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A confirmatory analysis of an exploratory factor analytic study of the fluid milk bottling firms in North Central Region

by

Emman O. Oyinlola

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I. INTRODUCTION

The fluid milk industry is continually undergoing changes in regard to the number, size, location and ownership of firms. The present pattern of fluid milk processing is the outgrowth of earlier tendency toward urban concentration of the consumers, technological development and more importantly, the recent growth of supermarket chains. The growth of modern cities and the resultant concentration of the population enhanced the decline and eventual demise of "backyard dairy barns". Various forms of technological changes facilitated the shift from local hauls of milk to hauling from a more distant supply area. Prior to the development of bottles, milk was handled by suppliers from horse-drawn carts pulled from door to door. The marketing of milk under these conditions require few specialized inputs. For this reason there were many firms involved in the functions of milk processing and distribution, but the numbers of specialized processing firms were relatively few.

By the turn of the century fluid milk marketing became more complicated and milk processing plants were started. This was followed by the introduction of pasteurization and distribution in glass bottles. Despite these developments, plant sizes were limited by the absence of mechanized equipment to handle milk and by a relatively small procurement area. Prior to the development of adequate highways, the most common method of farm-to-market transportation of fluid milk was by railroads; thus limiting commercial fluid milk production to the immediate market areas or areas adjacent to rail lines. The improvement of highways, the organization of an efficient trucking business and the development of refrigerated trucks have extended the procurement and distribution areas to the point where milk distribution has become a regional and national concern. The introduction of square glass bottles and paper containers have cut down weight and space requirement for hauling which in turn have reduced processing and distribution costs.

During the past two decades or so, the firms participating in fluid milk marketing, particularly the fluid milk processing industry and the retail food store industry, have demonstrated considerable structural changes. A trend toward fewer and larger firms is apparent in both industries. There are some evidences that the nature of competition is being affected by these structural changes. The number of fluid milk bottlers has been declining due, in part, to the relatively high volume of sales required to operate a modern plant efficiently. From 1948 to 1965, the number of bottling plants which were operated by the commercial processors in the United States declined by 53% - from 8,484 in 1948 to 3,981 in 1965. During the same period sales increased by 63%. These changes were not peculiar to this period. The number of plants declined from about 22,210 in 1920 to 8,484 in 1948 (Manchester, 1968). The main reason for these changes was that small firms left the business during these periods. In fact from 1948 to 1965, 85% of the small producer-dealer left the milk bottling industry. Major companies grew mostly through mergers and acquisitions, which were the main avenues of adjustment within the industry especially for those firms desiring to leave the industry. Where mergers and acquisitions had been stopped by the

Federal Trade Commission, custom packing has been used to circumvent the proscription on merger in their adjustment practices (Strain, 1963).

Other changes in the fluid milk industry are the declining number of large retailers, the expanding areas of economical distribution, and the importance of brand advertising. The development of private labels increased as retailers sought greater control over the product. Milk retailers may purchase their private label products from a fluid milk plant or may operate their own processing facilities. Most bottlers will not deny the fact that they supplied private label products in order to forestall entry of the supermarket chains into the processing industry, and to obtain and keep display space for their own brand on the retailers' shelves. Pressures for volume are felt by all processing firms but, as usual, the small operators are faced with several competitive handicaps in relation to the larger operators. Most of the problems faced by small operators are high costs, product acceptance, difficulty and high cost of procurement of raw materials and difficulties in innovation.

The growth of supermarket chains has affected the structure, conduct and performance of milk bottlers. Changes in the market structure of the food retailing sector have markedly influenced the buyer-seller relationships between food retailers and fluid milk processors.[1] An increasing number of food chains have started contracting for package milk supplies through central or district office purchasing program. This development meant that brand names of products sold under private

[1] For a detailed analysis of the effect of the growth of supermarket chains on the fluid milk processing industry see Gruebele, Williams and Fallert (1970).

label lose their meaning once the milk has reached the hands of the contracting agents. The supermarket chains took over the responsibility for guaranteeing quality of the product; and most milk procured from different processors was marketed under the merchandizing private label brands. This development gave the retailers a greater freedom to shift sources of supplies and thus placed the supermarket chains in such a position that contract negotiations were more in their favor. The threat that the chains might start their own processing plant further weaken the bargaining position of the processors. All these developments have influenced the behavior of fluid milk processors. Obvious changes have been noticed in the processing and distribution of milk. The competitive conditions which were essentially traditional to the fluid milk processing industry by mere facts of number and homogeneity of the product, have become changed or modified due to the presence of some problems faced by the milk bottling firms.

The technological developments experienced in fluid milk processing tended to smother out small operators, encouraged consolidation of existing plants and influenced the tendency toward mergers of small and moderately sized operators. The increasing size, the changing business organization and the conduct of food chains have affected the distribution of fluid milk products to wholesale merchants. As mentioned above, a number of large chains have started contracting for package milk supplies through a central or district office purchasing program. Thus, some processors delivered a high percentage of their total volume to a relatively few large national or regional supermarket chains with high or potentially high market power. The fact that business has to be executed in large volumes created additional problems for some processors.

The contract arrangements for milk delivery might have positive or negative effect on the processors. The gain or loss of a contract to supply the supermarket chains in a region could have considerable effect on the sales and financial well-being of a processor. When dealing with large national or regional chains, the processors were faced with the problem of losing any identity their products might have. Usually the supermarket chains, voluntary and cooperative grocery wholesalers obtaining milk under district office purchasing program were procuring and merchandizing private label brands of milk. Thus the effectiveness of product differentiation diminished and the power of food chains to change supply sources is greatly increased. Accordingly, the market structure of the retailing industry affects the well-being and survival of bottlers and thus the market structure of the milk bottling industry. These changes in the relationship between bottlers and retailers call for adjustments in the operations of the milk bottlers. Those who could not make these adjustments, either due to size limitation or financial bottlenecks were forced out of business, thus reducing the number in the industry.

Some managers have made the adjustments while others are planning to make the necessary changes required for coping effectively with their marketing problems. Those operators who are planning on making adjustments must consider many issues and conditions before making the final decisions. The contract arrangements for milk supply necessitate big volume business, thus there is the need for managers to change the size of their operation. The location of the contracting agents may also necessitate a relocation of plants. These decisions on size and location of operation may be influenced by other decisions regarding the line of products to carry, type and size of packages, flexibility to provide for in-plant operations and the marketing functions that are to be performed.

This research work has been organized to probe into the many problems faced by the fluid milk processors, the adjustments required in order to cope with these problems and finally, it is hoped that it would be possible to isolate the implications of these problems and adjustments and use these in developing a sound theoretical model of the fluid milk processors' market structure.

An exploratory analysis of the various marketing and adjustment problems in the fluid milk processing industry has been carried out by Oehrtman (1970) in his hierarchical factor analysis of the adjustment problems faced by the fluid milk bottling operators. This analysis was reported in a dissertation (Oehrtman, 1970) and will be published as a North Central Research Publication in the Iowa State Agricultural Experiment Station bulletin series (Ladd and Oehrtman, 1971). The present research is a follow up study of this exploratory analysis. The hierarchical factor analysis was aimed at determining some of the sociological and psychological values and economic variables which the operators in the fluid milk industry (from their own knowledge and experience) believed to be relevant to their marketing problems. The main objective of the current study is to test some hypotheses derived from the exploratory factor analysis. In order to meet this objective, statistical inference procedures must be developed. It is hoped that any accepted hypotheses will help us in developing a theoretical model of the fluid milk bottling industrial structure.

II. MARKET STRUCTURE ANALYSIS

Agricultural marketing can be defined as the performance of all business activities involved in the movement of farm goods from the farm gates to the hands of the ultimate consumer in the form, place and time he wants them. The performance of these business activities is affected by the structure of the market in which these activities are carried out. Market structure has some definite effect on the conduct of the firms and their performance; and sometimes performance has a feedback on structure.

Market structure analysis is a research method which is used for a comprehensive analysis of agricultural marketing systems. The basic unit for the analysis is the industry. An industry is a group of firms producing products which are reasonable, if not identical, substitutes as far as the buyers are concerned. In general, the higher the cross elasticity between the products of the industry, the more narrowly we have defined the industry. Thus when we speak of the dairy industry at the processing level, we have defined a much wider industry category than is the case when we speak of the fluid milk industry, or the cheese industry, etc.

Economists have placed heavy reliance on a prior relationship between structure and business conduct and performance as the main tool for providing a meaningful interpretation of the activities of the private industries of an economy. Number of firms has been used as a key variable in determining the nature of an industrial organization. When firms are many and no one firm controls a significant share of the appropriately defined market, economists will reasonably predict competitive pricing; when there are few firms (a case of concentrated oligopoly), the prediction is a less competitive pricing. Thus we usually presume that low concentration ratio, other things remaining the same, is a desirable structural goal on the grounds that competitive market organization is more likely to assure the attainment of certain performance such as price relationships compatible with efficient allocation of resources (Markham, 1965).

The concept of market power has been used to evaluate the structure, conduct and performance of an industry. Market power can be defined as an element of monopoly in the sense that the firm possessing it is less contrained in its market behavior than the firm operating under pure or perfect competition. A firm will be said to possess market power if price, production, marketing (sales promotion, advertizing, etc.) or purchasing decisions it might practically make can appreciably change the average price, total quantity, marketing or purchasing practices in a market in which it participates. When a firm or a group of firms have considerable market power, entry can be very difficult and the dominant firm or firms can institute a price policy in terms of long-run objectives and aim at higher current profits against the risk that high current profit will induce new entrants.

Market structure can be defined as the organizational characteristics of a market which seem to determine the competitive conduct of the firms, which in turn generates certain forms of industrial performance. In other words, market structure means those characteristics of the organization of the market which seem to influence strategically the nature of competition and pricing within the market. The important dimensions of market structure some of which are listed by (Bain, 1968; Needham, 1969; and Clodius and Mueller, 1961) are as follows:

1. The degree of seller concentration
   a. Number of sellers in the market
   b. Size distribution of sellers in the market; that is the percentage of the market controlled by each seller.
2. The degree of buyer concentration
   a. Number of buyers in the market
   b. Size distribution of buyers in the market
3. The degree of product and service differentiation
   a. Market knowledge of buyers and sellers
   b. Degree to which outputs of sellers are viewed as non-identical by buyers
      i) Advertizing
      ii) Manufacturers reputation
      iii) Sales and service operations
4. The condition of entry to the market, that is, the relative ease or difficulty with which new sellers may enter the market.
   a. Cost advantage of established firms
   b. Economies of scale of established firms: the higher the economies of scale, the more restrictive is the condition of entry
   c. Product advantage (product differentiation) of established firms
      i) Grant of patent
      ii) Reputation of the firm

d. Legal restrictions

Laws in favor of or against monopolies

e. Capital requirements

The amount of capital required for entry at the scale of a single efficient plant

f. Diversified product lines

The extent to which a firm provides different kinds of output not vertically related to one another

g. Research knowledge

5. A fifth dimension which is not mentioned explicitly by neither Bain nor Clodius and Mueller is vertical integration through ownership.

a. Cooperative, governmental and ordinary corporate organization

b. This dimension refers to the extent to which successive stages in the production of a particular product or the performance of a service are performed by a single firm.

Most of the points listed under conditions of entry are based upon the implicit assumption that potential entrants behave as though they expected established firms in the industry to maintain their output at the pre-entry level in the face of entry, and that these firms do behave in this manner if entry occurs (Needham, 1969). Given this postulate, entry will occur if price exceeds average cost of the marginal, or least efficient established firm by more than an amount that is directly related to the magnitude of scale economies and absolute cost differences between the established firms and new entrants.

Market conduct is defined as the pattern of behavior which entrepreneurs follow in adapting or adjusting to the market structure in which they buy or sell. Significant dimensions of market conduct include:

1. Method and principle employed by the firms in determining price and output
   a. Agreements among sellers
   b. Price leadership
   c. Tacit collusion
2. Means of coordination and cross-adaptation of price, product and sales-promotion policies among firms in the market
3. Presence or absence of predatory or exclusionary tactics directed against either established rivals or potential entrants.
   Frequency of price war
4. Policy of product variation overtime; this is a dimension in non-price conduct
5. Sales-promotion policy: another dimension in non-price conduct
   a. Advertizing expenditures
   b. Sales and service operations

Market conduct is the pattern of behavior that an enterprise follows in its marketing activities.

Market performance is defined as the results that flow from the industry as an aggregate of firms. The performance is the end result which enterprisers arrive at in any market as a consequence of pursuing what line of conduct they espouse. The main dimensions of market performance include:

1. The height of price relative to average cost of production or profits relative to long-run interest rate
2. The relative efficiency of production
   a. Scale or size of plants relative to the optimum scale and its aggregate cost
   b. Extent of excess capacity and its aggregate cost
3. Relative efficiency of distribution
   a. Scale or size of distribution facilities relative to optimum scale and its aggregate cost
   b. Extent of excess distribution facility capacity and its aggregate cost
4. Aggregate sales-promotion costs compared to costs of production and to consumer benefits
   a. Advertizing
   b. Sales and service operations
5. Characteristics of products in terms of consumer utility. Success should accrue to sellers who give buyers more of what they want
   a. Form utility
      i) Design of product
      ii) Quality or durability, reliability, etc.
      iii) Variety in the product
   b. Spatial or locational aspects of product utility
   c. Temporal or storage aspects of product utility
6. The rate of progressiveness of firms and the industry in developing both products and techniques of production and distribution relative to the cost of progress. Opportunities for better products and techniques should not be neglected.

7. Output and input should be consistent with a good allocation of resources.

Market performance is the result of market structure and market conduct.

The basic analytical framework for market structure analysis are narrower than what is sketched above. Some elements of market structure which may facilitate our understanding of market structure, conduct and performance are left out of the analysis. Many market analysts overplay the importance of concentration ratio as a determinant of market power. The concentration ratio - the measure of market power most frequently used in market structure research - is only one of the many possible points on the cumulative concentration ratio curve and cannot be treated as a summary index of the entire curve (Markham, 1965).

While market structure, conduct and performance have characteristics which are internal to the market, it should be recognized that additional factors affect market behavior and performance. Two important additional categories of market characteristics which should be taken into account include those influences which are internal to the firm and those external to the industry. Some influences from sources internal to the firm include: managerial goals, values and motivating forces of businessmen. The drive for growth may be stronger than the objective of profit maximization. Hence we should expect that managerial behavior, motivating forces and operating goals may lead to certain types of market structure changes overtime (Farris, 1963). Factors which are external to the market include government activities, technological developments, the structure of the factor and retail markets, physical properties of of the products and general economic conditions. The most important of these external factors is government policy which may alter the legal and economic environment within which the firms operate. The external factors are specially significant determinants of market structure changes in the long-run. In the light of the foregoing, the following elements should be considered in market structure research (Pritchard, 1969).

1. Structure of closely related industries
2. Contractual arrangements
3. Laws and regulations
4. Some basic economic and technological features of products and processes
5. Attitudes, knowledge, goals and the perceptions of the businessmen

Within the content of our received theory on market structure analysis and the theory of the firm, market structure analysis is usually static. Thus some important considerations are usually precluded. These considerations are:

6. Effect of conduct and performance on structure
7. Effect of conduct and performance on attitudes, knowledge, goals and perception of businessmen
8. Determination of the markets and industries in which a firm will sell
9. Firm growth and decline

The problem we face as researchers is how to incorporate these nine elements into our market structure analysis. The measurement of concentration ratios, conditions and barriers to entry, pricing policies and other elements from economic theory are no more than viewing the real world through "our own eye-glasses", that is through our body of economic theory. It is also important in market structure research to seek an understanding of industrial organization in the light of the businessmen's viewpoint; afterall, decisions are made from their own viewpoints, not ours.

III. OBJECTIVES

The main objective of this study is a market structure analysis through a factor analytic model. Oehrtman (1970) has laid the groundwork for the present research. His hierarchical factor analysis of the responses to a survey questionnaire determined some of the sociological, psychological and economical factors or influences which the fluid milk operators believed to be relevant to their marketing problems. Thus, what I attempt to do in this study is to make some inductive analysis of the milk bottling industry by an analysis of the responses, given by the bottlers themselves, to the questions that probed into many aspects of the problems the bottlers face.

The market analysis procedure followed here is aimed at a better understanding of the structure of the milk bottling industry. The procedure is divided into two parts. The first part was concerned with data collection from the bottlers and an exploratory analysis of these data. These data were not used to test prior hypotheses (there were few or no prior hypotheses to be tested on the economic perceptions of businessmen and the impact of these perceptions on their decisions) but the data were analyzed to develop hypotheses which could be tested at the second phase of the research. In the first phase (Oehrtman, 1970), the exploratory approach of factor analysis was used on the responses of entrepreneurs to a number of questions that probed into different aspects of their operations. This exploratory technique reduced the multitude of responses to the questions in the survey questionnaire to a smaller set of potential influences. These influences were derived by categorizing the responses and grouping together all those variables that tended to explain a common market problem. Each group was then given a name which was dictated by the content of the items in the group. The factor analytic model and the underlying assumptions, made the derivation of these influences easy. The items which were factor analyzed were related to each latent influence in a specific way. The coefficients which measure this association between items and influences were used to formulate testable hypotheses concerning the fluid milk bottling industry.

It should be expected that the derived potential influences are not observable. Thus in the second phase (that is, this thesis) of the market analysis we aim at quantifying these influences. A new sample of fluid milk bottlers is needed. The responses of this sample to the set of questions in the questionnaire are used in conjunction with the empirical results from the exploratory analysis to estimate these influences. The estimated latent influences and the responses in the second sample provide the basis for testing the hypotheses formulated from the results of the exploratory analysis. The extent to which the estimated influences and the second sample responses can be used to generate the coefficient of the relationship between the questionnaire items and the hypothetical influences will lead directly into the test of the hypotheses[1] which were derived from the exploratory analysis.

Hypotheses

From the results of Oehrtman's Solution IV, the following hypotheses were derived. It is hypothesized that the items associated with the numbers listed under each common factor affect the economic situation described by the name of the common factor than any other items do. All the items under the common factors have factor loadings which are greater than 0.14 in absolute value on the factor under which they are listed.

Group Factors

Group Factor 1: Market Area Structure

The following items are closely associated with this group factor: 2, 4, 5, 6, 8, 9, 13, 14, 15, 16, 17, 19, 20, 87, 148, 159 and 160.

Group Factor 2: Consequences of the Growth of Supermarket Chains

The items that load highly on this common factor are: 6, 7, 21, 22, 23, 24, 25, 26 and 27.

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[1]The hypotheses listed below only relate items to the common factor to which they are closely related as the exploratory factor solution showed. For worded statements of these hypotheses see Ladd and Oehrtman (1971). The items listed under each common factor correspond to the numbers in the questionnaire presented as Appendix B in Oehrtman (1971) and Appendix A in Ladd and Oehrtman (1971). This questionnaire is not reproduced in this thesis; whenever item numbers are mentioned below the corresponding statements in either of these appendices are intended.

Group Factor 3: Size of Discounts

The items that are closely related to the size of discounts granted to large wholesale customers are: 30, 31, 32, 33, 34, 35, 36, 37, 126, 150 and 250.

Group Factor 4: Competitors' Apparent Merchandising Practices

The following items are closely related to this factor: 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 56 and 57.

Group Factor 5: Wholesale Customers' Bargaining Power

The items that are closely associated with this factor are 58, 60, 61, 111, and 132.

Group Factor 6: Bottler's Bargaining Power

The items that have high loadings on this group factor are 63, 64, 65, 66, 67, 69, 70, 84, 94, 130, 163, 164 and 248.

Group Factor 7: Sales Procedure and Service

The following items are closely related to this group factor: 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83 and 144.

Group Factor 8: Supermarket Chain Policy

The following items are closely associated with this factor: 77, 86, 89, 91, 93, 96 and 149.

Group Factor 9: Wholesale Milk Drivers' Reputation

Items associated with this factor are: 98, 99, 100, 103, 104, 105 and 140.

Group Factor 10: Firm Dimension

The following items are closely related to this factor: 12, 28, 89, 106, 107, 108, 109, 111, 112, 113, 114, 118, 120, 121, 122, 123, 124, 129, 141, 167, 179, 242, 243, 246, 247, 249 and 251.

Group Factor 11: Management's Wholesale Merchandising Practices

The following items are closely related to this group factor: 62, 95, 161, 162, 163, 165, 166, 167, 168 and 249.

Group Factor 12: Cooperative Reputation

Items associated with this factor are: 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 245 and 251.

General Factors

General Factor A: Processors' Venture in the Market

The items that loaded heavily on this factor are those that are associated with group factors 1, 2, 3, 6, and 11.

General Factor B: Distribution and Merchandising Policy

Items that are closely related to this factor are those associated with group factors 7, 8 and 12.

General Factor C: Problem and Policies of Distribution

The items associated with this general factor are those associated with group factor 9.

General Factor D: Size

The items that affect the economic situation described by size are those items associated with group factor 10.

General Factor E: Illegal Trade Practices

The items that loaded heavily on group factors 4 and 5 affect the economic situation described by this general factor.

Adjustments

Items 131 to 155 deal with adjustment problems of fluid milk processors. In the exploratory analysis it was found out that the index[1] that indicates the proportion of the variation in each of these items explained by the common factors was low. Thus it was hypothesized that the common factors extracted in the exploratory factor analysis explained relatively little of the variation in bottlers' decision to make or not to make certain adjustments in their operations.

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[1]This index is known as the communality and its meaning and role in factor analysis will be explained in the next chapter.

IV. THEORETICAL CONSIDERATIONS: FACTOR ANALYSIS

As mentioned above, this study is a follow up of an exploratory factor analysis of the adjustment problems facing milk bottling firms (Oehrtman, 1970). The model employed in this exploratory analysis was the "Factor Analytic Model" which is a mathematical tool for explaining psychological theories of human behavior. It is a method in multivariate analysis involving m latent common factors and n unique factors; where n represents the number of variables under analysis. The two basic problems with which factor analysis is concerned are:

1. The linear resolution of a set of variables in terms of a small number of categories or hypothetical factors. This is the task of obtaining a parsimonious description of the observed data. This was the main concern of Oehrtman's study which we shall henceforth call the exploratory analysis. (A brief description of the factor analytic model is presented below. For detailed discussion on this model readers are referred to: Harman, 1967; Lawley and Maxwell, 1963; Morrison, 1967; Oehrtman, 1970.)
2. The second concern is the description of the latent factors in terms of the observed data; that is the problem of factors regression. This is the main concern of this present study and an elaborate discussion of this approach is presented below under factor regression, and also in Chapter V.

The Factor Model

Factor analysis is a mathematical model under which each response variate is represented as a linear function of a small number of unobservable latent common-factor variates and a single latent specific variate. The main goal of using the classical factor model is to maximally reproduce the correlations among variables. For an overview of this model, let us suppose that the multivariate system consists of n responses described by the observable random variables $X_1 \ldots, X_j \ldots, X_n$. Since the correlation structure or the covariance matrix will be of interest, we can, without loss of generality, standardize the responses by defining a new variate.

$$Z_{ji} = \frac{X_{ji} - \overline{X}_j}{s_{X_j}} \quad \ldots (1)$$

where

$$X_{ji} = i^{th} \text{ observation on the } j^{th} \text{ variable} \quad \ldots (2)$$

$$\overline{X}_j = \frac{1}{N} \sum_{i=1}^{N} X_{ji} \quad \ldots (3)$$

and

$$s^2_{X_j} = \frac{1}{N-1} \sum_{i=1}^{N} (X_{ji} - \overline{X}_j)^2 \quad \ldots (4)$$

N is the number of observations. Clearly Z_j becomes a variate of zero mean and unit variance.

The sample variance of the variable Z_j is

$$s^2_{Z_j} = \frac{1}{N} \sum_{i=1}^{N} Z^2_{ji} = 1 \quad \ldots (5)$$

and the sample covariance for any two variables Z_j and Z_k is given by

$$s_{Z_{jk}} = \frac{1}{N} \sum_{i=1}^{N} Z_{ji} Z_{ki} = r_{jk} \quad . . . (6)$$

Substituting equation (1) into (6) reduces equation (6) to

$$r_{jk} = \frac{1}{N} \sum_{i=1}^{N} Z_{ji} Z_{ki} = \frac{s_{X_{jk}}}{s_{X_j} s_{X_k}} = \frac{\sum_{i=1}^{N} (X_{ji} - \overline{X}_j)(X_{ki} - \overline{X}_k)}{\sqrt{\sum_{i=1}^{N} (X_{ji} - \overline{X}_j)^2 \sum_{i=1}^{N} (X_{ki} - \overline{X}_k)^2}} \quad . . . (7)$$

The intercorrelations among the variables of the study constitute the basic data for factor analysis.

The classical factor analytic model begins the quest for a more parsimonious explanation of the correlation structure of a given set of variates with the following model

$$Z_{ji} = \sum_{p=1}^{m} a_{jp} f_{pi} + \alpha_j U_{ji} \quad . . . (8)$$

for i = 1, 2, ... N; j = 1, 2, ... n. Z_{ji} is the standardized value of the i^{th} observation on the j^{th} variable. Each of the n observed variables is described linearly in terms of m ($m < n$) common factors f_p (p = 1, 2, ... m) and one unique factor U_j. The m common factors are such that they account for the correlations among the response variates while each of the unique factors accounts for the remaining variance (including error) of any particular variate. The coefficients $a_{jp's}$ of the factors are the parameters reflecting the importance of the p^{th} factor in the composition of the j^{th} variable. These parameters are called factor loadings. Thus a_{jp} is the loading of the j^{th} variable on the p^{th} factor. f_{pi} is the unobservable value of the p^{th} common factor for the i^{th} sample unit. Each of the m terms $a_{jp}f_{pi}$ represents the contribution of factor p to the linear composite. The term $\alpha_j U_{ji}$ is the residual error in the theoretical presentation of the observed measurement of Z_{ji} (Harman, 1967; Lawley and Maxwell, 1963).

For the matrix version of the factor model let us define the NXn matrix of response variates by

$$Z = [Z_1 \ldots Z_j \ldots Z_n] \quad . . . (9)$$

where Z_j is the NX1 vector of N observations on variable Z_j. The NXm matrix of hypothesized common factors is given by

$$f = [f_1 \ldots f_p \ldots f_m] \quad . . . (10)$$

where f_p is the NX1 vector of the values of the p^{th} common factor. The NXn matrix of unique factors is

$$U' = [U_1 \ldots U_j \ldots U_n] \quad . . . (11)$$

where U_j is the NX1 vector of the values of the j^{th} unique factor. The nXm matrix of factor loadings and nXn matrix of unique factor coefficients are defined in equations (12) and (13) respectively:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1p} & \cdots & a_{1m} \\ \vdots & & \vdots & & \vdots \\ a_{j1} & \cdots & a_{jp} & \cdots & a_{jm} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{np} & \cdots & a_{nm} \end{bmatrix} \quad . . . (12)$$

$$\alpha = \begin{bmatrix} \alpha_1 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & \alpha_j & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & \alpha_n \end{bmatrix} \quad . . . (13)$$

Then the factor model can be written as follows:

$$Z' = Af' + \alpha U \qquad . . . (14)$$

The following assumptions are usually made to facilitate factor analysis solutions:

$$E(f') = 0,\ E(\alpha) = 0,\ E(U) = 0 \qquad . . . (15)$$

$$F(f'f) = I_m,\ E(\alpha\alpha') = \alpha^2,\ E(UU') = I_n \qquad . . . (16)$$

$$U \text{ is independent of } f \qquad . . . (17)$$

The equations in (15) state that the common factors, the unique factor coefficients and the unique factors have a mean of zero. Equations in (16) state that each of the common factors, unique factors and unique factor coefficients have a variance of one and zero covariances between any two of each category. These assumptions facilitate the numerical solutions for matrices Λ and α.

Some concepts in factor analysis solution whose numerical values are relevant to the present study are given below. The total variance of the standardized response variate Z_j may be expressed in terms of the factors according to the factor analytic model given above. Thus the variance of Z_j is given by

$$s_j^2 = 1 = \sum_{p=1}^{m} a_{jp}^2 + \alpha_j^2 + 2\sum_{p=1}^{m-1}\sum_{q=p+1}^{m} a_{jp}a_{jq}r_{f_pf_q} + 2\alpha_j \sum_{p=1}^{m} a_{jp}r_{f_pu_j} \quad ..(18)$$

From equations (16) and (17) we saw that the common factors are uncorrelated among themselves; and the unique factors are uncorrelated with the common factors. These assumptions reduce equation (18) to

$$s_j^2 = 1 = \sum_{p=1}^{m} a_{jp}^2 + \alpha_j^2 \qquad . . . (19)$$

The m terms under the summation sign in equation (19) represent the portions of the unit variance of the variable Z_j ascribable to each of the m common factors. For example a_{j1}^2 is the contribution of the factor f_1 to the variance of Z_j (Harman, 1967).

Two important concepts in factor analysis follow from equation (19):

$$h_j^2 = \sum_{p=1}^{m} a_{jp}^2 \quad (j = 1, 2, \ldots n) \qquad \ldots (20)$$

is known as the communality and measures the percentage of the variance of Z_j explained by the m common factors partialled out in the analysis. The quantity

$$\alpha_j^2 = 1 - h_j^2 \qquad \ldots (21)$$

is the percentage of the variance of Z_j not explained by the m common factors, and is composed of unique variance and uncorrelated error of measurement of Z_j.

Another concept which is important in factor regression is the notion of factor pattern. Consider the systems of equations in equation (22) below:

$$\begin{aligned}
Z_{1i} &= a_{11}f_{1i} + a_{12}f_{2i} + \ldots + a_{1m}f_{mi} + \alpha_1 U_{1i} \\
Z_{2i} &= a_{21}f_{1i} + a_{22}f_{2i} + \ldots + a_{2m}f_{mi} + \alpha_2 U_{2i} \\
&\vdots \\
Z_{ni} &= a_{ni}f_{1i} + a_{n2}f_{21} + \ldots + a_{nm}f_{mi} \quad \alpha_n U_{ni}
\end{aligned} \qquad \ldots (22)$$

$$(i = 1, 2, \ldots N)$$

These equations which show the linear composition of the response variates in terms of factors is referred to as a factor pattern. When the common factors are uncorrelated, a factor pattern yields coefficients or loadings which are the correlation coefficients between the corresponding variables and factors, that is

$$a_{jp} = r_{Z_j f_p} \quad . . . (23)$$

The factor pattern (the system of equations in 22) can be used to reproduce the correlations between the response variates. To reproduce the correlation between any two variates multiply item by item the corresponding two equations in the system, then sum over all observations and then divide by the number of observations. Since the factors are in standard form, the reproduced correlation is

$$r^*_{jk} = \sum_{p=1}^{m} a_{jp} a_{kp} \quad . . . (24)$$

$$(j \neq k = 1, 2, ..., n)$$

Denote the observed correlation by r_{jk}. Then the residual correlation is defined as the difference between the observed correlation and the reproduced correlation. That is

$$\tilde{r}_{jk} = r_{jk} - r^*_{jk} \quad . . . (25)$$

This residual is used as an indicator to the maximum number of common factors that can be extracted from a given correlation matrix. When all of the common factors have been removed, the magnitude of the resulting residuals should be approximately zero. When $\tilde{r}_{jk}$ tends to zero then there is no further linkages between the response variates.

The coefficients of the factors in the factor pattern may be represented by the nX(n+m) partitioned matrix M:

$$M = [A \;\vdots\; \alpha] \quad . . . (26)$$

wherein the total pattern is made up of an nXm matrix A of factor loadings and the nXn diagonal matrix α of unique factor coefficients. The sets of factors may be represented by the partitioned matrix F:

$$F = \begin{bmatrix} f' \\ \cdots \\ U \end{bmatrix} \qquad . . . (27)$$

when f is the NXm matrix of common factors and U' is the NXn matrix of unique factors.

In addition to the factor pattern, factor analysis also yields a factor structure which is the matrix of correlations of the variables with the factors. If the correlations of the variables with the common factors are defined by

$$s_{jp} = r_{Z_j f_p} \quad (j = 1, 2, \ldots n; p = 1, 2, \ldots m) \qquad . . . (28)$$

and the correlations with the unique factors are identical to the unique factor coefficients of the factor pattern, then the complete factor structure may be represented by

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1p} & \cdots & s_{1m} & \alpha_1 & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ s_{j1} & \cdots & s_{jp} & \cdots & s_{jm} & 0 & \cdots & \alpha_j & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots & & \vdots \\ s_{n1} & \cdots & s_{np} & \cdots & s_{mm} & 0 & \cdots & 0 & \cdots & \alpha_n \end{bmatrix} = [s \vdots \alpha] \qquad . . . (29)$$

Given the factor pattern M and the factor structure S of a given factor analytic model, we can develop the relationship between M and S: The factor model may be expressed as follows:

$$Z' = Af' + \alpha U = [A \vdots \alpha] \cdot \begin{bmatrix} f' \\ \cdots \\ U \end{bmatrix} = MF \qquad . . . (30)$$

Postmultiplying equation (30) by F' and dividing by the scalar N (the number of observations on each response variate) yield:

$$N^{-1}\ Z'F' = N^{-1}\ MFF' = M\ [N^{-1}\ FF'] \quad . . . (31)$$

By definition the correlation matrix between the variables and the factors is given by

$$N^{-1}\ Z'F' = S \quad . . . (32)$$

The right-hand member of equation (31) is the correlation matrix among all factors (common factors and unique factors) premultiplied by the factor pattern M. Let Φ represent the correlation among all factors, then

$$\Phi = N^{-1}\ FF'$$

$$= N^{-1} \cdot \begin{bmatrix} f' \\ \cdots \\ U \end{bmatrix} \cdot [f \vdots U']$$

$$= \begin{bmatrix} N^{-1}\ f'f & N^{-1}f'U' \\ N^{-1}\ Uf & N^{-1}UU' \end{bmatrix} \quad . . . (33)$$

By assumption $f'U' = 0$, $Uf = 0$ and $N^{-1}\ UU' = I$.

Hence

$$\Phi = \begin{bmatrix} \emptyset & 0 \\ 0 & I \end{bmatrix} \quad . . . (34)$$

where $\emptyset$ is the mXm matrix of correlations among the common factors defined as

$$\emptyset = \begin{bmatrix} 1 & r_{f_1f_2} & . . . & r_{f_1f_m} \\ r_{f_2f_1} & & . . . & r_{f_2f_m} \\ \vdots & & & \vdots \\ r_{f_mf_1} & r_{f_mf_2} & . . . & 1 \end{bmatrix} \quad . . . (35)$$

It follows from equations (31) and (32) that

$$S = M\,\Phi \qquad \ldots (36)$$

Using the partitioned forms of S and M equation (36) reduces to

$$S = [s \,\vdots\, \alpha] = [A \,\vdots\, \alpha] \begin{bmatrix} \emptyset & 0 \\ 0 & I \end{bmatrix} = [A\emptyset \,\vdots\, \alpha] \qquad \ldots (37)$$

From equation (37) we have the obvious result

$$s = A\emptyset \qquad \ldots (38)$$

It will be seen in our discussion of factor regression later in this chapter that it is very convenient to replace the matrix of observed correlations with a matrix of reproduced correlations (with communalities in the main diagonal) plus the unique variances, α^2. By definition, the observed correlation matrix is given by

$$R = N^{-1}\, Z'Z \qquad \ldots (39)$$

Substituting $Z' = MF$ from equation (30), the observed correlation reduces to

$$R = N^{-1}\, MFF'M' \qquad \ldots (40)$$

From equation (33) $N^{-1}FF' = \Phi$, thus equation (40) reduces to

$$R = M\Phi M' = [A \,\vdots\, \alpha] \begin{bmatrix} \emptyset & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A' \\ \cdots \\ \alpha' \end{bmatrix} = A\emptyset A' + \alpha^2 = R^* + \alpha^2 \qquad \ldots (41)$$

where $R^* = A\emptyset A'$ is the matrix of reproduced correlation with communalities in the main diagonal instead of unities.

The last concept which will be used in our discussion of factor regression in the last section can be derived from equation (32):

$$S = [s \vdots \alpha] = N^{-1} Z'F'$$
$$= N^{-1}Z' [f \vdots U']$$
$$= [N^{-1}Z'f \vdots N^{-1}Z'U'] \quad . . . (42)$$

It follows from the last equation that aside from the definition of s given in equation (38), s can also be expressed as

$$s = N^{-1} Z'f \quad . . . (43)$$

and we can express the coefficient of unique factors by

$$\alpha = N^{-1}Z'U' \quad . . . (44)$$

Method of Solution

There are different methods of solution to the factor analytic model. The basic indeterminacy[1] in factor analysis can be seen in the fact that a given correlation matrix may yield different factor solutions. Thus factor analysts must find ways of obtaining a unique solution to a particular correlation matrix. A researcher working independently under a certain sets of assumptions and imposed restrictions will describe a given matrix of correlations uniquely in terms of a factorial reference system. As long as any other researcher works under these assumptions and restrictions the same description of the matrix will result. Any change in the assumptions and/or restrictions will lead to a different description of the same correlation matrix. Each method of factor analysis may have inherent desirable as well as undesirable characteristics. Some of the most desirable characteristics

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[1]This indeterminacy means that an infinitude of factorizations of a given correlation matrix may account for a given set of variates equally well.

which form the basis of the solution criteria in the exploratory analysis (Oehrtman, 1970) are:

a. Principle of parsimony - As in all theoretical developments, it is desirable that the model employed should be simpler than the data upon which the model is based. Thus the number of common factors extracted from the correlation matrix should be less than the number of variables and the complexity[1] should be low.

b. Contribution of factors - A distinction among different factor solution may be made on the basis of the contribution of each factor to the variances of the variates. We may postulate a decreasing contribution; that is, each successive factor contributes a decreasing amount to the total communality. Another possibility is the requirement that the contribution of each factor to the variance of the j^{th} variable be the same. A third criterion might be one large contribution by one factor and level contributions by the others.

c. Grouping of variables - Several methods require that variables be grouped by the magnitude of intercorrelations for the purpose of estimating the rank of the correlation matrix. There is a large variation in the precision among the various methods used to assign variables to one group or another.

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[1]Complexity is used in factor analysis to mean the number of common factors with non-zero coefficient in the description of a variable.

d. Frame of reference - A choice must be made between an orthogonal and oblique reference system; that is, whether the variates will be described in terms of uncorrelated or correlated common factors.

The procedure for estimating factor loadings by the method of maximum-likelihood principle is presented below. Several other methods are available and for a detailed discussion of these methods see Harman (1967); Oehrtman, (1970).

Maximum-likelihood Solution

Unlike any other factor solution, this method is based purely on statistical considerations and is credited to D. N. Lawley (1940). For the discussion of this method, let us assume that a sample of N independent observations has been drawn from a multinormal distribution with the mean vector μ and covariance matrix Σ. We assume that the covariance matrix has a full rank n and that the sample covariance matrix S has enough information for the estimation of the factor parameters. The fundamental likelihood function of S is given in terms of the Wishart density[1]:

$$L = h(S) = K/S/^{1/2(N-n-2)}/\Sigma/^{-1/2(N-1)} \exp\left\{-\frac{N-1}{2}\ \mathrm{tr}\ (\Sigma^{-1}S)\right\} \quad . . . (45)$$

[1] Wishart distribution is the matrix generalization of the Chi-square. Just as the Chi-square is the distribution of sums of squares of independent normal variables with mean zero and variance σ^2, so also is the Wishart the distribution of the sums of squares and cross products of the elements of mutually independent random variables each distributed as $N(0,\Sigma)$.

where N = number of observations

n = number of variates

K = a constant involving N and n.

We want to maximize L in order to obtain the maximum-likelihood estimates of matrices A and α (which are parameters of the factor model) such that

$$\Sigma = AA' + \alpha^2 \qquad \ldots (46)$$

Under the m-factor model, the logarithm of the likelihood function L is

$$\ln L = \ln K - 1/2(N-1)\ /\Sigma/ + 1/2(N-n-2)\ /S/ - 1/2(N-1)\mathrm{tr}\ (\Sigma^{-1}S) \ldots (47)$$

Maximizing L is the same as minimizing a transformation of lnL in equation (47). That is we want to minimize

$$g(\ln L) = (N-1)\left\{\ln/\Sigma/ - \ln/S/ + \mathrm{tr}\ (\Sigma^{-1}S) - n\right\} \qquad \ldots (48)$$

which may be written as follows:

$$g(\ln L) = (N-1)\left\{\ln/\Sigma/ + \mathrm{tr}\ (\Sigma^{-1}S) + \text{fnc ind. of } \Sigma\right\} \qquad \ldots (49)$$

The maximum-likelihood equations follow from setting the derivatives of g(lnL) with respect to n(m + 1) elements of matrices A and α^2 equal to zero. Expressing these derivatives in terms of matrices[1] we have, using equation (46)

$$\begin{aligned} \frac{\partial g(\ln L)}{\partial A} &= 2(N-1)\ \Sigma^{-1}A - (N-1)\ \Sigma^{-1}\frac{\partial \Sigma}{\partial A}\Sigma^{-1}S \\ &= 2(N-1)\left\{\Sigma^{-1}A - \Sigma^{-1}S\Sigma^{-1}A\right\} \qquad \ldots (50) \\ &= 2(N-1)\left\{\Sigma^{-1}\ [\Sigma - S]\ \Sigma^{-1}\right\} A \end{aligned}$$

---

[1]For a detailed presentation of vector and matrix differentiation see Goldberger, 1964; and Dwyer, 1967.

$$\frac{\partial g(\ln L)}{\partial \alpha^2} = (N-1)\,\Sigma^{-1} + (N-1)\,\frac{\partial}{\partial \alpha}\left\{ tr\,(\Sigma^{-1}S)\right\}$$

$$= (N-1)\,\Sigma^{-1} + (N-1)\,(-\Sigma^{-1}\,\frac{\partial \Sigma}{\partial \alpha^2}\,\Sigma^{-1}S) \qquad ...\ (51)$$

$$= (N-1)\left\{\Sigma^{-1}\,(\Sigma - S)\,\Sigma^{-1}\right\}$$

Equating equation (50) to zero and replacing A by its estimator $\hat{A}$ and Σ by $\hat{\Sigma}$ we obtain

$$(\hat{\Sigma}^{-1}\,\hat{A} - \hat{\Sigma}^{-1}S\,\hat{\Sigma}^{-1}\,\hat{A}) = 0 \qquad ...\ (52)$$

It follows from equation (52) that

$$\hat{A} = S\,\hat{\Sigma}^{-1}\,\hat{A} \qquad ...\ (53)$$

Since α^2 is a diagonal matrix an expansion of equation (51) yields

$$\frac{\partial g(\ln L)}{\partial \alpha^2} = \mathrm{diag}\left\{(N-1)\,\Sigma^{-1}\,(I - S\Sigma^{-1})\right\} \qquad ...\ (54)$$

Replacing Σ by its equivalence $AA' + \alpha^2$ we have:

$$\frac{\partial g(\ln L)}{\partial \alpha^2} = \mathrm{diag}\left\{(N-1)\,[(AA' + \alpha^2)^{-1}\,(I - S(AA' + \alpha^2)^{-1})]\right\} \ ..(55)$$

Setting this equation to zero results[1] in

$$\mathrm{diag}\left\{(\hat{A}\hat{A}' + \hat{\alpha}^2)^{-1}\,[I - S\,(\hat{A}\hat{A}' + \hat{\alpha}^2)^{-1}]\right\} = 0 \qquad ...\ (56)$$

i.e.

$$\mathrm{diag}\left\{\hat{\Sigma}^{-1}\,(I - S\,\hat{\Sigma}^{-1})\right\} = 0 \qquad ...\ (57)$$

and

$$\mathrm{diag}\ \hat{\Sigma}^{-1} = \mathrm{diag}\ \hat{\Sigma}^{-1}S\hat{\Sigma}^{-1} \qquad ...\ (58)$$

Pre- and post-multiplying both sides of (58) by

$$\hat{\alpha}^2 = \hat{\Sigma} - \hat{A}\hat{A}' \qquad ...\ (59)$$

yield

$$\mathrm{diag}\ [(\hat{\Sigma} - \hat{A}\hat{A}')\,\hat{\Sigma}^{-1}\,(\hat{\Sigma} - \hat{A}\hat{A}')] = \mathrm{diag}\ [(\hat{\Sigma}^{-1} - \hat{A}\hat{A}')\hat{\Sigma}^{-1}S\hat{\Sigma}^{-1}(\hat{\Sigma} - \hat{A}\hat{A}')] (60)$$

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[1] Note that diag(X) denotes the matrix of the diagonal part of X.

which reduces to

$$\text{diag}\ [\hat{\Sigma} - 2\hat{A}\hat{A}' + \hat{A}\hat{A}\hat{\Sigma}^{-1}\hat{A}\hat{A}'] = \text{diag}\ [S + \hat{A}\hat{A}\hat{\Sigma}^{-1}S\hat{\Sigma}^{-1}\hat{A}\hat{A}' - \hat{A}\hat{A}'\hat{\Sigma}^{-1}S - S\hat{\Sigma}^{-1}\hat{A}\hat{A}'] \quad \ldots (61)$$

Since Σ and S are symmetric matrices it follows that equation 54 may be written as

$$\hat{A}' = \hat{A}'\ \hat{\Sigma}^{-1}\ S \qquad \ldots (62)$$

Using equation (62) in (61) yields

$$\text{diag}\ [\hat{\Sigma} - 2\hat{A}\hat{A}' + \hat{A}\hat{A}'\hat{\Sigma}^{-1}\hat{A}\hat{A}'] = \text{diag}\ [S - \hat{A}\hat{A}' - \hat{A}\hat{A}' + \hat{A}\hat{A}'\hat{\Sigma}^{-1}\hat{A}\hat{A}'] \quad \ldots (63)$$

which is equivalent to

$$\text{diag}\ [\hat{\Sigma}] - \text{diag}\ [2\hat{A}\hat{A}' - \hat{A}\hat{A}'\hat{\Sigma}^{-1}\hat{A}\hat{A}'] = \text{diag}\ [S] - \text{diag}\ [2\hat{A}\hat{A}' - \hat{A}\hat{A}'\hat{\Sigma}^{-1}\hat{A}\hat{A}'] \quad \ldots (64)$$

and reduces to

$$\text{diag}\ [\hat{\Sigma}] = \text{diag}\ [S] = I \qquad \ldots (65)$$

This means that the estimated specific variance and communality of each response must sum to the sample variance. Therefore equations (66) and (67) and the imposed conditions[1] in equations (68) and (69)

$$\hat{\Sigma} = \hat{A}\hat{A}' + \hat{\alpha}^2 \qquad \ldots (66)$$

$$\hat{A} = S\hat{\Sigma}^{-1}\hat{A} \qquad \ldots (67)$$

$$\hat{\alpha}^2 = I - \text{diag}\ \hat{A}\hat{A}' \qquad \ldots (68)$$

$$\hat{A}'\ \hat{\Sigma}^{-1}\hat{A} \text{ is diagonal} \qquad \ldots (69)$$

---

[1]The condition specified in equation (68) follows directly from equation (21): $\alpha_j^2 = 1 - h_j^2 = 1 - \sum_{p=1}^{m} a_{jp}^2$; and condition in (69) is imposed solely to remove the inherent indeterminacy of the matrix of factor loadings A due to the arbitrariness of rotation of the solution obtained.

provide the basis for obtaining the maximum-likelihood estimates of the factor loadings.

The inversion of an nXn matrix $\hat{\Sigma}$ can be avoided by expressing $\hat{A} = S\hat{\Sigma}^{-1}\hat{A}$ in an alternative form. Thus premultiplying equation (66) by $\hat{A}'\hat{\alpha}^{-2}$ yields

$$\hat{A}'\hat{\alpha}^{-2}\hat{\Sigma} = (\hat{A}'\hat{\alpha}^{-2}\hat{A} + I)\hat{A}' \qquad . . . (70)$$

Postmultiplying equation (70) by $\hat{\Sigma}^{-1}$ yields

$$\hat{A}'\hat{\alpha}^{-2} = (\hat{A}'\hat{\alpha}^{-2}\hat{A} + I)\hat{A}'\hat{\Sigma}^{-1} \qquad . . . (71)$$

Taking the transpose of both sides we obtain

$$\hat{\alpha}^{-2}\hat{A} = \hat{\Sigma}^{-1}\hat{A}(I + \hat{A}'\hat{\alpha}^{-2}\hat{A}) \qquad . . . (72)$$

Postmultiply equation (72) by $(I + \hat{A}'\hat{\alpha}^{-2}\hat{A})^{-1}$ to give

$$\hat{\alpha}^{-2}\hat{A}(I + \hat{A}'\hat{\alpha}^{-2}\hat{A})^{-1} = \hat{\Sigma}^{-1}\hat{A} \qquad . . . (73)$$

Substituting equation (73) into (67) yields

$$\hat{A} = S\hat{\alpha}^{-2}\hat{A}(I + \hat{A}'\hat{\alpha}^{-2}\hat{A})^{-1} \qquad . . . (74)$$

Postmultiply both sides of equation (74) by $(I + \hat{A}'\hat{\alpha}^{-2}\hat{A})$ and taking the transpose of both sides of the resulting expression yield

$$(I + \hat{A}'\hat{\alpha}^{-2}\hat{A})\hat{A}' = \hat{A}'\hat{\alpha}^{-2}S \qquad . . . (75)$$

The factor loadings can be obtained by subjecting equation (75) to an iterative method of solution. Using the sample correlation matrix to replace the sample covariance matrix we have

$$(I + \hat{A}'\hat{\alpha}^{-2}\hat{A})\hat{A}' = \hat{A}'\hat{\alpha}^{-2}R \qquad . . . (76)$$

let $\hat{A}'\hat{\alpha}^{-2}\hat{A} = J \qquad . . . (77)$

then

$$(I + J)\hat{A}' = \hat{A}'\hat{\alpha}^{-2}R \qquad . . . (78)$$

It follows from equation (78) that

$$J\hat{A}' = \hat{A}\alpha^{-2}R-\hat{A}' \qquad \ldots (79)$$

Equation (79) can be solved by the iterative procedure developed by Lawley (1942) and reproduced in Harman (1967) and Oehrtman (1970). The mechanics of this method is as follows:

Let us assume we have an initial estimate of matrix A:

$$\hat{A} = (a_1 \ldots a_p \ldots a_m) \qquad \ldots (80)$$

where a_p ($p = 1, 2, \ldots, m$) is an n-component column vector. Corresponding to these trial values a_p, the values derived from the iterative process are denoted by c_p; with $\hat{C}$ regarded as the complete pattern matrix and $\hat{E}^2$ is regarded as the new uniqueness matrix. The equation corresponding to equation (68) becomes

$$\hat{E}^2 = I - \text{diag}\ \hat{C}\hat{C}' \qquad \ldots (81)$$

The iterative equations for the case of three factors are:

$$c_1 = (R\hat{\alpha}^{-2}a_1-a_1) \Big/ \sqrt{a_1'\hat{\alpha}^{-2}(R\hat{\alpha}^{-2}a_1-a_1)} \qquad \ldots (82)$$

$$c_2 = (R\hat{\alpha}^{-2}a_2-a_2-c_{.}c'_{.}\hat{\alpha}^{-2}a_2) \Big/ \sqrt{a_2'\hat{\alpha}^{-2}(R\hat{\alpha}^{-2}a_2-a_2-c_1c_1'\hat{\alpha}^{-2}a_2)} \qquad \ldots (83)$$

$$c_3 = \frac{R\hat{\alpha}^{-2}a_3-a_3-c_1c_1'\hat{\alpha}^{-2}a_3-c_2c_2'\hat{\alpha}^{-2}a_3}{\sqrt{a_3'\hat{\alpha}^{-2}(R\hat{\alpha}^{-2}a_3-a_3-c_1c_1'\hat{\alpha}^{-2}a_3-c_2c_2'\hat{\alpha}^{-2}a_3)}} \qquad \ldots (84)$$

When the a's and c's converge to the desired accuracy, replace all c's by the a's and then the matrix $\hat{A}$ contains the maximum-likelihood estimates of the factor loadings for the assumed number of common factors (Harman, 1967). Usually this method does not lead to a convergence between c's and a's when the model is large and $m > 3$. For the discussion of an iterative process that converges satisfactorily see Morrison, (1967).

Hierar-chical Factor Analysis

This method of factor solution was used by Oehrtman (1970) in estimating the matrix of exploratory factor loadings. The method depends upon successively obtaining high-order factor solutions. These higher-order solutions are the factorizations of the matrices of correlations among the oblique factors. Initially first-order factors are obtained from correlations among observed variables and then the second-order factors are obtained from the correlations among the first-order factors. Either the maximum-likelihood method of solution discussed above or the multiple-group method discussed in Oehrtman, (1970) can be used to obtain higher-order factors. The theoretical equations necessary for hierarchical factor analysis are presented in Oehrtman (1970; pp. 44-47).

Factor Rotation

The classical factorization of a given correlation matrix R of the response variates is not unique because postmultiplication of the matrix of factor coefficients by any comformable orthogonal matrix would yield an equally valid factorization. This indeterminacy was removed from the maximum-likelihood solution by the imposition of the condition in equation (69), that is the requirement that matrix $J = \hat{A}'\hat{\alpha}^{-2}\hat{A}$ be diagonal. However, the question still remains: given a particular factor loading matrix, could one or more orthogonal transformation matrices be found which could lead to a pattern of loadings which is more easily interpreted or identifiable with the subject matter nature of the variables under study? As will become evident in the next paragraph the answer to the question is positive. Since such transformations amount to rigid rotations of the coordinate axes of the m-dimensional factor space, they are commonly called rotations of loadings. Thus to lend more meaning to the exploratory factor analysis reported in Oehrtman (1970), the resulting factor loadings were rotated. The rotated factor loadings still retain their essential properties.

To illustrate this let us define the mXN matrix g as

$$g = T'f' \qquad \ldots (85)$$

where T is any mXm orthogonal matrix, and f is the NXm matrix of original common factors. Also define the nXm matrix C by

$$C = AT \qquad \ldots (86)$$

where A is the original matrix of factor loadings. From the factor model we can define the response variates in terms of the new variates:

$$Z' = Cg + \alpha U \qquad \ldots (87)$$

This "factor model" can be shown to be equivalent to the original model expressed in equation (14). Substituting for g and C from above we have:

$$Z' = ATT'f' + \alpha U \qquad \ldots (88)$$

Since T is an orthogonal matrix, $TT' = I$ and it follows that

$$Z' = Af' + \alpha U \qquad \ldots (89)$$

which is exactly the same as equation (14). By definition $R = N^{-1}Z'Z$ (correlation matrix). It can be easily verified that

$$R = CC' + \alpha^2 \qquad \ldots (90)$$

is the same as the expression for R (assuming orthogonal factors) in equation (41). Thus substituting the definition for C, it follows that

$$R = ATT'A' + \alpha^2$$
$$= AA' + \alpha^2 \qquad \text{. . . (91)}$$

An expansion of the right hand side of (91) shows that

$$\sum_{p=1}^{m} a_{jp}^2 + \alpha_j^2 = 1 \qquad \text{. . . (92)}$$

That is the same sets of communalities $h_j^2 = \sum_{p=1}^{m} a_{jp}^2$ can be obtained from the new model defined in equation (87). For more elaborate treatment of factor rotation see Harman, (1967); Morrison, (1967); Lawley and Maxwell, (1963).

Factor Measurement - Factor Regression

The second basic problem with which factor analysis is concerned is the description of the factors in terms of the observed data; that is the problem of factor regression. The development of this model will enable us to make some inferences about the market structure of the milk bottling industry as the bottlers themselves see it.

The first step in building a suitable expression for the unobserved factors is to see whether or not the factor regression model (equation 93) is consistent with the classical sets of linear regression model:

$$f = ZB + \varepsilon \qquad \text{. . . (93)}$$

where

f is an NXm matrix of common factors

Z is an NXn matrix of observed data

B is an nXm matrix of unknown coefficients

ε is an NXn matrix of disturbances

The specifications given below show that this model is in a form consistent with Goldburger's formulation of the classical sets of linear regression (Goldburger, 1964).

Suppose we have a set of N observations on each of m common factors $(f_1, \ldots, f_p, \ldots, f_m)$ and on each of the n variables $(Z_1, \ldots, Zj, \ldots Z_n)$. We may summarize the pattern of the observations by fitting, for each common factor, the equation:

$$f_{ip} = \beta_{1p}Z_{i1} + \ldots + \beta_{jp}Z_{ij} + \ldots + \beta_{np}Z_{in} + \varepsilon_{ip} \quad . . . (94)$$

We may then define the NXm regressand matrix of common factors by f

$$f = [f_1 \ldots f_p \ldots f_m] = \begin{bmatrix} f_{11} & \cdots & f_{1p} & \cdots & f_{1m} \\ \vdots & & \vdots & & \vdots \\ f_{i1} & & f_{ip} & & f_{im} \\ \vdots & & \vdots & & \vdots \\ f_{N1} & \cdots & f_{Np} & \cdots & f_{Nm} \end{bmatrix} \quad . . . (95)$$

The NXn matrix of standardized regressors is given by

$$Z = [Z_1 \ldots Z_j \ldots Z_n] = \begin{bmatrix} Z_{11} & \cdots & Z_{1j} & \cdots & Z_{1n} \\ \vdots & & \vdots & & \vdots \\ Z_{i1} & \cdots & Z_{ij} & \cdots & Z_{in} \\ \vdots & & \vdots & & \vdots \\ Z_{N} & \cdots & Z_{Nj} & \cdots & Z_{Nn} \end{bmatrix} \quad . . . (96)$$

The nXm coefficient matrix B is given by

$$B = [\beta_1 \ldots \beta_p \ldots \beta_m] = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1p} & \cdots & \beta_{1m} \\ \vdots & & \vdots & & \vdots \\ \beta_{j1} & \cdots & \beta_{jp} & \cdots & \beta_{jm} \\ \vdots & & \vdots & & \vdots \\ \beta_{n1} & \cdots & \beta_{np} & \cdots & \beta_{nm} \end{bmatrix} \quad \ldots (97)$$

Finally let the NXm disturbance matrix be defined by

$$\varepsilon = [\varepsilon_1 \ldots \varepsilon_p \ldots \varepsilon_m] = \begin{bmatrix} \varepsilon_{11} & \cdots & \varepsilon_{1p} & \cdots & \varepsilon_{1m} \\ \vdots & & \vdots & & \vdots \\ \varepsilon_{i1} & \cdots & \varepsilon_{ip} & \cdots & \varepsilon_{im} \\ \vdots & & \vdots & & \vdots \\ \varepsilon_{N1} & \cdots & \varepsilon_{Np} & \cdots & \varepsilon_{Nm} \end{bmatrix} \quad \ldots (98)$$

The Nm equations in expression (94) may thus be written compactly as $f = ZB + \varepsilon$ where each column refers to one of the m relations; the p^{th} relation being:

$$f_p = Z\,\beta_p + \varepsilon_p \quad (p = 1, 2, \ldots m) \qquad \ldots (99)$$

and we assume[1]

$$E\,(\varepsilon_p) = 0 \qquad \ldots (100)$$

$$E\,(\varepsilon_p \varepsilon_p') = \omega_{pp}\, I \qquad \ldots (101)$$

where ω_{pp} is defined in equation (104) below. For the same observation, we allow for correlation between ε_p and $\varepsilon_{p'}$; that is

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[1]Most text books add the assumption that the rank of Z is $n \leq N$. This assumption is not necessary here since Z'Z will be replaced by a matrix which is non-singular and of order m.

$$E\ (\varepsilon_p \varepsilon'_{p'}) = \omega_{pp'} I \qquad \ldots (102)$$

These specifications are consistent with classical formulation of the multivariate linear regression model. To collect the specifications for the m relations define the NXm matrix of disturbances as

$$\varepsilon = [\varepsilon_1 \ldots \varepsilon_p \ldots \varepsilon_m] = \begin{bmatrix} \varepsilon'(1) \\ \vdots \\ \varepsilon'(i) \\ \vdots \\ \varepsilon'(N) \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \ldots \varepsilon_{1p} & \ldots \varepsilon_{1m} \\ \vdots & \vdots & \vdots \\ \varepsilon_{i1} & \ldots \varepsilon_{ip} & \ldots \varepsilon_{im} \\ \vdots & \vdots & \vdots \\ \varepsilon_{N1} & \varepsilon_{Np} & \varepsilon_{Nm} \end{bmatrix} \qquad \ldots (103)$$

where $\varepsilon'(i)$ is the 1Xm row vector of disturbances in all equations at observation i; and the mXm disturbance covariance matrix is given by:

$$\Sigma = E\ [\varepsilon(i)\varepsilon'(i)] = \begin{bmatrix} \omega_{11} & \ldots \omega_{1p} & \ldots \omega_{1m} \\ \vdots & \vdots & \vdots \\ \omega_{p1} & \ldots \omega_{pp} & \ldots \omega_{pm} \\ \vdots & \vdots & \vdots \\ \omega_{m1} & \ldots \omega_{mp} & \ldots \omega_{mm} \end{bmatrix} \qquad \ldots (104)$$

The equations in (99), (100) and (101) specify the multivariate classical linear regression model of factor p on the observed variables. The model for the m common factors on the variables may be written as follows:

$$f = ZB + \varepsilon \qquad \ldots (105)$$

$$E(\varepsilon) = 0 \qquad \ldots (106)$$

$$E\ [\varepsilon(i)\ \varepsilon'(i')] = \begin{cases} \Sigma & \text{if } i=i' \\ 0 & \text{if } i \neq i' \end{cases} \qquad \ldots (107)$$

The rank of Z is not important in the present context.[1]

Our discussion so far has demonstrated that the factor regression model in equation (93) can be treated as a set of classical linear regression model. Thus the least-squares estimate of factor regression parameter, B, is given by

$$\hat{B} = (Z'Z)^{-1} Z'f \qquad \ldots (108)$$

As will be shown presently, this least-squares estimate is in fact a function of the estimated parameters of the exploratory factor analysis. By definition

$$N^{-1} Z'Z = R \qquad \ldots (109)$$

and from equation (44)

$$N^{-1} Z'f = s \qquad \ldots (110)$$

Substituting equations (109) and (110) into equation (108) we have

$$\hat{B} = N^{-1}R^{-1}Ns = R^{-1}s \qquad \ldots (111)$$

We have shown in equation (38) that $s = A\emptyset$. Using this in (111) yields

$$\hat{B} = R^{-1}s = R^{-1}A\emptyset \qquad \ldots (112)$$

From equation (41) we have

$$R = (A\emptyset A' + \alpha^2) \qquad \ldots (113)$$

Premultiplying both sides of equation (113) by $A'\alpha^{-2}$ we obtain

$$A'\alpha^{-2}R = A'\alpha^{-2} (A\emptyset A' + \alpha^2)$$

$$= (A'\alpha^{-2}A\emptyset + I) A' \qquad \ldots (114)$$

---

[1] Z'Z will be replaced by a non-singular matrix of order m.

Premultiply both sides of equation (114) by $(A'\alpha^{-2}A\emptyset + I)^{-1}$ to give

$$(A'\alpha^{-2}A\emptyset + I)^{-1} A'\alpha^{-2}R = A' \quad . . . (115)$$

Postmultiplying equation (115) by R^{-1} yields:

$$(A'\alpha^{-2}A\emptyset + I)^{-1} A'\alpha^{-2} = A'R^{-1} \quad . . . (116)$$

Taking the transpose of both sides of equation (116) we have

$$R^{-1}A = \alpha^{-2}A(A'\alpha^{-2}A\emptyset + I)^{-1} \quad . . . (117)$$

Substituting this expression for $R^{-1}A$ into equation (112) gives the estimating expression for the coefficient matrix B as

$$\hat{B} = R^{-1}A\emptyset = \alpha^{-2}A(A'\alpha^{-2}A\emptyset + I)^{-1}\emptyset \quad . . . (118)$$

In the more conventional form for orthogonal factors (that is when $\emptyset = I$) the estimating expression for the matrix B becomes

$$\hat{B} = \alpha^{-2}A(A'\alpha^{-2}A + I)^{-1} \quad . . . (119)$$

Hence using the result from the exploratory factor analysis, the estimate of B is a function of the estimated factor loadings and the unique factor coefficients. That is

$$\hat{B}_\circ = \alpha_\circ^{-2}A_\circ(A_\circ'\alpha_\circ^{-2}A_\circ + I)^{-1} \quad . . . (120)$$

where the subscript ° denotes the empirical values from the exploratory analysis reported in Oehrtman (1970) and Ladd and Oehrtman (1971).

Using the set of new observations[1] on the same variables for the confirmatory factor analysis we can compute the factor regression:

$$\hat{f}_s = Z_s\hat{B}_\circ \quad . . . (121)$$

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[1]This is the 39 observations used in this analysis and they constitute about 10% of the number of observations used in the exploratory analysis.

where $\hat{f}_s$ is the NXm matrix of estimated factors explaining the correlations among the variables in the analysis. N is the number of observations in the new sample, and m is the number of common factors partialled out in the exploratory analysis. Z_s is the NXn transformed response variates and $\hat{B}_o$ is the nXm estimated factor regression coefficients.

In the literature equation (121) is usually written in a form which is the exact transpose of equation (121):

$$\hat{f}_s' = \hat{B}_o' Z_s' \qquad \ldots (122)$$

where the element $\hat{\beta}_{pj}$ of $\hat{B}_o$ is the coefficient of the p^{th} factor on the j^{th} variable (Harman, 1967). The form used above (equation 121) is maintained in order to facilitate computation and to ensure that the classical multivariate linear regression model can be used to obtain an expression for the factor regression coefficient $\hat{B}_o$.

V. PROCEDURE AND EMPIRICAL RESULTS

Procedure: Data Used and Estimation Methods

The data used in this analysis were collected by the members of the North Central Regional Committee on Dairy Marketing Research, NCM-38 through a questionnaire developed by members of the committee and administered on a large percentage of the milk bottlers in thirteen North Central States. The questionnaire was presented (in the form administered) as Appendix A and (in the rearranged form for analytical purposes) as Appendix B in Oehrtman (1970), and it is not reproduced here. The survey questions are relatively simple and the questionnaire is designed in such a way as to obtain maximum amount of information from each processor without occupying an undue amount of his time. The required answers were easily determined by the participating processors. Most of the questions required that the participants assign numbers to a homogenous class of variables in such a way that the appropriately transformed values of these numbers were additive.

The survey questions were divided into many problem areas as can be seen in the headings of the pages of the questionnaire. For example, questions 1 to 11 (page 2 of the questionnaire) probed the competitive situation of the fluid milk market and the heading of this page is "Developments That Have Changed the Competitive Situation." Similarly the questions on page 3 were designed to investigate the variables that were important in determining the area and the market served by a particular bottler. Appropriately, this page is titled: "Factors That

Have Determined Areas and Markets You Serve." Similar statements can be made for other pages of the questionnaire. It is obvious that the survey questions are meant to probe several aspects of each problem area facing the bottlers. Each processor could consider the questions under any particular problem area and indicate how relevant each question was to the various marketing problems that he faced. For a more detailed discussion of method of data collection and description of the data, see Oehrtman, (1970); Ladd and Oehrtman, (1971).

The exploratory analysis on which this work is based used the factor analytic model, discussed above, to determine some of the sociological and psychological values and economic variables which the operators in the fluid milk industry (from their own knowledge and experience) believe to be relevant to their marketing problems. The factor structure (that is the matrix of correlations of the variables under analysis and the extracted common factors) of these economic variables and the proportion of the observed variance which is accounted for by the factors were determined. The section of the exploratory study which is used in this present analysis consisted of 242 observations on 195 variables. Twelve group factors and five general factors were partialled in Oehrtman's (1970) hierarchical factor solution IV. These common factors and the associated 195 x 17 factor loading matrix A_o were used to provide information that the fluid milk bottlers might use in deciding how to adjust to changes in their market conditions. This matrix of factor loadings provides a means for developing meaningful hypotheses that could help market analysts in understanding the market structure, conduct and performance of the fluid milk processing industry.

The results of the previous work were used in the derivation of testable and meaningful hypotheses. The derivation was effected by selecting an arbitrary boundary line between important and unimportant factor loadings. In this analysis 0.15 has been selected as the dividing line. There is nothing extraordinary about this figure; any other figure could serve our purpose equally well. It is noticeable, however, that as this limit increases in absolute value, the number of derived hypotheses decreases. Two different procedures may be followed in deriving the hypotheses: 1) For each item we formulate hypothesis concerning the factors closely related to that item. Thus, each row of the matrix[1] of exploratory factor loadings offers a hypothesis. 2) For each factor we derive hypotheses concerning items that are closely related to that factor. Thus each column of the matrix of exploratory factor loadings offers a hypothesis. The relationships between items and factors stated as hypotheses in Chapter III were based on the second method.

The methodology discussed below can be termed a confirmatory analysis of an exploratory factor analysis solution in the sense that a rejection of any group of hypotheses is a disaffirmation of some sections of the exploratory analysis results and a non-rejection of the hypotheses is a confirmation of these results. In confirmatory factor studies of

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[1]This matrix is not reproduced in this thesis; it is presented as Appendix B in Ladd and Oehrtman (1971) and as Appendix F in Oehrtman (1970)

psychological or sociological data, the researcher has already obtained certain amount of knowledge about the variables under study; thus he is in a position to formulate hypotheses that specify some of the factors involved (Jöreskog and Lawley, 1968; Jöreskog, 1969). Essentially, what the analyst is doing in this type of confirmatory analysis is just reaffirming (or rejecting) the sufficiency of the number of common factors derived in the exploratory factor analysis after some restrictions have been placed on certain elements of the parameters of a factor analytic model. Another type of confirmatory analysis, which has been used extensively in psychological research, comprises in taking a second sample from the same population and subjecting these observations to a factor analysis. From this analysis, a second matrix of factor loadings is estimated and the "test criterion" becomes a visual comparison of the two matrices of factor loadings. These methods have served well in psychological and sociological research but they are not elucidative enough to be of much use in economic analysis. In research of economic nature, the exploratory factor analysis should enable us to develop meaningful and testable hypotheses from the relationship between variables and the latent factors. The confirmatory analysis, in this respect, turns out to be the development of a procedure for testing the formulated hypotheses.

The first task, in this new approach to confirmatory factor analysis, is to test whether or not the estimated factors $\hat{f}_s$ in equation (96) of Chapter IV can be used to reproduce the factor loadings obtained in the exploratory factor solution. Thus we might treat the estimated factors as fixed variables and use them as regressors in the model:

$$Z_s = \hat{f}_s\theta + \varepsilon_s \qquad \ldots (1)$$

where Z_s is an NXn matrix of response variates

$\hat{f}_s$ is an NXm matrix of estimated factors

θ is an mXn matrix of factor coefficients

ε_s is an NXn matrix of residuals with

$E(\varepsilon_s) = 0$ and $E(\varepsilon_s\varepsilon_s') = \sigma^2 I$.

The estimated coefficient, $\hat{\theta}$, is the matrix of reproduced factor loadings.

There is an inherent problem in the model as it stands. By definition $\hat{f}_s$ was expressed in equation (121) of Chapter IV as follows:

$$\hat{f}_s = Z_s\hat{B}_\circ \qquad \ldots (2)$$

Substituting equation (2) into (1) results in the model

$$Z_s = (Z_s\hat{B}_\circ)\ \theta + \varepsilon_s \qquad \ldots (3)$$

It is clearly obvious from equation (3) that Z_s is being regressed on itself or on some function of itself. Under the classical assumptions of zero mean and constant variances, the least-squares estimator of θ is given by

$$\begin{aligned}\hat{\theta} &= [(Z_s\hat{B}_\circ)'(Z_s\hat{B}_\circ)]^{-1}\ (Z_s\hat{B}_\circ)'\ Z_s \\ &= (\hat{B}_\circ'Z_s'Z_s\hat{B}_\circ)^{-1}\ \hat{B}_\circ'Z_s'Z_s \qquad \ldots (4)\end{aligned}$$

It can be shown that this estimator is biased and the bias does not disappear as the sample size becomes infinitely large, that is the least-squares estimator in equation (4) is not consistent. These properties of biasedness and inconsistency can be demonstrated as follows:

Substitute $Z_s = (Z_s\hat{B}_\circ)\theta + \varepsilon_s$ for the last Z_s in equation (4), then the estimator becomes

$$\hat{\theta} = (\hat{B}_\circ' Z_s' Z_s \hat{B}_\circ)^{-1} \hat{B}_\circ' Z_s' [Z_s \hat{B}_\circ \theta + \varepsilon_s] \quad . . . (5)$$

Expanding equation (5) yields

$$\hat{\theta} = (\hat{B}_\circ' Z_s' Z_s \hat{B}_\circ)^{-1} (\hat{B}_\circ' Z_s' Z_s \hat{B}_\circ)\, \theta + (\hat{B}_\circ' Z_s' Z_s \hat{B}_\circ)^{-1} \hat{B}_\circ' Z_s' \varepsilon_s$$

$$= \theta + (\hat{B}_\circ' Z_s' Z_s \hat{B}_\circ)^{-1} \hat{B}_\circ' Z_s' \varepsilon_s \quad . . . (6)$$

The property of unbiasedness requires that

$$E(\hat{\theta}) = \theta \quad . . . (7)$$

Taking expectation of both sides of equation (6) yields

$$E(\hat{\theta}) = \theta + E\,[(\hat{B}_\circ' Z_s' Z_s \hat{B}_\circ)^{-1} \hat{B}_\circ' Z_s' \varepsilon_s] \quad . . . (8)$$

Since $Z_s = f_s\theta + \varepsilon_s$; the residuals are not independent of the Z_s. Thus the last term on the right hand side of equation (8) does not vanish. That is

$$E\,(\hat{\theta}) \neq \theta$$

An estimator $\hat{\theta}$ is said to be consistent if

$$\lim_{N \to \infty} P(/\hat{\theta} - \theta\, / < \varepsilon) = 1 \quad . . . (9)$$

for some $\varepsilon > 0$

This condition states that if one chooses any arbitrarily small quantity, $\varepsilon > 0$, it becomes more and more certain, as the sample size becomes infinitely large, that the absolute discrepancy between $\hat{\theta}$ and θ will be less than ε. The estimator converges stochastically to θ as $N \to \infty$ (Johnston, 1963), that is

$$\operatorname*{plim}_{N \to \infty} \hat{\theta} = \theta \quad . . . (10)$$

If equation (10) does not hold for any estimator then we say that the estimator is not consistent. Applying this condition to the least-squares estimator $\hat{\theta}$ we have

$$\underset{N \to \infty}{\text{plim}}\ \hat{\theta} = \underset{N \to \infty}{\text{plim}}\ \theta + \underset{N \to \infty}{\text{plim}}\ (\hat{B}_o' Z_s' Z_s \hat{B}_o)^{-1}\ \hat{B}_o' Z_s' \varepsilon_s$$

$$= \theta + \underset{N \to \infty}{\text{plim}}\ (\hat{B}_o' Z_s' Z_s \hat{B}_o)^{-1}\ \hat{B}_o' Z_s' \varepsilon_s \qquad \text{. . . (11)}$$

Again since Z_s and ε_s are not independent, it follows that

$$\underset{N \to \infty}{\text{plim}}\ Z_s' \varepsilon_s \neq 0 \qquad \text{. . . (12)}$$

and hence the last term on the right hand side of equation (11) does not vanish. That is

$$\underset{N \to \infty}{\text{plim}}\ \hat{\theta} \neq \theta \qquad \text{. . . (13)}$$

The problem of inconsistent estimator is not particularly crucial but the bias of the estimator may be very large. Hence it is necessary to adjust the method for estimating the factors.

The main interest in computing the regression in equation (1) is to investigate the extent to which the regression model can be used to reproduce the matrix of the exploratory factor loadings. Consider the relation between the variable Z_j and the estimated factors:

$$Z_j = \hat{f}_s \theta_j + \varepsilon_j \qquad \text{. . . (14)}$$

To get around the problem of regressing Z_j on itself, we can manipulate the estimating procedure for the factors to make $\hat{f}_s$ independent of Z_j. For the regression model to reproduce the exploratory matrix of factor loadings we must have equality between the j^{th} column of matrix $\hat{\theta}$ and the j^{th} column of A_o'. In effect we want to test H: $\theta_j = A_{o_j}$ (where A_{o_j} is the j^{th} column of the transpose of the matrix of factor loadings obtained in the exploratory analysis). Since our interest is in these hypotheses, it seems ideal that A_{o_j} should not enter the estimating formula for f_s. That is, we shall postulate from the start that Z_j does not load on any common factor:

$$a_{j1} = a_{j2} = \ldots = a_{jm} = 0 \qquad \ldots (15)$$

This implies that the j^{th} row of matrix A_o (the matrix of factor loading obtained in the exploratory analysis) is the zero vector. For each j, we have to find an expression similar to equation (120) in Chapter IV by making the necessary adjustments on the matrices A_o and α_o^2 using the assumption in equation (15). Thus we have

$$\hat{B}_o^j = \overset{j}{\alpha}{}_o^{-2} A_o^j \left(A_o^{j\prime} \overset{j}{\alpha}{}_o^{-2} A_o^j + I\right)^{-1} \qquad \ldots (16)$$

where A_o^j is the matrix A_o with each element in the j^{th} row replaced by zero and $\overset{j}{\alpha}{}_o^{-2}$ is obtained from α_o^{-2} by replacing the scalar quantity α_j^{-2} by unity. It can be shown, quite easily, that the j^{th} row of $\hat{B}_o^j$ as defined in equation (16) is the zero vector: By definition, the j^{th} row of $\overset{j}{\alpha}{}_o^{-2}$ can be expressed as

$$j^{th} \text{ row of } \overset{j}{\alpha}{}_o^{-2} = (0, \ldots 1, \ldots 0) \qquad \ldots (17)$$

and the j^{th} row of α_o^{j-2} is given by

$$j^{th} \text{ row of } (\overset{j}{\alpha}{}_o^{-2} A_o^j) = (j^{th} \text{ row of } \overset{j}{\alpha}{}_o^{-2})\, A_o^j$$

$$= (0, \ldots 1 \ldots 0) A_o^j$$

$$= (0 \ldots 1 \ldots 0) \begin{bmatrix} a_{11} & \cdots & a_{1p} & \cdots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{np} & \cdots & a_{nm} \end{bmatrix} = \underline{0} \qquad \ldots (18)$$

Also, the j^{th} row of $\alpha_o^{j2} A_o^j (A_o^{j'} \alpha_o^{j2} A_o^j + I)^{-1}$ is given by

$$(j^{th} \text{ row of } \alpha_o^{j2} A_o^j)(A_o^{j'} \alpha_o^{j2} A_o^j + I)^{-1} = \underline{0}\ (A_o^{j'} \alpha_o^{j2} A_o^j + I)^{-1} = \underline{0} \ \ldots (19)$$

This establishes that the j^{th} row of the "modified" factor regression coefficient, $\hat{B}_o^j$, is the zero vector.

Algebraic manipulations will show that

$$\hat{f}_s^j = Z_s \hat{B}_o^j \qquad \ldots (20)$$

is independent of Z_j. To see this $\hat{B}_o^j$ can be written as follows:

$$\hat{B}_o^j = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2p} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \ldots & 0 & \ldots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} & \cdots & b_{nm} \end{bmatrix} \qquad \ldots (21)$$

and

$$Z_s = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1j} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2j} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{i1} & Z_{i2} & \cdots & Z_{ij} & \cdots & Z_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nj} & \cdots & Z_{Nn} \end{bmatrix} \qquad \ldots (22)$$

Then

$$\hat{f}_s^j = Z_s\hat{B}_o^j = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{ij} & \cdots & Z_{1n} \\ Z_{21} & Z_{22} & \cdots & Z_{2j} & \cdots & Z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{i1} & Z_{i2} & \cdots & Z_{ij} & \cdots & Z_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nj} & \cdots & Z_{Nn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2p} & \cdots & b_{2m} \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} & \cdots & b_{nm} \end{bmatrix} . \quad (23)$$

It is obvious from this that all elements of $\hat{f}_s^j$ are independent of Z_{ij} for i = 1, 2, ... N. That is $\hat{f}_s^j$ is independent of the NX1 vector Z_j. Extracting the contribution of Z_j from the estimating expression for $\hat{f}_s^j$, we can now validly use these estimates as regressors in each of the relations implicit in the model specified in equation (1).

Let us now rewrite the regression model in such a way that the matrices Z_s, θ and ε_s used in equation (1) are now vectors. Therefore, define the NnX1 vector of regressand observations Z_{s*} by

$$Z_{s*} = \begin{bmatrix} Z_1 \\ \vdots \\ Z_j \\ \vdots \\ Z_n \end{bmatrix} = \begin{bmatrix} Z_{11} \\ \vdots \\ Z_{1N} \\ \vdots \\ Z_{j1} \\ \vdots \\ Z_{jN} \\ \vdots \\ Z_{n1} \\ \vdots \\ Z_{nN} \end{bmatrix} \quad . \; . \; . \; (24)$$

The Nn X mn regressor matrix f_{s*} is given by

$$f_{s*} = \begin{bmatrix} \hat{f}_s^1 & \dots & 0 & \dots & 0 \\ \vdots & & & & \\ 0 & \dots & \hat{f}_s^j & \dots & 0 \\ \vdots & & & & \\ 0 & \dots & 0 & \dots & \hat{f}_s^n \end{bmatrix} \qquad \text{. . . (25)}$$

where $\hat{f}_s^j$ is the NXm matrix defined below:

$$\hat{f}_s^j = [\hat{f}_1^j \dots \hat{f}_p^j \dots \hat{f}_m^j] = \begin{bmatrix} \hat{f}_{11} & \dots & \hat{f}_{1p} & \dots & \hat{f}_{1m} \\ \vdots & & \vdots & & \vdots \\ \hat{f}_{i1} & \dots & \hat{f}_{ip} & \dots & \hat{f}_{im} \\ \vdots & & \vdots & & \vdots \\ \hat{f}_{N1} & \dots & \hat{f}_{Np} & \dots & \hat{f}_{Nm} \end{bmatrix} \qquad \text{. . . (26)}$$

the mnX1 vector of coefficients Θ is defined by

$$\Theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_j \\ \vdots \\ \theta_n \end{bmatrix} = \begin{bmatrix} \theta_{11} \\ \vdots \\ \theta_{1m} \\ \vdots \\ \theta_{j1} \\ \vdots \\ \theta_{jm} \\ \vdots \\ \theta_{n1} \\ \vdots \\ \theta_{nm} \end{bmatrix} \qquad \text{. . . (27)}$$

the NnX1 vector of residuals ε is defined by

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_j \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_n \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{1N} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{j1} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{jN} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{n1} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{nN} \end{bmatrix} \qquad \ldots (28)$$

The model can then be written compactly following Goldberger (1964) and Zellner (1962) as

$$Z_{s*} = f_{s*}\,\Theta + \varepsilon \qquad \ldots (29)$$

where

Z_{s*} is a NnX1 vector of dependent variables

f_{s*} is a block diagonal matrix of order NnXmn

Θ is a mnX1 vector of unknown coefficients

ε is a NnX1 vector of residuals.

It is assumed that

$$E(\varepsilon) = 0 \qquad \ldots (30)$$

$$E(\varepsilon\varepsilon') = \sigma^2 V \qquad \ldots (31)$$

where V is an NnXNn diagonal matrix with the j^{th} diagonal block being $\alpha_j^2 I_N$ ($j = 1,2, \ldots n$) and σ^2 is unknown.

The rank of f_{s*} is $nm \leq Nn$ . . . (32)

It is also assumed that ε is normally distributed; this, together with assumptions (30) and (31) makes ε a non-spherical normal vector with $E(\varepsilon) = 0$ and $E(\varepsilon\varepsilon') = \sigma^2 V$. This specification is a special case of generalized least-squares model and the estimate of Θ is given by

$$\hat{\Theta} = (f'_{s*}V^{-1}f_{s*})^{-1}f'_{s*}V^{-1}Z_{s*} \quad \ldots (33)$$

that is

$$\begin{bmatrix} \hat{\theta}_1 \\ \vdots \\ \hat{\theta}_j \\ \vdots \\ \hat{\theta}_n \end{bmatrix} = \left\{ \begin{bmatrix} \hat{f}_s^1 & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & \hat{f}_s^j & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & \ldots & \hat{f}_s^n \end{bmatrix}' \begin{bmatrix} \alpha_1^2 I_N & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & \alpha_j^2 I_N & & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & \ldots & \alpha_n^2 I_N \end{bmatrix}^{-1} \begin{bmatrix} \hat{f}_s^1 & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & \hat{f}_s^j & & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & \ldots & \hat{f}_s^n \end{bmatrix} \right\}^{-1} X$$

$$\begin{bmatrix} \hat{f}_s^1 & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & \hat{f}_s^j & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & \ldots & \hat{f}_s^n \end{bmatrix}' \begin{bmatrix} \alpha_1^{-2} I_N & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & \alpha_j^{-2} I_N & & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & \ldots & \alpha_n^{-2} I_N \end{bmatrix} \begin{bmatrix} Z_1 \\ \vdots \\ Z_j \\ \vdots \\ Z_n \end{bmatrix} \quad \ldots (34)$$

It is obvious from (34) that the least squares estimator of θ_j is given by

$$\hat{\theta}_j = (\hat{f}_s^{j\prime} \alpha_j^{-2} I_N \hat{f}_s^j)^{-1} \; \hat{f}_s^{j\prime} \alpha_j^{-2} I_N \, Z_j \quad \ldots (35)$$

and this reduces to

$$\hat{\theta}_j = (\hat{f}_s^{j\prime} \hat{f}_s^j)^{-1} \hat{f}_s^{j\prime} Z_j \quad \ldots (36)$$

Collecting the n θ_j's into a single vector, expression (34) reduces to

$$\hat{\Theta} = \begin{bmatrix} \hat{\theta}_1 \\ \vdots \\ \hat{\theta}_j \\ \vdots \\ \hat{\theta}_n \end{bmatrix} = \left\{ \begin{bmatrix} \hat{f}_s^1 & \dots & 0 & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & \hat{f}_s^j & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & 0 & \dots & \hat{f}_s^n \end{bmatrix}' \begin{bmatrix} \hat{f}_s^1 & \dots & 0 & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & \hat{f}_s^j & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & 0 & \dots & \hat{f}_s^n \end{bmatrix} \right\}^{-1} X$$

$$\begin{bmatrix} \hat{f}_s^1 & \dots & 0 & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & \hat{f}_s^j & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & 0 & \dots & \hat{f}_s^n \end{bmatrix}' \begin{bmatrix} Z_1 \\ \vdots \\ Z_j \\ \vdots \\ Z_n \end{bmatrix} \qquad \dots (37)$$

That is

$$\hat{\Theta} = (f'_{s*}f_{s*})^{-1}\ f'_{s*}Z_{s*} \qquad \dots (38)$$

Because of the structure of the variance-covariance matrix V, the generalized least-squares estimate turns out to be identical with the ordinary least-squares estimate.

From equation (16), it follows that $\hat{f}_s^j$ is a function of the sample observations, and the matrices of factor coefficients A_o^j and α_o^j obtained in the exploratory analysis reported in Oehrtman, (1970).

That is

$$\hat{f}_s^j = h(Z_s,\ A_o^j,\ \alpha_o^j) \qquad \dots (39)$$

Given the sample observations used in the exploratory factor analysis and the numerical values of A_o and α_o; and for a given set of observations used in the estimation of factors, it follows that the value of $\hat{f}_s^j$ is fixed. Thus f_{s*} as defined in equation (25) is fixed and as such the least-squares estimate of Θ is just a linear transformation of normal independent variables in Z_{s*}. Given the assumption of fixed regressors, it can be shown that $\hat{\Theta}$ is an unbiased estimator:

$$\begin{aligned} E(\hat{\Theta}) &= E[(f'_{s*}f_{s*})^{-1}f'_{s*}Z_{s*}] \\ &= E\ (f'_{s*}f_{s*})^{-1}f'_{s*}\ [f_{s*}\Theta + \varepsilon] \\ &= \Theta + (f'_{s*}f_{s*})^{-1}f'_{s*}E(\varepsilon) \\ &= \Theta \qquad \qquad \qquad \qquad \qquad \ldots (40) \end{aligned}$$

The covariance matrix of $\hat{\Theta}$ can be expressed as follows

$$\begin{aligned} E[(\hat{\Theta}-\Theta)(\hat{\Theta}-\Theta)'] &= E[(f'_{s*}f_{s*})^{-1}f'_{s*}Z_{s*}-\Theta]\ [(f'_{s*}f_{s*})^{-1}f_{s*}Z_{s*}-\Theta]' \\ &= E[(f'_{s*}f_{s*})^{-1}f'_{s*}\varepsilon\varepsilon' f_{s*}(f'_{s*}f_{s*})^{-1}] \\ &= (f'_{s*}f_{s*})^{-1}f'_{s*}\ E(\varepsilon\varepsilon')f_{s*}(f'_{s*}f_{s*})^{-1} \\ &= (f'_{s*}f_{s*})^{-1}f'_{s*}\ \sigma^2 V f_{s*}(f'_{s*}f_{s*})^{-1} \qquad \ldots (41) \end{aligned}$$

Since f_{s*} is block diagonal and V is a diagonal matrix it trivially follows that the covariance matrix of the estimator $\hat{\Theta}$ is also block diagonal and the j^{th} diagonal block is the covariance matrix of $\hat{\theta}_j$ given by

$$E(\hat{\theta}_j-\theta_j)(\hat{\theta}_j-\theta_j)' = (\hat{f}_s^{j\,'}\hat{f}_s^j)^{-1}\hat{f}_s^j\sigma^2\alpha_j^2 I f_s^j(\hat{f}_s^j\hat{f}_s^j)^{-1} = \sigma^2(\hat{f}_s^j\alpha_j^{-2}\hat{f}_s^j)^{-1} \quad \ldots (42)$$

Collecting all the covariances of $\hat{\theta}_j$ ($j = 1, 2, \ldots n$) it follows that

$$E(\hat{\Theta}-\Theta)(\hat{\Theta}-\Theta)' = \sigma^2(f'_{s*}V^{-1}f_{s*})^{-1} \qquad \ldots (43)$$

It has been shown that $\hat{\Theta}$ is a linear transformation of normal independent variables in Z_{s*}. Thus the estimate $\hat{\Theta}$ is a normal, unbiased estimator of Θ; and the covariance matrix is given by $\sigma^2(f'_{s*}V^{-1}f_{s*})^{-1}$.

Statistical Inference

Given the distribution of $\hat{\Theta}$ we are now in a position to make statistical inference about Θ. Consider the null hypothesis on the entire vector Θ

$$H:\ \Theta = A_o^* \qquad \ldots (A)$$

where A_o^* is the mnX1 vector developed from the matrix of factor loadings (obtained from the exploratory factor analysis) by stringing out the columns of the transpose of A_o matrix defined in equation (12) in Chapter IV. Clearly this test is set up to confirm or reject the assumption that the estimated factors and new sets of response variates can be used to reproduce the matrix of the exploratory factor loadings (Oehrtman, 1970: Appendix F). Obviously this test will be based on $\hat{\Theta} - A_o^*$. Given the null hypothesis H: $\Theta = A_o^*$, it will be true that

$$\begin{aligned}
\hat{\Theta} - A_o^* &= \hat{\Theta} - \Theta \\
&= (f'_{s*}V^{-1}f_{s*})^{-1}f'_{s*}V^{-1}Z_{s*} - \Theta \\
&= (f'_{s*}V^{-1}f_{s*})^{-1}f'_{s*}V^{-1}(f_{s*}\Theta + \varepsilon) - \Theta \\
&= \Theta + (f'_{s*}V^{-1}f_{s*})^{-1}f'_{s*}V^{-1}\varepsilon - \Theta \\
&= (f'_{s*}V^{-1}f_{s*})^{-1}f'_{s*}V^{-1}\varepsilon \\
&= E_*^{-1}f'_{s*}V^{-1/2}V^{-1/2}\varepsilon \qquad \ldots (44)
\end{aligned}$$

where $E_* = f'_{s*}V^{-1}f_{s*}$

Consider the statistics

$$Q_1 = (\hat{\Theta} - \Theta)' \, E_*(\hat{\Theta} - \Theta) \qquad \text{. . . (45)}$$

when H is true we see that

$$Q_1 = [E_*^{-1} f_{s*}' V^{-1/2} V^{-1/2} \varepsilon]' E_* [E_*^{-1} f_{s*}' V^{-1/2} V^{-1/2} \varepsilon]$$
$$= \varepsilon' V^{-1/2} V^{-1/2} f_{s*} E_*^{-1} f_{s*}' V^{-1/2} V^{-1/2} \varepsilon \qquad \text{. . . (46)}$$

Define the idempotent matrix M as follows

$$M = I_{Nn} - V^{-1/2} f_{s*} E_*^{-1} f_{s*}' V^{-1/2} \qquad \text{. . . (47)}$$

Thus

$$I_{Nn} - M = V^{-1/2} f_{s*} E_*^{-1} f_{s*}' V^{-1/2} \qquad \text{. . . (48)}$$

Substituting (48) into (46) we obtain

$$Q_1 = \varepsilon' V^{-1/2} (I_{Nn} - M) \, V^{-1/2} \varepsilon \qquad \text{. . . (49)}$$

It can be easily shown that $(I_{Nn} - M)$ is idempotent and its rank is given by

$$tr(I_{Nn} - M) = tr(I_{Nn}) - tr(M)$$

but $tr(M) = tr(I_{Nn} - V^{-1/2} f_{s*} E_*^{-1} f_{s*}' V^{-1/2})$

$$= tr(I_{Nn}) - tr(V^{-1/2} f_{s*} E_*^{-1} f_{s*}' V^{-1/2}$$
$$= tr(I_{Nn}) - tr\,(E_*^{-1} f_{s*}' V^{-1} f_{s*})$$
$$= tr(I_{Nn}) - tr(I_{mn})$$

Since tr (AB) = tr (BA) and using $f_{s*}' V^{-1} f_{s*} = E_*$

Hence tr M = Nn - mn and

$$tr(I_{Nn} - M) = Nn - Nn + mn = mn \qquad \text{. . . (50)}$$

Thus Q_1 is an idempotent quadratic form of rank mn; and its distribution is given by $\sigma^2 \chi^2$ with mn degrees of freedom.

An estimate of the disturbance variance σ^2 will be based on the error sum of squares:

$$\hat{\varepsilon} = Z_{s*} - \hat{Z}_{s*} = f_{s*}\Theta + \varepsilon - f_{s*}\hat{\Theta}$$

$$= \varepsilon - f_{s*}E_*^{-1}f'_{s*}V^{-1}\varepsilon$$

$$= (I - f_{s*}E_*^{-1}f'_{s*}V^{-1})\varepsilon$$

$$= (I - V^{1/2}V^{-1/2}f_{s*}E_*^{-1}f'_{s*}V^{-1/2}V^{-1/2})\varepsilon \qquad . . . (51)$$

Again using the definition for M in equation (47) it follows that

$$\hat{\varepsilon} = [I_{Nn} - V^{1/2}(I_{Nn} - M)V^{-1/2}]\varepsilon \quad = V^{1/2}MV^{-1/2}\varepsilon \qquad . . . (52)$$

Therefore the error sum of squares (SSE) equals

$$SSE = \hat{\varepsilon}'V^{-1}\hat{\varepsilon} = \varepsilon'V^{-1/2}M'V^{1/2}V^{-1}V^{1/2}MV^{-1/2}\varepsilon = \varepsilon'V^{-1/2}M'MV^{-1/2}\varepsilon$$
$$= \varepsilon'V^{-1/2}MV^{-1/2}\varepsilon \qquad . . . (53)$$

The rank of M is equal to the trace of M which has been shown to be equal to (Nn - mn). In our model ε is assumed to be non-spherical and normally distributed. The error sum of squares is an idempotent quadratic form of rank (Nn - mn). Thus SSE is distributed as $\sigma^2\chi^2$ with (Nn - mn) degrees of freedom.

It can be easily demonstrated that the two quadratic forms Q_1 and SSE are independently distributed since

$$M(I-M) = M-M^2 = 0 \qquad . . . (54)$$

Given the independence of Q_1 and SSE, the ratio

$$F_c = \frac{Q_1/mn}{SSE/(Nn-mn)} \qquad . . . (55)$$

is distributed as F distribution with mn and (Nn - mn) degrees of freedom. With the distribution of F_c known it follows that

$$P\left\{\frac{(\hat{\Theta} - \Theta)'E_*(\hat{\Theta} - \Theta)/_{mn}}{SSE/(Nn - mn)} \leq F^{mn}_{(Nn-mn);\tau}\right\} = (1 - \tau) \qquad \text{. . . (56)}$$

where τ is the level of significance and $F^{mn}_{(Nn-mn);\tau}$ is the value of an F variable with mn and (Nn - mn) degrees of freedom which is exceeded 100τ% of the time; that is the upper $(1 - \tau)$ percentile of the F random variable with the specified degrees of freedom. Equation (56) may be written as

$$P\left\{(\hat{\Theta} - \Theta)'E_*(\hat{\Theta} - \Theta) \leq mns^2\ F^{mn}_{(Nn-mn);\tau}\right\} = (1 - \tau) \qquad \text{. . . (57)}$$

where s^2 is the unbiased estimate of σ^2 given by

$$s^2 = \frac{\hat{\varepsilon}'V^{-1}\hat{\varepsilon}}{n(N-m)} \qquad \text{. . . (58)}$$

It is obvious from this probability statement that the region in the mn parameter space enclosed by the hypersurface

$$(\hat{\Theta} - \Theta)'E_*(\hat{\Theta} - \Theta) = mns^2\ F^{mn}_{(Nn-mn);\tau} \qquad \text{. . . (59)}$$

defines a $100(1 - \tau)$% confidence region for Θ. This region is in actual fact an ellipsoid with center at point $\hat{\Theta}$ (Fuller, 1962; Durrand, 1954 ; Goldberger, 1963), and it provides the basis for testing H: $\Theta = A^*_o$ by considering the vector A^*_o and see whether it is contained in the region. H is rejected if the region does not contain A^*_o. This test amounts to substituting A^*_o for Θ in (59) and see whether the resulting scalar of the left hand side of (59) is less than $mns^2F^{mn}_{(Nn-mn);\tau}$. Alternatively we can use equation (55) and reject H if $F_c > F^{mn}_{(Nn-mn);\tau}$.

If H: $\Theta = A^*_o$ is not rejected then the regression model used in estimating the factor coefficient vector Θ reproduces the factor loadings obtained in the exploratory factor analysis. This implies that a_{jp}

element of A_o^*, for all j that did not load heavily on common factor p, should have a corresponding non-significant θ_{jp} element of Θ. In effect we need to test the validity of selecting $/a_{jp}/ = 0.15$ as the dividing line between important and unimportant factor loadings. Before establishing the procedure for this test and discussing the implications for analyzing the fluid milk bottling industry, an investigation of the options available to us in case the hypothesis H: $\Theta = A_o^*$ is rejected will be made.

I. In the event that the data lead to the conclusion that $\Theta \neq A_o^*$, then it is necessary to find out the cause for the discrepancy. As was evident from the discussion of theoretical considerations, $/a_{jp}/ < 1.00$. Thus one major cause for rejecting H might be that most elements of $\hat{\Theta}$ were much larger than unity. Or it might be that for some p's, the elements $\hat{\theta}_{jp}$ correspond very closely to a_{jp} for all j; and for all other p's the elements of the two vectors diverge considerably. For the later case where $\hat{\theta}_{jp}$ and a_{jp} are close in magnitude for some p's it is appropriate to isolate these p's for which θ_{jp} approximates a_{jp}. That is a test procedure is required for:

$$H_p\colon \theta_p = A_{o_p}^* \quad p = 1, 2, \ldots, m \qquad \ldots (B)$$

where $A_{o_p}^*$ is the p^{th} column of A_o matrix of exploratory factor loadings. These hypotheses can be used to isolate those columns (if any) of θ' (an nXm matrix defined in equation 1) and of the matrix of exploratory factor loadings which are equal. For each p for which H_p is not rejected we proceed to test the validity of $/a_{jp}/ = 0.15$ as the dividing line between important and unimportant factor loadings. From this it is possible to analyze the fluid milk industry on the basis of the factor name for factor p and the variables that load highly on that factor. For those p's for which H_p is rejected we test the element θ_{jp} for significance and make inferences concerning the fluid milk industry on the basis of the corresponding a_{jp} loading highly on factor p.

II. When the elements of $\hat{\theta}'$ (the transpose of the matrix of the estimated factor coefficients defined in equation 1) are substantially larger than unity in absolute terms and there is no possibility of identifying columns of A_o and θ' that are equal, then the problem of making conclusions about the relationships between factors and items becomes complicated. Several reasons may be advanced for the failure of the regression model to reproduce, either in part or totality, the matrix of exploratory factor loadings. The reason which is considered most crucial is the difference between the size of the sample used in the factor regression and the size of the sample used in the exploratory analysis. Because of the desirable large sample properties of consistency and asymptotic efficiency of least-squares estimates of regression coefficients, the vector $\hat{\Theta}$ will approach A_o^* as the size of the sample used in estimating the unobserved matrix of latent factors increases. When $\hat{\Theta}$ and A_o^* are both significantly different in statistical terms and in magnitude, it is possible to use some non-parametric techniques to measure the extent of agreement in the classifications of items based on the regression results on one hand and that based on the exploratory factor solution on the other. Two non-parametric statistics that are available for use in the present context are: i) the use of contingency tables and chi-square test and ii) the use of rank correlation. Each of these alternatives will now be considered in turn.

Case I: Regression model reproduces the matrix of exploratory factor loadings

When $\Theta = A_o^*$ there is a need to test the validity of using 0.15 as the dividing line between important and unimportant factor loadings. In this case, each group of items that load highly on a particular factor could be considered as constituting a subspace in the parameter space represented by the p^{th} column of θ', that is θ_p. Hence we have a multiple or joint statistical hypotheses. Simply constructing a 100 $(1 - \tau)$ percent confidence interval for each item will not ensure a level of significance of 100τ percent for all joint tests. The method to follow in this case would be to construct a joint confidence region for θ_p in m-dimensional parameter space that will cover the parameter point $100(1 - \tau)$ percent of the time[1]. The model used to obtain the generalized least-squares estimate of Θ is:

$$Z_{s*} = f_{s*}\Theta + \varepsilon \qquad \ldots (29)$$

$$E(\varepsilon) = 0 \qquad \ldots (30)$$

$$E(\varepsilon\varepsilon') = \sigma^2 V \qquad \ldots (31)$$

In testing the hypotheses H_p: $\theta_p = A_{op}^{**}$ $(p = 1, 2, \ldots 17)$ where A_{op}^{**} is the pth column of A_o with

$$a_{jp}^{**} = \begin{cases} a_{jp} & \text{if } /a_{jp}/ \geqq 0.15 \\ 0 & \text{otherwise} \end{cases}$$

---

[1]For a detailed discussion on the use of joint confidence regions in testing multiple hypotheses see Boles and Collins (1959).

there is hypothesis only on the vector θ_p without any hypothesis on the remaining elements of Θ. We can rearrange the relations in equation (29) as follows:

$$Z_{s*} = f^q_{s*}\Theta_q + f^p_{s*}\,\theta_p + \varepsilon \qquad \ldots (60)$$

where Z_{s*} is as defined in equation (24) above.

f^q_{s*} is the Nn X n(m-1) matrix defined by

$$f^q_{s*} = \begin{bmatrix} \hat{f}^1_q & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & \hat{f}^j_q & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & \ldots & \hat{f}^n_q \end{bmatrix} \qquad \ldots (61)$$

where $\hat{f}^j_q$ (j = 1, 2, ..., n) is the NX(m-1) matrix formed by eliminating the p^{th} column of $\hat{f}^j_s$ defined in equation (26). f^p_{s*} is the NnXn matrix defined by

$$f^p_{s*} = \begin{bmatrix} \hat{f}^1_p & \ldots & 0 & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & \hat{f}^j_p & \ldots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \ldots & 0 & \ldots & \hat{f}^n_p \end{bmatrix} \qquad \ldots (62)$$

where $\hat{f}^j_p$ is the NX1 column vector representing the p^{th} column of the NXm matrix $\hat{f}^j_s$. θ_p is the nX1 column vector and Θ_q is the n(m-1)X1 column vector formed from Θ(defined in equation 27) by eliminating the corresponding elements of θ_p.

It is obvious from equation (60) that the matrix f_{s*} has been partitioned into $[f^q_{s*} \vdots f^p_{s*}]$. Given the assumptions in equations (30) and (31), the least squares estimator of θ_p can be obtained from:

$$\begin{bmatrix} \hat{\theta}_q \\ \hat{\theta}_p \end{bmatrix} = \begin{bmatrix} f^{q'}_{s*}V^{-1}f^q_{s*} & f^{q'}_{s*}V^{-1}f^p_{s*} \\ f^{p'}_{s*}V^{-1}f^q_{s*} & f^{p'}_{s*}V^{-1}f^p_{s*} \end{bmatrix}^{-1} \begin{bmatrix} f^{q'}_{s*}V^{-1}Z_{s*} \\ f^{p'}_{s*}V^{-1}Z_{s*} \end{bmatrix} \quad \ldots (63)$$

For the inversion of the partitioned matrix in equation (63) let

$$f^{q'}_{s*}V^{-1}f^q_{s*} = E; \; f^{q'}_{s*}V^{-1}f^p_{s*} = F$$

$$f^{p'}_{s*}V^{-1}f^q_{s*} = G; \; f^{p'}_{s*}V^{-1}f^p_{s*} = H \quad \ldots (64)$$

then

$$\begin{bmatrix} f^{q'}_{s*}V^{-1}f^q_{s*} & f^{q'}_{s*}V^{-1}f^p_{s*} \\ f^{p'}_{s*}V^{-1}f^q_{s*} & f^{p'}_{s*}V^{-1}f^p_{s*} \end{bmatrix}^{-1} = \begin{bmatrix} E & F \\ G & H \end{bmatrix}^{-1} \quad \ldots (65)$$

An application of the formula for the inversion of partitioned matrices[1] will show that

$$\begin{bmatrix} E & F \\ G & H \end{bmatrix}^{-1} = \begin{bmatrix} E^{-1}(I + FD^{-1}GE^{-1}) & -E^{-1}FD^{-1} \\ -D^{-1}GE^{-1} & -D^{-1} \end{bmatrix} \quad \ldots (66)$$

where $D = H - GE^{-1}F$ $\quad \ldots (67)$

Substituting the matrices defined in equation (64) we have

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[1]Most texts on application of matrices to statistical problems contain this formula. For a brief discussion of the formula see Goldberger, 1964; p. 27.

$$D = f_{s*}^{p'}V^{-1}f_{s*}^{p} - f_{s*}^{p'}V^{-1}f_{s*}^{q}\,(f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}f_{s*}^{q'}V^{-1}f_{s*}^{p}$$

$$= f_{s*}^{p'}[I_{Nn} - V^{-1}f_{s*}^{q}(f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}f_{s*}^{q'}]\;V^{-1}f_{s*}^{p}$$

$$= f_{s*}^{p'}M_1V^{-1}f_{s*}^{p} \qquad \ldots (68)$$

where $M_1 = I_{Nn} - V^{-1}f_{s*}^{q}(f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}f_{s*}^{q'}$ ... (69)

Then

$$\begin{bmatrix} E & F \\ & \\ G & H \end{bmatrix}^{-1} = \begin{bmatrix} \{(f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}\;[I_{Nn} + f_{s*}^{q'}V^{-1}f_{s*}^{p} & - (f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}f_{s*}^{q'}V^{-1}f_{s*}^{p}D^{-1} \\ D^{-1}f_{s*}^{p'}V^{-1}f_{s*}^{q}(f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}]\} & \\ -D^{-1}f_{s*}^{p'}V^{-1}f_{s*}^{q}(f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1} & - D^{-1} \end{bmatrix} \qquad \ldots (70)$$

Using this result in equation (63) yields

$$\begin{bmatrix} \hat{\Theta}_q \\ \hat{\theta}_p \end{bmatrix} = \begin{bmatrix} (f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}f_{s*}^{q'}V^{-1}Z_{s*} - (f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}f_{s*}^{q'}V^{-1}f_{s*}^{p}D^{-1}f_{s*}^{p'}M_1V^{-1}Z_{s*} \\ D^{-1}f_{s*}^{p'}M_1V^{-1}Z_{s*} \end{bmatrix} \qquad \ldots (71)$$

It follows from equation (71) that

$$\hat{\theta}_p = D^{-1}f_{s*}^{p'}M_1V^{-1}Z_{s*}$$

$$= D^{-1}f_{s*}^{p'}M_1V^{-1}[f_{s*}^{q}\Theta_q + f_{s*}^{p}\theta_p + \varepsilon]$$

$$= D^{-1}f_{s*}^{p'}M_1V^{-1}f_{s*}^{q}\,\Theta_q + \theta_p + D^{-1}f_{s*}^{p'}M_1V^{-1}\varepsilon \qquad \ldots (72)$$

But

$$M_1V^{-1}f_{s*}^{q} = [I - V^{-1}f_{s*}^{q}(f_{s*}^{q'}V^{-1}f_{s*}^{q})^{-1}f_{s*}^{q'}]\;V^{-1}f_{s*}^{q}$$

$$= V^{-1}f_{s*}^{q} - V^{-1}f_{s*}^{q} = 0 \qquad \ldots (73)$$

Thus the first term of the right hand side of equation (72) vanishes and

$$\hat{\theta}_p = \theta_p + D^{-1}f_{s*}^{p'}M_1V^{-1} \qquad \ldots (74)$$

Hence we have the desired expression for the derivation of the required confidence region for θ_p:

$$\hat{\theta}_p - \theta_p = D^{-1} f^{p'}_{s*} M_1 V^{-1} \varepsilon \qquad \ldots (75)$$

$$\begin{aligned} E(\hat{\theta}_p - \theta_p)(\hat{\theta}_p - \theta_p)' &= D^{-1} f^{p'}_{s*} M_1 V^{-1} E(\varepsilon\varepsilon') V^{-1} M_1 f^{p'}_{s*} D^{-1} \\ &= \sigma^2 D^{-1} f^{p1}_{s*} M_1 V^{-1} M_1' f^{p'}_{s*} D^{-1} \\ &= \sigma^2 D^{-1} \qquad \ldots (76) \end{aligned}$$

since direct calculation will show that $M_1 V^{-1} M_1' = M_1 V^{-1}$

Consider the quadratic form

$$\begin{aligned} Q_p &= (\hat{\theta}_p - \theta_p)' \; D^{-1} \; (\hat{\theta}_p - \theta_p) \\ &= \varepsilon' V^{-1} M_1' f^{p}_{s*} D^{-1} f^{p'}_{s*} M_1 V^{-1} \varepsilon \\ &= \varepsilon' V^{-1/2} V^{-1/2} M_1' f^{p}_{s*} D^{-1} f^{p'}_{s*} M_1 V^{-1/2} V^{-1/2} \varepsilon \\ &= \varepsilon' V^{-1/2} P V^{-1/2} \varepsilon \qquad \ldots (77) \end{aligned}$$

where

$$P = V^{-1/2} M_1' f^{p}_{s*} D^{-1} f^{p'}_{s*} M_1 V^{-1/2} \qquad \ldots (78)$$

It is readily verified that $p^2 = P$, so that P is an idempotent matrix and that the trace of P is given by

$$\begin{aligned} trP &= tr(V^{-1/2} M_1 f^{p}_{s*} D^{-1} f^{p'}_{s*} M_1 V^{-1/2}) \\ &= tr(D^{-1} f^{p'}_{s*} M_1 V^{-1} M_1 f^{p}_{s*}) \\ &= tr(D^{-1} f^{p'}_{s*} M_1 V^{-1} f^{p}_{s*}) \\ &= tr(D^{-1} D) = tr\ I_m = m \qquad \ldots (79) \end{aligned}$$

Hence Q_p is an idempotent quadratic form of rank m; and is distributed as $\sigma^2 \chi^2$ with m degrees of freedom. From equation (53) it was shown that SSE is distributed as $\sigma^2 \chi^2$ with (Nn - mn) degrees of freedom. To show that Q_p and SSE are independent quadratic forms it is only necessary to establish that PM = 0 where M is the idempotent matrix of the expression

$$SSE = \varepsilon' V^{-1/2} M V^{-1/2} \varepsilon$$

It can be shown by direct computation that $PM_1 = P$ where M_1 and P are defined in equations (69) and (78) respectively. By substituting the expression for P from equation (78) we have

$$\begin{aligned} PM_1 &= V^{-1/2} M_1' f_{s*}^{p} D^{-1} f_{s*}^{p'} M_1 V^{-1/2} M_1 \\ &= V^{-1/2} M_1' f_{s*}^{p} D^{-1} f_{s*}^{p'} M_1 V^{-1/2} = P \qquad \text{. . . (80)} \end{aligned}$$

since $M_1 V^{-1/2} M_1 = M_1 V^{-1/2}$

Further it can be shown that $M = M_1 - P$. By definition in equation (47)

$$M = I_{Nn} - V^{-1/2} f_{s*} (f_{s*}' V^{-1} f_{s*})^{-1} f_{s*} V^{-1/2}$$

Partitioning f_{s*} into $[f_{s*}^{q} \vdots f_{s*}^{p}]$ and using this in equation (47) yield:

$$M = I_{Nn} - [V^{-1/2} f_{s*}^{q} \quad V^{-1/2} f_{s*}^{p}] \begin{bmatrix} f_{s*}^{q'} V^{-1} f_{s*}^{q} & f_{s*}^{q'} V^{-1} f_{s*}^{p} \\ f_{s*}^{p'} V^{-1} f_{s*}^{q} & f_{s*}^{p'} V^{-1} f_{s*}^{p} \end{bmatrix}^{-1} \begin{bmatrix} f_{s*}^{q'} V^{-1/2} \\ f_{s*}^{p'} V^{-1/2} \end{bmatrix} \qquad \text{. . . (81)}$$

An application of the formula for the inversion of a partitioned matrix and direct computation show that

$$M = I_{Nn} - V^{-1/2} W_1 V^{-1/2} + V^{-1/2} W_1 V^{-1} W_2 V^{-1/2} - V^{-1/2} W_2 V^{-1/2} \qquad \text{. . . (82)}$$

where

$$W_1 = f_{s*}^{q} (f_{s*}^{q'} V^{-1} f_{s*}^{q})^{-1} f_{s*}^{q'} \qquad \text{. . . (83)}$$

and

$$W_2 = f_{s*}^{p} D^{-1} f_{s*}^{p'} M_1 \qquad \text{. . . (84)}$$

By substituting the definitions of M_1 and P from equations (69) and (78) respectively and making use of W_1 and W_2 defined in equations (83) and (84) we have

$$\begin{aligned} M_1-P &= I_{Nn}-V^{-1/2}W_1V^{-1/2} - [I_{Nn}-V^{-1/2}W_1V^{-1/2}]V^{-1/2}W_2V^{-1/2} \\ &= I_{Nn}-V^{-1/2}W_1V^{-1/2} + V^{-1/2}W_1V^{-1/2}V^{-1/2}W_2V^{-1/2}-V^{-1/2}W_2V^{-1/2} \\ &= I_{Nn}-V^{-1/2}W_1V^{-1/2} + V^{-1/2}W_1V^{-1}W_2V^{-1/2}-V^{-1/2}W_2V^{-1/2} \qquad \ldots (85) \end{aligned}$$

A comparison of equations (82) and (85) shows that the right hand members of both equations are equal; thus establishing the condition $M = M_1-P$. Hence

$$PM = P(M_1-P) = P - P = 0 \qquad \ldots (86)$$

The independence of Q_p and SSE has been proved by establishing that $PM = 0$. Given this condition the ratio

$$F_p = \frac{Q_p/m}{SSE/n(N-m)} \qquad \ldots (87)$$

is distributed as F distribution with m and n(N-m) degrees of freedom. With the distribution of F_p known, it follows that

$$P\left\{ \frac{(\hat{\theta}_p - \theta_p)'D(\hat{\theta}_p - \theta_p)/m}{SSE/n(N-m)} \leq F^m_{n(N-m);\tau} \right\} = (1 - \tau) \qquad \ldots (88)$$

where τ is the level of significance and $F^m_{n(N-m);\tau}$ is the value of an F variable with m and n(N-m) degrees of freedom which is exceeded 100τ percent of the time, that is the upper $100(1 - \tau)$ percentile of the F random variable with the specified degrees of freedom. Equation (88) may be written as follows:

$$P\left\{ (\hat{\theta}_p - \theta_p)'D(\hat{\theta}_p - \theta_p) \leq ms^2 F^m_{n(N-m);\tau} \right\} = (1 - \tau) \qquad \ldots (89)$$

where s^2 is as defined in equation (58) above. From this probability statement the region in the m - dimensional parameter space enclosed by the hypersurface

$$(\hat{\theta}_p - \theta_p)'D(\hat{\theta}_p - \theta_p) = ms^2F^m_{n(N-m);\tau} \qquad \ldots (90)$$

defines an ellipsoid which is the 100(1 - τ)% confidence region with center at $\hat{\theta}_p$.

To test the relationships[1] between items and the factors which were stated in Chapter III it is enough to test the hypotheses

$$H_p: \theta_p = A^{**}_{\circ p} \qquad \ldots (c)$$

where the element a^{**}_{jp} of $A^{**}_{\circ p}$ (an nX1 vector) takes the value of the factor loadings on factor p for all j's that load highly on factor p and zero otherwise.[2] The confidence region which is expressed by equation (90) provides the basis for testing these relationships. H_p will be rejected if the vector $A^{**}_{\circ p}$ is not contained in the confidence region expressed in equation (90).

To test the hypothesis that the contents of items whose numbers are 2, 4, 5, 6, 8, 9, 13, 14, 15, 16, 17, 19, 20, 87, 148, 159, and 160 affect the economic situation described by Market Area Structure, define

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[1]These relationships were hypothesized on the basis of the results from the exploratory analysis. The items that were claimed to be closely associated with each factor were listed under the common factor in Chapter III.

[2]The vector $A^{**}_{\circ p}$ can be obtained from the p^{th} column of the matrix of exploratory factor loadings (Appendix F, Oehrtman, 1970; Appendix B, Ladd and Oehrtman, 1971) by replacing those elements in the p^{th} column whose values are less than 0.15 in absolute value by zero.

the vector $A_{o_1}^{**}$ as the sixth column of the matrix of exploratory factor loadings (the first five columns of this matrix correspond to the five general factors) except for the replacement of a_{j1} ($j = 1, 2, \ldots, 190$) by zero whenever $/a_{j1}/ < 0.15$. Then specify the null hypothesis

$$H_1: \ \theta_1 = A_{o_1}^{**} \qquad \ldots (C_1)$$

Using a significance level of τ, H_1 will be rejected if the vector $A_{o_1}^{**}$ is not contained in the confidence region defined in equation (90) when $p = 1$. That is substitute $A_{o_1}^{**}$ for θ_p in equation (90), compute the scalar quantity $(\hat{\theta} - A_{o_1}^{**})'D(\hat{\theta} - A_{o_1}^{**})$ and finally see whether or not this computed value is less than $ms^2F^m_{n(N-m);\tau}$. The vector $A_{o_1}^{**}$ is contained in the ellipsoid if the computed scalar is less than $ms^2F^m_{n(N-m);\tau}$. This procedure can be followed to establish the validity or deny the validity of the association between items and the remaining common factors.

Case II: Regression model failed to reproduce the matrix of exploratory factor loadings

1. If the original hypothesis, $H: \Theta = A_o^*$ is rejected there are two possibilities for this rejection. Either some elements of the two vectors correspond very closely while others diverge substantially; or all elements diverge substantially. In the case where some elements of both vectors are close in magnitude it is necessary to isolate the columns of the matrix θ' defined in equation (1) which are equal to the corresponding columns of A_o. The test required is

$$H_*: \theta_p = A_{o_p}^* \quad (p = 1, 2, \ldots, m) \qquad \ldots (D)$$

where $A_{o_p}^*$ is the column of A_o corresponding to factor p. H_* will be rejected if and only if the confidence ellipsoid expressed in equation (90) does not contain the vector[1] $A^{*}_{o_p}$. For any p for which this hypothesis is not rejected we need to test the validity of 0.15 as the dividing line between important and unimportant factor loadings. The procedure for testing the validity of 0.15 has been established when we were discussing the test procedure for H_p: $\theta = A^{**}_{o_p}$. For each p under which the validity of 0.15 is not rejected, the hypothesized relationship between items and factor p is not rejected. It then follows that the contents of the items and the name of the common factor under which the items were listed can be used to make definite statements about the fluid milk bottling industry.

If under any p, H_* is rejected, we should then investigate the individual elements of the vector θ_p for significance. For each p under which H_* is rejected it can be instructive to test

$$H_{**}: \theta_{pj} = 0 \quad (j = 1, 2, \ldots, n) \qquad \ldots \text{(E)}$$

It has been shown earlier that the error sum of squares is given by

$$SSE = \varepsilon' V^{-1/2} M V^{-1/2} \varepsilon$$

where M is an idempotent matrix (defined in equation 47) of rank (Nn-mn). Given the normality assumption on ε, it was shown that SSE is distributed as $\sigma^2\chi^2$ with (Nn-mn) degrees of freedom. In equation (43) above, the variance-covariance matrix of $\hat{\Theta}$ is given by

$$= E(\hat{\Theta} - \Theta)(\hat{\Theta} - \Theta)' = \sigma^2 E_*^{-1} \qquad \ldots (43)$$

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[1]This vector $A^{*}_{o_p}$ should not be confused with the truncated vector $A^{**}_{o_p}$. The latter vector is the former with the exception that all elements a_{jp} whose values are less than 0.15 in absolute value are replaced by zero.

The unbiased estimate of this matrix can be expressed as follows

$$\tilde{\Lambda} = s^2(f'_{s*}V^{-1}f_{s*})^{-1} \qquad \text{. . . (91)}$$

where

$$s^2 = \frac{\hat{\varepsilon}'V^{-1}\hat{\varepsilon}}{n(N-m)}; \text{ and } (f'_{s*}V^{-1}f_{s*}) = E_*$$

Let the diagonal elements of $\tilde{\Lambda}$ be represented by λ^{tt} (t = 1, 2, ..., mn); that is

$$V(\hat{\theta}_{pj}) = \lambda^{tt} \qquad \text{. . . (92)}$$

Thus the test statistics for H_{**} is

$$T_{pj} = \frac{\theta_{pj}}{\sqrt{\lambda^{tt}}} \qquad \text{. . . (93)}$$

This statistics is distributed as a student's t-distribution with (Nn-mn) degrees of freedom. H_{**} will be rejected if $T_{pj} > t_{(Nn-mn);\tau}$.

When all elements of θ_p has been tested, construct the vector $\tilde{\theta}_p = (\tilde{\theta}_{pj})$ where

$$\tilde{\theta}_{jp} = \begin{cases} \hat{\theta}_{pj} \text{ if } T_{pj} > t_{n(N-m);\tau} \\ 0 \text{ otherwise} \end{cases}$$

Hence to test the hypothesis under any common factor we only need to compare the vector $\tilde{\theta}_p$ with the p^{th} column of the matrix A_o. If the non-zero elements of $\tilde{\theta}_p$ correspond (item by item) to the elements of the p^{th} column of A_o whose values are greater than or equal to 0.15 in absolute value we do not reject the relationship hypothesized under common factor p. Otherwise we make conclusions on the basis of which items have non-zero enteries in vector $\tilde{\theta}_p$ and $/a_{jp}/ \geq 0.15$.

2. When the original hypothesis H: $\Theta = A_o^*$ is rejected and the magnitudes of the elements of $\hat{\Theta}$ are very large relative to the elements of A_o^*, it is not possible to follow any of the test procedures discussed above. In this case two options are available:

2a. It is possible to classify the items on the factors on the basis of t - ratios of the regression coefficients; that is all items for which θ_{jp} is significantly different from zero are assigned to common factor p. Given this classification we can test for agreement of the classifications of relevant and irrelevant items under the grouping of items in the exploratory analysis on one hand and regression analysis on the other. Relevance under the exploratory analysis is judged by $/a_{jp}/ \geq 0.15$ while relevance under regression analysis is judged by significant coefficients at τ level of significance. The items listed under each common factor in Chapter III are those whose loadings on the common factor is greater than 0.14 in absolute value. It follows that for each common factor p, there can be two classifications for the n items. The first can classify the items into "significant t-ratios" when $T_{jp} \geq t_{(Nn-mn);\tau}$ and "non-significant t-ratios" when $T_{jp} < t_{(Nn-mn);\tau}$; the other classifies the items into "loading highly" when $/a_{jp}/ \geq 0.15$ and "not loading highly" when $/a_{jp}/ < 0.15$. These classifications lead to a 2 x 2 contingency table. Thus for each p (p = 1,2, ... m) construct the contingency table

	$/a_{jp}/ \geq 0.15$	$/a_{jp}/ < 0.15$	
t-ratio significant	n_{11p}	n_{12p}	$n_{1.p}$
t-ratio not significant	n_{21p}	n_{22p}	$n_{2.p}$
	$n_{.1p}$	$n_{.2p}$	n

where n_{11p} = number of items that load highly on factor p and whose regression coefficients are significant at 100 τ percent.

n_{21p} = number of items that load highly on factor p but whose regression coefficients are not significant at 100τ% level.

n_{12p} = number of items which do not load highly on factor p and whose regression coefficients are significant at 100τ%.

n_{22p} = number of items which do not load on factor p and whose regression coefficients are not significant at 100τ% level.

$n_{1.p}$ = number of items whose regression coefficients are significant at 100τ% level; obtained by adding across row 1.

$n_{2.p}$ = number of items whose regression coefficients are not significant at 100τ% level; obtained by adding across the second row.

$n_{.1p}$ = number of items whose factor loadings are greater than 0.14 in absolute value; obtained by adding down the first column.

$n_{.2p}$ = number of items whose factor loadings are less than 0.15 in absolute value; obtained by adding down the second column.

n = total number of items.

To test the agreement between row and column classifications it is sufficient to test for independence between the rows and columns. This amounts to testing the null hypothesis that the row and column classifications are independent. For this test compute the expected number for each cell and the multinomial chi-square statistics can be used. The expected value in each cell can be obtained as follows:

$$E_{rcp} = \frac{n_{r.p}}{n} \cdot n_{.cp} \qquad \ldots (94)$$

$$r, c, = 1, 2$$

To perform the test, find the contribution of each cell to the multinomial chi-square. The contribution of the $(r, c)^{th}$ cell will be

$$\chi^2_{crp} = \frac{(n_{rcp} - E_{rcp})^2}{E_{rcp}} \qquad \ldots (95)$$

and the total contribution can be expressed as

$$\chi^2_p = \sum_{r,c}^{2} \frac{(n_{rcp} - E_{rcp})^2}{E_{rcp}} \qquad \ldots (96)$$

Since we are dealing with a 2X2 contingency table, the degrees of freedom of this chi-square distribution is 1. Using a 100τ% level of significance the hypothesis of independent classification will be rejected if and only if $\chi^2_p > \chi^2_{\tau,(1)}$ and conclude that there is close agreement in the two classifications otherwise the hypothesis is not rejected.

In terms of analyzing the fluid milk bottling industry, the non-rejection of the hypothesis above under any p, leads to accepting the association between the common factor p and the items that load heavily on it.

2b. The second approach that could be used when there is enough evidence to believe that $\Theta \neq A_o^*$ and the magnitudes of the elements of $\hat{\Theta}$ diverge greatly from those of A_o^*, is the use of non-parametric statistics known as rank-correlation. The mechanics of this method is very simple. Suppose there is a sample of n individuals and there are two measurements on each. We have n pairs of observations say (X_1, Y_1), (X_2, Y_2) ..., (X_n, Y_n). The X values can be arranged in order of size and a rank assigned to each value. The largest value is assigned a rank of 1, the second largest a rank of 2 and so on. The values of Y are similarly treated. Now take the difference between each (X_j) rank and (Y_j) rank and denote this value by d_j. Then the statistics

$$r_s = 1 - \frac{6 \sum_{j=1}^{n} d_j^2}{n(n^2-1)} \qquad \text{. . . (97)}$$

is called the rank-correlation coefficient (Kruskal and Wallis, 1952; Dixon and Massey, 1957; Kendall, 1955). The statistics r_s is similar to the correlation coefficient in that its value ranges between -1 and +1. A value of +1 indicates perfect agreement in the ranking of the two measurements and a value of -1 indicates perfect disagreement. The distribution of $\sum_{j=1}^{n} d_j^2$ is tabulated and the hypothesis that there is close agreement between the two rankings is rejected if r_s is greater than the tabulated value at the preassigned level of significance.

With this brief overview, the procedure can be applied to the problem at hand. The absolute value of the factor loading a_{jp} is a measurement of item j on factor p. Item j is assigned to factor p if and only if $/a_{jp}/ \geq 0.15$; thus it is possible to rank the n items on the basis of their loadings on factor p. The value of $\hat{\theta}_{jp}$ cannot be treated in the same vein as the value of a_{jp}. A large $\hat{\theta}_{jp}$ (in absolute value) does not suggest a strong relationship between item j and factor p in the same way that a large $/a_{jp}/$ suggests the importance of item j on factor p. A large $/\hat{\theta}_{jp}/$ may be associated with a large standard error of $\hat{\theta}_{jp}$, in which case the test statistics for significance may be very small. Thus we need a measure that will show the importance of item j in influencing factor p. Fortunately, the t-ratios (obtained by dividing the standard error of $\hat{\theta}_{jp}$ into $\hat{\theta}_{jp}$) provides an excellent proxy for this measure. Large value of t-ratio (T_{jp}) in absolute value is likely to lead to the acceptance of the hypothesis that item j is important in influencing factor p. Hence the second ranking on factor p can be based on the absolute value of the computed t-ratios.

For a given factor p, consider a_{jp} and T_{jp} as two measures of item j on factor p. Thus these two measures can be used on the n items as follows: $(a_{1p}, T_{1p}), (a_{2p}, T_{2p}), \ldots, (a_{np}, T_{np})$. Then arrange the "a" values in order of absolute value and then assign a rank to each value. The largest value has the rank of 1, and the smallest is given the rank of n. Similarly arrange the "T" values in order of absolute magnitudes and assign ranks in the same way. Now let r_{jap} be the ranking, based on "a" values of item j on factor p and r_{jtp} be the ranking based on "T" values; and define $d_{jp} = r_{jap} - r_{jtp}$ as the difference between the two rankings. Then the statistics

$$r_{sp} = 1 - 6 \frac{\sum_{j=1}^{n} d_{jp}^2}{n(n^2-1)} \quad (p = 1, 2, \dots m) \qquad \dots (98)$$

represents the rank-correlation coefficient for factor p. The hypothesis to be tested is "that there is agreement between the assignment of items to factor p under the exploratory factor solution and the assignment under the regression analysis using the t-ratios as the criteria for assignment." The hypothesis will be rejected if the computed r_{sp} is greater than the tabulated rank-correlation statistics with (n-2) degrees of freedom; and τ level of significance.

This type of ranking and tests for agreement are performed for all p = 1, 2, ..., m. For each p for which the correlation is significant, then the relationship between the items and common factor p which was postulated in the exploratory analysis is accepted. If the correlation is not significant, the relationship is rejected.

To test the hypothesis on adjustments consider the model

$$Z_j = \hat{f}_s^j \theta_j + \varepsilon_j \qquad \dots (99)$$

for j corresponding to the items that relate to adjustment problems - items 131 to 155 of the survey questionnaire - and assume that $E(\varepsilon_j) = 0$; $E(\varepsilon_j \varepsilon_j') = \sigma^2 \alpha_j^2 I_N$ and θ_j is a mX1 vector. The least-squares estimate of θ_j is given by

$$\hat{\theta}_j = (\hat{f}_s^{j\prime} \hat{f}_s^j)^{-1} \hat{f}_s^{j\prime} Z_j \qquad \dots (100)$$

In testing the hypothesis: $\theta_{j1} = \ldots = \theta_{jp} = \ldots = \theta_{j\ m} = 0$ it is usual to employ the test statistics

$$F = \frac{R_j^2/(m-1)}{(1 - R_j^2)/(N-m)} \sim F_{(n-m)}^{(m-1)} \qquad \ldots (101)$$

The hypothesis will be rejected if $F > F_{(N-m);\tau}^{(m-1)}$ where τ is the level of significance. It follows that the dividing line between the critical and non-critical regions is provided by the value of R_j^2 which satisfies the relation

$$\frac{R_j^2/(m-1)}{(1 - R_j^2)/(N-m)} = F_{(N-m);\tau}^{(m-1)} \qquad \ldots (102)$$

Solving this equation for R_j^2 we have:

$$(N-m)R_j^2 = (m-1)(1-R_j^2)\ F_{(N-m);\tau}^{(m-1)}$$

$$= (m-1)\ F_{(N-m);\tau}^{(m-1)} - (m-1)R_j^2\ F_{(N-m);\tau}^{(m-1)}$$

i.e.,

$$(N-m)R_j^2 + (m-1)R_j^2\ F_{(N-m);\tau}^{(m-1)} = (m-1)\ F_{(N-m);\tau}^{(m-1)}$$

This implies that

$$R_j^2 = \frac{(m-1)\ F_{(N-m);\tau}^{(m-1)}}{\left\{N-m + (m-1)\ F_{(N-m);\tau}^{(m-1)}\right\}} \qquad \ldots (103)$$

Thus in testing the hypothesis that the factors included in the exploratory study explain relatively little of the variation in bottlers' decisions to make or not to make certain adjustments in their operations it is enough to compare the value of R_j^2 (j = 131, 132, ..., 155) in the last column of Appendix II with that obtained in equation (103). The hypothesis will be rejected if R_j^2 (for all j) is greater than the computed value in equation (103).

Empirical Results

The results presented here are based on the empirical results of the exploratory factor solution reported in Oehrtman (1970) as Solution IV. The results are applicable to those fluid milk processors that supply supermarket chains with milk and expressed their reactions about fluid milk bargaining cooperatives. Solution IV was based on 242 observations on 195 items. The matrix of factor loadings, A_o, obtained in this solution was presented as Appendix F in Oehrtman (1970) and as Appendix B in Ladd and Oehrtman (1971). The diagonal matrix of uniqueness, α_o^2, which is important in the present analysis can be easily obtained from either of these Appendices since α_j^2 is equal to $(1 - h_j^2)$ where h_j^2 is given in the second to the last column of these Appendices.

The empirical results presented below are based on 190 variables instead of 195 variables as were used in the exploratory analysis. The reason for this arises from the fact that the communalities (h_j^2's) for five items are greater than unity. The value of $h_j^2 \geq 1.00$ leads to the corresponding $\alpha_j^2 \leq 0$. When α_j^2 is zero for at least one j, there arises problem of singularity in the matrix α_o^2. If any α_j^2 is strictly negative, the problem of complex value for the unique factor coefficient arises. Aside from these two problems, it is most unlikely that the m common factors will explain more than 100% of the variance of variable Z_j.

Thus any $h_j^2 \geq 1.00$ should lead to suspecting that the responses on item j are exaggerated. These five items are referred to as Suspect Variables. Table 2 gives the mean score, content and communalities of these items.

Table 1. Items with $h_j^2 \geq 1.00$: Suspect Variables

Items [a]	h_j^2	Mean Score	Content
55	1.00	234	Servicing display equipment free or below cost
65	1.00	418	Pointing out that your product is high quality
173	1.01	376	The cooperative is a dependable organization
175	1.01	386	The cooperative lives up to its agreement with processor
182	1.33	370	The cooperative serves a useful purpose

[a]The numbers assigned to the items correspond to the number on the questionnaire (Oehrtman, 1970; Appendix B).

An inclusion of these variables in the computations would have made it impossible to obtain an estimating expression for f_s^j (j = 1, 2, ..., 190). Hence these five items do not enter the computations.

The procedures discussed above are followed to obtain the results presented here:

Step I: Compute the factor regression coefficients $\hat{B}_o^j$ (j = 1, 2, ..., 190) using equation (16). These 190 X 17 matrices and the 39 X 190 matrix of second sample observations are used to compute the estimated values of the matrices of hypothetical common factors f_s^j associated with variable Z_j. Equation (20) is used to obtain $\hat{f}_s^j$ for all j.

Step II: The $\hat{f}_s^j$ (j = 1, 2, ..., 190) provide a valid set of regressor matrices that can be used in obtaining the least-squares estimates of the factor coefficients in

$$Z_j = \hat{f}_s^j \theta_j + \varepsilon_j \quad (j = 1, 2, \ldots 190)$$

Equation (36) was used to obtain the estimates $\hat{\theta}_j$ for all j. Combining these results, the vector of factor coefficients of the model in equation (29) can be obtained by using equation (38). For each j, the elements of vector $\hat{\theta}_j$ are the factor coefficients of item j on all seventeen common factors. Thus $\hat{\theta}_j$ is the reproduced vector of factor loadings of item j on the common factors. Thus we can develop the matrix $\hat{\theta}'$ from $\hat{\Theta}$ by placing $\hat{\theta}_j'$ as the j^{th} row of $\hat{\theta}'$. This matrix is presented in Appendix II. The last column of this appendix shows the coefficient of multiple correlations R_j^2 which measures the percentage of total variation in variable j which is explained by the common factors. R_j^2 corresponds to h_j^2 (the communality of the j^{th} item) in that h_j^2 measures the percentage of the unit variance of item j which is explained by the seventeen common factors.

Step III: Given the least-squares estimate of the factor regression coefficient matrix θ' it is necessary to investigate the extent to which the regression model reproduces the matrix of exploratory factor loadings. This implies the test of the hypothesis H: $\Theta = A_o^*$. Using equation (55) yielded the test statistics $F_c = 1.519$. The tabulated F-random variable with (190 X 17) and (190 X 22) degrees of freedom at 5% level of significance is 1.00. Since $F_c = 1.519 > F^{\infty}_{\infty,\ 0.05} = 1.00$ the hypothesis H will be rejected. This leads to the conclusion that the regression model failed to reproduce the matrix of exploratory factor loadings. This suggests that the information contained in the second sample is not fully consistent with the information contained in the exploratory sample. That is some or all of the hypotheses listed in Chapter III will not be supported by the information in the sample used in this analysis.

Step IV: Here it is necessary to investigate why the second sample failed to reproduce the matrix of exploratory factor loadings. A visual inspection of the elements of each row of Appendix II shows that many of these elements are outside of the open interval (-1, 1). From the theoretical considerations in Chapter IV, it was evident that the elements a_{jp} ($j = 1, 2, \ldots, 190$; $p = 1, 2, \ldots, 17$) of A_o are all in the interval (-1, 1). Thus, it seems reasonable to infer that the high departure of the elements of θ' (Appendix II) from unity in absolute terms, and the rejection of H: $\Theta = A_o^*$ reveal the fact that corresponding column of A_o and θ' cannot be equal. Thus the parametric procedures developed for testing individual columns of θ' against the corresponding column of A_o cannot be followed.[1] To test the hypotheses listed in Chapter III, the

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[1]Though the procedures for looking at individual columns of matrix θ' against the corresponding column of A_o are not used to obtain numerical results, they are included in this report because the author feels very strongly that given a sufficiently large sample for the confirmatory analysis, these procedures are the ideal ones to follow. (See remarks about sample size in the next chapter).

only alternative available is the use of non-parametric statistics. There are two non-parametric methods that can be employed: a) the use of contingency table and chi-square test, and b) the use of rank correlation coefficients. The second non-parametric approach will be employed in testing the relation between items and factors.

Step V: Tables 2-13 show the rankings of 190 items on factor p ($p = 1, 2, \ldots, 12$) based on two "measures". The first measure is the absolute magnitude of the exploratory factor loadings a_{jp}. This measure is denoted by r_{jap} in column 2 of each table. The second measure is the absolute magnitudes of the t-ratios of the regression coefficients θ_{jp}. This measure is denoted by r_{jtp} in column 3 of each table. The difference between r_{jap} and r_{jtp} is denoted by d_{jp} in column 4. The square of the differences - d_{jp}^2 - is given in column 5. For each table, we want to test the null hypothesis that the two rankings r_{jap} and r_{jtp} are independent. The logic behind this hypothesis is that a rejection of the hypothesis of independent ranking implies that there is close agreement in the rankings; which in turn leads to accepting the hypothesized relationship between items and common factor p.

At the bottom of each table is the value of rank correlation coefficients computed from equation (98) for factor p, ($p = 1, 2, \ldots 17$):

$$r_{sp} = 1 - 6 \frac{\sum_{j=1}^{190} d_{jp}}{190\,(190^2-1)}$$

The null hypothesis of independent ranking will be rejected if r_{sp} is larger than the tabulated rank correlation coefficient with 188 degrees of freedom and at 5% level of significance which is 0.15.

For the ranking on group factor 1 - Market Area Structure - the information in table 2 will be used to test for independence in the rankings based on the absolute values of the exploratory factor loadings on one hand and the absolute value of the t-ratios on the other. From this table compute equation (98) for p = 1 (group factor 1). In this case $r_{s1} = 0.0197$. Since this is less than the tabulated value of r_s with 188 degrees of freedom and 5% level of significance (r_s; 0.05 (188) = 0.15) we do not reject the hypothesis of independent ranking. The non-significance of the rank correlation r_{s1} implies that there is no close agreement in the classifications of items on group factor 1 on the basis of the exploratory factor solution on one hand and the regression analysis on the other. Hence the second data reject the grouping of items on group factor 1. Since the items listed under this group factor were assigned to it as a result of this grouping, any rejection of this grouping leads to the rejection of the hypothesized close relationship between items 2, 4, 5, 6, 8, 9, 13, 14, 15, 16, 19, 20, 87, 148, 159 and 160 and group factor 1. This result is evident in table 2. Most of these items have high rankings in column 2 whereas the corresponding rankings in column 3 are relatively low. All these items have non-significant t-ratios at 5% level. On the evidence provided by the regression analysis these items have no effect on the economic situation described by group factor 1 - Market Area Structure.

Table 2. Ranking of items on group factor 1 by the absolute values of exploratory factor loadings ($a_{jp's}$) and the absolute values of the t-ratios ($T_{jp's}$) of the regression coefficients $\theta_{jp's}$

Items	Column 2* r_{ja1}	Column 3** r_{jt1}	Column 4 d_{j1}	Column 5 d^2_{j1}
1	88	39	49	2401
2	11	91	-80	6400
3	7	159	-152	23104
4	16	147	-131	17161
5	27	124	-97	9409
6	8	68	-60	3600
7	74	30	44	1936
8	31	173	-142	20164
9	10	146	-136	18496
10	42	60	-18	324
11	30	162	-132	17424
12	26	137	-111	12321
13	5	132	-127	16129
14	9	45	-36	1296
15	3	89	-86	7396
16	1	165	-164	26896
17	12	75	-63	3969
18	6	118	-112	12544
19	4	163	-159	25281
20	2	16	-14	196
21	118	54	64	4096
22	72	86	-14	196
23	65	82	-17	289
24	22	183	-161	25921
25	86	160	-74	5476
26	106	169	-63	3969
27	85	74	11	121
28	89	50	39	1521
29	151	115	36	1296
30	43	117	-74	5476
31	28	12	16	256
32	24	148	-124	15376
33	135	73	62	3844
34	75	22	53	2809
35	121	41	80	6400
36	70	116	-46	2116
37	124	65	59	3481

*r_{a1} is the ranking of items on factor 1 using the absolute values of the exploratory factor loadings as a measure.

**r_{t1} is the ranking of items using the t-ratio as a measure.

Table 2. (Cont'd)

Items	Column 2* r_{ja1}	Column 3** r_{jt1}	Column 4 d_{j1}	Column 5 d^2_{j1}
38	84	109	-25	625
39	103	71	32	1024
40	183	5	178	31684
41	187	27	160	25600
42	77	46	31	961
43	49	128	-79	6241
44	165	178	-13	169
45	160	3	157	24649
46	59	19	40	1600
47	58	18	40	1600
48	177	15	162	26244
49	142	25	117	13689
50	61	14	47	2209
51	68	51	17	289
52	64	185	-121	14641
53	184	142	42	1764
54	154	79	75	5625
56	57	38	19	361
57	145	188	-43	1849
58	188	58	130	16900
59	87	190	-103	10609
60	125	69	56	3136
61	95	13	82	6724
62	189	127	62	3844
63	62	181	-119	14161
64	147	105	42	1764
66	38	47	-9	81
67	117	145	-28	784
68	83	187	-104	10816
69	67	149	-82	6724
70	41	129	-88	7744
71	56	23	33	1089
72	162	103	59	3481
73	82	70	12	144
74	153	35	118	13924
75	120	62	58	3364
76	110	161	-51	2601
77	98	184	-86	7396
78	109	26	83	6889
79	126	93	33	1089
80	113	21	92	8464
81	92	37	55	3025
82	33	52	-19	361

Table 2. (Cont'd)

Items	Column 2* r_{ja1}	Column 3** r_{jt1}	Column 4 d_{j1}	Column 5 d^2_{j1}
83	119	157	-38	1444
84	19	42	-23	529
85	163	168	-5	25
86	69	100	-31	961
87	20	92	-72	5184
88	173	158	15	225
89	55	143	-88	7744
90	76	107	-31	961
91	116	131	-15	225
92	111	139	-28	784
93	48	53	-5	25
94	155	49	106	11236
95	143	104	39	1521
96	91	99	-8	64
97	140	64	76	5776
98	104	176	-72	5184
99	146	122	24	576
100	60	57	3	9
101	172	90	82	6724
102	161	144	17	289
103	167	8	159	25281
104	136	150	-14	196
105	110	152	-52	2704
106	182	141	41	1681
107	90	85	5	25
108	52	11	41	1681
109	141	112	29	841
110	127	77	50	2500
111	178	1	177	31329
112	156	138	18	324
113	180	9	171	29241
114	131	164	-33	1089
115	149	123	26	676
116	114	32	82	6724
117	132	167	-35	1225
118	144	55	89	7921
119	175	106	69	4761
120	133	175	-42	1764
121	37	4	33	1089
122	18	36	-18	324

Table 2. (Cont'd)

Items	Column 2* r_{ja1}	Column 3** r_{jt1}	Column 4 d_{j1}	Column 5 d^2_{j1}
123	101	2	99	9801
124	181	61	120	14400
125	139	125	14	196
126	73	171	-98	9604
127	138	174	-36	1296
128	134	130	4	16
129	166	108	58	3364
130	21	96	-75	5625
131	152	111	41	1681
132	186	28	158	24964
133	50	140	-90	8100
134	66	67	-1	1
135	179	180	-1	1
136	158	114	44	1936
137	53	172	-119	14161
138	176	135	41	1681
139	150	113	37	1369
140	169	119	50	2500
141	51	10	41	1681
142	93	126	-33	1089
143	130	102	28	784
144	81	20	61	3721
145	168	153	15	225
146	190	83	107	11449
147	174	87	87	7569
148	17	186	-169	28561
149	94	166	-72	5184
150	46	88	-42	1764
151	159	134	25	625
152	63	17	46	2116
153	108	101	7	49
154	39	133	-94	8836
155	79	189	-110	12100
156	78	154	-76	5776
159	14	63	-49	2401
160	13	156	-143	20449
161	35	78	-43	1849
162	71	43	28	784
163	148	76	72	5184
164	99	136	-37	1369

Table 2. (Cont'd)

Items	Column 2* r_{ja1}	Column 3** r_{jt1}	Column 4 d_{j1}	Column 5 d_{j1}^2
165	36	72	-36	1296
166	29	7	22	484
167	123	6	117	13689
168	97	24	73	5329
169	137	40	97	9409
170	164	179	-15	225
171	157	95	62	3844
172	105	97	8	64
174	107	66	41	1681
176	128	121	7	49
177	122	59	63	3969
178	34	80	-46	2116
179	23	151	-128	16384
180	80	33	47	2209
181	54	81	-27	729
182	45	56	-11	121
184	47	31	16	256
242	15	98	-83	6889
243	25	110	-85	7225
244	185	48	137	18769
245	102	29	73	5329
246	129	182	-53	2809
247	170	177	-7	49
248	112	170	-58	3364
249	40	84	-44	1936
250	96	34	62	3844
251	171	155	16	256
252	44	44	0	0
253	115	94	21	441
254	32	120	-88	7744

$$\sum_{j=1}^{190} d_{j1}^2 = 1{,}120{,}660; \quad r_{s1} = 1 - \frac{6 \sum_{j=1}^{190} d_{j1}^2}{190(190^2-1)} = 0.0197$$

To test the relationship between items 6, 7, 21, 22, 23, 24, 25, 26 and 27; and group factor 2 - Consequences of the growth of Supermarket Chains - the information in table 3 is applicable. Computing equation (98) for p = 2; $r_{s2} = -0.0644$. The null hypothesis of independent rankings on this factor will not be rejected since the computed correlation is less than the tabulated value with 188 degrees of freedom and 5% level. Hence there is no close agreement between the two rankings in table 3. This conclusion leads to the rejection of the hypothesized close association between items 6, 7, 21-27 and group factor 2. Table 3 shows this clearly. Items 6, 7, 21-27 which load highly on this factor have high rankings in column 2 but very low ranking in column 3 and the t-ratios of these items on factor p are not significant at 5% level. Thus the claim made in the exploratory factor solution IV that these items show strong influence on the problems arising from the growth of supermarket chains is refuted by the evidence provided by the regression analysis. Item 7 - processing of milk by food distributors - is a very important item loading highly on group factor 2; its rank in column 2 of table 3 is 6^{th} but this item ranks 180^{th} in column 3.

In table 4 $r_{s3} = 0.0003$. This is not significant at 5% level. This means that there is no close agreement in the grouping of items using the absolute value of the factor loadings on one hand and the absolute value of the t-ratios on the other. The lack of agreement in the rankings leads to the rejection of the hypothesis that items 30 to 37, 126, 150 and 250 affect the economic situation described by group

Table 3. Ranking of items on group factor 2 by the absolute value of the exploratory factor loadings ($a_{j2's}$) and the absolute value of the t-ratios ($T_{j2's}$) of the regression coefficients $\theta_{j2's}$

Items	Column 2* r_{ja2}	Column 3** r_{jat}	Column 4 d_{j2}	Column 5 d_{j2}^2
1	26	157	-131	17161
2	121	93	28	784
3	180	107	73	5329
4	78	186	-108	11664
5	155	156	-1	1
6	16	168	-152	23104
7	6	180	-174	30276
8	73	19	54	2916
9	181	162	19	361
10	75	179	-104	10816
11	111	117	-6	36
12	132	67	65	4225
13	22	73	-51	2601
14	102	59	43	1849
15	135	109	26	676
16	186	98	88	7744
17	74	183	-109	11881
18	146	129	17	289
19	126	177	-51	2601
20	127	36	91	8281
21	2	78	-76	5776
22	14	79	-65	4225
23	1	149	-148	21904
24	7	83	-76	5776
25	4	190	-186	34596
26	3	154	-151	22801
27	5	137	-132	17424
28	87	167	-80	6400
29	31	23	8	64
30	129	88	41	1681
31	134	182	-48	2304
32	42	104	-62	3844
33	116	175	-59	3481
34	81	145	-64	4096
35	94	53	41	1681

*r_{a2} = ranking using $/a_{j2}/$ as a measure.

**r_{t2} = ranking using $/T_{j2}/$ as a measure.

Table 3. (Cont'd)

Items	Column 2* r_{ja2}	Column 3** r_{jat}	Column 4 d_{j2}	Column 5 d_{j2}^2
36	145	12	133	17689
37	189	105	84	7056
38	63	81	-18	324
39	133	102	31	961
40	137	68	69	4761
41	89	4	85	7225
42	43	114	-71	5041
43	33	6	27	729
44	176	37	139	19321
45	98	3	95	9025
46	38	171	-133	17689
47	77	2	75	5625
48	175	50	125	15625
49	125	20	105	11025
50	21	49	-28	784
51	107	184	-77	5929
52	70	25	45	2025
53	68	14	54	2916
54	138	136	2	4
56	105	96	9	81
57	188	1	187	34969
58	179	127	52	2704
59	36	16	20	400
60	101	18	83	6889
61	86	110	-24	576
62	90	132	-42	1764
63	67	133	-66	4356
64	95	90	5	25
66	144	38	106	11236
67	108	97	11	121
68	49	158	-109	11881
69	178	128	50	2500
70	149	69	80	6400
71	160	65	95	9025
72	40	7	33	1089
73	53	9	44	1936
74	37	48	-11	121
75	60	66	-6	36
76	185	172	13	169
77	32	121	-89	7921
78	65	57	8	64
79	184	45	139	19321
80	61	43	18	324

Table 3. (Cont'd)

Items	Column 2* r_{ja2}	Column 3** r_{jat}	Column 4 d_{j2}	Column 5 d_{j2}^2
81	172	174	-2	4
82	153	142	11	121
83	50	144	-94	8836
84	29	86	-57	3249
85	85	111	-26	676
86	148	62	86	7396
87	143	161	-18	324
88	128	135	-7	49
89	69	46	23	529
90	48	141	-93	8649
91	83	34	49	2401
92	80	122	-42	1764
93	112	89	23	529
94	59	70	-11	121
95	130	44	86	7396
96	91	152	-61	3721
97	110	32	78	6084
98	79	176	-97	9409
99	39	41	-2	4
100	99	80	19	361
101	41	40	1	1
102	71	140	-69	4761
103	165	8	157	24649
104	118	31	87	7569
105	187	22	165	27225
106	15	76	-61	3721
107	122	146	-24	576
108	106	60	46	2116
109	177	71	106	11236
110	55	119	-64	4096
111	156	24	132	17424
112	131	84	47	2209
113	120	64	56	3136
114	66	134	-68	4624
115	82	126	-44	1936
116	166	120	46	2116
117	139	173	-34	1156
118	47	153	-106	11236
119	100	159	-59	3481
120	152	148	4	16

Table 3. (Cont'd)

Items	Column 2* r_{ja2}	Column 3** r_{jat}	Column 4 d_{j2}	Column 5 d_{j2}^2
121	157	160	-3	9
122	154	99	55	3025
123	109	42	67	4489
124	142	92	50	2500
125	182	166	16	256
126	30	56	-26	676
127	34	165	-131	17161
128	88	123	-35	1225
129	171	33	138	19044
130	27	47	-20	400
131	136	108	28	784
132	117	91	26	676
133	62	61	1	1
134	72	189	-117	13689
135	163	87	76	5776
136	115	163	-48	2304
137	123	143	-20	400
138	167	116	51	2601
139	119	131	-12	144
140	158	21	137	18769
141	28	39	-11	121
142	10	51	-41	1681
143	84	155	-71	5041
144	150	82	68	4624
145	168	139	29	841
146	141	147	-6	36
147	159	27	132	17424
148	190	26	164	26896
149	17	185	-168	28224
150	13	113	-100	10000
151	169	130	39	1521
152	151	115	36	1296
153	183	17	166	27556
154	12	100	-88	7744
155	57	187	-130	16900
156	97	138	-41	1681
159	24	164	-140	19600
160	23	101	-78	6084

Table 3. (Cont'd)

Items	Column 2* r_{ja2}	Column 3** r_{jat}	Column 4 d_{j2}	Column 5 d_{j2}^2
161	170	55	115	13225
162	44	72	-28	784
163	164	118	46	2116
164	162	85	77	5929
165	54	35	19	361
166	11	11	0	0
167	76	10	66	4356
168	114	77	37	1369
169	64	29	35	1225
170	46	94	-48	2304
171	96	169	-73	5329
172	25	150	-125	15625
174	51	15	36	1296
176	104	95	9	81
177	93	13	80	6400
178	140	124	16	256
179	18	106	-88	7744
180	161	30	131	17161
181	20	5	15	225
182	174	58	116	13456
184	56	112	-56	3136
242	35	75	-40	1600
243	147	151	-4	16
244	103	52	51	2601
245	52	188	-136	18496
246	173	63	110	12100
247	92	103	-11	121
248	45	181	-136	18496
249	9	170	-161	25921
250	113	54	59	3481
251	58	74	-16	256
252	19	28	-9	81
253	8	178	-170	28900
254	124	125	-1	1

$$\sum_{j=1}^{190} d_{j2}^2 = 1{,}216{,}864; \quad r_{s2} = 1 - \frac{6\sum_{j=1}^{190} d_{j2}^2}{190(190^2-1)} = -0.0644$$

Table 4. Ranking of items on group factor 3 by the absolute values of the exploratory factor loadings ($a_{j3's}$) and the absolute value of the t-ratios ($T_{j3's}$) of the regression coefficients $\theta_{j4's}$

Items	Column 2* r_{ja3}	Column 3** r_{jt3}	Column 4 dj3	Column 5 d^2_{j3}
1	131	102	29	841
2	161	86	75	5625
3	45	47	-2	4
4	105	178	-73	5329
5	63	133	-70	4900
6	79	140	-61	3721
7	24	75	-51	2601
8	121	21	100	10000
9	86	125	-39	1521
10	92	190	-98	9604
11	46	94	-48	2304
12	30	135	-105	11025
13	23	104	-81	6561
14	36	65	-29	841
15	94	78	16	256
16	90	122	-32	1024
17	183	186	-3	9
18	38	113	-75	5625
19	176	164	12	144
20	84	101	-17	289
21	171	175	-4	16
22	118	181	-63	3969
23	11	148	-137	18769
24	163	71	92	8464
25	162	177	-15	225
26	185	132	53	2809
27	137	145	-8	64
28	20	179	-159	25281
29	66	13	53	2809
30	5	57	-52	2704
31	7	90	-83	6889
32	1	171	-170	28900
33	2	141	-139	19321
34	6	185	-179	32041
35	8	98	-90	8100

*r_{a3} = ranking on group factor 3 using $/a_{j3}/$ as a measure.

**r_{t3} = ranking on group factor 3 using $/T_{j2}/$ as a measure.

Table 4. (Cont'd)

Items	Column 2* r_{ja3}	Column 3** r_{jr3}	Column 4 d_{j3}	Column 5 d^2_{j3}
36	3	14	-11	121
37	4	146	-142	20164
38	150	66	84	7056
39	120	169	-49	2401
40	22	134	-112	12544
41	44	3	41	1681
42	87	83	4	16
43	64	8	56	3136
44	124	48	76	5776
45	154	1	153	23409
46	55	116	-61	3721
47	119	7	112	12544
48	172	18	154	23716
49	71	23	48	2304
50	168	106	62	3844
51	184	107	77	5929
52	135	20	115	13225
53	182	28	154	23716
54	164	183	-19	361
56	126	182	-56	3136
57	82	2	80	6400
58	181	172	9	81
59	132	15	117	13689
60	62	22	40	1600
61	136	167	-31	961
62	187	118	69	4761
63	68	162	-94	8836
64	186	67	119	14161
66	102	26	76	5776
67	53	142	-89	7921
68	180	184	-4	16
69	149	88	61	3721
70	52	11	41	1681
71	95	44	51	2601
72	47	6	41	1681
73	101	5	96	9216
74	167	34	133	17689
76	31	64	-33	1089
76	37	136	-99	9801
77	13	97	-84	7056
78	96	144	-48	2304

Table 4. (Cont'd)

Items	Column 2* r_{ja3}	Column 3** r_{jr3}	Column 4 d_{j3}	Column 5 d^2_{j3}
79	33	58	-25	625
80	26	180	-154	23716
81	98	69	29	841
82	155	147	8	64
83	35	110	-75	5625
84	144	76	68	4624
85	41	137	-96	9216
86	123	77	46	2116
87	48	173	-125	15625
88	145	127	18	324
89	39	39	0	0
90	49	165	-116	13456
91	83	31	52	2704
92	61	114	-53	2809
93	115	91	24	576
94	127	161	-34	1156
95	159	42	117	13689
96	134	49	85	7225
97	165	63	102	10404
98	152	168	-16	256
99	77	30	47	2209
100	29	43	-14	196
101	34	74	-40	1600
102	85	170	-85	7225
103	130	12	118	13924
104	142	27	115	13225
105	166	9	157	24649
106	14	92	-78	6084
107	177	152	25	625
108	111	112	-1	1
109	93	103	-10	100
110	129	79	50	2500
111	174	46	128	16384
112	78	70	8	64
113	104	51	53	2809
114	141	189	-48	2304
115	97	81	16	256
116	122	55	67	4489
117	89	156	-67	4489
118	76	187	-111	12321

Table 4. (Cont'd)

Items	Column 2* r_{ja3}	Column 3** r_{jr3}	Column 4 d_{j3}	Column 5 d_{j3}^2
119	27	163	-136	18496
120	110	124	-14	196
121	25	105	-80	6400
122	16	119	-103	10609
123	65	174	-109	11881
124	160	149	11	121
125	99	166	-67	4489
126	12	129	-117	13689
127	143	188	-45	2025
128	40	85	-45	2025
129	157	33	124	15376
130	51	56	-5	25
131	88	108	-20	400
132	81	157	-76	5776
133	158	53	105	11025
134	148	153	-5	25
135	138	82	56	3136
136	140	150	-10	100
137	170	158	12	144
138	69	109	-40	1600
139	173	120	53	2809
140	190	37	153	23409
141	56	59	-3	9
142	10	45	-35	1225
143	54	151	-97	9409
144	19	54	-35	1225
145	67	73	-6	36
146	114	84	30	900
147	58	41	17	289
148	103	24	79	6241
149	70	131	-61	3721
150	9	80	-71	5041
151	189	62	127	16129
152	188	139	49	2401
153	80	19	61	3721
154	43	61	-18	324
155	28	130	-102	10404
156	60	111	-51	2601

Table 4. (Cont'd)

Items	Column 2* r_{ja3}	Column 3** r_{jr3}	Column 4 d_{j3}	Column 5 d_{j3}^2
159	74	138	-64	4096
160	73	87	-14	196
161	112	121	-9	81
162	175	35	140	19600
163	133	123	10	100
164	169	89	80	6400
165	91	17	74	5476
166	15	25	-10	100
167	18	32	-14	196
168	109	99	10	100
169	57	40	17	289
170	59	96	-37	1369
171	151	95	56	3136
172	72	176	-104	10816
174	128	29	99	9801
176	147	72	75	5625
177	108	10	98	9604
178	125	50	75	5625
179	32	128	-96	9216
180	179	16	163	26569
181	116	4	112	12544
183	139	68	71	5041
184	107	160	-53	2809
242	106	52	54	2916
243	75	117	-42	1764
244	50	36	14	196
245	21	155	-134	17956
246	178	115	63	3969
247	42	93	-51	2601
248	146	159	-13	169
249	100	126	-26	676
250	17	100	-83	6889
251	156	60	96	9216
252	117	38	79	6241
253	113	154	-41	1681
254	153	143	10	100

$$\sum_{j=1}^{190} d_{j3}^2 = 1,143{,}526; \quad r_{s3} = 1 - \frac{6 \sum_{j=1}^{190} d_{j3}}{190(190^2-1)} \doteq -0.0003$$

factor 3 - Size of Discounts. In column 2 of table 4 the rankings on these items are high whereas in column 3 these items have low ranking and the t-ratios are not significant at 5% level.

To test the relationship between the content of the items listed under group factor 4 and the name of this factor compute equation (98) for p = 4. Using the data presented in table 5, r_{s4} = 0.1040. This is less than the tabulated value of 0.15. Hence the hypothesis of independent ranking of items according to the measure r_{ja4} on the one hand and r_{jt4} on the other will not be rejected. This means that there is no close agreement between the classification of items according to the absolute value of the exploratory factor loadings and the classification based on the absolute value of the regression t-ratios. It follows that the second data reject the assignment of items on group factor 4. Most of the items numbered 38 to 57 were hypothesized to be closely related to group factor 4 and as should be expected these items rank highly in column 2 of table 5. Of these items only 5 have relatively high rankings in column 3, the remaining items have low ranking and the t-ratios are not significant at 5% level. Therefore, given these contradictory rankings (which means conflicting classification of items on group factor 4) it is concluded that items 38 to 57 do not affect the economic situation described by group factor 4 - Competitors' Apparent Merchandising Practices.

Under group factor 5 - Wholesale Customers' Bargaining Power - the exploratory factor solution IV hypothesized that items 58, 60, 61,

Table 5. Ranking of items on group factor 4 by the absolute value of exploratory factor loadings ($a_{j4's}$) and the absolute value of the t-ratios ($T_{j4's}$) of the regression coefficients $\theta_{j4's}$.

Items	Column 2* r_{ja4}	Column 3** r_{jt4}	Column 4 d_{j4}	Column 5 d^2_{j4}
1	133	121	12	144
2	80	97	-17	289
3	63	89	-26	676
4	114	177	-63	3969
5	20	168	-148	21904
6	34	173	-139	19321
7	186	124	62	3844
8	163	17	146	21316
9	189	146	43	1849
10	177	164	13	169
11	175	98	77	5929
12	99	106	-7	49
13	161	70	91	8281
14	65	64	1	1
15	172	92	80	6400
16	158	99	59	3481
17	121	186	-65	4225
18	66	109	-43	1849
19	134	155	-21	441
20	104	52	52	2704
21	41	118	-77	5929
22	84	119	-35	1225
23	75	137	-62	3844
24	174	94	80	6400
25	120	176	-56	3136
26	60	162	-102	10404
27	181	140	41	1681
28	28	167	-139	19321
29	56	20	36	1296
30	49	66	-17	289
31	116	122	-6	36
32	168	117	51	2601
33	32	145	-113	12769
34	144	179	-35	1225
35	100	58	42	1764

*r^{a4} = ranking on group factor 4 using $/a_{j4}/$ as a measure.

**r_{t4} = ranking on group factor 4 using $/T_{j4}/$ as a measure.

Table 5. (Cont'd)

Items	Column 2* r_{ja4}	Column 3** r_{jt4}	Column 4 d_{j4}	Column 5 d_{j4}^2
36	124	11	113	12769
37	39	143	-104	10816
38	17	62	-45	2025
39	4	169	-165	27225
40	21	102	-81	6561
41	10	4	6	36
42	12	75	-63	3969
43	14	7	7	49
44	15	38	-23	529
45	6	2	4	16
46	7	128	-121	14641
47	2	3	-1	1
48	16	28	-12	144
49	1	13	-12	144
50	18	63	-45	2025
51	11	142	-131	17161
52	8	22	-14	196
53	9	15	-6	36
54	13	183	-170	28900
56	5	131	-126	15876
57	3	1	2	4
58	117	170	-53	2809
59	115	12	103	10609
60	173	19	154	23716
61	178	161	17	289
62	27	135	-108	11664
63	58	141	-83	6889
64	31	84	-53	2809
66	82	27	55	3025
67	96	107	-11	121
68	98	166	-68	4624
69	152	134	18	324
70	85	29	56	3136
71	86	87	-1	1
72	182	6	176	30976
73	154	8	146	21316
74	54	49	5	25
75	77	60	17	289
76	35	157	-122	14884
77	183	120	63	3969

Table 5. (Cont'd)

Items	Column 2* r_{ja4}	Column 3** r_{jt4}	Column 4 d_{j4}	Column 5 d_{j4}^2
78	102	101	1	1
79	55	51	4	16
80	23	79	-56	3136
81	145	133	12	144
82	43	163	-120	14400
83	22	136	-114	12996
84	132	65	67	4489
85	79	112	-33	1089
86	119	68	51	2601
87	76	154	-78	6084
88	26	127	-101	10201
89	185	42	143	20449
90	137	174	-37	1369
91	125	35	90	8100
92	112	123	-11	121
93	105	53	52	2704
94	167	111	56	3136
95	92	44	48	2304
96	188	105	83	6889
97	184	39	145	21025
98	151	190	-39	1521
99	97	31	66	4356
100	126	67	59	3481
101	150	47	103	10609
102	57	171	-114	12996
103	138	10	128	16384
104	94	30	64	4096
105	95	21	74	5476
106	103	76	27	729
107	140	147	-7	49
108	44	85	-41	1681
109	83	88	-5	25
110	176	83	93	8649
111	110	37	73	5329
112	187	71	116	13456
113	190	61	129	16641
114	46	149	-103	10609
115	47	114	-67	4489
116	147	74	73	5329
117	131	160	-29	841
118	170	185	-15	225

Table 5. (Cont'd)

Items	Column 2* r_{ja4}	Column 3** r_{jt4}	Column 4 d_{j4}	Column 5 d_{j4}^2
119	78	150	-72	5184
120	101	151	-50	2500
121	169	153	16	256
122	130	126	4	16
123	166	80	86	7396
124	113	95	18	324
125	53	180	-127	16129
126	91	57	34	1156
127	48	181	-133	17689
128	123	100	23	529
129	37	36	1	1
130	50	45	5	25
131	149	104	45	2025
132	19	138	-119	14161
133	111	48	63	3969
134	128	189	-61	3721
135	143	78	65	4225
136	118	156	-38	1444
137	139	148	-9	81
138	67	132	-65	4225
139	90	125	-35	1225
140	72	26	46	2116
141	42	46	-4	16
142	29	50	-21	441
143	148	159	-11	121
144	136	54	82	6724
145	180	108	72	5184
146	159	116	43	1849
147	24	32	-8	64
148	45	23	22	484
149	64	178	-114	12996
150	69	91	-22	484
151	40	113	-73	5329
152	51	184	-133	17689
153	25	18	7	49
154	38	73	-35	1225
155	61	188	-127	16129
156	109	129	-20	400
159	157	175	-18	324
160	156	96	60	3600

Table 5. (Cont'd)

Items	Column 2* r_{ja4}	Column 3** r_{jt4}	Column 4 d_{j4}	Column 5 d^2_{j4}
161	127	69	58	3364
162	171	43	128	16384
163	71	115	-44	1936
164	155	81	74	5476
165	62	25	37	1369
166	153	14	139	19321
167	73	16	57	3249
168	135	90	45	2025
169	59	40	19	361
170	107	103	4	16
171	33	165	-132	17424
172	162	172	-10	100
174	122	24	98	9604
176	87	72	15	225
177	88	9	79	6241
178	36	77	-41	1681
179	129	110	19	361
180	52	34	18	324
181	30	5	25	625
183	106	56	50	2500
184	146	158	-12	144
242	164	82	82	6724
243	179	139	40	1600
244	165	41	124	15376
245	108	130	-22	484
246	74	59	15	225
247	93	86	7	49
248	68	182	-114	12996
249	70	144	-74	5476
250	89	93	-4	16
251	160	55	105	11025
252	81	33	48	2304
253	141	187	-46	2116
254	142	152	-10	100

$$\sum_{j=1}^{190} d^2_{j4} = 1{,}024{,}310; \quad r_{s4} = 1 - \frac{6 \sum_{j=1}^{190} d^2_{j4}}{190(190^2-1)} = 0.1040$$

Table 6. Ranking of items on group factor 5 by the absolute values of the exploratory factor loadings ($a_{j5's}$) and the absolute value of the t-ratios ($T_{j4's}$) of the regression coefficients $\theta_{j4's}$.

Items	Column 2* r_{ja5}	Column 3** r_{jt5}	Column 4 d_{j5}	Column 5 d^2_{j5}
1	88	128	-40	1600
2	181	95	86	7396
3	18	101	-83	6889
4	176	173	3	9
5	25	174	-149	22201
6	114	190	-76	5776
7	113	141	-28	784
8	128	18	110	12100
9	55	156	-101	10201
10	90	170	-80	6400
11	119	94	25	625
12	28	99	-71	5041
13	144	67	77	5929
14	51	61	-10	100
15	127	102	25	625
16	111	90	21	441
17	166	181	-15	225
18	140	110	30	900
19	108	144	-36	1296
20	110	45	65	4225
21	99	108	-9	81
22	27	103	-76	5776
23	107	138	-31	961
24	45	112	-67	4489
25	174	178	-4	16
26	67	163	-96	9216
27	40	127	-87	7569
28	129	154	-25	625
29	80	22	58	3364
30	7	77	-70	4900
31	121	145	-24	576
32	11	107	-96	9216
33	159	133	26	676
34	73	162	-89	7921
35	172	57	115	13225

* r_{a5} = rankings based on $/a_{j5}/$.

** r_{t5} = rankings based on $/T_{j5}/$.

Table 6. (Cont'd)

Items	Column 2* r_{ja5}	Column 3** r_{jt5}	Column 4 d_{j5}	Column 5 d^2_{j5}
36	164	11	153	23409
37	101	140	-39	1521
38	132	74	58	3364
39	15	189	-174	30276
40	186	80	106	11236
41	54	4	50	2500
42	37	81	-44	1936
43	157	7	150	22500
44	70	40	30	900
45	175	3	172	29584
46	138	121	17	289
47	115	2	113	12769
48	21	33	-12	144
49	84	19	65	4225
50	35	59	-24	576
51	102	139	-37	1369
52	31	23	8	64
53	178	14	164	26896
54	33	187	-154	23716
56	162	106	56	3136
57	150	1	149	22201
58	2	155	-153	23409
59	87	16	71	5041
60	3	15	-12	144
61	1	149	-148	21904
62	26	125	-99	9801
63	19	123	-104	10816
64	17	100	-83	6889
66	117	27	90	8100
67	173	93	80	6400
68	190	160	30	900
69	112	152	-40	1600
70	152	32	120	14400
71	104	87	17	289
72	116	6	110	12100
73	58	8	50	2500
74	53	48	5	25
75	91	64	27	729
76	56	158	-102	10404
77	16	130	-114	12996

Table 6. (Cont'd)

Items	Column 2* r_{ja5}	Column 3** r_{jt5}	Column 4 d_{j5}	Column 5 d^2_{j5}
78	94	78	16	256
79	34	58	-24	576
80	123	62	61	3721
81	20	167	-147	21609
82	169	157	12	144
83	32	142	-110	12100
84	188	73	115	13225
85	83	104	-21	441
86	57	66	-9	81
87	50	136	-86	7396
88	124	129	-5	25
89	167	43	124	15376
90	42	183	-141	19881
91	62	36	26	676
92	156	115	41	1681
93	69	54	15	225
94	143	98	45	2025
95	6	42	-36	1296
96	165	120	45	2025
97	98	38	60	3600
98	170	184	-14	196
99	97	31	66	4356
100	78	71	7	49
101	126	47	79	6241
102	189	182	7	49
103	153	9	144	20736
104	105	34	71	5041
105	137	25	112	12544
106	22	75	-53	2809
107	120	131	-11	121
108	133	68	65	4225
109	46	86	-40	1600
110	85	84	1	1
111	5	29	-24	576
112	48	79	-31	961
113	59	116	-57	3249
114	43	143	-100	10000
115	66	124	-58	3364
116	68	91	-23	529
117	24	171	-147	21609
118	49	169	-120	14400

Table 6. (Cont'd)

Items	Column 2* r_{ja5}	Column 3** r_{jt5}	Column 4 d_{j5}	Column 5 d^2_{j5}
119	151	135	16	256
120	100	148	-48	2304
121	145	186	-41	1681
122	92	118	-26	676
123	163	53	110	12100
124	75	92	-17	289
125	10	165	-155	24025
126	118	46	72	5184
127	39	185	-146	21316
128	30	113	-83	6889
129	38	35	3	9
130	158	51	107	11449
131	61	111	-50	2500
132	4	122	-118	13924
133	74	44	30	900
134	63	175	-112	12544
135	29	69	-40	1600
136	36	147	-111	12321
137	125	151	-26	676
138	187	153	34	1156
139	155	119	36	1296
140	179	24	155	24025
141	130	39	91	8281
142	168	50	118	13924
143	86	161	-75	5625
144	139	60	79	6241
145	14	114	-100	10000
146	161	126	35	1225
147	147	28	119	14161
148	52	20	32	1024
149	12	180	-168	28224
150	44	96	-52	2704
151	134	132	2	4
152	160	164	-4	16
153	149	17	132	17424
154	177	172	5	25
155	76	179	-103	10609
156	182	134	48	2304
159	184	177	7	49
160	183	89	94	8836

Table 6. (Cont'd)

Items	Column 2* r_{ja5}	Column 3** r_{jt5}	Column 4 d_{j5}	Column 5 d_{j5}^2
161	71	65	6	36
162	122	49	73	5329
163	13	105	-92	8464
164	89	85	4	16
165	142	26	116	13456
166	141	12	129	16641
167	109	13	96	9216
168	23	76	-53	2809
169	81	41	40	1600
170	148	109	39	1521
171	135	176	-41	1681
172	82	168	-86	7396
174	154	21	133	17689
176	146	70	76	5776
177	72	10	62	3844
178	103	82	21	441
179	106	97	9	81
180	171	37	134	17956
181	60	5	55	3025
182	65	52	13	169
184	96	166	-70	4900
242	180	88	92	8464
243	41	137	-96	9216
244	64	56	8	64
245	131	117	14	196
246	8	63	-55	3025
247	9	72	-63	3969
248	79	159	-80	6400
249	47	146	-99	9801
250	93	83	10	100
251	95	55	40	1600
252	77	30	47	2209
253	185	188	-3	9
254	136	150	-14	196

$$\sum_{j=1}^{190} d_{j5}^2 = 1{,}206{,}538; \quad r_{s5} = 1 - \frac{6 \sum_{j=1}^{190} d_{j5}^2}{190(190^2-1)} = -0.0554$$

111, and 132 are closely associated with group factor 5. To test this hypothesis consider table 6. The rank correlation coefficient computed from this table is -0.0554. This is less than the tabulated value at 5% level of significance and 188 degrees of freedom; and leads to the acceptance of the null hypothesis of independent ranking. Since this implies no close agreement in the classifications of items according to exploratory factor loadings and t-ratios, the second data reject the grouping of items on group factor 5. Items 58, 60, 61, 111 and 132 rank highly in column 2; their ranks are between 1 and 5. The corresponding rankings in column 3 are over 100 for three of these items and the remaining two items have rankings over 14. These five items also have non-significant t-ratios at 5% level. It follows from these contradictory classification of items on group factor 5 that the content of items 58, 60, 61, 111 and 132 do not affect the economic situation described by the name of group factor 5.

For the ranking on group factor 6 - Bottler's Bargaining Power - the information in table 7 will be used to test for independence in the rankings based on the absolute values of the exploratory factor loadings and that based on the absolute value of the t-ratios. Using equation (98) r_{s6} = 0.0319, this is less than the tabulated value of 0.15. Hence the hypothesis of independent ranking is not rejected at 5% level of significance. This means that there is no close agreement in the classification of items according to the two measures r_{ja6} and r_{jt6}. That is the second data reject the grouping of items under the

Table 7. Ranking of items on group factor 6 by the absolute values of the exploratory factor loadings and the absolute values of the t-ratios

Items	Column 2* r_{ja6}	Column 3** r_{jt6}	Column 4 d_{j6}	Column 5 d^2_{j6}
1	189	180	9	81
2	22	103	-81	6561
3	25	163	-138	19044
4	76	169	-93	8649
5	106	186	-80	6400
6	93	137	-44	1936
7	44	129	-85	7225
8	14	29	-15	225
9	120	185	-65	4225
10	169	183	-14	196
11	141	101	40	1600
12	160	70	90	8100
13	113	66	47	2209
14	72	52	20	400
15	46	162	-116	13456
16	53	89	-36	1296
17	187	166	21	441
18	114	112	2	4
19	95	139	-44	1936
20	155	25	130	16900
21	126	53	73	5329
22	124	62	62	3844
23	81	145	-64	4096
24	90	143	-53	2809
25	173	179	-6	36
26	42	189	-147	21609
27	117	142	-25	625
28	69	100	-31	961
29	158	35	123	15129
30	190	134	56	3136
31	66	126	-60	3600
32	185	67	118	13924
33	56	146	-90	8100
34	121	88	33	1089
35	32	40	-8	64

*r_{a6} = ranking based on $/a_{j6}/$.

**r_{t6} = ranking based on $/T_{j6}/$.

Table 7. (Cont'd)

Items	Column 2* r_{ja6}	Column 3** r_{jt6}	Column 4 d_{j6}	Column 5 d^2_{j6}
36	92	15	77	5929
37	172	110	62	3844
38	37	96	-59	3481
39	170	60	110	12100
40	135	37	98	9604
41	45	4	41	1681
42	164	147	17	289
43	176	8	168	28224
44	41	45	-4	16
45	30	2	28	784
46	118	188	-70	4900
47	112	1	111	12321
48	148	107	41	1681
49	178	31	127	21609
50	125	32	93	8649
51	107	165	-58	3364
52	186	33	153	23409
53	165	12	153	23409
54	68	153	-85	7225
56	150	43	107	11449
57	58	3	55	3025
58	157	80	77	5929
59	108	26	82	6724
60	138	16	122	14884
61	98	73	25	625
62	73	117	-44	1936
63	11	97	-86	7396
64	8	150	-142	20164
66	4	57	-53	2809
67	2	95	-93	8649
68	174	144	30	900
69	6	168	-162	26244
70	1	106	-105	11025
71	59	86	-27	729
72	102	10	92	8464
73	86	14	72	5184
74	184	55	129	16641
75	74	69	5	25
76	34	175	-141	19881
77	134	159	-25	625

Table 7. (Cont'd)

Items	Column 2* r_{ja6}	Column 3** r_{jt6}	Column 4 d_{j6}	Column 5 d^2_{j6}
78	100	38	62	3844
79	154	54	100	10000
80	16	27	-11	121
81	151	94	57	3249
82	55	123	-68	4624
83	49	157	-108	11664
84	20	118	-98	9604
85	111	87	24	576
86	63	64	-1	1
87	51	104	-53	2809
88	123	141	-18	324
89	64	79	-15	225
90	146	172	-26	676
91	119	46	73	5329
92	137	105	32	1024
93	35	78	-43	1849
94	7	50	-43	1849
95	104	72	32	1024
96	159	171	-12	144
97	83	28	55	3025
98	183	182	1	1
99	142	58	84	7056
100	19	92	-73	5329
101	122	41	81	6561
102	24	170	-146	21316
103	82	7	75	5625
104	130	47	83	6889
105	77	49	28	784
106	94	76	18	324
107	132	102	30	900
108	177	36	141	19881
109	28	81	-53	2809
110	52	132	-80	6400
111	15	11	4	16
112	116	121	-5	25
113	161	125	36	1296
114	110	115	-5	25
115	21	173	-152	23104
116	167	190	-23	529
117	23	181	-158	24964
118	103	116	-13	169

Table 7. (Cont'd)

Items	Column 2* r_{ja6}	Column 3** r_{jt6}	Column 4 d_{j6}	Column 5 d^2_{j6}
119	168	124	44	1936
120	57	151	-94	8836
121	136	63	73	5329
122	143	90	53	2809
123	105	13	92	8464
124	181	93	88	7744
125	40	113	-73	5329
126	18	30	-12	144
127	84	167	-83	6889
128	60	158	-98	9604
129	12	39	-27	729
130	26	71	-45	2025
131	129	128	1	1
132	153	74	79	6241
133	62	51	11	121
134	182	161	21	441
135	78	85	-7	49
136	85	131	-46	2116
137	65	149	-84	7056
138	139	184	-45	2025
139	29	108	-79	6241
140	145	18	127	16129
141	75	24	51	2601
142	91	59	32	1024
143	80	160	-80	6400
144	36	164	-128	16384
145	27	152	-125	15625
146	144	187	-43	1849
147	99	21	78	6084
148	140	22	118	13924
149	188	133	55	3025
150	163	148	15	225
151	175	174	1	1
152	101	75	26	676
153	39	20	19	361
154	166	82	84	7056
155	50	130	-80	6400
156	115	155	-40	1600
159	128	177	-49	2401
160	127	111	16	256

Table 7. (Cont'd)

Items	Column 2* r_{ja6}	Column 3** r_{jt6}	Column 4 d_{j6}	Column 5 d_{j6}^2
161	31	42	-11	121
162	10	119	-109	11881
163	5	83	-78	6084
164	3	109	-106	11236
165	96	68	28	784
166	33	9	24	576
167	149	5	144	20736
168	133	48	85	7225
169	67	34	33	1089
170	79	122	-43	1849
171	109	135	-26	676
172	97	140	-43	1849
174	61	17	44	1936
176	17	99	-82	6724
177	147	19	128	16384
178	179	154	25	625
179	87	91	-4	16
180	54	56	-2	4
181	180	6	174	30276
182	43	44	-1	1
184	171	120	51	2601
242	13	114	-101	10201
243	38	156	-118	13924
244	152	138	14	196
245	162	136	26	676
246	70	61	9	81
247	71	84	-13	169
248	9	98	-89	7921
249	131	178	-47	2209
250	156	65	91	8281
251	89	77	12	144
252	88	23	65	4225
253	47	176	-129	16641
254	48	127	-79	6241

$$\sum_{j=1}^{190} d_{j6}^2 = 1{,}106{,}758, \quad r_{s6} = 1 - \frac{6 \sum_{j=1}^{190} d_{j6}^2}{190(190^2-1)} = 0.0319$$

exploratory factor solution. In table 7 it can be easily verified that items 63 to 70, 84, 94, 130, 163, 164 and 248 have high ranking in column 2 but very low ranking in column 3. This implies a contradictory classification of items on group factor 6. The conclusion is that the second data reject the hypothesis that items 63 to 70, 84, 94, 130, 163, and 248 affect the economic situation described by the name of group factor 6.

To test the relationship between items 71 to 83 and 144; and group factor 7 - Sales Procedure and Service - consider the data in table 8. Compute equation (98) for $p = 7$; $r_{s7} = 0.0484$. The null hypothesis of independent ranking on this factor will not be rejected since r_{s7} is less than the tabulated value with 188 degrees of freedom and 5% level. This leads to the conclusion that there is no close agreement between the rankings in table 8. This can be seen by considering columns 2 and 3 of table 8. Items 71 to 83 and 144 have very high rankings in column 2 but very low rankings in column 3. This implies contradictory classifications of items on group factor 7 and thus the conclusion that the regression analysis leads to the rejection of the hypothesized relationship between items 71 to 83 and 144; and group factor 7.

To test the hypothesis that items 77, 86, 89, 91, 93, 96 and 148 are closely related to group factor 8 - Supermarket Chain Policy - consider the rankings in table 9. Using equation (98) $r_{s8} = 0.0201$, this is less than the tabulated value. Thus the null hypothesis of independent ranking of items on group factor 8 according to the

Table 8. Ranking of items on group factor 7 by the absolute values of the exploratory factor loadings and the absolute values of the t-ratios

Items	Column 2* r_{ja7}	Column 3** r_{jt7}	Column 4 d_{j7}	Column 5 d_{j7}^2
1	20	71	-51	2601
2	87	178	-91	8281
3	68	14	54	2916
4	64	133	-69	4761
5	141	167	-26	676
6	171	179	-8	64
7	102	30	72	5184
8	35	5	30	900
9	48	174	-126	15876
10	167	23	144	20736
11	54	166	-112	12544
12	114	63	51	2601
13	58	163	-105	11025
14	34	99	-65	4225
15	21	112	-91	8281
16	95	25	70	4900
17	23	31	-8	64
18	60	45	15	225
19	169	160	9	81
20	107	130	-23	529
21	146	187	-41	1681
22	47	136	-89	7921
23	183	54	129	16641
24	151	143	8	64
25	108	73	35	1225
26	112	52	60	3600
27	175	20	155	24025
28	163	79	84	7056
29	180	59	121	14641
30	161	107	54	2916
31	66	90	-24	576
32	176	120	56	3136
33	69	93	-24	576
34	100	101	-1	1
35	130	162	-32	1024

*r_{a7} = ranking based on $/a_{j7}/$.

**r_{t7} = ranking based on $/T_{j7}/$.

Table 8. (Cont'd)

Items	Column 2* r_{ja7}	Column 3** r_{jt7}	Column 4 d_{j7}	Column 5 d^2_{j7}
36	50	42	8	64
37	143	95	48	2304
38	105	170	-65	4225
39	82	2	80	6400
40	36	3	33	1089
41	122	70	52	2704
42	133	39	94	8836
43	152	58	94	8836
44	103	151	-48	2304
45	120	128	-8	64
46	160	105	55	3025
47	31	97	-66	4356
48	53	137	-84	7056
49	157	155	2	4
50	170	8	162	26244
51	142	117	25	625
52	84	138	-54	2916
53	149	181	-32	1024
54	88	91	-3	9
56	185	53	132	17424
57	26	16	10	100
58	113	144	-31	961
59	154	61	93	8649
60	184	159	25	625
61	119	1	118	13924
62	83	125	-42	1764
63	74	78	-4	16
64	129	56	73	5329
66	49	13	36	1296
67	181	10	171	29241
68	150	175	-25	625
69	165	66	99	9801
70	139	153	-14	196
71	1	88	-87	7569
72	2	100	-98	9604
73	3	161	-158	24964
74	5	168	-163	26569
75	4	72	-68	4624
76	11	156	-145	21025
77	6	80	-74	5476

Table 8. (Cont'd)

Items	Column 2* r_{ja7}	Column 3** r_{jt7}	Column 4 d_{j7}	Column 5 d^2_{j7}
78	51	173	-122	14884
79	8	77	-69	4761
80	9	176	-167	27889
81	10	28	-18	324
82	7	184	-177	31329
83	24	123	-99	9801
84	118	177	-59	3481
85	121	87	34	1156
86	72	21	51	2601
87	67	172	-105	11025
88	61	139	-78	6084
89	18	18	0	0
90	178	127	51	2601
91	22	185	-163	26569
92	71	180	-109	11881
93	140	32	108	11664
94	117	11	106	11236
95	148	146	2	4
96	63	104	-41	1681
97	188	150	38	1444
98	182	126	56	3136
99	174	114	60	3600
100	164	22	142	20164
101	138	148	-10	100
102	187	165	22	484
103	132	36	96	9216
104	80	188	-108	11664
105	127	29	98	9604
106	38	57	-19	361
107	27	85	-58	3364
108	155	94	61	3721
109	46	157	-111	12321
110	104	182	-78	6084
111	28	47	-19	361
112	135	27	108	11664
113	116	149	-33	1089
114	19	40	-21	441
115	93	152	-59	3481
116	147	84	63	3969
117	125	86	39	1521
118	153	186	-33	1089

Table 8. (Cont'd)

Items	Column 2* r_{ja7}	Column 3** r_{jt7}	Column 4 d_{j7}	Column 5 d^2_{j7}
119	14	109	-95	9025
120	55	89	-34	1156
121	85	60	25	625
122	179	48	131	17161
123	43	189	-146	21316
124	77	121	-44	1936
125	172	51	121	14641
126	40	183	-143	20449
127	99	122	-23	529
128	37	17	20	400
129	32	111	-79	6241
130	81	81	0	0
131	17	41	-24	576
132	111	83	28	784
133	94	15	79	6241
134	110	115	-5	25
135	159	190	-31	961
136	190	142	48	2304
137	109	118	-9	81
138	73	147	-74	5476
139	90	141	-51	2601
140	15	131	-116	13456
141	101	158	-57	3249
142	186	116	70	4900
143	62	34	28	784
144	12	37	-25	625
145	52	103	-51	2601
146	168	44	124	15376
147	96	113	-17	289
148	45	98	-53	2809
149	106	9	97	9409
150	98	26	72	5184
151	173	145	28	784
152	57	4	53	2809
153	70	46	24	576
154	177	140	37	1369
155	137	135	2	4
156	144	110	34	1156
159	79	76	3	9
160	78	169	-91	8281

Table 8. (Cont'd)

Items	Column 2* r_{ja7}	Column 3** r_{jt7}	Column 4 d_{j7}	Column 5 d_{j7}^2
161	145	124	21	441
162	136	50	86	7396
163	41	134	-93	8649
164	134	38	96	9216
165	13	106	-93	8649
166	91	82	9	81
167	162	12	150	22500
168	123	102	21	441
169	128	49	79	6241
170	156	67	89	7921
171	30	64	-34	1156
172	89	65	24	576
174	65	35	30	900
176	86	129	-43	1849
177	33	7	26	676
178	189	75	114	12996
179	97	96	1	1
180	59	24	35	1225
181	76	6	70	4900
182	92	108	-16	256
184	126	69	57	3249
242	44	132	-88	7744
243	124	43	81	6561
244	42	119	-77	5929
245	39	55	-16	256
246	25	154	-129	16641
247	56	19	37	1369
248	115	92	23	529
249	131	171	-40	1600
250	158	164	-6	36
251	29	68	-39	1521
252	166	74	92	8464
253	16	33	-17	289
254	75	62	13	169

$$\sum_{j=1}^{190} d_{j7}^2 = 1{,}087{,}894; \quad r_{s7} = 1 - \frac{6 \sum_{j=1}^{190} d_{j7}^2}{190(190^2-1)} = 0.0484$$

exploratory factor solution on one hand and the regression results on the other will not be rejected. This means that there is no close agreement in the rankings in table 9 which in turn leads to the conclusion that the classification of items on group factor 8 according to the exploratory factor solution is not consistent with the data in the second sample. Thus we reject the hypothesis that the contents of items 77, 86, 89, 91, 93, 96 and 148 affect the economic situation described by the name of group factor 8.

To test the hypothesis from the exploratory factor solution that items 98, 99, 100, 103, 104, 105 and 140 affect the economic situation described by group factor 9 - Wholesale Milk Drivers' Reputation - consider the rankings in table 10. Using equation (98) the computed rank correlation coefficient is 0.0442. This is less than the tabulated value. Therefore the null hypothesis of independent rankings of items according to the measures r_{ja9} and r_{jt9} is not rejected. This implies that there is no close agreement in the classification of items on group factor 9 according to the exploratory factor solution on one hand and the regression analysis on the other. This leads to the rejection of the hypothesis that the items listed above are closely related to group factor 9. This conclusion is obvious in table 10. Items 98, 99, 100, 103 and 105 relating to reactions about wholesale milk driver's unions have rankings between 1 and 5 in column 2. These high rankings suggest very close relationship between the factor and items. In column 3 these items have very low rankings thus suggesting a lack of relationship between the factor and items. Moreover the

Table 9. Ranking of items on group factor 8 by the absolute values of the exploratory factor loadings and the absolute values of the t-ratios of the regression coefficients

Items	Column 2* r_{ja8}	Column 3** r_{jt8}	Column 4 d_{j8}	Column 5 d^2_{j8}
1	175	160	15	225
2	190	100	90	8100
3	88	134	-46	2116
4	158	171	-13	169
5	161	179	-18	324
6	92	185	-93	8649
7	160	187	-27	729
8	170	21	149	22201
9	109	163	-54	2916
10	113	169	-56	3136
11	37	103	-66	4356
12	10	74	-64	4096
13	181	69	112	12544
14	56	62	-6	36
15	32	118	-86	7396
16	178	83	95	9025
17	31	182	-151	22801
18	68	114	-46	2116
19	133	153	-20	400
20	138	31	107	11449
21	22	67	-45	2025
22	64	68	-4	16
23	30	132	-102	10404
24	124	117	7	49
25	57	178	-121	14641
26	173	181	-8	64
27	154	137	17	289
28	136	143	-7	49
29	188	27	161	25921
30	89	98	-9	81
31	125	184	-59	3481
32	23	82	-59	3481
33	87	145	-58	3364
34	62	133	-71	5041
35	74	49	25	625

*r_{a8} = Ranking based on $/a_{j8}/$.

**r_{t8} = ranking based on $/T_{j8}/$.

Table 9. (Cont'd)

Items	Column 2* r_{ja8}	Column 3** r_{jt8}	Column 4 d_{j8}	Column 5 d_{j8}^2
36	169	14	155	24025
37	176	119	57	3249
38	159	79	80	6400
39	174	127	47	2209
40	14	59	-45	2025
41	107	4	103	10609
42	42	113	-71	5041
43	148	5	143	20449
44	166	34	132	17424
45	168	3	165	27225
46	180	173	7	49
47	82	1	81	6561
48	120	57	63	3969
49	96	15	81	6561
50	126	44	82	6724
51	101	190	-89	7921
52	48	26	22	484
53	171	12	159	25281
54	121	147	-26	676
56	187	64	123	15129
57	39	2	37	1369
58	153	108	45	2025
59	117	19	98	9604
60	108	18	90	8100
61	77	96	-19	361
62	104	124	-20	400
63	127	111	16	256
64	36	112	-76	5776
66	52	36	16	256
67	150	84	66	4356
68	139	146	-7	49
69	142	170	-28	784
70	182	72	110	12100
71	86	92	-6	36
72	185	6	179	32041
73	115	11	104	10816
74	26	65	-39	1521
75	80	73	7	49
76	67	177	-110	12100
77	105	130	-25	625

Table 9. (Cont'd)

Items	Column 2* r_{ja8}	Column 3** r_{jt8}	Column 4 d_{j8}	Column 5 d_{j8}^2
78	165	50	115	13225
79	60	52	8	64
80	44	35	9	81
81	38	175	-137	18769
82	144	157	-13	169
83	122	158	-36	1296
84	184	86	98	9604
85	18	88	-70	4900
86	7	61	-54	2916
87	15	115	-100	10000
88	3	129	-126	15876
89	41	53	-12	144
90	55	139	-84	7056
91	4	40	-36	1296
92	70	116	-46	2116
93	1	63	-62	3844
94	69	60	9	81
95	54	46	8	64
96	2	186	-184	33856
97	81	28	53	2809
98	137	183	-46	2116
99	93	43	50	2500
100	129	90	39	1521
101	51	39	12	144
102	28	142	-114	12996
103	140	8	132	17424
104	143	41	102	10404
105	83	29	54	2916
106	50	76	-26	676
107	85	135	-50	2500
108	162	56	106	11236
109	47	80	-33	1089
110	128	107	21	441
111	61	24	37	1369
112	106	85	21	441
113	111	109	2	4
114	179	106	73	5329
115	118	148	-30	900
116	132	122	10	100
117	151	176	-25	625
118	123	156	-33	1089

Table 9. (Cont'd)

Items	Column 2* r_{ja8}	Column 3** r_{jt8}	Column 4 d_{j8}	Column 5 d^2_{j8}
119	53	136	-83	6889
120	130	154	-24	576
121	157	141	16	256
122	156	110	46	2116
123	141	30	111	12321
124	146	78	68	4624
125	73	144	-71	5041
126	45	38	7	49
127	21	172	-151	22801
128	13	126	-113	12769
129	19	37	-18	324
130	145	45	100	10000
131	134	105	29	841
132	102	91	11	121
133	94	54	40	1600
134	43	180	-137	18769
135	131	87	44	1936
136	63	155	-92	8464
137	58	138	-80	6400
138	119	140	-21	441
139	183	121	62	3844
140	17	17	0	0
141	91	33	58	3364
142	99	58	41	1681
143	20	165	-145	21025
144	79	174	-95	9025
145	110	152	-42	1764
146	172	161	11	121
147	46	23	23	529
148	189	22	167	27889
149	6	167	-161	25921
150	9	120	-111	12321
151	98	168	-70	4900
152	149	97	52	2704
153	35	20	15	225
154	65	189	-124	15376
155	40	166	-126	15876
156	186	150	36	1296
159	25	188	-163	26569
160	24	104	-80	6400

Table 9. (Cont'd)

Items	Column 2* r_{ja8}	Column 3** r_{jt8}	Column 4 d_{j8}	Column 5 d^2_{j8}
161	103	47	56	3136
162	116	81	35	1225
163	164	101	63	3969
164	84	95	-11	121
165	11	42	-31	961
166	34	10	24	576
167	33	9	24	576
168	155	77	78	6084
169	49	32	17	289
170	27	102	-75	5625
171	97	159	-62	3844
172	152	149	3	9
174	75	16	59	3481
176	66	89	-23	529
177	112	13	99	9801
178	177	125	52	2704
179	12	93	-81	6561
180	167	48	119	14161
181	90	7	83	6889
183	100	55	45	2025
184	8	131	-123	15129
242	78	99	-21	441
243	135	151	-16	256
244	72	70	2	4
245	147	123	24	576
246	59	51	8	64
247	29	94	-65	4225
248	16	75	-59	3481
249	5	164	-159	25281
250	76	66	10	100
251	114	71	43	1849
252	95	25	70	4900
253	163	162	1	1
254	71	128	-57	3249

$$\sum_{j=1}^{190} d^2_{j8} = 1{,}120{,}226; \quad r_{s8} = 1 - \frac{6 \sum_{j=1}^{190} d^2_{j8}}{190(190^2-1)} = 0.0201$$

Table 10. Ranking of items on group factor 9 by the absolute value of exploratory factor loadings and the absolute values of the t-ratios of the regression coefficients

Items	Column 2* r_{ja9}	Column 3** r_{jt9}	Column 4 d_{j9}	Column 5 d_{j9}^2
1	190	170	20	400
2	108	106	2	4
3	137	183	-46	2116
4	111	166	-55	3025
5	166	188	-22	484
6	55	138	-83	6889
7	95	100	-5	25
8	23	31	-8	64
9	139	185	-46	2116
10	70	182	-112	12544
11	65	114	-49	2401
12	15	62	-47	2209
13	189	65	124	15376
14	125	58	67	4489
15	170	167	3	9
16	176	80	96	9216
17	106	163	-57	3249
18	112	122	-10	100
19	79	142	-63	3969
20	96	21	75	5625
21	32	47	-15	225
22	20	51	-31	961
23	81	143	-62	3844
24	162	152	10	100
25	98	174	-76	5776
26	155	176	-21	441
27	173	145	28	784
28	62	99	-37	1369
29	93	43	50	2500
30	157	147	10	100
31	180	105	75	5625
32	130	60	70	4900
33	178	150	28	784
34	43	86	-43	1849
35	25	39	-14	196

*r_{a9} = ranking based on $/a_{j9}/$.

**r_{t9} = ranking based on $/T_{j9}/$.

Table 10. (Cont'd)

Items	Column 2* r_{ja9}	Column 3** r_{jt9}	Column 4 d_{j9}	Column 5 d^2_{j9}
36	82	14	68	4624
37	123	108	15	225
38	174	95	79	6241
39	54	75	-21	441
40	91	35	56	3136
41	85	4	81	6561
42	103	168	-65	4225
43	126	8	118	13924
44	92	44	48	2304
45	169	3	166	27556
46	152	169	-17	289
47	141	1	140	19600
48	12	140	-128	16384
49	177	29	148	21904
50	133	32	101	10201
51	172	148	24	576
52	119	42	77	5929
53	134	13	121	14641
54	136	141	-5	25
56	168	36	132	17424
57	167	2	165	27225
58	187	77	110	12100
59	146	30	116	13456
60	116	17	99	9801
61	117	48	69	4761
62	151	121	30	900
63	147	93	54	2916
64	105	156	-51	2601
66	39	63	-24	576
67	118	90	28	784
68	100	135	-35	1225
69	9	161	-152	23104
70	154	124	30	900
71	121	92	29	841
72	135	9	126	15876
73	182	18	164	26896
74	80	74	6	36
75	61	72	-11	121
76	164	189	-25	625
77	74	162	-88	7744

Table 10. (Cont'd)

Items	Column 2* r_{ja9}	Column 3** r_{jt9}	Column 4 d_{j9}	Column 5 d^2_{j9}
78	6	34	-28	784
79	42	57	-15	225
80	18	19	-1	1
81	8	83	-75	5625
82	101	123	-22	484
83	37	159	-122	14884
84	68	127	-59	3481
85	10	85	-75	5625
86	109	61	48	2304
87	69	101	-32	1024
88	165	137	28	784
89	7	84	-77	5929
90	59	151	-92	8464
91	60	53	7	49
92	31	110	-79	6241
93	67	82	-15	225
94	142	45	97	9409
95	11	73	-62	3844
96	160	139	21	441
97	19	27	-8	64
98	5	190	-185	34225
99	3	71	-68	4624
100	4	98	-94	8836
101	36	38	-2	4
102	94	154	-60	3600
103	1	6	-5	25
104	13	56	-43	1849
105	2	66	-64	4096
106	56	76	-20	400
107	132	111	21	441
108	122	37	85	7225
109	87	79	8	64
110	57	144	-87	7569
111	120	11	109	11881
112	47	112	-65	4225
113	38	133	-95	9025
114	127	97	30	900
115	83	186	-103	10609
116	188	178	10	100
117	78	181	-103	10609
118	35	104	-69	4761

Table 10. (Cont'd)

Items	Column 2* r_{ja9}	Column 3** r_{jt9}	Column 4 d_{j9}	Column 5 d^2_{j9}
119	159	120	39	1521
120	77	153	-76	5776
121	129	50	79	6241
122	30	91	-61	3721
123	71	12	59	3481
124	41	87	-46	2116
125	179	102	77	5929
126	153	28	125	15625
127	102	173	-71	5041
128	33	171	-138	19044
129	28	41	-13	169
130	53	70	-17	289
131	183	132	51	2601
132	124	68	56	3136
133	66	59	7	49
134	48	177	-129	16641
135	84	96	-12	144
136	163	131	32	1024
137	26	149	-123	15129
138	21	179	-158	24964
139	140	116	24	576
140	16	15	1	1
141	90	22	68	4624
142	145	64	81	6561
143	97	155	-58	3364
144	22	67	-45	2025
145	104	175	-71	5041
146	44	172	-128	16384
147	86	20	66	4356
148	186	26	160	25600
149	46	129	-83	6889
150	63	160	-97	9409
151	115	157	-42	1764
152	89	55	34	1156
153	72	23	49	2401
154	40	119	-79	6241
155	185	126	59	3481
156	45	164	-119	14161
159	149	187	-38	1444
160	148	115	33	1089

Table 10. (Cont'd)

Items	Column 2* r_{ja9}	Column 3** r_{jt9}	Column 4 d_{j9}	Column 5 d_{j9}^2
161	144	40	104	10816
162	175	136	39	1521
163	73	88	-15	225
164	64	109	-45	2025
165	114	78	36	1296
166	128	7	121	14641
167	50	5	45	2025
168	110	52	58	3364
169	51	33	18	324
170	17	128	-111	12321
171	184	117	67	4489
172	107	130	-23	529
174	113	16	97	9409
176	161	107	54	2916
177	158	25	133	17689
178	14	184	-170	28900
179	76	89	-13	169
180	52	69	-17	289
181	131	10	121	14641
183	49	46	3	9
184	156	113	43	1849
242	24	118	-94	8836
243	58	165	-107	11449
244	171	146	25	625
245	99	134	-35	1225
246	181	49	132	17424
247	143	94	49	2401
248	75	103	-28	784
249	27	180	-153	23409
250	88	54	34	1156
251	138	81	57	3249
252	29	24	5	25
253	34	158	-124	15376
254	150	125	25	625

$$\sum_{j=1}^{190} d_{j9}^2 = 1{,}092{,}656; \quad r_{s9} = 1 - \frac{6\sum_{j=1}^{190} d_{j9}^2}{190(190^2-1)} = 0.0442$$

t-ratios of the factor coefficients of these items are not significant at 5% level. Hence on the basis of the information in the sample used in the regression analysis, the relationship established between some items and group factor 9 is rejected.

To test the relationship between the items listed under group factor 10 and Firm Dimension, consider table 11. Using equation (98) $r_{s,10} = 0.0317$, this is less than the tabulated value. Thus the null hypothesis of independent ranking of items on group factor 10 according to the exploratory factor solution on one hand and the regression results on the other will not be rejected. This means that there is no close agreement in the rankings in table 11; and thus the conclusion that the classification of items on this factor according to the exploratory factor solution is not consistent with the data used in the regression analysis. Hence the hypothesis that items 12, 28, 89, 106 to 124, 129, 141, 167, 169, 242, 243, 246, 247, 249 and 251 affect the economic situation described by group factor 10 - Firm Dimension - is rejected. Table 11 reveals this conclusion by considering the rankings in columns 2 and 3. These items have high rankings in column 2 thus suggesting the close relationship between these items and group factor 10 as claimed in the exploratory analysis but the rankings in column 3 are low suggesting that there is no close relationship between the items and the factor. This shows that the regression analysis rejects the result from the exploratory analysis.

Table 11. Ranking of items on group factor 10 by the absolute value of the exploratory factor loadings and the absolute value of the t-ratio of the regression coefficients

Items	Column 2* r_{ja10}	Column 3** r_{jt10}	Column 4 d_{j10}	Column 5 d^2_{j10}
1	141	139	2	4
2	72	99	-27	729
3	78	115	-37	1369
4	16	171	-155	24025
5	173	176	-3	9
6	80	188	-108	11664
7	170	172	-2	4
8	113	19	94	8836
9	90	154	-64	4096
10	138	166	-28	784
11	71	107	-36	1296
12	25	78	-53	2809
13	39	72	-33	1089
14	116	66	50	2500
15	62	103	-41	1681
16	100	84	16	256
17	89	189	-100	10000
18	85	111	-26	676
19	179	152	27	729
20	149	38	111	12321
21	167	81	86	7396
22	160	82	78	6084
23	133	126	7	49
24	83	110	-27	729
25	104	179	-75	5625
26	67	181	-114	12996
27	81	134	-53	2809
28	21	158	-137	18769
29	127	25	102	10404
30	95	90	5	25
31	91	164	-73	5329
32	177	86	91	8281
33	171	141	30	900
34	106	150	-44	1936
35	66	56	10	100

*r_{a10} = ranking based on $/a_{j10}/$.

**r_{t10} = ranking based on $/T_{j10}/$.

Table 11. (Cont'd)

Items	Column 2* r_{ja10}	Column 3** r_{jt10}	Column 4 d_{j10}	Column 5 d^2_{j10}
36	124	13	111	12321
37	188	131	57	3249
38	144	69	75	5625
39	69	187	-118	13924
40	176	70	106	11236
41	142	4	138	19044
42	65	96	-31	961
43	97	7	90	8100
44	183	32	151	22801
45	120	3	117	13689
46	123	147	-24	576
47	45	1	44	1936
48	38	42	-4	16
49	189	15	174	30276
50	151	50	101	10201
51	174	177	-3	9
52	52	24	28	784
53	182	14	168	28224
54	186	157	29	841
56	77	85	-8	64
57	134	2	132	17424
58	140	129	11	121
59	51	16	35	1225
60	114	17	97	9409
61	165	114	51	2601
62	155	130	25	625
63	122	118	4	16
64	31	104	-73	5329
66	187	29	158	24964
67	157	93	64	4096
68	58	159	-101	10201
69	185	160	25	625
70	73	51	22	484
71	131	91	40	1600
72	178	6	172	29584
73	99	8	91	8281
74	109	63	46	2116
75	68	67	1	1
76	190	175	15	225
77	145	124	21	441

Table 11. (Cont'd)

Items	Column 2* r_{ja10}	Column 3** r_{jt10}	Column 4 d_{j10}	Column 5 d^2_{j10}
78	86	64	22	484
79	132	49	83	6889
80	121	45	76	5776
81	50	183	-133	17689
82	130	167	-37	1369
83	154	145	9	81
84	87	77	10	100
85	56	94	-38	1444
86	115	62	53	2809
87	150	121	29	841
88	118	127	-9	81
89	27	48	-21	441
90	162	142	20	400
91	37	36	1	1
92	76	119	-43	1849
93	43	60	-17	289
94	135	71	64	4096
95	47	47	0	0
96	128	162	-34	1156
97	143	31	112	12544
98	92	185	-93	8649
99	172	35	137	18769
100	82	87	-5	25
101	53	40	13	169
102	181	148	33	1089
103	110	9	101	10201
104	74	33	41	1681
105	139	26	113	12769
106	137	73	64	4096
107	1	140	-139	19321
108	5	68	-63	3969
109	14	79	-65	4225
110	63	101	-38	1444
111	30	27	3	9
112	3	75	-72	5184
113	2	98	-96	9216
114	18	117	-99	9801
115	6	128	-122	14884
116	8	102	-94	8836
117	57	173	-116	13456
118	12	170	-158	24964

Table 11. (Cont'd)

Items	Column 2* r_{ja10}	Column 3** r_{jt10}	Column 4 d_{j10}	Column 5 d^2_{j10}
119	166	136	30	900
120	24	151	-127	16129
121	4	186	-182	33124
122	19	120	-101	10201
123	7	43	-36	1296
124	26	76	-50	2500
125	41	161	-120	14400
126	148	41	107	11449
127	54	174	-120	14400
128	48	116	-68	4624
129	17	34	-17	289
130	34	44	-10	100
131	61	106	-45	2025
132	59	108	-49	2401
133	35	53	-18	324
134	55	182	-127	16129
135	125	88	37	1369
136	75	165	-90	8100
137	158	135	23	529
138	46	132	-86	7396
139	164	122	42	1764
140	96	21	75	5625
141	11	39	-28	784
142	84	57	27	729
143	88	169	-81	6561
144	98	155	-57	3249
145	112	137	-25	625
146	102	144	-42	1764
147	28	23	5	25
148	161	22	139	19321
149	159	184	-25	625
150	79	113	-34	1156
151	70	146	-76	5776
152	94	123	-29	841
153	32	18	14	196
154	23	58	-35	1225
155	60	180	-120	14400
156	136	138	-2	4
159	153	190	-37	1369
160	152	105	47	2209

Table 11. (Cont'd)

Items	Column 2* r_{ja10}	Column 3** r_{jt10}	Column 4 d_{j10}	Column 5 d^2_{j10}
161	44	52	-8	64
162	163	61	102	10404
163	169	109	60	3600
164	117	89	28	784
165	105	30	75	5625
166	146	11	135	18225
167	33	10	23	529
168	175	80	95	9025
169	36	37	-1	1
170	156	100	56	3136
171	126	178	-52	2704
172	101	163	-62	3844
174	103	20	83	6889
176	147	83	64	4096
177	64	12	52	2704
178	184	112	72	5184
179	29	95	-66	4356
180	180	46	134	17956
181	119	5	114	12996
183	129	59	70	4900
184	42	143	-101	10201
242	20	97	-77	5929
243	15	149	-134	17956
244	9	55	-46	2116
245	108	125	-17	289
246	13	54	-41	1681
247	10	92	-82	6724
248	168	156	12	144
249	93	153	-60	3600
250	40	74	-34	1156
251	22	65	-43	1849
252	111	28	83	6889
253	49	168	-119	14161
254	107	133	-26	676

$$\sum_{j=1}^{190} d^2_{j10} = 1{,}106{,}976; \quad r_{s10} = 1 - \frac{6 \sum_{j=1}^{190} d^2_{j10}}{190(190^2-1)} = 0.0317$$

To test the relationship between the contents of items 62, 95, 161, 162, 165, 166, 167, 168 and 249, and group factor 11 - Management's Wholesale Merchandising Practices - consider the information in table 12. Using equation (98) for p = 11, $r_{s,11} = 0.0217$. This is less than the tabulated correlation coefficient at 5% level of significance and 188 degrees of freedom. Hence the hypothesis of independent ranking of items according to the measure $r_{ja,11}$ on one hand and $r_{jt,11}$ on the other will not be rejected. This means that there is no close agreement between the classification of items according to the absolute value of the exploratory factor loadings and the classification based on the absolute values of the t-ratios of the regression coefficients. It follows that the second data reject the assignment of items on group factor 11. This conclusion can be seen in table 12. Items 161 to 168 which are related to elements "determining which supermarket chains a bottler supplies with milk" have high rankings in column 2 thus revealing the strong influence of these items on group factor 11 - Management's Wholesale Merchandising Practices. This strong influence is refuted by the regression results as seen in the low rankings in column 3 of table 12. Moreover these items have non-significant coefficients on this common factor. Therefore, given these conflicting classifications of items on group factor 11, it is concluded that the items 161 to 168 do not affect the economic situation described by group factor 11.

Under group factor 12 - Cooperative Reputation - the exploratory factor solution led to the hypothesis that items 164-184, 245 and 251

Table 12. Ranking of items on group factor 11 by the absolute value of the exploratory factor loadings and the absolute value of the t-ratio of the regression coefficients

Items	Column 2* $r_{ja,11}$	Column 3** $r_{jt,11}$	Column 4 $d_{j,11}$	Column 5 $d^2_{j,11}$
1	178	134	44	1936
2	16	110	-94	8836
3	15	172	-157	24649
4	119	177	-58	3364
5	76	160	-84	7056
6	168	95	73	5329
7	118	74	44	1936
8	143	46	97	9409
9	157	147	10	100
10	132	151	-19	361
11	104	120	-16	256
12	53	67	-14	196
13	99	60	39	1521
14	11	50	-39	1521
15	173	164	9	81
16	90	84	6	36
17	92	145	-53	2809
18	164	133	31	961
19	39	128	-89	7921
20	91	18	73	5329
21	110	49	61	3721
22	51	58	-7	49
23	131	155	-24	576
24	120	179	-59	3481
25	136	184	-48	2304
26	170	163	7	49
27	183	146	37	1369
28	100	78	22	484
29	111	54	57	3249
30	21	181	-160	25600
31	169	68	101	10201
32	45	56	-11	121
33	146	150	-4	16
34	141	62	79	6241
35	72	34	38	1444

*$r_{a,11}$ = ranking based on $/a_{j,11}/$.

**$r_{t,11}$ = ranking based on $/T_{j,11}/$.

Table 12. (Cont'd)

Items	Column 2* $r_{ja,11}$	Column 3** $r_{jt,11}$	Column 4 $d_{j,11}$	Column 5 $d^2_{j,11}$
36	87	17	70	4900
37	156	105	51	2601
38	125	127	-2	4
39	175	70	105	11025
40	13	25	-12	144
41	133	8	125	15625
42	116	161	-45	2025
43	66	11	55	3025
44	49	71	-22	484
45	105	2	103	10609
46	154	148	6	36
47	181	1	180	32400
48	163	170	-7	49
49	179	57	122	14884
50	93	24	69	4761
51	55	139	-84	7056
52	159	52	107	11449
53	135	13	122	14884
54	117	165	-48	2304
56	122	30	92	8464
57	70	3	67	4489
58	167	61	106	11236
59	128	43	85	7225
60	121	14	107	11449
61	147	40	107	11449
62	12	108	-96	9216
63	80	85	-5	25
64	46	183	-137	18769
66	64	101	-37	1369
67	71	97	-26	676
68	174	144	30	900
69	31	138	-107	11449
70	140	130	10	100
71	153	87	66	4356
72	50	12	38	1444
73	180	29	151	22801
74	23	53	-30	900
75	38	63	-25	625
76	69	190	-121	14641
77	98	189	-91	8281

Table 12. (Cont'd)

Items	Column 2* $r_{ja,11}$	Column 3** $r_{jt,11}$	Column 4 $d_{j,11}$	Column 5 $d^2_{j,11}$
78	65	32	33	1089
79	14	65	-51	2601
80	74	16	58	3364
81	187	48	139	19321
82	124	94	30	900
83	54	166	-112	12544
84	68	173	-105	11025
85	186	91	95	9025
86	25	66	-41	1681
87	134	113	21	441
88	165	141	24	576
89	9	111	-102	10404
90	81	158	-77	5929
91	43	77	-34	1156
92	58	98	-40	1600
93	75	102	-27	729
94	107	44	63	3969
95	6	99	-93	8649
96	123	116	7	49
97	102	28	74	5476
98	32	174	-142	20164
99	114	83	31	961
100	160	90	70	4900
101	52	42	10	100
102	19	182	-163	26569
103	151	9	142	20164
104	78	76	2	4
105	137	114	23	529
106	109	72	37	1369
107	161	93	68	4624
108	188	22	166	27556
109	172	89	83	6889
110	24	157	-133	17689
111	106	5	101	10201
112	138	159	-21	441
113	126	36	90	8100
114	177	118	59	3481
115	185	169	16	256
116	73	121	-48	2304
117	35	153	-118	13924
118	176	81	95	9025

Table 12. (Cont'd)

Items	Column 2* $r_{ja,11}$	Column 3** $r_{jt,11}$	Column 4 $d_{j,11}$	Column 5 $d^2_{j,11}$
119	148	122	26	676
120	28	168	-140	19600
121	36	23	13	169
122	101	75	26	676
123	37	6	31	961
124	158	125	33	1089
125	162	86	76	5776
126	83	27	56	3136
127	60	176	-116	13456
128	27	185	-158	24964
129	22	45	-23	529
130	84	92	-8	64
131	94	154	-60	3600
132	41	51	-10	100
133	62	59	3	9
134	34	126	-92	8464
135	63	96	-33	1089
136	129	112	17	289
137	33	178	-145	21025
138	189	142	47	2209
139	44	103	-59	3481
140	112	21	91	8281
141	79	15	64	4096
142	190	64	126	15876
143	29	152	-123	15129
144	155	119	36	1296
145	152	180	-28	784
146	30	149	-119	14161
147	97	26	71	5041
148	56	33	23	529
149	61	106	-45	2025
150	18	186	-168	28224
151	40	131	-91	8281
152	95	47	48	2304
153	20	31	-11	121
154	142	104	38	1444
155	59	107	-48	2304
156	48	162	-114	12996
159	86	156	-70	4900
160	85	115	-30	900

Table 12. (Cont'd)

Items	Column 2* $r_{ja,11}$	Column 3** $r_{jt,11}$	Column 4 $d_{j,11}$	Column 5 $d^2_{j,11}$
161	4	39	-35	1225
162	1	171	-170	28900
163	8	73	-65	4225
164	108	137	-29	841
165	7	129	-122	14884
166	2	7	-5	25
167	3	4	-1	1
168	5	35	-30	900
169	113	41	72	5184
170	47	143	-96	9216
171	171	100	71	5041
172	89	136	-47	2209
174	182	19	163	26569
176	103	124	-21	441
177	88	38	50	2500
178	139	188	-49	2401
179	166	88	78	6084
180	144	79	65	4225
181	145	10	135	18225
183	17	37	-20	400
184	150	117	33	1089
242	57	132	-75	5625
243	42	167	-125	15625
244	130	140	-10	100
245	77	109	-32	1024
246	67	69	-2	4
247	115	82	33	1089
248	26	123	-97	9409
249	10	175	-165	27225
250	127	55	72	5184
251	184	80	104	10816
252	82	20	62	3844
253	149	187	-38	1444
254	96	135	-39	1521

$$\sum_{j=1}^{190} d^2_{j,11} = 1{,}167{,}968; \quad r_{s,11} = 1 - \frac{6\sum_{j=1}^{190} d^2_{j,11}}{190(190^2-1)} = -0.0217$$

are closely associated with group factor 12. To test this hypothesis consider table 13. Compute equation (98) for p = 12. In this case $r_{s,12} = 0.0087$. This is less than the tabulated value at 5% level of significance and 188 degrees of freedom; and leads to the acceptance of the hypothesis of independent ranking of items on group factor 12 according to the measure $r_{ja,12}$ on one hand and $r_{jt,12}$ on the other. The acceptance of this hypothesis implies that there is no close agreement in the classifications of items according to the exploratory factor loadings and t-ratios. Therefore, the second data reject the grouping of items on group factor 12; and the hypothesis that items 164-184, 245 and 251 affect the economic situation described by this factor is not supported by the data.

Thus far decisions about the hypotheses under the 12 group factors have been made. All the hypotheses considered were rejected on the basis of limited information available in the second sample. Before making any statements about the hypotheses listed under the general factors A, B, C, D, and E it is worthwhile to note that these five general factors were extracted as second-order factors from the correlations between the twelve group factors (i.e., the first-order factors). Table 14 shows the names of the first-order factors and their loadings on the rotated second-order factors. The twelve group factors and the five general factors extracted from their correlation matrix are associated as follows:

General factor A: Processors' venture

Group factor 1: Market area structure

Table 13. Ranking of items on group factor 12 by the absolute values of the exploratory factor loadings and the absolute values of the t-ratios of the regression coefficients

Items	Column 2* $r_{ja,12}$	Column 3** $r_{jt,12}$	Column 4 $d_{j,12}$	Column 5 $d^2_{j,12}$
1	183	174	9	81
2	145	80	65	4225
3	41	157	-116	13456
4	63	128	-65	4225
5	73	142	-69	4761
6	48	140	-92	8464
7	177	129	48	2304
8	162	49	113	12769
9	97	184	-87	7569
10	31	136	-105	11025
11	24	90	-66	4356
12	88	34	54	2916
13	152	143	9	81
14	55	168	-113	12769
15	111	144	-33	1089
16	180	114	66	4356
17	185	138	47	2209
18	36	88	-52	2704
19	90	185	-95	9025
20	93	30	63	3969
21	127	25	102	10404
22	173	33	140	19600
23	89	79	10	100
24	110	105	5	25
25	56	175	-119	14161
26	174	169	5	25
27	166	172	-6	36
28	163	50	113	12769
29	43	102	-59	3481
30	190	133	57	3249
31	179	141	38	1444
32	92	82	10	100
33	101	98	3	9
34	96	190	-94	8836
35	158	149	9	81

*$r_{a,12}$ = ranking based on $/a_{j,12}/$.

**$r_{t,12}$ = ranking based on $/T_{j,12}/$.

Table 13. (Cont'd)

Items	Column 2* $r_{ja,12}$	Column 3** $r_{jt,12}$	Column 4 $d_{j,12}$	Column 5 $d^2_{j,12}$
36	131	170	-39	1521
37	184	156	28	784
38	64	66	-2	4
39	151	1	150	22500
40	80	40	40	1600
41	136	73	63	3969
42	22	165	-143	20449
43	81	17	64	4096
44	91	14	77	5929
45	178	137	41	1681
46	172	178	-6	36
47	130	2	128	16384
48	108	65	43	1849
49	103	117	-14	196
50	57	97	-40	1600
51	112	12	100	10000
52	21	41	-20	400
53	118	13	105	11025
54	69	11	58	3364
56	149	96	53	2809
57	181	6	175	30625
58	150	182	-32	1024
59	75	150	-75	5625
60	78	99	-21	441
61	105	125	-20	400
62	100	180	-80	6400
63	129	189	-60	3600
64	160	113	47	2209
66	161	63	98	9604
67	164	69	95	9025
68	54	70	-16	256
69	140	122	18	324
70	125	28	97	9409
71	86	26	60	3600
72	139	179	-40	1600
73	62	23	39	1521
74	113	9	104	10816
75	142	37	105	11025
76	84	112	-28	784
77	170	93	77	5929

Table 13. (Cont'd)

Items	Column 2* $r_{ja,12}$	Column 3** $r_{jt,12}$	Column 4 $d_{j,12}$	Column 5 $d^2_{j,12}$
78	188	39	149	22201
79	27	177	-150	22500
80	141	10	131	17161
81	44	115	-71	5041
82	52	64	-12	144
83	16	54	-38	1444
84	87	132	-45	2025
85	60	107	-47	2209
86	167	118	49	2401
87	165	130	35	1225
88	23	86	-63	3969
89	106	104	2	4
90	159	7	152	23104
91	104	95	9	81
92	59	58	1	1
93	83	152	-69	4761
94	135	84	51	2601
95	68	35	33	1089
96	53	22	31	961
97	71	27	44	1936
98	28	47	-19	361
99	147	71	76	5776
100	102	5	97	9409
101	67	67	0	0
102	155	21	134	17956
103	148	160	-12	144
104	38	147	-109	11881
105	42	111	-69	4761
106	126	155	-29	841
107	50	91	-41	1681
108	26	53	-27	729
109	169	55	114	12996
110	49	78	-29	841
111	61	31	30	900
112	182	183	-1	1
113	116	4	112	12544
114	15	46	-31	961
115	134	146	-12	144
116	133	77	56	3136
117	94	154	-60	3600
118	121	162	-41	1681

Table 13. (Cont'd)

Items	Column 2* $r_{ja,12}$	Column 3** $r_{jt,12}$	Column 4 $d_{j,12}$	Column 5 $d^2_{j,12}$
119	175	87	88	7744
120	76	159	-83	6889
121	144	124	20	400
122	115	163	-48	2304
123	79	68	11	121
124	187	16	171	29241
125	51	166	-115	13225
126	25	36	-11	121
127	30	110	-80	6400
128	122	120	2	4
129	123	109	14	196
130	189	108	81	6561
131	154	52	102	10404
132	20	103	-83	6889
133	33	164	-131	17161
134	128	24	104	10816
135	132	76	56	3136
136	29	85	-56	3136
137	66	101	-35	1225
138	47	51	-4	16
139	107	94	13	169
140	168	29	139	19321
141	40	171	-131	17161
142	138	134	4	16
143	98	139	-41	1681
144	95	176	-81	6561
145	119	18	101	10201
146	137	100	37	1369
147	171	74	97	9409
148	72	126	-54	2916
149	143	72	71	5041
150	74	92	-18	324
151	117	83	34	1156
152	45	116	-71	5041
153	120	123	-3	9
154	85	158	-73	5329
155	124	186	-62	3844
156	99	153	-54	2916
159	35	75	-40	1600
160	34	127	-93	8649

Table 13. (Cont'd)

Items	Column 2* $r_{ja,12}$	Column 3** $r_{jt,12}$	Column 4 $d_{j,12}$	Column 5 $d^2_{j,12}$
161	157	32	125	15625
162	77	148	-71	5041
163	156	60	96	9216
164	58	161	-103	10609
165	176	38	138	19044
166	70	145	-75	5625
167	186	48	138	19044
168	37	81	-44	1936
169	2	8	-6	36
170	10	62	-52	2704
171	4	173	-169	28561
172	9	181	-172	29584
174	1	42	-41	1681
176	8	119	-111	12321
177	6	131	-125	15625
178	7	56	-49	2401
179	13	188	-175	30625
180	3	3	0	0
181	5	19	-14	196
183	12	59	-47	2209
184	11	43	-32	1024
242	39	121	-82	6724
243	114	187	-73	5329
244	109	61	48	2304
245	14	15	-1	1
246	146	20	126	15876
247	32	57	-25	625
248	46	45	1	1
249	19	167	-148	21904
250	82	89	-7	49
251	17	135	-118	13924
252	18	106	-88	7744
253	65	44	21	441
254	153	151	2	4

$$\sum_{j=1}^{190} d_{j,12} = 1{,}133{,}022, \quad r_{s,12} = 1 - \frac{6 \sum_{j=1}^{190} d^2_{j,12}}{190(190^2-1)} = 0.0089$$

Group factor 2: Consequences of the growth of supermarket chains

Group factor 3: Size of discounts

Group factor 6: Management's bargaining power

Group factor 11: Management's merchandising practices

General factor B: Distribution and merchandising policy

Group factor 7: Sales procedure and service

Group factor 8: Supermarket chain policy

Group factor 12: Cooperative reputation

General factor c: Problems and policies of distribution

Group factor 9: Wholesale milk drivers' policy

General factor D: Size

Group factor 10: Firm dimension

General factor E: Illegal trade practices

Group factor 4: Competitors' apparent merchandising policy

Group factor 5: Wholesale customers' bargaining power

A consideration of the foregoing shows that each of the hypotheses under the five general factors was formulated on the basis of the group factors that load heavily on the general factors. This means that the justification for the hypotheses can also be seen in the items that load heavily on the group factors that are associated to a particular general factor. With this in mind, it follows that the test criteria for the hypothesized relationships between items and general factors are provided by the decisions made on the hypotheses under the group factors.

In testing the hypothesis that the items that are closely related to group factors 1, 2, 3, 6 and 11 affect the economic situation

Table*14. Names of first order factors and loadings of first order factors on rotated second order factors[a]

Number and name of first-order factors	Rotated second order factors A	B	C	D	E
1 Market Area Structure	-71	-02	10	24	13
2 Consequences of Growth of Supermarket Chains	-83	-03	-06	05	22
3 Size of Discounts	-73	00	-02	05	39
4 Competitors' Apparent Merchandising Practices	-49	00	01	-25	56
5 Wholesale Customers' Bargaining Power	-40	-27	07	09	74
6 Bottler's Bargaining Power	-37	-17	29	25	30
7 Sales Procedure and Service	-26	-50	-09	37	27
8 Supermarket-Chain Reputation	-23	-60	35	-50	36
9 Wholesale Milk Drivers' Reputation	-04	06	79	-13	01
10 Firm Dimension	-13	-11	-13	61	-03
11 Management's Wholesale Merchandising Practices	-47	-15	28	43	35
12 Cooperative Reputation	00	-33	-03	06	01

[a] Expressed as a percent, not as a decimal.

*Source: Ladd and Oehrtman, 1971.

described by general factor A - Processors' Venture in the Market - considers the conclusions reached in regard to the items that load heavily on these group factors. For each of the group factors 1, 2, 3, 6 and 11 it was established that the items assigned to each factor did not affect the economic situation described by the factor. Therefore, transitivity relation requires that these items should show little or no influence on general factor A. Hence we reject the hypothesis that items that are closely related to group factors 1, 2, 3, 6 and 11 affect the economic situation described by general factor A.

By a similar argument as given in the last paragraph, the relationships between items and general factor B, C, D and E will be rejected since the items that form the basis of these hypotheses have been refuted as having any relationship with the group factors that load heavily on these general factors.

On the adjustments problems, the hypothesis that the factors included in the exploratory study explain little of the variation in bottlers' decisions to make or not to make certain adjustments in their operations is supported by the second sample on which the regression analysis was based. Items 131-155 relate to these adjustments. The basis for testing this hypothesis is provided by equation (103) where $m = 17$, $N = 39$, $\tau = 0.05$ and $F^{16}_{22;\,0.05} = 2.13$; that is

$$R_j^2 = \frac{16 \times 2.13}{22 + 16 \times 2.13} = 0.61$$

It can be easily verified from the last column of Appendix II that the coefficient of multiple correlations, R_j^2, of these items (with the exception of item 152) are less than 0.61. For most of the items, $R_j^2 \leq 0.45$; the smallest being 0.26. Only item 152 has $R_j^2 = 0.67$ which is greater than the computed value. Therefore the claim made in the exploratory analysis that the twelve group factors and the five general factors explain very little of the variation in bottlers' decisions to make or not to make certain adjustments in their operations is supported by the regression analysis in this study.

VI. SUMMARY AND CONCLUSIONS

Summary

The objective of the present study was to analyze the marketing problems of the fluid milk bottling industry through a factor analytic model with a view to making definite statements about the structure of this industry. The analysis amounted to developing test procedures for the hypothesized relationship between factors and items from the exploratory analysis reported in Oehrtman (1970) and Ladd and Oehrtman (1971). This exploratory factor analysis determined the relevant psychological and sociological values and economic variables, and their underlying factor structure that account for the marketing problems that the fluid milk processors face. Many of these problems were the results of the vast changes that have occurred in the processing and retailing industries connected with dairy products. The exploratory analysis identified the relevant variables from the viewpoint of the processors through a factor analytic model applied to the data supplied by the fluid milk processors.

For the exploratory analysis, a detailed questionnaire was developed, pretested and administered to a sample of milk processors in 13 states in the North Central Region. The 281 processors in the sample supplied supermarket chains with milk and expressed their reactions about fluid milk bargaining cooperatives. Responses were collected on 195 variables. The sample size of 281 processors was divided into subsamples of 242 and 39 units respectively. The first subsample of size 242 was used in the exploratory analysis; and the remaining 39 units were used in the present study. The hierarchical factor solution of these 242 observations on 195 variables was reported as Solution IV in Oehrtman (1970). The empirical results of the exploratory analysis were highly important in the present study. The matrix of factor loadings and communalities obtained from this study were presented as Appendix F in Oehrtman (1970).

By placing an arbitrary dividing line between important and unimportant factor loadings from the exploratory analysis, hypotheses were formulated. For the present study a factor loading of magnitude 0.15 in absolute value was selected as the dividing line between the important and unimportant factor loadings. The method employed in formulating the hypotheses was as follows: For each factor, hypotheses were developed concerning items that were closely related to the factor. Thus each column of the matrix of exploratory factor loadings offered a hypothesis. In chapter III groups of items were listed under each factor; it was hypothesized that these items affect the economic situation described by the factor under which they were listed. More specific hypotheses were developed under each common factor by Ladd and Oehrtman (1971). One of the objectives of this research was to test these hypotheses. Before the test could be performed it was necessary to develop statistical test procedures. The analysis was based mainly on regression model. The second sample of 39 units on 190 variables was used in this analysis. The number of variables was 190 instead of 195 as was used in the exploratory analysis because of the five suspect[1] variables whose communalities were greater than or equal to 1. These suspect variables were removed from the analysis in order to avert the problem of singularity and/or complex elements in the matrix of uniqueness α_o^2.

In order to test these hypotheses it was necessary to quantify the extracted common factors. Factor regression model was applicable. Given the 39 observations on 190 variables in the second subsample and the hierarchical factor solution of the exploratory analysis, it was possible to quantify the hypothetical factors. As a starting point, the factor regression model turned out to be representable as a multivariate classical linear regression model. The estimates of the common factors, obtained by the use of the expression commonly found in the literature (see equation 122 of chapter IV), were not ideal for the analysis reported here. Thus adjustments were made in such a way that for each item a separate estimate of the matrix of common factors was obtained. The adjustments made purged the estimates $\hat{f}_s^j$ $(j = 1, 2, \ldots, 190)$ of the influence of the variable Z_j. Hence it was possible to use Z_j as the dependent variable and the associated quantified factors $\hat{f}_s^j$ as the independent variables. Using regression model, an attempt was made at reproducing the matrix of exploratory factor loadings. The reproduced factor coefficient was denoted by $\hat{\theta}'$ and was presented in Appendix II.

In the analysis it was seen that the regression model did not reproduce the matrix of factor loadings obtained in Oehrtman's hierarchical factor solution IV; that is $A_o \neq \hat{\theta}'$. As could be easily verified,

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[1]See the discussion on empirical results and particularly table 2 in chapter V, p. 95.

the elements of matrix A_o, were less than unity in absolute value, whereas those of Appendix II, $\hat{\theta}'$, were mostly greater than unity in absolute value. In fact many elements of $\hat{\theta}'$ were greater than 20.00 while some were above 100.00 in absolute value. The method of multivariate statistical inference developed for investigating the hypotheses in chapter III were not used because of the large discrepancies between the matrix of exploratory factor loadings and the matrix of reproduced factor coefficients. These methods, however, were reported in this thesis because it was felt they were the ideal methods to follow if $\hat{\theta}'$ had come closer to expectations.

In investigating the various hypotheses under the common factors, the nonparametric method of rank correlation coefficient was employed. For the most part the hypotheses were rejected for all the items listed under the twelve group factors. Since the correlation matrix among the group factors provided the basis for the extraction of the five general factors it followed that the rejection of all the hypotheses under the group factors led to the rejection of the hypotheses under the general factors. The hypothesis on adjustment problems faced by fluid milk bottling processors was investigated by a consideration of the multiple correlation coefficients of the regression model of those items relating to adjustment problems on all the common factors. This nonconventional test procedure was valid and in fact necessary considering the fact that the hypothesis concerning adjustments was formulated on the basis of the magnitude of the communalities of the items relating to adjustment problems in the fluid milk bottling industry.

Conclusions

The procedures developed in chapter V provide a tool through which factor analysis might be more widely used in economic analysis. Because of the limited information available in this study it was not possible to reproduce the matrix of exploratory factor loadings either wholly or in part. The observed disparity between the reproduced factor coefficient matrix $\hat{\theta}'$ and the matrix of factor loadings A_o was too large to be attributed solely to chance. The main reason for this disparity, in the author's opinion, was in the size of the sample used in the factor regression. The exploratory analysis was based on 242 observations while the present analysis was based on only 39 observations. The difference in these sample sizes was enough to account for a large proportion of the differences in the results of the exploratory factor analysis and the results of the present study.

The hypothetical factors were unobserved. Their values were estimated for use in the regression analysis. Thus it was necessary that as much information should be included in the estimating expression (equation 20, chapter V) of these factors as was used in the exploratory analysis.[1] Since $\hat{f}_s^j$ was an estimate of the matrix of hypothetical factors it was subject not only to the errors of measurements in the

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[1] By the time the problem of insufficient sample observations occurred to the author the analysis has been carried almost to the end and funds were also running low. Thus it was impossible to increase the sample size by administering the questionnaire to new processors and thereby increasing the sample observations to the desirable level.

responses of the sampled processors but also it was prone to be affected by the errors in the estimation of the exploratory factor loadings. The smaller the amount of information available for estimating the hypothetical factors, the less was the precision of the estimated factors; and more importantly, the accuracy of the regression model to reproduce the factor loadings depended, to a large extent, on the precision of the estimated factors.

In regression analysis it is desirable that the least-squares estimates of regression coefficients have the large sample properties of consistency and asymptotic efficiency. That is, the estimated regression coefficients should approach the true value as the sample size increases and the standard error of the estimate should be small. Thus if it is assumed that the exploratory factor loadings reflected the true relationship between the factors and the items, then the regression model would yield factor coefficients that would approach the exploratory factor loadings as the sample size increased. It was the conviction of the author that the ideal sample size for any confirmatory factor analysis that follows the methodology developed in chapter V should be at least as large as the sample size used in the exploratory factor analysis.

For reasons attributable to small sample size, the conclusions from this analysis should be regarded as tentative subject to an over-riding result from future research in this area. On the basis of the nonparametric test statistics used, the conclusions reached are presented below. Each of the hypotheses in chapter III was formulated on the basis of items that the exploratory analysis claimed to be important in influencing a particular common factor. Under each common factor below are listed the items that are hypothesized to be closely related to the common factor. The conclusion from this study was that none of these items under a particular common factor were closely related to the common factor:

Group factor 1: Market area structure

Item numbers: 2, 3, 4, 5, 6, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 87, 148, 159, 160

Group factor 2: Consequences of the growth of supermarket chains

Item numbers: 6, 7, 21, 22, 23, 24, 25, 26 and 27

Group factor 3: Size of discounts

Item numbers: 30, 31, 32, 33, 34, 35, 36 and 37

Group factor 4: Competitors' apparent merchandising practices

Item numbers: 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 56 and 57

Group factor 5: Wholesale customers' bargaining power

Item numbers: 58, 60, 61, 132 and 111

Group factor 6: Management's bargaining power

Item numbers: 63, 64, 65, 66, 67, 69, 70, 84, 94, 130, 163, 164 and 248

Group factor 7: Sales procedure and service

Item numbers: 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83 and 144

Group factor 8: Supermarket chain policy

Item numbers: 77, 86, 88, 91, 93, 96 and 148

Group factor 9: Wholesale milk drivers' policy

Item numbers: 98, 99, 100, 103, 104, 105 and 140

Group factor 10: Firm dimension

Item numbers: 12, 28, 59, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 118, 120, 121, 122, 123, 124, 129, 141, 167, 179, 242, 243, 244, 246, 247, 249 and 251

Group factor 11: Management's merchandising practices

Item number: 62, 95, 161, 162, 163, 165, 166, 167, 168 and 249

Group factor 12: Cooperative reputation

Item numbers: 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 245 and 251

General factor A: Processors' venture in the market

Items on group factors: 1, 2, 3, 6 and 11

General factor B: Distribution and merchandising policy

Items on group factors: 7, 8 and 12

General factor C: Problems and policies of distribution

Items on group factor: 9

General factor D: Size

Items on group factor: 10

General factor E: Illegal trade practices

Items on group factors: 4 and 5

On the adjustment problems it was the conclusion from this study that very little of the variation of bottlers' decisions to make or not to make certain adjustments in their operations was explained by the regression analysis used in the study reported here.

In general no items were found to show strong influence on the factors which the exploratory analysis showed to be important in the marketing problems of the processors. This is a rather startling result! Due to the limitations in the sample size it is believed that no valid statements could be made about the market structure of the fluid milk industry at this point.

In the discussion of market structure analysis in chapter II nine elements were included as new dimensions in market structure analysis. These elements were:[1]

1. Structure of closely related industries
2. Contractual arrangements
3. Laws and regulations
4. Some basic economic and technological features of products and processes
5. Attitudes, knowledge, goals and the perception of the businessmen
6. Effect of conduct and performance on structure
7. Effect of conduct and performance on attitudes, goals and perception of businessmen

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[1]See chapter II for more discussion on these elements.

8. Determination of the markets and industries in which a firm will sell
9. Firm growth and decline

No attempts were made to relate these elements to the factors. Each of these elements was suggested by the wording of the statements concerning problem areas and the questions in the survey questionnaire. It follows that the general rejection of the hypothesis on the association of items and common factors revealed that these elements could not be considered given the general conclusion that no items were important in the marketing problems faced by the fluid milk processors. It is the author's belief that future research, with adequate number of observations, will shed some light on how market analysts can include these nine elements in their market structure analysis.

VII. SUGGESTIONS FOR FURTHER STUDIES

There is a very severe limitation on the factor regression model used in this analysis. The sample observations applied to this model are too few. Hence the results of the present study are tentative. No generalizations can be made. It is hoped that future research in this area will overcome this limitation and thus yield results that will lead to the development of a sound working model for analyzing the industrial structure of the fluid milk bottling industry. It is the author's view that the sample size needed for the type of analysis used in this study should be at least as large as the sample size used in the exploratory analysis. It is therefore suggested that the hypotheses in chapter III be retested in future research effort in this area. The statistical inference procedures developed in chapter V are applicable and in fact the procedures should be used to serve as a test of the validity of the methods whenever adequate sample observations are used. With large enough sample observations, the methods discussed in this thesis should be able to reproduce factor coefficients that should be less than unity in absolute value and any difference between the reproduced factor coefficients and the matrix of exploratory factor loadings can be attributed to the incompatibility of the two samples.

The method of factor measurement given in this thesis should be reviewed with care. The method requires the estimation of n separate matrices for the n items under investigation. When n is large, the work of factor measurements may be very expensive even on high speed electronic computers. Thus it is necessary that any analysis that requires the use of this method should give a very careful consideration to costs and adjust the number of variables under investigation and the number of observations in the two analyses to the optimum level.

There are other methods for testing the hypotheses formulated from the exploratory factor analysis results aside from the procedure employed in this study. Two of these are: a) Use a new set of observations and compute new factor loadings from these new data then compare these new factor loadings with the ones obtained in the exploratory analysis. b) Use the hypotheses formulated from the exploratory analysis to construct a theoretical model and subject the predictions from this model to statistical analysis.

The questionnaire used to obtain responses from processors is not perfect. A number of suggestions for improving the questionnaire are presented in Oehrtman (1970).

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X. APPENDIX I:

ESTIMATED VALUES OF THE COMMON FACTORS

Table 15. . Estimated values[a] of the common factors

Obser-vations	A	B	C	Factors D	E	1	2	3
1	-2.18	2.15	1.97	0.53	0.57	-3.33	1.84	0.91
2	-1.44	-1.12	-0.72	0.63	-0.67	-0.88	0.71	-0.25
3	0.18	0.60	0.31	0.19	0.74	0.53	0.48	-1.60
4	-0.10	0.65	1.01	0.02	0.37	-1.27	0.63	-0.08
5	0.36	1.31	-0.16	0.52	0.28	-1.90	0.84	0.37
6	-0.31	-0.20	-0.12	-0.17	0.01	-0.50	0.29	0.73
7	0.07	-0.65	0.04	1.04	-0.65	0.66	-0.08	-0.22
8	0.64	-3.12	-2.21	-0.49	2.55	4.59	-4.87	0.35
9	1.50	-0.96	-0.90	-0.57	-1.43	1.73	-1.47	-0.34
10	-1.70	1.83	1.94	-0.96	-0.01	-4.63	2.78	1.28
11	0.26	1.22	1.04	0.53	-0.55	-0.76	0.10	-0.04
12	-0.01	1.86	1.23	0.21	-0.07	-2.60	1.28	0.91
13	-0.55	-2.58	-1.34	0.45	1.23	4.20	-2.56	-0.57
14	-0.68	-0.30	-1.16	-0.25	-0.57	0.74	0.07	-0.12
15	-1.88	3.26	3.06	0.79	-0.39	-5.60	4.19	1.09
16	-0.06	2.64	2.23	0.17	-0.02	-3.56	2.39	-0.74
17	-0.63	1.65	-0.05	1.12	-1.15	-2.24	1.87	-0.05
18	-0.42	0.02	-0.73	-0.28	0.09	-0.10	0.25	0.57
19	-0.15	-0.63	-0.78	-0.48	1.28	-0.03	-0.42	0.30
20	0.80	-0.23	0.95	·0.89	0.38	0.24	-0.01	-0.96
21	1.18	2.40	2.78	-0.55	0.79	-3.03	1.04	-0.18
22	-0.07	-0.94	-0.76	0.27	-0.47	1.28	-1.13	0.33
23	0.27	-0.68	-0.87	-0.51	-1.42	1.20	-0.36	0.10
24	-0.53	0.19	-0.50	0.91	-0.45	0.67	-0.46	0.34
25	1.34	0.04	2.26	-0.23	0.81	-2.41	0.90	-0.72
26	-1.46	1.51	0.76	-0.01	-0.97	-1.77	2.44	-0.12
27	0.61	-0.82	-0.25	0.46	-0.45	1.18	-0.62	-0.82
28	-0.76	-0.20	-0.82	-0.75	0.03	1.43	-0.29	-0.20
29	-0.29	0.01	0.05	0.22	0.95	-0.35	-0.21	-0.17
30	-0.11	0.40	-0.21	-0.05	-0.84	-0.57	0.83	-0.22
31	-0.47	0.50	0.80	0.26	0.03	-1.12	1.14	-0.72
32	0.69	-0.55	1.01	0.08	-1.06	1.36	-0.23	-0.80
33	0.23	-0.50	0.59	0.63	0.41	1.42	0.14	-2.14
34	0.34	-0.40	-0.71	-0.18	-0.31	0.92	-0.80	0.05
35	0.21	-0.70	-2.17	-0.64	1.10	2.68	-2.67	0.40
36	0.77	-0.85	-2.51	-0.86	-1.24	2.08	-1.14	0.21
37	-0.08	-2.18	-2.71	0.30	1.00	4.65	-3.43	0.63
38	1.92	-0.81	-1.10	0.07	-1.13	1.65	-1.88	0.26
39	-0.96	-0.45	-0.81	-0.65	0.06	0.17	0.19	-0.32

[a] This matrix is obtained through the use of equation (121), p.48, chapter IV.

Factors								
4	5	6	7	8	9	10	11	12
0.42	-1.38	-0.51	1.84	0.18	1.24	1.23	1.21	-4.68
1.47	-1.83	1.68	0.66	-0.39	-0.41	-0.16	-0.15	0.74
-0.63	1.48	0.21	-0.01	-0.50	-0.33	-1.48	0.84	0.36
0.62	-0.31	-0.38	1.54	-0.23	1.22	0.24	-0.41	-1.66
0.34	0.24	0.19	0.56	0.06	0.78	2.73	-1.58	-2.30
-0.31	-0.06	-0.42	0.37	0.06	0.20	0.16	-0.25	-0.31
-0.76	0.81	-0.73	-0.39	-0.01	0.52	1.10	-0.58	0.85
1.04	1.49	1.45	-3.59	0.29	-2.86	-1.87	1.41	7.54
-0.17	-0.22	-1.43	-0.25	0.38	-0.96	-0.76	1.50	2.35
0.63	-1.82	-0.81	3.20	0.46	2.15	2.33	-1.11	-6.25
0.77	-0.81	0.65	1.06	-0.40	1.17	0.39	-0.56	-0.80
-1.08	-0.05	-0.02	1.58	0.05	1.23	0.94	-0.76	-3.90
-0.93	1.31	0.63	-2.26	0.27	-2.56	-2.50	1.90	4.55
0.27	-0.10	0.00	-1.18	-0.32	-0.76	-0.85	0.34	2.38
-0.30	-1.62	-0.75	4.45	-0.12	3.36	2.68	-1.34	-8.13
1.66	-0.77	-0.58	2.64	-0.05	2.90	3.52	-2.06	-5.85
0.82	-1.02	-0.61	1.84	-0.42	1.37	2.15	-1.23	-3.02
-0.57	0.32	-1.10	-0.05	-0.25	-0.18	-1.02	0.42	0.48
-0.41	1.09	0.31	-1.39	0.65	-1.03	1.21	-0.07	0.27
-0.40	0.90	0.64	-0.58	0.42	-0.17	0.28	0.12	0.41
-0.18	0.17	-0.03	1.89	0.70	1.59	2.36	-0.11	-5.30
-0.62	-0.11	0.71	-1.35	-0.25	-1.26	-2.13	1.56	3.29
-0.60	-0.32	0.62	0.08	0.08	-0.75	-0.68	-0.55	1.60
-0.15	-0.15	-0.02	0.33	-0.08	-0.06	0.37	-0.30	0.12
-0.28	0.33	-0.26	2.25	0.36	1.72	1.16	-0.81	-3.70
0.07	-0.41	-0.76	1.39	-0.07	1.64	2.09	-1.91	-3.66
-0.43	0.56	0.52	-1.18	0.18	-0.56	0.35	-0.00	1.40
-0.12	0.54	0.21	-0.84	-0.08	-0.99	-1.01	0.39	1.87
0.69	0.37	-0.20	-0.23	-0.04	-0.26	0.33	1.32	1.03
0.11	-0.06	-1.75	0.17	-0.06	0.53	0.68	0.10	-0.73
0.22	0.01	-0.65	0.38	-0.22	0.68	0.13	0.55	-0.85
-0.83	0.46	0.96	-0.50	-0.07	0.55	-0.26	-1.25	1.00
0.32	0.99	0.81	-0.56	-0.61	0.17	-0.99	-0.04	1.21
-0.13	0.11	0.30	-0.49	-0.19	-0.53	-1.05	0.05	1.75
0.53	0.85	-1.06	-1.23	-0.06	-1.66	-1.88	1.22	3.38
-0.58	0.38	-0.91	-0.13	-0.16	-1.38	-1.63	-0.15	3.05
-1.49	1.62	0.94	-2.20	-0.08	-2.94	-3.34	0.97	5.58
0.22	0.34	0.71	-1.63	-0.59	-0.25	-0.90	-0.60	4.57
0.36	0.08	0.12	-1.23	-0.34	-0.81	-1.43	0.85	1.94

XI. APPENDIX II:

MATRIX OF FACTOR COEFFICIENTS OBTAINED FROM REGRESSION ANALYSIS AND THE MULTIPLE CORRELATION COEFFICIENTS

Table 16. Matrix of factor coefficients obtained from regression analysis and the multiple correlation coefficients

Items	A	B	C	D	E	1	2	3
1	-0.8	0.5	4.7	7.2	-7.4	-2.8	2.9	5.3
2	-0.5	1.6	-12.0	8.2	-8.3	1.2	7.5	5.9
3	0.7	0.5	0.1	8.9	-8.1	-0.3	6.2	8.3*
4	-1.2	0.9	-3.6	1.4	-1.1	-0.6	0.5	-0.6
5	0.5	0.1	1.1	2.9*	-1.0	0.9	2.8	2.8
6	-1.5	-0.5	- 9.2	-3.1	0.1	2.0	-2.0	-2.6
7	-0.3	-1.8	14.2	5.4	-6.8	-2.8	0.8	6.2
8	-0.1	10.1*	-41.1	33.6*	-28.0*	-0.2	27.1*	17.5*
9	0.2	1.4	1.1	5.8	-3.8	-0.6	2.5	3.2
10	-0.2	-1.8	1.5	-4.2	1.8	2.2	-1.0	-0.2
11	-1.1	1.8	-11.9	9.0	-9.6	0.3	6.8	5.8
12	-0.6	-4.4	20.0	-7.8	4.9	-0.6	-8.9	-2.2
13	0.4	-3.9	21.6	-9.3	9.1*	-0.7	-9.2	-4.1
14	-0.8	2.7	-20.4	9.1	-8.8	2.0	9.9	5.8
15	-0.2	0.6	-2.2	8.2	-6.7	-1.1	5.3	5.2
16	-1.1	2.4	-13.8	6.9	-5.7	0.2	5.3	2.4
17	-0.9	-0.3	-2.9	-0.0	-0.7	1.3	0.6	0.3
18	-0.4	-0.8	7.9	-5.7	6.0	0.7	-3.9	-3.1
19	-1.5	0.4	-7.3	3.1	-4.4	-0.3	1.0	1.3
20	-0.5	5.6	-30.4	11.4	-7.9	2.5	12.1	3.4
21	-1.0	3.5*	-22.2	7.3	-4.0	1.6	7.7	0.6
22	-1.4	4.7	-21.5	7.4*	-4.3	1.1	7.9*	0.3
23	-0.7	-1.2	5.1*	-3.8	2.8*	0.9	-2.1	-1.3
24	0.7	2.2	-3.6	7.2*	-6.0*	0.1	7.5	5.5
25	-0.2	-0.6	2.3	-1.1	1.2	0.3	0.0	-0.6
26	-0.3	0.7	2.0	1.9	-1.6	-0.2	2.3	2.1
27	-0.9	1.2	-5.9	4.6	-4.4	-1.4	3.6	1.9
28	0.2	-0.3	11.8	-0.7	2.7	-1.8	-1.6	-0.5
29	0.6	5.0	-29.0	25.6*	-23.7*	0.9	22.7*	17.7*
30	-0.6	1.9	- 5.5	10.2	-10.0	-0.9	7.4	7.2
31	-0.9	0.5	-10.3	-5.6	5.6	3.3	-0.6	-4.3
32	-1.2*	1.6	-14.0	4.8	-3.2	0.3	3.6	0.5
33	-1.7*	1.0	- 4.4	3.1	-4.4	-1.3	0.9	1.9
34	-1.0	0.6	-14.5	0.8	-0.3	2.4	2.8	-0.2
35	0.2	-3.6	28.6	-10.5	10.3	-2.7	-10.7	-4.4
36	-0.6	-5.0	29.3	-19.6	17.7**	-0.6	-16.2*	-10.3**
37	-0.9	-1.4	6.3	-2.8	2.5	-1.0	-3.6	-1.2
38	-0.1	2.4	-11.4	10.0	-6.9	-0.7	7.4	5.2
39	-0.4*	-0.8	-15.1	-4.5	2.0	4.7	1.8	-1.2
40	-0.4	-2.3	25.6	-6.2	5.8	-3.2*	-7.8	-1.8

*Significant at 5% level.

**Significant at 1% level.

4	5	6	7	8	9	10	11	12	R_j^2
12.0	12.5	-0.8	0.6	18.6	-5.5	-7.9	-3.9	-0.1	50
14.5	17.1*	8.1	-0.0	44.8*	20.6	-13.4	4.9	-0.9	69**
15.1*	15.6	2.4	1.1	29.0	1.7	-10.6	-1.0	0.3	68*
2.2	2.9	1.9	-0.2	12.9	6.6	-3.7	0.8	0.5	59
3.9	2.8	-0.2	0.1	6.8*	-1.1	-3.0	-1.5	-0.4	73**
-3.1	-0.0	5.7*	-0.1	3.9*	14.4*	0.9	6.5	-0.4	73**
10.3	8.9	-5.7	0.8	-2.9	-21.7	-3.3	-7.6	-0.5	76**
51.4*	59.7*	28.0	1.6*	175.0*	70.3	-50.4*	12.9	1.4	82**
7.8	6.8	0.2	-0.1	16.9	-1.5	-6.3	-2.4	-0.1	53
-4.1	-3.6	-0.3	1.0	-14.5	-2.1	5.2	2.1	-0.5	46
16.0	19.0	9.3	-0.1	48.7	20.9	-14.1	4.8	-1.0	65
-10.7	-13.5	-11.8	0.5	-57.1	-32.9	14.8	-7.5	-1.4	59
-16.4	-20.5	-13.4	-0.1	-63.9**	-34.9*	16.8*	-8.9	0.4	70**
16.1	20.4	13.5	-0.3	63.8**	35.6	-16.9	9.2	0.1	78**
12.5	13.1	2.3	-0.3	30.4**	4.9	-10.9	-1.1	0.3	71**
10.4	12.7	7.7	-0.7	43.7	23.2	-12.1	5.2	0.5	59
0.6	1.4	2.0	-0.7	4.7*	5.8*	-0.6	2.4	-0.4	70**
-9.5	-11.6	-5.8	0.5	-29.9	-13.1	9.0	-2.8	0.7	63*
5.6	8.7	5.1	0.1	20.4	12.4	-5.9	3.8	-0.0	45
15.7	20.9	18.0	-0.2	83.1	49.8	-20.8	12.3	1.3	30
8.7	11.7	12.4	0.0	55.8**	35.8*	-13.2	8.7*	1.5	78**
8.9	12.3	12.2	0.2	57.0**	35.2	-13.5	8.2	1.4	77**
-5.6	-6.4	-2.7	-0.4	-19.6*	-7.7*	6.0	-1.0	-0.6	86**
12.0*	12.1	3.8	0.2	30.5**	7.0	-9.7	0.6	-0.6	83**
-2.0	-2.1	-0.8	0.5	-7.1	-3.9	2.3	-0.4	0.1	73**
3.2	3.4	-0.1	0.5	4.3	-2.6	-1.9	-1.1	-0.1	85**
7.1	9.9**	4.3	0.9	24.8**	10.0	-7.1	2.2	0.1	85**
-3.1	-6.0	-7.0	0.4	-20.1	-19.2	4.6	-6.3	1.0	80**
43.5*	49.3*	21.5	0.6	129.4	50.8	-38.5	10.5	-0.7	72**
18.0	19.6	5.3	0.3	43.1	10.4	-14.4	0.6	-0.4	59
-8.3	-6.6	4.7	-0.3	-3.4	16.0	3.9	7.0	-0.3	74**
6.2	8.3	7.9	0.1	34.7	22.2	-8.8	5.7	0.5	71**
6.3	8.5	3.4	0.3	18.9	7.8	-5.7	1.7	-0.6	70**
1.2	3.8	8.0	-0.3	22.5	22.8	-4.6	7.4	-0.0	84**
-17.5	-23.3	-17.5	0.1	-78.0	-47.5	20.2	-12.5	0.3	71**
-30.9*	-37.1**	-19.6*	-0.4*	-107.8*	-49.7*	30.8*	-10.9	-0.1	81**
-4.4	-5.3	-4.3	0.2	-19.9	-11.3	4.8	-2.9	0.2	84**
14.8	16.1	7.3	-0.1	49.3	19.4	-14.8	3.0	0.8	78**
-3.0	-0.2	8.9	-0.4**	8.4	24.3	0.5	10.6	-1.9**	88**
-9.6	-13.9	-14.9	1.2**	-56.6	-42.0	13.9	-12.0	1.0	88**

Table 16. (Cont'd)

Items	A	B	C	D	E	1	2	3
41	0.2	6.1**	-32.1**	21.9**	-18.6**	1.4	19.6**	12.8**
42	-0.3	-0.7	2.3	-7.8	7.6	1.6	-4.2	-4.2
43	-0.3	9.2**	-41.9**	28.4**	-23.3**	0.6	24.0**	14.7**
44	-0.7	-3.0	15.2	-11.0	8.0	0.1	-8.8	-4.4
45	-0.3	7.4**	-43.7**	25.4**	-22.7**	2.3**	23.6**	14.7**
46	-0.1	0.5	-1.5	-2.2	4.0	1.6	-0.7	-1.8
47	0.5**	-6.5**	34.4**	-16.8**	13.2**	-0.8	-14.0**	-5.5**
48	-0.4	1.3	-3.9	13.2	-12.9	-2.4	8.0	9.8*
49	-0.9**	2.8*	-14.7	13.7**	-12.6*	-1.1	9.8*	8.0*
50	0.0	-3.7	30.4	-10.2	9.4	-3.1	-10.6	-3.7
51	0.1	0.4	-3.3	-1.9	3.8	1.0	0.3	-2.1
52	-0.5	-3.6	17.7	-15.8*	14.9*	0.0	-12.9*	-9.5*
53	-0.9	9.0**	-43.3*	23.5*	-18.3*	0.5	20.5*	10.0*
54	-1.0*	-0.5	3.5	-1.8	-0.1	-0.6	-1.8	0.2
56	-1.1*	2.3	-15.9	3.0	-1.8	1.2	3.4	-0.2
57	-0.7*	-7.3**	32.7**	-23.9**	19.1**	-0.0	-19.4**	-11.8**
58	0.2	-2.6	12.1	-1.5	1.8	-1.2	-3.0	0.5
59	-1.2	-9.2*	38.1	-31.0*	26.6*	0.0	-26.2*	-17.5*
60	-1.0	5.3	-39.5*	22.7*	-21.0*	1.5	19.9*	12.8*
61	-0.5	1.1	-13.5	1.9	0.0	1.9	3.2	-0.6
62	-0.5	0.9	-9.3	4.4	-4.3	0.8	4.3	3.3
63	-1.6	2.6	-17.5	5.3	-5.4	-0.1	5.0	1.5
64	-0.9	-1.7	3.4	-10.8	9.1	1.1	-7.8	-7.0
66	-0.2	-3.7	14.0	-14.6	13.2*	1.3	-10.4	-8.1
67	0.6	-3.0	10.4	-5.3	5.1	0.4	-4.5	-1.4
68	-1.0*	2.6	-9.3	2.9	-1.7	-0.1	2.9	0.4
69	-1.9	-1.0	-4.8	-6.2	4.0	0.5	-5.4	-6.0
70	-0.6**	0.3	2.5	-3.6	5.3*	0.2	-2.4	-3.8*
71	0.9	0.9	-8.9	6.1	-4.6	1.0	7.0	5.5
72	-0.6	9.0**	-42.9	31.6**	-26.9**	-0.7	24.8**	16.7**
73	0.1	7.3	-34.0*	29.7**	-27.3**	-1.3	22.8**	18.0**
74	0.5	1.5	-19.3	12.0	-13.3	2.4	12.2	10.6
75	-0.3	2.2	-22.9	11.9	-11.7	2.0	11.4	7.5
76	-0.0	-1.4	-0.5	-2.1	3.9	0.3	-1.6	-2.7
77	0.2	1.2	-3.3	7.6	-5.0	-0.1	5.5	4.8
78	-0.6	7.1	-39.0	11.1	-7.2	3.3	13.2	2.4
79	0.3	4.1	-26.8	15.7	-12.2	1.3	14.4	8.0
80	-2.0*	9.1*	-49.2*	14.3	-8.2	3.3	15.4	0.5
81	-1.7	0.0	-16.3	-4.4	3.7	2.1	-1.0	-5.5
82	-0.1	-0.9	-10.6	1.2	-2.2	2.2	4.0	2.1
83	0.5	0.0	-3.9	5.5	-5.5	0.4	4.1	4.6
84	-0.2	-3.8	11.3	-15.5	13.4	2.9	-9.5	-7.6

4	5	6	7	8	9	10	11	12	R_j^2
35.9**	41.4**	21.9**	0.2	121.5**	54.5**	-34.7**	11.9**	0.4	69**
-12.0	-13.2	-2.6	-0.5	-27.1	-3.8	9.7	0.9	-0.1	75**
44.3**	52.1**	27.7**	0.4	157.9**	70.5**	-44.3**	14.5*	1.5	80**
-15.1	-16.8	-9.2	-0.1	-58.3	-25.6	16.6	-4.2	-1.3	80**
42.4**	50.7**	29.2**	0.1	151.7**	73.5**	-42.2**	17.4**	0.2	79**
-4.4	-5.6	-0.1	-0.1	-5.1	2.4	2.9	1.0	-0.0	67*
-23.3**	-29.8**	-20.5**	-0.1	-106.2**	-56.9**	28.1**	-12.7**	-1.6**	76**
23.0	24.0	4.9	0.1	49.6	8.0	-17.3	-0.7	-0.7	81**
23.3*	25.6	10.4	-0.1	68.2*	26.0	-20.9*	4.5	-0.5	89**
-15.7	-21.0	-18.3	1.0	-76.8	-49.9	19.7	-13.5	0.7	90**
-3.7	-4.5	1.1	-0.1	0.3	5.0	1.3	1.4	1.3	63*
-25.0*	-29.0*	-13.0	0.1	-77.1	-30.3	23.0	-6.2	0.9	66*
35.4*	43.3*	26.8*	-0.0	144.8*	72.1*	-39.1*	15.4*	1.9	80**
-0.5	0.2	-1.4	0.2	-9.7	-5.5	2.5	-0.6	-1.1	85**
4.4	7.1	8.7	-0.3	34.1	26.1	-7.5	7.0	-0.4	63*
-34.7**	-40.7**	-21.0**	0.4	-127.5**	-55.7**	36.1**	-10.6**	-1.1*	84**
-1.8	-4.0	-6.8	0.1	-23.8	-19.8	5.0	-5.7	-0.0	88**
-49.2*	-56.0*	-26.4*	-0.6	160.8*	-65.0	46.8*	-12.3*	-0.3	60
37.7*	46.3*	25.7*	-0.1	132.7*	66.8*	-37.5*	16.2	-0.7	71**
2.1	4.0	7.1	-1.0**	25.9	22.4	-5.6	6.3	-0.3	81**
8.3	10.9	6.5	0.2	29.5	16.2	-8.1	4.6	-0.1	73**
8.8	13.3	10.3	-0.5	44.4	28.5	-11.5	7.5	0.0	51
-16.3	-16.6	-3.7	-0.6	-41.0	-7.6	13.2	0.5	-0.7	59
-22.9	-25.9	-9.5	-0.7	-69.4	-23.9	21.0	-3.2	-0.6	63*
-7.7	-10.4	-5.9	-0.8	-35.4	-16.9	9.0	-3.6	-0.6	73**
4.2	5.8	5.0	0.1	25.8	15.7	-6.2	3.1	1.1	43
-9.3	-7.6	2.0	-0.6	-12.5	7.0	5.5	3.2	-0.6	63*
-7.7	-8.5	-1.7	0.0	-15.3	-3.7	5.2	-0.9	0.4	79**
9.9	11.3	6.7	0.3	33.1	16.4	-9.5	4.3	-1.2	76**
48.5**	57.2**	28.9**	0.3	169.6**	74.7**	-48.6**	14.6	0.1	75**
48.3**	55.7**	24.5*	-0.1	149.9**	59.5*	-44.5**	11.3	-1.4	70**
22.3	27.4	14.6	-0.1	67.4	33.2	-19.2	9.7	-2.7*	61*
20.5	24.9	14.9	-0.5	72.3	38.0	-20.4	10.1	-1.6	56
-5.4	-6.2	-1.3	-0.2	-8.7	-1.0	2.9	-0.0	0.7	81**
10.4	10.1	2.8	0.4	28.8	6.3	-9.0	-0.0	0.8	73**
15.3	22.3	22.5	-0.1	97.1	64.2	-22.9	16.7	1.7	58
24.3	27.9	16.6	-0.5	91.1	44.2	-25.8	9.5	0.1	46
17.5	25.5	26.9	0.1	120.6	80.5	-28.1	20.1*	2.9	50
-7.4	-3.3	7.7	0.7	7.9	24.5	1.2	8.8	0.5	62*
3.9	6.4	6.6	-0.0	19.5	17.3	-4.5	6.3	-1.1	64*
9.6	10.1	3.4	-0.3	21.0	7.6	-7.3	1.5	-1.3	54
-23.0	-25.3	-8.3	0.1	-66.8	-19.2	21.4	-1.1	-0.5	60

Table 16. (Cont'd)

Items	A	B	C	D	E	1	2	3
85	-0.9	3.0	-20.4	8.8	-7.4	-0.3	7.0	2.9
86	-0.0	-5.5	25.7	-13.6	11.5	-1.2	-12.4	-6.8
87	-1.9	2.3	-15.5	5.1	-4.9	-1.3	2.6	-0.9
88	-0.6	0.9	-7.6	5.7	-5.1	0.4	4.4	2.9
89	-0.4	-4.5	18.2	-17.8	14.6	0.6	-14.1	-10.2
90	0.2	5.8	-7.0	6.9	1.2	-1.2	4.8	-1.6
91	-0.9	-7.2	24.8	-21.7	18.1	0.8	-17.7	-12.3
92	-0.2	-2.1	12.7	-6.3	7.2	-0.7	-6.4	-4.5
93	-2.0**	2.5	-13.8	10.3	-10.4	-1.7	6.1	4.3
94	0.6	-6.7	35.5	-11.9	6.1	-2.5	-12.6	-1.6
95	-0.3	6.1	-21.9	19.6	-15.6	-1.1	15.5	10.0
96	-0.8	1.3	-7.3	-4.4	7.0	0.9	-2.5	-7.0
97	-1.0	10.3*	-49.5	23.8	-17.6	2.3	22.0	8.7
98	-0.2	-1.6	0.6	-1.2	-0.8	0.2	-1.1	1.1
99	0.5	3.6	-15.0	15.7	-14.3	-0.6	11.9	9.3
100	-0.8	-1.3	15.9	-12.6	13.3	-1.8	-12.3	-10.8
101	0.0	-9.7	40.9	-21.6	14.8	-1.5	-19.9	-7.8
102	-0.7	-4.7	6.2	-5.8	-0.7	0.7	-4.6	1.2
103	0.1	-8.7**	47.4**	-25.1**	20.8**	-3.1	-23.7	-12.9
104	1.1	6.2	-26.7	23.6	-20.4	0.5	20.6	14.8
105	-1.3	-5.1	14.5	-19.0*	14.8*	0.3	-15.4*	-12.3**
106	0.5	-4.1	24.0	-13.3	11.1	-0.7	-11.7	-6.4
107	-0.9	0.3	-9.0	3.1	-3.5	1.0	2.6	1.3
108	-0.7	3.0	-29.5	8.2	-7.5	3.7	10.1	3.5
109	0.9	5.1	-22.4	12.9	-8.1	1.1	12.0	5.4
110	-0.6	1.4	-6.9	10.3	-11.4	-1.6	6.7	7.1
111	-0.4	6.8	-65.7*	19.8	-19.0	7.9**	24.6*	9.7
112	-0.2	-2.2	4.0	-4.9	0.3	0.3	-4.2	-3.2
113	0.8**	1.4	2.2	5.0	-1.1	-1.1	2.8	2.1
114	-1.2	3.4	-15.4	6.9	-0.3	-0.3	4.8	0.2
115	1.0	0.8	0.2	5.7	-0.8	-0.8	4.9	5.7
116	-0.8	-1.3	-2.6	-9.7	2.7	2.7	-5.4	-7.1
117	0.6	0.7	1.2	3.4	-0.3	-0.3	1.7	1.7
118	0.0	-1.0	13.4	-0.1	-2.3	-2.3	-3.3	-0.3
119	-1.3	1.1	-11.7	5.2	-1.2	-1.2	3.0	1.6
120	0.0	-1.6	5.2	-4.1	0.2	0.2	-4.1	-3.8
121	0.8	-0.3	21.2	3.2	-4.5	-4.5	-1.9	3.4
122	-0.0	1.1	-18.1	6.4	3.0	3.0	7.9	4.0
123	-1.3	5.4	-43.5*	8.7	4.6	4.6	12.0	0.6
124	1.1	-7.1	20.6	-14.1	2.3	2.3	-8.9	-2.2
125	-0.9	1.6	-16.0	1.6	1.0	1.0	2.3	-1.4
126	-2.7**	7.8	-44.3	15.5	0.3	0.3	13.0	3.0

4	5	6	7	8	9	10	11	12	R_j^2
13.4	17.2	11.8	-0.5	59.8	34.1	-16.5	7.2	0.8	43
-20.5	-25.2	-16.3	1.0	-82.4	-43.7	22.5	-9.8	0.6	70**
6.7	10.8	8.5	-0.1	40.7	23.9	-10.5	5.1	0.5	60
10.0	11.1	5.1	-0.2	31.0	13.9	-9.3	2.9	-0.9	74**
-28.0	-31.2	-12.7	-1.1	-88.3	-32.1	26.4	-5.1	-0.8	56
3.3	1.3	1.9	-0.3	34.1	11.3	-8.5	-1.7	3.9	39
-33.0	-37.8	-17.5	-0.0	-110.4	-43.6	32.1	-7.6	-0.8	62
-11.9	-15.6	-8.9	0.0	-41.8	-21.9	11.4	-6.3	1.3*	55
17.2	21.1	9.6	0.6	57.9	24.3	-17.0	4.2	0.2	70**
-13.9	-18.8	-19.2	1.4	-92.5	-59.4	22.8	-13.7	-1.2	53
28.9	33.2	15.0	0.2	99.0	38.9	-28.7	5.9	1.7	47
-10.6	-10.5	1.3	-0.3	-2.9	10.4	4.2	3.6	1.6	63
34.2	42.2	30.1	0.2	154.2	82.1	-40.2	18.8	2.3	59
-0.1	1.0	0.4	-0.3	-4.8	-0.8	1.1	0.9	-1.4	32
25.6	28.8	11.4	0.2	74.5	27.4	-23.0	5.1	-0.7	69**
-23.8	-26.7	-12.1	0.7	-65.7	-27.4	19.8	-7.2	2.6*	55
-30.3	-35.8	-24.3	0.2	-132.6	-68.6	35.4	-14.7	-1.2	60
-3.6	-1.7	-1.9	-0.1	-27.9	-9.3	7.2	0.6	-2.4	53
-39.2	-47.3**	-29.5	0.5	-154.0**	-79.3**	41.9	-18.5	-0.2	58
38.7	42.3	18.9	0.0	120.3	47.0	-35.9	8.6	0.4	63*
-28.9*	-30.8	-11.1	-0.5	-85.0	-25.8	25.6	-3.2	-0.4	59
-20.1	-24.5	-15.2	-0.7	-80.1	-41.6	21.7	-9.4	-0.4	65*
5.2	7.5	5.8	0.3	20.9	13.8	-5.0	4.3	-0.6	53
13.5	18.9	18.0	-0.3	69.2	48.6	-16.7	14.4	-1.1	39
17.6	20.6	13.4	0.2	73.6	36.0	-19.6	7.4	1.3	61
18.1	20.5	6.4	0.0	46.9	13.0	-14.9	1.6	-1.0	45
34.6	47.4	40.6*	-0.7	161.6	108.6*	-41.0	31.3**	-2.1	81**
-8.6	-9.1	-3.1	0.4	-26.8	-8.9	8.6	-0.7	0.0	80**
5.0	3.4	-1.6	-0.0	10.5	-3.6	-3.5	-3.3	1.0	50
7.3	9.7	8.0	-0.7	44.3	25.3	-11.3	4.6	1.5	42
10.7	11.0	1.4	-0.1	20.8	1.1	-8.2	-1.1	-0.3	52
-15.8	-15.6	0.0	-0.4	-30.3	2.1	11.9	3.8	-0.8	54
4.6	3.1	-0.8	0.5	9.4	-2.2	-3.4	-1.9	0.3	33
-1.0	-3.9	-7.9	0.0	-20.8	-22.8	4.2	-7.9	0.3	54
7.9	11.9	7.3	0.4	33.9	20.5	-9.2	4.9	1.0	62*
-7.8	-9.4	-4.1	-0.5	-24.1	-9.6	7.0	-1.5	0.3	48
4.7	0.4	-11.5	0.4	-18.2	-33.4	0.8	-12.7	0.4	49
11.2	15.0	11.6	0.7	45.6	29.4	-11.4	8.8	0.2	60
12.2	20.5	24.8*	-0.0	91.1*	70.2*	-20.1	20.2	0.7	71**
-17.3	-20.2	-11.7	-0.3	-78.4	-34.3	21.5	-4.7	-2.7	43
1.6	4.8	8.9	-0.7	28.4	25.8	-6.1	8.0	0.2	32
22.4	32.0	26.0	0.0	119.7	72.5	-30.7	17.2	1.7	78**

Table 16. (Cont'd)

Items	A	B	C	D	E	1	2	3
127	-0.6	-1.6	3.7	-1.7	-0.0	-0.2	-2.2	0.2
128	-0.5	-0.9	2.7	-10.2	9.5	1.0	-6.6	-7.1
129	-0.1	7.1	-33.3	22.0	-19.1	1.1	19.6	12.6
130	-0.4	6.3	-24.7	19.3	-14.5	-1.3	14.3	8.4
131	-0.0	3.7	-10.7	11.2	-8.2	-1.2	7.7	5.3
132	0.4	-4.1	25.1	-5.5	4.7	-3.3	-8.3	-1.6
133	1.1	-3.7	27.7	-16.1	16.3	-0.8	-13.3	-9.1
134	-0.7	-2.7	-1.8	-2.5	-1.5	1.8	-0.4	1.7
135	0.5	-2.8	17.6	-12.0	13.0	0.1	-9.5	-7.4
136	1.0	-0.7	10.1	-2.3	4.4	-1.1	-2.7	-2.1
137	0.0	-2.7	7.5	-6.7	4.0	0.3	-4.6	-1.8
138	-1.9	-3.1	0.1	-7.6	3.4	0.8	-6.7	-4.8
139	-0.2	-1.3	13.8	-6.6	7.4	-1.2	-5.9	-4.3
140	0.1	-10.3	46.5	-25.5	19.0	-0.9	-22.2	-9.8
141	0.8	-5.9	48.3	-16.2	15.8	-4.6	-18.6	-8.4
142	0.3	3.5	-27.2	16.6	-15.6	1.0	14.9	10.1
143	0.0	0.8	-5.1	3.7	-2.4	1.2	3.3	2.1
144	0.1	3.9	-48.6	-13.7	50.3	3.7	-17.4	-42.7
145	0.0	0.6	1.4	-6.5	9.6	0.5	-4.8	-7.7
146	0.7	-0.2	4.7	7.7	-8.0	-1.6	4.2	7.1
147	0.7	-9.3	46.7	-24.1	19.3	-1.4	-20.9	-9.6
148	0.6	-8.2	46.5	-29.7	27.5	-0.1	-25.0	-15.5
149	1.7	-1.4	10.6	0.8	0.1	-0.3	0.6	2.8
150	0.1	1.4	-3.9	9.9	-9.9	-1.5	7.0	7.0
151	-2.1	-0.2	-6.4	-6.9	6.9	0.9	-5.7	-8.2
152	0.0	-5.2	30.8	-3.1	-1.2	-4.2	-7.3	3.0
153	-0.4	-7.9	40.7	-27.3	24.5	-1.1	-23.6	-15.7
154	-0.5	0.0	-0.6	0.2	0.3	-0.3	-0.8	-1.1
155	1.5	-1.1	12.6	0.6	0.8	0.1	0.5	3.2
156	0.2	-0.2	-4.3	6.5	-6.7	-0.5	4.9	4.9
159	1.1	-1.6	-0.5	0.6	-1.3	2.2	2.6	2.9
160	-0.3	1.0	-12.2	10.2	-9.7	0.4	7.8	6.5
161	-1.1	4.8	-28.7	12.3	-8.3	1.3	10.9	3.0
162	0.1	1.8	-5.8	13.1	-12.2	-2.0	8.4*	8.5
163	0.1	-1.2	15.1	-5.0	6.6	-1.3	-5.2	-2.8
164	-1.1	-1.7	8.6	-6.9	5.9	0.5	-5.6	-3.8
165	-0.9	-3.6	14.9	-20.0	19.8*	1.4	-15.1	-14.4*
166	-0.2	9.7	-56.3	25.9	-21.2	3.9	25.5	11.7
167	-0.2	11.7	-68.6	29.4*	-22.4	4.3	28.9*	11.2
168	-0.8	1.6	-26.5	8.8	-8.8	3.0	9.7	4.7
169	0.1	1.4	-10.7	6.0	-4.2	0.8	6.1	3.0
170	0.8	2.5	-9.7	9.8	-6.9	-0.1	7.9	5.3

4	5	6	7	8	9	10	11	12	R_j^2
-1.3	-1.0	-2.3	-0.3	-12.6	-5.1	3.2	-0.8	-0.8	53
-16.8	-17.5	-3.6	-1.2	-37.1	-5.7	12.9	0.6	-0.7	42
34.9	41.3	22.3	-0.4	124.0	57.7	-34.7	12.9	-0.8	63*
27.7	31.4	15.5	-0.5	100.1	41.4	-29.2	6.8	0.8	45
15.9	18.0	7.2	0.8	52.1	17.6	-15.3	2.0	1.5	26
-9.1	-13.3	-15.1	0.5	-57.5	-41.5	13.6	-11.8	0.8	38
-27.5	-34.5	-18.6	-1.2	-95.1	-47.0	26.9	-11.6	0.2	34
0.5	2.6	2.8	0.3	-6.1	3.0	2.1	3.9	-2.0	43
-20.5	-26.2	-12.9	-0.0	-65.6	-29.8	19.2	-7.2	1.1	52
-6.3	-9.3	-6.6	-0.2	-23.2	-16.9	5.5	-5.5	1.0	30
-8.6	-8.9	-4.4	-0.3	-33.6	-12.6	9.6	-0.9	-0.9	27
-10.1	-8.1	-0.3	0.2	-28.7	-2.3	9.4	3.1	-1.4	50
-12.3	-15.7	-9.6	0.2	-41.6	-22.5	12.0	-6.4	1.0	49
-36.9	-44.4	-28.4	0.2	-155.4	-78.2	42.4	-16.2	-1.8	51
-28.0	-37.7	-30.0	-0.1	-128.2	-81.0	32.8	-21.9	0.2	50
27.5	34.0	18.3	0.3	93.8	46.0	-26.7	11.3	-0.5	31
5.4	6.0	3.4	0.9	17.8	8.6	-4.6	2.2	0.5	40
-63.4	-18.5	2.7	-25.0	-11.7	74.3	6.8	5.0	0.1	52
-14.5	-16.8	-3.9	-0.4	-24.5	-4.0	9.1	-0.8	2.6	33
13.6	13.8	-0.2	0.8	19.0	-5.7	-8.0	-2.6	-0.9	56
-35.9	-44.3	-28.6	0.3	-150.1	-78.1	41.0	-17.0	-1.0	42
-47.8	-57.8	-31.0	-0.4	-168.6	-78.5	47.5	-17.3	-0.6	56
1.9	-1.9	-5.9	-1.3	-14.2	-16.4	1.6	-5.2	-1.0	43
17.1	18.9	4.3	1.0	39.9	7.6	-12.8	0.5	1.0	42
-13.2	-11.9	1.6	-0.2	-15.3	8.2	7.3	4.2	1.0	41
-1.2	-5.0	-15.3	1.9*	-56.5	-50.1*	11.5	-13.3	-0.7	67*
-44.5	-52.3	-27.7	-0.6	-152.5	-69.4	43.9	-14.8	0.5	55
-0.5	- 0.1	-0.3	-0.1	-0.0	-0.1	-0.4	-0.3	-0.1	31
0.8	-2.5	-6.7	-0.2	-17.4	-18.7	2.9	-5.7	0.0	53
11.6	12.5	3.6	-0.4	25.9	7.6	-9.1	1.6	-0.4	56
2.7	2.6	1.1	-0.6	2.3	1.2	-0.3	1.7	-1.1	73**
16.3	19.7	8.6	-0.1	48.5	20.8	-14.3	5.1	-0.6	65*
16.6	21.2	17.0	0.2	82.5	47.5	-21.3	11.6	1.6	74**
21.9	23.6	5.6	0.5	52.0*	11.4	-17.5	0.9	-0.3	67*
-9.9	-13.3	-9.9	0.2	-38.5*	-24.7	10.5	-7.0	1.0	80**
-11.3	-12.4	-5.3	0.5	-35.1*	-13.8	10.7	-2.1	-0.2	79**
-34.4*	-37.6	-12.6	0.3	-90.7*	-27.7	29.1	-3.3	1.4	78**
40.2**	50.2**	34.8*	-0.4	170.0**	94.4**	-45.1	23.7*	0.3	63*
44.1*	55.2*	41.1	-1.0	199.7**	113.3**	-52.7**	27.6**	1.2	68*
14.7	20.3*	16.6	-0.3	65.8**	44.1**	-16.7	13.5	-0.9	81**
9.0	10.4	7.0	-0.2	37.5*	19.2*	-9.7	4.0	1.0*	82**
14.1	15.8	6.9	0.6	46.3	16.9	-14.0	2.9	1.1	50

Table 16. (Cont'd)

Items	A	B	C	D	E	1	2	3
171	0.2	-2.0	5.8	0.4	-1.1	-0.5	-0.8	2.2
172	-0.4	-1.7	8.9	-2.4	1.1	-1.2	-3.3	-0.7
174	-0.1	-8.8*	43.8*	-24.8*	19.4*	-1.7	-23.5*	-11.7
176	-0.8	2.0	-13.0	11.1	-11.6	-0.9	7.7	6.8
177	0.4	-5.6**	16.6	-14.5*	12.7**	0.8	-10.9*	-7.2*
178	-0.2	-0.2	-0.5	7.6	-10.0	-1.1	4.8	7.2
179	1.2	-3.4	17.1	-7.7	7.5	0.5	-6.4	-2.8
180	-0.8	-1.1	5.6	-5.7	5.4	-0.9	-6.0	-5.5
181	-0.0	4.6	-36.5*	26.6**	-27.4**	0.9	22.8**	18.2**
183	-1.0	3.6	-31.5	13.1	-13.8	2.4	13.1	7.5
184	0.9	2.1	-9.8	3.5	-0.1	2.5	5.4	1.2
242	-1.5	-1.7	8.7	-7.9	7.2	-1.0	-9.3	-7.3
243	-0.8	1.0	-3.2	5.4	-5.7	-1.0	3.3	3.8
244	1.0	3.6	-4.9	15.6	-11.5	-1.9	10.4	8.8
245	1.5	-0.5	2.0	-4.6	6.0	2.3	-0.3	-1.6
246	-1.1	6.5	-27.4	15.6	-10.4	-0.1	11.9	4.2
247	-0.8	1.3	-14.5	7.3	-9.8	-0.2	6.1	5.0
248	9.7	-5.2	56.7	2.8	3.5	0.3	4.4	13.9
249	-0.4	0.0	-1.4	5.1	-4.6	-1.4	1.7	3.0
250	0.5	5.0	-29.5	11.8	-7.6	3.3	14.2	5.7
251	0.8	-3.7	21.1	-14.6	15.2	0.4	-11.5	-8.5
252	-0.6	8.8	-48.7	24.6	-21.2	2.7	23.2	11.7
253	-0.4	1.7	-5.6	3.7	0.6	-1.3	1.1	-1.8
254	-0.6	-2.5	11.6	-5.8	3.9	-1.1	-6.5	-2.7

4	5	6	7	8	9	10	11	12	R_j^2
1.6	1.1	-2.4	0.3	-7.6	-7.9	1.2	-2.3	0.1	64*
-3.1	-4.1	-5.3	0.6	-22.2	-15.5	5.2	-3.4	0.1	45
-37.3*	-45.2*	-27.5*	-0.7	-149.1*	-73.8*	40.2*	-16.1	-1.4	53
18.8	22.5	9.1	0.2	55.4	21.2	-17.1	4.3	-0.6	43
-22.1**	-25.8**	-11.7*	-0.5	-74.8*	-28.7*	21.6*	-5.2	0.2	78
15.0	16.6	2.7	0.4	27.3	1.2	-10.0	-0.1	-1.0	64*
-12.2	-16.5	-10.2	-0.4	-52.2	-27.8	14.3	-6.5	0.0	54
-10.5	-11.7	-5.1	0.3	-29.0	-12.3	8.6	-2.4	1.3	66*
46.8**	56.2**	26.7**	0.9	143.5**	63.2**	-41.7	15.2*	-1.4	69**
23.4	30.9	20.4	0.4	90.6	52.6	-24.3	14.7	-1.2	25
4.0	3.5	5.8	-0.5	26.0	16.3	-5.9	3.8	1.3	58
-14.9	-16.3	-6.9	-0.2	-41.7	-16.2	12.8	-3.4	0.5	36
8.7	10.7	3.2	0.7	22.5	6.7	-6.6	1.4	-0.0	42
23.4	22.7	4.4	0.2	59.2	9.5	-19.5	-2.5	0.9	41
-8.2	-10.3	-1.7	-0.7	-18.5	-3.4	6.5	0.2	-0.6	55
20.6	24.9	16.0	-0.2	91.1	45.9	-24.3	9.1	2.2	40
14.5	19.6	10.1	0.9	47.5	23.9	-13.8	6.7	-1.1	50
5.1	-12.1	-29.2	-0.4	-78.4	-88.9	12.8	-27.6	-1.5	61
8.1	8.4	1.0	-0.1	15.6	2.1	-6.0	-0.8	-0.2	45
17.3	21.3	17.7	0.1	84.5	50.0	-21.5	12.2	1.0	58
-24.7	-30.7	-14.7	-0.6	-79.2	-35.5	23.0	-8.2	0.5	37
38.7	48.5	31.3	0.6	156.3	83.1	-41.8	19.7	0.8	60
0.8	0.4	1.3	-0.8	17.4	7.8	-4.7	-0.3	1.6	51
-8.0	-9.4	-7.4	0.7	-37.1	-19.9	10.0	-4.1	0.4	43