MODELING THE RADIATION OF FOCUSED AND UNFOCUSED ULTRASONIC TRANSDUCERS THROUGH PLANAR INTERFACES

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INTRODUCTION

Characterizing the wave field produced by an ultrasonic transducer is one important part of developing a complete measurement model of an ultrasonic NDE system. Within the paraxial approximation [1], this wave field can be expressed as a quasi plane wave field modified by a diffraction correction term to account for the 3-D effects of having a finite beam generated by the transducer. Previously, diffraction corrections have been obtained analytically in some simple cases [1] and numerically, using a Gauss-Hermite expansion [2]. Here, we will show that the radiation of both planar and spherically focused probes through a planar interface can be treated in a unified manner using angular plane wave spectrum integrals and the method of stationary phase. In the paraxial approximation, this unified approach will also be shown to produce diffraction corrections in the form of boundary diffraction waves [3]. These boundary diffraction wave diffraction corrections are in terms of at most 1-D integrations on the edge of the transducer, so that they can be easily evaluated numerically.

THE GENERAL MODEL

Consider the case shown in Fig. 1 where a spherically focused transducer of radius a and geometrical focal length R_0 radiates sound at oblique incidence through a planar fluid-solid interface. We will model the solid here as a second fluid medium. All the same procedures and results, for the actual fluid-solid case, however, are direct extensions of this fluid-fluid case, even for the mode-converted waves in the solid.

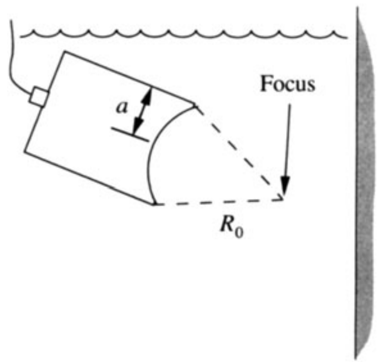

Fig. 1 Spherically focused transducer radiating at oblique incidence into a second medium.

To represent the waves generated in the first fluid by a spherically focused transducer, we will use the theory of O'Neil, which models the focused transducer as constant velocity (piston) sources acting on a spherical aperture, and incorporates these sources directly into the Rayleigh/Sommerfeld integral [4]. Although the O'Neil model contains a number of approximations, recent numerical comparisons with more exact models have shown that this theory works well except under rather extreme conditions [5]. Within this theory, the pressure at an arbitrary point **x** in the first fluid, therefore, is given (for harmonic waves of $\exp(-i\omega t)$ time dependency) by

$$p(\mathbf{x},\omega)=\frac{-i\omega\rho_1 v_0}{2\pi}\int_S \frac{\exp(i k_1 r)}{r}dS \tag{1}$$

where ρ_1 is the density of the fluid, v_0 is the constant normal velocity on the spherical surface S, $k_1=\omega/c_1$ is the wave number, and c_1 is the wave speed. Physically, the Rayleigh/Sommerfeld integral represents the wave field as a superposition of spherical waves (Huygens' principle). These spherical waves can be written in terms of an angular spectrum of plane waves [6] as

$$\frac{\exp(ikr)}{r}=\frac{i}{2\pi}\iint \frac{\exp(i\mathbf{q}\cdot\mathbf{r})}{q_z}dq_x dq_y \tag{2}$$

where

$$\begin{aligned}\mathbf{q} &= q_x\mathbf{e}_x+q_y\mathbf{e}_y+\sqrt{k_1^2-q_x^2-q_y^2}\,\mathbf{e}_z\\ \mathbf{r} &= \mathbf{x}-\mathbf{y}\end{aligned} \tag{3}$$

Thus, the pressure in the first fluid becomes

$$p(\mathrm{x},\omega)=\frac{\omega\rho_1 v_0}{4\pi^2}\int_S\left\{\iint \frac{\exp(i\mathrm{q}\cdot\mathrm{r})}{q_z}dq_x dq_y\right\}dS \tag{4}$$

The value of using this plane wave representation is that it is possible to propagate the plane wave terms in Eq. (4) across a planar interface by merely making the replacement

$$\exp(i\,\mathbf{q}\cdot\mathbf{r}) \rightarrow T_{12}\exp\left[i\left(\mathbf{q}_2\cdot\mathbf{r} - q_{2z}D + q_z D\right)\right] \tag{5}$$

with

$$\mathbf{q}_2 = q_x\mathbf{e}_x + q_y\mathbf{e}_y + q_{2z}\mathbf{e}_z$$

$$q_{2z} = \sqrt{k_2^2 - q_x^2 - q_y^2}$$

where T_{12} is the ordinary plane wave transmission coefficient (based on a pressure ratio) for the interface. The complex exponential on the right side of Eq. (5) is the phase term for the transmitted plane waves.

Using Eq. (5), the pressure at an arbitrary point **x** in the second fluid, therefore, can be written as

$$p(x,\omega) = \frac{\omega\rho_1 v_0}{4\pi^2}\int_s\left\{\iint \frac{T_{12}\exp\left[i\left(q_2\cdot\mathbf{r} - q_{2z}D + q_z D\right)\right]}{q_z}\,dq_x dq_y\right\}dS \tag{6}$$

The angular spectrum of plane waves integrals in Eq. (6) can be evaluated, at high frequencies, by the method of stationary phase [6]. For a general 2-D integral with amplitude term $A(q_x,q_y)$ and phase term $f(q_x,q_y)$, the method of stationary phase gives

$$\begin{aligned}&\iint A(q_x,q_y)\exp\left[i f(q_x,q_y)\right]dq_x dq_y\\ &= \frac{2\pi A(q_{xs},q_{ys})\exp\left[i f(q_{xs},q_{ys})\right]}{\sqrt{\lambda_1\lambda_2}}\exp\left[i(\operatorname{sgn}\lambda_1 + \operatorname{sgn}\lambda_2)\pi/4\right]\end{aligned} \tag{7}$$

where λ_i $(i = 1,2)$ are the eigenvalues of the $f_{ij} = \partial^2 f/\partial q_i \partial q_j$ matrix at the stationary phase point (q_{xs}, q_{ys}) and where $q_1 = q_x$, $q_2 = q_y$. Applying the general form of Eq. (7) to the specific integrals in Eq. (6), we find

$$p(\mathbf{x},\omega) = \frac{-i\omega\rho_1 v_0}{2\pi}\int_S \frac{T_{12}(\cos\theta_1)\exp\left[i(k_1 d_1 + k_2 d_2)\right]}{\sqrt{\Delta_x}\sqrt{\Delta_y}}dS \tag{8}$$

where

$$\begin{aligned}\Delta_x &= d_1 + \left(c_2\cos^2\theta_1 / c_1\cos^2\theta_2\right)d_2\\ \Delta_y &= d_1 + \left(c_2/c_1\right)d_2\end{aligned} \tag{9}$$

and the distances (d_1, d_2) and angles (θ_1, θ_2) are for a "ray" path that satisfies Snell's law and goes through the interface from a general point **y** on the transducer face to a general point **x** in the second medium (Fig. 2). The transmission coefficient $T_{12}(\cos\theta_1)$ is also evaluated in Eq. (8) for this same ray path.

Equation (8) is a general result for calculating the wave field in the second medium. It is also valid for the unfocused probe case by simply taking the surface S to be a planar

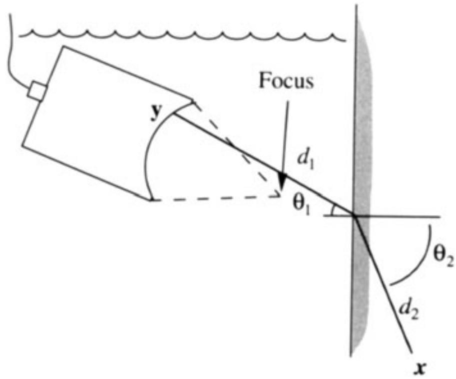

Fig. 2 Ray path for a wave traveling from the transducer face to a point in the second medium.

aperture. The approximations used in obtaining this result are merely those inherent in the original O'Neil theory itself (for the focused case), and the approximations introduced by the use of the method of stationary phase. As a result of the stationary phase approximation, only the waves which travel directly from the transducer face to the point **x** along geometric ray paths are retained in Eq. (8). Except at very high angles (near grazing incidence to the interface), however, it is these wave contributions that will be most significant. Thus, Eq.(8) should be able to be used under most practical testing situations.

THE PARAXIAL APPROXIMATION

Although Eq. (8) can model the wave fields in the second medium under rather general conditions, its numerical evaluation requires a double integration. However, if the point **x** is not too close to the transducer, most of the ray paths in the evaluation of these equations are not too different from the one fixed ray path that goes from point **x** to the transducer through the geometrical focus (Fig. 3). In this case, we approximate the amplitude in Eq. (8) as that along this fixed ray (all quantities associated with the fixed ray are denoted here by including an additional "0" subscript) and the phase can be shown to be given approximately by

$$k_1 d_1 + k_2 d_2 = k_1 d_{10} + k_2 d_{20} + \frac{k_1 \rho^2}{2}\left[\frac{p_{x0}\cos^2\phi}{\Delta_{x0}} + \frac{p_{y0}\sin^2\phi}{\Delta_{y0}}\right] \tag{10}$$

where

$$\begin{aligned} p_{x0} &= 1 - \Delta_{x0}/R_0 \\ p_{y0} &= 1 - \Delta_{y0}/R_0 \end{aligned} \tag{11}$$

and (ρ, ϕ) are polar coordinates in a plane that is perpendicular to the fixed ray at point $\mathbf{y}_0$.

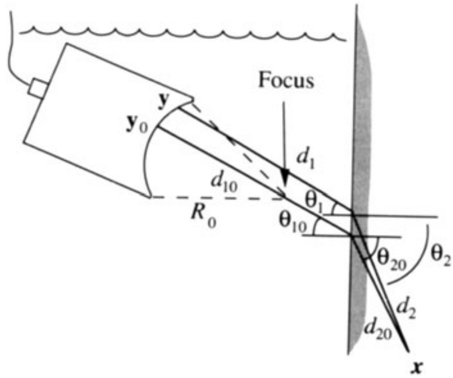

Fig. 3 A general ray path and a fixed ray path that goes from a point in the second medium to the transducer face, passing through the geometrical focus.

With these two approximations then the ρ integration can be performed and the pressure in medium two can be written as merely a plane wave term modified by an appropriate diffraction coefficient, C_1, i.e.

$$p(\mathbf{x},\omega) = \rho_1 c_1 v_0 T_{12}(\cos\theta_{10})\exp[i(k_1 d_{10} + k_2 d_{20})]C_1(\mathbf{x},\omega) \quad (12)$$

where C_1 is given by

$$C_1(\mathbf{x},\omega) = \left[\Theta \frac{\varepsilon}{\sqrt{|p_{x0}|}\sqrt{|p_{y0}|}} - \frac{1}{2\pi\sqrt{\Delta_{x0}}\sqrt{\Delta_{y0}}}\int_0^{2\pi}\frac{\exp\{ik_1\rho_e^2 f(\phi)/2\}}{f(\phi)}d\phi\right] \quad (13)$$

with

$$f(\phi) = \frac{p_{x0}\cos^2\phi}{\Delta_{x0}} + \frac{p_{y0}\sin^2\phi}{\Delta_{y0}} \quad (14)$$

and

$$\Theta = \begin{cases} 1 & \mathbf{y}_0 \text{ in } S \\ 1/2 & \mathbf{y}_0 \text{ on edge of } S \\ 0 & \mathbf{y}_0 \text{ outside } S \end{cases} \qquad \varepsilon = \begin{cases} 1 & \textit{if both } p_{x0}, p_{y0} > 0 \\ -i & \textit{if } p_{x0}p_{y0} < 0 \\ -1 & \textit{if both } p_{x0}, p_{y0} < 0 \end{cases} \quad (15)$$

and $\rho_e = \rho_e(\phi)$ is the radial distance to the edge of the transducer from point $\mathbf{y}_0$. If we simply let $R_0 \to \infty$ in Eq.(13) we obtain the corresponding diffraction coefficient for the planar transducer radiating at oblique incidence to the interface. In both cases, the diffraction correction term is composed of two terms. The first term can be shown to be a "direct" wave which travels from the transducer face to point **x** according to the laws of geometrical optics. This term only exists when a ray exists from point **x** to the face of the transducer that passes through the focus. In this situation, we will say that the point **x** lies in the "main beam" of the transducer. The second term is an edge wave (boundary diffraction wave term) which arises from a superposition of sources on the transducer edge. This term is present both in the main beam and outside it. In general this edge wave must be obtained numerically, but it is only a 1-D integral which can be done efficiently.

CONCLUSIONS

We have developed a unified model for the calculation of the wave fields from both focused and planar transducers that are radiating at oblique incidence to a planar interface. In the paraxial approximation, this unified model can be used to obtain the wave field in a quasi plane wave form with a diffraction correction that is given in terms of both direct and edge wave contributions. This paraxial theory, therefore, is in the form of a boundary diffraction wave theory that can be evaluated numerically in an efficient manner. Such numerical evaluations are shown explicitly for the unfocused probe case in a companion paper in this proceedings [7].

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