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ELUTION KINETICS OF THE COPPER AMMINE COMPLEX FROM A CATION EXCHANGE RESIN

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INTRODUCTION

Insoluble materials which are capable of exchanging cations or anions are commonly called ion exchangers. These materials may be of natural or synthetic origin and may be organic or inorganic in composition. An ion exchanger consists of a porous framework which contains a surplus electrical charge. This charge is compensated for by mobile ions of the opposite electrical charge which may be exchanged for other similarly charged mobile ions. The exchange of one species of ions for another species of ions is the process of ion exchange and it is generally reversible and stoichiometric.

The ion exchange process has been studied extensively and it is generally considered to take place in the following manner. Ions in the solution surrounding the resin bead must first reach the surface of the resin bead by diffusion and mixing. The ions then diffuse into the pores of the bead. When the diffusing ionic species has reached an exchange site in the resin which has another species attached to it, the exchange reaction takes place. In the exchange reaction the ion attached to the resin matrix is released and the other ion attaches itself to the resin site and is held there by ionic forces. The newly released species then diffuses out through the pores of the resin and into the solution surrounding the resin bead. In nearly all cases which have been studied the reaction of exchanging ions at the charged site is a very fast reaction. Thus this step is not generally considered to be a controlling step in kinetic investigations. The diffusion of ions into and out of the resin bead as well as the movement of ions to and from the surface of the bead are the controlling steps.

The chemical system studied in this work is the exchange of the hydrogen ion for the copper ammine complex as is represented in Equation 1.

$$R_2Cu(NH_3)_a + d\ H^+ \longrightarrow 2\ RH + a\ NH_4^+ + Cu^{++} \quad (1)$$

In Equation 1 symbol R represents the ion exchange resin, symbol a represents the moles of ammonia complexed to the copper ion per mole of copper, and symbol d represents the moles of hydrogen required to remove the copper ammine complex from the resin. This particular system has a very interesting characteristic which is most unique in ion exchange reactions. When the resin is put into the copper ammine form it changes color from its usually light yellow to the blue color which is characteristic of the copper ammine complex in aqueous solution. When acid is added to the resin in this form, the copper ammine complex is removed and replaced by the hydrogen ion. As this reaction proceeds the resin undergoes a color change from blue to yellow. The resin bead is sufficiently transparent so the color change may be observed within the resin. The color change during the course of the process occurs in the form of a moving boundary. At any time during the exchange process there is a spherical dark blue core surrounded by a yellow shell.

The unique color change of this ion exchange process was used in the work reported here to investigate the kinetics of the process. It was proposed in the beginning of this work that this moving boundary could be used to investigate several aspects of the ion exchange process. These are summarized in the following statements and described more completely in the paragraphs which follow.

1. This process should be described by a mathematical model which takes the moving boundary into account.

2. The rate controlling steps in this process should be determined and evaluated.

3. The influence of the resin diameter, resin cross-linkage, acid concentration, and acid flow rate on the process should be investigated experimentally.

4. Some information should be obtained about the homogeneity of the individual resin particles.

Experimental data were taken by contacting resin beads loaded with the copper ammine complex with sulfuric acid. For this elution the beads were placed in a cell which allowed the acid solution to flow past the beads. While the copper ammine complex was being removed from the resin, photographs were taken of the beads to record the position of the moving color boundary. The time of each photograph was recorded and thus it was possible to obtain data which related the position of the moving boundary to time.

Previous studies of ion exchange processes had used solutions of Fick's law of diffusion and the Nernst-Planck diffusion equation. These equations had been solved assuming that there is no moving boundary in the resin bead. To analyze the present system it was necessary to solve the diffusion equations with a moving boundary. A rigorous solution, either analytical or numerical, to this type of problem has long been sought by many workers without success. Therefore, approximate solutions were used to model this ion exchange process. Two approximate quasi steady state solutions were evaluated. Both solutions were developed by assuming the boundary to be at a fixed position and then solving for the concentration profile in the outer diffusion shell. The flux of material at the boundary was determined from the concentration profile and this was then used to develop an equation for the rate of movement of the boundary. In the first approximate solution the concentration profile was assumed to be the unsteady state profile evaluated at the time which had elapsed in the process. In the second approximate solution the steady state concentration profile was assumed and film diffusion was also included in the process. The diffusion coefficients in the second approximate or quasi steady state solution were determined by a non-linear regression analysis of the experimental data.

To test the equations and investigate the nature of the elution process, the experimental conditions were varied. The sulfuric acid used to contact the resin was varied from 0.05 to 5 N. The acid flow rate was varied from 0 to 2 ml./sec. while room temperature and pressure were used throughout the experimental work. The resin particle size was varied over a diameter range of about 0.06 to 1.2 cm. Most of the experimental work was carried out using Dowex HCR-M resin which is a sulfonated styrene-divinylbenzene copolymer. The particular resin used had 8 per cent divinylbenzene. Two other resins with 2 per cent and 12 per cent divinylbenzene were tested to determine if the resin cross-linkage has an effect upon the rate of the exchange process.

Data obtained by following the color boundary in the resin bead had another aspect which was quite different from other ion exchange studies. Other methods of studying ion exchange resins have involved an analysis of the material which is removed by the exchange process from a group of ion exchange beads. Thus the cumulative effect of a number of resin beads was measured and not the action of a single resin bead. In this work each resin bead was considered individually. The consideration of each bead individually allows the investigation into the homogeneity of the resin beads. That is, it was possible to determine if the resin beads are very similar to each other in their rate of exchange or if the bead manufacturing process produces some beads which react quite rapidly while others react slowly.

PREVIOUS WORK

There are two different approaches to ion exchange kinetics which appear in the literature. A number of investigators have developed expressions which related the rate of exchange to some function of the concentration of each ionic species in the solution and each ionic species in the resin. Helfferich (23) lists 13 different expressions which have been used in different systems. These expressions often fail if the experimental conditions are changed only slightly (59). Generally they are combined with the flow equations to describe column operations. For this purpose the rate laws have the advantage of being simple and therefore do not introduce too many mathematical complications into the flow equations (1,2,10,35,39,40,41,44,46,48,64,65).

The other approach to ion exchange kinetics is more general and the same expressions fit the experimental data from widely varying systems (23,24). The ion exchange bead is considered to be a porous network with a surplus electrical charge at various points along the walls of the pores of the resin bead. The exchangeable ions are of the opposite electrical charge as the charge of the resin and are bound to the resin. With this resin model the process of ion exchange is divided into the following five steps. For the purpose of this discussion let the resin be loaded initially with ion B while the surrounding solution contains ion A.

Step 1. Ion A is in the solution and must move to the surface of the resin. This can occur either by a mixing process or by a diffusion process or, generally, by some combination of both mechanisms. Since the actual mechanism is extremely complicated a simplifying assumption must be made in order to treat the problem at all. Usually it is assumed that the particle is surrounded by a stagnant layer of a definite thickness called the Nernst diffusion layer. The ions move through this layer by molecular diffusion. Outside of this spherical layer of solution it is assumed that the solution is perfectly mixed and the concentration of all species is the same at all points in the solution. To account for different rates of agitation or mixing, the thickness of the Nernst layer is adjusted. Thus for a slow mixing rate the Nernst layer is thicker than for a fast mixing rate.

Step 2. The second step in the process of ion exchange is the movement of ion A from the surface of the resin into the resin to some site where ion B is located. This is assumed to occur by diffusion through the pores of the resin matrix. It is further assumed that only the diffusion process is taking place and there is no mixing by any other mechanism.

Step 3. Once ion A has reached ion B located in the resin the exchange reaction takes place. Ion B is held to the resin by an ionic force between the resin matrix and ion B. When the reaction takes place, ion B will be freed from the resin matrix and ion A will take its place in the resin matrix.

Step 4. The fourth step in the ion exchange process is the movement of ion B from within the resin to the surface of the resin in the same manner that ion A came into the resin.

Step 5. The fifth step is the movement of the B ion from the surface of the resin to the bulk solution. This step is assumed to happen in the same manner as the first step when ion A moved to the surface of the resin.

For a complete mathematical model, all five of the above steps should be included. However, in practical problems simplification must be introduced in order to obtain a mathematical model which can be solved. The usual approach is to assume that one or more of the five steps of the process are so fast compared to the other steps that the effects of their rates may be neglected. Which steps to neglect must be verified either experimentally or theoretically in any practical case.

The exchange reaction or step 3 in the preceding discussion has been eliminated in all cases of ordinary ion exchange which have been studied (23). The exchange reaction has been found to be important in the study of chelating resins when the complexing reaction is a very slow reaction (62,63). However, a chelating resin is different than an ordinary resin since it has complexing or chelating functional groups attached to the resin matrix rather than a group which just provides for ionic exchange. For ordinary ion exchange the diffusion equations fit the experimental data much better than the kinetic expressions which would be used if the chemical reaction step was controlling (9,22,51).

The decision as to whether to eliminate the film diffusion or the particle diffusion is done on the basis of experimental data. Usually the exchange rate is measured at several different flow rates or mixing rates. If the rate is constant for different mixing rates, then film diffusion is eliminated since the film thickness varies with the velocity of the material flowing past the beads. If the process is controlled by film diffusion, then the rate should also increase with increasing velocity. Under the conditions of good agitation and moderate particle sizes most ion exchange processes have been found to be controlled by particle diffusion (32). With moderate agitation the film thickness is of the order of 0.005 cm. (23).

Two different forms of diffusion equations are in general use for modeling the ion exchange process. The simplier form is Fick's first law which describes the diffusion as being driven by a concentration gradient only.

The Nernst-Planck equations are also used to describe ion exchange processes and in these equations the effect of electrical coupling is included as well as the concentration gradient.

For describing ion exchange Fick's law is usually written as Equation 2.

$$J_A = -D \text{ grad } C_A \tag{2}$$

For Equation 2 it was assumed that activities are equal to concentrations, there are no diffusion-induced electrical forces, there are only two ionic species diffusing and the diffusion is equimolal, and that there is no flow or mixing other than that caused by diffusion. Equation 2 is combined with the unsteady state material balance across a differential element as given in Equation 3.

$$\frac{\partial C_A}{\partial t} = - \text{div } J_A \tag{3}$$

Combining Equations 2 and 3 gives Equation 4.

$$\frac{\partial C_A}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D \frac{\partial C_A}{\partial r} \right) \tag{4}$$

In writing Equation 4 it was assumed that only effects in the radial direction are important. Initial conditions and boundary conditions must be assumed in order to solve Equation 4. A number of solutions to Equation 4 are available (9,14,15,23) using various boundary conditions. However, none of these solutions were obtained with moving boundary conditions. Solutions have been obtained for both particle and film diffusion cases as well as one solution for a case of combined film and particle diffusion (21) with fixed boundary conditions.

In isotopic exchange a number of the assumptions listed above need not be made since only one element or molecular species is involved in the exchange. This situation has been widely used by a number of investigators (11,12,13,29, 49,53,56,57,58,60) to obtain diffusion coefficients of a number of different ions. For the experimental determination of the diffusion coefficient in isotopic exchange the ion exchange resin is first equilibrated with a solution containing the ion whose diffusion coefficient is to be determined. A solution containing the same ion plus an isotope tracer of the ion is then allowed to flow past the resin. The flow may be in the form of a small volume of solution which is recirculated past the resin or it may be a large volume of solution which flows past the resin only once. In either case the isotope concentration in the solution is determined by a radioactive counter as a function of time. The concentration of the isotope in the solution will decrease as the isotope is adsorbed on the ion exchange resin. This information is then used with the solution of Equation 4 to determine the diffusion coefficient of the ion in the resin.

The experimental method described above has also been extended to the situation where the resin has been equilibrated with one ionic species and the solution used to contact the resin contains another ionic species. In this case the concentration of one of the ions is measured by some appropriate means such as conductivity, pH, radioactivity, or polarography (20,22,30,31,42,43).

When the effect of diffusion-induced electric potential is included, the Nernst-Planck equations are used (4,5,6,28, 37,47,52). Equation 5 is then the flux equation.

$$J_i = -D_i \left(\text{grad } C_i + \frac{Z_i C_i f}{RT} \text{ grad } \phi \right) \qquad (5)$$

In the two-ion case two additional restrictions are placed on the process.

$$|Z_A| C_A + |Z_B| C_B = C_o \qquad (6)$$

$$Z_A J_A + Z_B J_B = 0 \qquad (7)$$

Equation 6 states that the total mobile ion concentration must remain constant in the resin bead. Equation 7 states that the equivalent fluxes are equal and opposite. Using Equations 6 and 7 and the two equations resulting from writing Equation 5 for each component, appropriate substitutions reduce the system of four equations to a single flux equation.

$$J_A = - \left[\frac{D_A D_B (Z_A^2 C_A + Z_B^2 C_B)}{D_A Z_A^2 C_A + D_B Z_B^2 C_B} \right] \text{grad } C_A \qquad (8)$$

Equation 8 is combined with the unsteady state material balance equation used previously (Equation 3). The result after assuming spherical symmetry is Equation 9.

$$\frac{\partial C_A}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D' \frac{\partial C_A}{\partial r} \right) \qquad (9)$$

The diffusion coefficient D' in Equation 9 is defined by Equation 10.

$$D' = \frac{D_A D_B (Z_A^2 C_A + Z_B^2 C_B)}{D_A Z_A^2 C_A + D_B Z_B^2 C_B} \qquad (10)$$

Equation 9 is the same as Equation 4 except that the diffusion coefficient is now a function of concentration. Equation 9 was integrated numerically for the case of constant solution concentration at the surface of the bead assuming that diffusion could proceed simultaneously throughout the bead (28,47). Thus the boundaries were assumed fixed as was done where Fick's law was used. The numerical integration of the Nernst-Planck equation produced concentration profiles in the resin which were somewhat different than the Fick's law case. For the hydrogen-sodium system it was found that the Nernst-Planck equations fit experimental data better than the Fick's law equations (25,61).

The problem of a moving boundary was treated mathematically for the case of the outer boundary of an ion exchange bead. The outer boundary moves when the resin swells or shrinks due to absorption of a solvent or due to a change in ionic form. Dickel (16) used a quasi steady state approach to treat this problem. He solved the diffusion equation (Equation 4) for the case of diffusion into a sphere. To obtain the solution for the concentration as a function of the radial coordinate and time he assumed that the sphere's radius was constant. This equation was then integrated with respect to the radial coordinate to obtain an equation relating the amount which entered the sphere with time. Using this equation he assumed that the position of the resin's radius was proportional to the amount of material which had entered the resin. He thus obtained an equation which related the radius of the resin to the time which had elapsed from the start of the diffusion process.

Helfferich (26) discussed circumstances in ion exchange processes where it is possible to develop a sharp boundary between the eluted and uneluted portion of the resin within an ion exchange resin bead. He cited the situation where a resin may have a very great preference for one ion over another ion. If the weakly held ion is attached to the resin and a solution containing the other ion is brought in contact with the resin, a sharp boundary will form. One example of this is a weak-acid ion exchanger loaded with sodium ions. This resin holds the hydrogen quite strongly and prefers it over the sodium ions. Thus when a solution containing acid comes in contact with the sodium form resin, the sodium is immediately replaced by the hydrogen. The concentration of unattached hydrogen ions is thus very low in the pores of the resin. As a hydrogen ion enters the resin it is "grabbed up" by the first available resin site loaded with sodium. This immediate removal of the ion from solution in the pores causes the boundary to remain sharp throughout the ion exchange process and the boundary will move towards the center of the resin bead. No experimental data was presented to confirm the hypothesis of a moving boundary.

Because of the lack of a method for solving Fick's law equations with a moving boundary condition, a quasi steady state solution was used by several investigators (3,36,67, 68,69) for the case of gas diffusion and chemical reaction in spherical particles. For the quasi steady state assumption it was assumed that the concentration profile is always at its steady state value. Then using the steady state concentration profile the equation describing the moving boundary was solved. This resulted in Equation 11 which relates the boundary condition to time for the case where diffusion in the particle controls the process.

$$t = \frac{PR^2}{6gDC_s} \left(1 - 3\,(r/R)^2 + 2\,(r/R)^3 \right) \qquad (11)$$

For the case where film diffusion controls the process, the quasi steady state solution provided Equation 12.

$$t = \frac{PR}{3gkC_s}\left[1 - (r/R)^3\right] \tag{12}$$

If it is assumed that the chemical reaction itself controls the process, it is not necessary to make the steady state assumption and the solution for a reaction with first order kinetics is given by Equation 13.

$$t = \frac{PR}{bk_sC}\left[1 - (r/R)\right] \tag{13}$$

The use of the quasi steady state approximation raises the question of when is it valid and when is it not valid. Bischoff (8) derived a method of answering this question. He took the steady state concentration profile (Equation 14) which was used for the quasi steady state solution and added another term to it.

$$C = A + \frac{BR}{r} \tag{14}$$

The new concentration profile is given in Equation 15 where A and B are now functions of time.

$$C = A(t) + \frac{B(t)R}{r} + \frac{r^2 \frac{dA(t)}{dt} + 3rR\frac{dB(t)}{dt}}{6D} \tag{15}$$

The new concentration profile was combined with the boundary conditions to obtain an equation which estimated the error of the quasi steady state approximation. The error equation is given as Equation 16 where X is the quasi steady state value and X^* is the corrected quasi steady state value.

$$X^* = X \left(1 + \frac{1}{6} \frac{C_s}{C_r} \frac{1-X}{X^2} \right) \qquad (16)$$

This approximate error equation was justified by noting its similarity to an equation obtained for the case of a moving boundary in a flat plane. For the flat plane case an analytical solution was available for the moving boundary equations without making the steady state approximation. Thus, from Equation 16, the accuracy of the quasi steady state approximation is dependent upon the relative concentrations of materials in the particle and in the bulk solution. Wen (69) published a similar method of determining the accuracy. He used the quasi steady state solution for an infinite plane surface and compared this with the analytical solution to the infinite plane.

Some work has been done using a microscope to determine various properties of ion exchange resins. Diameters of individual beads have been determined quite accurately using a microscope with an image-splitting ocular or a filar micrometer ocular. The determination of resin diameter has been very useful for the study of swelling characteristics of resins in various ionic forms and in various organic and water-organic solvent mixtures. Freeman (19) gives a good discussion of the use of the microscope for studying ion exchange resins. The practical use of the microscope for measurements and photomicrographs is well documented in several books (38,45,50,55).

EXPERIMENTAL INVESTIGATION

Materials and Equipment

Equipment

Data relating the movement of the boundary with time was obtained by photographing the resin beads during the elution process. For this purpose a cell was built to hold a small number of resin beads so that they could be in view of the camera and so that the acid could flow around the beads. The cell (see Figure 1) was constructed from a number 3, single oblique-bore, taper-ground stopcock body. A Saran screen was placed about half way down in the stopcock between two washers cut from Plexiglas plastic. The washers and screen were held in place with epoxy glue. The acid solution was introduced in one of the two ports of the valve and removed through the other port of the valve. The bottom of the valve body was plugged with a rubber stopper. On top of the stopper a thin disc of Teflon was placed to provide a white background for the photographs.

The acid solution was supplied from an overhead tank. A stopcock and a rotameter were placed in the line connecting the tank with the cell to control and measure the flow rate. The liquid level in the cell was controlled by the level of the outlet line. The level in the cell was maintained at about 1 cm. above the resin beads for all of the photographs.

A Model BVB-73 Stereo Zoom microscope, manufactured by

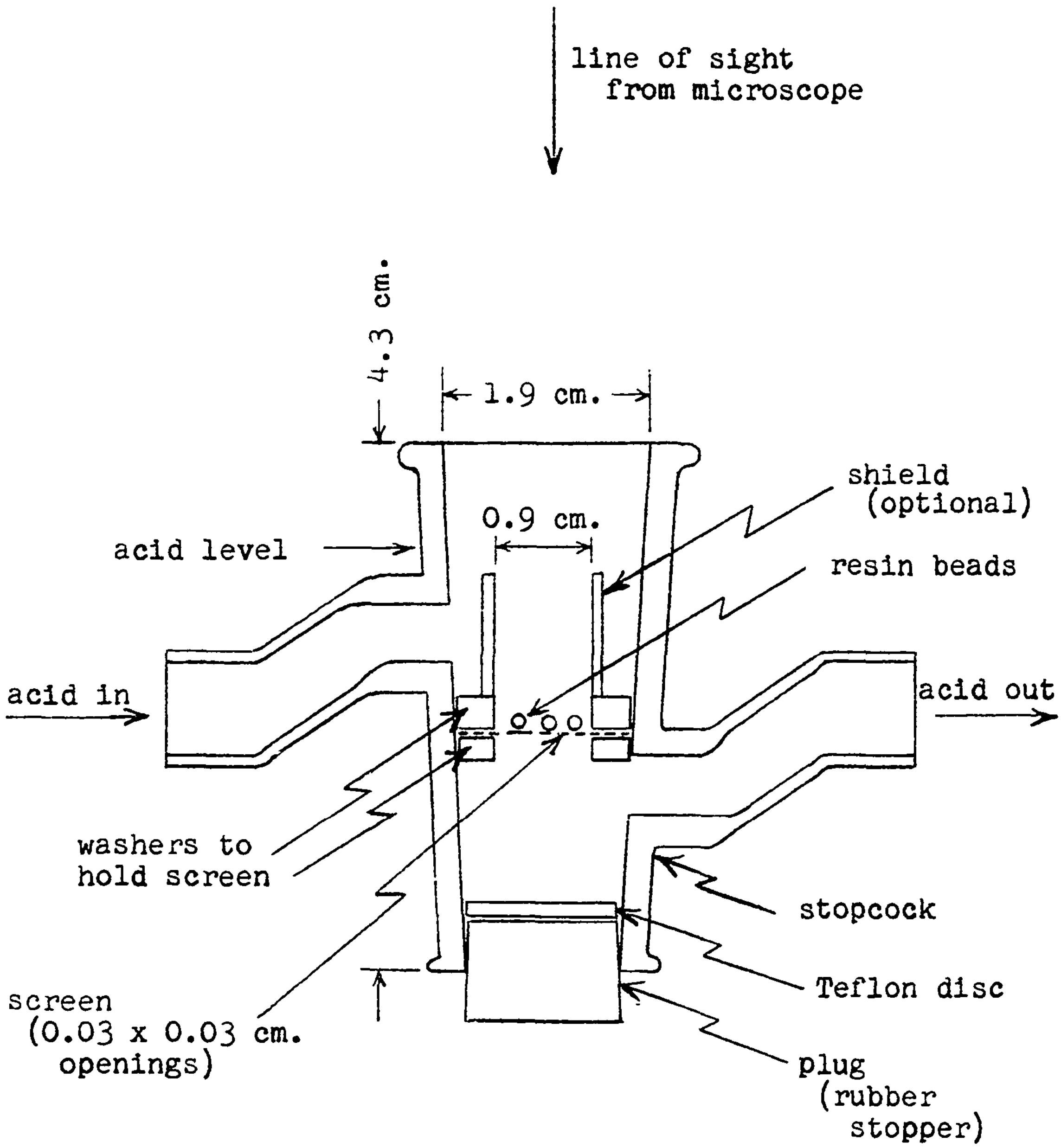


Figure 1. Drawing of the cell used for photographing the resin beads during the elution process

Bausch and Lomb, Inc., Rochester, New York, was available for the microscopic work. This microscope has a maximum magnification of 120X and the magnification is continuously variable. For microscopic measurement of objects a micrometer disc can be installed in the eyepiece of the microscope. The micrometer disc puts a scale into the field of view which can be compared with the object under observation. The micrometer disc is calibrated at any given magnification against a stage micrometer mounted on a glass slide. A Bausch and Lomb stereo camera was available for taking the pictures of the resin beads through the microscope. The stereo camera uses standard 35 mm. photographic film. Ektachrome Type "B" (Eastman Kodak Company, Rochester, New York) was used for the photographic work. The exposure time required depended upon the illumination and upon the magnification of the microscope. Most of the photographs were taken at a magnification of 15X to 20X. For this magnification an exposure time of about 0.5 sec. was required.

The light necessary for viewing the resin beads and for photographing the beads was supplied with a high intensity lamp. The light was placed above the cell and back of the microscope so that it would shine down on the beads.

The time when each photograph was taken was recorded with a strip chart recorder. The recorder was connected to the camera through a relay box by using the switch on the camera supplied for a flash attachment. A deflection of the recorder pen was obtained when the shutter was opened and a second deflection was obtained when the shutter was closed. Thus both the time of the photograph and the exposure time were recorded on the strip chart.

Materials

Chemicals All chemicals used in the experimental work were reagent grade chemicals. Distilled water was used throughout the experimental work.

Ion exchange resins Three resins of different cross-linkage were used for the experimental work. All of the resins were the strong-acid type manufactured by treating a styrene-divinylbenzene copolymer with sulfuric acid. The polymer was made by an emulsion polymerization process which produced nearly perfect spheres. The resin after treatment with sulfuric acid had the sulfate group attached to it which gave the resin a negative charge. Since the divinylbenzene cross-links the polymer the per cent divinylbenzene is also called the per cent cross-linkage.

All three of the resins were prepared for the kinetics experiments in the same manner. The resin was preconditioned by first backwashing it in a column with water. This removed the fine particles and most of the broken particles. The ionic form of the resin was changed several times by contacting it with the following solutions:

1. 2.0 N. sodium chloride
2. Distilled water
3. 2.0 N. sulfuric acid
4. Distilled water
5. 2.0 N. ammonium sulfate
6. Distilled water
7. 0.25 molar copper sulfate solution containing 2.5 moles/l. ammonia
8. Distilled water

In all cases twice the amount of solution required by the resin for complete stoichiometric exchange was used and the flow rates through the column were kept slow to insure a complete exchange of the ions. The resin was removed from the column and stored in distilled water in a tightly stoppered bottle.

The three ion exchange resins used are listed below. The first resin listed was used for all of the experiments except runs 299 and 300. The second resin was used for run 299 and the third resin was used for run 300.

1. Dowex HCR-M, 16-20 mesh, lot 02266-W1
2. Dowex 50W-X2, 20-50 mesh, lot 4673-17
3. Dowex 50W-X12, 16-50 mesh, lot number not given

All three resins are the sulfonated styrene-divinylbenzene copolymer. The Dowex HCR-M resin has 8 per cent cross-linkage while the Dowex 50W-X2 has 2 per cent and the Dowex

50W-X12 resin has 12 per cent cross-linkage. All of the resins were manufactured by the Dow Chemical Company, Midland, Michigan.

Experimental Procedures

Photographic procedure

For each experimental run a number of resin beads were selected which were not cracked, broken or deformed from the spherical shape. The selection was done under a microscope by putting a quantity of resin beads on a watch glass and covering them with distilled water. The light was reflected up from below the beads. Under these lighting conditions cracked or deformed beads could usually be detected even though the beads had a very deep blue color. When the light source was above the beads it was very difficult to detect cracked beads. The good beads were then removed from the watch glass with tweezers.

The procedure used for photographing the resin beads during elution was as follows: The microscope was adjusted to the desired power and one or two photographs were taken of the stage micrometer to record the exact magnification used in the particular run. The acid was turned on and allowed to fill the cell. Any air bubbles which formed in the cell were removed by an eyedropper or syringe. A single bead was then placed in the center of the cell on the screen and allowed to be partially eluted. The microscope was then focused on this bead. The bead was removed and about a dozen new resin beads were drawn into an eye-dropper. The rubber bulb on the eyedropper was squeezed until a drop of water started to form at the end of the eye-dropper. The resin beads contained in the eyedropper would fall into the drop of water. A single drop of water could contain about a dozen resin beads and still remain attached to the glass dropper tube. The drop of resin beads was placed over the cell containing the flowing acids and the dropper bulb was squeezed causing the water drop containing the beads to fall into the cell. At the same time that the beads fell into the cell a switch was opened to record the time on the strip chart recorder. Immediately after the beads had fallen into the cell their location was examined to determine if there were a sufficient number in view of the microscope. If the location of the beads was not adequate they were removed and a new group of beads were dropped into the cell. If the location of the beads was adequate, pictures were taken at about 20 to 30 sec. intervals. When the beads were eluted, the camera was removed, the recorder was stopped, and the acid flow was stopped. A map was then sketched to record the location of each bead which was in view of the microscope. Using a pair of tweezers the beads were removed and put into individual test tubes with a number corresponding to the location on the sketch.

After the film had been developed, the photographs were projected on a screen. Using the photographs of the stage micrometer the diameter of the beads could be determined. The diameter of the blue core was measured in each frame using a scale and a pair of dividers. The time when each photograph was taken was read from the strip chart recorder.

Chemical analysis

The sulfuric acid concentrations were determined by titration with standard sodium hydroxide using phenolphthalein as the indicator. The sodium hydroxide was standardized against potassium acid phthalate. The copper concentrations were determined by atomic absorption using a Perkin-Elmer atomic absorption spectrophotometer Model 290 (Perkin-Elmer Corporation, Norwalk, Connecticut). The standard copper solutions for the atomic absorption spectrophotometer were prepared by dissolving a weighed piece of copper wire in nitric acid and diluting to a known volume.

The determination of the ion exchange capacity of individual resin beads was accomplished by placing the bead in a vile or test tube of about 10 ml. capacity. Solutions were added to the test tube and removed with a syringe. The general procedure was to first wash the bead with about three treatments of distilled water. Then about 5 ml. of the copper tetrammine solution was added and allowed to stand at least an hour. This was repeated three times to insure a complete loading of the copper complex onto the resin. The resin bead was then washed with distilled water with five successive washes. The first two washes were removed rather quickly thus serving only to rinse the test tube of the copper solution. The remaining washes were allowed to remain in the tube for at least an hour each so that the pores of the resin bead could be washed. The last wash was removed with care so that as much of the water as possible could be removed. The tube and bead were dried in a low temperature oven (about 35° C.) for a few minutes to remove the last traces of water clinging to the walls of the test tube. The eluting acid was then added in the desired amount using a syringe. The bead was allowed to remain in the acid for at least two hours and then the acid solution was analyzed for copper content with the atomic absorption spectrophotometer.

The ammonia to copper ratio in the resin was determined by eluting about 10 ml. of resin with 5 N. sulfuric acid. The effluent solution was then analyzed for copper and ammonia. The ammonia analysis was made by the standard Kjeldahl method (17,66) with the sulfuric acid digestion step eliminated. After the ammonia analysis was completed the residual solution containing the precipitate was acidified with sulfuric acid which dissolved the copper precipitate and was then analyzed for copper with the atomic absorption spectrophotometer. The ammonia to copper ratio could then be calculated from the results of the analysis.

Optical measurements

The possible distortion due to the layer of acid covering the resin bead was investigated. Resin beads were photographed without any liquid over them and with a layer (about 1 cm.) of acid or water covering them. The photographs of the resin beads were then compared to determine if the solution changed the apparent size of the resin bead. This test was also made using the stage micrometer as the object instead of the resin beads. However, the stage micrometer was not photographed but its apparent size with and without acid solution of various strengths and depths was measured using a filar micrometer ocular.

Distortion due to possible refraction in the resin bead was investigated. A small amount of resin loaded with the copper tetrammine complex was put into a filtering crucible. The crucible was arranged for vacuum filtration using a water aspirator as the source of the vacuum. A small quantity of 5 N. sulfuric acid was added to the resin in the crucible thus starting the elution of the copper tetrammine from the resin. However, before the resin was entirely eluted the acid solution was removed with vacuum and distilled water was immediately added to the crucible.

The vacuum was left on and water was added continuously until all of the acid had been removed. The resin was removed and upon examination under the microscope the resin beads had a blue core with an outer yellow transparent shell. This resin appeared identical to resin during the course of elution except the movement of the core inward had been stopped. The resin diameter and the core diameter could then be measured with the microscope. The bead was split in half using a sharp razor blade and the core was measured from the open face. The measurements were made both photographically and with a filar micrometer ocular. A number of beads of various sizes and various degrees of elution were measured to determine the optical distortion of the bead.

Experimental Results

Resin loading

For analysis of the rate data it was necessary to determine the amount of copper and ammonia loaded onto the resin. The determination was made by analyzing individual beads for their copper content and then analyzing a bulk sample of resin for the ratio of ammonia to copper. This method was necessary since no analytical method was available which was sensitive enough to measure the ammonia in a single resin bead.

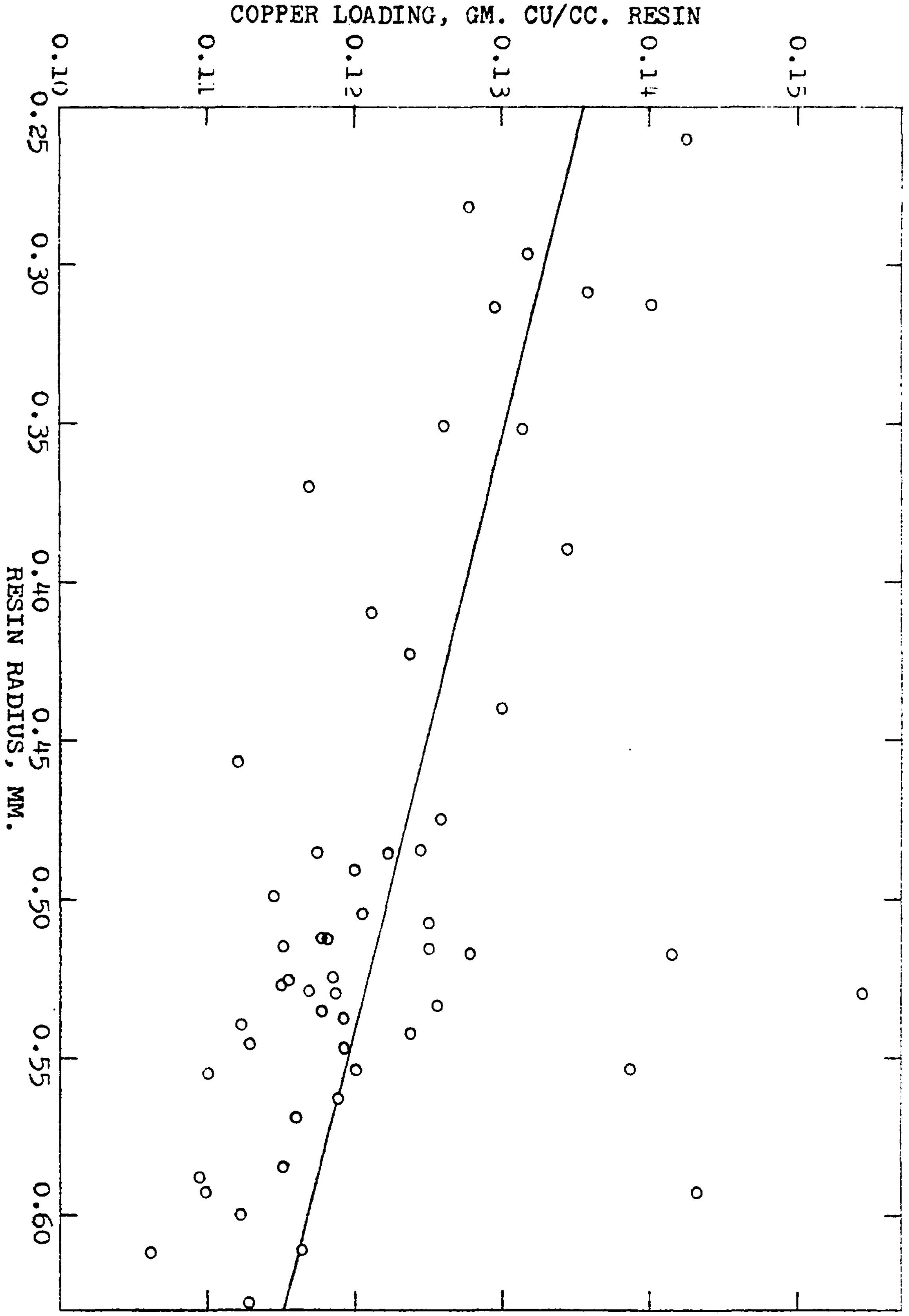
A number of resin beads were selected from the supply of Dowex HCR-M resin in the copper ammine form which were not cracked or deformed. After measuring the radius of a bead, the bead was placed in a test tube. To each test tube 5 ml. of 5 N. sulfuric acid was added. After several hours the acid solution was analyzed for copper with the atomic absorption spectrophotometer. After analysis several beads were washed free of sulfuric acid with distilled water. A second 5 ml. of 5 N. sulfuric acid was added to each resin bead and allowed to stand several hours. The acid was then analyzed with the atomic absorption spectrophotometer. No copper was detected in this analysis which indicates that all of the copper was removed from the beads during the first elution. The analysis was sensitive enough to detect a copper content equal to about 0.5 per cent of the amount measured on the first analysis. The results of the first analysis are presented in Appendix A and in Figure 2. The straight line on Figure 2 represents Equation 17.

$$M_1 = 0.1491 - 0.539\ R \qquad (17)$$

M represents the grams of copper per cubic centimeter of resin. The numerical coefficients of Equation 17 were found by a least squares regression analysis.

After the copper had been removed from the resin beads by the sulfuric acid the beads were individually reloaded with the copper ammine complex. Sulfuric acid was again

Figure 2. Results of the first analysis of the copper content of the resin beads as a function of the bead radius


COPPER LOADING, GM. CU/CC. RESIN
0.10
0.11
0.12
0.13
0.14
0.15
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
RESIN RADIUS, MM.

added and the solution was analyzed for copper. The results are presented in Appendix A and Figure 3. A least squares regression was again used to fit a linear equation to the data to give Equation 18.

$$M_2 = 0.4242 - 4.7978 R \tag{18}$$

The average loading of the first analysis was 0.1226 gm. Cu/cc. of resin while the average loading of the second analysis was 0.1878 gm. Cu/cc. resin. In both cases the copper solution which was used to put the resin into the copper form was the same concentration in both copper and ammonia.

Figure 4 is a plot of the loading obtained from the first analysis versus the loading obtained from the second analysis. Figure 4 demonstrates that there was a much greater variation or scatter in the results obtained by the second analysis as compared to the first analysis. It was originally planned in this work to recover the individual resin beads after the rate determination and determine their capacity individually by loading them with the copper ammine complex followed by elution and analysis. The original loading would then be predicted on the basis of this analysis. Figure 4 demonstrates why this procedure did not work well. There appeared to be little or no correlation between the two loadings. Perhaps the difference in the two loadings was due to the way in which they were made. Before the first

Figure 3. Results of the second analysis of the copper content of the resin beads as a function of the bead radius

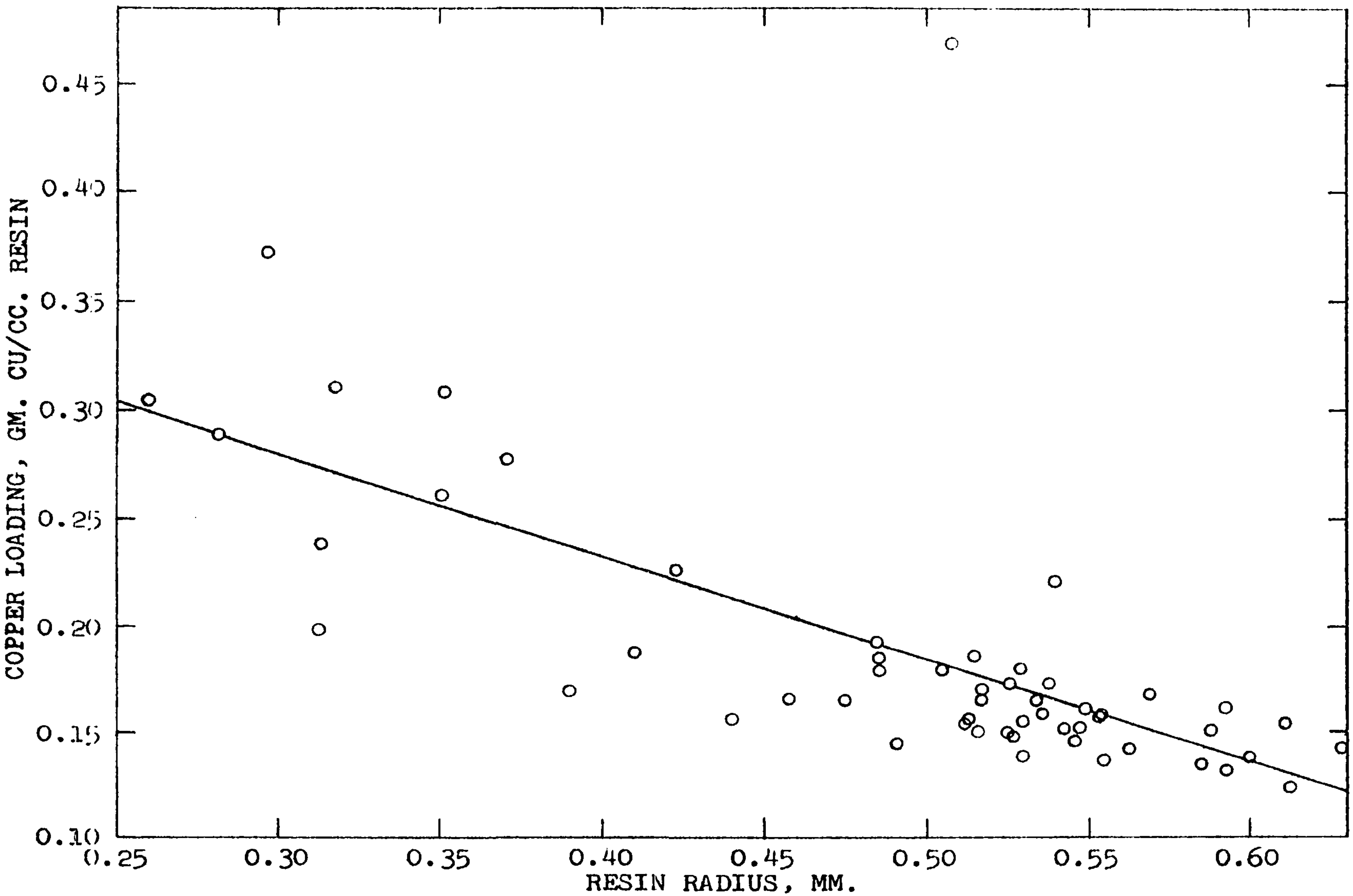
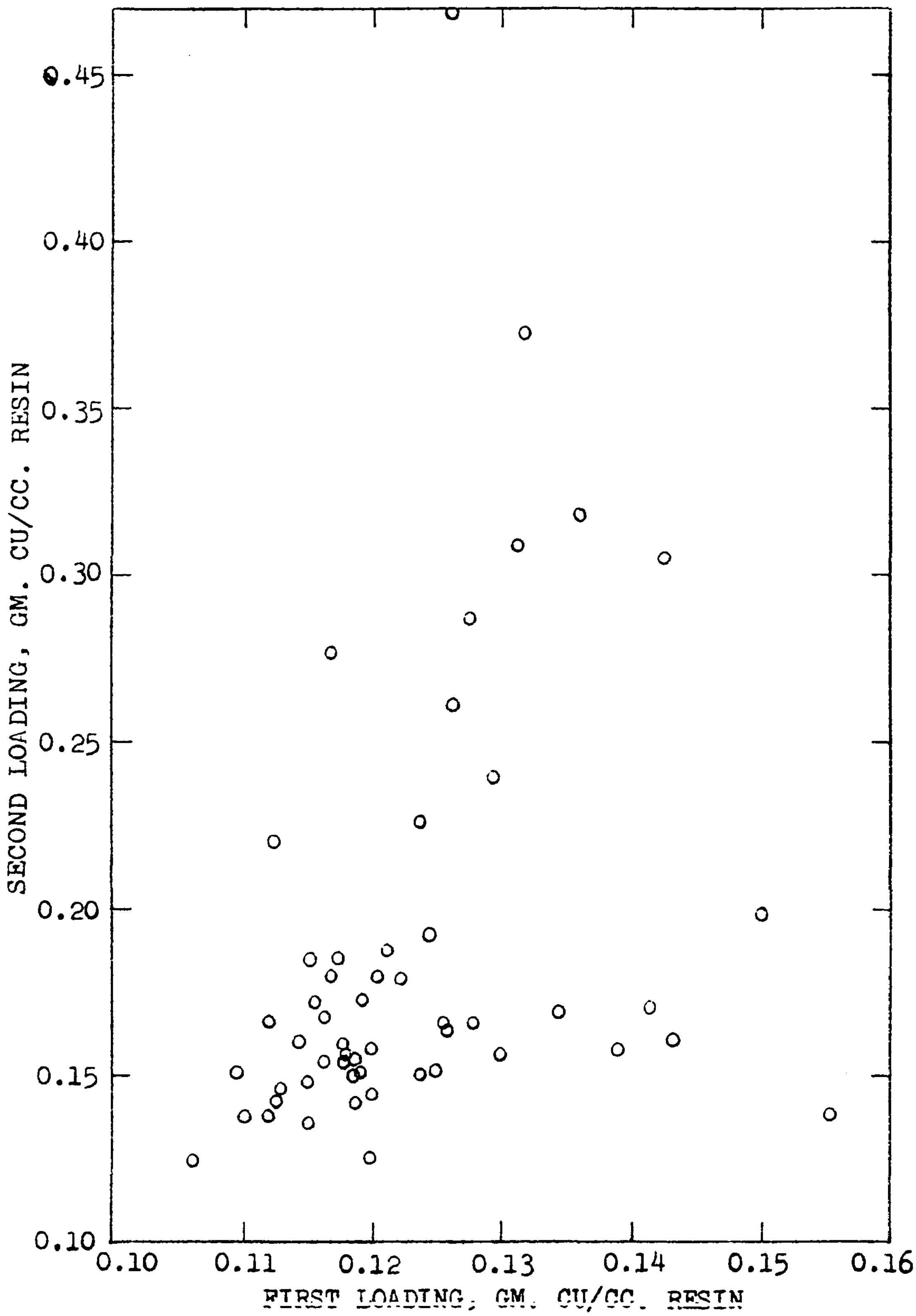

COPPER LOADING, GM. CU/CC. RESIN
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
RESIN RADIUS, MM.

Figure 4. Plot of the copper content from the first analysis versus the copper content from the second analysis for the individual resin beads


SECOND LOADING, GM. CU/CC. RESIN
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.10
0.11
0.12
0.13
0.14
0.15
0.16
FIRST LOADING, GM. CU/CC. RESIN

analysis the entire batch of resin had been stored for a period of time under water. Thus all of the resin beads had an opportunity to reach equilibrium with each other. Before the second elution the resin beads were kept separated and did not have a chance to equilibrate with each other. The other difference between the two situations is that the resin was originally loaded with copper in a column operation whereas in the second situation the resin beads were loaded individually in a batch type operation with a very great excess of solution. The equilibration time can probably explain the much more nearly constant loading in the first case and individual loading can probably explain the much higher average loading in the second case.

To determine the ammonia to copper ratio in the Dowex HCR-M resin, six samples were analyzed in the following manner: Approximately 20 ml. samples of the resin loaded with the copper ammine complex were eluted in a column with 100 ml. of 5 N. sulfuric acid. The effluent solution was diluted to 500 ml. and a 50 ml. sample of this solution was taken for analysis of copper and ammonia. The solution was analyzed for ammonia by the Kjeldahl method and copper was dissolved from the Kjeldahl residue and determined with the atomic absorption spectrophotometer. The results of these analyses are presented in Table 1. The quantities of copper and ammonia shown are for the 50 ml. samples.

Table 1. Results of copper and ammonia analysis used to determine the ammonia to copper ratio for 50 ml. samples

Sample number	Copper meq.	Ammonia meq.	NH_3/Cu ratio
268-1	2.153	7.642	3.5499
268-2	2.099	7.472	3.5605
268-3	2.130	7.213	3.3868
268-4	2.168	7.513	3.4661
268-5	2.076	7.228	3.4824
268-6	2.147	7.394	3.4442

In order to utilize the rate equations it was necessary to calculate the amount of hydrogen required to elute the copper from the resin. This was done by using Equation 17 to calculate the copper in the resin and then using the ammonia to copper ratio to calculate the ammonia in the resin. For this purpose, Equation 17 was rewritten in terms of C_r, the milliequivalents per cubic centimeter of resin. This is given as Equation 19.

$$C_r = 12.86 - 46.50 R \tag{19}$$

Equation 17 was used instead of Equation 18 because Equation 17 was developed from resin beads taken directly from storage.

The beads used in the rate studies were also drawn directly from storage.

Optical correction

Most of the data taken to determine the optical correction was taken by measuring the resin beads with the filar micrometer ocular rather than by photographing the beads. This method proved to be faster and less expensive. The data taken by photographing the beads and measurements with the micrometer ocular are presented in Figure 5. The data were fitted with Equation 20 using a least squares fit of the data.

$$X = 0.9003\ X_o \tag{20}$$

This equation is represented by a straight line drawn in Figure 5. No data could be taken for small core diameters because of the difficulty of stopping the process when it had proceeded to very near completion. Evidently there was sufficient acid within the resin when the core had nearly disappeared so that it could not be washed out to stop the reaction before the core had disappeared. It was also assumed that the line represented by Equation 20 would go through zero, that is, it was assumed that when the outside view of the core disappeared the core had also actually disappeared. Thus the regression line was forced to go through the origin.

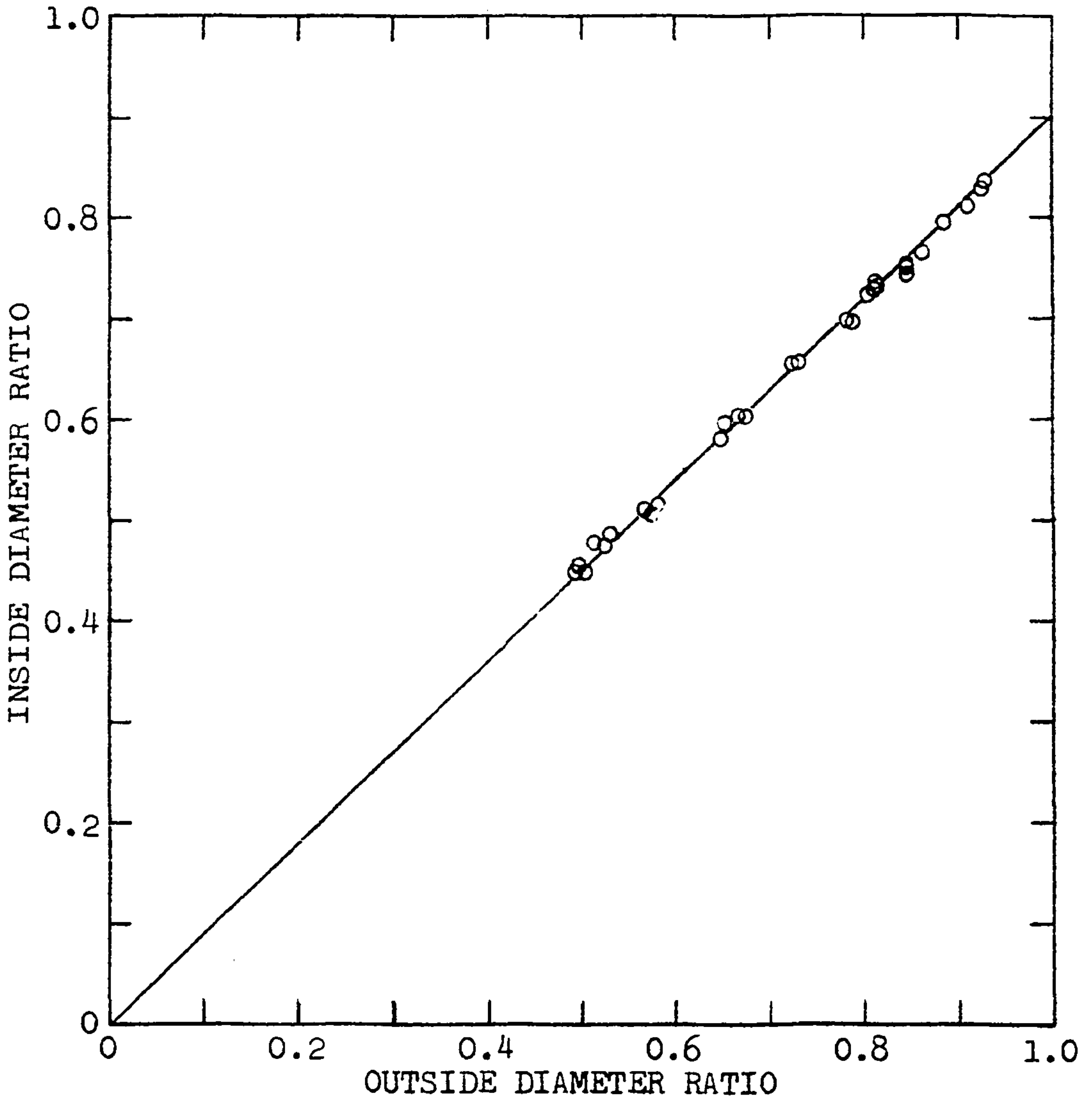


Figure 5. Plot of the diameter ratio observed from the outside of the resin beads versus the diameter ratio observed on the split resin bead

Experimental rate data

Experimental rate data were taken under a number of different acid concentrations and flow rates. In each experimental run about 10 to 20 resin beads were charged to the cell. It was nearly always the case that upon examination of the developed photographic film either the right lens or the left lens of the camera-microscope system gave sharper pictures. That is, one of the lenses was slightly out of focus. The data were taken from the better set of photographs. When neither lens was focused to give sharp pictures the experimental run had to be repeated. Usually at least some of the resin beads did not yield acceptable data. This occurred for several reasons. Generally some of the beads in the photographs were slightly out of focus while others were in focus. Sometimes beads were cracked and this had not been detected in the preliminary examination of them. Sometimes beads would touch each other or touch the wall of the cell and the blue core in the resin did not recede into the center uniformly.

A typical set of photographs is presented in Figure 6 (Run number 297). Only every other photograph is reproduced here. The number in the lower left of each frame corresponds to the photograph number. As mentioned in the Procedures Section the developed film was projected and the diameters of the bead and core were measured. Table 2 is a

Figure 6. Photographic print of the film from run number 297

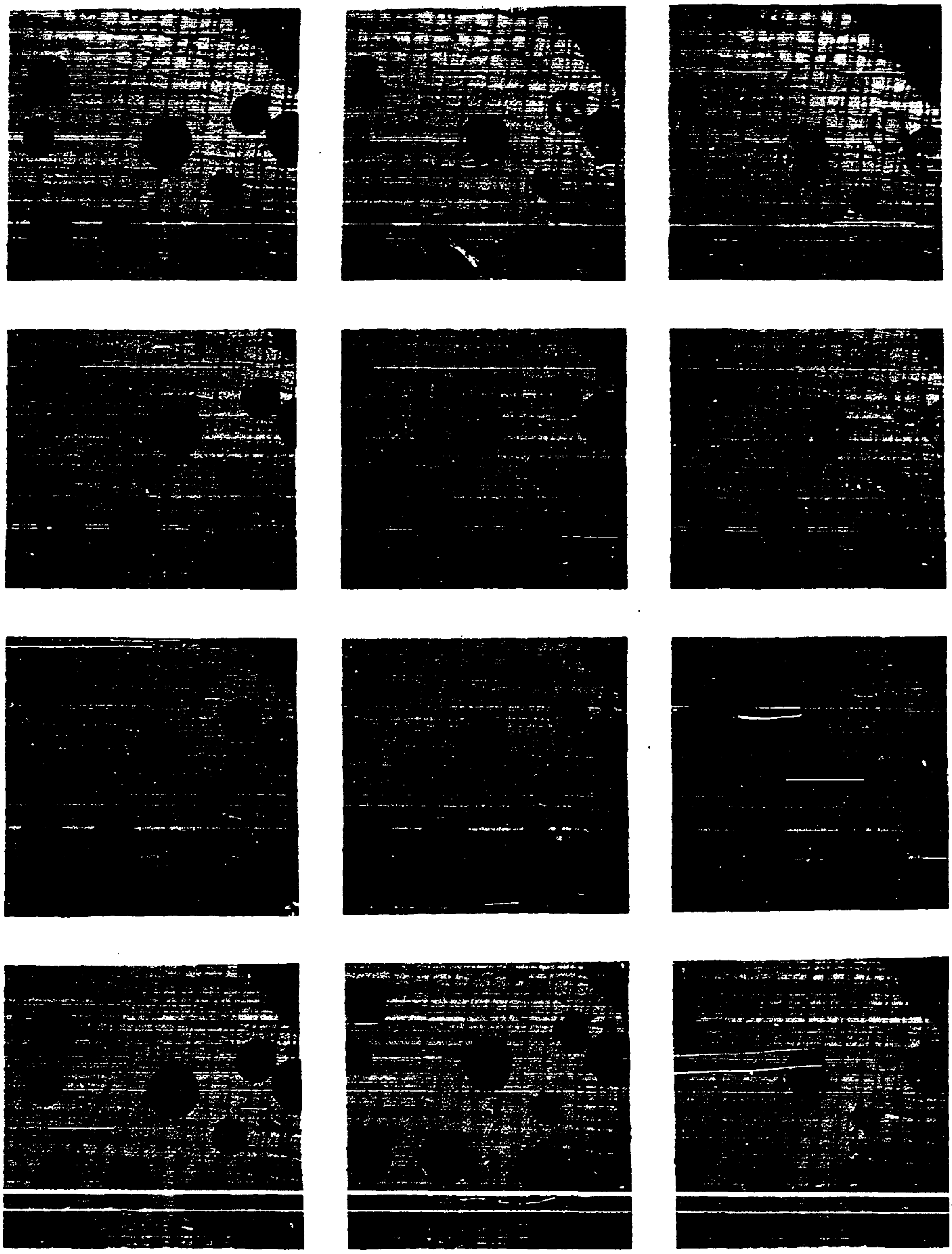


Table 2. Data from run number 297, bead number 5

Time sec.	Exposure sec.	Avg. time sec.	Core/dia.[a] cm.	X Uncorrected	X Corrected	τ
44.7	0.5	44.95	2.82	0.9870	0.8887	0.0447
83.5	0.6	83.80	2.82	0.9870	0.8887	0.0835
118.0	0.7	118.35	2.82	0.9870	0.8887	0.1178
159.5	0.6	159.80	2.81	0.9835	0.8855	0.1590
200.0	0.6	200.30	2.80	0.9800	0.8823	0.1993
243.2	0.7	243.55	2.70	0.9450	0.8508	0.2423
292.0	0.6	292.30	2.60	0.9100	0.8193	0.2908
343.4	0.5	343.65	2.50	0.8750	0.7878	0.3419
388.5	0.6	388.80	2.44	0.8540	0.7689	0.3869
431.1	0.6	431.40	2.31	0.8085	0.7279	0.4292
477.1	0.4	477.30	2.25	0.7875	0.7090	0.4749
530.4	0.6	530.70	2.12	0.7420	0.6681	0.5280
581.6	0.6	581.90	2.01	0.7035	0.6334	0.5790
630.8	0.7	631.15	1.89	0.6615	0.5956	0.6280
675.0	0.5	675.25	1.76	0.6160	0.5546	0.6719
721.6	0.6	721.90	1.63	0.5705	0.5137	0.7183
762.8	0.6	763.10	1.51	0.5285	0.4758	0.7593
801.7	0.6	802.00	1.36	0.4760	0.4286	0.7980
839.9	1.1	840.45	1.20	0.4200	0.3781	0.8363
869.6	0.7	869.95	1.11	0.3885	0.3498	0.8656
894.0	0.5	894.25	0.99	0.3465	0.3120	0.8898
927.0	0.6	927.30	0.82	0.2870	0.2584	0.9227
951.1	0.5	951.35	0.67	0.2345	0.2111	0.9466
972.4	0.5	972.65	0.50	0.1750	0.1576	0.9678
997.9	0.5	998.15	0.11	0.0385	0.0347	0.9932

[a]The diameter listed in the table is the actual measurement from the projected photograph. The average outside diameter was 2.857 cm. which was equal to an actual bead diameter of 0.1140 cm.

list of the data taken from one of the beads in Figure 6. This bead is identified in the first photograph by the arrow. The first column in Table 2 is the time measured from the instant the beads were placed in the cell. The second column is the exposure time of the photograph. Both of these time measurements were taken from the recorder attached to the camera. The average time of the photograph was used for all of the data analysis and this is presented in the third column. The fourth column gives the diameter of the blue core in the resin bead. To measure the diameter the photographs were projected on a screen and a pair of dividers was adjusted to the diameter of the core in the projected photograph. The dividers were then compared with a scale calibrated in 0.1 cm. increments. The outside diameter of the bead was measured in the same manner as the core diameter. The outside diameter was measured about every seventh photograph. These values were averaged to give the average outside diameter which is given at the bottom of Table 2. In the particular case of this bead three readings were taken. They ranged from 2.81 to 2.89 cm. and they increased with increasing time. These measurements are the diameters measured from the projected picture. However, the increase of the outside diameter with time was not a general pattern since other beads had a decrease in outside diameter and some showed no particular trend at all.

The fifth column in Table 2 is the core diameter divided by the outside diameter. The optical correction (Equation 20) for the distortion of light in the resin bead was applied to the diameter ratios in column five to get column six. The way in which the core receded through time is shown in Figure 7 where the dimensionless diameter X (corrected) is plotted against time T. The smooth curve drawn in Figure 7 is not a theoretical estimate but was drawn by eye with the aid of a French curve. This curve was extrapolated to estimate the total elution time (T_∞ = 1005 sec.). The last column in Table 2 is the average time divided by the total time T_∞ to give a dimensionless time τ. Figure 8 is a plot of the dimensionless core diameter X versus dimensionless time τ.

In both Figures 7 and 8 the first four points did not fall on the lines which were drawn through the data. This is the result of the optical properties of the beads. In the early stages of elution the boundary moved into the resin a very small distance. However, when viewed from outside this movement was not evident. The resin bead appeared to be entirely blue in color. The fact that the boundary had moved was checked by splitting some beads with the process stopped using the technique described previously. The split beads had a blue core surrounded by a narrow eluted portion. The first four points on Figures 7 and 8 do not

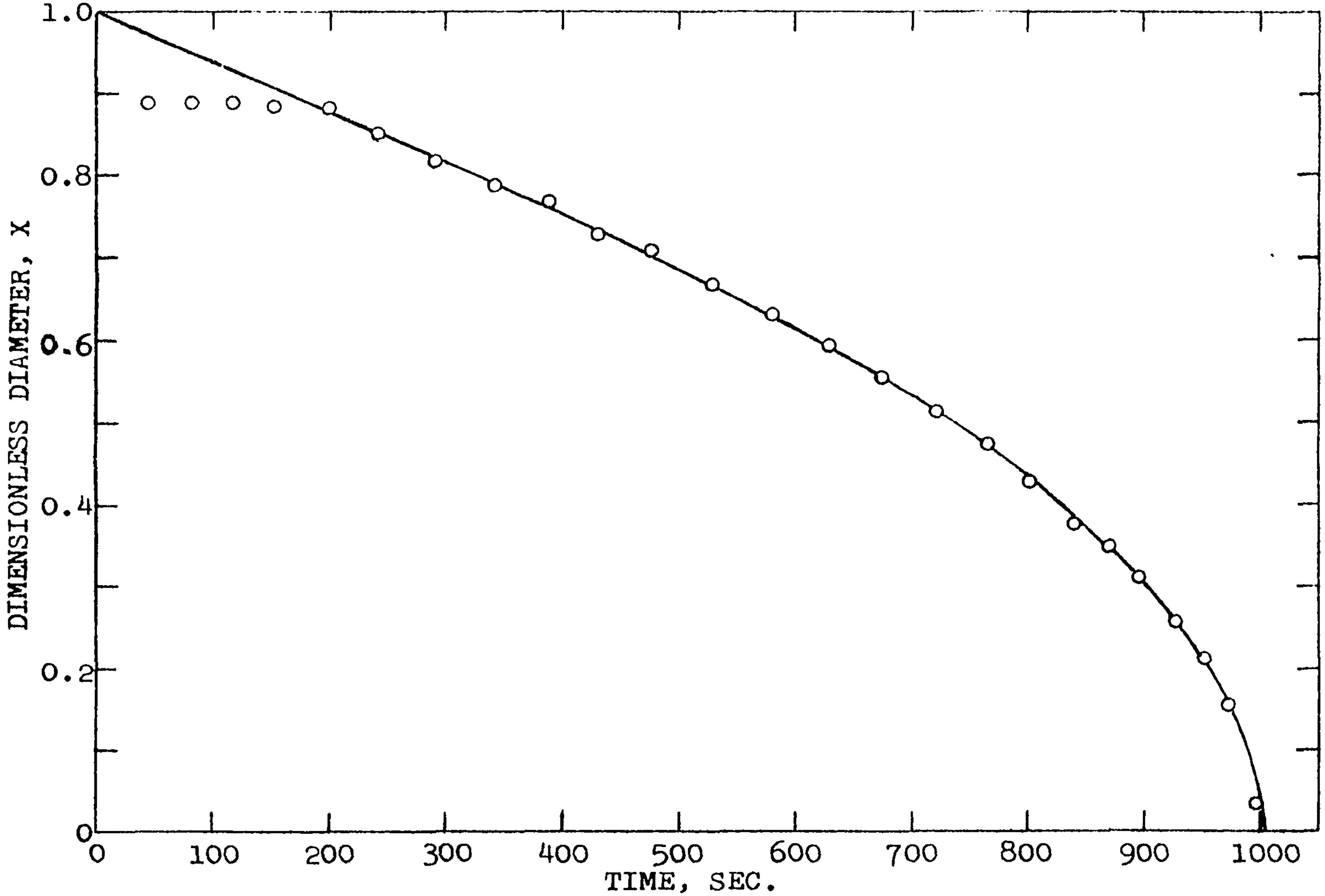


Figure 7. Plot of time versus the dimensionless position for run number 297, bead 5

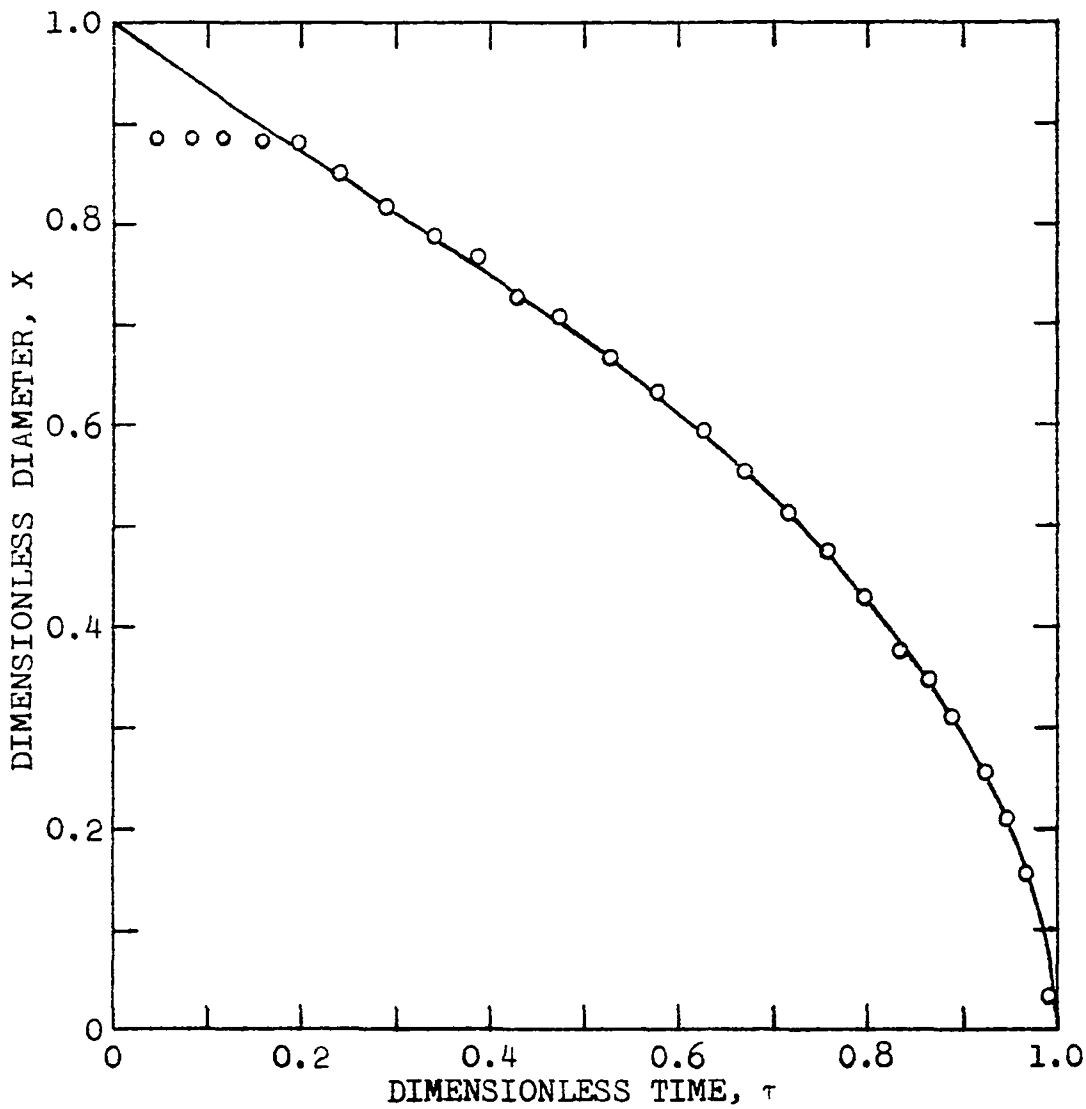


Figure 8. Plot of the dimensionless time versus the dimensionless position for run number 297, bead 5

have a dimensionless diameter of about 1.0 because the optical correction for distortion in the bead (Equation 20) has been applied to the data.

The results of run 297, bead number 5 which was carried out with an acid concentration of 0.0521 N. and a flow rate of 0.980 ml./sec. were quite typical. A number of runs were carried out with various acid concentrations, flow rates, and resin cross-linkages. The experimental conditions for these runs are presented in Table 3. In all of the experimental runs the system was at room temperature (77° F.) and pressure. A summary of all of the results is presented in Appendix B. Appendix B contains a list of the adjusted time T and the dimensionless diameter X (after adjusting for the optical distortion of the bead using Equation 20) for each run. Also given are the experimental conditions of bead radius, acid concentration, and flow rate for each run.

Figure 8 is quite typical of the results obtained from the runs made with acid concentrations of less than 1.0 N. Figure 9 is typical of the results obtained with higher concentrations. The data plotted in Figure 9 was obtained during run 232 with bead number 4 using an acid concentration of 2.46 N. and a flow rate of 1.97 ml./sec. The curve obtained with the higher acid concentration (Figure 9) is shifted to the left and is lower than the curve obtained

Table 3. Experimental conditions[a]

Run number	Dowex resin type	Acid normality	Flow rate ml./sec.
174	HCR-M	5.06	0.161
178	HCR-M	5.06	0.794
182	HCR-M	4.99	0.593
189	HCR-M	2.43	0.866
193	HCR-M	0.986	0.950
194	HCR-M	1.97	0.910
231	HCR-M	2.46	0.166
232	HCR-M	2.46	1.970
261	HCR-M	2.46	0.000
272	HCR-M	0.495	0.973
276	HCR-M	2.45	0.973
279	HCR-M	0.103	0.973
284	HCR-M	0.103	0.0911
287	HCR-M	0.103	2.020
295	HCR-M	0.204	0.980
297	HCR-M	0.0521	0.980
299	50W-X2	0.495	0.980
300	50W-X12	0.495	0.980

[a]All runs were made with sulfuric acid solutions at room temperature (77° F.) and atmospheric pressure.

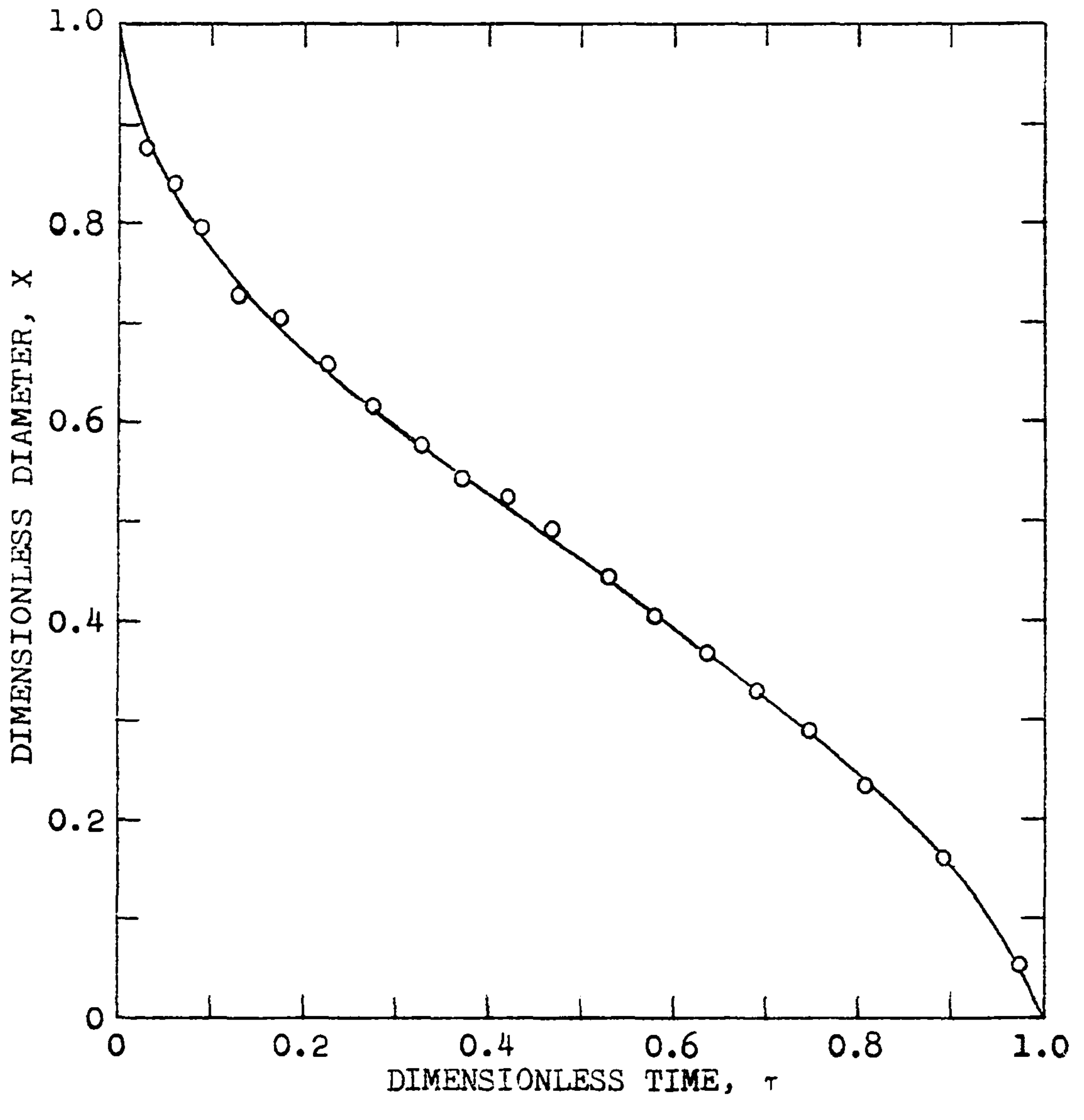


Figure 9. Plot of the dimensionless time versus the dimensionless position for run number 232, bead 4

with the lower acid concentration (Figure 8). With the higher concentration the initial rate of movement of the boundary was much faster than with the lower concentration.

Another general observation which was immediately evident from the experimental data is that the larger resin beads took longer to elute than the smaller resin beads. It was also very evident that the run made with Dowex 50W-X2 resin (run 299) was much faster than any other run. For example, in this run bead number 1 with a radius of 0.0402 cm. was eluted in 91 sec. Run number 272 was made at the same flow rate and acid concentration as run 299 but with Dowex HCR-W resin. In this run bead number 10 with a radius of 0.035 cm. took 310 sec. to be eluted.

Analysis of Data

Mathematical models

As discussed in the section on Previous Work the most general equation which has been proposed to describe diffusion in ion exchange processes is the Nernst-Planck Equation (Equation 21).

$$J_i = -D_i \left(\text{grad } C_i + \frac{Z_i C_i f}{RT} \text{ grad } \phi \right) \qquad (21)$$

Equation 21 is based on a number of assumptions about the ion exchange process taking place. The resin bead is considered to be a homogeneous phase without regard to the actual porous structure of the bead itself. It is assumed that the resin matrix has no influence upon the diffusion process (effects caused by swelling or shrinking of the bead are neglected). It is further assumed that each ionic species is not influenced by any other species present except by the electrical charge.

Some additional assumptions were made to reduce Equation 21 to an equation which could be solved with the moving boundary condition. Thus it was assumed that the effects of electrical coupling are not important. This assumption reduces the Nernst-Planck equation to Fick's law of diffusion. It was also assumed that the process could be considered as an equimolal diffusion process. This assumption involves assuming that one component is diffusing into the resin bead and that an equivalent number of moles of a second component is moving out of the resin. This assumption did not quite fit the experimental condition since 1.22 moles of acid was required for each mole of copper and ammonium moving out of the resin. This assumption also requires that the copper and ammonium ions be lumped into a single component.

Under the above assumptions and assuming that diffusion in directions other than the radial direction in the spherical coordinate system can be neglected reduces Equation 21 to Equation 22.

$$J = -D \frac{\partial C}{\partial r} \qquad (22)$$

Equation 22 can be combined with the unsteady state material balance given in Equation 23.

$$\frac{\partial C}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 J) \quad (23)$$

The result of combining these two equations is Equation 24.

$$\frac{\partial C}{\partial t} = \frac{D}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) \quad (24)$$

In Equation 24, C is the concentration of the hydrogen ion, D is the diffusion coefficient, t is time, and r is the radial coordinate. A schematic diagram of the resin bead is presented in Figure 10.

General solution The initial condition for Equation 24 is given below.

$$\text{at } t = 0,\ C = 0 \text{ for } 0 \leq r \leq R \quad (25)$$

At the surface of the bead it was assumed that the acid concentration is constant as given below:

$$\text{at } r = R,\ C = C_s \text{ for all } t \quad (26)$$

The boundary condition at the moving boundary may be derived in the following way. The flux N of material reaching the moving boundary can be evaluated by Equation 27.

$$N = D \left. \frac{\partial C}{\partial r} \right|_{r = b} \quad (27)$$

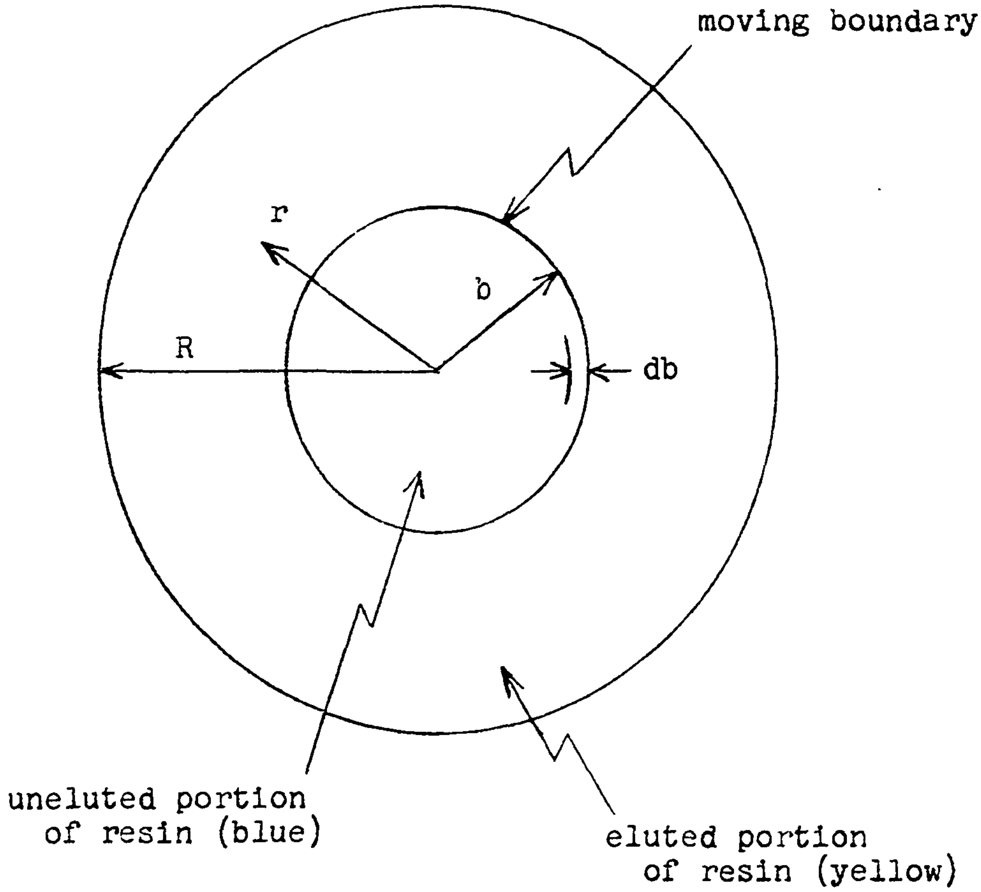


Figure 10. Schematic diagram of a resin bead

The movement of the boundary can also be defined by the amount of material required by the resin. The amount of hydrogen ions reaching the boundary in an element of time dt is given by Equation 28.

$$\alpha = N\,(4\,\pi\,b^2)\,dt \tag{28}$$

where $4\,\pi\,b^2$ is the area of the spherical boundary. The amount of hydrogen ions required for the breaking of the copper complex and the elution of the resin is given by Equation 29.

$$\alpha = -C_r\,(4\,\pi\,b^2)\,db \tag{29}$$

In Equation 29 the term $(4\,\pi\,b^2)\,db$ is the volume of resin eluted in a small element of time and C_r is the amount of hydrogen required for elution and breaking the complex per unit volume of resin. Equations 28 and 29 may be equated and rearranged to give Equation 30.

$$N = -C_r\,\frac{db}{dt} \tag{30}$$

N may be eliminated from Equations 27 and 30 to give the boundary condition at the moving boundary. This is given in Equation 31.

$$\text{at } r = b, \quad D\,\frac{\partial C}{\partial r}\bigg|_{r=b} = -C_r\,\frac{db}{dt} \quad \text{for} \quad t > 0 \tag{31}$$

No analytical solution to Equation 24 with these boundary conditions has been found. Others[1] are working on a numerical solution to this equation.

Quasi steady state solution (Model I) With the addition of one mathematical assumption a solution to Equation 24 has been obtained. It is first assumed that Equation 24 can be solved using a constant boundary instead of a moving boundary as given below.

$$C = 0 \text{ at } r = b \tag{32}$$

The solution to Equation 24 is given by Carslaw and Jaeger (14) and is presented here as Equation 33.

$$C = \frac{RC_s(r-b)}{r(R-b)} + \frac{2}{r\pi}\sum_{i=1}^{\infty} \frac{RC_s(-1)^i}{i} \sin\left[\frac{i\pi(r-b)}{R-b}\right] \exp\left[-\frac{Di^2\pi^2 t}{(R-b)^2}\right] \tag{33}$$

Using Equation 33 to define the concentration profile the derivative $\partial C/\partial r$ can be evaluated as given in Equation 34.

$$\left.\frac{\partial C}{\partial r}\right|_{r=b} = \frac{RC_s}{b(R-b)}\left[1 + 2\sum_{i=1}^{\infty} (-1)^i \exp\left[-\frac{Di^2\pi^2 t}{(R-b)^2}\right]\right] \tag{34}$$

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[1]Richard C. Seagrave and Sami M. Selim, Chemical Engineering Department, Iowa State University, Ames, Iowa, are working on a computer solution to the moving boundary problem in various coordinate systems.

This equation may be combined with Equation 31 to give Equation 35 which describes the movement of the boundary as a function of time.

$$\frac{db}{dt} = - \frac{DRC_s}{C_r b(R-b)} \left[1 + 2 \sum_{i=1}^{\infty} (-1)^i \exp\left[- \frac{Di^2\pi^2 t}{(R-b)^2} \right] \right] \quad (35)$$

Equation 35 was integrated using the IBM 360 model 65 computer located at the Iowa State University Computation Center. For computer programming the equation was put into dimensionless form using the variable changes given in Equations 36 and 37.

$$X = b/R \quad (36)$$

$$\tau = t/T \quad (37)$$

In Equation 37 T is the time required for the complete elution of the resin bead; thus at T the boundary is at the center of the bead (or b = 0). Using these substitutions, Equation 35 is transformed into Equation 38.

$$\frac{dX}{d\tau} = \frac{-C_s\beta}{C_r X(1-X)} \left[1 + 2 \sum_{i=1}^{\infty} (-1)^i \exp\left[\frac{-\beta i^2\pi^2\tau}{(1-X)^2} \right] \right] \quad (38)$$

where

$$\beta = \frac{DT}{R^2}$$

The boundary conditions for Equation 38 are given in Equations 39 and 40.

$$\text{at} \quad \tau = 0, \quad X = 1 \tag{39}$$

$$\text{at} \quad \tau = 1, \quad X = 0 \tag{40}$$

On the right hand side of Equation 38 the term X(1-X) appears in the denominator. This term goes to zero when X is equal to either 1 or 0. This in turn causes the right hand side of the equation to be undefined and difficult to handle at the boundary conditions. To facilitate the programming of the equation, Equation 38 was inverted to give Equation 41.

$$\frac{d\tau}{dX} = \frac{-C_r X(1-X)}{\beta C_s \left[1 + 2 \sum_{i=1}^{\infty} (-1)^i \exp\left(\frac{-\beta i^2 \pi^2 \tau}{(1-X)^2} \right) \right]} \tag{41}$$

A computer program was written to integrate Equation 41 with the boundary conditions given in Equations 39 and 40. The method used for the integration was a predictor-corrector method. The predictor-corrector subroutine was supplied by the Iowa State University Computation Center.[1] Since it was necessary to know the diffusion coefficient before Equation 41 could be integrated and the integration of the equation was being used to determine the diffusion coefficient, it was necessary to assume a diffusion coefficient and then integrate the equation to see if the final boundary condition had been

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[1]NODE: a Package of Fortran Subroutines to Solve Ordinary Differential Equations. Iowa State University Computation Center, Ames, Iowa. This program uses the predictor-corrector equations of R. L. Crane.

met. If the final boundary condition had not been met a new value was assumed and the equation was integrated again. This procedure was followed until the final boundary condition was within tolerance.

The numerical integration of Equation 41 gave the results presented in Figure 11. The various curves are plotted for various ratios of the concentration in the solution to the equivalent concentration in the resin (C_s/C_r).

Quasi steady state solution (Model II) A second model will be developed in a manner similar to the one just presented. The following derivation yields the quasi steady state equations given by Levenspiel (36) and others (3,67, 68,69). For this derivation it is assumed that the concentration profile in the bead and the film are always at the steady state value. Thus Equation 24 is reduced to Equation 42 by dropping the time dependent term.

$$\frac{d}{dr}\left(r^2 \frac{dc}{dr}\right) = 0 \tag{42}$$

The Model II equation will be developed for combined film diffusion and diffusion within the bead. It is assumed that there is a liquid film surrounding the resin bead of thickness S and the concentration at the surface of the bead is C^*. Under these conditions the boundary conditions for the solution of Equation 42 are given in Equations 43, 44, and 45.

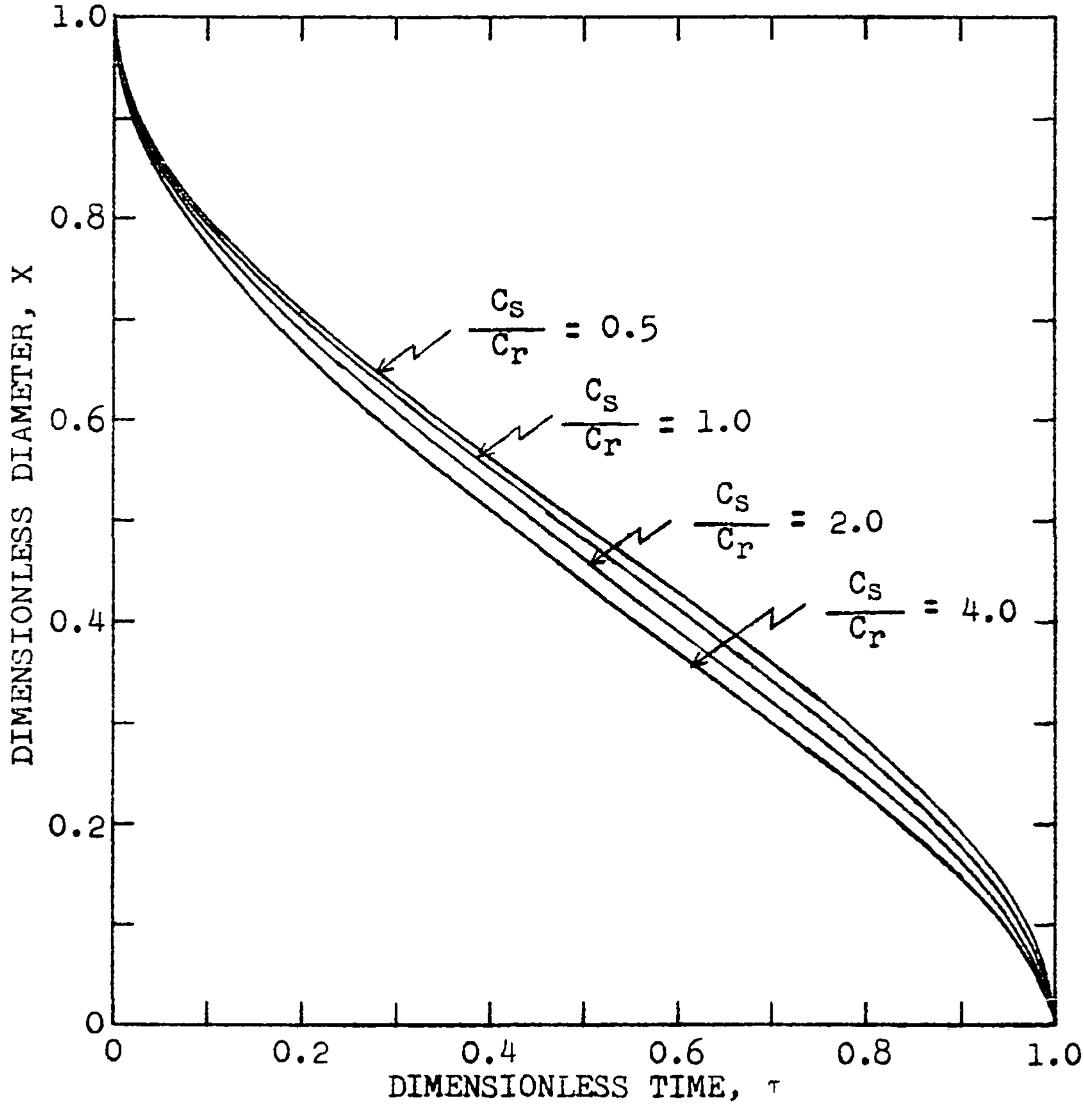


Figure 11. Plot of the dimensionless time versus the dimensionless position for Model I

$$\text{at } r = b, \quad C = 0 \tag{43}$$

$$\text{at } r = R, \quad C = C^* \tag{44}$$

$$\text{at } r = R + S, \quad C = C_s \tag{45}$$

For diffusion in the bead Equation 42 can be integrated using the boundary conditions in Equations 43 and 44 to give Equation 46.

$$C = \frac{C^* R}{R-b} \left(1 - \frac{r}{R} \right) \tag{46}$$

Equation 42 can be integrated for the film using the boundary conditions in Equations 44 and 45 to give Equation 47.

$$C_f = C^* + \frac{(C_s - C^*)(R+S)}{S} \left(1 - \frac{r}{R} \right) \tag{47}$$

At the surface of the resin bead the flux of material in the film must be equal to the flux of material in the bead. Equation 48 is the mathematical statement of this boundary condition.

$$D_f \left. \frac{dC_f}{dr} \right|_{r=R} = D \left. \frac{dC}{dr} \right|_{r=R} \tag{48}$$

The result of differentiating Equations 46 and 47 and combining with Equation 48 is given in Equation 49 after solving for C^*.

$$C^* = \frac{D_f C_s (R+S)(R-b)}{DbS + D_f (R+S)(R-b)} \tag{49}$$

This equation may then be used to eliminate C^* from Equation 46 to give Equation 50.

$$C = \frac{D_f C_s (R+S) R}{DbS + D_f (R+S)(R-b)} \left(1 - \frac{r}{R} \right) \qquad (50)$$

This equation describing the concentration profile in the bead may be combined with the boundary condition describing the moving boundary (Equation 31) giving Equation 51.

$$\frac{db}{dt} = \frac{-DD_f C_s (R+S) R}{C_r b \, [DSb + D_f (R+S)(R-b)]} \qquad (51)$$

The initial condition for the integration of Equation 51 is given in Equation 52.

$$\text{at} \quad t = 0, \quad b = R \qquad (52)$$

After integrating and making the substitutions given in Equations 53 and 54 the final result is given in Equation 55.

$$X = b/R \qquad (53)$$

$$k = \frac{D_f (R+S)}{2S} \qquad (54)$$

$$t = \frac{C_r R^2}{6C_s} \left[\frac{1-X^3}{k} + \frac{1-3X^2+2X^3}{D} \right] \qquad (55)$$

In Equation 55, D is the diffusion coefficient in the resin bead and k represents the film coefficient.

Equation 55 may be transformed to give the dimensionless equation given in Equation 56.

$$\tau = \frac{t\ 6C_s}{C_r R^2 k} = 1-X^3 + \frac{k}{D}\ (1-3X^2+2X^3) \qquad (56)$$

Equation 56 is plotted in Figure 12 for different ratios of the film coefficient to the diffusion coefficient. Film diffusion controls when k is small and D is large ($k/D \rightarrow 0$) and this situation is represented by the upper line in Figure 12. In the reverse situation when k is large and D is small ($k/D \rightarrow \infty$) the process is controlled by diffusion in the resin bead and this is represented by the lowest line in Figure 12. The lines in between these two extremes are for varying degrees of particle and film diffusion control.

A comparison can now be made between Model I and Model II. It is evident from Figure 11 that as the concentration ratio C_s/C_r approaches zero the dimensionless time versus dimensionless boundary position curve approaches the bead diffusion curve in Figure 12. Thus, in this respect, the two models give consistent results. Further comparison is difficult since any effects of film diffusion are not included in Model I.

From Figure 12 it is evident that in order to analyze data using Equation 55 the data must fall between the film diffusion curve in Figure 12 and the bead diffusion curve.

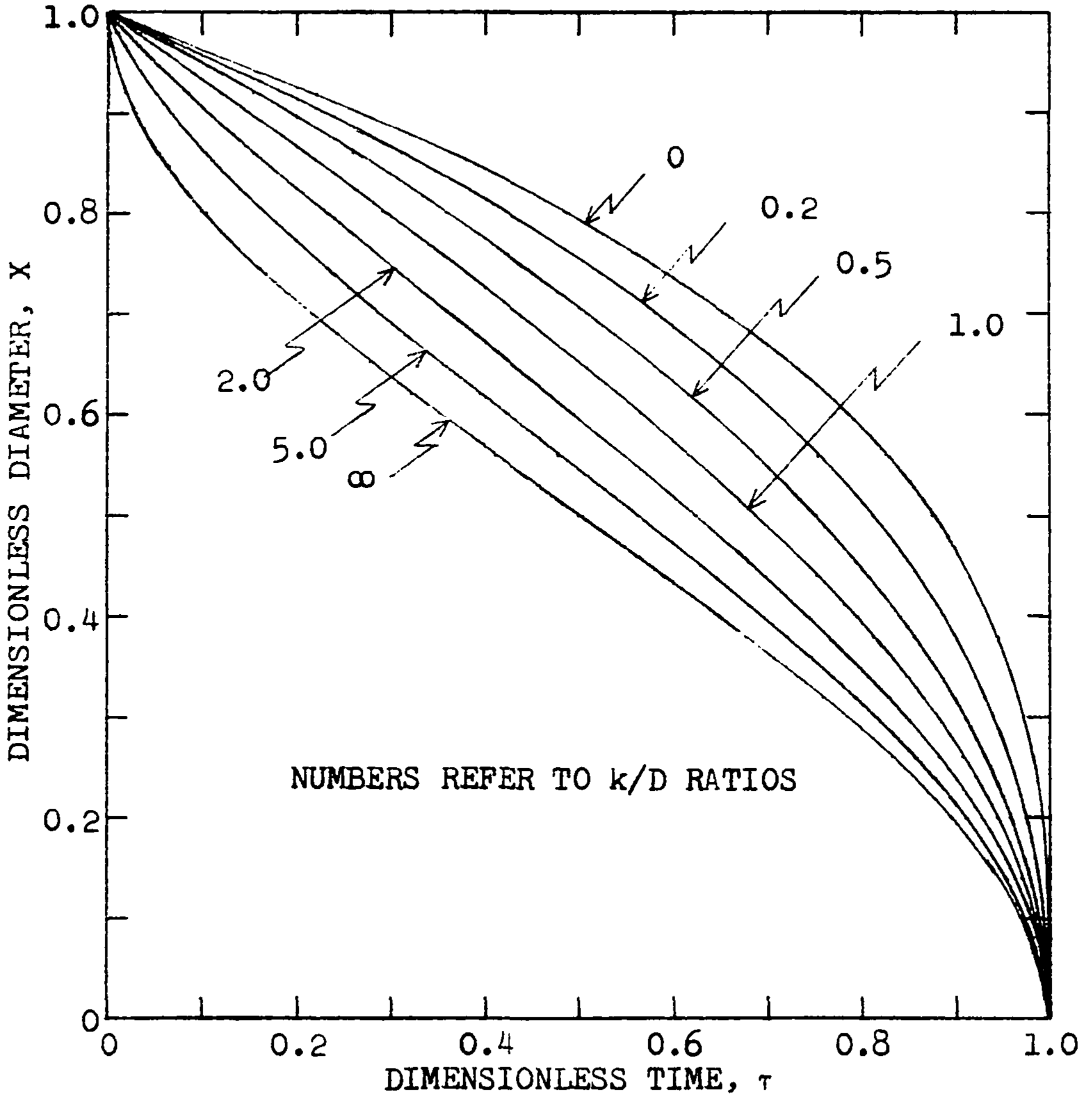


Figure 12. Plot of the dimensionless time versus the dimensionless position for Model II

The sample data for run 297, bead 5 (Figure 7) does have data which fall within this region. However, the data from run 232, bead 4 fell below and to the left of the bead diffusion curve in Figure 12. Thus, this data did not fall within the region where Equation 55 could be used. The runs made with an acid concentration of 1.0 N. or less yielded data which could be analyzed by Equation 55 but runs made with an acid concentration greater than 1.0 N. yielded data which could not be analyzed by Equation 55.

Integration of Model I gave curves which were below and to the left of the bead diffusion curve from Model II as shown in Figure 12. This is in general agreement with the high concentration data which also gave curves below and to the left of the bead diffusion curve. However, Model I could not be used to analyze the data from the higher concentration runs. It can be observed in Figure 11 that the curve from Model I for a concentration ratio (C_s/C_r) of 0.5 is only very slightly different from the bead diffusion curve from Model II. The data for a concentration ratio of about 0.5 showed a considerable deviation below and to the left of the bead diffusion curve from Model II in Figure 12. Thus Model I was not adequate to analyze the data but it did seem to show the same general deviation from the bead diffusion model (Model II) that the data did.

Non-linear regression

A large portion of the experimental data yielded curves which were very similar to those in Figure 12. A non-linear regression technique (18) was used to determine the best values of k_f and D for Model II. The time of each photograph was chosen as the independent variable since it was determined much more accurately than the measurement of core diameter from the photographs. The computation of the regression coefficients was done with a digital computer with the aid of a non-linear regression program.[1] The general program which was available required the user to write a subroutine to evaluate the function and its derivatives. The function (Equation 55) was not explicit in the core diameter X, the dependent variable, and therefore it was necessary to evaluate it by an iteration technique. A Newton iteration technique (34) was used for this purpose and the iteration was continued until two successive estimates of X were equal to five significant digits. Equation 55 was differentiated implicitly to give the derivatives required for the regression analysis. They are given here in Equations 57 and 58 and could be evaluated in

[1]TARSIER by J. D. Atkinson, Numerical Analysis Programming Series, No. 8, Statistical Laboratory, Iowa State University, Ames, Iowa, February 1966. This program is a general program for fitting non-linear regression functions by least squares, using the modified Gauss Newton method by H. O. Hartley.

the computer program once the value of X had been determined.

$$\frac{\partial X}{\partial k} = \frac{\frac{(1-X^3)}{k^2}}{\frac{(6X^2-6X)}{D} - \frac{3X^2}{k}} \tag{57}$$

$$\frac{\partial X}{\partial D} = \frac{\frac{(1-3X^2+2X^3)}{D^2}}{\frac{(6X^2-6X)}{D} - \frac{3X^2}{k}} \tag{58}$$

In order to use the computer program it was necessary to assume a value for k and D. The program then made new estimates of k and D and computed the sum of squares on each assumed value. The program then minimized the sum of squares. The process was continued until two successive estimates of k and D were within a tolerance level. The tolerance level was set at three significant digits for this work. The computer program was somewhat sensitive to the initial estimates which were chosen. In fact, for several of the data sets no solution was ever obtained.

It was observed previously that the first several data points in many data sets were distorted due to the nearness of the boundary to the outside of the resin bead. In many of the data sets there were from two to five data points with a dimensionless core diameter value of from 0.86 to 0.90. The regression analysis was run with these points removed as well as with these points present. For the regression analysis with the first several points removed all points with a dimensionless core diameter greater than about 0.86 were removed. The number of points which were removed for this regression analysis is given in Appendix B. Figure 13 is an example of the predicted regression curve with these points removed. The data used in Figure 13 is from run number 279, bead number 10. The first four points in Figure 13 were removed from the data for the regression analysis. It is evident from Figure 13 that there is a slight discrepancy between the model and the data. At shorter and longer times the data points fall below the regression curve while at intermediate times (about 300 to 500 sec.) the data points fall above the regression curve. This effect was quite typical of the results obtained from the other beads.

The results of the regression analysis are included in Appendix B. The coefficients determined for the various beads in a run by regression analysis were averaged and these results are presented in Tables 4 and 5. Also presented in Tables 4 and 5 is the variance of coefficients for the various beads within a given run as determined by the regression analysis. These variances are labeled σ_D and σ_K for the variance of the bead diffusion coefficient and the variance of the film coefficient respectively. As mentioned previously only the runs made with an acid concen-

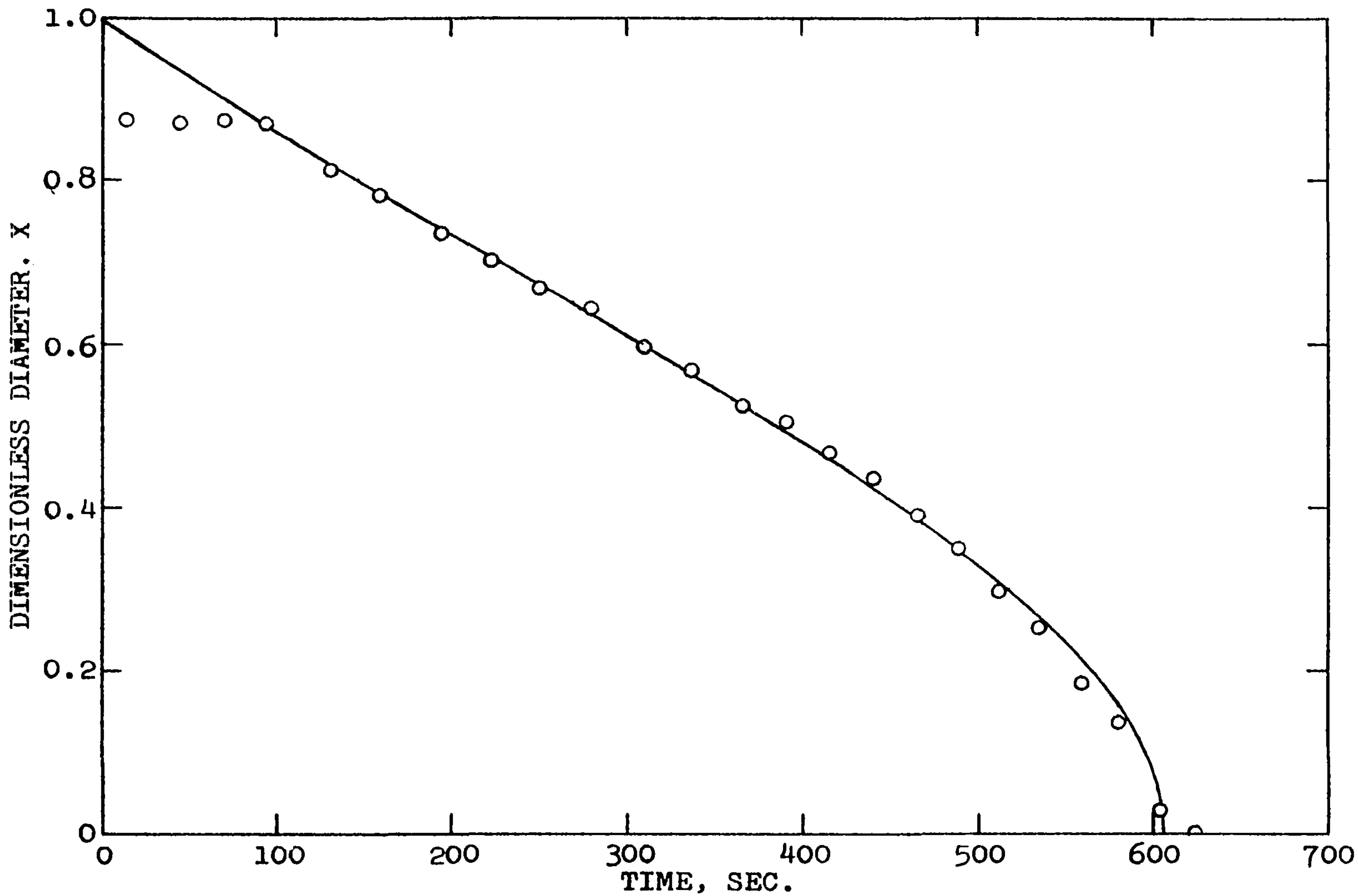


Figure 13. Plot of the time versus the dimensionless position using data from run number 279, bead 10 and the least squares regression curve for this data

Table 4. Average coefficients from the non-linear regression analysis with the first several points included

Number	Acid N.	Flow rate ml./sec.	k x 10^4 sq.cm./sec.	σ_k x 10^4 sq.cm./sec.	D x 10^4 sq.cm./sec.	σ_D x 10^4 sq.cm./sec.	Average σ
193	0.986	0.950	2.235	1.051	0.223	0.0187	0.02132
272	0.495	0.973	1.923	1.188	0.210	0.0063	0.01864
279	0.103	0.973	2.789	1.833	1.018	0.1482	0.02915
284	0.103	0.091	1.031	0.093	1.144	0.0850	0.01828
287	0.103	2.019	3.222	1.009	1.131	0.1582	0.02022
295	0.204	0.980	1.481	0.261	0.500	0.0449	0.02006
297	0.052	0.980	1.740	0.339	3.094	1.6767	0.02540
300[a]	0.495	0.980	1.030	1.811	0.220	0.0709	0.03152

[a]Dowex 50W-X12 resin used for this run.

Table 5. Average coefficients from the non-linear regression analysis with the first points left out

Number	Acid N.	Flow rate ml./sec.	$k \times 10^4$ sq.cm./sec.	$\sigma_k \times 10^4$ sq.cm./sec.	$D \times 10^4$ sq.cm./sec.	$\sigma_D \times 10^4$ sq.cm./sec.	Average σ
193	0.986	0.950	1.639	0.602	0.234	0.0238	0.01339
272	0.495	0.973	1.506	0.738	0.217	0.0075	0.01284
279	0.103	0.973	2.657	1.915	1.036	0.1570	0.01832
284	0.103	0.091	0.997	0.081	1.206	0.838	0.01104
287	0.103	2.019	3.165	0.917	1.139	0.1669	0.01940
295	0.204	0.980	1.419	0.263	0.512	0.0499	0.01688
297	0.052	0.980	1.705	0.338	3.271	1.8584	0.01320
300[a]	0.495	0.980	1.017	1.815	0.225	0.0754	0.02735

[a]Dowex 50W-X12 resin used for this run.

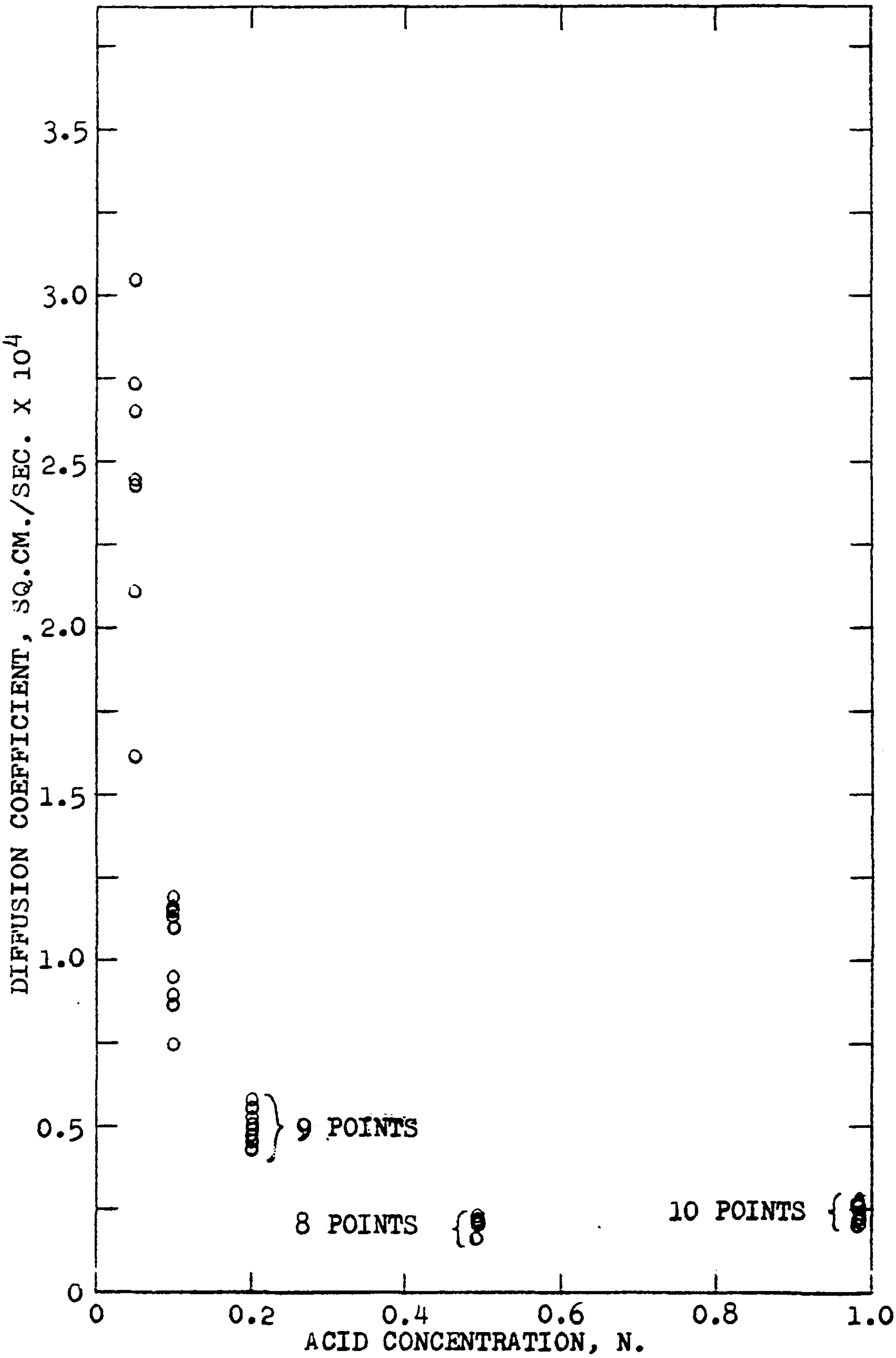
tration of 1.0 N. or less could be analyzed by the regression analysis and thus only these runs are presented in Tables 4 and 5. Table 4 is the results of the regression analysis with the first several points included and Table 5 is the results with the first several points removed. The removal of the initial points did not change the coefficients greatly. In general, the diffusion coefficient decreased slightly. The big difference was in the variance of points about the regression. This variance was calculated by Equation 59 where $\hat{X}_i$ is the dimensionless core diameter as estimated by the regression model.

$$\sigma = \frac{\sum_{i=1}^{n} (X_i - \hat{X}_i)^2}{n-2} \tag{59}$$

The sum of squares in Equation 59 was divided by n-2 because there were two degrees of freedom used in estimating the two unknown parameters k and D. The last column in Tables 4 and 5 is the average variance for each run. With the first several points removed the variance was about one-third the sum of squares with these points included. In the discussion of the other results from the regression analysis which follows all of the results are from the regression analysis with these points removed unless otherwise noted.

Effects of concentration and flow rate The results of regression analysis of runs with a flow rate of about 0.98 ml./sec. are plotted in Figure 14. Only the diffusion

Figure 14. Effect of acid concentration on the diffusion coefficient



DIFFUSION COEFFICIENT, SQ.CM./SEC. X 10^4
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
9 POINTS
8 POINTS
10 POINTS
0
0.2
0.4
0.6
0.8
1.0
ACID CONCENTRATION, N.

coefficients of the individual beads have been plotted in this figure. From Figure 14 and Table 5 it is evident that the variance of the points increases as the acid concentration decreases. The average diffusion coefficient decreases as the acid concentration increases but it appears to be nearly constant on the range between 0.5 and 1.0 N. acid solutions.

Figure 15 is a plot of both the film coefficient and the bead diffusion coefficient of the individual beads as determined by regression analysis. All of the data plotted in Figure 15 were obtained with an acid concentration of 0.103 N. with the flow rate varying from 0.091 to 2.019 ml./sec. The film and diffusion coefficients of the individual beads have been plotted against the radius of the resin beads. From Figure 15 it is evident the diffusion coefficients are about the same for all three flow rates. This result would be expected since the velocity of the fluid past the resin bead should not change the internal characteristics of the resin bead. This result indicates that the Model II equation is quite effective in separating the effects of film diffusion and particle diffusion. Figure 15 also indicates that the bead diffusion coefficients were almost constant with respect to the radius of the bead.

The film mass transfer coefficient k appears to increase with increasing fluid velocity and it also increases with

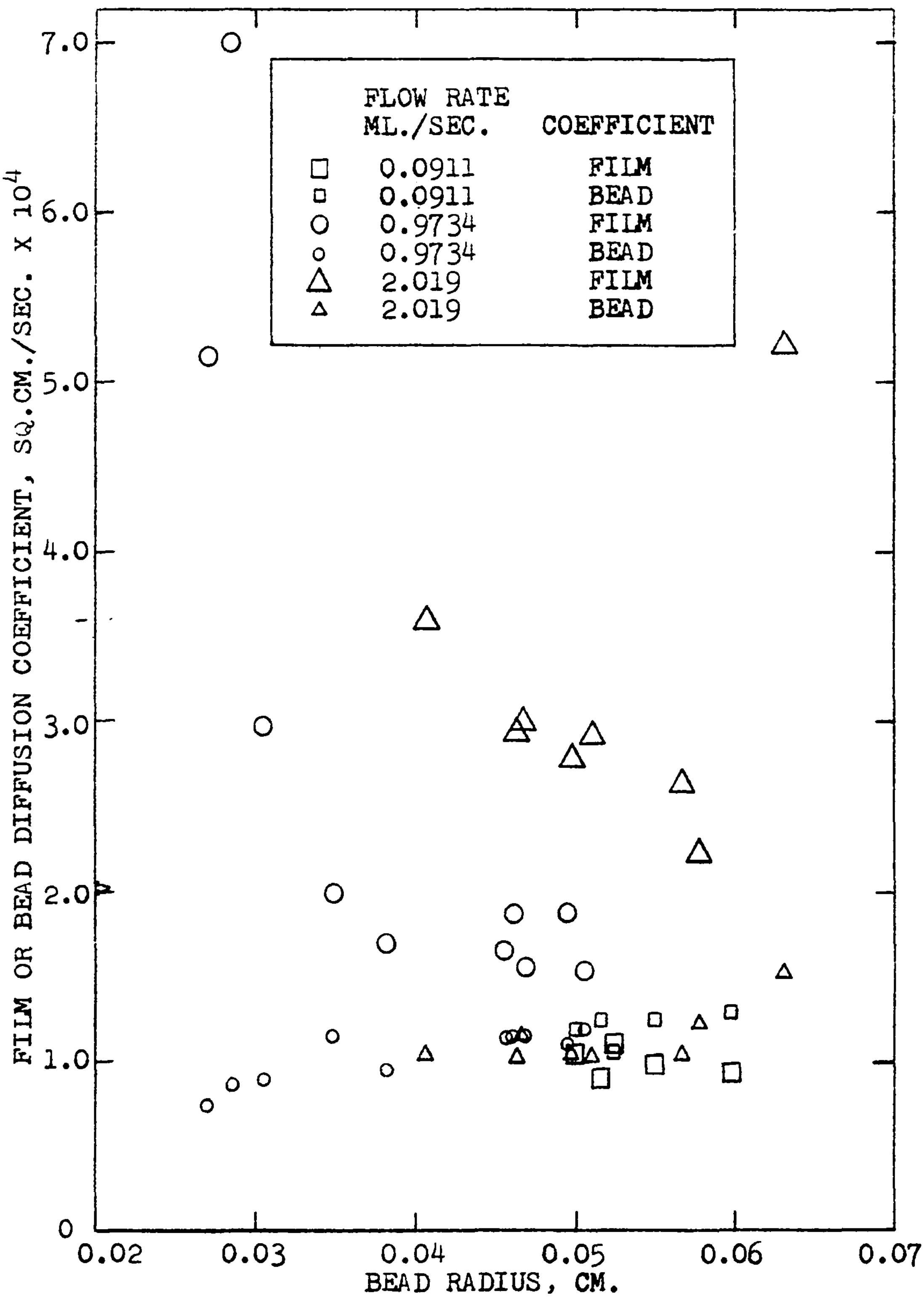


Figure 15. Effect of bead radius on the diffusion coefficient and the film coefficient

decreasing radius. These results are in agreement with general correlations of mass transfer. Bird, et al. (7) give Equation 60 as a correlation for mass transfer to single spherical particles from a fluid flowing past the sphere.

$$\frac{k\ D_p}{C\ D_f} = 2 + 0.6\left(\frac{D_p v_\infty\ \rho}{u}\right)^{1/2}\left(\frac{u}{\rho\ D_f}\right)^{1/3} \tag{60}$$

From Equation 60 it is evident that the mass transfer coefficient k increases as the velocity v_∞ past the sphere increases. It is also apparent from Equation 60 that as the diameter of the particle D_p decreases the mass transfer coefficient k will increase.

Figure 16 is a plot of the total time required for the blue core to shrink to zero divided by the radius of the bead squared versus the acid concentration. The data points in Figure 16 are for the individual resin beads. The flow rate past the bead was approximately 1.0 ml./sec. in all cases. For the data where the regression analysis had been used the total time was calculated using Equation 61.

$$T = \frac{C_r R^2}{6C_s}\left(\frac{1}{k} + \frac{1}{D}\right) \tag{61}$$

Equation 61 was derived by setting the dimensionless position equal to zero in Equation 55. For the runs where the regression analysis could not be used the total time was determined

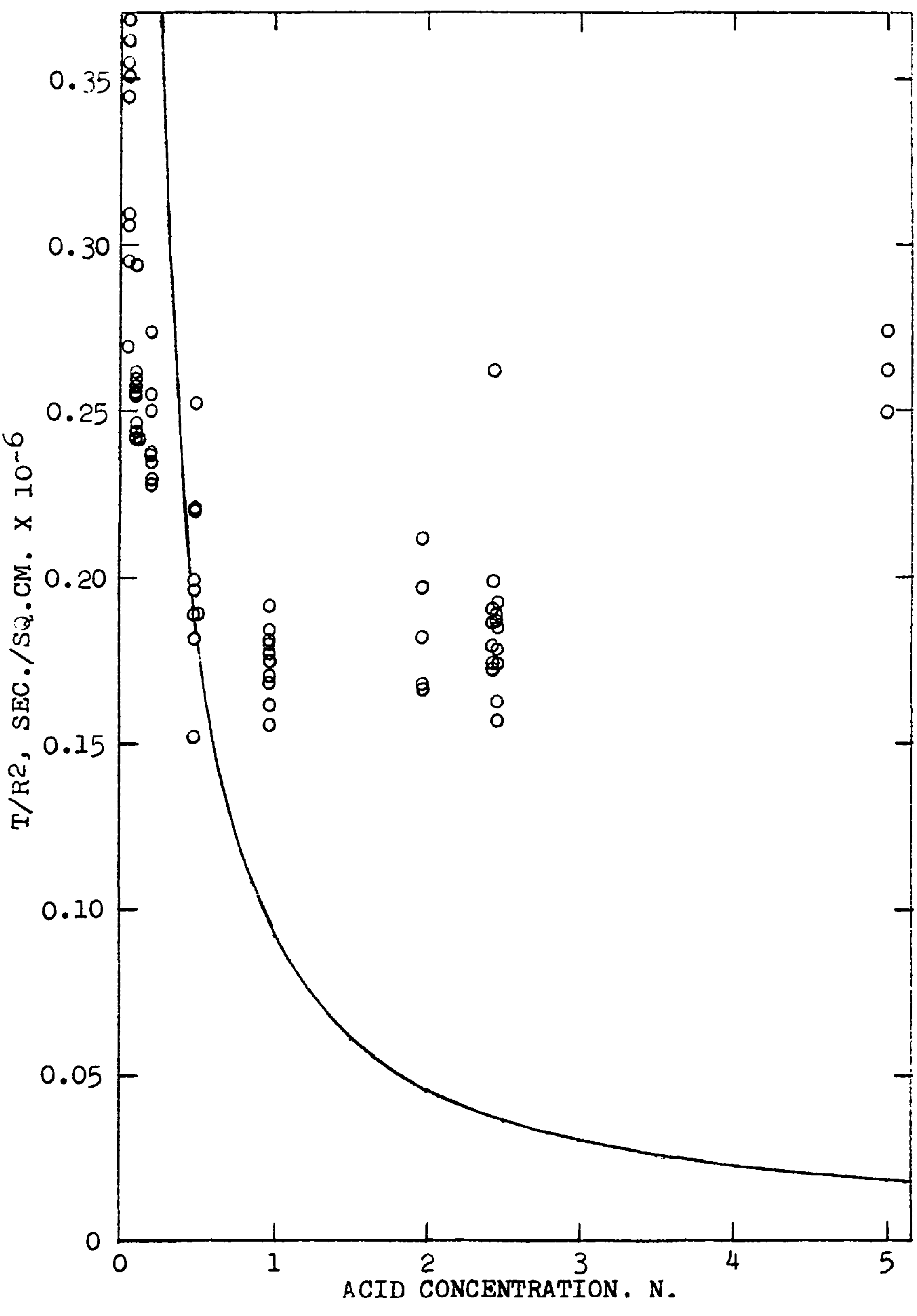


Figure 16. Effect of acid concentration on T/R^2

by extrapolating the dimensionless core diameter versus time curves. The total time was divided by the bead radius squared to remove the influence of the various sizes of resin particles. It will be observed that in both Model I (Equation 35) and Model II (Equation 55) the radius occurs as the square. In the region from 1 to 2.5 N. the time required for the boundary to reach the center of the bead seems to be constant and independent of the solution concentration. However, there are deviations at both higher and lower concentrations. The calculated curve drawn in Figure 16 is the predicted value of T/R^2 where it is assumed that the diffusion coefficient has a constant value of 0.22 x 10^{-4} sq.cm./sec. and the film coefficient has a constant value of 1.5 x 10^{-4} sq.cm./sec. From Table 5 it is evident that the film coefficient is relatively constant for flow rates from 0.05 to 1.0 ml./sec. At low concentrations it appears that the calculated curve may tend to fit the data but at high concentrations the curve does not fit the data at all well. The calculated curve was determined using Model II (Equation 61).

Figure 17 is a plot of T/R^2 versus the flow rate for data taken at an acid concentration of 2.45 N. At 2.45 N. the regression analysis and Model II could not be used for reasons explained previously. Figure 17 demonstrates that film diffusion is important at the higher acid concentrations just as it was at the lower concentrations. Flow rates

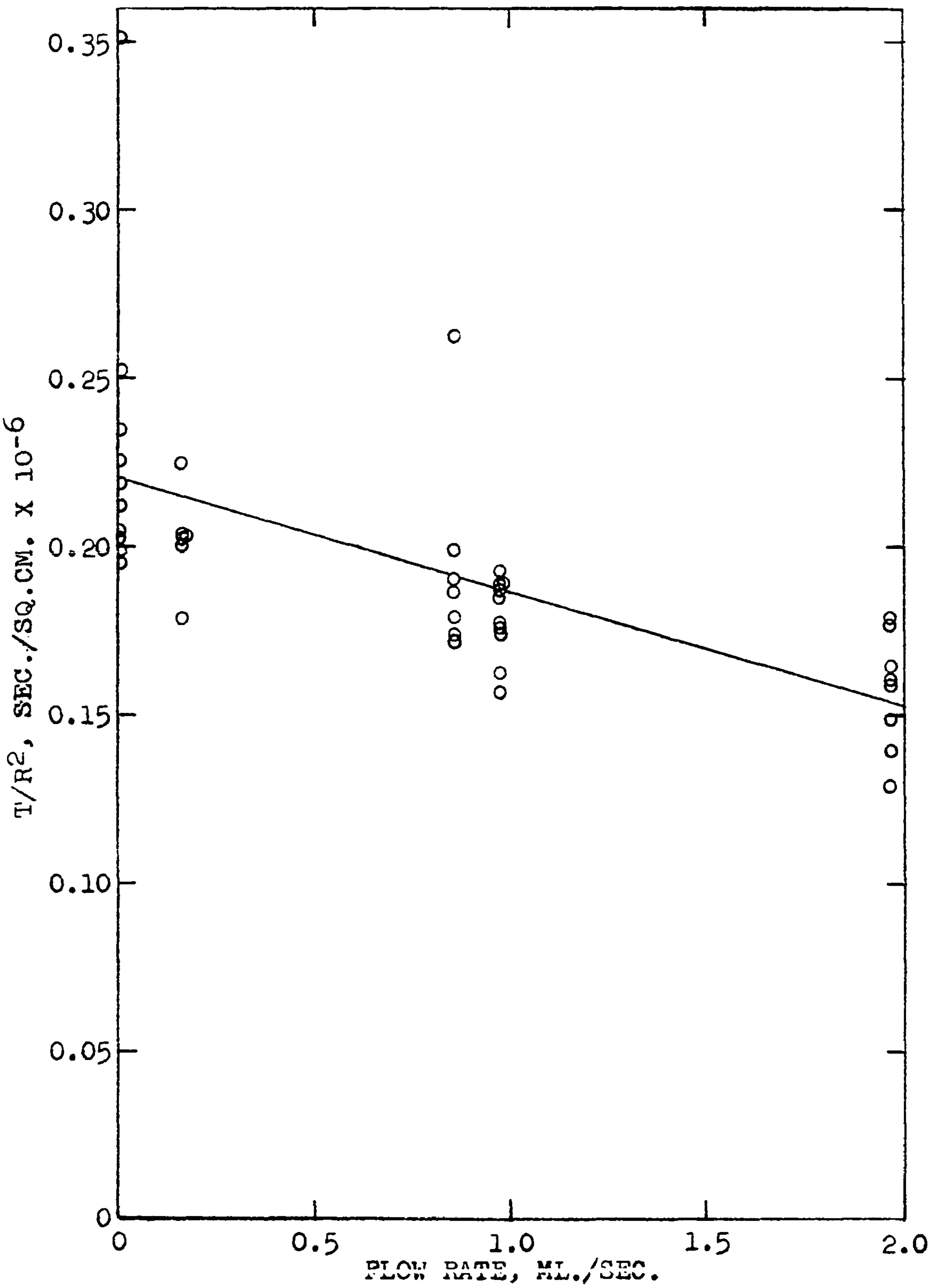


Figure 17. Effect of flow rate on T/R^2 for a constant acid concentration of 2.4 N.

greater than 2 ml./sec. were attempted but photographs could not be taken because the resin beads moved about in the cell. The line drawn in Figure 17 was obtained by a least squares fit of the data.

Effect of cross-linkage

Two runs were made with resins of different cross-linkages to get a general indication of how the cross-linkage affects the process. Runs were made using resin with a 2 per cent cross-linkage (run 299) and a 12 per cent cross-linkage (run 300). The runs were made with 0.495 N. acid and a flow rate of 0.98 ml./sec. These are the same conditions used in run 272 using the 8 per cent cross-linkage resin. Figure 18 gives a comparison of the results of these three runs. The value of T/R^2 for each bead in these runs has been plotted against the cross-linkage and the average of each run has been marked. The rate of the exchange process decreased with the increased cross-linkage. This result is consistent with the general results in the literature (23). The higher cross-linked resin has fewer and smaller pores in its structure than resin with a lower cross-linkage. Although the acid concentration was 0.495 N., run 299 could not be analyzed with the statistical regression of Model II because the data were out of the range of Model II. This is the same situation as was experienced at higher concentrations with Dowex HCR-M resin. Run 300 could be

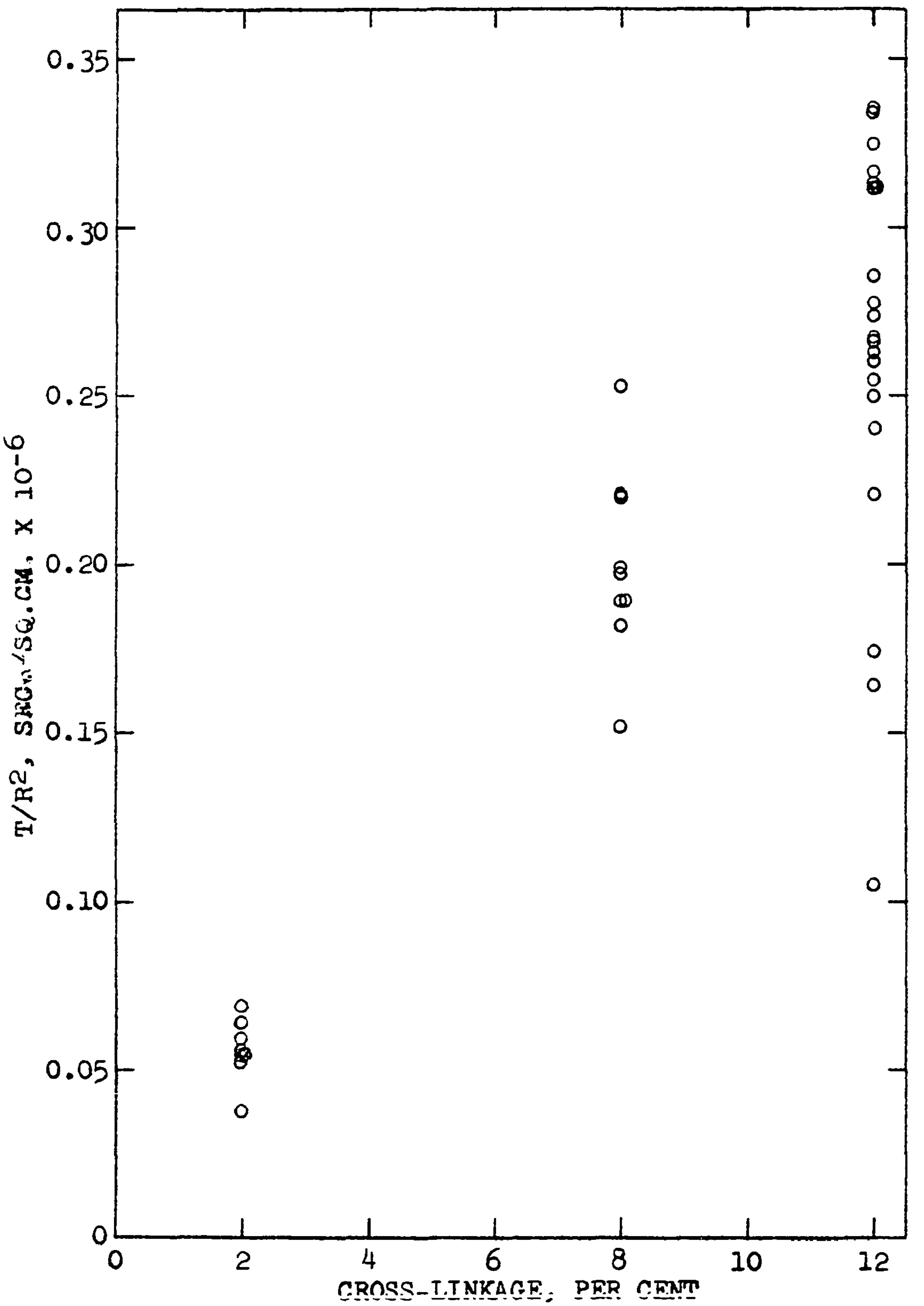


Figure 18. Effect of resin cross-linkage on T/R^2

analyzed by the statistical regression of Model II and the average diffusion coefficient was 0.225 x 10^{-4} sq.cm./sec. as compared with 0.217 x 10^{-4} sq.cm./sec. for run 272 with 8 per cent cross-linkage. This also indicates that the lower cross-linked resin reacts faster.

Linear regression

Non-linear regression analysis was used for the analysis of the data using Model II. Non-linear regression was used because the model (Equation 55) was not explicit in the dependent variable and it is certainly not a linear relationship. However, with a few assumptions, it is possible to use linear regression analysis. For linear regression Equation 55 was rearranged to give Equation 62.

$$\frac{6C_s t}{C_r R^2} = \frac{1}{k}(1-X^3) + \frac{1}{D}(1-3X^2+2X^3) \tag{62}$$

Equation 62 is of the same form as Equation 63 where b_1 = $1/k$ and $b_2 = 1/D$.

$$y = b_1 x_1 + b_2 x_2 \tag{63}$$

Equation 63 was the model for linear regression. The data were adjusted by Equations 64, 65, and 66.

$$y = \frac{6C_s t}{C_r R^2} \tag{64}$$

$$x_1 = 1-X^3 \tag{65}$$

$$x_2 = 1-3X^2+2X^3 \qquad (66)$$

To make this conversion to a linear regression model the dependent and independent variables were interchanged. In order to get some idea of what kind of error would result the data from two of the resin beads were analyzed by a linear regression. The results are presented in Table 6. It is evident that there are some differences between the two statistical methods. These differences were probably introduced by the interchanging of the dependent and independent variables.

Classical diffusion model

In the past it has been common practice to analyze ion exchange processes by means of a Fick's law diffusion model which assumes that diffusion and chemical reaction occur throughout an ion exchange bead. In other words diffusion is not confined to an outer shell surrounding an unreacted core. Boyd, et al. (9) have developed a mathematical expression for this model which is given in Equation 67.

$$F = 1 - \frac{6}{\pi^2} \sum_{i=1}^{\infty} \frac{1}{i^2} \exp\left[\frac{-D^* t \pi^2 i^2}{R^2}\right] \qquad (67)$$

In Equation 67 F is the fraction of material which has been removed at time t. Fraction F is computed on the basis of the amount of material which will be removed at very large times. The half time for the process is defined as the time

Table 6. Comparison of linear and non-linear regression

	Run 272, bead 6	Run 287, bead 3
k from linear regression	1.17×10^{-4} sq.cm./sec.	3.16×10^{-4} sq.cm./sec.
k from non-linear regression	1.29×10^{-4} sq.cm./sec.	2.99×10^{-4} sq.cm./sec.
% error in k	9.3 %	5.7 %
D from linear regression	2.18×10^{-5} sq.cm./sec.	9.89×10^{-5} sq.cm./sec.
D from non-linear regression	2.12×10^{-5} sq.cm./sec.	10.3×10^{-5} sq.cm./sec.
% error in D	2.8 %	4.0 %

required to remove one-half of the material from the resin. Helfferich (23) gives Equation 68 as the result of solving Equation 67 for the half time.

$$t_{\frac{1}{2}} = 0.030 \frac{R^2}{D^*} \tag{68}$$

Equations 67 and 68 can then be combined to give Equation 69.

$$F = 1 - \frac{6}{\pi^2} \sum_{i=1}^{\infty} \frac{1}{i^2} \exp \left[-i^2\pi^2 \left(\frac{t}{t_{\frac{1}{2}}} \right) \right] \tag{69}$$

A comparison can be made between Model II and the classical diffusion model if Model II is written in terms of the fraction removed from the resin. For this purpose film diffusion is neglected. Since the volume is proportional to the radius cubed, Equation 70 can be written for the fraction removed.

$$F = \left(\frac{b}{R} \right)^3 \tag{70}$$

Equation 70 can be substituted into Equation 55 after neglecting film diffusion to give Equation 71.

$$t = \frac{C_r R^2}{6 C_s D} \left[1 - 3F^{2/3} + 2F \right] \tag{71}$$

The half time for this model is given in Equation 72

$$t_{\frac{1}{2}} = 0.1101 \frac{C_r R^2}{6 C_s D} \tag{72}$$

which can be combined with Equation 71 to give Equation 73.

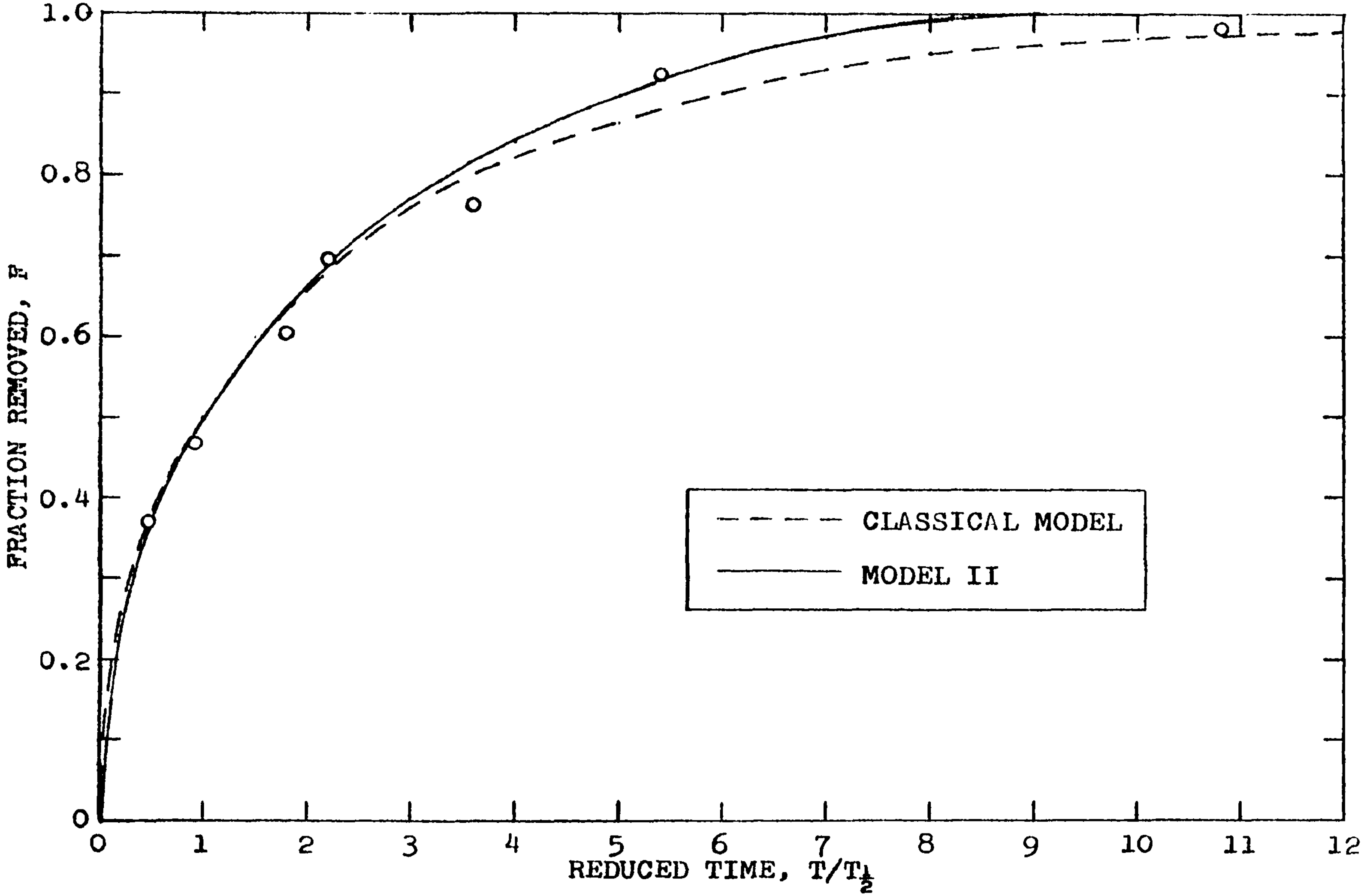
$$t/t_{\frac{1}{2}} = \frac{1 - 3F^{2/3} + 2F}{0.1101} \tag{73}$$

Equations 69 and 73 are plotted in Figure 19. Equation 73 was evaluated using a desk calculator and Equation 69 was plotted using data given by Helfferich (23). Boyd, et al. (9) presented some data they obtained for the isotopic exchange of sodium ions. They used a 0.1 molar solution and -60+70 mesh Amberlite IR-1 ion exchange resin. The steady state moving boundary model fits this data about as well as the classical diffusion model. Hence, we cannot rule out the possibility that the isotopic exchange of sodium ions is a moving boundary type of process.

Comparison of diffusion coefficients

No values were found in the literature for the chemical system studied in this work. However, the self-diffusion coefficient reported by Inczedy (30) for hydrogen is 5.4 (10^{-6}) sq.cm./sec. which can be used for an approximate comparison. In our work a diffusion coefficient of the order of 10^{-3} to 10^{-4} sq.cm./sec. was obtained from the quasi steady state model (Model II). The values reported by Inczedy (30) were obtained for small solution concentrations using the classical diffusion model (Equation 67). The two models can be compared by equating the half times which result from the two models. Thus, equating Equations

Figure 19. Comparison of Model II with the classical diffusion model


FRACTION REMOVED, F
REDUCED TIME, $T/T_{\frac{1}{2}}$
CLASSICAL MODEL
MODEL II
1.0
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8 9 10 11 12

68 and 72 gives Equation 74 where D is the quasi steady state diffusion coefficient (Model II) and D^* is the classical diffusion coefficient from Equation 67.

$$0.01835 \frac{C_r R^2}{C_s D} = \frac{0.030\ R^2}{D^*} \tag{74}$$

After rearrangement this equation becomes Equation 75.

$$D^* = \frac{D}{0.612} \frac{C_s}{C_r} \tag{75}$$

The average diffusion coefficients given in Table 4 with a flow rate of about 0.98 were converted using Equation 75. These results are given in Table 7.

Table 7. Average diffusion coefficients

Run number	Acid normality	$D \times 10^4$ sq.cm./sec.	$D^* \times 10^6$ sq.cm./sec.
193	0.968	0.234	3.50
272	0.495	0.217	1.66
295	0.204	0.512	1.61
272	0.103	1.036	1.65
297	0.052	3.271	2.63

It is evident from Table 6 that the diffusion coefficient D^* is of the same order of magnitude as the diffusion coefficient found in other ion exchange studies. It is also interesting that the values calculated for D^* are much more nearly constant than the D values.

Error in quasi steady state assumption (Model II)

Bischoff (8) presented Equation 16 which could be used to estimate the error of the quasi steady state assumption (Model II). Figure 20 is a graphical representation of this error. In Figure 20 the fractional error $(X_1-X)/X_1$, where X_1 is a corrected position, is plotted against the position X. The error is a function of the position and the relative concentrations in the resin and the solution. It is evident from Figure 18 that as the C_s/C_r ratio increases the error increases and as the boundary nears the center of the resin bead the error also increases. In Table 8 the C_s/C_r ratio is presented for the various experimental runs made in this work. Also presented in Table 8 is the dimensionless core diameter at which a 10 per cent error is encountered. The greatest error listed in Table 8 occurs in run 193 with an acid normality of 0.968. From Table 8 a 10 per cent error occurs at a dimensionless core diameter of 0.311. The regression analysis was run on this data after removing all of the data with a dimensionless core diameter of less than 0.311. The average diffusion coefficient with these data points removed was 0.202 (10^{-4}) sq.cm./sec. compared with 0.234 (10^{-4}) sq.cm./sec. when this data had been included. Thus, it would appear that in the situation where the error should be most serious Equation 16 predicts an error of

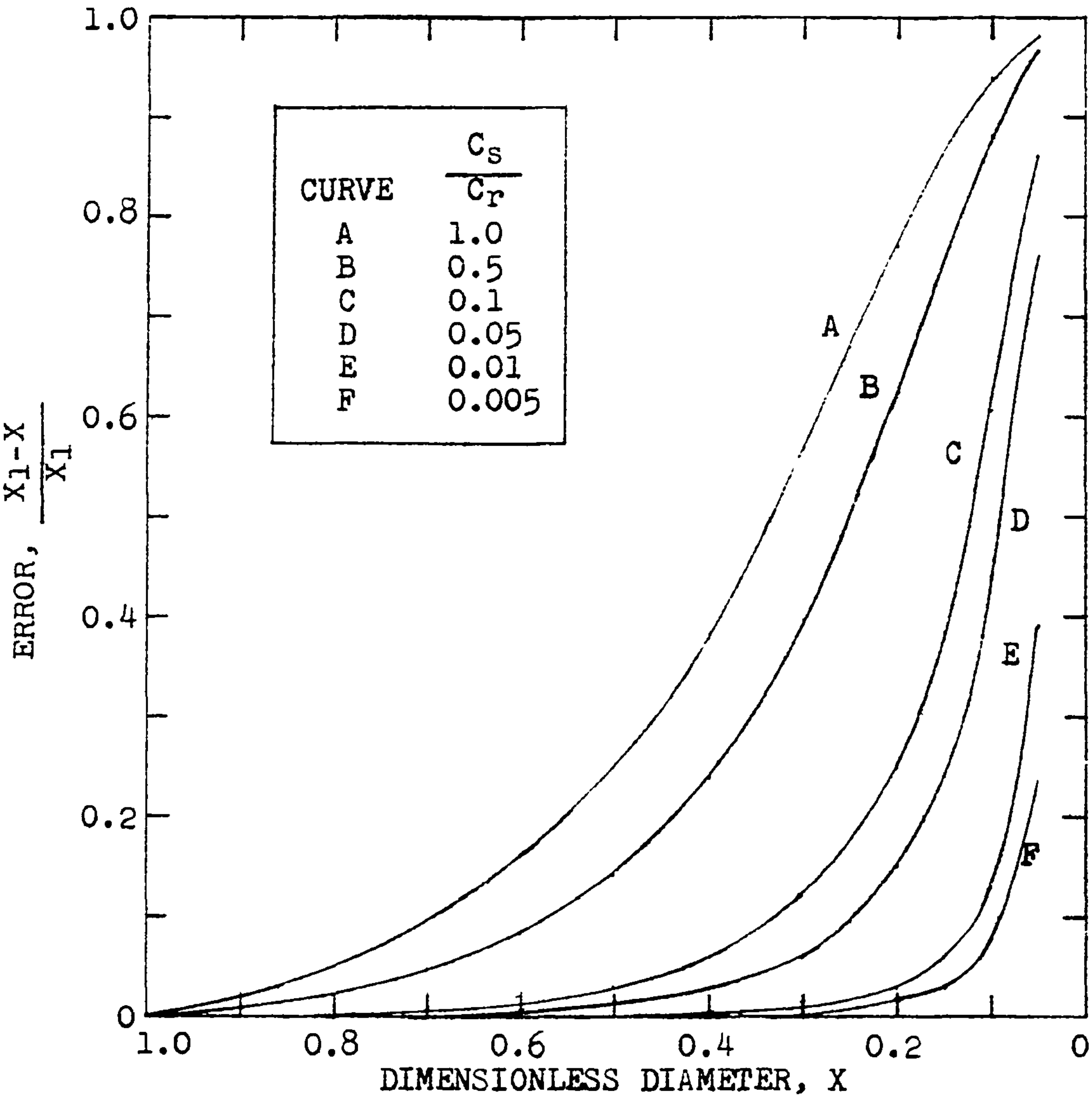


Figure 20. Plot of the error in the calculated boundary position as a function of the boundary position

Table 8. C_s/C_r ratios for the experimental runs

Run number	Acid normality	C_s/C_r ratio	X for 10% error
193	0.986	0.0933	0.311
272	0.495	0.0468	0.232
279	0.103	0.00974	0.114
284	0.103	0.00974	0.114
287	0.103	0.00974	0.114
295	0.204	0.0193	0.156
297	0.052	0.00492	0.082
300	0.495	0.0468	0.232

about 15 per cent in the diffusion coefficient when these data points were removed. Although this is a substantial error it is not great enough to account for the changes in the diffusion coefficient with concentration which are evident in Figure 14. The detailed results of this regression analysis are presented in Appendix C.

DISCUSSION OF RESULTS

A visible sharp moving boundary was observed in the ion exchange system investigated in this work. Such a sharp visible boundary is unusual and this unusual feature made it possible to investigate the kinetics of the system by photographing the resin beads and following the boundary position as a function of time. The photographic techniques were developed for taking this data and they worked well except for two problems. The movement of the boundary could not be detected in the very first portion of its movement because of optical problems in the resin. In addition, the cell and acid flow system used in this work did not allow data to be taken at a flow rate sufficiently high to eliminate the effects of film resistance.

Experimental measurements were taken of the boundary position as a function of time. These experiments were carried out at various conditions of acid concentration, flow rate, bead radius, and resin cross-linkage. When the boundary position versus time data were examined, it was found that the data obtained at acid concentrations of 1 N. or less could be analyzed using a quasi steady state mathematical model for combined film and bead diffusion. The coefficients in the mathematical model were fit to the data by using a least squares, non-linear regression technique.

The combined film diffusion and bead diffusion model appeared to fit the data quite well. This would indicate that the rate controlling mechanism is a combination of both a film resistance and a resistance due to diffusion in the bead itself. The total time required by the process also increased as the flow rate decreased. This would also indicate that there is a resistance due to a liquid film surrounding the bead. This was found to be true at acid concentrations greater than 1 N. as well as concentrations less than 1 N. The general trends of the resistance due to flow rate were in general agreement with the general theory in both flow rate changes as well as the particle radius. A precise test of this theory could not be made because the design of the equipment was such that accurate measurements of the flow rate past individual beads could not be made.

The bead diffusion coefficients obtained with the quasi steady state model were nearly constant for a fixed acid concentration but a variable flow rate. This would indicate that the model provides a good separation of the effects due to film resistance from the effects due to bead resistance. The diffusion coefficients did display more scatter or variance when they were determined from data taken at low concentrations (0.05 N.) than at moderate concentrations (0.5 or 1 N.). The reason for this effect is not known, especially in light of the fact that the quasi steady state model should show its greatest deviation or error at higher concentrations.

The use of the quasi steady state model predicted diffusion coefficients which were considerably higher than others using the classical non-moving boundary model. When the classical model was used to analyze the data, the diffusion coefficients which were obtained were nearly in agreement with diffusion coefficients obtained by others using systems which were similar but not identical to the system used here. Thus it would appear that the difference in diffusion coefficients is due to the mathematical model used to analyze the data and not due to differences in the data.

A second quasi steady state model (Model I) was derived and integrated on a digital computer. The computer solution of this model did not agree well with the experimental data. It did indicate that the initial rate of movement of the boundary should increase as the concentration of the solution increases. This effect was not predicted by Model II. The high concentration data also predicted an initial rate increase, however the data had a greater initial rate increase than the model predicted and thus the model could not be used.

CONCLUSIONS

Several conclusions can be reached from the results of this study. They are listed below.

1. When the copper ammine complex is removed from sulfonated styrene-divinylbenzene cation exchange resin by sulfuric acid, a visible sharp moving color boundary develops. This moving boundary can be photographed and boundary position versus time data can be obtained.

2. Over the entire concentration range and flow rate range studied film diffusion appears to have a significant influence on the rate of movement of the boundary.

3. For concentrations less than 1 N. the position versus time data could be represented by a quasi steady state mathematical model for combined film and particle diffusion. For acid concentrations greater than 1 N. an adequate mathematical model was not found which would accurately represent the position versus time data.

4. The internal diffusion coefficient appeared to be quite constant for various beads at acid concentrations of 0.5 and 1 N. but displayed increasing variance among particles as the concentration decreased.

SUGGESTED FUTURE WORK

Future work should be carried out both experimentally and mathematically. Experimentally there are several methods which could be used to further test the mathematical models. In our work the concentration of the acid solution was varied but the concentration of the copper ammine complex loaded on the resin was not varied. This could be varied by using different concentrations of the copper ammine solution used in preparing the resin for the experimental work. This parameter is of particular interest because it appears in the quasi steady state equation (Model II). It would also be of interest to add some ammonium ions to the acid solution to see if the diffusion out of the ion exchange beads is affecting or possibly controlling the boundary movement. It would also be of interest to add copper ions to the eluting solution for the same reason. However, the addition of copper ions to the eluting solution could lead to problems in seeing and photographing the resin beads if the solution becomes too deeply colored. It would also be desirable to develop a cell which could be used with higher flow rates so that the effects of film diffusion resistance could be removed.

Most ion exchange systems do not have a color change associated with the exchange of one ion for another in the resin. Thus, there is no information available on whether a boundary exists in other ion exchange chemical systems or not. Perhaps other methods could be developed to establish whether they have a sharp moving boundary or not. It might also be possible to find other systems which have a color change associated with the ion exchange process.

More work is needed on the mathematical solution of the moving boundary diffusion problem without making a steady state assumption. The same mathematical equations come up in other physical situations and better solutions could find applications in other areas such as liquid-solid and gas-solid reactions. With a more rigorous solution it might be possible to better analyze and understand the high concentration data.

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LIST OF SYMBOLS

A	arbitrary constant
a	stoichiometric constant
B	arbitary constant
b	position of boundary in bead, cm.
b_1	regression constant (Equation 63)
b_2	regression constant (Equation 63)
C	concentration of H^+ ion, meq./cc.
C_A	concentration of ion A, meq./cc.
C_B	concentration of ion B, meq./cc.
C_i	concentration of i'th ionic species, meq./cc.
C_f	concentration of H^+ in film, meq./cc.
C_o	total electrical chemical concentration, meq./cc.
C_r	concentration in resin in equivalents of H^+ ion, meq. H^+/cc.
C_s	concentration in solution, N. (meq. H^+/cc.)
C^*	concentration of H^+ ions at resin surface, meq./cc.
D	diffusion coefficient of H^+ ion, sq.cm./sec.
D_A	diffusion coefficient of ion A, sq.cm./sec.
D_B	diffusion coefficient of ion B, sq.cm./sec.
D_f	diffusion coefficient of H^+ ion in film, sq.cm./sec.
D_i	diffusion coefficient, sq.cm./sec.
D_p	particle diameter
D^*	diffusion coefficient by Equation 67, sq.cm./sec.
d	stoichiometric constant

F fractional loading, dimensionless

f Faraday's constant

g stoichiometric constant

i summation index

J diffusion flux of H^+ ion, meq./sq.cm.sec.

J_A diffusion flux of ion A, meq./sq.cm.sec.

J_i diffusion flux of i'th ionic species, meq./sq.cm.sec.

k film mass transfer coefficient, sq.cm./sec.

k_s reaction rate constant, cm./sec.

M_1 resin loading, g. Cu/cc. resin

M_2 resin loading, g. Cu/cc. resin

N flux of H^+ ions at boundary, meq./cc.sec.

n number of points in data set

P density of reacting material in particle, meq./cc.

R radius of resin bead in cm. in all equations except Equations 1, 5, and 21

R designates the resin in Equation 1

R designates the ideal gas constant in Equations 5 and 21

r radial coordinate in bead, cm.

S boundary layer thickness, cm.

T absolute temperature in Equations 5 and 21

T total time of elution, sec. (except in Equations 5 and 21)

t time, sec.

$t_{\frac{1}{2}}$ half time of elution process, sec.

v_∞ fluid velocity past particle, cm./sec.

X	dimensionless boundary position
X_i	dimensionless boundary position of data point i
$\hat{X}_i$	dimensionless boundary position estimated from Equation 55 of data point i
X_o	dimensionless boundary position, uncorrected
X^*	dimensionless boundary position, corrected
x_1	dimensionless parameter (Equation 65)
x_2	dimensionless parameter (Equation 66)
y	regression parameter, sec./sq.cm. (Equation 64)
Z_A	electrochemical valence of ion A
Z_B	electrochemical valence of ion B
Z_i	electrochemical valence of i'th ion
α	total flux of H^+ ions at boundary, meq.
β	dimensionless constant (Equation 38)
ϕ	electrical potential
ρ	fluid density, g./cc.
σ	variance
τ	dimensionless time
μ	viscosity, cp.
div	mathematical divergence operator
grad	mathematical gradient operator

APPENDIX A. INDIVIDUAL BEAD ANALYSIS

Bead number	Bead radius cm.	First analysis g.Cu./cc.	Second analysis g.Cu./cc.
210-2	0.0593	0.143	0.161
210-3	0.0508	0.125	0.469
210-6	0.0313	0.140	0.199
210-7	0.0530	0.155	0.139
210-8	0.0516	0.125	0.150
210-9	0.0390	0.135	0.169
210-10	0.0352	0.126	0.261
210-11	0.0555	0.110	0.137
210-12	0.0297	0.132	0.373
210-13	0.0486	0.118	0.185
210-14	0.0588	0.110	0.151
210-15	0.0554	0.139	0.158
210-16	0.0540	0.112	0.220
210-17	0.0440	0.130	0.156
210-18	0.0612	0.106	0.124
210-19	0.0517	0.142	0.170
210-20	0.0549	0.114	0.160
219-1	0.0525	0.118	0.150
219-2	0.0515	0.115	0.186
219-3	0.0485	0.124	0.192
219-4	0.0530	0.119	0.155
219-5	0.0529	0.117	0.180
219-6	0.0628	0.113	0.142
219-7	0.0526	0.116	0.173
219-8	0.0371	0.117	0.278
219-9	0.0282	0.128	0.290
219-11	0.0260	0.143	0.306
219-12	0.0486	0.122	0.179
219-13	0.0611	0.116	0.154
219-14	0.0505	0.120	0.180
219-15	0.0569	0.116	0.168
219-16	0.0410	0.121	0.188
219-18	0.0527	0.115	0.148
219-19	0.0554	0.120	0.159
219-20	0.0563	0.119	0.142
219-21	0.0491	0.120	0.145
219-22	0.0593	0.110	0.125
219-23	0.0513	0.118	0.156
219-24	0.0352	0.131	0.309
219-25	0.0314	0.129	0.239

Bead number	Bead radius cm.	First analysis g.Cu./cc.	Second analysis g.Cu./cc.
219-26	0.0585	0.115	0.136
219-27	0.0547	0.119	0.152
219-29	0.0546	0.113	0.146
219-30	0.0423	0.124	0.226
219-31	0.0538	0.119	0.173
219-32	0.0534	0.126	0.165
219-33	0.0600	0.112	0.138
219-34	0.0517	0.128	0.166
219-35	0.0536	0.118	0.159
219-36	0.0543	0.124	0.151
219-37	0.0475	0.126	0.165
219-38	0.0457	0.112	0.166
219-39	0.0309	0.136	0.318
219-40	0.0512	0.118	0.155

APPENDIX B. EXPERIMENTAL DATA

The following pages contain a complete listing of the experimental data. The listing was prepared by a computer output from the punched cards. The first line in each data set contains the run number, bead number, and which microscope lens was used to obtain the data. The rest of the data is identified by the symbols given below.

R radius of bead in cm.

C concentration of acid, N.

F flow rate of acid in ml./sec.

TE time in min. estimated for the disappearance of the blue core. For the runs where the regression analysis was used this time was estimated by the regression equation. For the remaining runs this time was estimated by plotting the data

N photograph number

T time in sec. from the start of the experiment to the start of the photograph plus one-half of the exposure time

X radius of blue core divided by the radius of the bead after optical correction by Equation 20

Results from the regression analysis.

K film coefficient for regression with all data included (sq.cm./sec.)

D	diffusion coefficient for regression with all data included(sq.cm./sec.)
S	sum of squares of error for regression with all of the data included
V	variance for regression with all of the data included (Equation 59)
I	number of points removed
KA	film coefficient for regression with first points removed (sq.cm./sec.)
DA	diffusion coefficient for regression with first points removed (sq.cm./sec.)
SA	sum of squares for regression with first points removed
VA	variance for regression with the first points removed (Equation 59)

DATA FROM RUN NUMBER IV-174, LEFT LENS

R = 0.05100 C = 5.0600 F = 0.1610 TE = 905.0

N	T	X	N	T	X	N	T	X
1	22.35	0.9003	10	239.45	0.6260	19	598.55	0.3645
2	32.30	0.8690	11	279.45	0.6057	20	638.25	0.3314
3	41.65	0.8561	12	319.40	0.5542	21	678.20	0.3075
4	50.00	0.8432	13	359.25	0.5229	22	718.10	0.2670
5	58.75	0.8156	14	399.10	0.5008	23	758.20	0.2228
6	88.60	0.7806	15	439.00	0.4676	24	798.50	0.1823
7	120.75	0.7457	16	479.00	0.4419	25	838.35	0.1362
8	159.35	0.7015	17	518.75	0.4124	26	878.45	0.0718
9	199.55	0.6702	18	558.70	0.3903			

DATA FROM RUN IV-178, BEAD NUMBER 6

R = 0.05040 C = 5.0600 F = 0.7940 TE = 696.0

N	T	X	N	T	X	N	T	X
1	16.30	0.9003	8	211.75	0.5759	15	488.25	0.3219
2	27.00	0.9003	9	251.95	0.5533	16	527.45	0.2741
3	39.60	0.8324	10	292.00	0.5030	17	566.65	0.2414
4	51.90	0.7922	11	331.80	0.4577	18	606.35	0.1936
5	91.90	0.7293	12	370.70	0.4175	19	640.30	0.1509
6	132.05	0.6765	13	409.85	0.3948	20	673.95	0.0754
7	171.60	0.6262	14	449.40	0.3470	21	706.10	0.0

DATA FROM RUN IV-178, BEAD NUMBER 7

R = 0.05385 C = 5.0600 F = 0.7940 TE = 762.0

N	T	X	N	T	X	N	T	X
1	16.30	0.9003	9	251.95	0.5994	17	566.65	0.3220
2	27.00	0.9003	10	292.00	0.5642	18	606.35	0.2750
3	39.60	0.8603	11	331.80	0.5266	19	640.30	0.2351
4	51.90	0.8368	12	370.70	0.4913	20	673.95	0.1951
5	91.90	0.7710	13	409.85	0.4584	21	706.10	0.1316
6	132.05	0.7099	14	449.40	0.4231	22	735.85	0.0752
7	171.60	0.6770	15	488.25	0.3926	23	764.60	0.0
8	211.75	0.6253	16	527.45	0.3526			

DATA FROM RUN IV-178, BEAD NUMBER 8

R = 0.05530 C = 5.0600 F = 0.7940 TE = 766.0

N	T	X	N	T	X	N	T	X
1	16.30	0.9003	9	251.95	0.5796	17	566.65	0.3001
2	27.00	0.9003	10	292.00	0.5429	18	606.35	0.2566
3	39.60	0.8499	11	331.80	0.5040	19	640.30	0.2222
4	51.90	0.8087	12	370.70	0.4719	20	673.95	0.1947
5	91.90	0.7629	13	409.85	0.4330	21	706.10	0.1283
6	132.05	0.6941	14	449.40	0.3963	22	735.85	0.0825
7	171.60	0.6644	15	488.25	0.3711	23	764.60	0.0115
8	211.75	0.6117	16	527.45	0.3230			

DATA FROM RUN IV-182, BEAD NUMBER 3, RIGHT LENS
R = 0.04210 C = 4.9920 F = 0.5930 TE = 782.0

N	T	X	N	T	X	N	T	X
1	16.10	0.9003	11	208.80	0.6223	21	477.20	0.3849
2	32.40	0.8789	12	236.00	0.5988	22	504.90	0.3657
3	46.20	0.8340	13	262.05	0.5774	23	533.60	0.3422
4	56.05	0.8126	14	288.50	0.5539	24	560.55	0.3122
5	65.25	0.7891	15	314.85	0.5346	25	587.45	0.2908
6	83.40	0.7699	16	341.45	0.5025	26	614.05	0.2566
7	101.90	0.7378	17	367.80	0.4854	27	641.40	0.2245
8	127.45	0.7057	18	394.35	0.4576	28	668.25	0.1925
9	154.95	0.6843	19	421.70	0.4341	29	781.60	0.0
10	182.15	0.6480	20	449.20	0.4085			

DATA FROM RUN IV-182, BEAD NUMBER 4, RIGHT LENS
R = 0.04550 C = 4.9920 F = 0.5930 TE = 842.0

N	T	X	N	T	X	N	T	X
1	16.10	0.9003	11	208.80	0.6332	21	477.20	0.4155
2	32.40	0.8766	12	236.00	0.6035	22	504.90	0.3957
3	46.20	0.8390	13	262.05	0.5897	23	533.60	0.3760
4	56.05	0.8172	14	288.50	0.5659	24	560.55	0.3502
5	65.25	0.8014	15	314.85	0.5362	25	587.45	0.3225
6	83.40	0.7737	16	341.45	0.5224	26	614.05	0.2968
7	101.90	0.7519	17	367.80	0.4967	27	641.40	0.2750
8	127.45	0.7183	18	394.35	0.4828	28	668.25	0.2473
9	154.95	0.6846	19	421.70	0.4591	29	842.40	0.0
10	182.15	0.6569	20	449.20	0.4353			

DATA FROM RUN IV-182, BEAD NUMBER 5, RIGHT LENS
R = 0.04680 C = 4.9920 F = 0.5930 TE = 885.0

N	T	X	N	T	X	N	T	X
1	16.10	0.9003	11	208.80	0.6695	21	477.20	0.4598
2	32.40	0.8888	12	236.00	0.6406	22	504.90	0.4425
3	46.20	0.8503	13	262.05	0.6175	23	533.60	0.4232
4	56.05	0.8388	14	288.50	0.5983	24	560.55	0.4021
5	65.25	0.8176	15	314.85	0.5790	25	587.45	0.3751
6	83.40	0.7926	16	341.45	0.5579	26	614.05	0.3578
7	101.90	0.7733	17	367.80	0.5386	27	641.40	0.3290
8	127.45	0.7445	18	394.35	0.5194	28	668.25	0.3059
9	154.95	0.7118	19	421.70	0.5021	29	884.90	0.0
10	182.15	0.6906	20	449.20	0.4809			

DATA FROM RUN NUMBER IV-189, BEAD NUMBER 3, RIGHT LENS
R = 0.05855 C = 2.4320 F = 0.8660 TE = 598.0

N	T	X	N	T	X	N	T	X
1	19.70	0.8929	10	219.90	0.5922	19	470.85	0.2896
2	31.15	0.8837	11	248.20	0.5608	20	490.75	0.2749
3	41.95	0.8505	12	281.05	0.5332	21	510.80	0.2417
4	51.95	0.8320	13	309.95	0.4981	22	530.90	0.2029
5	73.55	0.7933	14	340.95	0.4631	23	550.40	0.1642
6	98.10	0.7490	15	370.40	0.4354	24	563.20	0.1310
7	130.90	0.7048	16	400.70	0.4022	25	576.30	0.0922
8	160.85	0.6642	17	431.00	0.3542	26	588.85	0.0332
9	190.80	0.6273	18	450.75	0.3321	27	600.10	0.0

DATA FROM RUN NUMBER IV-189, BEAD NUMBER 5 , RIGHT LENS
R = 0.04155 C = 2.4320 F = 0.8660 TE = 344.0

N	T	X	N	T	X	N	T	X
1	19.70	0.8504	6	98.10	0.6326	11	248.20	0.3176
2	31.15	0.7953	7	130.90	0.5643	12	281.05	0.2625
3	41.95	0.7638	8	160.85	0.5118	13	309.95	0.1680
4	51.95	0.7349	9	190.80	0.4488	14	340.95	0.0289
5	73.55	0.6825	10	219.90	0.3780	15	370.40	0.0

DATA FROM RUN NUMBER IV-189, BEAD NUMBER 6 , RIGHT LENS
R = 0.05340 C = 2.4320 F = 0.8660 TE = 544.0

N	T	X	N	T	X	N	T	X
1	19.70	0.8819	9	190.80	0.5995	17	431.00	0.2906
2	31.15	0.8635	10	219.90	0.5586	18	450.75	0.2455
3	41.95	0.8287	11	248.20	0.5259	19	470.85	0.2251
4	51.95	0.8103	12	281.05	0.4788	20	490.75	0.1842
5	73.55	0.7734	13	309.95	0.4481	21	510.80	0.1269
6	98.10	0.7305	14	340.95	0.4113	22	530.90	0.0614
7	130.90	0.6793	15	370.40	0.3663	23	550.40	0.0
8	160.85	0.6425	16	400.70	0.3315			

DATA FROM RUN NUMBER IV-189, BEAD NUMBER 8, RIGHT LENS
R = 0.05555 C = 2.4320 F = 0.8660 TE = 553.0

N	T	X	N	T	X	N	T	X
1	19.70	0.8906	9	190.80	0.6067	17	431.00	0.2956
2	31.15	0.8692	10	219.90	0.5756	18	450.75	0.2703
3	41.95	0.8478	11	248.20	0.5386	19	470.85	0.2353
4	51.95	0.8148	12	281.05	0.4861	20	490.75	0.2120
5	73.55	0.7778	13	309.95	0.4511	21	510.80	0.1672
6	98.10	0.7292	14	340.95	0.4258	22	530.90	0.1089
7	130.90	0.6825	15	370.40	0.3850	23	550.40	0.0
8	160.85	0.6436	16	400.70	0.3461			

DATA FROM RUN NUMBER IV-189, BEAD NUMBER 10, RIGHT LENS

R = 0.05605 C = 2.4320 F = 0.8660 TE = 544.0

N	T	X	N	T	X	N	T	X
1	19.70	0.8906	9	190.80	0.6041	17	431.00	0.2923
2	31.15	0.8633	10	219.90	0.5651	18	450.75	0.2533
3	41.95	0.8418	11	248.20	0.5242	19	470.85	0.2144
4	51.95	0.8165	12	281.05	0.4872	20	490.75	0.1754
5	73.55	0.7697	13	309.95	0.4482	21	510.80	0.1208
6	98.10	0.7347	14	340.95	0.4112	22	530.90	0.0507
7	130.90	0.6898	15	370.40	0.3742	23	550.40	0.0
8	160.85	0.6431	16	400.70	0.3313			

DATA FROM RUN NUMBER IV-189, BEAD NUMBER 11, RIGHT LENS

R = 0.03425 C = 2.4320 F = 0.8660 TE = 309.0

N	T	X	N	T	X	N	T	X
1	19.70	0.8814	6	98.10	0.6413	11	248.20	0.2843
2	31.15	0.8308	7	130.90	0.5686	12	281.05	0.1548
3	41.95	0.7992	8	160.85	0.5054	13	309.95	0.0
4	51.95	0.7740	9	190.80	0.4328			
5	73.55	0.6950	10	219.90	0.3506			

DATA FROM RUN NUMBER IV-189, BEAD NUMBER 13, RIGHT LENS

R = 0.05490 C = 2.4320 F = 0.8660 TE = 563.0

N	T	X	N	T	X	N	T	X
1	19.70	0.8943	9	190.80	0.6042	17	431.00	0.3081
2	31.15	0.8844	10	219.90	0.5764	18	450.75	0.2782
3	41.95	0.8506	11	248.20	0.5366	19	470.85	0.2385
4	51.95	0.8327	12	281.05	0.4988	20	490.75	0.2186
5	73.55	0.7791	13	309.95	0.4631	21	510.80	0.1789
6	98.10	0.7413	14	340.95	0.4313	22	530.90	0.1192
7	130.90	0.6956	15	370.40	0.3975	23	550.40	0.0775
8	160.85	0.6519	16	400.70	0.3478	24	563.20	0.0

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 2, LEFT LENS

R = 0.05140 C = 0.9863 F = 0.9500 TE = 458.4

N	T	X	N	T	X	N	T	X
1	12.50	0.8814	7	171.10	0.6247	13	350.40	0.3303
2	25.60	0.8709	8	201.45	0.5680	14	380.85	0.2693
3	40.80	0.8267	9	231.20	0.5238	15	409.90	0.1956
4	71.00	0.7762	10	260.75	0.4712	16	440.20	0.1136
5	100.15	0.7026	11	290.90	0.4228	17	470.45	0.0
6	138.60	0.6668	12	320.70	0.3786			

K = 0.414E-03 D = 0.214E-04 S = 0.355E-02 V = 0.159E-01

I-2 KA=0.277E-03 DA=0.221E-04 SA=0.148E-02 VA=0.111E-01

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 5, LEFT LENS
R = 0.06240 C = 0.9863 F = 0.9500 TE = 633.6

N	T	X	N	T	X	N	T	X
1	12.50	0.8956	9	231.20	0.6462	17	470.45	0.3811
2	25.60	0.8904	10	260.75	0.6237	18	500.95	0.3465
3	40.80	0.8835	11	290.90	0.5890	19	530.80	0.2980
4	71.00	0.8454	12	320.70	0.5699	20	560.55	0.2443
5	100.15	0.7796	13	350.40	0.5266	21	589.85	0.1594
6	138.60	0.7622	14	380.85	0.5007	22	620.45	0.0624
7	171.10	0.7103	15	409.90	0.4591	23	650.35	0.0
8	201.45	0.6826	16	440.20	0.4262			

K = 0.116E-03 D = 0.251E-04 S = 0.144E-01 V = 0.269E-01
I=3 KA=0.100E-03 DA=0.262E-04 SA=0.575E-02 VA=0.184E-01

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 6, LEFT LENS
R = 0.04994 C = 0.9863 F = 0.9500 TE = 451.5

N	T	X	N	T	X	N	T	X
1	12.50	0.8657	7	171.10	0.6146	13	350.40	0.3160
2	25.60	0.8657	8	201.45	0.5649	14	380.85	0.2510
3	40.80	0.8397	9	231.20	0.5216	15	409.90	0.1645
4	71.00	0.7921	10	260.75	0.4545	16	440.20	0.1060
5	100.15	0.7272	11	290.90	0.4285	17	470.45	0.0
6	138.60	0.6731	12	320.70	0.3636			

K = 0.319E-03 D = 0.209E-04 S = 0.651E-02 V = 0.216E-01
I=2 KA=0.207E-03 DA=0.218E-04 SA=0.185E-02 VA=0.124E-01

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 7, LEFT LENS
R = 0.04913 C = 0.9863 F = 0.9500 TE = 433.6

N	T	X	N	T	X	N	T	X
1	12.50	0.8799	7	171.10	0.5983	13	350.40	0.2970
2	25.60	0.8755	8	201.45	0.5521	14	380.85	0.2200
3	40.80	0.8183	9	231.20	0.5169	15	409.90	0.1320
4	71.00	0.7897	10	260.75	0.4597	16	440.20	0.0
5	100.15	0.7039	11	290.90	0.3981			
6	138.60	0.6797	12	320.70	0.3519			

K = 0.269E-03 D = 0.214E-04 S = 0.533E-02 V = 0.202E-01
I=2 KA=0.194E-03 DA=0.223E-04 SA=0.251E-02 VA=0.151E-01

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 8, LEFT LENS
R = 0.06000 C = 0.9863 F = 0.9500 TE = 611.0

N	T	X	N	T	X	N	T	X
1	12.50	0.8972	9	231.20	0.6486	17	470.45	0.3567
2	25.60	0.8828	10	260.75	0.6126	18	500.95	0.3153
3	40.80	0.8828	11	290.90	0.5747	19	530.80	0.2540
4	71.00	0.8396	12	320.70	0.5549	20	560.55	0.1928
5	100.15	0.7982	13	350.40	0.5081	21	589.85	0.0919
6	138.60	0.7603	14	380.85	0.4793	22	620.45	0.0
7	171.10	0.7207	15	409.90	0.4450			
8	201.45	0.6864	16	440.20	0.4036			

K = 0.106E-03 D = 0.247E-04 S = 0.124E-01 V = 0.255E-01
I=3 KA=0.908E-04 DA=0.259E-04 SA=0.288E-02 VA=0.134E-01

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 9, LEFT LENS
R = 0.05665 C = 0.9863 F = 0.9500 TE = 552.7

N	T	X	N	T	X	N	T	X
1	12.50	0.8952	7	171.10	0.7043	13	350.40	0.4772
2	25.60	0.8856	8	201.45	0.6680	14	380.85	0.4428
3	40.80	0.8780	9	231.20	0.6299	15	409.90	0.3817
4	71.00	0.8207	10	260.75	0.5936	16	440.20	0.3073
5	100.15	0.7825	11	290.90	0.5573	17	470.45	0.2100
6	138.60	0.7501	12	320.70	0.5306	18	500.95	0.0

K = 0.102E-03 D = 0.249E-04 S = 0.171E-01 V = 0.337E-01
I=3 KA=0.974E-04 DA=0.278E-04 SA=0.772E-02 VA=0.254E-01

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 15, LEFT LENS
R = 0.05220 C = 0.9863 F = 0.9500 TE = 477.4

N	T	X	N	T	X	N	T	X
1	12.50	0.8858	7	171.10	0.6230	13	350.40	0.3518
2	25.60	0.8796	8	201.45	0.5857	14	380.85	0.3105
3	40.80	0.8361	9	231.20	0.5402	15	409.90	0.2484
4	71.00	0.7761	10	260.75	0.4947	16	440.20	0.1718
5	100.15	0.7265	11	290.90	0.4533	17	470.45	0.0621
6	138.60	0.6830	12	320.70	0.4119	18	500.95	0.0

K = 0.296E-03 D = 0.216E-04 S = 0.362E-02 V = 0.155E-01
I=3 KA=0.207E-03 DA=0.224E-04 SA=0.833E-03 VA=0.833E-02

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 16, LEFT LENS
R = 0.05690 C = 0.9863 F = 0.9500 TE = 547.5

N	T	X	N	T	X	N	T	X
1	12.50	0.8873	8	201.45	0.6287	15	409.90	0.3534
2	25.60	0.8743	9	231.20	0.5822	16	440.20	0.3106
3	40.80	0.8501	10	260.75	0.5543	17	470.45	0.2418
4	71.00	0.7961	11	290.90	0.5078	18	500.95	0.1786
5	100.15	0.7608	12	320.70	0.4743	19	530.80	0.0874
6	138.60	0.7031	13	350.40	0.4353	20	560.55	0.0
7	171.10	0.6659	14	380.85	0.3944			

K = 0.260E-03 D = 0.221E-04 S = 0.610E-02 V = 0.189E-01
I=3 KA=0.174E-03 DA=0.233E-04 SA=0.128E-02 VA=0.954E-02

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 17, LEFT LENS

R = 0.05560 C = 0.9863 F = 0.9500 TE = 575.0

N	T	X	N	T	X	N	T	X
1	12.50	0.9016	8	201.45	0.6520	15	409.90	0.3856
2	25.60	0.8923	9	231.20	0.6147	16	440.20	0.3390
3	40.80	0.8755	10	260.75	0.5793	17	470.45	0.2943
4	71.00	0.8066	11	290.90	0.5402	18	500.95	0.2347
5	100.15	0.7619	12	320.70	0.5067	19	530.80	0.1490
6	138.60	0.7321	13	350.40	0.4694	20	560.55	0.0
7	171.10	0.6911	14	380.85	0.4266			

K = 0.156E-03 D = 0.212E-04 S = 0.673E-02 V = 0.199E-01

I=2 KA=0.128E-03 DA=0.220E-04 SA=0.299E-02 VA=0.141E-01

DATA FROM RUN NUMBER IV-193, BEAD NUMBER 18, LEFT LENS

R = 0.05560 C = 0.9863 F = 0.9500 TE = 592.3

N	T	X	N	T	X	N	T	X
1	12.50	0.8947	9	231.20	0.6095	17	470.45	0.3150
2	25.60	0.8910	10	260.75	0.5629	18	500.95	0.2628
3	40.80	0.8761	11	290.90	0.5368	19	530.80	0.2050
4	71.00	0.8146	12	320.70	0.5070	20	560.55	0.1435
5	100.15	0.7661	13	350.40	0.4753	21	589.85	0.0354
6	138.60	0.7288	14	380.85	0.4287	22	620.45	0.0
7	171.10	0.6822	15	409.90	0.3914			
8	201.45	0.6468	16	440.20	0.3560			

K = 0.197E-03 D = 0.199E-04 S = 0.422E-02 V = 0.149E-01

I=3 KA=0.164E-03 DA=0.204E-04 SA=0.716E-03 VA=0.669E-02

DATA FROM RUN NUMBER IV-194, BEAD NUMBER 1, RIGHT LENS

R = 0.05620 C = 1.9739 F = 0.9100 TE = 529.0

N	T	X	N	T	X	N	T	X
1	16.10	0.8812	10	203.55	0.5536	19	404.65	0.2931
2	31.60	0.8543	11	225.45	0.5344	20	430.55	0.2529
3	54.70	0.8026	12	247.45	0.5019	21	446.50	0.2299
4	75.70	0.7662	13	269.75	0.4789	22	468.25	0.1916
5	96.65	0.7049	14	292.20	0.4463	23	487.05	0.1571
6	117.85	0.6819	15	314.75	0.4157	24	506.55	0.0996
7	138.95	0.6513	16	336.80	0.3984	25	526.60	0.0192
8	159.70	0.6206	17	359.85	0.3563	26	557.20	0.0
9	181.00	0.5938	18	382.20	0.3295			

DATA FROM RUN NUMBER IV-194, BEAD NUMBER 2, RIGHT LENS

R = 0.03205 C = 1.9739 F = 0.9100 TE = 203.0

N	T	X	N	T	X	N	T	X
1	16.10	0.7827	5	96.65	0.4535	9	181.00	0.1041
2	31.60	0.7055	6	117.85	0.3762	10	203.55	0.0
3	54.70	0.6282	7	138.95	0.2766			
4	75.70	0.5173	8	159.70	0.2049			

DATA FROM RUN NUMBER IV-194, BEAD NUMBER 3, RIGHT LENS
R = 0.03040 C = 1.9739 F = 0.9100 TE = 196.0

N	T	X	N	T	X	N	T	X
1	16.10	0.8117	5	96.65	0.4572	9	181.00	0.0603
2	31.60	0.7195	6	117.85	0.3934	10	203.55	0.0
3	54.70	0.6167	7	138.95	0.2907			
4	75.70	0.5352	8	159.70	0.2056			

DATA FROM RUN NUMBER IV-194, BEAD NUMBER 5, RIGHT LENS
R = 0.03550 C = 1.9739 F = 0.9100 TE = 231.0

N	T	X	N	T	X	N	T	X
1	16.10	0.8201	5	96.65	0.5175	9	181.00	0.2633
2	31.60	0.7293	6	117.85	0.4206	10	203.55	0.1816
3	54.70	0.6597	7	138.95	0.3904	11	225.45	0.0454
4	75.70	0.5720	8	159.70	0.3268	12	247.45	0.0

DATA FROM RUN NUMBER IV-194, BEAD NUMBER 11, RIGHT LENS
R = 0.05505 C = 1.9739 F = 0.9100 TE = 505.0

N	T	X	N	T	X	N	T	X
1	16.10	0.8841	9	181.00	0.5809	17	359.85	0.3247
2	31.60	0.8489	10	203.55	0.5477	18	382.20	0.2953
3	54.70	0.7843	11	225.45	0.5164	19	404.65	0.2543
4	75.70	0.7413	12	247.45	0.4890	20	430.55	0.2034
5	96.65	0.7022	13	269.75	0.4518	21	446.50	0.1760
6	117.85	0.6650	14	292.20	0.4264	22	468.25	0.1232
7	138.95	0.6396	15	314.75	0.3912	23	487.05	0.0782
8	159.70	0.6063	16	336.80	0.3599	24	506.55	0.0

DATA FROM RUN NUMBER IV-231, BEAD NUMBER 3, LEFT LENS
R = 0.05090 C = 2.4600 F = 0.1660 TE = 526.0

N	T	X	N	T	X	N	T	X
1	14.60	0.8832	10	202.10	0.5766	19	404.20	0.3044
2	26.60	0.8660	11	226.40	0.5359	20	420.55	0.2787
3	48.80	0.8124	12	248.60	0.5166	21	440.95	0.2551
4	68.60	0.7781	13	275.30	0.4716	22	457.45	0.2165
5	91.30	0.7288	14	299.20	0.4416	23	475.45	0.1779
6	111.25	0.7052	15	323.85	0.4137	24	494.40	0.1479
7	134.15	0.6688	16	350.50	0.3837	25	509.85	0.0986
8	156.70	0.6238	17	369.90	0.3580	26	529.25	0.0
9	179.35	0.6002	18	386.45	0.3280			

DATA FROM RUN NUMBER IV-231, BEAD NUMBER 5, LEFT LENS

R = 0.05495 C = 2.4600 F = 0.1660 TE = 540.0

N	T	X	N	T	X	N	T	X
1	14.60	0.8844	10	202.10	0.6048	19	404.20	0.3629
2	26.60	0.8884	11	226.40	0.5771	20	420.55	0.3550
3	48.80	0.8488	12	248.60	0.5612	21	440.95	0.3153
4	68.60	0.7952	13	275.30	0.5334	22	457.45	0.2975
5	91.30	0.7575	14	299.20	0.5116	23	475.45	0.2598
6	111.25	0.7179	15	323.85	0.4740	24	494.40	0.2360
7	134.15	0.6961	16	350.50	0.4383	25	509.85	0.1963
8	156.70	0.6723	17	369.90	0.4164	26	529.25	0.1487
9	179.35	0.6385	18	386.45	0.3966			

DATA FROM RUN NUMBER IV-231, BEAD NUMBER 9, LEFT LENS

R = 0.03465 C = 2.4600 F = 0.1660 TE = 270.0

N	T	X	N	T	X	N	T	X
1	14.60	0.9003	6	111.25	0.5383	11	226.40	0.2204
2	26.60	0.8153	7	134.15	0.4911	12	248.60	0.1574
3	48.80	0.7240	8	156.70	0.4313	13	275.30	0.0
4	68.60	0.6768	9	179.35	0.3746			
5	91.30	0.6013	10	202.10	0.3148			

DATA FROM RUN NUMBER IV-231, BEAD NUMBER 10, LEFT LENS

R = 0.03310 C = 2.4600 F = 0.1660 TE = 222.0

N	T	X	N	T	X	N	T	X
1	14.60	0.8230	5	91.30	0.5300	9	179.35	0.2732
2	26.60	0.7505	6	111.25	0.4740	10	202.10	0.1745
3	48.80	0.6617	7	134.15	0.4082	11	226.40	0.0
4	68.60	0.6090	8	156.70	0.3391			

DATA FROM RUN NUMBER IV-231, BEAD NUMBER 11, LEFT LENS

R = 0.05215 C = 2.4600 F = 0.1660 TE = 546.0

N	T	X	N	T	X	N	T	X
1	14.60	0.8747	10	202.10	0.5678	19	404.20	0.3194
2	26.60	0.8601	11	226.40	0.5470	20	420.55	0.2944
3	48.80	0.8163	12	248.60	0.5240	21	440.95	0.2714
4	68.60	0.7724	13	275.30	0.4823	22	457.45	0.2463
5	91.30	0.7202	14	299.20	0.4572	23	475.45	0.2192
6	111.25	0.6910	15	323.85	0.4321	24	494.40	0.1942
7	134.15	0.6681	16	350.50	0.3946	25	509.85	0.1649
8	156.70	0.6472	17	369.90	0.3737	26	529.25	0.1169
9	179.35	0.6096	18	386.45	0.3486			

DATA FROM RUN NUMBER IV-231, BEAD NUMBER 13, LEFT LENS

R = 0.03110 C = 2.4600 F = 0.1660 TE = 196.0

N	T	X	N	T	X	N	T	X
1	14.60	0.7862	5	91.30	0.4774	9	179.35	0.1404
2	26.60	0.7090	6	111.25	0.4177	10	202.10	0.0
3	48.80	0.6318	7	134.15	0.3334			
4	68.60	0.5476	8	156.70	0.2422			

DATA FROM RUN NUMBER IV-232, BEAD NUMBER 1, LEFT LENS
R = 0.04820 C = 2.4600 F = 1.9700 TE = 370.0

N	T	X	N	T	X	N	T	X
1	12.90	0.8711	8	132.60	0.5583	15	280.30	0.2865
2	24.90	0.8185	9	150.70	0.5262	16	302.75	0.2368
3	36.60	0.7805	10	170.45	0.4911	17	327.40	0.1754
4	52.40	0.7278	11	190.50	0.4472	18	362.95	0.0585
5	71.15	0.6782	12	214.50	0.4092	19	394.90	0.0
6	91.65	0.6402	13	234.70	0.3508			
7	111.90	0.5846	14	257.85	0.3245			

DATA FROM RUN NUMBER IV-232, BEAD NUMBER 2, LEFT LENS
R = 0.06695 C = 2.4600 F = 1.9700 TE = 578.0

N	T	X	N	T	X	N	T	X
1	12.90	0.8919	10	170.45	0.6858	19	394.90	0.4712
2	24.90	0.8835	11	190.50	0.6710	20	434.05	0.4186
3	36.60	0.8688	12	214.50	0.6479	21	473.10	0.3155
4	52.40	0.8351	13	234.70	0.6311	22	504.00	0.2671
5	71.15	0.8162	14	257.85	0.6121	23	531.70	0.1893
6	91.65	0.7783	15	280.30	0.5827	24	554.5[illegible]	0.1472
7	111.90	0.7552	16	302.75	0.5680	25	573.20	0.0421
8	132.60	0.7341	17	327.40	0.5532	26	591.40	0.0
9	150.70	0.7110	18	362.95	0.4964			

DATA FROM RUN NUMBER IV-232, BEAD NUMBER 3, LEFT LENS
R = 0.03740 C = 2.4600 F = 1.9700 TE = 250.0

N	T	X	N	T	X	N	T	X
1	12.90	0.8250	6	91.65	0.5311	11	190.50	0.2637
2	24.90	0.7647	7	111.90	0.4746	12	214.50	0.1808
3	36.60	0.7157	8	132.60	0.4294	13	234.70	0.0979
4	52.40	0.6479	9	150.70	0.3767	14	257.85	0.0
5	71.15	0.6027	10	170.45	0.3315			

DATA FROM RUN NUMBER IV-232, BEAD NUMBER 4, LEFT LENS
R = 0.04790 C = 2.4600 F = 1.9700 TE = 405.0

N	T	X	N	T	X	N	T	X
1	12.90	0.8768	8	132.60	0.5767	15	280.30	0.3295
2	24.90	0.8415	9	150.70	0.5414	16	302.75	0.2913
3	36.60	0.7944	10	170.45	0.5237	17	327.40	0.2354
4	52.40	0.7267	11	190.50	0.4913	18	362.95	0.1618
5	71.15	0.7061	12	214.50	0.4443	19	394.90	0.0530
6	91.65	0.6591	13	234.70	0.4060	20	434.05	0.0
7	111.90	0.6179	14	257.85	0.3678			

DATA FROM RUN NUMBER IV-232, BEAD NUMBER 5, LEFT LENS
R = 0.03035 C = 2.4600 F = 1.9700 TE = 148.0

N	T	X	N	T	X	N	T	X
1	12.90	0.7657	4	52.40	0.5105	7	111.90	0.2692
2	24.90	0.6822	5	71.15	0.4223	8	132.60	0.1392
3	36.60	0.6033	6	91.65	0.3341	9	150.70	0.0

DATA FROM RUN NUMBER IV-232, BEAD NUMBER 8, LEFT LENS

R = 0.05445 C = 2.4600 F = 1.9700 TE = 490.0

N	T	X	N	T	X	N	T	X
1	12.90	0.8770	9	150.70	0.6261	17	327.40	0.3674
2	24.90	0.8615	10	170.45	0.6028	18	362.95	0.3260
3	36.60	0.7994	11	190.50	0.5743	19	394.90	0.2665
4	52.40	0.7684	12	214.50	0.5252	20	434.05	0.2018
5	71.15	0.7528	13	234.70	0.4967	21	473.10	0.1035
6	91.65	0.7166	14	257.85	0.4734	22	504.00	0.0
7	111.90	0.6752	15	280.30	0.4398			
8	132.60	0.6468	16	302.75	0.4139			

DATA FROM RUN NUMBER IV-232, BEAD NUMBER 9, LEFT LENS

R = 0.04730 C = 2.4600 F = 1.9700 TE = 333.0

N	T	X	N	T	X	N	T	X
1	12.90	0.8491	7	111.90	0.5721	13	234.70	0.3252
2	24.90	0.8100	8	132.60	0.5330	14	257.85	0.2710
3	36.60	0.7317	9	150.70	0.4878	15	280.30	0.2108
4	52.40	0.6805	10	170.45	0.4667	16	302.75	0.1506
5	71.15	0.6594	11	190.50	0.4005	17	327.40	0.0602
6	91.65	0.6112	12	214.50	0.3613	18	362.95	0.0

DATA FROM RUN NUMBER IV-232, BEAD NUMBER 10, LEFT LENS

R = 0.04100 C = 2.4600 F = 1.9700 TE = 235.0

N	T	X	N	T	X	N	T	X
1	12.90	0.8591	6	91.65	0.5842	11	190.50	0.2612
2	24.90	0.7903	7	111.90	0.5120	12	214.50	0.1649
3	36.60	0.7285	8	132.60	0.4570	13	234.70	0.0
4	52.40	0.6873	9	150.70	0.3986			
5	71.15	0.6460	10	170.45	0.3402			

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 1

R = 0.04700 C = 2.4600 F = 0.0 TE = 453.0

N	T	X	N	T	X	N	T	X
1	12.05	0.8868	8	158.15	0.5852	15	345.75	0.2971
2	35.45	0.8463	9	185.65	0.5492	16	363.25	0.2656
3	45.15	0.8013	10	210.30	0.5042	17	380.00	0.2296
4	65.65	0.7473	11	235.60	0.4817	18	399.70	0.1846
5	88.00	0.7067	12	262.65	0.4412	19	423.85	0.0990
6	112.85	0.6797	13	294.50	0.3736	20	445.00	0.0495
7	138.25	0.6257	14	316.80	0.3511	21	470.90	0.0

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 3

R = 0.05875   C = 2.4600   F = 0.0   TE = 672.0

N	T	X	N	T	X	N	T	X
1	12.05	0.9111	11	235.60	0.6410	21	470.90	0.3853
2	35.45	0.8967	12	262.65	0.6014	22	495.15	0.3601
3	45.15	0.8823	13	294.50	0.5726	23	518.70	0.3241
4	65.65	0.8463	14	316.80	0.5402	24	545.15	0.2881
5	88.00	0.8175	15	345.75	0.5042	25	570.65	0.2449
6	112.85	0.7815	16	363.25	0.4970	26	591.10	0.2053
7	138.25	0.7455	17	380.00	0.4754	27	615.25	0.1693
8	158.15	0.7166	18	399.70	0.4682	28	643.45	0.1044
9	185.65	0.6806	19	423.85	0.4357	29	659.80	0.0468
10	210.30	0.6554	20	445.00	0.4105			

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 4

R = 0.04324   C = 2.4600   F = 0.0   TE = 398.0

N	T	X	N	T	X	N	T	X
1	12.05	0.8758	7	138.25	0.5872	13	294.50	0.3180
2	35.45	0.8122	8	158.15	0.5480	14	316.80	0.2789
3	45.15	0.7584	9	185.65	0.5040	15	345.75	0.2251
4	65.65	0.7291	10	210.30	0.4697	16	363.25	0.1664
5	88.00	0.6850	11	235.60	0.4306	17	380.00	0.1076
6	112.85	0.6410	12	262.65	0.3719	18	399.70	0.0

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 5

R = 0.02890   C = 2.4600   F = 0.0   TE = 294.0

N	T	X	N	T	X	N	T	X
1	12.05	0.9003	6	112.85	0.6222	11	235.60	0.3001
2	35.45	0.8564	7	138.25	0.5343	12	262.65	0.2049
3	45.15	0.7612	8	158.15	0.5124	13	294.50	0.0
4	65.65	0.7246	9	185.65	0.4392			
5	88.00	0.6588	10	210.30	0.3660			

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 6

R = 0.03948   C = 2.4600   F = 0.0   TE = 316.0

N	T	X	N	T	X	N	T	X
1	12.05	0.8521	6	112.85	0.5788	11	235.60	0.3108
2	35.45	0.7663	7	138.25	0.5252	12	262.65	0.2144
3	45.15	0.7127	8	158.15	0.4823	13	294.50	0.1286
4	65.65	0.6699	9	185.65	0.4234	14	316.80	0.0
5	88.00	0.6324	10	210.30	0.3644			

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 7
R = 0.04441 C = 2.4600 F = 0.0 TE = 392.0

N	T	X	N	T	X	N	T	X
1	12.05	0.8717	7	138.25	0.5907	13	294.50	0.3001
2	35.45	0.8241	8	158.15	0.5573	14	316.80	0.2620
3	45.15	0.8003	9	185.65	0.5097	15	345.75	0.2048
4	65.65	0.7241	10	210.30	0.4716	16	363.25	0.1477
5	88.00	0.6764	11	235.60	0.4240	17	380.00	0.0810
6	112.85	0.6574	12	262.65	0.3811	18	399.70	0.0

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 8
R = 0.02679 C = 2.4600 F = 0.0 TE = 169.0

N	T	X	N	T	X	N	T	X
1	12.05	0.8450	4	65.65	0.5607	7	138.25	0.2369
2	35.45	0.6950	5	88.00	0.4659	8	158.15	0.1185
3	45.15	0.6476	6	112.85	0.3554	9	185.65	0.0

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 9
R = 0.03078 C = 2.4600 F = 0.0 TE = 214.0

N	T	X	N	T	X	N	T	X
1	12.05	0.8659	5	88.00	0.5429	9	185.65	0.1993
2	35.45	0.7147	6	112.85	0.4880	10	210.30	0.0344
3	45.15	0.6941	7	138.25	0.3780	11	235.60	0.0
4	65.65	0.6185	8	158.15	0.3093			

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 12
R = 0.02867 C = 2.4600 F = 0.0 TE = 207.0

N	T	X	N	T	X	N	T	X
1	12.05	0.8413	5	88.00	0.5166	9	185.65	0.0959
2	35.45	0.6863	6	112.85	0.4428	10	210.30	0.0
3	45.15	0.6642	7	138.25	0.3026			
4	65.65	0.5904	8	158.15	0.2361			

DATA FROM RUN NUMBER IV-261, LEFT LENS, BEAD NUMBER 13
R = 0.04512 C = 2.4600 F = 0.0 TE = 433.0

N	T	X	N	T	X	N	T	X
1	12.05	0.8722	8	158.15	0.5768	15	345.75	0.2860
2	35.45	0.8440	9	185.65	0.5580	16	363.25	0.2438
3	45.15	0.8347	10	210.30	0.5158	17	380.00	0.2110
4	65.65	0.7596	11	235.60	0.4689	18	399.70	0.1547
5	88.00	0.7081	12	262.65	0.4220	19	423.85	0.0656
6	112.85	0.6752	13	294.50	0.3751	20	445.00	0.0
7	138.25	0.6096	14	316.80	0.3282			

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 3

R = 0.03850 C = 0.4946 F = 0.9734 TE = 326.0

N	T	X	N	T	X	N	T	X
1	10.55	0.8886	7	115.25	0.6782	13	257.00	0.3274
2	20.45	0.8769	8	136.35	0.6138	14	279.20	0.2689
3	30.45	0.8769	9	156.70	0.5846	15	299.70	0.1871
4	54.35	0.7951	10	180.20	0.5262	16	318.40	0.0877
5	79.30	0.7132	11	207.35	0.4502	17	333.50	0.0
6	100.60	0.6957	12	237.10	0.3800			

K = 0.101E-03 D = 0.204E-04 S = 0.701E-02 V = 0.224E-01

I=3 KA=0.869E-04 DA=0.212E-04 SA=0.218E-02 VA=0.141E-01

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 4

R = 0.04150 C = 0.4946 F = 0.9734 TE = 317.1

N	T	X	N	T	X	N	T	X
1	10.55	0.8678	7	115.25	0.6020	13	257.00	0.3037
2	20.45	0.8569	8	136.35	0.5695	14	279.20	0.2169
3	30.45	0.8190	9	156.70	0.5369	15	299.70	0.1193
4	54.35	0.7430	10	180.20	0.4718	16	318.40	0.0
5	79.30	0.7051	11	207.35	0.4230			
6	100.60	0.6563	12	237.10	0.3417			

K = 0.447E-03 D = 0.210E-04 S = 0.454E-02 V = 0.187E-01

I=1 KA=0.305E-03 DA=0.216E-04 SA=0.249E-02 VA=0.144E-01

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 5

R = 0.02725 C = 0.4946 F = 0.9734 TE = 187.4

N	T	X	N	T	X	N	T	X
1	10.55	0.8755	5	79.30	0.5534	9	156.70	0.1487
2	20.45	0.8425	6	100.60	0.4873	10	180.20	0.0
3	30.45	0.7599	7	115.25	0.4130			
4	54.35	0.6608	8	136.35	0.3056			

K = 0.130E-03 D = 0.201E-04 S = 0.201E-02 V = 0.169E-01

I=1 KA=0.117E-03 DA=0.205E-04 SA=0.146E-02 VA=0.156E-01

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 6

R = 0.05225 C = 0.4946 F = 0.9734 TE = 534.4

N	T	X	N	T	X	N	T	X
1	10.55	0.8960	10	180.20	0.6677	19	370.45	0.4049
2	20.45	0.8917	11	207.35	0.6160	20	401.80	0.3446
3	30.45	0.8960	12	237.10	0.5902	21	414.10	0.3102
4	54.35	0.8658	13	257.00	0.5600	22	436.40	0.2757
5	79.30	0.8185	14	279.20	0.5212	23	461.30	0.2455
6	100.60	0.7797	15	299.70	0.4997	24	490.15	0.1852
7	115.25	0.7668	16	318.40	0.4738	25	502.35	0.1508
8	136.35	0.7280	17	333.50	0.4480	26	528.40	0.0775
9	156.70	0.6892	18	352.25	0.4394	27	549.45	0.0

K = 0.127E-03 D = 0.209E-04 S = 0.844E-02 V = 0.188E-01

I=3 KA=0.113E-03 DA=0.214E-04 SA=0.202E-02 VA=0.980E-02

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 7
R = 0.04500   C = 0.4946   F = 0.9734   TE = 383.1

N	T	X	N	T	X	N	T	X
1	10.55	0.8903	8	136.35	0.6402	15	299.70	0.3101
2	20.45	0.8803	9	156.70	0.6052	16	318.40	0.2801
3	30.45	0.8603	10	180.20	0.5802	17	333.50	0.2451
4	54.35	0.8153	11	207.35	0.5202	18	352.25	0.1801
5	79.30	0.7553	12	237.10	0.4602	19	370.45	0.1050
6	100.60	0.7052	13	257.00	0.4101	20	401.80	0.0
7	115.25	0.6752	14	279.20	0.3651			

K = 0.162E-03   D = 0.218E-04   S = 0.528E-02   V = 0.176E-01
I=3   KA=0.129E-03   DA=0.227E-04   SA=0.104E-02   VA=0.860E-02

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 8
R = 0.03300   C = 0.4946   F = 0.9734   TE = 210.3

N	T	X	N	T	X	N	T	X
1	10.55	0.8594	5	79.30	0.6207	9	156.70	0.3410
2	20.45	0.8321	6	100.60	0.5525	10	180.20	0.2251
3	30.45	0.7844	7	115.25	0.4979	11	207.35	0.0750
4	54.35	0.7366	8	136.35	0.4229	12	237.10	0.0

K = 0.224E-03   D = 0.217E-04   S = 0.432E-02   V = 0.219E-01
I=1   KA=0.181E-03   DA=0.223E-04   SA=0.247E-02   VA=0.176E-01

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 9
R = 0.05250   C = 0.4946   F = 0.9734   TE = 518.2

N	T	X	N	T	X	N	T	X
1	10.55	0.8960	10	180.20	0.6474	19	370.45	0.3773
2	20.45	0.8746	11	207.35	0.6259	20	401.80	0.3344
3	30.45	0.8660	12	237.10	0.5831	21	414.10	0.3001
4	54.35	0.8532	13	257.00	0.5445	22	436.40	0.2615
5	79.30	0.7931	14	279.20	0.5316	23	461.30	0.2144
6	100.60	0.7588	15	299.70	0.4802	24	490.15	0.1586
7	115.25	0.7288	16	318.40	0.4630	25	502.35	0.0900
8	136.35	0.7117	17	333.50	0.4544	26	528.40	0.0
9	156.70	0.6860	18	352.25	0.4159			

K = 0.155E-03   D = 0.213E-04   S = 0.110E-01   V = 0.218E-01
I=3   KA=0.122E-03   DA=0.222E-04   SA=0.262E-02   VA=0.114E-01

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 10
R = 0.03500   C = 0.4946   F = 0.9734   TE = 310.0

N	T	X	N	T	X	N	T	X
1	10.55	0.8296	7	115.25	0.6045	13	257.00	0.2637
2	20.45	0.8231	8	136.35	0.5788	14	279.20	0.1929
3	30.45	0.8103	9	156.70	0.5145	15	299.70	0.1158
4	54.35	0.7524	10	180.20	0.4502	16	318.40	0.0
5	79.30	0.6688	11	207.35	0.4051			
6	100.60	0.6366	12	237.10	0.3344			

REGRESSION DID NOT CONVERGE
I=2   KA=0.326E-03   DA=0.157E-04   SA=0.155E-02   VA=0.119E-01

DATA FROM RUN NUMBER IV-272, RIGHT LENS, BEAD NUMBER 11
R = 0.02525 C = 0.4946 F = 0.9734 TE = 97.0

N	T	X	N	T	X	N	T	X
1	10.55	0.7220	3	30.45	0.5527	5	79.30	0.2318
2	20.45	0.6685	4	54.35	0.4011	6	100.60	0.0

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 1
R = 0.05540 C = 2.4540 F = 0.9734 TE = 581.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8902	8	234.70	0.5503	15	436.00	0.3029
2	62.15	0.8015	9	261.70	0.5171	16	469.25	0.2586
3	89.85	0.7424	10	290.40	0.4765	17	494.50	0.2179
4	116.90	0.6944	11	320.30	0.4469	18	520.15	0.1810
5	144.60	0.6575	12	349.40	0.4137	19	547.60	0.1182
6	172.00	0.6205	13	378.45	0.3731	20	570.40	0.0517
7	205.75	0.5873	14	407.45	0.3398	21	596.50	0.0

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 2
R = 0.05475 C = 2.4540 F = 0.9734 TE = 564.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8854	8	234.70	0.5566	15	436.00	0.2989
2	62.15	0.7957	9	261.70	0.5230	16	469.25	0.2540
3	89.85	0.7471	10	290.40	0.4856	17	494.50	0.2092
4	116.90	0.7061	11	320.30	0.4483	18	520.15	0.1420
5	144.60	0.6650	12	349.40	0.4109	19	547.60	0.0971
6	172.00	0.6351	13	378.45	0.3736	20	570.40	0.0
7	205.75	0.5940	14	407.45	0.3362			

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 3
R = 0.05610 C = 2.4540 F = 0.9734 TE = 608.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8793	8	234.70	0.5364	15	436.00	0.3028
2	62.15	0.7918	9	261.70	0.5108	16	469.25	0.2554
3	89.85	0.7334	10	290.40	0.4816	17	494.50	0.2262
4	116.90	0.6969	11	320.30	0.4378	18	520.15	0.1861
5	144.60	0.6531	12	349.40	0.4014	19	547.60	0.1459
6	172.00	0.6203	13	378.45	0.3649	20	570.40	0.1131
7	205.75	0.5801	14	407.45	0.3357	21	596.50	0.0438

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 4
R = 0.05865 C = 2.4540 F = 0.9734 TE = 612.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8933	8	234.70	0.5583	15	436.00	0.3455
2	62.15	0.8166	9	261.70	0.5304	16	469.25	0.2861
3	89.85	0.7537	10	290.40	0.5095	17	494.50	0.2547
4	116.90	0.7154	11	320.30	0.4711	18	520.15	0.2233
5	144.60	0.6805	12	349.40	0.4362	19	547.60	0.1745
6	172.00	0.6281	13	378.45	0.4048	20	570.40	0.1361
7	205.75	0.6002	14	407.45	0.3664	21	596.50	0.0628

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 6
R = 0.05720 C = 2.4540 F = 0.9734 TE = 515.0

N	T	X	N	T	X	N	T	X
1	28.60	0.9012	7	205.75	0.6544	13	378.45	0.4327
2	62.15	0.8225	8	234.70	0.6187	14	407.45	0.3827
3	89.85	0.7868	9	261.70	0.5937	15	436.00	0.3076
4	116.90	0.7546	10	290.40	0.5615	16	469.25	0.2146
5	144.60	0.7188	11	320.30	0.5257	17	494.50	0.1323
6	172.00	0.6902	12	349.40	0.4864	18	520.15	0.0

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 7
R = 0.05685 C = 2.4540 F = 0.9734 TE = 610.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8850	8	234.70	0.5540	15	436.00	0.3238
2	62.15	0.7879	9	261.70	0.5217	16	469.25	0.2770
3	89.85	0.7519	10	290.40	0.4785	17	494.50	0.2374
4	116.90	0.6871	11	320.30	0.4569	18	520.15	0.2123
5	144.60	0.6548	12	349.40	0.4137	19	547.60	0.1691
6	172.00	0.6260	13	378.45	0.3849	20	570.40	0.1331
7	205.75	0.5828	14	407.45	0.3346	21	596.50	0.0612

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 8
R = 0.05635 C = 2.4540 F = 0.9734 TE = 588.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8713	8	234.70	0.5518	15	436.00	0.3013
2	62.15	0.7841	9	261.70	0.5228	16	469.25	0.2614
3	89.85	0.7333	10	290.40	0.4792	17	494.50	0.2287
4	116.90	0.6934	11	320.30	0.4502	18	520.15	0.1815
5	144.60	0.6607	12	349.40	0.4066	19	547.60	0.1089
6	172.00	0.6280	13	378.45	0.3812	20	570.40	0.0799
7	205.75	0.5881	14	407.45	0.3412	21	596.50	0.0

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 9
R = 0.05680 C = 2.4540 F = 0.9734 TE = 560.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8931	8	234.70	0.5438	15	436.00	0.2917
2	62.15	0.7851	9	261.70	0.5150	16	469.25	0.2449
3	89.85	0.7311	10	290.40	0.4754	17	494.50	0.1945
4	116.90	0.6950	11	320.30	0.4394	18	520.15	0.1440
5	144.60	0.6518	12	349.40	0.4105	19	547.60	0.0684
6	172.00	0.6194	13	378.45	0.3889	20	570.40	0.0
7	205.75	0.5834	14	407.45	0.3349			

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 10

R = 0.C5820 C = 2.4540 F = 0.9734 TE = 553.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8792	8	234.70	0.5557	15	436.00	0.3130
2	62.15	0.7983	9	261.70	0.5205	16	469.25	0.2462
3	89.85	0.7561	10	290.40	0.4888	17	494.50	0.2075
4	116.90	0.7034	11	320.30	0.4572	18	520.15	0.1547
5	144.60	0.6682	12	349.40	0.4255	19	547.60	0.0528
6	172.00	0.6330	13	378.45	0.3869	20	570.40	0.0
7	205.75	0.5908	14	407.45	0.3446			

DATA FROM RUN NUMBER IV-276, RIGHT LENS, BEAD NUMBER 11

R = 0.C5880 C = 2.4540 F = 0.9734 TE = 614.0

N	T	X	N	T	X	N	T	X
1	28.60	0.8733	8	234.70	0.5637	15	436.00	0.3445
2	62.15	0.7968	9	261.70	0.5289	16	469.25	0.2992
3	89.85	0.7516	10	290.40	0.4976	17	494.50	0.2575
4	116.90	0.6994	11	320.30	0.4593	18	520.15	0.2157
5	144.60	0.6611	12	349.40	0.4384	19	547.60	0.1775
6	172.00	0.6298	13	378.45	0.4106	20	570.40	0.1496
7	205.75	0.5915	14	407.45	0.3827	21	596.50	0.0765

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 1

R = 0.03495 C = 0.1028 F = 0.9734 TE = 305.6

N	T	X	N	T	X	N	T	X
1	14.95	0.8938	5	131.65	0.6654	9	251.20	0.3458
2	46.90	0.8351	6	160.10	0.6067	10	279.80	0.1957
3	70.95	0.7829	7	195.10	0.5154	11	310.40	0.0
4	95.60	C.7503	8	223.20	0.4110			

K = 0.209E-03 D = 0.112E-03 S = 0.724E-02 V = 0.301E-01

I=1 KA=C.200E-03 DA=0.116E-03 SA=0.341E-02 VA=0.221E-01

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 2

R = 0.02710 C = 0.1028 F = 0.9734 TE = 212.4

N	T	X	N	T	X	N	T	X
1	14.95	0.8498	4	95.60	0.6058	7	195.10	0.0
2	46.9C	0.7404	5	131.65	0.4291			
3	70.95	0.6731	6	160.10	0.3366			

K = 0.517E-03 D = 0.744E-04 S = 0.348E-02 V = 0.295E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 3

R = 0.02860 C = 0.1028 F = 0.9734 TE = 197.5

N	T	X	N	T	X	N	T	X
1	14.95	0.8366	4	95.60	0.5497	7	195.10	0.0558
2	46.90	0.7091	5	131.65	0.4382	8	223.20	0.0
3	70.95	0.6374	6	160.10	0.3346			

K = 0.7C0E-03 D = 0.870E-04 S = 0.424E-02 V = 0.291E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 4
R = 0.03065 C = 0.1028 F = 0.9734 TE = 252.8

N	T	X	N	T	X	N	T	X
1	14.95	0.8631	4	95.60	0.6697	7	195.10	0.3720
2	46.90	0.7887	5	131.65	0.5952	8	223.20	0.2009
3	70.95	0.7069	6	160.10	0.4985	9	251.20	0.0

K = 0.298E-03 D = 0.897E-04 S = 0.105E-01 V = 0.418E-01
NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 5
R = 0.03825 C = 0.1028 F = 0.9734 TE = 432.4

N	T	X	N	T	X	N	T	X
1	14.95	0.8765	7	195.10	0.6499	13	367.55	0.3100
2	46.90	0.8705	8	223.20	0.5962	14	391.85	0.2325
3	70.95	0.8347	9	251.20	0.5485	15	416.90	0.1192
4	95.60	0.8049	10	279.80	0.4949	16	442.15	0.0
5	131.65	0.7393	11	310.40	0.4293			
6	160.10	0.6976	12	337.60	0.3756			

K = 0.181E-03 D = 0.914E-04 S = 0.109E-01 V = 0.289E-01
I=2 KA=0.171E-03 DA=0.951E-04 SA=0.151E-02 VA=0.117E-01

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 9
R = 0.04685 C = 0.1028 F = 0.9734 TE = 568.0

N	T	X	N	T	X	N	T	X
1	14.95	0.8857	9	251.20	0.6667	17	466.75	0.3455
2	46.90	0.8906	10	279.80	0.6278	18	489.20	0.3017
3	70.95	0.8954	11	310.40	0.6035	19	513.50	0.2531
4	95.60	0.8614	12	337.60	0.5499	20	535.70	0.1947
5	131.65	0.8224	13	367.55	0.5159	21	560.10	0.0925
6	160.10	0.7835	14	391.85	0.4867	22	581.45	0.0
7	195.10	0.7397	15	416.90	0.4477			
8	223.20	0.7105	16	442.15	0.3893			

K = 0.162E-03 D = 0.114E-03 S = 0.117E-01 V = 0.248E-01
I=3 KA=0.157E-03 DA=0.116E-03 SA=0.800E-03 VA=0.707E-02

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 10
R = 0.04965 C = 0.1028 F = 0.9734 TE = 605.7

N	T	X	N	T	X	N	T	X
1	14.95	0.8727	9	251.20	0.6706	17	466.75	0.3904
2	46.90	0.8682	10	279.80	0.6431	18	489.20	0.3491
3	70.95	0.8727	11	310.40	0.5971	19	513.50	0.2986
4	95.60	0.8682	12	337.60	0.5696	20	535.70	0.2526
5	131.65	0.8130	13	367.55	0.5236	21	560.10	0.1837
6	160.10	0.7809	14	391.85	0.5053	22	581.45	0.1378
7	195.10	0.7349	15	416.90	0.4685	23	604.45	0.0276
8	223.20	0.7028	16	442.15	0.4364	24	626.05	0.0

K = 0.197E-03 D = 0.108E-03 S = 0.174E-01 V = 0.288E-01
I=4 KA=0.189E-03 DA=0.110E-03 SA=0.171E-02 VA=0.100E-01

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 12

R = 0.05065   C = 0.1028   F = 0.9734   TE = 650.9

N	T	X	N	T	X	N	T	X
1	14.95	0.8958	10	279.80	0.6842	19	513.50	0.3871
2	46.90	0.8913	11	310.40	0.6662	20	535.70	0.3421
3	70.95	0.9003	12	337.60	0.6212	21	560.10	0.2836
4	95.60	0.8868	13	367.55	0.5897	22	581.45	0.2566
5	131.65	0.8508	14	391.85	0.5492	23	604.45	0.2161
6	160.10	0.8013	15	416.90	0.5042	24	626.05	0.1395
7	195.10	0.7833	16	442.15	0.4817	25	648.30	0.0540
8	223.20	0.7608	17	466.75	0.4547	26	670.50	0.0
9	251.20	0.7202	18	489.20	0.4321			

K = 0.157E-03   D = 0.117E-03   S = 0.129E-01   V = 0.237E-01

I=4   KA=0.154E-03   DA=0.119E-03   SA=0.223E-02   VA=0.108E-01

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 14

R = 0.04610   C = 0.1028   F = 0.9734   TE = 516.2

N	T	X	N	T	X	N	T	X
1	14.95	0.8756	8	223.20	0.6579	15	416.90	0.3562
2	46.90	0.8756	9	251.20	0.6035	16	442.15	0.3067
3	70.95	0.8805	10	279.80	0.5837	17	466.75	0.2424
4	95.60	0.8409	11	310.40	0.5540	18	489.20	0.1583
5	131.65	0.7915	12	337.60	0.4996	19	513.50	0.0495
6	160.10	0.7420	13	367.55	0.4551	20	535.70	0.0
7	195.10	0.6975	14	391.85	0.4007			

K = 0.196E-03   D = 0.113E-03   S = 0.138E-01   V = 0.285E-01

I=3   KA=0.188E-03   DA=0.115E-03   SA=0.178E-02   VA=0.113E-01

DATA FROM RUN NUMBER IV-279, RIGHT LENS, BEAD NUMBER 17

R = 0.04585   C = 0.1028   F = 0.9734   TE = 540.7

N	T	X	N	T	X	N	T	X
1	14.95	0.8904	8	223.20	0.6864	15	416.90	0.3979
2	46.90	0.8705	9	251.20	0.6367	16	442.15	0.3681
3	70.95	0.8953	10	279.80	0.6068	17	466.75	0.2885
4	95.60	0.8605	11	310.40	0.5670	18	489.20	0.2388
5	131.65	0.8108	12	337.60	0.5322	19	513.50	0.1691
6	160.10	0.7710	13	367.55	0.4924	20	535.70	0.0746
7	195.10	0.7113	14	391.85	0.4327	21	560.10	0.0

K = 0.172E-03   D = 0.112E-03   S = 0.124E-01   V = 0.262E-01

I=4   KA=0.167E-03   DA=0.114E-03   SA=0.128E-02   VA=0.957E-02

DATA FROM RUN NUMBER IV-284, BEAD NUMBER 1,RIGHT LENS

R = 0.05500 C = 0.1028 F = 0.0911 TE = 916.5

N	T	X	N	T	X	N	T	X
1	100.00	0.8925	10	344.55	0.7451	19	635.00	0.5278
2	113.00	0.8887	11	371.55	0.7334	20	666.85	0.5045
3	135.75	0.8848	12	406.80	0.7179	21	697.60	0.4385
4	161.50	0.8848	13	441.10	0.6869	22	734.75	0.4152
5	196.55	0.8654	14	468.85	0.6597	23	779.95	0.3570
6	225.45	0.8382	15	500.20	0.6325	24	818.80	0.3027
7	253.40	0.8033	16	533.30	0.6170	25	853.70	0.2290
8	279.75	0.7839	17	568.20	0.5821	26	889.55	0.1319
9	312.95	0.7684	18	598.20	0.5472	27	925.00	0.0

K = 0.101E-03 D = 0.121E-03 S = 0.671E-02 V = 0.167E-01

I=4 KA=0.996E-04 DA=0.124E-03 SA=0.325E-02 VA=0.127E-01

DATA FROM RUN NUMBER IV-284, BEAD NUMBER 2,RIGHT LENS

R = 0.05260 C = 0.1028 F = 0.0911 TE = 870.2

N	T	X	N	T	X	N	T	X
1	100.00	0.8679	10	344.55	0.7097	19	635.00	0.4785
2	113.00	0.8638	11	371.55	0.6975	20	666.85	0.4137
3	135.75	0.8516	12	406.80	0.6732	21	697.60	0.3934
4	161.50	0.8516	13	441.10	0.6489	22	734.75	0.3244
5	196.55	0.8314	14	468.85	0.6164	23	779.95	0.2595
6	225.45	0.8111	15	500.20	0.5921	24	818.80	0.1703
7	253.40	0.7786	16	533.30	0.5678	25	853.70	0.0
8	279.75	0.7665	17	568.20	0.5272			
9	312.95	0.7421	18	598.20	0.5069			

K = 0.115E-03 D = 0.100E-03 S = 0.960E-02 V = 0.209E-01

I=4 KA=0.110E-03 DA=0.107E-03 SA=0.285E-02 VA=0.126E-01

DATA FROM RUN NUMBER IV-284, BEAD NUMBER 3,RIGHT LENS

R = 0.05980 C = 0.1028 F = 0.0911 TE = 1099.0

N	T	X	N	T	X	N	T	X
1	100.00	0.8967	10	344.55	0.8038	19	635.00	0.6109
2	113.00	0.8967	11	371.55	0.7788	20	666.85	0.6002
3	135.75	0.8932	12	406.80	0.7610	21	697.60	0.5716
4	161.50	0.8896	13	441.10	0.7431	22	734.75	0.5430
5	196.55	0.8896	14	468.85	0.7145	23	779.95	0.5109
6	225.45	0.8574	15	500.20	0.7074	24	818.80	0.4644
7	253.40	0.8503	16	533.30	0.6788	25	853.70	0.4287
8	279.75	0.8431	17	568.20	0.6502	26	889.55	0.3930
9	312.95	0.8217	18	598.20	0.6395	27	925.00	0.3251

K = 0.973E-04 D = 0.117E-03 S = 0.731E-02 V = 0.175E-01

I=6 KA=0.934E-04 DA=0.129E-03 SA=0.167E-02 VA=0.964E-02

DATA FROM RUN NUMBER IV-284, BEAD NUMBER 5,RIGHT LENS

R = 0.05170 C = 0.1028 F = 0.0911 TE = 870.1

N	T	X	N	T	X	N	T	X
1	100.00	0.8920	10	344.55	0.7434	19	635.00	0.5038
2	113.00	0.8755	11	371.55	0.7269	20	666.85	0.4584
3	135.75	0.8714	12	406.80	0.7103	21	697.60	0.4171
4	161.50	0.8755	13	441.10	0.6649	22	734.75	0.3676
5	196.55	0.8673	14	468.85	0.6484	23	779.95	0.2973
6	225.45	0.8301	15	500.20	0.6195	24	818.80	0.2065
7	253.40	0.8177	16	533.30	0.5906	25	853.70	0.1156
8	279.75	0.7888	17	568.20	0.5493	26	889.55	0.0
9	312.95	0.7723	18	598.20	0.5369			

K = 0.922E-04 D = 0.120E-03 S = 0.657E-02 V = 0.169E-01

I=5 KA=0.903E-04 DA=0.124E-03 SA=0.137E-02 VA=0.873E-02

DATA FROM RUN NUMBER IV-284, BEAD NUMBER 6,RIGHT LENS

R = 0.05010 C = 0.1028 F = 0.0911 TE = 768.5

N	T	X	N	T	X	N	T	X
1	100.00	0.8619	9	312.95	0.7254	17	568.20	0.4864
2	113.00	0.8619	10	344.55	0.6870	18	598.20	0.4267
3	135.75	0.8576	11	371.55	0.6656	19	635.00	0.3840
4	161.50	0.8619	12	406.80	0.6400	20	666.85	0.3115
5	196.55	0.8320	13	441.10	0.6059	21	697.60	0.2645
6	225.45	0.7894	14	468.35	0.5803	22	734.75	0.1621
7	253.40	0.7680	15	500.20	0.5504	23	779.95	0.0
8	279.75	0.7595	16	533.30	0.5120			

K = 0.110E-03 D = 0.114E-03 S = 0.783E-02 V = 0.198E-01

I=4 KA=0.105E-03 DA=0.119E-03 SA=0.231E-02 VA=0.120E-01

DATA FROM RUN NUMBER IV-287, RIGHT LENS, BEAD NUMBER 1

R = 0.05680 C = 0.1028 F = 2.0190 TE = 705.4

N	T	X	N	T	X	N	T	X
1	48.25	0.8965	10	304.00	0.6194	19	535.55	0.3924
2	72.80	0.8811	11	325.40	0.6079	20	565.50	0.3578
3	107.20	0.8311	12	350.30	0.5771	21	587.00	0.3193
4	136.85	0.8157	13	384.30	0.5579	22	611.50	0.2847
5	160.50	0.8041	14	398.25	0.5386	23	633.90	0.2578
6	203.35	0.7233	15	423.65	0.5156	24	656.45	0.1924
7	219.15	0.6964	16	446.60	0.5002	25	680.85	0.1193
8	241.20	0.6810	17	474.70	0.4617	26	702.75	0.0308
9	265.00	0.6579	18	509.40	0.4194			

K = 0.270E-03 D = 0.105E-03 S = 0.691E-02 V = 0.173E-01

I=2 KA=0.264E-03 DA=0.106E-03 SA=0.579E-02 VA=0.166E-01

DATA FROM RUN NUMBER IV-287, RIGHT LENS, BEAD NUMBER 3

R = 0.05095 C = 0.1028 F = 2.0190 TE = 581.9

N	T	X	N	T	X	N	T	X
1	48.25	0.8960	8	241.20	0.6388	15	423.65	0.4073
2	72.80	0.8489	9	265.00	0.6088	16	446.60	0.3816
3	107.20	0.7803	10	304.00	0.5916	17	474.70	0.3344
4	136.85	0.7631	11	325.40	0.5316	18	509.40	0.2615
5	160.50	0.7245	12	350.30	0.5102	19	535.55	0.2058
6	203.35	0.6817	13	384.30	0.4673	20	565.50	0.0900
7	219.15	0.6559	14	398.25	0.4287	21	587.00	0.0

K = 0.297E-03 D = 0.102E-03 S = 0.565E-02 V = 0.177E-01

I=1 KA=0.293E-03 DA=0.103E-03 SA=0.551E-02 VA=0.180E-01

DATA FROM RUN NUMBER IV-287, RIGHT LENS, BEAD NUMBER 4

R = 0.06310 C = 0.1028 F = 2.0190 TE = 544.8

N	T	X	N	T	X	N	T	X
1	48.25	0.8449	8	241.20	0.6060	15	423.65	0.3497
2	72.80	0.8345	9	265.00	0.5610	16	446.60	0.3186
3	107.20	0.7583	10	304.00	0.5471	17	474.70	0.2632
4	136.85	0.7341	11	325.40	0.4917	18	509.40	0.1731
5	160.50	0.7202	12	350.30	0.4779	19	535.55	0.0693
6	203.35	0.6614	13	384.30	0.4259	20	565.50	0.0
7	219.15	0.6233	14	398.25	0.3982			

K = 0.554E-03 D = 0.149E-03 S = 0.680E-02 V = 0.200E-01

I=1 KA=0.522E-03 DA=0.152E-03 SA=0.455E-02 VA=0.169E-01

DATA FROM RUN NUMBER IV-287, RIGHT LENS, BEAD NUMBER 5

R = 0.04830 C = 0.1028 F = 2.0190 TE = 487.1

N	T	X	N	T	X	N	T	X
1	48.25	0.8596	7	219.15	0.6108	13	384.30	0.3574
2	72.80	0.8189	8	241.20	0.5927	14	398.25	0.3167
3	107.20	0.7601	9	265.00	0.5474	15	423.65	0.2669
4	136.85	0.7193	10	304.00	0.4977	16	446.60	0.2217
5	160.50	0.7012	11	325.40	0.4479	17	474.70	0.0860
6	203.35	0.6696	12	350.30	0.4117	18	509.40	0.0

K = 0.294E-03 D = 0.103E-03 S = 0.593E-02 V = 0.199E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-287, RIGHT LENS, BEAD NUMBER 7

R = 0.04077 C = 0.1028 F = 2.0190 TE = 367.5

N	T	X	N	T	X	N	T	X
1	48.25	0.8092	6	203.35	0.5520	11	325.40	0.2304
2	72.80	0.7556	7	219.15	0.5198	12	350.30	0.1125
3	107.20	0.6913	8	241.20	0.4609	13	384.30	0.0
4	136.85	0.6699	9	265.00	0.4073			
5	160.50	0.6163	10	304.00	0.3323			

K = 0.360E-03 D = 0.104E-03 S = 0.876E-02 V = 0.296E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-287, RIGHT LENS, BEAD NUMBER 8

R = 0.04951 C = 0.1028 F = 2.0190 TE = 543.5

N	T	X	N	T	X	N	T	X
1	48.25	0.8694	8	241.20	0.6179	15	423.65	0.3575
2	72.80	0.8385	9	265.00	0.6046	16	446.60	0.3089
3	107.20	0.7988	10	304.00	0.5693	17	474.70	0.2604
4	136.85	0.7547	11	325.40	0.5119	18	509.40	0.1677
5	160.50	0.7105	12	350.30	0.4810	19	535.55	0.0662
6	203.35	0.6752	13	384.30	0.4413	20	565.50	0.0
7	219.15	0.6532	14	398.25	0.3928			

K = 0.278E-03 D = 0.107E-03 S = 0.551E-02 V = 0.180E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-287, RIGHT LENS, BEAD NUMBER 9

R = 0.04684 C = 0.1028 F = 2.0190 TE = 455.7

N	T	X	N	T	X	N	T	X
1	48.25	0.8583	7	219.15	0.6064	13	384.30	0.3032
2	72.80	0.8303	8	241.20	0.5598	14	398.25	0.2659
3	107.20	0.7464	9	265.00	0.5178	15	423.65	0.1866
4	136.85	0.7184	10	304.00	0.4758	16	446.60	0.0886
5	160.50	0.6997	11	325.40	0.4338	17	474.70	0.0
6	203.35	0.6298	12	350.30	0.3732			

K = 0.300E-03 D = 0.115E-03 S = 0.426E-02 V = 0.174E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-287, RIGHT LENS, BEAD NUMBER 10

R = 0.05775 C = 0.1028 F = 2.0190 TE = 702.8

N	T	X	N	T	X	N	T	X
1	48.25	0.8738	10	304.00	0.6733	19	535.55	0.4010
2	72.80	0.8663	11	325.40	0.6393	20	565.50	0.3707
3	107.20	0.8322	12	350.30	0.6053	21	587.00	0.3367
4	136.85	0.8171	13	384.30	0.5712	22	611.50	0.3026
5	160.50	0.8020	14	398.25	0.5485	23	633.90	0.2459
6	203.35	0.7641	15	423.65	0.5296	24	656.45	0.1891
7	219.15	0.7301	16	446.60	0.5107	25	680.85	0.1173
8	241.20	0.7074	17	474.70	0.4842	26	702.75	0.0303
9	265.00	0.6809	18	509.40	0.4502			

K = 0.225E-03 D = 0.120E-03 S = 0.118E-01 V = 0.226E-01

I=1 KA=0.221E-03 DA=0.121E-03 SA=0.763E-02 VA=0.186E-01

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 1

R = 0.04147 C = 0.2036 F = 0.9800 TE = 394.8

N	T	X	N	T	X	N	T	X
1	24.45	0.8947	6	169.60	0.6428	11	296.80	0.3865
2	48.55	0.8773	7	193.85	0.6037	12	328.80	0.3127
3	87.20	0.7948	8	218.25	0.5429	13	355.30	0.2215
4	119.40	0.7209	9	243.60	0.4951	14	381.20	0.1303
5	142.35	0.6992	10	271.50	0.4386	15	405.30	0.0

K = 0.133E-03 D = 0.550E-04 S = 0.226E-02 V = 0.137E-01

I=2 KA=0.129E-03 DA=0.560E-04 SA=0.562E-03 VA=0.750E-02

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 2
R = 0.04693   C = 0.2036   F = 0.9800   TE = 521.6

N	T	X	N	T	X	N	T	X
1	24.45	0.8861	8	218.25	0.6483	15	405.30	0.3874
2	48.55	0.8784	9	243.60	0.6138	16	430.30	0.3299
3	87.20	0.8286	10	271.50	0.5792	17	456.80	0.2532
4	119.40	0.7672	11	296.80	0.5409	18	481.10	0.1995
5	142.35	0.7595	12	328.80	0.4910	19	505.70	0.1074
6	169.60	0.7250	13	355.30	0.4450	20	530.05	0.0
7	193.85	0.6790	14	381.20	0.4143			

K = 0.126E-03 D = 0.521E-04 S = 0.737E-02 V = 0.208E-01
I=2 KA=0.119E-03 DA=0.536E-04 SA=0.233E-02 VA=0.125E-01

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 4
R = 0.05405   C = 0.2036   F = 0.9800   TE = 670.3

N	T	X	N	T	X	N	T	X
1	24.45	0.8960	10	271.50	0.6662	19	505.70	0.4064
2	48.55	0.8960	11	296.80	0.6328	20	530.05	0.3797
3	87.20	0.8727	12	328.80	0.6162	21	553.10	0.3364
4	119.40	0.8294	13	355.30	0.5729	22	576.10	0.2864
5	142.35	0.8060	14	381.20	0.5629	23	600.00	0.2365
6	169.60	0.7694	15	405.30	0.5396	24	620.20	0.1932
7	193.85	0.7561	16	430.30	0.5029	25	635.55	0.1399
8	218.25	0.7228	17	456.80	0.4796			
9	243.60	0.7061	18	481.10	0.4596			

K = 0.107E-03 D = 0.564E-04 S = 0.118E-01 V = 0.231E-01
I=3 KA=0.103E-03 DA=0.580E-04 SA=0.515E-02 VA=0.165E-01

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 5
R = 0.05080   C = 0.2036   F = 0.9800   TE = 585.0

N	T	X	N	T	X	N	T	X
1	24.45	0.8897	9	243.60	0.6664	17	456.80	0.3615
2	48.55	0.8897	10	271.50	0.6345	18	481.10	0.3474
3	87.20	0.8542	11	296.80	0.5990	19	505.70	0.2907
4	119.40	0.8188	12	328.80	0.5529	20	530.05	0.2410
5	142.35	0.7798	13	355.30	0.5175	21	553.10	0.1808
6	169.60	0.7231	14	381.20	0.4679	22	576.10	0.0780
7	193.85	0.7089	15	405.30	0.4324	23	600.00	0.0
8	218.25	0.6805	16	430.30	0.4112			

REGRESSION DID NOT CONVERGE
I=2 KA=0.119E-03 DA=0.556E-04 SA=0.289E-02 VA=0.127E-01

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 6

R = 0.04046 C = 0.2036 F = 0.9800 TE = 385.4

N	T	X	N	T	X	N	T	X
1	24.45	0.8678	6	169.60	0.6186	11	296.80	0.3560
2	48.55	0.8367	7	193.85	0.5741	12	328.80	0.2759
3	87.20	0.7566	8	218.25	0.5296	13	355.30	0.1869
4	119.40	0.7032	9	243.60	0.4806	14	381.20	0.0668
5	142.35	0.6720	10	271.50	0.4361	15	405.30	0.0

K = 0.166E-03 D = 0.496E-04 S = 0.532E-02 V = 0.211E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 7

R = 0.03170 C = 0.2036 F = 0.9800 TE = 256.0

N	T	X	N	T	X	N	T	X
1	24.45	0.8747	5	142.35	0.5453	9	243.60	0.1363
2	48.55	0.7952	6	169.60	0.4658	10	271.50	0.0
3	87.20	0.6930	7	193.85	0.3863			
4	119.40	0.6078	8	218.25	0.2954			

K = 0.135E-03 D = 0.502E-04 S = 0.201E-02 V = 0.169E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 8

R = 0.04300 C = 0.2036 F = 0.9800 TE = 437.8

N	T	X	N	T	X	N	T	X
1	24.45	0.8668	7	193.85	0.6030	13	355.30	0.2973
2	48.55	0.8501	8	218.25	0.5569	14	381.20	0.2512
3	87.20	0.7872	9	243.60	0.5276	15	405.30	0.1759
4	119.40	0.7328	10	271.50	0.4983	16	430.30	0.0837
5	142.35	0.6993	11	296.80	0.4313	17	456.80	0.0
6	169.60	0.6449	12	328.80	0.3769			

K = 0.178E-03 D = 0.476E-04 S = 0.549E-02 V = 0.198E-01

I=1 KA=0.166E-03 DA=0.486E-04 SA=0.180E-02 VA=0.118E-01

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 9

R = 0.03780 C = 0.2036 F = 0.9800 TE = 357.6

N	T	X	N	T	X	N	T	X
1	24.45	0.8622	6	169.60	0.5907	11	296.80	0.3144
2	48.55	0.8193	7	193.85	0.5335	12	328.80	0.1905
3	87.20	0.7431	8	218.25	0.4811	13	355.30	0.0476
4	119.40	0.6812	9	243.60	0.4478	14	381.20	0.0
5	142.35	0.6383	10	271.50	0.3858			

K = 0.172E-03 D = 0.460E-04 S = 0.527E-02 V = 0.219E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-295, RIGHT LENS, BEAD NUMBER 10

R = 0.02960 C = 0.2036 F = 0.9800 TE = 240.2

N	T	X	N	T	X	N	T	X
1	24.45	0.8516	4	119.40	0.6083	7	193.85	0.3163
2	48.55	0.7482	5	142.35	0.5049	8	218.25	0.1825
3	87.20	0.6448	6	169.60	0.4258	9	243.60	0.0

K = 0.168E-03 D = 0.430E-04 S = 0.600E-02 V = 0.316E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 1

R = 0.03543 C = 0.0521 F = 0.9800 TE = 462.0

N	T	X	N	T	X	N	T	X
1	44.95	0.8724	5	200.30	0.6695	9	388.80	0.2942
2	83.80	0.8471	6	243.55	0.6137	10	431.40	0.0
3	118.35	0.8014	7	292.30	0.5326			
4	159.80	0.7304	8	343.65	0.4362			

K = 0.244E-03 D = 0.162E-03 S = 0.413E-02 V = 0.243E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 3

R = 0.05870 C = 0.0521 F = 0.9800 TE = 927.9

N	T	X	N	T	X	N	T	X
1	44.95	0.8881	9	388.80	0.7901	17	763.10	0.4532
2	83.80	0.8881	10	431.40	0.7380	18	802.00	0.4012
3	118.35	0.8850	11	477.30	0.7104	19	840.45	0.3185
4	159.80	0.8850	12	530.70	0.6706	20	869.95	0.2572
5	200.30	0.8819	13	581.90	0.6308	21	894.25	0.1837
6	243.55	0.8360	14	631.15	0.5849	22	927.30	0.0
7	292.30	0.8238	15	675.25	0.5512			
8	343.65	0.8023	16	721.90	0.5022			

K = 0.169E-03 D = 0.420E-03 S = 0.190E-01 V = 0.316E-01

I=5 KA=0.165E-03 DA=0.456E-03 SA=0.232E-02 VA=0.129E-01

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 4

R = 0.04272 C = 0.0521 F = 0.9800 TE = 641.1

N	T	X	N	T	X	N	T	X
1	44.95	0.8835	6	243.55	0.7531	11	477.30	0.4544
2	83.80	0.8793	7	292.30	0.7026	12	530.70	0.3702
3	118.35	0.8540	8	343.65	0.6311	13	581.90	0.2398
4	159.80	0.7993	9	388.80	0.5932	14	631.15	0.0
5	200.30	0.7867	10	431.40	0.5259			

K = 0.199E-03 D = 0.198E-03 S = 0.930E-02 V = 0.291E-01

I=2 KA=0.189E-03 DA=0.211E-03 SA=0.308E-02 VA=0.185E-01

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 5

R = 0.05700 C = 0.0521 F = 0.9800 TE = 1005.0

N	T	X	N	T	X	N	T	X
1	44.95	0.8887	10	431.40	0.7279	19	840.45	0.3781
2	83.80	0.8887	11	477.30	0.7090	20	869.95	0.3498
3	118.35	0.8887	12	530.70	0.6681	21	894.25	0.3120
4	159.80	0.8855	13	581.90	0.6334	22	927.30	0.2584
5	200.30	0.8823	14	631.15	0.5956	23	951.35	0.2111
6	243.55	0.8508	15	675.25	0.5546	24	972.65	0.1576
7	292.30	0.8193	16	721.90	0.5137	25	998.15	0.0347
8	343.65	0.7878	17	763.10	0.4758			
9	388.80	0.7689	18	802.00	0.4286			

REGRESSION DID NCT CCNVERGE

I=5 KA=0.173E-03 DA=0.273E-03 SA=0.642E-03 VA=0.614E-02

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 7

R = 0.05675 C = 0.0521 F = 0.9800 TE = 984.1

N	T	X	N	T	X	N	T	X
1	44.95	0.8930	10	431.40	0.7284	19	840.45	0.3737
2	83.80	0.8962	11	477.30	0.7030	20	869.95	0.3420
3	118.35	0.8867	12	530.70	0.6650	21	894.25	0.3072
4	159.80	0.8899	13	581.90	0.6334	22	927.30	0.2280
5	200.30	0.8804	14	631.15	0.5985	23	951.35	0.1837
6	243.55	0.8550	15	675.25	0.5605	24	972.65	0.0855
7	292.30	0.8234	16	721.90	0.5130	25	998.15	0.0
8	343.65	0.7885	17	763.10	0.4718			
9	388.80	0.7569	18	802.00	0.4370			

K = 0.168E-03 D = 0.294E-03 S = 0.152E-01 V = 0.263E-01

I=5 KA=0.165E-03 DA=0.305E-03 SA=0.191E-02 VA=0.106E-01

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 8

R = 0.04591 C = 0.0521 F = 0.9800 TE = 727.9

N	T	X	N	T	X	N	T	X
1	44.95	0.8690	7	292.30	0.7516	13	581.90	0.4502
2	83.80	0.8807	8	343.65	0.7046	14	631.15	0.3601
3	118.35	0.8729	9	388.80	0.6654	15	675.25	0.2505
4	159.80	0.8690	10	431.40	0.6224	16	721.90	0.0705
5	200.30	0.8220	11	477.30	0.5793	17	763.10	0.0
6	243.55	0.7868	12	530.70	0.5128			

K = 0.163E-03 D = 0.254E-03 S = 0.155E-01 V = 0.333E-01

I=3 KA=0.159E-03 DA=0.265E-03 SA=0.172E-02 VA=0.125E-01

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 9

R = 0.05870 C = 0.0521 F = 0.9800 TE = 1014.9

N	T	X	N	T	X	N	T	X
1	44.95	0.8881	10	431.40	0.7717	19	840.45	0.4961
2	83.80	0.8942	11	477.30	0.7594	20	869.95	0.4624
3	118.35	0.8911	12	530.70	0.7349	21	894.25	0.4287
4	159.80	0.8942	13	581.90	0.7074	22	927.30	0.3613
5	200.30	0.8942	14	631.15	0.6676	23	951.35	0.3093
6	243.55	0.8789	15	675.25	0.6431	24	972.65	0.2695
7	292.30	0.8574	16	721.90	0.6063	25	998.15	0.1439
8	343.65	0.8268	17	763.10	0.5757			
9	388.80	0.7962	18	802.00	0.5451			

K = 0.131E-03 D = 0.678E-03 S = 0.244E-01 V = 0.333E-01

I=5 KA=0.130E-03 DA=0.734E-03 SA=0.456E-02 VA=0.164E-01

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 10

R = 0.04391 C = 0.0521 F = 0.9800 TE = 699.9

N	T	X	N	T	X	N	T	X
1	44.95	0.8921	7	292.30	0.7366	13	581.90	0.3888
2	83.80	0.8962	8	343.65	0.6916	14	631.15	0.2865
3	118.35	0.8921	9	388.80	0.6343	15	675.25	0.1637
4	159.80	0.8594	10	431.40	0.6057	16	721.90	0.0
5	200.30	0.8185	11	477.30	0.5566			
6	243.55	0.7775	12	530.70	0.4706			

K = 0.163E-03 D = 0.230E-03 S = 0.741E-02 V = 0.239E-01

I=3 KA=0.159E-03 DA=0.239E-03 SA=0.102E-02 VA=0.101E-01

DATA FROM RUN NUMBER IV-297, RIGHT LENS, BEAD NUMBER 11

R = 0.05255 C = 0.0521 F = 0.9800 TE = 979.3

N	T	X	N	T	X	N	T	X
1	44.95	0.8959	9	388.80	0.7523	17	763.10	0.4479
2	83.80	0.9061	10	431.40	0.7249	18	802.00	0.4035
3	118.35	0.8993	11	477.30	0.6941	19	840.45	0.3488
4	159.80	0.9027	12	530.70	0.6565	20	869.95	0.3146
5	200.30	0.8993	13	581.90	0.6257	21	894.25	0.2701
6	243.55	0.8583	14	631.15	0.5813	22	927.30	0.2052
7	292.30	0.8206	15	675.25	0.5471	23	951.35	0.1539
8	343.65	0.7864	16	721.90	0.5061	24	972.65	0.0

K = 0.155E-03 D = 0.239E-03 S = 0.105E-01 V = 0.224E-01

I=5 KA=0.153E-03 DA=0.245E-03 SA=0.780E-03 VA=0.698E-02

DATA FROM RUN NUMBER IV-299, RIGHT LENS, BEAD NUMBER 1

R = 0.04020 C = 0.4946 F = 0.9800 TE = 91.0

N	T	X	N	T	X	N	T	X
1	6.80	0.8331	6	40.60	0.4524	11	73.60	0.1881
2	12.90	0.7570	7	47.20	0.4031	12	78.55	0.1433
3	19.80	0.6495	8	53.20	0.3494	13	83.40	0.0896
4	26.45	0.5912	9	59.80	0.3001	14	87.95	0.0448
5	33.25	0.5285	10	68.00	0.2284	15	93.95	0.0

DATA FROM RUN NUMBER IV-299, RIGHT LENS, BEAD NUMBER 2
R = 0.03353 C = 0.4946 F = 0.9800 TE = 78.4

N	T	X	N	T	X	N	T	X
1	6.80	0.8107	5	33.25	0.4939	9	59.80	0.2255
2	12.90	0.7301	6	40.60	0.4134	10	68.00	0.1503
3	19.80	0.6442	7	47.20	0.3543	11	73.60	0.0859
4	26.45	0.5798	8	53.20	0.2953	12	78.55	0.0

DATA FROM RUN NUMBER IV-299, RIGHT LENS, BEAD NUMBER 3
R = 0.04400 C = 0.4946 F = 0.9800 TE = 106.1

N	T	X	N	T	X	N	T	X
1	6.80	0.8635	7	47.20	0.4583	13	83.40	0.2046
2	12.90	0.7939	8	53.20	0.4420	14	87.95	0.1637
3	19.80	0.7080	9	59.80	0.3642	15	93.95	0.1228
4	26.45	0.6548	10	68.00	0.3233	16	99.35	0.0655
5	33.25	0.5852	11	73.60	0.2946	17	104.90	0.0164
6	40.60	0.5320	12	78.55	0.2414	18	111.90	0.0

DATA FROM RUN NUMBER IV-299, RIGHT LENS, BEAD NUMBER 4
R = 0.04606 C = 0.4946 F = 0.9800 TE = 112.1

N	T	X	N	T	X	N	T	X
1	6.80	0.8679	8	53.20	0.4691	15	93.95	0.1642
2	12.90	0.8053	9	59.80	0.3948	16	99.35	0.1251
3	19.80	0.7271	10	68.00	0.3557	17	104.90	0.0586
4	26.45	0.6802	11	73.60	0.3206	18	111.90	0.0195
5	33.25	0.6216	12	78.55	0.2815	19	119.70	0.0
6	40.60	0.5512	13	83.40	0.2619			
7	47.20	0.5043	14	87.95	0.2033			

DATA FROM RUN NUMBER IV-299, RIGHT LENS, BEAD NUMBER 5
R = 0.03600 C = 0.4946 F = 0.9800 TE = 76.8

N	T	X	N	T	X	N	T	X
1	6.80	0.8003	5	33.25	0.4752	9	59.80	0.2151
2	12.90	0.7252	6	40.60	0.4001	10	68.00	0.1200
3	19.80	0.6352	7	47.20	0.3251	11	73.60	0.0600
4	26.45	0.5602	8	53.20	0.2751	12	78.55	0.0

DATA FROM RUN NUMBER IV-299, RIGHT LENS, BEAD NUMBER 6
R = 0.03890 C = 0.4946 F = 0.9800 TE = 58.1

N	T	X	N	T	X	N	T	X
1	6.80	0.8286	4	26.45	0.4907	7	47.20	0.1666
2	12.90	0.6897	5	33.25	0.3796	8	53.20	0.0972
3	19.80	0.5786	6	40.60	0.2731	9	59.80	0.0

DATA FROM RUN NUMBER IV-299, RIGHT LENS, BEAD NUMBER 7
R = 0.03754 C = 0.4946 F = 0.9800 TE = 90.5

N	T	X	N	T	X	N	T	X
1	6.80	0.8730	6	40.60	0.4892	11	73.60	0.2110
2	12.90	0.7674	7	47.20	0.4317	12	78.55	0.1871
3	19.80	0.7099	8	53.20	0.3741	13	83.40	0.1391
4	26.45	0.6331	9	59.80	0.3310	14	87.95	0.0719
5	33.25	0.5708	10	68.00	0.2350	15	93.95	0.0

DATA FROM RUN NUMBER IV-299, RIGHT LENS, BEAD NUMBER 8
R = 0.04386 C = 0.4946 F = 0.9800 TE = 107.3

N	T	X	N	T	X	N	T	X
1	6.80	0.8703	7	47.20	0.4803	13	83.40	0.2176
2	12.90	0.7800	8	53.20	0.4352	14	87.95	0.1930
3	19.80	0.7102	9	59.80	0.4023	15	93.95	0.1560
4	26.45	0.6528	10	68.00	0.3366	16	99.35	0.0903
5	33.25	0.5994	11	73.60	0.2833	17	104.90	0.0411
6	40.60	0.5296	12	78.55	0.2545	18	111.90	0.0

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 1
R = 0.04860 C = 0.4946 F = 0.9800 TE = 521.8

N	T	X	N	T	X	N	T	X
1	13.85	0.8929	10	191.50	0.7416	19	371.45	0.5498
2	35.85	0.8929	11	208.80	0.7269	20	393.80	0.5166
3	57.45	0.8929	12	227.25	0.7084	21	422.15	0.4502
4	82.40	0.8782	13	249.50	0.6937	22	441.05	0.3837
5	104.20	0.8450	14	271.20	0.6715	23	450.40	0.3653
6	125.20	0.8154	15	288.80	0.6568	24	464.50	0.3026
7	143.20	0.7970	16	309.00	0.6273	25	476.35	0.2546
8	160.25	0.7822	17	327.35	0.6088	26	491.60	0.1587
9	175.25	0.7638	18	352.85	0.5756			

K = 0.275E-04 D = 0.390E-04 S = 0.381E-01 V = 0.407E-01
I=3 KA=0.269E-04 DA=0.409E-04 SA=0.230E-01 VA=0.339E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 2
R = 0.03526 C = 0.4946 F = 0.9800 TE = 393.7

N	T	X	N	T	X	N	T	X
1	13.85	0.9054	8	160.25	0.6409	15	288.80	0.3967
2	35.85	0.8952	9	175.25	0.6104	16	309.00	0.3510
3	57.45	0.8189	10	191.50	0.5849	17	327.35	0.3103
4	82.40	0.7681	11	208.80	0.5493	18	352.85	0.2391
5	104.20	0.7223	12	227.25	0.5086	19	371.45	0.1679
6	125.20	0.7019	13	249.50	0.4680	20	393.80	0.0203
7	143.20	0.6612	14	271.20	0.4476	21	422.15	0.0

K = 0.490E-04 D = 0.146E-04 S = 0.500E-02 V = 0.167E-01
I=2 KA=0.479E-04 DA=0.147E-04 SA=0.262E-02 VA=0.128E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 3

R = 0.04522 C = 0.4946 F = 0.9800 TE = 510.3

N	T	X	N	T	X	N	T	X
1	13.85	0.8924	10	191.50	0.7099	19	371.45	0.4958
2	35.85	0.8963	11	208.80	0.6980	20	393.80	0.4601
3	57.45	0.8805	12	227.25	0.6742	21	422.15	0.3807
4	82.40	0.8408	13	249.50	0.6425	22	441.05	0.3094
5	104.20	0.8012	14	271.20	0.6306	23	450.40	0.2856
6	125.20	0.7932	15	288.80	0.6029	24	464.50	0.1983
7	143.20	0.7615	16	309.00	0.5791	25	476.35	0.1269
8	160.25	0.7575	17	327.35	0.5632	26	491.60	0.1705
9	175.25	0.7218	18	352.85	0.5156			

K = 0.321E-04 D = 0.257E-04 S = 0.397E-01 V = 0.415E-01

I=3 KA=0.312E-04 DA=0.265E-04 SA=0.279E-01 VA=0.373E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 4

R = 0.04123 C = 0.4946 F = 0.9800 TE = 453.9

N	T	X	N	T	X	N	T	X
1	13.85	0.9134	9	175.25	0.7220	17	327.35	0.5219
2	35.85	0.9047	10	191.50	0.6959	18	352.85	0.4697
3	57.45	0.8786	11	208.80	0.6872	19	371.45	0.4262
4	82.40	0.8351	12	227.25	0.6524	20	393.80	0.3479
5	104.20	0.8090	13	249.50	0.6176	21	422.15	0.2479
6	125.20	0.7872	14	271.20	0.6046	22	441.05	0.0870
7	143.20	0.7611	15	288.80	0.5828	23	450.40	0.0
8	160.25	0.7394	16	309.00	0.5567			

K = 0.254E-04 D = 0.281E-04 S = 0.300E-01 V = 0.387E-01

I=2 KA=0.250E-04 DA=0.286E-04 SA=0.234E-01 VA=0.361E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 5

R = 0.04163 C = 0.4946 F = 0.9800 TE = 453.4

N	T	X	N	T	X	N	T	X
1	13.85	0.8960	9	191.50	0.6763	17	327.35	0.4954
2	35.85	0.8960	10	175.25	0.6935	18	352.85	0.4523
3	57.45	0.8702	11	208.80	0.6462	19	371.45	0.3963
4	82.40	0.8271	12	227.25	0.6203	20	393.80	0.3403
5	104.20	0.7840	13	249.50	0.5988	21	422.15	0.2154
6	125.20	0.7538	14	271.20	0.5686	22	441.05	0.0603
7	143.20	0.7323	15	288.80	0.5557	23	450.40	0.0
8	160.25	0.7194	16	309.00	0.5169			

K = 0.320E-04 D = 0.237E-04 S = 0.342E-01 V = 0.413E-01

I=2 KA=0.312E-04 DA=0.242E-04 SA=0.258E-01 VA=0.378E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 6
R = 0.03038 C = 0.4946 F = 0.9800 TE = 309.7

N	T	X	N	T	X	N	T	X
1	13.85	0.8619	7	143.20	0.5727	13	249.50	0.2952
2	35.85	0.8147	8	160.25	0.5313	14	271.20	0.2302
3	57.45	0.7557	9	175.25	0.4782	15	288.80	0.1594
4	82.40	0.6848	10	191.50	0.4546	16	309.00	0.0
5	104.20	0.6612	11	208.80	0.4014			
6	125.20	0.5904	12	227.25	0.3483			

K = 0.125E-03 D = 0.116E-04 S = 0.385E-02 V = 0.172E-01
NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 7
R = 0.03686 C = 0.4946 F = 0.9800 TE = 222.8

N	T	X	N	T	X	N	T	X
1	13.85	0.8273	5	104.20	0.5499	9	175.25	0.2969
2	35.85	0.7446	6	125.20	0.4867	10	191.50	0.2385
3	57.45	0.6910	7	143.20	0.4283	11	208.80	0.1411
4	82.40	0.6035	8	160.25	0.3553	12	227.25	0.0

K = 0.709E-03 D = 0.224E-04 S = 0.291E-02 V = 0.180E-01
NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 8
R = 0.03824 C = 0.4946 F = 0.9800 TE = 458.0

N	T	X	N	T	X	N	T	X
1	13.85	0.8956	9	175.25	0.6659	17	327.35	0.4689
2	35.85	0.8909	10	191.50	0.6565	18	352.85	0.4033
3	57.45	0.8675	11	208.80	0.6096	19	371.45	0.3751
4	82.40	0.8440	12	227.25	0.6096	20	393.80	0.2907
5	104.20	0.7925	13	249.50	0.5721	21	422.15	0.1313
6	125.20	0.7503	14	271.20	0.5486	22	441.05	0.0
7	143.20	0.7127	15	288.80	0.5205			
8	160.25	0.6940	16	309.00	0.4783			

K = 0.348E-04 D = 0.169E-04 S = 0.211E-01 V = 0.333E-01
I=2 KA=0.332E-04 DA=0.175E-04 SA=0.142E-01 VA=0.289E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 9
R = 0.03745 C = 0.4946 F = 0.9800 TE = 437.7

N	T	X	N	T	X	N	T	X
1	13.85	0.8955	9	175.25	0.6561	17	327.35	0.3927
2	35.85	0.8955	10	191.50	0.6273	18	352.85	0.3352
3	57.45	0.8189	11	208.80	0.5890	19	371.45	0.2969
4	82.40	0.7950	12	227.25	0.5555	20	393.80	0.2299
5	104.20	0.7614	13	249.50	0.5268	21	422.15	0.0910
6	125.20	0.7231	14	271.20	0.4885	22	441.05	0.0
7	143.20	0.6992	15	288.80	0.4645			
8	160.25	0.6704	16	309.00	0.4310			

K = 0.458E-04 D = 0.152E-04 S = 0.956E-02 V = 0.224E-01
I=2 KA=0.438E-04 DA=0.155E-04 SA=0.507E-02 VA=0.173E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 10
R = 0.03456 C = 0.4946 F = 0.9800 TE = 208.2

N	T	X	N	T	X	N	T	X
1	13.85	0.8510	5	104.20	0.5241	9	175.25	0.2491
2	35.85	0.7369	6	125.20	0.4566	10	191.50	0.1712
3	57.45	0.6642	7	143.20	0.3944	11	208.80	0.0
4	82.40	0.5812	8	160.25	0.3113			

K = 0.561E-03 D = 0.212E-04 S = 0.107E-02 V = 0.116E-01
NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 11
R = 0.04312 C = 0.4946 F = 0.9800 TE = 473.6

N	T	X	N	T	X	N	T	X
1	13.85	0.8774	10	191.50	0.7111	19	371.45	0.4657
2	35.85	0.8858	11	208.80	0.6903	20	393.80	0.4450
3	57.45	0.8816	12	227.25	0.6737	21	422.15	0.3368
4	82.40	0.8359	13	249.50	0.6404	22	441.05	0.2537
5	104.20	0.8151	14	271.20	0.6238	23	450.40	0.1996
6	125.20	0.7818	15	288.80	0.6155	24	464.50	0.0499
7	143.20	0.7568	16	309.00	0.5739	25	476.35	0.0
8	160.25	0.7402	17	327.35	0.5489			
9	175.25	0.7153	18	352.85	0.5073			

K = 0.259E-04 D = 0.304E-04 S = 0.533E-01 V = 0.492E-01
I=3 KA=0.251E-04 DA=0.317E-04 SA=0.358E-01 VA=0.434E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 12
R = 0.04293 C = 0.4946 F = 0.9800 TE = 491.5

N	T	X	N	T	X	N	T	X
1	13.85	0.8899	10	191.50	0.7060	19	371.45	0.4721
2	35.85	0.8940	11	208.80	0.6935	20	393.80	0.4428
3	57.45	0.8857	12	227.25	0.6643	21	422.15	0.3468
4	82.40	0.8439	13	249.50	0.6308	22	441.05	0.2799
5	104.20	0.7980	14	271.20	0.6100	23	450.40	0.2423
6	125.20	0.7729	15	288.80	0.5932	24	464.50	0.0836
7	143.20	0.7478	16	309.00	0.5682	25	476.35	0.0
8	160.25	0.7311	17	327.35	0.5515			
9	175.25	0.7102	18	352.85	0.5097			

K = 0.295E-04 D = 0.244E-04 S = 0.480E-01 V = 0.467E-01
I=2 KA=0.287E-04 DA=0.251E-04 SA=0.371E-01 VA=0.431E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 13

R = 0.04532 C = 0.4946 F = 0.9800 TE = 495.2

N	T	X	N	T	X	N	T	X
1	13.85	0.8785	10	191.50	0.7202	19	371.45	0.4986
2	35.85	0.8785	11	208.80	0.6925	20	393.80	0.4591
3	57.45	0.8785	12	227.25	0.6728	21	422.15	0.3878
4	82.40	0.8588	13	249.50	0.6569	22	441.05	0.2849
5	104.20	0.8152	14	271.20	0.6332	23	450.40	0.2414
6	125.20	0.7954	15	288.80	0.6174	24	464.50	0.1266
7	143.20	0.7598	16	309.00	0.5857	25	476.35	0.0
8	160.25	0.7479	17	327.35	0.5620			
9	175.25	0.7440	18	352.85	0.5224			

K = 0.292E-04 D = 0.299E-04 S = 0.490E-01 V = 0.472E-01

I=3 KA=0.280E-04 DA=0.317E-04 SA=0.306E-01 VA=0.401E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 14

R = 0.03048 C = 0.4946 F = 0.9800 TE = 336.5

N	T	X	N	T	X	N	T	X
1	13.85	0.8768	7	143.20	0.5943	13	249.50	0.3531
2	35.85	0.8709	8	160.25	0.5767	14	271.20	0.2942
3	57.45	0.7885	9	175.25	0.5355	15	288.80	0.2530
4	82.40	0.7238	10	191.50	0.5061	16	309.00	0.2177
5	104.20	0.6944	11	208.80	0.4708	17	327.35	0.0941
6	125.20	0.6355	12	227.25	0.4119	18	352.85	0.0

K = 0.617E-04 D = 0.117E-04 S = 0.547E-02 V = 0.191E-01

I=2 KA=0.576E-04 DA=0.119E-04 SA=0.206E-02 VA=0.126E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 15

R = 0.03427 C = 0.4946 F = 0.9800 TE = 393.4

N	T	X	N	T	X	N	T	X
1	13.85	0.8898	8	160.25	0.6334	15	288.80	0.3926
2	35.85	0.8846	9	175.25	0.6124	16	309.00	0.3507
3	57.45	0.8270	10	191.50	0.5810	17	327.35	0.3088
4	82.40	0.7799	11	208.80	0.5391	18	352.85	0.2408
5	104.20	0.7328	12	227.25	0.5130	19	371.45	0.1570
6	125.20	0.7014	13	249.50	0.4711	20	393.80	0.0157
7	143.20	0.6752	14	271.20	0.4292	21	422.15	0.0

K = 0.463E-04 D = 0.138E-04 S = 0.537E-02 V = 0.173E-01

I=2 KA=0.445E-04 DA=0.139E-04 SA=0.111E-02 VA=0.831E-02

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 16

R = 0.03326 C = 0.4946 F = 0.9800 TE = 359.7

N	T	X	N	T	X	N	T	X
1	13.85	0.8734	8	160.25	0.5822	15	288.80	0.3181
2	35.85	0.8626	9	175.25	0.5445	16	309.00	0.2642
3	57.45	0.7817	10	191.50	0.5337	17	327.35	0.2156
4	82.40	0.7440	11	208.80	0.4852	18	352.85	0.0701
5	104.20	0.7008	12	227.25	0.4421	19	371.45	0.0
6	125.20	0.6523	13	249.50	0.4097			
7	143.20	0.6200	14	271.20	0.3396			

K = 0.761E-04 D = 0.128E-04 S = 0.543E-02 V = 0.184E-01

I=2 KA=0.685E-04 DA=0.131E-04 SA=0.172E-02 VA=0.111E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 17

R = 0.03625 C = 0.4946 F = 0.9800 TE = 138.2

N	T	X	N	T	X	N	T	X
1	13.85	0.8657	5	104.20	0.5837	9	175.25	0.3265
2	35.85	0.7816	6	125.20	0.5046	10	191.50	0.2473
3	57.45	0.7272	7	143.20	0.4403	11	208.80	0.1385
4	82.40	0.6381	8	160.25	0.3858	12	227.25	0.0

K = 0.137E-03 D = 0.250E-04 S = 0.300E-02 V = 0.183E-01

NO POINTS COULD BE DROPPED

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 18

R = 0.04073 C = 0.4946 F = 0.9800 TE = 461.7

N	T	X	N	T	X	N	T	X
1	13.85	0.8893	9	175.25	0.7088	17	327.35	0.5063
2	35.85	0.8849	10	191.50	0.6824	18	352.85	0.4711
3	57.45	0.8717	11	208.80	0.6604	19	371.45	0.4402
4	82.40	0.8365	12	227.25	0.6252	20	393.80	0.3874
5	104.20	0.7880	13	249.50	0.6119	21	422.15	0.2686
6	125.20	0.7704	14	271.20	0.5811	22	441.05	0.1321
7	143.20	0.7440	15	288.80	0.5679	23	450.40	0.0704
8	160.25	0.7132	16	309.00	0.5327	24	464.50	0.0

K = 0.284E-04 D = 0.233E-04 S = 0.425E-01 V = 0.450E-01

I=3 KA=0.273E-04 DA=0.243E-04 SA=0.296E-01 VA=0.405E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 19

R = 0.04263   C = 0.4946   F = 0.9800   TE = 478.5

N	T	X	N	T	X	N	T	X
1	13.85	0.8919	10	191.50	0.6984	19	371.45	0.4586
2	35.85	0.8835	11	208.80	0.6815	20	393.80	0.4165
3	57.45	0.8793	12	227.25	0.6395	21	422.15	0.3197
4	82.40	0.8498	13	249.50	0.6311	22	441.05	0.2188
5	104.20	0.8078	14	271.20	0.5890	23	450.40	0.1767
6	125.20	0.7783	15	288.80	0.5806	24	464.50	0.0926
7	143.20	0.7531	16	309.00	0.5469	25	476.35	0.0
8	160.25	0.7362	17	327.35	0.5385			
9	175.25	0.7194	18	352.85	0.5048			

K = 0.292E-04 D = 0.252E-04 S = 0.368E-01 V = 0.409E-01

I=3 KA=0.282E-04 DA=0.261E-04 SA=0.237E-01 VA=0.353E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 20

R = 0.03994   C = 0.4946   F = 0.9800   TE = 455.6

N	T	X	N	T	X	N	T	X
1	13.85	0.8936	9	175.25	0.7005	17	327.35	0.4850
2	35.85	0.8981	10	191.50	0.6691	18	352.85	0.4356
3	57.45	0.8532	11	208.80	0.6511	19	371.45	0.3951
4	82.40	0.8127	12	227.25	0.6242	20	393.80	0.3233
5	104.20	0.7903	13	249.50	0.5882	21	422.15	0.2245
6	125.20	0.7634	14	271.20	0.5703	22	441.05	0.1257
7	143.20	0.7454	15	288.80	0.5343	23	450.40	0.0449
8	160.25	0.7140	16	309.00	0.5029	24	464.50	0.0

K = 0.305E-04 D = 0.211E-04 S = 0.248E-01 V = 0.344E-01

I=2 KA=0.299E-04 DA=0.215E-04 SA=0.165E-01 VA=0.294E-01

DATA FROM RUN NUMBER IV-300, RIGHT LENS, BEAD NUMBER 21

R = 0.04163   C = 0.4946   F = 0.9800   TE = 475.8

N	T	X	N	T	X	N	T	X
1	13.85	0.8745	10	191.50	0.6978	19	371.45	0.4566
2	35.85	0.8745	11	208.80	0.6677	20	393.80	0.3877
3	57.45	0.8745	12	227.25	0.6462	21	422.15	0.3102
4	82.40	0.8486	13	249.50	0.6375	22	441.05	0.2240
5	104.20	0.7883	14	271.20	0.6031	23	450.40	0.1723
6	125.20	0.7754	15	288.80	0.5945	24	464.50	0.0732
7	143.20	0.7625	16	309.00	0.5600	25	476.35	0.0
8	160.25	0.7452	17	327.35	0.5471			
9	175.25	0.7022	18	352.85	0.5083			

K = 0.271E-04 D = 0.249E-04 S = 0.458E-01 V = 0.456E-01

I=3 KA=0.262E-04 DA=0.260E-04 SA=0.273E-01 VA=0.379E-01

APPENDIX C. REGRESSION ANALYSIS OF RUN 193[a]

Bead number	Film coefficient sq.cm./sec. x 10^3	Diffusion coefficient sq.cm./sec. x 10^4	Error sum of squares x 10^2
2	0.906	0.202	0.225
5	0.268	0.213	0.608
6	0.438	0.201	0.515
7	0.484	0.197	0.404
8	0.176	0.220	0.739
9	0.225	0.196	0.655
15	0.693	0.199	0.199
16	0.636	0.206	0.297
17	0.258	0.195	0.367
18	0.306	0.191	0.303
Average	0.439	0.202	0.431

[a]Data with a dimensionless core diameter of less than 0.311 was removed for this regression analysis.