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On Myopia as Rationale for Social Security

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Abstract. This paper revisits the role played by myopia in generating a theoretical rationale for pay-as-you-go social security in dynamically efficient economies. Contrary to received wisdom, if the real interest rate is exogenously fixed, enough myopia may justify public pensions but never alongside positive private savings. With sufficient myopia, co-existence of positive optimal pensions and positive private saving is possible if the real interest rate on saving evolves endogenously, as in a model with a neoclassical technology.

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1. Introduction

Consider a frictionless economy in which agents care about future consumption and have unfettered access to a saving instrument. In such a world, as originally shown by Aaron (1966), there can be no role for unfunded public pensions or pay-as-you-go (PAYG) social security unless the gross real return on the saving technology (call it R) is dominated by the return on social security – Samuelson's (1958) biological interest rate or the gross population growth rate, n. If saving is done via capital accumulation and capital is productive, then $R < n$ implies the economy is overaccumulating capital – it is dynamically inefficient. In that case, as Samuelson (1975) and Blanchard and Fischer (1989) have explained, introducing unfunded public pensions may be welfare improving if they crowd out private saving, hence diminishing the incentive to overaccumulate capital. Realistically speaking, though, since most real-world economies are very likely dynamically efficient (where $R > n$) and yet have supported PAYG social security historically, the question remains: what *is* the rationale for PAYG social security in dynamically efficient economies, *if any*?

Myopia or shortsightedness is often offered as a rationale for public pensions. As Kotlikoff (1987) puts it, "There seems to be an unstated belief that, left to their own devices, a sizeable fraction of households would inadequately save and insure." Presumably, insufficient foresight is to blame here. In that case, it seems intuitively plausible that a paternalistic government could improve the welfare of such shortsighted people via the forced-saving element present in PAYG social security. In fact, such "paternalistically motivated forced savings constitutes an important, and to some the most important, rationale for social security retirement systems." (Kaplow, 2008).

The seminal, formal contribution in this area is Feldstein (1985) with a follow-up in Feldstein and Leibman (2002). Feldstein (1985) studies a simple two-period overlapping generations (OG) model with productive capital but an exogenously-fixed marginal product of capital (a fixed return-rate R). Therein, he poses the problem of optimal social security system design – choosing the parameters of a PAYG program by maximizing the lifetime welfare of agents who take those parameters as given. He simultaneously introduces two types of deviations from perfect foresight, namely, i) myopia (or shortsightedness): agents attach lower weight to future utility compared to what their own "true" preferences would suggest (alternatively, a setting in which the agent uses a discount rate of $\rho < 1$ compared to no discounting by a benevolent, yet paternalistic government), and ii) pension pessimism: agents perceive they will get only a fraction α of the pensions due to them. Feldstein's original question of optimal pension design can be reformulated as, when are optimal public pensions positive? He concludes that substantial myopia may justify positive public pensions even if $R > n$, but the optimal level of social benefits may be low once it is recognized that private savings are reduced in anticipation of the pension.[1] It is this celebrated result that underlies our faith in myopia as the "most important rationale" for PAYG social security.

A potential lacuna in Feldstein's analysis, one that seems to have gone unnoticed, is his failure to impose a non-negativity constraint on private savings. This turns out to have several profound implications. In Section 2, the current paper revisits the issue of optimal pensions in an OG model *without* productive capital, but with a linear storage technology and exogenously-fixed return-rate[2] R. Therein, we show that with myopia alone, an equilibrium with positive private saving *and* positive public benefits is not possible – either a positive public pension is optimal with $R > n > R\rho$ (enough myopia), but then private saving is zero (at a corner), or if private saving is positive ($R\rho > n$), the optimal public pension is zero. In other words, insufficient foresight can offer a rationale for PAYG social security in a dynamically efficient economy, but then private saving is fully crowded out. A fairly broad intuition for this is that agents perceive private saving and PAYG social security as two competing "assets" and exclusively choose the one with the dominant return.

[1] Feldstein (1985) seems unaware that sufficiently strong pension pessimism along with enough myopia is needed for this result – a point we elaborate below.

[2] This may be interpreted as a small open economy case.

A similar one-or-the-other result holds with pension pessimism. Indeed, we show that pension pessimism alone is not sufficient to produce an equilibrium with interior private savings and public pension – a point missed by Feldstein (1985) and Feldstein and Leibman (2002).[3] More generally, we explore the exact relation between myopia and pension pessimism that is needed to ensure an interior solution (positive optimal benefits and positive private savings).[4] We prove that a necessary condition for such a solution is that for a given level of pension pessimism, myopia can neither be too weak, and somewhat surprisingly, nor too strong. Is heterogeneity with respect to time preference sufficient to ensure an equilibrium with positive savings and positive public pensions? We show that it is not.

The above results had a discomforting bang-bang feel to them, the upshot being that coexistence of positive optimal public pensions and private saving was not possible. It turns out that the assumption of a exogenous return-rate was crucial to generating this outcome. In Section 3, we go on to show that with a concave neoclassical technology (and hence an endogenous return-rate), the bang-bang feature does not arise. More precisely, we prove that at a stable dynamically-efficient steady state in the standard Diamond (1965) OG model, under sufficiently high myopia, coexistence of interior public pensions and private saving is possible. Heuristically speaking, this happens because the return-rate (the marginal product of capital) rises as capital gets crowded out by pensions, preventing future private saving from plummeting to zero. It also deserves mention here that with an endogenous return-rate, pension pessimism is not necessary to generate an equilibrium with both positive private savings and public pension – myopia is enough.

[3] Ironically, Feldstein (1985) requires enough pension pessimism to ensure positive public benefits and sufficiently high equilibrium private savings, and yet, he offers myopia (and insufficient savings) as the rationale for a PAYG pension scheme.

[4] This issue gets clouded in Feldstein (1985) because he assumes specific functional forms, relies on numerical computations, and does not impose a non-negativity constraint on private savings.

2. The model with a linear storage technology

Consider a simple two-period overlapping generations model. There is a continuum of agents of unit measure born at each date $(n = 1)$, and they each live two periods. Agents are endowed with $w > 0$ units of the good only when young. All agents also have access to a simple linear storage technology: 1 unit invested in this technology in any period yields an exogenously-specified $R \lesseqgtr 1$ units of output the following period.

There is a government that runs a simple PAYG social security program: it imposes a proportional tax τ on young-age endowments and uses the proceeds to finance a lump-sum pension $B \geq 0$ to the current old. The government budget constraint is given by $\tau w = B$. The government is benevolent in the sense that it chooses B by maximizing lifetime utility of a representative agent; it then picks the τ to generate the appropriate revenue.

Let c_i denote consumption of the single perishable good as young $(i = 1)$ and old $(i = 2)$, respectively. We make a distinction between the true and choice utility of agents, implying that agents act myopically.[5] Agents' behavior is dictated by their choice utility, but their actual well-being, the yardstick for calculations of social welfare, is governed by their true utility. All agents have "true" preferences over consumption in each period of life that are summarized by a utility function $U(c_1, c_2) = u(c_1) + \rho^g v(c_2)$, where $\rho^g \in [0, 1]$, u and v are twice continuously differentiable, strictly increasing, strictly concave functions with $u'(0) = v'(0) = \infty$.[6] Agents make their savings decision based on the utility function $U(c_1, c_2) = u(c_1) + \rho^p v(c_2)$, where $\rho^p \in [0, 1]$. When $\rho^p < \rho^g$ choices are made with insufficient weight on second period utility, i.e. agents are myopic. If $\rho^p = 0$, they are "completely myopic" (place no weight on the future) and have no reason to save ("hand-to-mouth"); if $\rho^p = \rho^g$, they are not myopic. We assume $\rho^p \leq \rho^g \leq 1$.

[5] This follows recent developments in the so-called behavioral public finance literature, see e.g., McCaffery and Slemrod (2004) and Gul and Pesendorfer (2007).

[6] In the parlance of behavioral economics, myopia is really a problem of self control or hyperbolic discounting. To see this, following Kaplow (2006), one can write the choice utility of agents as

$$U(c_1, c_2) = \beta u(c_1) + \delta v(c_2)$$

where $\beta >> 1$ and $\delta \in (0, 1)$; the true utility sets $\beta = 1$.

A myopic private agent's saving decision is encapsulated in the following program:

$$\max_{c_1, c_2} \quad u(c_1) + \rho^p v(c_2)$$

subject to

$$c_1 = (1-\tau) w - S$$

$$c_2 = RS + B$$

$$c_1 > 0, \ c_2 > 0, \ S > 0$$

where S denotes saving. Given a B, an interior solution to this problem is described by $S \equiv S(B) > 0$ and $S < (1-\tau) w$. Since $u'(0) = \infty$, we are assured that any solution will satisfy $S < (1-\tau) w$; henceforth, in all that we do below, we will ensure $S > 0$ (the non-negativity constraint on saving) holds in any equilibrium. The first order condition that determines an interior level of S is given by

$$u'(c_1) = R\rho^p v'(c_2) \Leftrightarrow u'((1-\tau) w - S(B)) = R\rho^p v'(RS(B) + B). \tag{1}$$

The government does not share the myopia of private agents, and hence it may be labeled "paternalistic". The government is assumed to be benevolent, interested in maximizing the stationary lifetime utility of a representative agent. It recognizes that a change in the amount of pension benefits has two effects on the private agent's saving: it influences the agent's after-tax endowment and also his future income. The government takes the agent's optimal saving response to its pension as given and solves the following program in order to compute the optimal level for B :

$$\max_B U(B) \equiv u(w - B - S(B)) + \rho^g v(RS(B) + B)$$

The optimal level of B, if positive, is defined as the solution to

$$U'(B) = -u'(c_1)\left[1 + \frac{\partial S(B)}{\partial B}\right] + \rho^g v'(c_2)\left[R\frac{\partial S(B)}{\partial B} + 1\right] = 0. \tag{2}$$

We assume that if (2) has a solution, it is a unique solution.

To foreshadow, below we show that in the absence of myopia, the optimal B is never positive if $R > 1$. If enough myopia is present, the optimal B may be positive but never alongside $S > 0$.

2.1. Benchmark case: no myopia. In this case, $\rho^p = \rho^g$. The following is a restatement of a classic result due originally to Aaron (1966) and expounded in Blanchard and Fischer (1989).

Proposition 1. *If private agents do not suffer from myopia, the government would provide a positive level of pension benefits if and only if $R < 1$.*

Proof. Let $\rho^p = \rho^g = \rho$. Using the first order condition to the agent's problem, $u'(c_1) = R\rho\, v'(c_2)$, we have from (2) that

$$U'(B) = \rho\, v'(c_2) \left\{ -R - R\frac{\partial S(B)}{\partial B} + R\frac{\partial S(B)}{\partial B} + 1 \right\} = \rho\, v'(c_2)(1 - R).$$

Hence $U'(B = 0) \gtreqless 0$ for $R \lesseqgtr 1$. ■

Proposition 1 provides a simple condition: PAYG social security is justified on welfare grounds if and only if $R < 1$ (dynamic inefficiency). Since the point of this paper is to investigate the role of myopia in rationalizing social security in dynamically efficient economies, from here on, we assume:

Assumption (Dynamic efficiency)

$$R > 1.$$

For future reference, we have that the private savings response to a change in public benefits (using the first order condition (1) and the government budget constraint) can be written

$$-u''(c_1)\left[1 + \frac{\partial S(B)}{\partial B}\right] = R\rho^p v''(c_2)\left[R\frac{\partial S(B)}{\partial B} + 1\right] \Leftrightarrow \frac{\partial S(B)}{\partial B} = -\frac{R\rho^p v''(c_2) + u''(c_1)}{\left[u''(c_1) + R^2\rho^p v''(c_2)\right]} < 0$$

(3)

This describes the extent of equilibrium crowding out of private savings by public pensions. Note that

$$\frac{\partial S(B)}{\partial B} \geq -1 \text{ if } R \geq 1.$$

2.2. Myopic agents. In this case, agents attach a strictly lower weight to their future than what their "true" preferences imply, i.e., $\rho^p < \rho^g$. Using (2) and (1), it is easily checked that

$$U'(B) = v'(c_2)\left\{\rho^g - R\rho^p + R\frac{\partial S(B)}{\partial B}(\rho^g - \rho^p)\right\},$$

implying, at an interior, that the optimal B satisfies

$$U'(B) = 0 \Leftrightarrow \frac{\partial S(B)}{\partial B} = -\frac{\rho^g - R\rho^p}{R(\rho^g - \rho^p)}. \tag{4}$$

From the point of view of the government, the optimal level of public benefits requires that the optimal saving response to an increase in pension is given by (4). But, to the individual, at an interior S, it follows from (3) that the optimal equilibrium response is given by

$$\frac{\partial S(B)}{\partial B} = -\frac{R\rho^p v''(c_2) + u''(c_1)}{[u''(c_1) + R^2\rho^p v''(c_2)]}. \tag{5}$$

If an equilibrium with both an interior S and an interior value for B is to exist, these two saving responses must be identical.

Proposition 2. *Under myopia, i.e., when $\rho^p < \rho^g$, there does not exist an equilibrium with $S > 0$ and $B > 0$.*

Proof. Suppose $S > 0$ and $B > 0$. Then, it follows from the equality of (5) and (4) that

$$\frac{\rho^g - R\rho^p}{R(\rho^g - \rho^p)} = \frac{R\rho^p v''(c_2) + u''(c_1)}{[u''(c_1) + R^2\rho^p v''(c_2)]}$$

must hold. Cross multiplying, this reduces to

$$\left[\rho^g - R\rho^p\right]\left[u''\left(c_1\right) + R^2\rho^p v''\left(c_2\right)\right] = R\left(\rho^g - \rho^p\right)\left[R\rho^p v''\left(c_2\right) + u''\left(c_1\right)\right]$$

which upon some simplification yields

$$\rho^g\left(1 - R\right)u''\left(c_1\right) = R^2\left(\rho^p\right)^2 v''\left(c_2\right)\left(R - 1\right).$$

Since $R > 1$ and $u'', v'' < 0$, the left hand side of the above equation is positive while the right hand side is negative. Hence, the contradiction. ■

As the following corollary makes clear, the optimal pension is never positive when private saving is positive.

Corollary 1. *Under myopia, i.e., when $\rho^p < \rho^g$, the optimal pension is positive when $R\rho^p < \rho^g$, but then private saving is zero.*

Proof. At an interior $S > 0$,

$$u'\left(w - B - S\left(B\right)\right) = R\rho^p v'\left(RS^p\left(B\right) + B\right)$$

holds, which implies that at the corner, $S = 0$, we must have

$$u'\left(w - B\right) > R\rho^\rho v'\left(B\right).$$

In that case, the optimal B is the solution to the program

$$\max_B U\left(B\right) \equiv u\left(w - B\right) + \rho^g v\left(B\right)$$

or, the optimal B when $S = 0$ satisfies

$$u'\left(w - B\right) = \rho^g v'\left(B\right)$$

From the agent's problem, it follows that

$$u'\left(w - B\right) > R\rho^p v'\left(B\right) = \frac{R\rho^p}{\rho^g}u'\left(w - B\right) \Leftrightarrow \rho^g > R\rho^p.$$

■

Notice that optimal pension is positive only when myopia is strong enough so as to drive $R\rho^p$ (recall $R > 1$) below ρ^g. The intuition for Corollary 1 is somewhat like this. With exogenous wage income, the proportional tax does not distort. Agents perceive two alternative means of transferring income to their retirement years: they can either save 1 unit on their own at a perceived (effective) return of $R\rho^p < R$, the latter being the true return, *or* they can pay 1 unit in taxes and receive an effective return of $\rho^g \times 1$ (since there is no population growth) generated by a PAYG system. If $R\rho^p < \rho^g$, the agent prefers to go with the pension system and saves nothing on his own.

The rationale for public pensions is that agents save too little due to myopia. However, once a public pension is offered, private agents lower their savings since they, given their choice utility, find that the sum of private savings and public pensions leave too high a consumption level in the future. This drives private savings to a corner where they are fully crowded out.[7] Clearly, agents are better off under the public pension scheme evaluated in terms of the true utility, although they are worse off assessed in terms of their choice utility. The bottom line is this. Myopia of agents has been touted as the main reason for why public pensions are a good idea. The above results indicate that while enough myopia may generate a rationale for public pensions it also implies that private savings are fully crowded out.[8]

2.3. Pension pessimism. Feldstein (1985) introduced the idea that individuals may "myopically" believe they will get only a fraction $\alpha \in [0, 1]$ of benefits due to them. Below, we investigate the role played by pension pessimism in generating a rationale for public pensions.

[7] A mandatory level of pension savings may overcome this problem. However, as shown in Cremer et.al. (2008) this is not necessarily optimal once agent heterogeneity and labor supply distortions are allowed.

[8] This result generalizes to a very general form of myopia/paternalism. Private agents perceive their lifetime utility to be given by $u(c_1)+z(c_2)$ while their true preferences are captured instead by $u(c_1)+v(c_2)$. It is easy to verify that the problem of co-existence of positive pensions and interior private saving would remain even for this very general specification.

Henceforth, without any loss of generality, assume $\rho^g = 1$ and $\rho \equiv \rho^p$. Then, the problem of the private agent – who takes B parametrically given – is given by

$$\max_{c_1, c_2} \quad u(c_1) + \rho v(c_2)$$

subject to

$$c_1 = (1-\tau)\, w - S$$

$$c_2 = RS + \alpha B$$

$$c_1 > 0, \; c_2 > 0.$$

Analogous to what was discussed above, the first order condition to the private agent's problem that determines an interior choice of S is given by

$$u'(c_1) = R\rho\, v'(c_2) \Rightarrow u'((1-\tau)\, w - S) = R\rho v'(RS + \alpha B),$$

using which we can get

$$\frac{\partial S(B,\alpha)}{\partial B} = -\frac{R\rho\alpha v''(c_2) + u''(c_1)}{[u''(c_1) + R^2 \rho v''(c_2)]} < 0$$

and

$$\frac{\partial S(B,\alpha)}{\partial \alpha} = -\frac{R\rho B v''(c_2)}{[u''(c_1) + R^2 \rho v''(c_2)]} < 0.$$

i.e., the more pessimistic agents are about future pensions, the more they save.

Turning to the government's optimal interior choice of B (for $\rho^p \equiv \rho$ and $\rho^g = 1$ and $\alpha = 1$), we have that it is defined by

$$U'(B) = -u'(c_1)\left[1 + \frac{\partial S(B)}{\partial B}\right] + v'(c_2)\left[R\frac{\partial S(B)}{\partial B} + 1\right] = 0$$

which, using the agent's first order condition, reduces to

$$\frac{\partial S(B)}{\partial B} = -\frac{1 - R\rho}{R(1-\rho)}.$$

If there is to exist an interior optimal S and B, the saving response of an increase in B from the point of view of the agent must coincide with that of the government, or

$$\frac{R\rho\alpha v''(c_2) + u''(c_1)}{[u''(c_1) + R^2\rho v''(c_2)]} = \frac{1 - R\rho}{R(1-\rho)} \tag{6}$$

must hold.

Proposition 3. *If $\rho = 1$ and $\alpha < 1$, then for $R > 1$, there does not exist an interior optimal choice for B and S. An interior solution requires $\rho \in [\underline{\rho}, \overline{\rho}]$ where $\underline{\rho} > 0$ and $\overline{\rho} < \frac{1-\alpha}{R-\alpha} < 1$.*

Proof. It is easily checked that (6) reduces to

$$0 = u''(c_1)(1-R) + R^2\rho v''(c_2)[(1-\alpha) + \rho(\alpha - R)]. \tag{7}$$

The first term on the r.h.s of (7) is positive; hence, for an interior solution, the second term has to be negative. Clearly, for $\rho = 1$, the second term reduces to $R^2\rho v''(c_2)(1-R) > 0$. This proves that there does not exist an interior optimal choice for B and S when $\rho = 1$. Similarly for $\rho = 0$, the second term is zero. Hence, an interior solution requires

$$\rho[(1-\alpha) + \rho(\alpha - R)] = \frac{u''(c_1)(R-1)}{R^2\rho v''(c_2)} > 0. \tag{8}$$

It follows that a necessary condition for an interior optimal choice for B and S is that

$$(1-\alpha) + \rho(\alpha - R) > 0 \Leftrightarrow \rho < \frac{1-\alpha}{R-\alpha}.$$

Since the l.h.s. of (8) is a quadratic function in ρ and $\alpha - R < 0$, it follows that (8) only holds for $\rho \in [\underline{\rho}, \overline{\rho}]$ where $\underline{\rho} > 0$ and $\overline{\rho} < \frac{1-\alpha}{R-\alpha} < 1$. ■

Notice from (7) above that neither for $\rho = 0$ nor for $\rho = 1$ is an interior solution to B and S possible! Hence for a given α, ρ can neither be too high nor too low for an interior solution with positive public benefits and private savings to exist.

The implication is clear. Pension pessimism alone (i.e., without accompanying myopia) is not enough to generate an equilibrium with positive savings and positive public pensions.

As we show below, the assumption of neoclassical technology can generate such an equilibrium without need for pension pessimism. In passing, also note that Feldstein (1985) needs a sufficiently low α in order to ensure positive savings in equilibrium, i.e., enough pension pessimism strengthens the motive for private savings. Ironically, this keeps the size of the pension program "modest" – if agents are pessimistic and do not expect any pension, there is no need to give them one!

2.4. Heterogenous agents. In some parts of his analysis, Feldstein (1985) allows for the possibility of time-preference heterogeneity. Specifically, he considers an economy in which a fixed fraction of agents is completely myopic ($\rho = 0$). In the literature, these are referred to as "hand-to-mouth" consumers, who save nothing. The remaining fraction of agents is assumed to be non-myopic. Below, we study a slightly more general setting that nests the aforediscussed Feldstein formulation: we assume that a fraction (λ) of agents is myopic with a time preference ρ^m, and the remaining fraction has time preference ρ^{nm}, where $\rho^{nm} > \rho^m \geq 0$. We allow for the possibility that $\rho^{nm} \leq \rho^g$.[9] We ask, is heterogeneity with respect to time preferences sufficient to ensure an equilibrium with positive savings and positive public pensions? [10]

For agents with time preference ρ^m, the first order condition that determines an *interior* level of S is given by

$$u'\left(c_1^m\right) = R\rho^m v'\left(c_2^m\right) \Leftrightarrow u'\left(\left(1-\tau\right)w - S^m\left(B\right)\right) = R\rho^m v'\left(RS^m\left(B\right) + B\right); \tag{9}$$

[9] The Feldstein formulation assumes $\rho^{nm} = \rho^g = 1$ and $\rho^m = 0$.

[10] This is interesting not only as a generalization of Feldstein, but also because the literature features models with a distinction between so-called constrained and unconstrained households, see e.g. Jappelli and Pagano (1989) and Campell and Mankiw (1991), where the former are defined as liquidity constrained households and therefore they are "hand in mouth consumers"; i.e. they consume their income, and their marginal propensity to consume/save is one/zero. The unconstrained are forward looking. In a macro-context this is important since it introduces Keynesian type demand effects, and implies also that Ricardian equivalence does not hold. Empirical estimates find the fraction of constrained households to be quite high (70%). Our result shows that this corner (no savings) may arise endogenously due to social security; it does not have to rely on complete myopia ($\rho^m = 0$) or some other form of irrationality.

analogous to (5), we have

$$\frac{\partial S^m(B)}{\partial B} = -\frac{R\rho^m v''(c_2^m) + u''(c_1^m)}{[u''(c_1^m) + R^2\rho^m v''(c_2^m)]}.$$

The analogous expressions for those with time preference ρ^{nm} are given by

$$u'(c_1^{nm}) = R\rho^{nm} v'(c_2^{nm}) \Leftrightarrow u'((1-\tau)w - S^{nm}(B)) = R\rho^{nm} v'(RS^{nm}(B) + B). \quad (10)$$

and

$$\frac{\partial S^{nm}(B)}{\partial B} = -\frac{R\rho^{nm} v''(c_2^{nm}) + u''(c_1^{nm})}{[u''(c_1^{nm}) + R^2\rho^{nm} v''(c_2^{nm})]}.$$

The optimal benefit level for the government solves

$$\begin{aligned}&\max_B U(B) \equiv \lambda\left[u(w - B - S^m(B)) + \rho^g v(RS^m(B) + B)\right]\\ &+(1-\lambda)\left[u(w - B - S^{nm}(B)) + \rho^g v(RS^{nm}(B) + B)\right].\end{aligned}$$

The optimal level of B, if positive, is defined as the solution to

$$\begin{aligned}U'(B) &= \lambda\left\{-u'(c_1^m)\left[1 + \frac{\partial S^m(B)}{\partial B}\right] + \rho^g v'(c_2^m)\left[R\frac{\partial S^m(B)}{\partial B} + 1\right]\right\}\\ &+(1-\lambda)\left\{-u'(c_1^{nm})\left[1 + \frac{\partial S^{nm}(B)}{\partial B}\right] + \rho^g v'(c_2^{nm})\left[R\frac{\partial S^{nm}(B)}{\partial B} + 1\right]\right\} = 0. \quad (11)\end{aligned}$$

Lemma 1. *$\left(1 + \frac{\partial S^m(B)}{\partial B}\right) > 0$ and $\left(R\frac{\partial S^m(B)}{\partial B} + 1\right) < 0$ for $R > 1$; $1 + \frac{\partial S^{nm}(B)}{\partial B} > 0$ and $R\frac{\partial S^{nm}(B)}{\partial B} + 1 < 0$ for $R > 1$.*

Proof. First,

$$\begin{aligned}&\left(1 + \frac{\partial S^m(B)}{\partial B}\right) > 0 \Leftrightarrow -\frac{R\rho^m v''(c_2^m) + u''(c_1^m)}{[u''(c_1^m) + R^2\rho^m v''(c_2^m)]} > -1\\ &\Leftrightarrow R\rho^m v''(c_2^m) + u''(c_1^m) > u''(c_1^m) + R^2\rho^m v''(c_2^m) \Leftrightarrow 1 < R\end{aligned}$$

and second,

$$\begin{aligned}&1 + R\frac{\partial S^m(B)}{\partial B} = \frac{u''(c_1^m) + R^2\rho^m v''(c_2^m) - R^R\rho^m v''(c_2^m) - Ru''(c_1^m)}{[u''(c_1^m) + R^2\rho^m v''(c_2^m)]}\\ &= \frac{u''(c_1^m)(1-R)}{[u''(c_1^m) + R^2\rho^m v''(c_2^m)]} < 0 \text{ for } R > 1\end{aligned}$$

The rest follows. ■

The next result states that heterogeneity in time preference is not enough to produce an equilibrium with interior private savings and positive public pensions.

Proposition 4. *Irrespective of* λ, *there does not exist an equilibrium with* $S^m > 0$, $S^{nm} > 0$, *and* $B > 0$ *if* $R > 1$.

Proof. It follows from (11) and Lemma 1 that, for interior private savings $S^m > 0$ and $S^{nm} > 0$,

$$U'(B=0) = \lambda v'(c_2^m)\left\{-R\rho^m\left(1+\frac{\partial S^m(B)}{\partial B}\right)+\rho^g\left(R\frac{\partial S^m(B)}{\partial B}+1\right)\right\}$$
$$+(1-\lambda)v'(c_2^{nm})\left\{-R\rho^{nm}\left(1+\frac{\partial S^{nm}(B)}{\partial B}\right)+\rho^g\left(R\frac{\partial S^{nm}(B)}{\partial B}+1\right)\right\}<0.$$

■

Note, in passing, that this result does not require that $\rho^{nm} = \rho^g$ holds. We close this section by proving a corollary to Proposition 4 that states that coexistence of positive private saving by the non-myopic agents, zero saving by myopic agents, and positive optimal pensions is possible if the myopic agents are sufficiently myopic.

Corollary 2. *Suppose* $\rho^g = \rho^{nm}$. *A necessary condition for an equilibrium with* $S^m = 0$, $S^{nm} > 0$ *and* $B > 0$ *is that* $\rho^m < \widetilde{\rho}^m(\lambda, R)$ *where* $\widetilde{\rho}^m(\lambda, R) > \frac{\rho^g}{R}$ *and* $\widetilde{\rho}^m_\lambda(\lambda, R) < 0$, $\widetilde{\rho}^m_R(\lambda, R) \lesseqgtr 0$.

Proof. The optimal benefit level is determined from

$$U'(B) = \lambda v'(c_2^m)\left[\rho^g - R\rho^m + R\frac{\partial S^m(B)}{\partial B}(\rho^g-\rho^m)\right]$$
$$+(1-\lambda)v'(c_2^{nm})\left[\rho^g - R\rho^{nm} + R\frac{\partial S^m(B)}{\partial B}(\rho^g-\rho^{nm})\right]=0.$$

or

$$U'(B) = \lambda\left[\rho^g v'(c_2^m) - u'(c_1^m)\right] + \lambda v'(c_2^m)R\frac{\partial S^m(B)}{\partial B}(\rho^g-\rho^m)$$
$$+(1-\lambda)v'(c_2^{nm})\left[\rho^g - R\rho^{nm} + R\frac{\partial S^m(B)}{\partial B}(\rho^g-\rho^{nm})\right]=0.$$

If savings by myopic agents are zero, it follows that $\frac{\partial S^m(B)}{\partial B} = 0$; hence the optimal benefit level, assuming $\rho^g = \rho^{nm}$, is determined by

$$U'(B) = \lambda\left[-u'(c_1^m) + \rho^g v'(c_2^m)\right] + (1-\lambda)\left[v'(c_2^{nm})\,\rho^g(1-R)\right] = 0,$$

which can be restated as

$$u'(c_1^m) = \rho^g v'(c_2^m) + \frac{(1-\lambda)}{\lambda}\left[\rho^g v'(c_2^{nm})(R-1)\right]. \tag{12}$$

Myopic agents do not save if

$$u'(c_1^m) > R\rho^m\, v'(c_2^m)$$

which, by use of (12) above holds, iff

$$\begin{aligned} &\rho^g v'(c_2^m) + \frac{(1-\lambda)}{\lambda}\left[\rho^g v'(c_2^{nm})(R-1)\right] > R\rho^m\, v'(c_2^m) \\ \Leftrightarrow\ &\frac{(1-\lambda)}{\lambda}\left[\rho^g v'(c_2^{nm})(R-1)\right] > (R\rho^m - \rho^g)\, v'(c_2^m) \end{aligned} \tag{13}$$

A sufficient condition for (13) to hold is $R\rho^m - \rho^g < 0$. Suppose $R\rho^m - \rho^g > 0$ instead. Since $S^m = 0$ and $S^{nm} > 0$, we know $v'(c_2^{nm}) < v'(c_2^m)$. Then, (13) implies

$$\frac{(1-\lambda)}{\lambda}\left[\frac{\rho^g(R-1)}{(R\rho^m - \rho^g)}\right] > \frac{v'(c_2^m)}{v'(c_2^{nm})} > 1.$$

implying that a necessary condition for an equilibrium is[11]

$$\frac{(1-\lambda)}{\lambda}\frac{\rho^g(R-1)}{R\rho^m - \rho^g} > 1 \Leftrightarrow \widetilde{\rho}^m \equiv \left[(\frac{1}{\lambda} - 1)(1 - \frac{1}{R}) + \frac{1}{R}\right]\rho^g > \rho^m$$

■

It is useful to summarize our results thus far. Myopia on the part of agents has been used to rationalize the existence of public pensions in "high interest" dynamically efficient economies. We show that while enough myopia may provide a justification for public pensions in such economies, private savings would be fully crowded out. Pension pessimism

[11] Note that this condition also includes $R\rho^m - \rho^g < 0$.

alone is not enough to ensure an interior solution (positive optimal benefits and positive private savings). Indeed, a necessary condition for such a solution is that there is some pension pessimism, but myopia can neither be too weak nor too strong. Finally, heterogeneity in time preference is not enough either. In the next section, we show that sufficient myopia and a concave neoclassical technology can produce an equilibrium with positive optimal benefits and positive private savings.

3. The model with neoclassical technology

In this section, we make one important change to the structure developed thus far: we replace the linear storage technology with a neoclassical technology. Specifically, assume the single final good is produced using a standard neoclassical production function $F(K_t, L_t)$ where K_t denotes the capital input and L_t denotes the labor input at t. The final good can either be consumed in the period it is produced, or it can be saved to yield capital at the very beginning of the following period. Capital is assumed to depreciate 100% between periods. Let $k_t \equiv K_t/L_t$ denote the capital-labor ratio (capital per young agent). Then, output per young agent at time t may be expressed as $f(k_t)$ where $f(k_t) \equiv F(K_t/L_t, 1)$ is the intensive production function. We assume that $f(0) = 0$, $f' > 0 > f''$, and that the usual Inada conditions hold.

There are two implications of moving to a model with neoclassical technology: the exogenous endowment w is replaced by a wage income, and the return on saving, R, is replaced by the marginal product of capital, both of which endogenously evolve with the economy.[12] For convenience, assume that agents are endowed with one unit of labor when young and are retired when old. Since agents do not value leisure, they supply their labor endowment inelastically to the labor market at the market determined wage rate, w. We assume that factor markets are perfectly competitive, and thus factors of production are

[12] For future use, bear in mind that Feldstein (1985) assumes R to be exogenously fixed – in effect, he assumes $R'(k) = 0$.

paid their marginal product in each period. Then

$$R_{t+1} = f'(k_{t+1}) \tag{14}$$

and

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t), \tag{15}$$

where $w'(k) = -kR'(k) > 0, \forall k$.

Since the private agent takes R and w as given, the first order condition to the myopic agent's problem remains the same as in (1) above, reproduced below as:

$$u'(c_{1,t}) = R_{t+1}\rho\, v'(c_{2,t+1}) \Rightarrow u'((1-\tau) w_t - S_t) = R_{t+1}\rho v'(R_{t+1} S_t + B). \tag{16}$$

In equilibrium, $S_t = k_{t+1}$; then using (14)-(15) and the government budget constraint, $\tau w_t = B_t$, eq. (16) may be written as

$$u'(w(k_t) - B - k_{t+1}) = R(k_{t+1})\rho v'(R(k_{t+1}) k_{t+1} + B). \tag{17}$$

Eq. (17) characterizes the equilibrium law of motion for the capital-labor ratio. All competitive equilibria, both stationary and non-stationary, are characterized by (17). A steady state k is a time-invariant solution to (17). Note that (17) may be "solved" to yield

$$k_{t+1} = S[w(k_t), R(k_{t+1})]. \tag{18}$$

For future reference, note that at any steady state k,

$$S_w \equiv \frac{\partial S}{\partial w} = \frac{u''(\cdot)}{\mathcal{F}}; \quad S_R = -\frac{\rho R S[\cdot] v''(\cdot) + v'(\cdot)}{\mathcal{F}} \tag{19}$$

where $\mathcal{F} \equiv u''(\cdot) + R^2 \rho v''(\cdot) < 0$. It is clear $S_w > 0$. Henceforth, assume $S_R \geq 0$. Such an assumption is warranted for two reasons: a) it has very strong empirical support, and b) it rules out the possibility of endogenous cycles in k (see Azariadis, 1993).

Straightforward differentiation of (17) yields

$$\frac{\partial k}{\partial B}=\frac{u''(c_1)+R(k)\rho v''(c_2)}{u''(c_1)w'(k)-u''(c_1)-R'(k)\rho v'(c_2)-R(k)\rho v''(c_2)\left[R'(k)k+R(k)\right]}. \tag{20}$$

The next result is a short detour; it uses Samuelson's correspondence principle to establish a well-known proposition regarding crowding out of private saving by public pensions.

Lemma 2. *At a locally stable steady state,* $\frac{\partial k}{\partial B}<0$.

Proof. The numerator in (20) has negative sign. We now use a stability condition to sign the denominator. Using (18), it is easily checked that, at a locally stable steady state,

$$\frac{dk_{t+1}}{dk_t}|_k=\frac{-S_w(\cdot)SR'(k)}{1-S_R(\cdot)R'(k)}<1\Leftrightarrow 0<1+R'(k)\left[SS_w-S_R\right] \tag{21}$$

must hold. Using (19), and $S=k$, it follows that

$$\begin{aligned}&1+R'(k)\left[SS_w-S_R\right]=1+\frac{R'(k)}{\mathcal{F}}\left[Su''(c_1)+\rho R(k)Sv''(c_2)+\rho v'(c_2)\right]>0\\ &\Leftrightarrow\frac{1}{\mathcal{F}}\left\{u''(c_1)+\rho R^2v''(c_2)+R'(k)Su''(c_1)+R'(k)\rho^2RSv''(c_2)+\rho R'(k)v'(c_2)\right\}>0\\ &\Leftrightarrow\frac{1}{\mathcal{F}}\left\{u''(c_1)+\rho R^2v''(c_2)+R'(k)ku''(c_1)+R'(k)\rho^2Rkv''(c_2)+\rho R'(k)v'(c_2)\right\}>0\end{aligned} \tag{22}$$

The denominator of (20) may be written as

$$-\left[u''kR'(k)+u''+R'(k)\rho v'(.)+R\rho v''(.)R'(k)k+R^2\rho v''(.)\right]$$

which has the same sign as the expression in (22). This proves $\frac{\partial k}{\partial B}<0$ at a stable steady state. ■

We now return to the central focus of the paper. Is there a role for public pensions in this economy with myopic agents if the starting point is a dynamically efficient steady state? As a benchmark, consider first the case of no myopia, i.e., $\rho^p=\rho^g=\rho$. The government, in this case, chooses the optimal B by solving

$$\max_B U(B)=u(w(k)-B-k(B))+v(R(k)k(B)+B).$$

Using (16), we get

$$\begin{aligned} U'(B) &= u'(c_1)\left[w'(k)\frac{\partial k}{\partial B} - 1 - \frac{\partial k}{\partial B}\right] + \rho v'(c_2)\left[R'(k)\frac{\partial k}{\partial B}k + R(k)\frac{\partial k}{\partial B} + 1\right] \\ &= \rho v'(c_2^p)(1 - R(k))\left\{1 + kR'(k)\frac{\partial k}{\partial B}\right\}. \end{aligned} \tag{23}$$

The importance of neoclassical technology is now apparent. For in its absence, $R'(k) = 0$ would obtain and then $U'(B) \leq 0$ would hold at a dynamically efficient ($R(k) > 1$) steady state. In other words, we would get the same result as Proposition 1. As an aside, note that Lemma 2 ensures that at a locally stable steady state, $R'(k)\frac{\partial k}{\partial B} > 0$ holds, which implies that the neoclassical technology in the absence of myopia is not sufficient to generate a role for public pensions. This last remark is really to be understood as a re-statement of the main result in Samuelson (1975).

Proposition 5. *(Samuelson, 1975) At a locally-stable, dynamically-efficient, initial steady state, there is no role for public pensions.*

Now suppose agents are myopic, i.e., $\rho < 1$. Can myopia along with neoclassical technology restore a role for publicly-funded pensions? Assuming an interior solution, we find that the optimal B is characterized by $U'(B) = 0$ where

$$\begin{aligned} U'(B) &= u'(c_1)\left[w'(k)\frac{\partial k}{\partial B} - 1 - \frac{\partial k}{\partial B}\right] + v'(c_2)\left[R'(k)\frac{\partial k}{\partial B}k + R(k)\frac{\partial k}{\partial B} + 1\right] \\ &= \rho v'(c_2)\left\{\left[1 + kR'(k)\right](1 - R\rho) + R\frac{\partial k}{\partial B}(1 - \rho)\right\}, \end{aligned} \tag{24}$$

an expression analogous to (23). We are now ready to state our main result.

Proposition 6. *Under sufficiently high myopia ($\rho < \frac{1}{R(k)}$), a necessary condition for the existence of a dynamically efficient steady state equilibrium with positive savings and positive public pensions is $R'(k) \neq 0$.*

Proof. It follows from (24) that

$$U'(B) = 0 \Leftrightarrow \frac{\partial k}{\partial B} = \frac{R\rho - 1}{(1 - R\rho)kR'(k) + R(1 - \rho)} \tag{25}$$

implying that from the point of view of the government, the optimal response of capital to a change in pensions is given by (25). As before, compare this to the same from the point of view of the agent – captured by (20). At an equilibrium with interior k and B, these two responses must be equal; hence

$$\frac{(R\rho - 1)}{(1 - R\rho)\, kR'(k) + R(1-\rho)} = \frac{u''(c_1) + R(k)\,\rho v''(c_2)}{u''(c_1)\, w'(k) - u''(c_1) - R'(k)\,\rho v'(c_2) - R(k)\,\rho v''(c_2)\,[R'(k)\,k + R(k)]}$$

must hold. Routine manipulation and re-arrangement of terms in (**??**) yield

$$-[R(k)\,\rho - 1]\, R'(k)\,\rho v'(c_2) = (R(k) - 1)\left\{[R(k)]^2 \rho^2 v''(c_2) + u''(c_1)\right\} \tag{27}$$

It is clear that at a dynamically efficient steady state, the r.h.s of (27) is negative; in that case, $R'(k) = 0$ would present a contradiction. In fact, since $R'(k) < 0$, for (27) to hold, it is necessary that $R\rho - 1 < 0$ holds. Combining $R\rho < 1$ with the requirement that $R > 1$ implies ρ should be small enough. ■

Notice that Proposition 6 goes through for a constant $R'(k)$. Also note that $R'(k) \neq 0$ is necessary but not sufficient for eq. (27) to have a unique positive solution in B.

Finally, we offer some intuition for our main result. Suppose the return to saving is identical to the marginal product of capital and that the latter is falling in capital. Public pensions have two effects on private saving (in the form of capital). By raising future income, they depress the need for private saving – the income effect. But, as private saving falls, the return to saving rises – the interest-rate effect – and this prevents private saving from getting completely crowded out, as in Corollary 1.

4. Concluding remarks

It is well understood that PAYG social security, a dominant and long-standing institution in developed economies, has no welfare role in dynamically efficient economies. In fact, the most common justification for public pensions in such economies relies on the notion of myopia – agents discounting near dates much less than distant dates. In this paper, we revisit the theoretical underpinnings of myopia as a rationale for social security. We find that in a dynamically efficient OG economy with a linear exogenous-return storage technology and exogenous young-age endowments, positive levels of optimal public pensions and private saving cannot coexist. In fact, a necessary condition for such coexistence is that capital be available as a productive resource and that the production technology be neoclassical.

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