optimal levels of qualified patronage refunds, stock dividends, revolving fund period, percent cash patronage refund, and the amount of cooperative debt by maximizing cooperative profits. The levels of pricing and production were assumed exogenous in the financial model of profit maximization.

In a vein similar to VanSickle, Fischer [18] developed a model that consolidates the optimal production, pricing, and financing decisions of the cooperative decision-maker. Fischer goes further than Royer's and VanSickle's

normative-prescriptive study in that in addition to determining how cooperatives should be financed, a look is taken at how they are financed and why there are any discrepancies. Using an objective function similar to Royer [52], Fischer assumed that a typical cooperative member would maximize his expected after-tax "profit". After-tax profit for a member of a single-product farm supply cooperative is represented as:

$$\pi^* = (P_y^* y - r(lf)d - p_m q_p - p_o q_c)(1-T_p) + pvpr^* q_c$$

$$(1.5)$$

where π^* = expected after-tax "profit"

P_y^* = expected farm product price

y = output produced and sold by member

$r(lf)$ = interest rate on farm debt, a function of

lf = farm leverage (=d/e)

d = farm debt

e = farmer's adjusted net worth (= eb - ea)

eb = farmer's book net worth

ea = allocated equity owned by farmer

p_m = market price of input q when purchased outside the cooperative

p_o = initial cooperative price of input

q_p = amount of q bought from non-cooperative sources

q_c = amount of q bought from cooperative

$$T_p = \text{producer's personal tax rate}$$

$$\text{pvpr}^* = \text{expected after-tax present value of per-unit patronage refunds}$$

Patronage refunds were specified in greater detail than by Royer [52] or VanSickle [64]. The specification of after-tax patronage refunds included terms to take into account dividends paid on allocated equity. Expected after-tax qualified patronage refunds were given as:

$$\text{pvpr}^* = \text{pr}^*\{s-T_p+(1-s)[[r_c^*(1-T_p)/k_{ae}] + [k_{ae}-r_c^*(1-T_p)]]/k_{ae}(1+k_{ae})^\tau\} \tag{1.6}$$

where $\text{pr}^* = $ expected book value of per unit refund

$s = $ percent of refund paid in cash

$(1-s) = $ fraction of refund retained as allocated equity

$T_p = $ member's marginal personal tax rate

$\tau = $ length of revolving fund period

$r_c^* = $ expected dividend rate on allocated equity

$k_{ae} = $ discount rate for expected after-tax return on allocated equity

Assuming no dividends were paid on allocated equity, the after-tax patronage refund equation is similar to the one used by Royer [52] and VanSickle [64] as represented by (1.7).

$$\text{pvpr}^*|_{rc=0} = \text{pr}^*[s-T_p+(1-s)/(1+k_{ae})^\tau] \tag{1.7}$$

Using q_p, q_c and d as choice variables the members could determine their optimal production and pricing decisions. With two markets for the producers' product, the determination of q_c and q_p depend on the prices between the two markets. Depending on the prices, if $p_m(1-T_p)$ is greater (less) than $p_o(1-T_p)-pvpr^*$, the producer will patronize the private (cooperative) suppliers exclusively. Fischer then proceeds to work through the farmers' profit function using the minimum price of the good for the two markets and sets up optimality conditions concerning the usage of farm inputs between the two sources. The member's demand for an input is said to be a function of the expected price, the farmer's adjusted net worth and the producer's personal tax rate. Risk is then incorporated into the model by assuming that a producer's expected utility is represented as: $EU = \pi^* +(a/2) Var(\pi^*)$, where "a" represents a risk attitude measure. When "a" is less than zero the patron is said to be risk averse.

A model of a single product farm supply cooperative is used to look at the cost of the cooperative's capital. Capital costs are first determined for a risk neutral member on a pre-tax and after-tax basis, then for a risk averse member on an after-tax basis. Similar to other studies, Fischer found that the cooperative's cost of capital was a decreasing function of leverage and that most cooperatives

should utilize more debt capital. A dynamic stochastic simulation analysis showed that increasing leverage would be both feasible and profitable for cooperatives that are earning an adequate return on assets. While Royer's model concentrates on the pricing and production decisions, Fischer as well as VanSickle also consider financial matters.

VanSickle and Ladd [65] extend the work of VanSickle [64] by developing a simulation model to find the optimal financial decisions for the cooperative. Jones [30] thought the results of the VanSickle and Ladd model were different than would be expected and scrutinized their study. The problem of how to specify the cooperative objective function was pointed out. Jones specified an objective function that was different from that used by VanSickle and Ladd. The question is, which one is correct? VanSickle and Ladd [66] respond to Jones' concerns by developing a new model that recognizes the complementarity of the deferment period (τ) and the percent patronage refund paid in cash (s). Recognizing this complementarity, a two-stage synthesis of the VanSickle and Ladd model is formulated. The model first solves for the level of H, a composite variable defined as $\tau(1-s)$, then the second stage solves for the actual values of τ and s. Other possible models are also suggested. The two-stage model has a computational advantage over the

others. The model in this two-stage process involves solving a set of simultaneous equations that has at most one non-linear equation. The other models require solving a set of non-linear simultaneous equations. Noting the computational advantage of this two-stage synthesis of the VanSickle, Ladd and Jones model, the differential treatment model presented in this work will utilize this type of a procedure.

Using the work of Royer, VanSickle, Fischer, VanSickle and Ladd, and Jones as the current status of cooperative modelling, a cooperative model of differential member treatment can be formulated. Each study provides an integral understanding of the cooperative that should be realized when formulating a model of cooperative behavior.

Statement of Objectives

The existing literature does a satisfactory job of explaining the optimizing behavior of cooperative and typical member patron decision-makers. Most cooperative decision-makers base their decisions on the assumption of a homogeneous rather than a heterogeneous membership. The existing literature is fitting when members are basically similar. However a significantly diverse membership exposes deficiencies in the literature. The objectives of this study are to provide decision-makers with a cooperative

model that allows for different types of members to be treated differently and to look at the feasibility, effects, and methods of differential treatment. Recognizing that Royer's [52] short run production-pricing model, and the production-pricing and finance models of VanSickle [64], and Fischer [18] had different objectives than this study, some aspects of their models will be used while others will not. The general model presented in this study will: (a) represent a multi-product marketing and supply cooperative, (b) allow members to patronize other businesses, (c) permit the cooperative to perform non-member business on a for-profit basis, and (d) include patronage refunds. This general model will be used to arrive at the optimality conditions that the cooperative decision-maker should strive for in setting production, pricing, and financial policies. The optimality conditions arrived at will yield the values of the choice variables that will maximize member profits which may or may not be equilibrium values. These optimality conditions will be similar to those presented by Royer [52] and VanSickle [64]. Both Royer's and VanSickle's models required extensive amounts of information and yielded complex optimality conditions. The Kuhn-Tucker conditions afforded by this model allowing for differential treatment among members will require even more detailed information from the cooperatives. Even though it may not be possible

to solve for all the optimality conditions of a specific
cooperative, the general Kuhn-Tucker conditions can be
solved which will provide general operating guidelines. The
problem of needing large amounts of data can be partially
circumvented by using simulation analysis which lends itself
to this type of situation. Previous models use marginal
analysis and linear programming methods to analyze
cooperatives while only a few have used simulation. The
work that has been done using simulation analysis has
involved cooperative finance, but not the complete operation
of a cooperative. A simulation model will be developed in
Chapter V that will call for specific information from
cooperative decision-makers and then proceed to show the
effects of differential patronage refund and stock
requirement policies. To set up a model that would
determine a level of optimal differential treatment of
members would require a large volume of information as well
as subjective decisions on the allocation of member welfare.
Therefore, these models are meant to give cooperative
decision-makers a guide to help make comparisons concerning
potential situations that could arise. For example, a
cooperative contemplating a policy of differential treatment
could use the model to simulate both the current and
anticipated situations and compare them. The study attempts

to show the internal production, pricing and financial
details of a cooperative that treats members differently.

Although cooperatives may desire to treat members
differently, a theoretical model that allows it to do so may
be of no importance if the particular policy is not allowed
for one reason or another. An examination of the practical
side of differential member treatment must be done to
determine the feasibility of such a policy. To determine
the practicality of such a procedure the heterogeneity of
cooperative members is analyzed. This study examines how
members are different and if there is a basis for
differential treatment. Even if members are different,
other obstacles such as legality, compatibility with the
Rochdale Principles, and the acceptability of the policy
must be confronted.

Following Chapters

In Chapter II, the empirical justification for a
cooperative model allowing differential treatment is given.
In this chapter, emphasis is on the practicality of
differential treatment.

In Chapter III, a general theoretical model allowing
cooperative member differential treatment is presented. A
model of an individual producer with an assumed goal of

profit maximization is given as well as a cooperative model which maximizes the total profits of all members.

In Chapter IV, the Kuhn-Tucker conditions for the general model are derived and interpreted.

In Chapter V, a simulation model is presented which allows a cooperative manager to specify information that is appropriate for their cooperative. These models are intended to aid the cooperative decision-makers by providing different scenarios which can be compared.

Finally, Chapter VI consists of a summary, conclusions, and suggestions for future research in the area of differential treatment of cooperative members.

CHAPTER II.   THE BASIS FOR A THEORETICAL MODEL THAT

ALLOWS DIFFERENTIAL TREATMENT

OF COOPERATIVE MEMBERS

Before building a cooperative model that allows differential treatment of members, some attention should be given to potential problems that may arise as a result of differential treatment. This chapter discusses some factors that establish the relevance and feasibility of such a model. Patron homogeneity, methods and effects of differential treatment, and program feasibility will be discussed.

Patron Homogeneity

The premise of a cooperative model that allows differential treatment of members is that patrons are heterogeneous rather than homogeneous. As stated earlier, most cooperative research has assumed that members are so similar as to be indistinguishable. In recent years the American agricultural sector as well as individual producers have become more specialized and diverse. Numerous studies [23, 40, 41] have shown that agricultural producers are non-homogeneous. Reports by Lasley and Goudy [40, 41] show

that the total number of Iowa farms has decreased during the period from 1969-1982, but that the decrease was not uniform across all farm sizes. Although mid-sized farms (50-499 acres) are still the dominant size, they are declining in number while small (<50 acres) and large (>500 acres) farms are increasing in number. During the eight year span from 1974 to 1982, mid-sized farms in Iowa decreased in number by 20.2% while the number of small and large farms increased 35.9% and 25.0% respectively. The trend for the United States as a whole parallels the situation in Iowa. In 1974, mid-sized farms represented 47.8% of all farms whereas in 1982 they account for only 43.7%. The change in farm numbers and size indicates that agribusiness firms must acknowledge that the majority of producers are mid-sized but they must also be able to service an increasing number of both smaller and larger producers. Traditionally firms have aimed the majority of their services at a "typical" or mid-sized patron. However, the "typical" patron may not be so typical now.

Even though farms having small acreages are increasing in number, the number of farms with small sales volume is decreasing. In general, the number of Iowa farms with sales volume greater than $40,000 are increasing and those with less than $40,000 are declining [61]. The same trend can be seen in the U.S., from 1978 to 1982 the number of farms with

sales volume over $100,000 has increased 132.8% while
smaller sales volume farms had a relatively slower increase
or a decline. The trend is for farms to market higher
dollar volumes of agricultural products. Many firms now
sell over $200,000 of products per year where only a few had
this volume 10 years ago. This gives agribusiness firms a
very broad spectrum of clients. Although many individual
producers have high sales volumes, a market may still have
to be provided to a declining number of low sales volume
producers. Both the age distribution and amount of off-farm
employment of United States farmers have remained rather
stable in recent years [60]. Even though the age
distribution is stable, agribusinesses must still serve
producers ranging from beginning farmers to older
established farmers who will soon retire.

The age distribution of producers can have an impact on
cooperatives through patronage refund policies. In Iowa
approximately 40% of all cooperative members are over 55
years of age with most still having considerable invested
cooperative equity that should be retired as soon as
possible after they stop patronizing the cooperative [21].
Depending on the cooperative's policy of retiring retained
equity capital of past patrons, the cooperative could run
into financial trouble if a large proportion of members
retire within a short time span. The heavy concentration of

older members in cooperatives has the potential to cause
such problems. Looking at a typical Iowa cooperative the
diversity of producer patrons can be seen. An equity-age
distribution obtained from a local cooperative, assuming it
is representative, shows that there is substantial overall
diversity of American producers in individual
cooperatives [21]. Data from this cooperative showed that
their membership age ranged from less than 15 to over 100
years old. The analysis indicated that 25.6% of the members
were less than 40 years of age and provided less than 13.5%
of the total cooperative equity, 53.9% of the members were
40 - 65 years old and supplied 69.9% of the total equity,
and 19.3% of the members were over 65 and provided 13.7% of
the cooperative's equity. If the percent of total
cooperative equity supplied by a patron group can be used as
a proxy for the amount of business done by that group, then
it shows that the average business volume of patrons 40 - 65
years old is greater than that for other age categories.
This supports the notion that cooperatives do have diverse
memberships that cover a large spectrum of age groups and
patronage volumes.

The degree to which farmers have off-farm employment
can also divide producers into different classifications.
In 1982, the number of U.S. producers who had no off-farm
employment and those who worked off-farm over 200 days a

year were nearly equal [61]. The existence of full-time
farmers and part-time farmers, again, leads to heterogeneity
of producers. Another difference among producers is their
location and distance from the cooperative or other market
outlet. Members can be divided into groups based on their
location relative to the cooperative. This is another
possible criterion which could provide a basis for
differential treatment.

Assuming that cooperative patrons are typical
agricultural producers, cooperatives could use farm size,
sales volume, age, amount of off-farm employment, as
differentiating characteristics of producers. The key issue
in trying to group producers is to categorize them into
groups that cost the same to service. This implies that not
all ways of categorizing producers should be used for
applying differential treatment. The use of a base capital
finance plan utilized currently by some cooperatives bases
differential treatment on use of the cooperative assets.
This differentiating method will be discussed further in the
section covering the Rochdale Principles. Even though
producers can be divided into classes by these
characteristics, some of these classification schemes can
not legally or perhaps should not, on the basis of
cooperative principles, be used as a basis for differential
treatment. The model presented in Chapter III utilizes

patron classifications based on sales or patronage volume to form a model of differential treatment of cooperative members.

Methods of Applying Differential Treatment

The cooperative decision-maker has several possible methods of differentially treating patrons. Looking at the pricing and production side first, immediate and delayed methods are available. The most obvious method of differentially treating patrons is to offer different prices to members in the various classes of patrons. This immediate method of applying differential treatment requires knowledge about the producer by the clerk at the time of the sale. By using various patronage refund policies, the actual differential treatment can occur after the time of sale and would require less information by the clerk who makes the sale. The cooperative decision-maker can classify patrons and assign differing patronage refund policies. By allotting different deferment periods and different levels of cash patronage returned, the cooperative decision-maker can adjust the refund policy.

Accounts receivable policies can be varied across different groups of patrons. This type of policy would allow the cooperative decision-maker to take into account

different levels of risk in extending credit to patrons.
Differential treatment by requiring varying contributions of
equity capital, such as base capital plans, is another
method to consider. This is elaborated on in the section
discussing cooperative principles. Other differential
treatment plans could be devised and are most likely already
being used by some cooperatives. The extent of this
practice is not known since cooperative managers may not
readily admit doing this. Any method of differential
treatment must abide by the laws governing cooperative
operation, adhere to cooperative principles, and be operable
on a business basis.

Feasibility of a Cooperative Using Differential Treatment

Legality

If a hypothesized plan to differentiate members is to
be employed, it must be legal. The plan must be able to
differentiate patrons and meet certain legal requirements.
Otherwise, a plan to treat members differently may turn into
an unjustified policy of patron discrimination.

The practice of differential member treatment has not
been explicitly tested in the legal system. There have been
no edicts or court rulings on this issue. Harl [25b] makes
no mention of this subject. However, existing statutes may

yield some legal precedent. The statute that probably has
the most impact on cooperative member differential treatment
is found in the United States Code 135.02 §(3)
IRC §1388 (a) (1) which requires that dividends must be paid
to the patron "on the basis of quality of the value of
business done with or for such patron" [62]. This can be
interpreted as meaning the cooperative must pay dividends in
a uniform manner to patrons who have similar cooperative
business value. That is, a cooperative can differentially
treat dissimilar patrons if it is "value" or cost-justified.
Another potential legal problem exists if there is a
requirement that the percent cash patronage refund and the
length of the deferment period for a given product must be
equal for all members. Additionally, if a cooperative
wishes to qualify under the section 521 of the Internal
Revenue Code, then the cooperative must treat ALL patrons,
members and non-members "alike" [25b]. An argument can be
made that even though the product may be the same for all
patrons, the cooperative has different costs associated with
the provision of the product and can charge different
prices. The average cost of servicing a very small volume
patron, or an infrequent patron, may differ from the cost of
servicing a large volume patron with regular business. The
argument is that each patron should be responsible for
paying the full cost of providing the goods or services that

he demands. By recognizing that different groups of patrons
may cost the cooperative different amounts to service,
varying levels of percent cash patronage refunds and lengths
of deferment periods may legally be used for such patrons.
The model presented in Chapter III allows the cooperative to
assign different percentage cash patronage refunds and
length of deferment period to different groups of members.

Adherence to cooperative principles

Another concern related to differential treatment of
members is its consistency with cooperative ideals as
established by such cooperatives as the Rochdale Pioneers.
If cooperative principles are grossly violated, the
cooperative way of doing business would be breached raising
concerns about their legal existence or protection under
current laws including the Capper-Volstead Act of 1922. The
original Rochdale Principles of 1884 have evolved over time
with some of the principles becoming obsolete and
disregarded while others have been modified to remain
current. A brief review of the cooperative principles as
established by the original pioneers and their original
intent is presented to discern the implications for
differential treatment of cooperative members.

Open membership   When the Rochdale cooperative was
formed in 1844 the founders wanted to have an open society

without any form of discrimination. These pioneers were interested in equality for all members. The Chartists, a political group whose main goal was to provide voting rights to all people, may have supplied the inspiration for this ideal [51]. Even though they had an open membership policy, they did have several qualifications regarding this principle. First, a maximum number of members (250) was allowed into the cooperative (possibly to enhance the social structure of the organization). Secondly, although open membership was a stated policy, membership could still be denied or revoked for those who had "bad character or habits" and/or who did not meet their social and economic responsibilities for the cooperative. Currently, this principle has been modified or altogether relaxed. Today cooperatives can have either a closed or open membership policy with most having the latter. The closed membership policy is used mainly when a cooperative needs to control the supply of its members' product. The open membership policy as seen today must be qualified. Anybody can apply, however membership may not be automatic. The patron may have to apply and/or prove his qualifications to become a member. On the other hand, any member can exit the cooperative membership by simply discontinuing patronage with the business. This principle was originally fashioned with a sociological goal in mind, "equality of members", yet

with a fixed maximum membership it could have some economic
effects on the cooperative operations. Today most
cooperative scholars emphasize the sociological aspects of
open membership while de-emphasizing its importance as an
economic principle.

Differential treatment would have no direct effect on
the open membership principle. No members would be excluded
with such a policy. In fact a situation could exist where
differential member treatment would support an open
membership policy. For example, some members may feel they
are "mistreated" if they are treated in the same way others
are. These members may be economically coerced into leaving
the cooperative to do business elsewhere. Differential
treatment could also coerce some members to discontinue
patronizing the cooperative, but it still stands that
differential member treatment does not directly violate the
open membership principle.

One man - one vote   The idea of one man - one vote
was so basic to the Rochdale founders that it was not even
mentioned in their original statutes of 1844. By 1845,
however, this idea was made more than a common assumption
when the statutes were amended. It was believed that voting
should not be based on the amounts of an individual's
capital stock, but rather on the individual person.

Cooperatives utilizing the one man - one vote concept did more than create member equality. By operating in this manner, member interest, control, and participation in the cooperative was stimulated. Charles Fourier, Robert Owens and the Chartists had earlier promoted this form of democratic control. Torgerson [57, p. 11] stated the importance of this principle as, "if liberty, equality, & freedom are indispensable in democracy, then it must also be needed in economic organizations".

The change from a strict one man - one vote policy to a system of voting based on patronage exhibits a parallel to cooperative member differential treatment. Realizing that all members may not be equal, cooperatives may give more voting rights to those with greater patronage levels. From an economic viewpoint, a cooperative can treat various groups of cooperative members differently while retaining the one man - one. vote principle.

Cash trading   Cash trading as an economic operating principle has almost been completely abrogated. Many cooperatives have even begun issuing their own credit cards. When the Rochdale Pioneers instituted this principle, the economic environment imposed the concept of assuming minimal levels of risk. At this time in England (1840-1850), credit sales were running rampant and caused many over extended and

unstable businesses. Credit difficulties led to the failure of the original cooperative at Rochdale in 1835 [1, p. 49]. The fate of this cooperative must have lingered in the minds of these founding pioneers because they decided that a strict cash policy was an appropriate method to control cash flow problems. Although cash flow problems are still major concerns, other precautions such as better business projections and cautious credit extension can alleviate this problem. Non-cooperative forms of business may also have led to cooperatives adopting a credit policy. If cooperatives failed to extend credit, non-cooperatives with credit policies would have a competitive economic edge in obtaining a potential patron's business. Abandoning the cash trading principle has not caused much concern. As mentioned earlier, one potential method of differential treatment is allowing various levels of credit to patrons. Originally any type of credit policy was in violation of the Rochdale Principles. Since this principle is not strictly followed currently, further infringement of offering various levels of credit to patrons is a moot point.

Membership education   Both Robert Owens and William King were staunch believers in education of their society members [51]. The Rochdale Pioneers, realizing that they were a special type of organization, knew that to succeed

they had to educate their members on cooperative philosophy. To begin with, few members knew of the advantages and disadvantages of a cooperative, let alone the operating principles of a cooperative. To alleviate this ignorance, a fixed percentage of profits (2.5%) was earmarked for educational purposes [1, p. 61]. Slowly a strong educational program developed which not only was directed towards members, but also managers, employees, boards of directors, and the general public. Today the need for cooperative education is as crucial as ever. Every cooperative member needs to know his rights and responsibilities within their cooperative. Non-members also need to understand the cooperative ideals. As cooperatives are becoming larger and more complex, the training of managers and board members becomes an even more crucial task. Cooperative education was not stressed in the United States until the latter 1920s [37]. Many people see it as a noble objective, however not as a cooperative principle. Without proper cooperative education programs, this type of business form could die a slow death [68, p. 11]. Members, directors, management, and employees need to know how their cooperative operates while the public has to at least have a general understanding of the cooperative philosophy.

Membership education may provide the key to a successful program of differential treatment of members. If

potential and present patrons do not understand the reasons
for treating certain patrons differently, confusion and loss
of cooperative goodwill may result. A strong education
program that explains why patrons may receive different
prices, refunds, or treatment, would help minimize the
potential turmoil of such a policy. The lack of a capable
education program prior to enactment and thereafter could
spell the doom of even the best policy of member
differential treatment.

Political and religious neutrality   A goal closely
related to equality of members is that of political and
religious neutrality. Even though they seem similar, they
were set up for different reasons. Being a very
heterogeneous group, the cooperative founders wanted to
avoid any political or religious squabbles among themselves
which would adversely affect their business. This principle
can be traced back to a resolution on neutrality passed by
the 1832 English Cooperative Congress [51]. The Chartists,
being atheists, also were decisive on the inclusion of this
idea. By the nature of this principle, it would probably be
better called a recommendation. Although it may be a
commendable practice to follow, even from the start it has
not been piously upheld. Violations have been blatant.
Several times cooperative political parties have been formed

to serve member interests. Today many cooperatives are represented by special interest lobbyist groups. The cohesion of members for a united effort is important. Any division of the membership could jeopardize the economic well-being of the cooperative. The issue of cooperative political involvement is not clear cut. There are arguments for and against. It may segregate the members but, on the other hand, one duty the cooperative has is to represent its members as a group, even if it is in the political arena. The issue of neutrality has always been controversial and difficult to abide by, and in today's cooperative it may be even more so. If the intent of the political and religious neutrality principle was to ensure cohesion of the cooperative membership, differential treatment of members certainly may cause disunity. Maintaining harmony among members when the cooperative uses a differential treatment may depend on how the policy is initially set up and the quality of the educational program. Differential member treatment based on political or religious leanings of patrons is an obvious violation of the Rochdale Principles. Treating members differently based on economic traits such as patronage volume, credit risk, or location may violate the intent of this principle if proper educational programs are neglected and member cohesion is lost.

No unusual risk assumption   When the Rochdale
cooperative was formed the original members had only a
limited source of capital. Since the initial amount of
capital was minimal, extreme care regarding the financial
operations was crucial to survival. Small amounts of
capital coupled with an initial condition of unlimited
liability created an atmosphere of extreme caution. Even
though cooperative members gained limited liability status
in 1852, financial risks were still a primary concern. This
principle is very difficult to justify as a bona fide
cooperative principle. How is unusual risk defined and how
is this risk measured? Cooperatives may be no different
from non-cooperatives in that both may generally prefer less
risk. One way to interpret this principle is to say that
cooperatives should tend to be a conservative form of
business.

There is no clear violation of this questionable
principle with differential treatment. One could also look
at the risk involved when a cooperative does not treat
members differently. Will treating preferred patrons more
favorably be worth the risk of losing patronage of the
relatively less preferred patrons? On the other hand, is
the risk involved in treating all members similarly unusual
considering that this may cause some preferred patrons to
exit the cooperative? Prima facie, differential treatment

of cooperative members does not violate the principle that
no unusual risk should be taken. There may be risk involved
in utilizing a policy of differential treatment yet with the
problems in defining "unusual risk" and measuring it, its
use is not precluded.

    <u>Limited interest on stock</u>   Unlike other types of
corporations, cooperatives are formed to provide services to
patron members rather than to benefit investors in the
business. Cooperatives were never meant to provide an
outlet for capital investment. Their sole purpose is to
serve members. Capital should be supplied by members to run
the business without any expectations of benefiting greatly
from interest on capital stock. The benefit of supplying
cooperative capital comes from the services provided by the
cooperative. Looking at Emelianoff's theory on equality, if
all members were equal, no interest would have to be paid to
maintain fairness. The reason the Rochdale Pioneers
accepted this principle of Robert Owen's was to show that a
member's capital contribution was only a means to an end,
that of providing a service. Originally it was limited
interest on "stock" while today it is interpreted as limited
interest on "capital". Returns to capital investment are
limited in order to maintain cooperative control based on
patronage rather than investment. For this principle to be

meaningful, the limit must be binding. Since the capital

market conditions have changed since the inception of the

Rochdale cooperative, the limit has also changed from 5% to

8%, the value of 8% is obtained from the Capper-Volstead Act

of 1922. The Capper-Volstead Act enabled the cooperative

form of business to exist, however several operating

conditions had to be met. One of these requirements is that

either the one man - one vote principle or the limitation of

annual dividends to 8% be used. Some states require both of

these conditions to be met.

There is the possibility that members can be paid

varying rates of return on capital stock. However, some

minor difficulties may arise. When cooperatives already pay

the maximum allowed rate, only by reducing stock dividends

of some can members be differentially treated. It may be

more appealing to cooperative members if some return rates

were increased and others held constant rather than lowering

some and holding others constant. Differential treatment

via capital stock returns would not be applicable to

non-stock cooperatives or those not paying a return on

capital stock. Even though varying rates of capital stock

returns may not work for all cooperatives, it may be a

feasible method for most.

The intent of this principle is to preserve patron

control of the cooperative by discouraging outside

investment and is unaffected by differential treatment
policies. A type of differential policy which violates this
principle is one that allows voting privileges
disproportionate to member patronage. This would also be in
violation of the one man - one vote principle discussed
earlier.

Goods should be sold at regular retail prices   One
risk that the Rochdale cooperative did not want to incur was
that of starting a price war. If they sold their goods at a
price less than the going retail price, competitors could
have retaliated by lowering their short-run price, possibly
in an attempt to drive the cooperative out of business.
Goods were priced at the going market value and any profit
was returned to the members. Today, there are two
cooperative pricing schemes. One is selling at the same
price other retailers charge and returning a refund later.
The second is selling at the cooperative's true cost. The
former method is used primarily when management can not
accurately predict cost of operations in advance, when
patronage refunds are paid in a lump sum at the year's end
to provide financial stability, and when management believes
only members should receive the benefits of the cooperative
prices. The latter pricing method may be preferred when the
cooperative desires to disrupt the market pricing system,

encourage new membership, or when costs are reasonably predictable. With the original intent of this principle stated as "to avoid price wars", obedience to this principle may not be crucial today. Individual cooperatives need to devise their own business strategies and if it calls for stiff price competition, then they should act in their best interest. If the price being charged by competitors is excessively high, there is no reason why a cooperative should have to do likewise. Charging varying "regular retail prices" is one obvious way to apply a policy of differential treatment. Cooperatives are independent business firms that can make pricing pricing decisions any way the cooperative members deem reasonable. "Regular retail prices" for individual patrons can be different if they are cost justified.

Limitation on the number of shares owned   Even though a limited return was established on capital stock, the Rochdale Pioneers also desired to control the numbers of shares owned by individual members. The intent of this principle was to yield equality of control to the members. Even though there are criticisms of this principle, it is widely accepted today. The primary criticism is that it is not needed. If a strict policy of one man – one vote and limited returns on invested capital are followed, the number

of shares owned by a single member is unimportant in the
control of the cooperative. The second criticism is that it
is meaningless for non-stock cooperatives. These criticisms
show the non-necessity of this principle.

Some cooperatives use a base capital financing plan
which requires each patron to supply a certain amount of
capital based on his patronage level. This base capital
plan is a type of differential treatment since those who
utilize the capital assets of the cooperative more provide
proportionately more equity. With no differential treatment
policy, an interesting predicament arises for the
cooperative decision-maker when he considers the fair price
and financing responsibilities of patrons. For example, it
could be argued large volume patrons should receive a higher
price for their products and should supply relatively more
equity capital. It follows that when all members finance
the cooperative equally and receive the same prices, large
volume producers realize benefits with respect to financing
since they utilize the cooperative equity capital more fully
while on the other hand they realize a "loss" in the product
price they receive. Conversely small volume patrons benefit
from higher product prices received but must pay
disproportionately more for the use of the cooperative
capital. The trade off between these pricing and financing
gains (losses) should be considered when devising a

differential treatment policy. If larger quantities of stock are required for some patrons the complete picture of the patron must be considered. Some cooperatives have already used a type of differential member treatment by using a base equity capital plan. There does not seem to be any discrepancies between using differential financing treatment and limiting the number of shares owned. Considering that the limit on the number of stock shares owned by each patron was intended to keep the cooperative control in the hands of the users, differential financing treatment should not be precluded.

Net margins are distributed according to patronage

The Lennox Town Society in Scotland (1812) was the first verified consumer's cooperative to use patronage refunds [51]. The Rochdale Pioneers perceived their cooperative only as an extension of their individual businesses. The net earnings of the cooperative belonged to the members and was distributed back to the members according to their patronage so no one would gain at another's expense. The cooperative wanted to be fair to all members no matter what their volume of business with the cooperative was. Emelianoff emphasized this in his theory of cooperation by saying that cooperatives operated not necessarily on the principle of equality, but more so on the

idea of proportionality. Another crucial aspect of this
principle is the idea that a cooperative earns no profit for
itself. It operates at cost and returns the net margins to
the patrons. Since not all patronage comes from members,
the cooperative can decide whether or not to distribute
patronage refunds to these non-members. This is not always
an easy decision. If the cooperative wants to obtain the
benefits of being organized under Chapter 521 as outlined by
VanSickle and Ladd [67], patronage refunds need to be
distributed to members and non-members alike.

Net margins are distributed back to patrons in the form
of patronage refunds. The value of each member's patronage
refund depends on the level of patronage (member
determined), the amount received in cash for the year's
trading, and the length of the revolving fund used by the
cooperative. The cooperative decision-maker could vary the
percentage cash patronage refund or the length of the
deferment period to differentially treat members. As long
as the cooperative distributes net margins, the
permissibility of differential treatment should not be
questioned with respect to this principle.

Even though the present principles of cooperation are
not always well-defined, or agreed upon, the principles of
operation at cost, member ownership and control, and limited
returns on equity are universally accepted as the

distinguishing characteristics of a cooperative firm. With some care and planning, a policy of differential treatment of cooperative members can be used while upholding these predominant principles as well as the intent of all the Rochdale Principles.

Acceptance by members

In addition to considering the legality and adherence to cooperative ideology of differential treatment of cooperative members, the policy must be acceptable to the members. The policy must be acceptable to the majority of the patrons and be able to be implemented. In the decision to implement a policy of differential treatment, the decision-maker must acknowledge the consequences that it will have on the cooperative. The decision-maker should acknowledge membership implications, program equitability, and legal issues. The implications of a differential treatment policy on membership is probably the most important issue the decision-maker must consider. Since cooperative membership is voluntary, patrons can join or leave any time they wish. The cooperative will try to satisfy patron needs and most likely try to maintain or increase its membership. When members are homogeneous, a single operational strategy will attract or deter all patrons and potential patrons alike. Cooperatives with a

differential treatment policy because of a heterogeneous membership must simultaneously try to cater to the needs of various patron types in order to maintain the current membership. Problems arise when some members perceive others to be treated more favorably for no apparent reason. The cooperative decision-maker must be in touch with the total membership because when a group of members feels slighted too much, they may not mesh together with other groups and one may secede and start a new organization. It should be noted that this type of tension can arise even without a policy of differential treatment. With a policy of differential treatment, some members may be treated "more favorably" than others because the value of business done with the cooperative is greater. Since cooperative membership is voluntary, with or without a policy of differential treatment, changes in the membership will depend to some extent on cooperative operating practices. The sole purpose of differential treatment is to eliminate or reduce "unjust" treatment of patrons. However, in this process previously "favored" members may be slighted and vice versa. The cooperative decision-maker should reflect upon the change of membership stratification when implementing a policy of differential treatment. An obvious factor in determining who will retain membership and who will leave rests on the loyalty of the members. Loyalty is

a function of many factors ranging from the years of
cooperative patronage to the existence of alternative
markets. The greater the degree of cooperative loyalty, the
less apt a member will consider terminating membership.
When contemplating policy changes the cooperative with a
high degree of member loyalty will have less fluctuation in
membership as compared to those with very fickle patrons.
French et al. [20, p. 237] predict that member commitment
(loyalty) will increase during the period of 1978-1987 since
more cooperatives are making use of marketing agreements and
contracts. To assess the membership implications of
converting to a policy of differential treatment of members,
membership loyalty should be considered.

As discussed earlier there are some legal questions to
consider with differential treatment policies. If the
cooperative must pay equal proportions of cash patronage
refunds, would some members voluntarily accept less and
receive other remunerations? In using the cost differences
in providing goods to members as the basis for differential
treatment, can this cost schedule be ascertained? These
questions and others should be considered when the
cooperative decision-makers contemplate a differential
treatment policy.

The feasibility of differential treatment of members
rests on its adherence to cooperative principles, its

compliance with legal regulations, and its acceptance by members. These three inter-related concerns must be confronted in implementing a differential treatment policy. Each area may generate barriers to successful differential treatment, yet each might be overcome. Despite the potential problems, a program of differential treatment is a possible strategy that should be considered by all cooperatives.

CHAPTER III. DEVELOPMENT OF THE ECONOMIC MODEL

The model developed here is intended to provide cooperative decision-makers a framework with which to allow differential treatment of members and analyze its consequences. Using models developed by Royer [52], VanSickle [64], and VanSickle and Ladd [66] as a foundation, mechanisms allowing members to be treated dissimilarly along with ideas from Fischer's [18] and Jones' [30] work are combined to form a new model. The major difference between the previous works and this model is in their ability to look at a typical individual member versus looking at different groups or types of members. Unlike previous models which are based on the assumption that members are homogeneous and therefore should all receive the same prices and patronage refunds, this model is based on the fact that cooperative members are heterogeneous and could receive different prices, patronage refunds, or dividends on stock. The procedure to be followed in building the model will be to look at the individual producer first and then construct a cooperative firm model which is a summation of individual objective functions that incorporates some additional constraints. The model extends the previous work by integrating the production-pricing and the financial models

developed by others. To integrate the two models the timing of the production-pricing and financial decisions must be considered. Chronologically the first decisions made are those for the production and price levels of the cooperatives inputs and outputs. These are made at the beginning of the fiscal year. These decisions will affect the values concerning quantity and net savings. The cooperative then operates for its fiscal year with these production-pricing levels. At the end of the fiscal year the cooperative will make decisions that determine the financial structure and how patronage refunds are allocated. These decisions ultimately affect membership numbers. The next step is to determine the next year's production-pricing decisions using the changes in the cooperative's financial structure from the previous fiscal period's financial decisions.

The Individual Producer

Assuming individual producers wish to maximize profits subject to production constraints, a Lagrangian function can be set up for each cooperative patron. The producer's objective (profit) function and production function are set up in a general form to be applicable for both member and non-member patrons. These functions are general enough to allow for differences among patrons yet do not force

heterogeneity of members on the cooperative decision-maker.
The model will allow all patrons to be different or they can
all be similar. The model assumes that there are "m"
different patron groups in the cooperative where "m" is
greater than or equal to one but less than or equal to the
total number of cooperative members.

Objective (profit) function

A given member or group of similar members will have a
profit function represented as;

$$\pi_n = (1-Tp_n)[\sum_{i \in X} p_{in} q_{in} - \sum_{i \in Y} p_{in} q_{in} - fc_n + ds_n] + pvpr_n \tag{3.1}$$

where Tp_n is the marginal tax rate for the n-th patron
group, p_{in} and q_{in} are the price and average quantity of the
i-th product for the n-th group of patrons, where X is the
set of outputs produced by the members and non-members and
Y is the set of variable inputs purchased by the members and
non-members, fc_n is the fixed costs of the average producer
in the n-th member or non-member group, ds_n is the dividends
on stock held by the average producer in the n-th member or
non-member group, and $pvpr_n$ is the present value of the
patronage refunds allocated to the average patron in the
n-th group. The dividends on stock can be defined as;

$$ds_n = SH_n IC_n PS_n \qquad\qquad (3.2)$$

where SH_n and PS_n represent the number of shares of stock owned by a member of the n-th group and its corresponding price. This variable can represent voting, non-voting, preferred stock or any other types. IC_n represents the corresponding dividend rate as a percent of the stock price. There are certain limitations on these stock dividends depending on the cooperative's status. A cooperative organized under Chapter 521 must not exceed 8 percent per annum or the State of incorporation's maximum rate. Unlike 521 cooperatives, a non-exempt cooperative can pay rates greater than 8 percent per annum as long as the applicable State does not impose a limit, and a one man - one vote system is used.

Since all cooperatives can be set up differently, there are many classes of stock that could be employed. For example, a cooperative may have voting stock which may or may not pay any monetary rate of return. Other types of stocks with or without voting privileges may also be offered by the cooperative. The model presented here utilizes only one type of stock; however, any number of stock types can be added to accommodate each individual situation.

The present valve of the patronage refunds ($pvpr_n$) can be further defined as;

$$pvpr_n = s_n' \sum_{i \in C} r_{in}^* q_{in} \qquad (3.3)$$

where

$$s_n' = [s_n - Tp_n + \frac{(1-s_n)}{(1+d_n)^{\tau_n}}] \qquad (3.4)$$

where s_n is the proportion of allocated patronage refunds paid in cash for the n-th patron group leaving the proportion $(1-s_n)$ deferred into a revolving fund of length τ_n for the n-th patron group, and d_n is the discount rate for after-tax cash flows of the n-th patron group. C is the set of outputs (from set X) sold to and the set of variable inputs (from set Y) purchased from the cooperative by the patrons, and r_{in}^* is the n-th patron group's individual expected per-unit patronage refund. The expected per-unit patronage refund for each product for each patron (r_{in}^*) can be represented solely as a function of past per-unit patronage refunds ($r_{in,t-1}$, $r_{in,t-2}$,...) as done by Royer [52] and VanSickle [64], or by these past per-unit refunds and other criteria as done by Fischer [18]. Fischer argues that Royer's extrapolative assumption on expected refunds is not proper since they are not rational. It was argued that producers have more information available to them in deriving this expectation and will use it to get a more refined valve for r_{in}^* [18, p. 64]. Using this additional information to arrive at an expected per-unit

return is theoretically correct yet may become computationally overly burdensome if r_{in}^{*} is a function of many other factors. Treating r_{in}^{*} as an extrapolative expectation may not be totally correct theoretically, however it is plausible and certainly more convenient.

This analysis assumes that all patronage refunds are allocated in a qualified form, therefore all tax liability rests on the patron. Per-unit capital retains, qualified and unqualified, along with unqualified patronage refunds are not incorporated into the model for several reasons. A model incorporating these ideas is developed by Fischer [18, p. 112]. Per-unit capital retains are similar in nature to deferred patronage refunds. Both are based on levels of patronage and have analogous tax consequences. Unqualified patronage refunds and unqualified per-unit retains yield an initial tax liability to the cooperative which is later transferred to the patron when they are redeemed. Unlike their qualified counterparts, unqualified patronage refunds and per-unit retains have no certain redemption date which would make them difficult to model. Finally, and maybe most importantly, allocated patronage refunds are the most typical form of raising equity capital through patronage. Understanding differential treatment of them can be generalized to other forms of equity financing.

By looking at specific member's rather than typical member's prices, quantities, fixed costs, dividends on stock and present value of patronage refunds, some of the ramifications of differential treatment can be analyzed. The cooperative decision-maker's choice of p_{in}, ds_n, s_n, and τ_n for all products for all member types and the resulting effects can be evaluated. The method of determining the levels for these choice variables should be communicated to the patrons once it is established.

Constraints (production function)

With a knowledge of their profit function, the producer can look at their own specific production function to determine optimal levels of production. The individual producer's production function is assumed to be a single-valued continuous function with continuous first-order and second-order partial derivatives. This strictly concave production function for member and non-member type "n" is specified in the implicit form as;

$$\phi_n = \phi_n(q_{Xn}, q_{Yn}, q_{Wfn}) = 0 \tag{3.5}$$

where q_{Xn}, q_{Yn}, and q_{Wfn} are respectively the vector of quantities of outputs in set X produced, variable inputs in set Y used, and fixed inputs in set Wf used by the n-th patron group. Another set of constraints related to the

production function comes from the fact that each producer
can utilize only the amount of fixed inputs that is
available. This availability constraint is given as;

$$q_{in} \leq \bar{q}_{in} \qquad i \epsilon Wf \qquad (3.6)$$

where $\bar{q}_{in}$ is the stock of the i-th fixed factor available to
the n-th patron group. Using this well-behaved production
function and fixed factor availability constraint the
optimal levels of output and variable input usage for
individual patrons can be determined.

Lagrangian function

With both the objective function and constraints
specified for individual producers the Lagrangian function
can be set up. The Lagrangian function will be represented
as a maximization of a producer's profit function subject to
both production and fixed factor availability constraints.
This is given as;

$$
\begin{aligned}
\Lambda_n &= (1-Tp_n)[\sum_{i \epsilon X} p_{in}q_{in} - \sum_{i \epsilon Y} p_{in}q_{in} + ds_n - fc_n] \\
&\quad + [s_n - Tp_n + (1-s_n)/(1+d_n)^{\tau n}] \sum_{i \epsilon C} r_{in}{}^{*} q_{in} \\
&\quad + \psi_{1n} \phi_n(q_{Xn}, q_{Yn}, q_{Wfn}) \\
&\quad + \sum_{i \epsilon Wf} \psi_{2in}(q_{in} - \bar{q}_{in})
\end{aligned}
\qquad (3.7)
$$

where ψ_{1n} and ψ_{2in} are Lagrangian multipliers corresponding to the production function of the n-th patron group and the i-th fixed factor constraint of the n-th patron group. To find the optimal levels of production and factor usage, the producer must choose which markets to use. The producer, unless bound by a marketing agreement, can use both non-cooperative and cooperative markets to sell products or to buy variable inputs.

The notation X_c, X_o, Y_c, and Y_o represent subsets of X and Y where the subscript "c" denotes patronage of cooperatives and the subscript "o" represents business done with other firms. The producer can utilize either cooperative markets, non-cooperative markets, or both. Solutions can be found for q_{in} where the subscript "i" represents products in sets X and Y. The Kuhn-Tucker conditions for this model are similar to those of Royer [52] and are only briefly presented in this study. Having developed the producer submodel, the next step is to develop the cooperative submodel in a similar fashion.

The Cooperative

Since the cooperative's sole purpose is to operate for the benefit of its patrons, the construction of the cooperative submodel is similar to the individual producer's

submodel. In addition to specifying an objective function
and constraints, other factors must be considered in
specifying the cooperative model. The cooperative
decision-maker must consider the determination and
distribution of net savings which will affect the financing
of the cooperative. The most convenient way to understand
the patron-cooperative relationship is to use a diagram
showing the flow of products, cash, and patronage refunds
within the model. Figure 3.1 illustrates the model of a
cooperative association and the relationship between the
cooperative and its patrons. It should be noted that this
figure is the same as VanSickle's Figure 2.1 [64, p. 17]
except that it explicitly states that there are various
groups of member and non-member patrons.

In Figure 3.1, Y represents a set of variable inputs
purchased by the producer, X is a set of products sold by
the producer, V is a set of inputs purchased by the
cooperative from outside markets, Z is a set of products
sold by the cooperative to outside markets, and Wf is a set
of fixed inputs available to the cooperative. Figure 3.1
shows the general flows of goods. However, it does not show
the flows to different types of members. Figure 3.2 looks
at a simple corn marketing cooperative that details the
movement of products, prices, and patronage refunds to four
different individual producer groups.

Key:

goods
========>

cash or credit
--------->

patronage refunds
- - - - ->
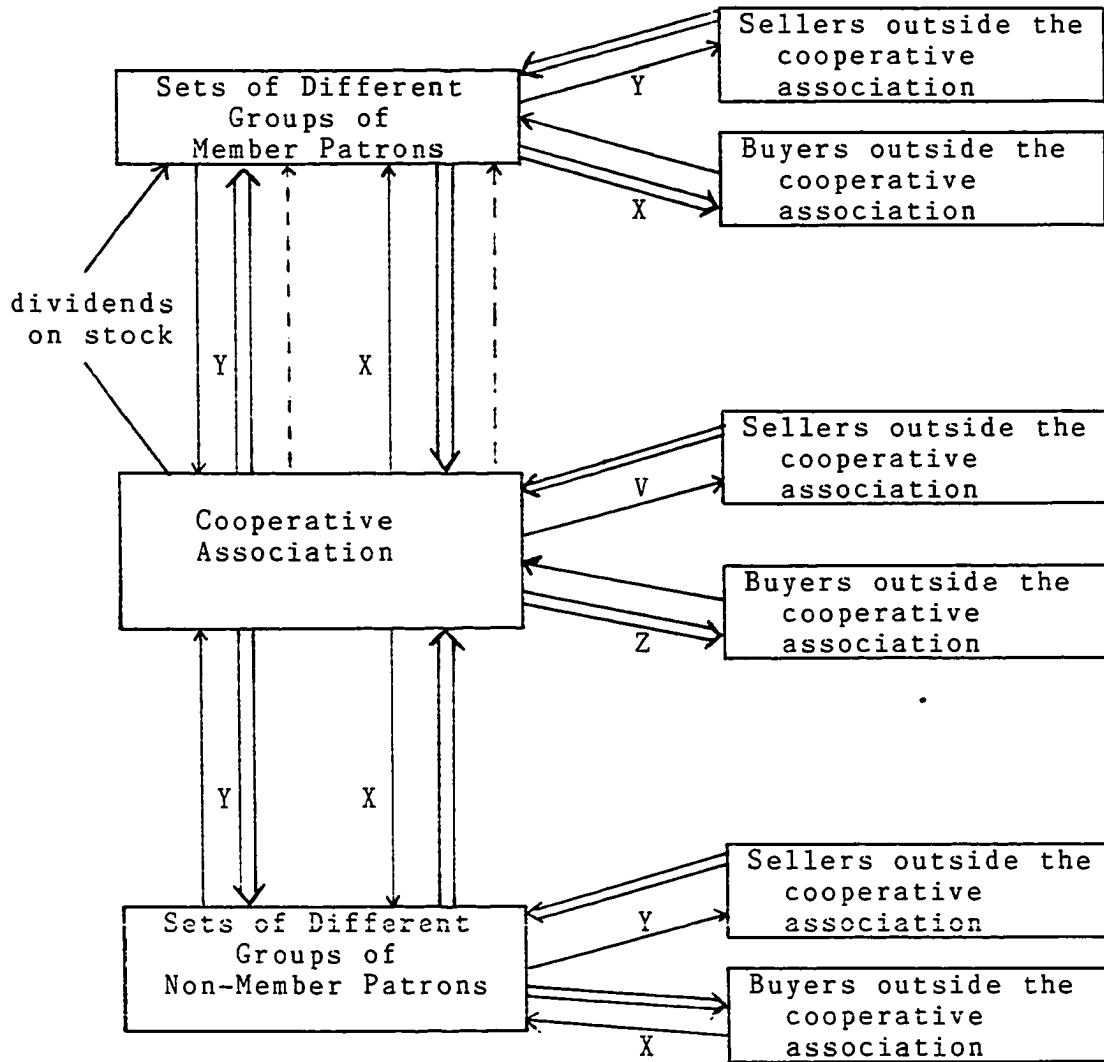
Figure 3.1. Model of the cooperative association
(adapted from VanSickle, Fig. 2.1, p. 17)

The groups of patrons in this example are based on the two factors of volume and membership. The four categories of producers are large volume members, small volume members, large volume non-members, and small volume non-members.

These groups can sell their corn in either cooperative or non-cooperative markets, however they must purchase their inputs from non-cooperative firms. The only patronage refunds paid in this model are when member patrons sell corn to the cooperative. This is similar to previous models except the member patrons fall into two distinct groups and may receive different patronage refunds, either in the immediate cash portion (imm) or the amount held in the revolving fund (def). These members must also provide initial capital stock or membership fees which may or may not return dividends.

The cooperative purchases corn from these four producer groups, pays the going market price, and provides patronage refunds to members. In turn the cooperative sells the corn, or some processed form of it, to other firms for direct cash payments. These other firms could conceivably be other cooperatives which could differentially treat its members, the locals, via its patronage refund policy. To operate the cooperative, labor is hired from outside firms and paid market wages. Depending on the financial status of the cooperative, outside debt sources can be utilized.
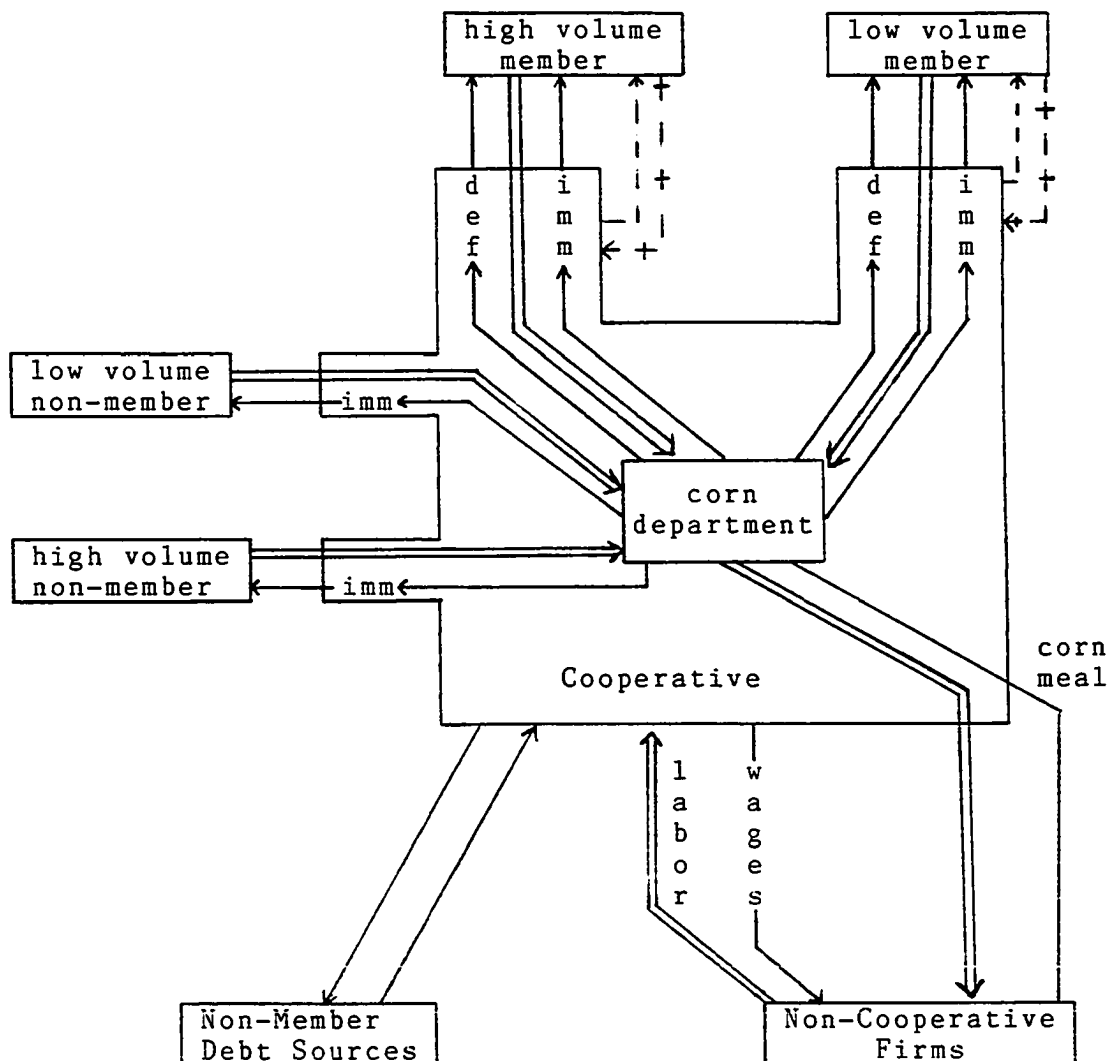
Figure 3.2. Model of a simple corn marketing cooperative

In the context of Figure 3.1, the sets of goods in this simple model are: X_c = {corn}, X_o = {corn}, Y_c = {null}, Y_o = {corn production inputs}, V = {labor to operate cooperative}, Z = {corn meal}, and Wf = {fixed processing facilities owned by the cooperative}. This illustration is for a very simple cooperative yet it does show the product, cash, and patronage refund flows. The following section attempts to generalize the flows through a multi-product marketing and supply cooperative model that has "m" different homogeneous groups of patrons.

Production-pricing objective (profit) function

The cooperative decision-maker's objective function is the maximization of the profits of all members. This is not saying that the cooperative maximizes its own profit. When all members are homogeneous, maximizing total member profit is obviously consistent with the maximizing a single member's objective function. When members are heterogeneous, maximizing total profit is still the cooperative decision-maker's goal even though individual members' profits will vary according to their value to the cooperative. Members who cost less to serve, may receive relatively higher prices than others who cost more to serve. The cooperative decision-maker offers prices for products according to the value of the patron's business which in

turn provides producer's with corresponding revenues. Even though each member may receive different prices, maximizing total profits of all members is a reasonable objective. Following this logic the profits of all cooperative members can be defined as:

$$\Pi = \sum_{i \in X} \sum_{n \in A} N_n (1-Tp_n) p_{in} q_{in} - \sum_{i \in Y} \sum_{n \in A} N_n (1-Tp_n) p_{in} q_{in}$$

$$- \sum_{n \in A} N_n (1-Tp_n) [fc_n - ds_n] + \sum_{n \in A} N_n pvpr_n \qquad (3.8)$$

where N_n is the number of homogeneous members in the n-th group of patrons, A is the subset of patrons who are full members, (B is the set of all patrons), fc_n is the total fixed costs of the average member patron in the n-th group, $\dot{ds}_n$ is the before tax dividends on member stock for an individual in the n-th patron group, and $pvpr_n$ is the after-tax present value of the allocated patronage refunds for the average producer in the n-th patron group. Total fixed costs, dividends, and after-tax present value of allocated patronage refunds (FCM, DS, and PVPR) are found by summing fc_n, ds_n, and $pvpr_n$ over all patron groups in set A.

Production-pricing model constraints

There are several constraints for the production-pricing model. The cooperative's production function is the first constraint that will be considered.

The cooperative production function is analagous to the individual producer's. In implicit form, the production function can be stated as:

$$\Phi(Q_Z, Q_Y, Q_X, Q_V, Q_{Wf}) = 0 \qquad (3.9)$$

where Q_Z is a vector of quantities of the outputs in set Z produced by the cooperative and sold to buyers outside the cooperative association, Q_Y is a vector of quantities of the variable inputs in set Y purchased by the individual producers. Q_X is a vector of quantities of the outputs in set X produced by individual producers and used by the cooperative, Q_V is a vector of quantities of each of the variable inputs in set V used by the cooperative and purchased from outside the cooperative association, and, Q_{Wf} is a vector of quantities of the fixed inputs in set Wf used by the cooperative.

The production function is only one of the constraints in the production and pricing model. Others pertaining to full use of purchased goods, availability of fixed inputs, member patronage agreements, and linkage to a financial model are needed. All products purchased by the cooperative from patrons, set X, must be fully utilized. These goods must either be sold back to patrons (set Y) or sold to outside cooperative markets (set Z). This full utilization constraint is represented as:

$$\sum_{n \epsilon B} N_n q_{in} - \sum_{j \epsilon Y, Z} q_{ij} = 0 \qquad \text{for all } i \epsilon X \qquad (3.10)$$

The availability of fixed factor constraints assures that the cooperative does not use more of these inputs than it owns. This constraint is given by:

$$q_i - \sum_{j \epsilon Y, Z} q_{ij} = 0 \qquad \text{for all } i \epsilon Wf \qquad (3.11)$$

where q_i; $i \epsilon Wf$, is the level of fixed factor "i" available to the cooperative.

Cooperatives must limit the amount of non-member patronage. The Capper-Volstead Act restricts non-member business for marketing cooperatives to less than one-half of total cooperative business. There may be state statutes that also limit the amount of non-patron business. Because there are so many different rules involved in this modelling non-patronage level, this type of constraint will not be used. This study will not address this non-patronage limit. If the cooperative utilized the non-patronage limit as defined by the Capper-Volstead Act, the constraint would be given as:

$$\sum_{i \epsilon C} \sum_{n \epsilon B} N_n P_{in} q_{in} - \sum_{i \epsilon C} \sum_{n \epsilon A} N_n P_{in} q_{in} \leq \sum_{i \epsilon C} \sum_{n \epsilon A} N_n P_{in} q_{in}$$

$$(3.12)$$

The final constraint deals with linking the
production-pricing model and the finance model. This
constraint specifies the amount of net savings that will go
into a revolving fund to provide operating capital for the
cooperative. The constraint is expressed as:

$$N = \sum_{n \in A} (1-s_n)[\alpha_n(NS - rD) - N_n ds_n] - rD \qquad (3.13)$$

where N is the amount of net savings to be used in the
revolving fund, α_n is the proportion of cooperative
operating income (NS-rD) allocated to the n-th patron group,
NS is the cooperative's net savings, r is the average
interest rate of cooperative debt, D is the level of
cooperative debt, and ds_n is dividends paid on stock for
each member of the n-th patron group. Since cooperative net
savings are used in the determination of patronage refunds,
NS will be considered in more depth.

Determination of net savings

The determination of net savings is a result of the
concept of cooperatives operating at cost. Net savings
before taxes from cooperative operations can be used as
patronage refunds, dividends on stock, unallocated reserves,
educational funds, or payment of income taxes. The
allocation of net savings is determined at the end of the

fiscal year however the level to be distributed is determined from the production and pricing decisions made at the beginning of the fiscal year.

Since most cooperatives have several departments which may or may not be independent, it is necessary to determine net savings for each of these departments. The two types of departments in this model, marketing and supply, have different methods to determine net savings because of their innate nature. Equations representing the net savings of the individual marketing and supply departments are given in Appendix B. The net savings equations presented here and in Appendix B are directly analagous to Royer's [52] work except for the inclusion of varying prices and quantities of products pertaining to different groups of members. The net savings of a cooperative as a whole is the sum of the net savings for all supply and marketing departments and is given by:

$$NS = k + \sum_{n \varepsilon B} N_n [\sum_{i \varepsilon Y_c} p_{in} q_{in} - \sum_{i \varepsilon X_c} p_{in} q_{in}] \qquad (3.14)$$

where
$$k = \sum_{i \varepsilon Z} p_i q_i - \sum_{i \varepsilon V} p_i q_i - FCC \qquad (3.14a)$$

where FCC is the total fixed costs of the cooperative. NS is equal to the sum of the value of products sold outside the cooperative, the value of products sold to patrons, less

the value of products purchased from the patrons, less the value of products purchased from outside the cooperative, and less the total fixed costs incurred. The relationship between net savings and patronage refunds is the next step. Equation (3.15) represents this relationship.

$$N_n PR_n = \alpha_n (NS - rD) - N_n ds_n \qquad (3.15)$$

where PR_n is the amount of patronage refunds allocated to each member of the n-th group. Equations (3.16) discounts the patronage refunds and (3.17) sums these discounted patronage refunds across all member groups.

$$N_n pvpr_n = (NS - rD) \, \alpha_n s_n' - N_n ds_n \qquad (3.16)$$

$$\sum_{n \varepsilon A} N_n pvpr_n = (NS - rD) \sum_{n \varepsilon A} \alpha_n s_n' - \sum_{n \varepsilon A} N_n ds_n \qquad (3.17)$$

It should be noted that NS and N_n are the only terms that can vary in the right hand side of (3.12) and (3.17), the other terms are fixed. The levels of these fixed terms are determined by the previous year's financial decisions or simply as givens for the initial run. Substituting (3.17) into (3.8), the cooperative's production-pricing objective function becomes

$$\Pi = \sum_{n \varepsilon A} N_n (1 - Tp_n) [\sum_{i \varepsilon X} p_{in} q_{in} - \sum_{i \varepsilon Y} p_{in} q_{in}]$$

$$- \sum_{n \epsilon A} [N_n(1-Tp_n) \ (fc_n - ds_n) + \sum_{n \epsilon A} N_n ds_n] + (NS-rD) \sum_{n \epsilon A} \alpha_n s_n^{'}$$

$$(3.18)$$

Using (3.18) as the relevant objective function, the production-pricing Lagrangian can be formed.

Production-pricing model Lagrangian

$$L= \sum_{n \epsilon A} N_n(1-Tp_n) [\sum_{i \epsilon X} p_{in} q_{in} - \sum_{i \epsilon Y} p_{in} q_{in}]$$

$$- \sum_{n \epsilon A} [N_n(1-Tp_n) \ (fc_n - ds_n) + \sum_{n \epsilon A} N_n ds_n] + (NS-rD) \sum_{n \epsilon A} \alpha_n s_n^{'}$$

$$+ \lambda_1 [\Phi(Q_Z, \ Q_Y, \ Q_X, \ Q_V, \ Q_{Wf})]$$

$$+ \sum_{i \epsilon X} \lambda_{2i} [\sum_{n \epsilon B} N_n q_{in} - \sum_{j \epsilon Y, Z} q_{ij}]$$

$$+ \sum_{i \epsilon Wf} \lambda_{3i} [q_i - \sum_{j \epsilon Y, Z} q_{ij}]$$

$$+ \lambda_4 [N - \sum_{n \epsilon A} (1-s_n)[\alpha_n(NS-rD) - N_n ds_n] - rD] \qquad (3.19)$$

Using $\{p_{in}, \ i \epsilon X, Y\}$, $\{q_i, \ i \epsilon Z\}$, and $\{q_{ij} \ i \epsilon V, \ Wf; \ j \epsilon Y_c, Z\}$ as instrument variables, the Kuhn-Tucker conditions can be derived. The Kuhn-Tucker conditions for the cooperative production-pricing model are given and interpreted in Chapter IV.

Financial model objective (profit) function

The objective function for the financial model, like the production-pricing model, is the cooperative's profit

function. The financial and production-pricing objective functions are different in that the former has financial variables directly incorporated. Two notable differences are in the determination of qualified patronage refunds and the inclusion of stock activities. The objective function for the financial model is presented by:

$$\Pi = T(M,K)$$

$$+ (NS - rD) \sum_{n \in A} \alpha_n [s_n - Tp_n + \frac{(1-s_n)}{(1+d_n)^{\tau_n}}]$$

$$- \sum_{n \in A} Tp_n N_n IC_n SH_n PS \tag{3.20}$$

where $T(M,K)$ is the total net revenues generated by the cooperative exclusive of the revenue from patronage refunds and stock dividends. $T(M,K)$ depends on the total membership (M) and the total amount of capital used (K). NS is the cooperative's net savings as defined by (3.14). The terms r and D represent the average interest rate of all cooperative debt sources and the total debt employed by the cooperative. The terms IC_n, SH_n and PS_n are respectively the dividend rate, the number of shares of stock required, and the purchase price of the stock for each member in the n-th patron group. Noting the complementarity of the number of shares required and the price of that stock, further

references to stock prices will not be differentiated between members. T(M,K) is defined by equation (3.21).

$$T(M,K) = \sum_{n \in A} N_n(1-Tp_n)[\sum_{i \in X} p_{in}q_{in} - \sum_{i \in Y} p_{in}q_{in}] - \sum_{n \in A} N_n(1-Tp_n)fc_n$$

(3.21)

The second and third line of (3.20) represent the discounted present value of cooperative member patronage refunds and stock dividends.

Financial constraints

The production function represents one constraint on the cooperative however other constraints are needed for the cooperative financial model. Several sources of capital are available to the cooperative including sales of stock, funds held back from operations, and debt sources. Capital obtained through operations such as patronage refunds and per-unit retains is the most common method of cooperative financing. VanSickle and Ladd [67] state that nearly 85 percent of cooperative equity capital is obtained this way. Even though it is commonly used, sales of stock and the use of debt sources may be more preferred by members [14]. A use and source of capital constraint must be formed. That is, the amount of capital used must equal the level that is

provided. An approach looking at changes in the sources and uses of capital each year is used.

The amount of capital utilized by the cooperative each fiscal year is represented as:

$$K = CS + TKQP + D \qquad (3.22)$$

with

$$CS = \sum_{n \in A} N_n (SH_n PS) \qquad (3.23)$$

$$TKQP = PKQP + KQP \qquad (3.24)$$

$$KQP = \sum_{n \in A} N_n (1-s_n) PR_n \qquad (3.25)$$

where D is the level of cooperative debt for the year, CS is the total value of stock employed by the cooperative for the year, TKQP is the total capital supplied by retained patronage refunds from the current (KQP) and all previous year's (PKQP), KQP is the total value of the capital supplied by qualified patronage refunds for the given year, s_n is the proportion of allocated patronage refund paid in cash for the n-th patron group, and PR_n is the amount of patronage refunds allocated to the n-th patron group. The patronage refund term (PR_n) can be represented as:

$$PR_n = [\alpha_n/N_n][NS - rD] - IC_n SH_n PS \qquad (3.26)$$

Substituting (3.23), (3.24), (3.25) and (3.26) into (3.22) yields:

$$K = \sum_{n \varepsilon A} N_n SH_n PS[1-(1-s_n)IC_n] + PKQP + \sum_{n \varepsilon A} (1-s_n)\alpha_n[NS-rD] + D$$

$$(3.27)$$

Other financial constraints that must be dealt with pertain to the percent of patronage refunds paid in cash, the allowable interest rates paid on capital stock. These constraints are represented by:

$$s_n \geq 0.2 \qquad (3.28)$$

$$s_n \leq 1.0 \qquad (3.29)$$

$$IM \geq IC_n \qquad (3.30)$$

A final constraint for the model is to fix the level of net savings at the amount determined by the production-pricing model. This constraint is taken care of by retaining the terms of the production-pricing model that determine this value, with solution values that are already known. Using all these constraints, the Lagrangian function for the finance model can be formed.

Financial model Lagrangian function

Given the objective of maximizing total collective member after-tax profits subject to the aforementioned constraints, the Lagrangian function can be written:

$$\Pi = T(M,K)$$

$$+ (NS - rD) \sum_{n \in A} \alpha_n [s_n - Tp_n + \frac{(1-s_n)}{(1+d_n)^{\tau_n}}]$$

$$- \sum_{n \in A} Tp_n N_n IC_n SH_n PS$$

$$+ \delta_1 \{K - \sum_{n \in A} N_n SH_n PS [1-(1-s_n)IC_n] - PKQP$$

$$- \sum_{n \in A} (1-s_n)_n [NS-rD] - D\}$$

$$+ \sum_{n \in A} \delta_{2n} [s_n - 0.2]$$

$$+ \sum_{n \in A} \delta_{3n} [1.0 - s_n]$$

$$+ \sum_{n \in A} \delta_{4n} [IM-IC_n] \tag{3.31}$$

It should be noted that if τ_n equals zero, then s_n must equal one.

The instrument variables used by the cooperative decision-maker are: s_n, τ_n, stock prices, and stock dividends. These instruments yield Kuhn-Tucker conditions that are presented in Chapter IV. The theoretical model developed above is slightly different than the application model discussed later. The application model utilizes the most current literature on modelling cooperative finances. The previous theoretical model was developed before this literature was available. The generalized theoretical model used for the application will be presented however the

resulting Kuhn-Tucker conditions will not. These
Kuhn-Tucker conditions can be found in VanSickle and
Ladd [66]. The procedure used to solve this model involves
maximizing two separate objective functions given by (3.32)
and (3.33).

$$
\begin{aligned}
L_2 &= T(M,K) + \sum_{n \in A} (1-Tp_n)[N_n QPR_n + IC_n PS_n SH_n] \\
&+ \sum_{n \in A} d_n(1-Tp_n)[N_n H_n QPR_n + PS_n SH_n] \\
&+ \delta_5 \{K - \sum_{n \in A} [H_n QPR_n - PS_n SH_n] - D\} \\
&+ \delta_6 \{O(M,K) - \sum_{n \in A} [QPR_n - IC_n PS_n SH_n] - rD\} \\
&+ \delta_{7n}' \sum_{n \in A} \{IM-IC_n\}
\end{aligned}
\tag{3.32}
$$

$$
\begin{aligned}
L_3 &= \sum_{n \in A} [s_n - Tp_n \frac{(1-s_n)}{(1+d_n)^{\tau n}}]QPR_n \\
&+ \sum_{n \in A} \delta_{7n} [H_n - \alpha_n(1-s_n)] \\
&+ \sum_{n \in A} \delta_{8n} [1-s_n] + \sum_{n \in A} \delta_{9n} [s_n-0.2]
\end{aligned}
\tag{3.33}
$$

Equation (3.32) maximizes the sum of cooperative member's
profits. The instrument variables for this Lagrangian
include QPR_n, IC_n, X_n, and D. The second equation (3.33)
maximizes the present value of qualified patronage refunds
using the values of QPR_n and H_n obtained from solving
(3.32). The choice variables used in (3.33) are τ_n and s_n.
This two-stage model is discussed again in Chapter V.

CHAPTER IV. ANALYSIS OF THE THEORETICAL MODEL

The preceding chapter presented the theoretical models
for a cooperative enterprise but does not provide any
interpretation or implications for cooperative behavior.
The purpose of this chapter is to derive the model's
implications for optimizing behavior by deriving and
interpreting the profit maximizing Kuhn-Tucker conditions.
The interpretation of the Kuhn-Tucker conditions are
basically similar to those of VanSickle [64] with some
differences arising because of the possibility of
differential treatment of members. The existence of
differing groups of patrons who can be treated dissimilarly
gives the decision-maker more instrument variables which can
be used in determining the optimizing behavior of the
cooperative.

Individual Patron Model

The model of an individual cooperative member or
non-member patron presented in this work is basically the
same as put forth by Royer [52]. This work considers that
specific patrons can and will typically have different
operating conditions and optimizing behavior to maximize
profits. Each patron will use the quantities supplied and
demanded of products in sets X_c, X_o, Y_c, Y_o, and Wf to

maximize their profits. Using these quantities as choice variables the following set of Kuhn-Tucker conditions can be derived.

for all $i \varepsilon X_c$:

$$\frac{\partial \Lambda_n}{\partial q_{in}} = (1-Tp_n) \ p_{in} + [s_n-Tp_n+\frac{(1-s_n)}{(1+d_n)^{\tau_n}}] \ r_{in}^* + \psi_{ln} \frac{\partial \phi_n}{\partial q_{in}} \leq 0$$

(4.1a)

$$\frac{\partial \Lambda_n}{\partial q_{in}} \ q_{in} = 0$$

(4.1b)

$$q_{in} \geq 0$$

(4.1c)

for all $i \varepsilon X_o$:

$$\frac{\partial \Lambda_n}{\partial q_{in}} = (1-Tp_n) \ p_{in} + \psi_{ln} \frac{\partial \phi_n}{\partial q_{in}} \leq 0$$

(4.2a)

$$\frac{\partial \Lambda_n}{\partial q_{in}} \ q_{in} = 0$$

(4.2b)

$$q_{in} \geq 0$$

(4.2c)

for all $i \varepsilon Y_c$:

$$\frac{\partial \Lambda_n}{\partial q_{in}} = (Tp_n-1) \ p_{in} + [s_n-Tp_n+ \frac{(1-s_n)}{(1+d_n)^{\tau_n}}] \ r_{in}^* + \psi_{ln}\frac{\partial \phi_n}{\partial q_{in}} \leq 0$$

(4.3a)

$$\frac{\partial \Lambda_n}{\partial q_{in}} \ q_{in} = 0$$

(4.3b)

$$q_{in} \geq 0$$

(4.3c)

for all $i \epsilon Y_o$:

$$\frac{\partial \Lambda_n}{\partial q_{in}} = (Tp_n - 1) \, p_{in} + \psi_{1n} \frac{\partial \phi_n}{\partial q_{in}} \leq 0 \qquad (4.4a)$$

$$\frac{\partial \Lambda_n}{\partial q_{in}} \, q_{in} = 0 \qquad (4.4b)$$

$$q_{in} \geq 0 \qquad (4.4c)$$

for all $i \epsilon Wf$:

$$\frac{\partial \Lambda_n}{\partial q_{in}} = \psi_{1n} \frac{\partial \phi_n}{\partial q_{in}} \leq 0 \qquad (4.5a)$$

$$\frac{\partial \Lambda_n}{\partial q_{in}} \, q_{in} = 0 \qquad (4.5b)$$

$$q_{in} \geq 0 \qquad (4.5c)$$

for ψ_{1n}:

$$\frac{\partial \Lambda_n}{\partial \psi_{1n}} = \phi_n(q_{Xn}, \, q_{Yn}, \, q_{Wfn}) = 0 \qquad (4.6)$$

for ψ_{2in}, $i \epsilon Wf$:

$$\frac{\partial \Lambda_n}{\partial \psi_{2in}} = q_{in} - \bar{q}_{in} \geq 0 \qquad (4.7a)$$

$$\frac{\partial \Lambda_n}{\partial \psi_{2in}} \, q_{in} = 0 \qquad (4.7b)$$

$$\psi_{2in} \geq 0 \qquad (4.7c)$$

Interpreting (4.1a), (4.1b), and (4.1c) implies that the patron's marginal cost of producing each output sold to the cooperative should equal the price paid plus the

discounted present value of expected patronage refunds associated with cooperative patronage. This condition holds for members and non-members alike but the latter will simply not expect any patronage refunds. The interpretation of (4.2a), (4.2b), and (4.2c) is analagous to (4.1a), (4.1b), and (4.1c) except now patrons will equate the marginal cost of production to the price paid by other firms. The Kuhn-Tucker conditions for all $i \varepsilon Y_c$ and all $i \varepsilon Y_o$ are interpreted as setting the marginal value product of each input equal to the net price paid for the input. Expected patronage refunds enter into (4.3a), (4.3b), and (4.3c) and can lower the net cost of the input, (4.4a), (4.4b), and (4.4c) are similar except there are no patronage refunds received. The level of fixed factor usage is determined by (4.5a), (4.5b), and (4.5c) and indicates that; if the i-th fixed factor is used, its imputed value is equal to its marginal value product. Conditions (4.6), (4.7a), (4.7b), and (4.7c) yield the production and fixed factor constraints.

The results for the individual are simple extensions of previous work whereas the next step, the cooperative model, is where more interesting results arise. By summing across all of the individual patrons, the cooperative production-pricing model and financial model can be discussed and interpreted.

Production-Pricing Model

Previous production-pricing models used prices and quantities of goods as decision variables but do not enable the decision-maker to select different values for dissimilar patron groups. Realizing that the decision-maker can set varying prices and quantities for specific groups of patrons, the instrument variables for the production-pricing model are:

p_{jn} for all $j \varepsilon X$, $n \varepsilon B$

p_{jn} for all $j \varepsilon Y$, $n \varepsilon B$

q_j for all $j \varepsilon Z$

q_{ij} for all $i \varepsilon V$, $j \varepsilon Y, Z$

q_{ij} for all $i \varepsilon Wf$, $j \varepsilon Y, Z$

In the following Kuhn-Tucker conditions it should be noted that NS is represented by equation (3.14). The Kuhn-Tucker conditions for the pricing-production model are:

for all $j \varepsilon X_c$, $n \varepsilon B$

$$\frac{\partial L}{\partial p_{jn}} = N_n(1-Tp_n) \left[q_{jn} + \sum_{i \varepsilon X} p_{in} \frac{\partial q_{in}}{\partial p_{jn}} - \sum_{i \varepsilon Y} p_{in} \frac{\partial q_{in}}{\partial p_{jn}} \right]$$

$$+ \sum_{m \varepsilon A} (1-Tp_n) \left(\sum_{i \varepsilon X} p_{in} q_{in} - \sum_{i \varepsilon Y} p_{in} q_{in} \right] \frac{\partial N_m}{\partial p_{jn}}$$

$$+ \sum_{m \varepsilon A} [ds_n + (1-Tp_n)(fc_n - ds_n)] \frac{\partial N_m}{\partial p_{jn}}$$

$$+ \alpha_n s_n \hat{} N_n [\sum_{i \epsilon Y_c} p_{in} \frac{\partial q_{in}}{\partial p_{jn}} - \sum_{i \epsilon X_c} p_{in} \frac{\partial q_{in}}{\partial p_{jn}} - q_{in}]$$

$$+ \sum_{n \epsilon A} \alpha_n s_n \hat{} \{ \sum_{m \epsilon B} [\sum_{i \epsilon Y_c} p_{im} q_{im} - \sum_{i \epsilon X_c} p_{in} q_{in}] \frac{\partial N_m}{\partial p_{jn}} \}$$

$$+ \lambda_1 \sum_{i \epsilon X, Y} \frac{\partial \Phi}{\partial q_{in}} [\sum_{m \epsilon B} q_{im} \frac{\partial N_m}{\partial p_{jn}} + N_n \frac{\partial q_{in}}{\partial p_{jn}}]$$

$$+ \sum_{i \epsilon X} \lambda_{2i} [N_n \frac{\partial q_{in}}{\partial p_{jn}} + \sum_{m \epsilon B} q_{im} \frac{\partial N_m}{\partial p_{jn}}]$$

$$+ \lambda_4 \{ -(1-s_n) \alpha_n [N_n (\sum_{i \epsilon Y} p_{in} \frac{\partial q_{in}}{\partial p_{jn}} - q_{jn} \sum_{i \epsilon X} p_{in} \frac{\partial q_{in}}{\partial p_{jn}})$$

$$+ \sum_{n \epsilon A} (1-s_n) \alpha_n [\sum_{m \epsilon B} (\sum_{i \epsilon Y_c} p_{im} q_{im} - \sum_{i \epsilon X_c} p_{im} q_{im}) \frac{\partial N_m}{\partial p_{jn}}]$$

$$- \sum_{m \epsilon A} d s_m \frac{\partial N_m}{\partial p_{jn}} \} \leq 0 \tag{4.8a}$$

$$\frac{\partial L}{\partial p_{jn}} p_{jn} = 0 \tag{4.8b}$$

$$p_{jn} \geq 0 \tag{4.8c}$$

for all $j \epsilon Y$, $n \epsilon B$

$$\frac{\partial L}{\partial p_{jn}} = N_n (1-Tp_n) [\sum_{i \epsilon X} p_{in} \frac{\partial q_{in}}{\partial p_{jn}} - q_{jn} - \sum_{i \epsilon Y} p_{in} \frac{\partial q_{in}}{\partial p_{jn}}]$$

$$+ \sum_{m \epsilon A} (1-Tp_n) [\sum_{i \epsilon X} p_{in} q_{in} - \sum_{i \epsilon Y} p_{in} q_{in}] \frac{\partial N_m}{\partial p_{jn}}$$

$$+ \sum_m [ds_m + (1-Tp_m)(fc_m + ds_m)] \frac{\partial N_m}{\partial p_{jn}}$$

$$+ \alpha_n s_n' N_n [q_{jn} + \sum_{i \varepsilon Y_c} p_{in} \frac{\partial q_{in}}{\partial p_{jn}} - \sum_{i \varepsilon X_c} p_{in} \frac{\partial q_{in}}{\partial p_{jn}}]$$

$$+ \sum_{n \varepsilon A} \alpha_n s_n' \{ [\sum_{m \varepsilon B} [\sum_{i \varepsilon Y_c} p_{im} q_{im} - \sum_{i \varepsilon X_c} p_{im} q_{im}] \frac{\partial N_m}{\partial p_{jn}} \}$$

$$+ \lambda_1 \sum_{i \varepsilon X, Y} \frac{\partial \Phi}{\partial q_{in}} [\sum_{m \varepsilon B} q_{im} \frac{\partial N_m}{\partial p_{jn}} + N_n \frac{\partial q_{in}}{\partial p_{jn}}]$$

$$+ \sum_{i \varepsilon X} \lambda_{2i} [N_n \frac{\partial q_{in}}{\partial p_{jn}} + \sum_m q_{im} \frac{\partial N_m}{\partial p_{jn}}]$$

$$+ \lambda_4 \{ -(1-s_n) \alpha_n N_n (q_{jn} + \sum_{i \varepsilon Y} p_{in} \frac{\partial q_{in}}{\partial p_{jn}} - \sum_{i \varepsilon X} p_{in} \frac{\partial q_{in}}{\partial p_{jn}})$$

$$+ \sum_{m \varepsilon A} (1-s_n) \alpha_n [\sum_{m \varepsilon B} (\sum_{i \varepsilon Y_c} p_{in} q_{in} - \sum_{i \varepsilon X_c} p_{in} q_{in}) \frac{\partial N_m}{\partial p_{jn}}$$

$$- \sum_{m \varepsilon A} ds_m \frac{\partial N_m}{\partial p_{jn}} \} \leq 0 \qquad (4.9a)$$

$$\frac{\partial L}{\partial p_{jn}} p_{jn} = 0 \qquad (4.9b)$$

$$p_{jn} \geq 0 \qquad (4.9c)$$

for all $j \varepsilon Z$

$$\frac{\partial L}{\partial q_j} = \sum_{n \varepsilon A} \alpha_n s_n' p_j + \lambda_1 \frac{\partial \Phi}{\partial q_j} - \lambda_4 \sum_{n \varepsilon B} (1-s_n)(\alpha_n p_j) \leq 0$$

$$\qquad (4.10a)$$

$$\frac{\partial L}{\partial q_j} q_j = 0 \qquad (4.10b)$$

$$q_j \geq 0 \qquad (4.10c)$$

for all $i \epsilon V$, $j \epsilon Y, Z$

$$\frac{\partial L}{\partial q_{ij}} = \sum_{n \epsilon A} \alpha_n s_n'(-p_i) + \lambda_1 \frac{\partial \Phi}{\partial q_{ij}} + \lambda_4 \sum_{n \epsilon A} (1-s_n)(\alpha_n p_j) \leq 0 \qquad (4.11a)$$

$$\frac{\partial L}{\partial q_{ij}} q_{ij} = 0 \qquad (4.11b)$$

$$q_{ij} \geq 0 \qquad (4.11c)$$

for all $i \epsilon Wf$, $j \epsilon Y, Z$

$$\frac{\partial L}{\partial q_{ij}} = \lambda_1 \frac{\partial \Phi}{\partial q_{ij}} - \lambda_{3i} \leq 0 \qquad (4.12a)$$

$$\frac{\partial L}{\partial q_{ij}} q_{ij} = 0 \qquad (4.12b)$$

$$q_{ij} \geq 0 \qquad (4.12c)$$

for λ_1

$$\frac{\partial L}{\partial \lambda_1} = \Phi(Q_Z, Q_Y, Q_X, Q_V, Q_{Wf}) = 0 \qquad (4.13)$$

for λ_{2j} all $j\varepsilon X$

$$\frac{\partial L}{\partial \lambda_{2j}} = \sum_{n\varepsilon A} N_n q_{jn} - \sum_{i\varepsilon Y,Z} q_{ij} = 0 \qquad (4.14)$$

for λ_{3j} all $j\varepsilon Wf$

$$\frac{\partial L}{\partial \lambda_{3j}} = q_{jo} - \sum_{i\varepsilon Y,Z} q_{ij} = 0 \qquad (4.15)$$

for λ_4

$$\frac{\partial L}{\partial \lambda_4} = N - \sum_{n\varepsilon A} (1-s_n)[\alpha_n(NS-rD) - N_n ds_n] - rD = 0 \qquad (4.16)$$

The Kuhn-Tucker conditions in these forms are very detailed and show all the considerations which must be dealt with in optimizing cooperative profits. Using the procedure employed by Royer [52] these Kuhn-Tucker conditions can be put into a form that is more concise and comprehendible. In (4.8a), (4.8b), and (4.8c) the price of the j-th output sold by the n-th patron group is used as the instrument variable. It follows that the number of Kuhn-Tucker conditions of this form is the number of products in set X multiplied by the number of different member groups. This work differs from Royer's [52] and VanSickle's [64] work since they considered only a one-dimensional array of Kuhn-Tucker conditions ($j\varepsilon X$) whereas this work considers a two-dimensional array ($j\varepsilon X$ and $n\varepsilon B$).

All the Kuhn-Tucker conditions that are in the form of (4.8a) can be re-written as:

for all $j\epsilon X$, $n\epsilon A$

$$\frac{\partial L}{\partial p_{jn}} = N_n(1-Tp_n)\{(p_{jn}+q_{jn}\frac{\partial p_{jn}}{\partial q_{jn}})\frac{\partial q_{jn}}{\partial p_{jn}}+ \sum_{\substack{i\epsilon X \\ i\neq j}} p_{in}\frac{\partial q_{jn}}{\partial p_{jn}} - \sum_{i\epsilon Y} p_{in}\frac{\partial q_{in}}{\partial p_{jn}})\}$$

$$+ \sum_{m\epsilon A}(1-Tp_m)[\sum_{i\epsilon X}(1-Tp_m)p_{im}q_{im} - \sum_{i\epsilon Y}p_{im}q_{im}]\frac{\partial N_m}{\partial p_{jn}}$$

$$+ \sum_{m\epsilon A}[ds_m + (1-Tp_m)(fc_m - ds_m)]\frac{\partial N_m}{\partial p_{jn}}$$

$$+ \alpha_n s_n' N_n[\sum_{i\epsilon Y_c} p_{in}q_{in} -(p_{jn} +q_{jn}\frac{\partial p_{jn}}{\partial q_{jn}})\frac{\partial q_{jn}}{\partial p_{jn}} - \sum_{\substack{i\epsilon X_c \\ i\neq j}} p_{in}\frac{\partial p_{in}}{\partial p_{jn}}]$$

$$+ \alpha_n s_n'[\sum_{m\epsilon A}[\sum_{i\epsilon Y_c} p_{im}q_{im} - \sum_{i\epsilon X_c} p_{im}q_{im}]\frac{\partial N_m}{\partial p_{jn}}$$

$$+ \sum_{i\ X,Y} \lambda_1 \frac{\partial\Phi}{\partial q_{in}} [\sum_m q_{im}\frac{\partial N_m}{\partial p_{jn}} + N_n\frac{\partial q_{in}}{\partial p_{jn}}]$$

$$+ \sum_{i\epsilon X} \lambda_{2i} [N_n\frac{\partial q_{in}}{\partial p_{jn}} + \sum_m q_{im}\frac{\partial N_m}{\partial p_{jn}}]$$

$$+ \lambda_4\{(1-s_n)\alpha_n[(p_{jn} + q_{jn}\frac{\partial p_{jn}}{\partial q_{jn}})\frac{\partial q_{jn}}{\partial p_{jn}} + \sum_{\substack{i\epsilon X \\ i\neq j}} p_{in}\frac{\partial q_{in}}{\partial p_{jn}}$$

$$- \sum_{i\epsilon Y} p_{in}\frac{\partial q_{in}}{\partial p_{jn}}]$$

$$+ \sum_{n\epsilon A}(1-s_n)\alpha_n[\sum_{m\epsilon B}(\sum_{i\epsilon Y_c} p_{im}q_{im} - \sum_{i\epsilon X_c} p_{im}q_{im})\frac{\partial N_m}{\partial p_{jn}}]$$

$$- \sum_{m \in A} ds_m \frac{\partial N_m}{\partial p_{jn}} \} \leq 0 \qquad\qquad (4.17)$$

This form may not appear any more comprehendible; however by looking at each term this Kuhn-Tucker condition can be explained more easily or with less difficulty. The first two lines can be interpreted as the variation in revenues received by the n-th patron group from the j-th product caused by patron's output shifts induced by changing the corresponding price paid the member, less the variation in costs for the j-th product of the n-th patron group caused by shifts in factor usage of the n-th patron group which was induced by changes in the price of the j-th factor for the n-th patron group plus the change in revenues caused by changes in the number of patrons in the different groups. The third, fourth, and fifth lines deal with the net savings of the cooperative and can be interpreted as the change in the present value of net savings associated with the costs and revenues in the first and second lines. Before analyzing the remaining lines, the meaning of the Lagrangian multipliers must be determined. Care must be used when interpreting λ_1, when $j \varepsilon X$, it is the marginal variation in profit arising from the change in the quantity of the j-th input used by the cooperative. Conversely, λ_1 can be interpreted as the marginal variation in profit arising from the change in the quantity of the j-th cooperative output

when $j \varepsilon Y$. The term on the sixth line is interpreted as the variation in cooperative profits caused by changes in the quantities of the i-th output (factor) caused by the change in the price of the j-th output (factor) for the n-th patron group, summed over all patrons. The seventh line represents the variation in cooperative profits from the transformation of products in set X caused by a change in the price of the j-th good for the n-th patron group, summed over all patrons. The Lagrangian multiplier λ_4 would be interpreted as the marginal variation in cooperative profit arising from a change in the net savings constraint. The remaining lines are interpreted as the variation in cooperative member profits from a change in the amount of deferred patronage dividends arising from output shifts caused by changing the price of the j-th product for the n-th patron group.

The affects of changing the price of the j-th output for the n-th patron group can be seen in Total Private Revenues (TPR), Total Private Costs (TPC), Total Collective Revenues (TCR), Total Collective Costs (TCC), and Total Member Profits (TMP). Total Private Revenues are the revenues obtained from the sale of all goods in set X by the entire group of member patrons. The effects on TPC of changing the price of the j-th good in set X for the n-th patron group in set A can be divided into three aspects: (a) the own effect, $\partial q_{jn} / \partial p_{jn}$, (b) the cross product effect

for the n-th member group, $\partial q_{in}/\partial p_{jn}$, where $i\epsilon X$, $i\neq j$, and

(c) the change in membership numbers, $\partial N_m/\partial p_{jn}$. Total

Private Costs are the costs associated with the purchase by

member patrons only of all goods in set Y. The effects on

TPC of changing the price of the j-th good in set X for the

n-th member group in set A deal only with the cross effect

of the n-th member group, $\partial q_{in}/\partial p_{jn}$ where $i\epsilon Y$, and the

change in membership numbers, $\partial N_m/\partial p_{jn}$. Total Collective

Revenues include all revenue obtained from the sale of all

goods in set X by the entire group of patrons in set B. The

effects on TCR of changing the price of the j-th good in set

X for the n-th patron group in set A is again divided into

three parts: (a) the own effect $\partial q_{in}/\partial p_{jn}$, (b) the cross

product effect for the n-th member group, $\partial q_{in}/\partial p_{jn}$ where

$i\epsilon X$, and $i\neq j$, and (c) the change in membership numbers,

$\partial N_m/\partial p_{jn}$. Total Collective Costs are those costs associated

with the purchase of factors from set Y by all patrons in

set B. The effects on TCC of changing the price of the j-th

good in set X for the n-th member group in set A include

only the cross effect of the n-th member group, $\partial q_{in}/\partial p_{jn}$,

where $i\epsilon Y$. Total Member Profits are affected by (a) changes

in cooperative production via $\partial q_{in}/\partial p_{jn}$ where $i\epsilon X,Y$, $n\epsilon B$,

and membership changes (b) the transformation of factors

into products in set X by all patrons and (c) the change in

the amount of deferred patronage dividends (DP) via

$\partial q_{in}/\partial p_{jn}$ where $i \varepsilon X,Y$, $n \varepsilon B$ and membership changes.

The Kuhn-Tucker condition (4.17) indicates that in order for a decision-maker to maximize member profits, the sum of the variation in TPR of all members from sale of all goods in set X, the variation in TPC of all members from purchases of all goods in set Y, the variation in TCR of all patrons from purchases of all goods in set Y, the variation in TCC of all patrons from sale of all goods in set X, the variation in TMP caused by the change in levels of outputs and factors in the production function, the variation in TMP caused by the change in the transformation ratio, and the variation in TMP caused by the change in amount of deferred patronage dividends by changing levels of output and factor usage by the n-th patron group should be set equal to zero. Using the above interpretation (4.17) can be re-written as:

for $j \varepsilon X$, $n \varepsilon A$

$$\frac{\partial L}{\partial p_{jn}} = \Sigma\{ \underset{i \varepsilon X}{\Sigma} [\frac{\partial TPR}{\partial q_{in}} \frac{\partial q_{in}}{\partial p_{jn}} + \frac{\partial TPR}{\partial N_m} \frac{\partial N_m}{\partial p_{jn}}] - \underset{i \varepsilon Y}{\Sigma} [\frac{\partial TPC}{\partial q_{in}} \frac{\partial q_{in}}{\partial p_{jn}} + \frac{\partial TPC}{\partial N_m} \frac{\partial N_m}{\partial p_{jn}}]\}$$

$$+\Sigma\{ \underset{i \varepsilon Y}{\Sigma} [\frac{\partial TCR}{\partial q_{in}} \frac{\partial q_{in}}{\partial p_{jn}} + \frac{\partial TCR}{\partial N_m} \frac{\partial N_m}{\partial p_{jn}}] - \underset{i \varepsilon X}{\Sigma} [\frac{\partial TCC}{\partial q_{in}} \frac{\partial q_{in}}{\partial p_{jn}} + \frac{\partial TCC}{\partial N_m} \frac{\partial N_m}{\partial p_{jn}}]\}$$

$$+ \{ \underset{i \varepsilon X}{\Sigma} \frac{\partial TMP}{\partial q_{in}} \frac{\partial q_{in}}{\partial p_{jn}} - \underset{i \varepsilon Y}{\Sigma} \frac{\partial TMP}{\partial q_{in}} \frac{\partial q_{in}}{\partial p_{jn}} \}$$

$$- \{ \underset{i \varepsilon X}{\Sigma} \frac{\partial TMP}{\partial q_{in}} \frac{\partial q_{in}}{\partial p_{jn}} \} + \underset{i \varepsilon X,Y}{\Sigma} \underset{m}{\Sigma} \frac{\partial TMP}{\partial N_m} \frac{\partial N_m}{\partial p_{jn}}$$

$$- \left\{ \sum_{i \in X, Y} \frac{\partial TMP}{\partial DP} \left[\frac{\partial DP}{\partial q_{in}} \frac{\partial q_{in}}{\partial p_{jn}} + \frac{\partial DP}{\partial N_m} \frac{\partial N_m}{\partial p_{jn}} \right] \right\} \leq 0 \qquad (4.18)$$

Since this analysis is similar to that used by Royer [52] and VanSickle [64], only the methodology relevant to this model is repeated here. An indepth explanation of this methodology can be found in these two studies.

The procedure used to interpret (4.8a) can also be used to analyze (4.9a), (4.10a), (4.11a), and (4.12a). Instead of working through each step here, the final form and the interpretation for the decision-maker will be given. Equation (4.9a) can be restated the same way as (4.8a), however it is interpreted differently. The interpretation differs in how the changes in TPR, TPC, TCR, and TCC are initially derived. For (4.8a) the changes arise from a change in the price of the j-th good in set X paid to the n-th patron group, whereas for (4.9a) the impetus for change is derived from varying the price of the j-th factor in set Y paid by the n-th patron group.

The interpretation of (4.10a) can be more easily understood by writing it in the form given by (4.19).

for $j \in Z$

$$\frac{\partial L}{\partial q_j} = \frac{\partial TPR}{\partial q_j} + \frac{\partial TMP}{\partial q_j} + \frac{\partial TMP}{\partial DP} \frac{\partial DP}{\partial q_j} = 0 \qquad (4.19)$$

In this concise form, the decision-maker will maximize profits by setting equal to zero the sum of the following

terms: the sum of individual members' present value of the total marginal revenue of the j-th product in the set of outputs produced by the cooperative and sold to non-cooperative buyers, the direct variation in TMP caused by the change in the level of the j-th good in set Z sold, and the indirect variation in TMP caused by a change in DP which is caused by the change in the amount of the j-th good in set Z.

It should be noted that when an input is used to produce a product in sets Y or Z ($q_{ij} > 0$ for $i\epsilon X$; $j\epsilon Y$, Z), the marginal profit of using the input must be equal for all outputs since λ_{2i} is a constant for all "j" outputs.

By rearranging (4.11a), it can be written as:

for all $i\epsilon V$; $j\epsilon Y$, Z

$$\frac{\partial L}{\partial q_{ij}} = \frac{\partial TPC}{\partial q_{ij}} + \frac{\partial TMP}{\partial q_{ij}} + \frac{\partial TMP}{\partial DP}\frac{\partial DP}{\partial q_{ij}} = 0 \qquad (4.20)$$

To maximize profits the decision-maker should set equal to zero; the total marginal factor cost to the cooperative of using the i-th variable input (purchased outside the cooperative association) taking into account its present value assuming each of the M patrons can have varying τ_n, s_n, and d_n, the variation in member profits from changing q_{ij} ($i\epsilon V$; $j\epsilon Y$, Z), and the variation in member profits from a change in DP caused by the change in q_{ij} ($i\epsilon V$; $j\epsilon Y$, Z). Similar to the interpretation of (4.11a), when an input

purchased from outside the cooperative is used to produce a product in set Y or Z, the marginal cost of using that factor should equal the marginal profit gained. Again, this marginal profit should be equal for all outputs produced with inputs from set V. The interpretation of (4.12a) indicates that, for a maximum level of profit, the decision-maker should set the variation in profit caused by the change in the amount of the i-th factor in set Wf to produce outputs in sets X or Z equal to the shadow price of the i-th fixed factor.

The Kuhn-Tucker conditions represented by (4.13), (4.14), (4.15), and (4.16) recreate the original constraints imposed on the objective function. The production constraint is given by (4.13) which is in implicit form. Equations (4.14) and (4.15) respectively specify that all the unprocessed products in set X purchased from patrons is transformed into final products and that all fixed factors of production are exhausted in the production process. The last constraint is represented by (4.16) and states that a specified amount of capital from net savings is deferred in a revolving fund while some can be allocated as stock dividends.

Financial Model

The instrument variables for the financial model are different from those for the production-pricing model. To maximize profits of all members the cooperative decision-maker can utilize several different instrument variables including the proportion of patronage refunds paid in cash to individual patrons (s_n), the length of the deferment period for each patron (τ_n), the price of stock sold to each patron group by the cooperative (PS), the level of dividends paid to each patron group for stock (IC_n), and the percentage of total operating income allocated to each member group (α_n). This model is not meant to be a model of cooperative investment so the the level of total capital employed by the cooperative (K) is used as a parameter not as an instrument. Since there are different choice variables for the production-pricing and the financial models, these two models are built in a step-wise manner. The decision variables in one model may be fixed in the other.

The Financial model Lagrangian function is stated as:

$$V = T(M,K)$$

$$+ (NS - rD) \sum_{n \in A} \alpha_n [s_n - Tp_n + \frac{(1-s_n)}{(1+d_n)^{\tau_n}}]$$

$$- \sum_{n \in A} Tp_n N_n IC_n SH_n PS$$

$$+ \delta_1 \{ K - \sum_{n \in A} N_n SH_n PS \ [1-(1-s_n)IC_n] - PKQP$$

$$- \sum_{n \in A} (1-s_n)\alpha_n [NS-rD] - D \}$$

$$+ \sum_{n \in A} \delta_{2n} [s_n - 0.2]$$

$$+ \sum_{n \in A} \delta_{3n} [1.0 - s_n]$$

$$+ \sum_{n \in A} \delta_{4n} [IM-IC_n] \qquad\qquad (3.31)$$

The Kuhn-Tucker conditions for the financial model are:

$$\frac{\partial V}{\partial s_n} = - \sum_{m \in A} Tp_m IC_m SH_m PS \ \frac{\partial N_m}{\partial s_n}$$

$$+ (NS-rD)\alpha_n [1 - \frac{1}{(1+d_n)^{\tau_n}}]$$

$$- \delta_1 \{ \sum_m \frac{\partial N_m}{\partial s_n} [SH_m PS - [1-(1-s_m)IC_m]]$$

$$+ N_n IC_n SH_n PS + \alpha_n (NS-rD) \}$$

$$+ \delta_{2n} - \delta_{3n} \geq 0 \qquad\qquad (4.21a)$$

$$\frac{\partial V}{\partial s_n} s_n = 0 \qquad\qquad (4.21b)$$

$$s_n \geq 0 \qquad\qquad (4.21c)$$

$$\frac{\partial V}{\partial \tau_n} = - \sum_{m \in A} Tp_m IC_m SH_m PS \ \frac{\partial N_m}{\partial \tau_n}$$

$$- (NS-rD)\alpha_n[\frac{(1-s_n)}{(1+d_n)^{\tau_n}} \ln(1+d_n)] \geq 0 \qquad (4.22a)$$

$$\frac{\partial V}{\partial \tau_n} \tau_n = 0 \qquad (4.22b)$$

$$\tau_n \geq 0 \qquad (4.22c)$$

$$\frac{\partial V}{\partial D} = \sum_{n \in A} \alpha_n s_n'[- D \frac{\partial r}{\partial D} - r]$$

$$+ \delta_1\{-\sum_{n \in A} [(1-s_n)(\alpha_n[- D \frac{\partial r}{\partial D} - r] + 1\} \geq 0$$

$$(4.23a)$$

$$\frac{\partial V}{\partial D} D = 0 \qquad (4.23b)$$

$$D \geq 0 \qquad (4.23c)$$

$$\frac{\partial V}{\partial PS} = - \sum_{n \in A} N_n Tp_n IC_n SH_n - \sum_{m \in A} Tp_m IC_m SH_m PS \frac{\partial N_m}{\partial PS}$$

$$- \delta_1\{ \sum_{m \in B} \frac{\partial N_m}{\partial PS} [SH_m PS [1 - (1-s_m)IC_m]]$$

$$+ \sum_{m \in A} N_n[SH_n - (1-s_n)IC_n SH_n]\} \geq 0 \qquad (4.24a)$$

$$\frac{\partial V}{\partial PS} PS = 0 \qquad (4.24b)$$

$$PS \geq 0 \qquad (4.24c)$$

$$\frac{\partial V}{\partial IC_n} = - \sum_{n \varepsilon A} N_n Tp_n SH_n PS - \sum_{m \varepsilon A} TP_m IC_m SH_m PS \frac{\partial Nm}{\partial IC_n}$$

$$- \delta_1 \{ \sum_m \frac{\partial N_m}{\partial IC_n} [SH_m PS [1 - (1-s_m)IC_m]]$$

$$+ N_n(1-s_n)SH_n PS \} - \delta_{4n} \geq 0 \qquad (4.25a)$$

$$\frac{\partial V}{\partial IC_n} IC_n = 0 \qquad (4.25b)$$

$$IC_n \geq 0 \qquad (4.25c)$$

$$\frac{\partial V}{\partial \alpha_n} = - \sum_{m \varepsilon A} Tp_m IC_n SH_n PS \frac{\partial N_m}{\partial \alpha_n}$$

$$+ (NS - rD) s_n^{'}$$

$$- \delta_1 \{ \sum_m \frac{\partial N_m}{\partial \alpha_n} [SH_m PS [1 - (1-s_m)IC_m]]$$

$$+ (1-s_n)(NS-rD) \} \geq 0 \qquad (4.26a)$$

$$\frac{\partial V}{\partial \alpha_n} \alpha_n = 0 \qquad (4.26b)$$

$$\alpha_n \geq 0 \qquad (4.26c)$$

$$\frac{\partial V}{\partial \delta_1} = K - \sum_{n \varepsilon A} N_n SH_n PS [1 - (1-s_n)IC_n] - PKQP$$

$$- \sum_{n \varepsilon A} (1-s_n)\alpha_n(NS-rD) - D \} = 0 \qquad (4.27)$$

$$\frac{\partial V}{\partial \delta_{2n}} = s_n - 0.2 \geq 0 \qquad (4.28a)$$

$$\frac{\partial V}{\partial \delta_{2n}} \; \delta_{2n} = 0 \tag{4.28b}$$

$$\delta_{2n} \geq 0 \tag{4.28c}$$

$$\frac{\partial V}{\partial \delta_{3n}} = 1.0 - s_n \geq 0 \tag{4.29a}$$

$$\frac{\partial V}{\partial \delta_{3n}} \; \delta_{3n} = 0 \tag{4.29b}$$

$$\delta_{3n} \geq 0 \tag{4.29c}$$

$$\frac{\partial V}{\partial \delta_{4n}} = IM - IC_n \geq 0 \tag{4.30a}$$

$$\frac{\partial V}{\partial \delta_{4n}} \; \delta_{4n} = 0 \tag{4.30b}$$

$$\delta_{4n} \geq 0 \tag{4.30c}$$

Before analyzing the financial model Kuhn-Tucker conditions, the meaning of the Lagrangian multipliers must be determined. In general, the Lagrangian multiplier will represent the change in the objective function resulting from a one unit change in the constraint constant. The first Lagrangian multiplier, δ_1, shows the change in the objective function caused by a unit change in capital employed. Similarly, δ_{4n} represents the change in the

objective function caused by a one unit change in the
maximum value of dividends payable to stock. The
multipliers δ_{2n} and δ_{3n} together represent the change in the
objective function from either a one unit change in the
maximum or minimum value of s_n. It should be noted that
only one of the partial derivatives with respect to δ_{2n} or
δ_{3n} can equal zero for a maximum.

Noting that s_n must be a positive value, since its
value must be confined to the range of 0.2 and 1.0, (4.21a)
can be written as an equality. The interpretation of
(4.21a) suggests that a cooperative decision-maker who wants
to maximize member profits should set the following sum
equal to zero: (a) the change in member profits by a change
in membership numbers caused by a change in s_n via changes
in total private sales revenues, present value of patronage
dividends by changing the amount distributed, dividends on
stock, and total collective profits from a change in
capital, (b) the change in present value of net savings
caused by a change in the present value factor, (c) the
variation in total collective profits from a change in
capital induced by a change in s_n, and (d) the shadow price
of s_n. Written in equation form as:

$$\frac{\partial V}{\partial s_n} = \sum_m \frac{\partial TCP}{\partial N_m} \frac{\partial N_m}{\partial s_n} + \alpha_n \frac{\partial PVNS}{\partial s_n} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial s_n} + SPs_n = 0 \qquad (4.31)$$

where TCP is the total collective profits, PVNS is the

present value of net savings, TCP is the total collective

profits, and SPs_n is the shadow price for s_n. This

condition should be met for all members in set B since the

percent of patronage refunds paid in cash can be different

for each patron group. Similarly the deferment period for

each member group can be different so (4.22a) must hold for

all patron groups in set B. Since (4.22c) does not need to

hold as a strict equality, (4.22a) also does not need to be

a strict equality. Some patrons can conceivably have a

deferment period equal to zero (100% patronage refund paid

in cash) which is the only case where (4.24a) would be a

strict equality.

Equation (4.22a) can be interpreted that the

decision-maker should set equal to or greater than zero the

sum of: (a) the change in member profits induced by a

change in membership caused by a change in τ_n from changes

in total private sales revenue, a change in the present

value of patronage dividends caused by a change in the

amount distributed, dividends on stock, and total collective

profits from a change in capital, (b) the change in the

present value of net savings caused by a change in the

present value factor, and (c) the variation in total

collective profits of members from a change in capital which

was induced by a change in τ_n. This is written in equation form as:

$$\frac{\partial V}{\partial \tau_n} = \sum_m \frac{\partial TCP}{\partial N_m} \frac{\partial N_m}{\partial \tau_n} + \alpha_n \frac{\partial PVNS}{\partial \tau_n} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial \tau_n} \geq 0 \qquad (4.32)$$

Using Debt (D) as an instrument variable to maximize profits indicates that the cooperative decision-maker should employ debt to the point where the marginal profit of debt equals or exceeds the marginal cost of debt. This is seen by acknowledging that the first term in (4.23a) is the present value of the marginal interest cost of debt and the second term is the variation in total collective profits arising from a change in K caused by a change in D. In short-hand notation, (4.23a) can be rewritten as:

$$\frac{\partial V}{\partial D} = \frac{\partial PVNS}{\partial D} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial D} \geq 0 \qquad (4.33)$$

The model allows for any type of stock and yields the Kuhn-Tucker conditions using stock prices (4.24a) and dividends paid on stock (4.25a) as instrument variables. Since the price of stock (value) will always be positive, (4.24a) can be written in strict equality form. It should be noted that the price of stock can be varied however it will typically be held constant to avoid the potential accounting problems. It should be realized that different

sized memberships may require varying the amounts of stock, which is not included in this model. The interpretation of (4.24a) shows that: the decision-maker should set stock prices to maximize profits by setting equal to zero the sum of: (a) the change in total collective profits caused by changing membership by changing the price of stock from changes in total private sales revenues, the change in present value of patronage dividends caused by changing the amount distributed and the level of stock dividends, (b) the variation in the present value of net savings caused by changing the stock price, and (c) the variation in total collective profits from a change in capital caused by a change in the price of the stock. This Kuhn-Tucker condition can be re-stated in shorthand notation as:

$$\frac{\partial V}{\partial PS} = \sum_m \frac{\partial TCP}{\partial N_m} \frac{\partial N_m}{\partial PS} + \frac{\partial PVNS}{\partial PS} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial PS} = 0 \qquad (4.34)$$

Equation (4.25a) can be written as:

$$\frac{\partial V}{\partial IC_n} = \sum_m \frac{\partial TCP}{\partial N_m} \frac{\partial N_m}{\partial IC_n} + \frac{\partial PVNS}{\partial IC_n} + \frac{\partial TCP}{\partial K} \frac{\partial K}{\partial IC_n} + SPic_n \geq 0 \qquad (4.35)$$

Since some dividend rates for certain stock types or members may be zero; (4.35) is still in the weak inequality form. The interpretation for the Kuhn-Tucker condition using the stock dividend rate as an instrument variable shows that to

maximize cooperative member profits, the decision-maker should set equal to or greater than zero the sum of: (a) the variation in total collective profits from changing membership via changing the dividend rate paid, (b) the variation in the present value of net savings by changing the dividend rate, (c) the variation in total collective profits from a change in K induced by a change in the dividend rate, and (d) the shadow price of the dividend rate paid.

Allocating operating income, debt costs, and net savings is one method to differentially treat members. By using α_n as an instrument the cooperative decision-maker can see how to make these allocations to maximize profits. If some net operating income, debt costs, or net savings is allocated to each member group, (4.27a) must be a strict inequality, that is, for all $n \varepsilon B$, $\alpha_n > 0$.

Another variable that is seemingly available to the cooperative decision-maker is the amount of stock that must be held by various members (SH_n). It would not be correct to utilize these as instrument variables in this study since the levels would remain constant once they are set. To change these periodically would cause enormous accounting and recordkeeping problems. Although they are not changed, members may realize different levels by movement between the different patron groups.

The remaining Kuhn-Tucker conditions (4.27), (4.28a), (4.28b), (4.28c), (4.29a), (4.29b), (4.29c), (4.30a), (4.30b), and (4.30c) represent the constraints on the financial model objective function. The condition (4.27) reconstructs the capital usage constraint of the cooperative that states the total capital employed is derived from the sale of stock, funds obtained from a revolving fund, and debt sources. The constraint that the proportion of patronage refunds paid in cash to each patron must be greater that 20% and less than 100% is given by (4.28a), 4.28b), (4.28c), (4.29a), (4.29b), and (4.29c). The remaining conditions (4.30a), (4.30b), and (4.30c) restrict the level of stock dividend rates that the cooperative can pay on stock. Even though these Kuhn-Tucker conditions can be interpreted, it is highly unlikely that any cooperative decision-maker could possibly have enough information to follow them. Royer [52] and VanSickle [64] both concede that their models are complex to the point that practical use is questionable. The Kuhn-Tucker conditions here fall into this same pitfall since a second level of information is also needed. For example, the cooperative decision-maker must now derive prices for each member group rather than one overall price. As with these previous models, even though the Kuhn-Tucker conditions are complicated and require extensive amounts of information, they still provide

worthwhile information about the optimal actions of a profit
maximizing cooperative decision-maker. The vast amount of
information that must be known by the decision-maker is one
major constraint in using this model. To look at the
applicability of this general theoretical model, a
simplified cooperative situation will be modelled in
Chapter V. This simulation shows an application of the
general model and at the same time tests the theoretical
model.

CHAPTER V. APPLICATION OF THE THEORETICAL MODEL

Previous chapters outlined the basis for a model allowing for differential member treatment, presented a general model, and derived the set of Kuhn-Tucker conditions that pertain to a cooperative decision-maker interested in maximizing profits. This work, similar to earlier studies, concluded that extensive information must be available to the decision-maker to fully utilize the proposed Kuhn-Tucker conditions. This chapter presents an application of the general model that can be used to test and analyze the model and Kuhn-Tucker conditions that were determined in Chapters III and IV. The financial submodel used in this application corresponds to the model presented on page 83. The model is solved in two stages (the financial model is solved after solving the production model). The application model also includes steps to make the model iterative with solutions to initial runs being used as feedback to successive runs. There are two reasons why the first theoretical model presented in Chapter III and the application model differ. First, a new procedure to solve the financial model was developed after the theoretical model was formulated. The Jones [30] and VanSickle and Ladd [66] articles outline a procedure that was not available when the initial

theoretical model was developed. The second reason relates to the computational advantage in solving the model with the two-step procedure. Although the application financial model is solved in two steps it can be solved much easier since it involves a set of equations with at most one non-linear equation. The theoretical financial model could be solved numerically however it would involve solving a set of non-linear equations. Even though the models would yield a different set of Kuhn-Tucker conditions, a priori it is impossible to say whether the results would be the same. The set of Kuhn-Tucker conditions is not explicitly derived in the application model. The remaining part of this chapter will provide a brief review of previous applications of cooperative models, objectives for this application, the formulation of a mathematical and computer model, and some conclusions arrived at with this model application.

Royer [52] designed his short-run theoretical model to find the optimal production and pricing decisions but did not make any attempt to empirically test the model. Eversull [17] proceeded to make an empirical test of Royer's work by modelling a simplified cooperative having four activities that included sales of bag and bulk fertilizer to patrons and the purchase of corn and soybeans from the patrons. Royer's work specified individual patron and

non-patron supply and demand functions in functional form, whereas in Eversull's hypothetical grain cooperative model these were assumed linear with known parameters. Eversull's study allowed for interrelated supply and demand functions and the possibility of cooperative members patronizing other firms. The nature of the model necessitated a quadratic programming approach to account for supply and demand being quadratic in basis values (the model used basis values in lieu of prices). After specifying the member and non-member supply and demand functions, total cooperative member profit was maximized subject to capacity constraints using basis values as the instrument variables. This approach solved for the values of basis (prices) and production but did not provide the decision-maker with any insight on financial decisions (deferment period, cash patronage refund, debt, and qualified patronage refunds).

VanSickle [64] proceeded with Royer's pricing-production model and developed a long-run financial model to accompany it. In addition to devising a financial model, VanSickle empirically estimated the price instrument variables for several commodities as represented by the structural equations of the pricing-production model. Using 1979 data from Iowa cooperatives, the pricing functions for fertilizer, feed, corn, and soybeans were estimated. Of

particular interest to the model presented in Chapter III,
it was found that both the percent of patronage refund paid
in cash and the length of the deferment period were not
statistically significant in determining the price of the
commodity. This phenomenon is interesting since it suggests
that s and τ are not influential in the pricing-production
decisions. It is obvious however that s and τ are very
important to the financial concerns of the cooperative.
VanSickle and Ladd [66] used the financial model of
VanSickle to derive the optimal levels of qualified
patronage refunds (QPR), dividend rates (i_c), deferment
period (τ), percent cash patronage refund (s), and the
amount of debt to employ (D). The procedure used involved
maximizing profits via the solution of a set of non-linear
equations and a simulation process. Since the VanSickle and
Ladd work was aimed at refining a financial model, price and
production levels were assumed exogenous and there was no
feedback between the models. The VanSickle and Ladd work
was scrutinized by Jones [30] who believed that their
specification of the cooperative objective function was in
error. Using the insight provided by Jones, VanSickle and
Ladd [66] developed several alternative models, one of which
was a two-stage synthesis of the two works. This two-stage
synthesis involved recognizing the complementarity of τ

and s. This complementarity relationship exists since the changes in one can be offset by changing the value of the other. The first step involves solving for the decision variables QPR, i_c, D, and H (where H = $\tau[1-s]$). The second step utilizes the optimal values of H and QPR determined in the first step and solves for the optimal values of τ and s. The procedure used in this two-stage procedure has a computational advantage over the other models in that it requires solving a set of simulataneous equations where at most one equation is non-linear, not the full set as in the other models. Recognizing this computational advantage, the application of the general model will use this procedure for determining the effects of cooperative member differential treatment.

Procedure Used for the Specific Model

The primary objective of this chapter is to set up and simulate an application of the general model proposed in Chapter III. The application will use the general model as a foundation and will be built using procedures employed in previous applications. The process of solving for the optimal levels of the decision variables is a four-step, iterative procedure. The basic procedure involves determining the pricing-production decisions using prices as

choice variables (Stage 1), solving the financial model in a two-step procedure as done by VanSickle and Ladd [66] using the solution price values from step one (Stages 2 and 3), and updating variables to solve for the following year's pricing-production model (Stage 4).

Formulation of a Computer Simulation Model

In setting up a mathematical model, special care needs to be taken in classifying variables and specifying the complexity of the cooperative enterprise to model. Since the model is solved in a four-step process, some variables will be endogenous (determined by the model) in one stage and exogenous (predetermined) in another. Prices and quantities are good examples of this phenomenon since they are endogenous in Stage I, the pricing-production model, and exogenous in Stage II, the first part of the financial model. Other variables which are of a similar nature are QPR, H, s, and τ. Table 5.1 provides a key for the variables that will be used in this chapter's simulation model and can be used to compare notation with the previous general model's notation.

TABLE 5.1. Application model key

Variables

       QPR = level of qualified patronage refunds
       IC = dividend rate
       τ = deferment period
       s   = percent cash patronage refund
       D   = level of cooperative debt
       H   = $\tau(1-s)$
       q   = quantity demanded/supplied by member patron
       Q   = quantity demanded/supplied by non-member patron
       p   = price, without superscript represents the price
             the cooperative pays for inputs (fertilizer) and
             receives for outputs (corn)
       r   = with subscripts this is the level of expected per
             unit patronage refund
           = without subscripts this represents the average
             cost of cooperative debt
       N   = the number of patrons
       K   = the level of capital used by the cooperative
       PS = the price of cooperative stock
       SH = the number of shares of stock held by a member
       Tp = the individual's income tax rate   •
       DS = the dividends paid on cooperative stock
Subscripts
       CH = corn high volume (over 5000 bushels)
       CL = corn low volume (under 5000 bushels)
       FH = fertilizer high volume (over 400 units)
       FL = fertilizer low volume (under 400 units)
       S   = field services (only one level)
Superscripts
       c   = cooperative
       o   = other firms
       T   = total firms (cooperative + other firms)
Operators and coefficients
       L   = logarithm of
       aij, bij = elasticity coefficients for members and
                  non-members

A consideration in the formulation of a specific

simulation model is its complexity. The model should be

complete enough to be realistic and usable, yet not too

cluttered and complex to be unmanageable. The general model
in this paper allows each patron to be afforded a different
value for prices, s, τ, stock shares, stock dividends, and
stock prices, rather than a single value that is typical
with previous models. Realizing these concerns, the model
in this chapter will include only two departments, a corn
marketing department and a fertilizer supply department.
The corn marketing department contains two activities,
marketing corn for high volume (CH) patrons and low volume
(CL) patrons. The supply department has three activities,
supplying fertilizer to a group of high volume (FH) patrons
and low volume (FL) patrons, along with the provision of a
set of field services (S) to these fertilizer patrons. The
first stage of the process is analogous to Eversull's work.
That is, the cooperative pricing and production decisions
are solved by maximizing profit subject to capacity
constraints with specified linear functions for member
supply, member demand, non-member supply, and non-member
demand as given. The following notation is used to specify
these functions: $\{Lq_{CH}^C, Lq_{CL}^C\}$ are the logarithms of the
levels of corn marketed through the cooperative by a member
in the high-corn-volume group and by a member in the
low-corn-volume corn group; $\{Lq_{CH}^O, Lq_{CL}^O\}$ are the logarithms
of the levels of corn marketed through other firms by a
member in the high-corn-volume group and a member in the

low-corn-volume group; $\{Lq^C_{FH}, Lq^C_{FL}, Lq^C_S\}$ are the logarithms of the levels of fertilizer purchased from the cooperative by a high-fertilizer-volume member, a low-fertilizer-volume member, and a level of field services demanded from the cooperative by all members; $\{Lq^O_{FH}, Lq^O_{FL}, Lq^O_S\}$ are the logarithms of the levels of fertilizer purchased from other firms by a high-fertilizer-volume member, a low-fertilizer-volume member, and a level of field services demanded from other firms by all members; $\{LQ^C_{CH}, LQ^C_{CL}\}$ are the logarithms of the levels of corn marketed through the cooperative by non-members in the high volume group and by non-members in the low volume group; $\{LQ^C_{FH}, LQ^C_{FL}\}$ are the logarithms of the levels of fertilizer demanded from the cooperative by non-members in a high volume fertilizer group and by non-members in a low volume group; $\{Lp^C_{CH}, Lp^C_{CL}, Lp^C_{FH}, Lp^C_{FL}, Lp^C_S\}$ and $\{Lp^O_{CH}, Lp^O_{CL}, Lp^O_{FH}, Lp^O_{FL}, Lp^O_S\}$ are the logarithms of the respective cooperative prices and other firm prices for CH, CL, FH, FL, and S. $\{Lr_{CH}, Lr_{CL}, Lr_{FH}, Lr_{FL}, Lr_S\}$, $\{L\tau_{CH}, L\tau_{CL}, L\tau_{FH}, L\tau_{FL}, L\tau_S\}$ and $\{Ls_{CH}, Ls_{CL}, Ls_{FH}, Ls_{FL}, Ls_S\}$ are respectively the logarithms of the expected per unit patronage refund, the length of the deferment period, and the percent patronage refund paid in cash for CH, CL, FH, FL, and S. These supply and demand functions are specified for all products for each patron and are given as follows.

Individual Member Supply Functions (to cooperative):

$$Lq_{CH}^{C} = a_{11}Lp_{CH}^{C} - a_{12}Lp_{CH}^{O} + a_{13}Lr_{CH} - a_{14}L\tau_{CH} + a_{15}Ls_{CH} \tag{5.1}$$

$$Lq_{CL}^{C} = a_{21}Lp_{CL}^{C} - a_{22}Lp_{CL}^{O} + a_{23}Lr_{CL} - a_{24}L\tau_{CL} + a_{25}Ls_{CL} \tag{5.2}$$

Individual Member Supply Functions (to other firms):

$$Lq_{CH}^{O} = - a_{31}Lp_{CH}^{C} + a_{32}Lp_{CH}^{O} - a_{33}Lr_{CH} + a_{34}L\tau_{CH} - a_{35}Ls_{CH} \tag{5.3}$$

$$Lq_{CL}^{O} = - a_{41}Lp_{CL}^{C} + a_{42}Lp_{CL}^{O} - a_{43}Lr_{CL} + a_{44}L\tau_{CL} - a_{45}Ls_{CL} \tag{5.4}$$

Individual Member Demand Functions (from cooperative):

$$Lq_{FH}^{C} = - a_{51}Lp_{FH}^{C} + a_{52}Lp_{FH}^{O} + a_{53}Lr_{FH} - a_{54}L\tau_{FH} + a_{55}Ls_{FH} - a_{56}Lp_{S}^{C} + a_{57}Lp_{S}^{O} \tag{5.5}$$

$$Lq_{FL}^{C} = - a_{61}Lp_{FL}^{C} + a_{62}Lp_{FL}^{O} + a_{63}Lr_{FL} - a_{64}L\tau_{FL} + a_{65}Ls_{FL} - a_{66}Lp_{S}^{C} + a_{67}Lp_{S}^{O} \tag{5.6}$$

$$Lq_{S}^{C} = - a_{91}Lp_{S}^{C} + a_{92}Lp_{S}^{O} + a_{93}Lr_{S} - a_{94}L\tau_{S} + a_{95}Ls_{S} \tag{5.7}$$

Individual Member Demand Functions (from other firms):

$$Lq_{FH}^{O} = a_{71}Lp_{FH}^{C} - a_{72}Lp_{FH}^{O} - a_{73}Lr_{FH} + a_{74}L\tau_{FH} - a_{75}Ls_{FH} + a_{76}Lp_{S}^{C} - a_{77}Lp_{S}^{O} \tag{5.8}$$

$$Lq_{FL}^{O} = a_{81}Lp_{FL}^{C} - a_{82}Lp_{FL}^{O} - a_{83}Lr_{FL} + a_{84}L\tau_{FL} - a_{85}Ls_{FL} + a_{86}Lp_{S}^{C} - a_{87}Lp_{S}^{O} \tag{5.9}$$

$$Lq_{S}^{O} = a_{10.1}Lp_{S}^{C} - a_{10.2}Lp_{S}^{O} - a_{10.3}Lr_{S} + a_{10.4}L\tau_{S} - a_{10.5}Ls_{S}^{C} \tag{5.10}$$

Individual Non-member Supply Functions (to cooperative):

$$LQ_{CH}^C = b_{11}Lp_{CH}^C - b_{12}Lp_{CH}^O + b_{13}Lr_{CH} - b_{14}L\tau_{CH}$$
$$+ b_{15}Ls_{CH} \hspace{3cm} (5.11)$$

$$LQ_{CL}^C = b_{21}Lp_{CL}^C - b_{22}Lp_{CL}^O + b_{23}Lr_{CL} - b_{24}L\tau_{CL}$$
$$+ b_{25}Ls_{CL} \hspace{3cm} (5.12)$$

Individual Non-member Demand Functions (from cooperative):

$$LQ_{FH}^C = - b_{31}Lp_{FH}^C + b_{32}Lp_{FH}^O + b_{33}Lr_{FH} - b_{34}L\tau_{FH} + b_{35}Ls_{FH}$$
$$+ b_{36}Lp_S^C - b_{37}Lp_S^O \hspace{2.5cm} (5.13)$$

$$LQ_{FL}^C = - b_{41}Lp_{FL}^C + b_{42}Lp_{FL}^O + b_{43}Lr_{FL} - b_{44}L\tau_{FL} + b_{45}Ls_{FL}$$
$$+ b_{46}Lp_S^C - b_{47}Lp_S^O \hspace{2.5cm} (5.14)$$

Assuming that all producers within each group are homogeneous, the total quantity of product supplied (demanded) by the n-th group is found by summing across the individual supply (demand) functions or by multiplying by N_n (the number of patrons in that group). The number of producers with member patronage to the cooperative, number of members with patronage to other firms, and total number of producers are represented by the following sets $\{N_{CH}^C,$ $N_{CL}^C, N_{FH}^C, N_{FL}^C, N_S^C\}$, $\{N_{CH}^O, N_{CL}^O, N_{FH}^O, N_{FL}^O, N_S^O\}$, and $\{N_{CH}^T, N_{CL}^T,$ $N_{FH}^T, N_{FL}^T, N_S^T\}$. The a_{ij} and b_{ij} (all i,j) are predetermined parameters of the model which should vary among producer groups since the members are assumed to have different business volumes. If the patrons were homogeneous, the parameters for Lq_{CH}^C and Lq_{CL}^C, Lq_{FH}^C and Lq_{FL}^C, Lq_{CH}^O and Lq_{CL}^O,

Lq_{FH}^O and Lq_{FL}^O, LQ_{CH}^C and LQ_{CL}^C, LQ_{FH}^C and LQ_{FL}^C would be the same. The supply and demand functions are in log-linear form which allows the a_{ij} and b_{ij} to be interpreted as supply and demand elasticities.

The a_{i1}, for i = 1 to 10, are the own price elasticities for members. For example, a_{11} is the percent change in the quantity of corn marketed through the cooperative by high volume members caused by a one percent change in the cooperative's respective corn price for those members. The a_{i2}, a_{i3}, a_{i4}, and a_{i5} for i = 1 to 10 are respectively the elasticities of patron supply or demand with respect to the competitor's price, expected per unit patronage refunds, length of deferment, and the percent patronage refund paid in cash. The a_{i6} and a_{i7} are cross price elasticities with respect to the level of cooperative and competitor field services provided. The interpretation of b_{ij} (all i,j), own price elasticities for non-members, is analogous to the a_{ij}'s. The supply and demand functions are set up in this manner to determine the effects of relative or percentage changes in the variables r, τ, and s rather than absolute changes. In obtaining the values for these parameters, the relevant questions to be answered are of the following form: if a given decision variable increases (decreases) X percent, what would the percentage change in

supply (demand) be? For example, to arrive at a value for a_{14} the relevant question is: If τ_{CH} increases 1%, what percentage change in q_{CH}^C would result? Values for the parameters a_{ij} and b_{ij} (all i,j) for this type of cooperative were determined after personal and telephone interviews with ten marketing and supply cooperative managers in Northeast Iowa and Southwest Wisconsin. The responses obtained from these interviews varied from cooperative to cooperative. The values used in the application base model are the averages of these responses and are presented in later in Table 5.2. After specifying the supply and demand function parameters, the values for the fully exogenous variables $\{p_{CH}^O, p_{CL}^O, p_{FH}^O, p_{FL}^O, p_S^O$ and $K\}$ and initial values for the short-run exogenous variables (levels of per-unit patronage refunds, lengths of deferment periods, and producer numbers) the first stage of the model can be solved. Stage I can now be completed by maximizing total member profits subject to the capacity constraints using cooperative prices as the decision variables. Once the optimal pricing (production) decisions are made, the net savings generated for each product can be determined. It should be noted that the variables representing expected patronage refunds, deferment period, percent patronage refund paid in cash, and membership breakdown are fixed

(exogenous) in Stage I yet are endogenous in latter stages. Even though the parameters a_{ij} (all i,j), b_{ij} (all i,j), other firm prices, capital needs (K), and capacity limits are fixed through all stages, they can be altered to consider various cooperative circumstances. This mathematical model for this first stage is represented by equations (5.15) through (5.18).

The notation for the first stage of the model is as follows. L_1 is the after-tax profits of all cooperative members; $\{Tp_{CH}, Tp_{CL}, Tp_{FH}, Tp_{FL}, Tp_S\}$ are the respective marginal tax rates for each type of member; $\{p^C_{CH}, p^C_{CL}, p^C_{FH}, p^C_{FL}, p^C_S\}$ are the cooperative prices for patrons; $\{q^C_{CH}, q^C_{CL}, q^C_{FH}, q^C_{FL}, q^C_S\}$ are the amounts of business patronage by cooperative producers; $\{p^O_{CH}, p^O_{CL}, p^O_{FH}, p^O_{FL}, p^O_S\}$ are the prices of the various commodities for other firms; $\{q^O_{CH}, q^O_{CL}, q^O_{FH}, q^O_{FL}, q^O_S\}$ are the amounts of member business patronage with other firms; $\{s_{CH}, s_{CL}, s_{FH}, s_{FL}, s_S\}$ are the respective percent of patronage refunds paid in cash; $\{d_{CH}, d_{CL}, d_{FH}, d_{FL}, d_S\}$ are the appropriate discount rates for each member group type; $\{\tau_{CH}, \tau_{CL}, \tau_{FH}, \tau_{FL}, \tau_S\}$ are the lengths of the deferment period for each group; $\{P_{CH}, P_{CL}\}$ are the prices the cooperative receives from outside buyers for corn; $\{P_{FH}, P_{FL}, P_S\}$ are the average total costs incurred by the cooperative to supply these goods and

services; $\{IC_{CH}, IC_{CL}, IC_{FH}, IC_{FL}, IC_S\}$ are the dividend rates paid on stock held by each group; $\{PS_{CH}, PS_{CL}, PS_{FH}, PS_{FL}, PS_S\}$ are the prices per share of stock for each patron group; $\{SH_{CH}, SH_{CL}, SH_{FH}, SH_{FL}, SH_S\}$ are the levels of stock held by each member in a group; and $\{q_C, q_F, q_S\}$ are the respective capacity limits for the cooperative for corn, fertilizer and field services.

Maximize

$$
\begin{aligned}
L_1 = &\ (1-Tp_{CH})\{N_{CH}^C(p_{CH}^C q_{CH}^C)+N_{CH}^O(p_{CH}^O q_{CH}^O)\} \\
&+ (1-Tp_{CL})\{N_{CL}^C(p_{CL}^C q_{CL}^C)+N_{CH}^O(p_{CL}^O q_{CL}^O)\} \\
&- (1-Tp_{FH})\{N_{FH}^C(p_{FH}^C q_{FH}^C)+N_{FH}^O(p_{FH}^O q_{FH}^O)\} \\
&- (1-Tp_{FL})\{N_{FL}^C(p_{FL}^C q_{FL}^C)+N_{FL}^O(p_{FL}^O q_{FL}^O)\} \\
&- (1-Tp_S)\{N_S^C(p_S^C q_S^C)+N_S^O(p_S^O q_S^O)\} \\[6pt]
&+ \left[s_{CH} - Tp_{CH} + \frac{(1-s_{CH})}{(1+d_{CH})^{\tau_{CH}}}\right]\{(p_{CH}-p_{CH}^C) \\
&\quad [N_{CH}^C\ q_{CH}^C + (N_{CH}^T-N_{CH}^O-N_{CH}^C)\ Q_{CH}^C] - IC_{CH}\ PS_{CH}\ SH_{CH}\} \\[6pt]
&+ \left[s_{CL} - Tp_{CL} + \frac{(1-s_{CL})}{(1+d_{CL})^{\tau_{CL}}}\right]\{(p_{CL}-p_{CL}^C) \\
&\quad [N_{CL}^C\ q_{CL}^C + (N_{CL}^T-N_{CL}^O-N_{CL}^C)\ Q_{CL}^C] - IC_{CL}\ PS_{CL}\ SH_{CL}\} \\[6pt]
&+ \left[s_{FH} - Tp_{FH} + \frac{(1-s_{FH})}{(1+d_{FH})^{\tau_{FH}}}\right]\{(p_{FH}^C-p_{FH}) \\
&\quad [N_{FH}^C\ q_{FH}^C + (N_{FH}^T-N_{FH}^O-N_{FH}^C)\ Q_{FH}^C] - IC_{FH}\ PS_{FH}\ SH_{FH}\} \\[6pt]
&+ \left[s_{FL} - Tp_{FL} + \frac{(1-s_{FL})}{(1+d_{FL})^{\tau_{FL}}}\right]\{(p_{FL}^C-p_{FL}) \\
&\quad [N_{FL}^C\ q_{FL}^C + (N_{FL}^T-N_{FL}^O-N_{FL}^C)\ Q_{FL}^C] - IC_{FL}\ PS_{FL}\ SH_{FL}\}
\end{aligned}
$$

$$+[s_S - Tp_S + \frac{(1-s_S)}{(1+d_S) \, S}]\{(p_S^C - p_S)$$

$$[N_S^C \, q_S^C + (N_S^T - N_S^O - N_S^C) \, Q_S^C] - IC_S \, PS_S \, SH_S\}$$

$$(5.15)$$

such that

$$N_{CH}^C \, q_{CH}^C + (N_{CH}^T - N_{CH}^O - N_{CH}^C) \, Q_{CH}^C$$

$$+ \, N_{CL}^C \, q_{CL}^C + (N_{CL}^T - N_{CL}^O - N_{CL}^C) \, Q_{CL}^C < q_C \qquad (5.16)$$

$$N_{FH}^C \, q_{FH}^C + (N_{FH}^T - N_{FH}^O - N_{FH}^C) \, Q_{FH}^C$$

$$+ \, N_{FL}^C \, q_{FL}^C + (N_{FL}^T - N_{FL}^O - N_{FL}^C) \, Q_{FL}^C < q_F \qquad (5.17)$$

$$N_S^C \, q_S^C + (N_S^T - N_S^O - N_S^C) \, Q_S^C < q_S \qquad (5.18)$$

During this first stage the volume of business done
during a fiscal year is determined by the optimizing values
of the choice variables. These optimizing values are the
decisions made at the beginning of the fiscal year and
affect the values of number of total producers (N^T), the
number of exclusive cooperative patrons (N^C), and number of
members who patronize other firms completely (N^O) for that
year. The model allows producers to patronize both the
cooperative and other firms. Even though a member may
patronize other firms they will be treated similar to other
members with like patronage levels. At this point the first
stage is complete and the next step involves the financial
aspects of the model. The second stage in this simulation

process involves determining the optimal values for the decision variables of the financial model. This stage uses the solution values for prices and quantities obtained from Stage I as exogenous variables and considers qualified patronage refunds (QPR), Debt (D), and H (where H is defined as $\tau[1-s]$) as financial choice variables. The variable H, a composite measure of τ and s, is endogenously determined in Stage II and treated as exogenous in Stage III. At the end of a fiscal year, the cooperative determines its net savings for that year. The net savings must be allocated with this allocation affecting the cooperative's financial structure and the present value of member's income. The financial structure is also affected by the other decisions made at this time. The objective function for Stage II maximizes profits of all members. The mathematical model for Stage II is represented by equations (5.19) through (5.26).

Maximize

$$
\begin{aligned}
L_2 = \ & (1-Tp_{CH})\{N_{CH}^c(p_{CH}^c q_{CH}^c)+N_{CH}^o(p_{CH}^o q_{CH}^o)\} \\
& +(1-Tp_{CL})\{N_{CL}^c(p_{CL}^c q_{CL}^c)+N_{CL}^o(p_{CL}^o q_{CL}^o)\} \\
& -(1-Tp_{FH})\{N_{FH}^c(p_{FH}^c q_{FH}^c)+N_{FH}^o(p_{FH}^o q_{FH}^o)\} \\
& -(1-Tp_{FL})\{N_{FL}^c(p_{FL}^c q_{FL}^c)+N_{FL}^o(p_{FL}^o q_{FL}^o)\} \\
& -(1-Tp_{S})\{N_{S}^c(p_{S}^c q_{S}^c)+N_{S}^o(p_{S}^o q_{S}^o)\} \\
& +N_{CH}^c(1-Tp_{CH})QPR_{CH} + (1-Tp_{CH})IC_{CH}\ PS_{CH}\ SH_{CH} \\
& -d_{CH}(1-Tp_{CH})[N_{CH}^c\ H_{CH}\ QPR_{CH} + PS_{CH}\ SH_{CH}] \\
& +N_{CL}^c(1-Tp_{CL})QPR_{CL} + (1-Tp_{CL})IC_{CL}\ PS_{CL}\ SH_{CL}
\end{aligned}
$$

$$-d_{CL}(1-Tp_{CL})[N^C_{CL} \; H_{CL} \; QPR_{CL} + PS_{CL} \; SH_{CL}]$$

$$+N^C_{FH}(1-Tp_{FH})QPR_{FH} + (1-Tp_{FH})IC_{FH} \; PS_{FH} \; SH_{FH}$$

$$-d_{FH}(1-Tp_{FH})[N^C_{FH} \; H_{FH} \; QPR_{FH} + PS_{FH} \; SH_{FH}]$$

$$+N^C_{FL}(1-Tp_{FL})QPR_{FL} + (1-Tp_{FL})IC_{FL} \; PS_{FL} \; SH_{FL}$$

$$-d_{FL}(1-Tp_{FL})[N^C_{FL} \; H_{FL} \; QPR_{FL} + PS_{FL} \; SH_{FL}]$$

$$+N^C_S(1-Tp_S)QPR_S + (1-Tp_S)IC_S \; PS_S \; SH_S$$

$$-d_S(1-Tp_S)[N^C_S \; H_S \; QPR_S + PS_S \; SH_S] \tag{5.19}$$

such that

$$0 = K - H_{CH}QPR_{CH} - H_{CL}QPR_{CL} - H_{FH}QPR_{FH} - H_{FL}QPR_{FL}$$

$$- H_S QPR_S - PS_{CH}SH_{CH} - PS_{CL}SH_{CL} - PS_{FH}SH_{FH}$$

$$- PS_{FL}SH_{FL} - PS_S SH_S - D \tag{5.20}$$

$$0 = (P_{CH} - P^C_{CH})(q^C_{CH} + Q^C_{CH}) + (P_{CL} - P^C_{CL})(q^C_{CL} + Q^C_{CL})$$

$$-(P_{FH} - P^C_{FH})(q^C_{FH} + Q^C_{FH}) - (P_{FL} - P^C_{FL})(q^C_{FL} + Q^C_{FL})$$

$$-(P_S - P^C_S)(q^C_S + Q^C_S) - QPR_{CH} - QPR_{CL} - QPR_{FH} - QPR_{FL}$$

$$- QPR_S - IC_{CH}PS_{CH}SH_{CH} - IC_{CL}PS_{CL}SH_{CL} - IC_{FH}PS_{FH}SH_{FH}$$

$$- IC_{FL}PS_{FL}SH_{FL} - IC_S PS_S SH_S - rD \tag{5.21}$$

$$IC_{CH} \leq 0.08 \tag{5.22}$$

$$IC_{CL} \leq 0.08 \tag{5.23}$$

$$IC_{FH} \leq 0.08 \tag{5.24}$$

$$IC_{FL} \leq 0.08 \tag{5.25}$$

$$IC_S \leq 0.08 \tag{5.26}$$

where $\{N^C_{CH}, N^C_{CL}, N^C_{FH}, N^C_{FL}, N^C_S\}$ are numbers of cooperative members of a given group who patronize the cooperative in

the respective departments, $\{H_{CH}, H_{CL}, H_{FH}, H_{FL}, H_S\}$ are intermediate variables defined as $\tau_j(1-s_j)$ for all "j" departments, K is the amount of capital needed to operate the cooperative, D is the amount of debt employed by the cooperative, and r is the average cost of cooperative debt sources.

The third stage involves solving for specific values of τ and s using the maximization of the present value of qualified patronage refunds as the objective function. The previously determined values of QPR and H are used to determine the optimal values of s and τ. At the completion of Stage III, all of the pricing-production and financial decision variables are determined. Equations (5.27) through (5.37) represent the mathematical model for Stage III.

Maximize

$$
\begin{aligned}
L_3 = \ & [s_{CH} - Tp_{CH}\frac{(1-s_{CH})}{(1+d_{CH})^{\tau_{CH}}}]QPR_{CH} \\
& +[s_{CL} - Tp_{CL}\frac{(1-s_{CL})}{(1+d_{CL})^{\tau_{CL}}}]QPR_{CL} \\
& +[s_{FH} - Tp_{FH}\frac{(1-s_{FH})}{(1+d_{FH})^{\tau_{FH}}}]QPR_{FH} \\
& +[s_{FL} - Tp_{FL}\frac{(1-s_{FL})}{(1+d_{FL})^{\tau_{FL}}}]QPR_{FL} \\
& +[s_S - Tp_S\frac{(1-s_S)}{(1+d_S)^{\tau_S}}]QPR_S
\end{aligned} \tag{5.27}
$$

such that

$$H_{CH} = \tau_{CH}(1-s_{CH}) \tag{5.28}$$

$$H_{CL} = \tau_{CL}(1-s_{CL}) \tag{5.29}$$

$$H_{FH} = \tau_{FH}(1-s_{FH}) \tag{5.30}$$

$$H_{FL} = \tau_{FL}(1-s_{FL}) \tag{5.31}$$

$$H_{S} = \tau_{S}(1-s_{S}) \tag{5.32}$$

$$0.2 \leq s_{CH} \leq 1.0 \tag{5.33}$$

$$0.2 \leq s_{CL} \leq 1.0 \tag{5.34}$$

$$0.2 \leq s_{FH} \leq 1.0 \tag{5.35}$$

$$0.2 \leq s_{FL} \leq 1.0 \tag{5.36}$$

$$0.2 \leq s_{S} \leq 1.0 \tag{5.37}$$

Stage IV involves updating several variables to enable successive runs of the four stage process. This stage allows several iterations of the model to be made giving it a time dimension. The cooperative then fully allocates net savings. In addition, decisions affecting the next fiscal year's prices must be made. This brings us back to Stage I for the upcoming year. The exogenous variables in Stage I can be updated based on the values of decision variables in the latter stages. The list of updated variables include the expected per unit patronage refund (the actual value becomes the expected value for successive runs) and the producer number breakdown. This update takes place after the short-run optimization of the pricing-production model but before the run of the next iteration. The total number

of producers in each class $(N_{CH}^T$, N_{CL}^T, N_{FH}^T, N_{FL}^T, $N_S^T)$ can be updated as well as the breakdown in patronage location (cooperative or other) and non-member patronage level. Assuming that patronage level (of members and non-members) is a function of financial policies, the change in patronage can be estimated for the next run. The number of patrons in a group is assumed to be positively related to the price paid, the expected per-unit cash refund and the percent of cash patronage refund while negatively related to the length of the deferment period. The general patron number elasticities that are relevant to this simulation model (that have to be specified) are given in Table 5.2.

TABLE 5.2. General patron number elasticities used in the base model in this study

	r	τ	s	p
N_{CH}^c	$\xi_{CH,r}$ (0.60)	$\xi_{CH,\tau}$ (-0.50)	$\xi_{CH,s}$ (0.80)	$\xi_{CH,p}$ (0.85)
N_{CL}^c	$\xi_{CL,r}$ (0.55)	$\xi_{CL,\tau}$ (-0.45)	$\xi_{CL,s}$ (0.70)	$\xi_{CL,p}$ (0.85)
N_{FH}^c	$\xi_{FH,r}$ (0.60)	$\xi_{FH,\tau}$ (-0.50)	$\xi_{FH,s}$ (0.80)	$\xi_{FH,p}$ (0.85)
N_{FL}^c	$\xi_{FL,r}$ (0.55)	$\xi_{FL,\tau}$ (-0.45)	$\xi_{FL,s}$ (0.70)	$\xi_{FL,p}$ (0.85)
N_S^c	$\xi_{S,r}$ (0.55)	$\xi_{S,\tau}$ (-0.45)	$\xi_{S,s}$ (0.70)	$\xi_{S,p}$ (0.75)

The elasticities of Table 5.2 are used to change the level of cooperative member patronage. For example, if $\xi_{CH,s}$ was equal to 0.90, this means that for a 10% change in s_{CH}, there would be a 9% change in the level of N^c_{CH}. For simplicity, this work assumes that patron number elasticities for a given group of patrons are constant over all iterations of the model. In reality, these elasticities will most likely vary as extreme levels of the instrument variables are reached.

After making any desired changes in parameters and/or exogenous variables, the model can be run again by returning to Stage I and proceeding through the successive stages. A brief outline and flow diagram clarify this simulation procedure and leads into the development of the computer model.

Stage I.

    a) obtain values of parameters for the supply and demand functions $\{a_{ij}$ and b_{ij} (all i,j)$\}$

    b) obtain values of the exogenous variables $\{K$ and the vector of $p^o\}$

    c) specify initial values for the vectors of variables exogenous in the production-pricing model $\{$ r, τ, s, N^T, N^O, $N^C\}$

    d) maximize total cooperative profits using the vector of cooperative prices as choice variables

    e) determine values for the vectors $\{$NS, QPR, DS$\}$

Stage II.

    a) use the pricing and production levels as determined in Stage I as exogenous variables.

    b) maximize members' total after-tax net profits using QPR, D, and H as the cooperative's choice variables.

Stage III.

    a) use the endogenously determined values of QPR, D and H from Stage II as exogenous variables

    b) maximize the discounted present value of patronage refunds using s and τ as the cooperative's choice variables

Stage IV

    a) using the endogenously determined variables from the previous stages calculate the actual per-unit patronage refunds

    b) compare values of s and τ that are exogenous in Stage I to those endogenously determined in Stage III and decide if a change in membership is necessary (based on patron number elasticities)

    c) make adjustments to patronage levels if necessary

    d) change values of parameters and/or exogenous variables if desired

    e) proceed to Stage I to re-run the model

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┌──────────────────────────────────────────────────────┐
│ Determine the appropriate                            │
│   1) set of producer behavioral functions            │◄─────┐
│   2) set of parameters and exogenous variables       │      │
│      (other firm prices and a_{ij}, b_{ij} a i,j).   │      │
└──────────────────────────────────────────────────────┘      │
                        │                                      │
                        ▼                                      │
┌──────────────────────────────────────────────────────┐      │
│ Select values for the short-run exogenous variables  │      │
│  (deferment period lengths, percent cash patronage    │      │
│  refund) and initial values for membership numbers,   │      │
│  and expected per-unit patronage refunds.             │      │
└──────────────────────────────────────────────────────┘      │
                        │                                      │
                        ▼                                      │
┌──────────────────────────────────────────────────────┐      │
│ Maximize cooperative profits via a non-linear         │      │
│  optimization routine using cooperative prices as     │      │
│  choice variables.                                    │      │
└──────────────────────────────────────────────────────┘      │
                        │                                      │
                        ▼                                      │
┌──────────────────────────────────────────────────────┐      │
│ Solve (determine) for the values of other response    │      │
│  variables (profits for each group and net            │      │
│  savings) and the allocation of net savings.          │      │
└──────────────────────────────────────────────────────┘      │
                        │                                      │
                        ▼                                      │
┌──────────────────────────────────────────────────────┐      │
│ Maximize cooperative member's profits using           │      │
│  values of qualified patronage refunds, level of      │      │
│  cooperative debt, and H (= $\tau[1-s]$) as choice    │      │
│  variables.                                           │      │
└──────────────────────────────────────────────────────┘      │
                        │                                      │
                        ▼                                      │
┌──────────────────────────────────────────────────────┐      │
│ Maximize the discounted present value of              │      │
│  patronage refunds using the percent of patronage     │      │
│  refund paid in cash and deferment period length      │      │
│  as choice variables.                                 │      │
└──────────────────────────────────────────────────────┘      │
                        │                                      │
                        ▼                                      │
┌──────────────────────────────────────────────────────┐      │
│ Determine the actual per-unit patronage refund        │      │
│  level for each department.                           │      │
└──────────────────────────────────────────────────────┘      │
                        │                                      │
                        ▼                                      │
                  ╱──────────╲          ┌────────────────────┐ │
                 ╱  another    ╲   YES   │ Update expected    │ │
                ╱  iteration?   ╲───────►│  per-unit patronage│─┘
                 ╲             ╱         │  refund values and │
                  ╲──────────╱          │  membership numbers │
                        │               └────────────────────┘
                       NO
                        ▼
                     ┌─────┐
                     │ END │
                     └─────┘
```
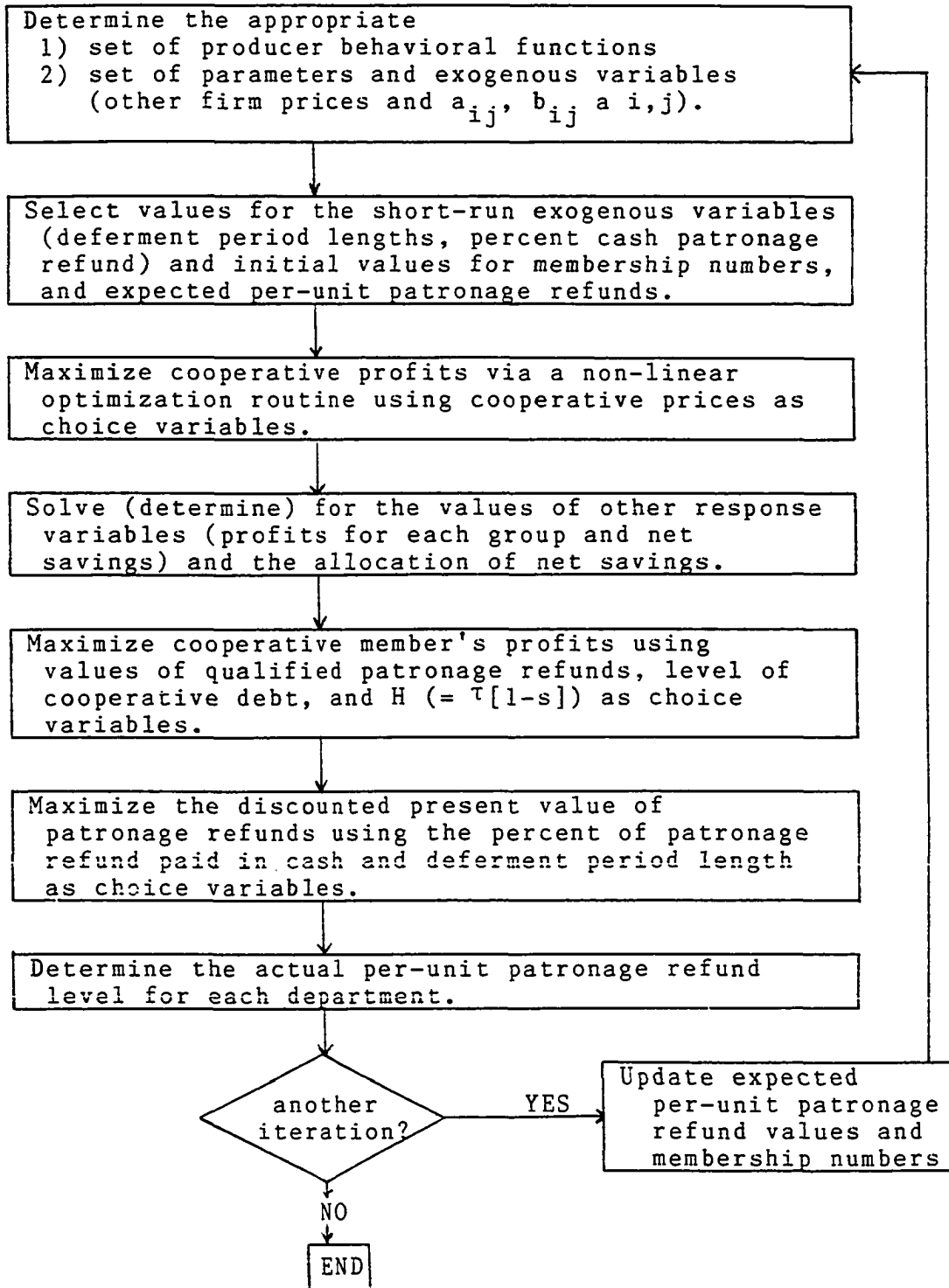
Figure 5.1. Flow chart of the cooperative decision process

Formulation of the Computer Model

The computer model follows the stages set forth for the mathematical formulation and solves for the optimal level of the decision variables. The computer modelling was done using a non-linear optimization program called GINO/PC. This software package uses a reduced gradient algorithm to solve the non-linear set of equations. This optimization package was chosen over other programs since the model could be solved on a microcomputer. This enables the model to be solved using a personal computer that many cooperative managers have access to. Equipped with the appropriate software and personal computer, the decision-maker can individualize the program to arrive at specific recommendations for a specific cooperative. A drawback with the GINO/PC program is that it can only handle 30 equations and 50 variables at one time. With just five product groups the full complement of equations is used in Stage I. Other stages also bump into this constraint of 30 equations. To deal with these limitations, each stage is solved in a single model with the results physically entered into the next stage. This is a burdensome process yet is worth the cost since it allows the flexibility of use by cooperative decision-makers at their location and convenience.

Application Results

In analyzing the results from the various computer runs, the parameters used for the different cooperative scenarios play an important role. The nature of the cooperative and some general operating guidelines are determined in steps a, b, and c of Stage I (as described above). Through simulation, the possible impacts of differential treatment can be estimated. First a base model with equal member treatment is constructed, then four sets of differential treatment policies are simulated. The first two sets involve changing prices and using varying membership elasticities. The third set involves changing the levels of s and $\dot{\tau}$ using four levels of membership elasticities. The last set allows prices, s, and τ to vary.

Base model

The first model constructed is one in which all members are treated equally. That is, high volume (HV) patrons receive the exact same treatment (with respect to prices, percent cash patronage refund, and deferment period) as low volume (LV) patrons. In this situation, the cooperative operations and membership are stable (relatively constant variable values) since the actual and expected per-unit patronage refunds, deferment periods, and percentage of patronage refund paid in cash are equal. The model

stabilizes after three iterations. Column 1 of Table 5.3 presents the average results of the base model with no differential treatment. The terms in the table are not superscripted or subscripted however the meanings are the same as discussed earlier.

With the base model, the decision-maker sets the choice variables at the same values for all patrons. Only one corn price and one fertilizer price exists for all patrons. The deferment period and the percent patronage refund paid in cash are also the same for all patrons. Of the 200 corn producers, 100 patronize the cooperative exclusively and 100 patronize other firms exclusively. Similarly, half of the fertilizer buyers patronize the cooperative and half do business with other firms. It is further noted that for both the cooperative and other firms, 75 out of 100 patrons for both commodities are classified as low volume patrons. Maximizing the total member profits with this set of decisions yields a profit of $35,340. To be meaningful this level must be compared with profit levels from other simulations. This application assumes that the total number of producers is 200. When membership numbers change this base value must be considered. A drop in membership of 10 may seem small however it represents 5% of the cooperative's patrons.

Differential treatment models

Patrons can be treated differentially through the prices paid/received and/or through τ or s which are used in determining the net discounted present value of patronage refunds. As discussed in Chapter II, the profitability to the cooperative of an individual patron's business can vary. For example, the cooperative may be able to offer a higher price to HV corn patrons since they market larger quantities than LV corn patrons. Allowing the cooperative to offer different prices for the same product may be justified if and only if a cost differential exists. By considering HV and LV patronage of corn and fertilizer as separate products, effectively the product line has doubled while the same physical product line remains the same. The product line now consists of HV corn, LV corn, HV fertilizer, LV fertilizer, and field services. Even though there is only one physical corn product, there are still two product lines that can each operate independently from each other. Each line can have different prices, deferment periods, and percentage cash patronage refunds. The decision-maker may have to differentially treat the HV and LV patrons to maintain the cooperative membership. By offering different prices, deferment periods, and/or percent cash patronage refunds, the cooperative membership can be affected.

<u>Different prices</u> The first set of differential treatment simulation involves offering different prices to high volume and low-volume corn patrons, PCCH = $2.50 and PCCL = $2.40. This cooperative scenario leads to a situation that changes membership numbers. Model #1 assumes membership responses to prices are inelastic, that is, $\xi_{CH,p}$ = 0.8 and $\xi_{CL,p}$ = 0.6. Model #2 uses unitary responses, $\xi_{CH,p}$ = $\xi_{CL,p}$ = 1.0. Model #3 assumes $\xi_{CH,p}$ = 2.2 and $\xi_{CL,p}$ = 2.0, both represent elastic responses. The second set of models involve changing prices by $0.10. Models #4, #5, and #6 use the same membership elasticities as Models #1, #2, and #3 respectively. The results of these changes in price to corn patrons are given in last six columns of Table 5.3.

Table 5.3. Results for the base model and differential treatment with prices

	Base Model	Model #1	Model #2	Model #3	Model #4	Model #5	Model #6
τCH	5	5	5	5	5	5	5
τCL	5	5	5	5	5	5	5
sCH	.3	.3	.3	.3	.3	.3	.3
sCL	.3	.3	.3	.3	.3	.3	.3
PCCH	2.45	2.50	2.50	2.50	2.55	2.55	2.55
PCCL	2.45	2.40	2.40	2.40	2.35	2.35	2.35
NCCH	25	25	26	26	26	26	27
NCCL	75	74	73	72	73	72	69
NOCH	25	25	24	24	24	24	23
NOCL	75	76	77	78	77	78	81
πCH	$17670	$17761	$18472	$18472	$18567	$18567	$19281
πCL	$17670	$17343	$17109	$16875	$17020	$16787	$16087
Π	$35340	$35104	$35581	$35347	$35587	$35354	$35368

These pricing schemes are used to encourage additional HV corn cooperative patronage. The levels of cooperative patronage for LV corn decline with respect to the base model. It should be noted that the magnitude of the price changes for corn between the different scenarios greatly influences the changes in membership. In the base situation only one price is paid for corn (PCCH = PCCL = $2.45) whereas the differential treatment model has PCCH = $2.50 ($2.55) and PCCL = $2.40 ($2.35) where PCCH and PCCL are respectively the prices for corn sold to the cooperative by HV and LV patrons respectively. These new price values are not the profit maximizing levels. These price changes are exogenously determined. The price changes were set to allow the revenue gained by one group be equal to that lost by the other. The price per bushel gained by the HV patrons equals the price per bushel lost by LV patrons. Figure 5.2. shows the distribution of profit between these two types of patrons.

In the first six models the profits of the HV patron increase whereas those of the LV patrons decrease. The total profit of the cooperative may increase or decrease. In these situations the cooperative decision-maker may have a difficult time justifying this type of differential treatment to the LV patrons. If the cooperative can receive a higher price for the corn that it sells to outside
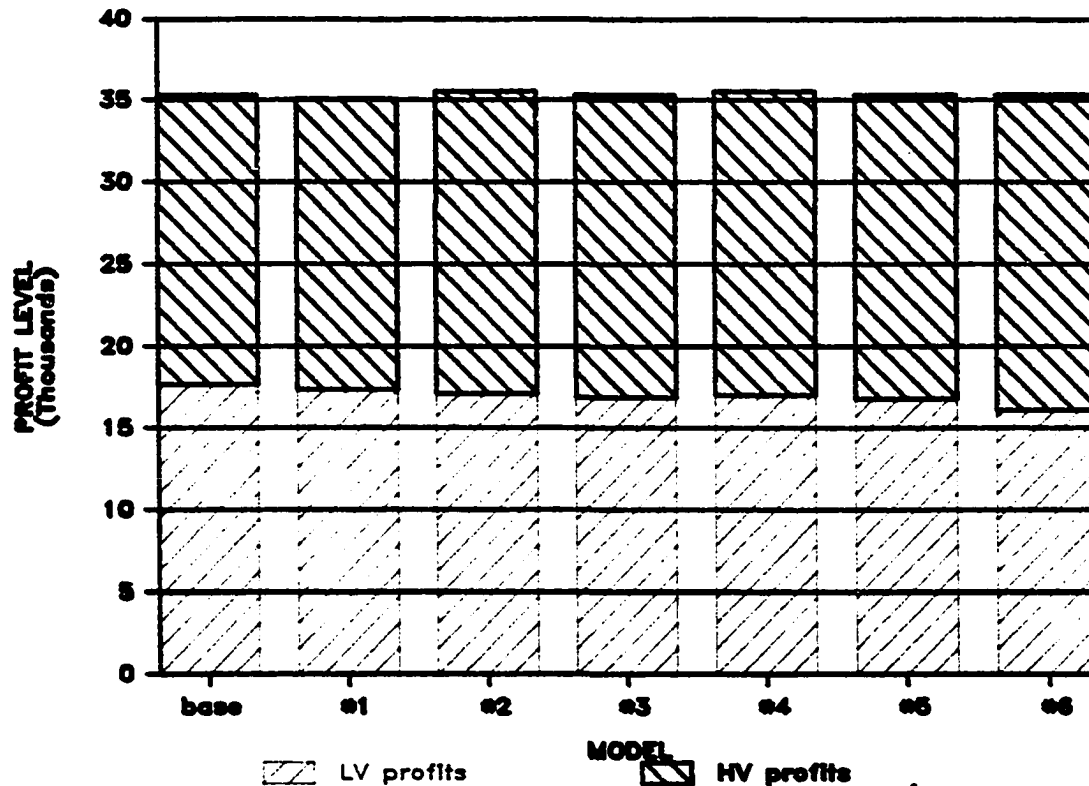
Figure 5.2. Profit levels for the base model and
differential treatment with prices

buyers, both HV and LV patrons could benefit. Model #7

looks at this scenario where economies of scale are realized

by the cooperative since it has a larger total business

volume. This model is the same as Model #6 except the

cooperative now has an additional margin of $0.05 per bushel

to distribute to all patrons. Model #7 illustrates that by

differentially treating members, the resulting increased

margins could make both LV and HV patrons better off

compared to a model when the margins remain constant

(Model #6).

Even though an increase in HV patronage was realized, by definition of the membership adjustment function in Stage III, the total cooperative membership could fall if some members are dissatisfied with their levels of patronage refunds paid in cash and their deferment period. This brings up a concern of how to define the membership adjustment function since it does not allow for any deviation in actual and expected levels of the adjustment variables without changing member numbers. In actuality there may exist tradeoffs between various levels of prices, percent patronage refunds paid in cash, deferment periods, and the per-unit patronage refunds; however, it is beyond the scope of this research to develop a membership function with these interaction terms. This model has distinct patron response functions with each variable operating independently; in reality they may be interrelated. A method that can be used by the cooperative to reduce the number of LV patrons who might discontinue doing business with the cooperative as determined from the previous scenario would be to educate members so they perceive the benefits to them of the pricing program. They should realize that the cooperative is trying to maintain the HV patrons which in turn may benefit the LV patrons. The procedure used in the model assumes the patron's expected level of these variables are the previous year's (previous

run's) value. This implies that the cooperative decision-maker is able to change the levels of τ_n, s_n, and r_{in} each year, however in reality these are relatively constant with only small changes each year. Although the levels of τ_n, s_n, and r_{in} are changed for the different groups of patrons, within each group the levels are invariant. By offering various levels of τ_n, s_n, and r_{in} to the different patrons, in effect the discount factor for the present value of patronage refunds is altered for each group. The discount factor $s_n - Tp_n - [1-s_n]/[1+d_n]^{\tau_n}$ is already different for each group if it is assumed that marginal income tax rates (Tp_n) and discount rates (d_n) vary for each group. By the nature of the supply and demand functions for the members and non-members, changing the levels of τ_n, s_n, and r_{in} affects both the quantities marketed and the membership adjustment function.

After differential treatment is incorporated into the model by offering different prices, the number of HV patrons increase while the number of LV patrons decrease slightly. The exit and entrance of the HV and LV patrons and/or potential patrons is determined by membership adjustment functions specified in Table 5.2. To entice more HV patronage and to slow the exit of LV patrons, other models allowing for differential treatment can be devised.

Different τ and s As noted in VanSickle and Ladd [66], the change in levels of s_n and τ_n can be adjusted to cancel the effect of each other. This creates an interesting situation for the cooperative decision-maker when the actual values of s_n and τ_n are determined. Changing the levels of τ_n and/or s_n will affect the model results through both the individual producer and the cooperative. The individual producer is affected directly by the change in the discounted value of patronage refunds. The cooperative will be affected directly through the change in the status of the revolving fund and indirectly via changes in membership and quantities marketed through the cooperative. In determining the values of τ_n and s_n, the cooperative must generate a certain amount of financing through the revolving fund but may not have a preference for specific levels of these variables. The individual producer may have likes and dislikes for high or low values of τ_n and s_n. These preferences are given to the cooperative decision-maker as elasticities through the coefficients of the log-linear supply and demand functions of members and non-members. Since each cooperative will have memberships with varying loyalties and preferences, the coefficients for the supply and demand functions will be different for each. For illustrative purposes, four scenarios were hypothesized, $\xi_{CH,s} = 0.8$ and $\xi_{CH,\tau} = 0.6$, $\xi_{CH,s} = \xi_{CH,\tau} = 1.0$, $\xi_{CH,s} = 2.0$

and $\xi_{CH,\tau} = 1.0$, and $\xi_{CH,s} = 3.0$ and $\xi_{CH,\tau} = 1.0$. Models #8

to #11 respectively use these elasticities. Model #12

assumes economies of scale, that is, the cooperative has

$0.05 per bushel more to return to the members. This model

assumes that it is paid immediately in the form of higher

prices, PCCH = PCCL = $2.50. These models assume a 20%

increase in sCH and a 40% increase in τCH. The results of

these models are given in Table 5.4.

Table 5.4. Results for the base model and differential
treatment with τ and s

	Base Model	Model #7	Model #8	Model #9	Model #10	Model #11	Model #12
τCH	5	5	6	6	6	6	6
τCL	5	5	5	5	5	5	5
sCH	.3	.3	.42	.42	.42	.42	.42
sCL	.3	.3	.3	.3	.3	.3	.3
PCCH	2.45	2.55	2.45	2.45	2.45	2.45	2.50
PCCL	2.45	2.35	2.45	2.45	2.45	2.45	2.50
NCCH	25	26	23	20	25	30	30
NCCL	75	72	75	75	75	75	75
NOCH	25	24	27	30	26	20	20
NOCL	75	78	75	75	75	75	75
πCH	$17670	$18831	$16286	$14146	$17683	$21220	$21634
πCL	$17670	$17030	$17670	$17670	$17670	$17670	$18014
Π	$35340	$35861	$33938	$31816	$35353	$38890	$39648

These results show that cooperative member profits can

be influenced by changing the levels of τ_n and s_n without

adversely affecting the financial situation of the

cooperative. Only in the case where $\xi_{CH,s}$ is more than

twice $\xi_{CH,\tau}$ are HV patrons profits increased. At this point

the gains realized by offering a 20% increase in sCH

outweigh the loss caused by increasing $^\tau CH$ 40%. Both τ and

s must be changed to maintain the level of H. The

cooperative decision-maker must consider the relative

elasticities when contemplating a differential treatment

program using s and $^\tau$. Model #12 shows that both the profit

levels of the HV and LV patrons could be increased when

economies of scale are realized. This again illustrates how

LV patrons can benefit by allowing the cooperative

decision-maker to use differential treatment. Figure 5.3.

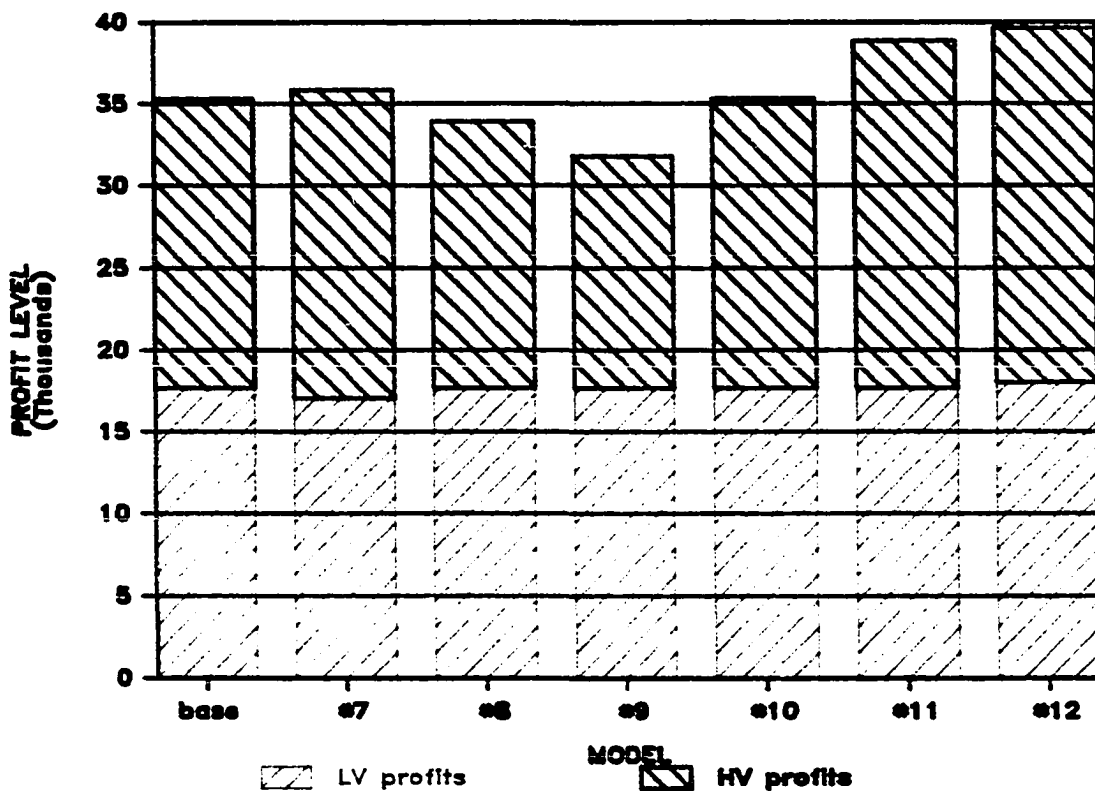shows the profit levels of each group for Models #7 to #12.

Figure 5.3. Profit levels for the base model and
differential treatment with τ and s

In Models #8 to #12 the patrons are less responsive to changes in the deferment period as compared to the percent patronage refund paid in cash. It is advantageous for the cooperative to increase the deferment period and increase the level of patronage refunds paid in cash. The gains realized by the increase in s_n outweigh the loss from increasing τ_n. The levels of cooperative membership and patronage increase which lead to higher cooperative member profits. As the relative difference in elasticities (responsiveness) become greater, the potential for larger gains increases. When the relative elasticities are very close or equal, little if any potential gains exist through altering the levels of τ_n and s_n. Allowing the five different products in the model to have varying relative elasticities would result in each having their own set of financial policy values. When the cooperative utilizes more product lines each having independent levels of τ_n and s_n, the potential for a higher level of member profit increases. As the elasticities with respect to deferment period and percent patronage refund paid in cash become more uneven the opportunity for increased member profits also increase. The profit maximizing levels of τ_n and s_n are dependent on the relative elasticities and are given by a general rule involving the changes in patronage (quantity per patron) and

the membership (number of patrons) function. The change in
profits caused by changes in membership and quantities
marketed/supplied as a result of changes in τ_n and s_n should
be equated.

Different τ, s, and prices The last set of models
actually combines the previous trials. Prices, s, and τ are
all simultaneously altered. Models #13 to #15 are the same
as Models #4 to #6 with the changes of Model #11 added. The
results of these models are presented in Table 5.5. and
Figure 5.4.

Table 5.5. Results for the base model and differential
treatment with prices, τ, and s

	Base Model	Model #13	Model #14	Model #15
τCH	5	6	6	6
τCL	5	5	5	5
sCH	.3	.42	.42	.42
sCL	.3	.3	.3	.3
PCCH	2.45	2.55	2.55	2.55
PCCL	2.45	2.35	2.35	2.35
NCCH	25	31	31	32
NCCL	75	73	72	69
NOCH	25	19	19	18
NOCL	75	77	78	81
πCH	$17670	$22143	$22143	$22857
πCL	$17670	$17020	$16787	$16087
Π	$35340	$39163	$38930	$38944
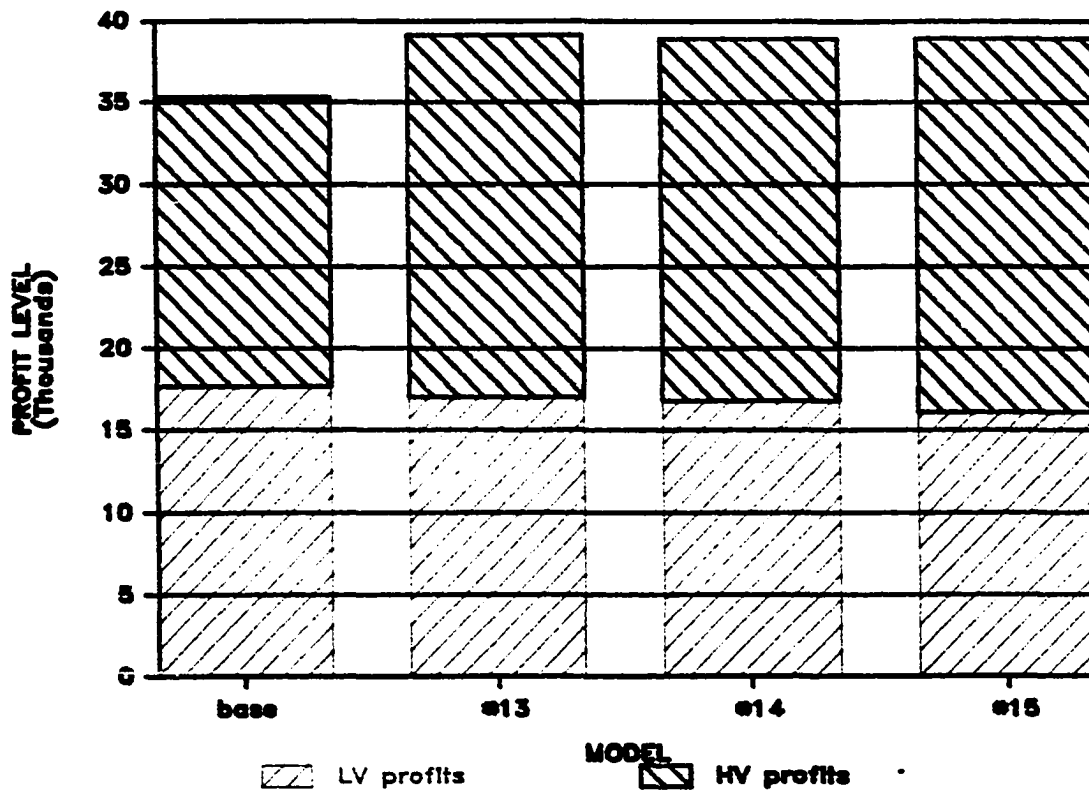
Figure 5.4.   Profit levels for differential treatment
with prices, τ and s

In Models #13 to #15 the level of profits are lower

than those in Models #4 to #6 where the levels of τ and s

are equal for all patrons.   Again, as the membership

elasticities become more elastic, LV patron's profits

decline.   The profits for HV patrons on the other hand

increase and are larger than without the changes in sCH and

τCH.   These models illustrate that the cooperative

decision-maker has many factors to consider when

differentially treating members.   The membership

elasticities play a crucial role in determining the level of profits for the different patron groups.

Other potential models

All of the models assume that maximization of members' profit is the primary objective although several other goals could be considered. One such goal would be obtaining and maintaining a given level of membership and/or patronage (member and non-member) in the cooperative. The values of τ_n and s_n could be set to achieve this goal in all the product lines or just in certain producer groups. The desire to change profit and member levels for specific product lines can be accomplished by both direct and indirect means. For example, to increase the number of HV fertilizer patrons, favorable levels of τ_n and s_n could be afforded this group of producers for the direct product, fertilizer, and indirectly through field services which can influence the quantity of fertilizer demanded. The field service product, as set up in the model, could act as a loss-leader. That is, it could be provided with the primary purpose of attracting potential members into the fertilizer product lines. By offering the field service at a low cost, potential patrons may be enticed to patronize the cooperative. The success of increasing patronage in the fertilizer product lines by indirectly manipulating τ_n and s_n of the loss-leader product is determined by the

complementarity of products and the allowable level of loss
for this product line. Although the use of the loss-leader
product to influence patronage of other products may not be
as effective as using direct methods, it may turn out to be
the most feasible instrument and should be considered.

The predominant conclusion resulting from the various
cooperative scenarios run through the model is that the
levels of τ_n and s_n for each product line can be used to
favorably affect the operations of the cooperative. The
magnitude of the effects is determined by the specification
of the supply and demand function parameters. By allowing
more product lines to offer independent levels of τ_n, s_n and
prices, the potential for gain increases. These results
suggest that the cooperative decision-maker should determine
the various types of patrons groups in the cooperative and
utilize any patron specific characteristics to attempt the
intricate task of maximizing the profits of these members.

CHAPTER VI. SUMMARY AND CONCLUSIONS

Summary

The success or failure of any business organization
depends on its ability to adapt to the changing environment
in which it operates. One predicament that cooperatives are
increasingly becoming aware of is the difficulty of serving
a vastly diverse membership. Cooperative members are
becoming increasingly heterogeneous. This creates a variety
of problems for the cooperative decision-maker including,
most notably, disgruntled patrons who perceive that they are
being treated "inequitably" by the cooperative business.
The heterogeneity of cooperative members can be seen by
looking at the many different sizes, financial situations,
and ages of the patrons. It is important to realize that
not all of these patron differences can or should be used to
differentially treat members. If the heterogeneity of
members creates a difference in the cost of servicing the
patrons, differential treatment could be used. A
heterogeneous membership could cause operational problems
for the cooperative such as retaining patronage of members
with high "quality of value of business". This research
suggests that a system of differential treatment of

members may provide the decision-maker with a tool to minimize these problems.

Although the cooperative membership can be categorized into many different groups, not all classifications can and/or should be used as a basis for differential treatment. The key to which classification systems can legally be used for differentially treating members depends on the existence of a definite cost difference between the groups. The number of different groups within the cooperative can range from one (no differentiation) to M, the total number of patrons. The general model presented in Chapter III allows for both extremes whereas the model application in Chapter V assumes that there are only two groups patronizing each of two departments. With fewer groups to be differentially treated, the system will be less complicated and more feasible to implement in actual cooperatives. After deciding how patrons are to be categorized, the method of applying differential treatment must be determined. Several alternative systems are possible with the most obvious being through prices and patronage refund parameters (τ_n, s_n, and r_{in}). However other instruments such as stock policy parameters can be used.

Although many different combinations of patron categories and differential treatment methods are possible, not all are feasible options. The feasibility of any

program of member differential treatment depends on its adherence to the Rochdale Principles, legality, and acceptability by members. Through an examination of the original intent of each of the Rochdale Principles it was concluded that a policy of differential treatment could be devised that would be consistent with cooperative philosophy. Some principles play more crucial roles than others in permitting a differential treatment policy and in determining its ultimate success. The keystone to the permissibility of cooperative differential treatment is the principle of "operation at cost". If each member is expected to pay only for the actual costs incurred by the cooperative in providing the service, then this infers that patrons can be treated distinctly. The member education principle does not outwardly advocate or dispel the notion of this type of policy. However, it could play a very significant role. In commencing a program that treats patrons differently, it is crucial that each patron knows the reasons for differential treatment. A sound education program is therefore a decisive element in any policy of differential treatment.

With no outright violation of the cooperative principles, two other areas of concern are the legality and acceptability by members of the policy. The legality of the issue revolves around the legal requirement that for a given

product, all members should be treated exactly the same.
This may come in conflict with the cooperative operating at
cost. This cost basis justification must be made if the
cooperative uses differential treatment. This cost
differential in turn enables the decision-maker to establish
different prices and different financial policies for the
different patrons. The key to achieving an operable policy
is having members accept the program, which in turn relies
heavily on the educational system of the cooperative. It is
apparent that the feasibility of differentially treating
members is closely related to the operating principles of
cooperatives, legality issues, and the acceptance of the
policy by the member patrons themselves.

In building the general model, previous works were
utilized but additional features were added to allow
individual members to be treated differently. The work by
Royer [52] and VanSickle [64] established models that looked
at the production and financial aspects of cooperative
operations. Both of these however dealt with only a
"typical" member patron and did not allow differential
treatment. As an extension to these works, the general
model presented in Chapter III allows differential treatment
of patrons. The Kuhn-Tucker conditions yielded by the model
are similar to those for previous models but provide more
detailed information for the cooperative decision-maker.

The model provides operating guidelines for the decision-maker with respect to pricing, patronage refund policies, and stock policies. The development of the model involved constructing an individual member objective function, a cooperative production-pricing objective function, and a cooperative financial objective function. The simulated application of the model in Chapter V is based on the general model but only considers a limited product line and only two patron groups. Similar to Royer's and VanSickle's research, the Kuhn-Tucker conditions for the general model are very complex and require that extensive amounts of information about each patron be available to the decision-maker. The computer model utilized three product lines with each being purchased or supplied by two distinct patron groups. A four stage procedure was used in the computer simulation model. The simulation model would yield varying results depending on the member and non-member supply and demand functions as exogenously specified. The steps employed in the application procedure involved (a) specifying the member and non-member supply and demand functions (distinct for each cooperative), (b) solving a pricing-production model to determine the price and quantity for each product for each patron group, (c) solving a financial model through a two-step procedure to ascertain the optimal values of τ_n and s_n, and (d) updating the

exogenous parameters of the pricing-production model with the new values for membership, patronage, τ_n, and s_n.

Conclusions

The simulation model was set up on a microcomputer with the intent that it could be used by cooperative decision-makers to make operating decisions. It is important to realize that the results of the various application runs are dependent on the specified supply and demand functions. Specific cooperative decision-makers can benefit from this model if they can supply the appropriate parameters. Even though the results vary depending on the exogenously specified parameters, several general conclusions can be derived. The first result involves the relative supply (demand) elasticities of the deferment period and percent patronage refund paid in cash. Depending on these relative elasticities, the cooperative decision-maker can increase aggregate member patrons' profits by adjusting the levels of τ_n and s_n. The second conclusion relates to the last stage of the computer model, the variable updating process. The decision-maker can use τ_n and s_n to encourage membership in certain groups which may in turn benefit both the patron and the cooperative as a whole. The most interesting result revolves around the idea that under certain conditions, the cooperative

decision-maker can increase the profits of both groups of patrons by using differential treatment. The membership function in the updating stage is also cooperative specific and can heavily influence the results. Even though the membership adjustment function in this work was specified in a general way, it still provides insight into the effects of differential treatment policies on membership. These conclusions suggest that the cooperative manager has many factors to consider when making their decisions. They must be aware of the cost of providing a product and the responsiveness of patrons to cooperative decisions or policies.

Suggestions for Future Research

In running the various cooperative scenarios in the simulation model, several areas of further research are evident. One obvious area is to look at the effects of alternative specifications of the member adjustment function. That is, how do various combinations or interactions of τ_n, s_n and price affect membership? The model used in this study assumes that membership decisions are based on a comparison of the previous year's values of these factors to the current values assuming no interaction effects. A second suggestion for future work is to apply the general simulation model to an actual cooperative. This

research applies the theoretical model by simulating various scenarios for a hypothetical cooperative. It would be interesting to work with a specific cooperative decision-maker and try to project (or determine) the results from converting (or already having converted) the firm to one that differentially treats patrons. Another area of further research involves obtaining empirical evidence to support the theoretical model. This would involve involve identifying the responsiveness of patrons to changes in the cooperative operating decisions and the determination of the actual cooperative cost functions.

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APPENDIX A. LIST OF THEORETICAL MODEL SYMBOLS

A   = The set of member patrons

B   = The set of all patrons

C   = The set of outputs sold to and variable inputs purchased from the cooperative by the member and non-member patrons

C_L = The amount of indirect costs allocated to the L-th department

CS   = The total value of stock employed by the cooperative

D   = The total amount of debt employed by the cooperative

D_L = The subset X_L if L is a marketing department and Y_L if L is a supply department

d_n = The discount rate of the n-th member

DS   = The total dividends on member stock (the sum over all "n" in ds_n)

ds_n = The dividends on stock held by the n-th member patron

E   = The set B if the cooperative meets 521 status requirements, otherwise set E is defined as set A

FCM   = The total fixed costs of the member patrons (the sum over all "n" in fc_n)

fc_n = The fixed costs of the n-th member or non-member

H_n = $\tau_n(1-s_n)$

IC_n = The dividend rates paid on stock

IM   = The maximum allowable dividend rate that can be paid on stock

K   = The amount of equity capital employed by the cooperative

L = The Lagrangian function corresponding to the cooperative's production-pricing model

M = The number of member patrons in the cooperative association

MNS_K = The marginal net savings of the cooperative caused by a change in the level of K

N = The amount of net savings used in the revolving fund of the cooperative

N_n = The number of patrons in the n-th group of producers

NS = The total net savings of the cooperative

NS_{L1} = The net savings of the L-th marketing department

NS_{L2} = The net savings of the L-th supply department

O(M,K) = The total net operating income of the cooperative

P_{Xn} = The vector of prices of outputs in set X produced by all patrons (for the n-th member)

P_{Yn} = The vector of prices of variable inputs in set Y used by all patrons (for the n-th member)

P_i = The price of the i-th good for sets V and Z

P_{in} = The price of the i-th product for the n-th patron

PKQP = The level of capital supplied by retained patronage refunds from previous years

PR = The amount of qualified patronage refunds allocated by the cooperative

PR_n = The amount of patronage refunds allocated to the n-th cooperative patron

PS_n = The price of stock for the n-th member group (the subscript 'n' is eliminated if the price is equal for all member groups)

PVPR = The present value of the allocated patronage refunds

$pvpr_n$ = The present value of the expected patronage refunds allocated to the n-th member patron

Q_C = A vector of quantities of each of the products in set C

Q_V = A vector of quantities of each variable input in set V used by the cooperative and purchased from outside the cooperative association

Q_{Wf} = A vector of quantities of the fixed inputs in Set Wf used by the cooperative

Q_X = A vector of quantities of the outputs in set X produced by the member and non-member patrons and used by the cooperative

Q_Y = A vector of quantities of the variable inputs in set Y purchased by the member and non-member patrons

Q_Z = A vector of quantities of the outputs in set Z produced by the cooperative and sold to buyers outside the cooperative association

q_i = The quantity of the i-th good for sets V and Z

q_{in} = The average quantity of the i-th product for a member in the n-th group

q_{ij} = The quantity of the i-th product used in the production of the j-th product

q_{ijn} = the quantity of the i-th product used in the production of the j-th product bought by the n-th member group

q_{in} = The stock of the i-th fixed factor available to the n-th patron group

q_{io} = The quantity of the i-th product purchased or sold by the non-member patrons

q_{Wfn} = A vector of the quantities of each of the fixed inputs in set Wf used by the n-th member patron group

q_{Xn} = A vector of the quantities of each of the outputs in set X produced by the n-th member patron group

q_{Yn} = A vector of the quantities of each of the variables in set Y used by the n-th member patron group

r $\quad$ = The average interest rate of all cooperative debt sources

r_{in} = The per-unit patronage refund on the i-th product for the n-th patron group

r_{in}^{*} = The n-th member patron group's expected per-unit patronage refund on the i-th product

SH_n = The n-th member group's number of shares of stock held

SPs_n = The shadow price associated with the percent cash patronage refund of the n-th member

$SPic_n$ = The shadow price associated with the actual dividend rate for stock for the n-th member group

s_n = The proportion of allocated patronage refunds paid in cash for the n-th member group

$s_n^{'} = s_n - Tp_n + (1-s_n)/(1+d_n)^{T_n}$

T(M,K) = The total net revenue generated by the cooperative from member business in sets X and Y.

TCC $\quad$ = $\quad$ Total collective costs

TCP $\quad$ = $\quad$ Total collective profits

TCR $\quad$ = $\quad$ Total collective revenue

TKQP = The total value of the capital supplied to the cooperative by qualified patronage refunds

TMP $\quad$ = $\quad$ Total member profits

TPC $\quad$ = $\quad$ Total private costs

TPP $\quad$ = $\quad$ Total private profits

TPR $\quad$ = $\quad$ Total private revenue

Tp_n $\quad$ = $\quad$ The marginal tax rate for the n-th patron

t $\quad$ = $\quad$ The cooperative's marginal tax rate

UR $\quad$ = $\quad$ The unallocated reserves of the cooperative

V = The set of variable inputs used by the cooperative and purchased from outside the cooperative association

W_f = The set of fixed inputs available to the cooperative

X = The set of outputs produced by the member and non-member patrons

X_c = The subset of goods in set X that represents business through the cooperative

X_L = The subset of products in set X handled by the L-th department

X_o = The subset of goods in set X that represents business through non-cooperative firms

Y = The set of variable inputs purchased by the member and non-member patrons

Y_c = The subset of products in set Y that represents business through the cooperative

Y_L = The subset of products in set Y handled by the L-th department

Y_o = The subset of products in set Y that represents business through non-cooperative firms

Z = The set of outputs produced by the cooperative and sold to buyers outside the cooperative association

Z_L = The set of outputs produced by the L-th department of the cooperative and sold to buyers outside the cooperative association

α_n = The proportion of the cooperative's operating income that is allocated to the n-th patron group

∇ = The Lagrangian function corresponding to the cooperative's financial model

δ_1 = The Lagrangian multiplier corresponding to the financial status of cooperative

δ_{2n} and δ_{3n} = The Lagrangian multipliers corresponding to the specified minimum and maximum values of the amount of patronage refund paid in cash

δ_{4n} = The Lagrangian multipliers corresponding to the maximum dividend rates on member held stock

Λ_n = The Lagrangian function corresponding to the problem of a member patron

λ_1 = The Lagrangian multipler corresponding to the production function

λ_{2i} = The Lagrangian multiplier corresponding to the production-pricing model for full usage of capital

λ_{3i} = The Lagrangian multiplier corresponding to the production-pricing model for full usage of fixed factors.

λ_4 = The Lagrangian multiplier corresponding to the distribution of net savings within the cooperartive

Π = The total profits of the member patrons

π_n = The profit of the n-th member patron

τ_n = The length of the cooperative's revolving fund for the n-th member group

Φ = The implicit form of the production function of the cooperative

ϕ_n = The implicit form of the production function of the n-th member or non-member patron group

ψ_{1n} = The Lagrangian multiplier corresponding to the individual member's production function

ψ_{2in} = The Lagrangian multiplier corresponding to the individual member's i-th fixed factor constraint

APPENDIX B.   DEPARTMENTAL NET SAVINGS

$$NS_{L1} = \sum_{j \epsilon Z_L} P_j q_j - \sum_{i \epsilon X_L} \sum_{n \epsilon B} N_n P_{in} (q_{in} - \sum_{j \epsilon Y_c} \sum_{n \epsilon B} q_{ijn}) - C_L$$

$$- \sum_{i \epsilon Y} \sum_{j \epsilon Z_L} \sum_{n \epsilon B} N_n P_{in} q_{ijn} - \sum_{i \epsilon V} \sum_{j \epsilon Z_L} P_i q_{ij} - \sum_{i \epsilon Wf} \sum_{j \epsilon Z_L} P_i q_{ij}$$

where NS_{L1} = the net savings of the L-th marketing department

Z_L = the subset of products in set Z, produced in the L-th department

X_L = the subset of products in set X handled by the L-th department

N_n = the number of patrons in the n-th group

P_i = the price of the i-th product (for goods in sets V or Z)

P_{in} = the price of the i-th product for the n-th patron

q_i = the quantity of the i-th product (for goods in sets V or Z)

q_{in} = the average quantity of the i-th product for a member in the n-th group

q_{ijn} = the quantity of the i-th product used in the production of the j-th product bought by the n-th member group

C_L = the amount of indirect cost allocated to the L-th department

P_i, $i \epsilon Wf$ = the price charged each department for the use of the i-th fixed factor

B = the set of all patrons, (members and non-members)

The net savings of the L-th marketing department of the cooperative can be explained as: The sales of products sold to markets outside the cooperative by department L; less a charge for all products marketed through the L-th department of the cooperative which were not used in production of variable inputs purchased from all patrons; less a charges for variable inputs purchased by patrons which were used in the production of goods sold to markets outside the cooperative by the L-th department; less a charge for variable inputs used by the cooperative and purchased from outside the cooperative by the L-th department for the production of goods sold to markets outside the cooperative; less a charge for all fixed factors used by the L-th department for production of goods sold to markets outside the cooperative; and less an amount of indirect cost allocated to the L-th marketing department.

$$NS_{L2} = \sum_{j \epsilon Z_L} p_j q_j - \sum_{i \epsilon X} \sum_{j \epsilon Y_L} \sum_{n \epsilon B} N_n p_{in} q_{ijn} - \sum_{\substack{i \epsilon Y \\ i \notin Y_L}} \sum_{j \epsilon Y_L} \sum_{n \epsilon B} p_{in} q_{in}$$

$$- \sum_{i \epsilon V} \sum_{j \epsilon Y_L} \sum_{n \epsilon B} N_n p_{in} q_{ijn} - \sum_{i \epsilon Wf} \sum_{j \epsilon Y_L} \sum_{n \epsilon B} N_n p_{in} q_{ijn} - C_L$$

where NS_{L2} = the net savings of the L-th supply department

Y_L = the subset of products in set Y, produced in the L-th department

The net savings of the L-th supply department of the cooperative can be described as: The value of variable inputs purchased by patrons that are sold by the L-th department; less the value of the set of outputs produced by patrons which are used in the production of the set of variable inputs purchased by patrons, sold by the L-th department; less the value of the set of variable inputs purchased by patrons that are not produced in the L-th department which are used in the production of the variable inputs purchased by patrons, sold by the L-th department; less the value of the set of variable inputs used by the cooperative which are purchased from outside the cooperative and are used in the production of variable inputs purchased by patrons sold by the L-th department; less the value of the set of fixed inputs available to the cooperative used in the production of the variable inputs purchased by patrons sold by the L-th department; and less an amount of indirect cost allocated to the L-th supply department.