ECONOMIC POWER SYSTEM LOADING

by

John Emil Lagerstrom

A Dissertation Submitted to the

Graduate Faculty in Partial Fulfillment of

The Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Major Subject: Electrical Engineering

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State College

1958

TABLE OF CONTENTS

I. INTRODUCTION

Electric power systems with two or more generating stations supplying power at different costs require analysis to determine the operating schedule which will result in the least total cost of power supplied to the system. In the early days of the electric industry a typical system consisted of a single generator feeding its load distribution system radially. In the next phase of development, generators were added in parallel to increase the capacity of the plant, thereby posing the problem of finding the most economic loading combination of boilers and generators connected to a plant bus. In general, newer units would be more efficient and probably should be loaded before the older units were placed in service. The problem is dynamic, in that the system load would vary throughout the day requiring a continuing check that the combination of generator loadings at any moment is the most economic combination.

Modern systems consist of several plants connected by long transmission lines or networks of lines. It is possible that a relatively new plant of high efficiency is more remote from a load area than an older, less efficient plant. The problem of most economic loading now involves not only the efficiency of the plants themselves, but also the cost of the losses associated with transmitting power through the network of lines to the load centers. A high-efficiency plant might be so remote that the cost of losses would make it impossible for it to compete economically with a low-efficiency plant near the load center.

The problem is further complicated by the integration of hydro-electric generation into the power system. The cost of hydro generation cannot be assessed in the same manner as the costs of fuel-burning plants. In the case of hydro plants there is a certain amount of energy in the stored water of the stream or reservoir which must either be utilized in the electric system or else lost altogether. The amount of energy available in a unit of time must be converted to a value of available power output throughout that interval, and if necessary a value of power cost must be assigned to make that plant competitive with the fuel-burning plants in the system. The output of the hydro plant must be assigned on the basis of available water and that value of output held in the solution for the most economic combination of the remaining fuel-burning plants in the system.

The purpose of this thesis is to demonstrate an iterative method for determining the most economic operating condition of a system of plants connected by a network. Each trial solution will be analyzed to determine the direction each plant should be moved in order to approach a minimum value of the total cost input to the system. The method includes a criterion for recognizing that this minimum value has been achieved in the final solution.

A direct solution of the economic loading criteria seems practically impossible. It will be shown that these criteria are trigonometric in nature, and that the solution requires two operations: the solution of the electrical network to determine the loading of each of the plants, and then the insertion of the plant loading conditions into the criteria.

To describe the network, it is necessary to know the transfer impedances among all pairs of generators. These may be obvious in simple systems with only a few generators. In complex systems it will be necessary to make use of a network analyzer to measure these transfer impedances.

They should be measured with the system loads trimmed to the proper values at a practical profile of voltages, and a network analyzer is appropriate for this type of work. Having trimmed the loads, they will thereafter be represented by constant shunt impedances, and if they were originally placed at points in the network other than the generator terminals they will affect the values of the transfer impedances. Thus good engineering judgment is necessary in trimming the system, and the schedule of system generation should be selected after considering the fuel rates of the various plants and assigning power outputs that will approximate the economic schedule sought. This can be done by assuming, as a first approximation, that all generators will operate at the same incremental rate. The incremental rate of a plant is defined as the slope of its fuel input vs. output characteristic.

It is also necessary to know the shunt impedances which represent those loads not originally located at the generator terminals. These values may be determined at the same time that the transfer impedances are measured, and they will become constant values applied at each of the generator terminals. They may be combined with the original generator local loads, so that the resulting network representation will be a system of transfer impedances among n generators together with n equivalent local loads at the terminals.

In applying the economic loading criteria, the network analyzer is not capable of measuring either power or voltage phase angles to the degree of precision required. Hence it is necessary to resort to digital computations. This digital technique will be illustrated by means of examples calculated with a desk calculator. For more extensive systems the technique may be used to program an automatic digital computer.

The only restriction placed on the operation of the system is that a schedule of voltages about the system be chosen and adhered to. This may appear to be idealistic, but it must be considered a goal to be achieved by those responsible for system planning and development. If experience proves that some point in a system has less than a satisfactory level of voltage, then it is necessary to make some correction in the system to provide adequate voltage. Sometimes this is done by building new lines into that load area, sometimes by increasing the conductor size of present construction, or perhaps by installing capacitors at that point. Such remedies are justified either by reducing the losses associated with transmission into that area or by improved service and better customer satisfaction. Having selected a voltage profile, the criteria for economic loading may be applied with the assurance that the system operating schedule obtained will be the most economic under the assumption of voltages chosen. A schedule of var generation may be obtained as a secondary consideration from the solution, and it is possible that the required generation of some generators will be relatively few watts and a large output of vars. The criteria do not look at vars explicitly, but the effect of var flows throughout the system is accounted for by virtue of the fact that they are functions of the same independent variables as the real power flows. Any additional loss incurred by var flow through a line element is accounted for in the calculations for real power flows through the line. Certain undesirable operating conditions can be avoided only by studying the preliminary representation on a network analyzer and making appropriate corrections before proceeding to measure transfer and shunt impedances. Vars circulating around a loop might be removed by making tap changer adjustments. Unsatisfactory watt/var ratios at certain generators might be corrected by revising the voltage profile of the system, or the heavy flow of vars to a load center might be corrected by installing capacitors near that load.

II. REVIEW OF LITERATURE

In 1943, Steinberg and Smith (29) published a book on the division of load among plants or units of different fuel economies. This work included considerable material on the performance of the boilers and turbines themselves. The concept of equal incremental rates was used as a criterion for the most economical performance of two or more units. The incremental rate of a unit is defined as the derivative of the input with respect to the output, and is given the connotation of the increment of input required to supply an increment of output. It was assumed that most units would have continuous input-output characteristics and that their incremental rate values would increase with output. The combined input to two or more machines whose individual characteristic curves have these properties would be a minimum when the individual machine outputs correspond to the same incremental rate value.

For any combined output there can be only one combination of machine outputs for which the incremental rates of the individual machines are equal, with the exception that if, for two or more machines, there are ranges of output for which the incremental rates of the individual machines are equal and constant, then there may be an indefinite number of combinations of machine outputs for which the combined input will remain constant and be at a minimum value. The method fails if one or more of the incremental rate curves corresponding to continuous input-output curves have decreasing incremental values with increase in output, or decreasing values at one or more points of discontinuity. Under these conditions, operation at outputs corresponding to equal incremental values does not necessarily result in the best overall efficiency, and incremental loading is not the sole criterion for proper load division.

Economical division of load between two or more interconnected generating stations, when tie-line losses are negligible, becomes merely a problem of operating the stations at loads which correspond to the same incremental rate value. This is the prevailing condition for metropolitan areas served by stations located near the load centers. For stations interconnected through long lines, the losses are appreciable and the load-division procedure should be modified to take them into account. Steinberg and Smith considered the case of two stations connected by a single line. They defined the incremental efficiency of the transmission line as the derivative of the received power with respect to the power at the sending end of the line. This concept has the connotation of the efficiency with which an increment of power can be transmitted over the line. The criterion for minimum input under these conditions is that the incremental rate at the sending end of the line must be equal to the product of the receiving end incremental rate and the incremental efficiency of the line. This criterion can be applied to all the possible combinations of system loads by plotting curves of incremental rates and adjusting them to obtain combined station rate curves as functions of total system load. This method, however, is prohibitively complicated for more than two machines. In addition, this method fails to consider the effect on losses of var flows on the line. It is implied that vars have a negligible effect which would be true only if the power factor of the line were very close to unity.

George (12) described a method of calculating transmission losses within power systems. The method is based on the principle of superimposing the load distribution from each source, determining the current in each line as the algebraic sum of the individual load flows, and setting up an equation for losses in terms of the power generation at the various plants and of power flows at each interchange point. This procedure results in deriving a special loss formula for a given power system. The method assumes that all the loads in the system vary throughout the load cycle in the same manner, with the exception that any load whose variations differ from the daily load cycle must be considered a special case and treated as if it were an interchange point. George implied that the method could be applied without the aid of a network analyzer, for he suggested that lines be represented in the solution for power flows by the magnitude of their impedances. This requires that the impedance angles of the lines of the network have similar values in order to minimize errors in specifying the power flows caused by each generator. Having determined the power flows due to each generator or interconnection, the loss equation for the system can be expressed as a function of these terminal power flows with constant coefficients. This loss equation is of second order in these power flows, and the equation will contain $\frac{n(n + 1)}{2}$ terms for an n-terminal system.

The method is recommended for any combination of generator or interchange loadings, but any change in the network such as the loss or addition of a line element requires the application of a new loss formula. George estimated the accuracy of the loss equation to be within ten percent for single readings, or within five percent for daily totals. The possible sources of error include variations in the power factor of line flows, or the distribution of vars throughout the network. On systems with large charging currents, the uncorrected calculated losses would be too high at times of light generation and low interchange. Also, the principle of superposition applies to a-c load flows only if the line power factor is uniform and constant. Losses are neglected in applying superposition to determine the individual generator contributions to the load flows. Less serious causes of error include variations in the distribution of loads among the substations during the daily load cycle, the combining of small substations with larger ones to simplify the network, and the assumption that the current through a line element is uniform.

Frampton and Floyd (10) enumerated the factors which affect the economy of energy generation in the hydro system. These are factors in the design and operation of the plant, and in the characteristics of the load. Included in the first are storage problems, efficiency of plant arrangement, unit efficiency, and the design of the new plant to combine most effectively with existing resources. Factors in operation include operation of units in a plant to obtain maximum energy delivery, economic operation of plants in parallel, effect of maintenance, and of load and frequency control. The factors in the load include the effect of seasonal variation of energy demand, provision of peak reserve capacity, development of off-peak consumption, and the effect of surplus energy in reducing the cost of primary generation.

George, Page, and Ward (13) described a technique for combining the loss equation of George with plant fuel cost data to determine the most economical division of plant loadings from the standpoint of minimum total cost. Ward was credited with developing a procedure for using the network analyzer to replace the trial and error determinations of power flows from each generating plant and interconnection previously needed in setting up the loss coefficients. These analyzer data were required to determine the total system loss equation. This equation was partially differentiated with respect to each of the plant power outputs to obtain a set of equations for incremental transmission loss. Each of these equations was combined with the incremental costs for each plant, resulting in a set of simultaneous equations equal in number to the number of plants to be coordinated. These equations were linear. Since the loss equation of George was of second order, and since the coefficients were constant, the partial derivatives were of first order and with constant coefficients. It was recommended that the cost data for the various plants be linearized, either graphically or arithmetically. The resulting equations were then applied to the network analyzer where an electric analog technique was used to produce solutions for minimum total cost. If λ is the incremental delivered cost of power, then each of the plant equations is of the form,

$$\begin{matrix}\text{Station Incremental} \\ \text{Production Cost}\end{matrix} + \lambda \begin{matrix}\text{Incremental} \\ \text{Transmission Loss}\end{matrix} = \lambda. \qquad (1)$$

Ward, Eaton, and Hale (32) reported their development of George's transmission loss formula, together with their technique for using a network analyzer to obtain the data necessary to evaluate the loss formula coefficients. The loss formula was derived by applying the principle of superposition, and by application of two approximations: each substation load is represented as a certain current which has a constant magnitude and fixed relative angular position with respect to other load currents, and each substation load varies proportionately and maintains its fixed relative angular position with respect to other load currents as the total system load changes. The loss formula developed expresses the total system loss in terms of four variables for each source: P, power; Q, var output; V, voltage; and Ø, voltage phase angle. It was recommended, however, that this formula was too complicated for practical applications, and a set of simplifying assumptions was applied to obtain a more practical expression. The simplifying assumptions were:

1. Ø values remain fixed
2. V values remain fixed
3. Load power factors are uniform throughout the network
4. Line R/X ratios are uniform throughout the network
5. No loop has an excess transformer turns ratio
6. Each source operates at a constant ratio Q/P.

The resulting simplified formula for total system loss was then of the form,

$$P_L = \sum_{m=1}^{s} \sum_{n=1}^{s} P_m P_n B_{mn} \tag{2}$$

where the coefficients B_{mn} are constants, and m and n refer to two of the s sources.

Kirchmayer and Stagg (23) reported a similar loss formula, except they recommended a different derivation of the method based on concepts of tensor analysis as presented by Kron (24). Glimn, Kirchmayer, and Stagg (16) reported application of this loss formula to determine the losses associated with interconnecting power systems, and presented graphical evidence of the accuracy of loss determination by means of the formula as compared with calculated losses from network analyzer data. A theoretical development of their method was given in a companion paper by Kron (25).

Glimn, Habermann, Kirchmayer, and Stagg (15) presented further simplifications of the loss formula by proposing improvements in the calculation of constants from analyzer data. The method described does not require the measurement of generator or load currents, and it results in an exact check between the actual base case losses and the losses obtained from the loss formula. The method is recommended for application to automatic digital computing machines. In the discussion accompanying this paper, Bills, Smith, and Wilgus of the Bonneville Power Administration point out that this simplified loss formula cannot be applied to their system because of the large spread in generator angles (as much as 60 degrees) and the fact that the load voltage profile is not flat.

Ward (31) described how incremental transmission loss characteristics and incremental plant characteristics must be considered together in achieving optimum system economy. Considerations of equipment ratings, voltage limitations, reactive power limitations, and stability limitations were omitted. The reactive power component of loads and generation need not be dealt with explicitly. It was assumed that as the customers' demands are satisfied by some particular generating schedule, the reactive power generation at plants and synchronous condensers, and also the transformer taps, are adjusted as a matter of course to obtain desired voltage levels. The problem of dispatching reactive power in such a way as to reduce losses and thus reduce fuel costs was not considered. Rather, the reactive flow around the network was assumed to depend on the power flow, and only the problem of dispatching power to achieve optimum fuel economy was studied.

Ward defined P_1, P_2,...P_n as the power output of the n generators of the system and C_1, C_2,...C_n as the fuel input to these plants. The loads were assumed to remain constant, and the total system loss was designated L. To determine the effect of exchanging generation between two plants, it was assumed that a small increment ΔP_1 was added to the output of generator 1, and a correspondingly small amount ΔP_2 was dropped from generator 2 so as to keep all the remaining generators at the values they originally produced. Corresponding to these incremental changes there would be changes ΔC_1 and ΔC_2. If the transfer produces no change in total fuel cost, then the original division of power between these two plants must have been satisfactory within the limits of the method. Either a decrease or an increase in the total cost input would have indicated whether the increment to generator 1 was desirable or not. To determine the best system operating condition, it is necessary to evaluate all the possible interchanges between pairs of generators. At the optimum condition, all possible interchanges will yield no change in the total fuel cost.

If generators 1 and 2 are at the optimum condition, then

$$\Delta C_1 + \Delta C_2 = 0 \tag{3}$$

and it must follow that

$$\frac{\Delta C_1}{\Delta P_1} = -\frac{\Delta C_2}{\Delta P_1}. \tag{4}$$

For a system without losses, an interchange between generators 1 and 2 would result in

$$\Delta P_1 + \Delta P_2 = 0 \tag{5}$$

or

$$\frac{\Delta P_2}{\Delta P_1} = -1. \tag{6}$$

It then follows that

$$\frac{\Delta C_1}{\Delta P_1} = - \frac{\Delta C_2}{\Delta P_2}\frac{\Delta P_2}{\Delta P_1} = \frac{\Delta C_2}{\Delta P_2}. \tag{7}$$

If the increments ΔP_1 and ΔP_2 are allowed to approach vanishingly small values these terms become derivatives, and the expression becomes the criterion of Steinberg and Smith that the optimum operating condition exists when the incremental rates of the various pairs of generators are equal.

For a system in which the losses are appreciable, an interchange between generators 1 and 2 would result in a change in the losses given by

$$\Delta P_1 + \Delta P_2 = \Delta L_{1,2}. \tag{8}$$

From this equation it follows that

$$\frac{\Delta P_2}{\Delta P_1} = \frac{-1}{1 - \frac{\Delta L_{1,2}}{\Delta P_2}}. \tag{9}$$

If the value $\Delta C_1/\Delta P_1$ is replaced by R_1, the incremental rate of generator 1, and $\Delta C_2/\Delta P_2$ is replaced by R_2, then these incremental rates may be equated

$$R_1 = \frac{R_2}{1 - \frac{\Delta L_{1,2}}{\Delta P_2}}. \tag{10}$$

Expansion of this criterion to all pairs of machines leads to the interpretation that optimum loading of plants is obtained when suitably penalized plant incremental rates are all equal. The penalty factor is the multiplier on R_2 in this equation. If the ratio $\Delta L_{1,2}/\Delta P_2$ is positive, transfer of increment generation from plant 2 to plant 1 causes a decrease in loss, then this weighting or penalty factor will be a number greater than unity. Thus plant 2 is placed at a disadvantage by loss characteristics relative to plant 1 and any other plants with smaller penalty factors. As a result, plant 2 will receive a smaller share of generation than it would if transmission losses were neglected.

Criteria for optimum allocation of generation can also be based on incremental loss characteristics which are determined by small changes initiated in the loads. A general expression may be written in which n denotes any one of the generators and K_b is the incremental cost of supplying increasing load at load point b.

$$\frac{R_n}{1 - \frac{\Delta L_{n,b}}{\Delta P_n}} = K_b \tag{11}$$

If optimum allocation of generation exists, then this incremental cost of power delivered to load b is the same regardless of which plant produces the additional power. This equation may be rewritten

$$R_n + K_b \frac{\Delta L_{n,b}}{\Delta P_n} = K_b . \tag{12}$$

This equation is similar to equation 1 published by George, Page, and Ward in which they used the derivative form for incremental transmission losses and the incremental cost of power was denoted λ.

Harker, Jacobs, Ferguson, and Harder (18, 17) reported in two papers their results of a study of loss evaluation methods sponsored jointly by the Westinghouse Electric Corporation, Consumers Power Company, and Commonwealth Associates. They described three methods which they designated the in-phase method, the current-form method, and the power-form method. The in-phase method had been used for a number of years by the Consumers Power Company and has the advantages that no computer is required either in the development of the formula or in the construction and use of the loss charts. The method is extremely flexible and provides direct chart reading of the losses for a wide variety of system load and sale conditions. It is applicable to many systems which can be represented by a simplified network. It has the disadvantage that it cannot be applied to a network which has internal loops, nor is it applicable to economic dispatch studies. The method is based on four assumptions: the system can be represented as a transmission system without loops and without mutual impedances between branches, the currents in all branches are in-phase and proportional to the power, the distribution of sale power through the ties is fixed for sale from a given generating station, and generation dispatches for various system loads are known and fixed. The accuracy of the method depends on the validity of these assumptions. The method ignores the flow of vars and consequently the results are always lower than the actual losses in the system. As long as the operation of the system is near unity power factor this method provides the easiest evaluation of the total system loss.

The current-form and power-form methods are more accurate because they are capable of including the effect of out-of-phase components of load current. The current-form loss formula does not require the assumption of fixed var/watt ratios and has made it possible to study the effect of vars transferred over interconnections on system transmission loss. In economic dispatch studies, transmission loss is best expressed in terms of the same variable as the incremental production cost to which it must be added.

Thus both are expressed in terms of station output power. This facilitates the operation of minimizing total cost. However, if the loss formula is to be used to determine the losses associated with the sale of power, for example, it is more desirable to divide the calculation into two steps: first, the conversion from generator and tie line watt and var values to currents, and then the determination of the losses from a loss formula. This current-form method has at least three advantages: it does not require fixing the var/watt ratio of either generators or ties, the loss formula is simpler to determine, and the total computation on an automatic computer is shorter.

The assumptions on which the current-form loss formula is based are:

1. All load currents are fixed vector fractions of the total load current. These fractions are determined in an a-c calculating board study for a "base case" well centered in the contemplated field of applicability of the formula. The load current components out-of-phase with the total load current are not neglected in this approximation.

2. The voltage magnitudes and angles at all generator and tie points are the same as for the base case. The spread in generator angles across the system is less than 25 or 30 degrees.

3. All line-charging and synchronous-condenser loads are lumped with the system loads.

4. Transformation ratios are unity around each closed loop of the network.

5. The generator and tie line var/watt ratios can be kept the same as the base case or changed in any desired manner.

Two loss formulas were reported for the Consumers Power system, one using 85 percent load as the base case and the other using 55 percent. Whereas the first predicted a loss of 15.8 mw. at the low load compared with 14.8 mw. calculated by summing the I^2R losses of the elements of the system, the second predicted a loss of 28.8 mw. at the high load compared with 22.2 mw. by I^2R. These results reveal the danger of extending application of these loss formulae too far from the base case. The first check was deemed satisfactory, the second was not. To guard against errors in the use of the formulae it is necessary to check each step of the work in obtaining network analyzer data and in converting these data into loss constants, and to run check cases diverging from the base case.

The power-form loss formula reported is similar to that of George and others, and is based on the same assumptions as the current-form equation except that the var/watt ratio of the generators and ties is fixed at the values of the base case. The total loss is given in terms of only the generator P values in order to be compatible with expressions for station outputs in economic dispatch studies. Two choices are offered in evaluating the loss coefficients: either to consider the out-of-phase components of load currents or to neglect them. The effort in preparing the coefficients to consider these out-of-phase components is approximately twice that involved in neglecting them, and it is recommended that they be neglected if it is possible to forecast that the loss formula will be sufficiently accurate throughout the range of application anticipated. In either case, the formula is made to give exactly the correct system loss for the base case.

Travers, Harker, Long, and Harder (30) published a third paper on loss evaluation in which they reported the results of an economic dispatch study for the Ohio Edison Company. The basis of the study was a set of economic dispatch equations such as those reported by George, Page and Ward. The loss coefficients were prepared neglecting out-of-phase load currents, and the base case was taken as 80 percent of normal system load. It was reported that the first attempt to establish a power-form loss formula failed due to the fact that at light system loads the smaller generators in the system were acting practically as synchronous condensers, thereby having digressed from the values of var/watt ratios they would have when operating near normal power. In order to correct this fault it was necessary to revise the loss formula to correlate with the var/watt ratios of these small machines at light system load. It was recommended that, if in the base case certain small plants are unloaded and the reactive power being supplied is similar to that of a synchronous condenser, it should be so treated and the reactive power lumped with the load. The var/watt ratio associated with the unit should correspond to the average incremental ratio as the unit loads up. To verify the procedure used, at least one of the check cases should be at maximum load when the small, inefficient plants are loaded up.

Eleven plants were included in this study. The station incremental cost curve of each of these plants was approximated by straight line segments, and the slope and intercept of each segment was evaluated in order that the resulting dispatch equations might be linear in the power outputs of the various plants. These simultaneous equations were solved by an iterative process by means of an International Business Machine card-programmed calculator. Values were chosen for the incremental cost of delivered power λ and the resulting power output of each plant determined. The sum of these powers is the total system generation, and the complete study consisted of choosing values of λ to determine a corresponding range of system generation values. The results of the study were summarized by plotting each plant output as a function of total system generation. By placing these curves on a single sheet the system dispatcher can assign generation in the most economic combination for any value of total system load. This dispatch was based on the simplification of zero power flow in the five tie lines to neighboring companies. It was shown that the dispatch thus obtained applied very closely for the normal power flows in the ties. It was found that the system losses changed considerably, but the dispatch of generation over the system changed very little and could be ignored. It was concluded that the cost of transferring power depended on whether or not the total system loss was increased, and the value assigned was determined by plotting the incremental cost vs. power flow at the interconnection point with λ as a parameter.

It was also found that if a dispatch which is obtained for all units in service is arbitrarily altered by eliminating from a given station's dispatch the output of a unit which is down, the resulting dispatch among the other stations will check very closely with an exact solution for that particular condition. The temporary solution is found by moving upward on the total generation scale from the original total load by an amount equal to the output the unit out of service should have supplied. The errors in this temporary solution are comparable with the errors in the telemetering system with which the dispatcher operates the system.

Brownlee (2) approached the economic dispatch problem from the viewpoint that incremental transmission losses can be expressed as functions of voltage phase angles. He started with the following expressions for the sending-end and receiving-end power flows of a short transmission line:

$$P_s = \frac{E_s^2 R}{Z^2} - \frac{E_s E_R}{Z^2} (R \cos \theta - X \sin \theta) \tag{13}$$

$$P_R = - \frac{E_R^2 R}{Z^2} + \frac{E_s E_R}{Z^2} (R \cos \theta + X \sin \theta) \tag{14}$$

where P_s and P_R are the sending-end and receiving-end powers; E_s and E_R are the sending-end and receiving-end voltages; R, X, and Z are the resistance, reactance, and impedance of the line; and θ is the phase angle between E_s and E_R. He observed that the power loss L on this line is the difference between these two expressions, and he proceeded to derive the incremental loss on this line as

$$\frac{dL}{dP_{12}} = \frac{2 \tan \theta_{12}}{K + \tan \theta_{12}} \tag{15}$$

dP_{12} is the limit of a small increase of generation at plant 1 with a reduction of the proper amount of generation at plant 2 to retain constant loads, and θ_{12} is the angle by which the bus voltage at plant 1 leads that of plant 2. K is the ratio of the reactance to the resistance of the line. The incremental loss is independent of the magnitude of the transfer impedance between plants, of the magnitudes of bus voltages, and of the var/watt ratio of the respective generators. This incremental loss expression was used to derive the condition for economic balance between two plants whose fuel costs are F_1 and F_2. The resulting expression is

$$\frac{dF_2/dP_2}{dF_1/dP_1} = \frac{K + \tan \theta_{12}}{K - \tan \theta_{12}}. \tag{16}$$

A series of curves relating fuel cost ratios to the angle θ_{12} for various values of K was presented for the rapid adjustment of network analyzer generating values. It was recommended that this method would be applicable to analyzer studies in which it might be desirable to investigate the economics of proposed generation while still in the board planning stages.

To determine the economic dispatch for a whole system, it was recommended that the fuel cost ratios of pairs of plants be compared until all the plants of the system were in balance. This requires for n plants that (n - 1) comparisons be made.

Brownlee compared the results of calculations of incremental losses by his angular functions with results obtained by the B-constant power-form loss formula of George and others. He also checked these results by calculating I^2R losses, and concluded that his formula was giving much closer correlation than the B-constant method. He advised caution in the use of these constants, arguing that the forcing adjustments used to make the constants yield the correct total system loss for the base case may make a substantial distortion in the incremental losses.

This paper drew considerable discussion. Early investigated the discrepancy in incremental losses obtained by Brownlee's method and by B-constants and compared results with what he purported to be a more exact although more laborious method. He concluded that Brownlee was justified in questioning the accuracy of the B-constant method, and he commended Brownlee's method for its ability to determine incremental losses directly from the network analyzer study. Glimn and Kirchmayer questioned the adequacy of Brownlee's development to represent incremental losses for the case of either loads or generators located between two machines under consideration, and they also commented on refinements of the B-constant loss formula by including out-of-phase load currents and correcting for digressions of generator reactive flow from base case values. Stagg reported he had studied the ability of Brownlee's method to schedule generation for most economic dispatch and found that the B-constant method showed additional savings over Brownlee's method. Watson reported that he also found better agreement between Brownlee's calculations for incremental losses and a basic formula than for the B-constant method. Ward and Hale questioned one of Brownlee's two basic assumptions, namely that multiple transmission paths between any two generating plants may be represented by a single transfer impedance. This apparently had led Brownlee to consider only systems without loops and with all generators tapped along a line, and this was objected to on the grounds that a load or another generator located between a pair of generators under consideration is not properly represented.

Cahn (4) gave further consideration to the work of Brownlee for the purpose of providing a more solid foundation for the results he believed Brownlee had obtained by heuristic reasoning. He concluded that the Brownlee theory and the new loss formulas are applicable to practical systems, and they appear to be about as accurate in most cases as the B-constant method.

Calvert and Sze (5, 6) reported in two papers the first results of their project to study the minimization of losses in power systems. Feeling that the methods based on B-constants introduce errors in the magnitude of losses which are difficult to bound, they chose to approach the problem by defining the power inputs to the network in terms of the vector voltages and currents at every load and generator terminal. Most of these voltages and currents are fixed by the operating conditions imposed on the network, but either the voltage or the current at the generators to be controlled is variable. The method of solution is to set partial derivatives of the total system input with respect to the variable generator currents (or voltages) equal to zero and to solve simultaneously the resulting linear equations. These equations include terms expressing restrictions on the watt and var flows of the loads and fixed generators inserted by means of Lagrange multipliers. Evaluation of this work depends on further application of the method to practical examples. The method is subject to some of the same criticisms as other methods, in that the behavior of the machines must be linearized and it is assumed that the phase angle position of the various buses remains fixed. Calvert and Sze offer a three-machine example with comparison of results previously obtained by Dandeno, but no other investigators have used or commented on this method.

III. THEORETICAL CONSIDERATIONS

A. Development of Criteria for Minimum Power Input

To develop criteria for minimum system input, consider the system to be a network of constant impedances with generators applied to each of n terminals. At each of these terminals the magnitude of voltage will be specified by considerations of the voltages required at the loads, or by adjustments from these values to compensate for the inability of certain generators to supply enough vars to maintain the desired voltage levels. Assume that the fuel input to each generator unit or plant can be expressed as a function of the generator power output, and this relationship is not a function of the var output. This will be necessary to permit scheduling the var output of a generator to meet system voltage requirements, and it will be tenable if it may be assumed that the I^2R losses in the generator are small in comparison with the losses in the mechanical portion of the plant.

Among the n terminals of the network there are $\frac{n(n-1)}{2}$ transfer impedances which may be represented as shown in Figure 1. The transfer impedance is the element Z, and the elements Y_1 and Y_2 are the shunt admittances resulting from a reduction of the original network shunt capacitors, line-charging capacitances, and loads. These two elements will, in general, not be equal.

The expressions for transmitted watts and vars are conveniently expressed in terms of the ABCD constants for the equivalent line section of Figure 1. The conversion from the constants shown to ABCD values is given by the following equations:

$$A = 1 + ZY_2 \qquad = A \underline{/\alpha} \tag{17}$$

$$B = Z \qquad = B \underline{/\beta} \tag{18}$$

$$C = (Y_1 + Y_2) + ZY_1Y_2 = C \underline{/\gamma} \tag{19}$$

$$D = 1 + ZY_1 \qquad = D \underline{/\Delta}. \tag{20}$$

If E_S and E_R designate the voltages at the sending end and receiving end of the line, then the expressions for sending-end and receiving-end power and reactive power may be written in terms of these voltages, the ABCD constants, and the angle δ representing the voltage phase angle by which E_S leads E_R.

$$P_S = -\frac{E_S E_R}{B} \cos(\beta + \delta) + \frac{DE_S^2}{B} \cos(\beta - \Delta) \tag{21}$$

$$P_R = \frac{E_S E_R}{B} \cos(\beta - \delta) - \frac{AE_R^2}{B} \cos(\beta - \alpha) \tag{22}$$

$$Q_S = -\frac{E_S E_R}{B} \sin(\delta + \beta) + \frac{DE_S^2}{B} \sin(\beta - \Delta) \tag{23}$$

$$Q_R = -\frac{E_S E_R}{B} \sin(\delta - \beta) - \frac{AE_R^2}{B} \sin(\beta - \alpha). \tag{24}$$

The signs in the expressions for Q_S and Q_R are such that lagging vars are positive.

The power flows at terminal i may be designated as shown in Figure 2. P_{Li} is the equivalent local load at station i, and this load may be defined in such a way that the handling of the shunt admittances shown in Figure 1 may be simplified. It will be seen that the significant information arising from the equivalent circuit of Figure 1 is the amount of power actually transferred between pairs of terminals, and the sending-end shunt

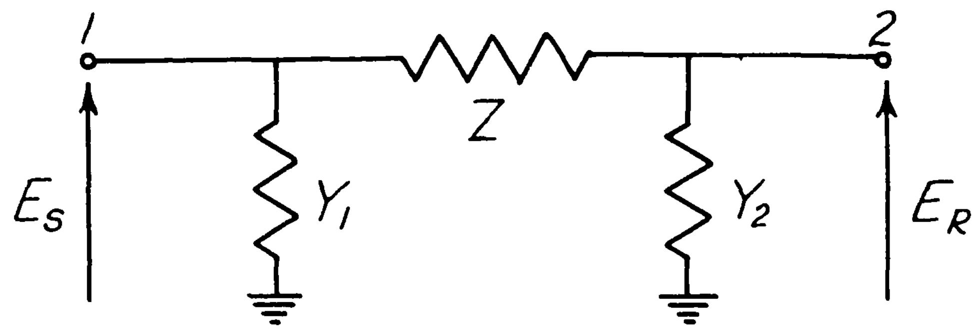


Figure 1. Equivalent circuit resulting from reduction of network between terminals 1 and 2.

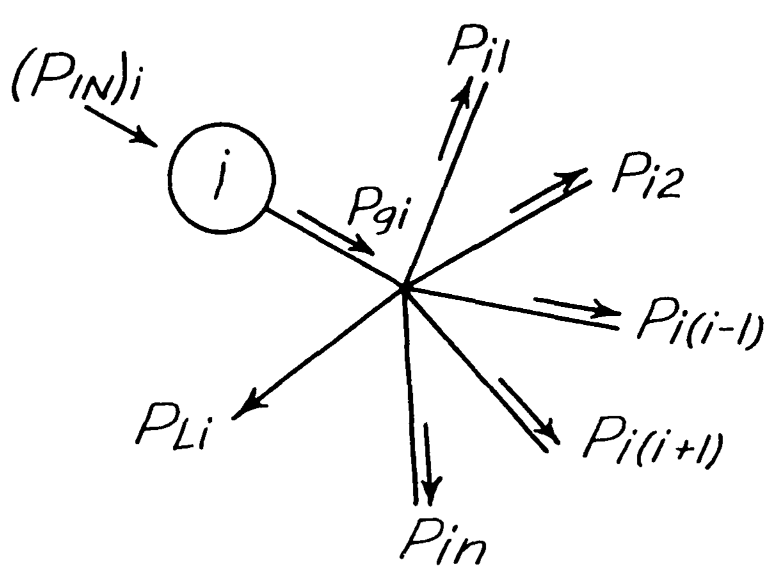


Figure 2. Power flows at terminal i of an n-terminal network.

elements of all the lines terminating at i may be combined with the admittance of the actual local load, giving an equivalent local load P_{Li} which includes the local shunt admittances of the lines terminating at i.

This definition of P_{Li} is consistent with the obvious method of measuring transfer and shunt impedances on a network analyzer. If all foreign terminals are short-circuited and a voltage applied to terminal i, then each of the transfer impedances is given by measuring the ratio of this test voltage to the short-circuit currents at each of the foreign terminals. At the same time, the amount by which the current drawn from the test generator exceeds the sum of the short-circuit currents divided by the test voltage is the admittance of the equivalent local load P_{Li}.

The generator output of station i is P_{gi}. The cost of operating this plant consists of fixed charges for overhead, maintenance, and depreciation plus variable charges for fuel depending upon the type of fuel, the cost of this fuel, and the efficiency with which the plant utilizes its fuel at varying loads. Only the variable fuel costs affect the economic loading problem. The rate at which money is spent for fuel input is denoted $(P_{IN})_i$. The units of this fuel input are monetary units per unit of time. The relationship between this quantity and the generator output is

$$(P_{IN})_i = C_i P_{gi} \tag{25}$$

where C_i is a cost function in monetary units per unit of power per unit of time.

The symbol for fuel input has been given capital subscripts and has been placed in parenthesis to distinguish it from the power flow P_{in}. P_{in} is the power transmitted through the transfer impedance B_{in} from plant i to plant n. This may be written from the general formulae given above as

$$P_{in} = -\frac{E_i E_n}{B_{in}} \cos(\beta_{in} + \delta_{in}) + \frac{D_{in} E_i^2}{B_{in}} \cos(\beta_{in} - \Delta_{in}) \tag{26}$$

where δ_{in} is defined as $\delta_i - \delta_n$. It follows that

$$\delta_{ni} = -\delta_{in}, \tag{27}$$

but β_{in} is the impedance angle of B_{in} and

$$\beta_{ni} = \beta_{in}. \tag{28}$$

Let

$$P_i = P_{i1} + P_{i2} + \cdots + P_{in} \tag{29}$$

represent the sum of all line flows away from station i. Note that there are (n - 1) terms since P_{ii} is not defined. P_{Li} is the local load at station i, hence

$$P_{gi} - P_{Li} = P_i \tag{30}$$

and

$$(P_{IN})_i = C_i(P_{Li} + P_i). \tag{31}$$

The total input to the system is the sum of these $(P_{IN})_i$, or

$$P_t = \sum_{i=1}^{n} (P_{IN})_i = \sum_{i=1}^{n} C_i(P_{Li} + P_i). \tag{32}$$

To find a minimum value of P_t, it is necessary that the partial derivatives of P_t with respect to each of the variables on which it depends be equal to zero. P_t can be expressed in terms of E_j and δ_j, hence it is necessary that

$$\frac{\partial P_t}{\partial E_j} = 0 \tag{33}$$

$$\frac{\partial P_t}{\partial \delta_j} = 0. \tag{34}$$

The values of E_j, however, are usually held within narrow limits. Unless there are system losses that vary with E_j, it seems reasonable that P_t would have a minimum value for very high values of E_j. This assumes the loads demand constant power. Since E_j is restricted, it will not generally be possible to find a true minimum for P_t, but only a relative minimum with respect to δ_j. This will, in general, be the most practical approach to the problem, since the variations of power depend on angular variations to the first degree for small changes in angle when the levels of voltage have been determined. It is true, within the range of angles permitted, that the power flow between two units will be increased by increasing their angular separation. Thus an inspection of the value and sign of the partial derivatives of P_t with respect to each angle will indicate an approximate amount and the direction of change to be made in the angle for a better relative minimum for P_t. Should this partial derivative be negative, for example, an increase in the value of angle is indicated.

The desired partial derivatives may be developed by expanding the following expression:

$$\frac{\partial P_t}{\partial \delta_j} = \frac{\partial}{\partial \delta_j} \sum_{i=1}^{n} C_i (P_{Li} + P_i)$$

$$= \sum_{i=1}^{n} \left[C_i \frac{\partial P_i}{\partial \delta_j} + (P_{Li} + P_i) \frac{\partial C_i}{\partial \delta_j} \right]. \tag{35}$$

C_i, however, is not a function of δ_j, but it is a function of P_{gi} which is in turn a function of δ_j. Hence it is necessary to write

$$\frac{\partial C_i}{\partial \delta_j} = \frac{dC_i}{dP_{gi}} \frac{\partial P_{gi}}{\partial \delta_j} \tag{36}$$

but

$$\frac{\partial P_{gi}}{\partial \delta_j} = \frac{\partial P_i}{\partial \delta_j} \tag{37}$$

since P_{Li} does not depend on δ_j.

The partial derivatives of P_t may now be written:

$$\frac{\partial P_t}{\partial \delta_j} = \sum_{i=1}^{n} \left[C_i \frac{\partial P_i}{\partial \delta_j} + P_{gi} \frac{dC_i}{dP_{gi}} \frac{\partial P_i}{\partial \delta_j} \right]$$

$$= \sum_{i=1}^{n} \frac{\partial P_i}{\partial \delta_j} \left[C_i + P_{gi} \frac{dC_i}{dP_{gi}} \right]$$

$$= \sum_{i=1}^{n} \xi_i \frac{\partial P_i}{\partial \delta_j} \tag{38}$$

where ξ_i is the sum in the brackets. This ξ_i is the incremental production rate of station i as defined in the literature. That this is so can be shown by differentiating the relationship between $(P_{in})_i$ and P_{gi}:

$$(P_{IN})_i = C_i P_{gi} \tag{39}$$

$$\xi_i = \frac{d(P_{IN})_i}{dP_{gi}} = C_i + P_{gi} \frac{dC_i}{dP_{gi}}. \tag{40}$$

Expansion of the derivatives, $\frac{\partial P_t}{\partial \delta_j}$, will reveal the technique to be recommended for analyzing the system and detecting the desired minimum condition. This can be done first for a three-machine system, and later generalities can be drawn for more than three machines. As an example of the expansion of $\frac{\partial P_i}{\partial \delta_j}$, let i = 1, 2, 3 in turn and let j = 1. The expansion is:

$$\frac{\partial P_1}{\partial \delta_1} = \frac{E_1 E_2}{B_{12}} \sin (\beta_{12} + \delta_{12}) + \frac{E_1 E_3}{B_{13}} \sin (\beta_{13} + \delta_{13}) \tag{41}$$

$$\frac{\partial P_2}{\partial \delta_1} = - \frac{E_2 E_1}{B_{21}} \sin (\beta_{21} + \delta_{21}) \tag{42}$$

$$\frac{\partial P_3}{\partial \delta_1} = - \frac{E_3 E_1}{B_{31}} \sin (\beta_{31} + \delta_{31}). \tag{43}$$

A similar set of derivatives may be obtained for $j = 2, 3$. Note that when $i = j$ all the terms in the expansion are positive, and there is a term relating plant i to each of the others. When $i \neq j$ the terms are negative as a result of the definition $\delta_{ni} = -\delta_{in}$. Also, there is only one term relating each plant j to the reference plant i.

The set of derivatives, $\frac{\partial P_t}{\partial \delta_j}$, for $j = 1, 2, 3$ may now be written out and each equated to zero:

$$\begin{aligned} \frac{\partial P_t}{\partial \delta_1} = 0 = {} & \xi_1 \left[\frac{E_1 E_2}{B_{12}} \sin (\beta_{12} + \delta_{12}) + \frac{E_1 E_3}{B_{13}} \sin (\beta_{13} + \delta_{13}) \right] \\ & - \xi_2 \frac{E_2 E_1}{B_{21}} \sin (\beta_{21} + \delta_{21}) \\ & - \xi_3 \frac{E_3 E_1}{B_{31}} \sin (\beta_{31} + \delta_{31}) \end{aligned} \tag{44}$$

$$\begin{aligned} \frac{\partial P_t}{\partial \delta_2} = 0 = {} & - \xi_1 \frac{E_1 E_2}{B_{12}} \sin (\beta_{12} + \delta_{12}) \\ & + \xi_2 \left[\frac{E_2 E_1}{B_{21}} \sin (\beta_{21} + \delta_{21}) + \frac{E_2 E_3}{B_{23}} \sin (\beta_{23} + \delta_{23}) \right] \\ & - \xi_3 \frac{E_3 E_2}{B_{32}} \sin (\beta_{32} + \delta_{32}) \end{aligned} \tag{45}$$

$$\begin{aligned} \frac{\partial P_t}{\partial \delta_3} = 0 = {} & - \xi_1 \frac{E_1 E_3}{B_{13}} \sin (\beta_{13} + \delta_{13}) \\ & - \xi_2 \frac{E_2 E_3}{B_{23}} \sin (\beta_{23} + \delta_{23}) \\ & + \xi_3 \left[\frac{E_3 E_1}{B_{31}} \sin (\beta_{31} + \delta_{31}) + \frac{E_3 E_2}{B_{32}} \sin (\beta_{32} + \delta_{32}). \right] \end{aligned} \tag{46}$$

Simultaneous solution of these three equations will yield the critical point or desired minimum value of P_t. These are the criteria for economic loading of the system.

B. Method of Solution for Critical Point

For the purpose of analyzing equations 44, 45, and 46, let

$$X_{12} = \xi_1 \frac{E_1 E_2}{B_{12}} \sin (\beta_{12} + \delta_{12}) \tag{47}$$

$$X_{13} = \xi_1 \frac{E_1 E_3}{B_{13}} \sin (\beta_{13} + \delta_{13}) \tag{48}$$

$$X_{21} = \xi_2 \frac{E_2 E_1}{B_{21}} \sin (\beta_{21} + \delta_{21}) \tag{49}$$

$$X_{23} = \xi_2 \frac{E_2 E_3}{B_{23}} \sin (\beta_{23} + \delta_{23}) \tag{50}$$

$$X_{31} = \xi_3 \frac{E_3 E_1}{B_{31}} \sin (\beta_{31} + \delta_{31}) \tag{51}$$

$$X_{32} = \xi_3 \frac{E_3 E_2}{B_{32}} \sin (\beta_{32} + \delta_{32}). \tag{52}$$

This set of substitutions will permit rewriting the minimum conditions in this form:

$$X_{12} + X_{13} - X_{21} - X_{31} = 0 \tag{53}$$

$$-X_{12} + X_{21} + X_{23} - X_{32} = 0 \tag{54}$$

$$- X_{13} + X_{31} - X_{23} + X_{32} = 0. \tag{55}$$

Note that there are six terms to be dealt with, and each of them appears in each of two equations. These terms may be paired in a particular way on the basis of the interpretation that may be placed on them. Terms X_{12} and X_{21}, for example, refer to the relationship existing between machines 1 and 2, X_{13} and X_{31} refer to a similar relationship between 1 and 3, and X_{23} and X_{32} to 2 and 3. The next step is to write these equations in this form:

$$(X_{12} - X_{21}) - (X_{31} - X_{13}) = 0 \tag{56}$$

$$-(X_{12} - X_{21}) + (X_{23} - X_{32}) = 0 \tag{57}$$

$$(X_{31} - X_{13}) - (X_{23} - X_{32}) = 0. \tag{58}$$

The original equations 44, 45, and 46, are now represented as a set of three linear homogeneous equations in three unknowns. The rank of the matrix of the coefficients of these unknowns is one less than the number of equations, indicating that there are an infinity of solutions of these equations other than the trivial one. Further, any two of the unknowns may be expressed in terms of the third. Explicitly,

$$(X_{12} - X_{21}) = (X_{31} - X_{13}) = (X_{23} - X_{32}). \tag{59}$$

If the trivial solution were the correct choice, then each of the unknowns would be zero and the criteria for a minimum of P_t could be simplified by cancelling out the system voltages and transfer impedance magnitudes, giving the following set of conditions to be met:

$$\xi_1 \sin (\beta_{12} + \delta_{12}) = \xi_2 \sin (\beta_{21} + \delta_{21}) \tag{60}$$

$$\xi_2 \sin (\beta_{23} + \delta_{23}) = \xi_3 \sin (\beta_{32} + \delta_{32}) \tag{61}$$

$$\xi_3 \sin (\beta_{31} + \delta_{31}) = \xi_1 \sin (\beta_{13} + \delta_{13}). \tag{62}$$

These criteria are similar to the results obtained by Brownlee for a two-machine system. For two machines, only the first expression would have any significance and the other two would not exist. The equivalence of the first criterion with Brownlee's formula, equation 16, may be demonstrated by means of trigonometric identities.

If these criteria were valid for a three-machine system, then application of the first and second would give economic ratios $\frac{\xi_1}{\xi_2}$ and $\frac{\xi_2}{\xi_3}$ without any consideration of the third criterion. The third ratio $\frac{\xi_3}{\xi_1}$, however, has been fixed by the first two criteria, but this will not necessarily satisfy the third criterion since it contains the impedance angle β_{13} while the first two criteria do not.

Table I shows calculated values of $\frac{\xi_1}{\xi_2} = \frac{\sin (\beta_{21} + \delta_{21})}{\sin (\beta_{12} + \delta_{12})}$ for various values of δ_{12} with β_{12} as a parameter. These values are plotted in Figure 3. Each of the curves becomes zero when $\beta_{12} = \delta_{12}$, which is the condition for maximum received power through β_{12}. The curve for $\beta_{12} = 90°$ is a horizontal line since this condition implies a lossless line and the two plants should operate at the same incremental rate for any angle δ_{12}. All the curves pass through $\frac{\xi_1}{\xi_2} = 1$, $\delta_{12} = 0$, implying the special case that the two plants should operate at the same incremental rate if there is no tie-line flow regardless of the value β_{12} of the line.

The possibility of satisfying the criteria for three machines can be further explored by noting that

$$\frac{\xi_1}{\xi_2} \times \frac{\xi_2}{\xi_3} = \frac{\xi_1}{\xi_3} \tag{63}$$

and

$$\delta_{12} + \delta_{23} + \delta_{31} = 0 \tag{64}$$

must hold for any selected values of β_{12}, β_{23}, and β_{31}. Suppose, for

Table I. Economic incremental fuel rate ratios $\frac{\xi_1}{\xi_2}$ for a two-machine system

δ_{12}	β_{12}							
	30°	40°	50°	60°	70°	80°	85°	90°
0°	1	1	1	1	1	1	1	1
2°	0.885	0.920	0.945	0.960	0.975	0.990	0.995	1
4°	0.785	0.845	0.890	0.922	0.950	0.975	0.990	1
6°	0.692	0.779	0.838	0.885	0.927	0.965	0.982	1
8°	0.608	0.712	0.788	0.850	0.903	0.950	0.976	1
10°	0.532	0.653	0.742	0.815	0.880	0.940	0.970	1
15°	0.366	0.515	0.633	0.732	0.822	0.910	0.955	1
20°	0.201	0.395	0.532	0.652	0.765	0.880	0.936	1
25°	0.096	0.285	0.437	0.575	0.710	0.848	0.922	1
30°	0	0.185	0.347	0.500	0.652	0.815	0.905	1
40°		0	0.174	0.348	0.532	0.742	0.855	1
50°			0	0.185	0.395	0.653	0.811	1
60°				0	0.227	0.532	0.737	1
70°					0	0.348	0.612	1
80°						0	0.336	1
85°							0	1
90°								1

example, that

$$\beta_{12} = 50° \qquad \delta_{12} = 8°$$

$$\beta_{23} = 60° \qquad \delta_{23} = 11°$$

$$\beta_{31} = 70° \qquad \delta_{31} = -19°.$$

From the curves,

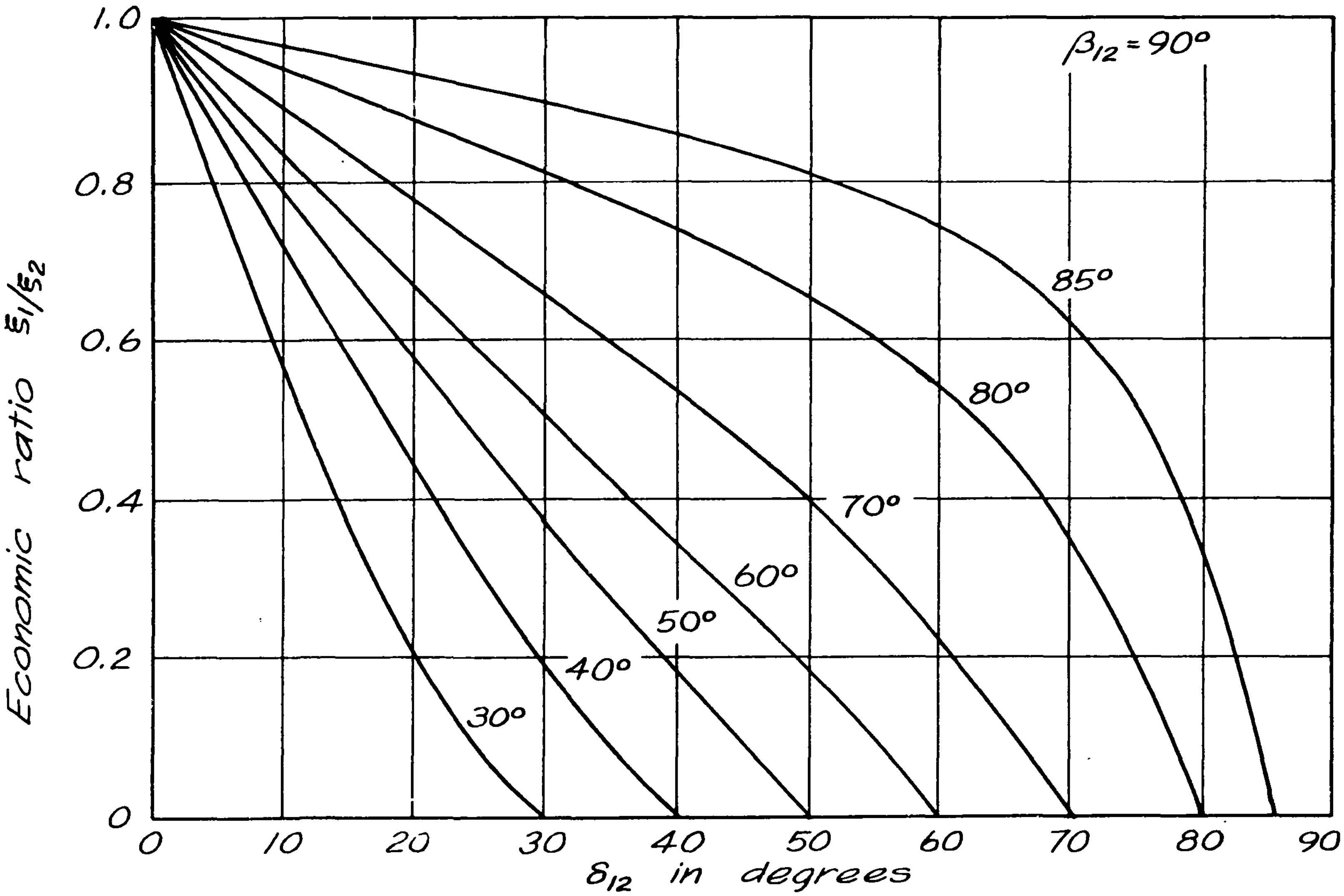


Figure 3. Economic incremental fuel rate ratio for a two-machine system.

$$\frac{\xi_1}{\xi_2} = \frac{\xi_2}{\xi_3} = 0.80.$$

It would follow from these values that $\frac{\xi_1}{\xi_3}$ should be 0.64, but the curve for the assumed value of β_{31} at the value $\delta_{13} = 1.90$ gives 0.78, which is a contradiction.

It must be concluded that the criteria given by equations 60, 61, and 62 will not hold in general, but they will give the conditions for minimum P_t in special cases. If all of the elements B_{in} have the same value β_{in}, then these criteria will hold for small angles δ_{in}. Inspection of the data in Table I will show that they will hold for values of δ_{in} less than 10 degrees for any value of β_{in}. For increasing values of β_{in} the range of values of δ_{in} increases until at the limit $\beta_{in} = 90°$ any value of δ_{in} may be used.

In general, the desired solution of equations 56, 57, and 58 is non-trivial, and each of the terms of equation 59 has a value not equal to zero. Therefore equations 44, 45, and 46 must be used to determine whether a minimum value of P_t has been achieved, or to indicate the direction of change of each δ_j to move toward the minimum. Equation 59 gives assurance that all three of the derivatives can be set equal to zero at the minimum.

C. Nature of the Critical Point

Evidence that the condition obtained is a minimum can be obtained by examining the second derivatives of P_t. By differentiating equation 38,

$$\frac{\partial^2 P_t}{\partial \delta_j^{\,2}} = \sum_{i=1}^{n} \left[\xi_i \frac{\partial^2 P_i}{\partial \delta_j^{\,2}} + \frac{\partial \xi_i}{\partial \delta_j} \frac{\partial P_i}{\partial \delta_j} \right]$$

$$= \sum_{i=1}^{n} \left[\xi_i \frac{\partial^2 P_i}{\partial \delta_j^{\,2}} + \frac{d \xi_i}{dP_{gi}} \frac{\partial P_{gi}}{\partial \delta_j} \frac{\partial P_i}{\partial \delta_j} \right.$$

$$= \sum_{i=1}^{n} \left[\xi_i \frac{\partial^2 P_i}{\partial \delta_j^{\,2}} + \frac{d \xi_i}{dP_{gi}} \left(\frac{\partial P_i}{\partial \delta_j} \right)^2 \right]. \tag{65}$$

The second term in the brackets will always be positive since ξ_i always increases with P_{gi}. To determine the nature of the first term, perform the indicated operations for j = 1 and n = 3:

$$\sum_{i=1}^{n} \xi_i \frac{\partial^2 P_i}{\partial \delta_1^{\,2}} = \xi_1 \left[\frac{E_1E_2}{B_{12}} \cos\,(\beta_{12} + \delta_{12}) + \frac{E_1E_3}{B_{13}} \cos\,(\beta_{13} + \delta_{13}) \right]$$

$$+ \xi_2 \frac{E_2E_1}{B_{21}} \cos\,(\beta_{21} + \delta_{21})$$

$$+ \xi_3 \frac{E_3E_1}{B_{31}} \cos\,(\beta_{31} + \delta_{31}). \tag{66}$$

The expressions for j = 2 or 3 will be similar. The first and third and the second and fourth terms should be compared to show that

$$\frac{E_1E_2}{B_{12}} \left[\xi_1 \cos\,(\beta_{12} + \delta_{12}) + \xi_2 \cos\,(\beta_{21} + \delta_{21}) \right] > 0 \tag{67}$$

$$\frac{E_1E_3}{B_{13}} \left[\xi_1 \cos\,(\beta_{13} + \delta_{13}) + \xi_3 \cos\,(\beta_{31} + \delta_{31}) \right] > 0. \tag{68}$$

Power will be transmitted from the plant with the lowest value of ξ toward those with higher values, and this requires that the angular position of the most efficient plant be ahead of the others. Thus, if ξ_1 is

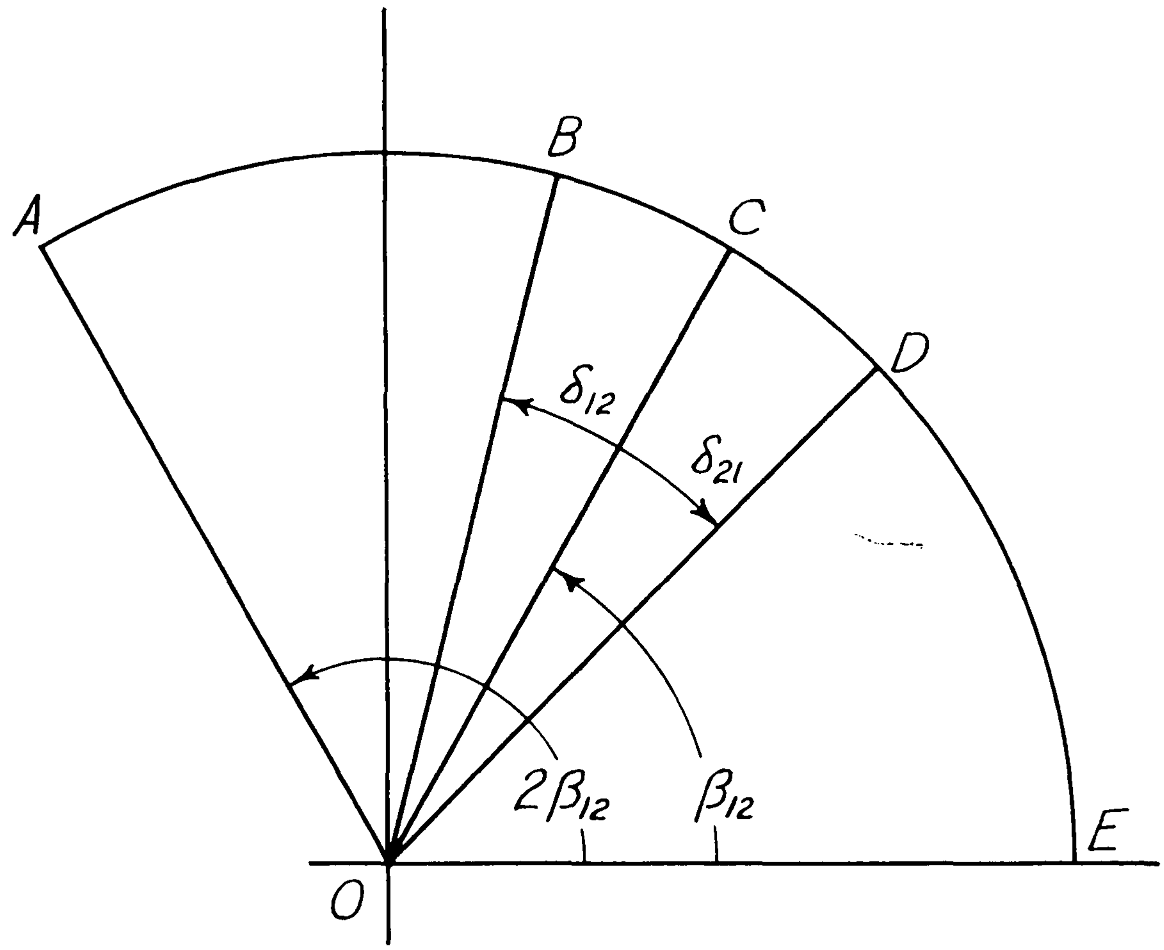


Figure 4. Permissible angles and related functions.

less than ξ_2, then δ_{12} is a positive angle and power flows toward plant 2. The limiting value of δ_{12} is equal to β_{12}, giving maximum received power at plant 2.

Figure 4 shows the permissible values of the cosine functions in equation 71. If the circle has unit radius, the projections of OB and OD on OE are the required cosine terms. OA is the limit for increasing values of δ_{12}, and if β_{12} approaches 90 degrees the angle $(\beta_{12} + \delta_{12})$ may approach 180 degrees as a limit. Even though OB might lie in the second quadrant it is obvious that

$$|\cos\ (\beta_{12} + \delta_{12})| < |\cos\ (\beta_{21} + \delta_{21})| \tag{69}$$

for all permissible angles, δ_{12}. It follows that

$$|\xi_1 \cos\ (\beta_{12} + \delta_{12})| < |\xi_2 \cos\ (\beta_{21} + \delta_{21})| \tag{70}$$

since it was assumed that

$$\xi_1 < \xi_2. \tag{71}$$

As a consequence of equation 70, equation 67 must be true, and by similar reasoning all such pairs of terms in 65 are always positive. Thus the second partial derivatives of P_t are always positive for all permissible angles. This is a necessary condition that the function P_t be at a minimum at the critical point, although it is not sufficient except for a two-machine system. For more machines it will be necessary, in a practical sense, to demonstrate that any digression from the critical point always gives higher values of P_t, regardless of the direction of the digression. This can be seen in the examples of the experimental work.

D. Generalization for More than Three Machines

The economic loading criteria as given by equations 44, 45, and 46 for three machines may be extended to apply to any number of machines. If there are n machines, there will be n partial derivatives of P_t, one with respect to each of the n voltage phase angles δ_j. All of the terms in these derivatives are of the general form

$$X_{ji} = \xi_j \frac{E_j E_i}{B_{ji}} \sin (\beta_{ji} + \delta_{ji}) \tag{72}$$

with the restriction that j can never be equal to i. Equations 47 through 52 are expansions of this form for three machines, and equations 56 through 58 demonstrate the manner in which each of these terms appears twice in a set of criteria. Furthermore, they may always be gathered together in such a way as to permit the generalization:

$$\frac{\partial P_t}{\partial \delta_j} = \sum_{\substack{i=1 \\ i \neq j}}^{n} (X_{ji} - X_{ij}) = 0. \tag{73}$$

IV. EXPERIMENTAL RESULTS

A. Preparation of Network Data

The literature contains several references to the question of the disposition of the loads in the network in determining transfer impedances or admittances. It has been suggested that these impedances be measured with all loads and shunt line elements removed from ground, but it can be shown that this is not correct. The discussion of Brownlee's work included criticism that his method did not properly account for loads between generators, or in the middle of line sections. The question of how loads should be represented has also been raised, since it is common practice to assume that power system loads will demand constant values of watts and vars regardless of the voltage level at which they operate.

The most practical method of determining transfer impedances is to measure them on a network analyzer. This should be done after the system has been balanced for a particular load condition and all loads trimmed to the desired levels of watts and vars. After this balance has been achieved, it must be assumed that the loads will thereafter be represented by the board values of admittance. The entire network must be considered passive if the transfer impedances are to have any meaning. The transfer impedances are the ratios of the various generator voltages applied one at a time to the short-circuit currents resulting at all the other generator terminals in the network. Successive application of generators to the network will result in every impedance being measured twice which will provide a check on the measurements.

To illustrate the effect of a load at the center of a line, consider a simple system consisting of two short lines of impedance Z with a load of admittance Y between them. Refer to Figure 5.

This system may be considered to be three four-terminal networks in cascade, and the ABCD matrix of the combination may be found by multiplying the ABCD matrices of the three parts, thus:

$$\begin{Vmatrix} 1 & Z \\ 0 & 1 \end{Vmatrix} \times \begin{Vmatrix} 1 & 0 \\ Y & 1 \end{Vmatrix} \times \begin{Vmatrix} 1 & Z \\ 0 & 1 \end{Vmatrix} = \begin{Vmatrix} 1 + ZY & 2Z + Z^2Y \\ Y & 1 + ZY \end{Vmatrix} . \tag{74}$$

The transfer impedance of the combination is the element

$$B = 2Z + Z^2Y \tag{75}$$

and this differs from 2Z, the transfer impedance in the absence of Y, by the amount Z^2Y. Thus not only the amount but also the direction of the change in transfer impedance is affected by Y. For the case of a resistive load, Y would be real and positive, and the transfer impedance would be greater than for no load. Should Z be inductive and Y capacitive, then it is possible that the transfer impedance might be less than the value without Y connected. Since Y does have an effect the measurement technique must encompass the shunt elements, and there can be no doubt that they should remain in the network while measurements are being made.

The effect of a tap changer on transfer impedance is illustrated in Figure 6. The tap changer and the line element Z may each be considered a four-terminal network element, and they may be connected in cascade in either order as shown. The ABCD matrix of each connection is given by

$$\begin{Vmatrix} \frac{1}{a} & 0 \\ 0 & a \end{Vmatrix} \times \begin{Vmatrix} 1 & Z \\ 0 & 1 \end{Vmatrix} = \begin{Vmatrix} \frac{1}{a} & \frac{Z}{a} \\ 0 & a \end{Vmatrix} \tag{76}$$

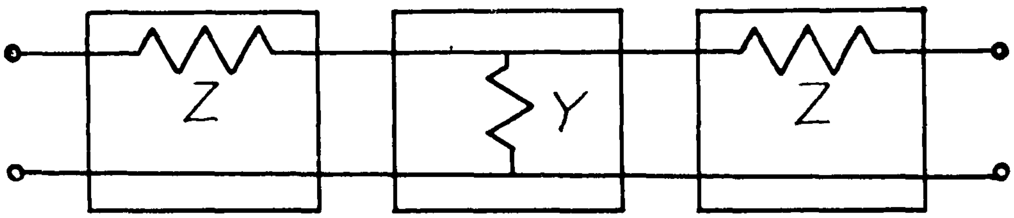


Figure 5. Equivalent circuit for a transmission line loaded at its midpoint.

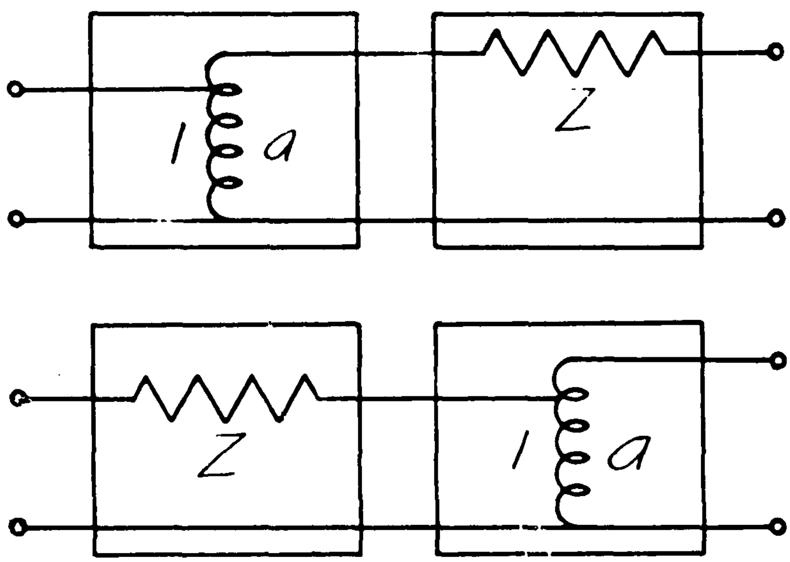


Figure 6. Equivalent circuit for a line and tap-changer in cascade.

$$\begin{Vmatrix} 1 & Z \\ & \\ 0 & 1 \end{Vmatrix} \times \begin{Vmatrix} \frac{1}{a} & 0 \\ & \\ 0 & a \end{Vmatrix} = \begin{Vmatrix} \frac{1}{a} & aZ \\ & \\ 0 & a \end{Vmatrix} . \tag{77}$$

Thus the transfer impedance of the combination is either Z/a of aZ, depending on the relative location of the two elements. It can be shown, however, that a measurement of applied voltage divided by short-circuit current will give the correct transfer impedance in every case, including the possibility of looking into either end of either combination. This leads to the generalization that tap changers should be left in the network while measurements are being made, for the results will reflect the ability of these units to improve system operation.

While data are being collected to evaluate the transfer impedances, data should be obtained from the same analyzer representation to evaluate the shunt impedances (or admittances) to represent the loads on the system. The amount by which the current supplied by each generator exceeds the sum of the short-circuit currents at all other terminals is a measure of the load to be applied at that generator terminal. This excess current divided by the test voltage is the admittance of the load, or its reciprocal is the shunt impedance to be used to represent the load at that terminal. These local loads do not appear explicitly in the economic criteria, but they are required to determine the incremental rate ξ_i of each generator. In general, if a plant has a relatively large local load its incremental rate will be increased so that it cannot economically transmit as much power toward other terminals in the system as it could if the local load were not present.

If a load were located at the mid-point of a uniform line, then half of that load would be placed at each end of the equivalent line. In any other case, more of the load will appear at whichever terminal was electrically closer to the load. In simple cases, the determination of both transfer and shunt impedances can be accomplished by means of wye-delta network conversions.

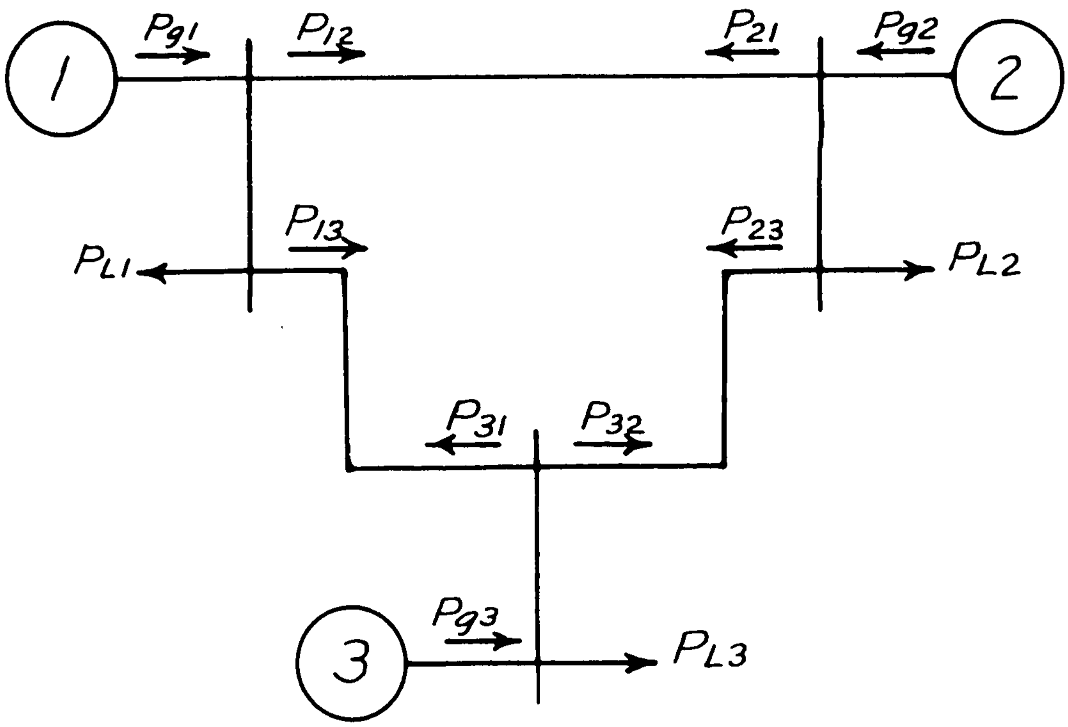
For the purposes of this thesis, it will be assumed that the impedances of the network are known. Figure 7 shows the system assumed for examples 1, 2, and 3, and Figure 8 shows the additional data assumed for examples 4 through 7. The data for these examples are all per unit values.

B. Preparation of Fuel Input Data

To describe each of the plants, it is necessary to evaluate the cost input to the plant as a function of the generator output. This cannot be done graphically because of the degree of precision required in applying the economic criteria. Although there are errors in the determination of the input, it must be assumed that the input data are as accurate as possible. Because of the sensitivity of the criteria the input-output function must include more significant figures than might be warranted in other problems. The best method is to perform a curve-fitting technique to selected data points along the input-output curve. Since a typical input-output curve increases with output at an increasing rate the best functional relationship to apply is a power series. In general, there will be as many terms in this power series as there are data points for its determination, including a constant term to represent the input at zero output. In practice, a third-order power series should be adequate for most plants, re-

$$B_{12} = 0.10 + j0.20 = 0.22361\angle 63^\circ 26'$$

Figure 7. Two-machine system of examples 1, 2, and 3.



$$B_{12} = 0.10 + j0.20 = 0.22361\underline{/63^\circ 26'}$$
$$B_{23} = 0.15 + j0.25 = 0.29155\underline{/59^\circ 2'}$$
$$B_{31} = 0.20 + j0.30 = 0.36056\underline{/56^\circ 19'}$$

Figure 8. Three-machine system of examples 4, 5, 6, and 7.

quiring three carefully-evaluated data points along the curve in addition to the intercept at zero output. By the nature of curve-fitting, the power series will give exactly the input when the output corresponds to one of the data points, and it will give the input to a high degree of precision and reasonable accuracy at any intermediate value of output.

Having expressed the input-output curve by means of a power series, the incremental rate of that plant can be determined by differentiating this series with respect to the output. The result will be another power series of order one less than the input curve, and of the same degree of precision. This technique is much more precise than any graphical method. There can be no doubt that the input-output curve and the incremental rate curve derived from it correspond.

Input-output curves and the corresponding incremental rate curves for three plants selected for this thesis are shown in Figure 9. Plant 1 has the highest no-load input, but it has the lowest incremental rate curve. This will serve to accent, in the examples, that the incremental rate is of vital importance in predicting how much load a plant should be assigned while the input curve itself will not reveal how to apportion the load.

The expressions for input and incremental rate as functions of output for these three plants are:

$$(P_{IN})_1 = 2.28 + 0.520P_{g1} + 0.380P_{g1}^{\ 2} + 0.040P_{g1}^{\ 3} \tag{78}$$

$$(P_{IN})_2 = 1.59 + 0.75333P_{g2} + 0.440P_{g2}^{\ 2} + 0.02667P_{g2}^{\ 3} \tag{79}$$

$$(P_{IN})_3 = 1.04 + 1.16333P_{g3} + 0.84002P_{g3}^{\ 2} - 0.01333P_{g3}^{\ 3} \tag{80}$$

$$\xi_1 = 0.520 + 0.760P_{g1} + 0.120P_{g1}^{\ 2} \tag{81}$$

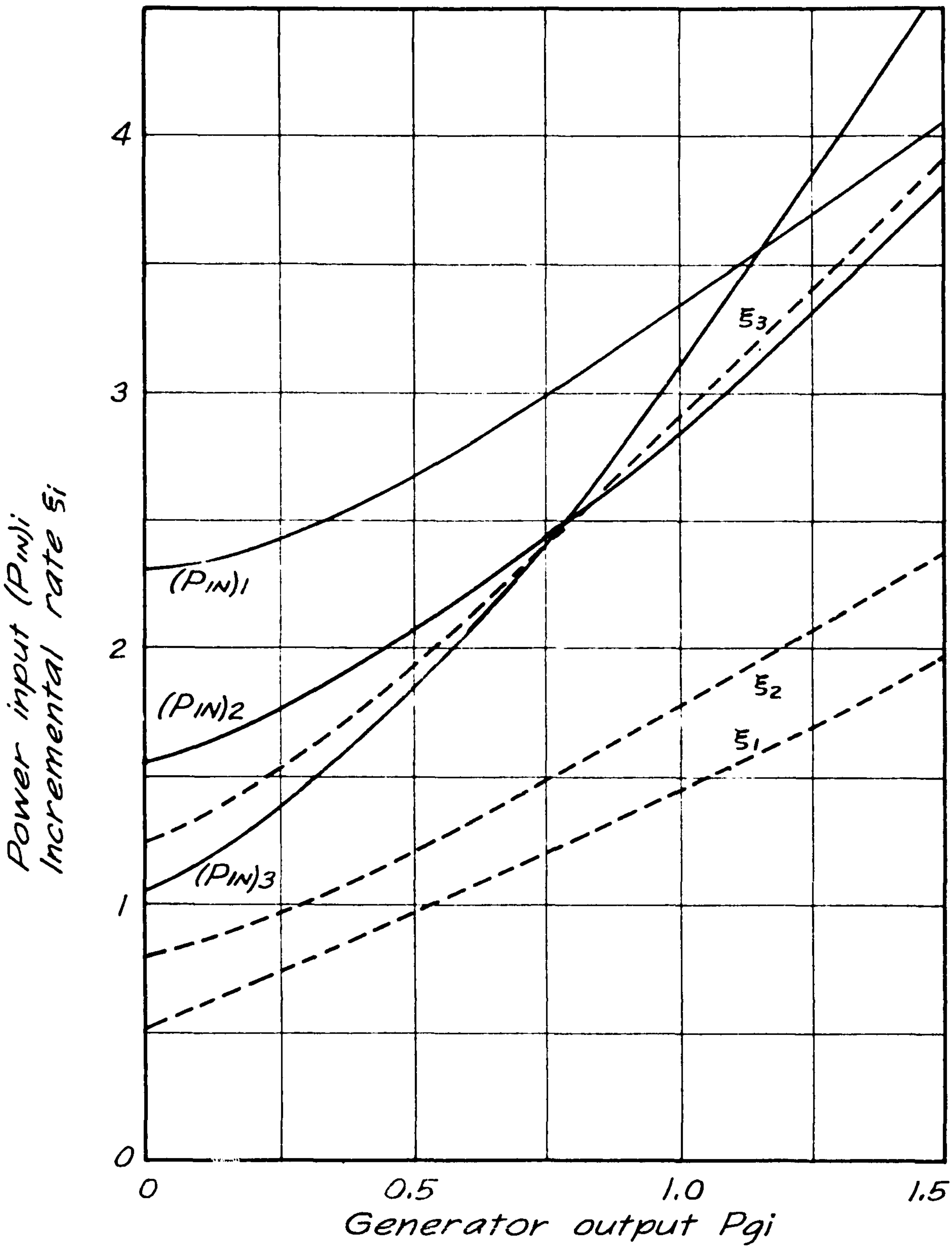


Figure 9. Input-output and incremental rate curves.

$$\xi_2 = 0.75333 + 0.880P_{g2} + 0.080P_{g2}^2 \tag{82}$$

$$\xi_3 = 1.16333 + 1.68003P_{g3} - 0.040P_{g3}^2. \tag{83}$$

C. Technique of Digital Computation

The practical solution of the criteria, equations 44, 45, and 46, is a series of trial evaluations of the partial derivatives, $\frac{\partial P_t}{\partial \delta_j}$. The criteria are not a set of independent equations. In general, for an n-generator system there will be n equations, but only (n - 1) of them will be independent. The necessary nth equation is always obtained from the condition that the sum of all the angular differences, e.g. $\delta_{12} + \delta_{23} + \delta_{31}$, must always be zero. This condition should be applied first, which follows from the fact that the angles δ_j are the independent variables in the problem. These angles applied to equation 21 will, in effect, solve the electrical network for all of the flows, P_{in}. As a secondary effect of this solution, it is possible to determine the electrical loss in the network for each trial solution, although this is not necessary in applying the economic loading criteria. Having determined P_{in}, it is possible to evaluate P_{gi}. Next it is possible to evaluate ξ_i by substitution into the appropriate incremental rate equation. The next step is to form the combinations X_{in} from the definitions given by equations 47 through 52. Substitution of these values into equations 44, 45, and 46 will yield values of the partial derivatives $\frac{\partial P_t}{\partial \delta_j}$, which may be investigated to determine whether a better solution may be obtained by another trial.

Theoretically, any choice of angles for the first approximation is permissible and will yield derivatives which will point toward the desired solution. Practically, the number of trial solutions can be reduced if the first choice of angles is near the true values. A network analyzer study can give such a set of angles, provided a schedule of generation approximating the economic schedule was applied. To do this, schedule the generators so that they all operate at the same incremental rate. Lacking an analyzer study, the next best choice is to assign angles in proportion to the output of each plant if all plants were to operate at the same incremental rate.

The algebraic sum of the partial derivatives will always be zero. This can be seen by inspection of equations 56, 57, and 58. Thus at least one of the derivatives will be positive and at least one of them negative for each trial solution. This implies that if a correction were applied whereby one plant output is increased, there is always at least one plant available to yield a corresponding decrease and permit the system to approach the desired minimum.

Having obtained the partial derivatives, it is necessary to interpret them to form a decision for the next trial. The solution will converge rapidly enough if only one angle is changed to obtain the new trial, provided that angle is the one for which the partial derivative of P_t has the largest magnitude. If that derivative is negative, the angle should be increased, and vice versa. The magnitude of such an increase must be based on experience with the system, implying that the first change might not be satisfactory. If the sign of the derivative should change, then the change was too large and another trial can be attempted with an intermediate value by assuming that the value of the derivative is a linear function of the angle. If the sign should not change, then obviously the amount of the change was too small, and again another trial may be formed by extending the angle in proportion to the values of the derivative. In successive trials the magnitudes of all the derivatives will decrease, implying that a minimum is being approached. When the minimum has been reached the values of the derivatives will either all be zero, or else a trial will produce values of the derivatives indicating that the next change should be the opposite of the change just made. This latter eventuality is the one to be expected, and it implies that the succession of trials has approached as near to the minimum as possible within the degree of precision allowed by the permissible changes of angles. In this thesis, this change of angle is one minute. In the examples chosen for this thesis, the largest difference between a specified generator output and that of the next best trial one minute removed was eight parts in 1280, or 0.625 percent, and this occured for the smallest value specified for any generator. In the same case, the change in system input was detected in the sixth significant figure, implying that considerable tolerance can be allowed in the schedule of individual generators without affecting the system input significantly. P_t changes very slowly for small digressions away from the minimum point.

D. Solution for Two Machines

For example 1, generators 1 and 2 were assumed to be located at opposite ends of a single transmission line with 1.0 per unit load applied at each end of the line. Reference to Figure 9 reveals that generator 1 would carry more of the load than generator 2 on the basis of equal incremental rates by approximately the ratio 1.15:0.85. Trial 1 was formulated by assigning δ_{12} the value 2°0' to yield this ratio. The results of this trial and the three succeeding trials are shown in Table II. Since the derivative of P_t was positive as a result of the first trial, δ_{12} was decreased for the second. The second trial indicated a further decrease, but the third trial produced a negative derivative implying that the correct angle was between those chosen for the second and third trials. The angle for the fourth trial was one minute greater than for the third and the derivative was positive. Thus the correct angle is between 1°50' and 1°51' and is probably closer to 1°51' because the magnitude of the derivative is less for this angle. Interpolation does not seem to be justified since the calculated values of P_t for these two trials do not differ in six significant figures.

The line loss in this example is 0.0022, or 0.11 percent of the total system load. Although this loss is extremely small, it is effective in reducing the output of generator 1 from the value it would have at equal incremental rates. If the system were to be operated without loss, then each generator should supply exactly its own local load. This would require a total system input of 6.03000 units, but the input for best economy is only 6.00984, or a saving of 0.334 percent of the no-load case.

For example 2, 2.0 per unit load was applied at the bus of generator 2. This required generator 1 to supply its share of the load over the transmission line with considerable loss. The loss calculated for the most economic condition was 0.1169, or 5.845 percent of the total system load. The results of the four trials required to find the minimum input are tabulated in Table III. Comparison of these results with those of example 1 shows that generator 1 has been penalized more heavily by transmission

Table II. Summary of results of example 1

$E_1 = 1.0$ $E_2 = 1.0$ $P_{L1} = 1.0$ $P_{L2} = 1.0$

Trial	δ_{12}	P_{g1}	P_{g2}	ξ_1	ξ_2	$\frac{\partial P_t}{\partial \delta_1}$	P_t
1	2°0'	1.1408	0.8616	1.54318	1.57093	0.1064	6.01000
2	1°55'	1.1349	0.8674	1.53708	1.57683	0.0493	6.00991
3	1°50'	1.1290	0.8731	1.53100	1.58264	-0.0073	6.00984
4	1°51'	1.1302	0.8719	1.53223	1.58142	0.0044	6.00984

Table III. Summary of results of example 2

$E_1 = 1.0$ $E_2 = 1.0$ $P_{L1} = 0$ $P_{L2} = 2.0$

Trial	δ_{12}	P_{g1}	P_{g2}	ξ_1	ξ_2	$\frac{\partial P_t}{\partial \delta_1}$	P_t
1	15°0'	1.1034	1.0329	1.50468	1.74763	0.7450	6.35884
2	12°0'	0.8753	1.2121	1.27717	1.93752	-1.2468	6.25016
3	13°52'	1.0169	1.0997	1.41693	1.81782	-0.0062	6.22982
4	13°53'	1.0182	1.0987	1.41824	1.81676	0.0051	6.22983

Table IV. Summary of results of example 3

$E_1 = 1.09$ $E_2 = 1.0$ $P_{L1} = 0$ $P_{L2} = 2.0$

Trial	δ_{12}	P_{g1}	P_{g2}	ξ_1	ξ_2	$\frac{\partial P_t}{\partial \delta_1}$	P_t
1	13°53'	1.3060	0.8376	1.71723	1.54654	2.2292	6.24171
2	12°0'	1.1503	0.9612	1.55301	1.67310	0.8718	6.18615
3	10°48'	1.0518	1.0417	1.45212	1.75684	0.0054	6.17615
4	10°47'	1.0504	1.0428	1.45070	1.75798	-0.0066	6.17615

losses, and there is a wider discrepancy in the incremental rates of the two machines due to these heavy losses.

Investigation of the no-loss case shows marked savings in the economic solution. If it might be assumed that generator 2 supplied all the load, and that generator 1 was operating at no load, then the total system input would be 7.34002. From Table III, the minimum input is 6.22983, or the savings is 15.15 percent of the no-loss case.

Application of equation 23 revealed that, for trial 4, the var output of generator 1 was -0.3632, or this generator was operating at a power factor of approximately 0.942, lead. This would cause the losses to be approximately 11 percent higher than they would be if these vars were zero. To reduce these vars to zero, the voltage of generator 1 should be raised to approximately 1.09 per unit, which would be possible in a practical system if there were no local load on the bus to prohibit it. Example 3 was formulated on this assumption.

The results of example 3 are tabulated in Table IV. Again, four trials were required to find the minimum input. The loss for this case was 0.0932, which is less than the loss of the previous example by an amount greater than the 11 percent saving due to the removal of the var flow. This is true in spite of the fact that the output of generator 1 was greater than for example 1. The additional savings are due to the increased voltage at generator 1, which follows from the generality that transmission may always be accomplished with less loss at higher voltages. The total system input for this case is less than for example 2, from which it may be generalized that a voltage profile for a system should always be chosen as high as possible.

E. Solution for Three Machines

The results of examples 4 and 5 are tabulated in Tables V and VI. In both of these examples, equal loads were applied at the three generator buses. Six trials were necessary to ascertain the minimum input for each example within one minute for each of the three angles. For each trial, only one generator angle was changed from the preceding trial, and that the one for which the derivative had the largest magnitude. In example 4, trial 1 showed that δ_1 should be increased, but the increment added was too large as evidenced by the large positive value of $\frac{\partial P_t}{\partial \delta_1}$ resulting from the second trial. Interpolation for the third trial reduced this derivative to a very low value and indicated that the next change should be an increase in δ_3. Trial 4 indicated another increase in δ_1, and trial 5 an increase in δ_2. Trial 6 indicated the opposite change in δ_2 from trial 5 which signified that the minimum was between trials 5 and 6. Since the values of the derivatives were less for trial 6, it was concluded that the minimum was nearer to trial 6, and it was accepted as a solution.

The results of example 5 were obtained by the same technique. In this example, however, the sixth trial demonstrated that the minimum lay between trial 5 and trial 6, but trial 5 was considered the solution since the derivatives were smaller for this trial.

The share of the total load assigned to each generator was not the same in these two examples. This is illustrated in Figure 10. Generator 3 may be assigned an increasing share of the load as the total system load increases, while generator 1 received less. This is due partly to the differences in the slopes of the incremental rate curves, and partly to the

Table V. Summary of results of example 4

$E_1 = 1.0$ $\quad$ $P_{L1} = 0.5$

$E_2 = 1.0$ $\quad$ $P_{L2} = 0.5$

$E_3 = 1.0$ $\quad$ $P_{L3} = 0.5$

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6
δ_{12}	1°30'	2°0'	1°39'	1°39'	1°41'	1°40'
δ_{23}	3°30'	3°30'	3°30'	3°25'	3°25'	3°26'
δ_{31}	-5°0'	-5°30'	-5°9'	-5°4'	-5°6'	-5°6'
P_{g1}	0.8124	0.8691	0.8294	0.8258	0.8297	0.8285
P_{g2}	0.5788	0.5444	0.5685	0.5641	0.5618	0.5638
P_{g3}	0.1284	0.1096	0.1228	0.1300	0.1288	0.1280
ξ_1	1.21662	1.27116	1.23289	1.22944	1.23318	1.23203
ξ_2	1.28947	1.25711	1.27947	1.27520	1.27296	1.27490
ξ_3	1.37839	1.34698	1.36904	1.38105	1.37906	1.37771
$\frac{\partial P_t}{\partial \delta_1}$	-0.1841	0.4444	0.0048	-0.0326	0.0100	-0.0037
$\frac{\partial P_t}{\partial \delta_2}$	0.0836	-0.2159	0.0640	0.0073	-0.0208	0.0042
$\frac{\partial P_t}{\partial \delta_3}$	0.1005	-0.2285	-0.0688	0.0253	0.0108	-0.0005
P_t	6.35649	6.35760	6.36639	6.36074	6.35645	6.35641

penalty applied to generator 1 for having to encounter increasing network losses in supplying power to terminals 2 and 3. The line-segment graphs of Figure 10 illustrate the tendencies to be expected in sharing load, but for a practical system these curves should be refined by taking intermediate values of total system load. This would result in smooth curves of the

Table VI. Summary of results of example 5

$E_1 = 1.0$ $\quad$ $P_{L1} = 1.0$

$E_2 = 1.0$ $\quad$ $P_{L2} = 1.0$

$E_3 = 1.0$ $\quad$ $P_{L3} = 1.0$

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6
δ_{12}	2°0'	1°30'	1°30'	1°39'	1°37'	1°36'
δ_{23}	5°0'	5°0'	4°40'	4°31'	4°33'	4°34'
δ_{31}	-7°0'	-6°30'	-6°10'	-6°10'	-6°10'	-6°10'
P_{g1}	1.4335	1.3766	1.3622	1.3728	1.3704	1.3693
P_{g2}	1.1246	1.1590	1.1411	1.1227	1.1268	1.1288
P_{g3}	0.4806	0.4991	0.5275	0.5348	0.5332	0.5324
ξ_1	1.85605	1.79362	1.77794	1.78948	1.78686	1.78567
ξ_2	1.84416	1.88071	1.86167	1.84215	1.84648	1.84861
ξ_3	1.96151	1.99187	2.03842	2.05037	2.04775	2.04644
$\frac{\partial P_t}{\partial \delta_1}$	0.7799	0.0488	-0.1112	0.0344	0.0016	-0.0138
$\frac{\partial P_t}{\partial \delta_2}$	-0.0643	0.4258	0.1862	-0.0679	-0.0115	0.0160
$\frac{\partial P_t}{\partial \delta_3}$	-0.7156	-0.4746	-0.0750	0.0335	0.0099	-0.0022
P_t	8.74736	8.74417	8.74219	8.74195	8.74195	8.74204

general shape indicated.

Example 6 illustrates a ramification of the technique described to permit holding the value of generated power at some terminal at a chosen value. This possibility includes either showing that a terminal is an interconnection point to a foreign system, or that a generator must have a

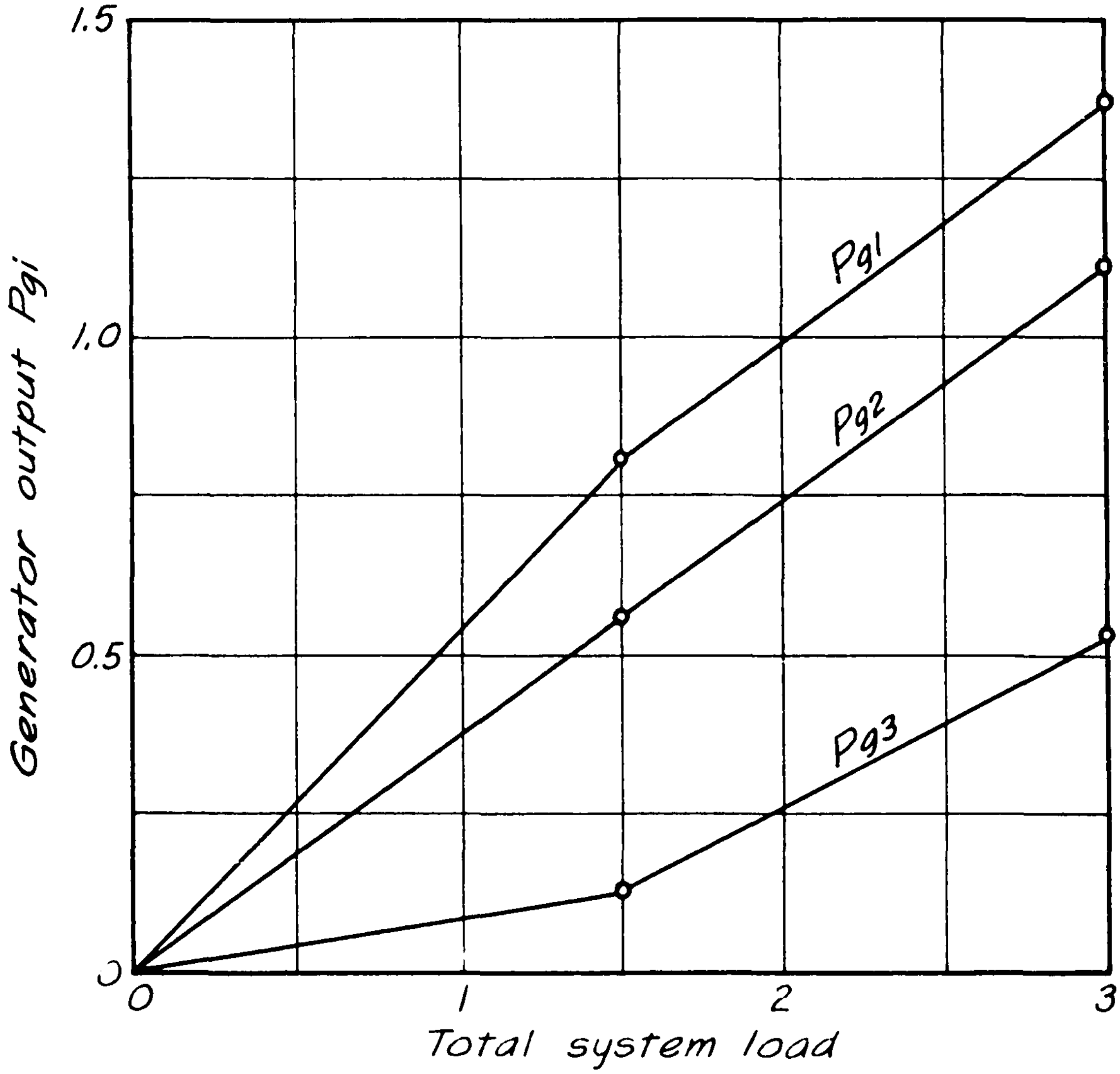


Figure 10. Generator schedule as a function of total system load from examples 4 and 5.

fixed value as in the case of hydro generation. If the terminal is an interconnection, then a load may be applied to represent the interchange power and generation there set equal to zero. If the terminal has fixed generation, there may also be a load if appropriate. In either case, fixing the value of generation imposes a new restriction on the system and requires a different method of solution.

For example 6, 2.0 per unit load was applied at terminal 3 and generator 3 set at zero. This required that, for all possible solutions,

$$P_{31} + P_{32} = -2.0. \tag{84}$$

This is effectively a restriction among the angles. For example, if δ_1 and δ_3 are chosen arbitrarily, then δ_2 must be given a value to satisfy this equation. The choice of δ_2 is no longer free. δ_1 and δ_3 will determine P_{31}, but P_{32} can have the correct value for only a certain value of δ_2.

Only the inputs to generators 1 and 2 may be varied to find a minimum, and the system input becomes

$$P_t = C_1 P_{g1} + C_2 P_{g2}. \tag{85}$$

The problem may now be characterized as one in which it is necessary to minimize a function in the presence of side conditions. The method of solution of such problems is the application of Lagrange multipliers. If the function may be denoted by f and the side conditions by g, then a minimum of f occurs when

$$\frac{\partial}{\partial \delta_j} (f + \lambda g) = 0 \tag{86}$$

where λ is the Lagrange multiplier and is a constant.

Applying this method to example 6, the criteria for minimum system input are:

$$X_{12} + X_{13} - X_{21} + \lambda \frac{\partial g}{\partial \delta_1} = 0 \tag{87}$$

$$X_{21} + X_{23} - X_{12} + \lambda \frac{\partial g}{\partial \delta_2} = 0. \tag{88}$$

The partial derivatives of g with respect to δ_1 and δ_2 are

$$\frac{\partial g}{\partial \delta_1} = - \frac{E_3 E_1}{B_{31}} \sin (\beta_{31} + \delta_{31}) \tag{89}$$

$$\frac{\partial g}{\partial \delta_2} = - \frac{E_3 E_2}{B_{32}} \sin (\beta_{32} + \delta_{32}). \tag{90}$$

If λ may be replaced by ξ_3, then the criteria may be given an interpretation which may be rationalized with the method of solution of the previous examples. ξ_3 should be considered the incremental rate that a ficticious generator at terminal 3 would have to have in order that the solution require its output to be zero. A generator at this terminal capable of supplying power at a lower incremental rate would be allowed to do so, and similarly a higher rate would prevent such a generator from competing economically. Having found a solution, ξ_3 is the rate which should be charged for power supplied at this terminal. In example 6 this rate is considerably higher than the rates of either of the generators due to the losses associated with transmission of power to terminal 3.

The technique of solution differs from that of the previous examples. The first step is to assume two angles and calculate the power received at terminal 3 via one line. The power received via the second line is then fixed, and as a result the third angle in the problem may be determined from equation 22. It is then possible to calculate P_{g1} and P_{g2}, and from these values calculate ξ_1 and ξ_2. A value of ξ_3 may be determined by taking from equation 59 the relationship that

$$(X_{23} - X_{32}) = (X_{31} - X_{13}). \tag{91}$$

This may be transposed to give

$$X_{31} + X_{32} = X_{13} + X_{23}. \tag{92}$$

If this is expanded it may be seen that a value of ξ_3 may be determined by solving

$$\xi_3 \left[\frac{E_3E_1}{B_{31}} \sin\,(\beta_{31} + \delta_{31}) + \frac{E_3E_2}{B_{32}} \sin\,(\beta_{32} + \delta_{32}) \right] = X_{13} + X_{23}. \tag{93}$$

All the quantities in this expression are known except ξ_3. The result will be that $\frac{\partial P_t}{\partial \delta_3}$ will be zero, implying that δ_3 is not to be changed, but this is consistent with the restriction of equation 84 that one angle is not independent.

The results of example 6 are tabulated in Table VII. The first two trials resulted in unreasonable values of generator output, but they were useful in formulating trial 3. Two more trials were needed to reduce the input to its minimum, and trial 5 was considered to be the solution. The loss associated with transmission to terminal 3 is very high: 0.7090 per unit, or 35.45 percent of the system load. This accounts for the very high value assigned to ξ_3, for the load is so far electrically from the generators that a relatively inefficient plant located at bus 3 can compete economically.

Figure 11 shows the power flows throughout the network from the minimum input solution, and values of var flow were calculated and added to

Table VII. Summary of results of example 6

$E_1 = 1.0$ $P_{L1} = 0$

$E_2 = 1.0$ $P_{L2} = 0$

$E_3 = 1.0$ $P_{L3} = 2.0$

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
δ_{12}	-12°29'	20°36'	4°9'	2°26'	2°28'
δ_{23}	32°29'	19°24'	24°51'	25°34'	25°33'
δ_{31}	-20°0'	-40°0'	-29°0'	-28°0'	-28°1'
P_{g1}	0.0346	3.3785	1.6064	1.4351	1.4383
P_{g2}	2.7675	-0.2023	1.1151	1.2740	1.2707
P_{g3}	0	0	0	0	0
ξ_1			2.05052	1.85782	1.86134
ξ_2			1.83410	2.00430	2.00072
ξ_3			3.72807	3.73287	3.73185
$\frac{\partial P_t}{\partial \delta_1}$			2.3487	-0.0409	0.0056
$\frac{\partial P_t}{\partial \delta_2}$			-2.3487	0.0409	-0.0056
$\frac{\partial P_t}{\partial \delta_3}$			0	0	0
P_t			7.27588	7.24612	7.24549

this figure. This figure reveals that a large block of reactive power must be supplied at terminal 3 to meet the voltage schedule demanded, and both generators operate at leading power factors. Any correction applied to reduce these vars will reduce the losses in the system and produce a better minimum condition. As in example 3, the voltages of both generators may be

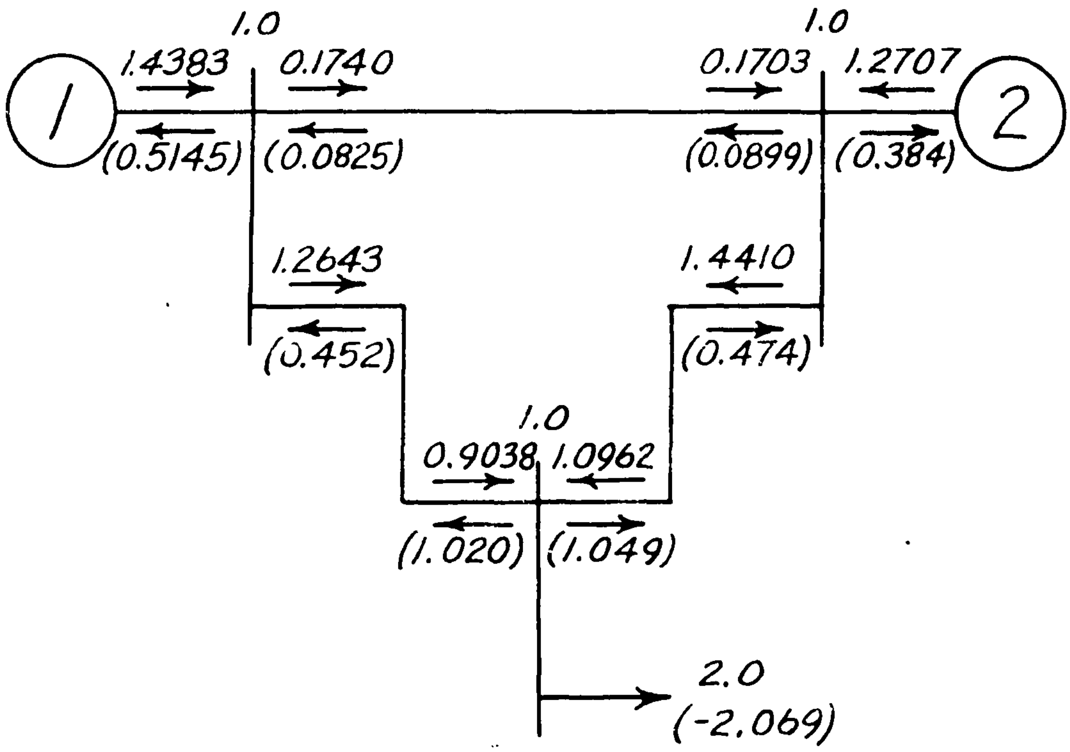


Figure 11. Power and reactive power flows throughout the network for example 6.

Table VIII. Summary of results of example 7

$E_1 = 1.0$ $\quad P_{L1} = 0$

$E_2 = 1.0$ $\quad P_{L2} = 0$

$E_3 = 0.9$ $\quad P_{L3} = 2.0$

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
δ_{12}	9°58'	0°30'	3°5'	2°22'	2°27'
δ_{23}	19°2'	24°30'	23°25'	23°43'	23°41'
δ_{31}	-29°0'	-25°0'	-26°30'	-26°5'	-26°8'
P_{g1}	2.0569	1.1964	1.4442	1.3750	1.3830
P_{g2}	0.4644	1.3841	1.1470	1.2118	1.2043
P_{g3}	0	0	0	0	0
ξ_1			1.86787	1.79187	1.80060
ξ_2			1.86794	1.93719	1.92914
ξ_3			3.40329	3.40818	3.40765
$\frac{\partial P_t}{\partial \delta_1}$			0.8482	-0.1061	0.0041
$\frac{\partial P_t}{\partial \delta_2}$			-0.8482	0.1061	-0.0041
$\frac{\partial P_t}{\partial \delta_3}$			0	0	0
P_t		7.06188	7.01723	7.01389	7.01376

raised. In this case, the voltage at terminal 3 was scheduled at a lower value, and example 7 was formulated on this premise. The voltage at terminal 3 was arbitrarily reduced to 0.9 per unit, and a solution for minimum input was sought as tabulated in Table VIII. The input at the minimum was, in fact, less than for example 6, and Figure 12 illustrates the resulting

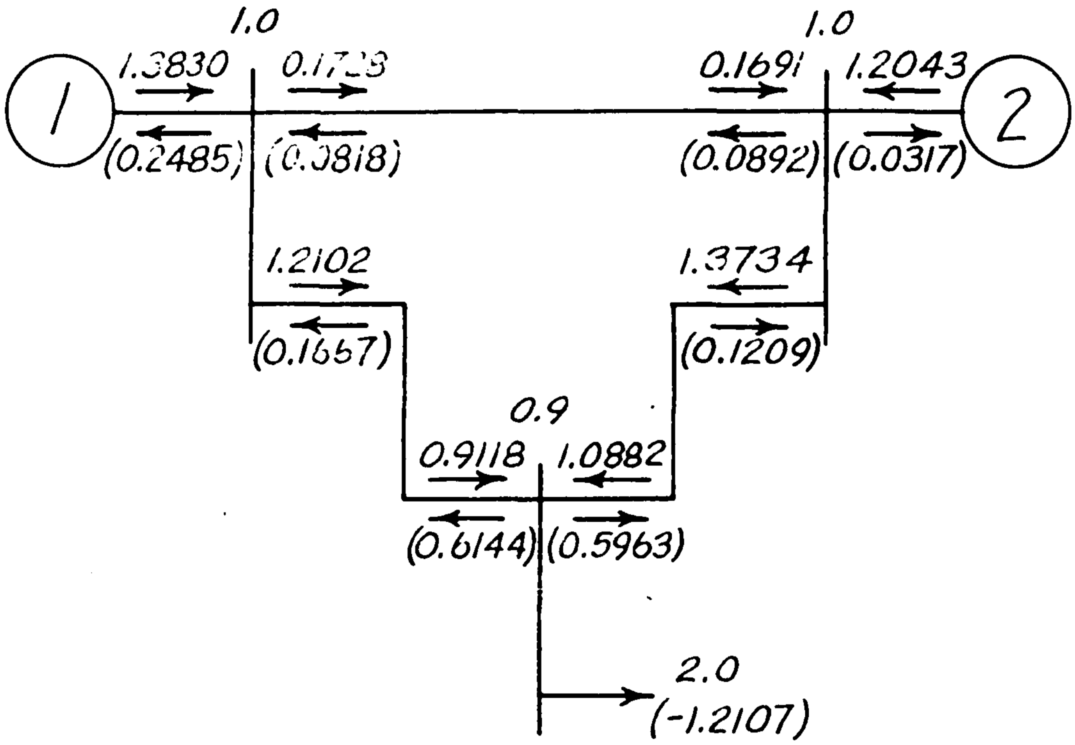


Figure 12. Power and reactive power flows throughout the network for example 7. The system is the same as for example 6 except $E_3 = 0.9$.

power flows at the minimum as well as the calculated values of vars throughout the network. The vars required at terminal 3 have been reduced to approximately 60 percent of the amount required for example 6.

V. DISCUSSION

This thesis describes a method for determining the operating conditions which will result in the minimum fuel input to an electrical system with plants of various fuel economies. The premise on which the analysis was based is that the profile of voltages about the system will be specified by considerations of customer requirements at the various load points, or by limitations imposed by system insulation or lightning arresters, and as a result the voltages about the system must be fixed in magnitude and cannot be used as independent variables in seeking the conditions for minimum input. The phase angles among these voltages, however, are not fixed, and they are the independent variables in solving for minimum input. Since the flows of both real and reactive power throughout a network can be specified as functions of voltages and angles, the solution for the minimum value of input becomes a problem in partial differentiation with respect to these phase angles. Equation 73 gives the resulting partial derivatives in general form.

Implied in equation 73 is the criterion that each generating plant in the system must be in economic balance with all the other plants in the system. This is not a comparison of conditions between only two plants taken at random as implied by other investigators, but rather that each plant be considered with all the other plants simultaneously. The transfer impedance between pairs of plants appears in the denominator of equation 73, which implies that if two plants are electrically relatively remote from each other there will be little effect in the economic criterion for the plant under consideration. Similarly, if two plants have a negligibly small transfer impedance between them they would become as one plant and would have to operate at identical incremental rates.

Examples 6 and 7 demonstrate how an incremental rate may be evaluated at a transfer or interconnection point. Again, this evaluation must, in general, consider all the other plants in the system. The rate determined for this point is the rate below which a generator applied at this terminal would be able to compete economically and supply power to the system. If a generator were applied to this point with the rate calculated for the interchange, then its output would be predicted to be zero by equation 73, and for any higher rate such a generator would not be able to compete and it would be more economical to buy power from other stations in spite of system losses.

Most of the work done in this field has been predicated on the method of George for expressing the losses in the system in terms of the real power flows at the terminals. The method of this thesis does not involve the losses directly, although they have been taken into account since P_{in} and P_{ni} both appear in the development and the difference between these quantities is the loss in this line. Furthermore, the effect of any var flow in such an element is taken into account without making any assumptions about the limits within which these vars must lie.

Other methods assume that the incremental rate characteristics are linear, at least in certain regions, but this method assumes that a power series gives a better fit for the fuel input curve and as a result the incremental rate curve corresponds exactly to the assumptions made about the fuel input curve.

The data required by the method of this thesis to characterize the electrical network are the set of transfer impedances among machines, together with values of shunt impedance to be applied at the terminals to represent the loads of the system. These data are easily compiled from a network analyzer, or they may be apparent without reduction in simple systems. It is not necessary to reduce analyzer measurements to a set of loss formula coefficients as in other methods, and there should be less difficulty in preparing to run special cases such as a certain transmission line open or a unit out of service. It is necessary to evaluate the transfer impedances in the presence of the loads, for loads intermediate between stations will have an effect on these transfer impedances. A set of system impedances is necessary for each loading condition, but they need not be analyzed by means of a computer to reduce them to coefficients for use in the economic loading criteria. The assumption that loads be represented by constant impedance is necessary to permit the reduction to transfer impedances, but this may be justified if the reduction is performed from a network analyzer case which has a generation schedule in the neighborhood of the economic schedule sought. The loads will then be trimmed to the proper impedance values. Loads which were originally located at or near the generator terminals will not be affected by the reduction. A relatively heavy load may be transferred with small error, but if it is felt that a heavy load is too remote from any generator it may have its identity preserved by calling its bus an interconnection point.

Brownlee also attempted to find the criteria for economic loading by means of voltage phase angles, but his conclusions can be applied only to a pair of machines and not to three or more machines or to systems which contain loops. Brownlee approached the problem through the concept of incremental losses and did not attempt to express the total system input or to differentiate it to find the economic loading criteria. The method of this thesis reduces to the same result found by Brownlee for just two machines, but the extension of Brownlee's work will not yield the method of this thesis for three or more machines.

For only two machines, the curves of Figure 3 may be used with relative ease to determine economic ratios. If more accuracy is desired than the curves can yield, then application of equation 60 is recommended. For three machines, a desk calculator is adequate from the standpoint of time required and of accuracy desired. It is possible to develop skill in handling the criteria so that a change in angle may be estimated with accuracy and a minimum number of trials required to find the desired solution. For four or more machines, however, the degree of complexity of the problem is such that it is not feasible to use desk calculators, and it is recommended that the problem be programmed for a digital computer.

It is not feasible to depend entirely on a network analyzer for the solution to this problem because of the precision with which angles must be measured. It is possible that a method might be developed for the analyzer which obviates this requirement, but equipment would have to be added to an analyzer to detect small differences in input due to angular changes without actually providing a visual indication of what these angles might be. Analyzer auxiliary equipment to evaluate equation 73 might be the subject of further research.

Another area for further research is the question of the effect of var flows on losses and economy as opposed to the cost of providing sources for these vars. Considerable work has been done showing the economy of providing capacitors for power factor correction on radial lines or feeders, but not in network problems where the effect of a specific capacitor application is more obscure.

VI. SUMMARY

There are two phases in the determination of the minimum input operating condition for a system. The first is the preparation of data, and the second is the evaluation of several trial solutions which will converge to the desired operating condition.

Phase 1:

1. Either by inspection of the system or by setting it up on a network analyzer, determine the transfer impedances among all possible pairs of generator terminals, denoting any interconnection points as generator terminals. These transfer impedances should be expressed in polar form, giving magnitude B_{in} and angle β_{in}.

2. Determine the shunt load impedances to be applied at these generator terminals. It is less vital to know the values of these impedances than to know the amount of power which they will demand from the network.

3. Determine the input-output fuel characteristic of each plant. By a curve-fitting process, reduce this characteristic to a power series with the plant generated output as the variable.

4. Differentiate the input-output characteristic to obtain a power series expression for the incremental fuel rate of each plant.

Phase 2:

1. Assign values to the voltage phase angles δ_j, and form the differences δ_{12}, δ_{23}, etc.

2. Evaluate the sums of the angles $(\beta_{ji} + \delta_{ji})$ for all possible values of j and i. j can never be equal to i.

3. Evaluate the cosines of the sums formed in step 2 (or look them up

in a trigonometric table) and apply them to equation 21 to determine P_{in} throughout the network.

4. From the values P_{in} and the local loads at each generator terminal, evaluate the generator output powers P_{gi}.

5. Determine the value ξ_i of each generator by substitution of the appropriate value of P_{gi} into the incremental rate expressions.

6. Evaluate the sines of the sums formed in step 2, and apply them together with the appropriate value of ξ_i to determine the products X_{ji} as defined by equation 72.

7. Combine the products X_{ji} as required by equation 73 to determine the various partial derivatives of P_t with respect to each angle δ_j.

8. For the next trial, move one of the angles δ_j according to whichever of the partial derivatives of equation 73 has the largest magnitude, and in a direction indicated by the sign of this derivative. If the derivative is negative, increase the angle, and vice versa.

9. The steps outlined in Phase 2 should be repeated until a trial yields derivatives which indicate that the next change should be the opposite of the change last made. This implies that the solution lies between the last two trials, and nearer that one for which the derivatives are less.

VII. SELECTED REFERENCES

1. Bills, G. W. Digital computers to speed studies. Elec. World. 143, No. 16: 115-117. 1955.

2. Brownlee, W. R. Coordination of incremental fuel costs and incremental transmission losses. Am. Inst. Elec. Engr. Transactions. 73, Pt. 3: 529. 1954.

3. ________. Coordination of losses is vital. Elec. World. 143, No. 16: 109-111. 1955.

4. Cahn, C. R. The derivation of incremental and total loss formulas from functions of voltage phase angles. Am. Inst. Elec. Engr. Transactions. 74, Pt. 3: 161-176. 1955.

5. Calvert, J. F. and Sze, T. W. Some general theorems on power flow in linear networks. Am. Inst. Elec. Engr. Transactions. 75, Pt. 3: 984-988. 1956.

6. ________ and ________. A new approach to loss minimization in electric power systems. Am. Inst. Elec. Engr. Transactions. 77, Pt. 3: 1439-1446. 1958.

7. Cameron, D. H. and Burt, W. S. Load scheduling goes automatic. Elec. World. 143, No. 16: 125. 1955.

8. Dunstan, L. A. Machine computation of power network performance. Am. Inst. Elec. Engr. Transactions. 66: 610-620. 1947.

9. ________. Digital load flow studies. Am. Inst. Elec. Engr. Transactions. 73, Pt. 3: 825-832. 1954.

10. Frampton, A. H. and Floyd, G. D. Factors in the economic supply of energy in hydroelectric systems. Am. Inst. Elec. Engr. Transactions. 66: 1117. 1947.

11. Fugill, A. P. Evaluation of energy differences in the economic comparison of alternative facilities. Am. Inst. Elec. Engr. Transactions. 70: 1854-1860. 1951.

12. George, E. E. Intrasystem transmission losses. Am. Inst. Elec. Engr. Transactions. 62: 153-158. 1943.

13. ________, Page, H. W. and Ward, J. B. Coordination of fuel cost and transmission loss by use of the network analyzer to determine plant loading schedules. Am. Inst. Elec. Engr. Transactions. 68: 1152-1163. 1949.

14. _______ and Stewart, E. C. Gas prices spur economic dispatch. Elec. World. 143, No. 16: 118-119. 1955.

15. Glimn, A. F., Habermann, R., Jr., Kirchmayer, L. K. and Stagg, G. W. Loss formulas made easy. Am. Inst. Elec. Engr. Transactions. 72, Pt. 3: 730-737. 1953.

16. _______, Kirchmayer, L. K. and Stagg, G. W. Analysis of losses in interconnected systems. Am. Inst. Elec. Engr. Transactions. 71, Pt. 3: 796-808. 1952.

17. Harder, E. L., Ferguson, R. W., Jacobs, W. E. and Harker, D. C. Loss evaluation, part II. Am. Inst. Elec. Engr. Transactions. 73, Pt. 3: 716-731. 1954.

18. Harker, D. C., Jacobs, W. E., Ferguson, R. W. and Harder, E. L. Loss evaluation, part I. Am. Inst. Elec. Engr. Transactions. 73, Pt. 3: 709-716. 1954.

19. _______ and Westrate, M. C. Modern power dispatch evaluates line losses. Elec. World. 142, No. 22: 80. 1954.

20. _______ and _______. Management faces decisions on tools for economic dispatch. Elec. World. 143, No. 16: 112-114. 1955.

21. Imburgia, C. A., Kirchmayer, L. K. and Stagg, G. W. A transmission-loss penalty factor computer. Elec. Engr. 73: 649-652. 1954.

22. Kearney, J. J. Ninth steam station cost survey. Elec. World. 144, No. 16: 127-146. 1955.

23. Kirchmayer, L. K. and Stagg, G. W. Analysis of total and incremental losses in transmission systems. Am. Inst. Elec. Engr. Transactions. 70: 1197-1205. 1951.

24. Kron, Gabriel. Tensorial analysis of integrated transmission systems. Part I, the six basic reference frames. Am. Inst. Elec. Engr. Transactions. 70: 1239-1248. 1951.

25. _______. Tensorial analysis of integrated transmission systems. Part III, the "primitive" division. Am. Inst. Elec. Engr. Transactions. 71, Pt. 3: 814-822. 1952.

26. McKinley, J. L. and Ballinger, D. O. EGAD is five-function computer. Elec. World. 147, No. 14: 87-90. 1957.

27. Osterle, W. H. and Harder, E. L. Automatic dispatch pays off. Elec. World. 143, No. 16: 123-124. 1955.

28. Sporn, Philip. The integrated power system. New York, McGraw-Hill Book Co., Inc. 1950.

29. Steinberg, M. J. and Smith, T. H. Economy loading of power plants and electric systems. New York, John Wiley and Sons, Inc. 1943.

30. Travers, R. H., Harker, D. C., Long, R. W. and Harder, E. L. Loss evaluation, part III. Am. Inst. Elec. Engr. Transactions. 73, Pt. 3: 1091-1104. 1954.

31. Ward, J. B. Economy loading simplified. Am. Inst. Elec. Engr. Transactions. 72, Pt. 3: 1306-1312. 1953.

32. ________, Eaton, J. R. and Hale, H. W. Total and incremental losses in power transmission networks. Am. Inst. Elec. Engr. Transactions. 60, Pt. 1: 626-632. 1950.

33. Wellennar, A. H. and Stagg, G. W. Penalty factor computer teams with slide rule. Elec. World. 143, No. 16: 120-122. 1955.

VIII. ACKNOWLEDGEMENTS

The author wishes to thank his major professor, Dr. W. B. Boast, for his help and encouragement throughout the work on this thesis. The author is also indebted to Dr. E. W. Anderson for his advice on the mathematical aspects of this work, and to Dr. H. J. Weiss for his advice regarding the technique of computation. Several members of the Department of Electrical Engineering contributed valuable advice and assistance, including Dr. W. L. Hughes, Dr. R. G. Brown, Dr. J. W. Nilsson, and Mr. P. M. Anderson.