On viscous flow and azimuthal anisotropy of quark-gluon plasma in strong magnetic field

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We calculate the viscous pressure tensor of the quark-gluon plasma in strong magnetic field. It is azimuthally anisotropic and is characterized by five shear viscosity coefficients, four of which vanish when the field strength eB is much larger than the plasma temperature squared. We argue, that the azimuthally anisotropic viscous pressure tensor generates the transverse flow with asymmetry as large as 1/3, even not taking into account the collision geometry. We conclude, that the magnitude of the shear viscosity extracted from the experimental data ignoring the magnetic field must be underestimated.

I. INTRODUCTION

Strong magnetic field produced in relativistic heavy-ion collisions [1, 2] has a strong impact on phenomenology of the quark-gluon plasma (QGP). It induces energy loss by fast quarks and charged leptons via the synchrotron radiation [3] and polarization of the fermion spectra [3]. It contributes to the enhancement of the dilepton production [4] and azimuthal anisotropy of the quark-gluon plasma (QGP) [5]. It causes dissociation of the bound states, particularly charmonia, via ionization [6, 7]. Additionally, the magnetic field drives the Chiral Magnetic Effect (CME) [1, 8–11], which is the generation of an electric field parallel to the magnetic one via the axial anomaly in the hot nuclear matter.

It has been argued recently in [5] that the magnetic field of strength $eB \simeq m_\pi^2$ [1, 2] is able to induce the azimuthal anisotropy of the order of 30% on produced particles. This conclusion was reached by utilizing the solution of the magneto-hydrodynamic equations in weak magnetic field. In this paper we discuss the magneto-hydrodynamics of the QGP in the limit of strong magnetic field. Our goal is to calculate the effect of the magnetic field on viscosity of the plasma. It is well-known that the viscous pressure tensor of magnetoactive plasma is characterized by seven viscosity coefficients, among which five are shear viscosities and two are bulk ones. Generally, calculation of the viscosities requires knowledge of the strong interaction dynamics of the QGP components. However, in strong enough magnetic field these interactions can be considered as a perturbation and viscosities can be analytically calculated using the kinetic equation. Application of this approach

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to the non-relativistic electro-magnetic plasma is discussed in [12]. A general relativistic approach was developed in [13]. We apply it in Sec. II to derive the viscosity coefficients of QGP, which are given by (22) and (33). As in the non-relativistic case, we found that four viscosities vanish as the magnetic field strength increases.

A characteristic feature of the viscous pressure tensor in magnetic field is its azimuthal anisotropy. This anisotropy is the result of suppression of the momentum transfer in QGP in the direction perpendicular to the magnetic field. Its macroscopic manifestation is decrease of the viscous pressure tensor components in the plane perpendicular to the magnetic field, which coincides with the "reaction plane" in the heavy-ion phenomenology. Since Lorentz force vanishes in the direction parallel to the field, viscosity along that direction is not affected at all. In fact, the viscous pressure tensor component in the reaction plane is twice as small as the one in the field direction. As the result, transverse flow of QGP develops azimuthal anisotropy in presence of the magnetic field. Clearly, this anisotropy is completely different from the one generated by the anisotropic pressure gradients and exists even if the later are absent.

In Sec. III we discuss QGP transverse flow in strong magnetic field using the Navie-Stokes equations. At later times after the heavy-ion collision, flow velocity is proportional to $\eta^{-1/2}$, see (40a) and (40b). If the system is such that in absence of the magnetic field it were azimuthally symmetric, then the magnetic field induces azimuthal asymmetry of $1/3$, see (44). This is surprisingly close to the weak field limit recently reported in [5]. The effect of the magnetic field on flow is strong and must be taken into account in phenomenological applications. Neglect of the contribution by the magnetic field leads to underestimation of the phenomenological value of viscosity extracted from the data [14–16]. In other words, more viscous QGP in magnetic field produces the same azimuthal anisotropy as a less viscous QGP in vacuum.

II. VISCOUS PRESSURE IN STRONG MAGNETIC FIELD

A. Kinetic equation

Kinetic equation for the distribution function f of a quark flavor of charge ze is

$$p^\mu \partial_\mu f = zeB^{\mu\nu}\frac{\partial f}{\partial u^\mu}u_\nu + \mathcal{C}[f,\dots] \tag{1}$$

where $\mathcal{C}$ is the collision integral and $B^{\mu\nu}$ is the electro-magnetic tensor, which contains only magnetic field components in the laboratory frame. Ellipsis in the argument of $\mathcal{C}$ indicates the distribution functions of other quark flavors and gluons (we will omit them below). The equilibrium

distribution:

$$f_0 = \frac{\rho}{4\pi m^3 T K_2(\beta m)} e^{-\beta\, p\cdot U(x)} \tag{2}$$

where $U(x)$ is the macroscopic velocity of fluid, $p^\mu = mu^\mu$ is particle momentum, $\beta = 1/T$ and ρ is the mass density. Since $\frac{\partial f_0}{\partial u^\mu} \propto u_\mu$, the first term on the r.h.s. of (1) vanishes in equilibrium as well as the collision integral. Therefore, we can write the kinetic equation as an equation for δf

$$p^\mu \partial_\mu f_0 = zeB^{\mu\nu}\frac{\partial(\delta f)}{\partial u^\mu} u_\nu + \mathcal{C}[\delta f] \tag{3}$$

where δf is a deviation from equilibrium. Differentiating (2) we find

$$\partial_\mu f_0 = -f_0 \frac{1}{T}\, p^\lambda \partial_\mu U_\lambda(x) \tag{4}$$

Since $U^\lambda = (\gamma_V, \gamma_V \underline{\mathrm{V}})$ and $p^\lambda = (\varepsilon, \underline{\mathrm{p}}) = (\gamma_v m, \gamma_v m \underline{\mathrm{v}})$ it follows

$$p\cdot U = \frac{m}{\sqrt{1-v^2}\sqrt{1-V^2}}(1 - \underline{\mathrm{v}}\cdot\underline{\mathrm{V}}) \tag{5}$$

Thus, in the comoving frame

$$\partial_\mu f_0|_{\underline{\mathrm{V}}=0} = f_0\, \frac{1}{T}\, p_\nu \partial_\mu V^\nu \tag{6}$$

Substituting (6) in (3) yields

$$-\frac{f_0}{T} p^\mu p^\nu V_{\mu\nu} = zeB^{\mu\nu}\frac{\partial(\delta f)}{\partial u^\mu} u_\nu + \mathcal{C}[\delta f] \tag{7}$$

where we defined

$$V_{\mu\nu} = \frac{1}{2}(\partial_\mu V_\nu + \partial_\nu V_\mu) \tag{8}$$

and used $u^\mu u^\nu \partial_\mu V_\nu = u^\mu u^\nu V_{\mu\nu}$.

Since the time-derivative of f_0 is irrelevant for the calculation of the viscosity we will drop it from the kinetic equation. All indexes thus become the usual three-vector ones. To avoid confusion we will label them by Greek letters from the beginning of the alphabet. Introducing $b_{\alpha\beta} = B^{-1}\varepsilon_{\alpha\beta\gamma}B_\gamma$ we cast (7) in the form

$$\frac{1}{T} p^\alpha u^\beta V_{\alpha\beta} f_0 = -zeBb_{\alpha\beta}v^\beta \frac{\partial(\delta f)}{\partial v_\alpha}\frac{1}{\varepsilon} - \mathcal{C}[\delta f]\,. \tag{9}$$

The viscous pressure generated by a deviation from equilibrium is given by the tensor

$$-\Pi_{\alpha\beta} = \int p_\alpha p_\beta\, \delta f\, \frac{d^3p}{\varepsilon} \tag{10}$$

Effectively it can be parameterized in terms of the viscosity coefficients as follows (we neglect bulk viscosities)

$$\Pi_{\alpha\beta} = \sum_{n=0}^{4} \eta_n \, V^{(n)}_{\alpha\beta} \tag{11}$$

where the linearly independent tensors $V^{(n)}_{\alpha\beta}$ are given by

$$V^{(0)}_{\alpha\beta} = (3b_\alpha b_\beta - \delta_{\alpha\beta}) \left(b_\gamma b_\delta V_{\gamma\delta} - \frac{1}{3} \nabla \cdot \underline{\mathrm{V}} \right) \tag{12a}$$

$$V^{(1)}_{\alpha\beta} = 2V_{\alpha\beta} + \delta_{\alpha\beta} V_{\gamma\delta} b_\gamma b_\delta - 2V_{\alpha\gamma} b_\gamma b_\beta - 2V_{\beta\gamma} b_\gamma b_\alpha + (b_\alpha b_\beta - \delta_{\alpha\beta}) \nabla \cdot \underline{\mathrm{V}} + b_\alpha b_\beta V_{\gamma\delta} b_\gamma b_\delta \tag{12b}$$

$$V^{(2)}_{\alpha\beta} = 2(V_{\alpha\gamma} b_{\beta\gamma} + V_{\beta\gamma} b_{\alpha\gamma} - V_{\gamma\delta} b_{\alpha\gamma} b_\beta b_\delta) \tag{12c}$$

$$V^{(3)}_{\alpha\beta} = V_{\alpha\gamma} b_{\beta\gamma} + V_{\beta\gamma} b_{\alpha\gamma} - V_{\gamma\delta} b_{\alpha\delta} b_{\alpha\gamma} b_\beta b_\delta - V_{\gamma\delta} b_{\beta\gamma} b_\alpha b_\delta \tag{12d}$$

$$V^{(4)}_{\alpha\beta} = 2(V_{\gamma\delta} b_{\alpha\delta} b_{\alpha\gamma} b_\beta b_\delta + V_{\gamma\delta} b_{\beta\gamma} b_\alpha b_\delta) \, . \tag{12e}$$

For calculation of shear viscosities η_n, $n = 1, \ldots, 4$ we can set $\nabla \cdot \underline{\mathrm{V}} = 0$ and $V_{\alpha\beta} b_\alpha b_\beta = 0$.

Let us expand δf to the second order in velocities in terms of the tensors $V^{(n)}_{\alpha\beta}$ as follows

$$\delta f = \sum_{n=0}^{4} g_n V^{(n)}_{\alpha\beta} v^\alpha v^\beta \tag{13}$$

Then, substituting (13) into (11) and requiring consistency of (10) and (11) yields

$$\eta_n = -\frac{2}{15} \int \varepsilon v^4 g_n d^3 p \tag{14}$$

This gives the viscosities in the magnetic field in terms of deviation of the distribution function from equilibrium. Transition to the non-relativistic limit in (14) is achieved by the replacement $\varepsilon \to m$ [12].

B. Viscosity of collisionless plasma

In strong magnetic field we can determine g_n by the method of consecutive approximations. Writing $\delta f = \delta f^{(1)} + \delta f^{(2)}$ and substituting into (9) we find

$$\frac{1}{T} p^\alpha v^\beta V_{\alpha\beta} f_0 = -zeB b_{\alpha\beta} v^\beta \frac{\partial(\delta f^{(1)} + \delta f^{(2)})}{\partial v_\alpha} \frac{1}{\varepsilon} + \mathcal{C}[\delta f^{(1)}] \, . \tag{15}$$

Here we assumed that the deviation from equilibrium due to the strong magnetic field is much larger than due to particle collisions. The explicit form of $\mathcal{C}$ is determined by the strong interaction

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dynamics but drops off the equation in the leading oder. The first correction to the equilibrium distribution obeys the equation

$$\frac{1}{T}p_\alpha v_\beta V_{\alpha\beta} f_0 = -zeBb_{\alpha\beta}v_\beta \frac{\partial \delta f^{(1)}}{\partial v_\alpha}\frac{1}{\varepsilon}\,. \tag{16}$$

Using (13) we get

$$b_{\alpha\beta}v_\beta \frac{\partial \delta f^{(1)}}{\partial v_\alpha} = 2b_{\alpha\beta}v_\beta \sum_{n=0}^{4} g_n\, V^{(n)}_{\alpha\gamma} v_\gamma \tag{17}$$

Substituting (17) into (16) and using (12) yields:

$$\begin{aligned}\frac{\varepsilon}{TzeB}\, p_\alpha v_\beta V_{\alpha\beta} f_0 \;=\; & -2b_{\beta\nu}v_\alpha v_\nu [g_1(2V_{\alpha\beta} - 2V_{\beta\gamma}b_\gamma b_\alpha) + 2g_2 V_{\beta\gamma}b_\gamma b_\alpha \\ & + g_3(V_{\alpha\gamma}b_{\beta\gamma} + V_{\beta\gamma}b_{\alpha\gamma} - V_{\gamma\delta}b_\alpha b_\delta) + 2g_4 V_{\gamma\delta}b_{\beta\gamma}b_\alpha b_\delta)]\end{aligned} \tag{18}$$

where we used the following identities $b_{\alpha\beta}b_\alpha = b_{\alpha\beta}b_\beta = b_{\alpha\beta}v_\alpha v_\beta = 0$. Clearly, (18) is satisfied only if $g_1 = g_2 = 0$. Concerning the other two coefficients, we use the identities

$$b_{\alpha\beta}b_{\beta\gamma} \;=\; b_\gamma b_\alpha - \delta_{\alpha\gamma}b^2\,, \tag{19a}$$

$$\varepsilon_{\alpha\beta\gamma}\varepsilon_{\delta\epsilon\zeta} \;=\; \delta_{\alpha\delta}\left(\delta_{\beta\epsilon}\delta_{\gamma\zeta} - \delta_{\beta\zeta}\delta_{\gamma\epsilon}\right) - \delta_{\alpha\epsilon}\left(\delta_{\beta\delta}\delta_{\gamma\zeta} - \delta_{\beta\zeta}\delta_{\gamma\delta}\right) + \delta_{\alpha\zeta}\left(\delta_{\beta\delta}\delta_{\gamma\epsilon} - \delta_{\beta\zeta}\delta_{\gamma\delta}\right) \tag{19b}$$

that we substitute into (18) to derive

$$-\frac{\varepsilon}{2TzeB}\, p^\alpha v^\beta V_{\alpha\beta} f_0 = g_3[2V_{\alpha\beta}b_\alpha b_\beta - 4V_{\alpha\beta}v_\alpha b_\beta(\underline{\mathrm{b}}\cdot\underline{\mathrm{v}})] + 2g_4 V_{\alpha\beta}v_\alpha b_\beta(\underline{\mathrm{b}}\cdot\underline{\mathrm{v}})\,. \tag{20}$$

Since $p_\alpha = \varepsilon v_\alpha$ we obtain

$$g_3 = \frac{g_4}{2} = -\frac{\varepsilon^2 f_0}{4TzeB} \tag{21}$$

Using (2), (21) in (14) in the comoving frame (of course η_n's do not depend on the frame choice) and integrating using 3.547.9 of [21] we get

$$\eta_3 = \frac{K_3(\beta m)}{K_2(\beta m)}\frac{\rho T}{2zeB} \tag{22}$$

The non-relativistic limit corresponds to $m \gg T$ in which case we get

$$\eta_3^{\mathrm{NR}} = \frac{\rho T}{2zeB}\,. \tag{23}$$

In the opposite ultra-relativistic case $m \ll T$ (high-temperature plasma)

$$\eta_3^{\mathrm{UR}} = \frac{2nT^2}{zeB}\,. \tag{24}$$

where $n = \rho/m$ is the number density.

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C. Contribution of collisions

In the relaxation-time approximation we can write the collision integral as

$$\mathcal{C}[\delta f] = -\nu\,\delta f \tag{25}$$

where ν is an effective collision rate. Strong field limit means that

$$\omega_B \gg \nu \tag{26}$$

where $\omega_B = zeB/\varepsilon$ is the synchrotron frequency. Whether ν itself is function of the field depends on the relation between the Larmor radius $r_B = v_T/\omega_B$, where v_T is the particle velocity in the plane orthogonal to $\underline{B}$ and the Debye radius r_D. If

$$r_B \gg r_D \tag{27}$$

then the effect of the field on the collision rate ν can be neglected [12]. Assuming that (27) is satisfied the collision rate reads

$$\nu = nv\sigma_t \tag{28}$$

where σ_t is the transport cross section, which is a function of the saturation momentum Q_s [19, 20]. We estimate $\sigma_t \sim \alpha_s^2/Q_s^2$, with $Q_s \sim 1$ GeV and $n = P/T$ with pressure $\alpha_s^2 P \sim 1$ GeV/fm^3 we get $\nu \sim 40$ MeV. Inequality (26) is well satisfied since $eB \simeq m_\pi^2$ [1, 2] and m is in the range between the current and the constituent quark masses. On the other hand, applicability of the condition (27) is marginal and is very sensitive to the interaction details. In this section we assume that (27) holds in order to obtain the analytic solution. Additionally, the general condition for the applicability of the hydrodynamic approach $\ell = 1/\nu \ll L$, where ℓ is the mean free path and L is the plasma size is assumed to hold. Altogether we have $r_D \ll r_B \ll \ell \ll L$.

Equation for the second correction to the equilibrium distribution $\delta f^{(2)}$ follows from (15) after substitution (25)

$$\frac{zeB}{\varepsilon} b_{\alpha\beta} v_\beta \frac{\partial \delta f^{(2)}}{\partial v_\alpha} = -\nu \delta f^{(1)} \tag{29}$$

Now, plugging

$$\delta f^{(1)} = [g_3 V^{(3)}_{\alpha\beta} + g_4 V^{(4)}_{\alpha\beta}] v_\alpha v_\beta\,, \tag{30a}$$

$$\delta f^{(2)} = [g_1 V^{(1)}_{\alpha\beta} + g_2 V^{(2)}_{\alpha\beta}] v_\alpha v_\beta \tag{30b}$$

into (29) yields

$$\frac{2zeB}{\varepsilon}\left\{g_1[2V_{\beta\alpha}b_{\alpha\gamma}v_\beta v_\gamma - 2V_{\beta\alpha}b_{\alpha\gamma}v_\beta v_\gamma(\underline{\mathrm{v}}\cdot\underline{\mathrm{b}})] + 2g_2 V_{\beta\alpha}b_{\alpha\gamma}v_\beta v_\gamma(\underline{\mathrm{v}}\cdot\underline{\mathrm{b}})\right\}$$
$$= -\nu g_3\left\{-2V_{\beta\alpha}b_{\alpha\gamma}v_\beta v_\gamma - 6V_{\beta\alpha}b_{\alpha\gamma}v_\beta v_\gamma(\underline{\mathrm{v}}\cdot\underline{\mathrm{b}})\right\} \tag{31}$$

where we used $g_4 = 2g_3$. It follows that

$$g_1 = \frac{g_2}{4} = \frac{\nu\gamma_v g_3}{2\omega_B} \tag{32}$$

With the help of (28),(2),(14) we obtain

$$\eta_1 = \frac{\eta_2}{4} = \frac{8}{5\sqrt{2\pi}}\frac{\rho^2\sigma_t\, T^{3/2}}{(zeB)^2 m^{1/2}}\frac{K_{7/2}(\beta m)}{K_2(\beta m)} \tag{33}$$

III. TRANSVERSE FLOW

To illustrate the effect of the magnetic field on the viscous flow of the electrically charged component of the quark-gluon plasma we will assume that the flow is non-relativistic and use the Navie-Stokes equations that read

$$\rho\left(\frac{\partial V_\alpha}{\partial t} + V_\beta\frac{\partial V_\alpha}{\partial x_\beta}\right) = -\frac{\partial P}{\partial x_\alpha} + \frac{\partial\Pi_{\alpha\beta}}{\partial x_\beta} \tag{34}$$

where $\Pi_{\alpha\beta}$ is the viscous pressure tensor, $\rho = mn$ is mass-density and P is pressure. We will additionally assume that the flow is non-turbulent and that the plasma is non-compressible. The former assumption amounts to dropping the non-linear in velocity terms, while the later implies vanishing divergence of velocity

$$\nabla\cdot\underline{\mathrm{V}} = 0 \tag{35}$$

Because of the approximate boost invariance of the heavy-ion collisions, we can restrict our attention to the two dimensional flow in the xz plane corresponding to the central rapidity region.

The viscous pressure tensor in vanishing magnetic field is isotropic in the xz-plane and is given by

$$\Pi^0_{\alpha\beta} = \eta\left(\frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial V_\beta}{\partial x_\alpha}\right) = 2\eta\begin{pmatrix} V_{xx} & V_{xz} \\ V_{zx} & V_{zz}\end{pmatrix} \tag{36}$$

where the superscript 0 indicates absence of the magnetic field. In the opposite case of very strong magnetic field the viscous pressure tensor has a different form (11). Neglecting all η_n with $n \geq 1$ we can write

$$\Pi^\infty_{\alpha\beta} = \eta_0\begin{pmatrix} -V_{zz} & 0 \\ 0 & 2V_{zz}\end{pmatrix} = 2\eta_0\begin{pmatrix} \frac{1}{2}V_{xx} & 0 \\ 0 & V_{zz}\end{pmatrix} \tag{37}$$

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where we also used (35). Notice that $\Pi^{\infty}_{xx} = \frac{1}{2}\Pi^{\infty}_{zz} = \frac{1}{2}\Pi^{0}_{xx}$ indicating that the plasma flows in the direction perpendicular to the magnetic field with twice as small viscosity as in the direction of the field. The later is not affected by the field at all, because the Lorentz force vanishes in the field direction. Substituting (37) into (34) we derive the following two equations characterizing the plasma velocity in strong magnetic field

$$\rho\frac{\partial V_x}{\partial t} = -\frac{\partial P}{\partial x} + \eta_0\frac{\partial^2 V_x}{\partial x^2}\,, \qquad \rho\frac{\partial V_z}{\partial t} = -\frac{\partial P}{\partial z} + 2\eta_0\frac{\partial^2 V_z}{\partial z^2} \tag{38}$$

Additionally we need to set two initial conditions

$$V_x\big|_{t=0} = \varphi_1(x,z)\,, \qquad V_z\big|_{t=0} = \varphi_2(x,z) \tag{39}$$

The solution to the the problem (38),(39) is

$$V_x(x,z,t) = \int_{-\infty}^{\infty} dx'\varphi_1(x',z)G_{\frac{1}{2}}(x-x',t) - \frac{1}{\rho}\int_0^t dt'\int_{-\infty}^{\infty} dx' G_{\frac{1}{2}}(x-x',t-t')\frac{\partial P(x',z,t')}{\partial x'} \tag{40a}$$

$$V_z(x,z,t) = \int_{-\infty}^{\infty} dz'\varphi_2(x,z')G_1(z-z',t) - \frac{1}{\rho}\int_0^t dt'\int_{-\infty}^{\infty} dz' G_1(z-z',t-t')\frac{\partial P(x,z',t')}{\partial z'} \tag{40b}$$

Here the Green's function is given by

$$G_k(z,t) = \frac{1}{\sqrt{4\pi a^2 kt}}e^{-\frac{z^2}{4a^2kt}} \tag{41}$$

and the diffusion coefficient by

$$a^2 = \frac{2\eta_0}{\rho} \tag{42}$$

Suppose that the pressure is isotropic, i.e. it depends on the coordinates x,z only via the radial coordinate $r = \sqrt{x^2+z^2}$; accordingly we pass from the integration variables x' and z' to r in (40a) and (40b) correspondingly. At later times we can expand the Green's function (41) in inverse powers of t. The first terms in the r.h.s. of (40a) and (40b) are subleasing and we obtain

$$\begin{aligned} V_x(x,z,t) &\approx -\frac{1}{\rho}\int_0^t ds\int_{-\infty}^{\infty} dr\,\frac{1}{\sqrt{2\pi a^2 s}}\frac{\partial P(r,t-s)}{\partial r} \\ &= -\frac{1}{\rho}\int_0^t ds\frac{1}{\sqrt{2\pi a^2 s}}\left[P(R(s),t-s) - P(0,t-s)\right] \end{aligned} \tag{43a}$$

and by the same token

$$V_z(x,z,t) \approx -\frac{1}{\rho}\int_0^t ds\,\frac{1}{\sqrt{4\pi a^2 s}}\left[P(R(s),t-s) - P(0,t-s)\right] \tag{43b}$$

where $R(t)$ is the boundary beyond which the density of the plasma is below the critical value. We observe that $V_x/V_z = \sqrt{2}$. Consequently, the azimuthal anisotropy of the hydrodynamic flow is

$$\frac{V_x^2 - V_z^2}{V_x^2 + V_z^2} = \frac{1-\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3} \tag{44}$$

Since we assumed that the initial conditions and the pressure are isotropic, the azimuthal asymmetry (44) is generated exclusively by the magnetic field.

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IV. SUMMARY

The structure of the viscous stress tensor in a very strong magnetic field (37) is general, model independent. However the precise amount of the azimuthal anisotropy that it generates is of course model dependent. We however draw the reader's attention to the fact that analysis of [5] using quite different arguments arrived at a very similar estimate. Although a more quantitive numerical calculation is certainly required before a final conclusion can be made, it looks very plausible that the QGP viscosity is significantly higher than the presently accepted value extracted without taking into account the magnetic field effect [14–16] and is perhaps closer to the value calculated using the perturbative theory [17, 18].

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