DEPENDENCE OF THE ACCURACY OF THE BORN INVERSION ON NOISE AND BANDWIDTH

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ABSTRACT

The Born Inversions are a set of techniques for reconstructing the shape of a flaw based on the scattering of ultrasound from the flaw. One technique is the one-dimensional Born Inversion, which estimates the radius of a flaw in one direction based on one pulse-echo (i.e., backscattering) measurement in that direction. The robustness of this technique with respect to limitations on the available bandwidth and with respect to the presence of noise in the data have been investigated. The Born Inversion requires a bandwidth sufficient to include at least the range $0.5 < ka < 2$ to give accurate estimates. The estimates continue to be accurate even when the amount of noise energy is comparable to the amount of flaw signal energy in the measurement.

INTRODUCTION

The requirements on the design of structures and machinery are coming more and more to stress maximum performance combined with minimum weight, minimum use of expensive materials and minimum life cycle costs. It therefore becomes more and more important to be able to predict the remaining lifetime of a component. Accurate nondestructive measurements of flaw properties, combined with the discipline of fracture mechanics, can provide these predictions.

Inversion algorithms may be classified in terms of how complete a description of a flaw they attempt to provide. Generally, an algorithm which can provide a detailed description of a flaw will require a large amount of measured data in order for it to work successfully. This data may include measurements from a wide range of angles of inspection about the flaw and/or a wide range of frequencies. Imaging is an example of a technique which requires measurements over a substantial range of angles. Obtaining large amounts of measured data from a flaw will be time-consuming and may require specialized equipment, such as an array transducer or an automated manipulator arm. In some cases, the data may not be available due to geometric limitations on what angles of inspection are possible or limitations on the available range of frequencies due to grain scattering or ultrasonic attenuation. These limitations will restrict the use of such data-intensive inversion algorithms to special inspection problems where the expense is justified by the results which the algorithm provide.

On the other hand, if an inversion algorithm determines only few properties of a flaw, then it may need only a limited amount of data to do so. This will make the algorithm much easier to implement in practical testing situations. Therefore, there is a need for inversion algorithms which:

1. Measure flaw properties that are valuable in predicting the remaining lifetime of a part, and

2. Require only a minimal amount of data to do so.

The properties of a flaw which are important in predicting the remaining lifetime of a part are, in many cases, the overall dimensions of the flaw. For example, the growth of a crack in a metal is largely determined by the cross-sectional dimensions of the crack perpendicular to the direction of the applied stress in the part. It is fortunate that the overall dimensions of the flaw are the properties which are of importance because high frequencies are not required to measure them and high frequencies are the most difficult to generate and propagate through materials. For measuring the overall dimensions of a flaw, the best frequencies are those for which the wavelength of sound is comparable to the dimensions of the flaw.

In ultrasonic nondestructive testing, the minimum amount of data that will be available from a flaw is the reflection of a single ultrasonic pulse from the flaw. This is the data that would be available, for example, if a large part were being scanned by a single transducer operated in the pulse-echo mode. The pulse contains frequency components over, typically, two octaves.

The Born inversion[1,2] is a technique which determines one dimension of a flaw by the use of one pulse-echo waveform reflected from the flaw. The dimension which it measures is the radius of the flaw in the direction of inspection. It does this without making use of the absolute amplitude of the pulse, as is done in conventional ultrasonic testing. Rather, it makes use of the shape of the reflected pulse. Another way of saying this is that it makes use of the relative amplitude and phase of the various frequency components which make up the reflected pulse. This lack of dependence on the absolute amplitude of the pulse makes the technique independent of several factors which disrupt conventional ultrasonic measurements, including coupling, variable attenuation and instrumental variations. This allows the technique to provide an absolute measurement of flaw size, that is, one which does not need a reference sample on which to calibrate.

The Born inversion was first derived for the case of weak scattering flaws. These are flaws whose density and sound velocity differ only a little from those of the host material in which they are located. Cracks, voids and some inclu-

sions do not fall in this class. However, some inclusions, such as Si in Si$_3$N$_4$ are weak scatterers.

In this weak scattering limit, the scattering of ultrasound from the flaw can be calculated by an approximate method known as the Born approximation. The scattering from the flaw, expressed as a function of frequency and angle of interrogation, is proportional to the Fourier transform of a quality called the characteristic function of the flaw. The characteristic function is defined as follows:

$$C(\vec{r}) = 1 \quad \text{inside the flaw}$$
$$= 0 \quad \text{outside the flaw}$$

If this function could be calculated, one would then know the location of the boundaries of the flaw, and therefore know its size, shape and orientation. The Born approximation scattering calculation shows that in the weak scattering regime the characteristic function can be obtained by performing a three-dimensional Fourier transform of the scattering amplitude measured as a function of frequency and angle. This procedure is called the three-dimensional Born inversion.

This procedure still requires the measurement and processing of a large amount of data. However, there is a simplification which can be made which produces a one-dimensional Born inversion-algorithm. Consider the case of a spherical scatterer. The scattering amplitude is the same in all directions and the three-dimensional Fourier transform reduces to a one-dimensional Fourier sine transform

$$C(r) = \frac{1}{r} \int_{-\infty}^{\infty} \frac{A(\omega)}{\omega} \sin \omega r \, d\omega \qquad (1)$$

The lower curve in Fig. 1 shows a one-dimensional cross section through the characteristic function of a flaw, as it might be reconstructed from scattering data with limited bandwidth. An estimate of the radius of the flaw can be obtained from the characteristic function by any of several estimators, such as the radius at which the characteristic function drops to 1/2 of its peak value.

In order to understand how the Born inversion works for flaws other than weakly scattering spheres, it is useful to recast Eq. (1) into the time domain. The result is

$$C(r) = C(vT) = \frac{1}{T} \int_{-T}^{T} a(t) \, dt \qquad (2)$$

where v is the sound velocity of the host material, and a(t) is the response of the flaw to an incident δ-function impulse. a(t) is the Fourier transform of the scattering amplitude A(ω). In what follows, "a" will also be used to indicate the radius of a flaw. Confusion can be avoided by noting that the impulse response will always appear with an argument: a(t), while the radius will appear without an argument. The upper curve in Fig. 1 shows a sketch of the impulse response for a weakly scattering flaw. It consists of (from left to right) a front surface echo, a constant region due scattering from the body of the

IMPULSE RESPONSE R(t)

CHARACTERISTIC FUNCTION C(r)

RADIUS

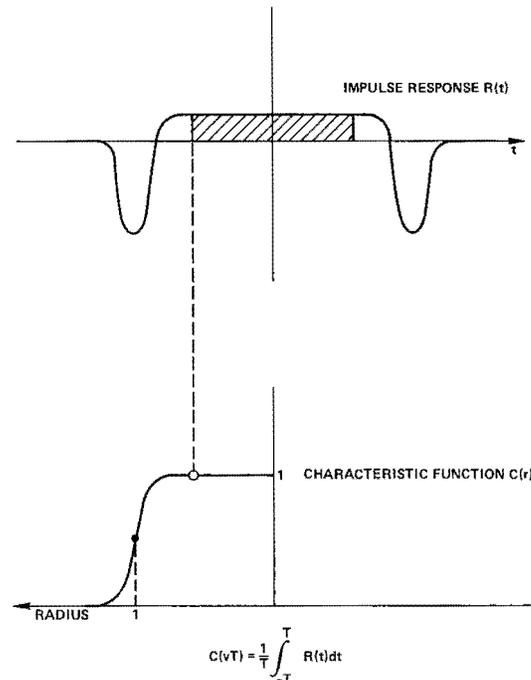$$C(vT) = \frac{1}{T} \int_{-T}^{T} R(t)dt$$

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Fig. 1   a) Impulse response of a weak scattering flaw, b) characteristic function derived by Born Inversion.

flaw and a rear surface echo. Equation (2) has a very simple interpretation in terms of this figure. Each point on the characteristic function is calculated by integrating over a central portion of a(t). This portion is shown cross-hatched in Fig. 1. This is called the "expanding window" method of calculating the characteristic function, because the characteristic function can be generated by performing the integration (2) over an ever expanding window of which the hatched area is one example.

Because the impulse response a(t) of a weak scatterer can be calculated by a simple construction based on its shape, it is easy to show that for any ellipsoidal weak scattering flaw, the characteristic function has the same shape as it does for a sphere and gives a measure of the radius of the ellipsoid along the direction of inspection. If a number of these one-dimensional Born inversions are performed from a variety of angles, the shape of the flaw can be traced out.

Thus far, we have considered only weak scatterers. The Born inversion has been shown to work on strong scatterers as well. The reasons why it does can be understood by noting that the Born inversion is essentially measuring the distance from the center of the flaw to the front surface tangent plane in the direction of observation. Note that the rapid fall-off of the characteristic function at r = 1 is primarily due to the sharp front surface echo. Strong scatterers have even more pronounced front surface echoes than do weak scatterers and it is found that their characteristic functions are enough like Fig. 1 to provide good radius estimates.

BANDWIDTH REQUIREMENTS

The Born inversion was found to require a bandwidth of

in order to give radius estimates accurate to within 20%. If insufficient low frequency data is available, then an underestimate results and if insufficient high frequency data is available, an overestimate results.

Figure 2 shows radius estimates obtained using the calcualted scattering amplitude for a spherical void when only limited bandwidth is available. The dashed curve shows the effect of insufficient low ka data. Underestimates of >20% occurs when $ka_{min} > 0.5$. The solid curve shows the effect of insufficient high ka data. Overestimates of >20% occur when $ka_{max} < 2$.
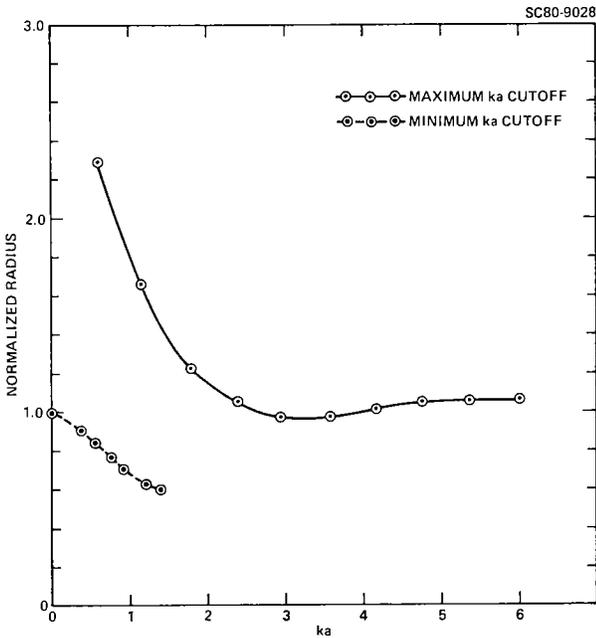

Fig. 2 Effect of limited bandwidth on the accuracy of the Born Inversion.

Figure 3 shows the effect of limiting both low and high frequency bandwidths. The figure shows the estimated radius â divided by the true radius a vs the average wavenumber k of a transducer multiplied by the flaw radius a. Each curve is for a transducer of a different relative bandwidth, expressed in terms of the ratio of the maximum ka of the transducer to the minimum ka.

Note that for the 6:1 transducer, measurements will be accurate to within 20% for a 1.7:1 range of flaw sizes, while for the 10:1 transducer, the range of flaw sizes is about 2.5:1. A good broadband commercial transducer might have a 10:1 range of usable k.

SIGNAL TO NOISE REQUIREMENTS

The sensitivity of the Born Inversion to the presence of noise in the data has been investigated by creating simulated experimental waveforms. These waveforms consist of the calculated scattering from a spherical void, with simulated noise added. The noise is selected to model the scattering of ultrasound by grains in the host material. Grain scattering has a power spectrum proportional to f^4 (f = frequency).
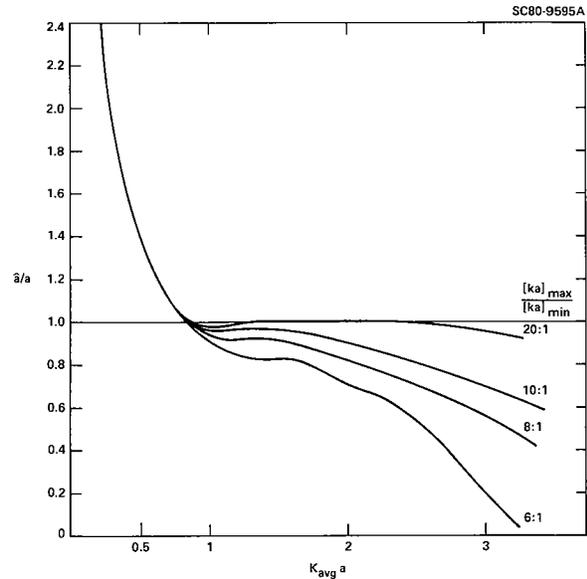

Fig. 3 Accuracy of Born Inversion vs transducer center frequency and bandwidth.

The simulated waveforms are calculated in the frequency domain, then multiplied by the measured spectrum of an ultrasonic transducer and finally put into the time domain.

$$a_\chi(t) = \mathscr{F}^{-1}\{X(f) \cdot [A(f) + N_g(f)]\}$$

where $a_\chi(t)$ is the simulated flaw waveform, $A(f)$ is the scattering amplitude of the flaw, $N_g(f)$ is grain scattering noise, $X(f)$ is the transducer spectrum and $\mathscr{F}^{-1}$ refers to a Fourier transform from frequency to time domains. The waveforms are then input to the Born inversion algorithm as experimental signals would be.

The noise waveform has the form

$$N_g(f) = (\frac{f}{f_0})^4 N(f)$$

where f_0 is a reference frequency and $N(f)$ is a complex valued white Gaussian noise waveform consisting of samples of a Gaussian random variable for which

$$E \, \nu(f) \, \nu(f')^* = \sigma_f^2 \, \delta_{ff'}$$

f_0 is chosen arbitrarily to be the maximum frequency of the sampled waveform. The mean energy in N_g is therefore

$$U_g = \Delta f \sum_{f=-f_0}^{f_0} \sigma_f^2 \, (\frac{f}{f_0})^4$$

$$= \frac{2}{5} f_0 \, \sigma_f^2$$

where Δf is the sample interval of N_g.

The signal to noise ratio of the simulated experimental waveforms is defined to be

$$(S/N)^2 = \frac{\text{energy in flaw signal}}{\text{mean energy in noise signal}}$$

$$= \frac{\Delta f \ \Sigma |\dot{A}(f)|^2}{U_g}$$

Figure 4 shows simulated and experimental flaw waveforms. The upper curve is an experimental waveform recorded for an 800 μm diameter spherical void. The middle and lower curves are simulated flaw waveforms. The middle curve contains no noise and the lower curve contains enough noise to produce a 0 dB signal to noise ratio. The similarity between the simulated and the measured waveforms is very good.
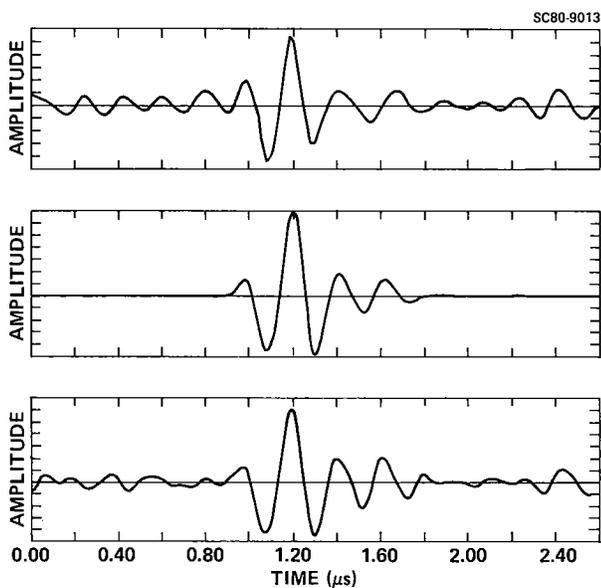

Fig. 4 Experimental and simulated flaw waveforms including grain scattering noise.

Figure 5 shows the radius estimates obtained for ensembles of noised up signals at various signal-to-noise ratios. The flaw has a diameter of 800 μm. For large signal-to-noise, the algorithm correctly estimates the radius to be 400 μm. The dashed curve shows the mean radius estimate for an ensemble of signals at each signal-to-noise ratio. The solid curves show the 95% confidence levels for the ensemble. As the signal-to-noise ratio decreases, the uncertainty of the estimates increases and the mean of the estimates eventually becomes inaccurate too. However, the 95% confidence level is within 20% of the correct answer down to a signal-to-noise ratio of 0 dB. The upper and lower curves in Fig. 4 show flaw signals in the presence of this level of grain noise.

DESCRIPTION OF THE ALGORITHM

An algorithm has been developed to perform the one-dimensional Born Inversion reliably on experimental data. The algorithm is written in Rockwell's ISP signal processing language and is executed on a mini- or a microcomputer. Figure 6 is a block diagram of the algorithm.
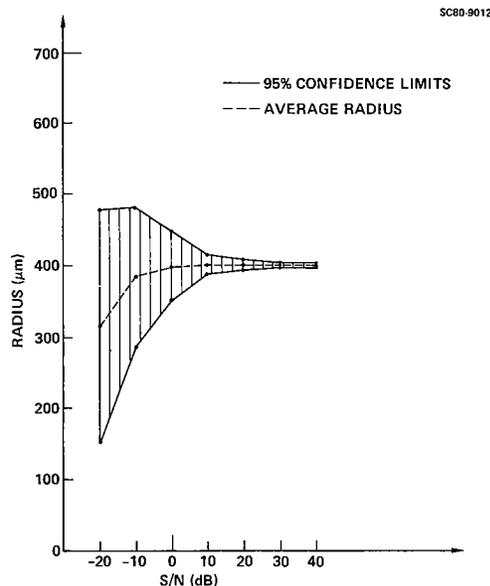

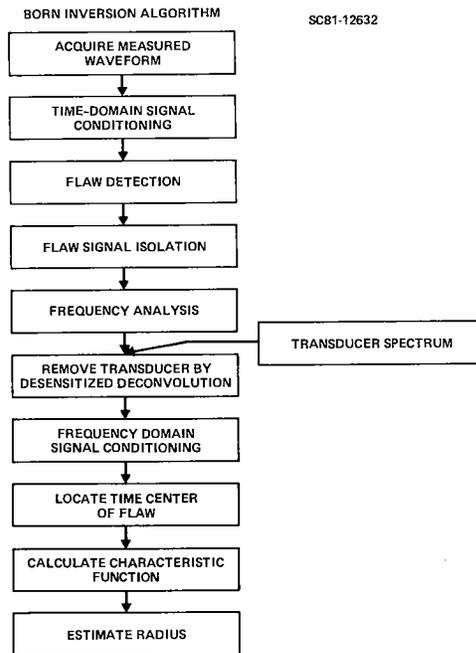Fig. 5 Radius estimates for ensembles of noisy flaw waveforms vs signal-to-noise ratio.

Fig. 6 Block diagram of Born Inversion algorithm.

The implementation of the Born inversion in a working algorithm is most readily done in the frequency domain using Eq. (1). The reason why a frequency domain implementation is better than a time domain implementation is because a measured ultrasonic signal consists of the impulse response of the flaw convolved with the impulse response of

the transducer and other measurement system components (i.e., the incident pulse). This transducer system response must be removed in order to isolate the flaw scattering data for use in the inversion. In particular, the incident pulse has very little energy at low frequencies and at high frequencies. Because there is some noise present in the measurements, a direct time-domain deconvolution of the measurement system response out of the measured signal is vulnerable to instabilities. Instead, it is better to do the signal processing in the frequency domain, where it is possible to make specific corrections in order to desensitize the results to the presence of noise.

After acquisition of the measured waveform, time-domain signal conditioning operations are performed. The primary signal conditioning operation is the subtraction of a measured signal form a flaw free region of the same or a similar part. This subtraction can remove, for example, the tail of a front surface echo or the recovery of the receiver from overload. Due to mechanical and geometrical uncertainties in the measurement process, subtraction usually does more harm than good at high frequencies, so that low passing of the subtractive reference is advisable.

Detection of a flaw signal in the received waveform is usually done simply by noting points at which the signal rises significantly above the background level. Matched filtering of the measured waveform with respect to a prototype flaw signal has been explored. However, because flaws covering a range of sizes and shapes are being searched for and because changes in the size and shape of a flaw cause large changes in the frequency spectrum of the scattering from the flaw, matched filtering did not produce a significant improvement in detectability unless a specific size and shape flaw was being searched for.

For further analysis, the portion of the received waveform containing the flaw signal is now isolated. This is done by multiplying the waveform by a window function of length comparable to the length of the impulse response of the flaw. A shaped window such as a Hanning or Kaiser-Bessel window is often a good choice. However, the convolution of such windows with the spectrum of the flaw signal distorts the important low frequency portion of the spectrum. Therefore a rectangular window is often used.

After frequency analysis, the properties of the transducer and other measurement system components are deconvolved out by division of the flaw spectrum by the spectrum of the measurement system (obtained from the measurement of the reflection from a flat surfaced target). This deconvolution is done in a desensitized manner in order to avoid noise-dominated behavior at high and low frequencies where the transducer has insufficient energy. The algorithm used is the following:

$$\hat{A}(\omega) = \frac{A_x(\omega)/X(\omega)}{1 + C\left|\frac{X_{pk}}{X(\omega)}\right|^2}$$

where $\hat{A}$ is the estimated flaw spectrum, A_x is the spectrum of the measured signal, X is the trans-

ducer (and measurement system) spectrum, X_{pk} is the peak value of the transducer spectrum and C is a factor (typical value = 0.01) which determines the degree of desensitization. Note that at any frequency where the transducer has very little energy, $\hat{A}$ is forced to zero to avoid wildly fluctuating results. If additional bandwidth is required, data from several transducers can be simultaneously deconvolved in a noise resistant manner. This technique is described elsewhere in this proceedings.[3]

Next, frequency domain signal conditioning can be performed. In particular, the low frequency portion of the frequency spectrum which was forced to zero by the deconvolution can be fit to f^2 to match the known frequency dependence of the scattering from small flaws.

The location of the center of the flaw in time must now be determined before proceeding with the inversion. This is best done by measuring the phase of the low frequency (ka < 0.5) portion of the spectrum. The slope of the phase at low frequencies is a measure of the time location of the center of the flaw with respect to the time coordinate system used. Several methods have been developed for estimating this time center, and are described elsewhere in these proceedings.[4] In some cases, there may be no data available in the measured signal at frequencies below ka = 0.5. (Note that the accuracy of the Born Inversion will begin to deteriorate if the minimum ka is much above 0.5). It has been found to be possible in some cases to empirically find the time center even when the low frequency data is not present. One such technique is to select that time center which causes the characteristic function to have zero slope at zero radius. Another is to measure the estimated flaw radius at various assumed flaw center positions τ and select the center position by reference to the radius vs τ curve.

The characteristic function of the flaw is then calcualted by Eq. (1) and the flaw radius is estimated from it. Several estimators of flaw radius were tried. The one which was found to work best is

$$\hat{a} = \frac{\text{area under } C(r)}{\text{peak value of } C(r)}$$

This estimator is insensitive to small errors in the determination of the flaw center and performs remarkably well for inclusions which have very long and complicated characteristic functions due to the internal sound paths within the inclusions.

The results which are presented below were obtained using this algorithm.

EXPERIMENTAL RESULTS

The results of the application of the Born inversion to a variety of flaws is shown in Table I. The first group of flaws are in Ti-6Al-4V and the second group are in hot-pressed silicon nitride. The table lists, in addition to the true and estimated radii of the flaws in the direction of observation, the range of ka (i.e., frequency) available in the experiment and an

Flaws in Ti-6Al-4V

Flaw	S/N (dB)	ka_{min}	ka_{max}	True Radius (μm)	Estimated Radius (μm)	Comments
1. Void Sphere	10	.2	3	400	388	0° Incidence
					389 ± 4	30° Incidence
					402 ± 4	52° Incidence
2. Void Prolate 415 × 806 μm	-10	.5	2.5	415	443	
3. Void Sphere	20	.5	3	400	374	
4. WC Sphere	10	.5	3	400	325	
5. Void Sphere	-13	.6	2.5	400	347	
6. Void Sphere	10	1.2	4.2	600	361	No low ka
7. Void Oblate 390 × 130 μm	-10	.6	1.1	225	330	No high ka

Flaw in Si_3N_4

Flaw	S/N (dB)	ka_{min}	ka_{max}	True Radius (μm)	Estimated Radius (μm)	Comments
1. Fe Sphere	25	.5	4	200	190	
2. Si "Sphere"	10	.5	3	"50"	38	Highly Distorted
3. Void "Sphere"	10	1.0	5.5	250	180	No low ka
4. Void "Sphere"	0	.5	2.8	125	132	

estimate of the signal-to-noise of the measurements.

The comparison between estimated and true radius is seen to be good for most of the flaws and for the others, the reason for the error is evident.

In the titanium samples, flaws 1-5 are estimated to within 20%. Flaw 1 was observed from three different angles of incidence. Flaws 6 and 7 were inspected with insufficient bandwidth for the sizes of flaw being inspected. Flaw 6 was inspected with too little low frequency energy and so the expected underestimate resulted. Similarly, the overestimate of flaw 7 is due the availability of too little high frequency energy.

In the ceramic specimens, flaws 1 and 4 are accurately estimated. Flaw 3 had insufficient low frequency data and was therefore underestimated. Flaw 2 consisted of a diffuse area of mixed chemical makeup and there is therefore no single "true" radius. However, the radius estimate is certainly consistant with the observed size.

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3. R. K. Elsley, L. Ahlberg and J. Richardson, "Low Frequency Characterization of Flaws in Ceramics," these proceedings.

Don Thompson (Ames Laboratory): On your next-to-the-last slide, Dick, what is your reference point of saying - how are you taking KA minimum and KA maximum? Are those zeros?

Dick Elsley (Rockwell Science Center): We know the size of the flaw. And so the frequency band for the transducer gave decent data when converted to K in the numbers that we have used for KA minimum and KA maximum.

Mr. Schmitz (Germany): Let us consider again the problem of sizing to the problem of rattlesnakes, if rattlesnakes are traveling in pairs, could you do it, handle this problem, too? Learning pairs closely related or different in sizes?

Dick Elsley: Both problems would be more difficult. If they are very close together, we get something like a circumscribed flaw, I think. To be able to time gate the two, of course, would be a very convenient way to avoid the problem.

Mr. Schmitz: In terms of wavelengths, two of the wavelengths?

James Krumhansl, Chairman (NSF): If your upper KA is less than the separation K separation instead of KA, something like that, a band width. There is a certain separation, talking about pairs of flaws. Now the critical sort of crossover in data is K times the separation. And if that's much less than one, you don't have a chance of resolution.

Dick Elsley: If the two flaws are the same distance from the transducer and within the beam of the transducer, there will be no way to separate them there. If they're in front of one another and you can time gate, you will be successful.

Norm Bleistein (Denver Applied Analytics): The earlier slide when you were dealing with synthetics, you showed some results from band widths with a ratio of 6 to 1 and 10 to 1. You look at ratios here and they are nowhere near that ratio. I guess it's less than 3 to 1, the kind of ratios we have for KA minimum and KA maximum. What kind of estimates do you have for synthetics? Can you run down things like that?

Dick Elsley: The curve that showed 6 to 1, 10 to 1, 20 to 1, that is where the artificial transducer went out to zero. The artificial transducer is a mathematical formula, and was identical to zero. The 6 to 1 like that would be something smaller, or 5 to 1. Using multiple transducers to get more band width is the way to get around that problem.

Norm Bleistein: What happens when it goes down to 3 to 1?

Dick Elsley: If the center frequency is not around KA equal to one, you will get a series of overestimates.

William Reynolds (Aere Harwell): Could I ask if this technique would be useful in distinguishing in the case of inclusions in silicon nitride between ions, say, and silicon particles of less than 100 microns diameter when they are less than spherical?

Dick Elsley: In the ceramics talk we gave last year, we did essentially that. We combined the Born inversion with the low frequency measurements, measuring the coefficient A2 and got how likely a flaw was to be either iron or something else. In the case of iron and silicon, you might be able to distinguish just by the sign of the selected pulse.

James Krumhansl, Chairman: Thank you, Dick. Let's move on to the next paper.

RAMP WAVE PROCESSING OF LONG WAVELENGTH ULTRASONIC SCATTERING INFORMATION

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ABSTRACT

A method of extracting size and orientation of flaws, voids and inclusions, from scattered ultrasonic signals is under investigation. The novel feature of this method is that the time domain ramp function response is used for interpretation. Consider a time domain ramp signal imposing on flaw. The spectral content of this signal decreases inversely with frequency squared. The back-scattered signal from a void, for example increases with the frequency squared. As a consequence, the echo of the ramp function from the flaw is rich in long wavelength information whereas the short wavelength information is de-emphasized. The net result is that the time domain ramp response has a height proportional to cross-sectional area of the flaw and has a length proportional to the flaw depth.

The results of scattering of ramp signals from acoustic targets illustrate the promise of this technique.

James Krumhansl, Chairman (NSF): Was There any particular reason for choosing a ramp function?

Bill Cook (University of Houston): I'll tell you why. The ramp function, as Jim Rose mentioned - he had an answer which was the second derivative of this area you go back to, and the ramp function does two inversions for you.

James Krumhansl, Chairman: Any other questions? Then thank you very much.