Particle Movement and Separation Phenomena for a Gravity Separator: I. Development of a Markov Probability Model and Estimation of Model Parameters

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ABSTRACT

The complexity of gravity separator mechanics precludes the use of a deterministic model for particle movement on a gravity separator. Particle movement is examined as a stochastic process; a distance-transition Markov probability model for particle movement is proposed. A linear programming method for estimation of the Markov model parameters is explained.

INTRODUCTION AND OBJECTIVE

The density separation principle has been used in the mineral-processing and seed-conditioning industries for many years. In the seed-conditioning industry, the process is known as gravity separation, but in other applications such as mineral processing, it is often referred to as "dry tabling," "jigging," or "gravity concentration." In the seed industry, gravity separators are used to clean grains such as corn, wheat, and soybean. Misra (1983) used a gravity separator to remove shrivelled black nightshade berries from soybeans. The gravity separator has also been used to improve the viability of seed lots by removing damaged or otherwise substandard seed (Misra 1982).

The gravity separation principle is based upon the segregation phenomena characteristic of pneumatically fluidized beds of particles. Vibration and gravity table geometry are employed to segregate particles having similar density or size. Deck slope, vibration, and upward airflow combine to produce the differential movement of light and heavy particles (Fig. 1).

The particle mixture is fed onto the gravity table deck near the lower left-hand corner (Fig. 2). Air is forced up through the perforated deck and through the particle mixture. Vibration agitates the particles, and the lighter or smaller particles, supported by a rising air current, move to the top of the particle mixture and float on its surface. After Rowe et al. (1972), the smaller or less dense particle fraction hereinafter will be referred to as "flotsam", and the larger or denser particle fraction will be referred to as "jetsam." The jetsam remains in contact with the deck surface and is transported up the cross-slope by the deck vibration. The flotsam, floating in a fluidized condition, flows down the cross-slope. The elevation of the deck shown in Fig. 2 (Oliver Manufacturing Co., Inc., 1980) increases from the bottom of the figure to the top (y direction). The elevation of the deck decreases from the left to right (x direction). Thus, the lower right-hand corner is the lowest point on the gravity table deck and is the location where the flotsam concentrate.

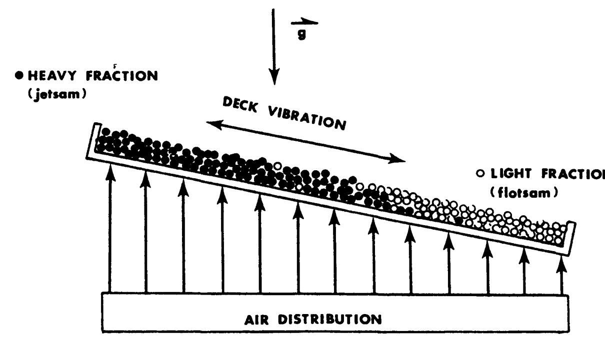

Fig. 1—View of gravity table deck and particle bed cross section. Net particle flow is perpendicular to the page.

A better understanding of gravity separator phenomena would aid in machine design and operation. To understand the process by which particles are separated and to use this knowledge to predict gravity separator performance, it is necessary to develop a model that describes movement of individual particles on the gravity table deck. However, dimensional analysis yields as many as eight classes of dimensionless parameters that influence particle separation (Balascio, 1985).

Developing a deterministic model for particle movement based on the equations of motion is infeasible because thousands of particles occupy the gravity table deck. Each particle can move in three directions, and this movement is influenced by adjacent particles.

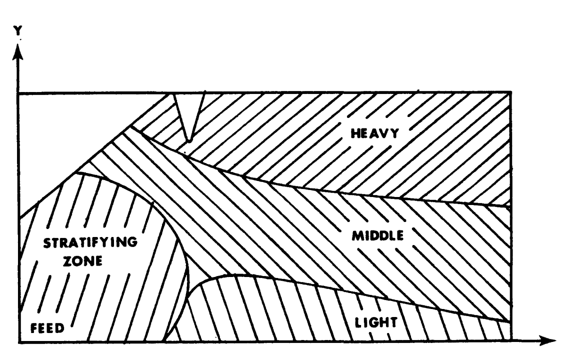

Fig. 2—Ideal operation of a gravity separator.

Article was submitted for publication in December, 1986; reviewed and approved for publication by the Food and Process Engineering Div. of ASAE in June, 1987.

Journal Paper No. J-12468 of the Iowa Agriculture and Home Economics Experiment Station, Ames, Iowa. Project No. 2591.

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Hence, the equations of motion would need to be solved as a system having thousands of degrees of freedom. Merely specifying the initial conditions would be a formidable task.

In view of the difficulties associated with the use of a deterministic model of particle movement, a stochastic model is an alternative. The objective of the research reported in this paper is to develop a stochastic model to describe the movement of particles through a gravity separator.

DEVELOPMENT OF A STOCHASTIC MODEL OF PARTICLE MOVEMENT

Observation of gravity separation processes indicates that a particle's previous movement has very little influence on its subsequent movement. For example, during the separation operation, a light particle may randomly sink to the bottom and be carried up the slope, and in the next instant, the same particle may float to the surface and slide down the slope. A particle's movement is, however, strongly influenced by its location on the deck.

This suggests the use of a Markov probability model (Parzen, 1960). Particle movement that is governed by a Markov process is independent of previous movement; it is a function of the particle's present location. For the purposes of modeling particle movement, the states of a markov process can be thought of as being associated with particle positions. Movements between the states (positions) are known as transitions. Transition probabilities are the probabilities of certain movements (transitions) occurring during a specified interval known as a transition period.

There are two fundamentally different ways of defining a transition period. They are: time transition and distance transition.

Time-Transition Markov Model

Fan and Chang (1979) proposed a stochastic model for the mixing of large particles in gas-fluidized beds. They developed a nonstationary random walk model to describe particle mixing and segregation. Random walk models belong to the general class of Markov models.

The use of Fan and Chang's model requires that the gravity table deck (Fig. 2) be divided into nonintersecting regions (Fig. 3). These regions correspond to states in a Markov chain. In general, each particle on the deck could have its movement controlled by its own unique Markov process. For practical purposes, however, it will be assumed that the movements of all particles belonging to the same class of particles (e.g., flotsam or jetsam) are governed by the same Markov process. The system is defined as the gravity table deck, the particle mixture flowing on the deck, and the specific particle whose movement is being studied. The system is in state j if the particle occupies the area of the deck associated with state j.

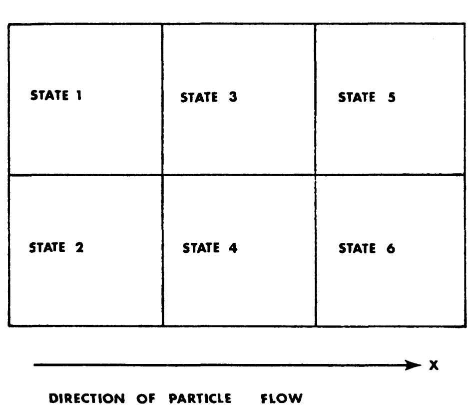

Fig. 3—Rectangular deck with six states.

If we confine our study to the steady-state operation of the gravity table, the stationary Markov model is appropriate. For a uniform mixture of particles fed at a constant rate, the time-averaged concentrations of particle fractions in various regions on the deck would be expected to change only with position and not with time. With the gravity table deck divided into a number of sections or "states" as is shown in the simplified rectangular deck in Fig. 3, each class of particle in the mixture would have associated with it a transition matrix for some specified time-transition period ΔT. A natural choice for the transition period might be some integer multiple of the deck vibration period. The transition matrix for a specific particle type is composed of the state-to-state transition probabilities for transition periods of length ΔT.

For example, suppose the particle mixtures were composed of jetsam and flotsam particles. The transition matrix for the light flotsam particles on the deck in Fig. 3 would be, say:

$$P = \begin{matrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} \\ P_{41} & P_{42} & P_{43} & P_{44} & P_{45} & P_{46} \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} & P_{56} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{65} & P_{66} \end{matrix}$$

P_{ij} is the probability that a flotsam particle located in state i would move to state j during the transition period T. For example, P_{66} is the probability that a particle located in state 6 would remain in state 6. P_{21} is the probability that a flotsam particle located in state 2 would move to state 1 during the span of one transition period. Clearly, for any i, we must have:

$$\sum_{j=1}^{6} P_{ij} = 1.0$$

i.e., the row sum must equal 1.

There are, however, a number of problems associated with the time-transition model. The most important difficulty is to collect data on movement of individual particles. Particles tend to stratify in layers; it is likely that an individual particle would not be visible for a portion of its journey. See Balascio (1985) for a more detailed discussion of the shortcomings of a time-transition model.

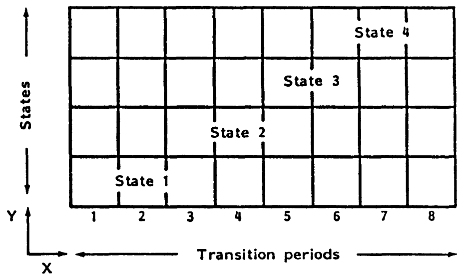

Fig. 4—Distance-transition scheme for a rectangular gravity separator deck.

Distance-Transition Markov Model

A way to circumvent the problems of the time-transition model is to adopt an alternate definition of transition period. A convenient approach is to consider transitions based upon position or movement in the longitudinal direction. Observation of the deck in operation indicates that movement in the x-direction is basically uniform across the width. All particles on the rectangular portion of the deck appear to move toward the output end at roughly the same x-velocity. Thus, we let longitudinal distance, x, become a "psuedo" time dimension.

Consider the rectangular portion of a gravity table deck shown in Fig. 4. The deck width is divided into discrete sections which corresponds to states. In this example, four states states divide the deck. The particle mixture is fed on the left at time step (position step) 1 and traverses the deck through a series of time (position) transition periods until it leaves on the right at the final transition period, which in Fig. 4 is period 8. Thus, the length of the deck is divided into 8 discrete sections so that x-position may be thought of as a discrete time variable. That is, one transition period corresponds to the movement of one discrete step in the x-direction.

For a stationary process, a single 4 by 4 transition probability matrix describes the particle movement: $P = [P_{ij}]$; i, j = 1, 2, 3, 4.

For example, P_{32} is the probability that a particle in state 3 will move to state 2 after moving forward one transition period in the x-direction (from say transition period t = 3 to transition period t = 4). Again, it is obvious that the row sums of matrix P must equal 1, i.e.,

$$\sum_{j=1}^{4} P_{ij} = 1.0$$

Using x as a transition variable has had two notable effects upon the model. First, it is possible to use fewer states; fewer states will reduce the number of transition probabilities which need to be computed. Second, since position on the deck changes with each transition period, it is possible that if conditions on the deck change greatly with x-position, we may have sacrificed the stationary property of the time-transition model which was discussed ealier. That is, the transition matrix may change from one transition period to the next. It is much preferable to use a stationary model; the introduction of nonstationarity greatly greatly complicates the analysis.

For simplicity, we will confine our study to the movement of particles on the rectangular portion of the gravity table deck. Since we will not be able to collect individual particle data or "micro data," it will be necessary to collect so-called aggregate data or "macro data." Aggregate date are in the form of particle distributions by states within the transition periods. These, particle distributions change with transition period. We have noted that x-velocities of particles are approximately equal across the width of the gravity table. With x-velocities for particles nearly uniform across the deck width, it is easy to relate the particle distributions in the various transition periods to one another by use of the Markov transition probability matrix. Uniform x-velocity is not a necessity, however, Balascio (1985) discusses the case of nonuniform x-velocity.

Let W be the quantity of particles of a specific type in a lot which is passed through a gravity separator. All particles in the lot pass over the deck shown in Fig. 4. Let $y_i(t)$ be the true fraction of particles from the population of size W which pass through state i in transition period t. Then, if r is the number of states, we have for all transition periods t:

$$\sum_{i=1}^{r} y_i(t) = 1.0.$$

If $W_i(t)$ is the number of particles which pass through state i in transition period t, then $y_i(t) = W_i(t)/W$. We cannot measure the $[y_i(t)]$ directly because we do not have data for the entire population of particles which pass through the gravity separator. By using the uniform x-velocity assumption, however, we can estimate the true fractions $[y_i(t)]$ with the aggregate data $[\gamma_i(t)]$ which are determined from samples of the larger population. That is, for the particles of the specific class we are studying, the $[\gamma_i(t)]$ are the fractions of those particles by state in transition period t. These fractions are determined from a sample of size w(t) from the larger population of size W. Thus, with $\hat{y}_i(t)$ the estimate of $y_i(t)$ we have

$$\hat{y}_i(t) = \gamma_i(t) = w_i(t)/w(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

Here, $w_i(t)$ is the portion of the sample from state i; and

$$w(t) = \sum_{i=1}^{r} w_i(t)$$

After Lee et al. (1977), we relate the fractions $[y_j(t+1)]$ to the fractions $[y_i(t)]$ with the following equation:

$$y_j(t+1) = \sum_{i=1}^{r} P_{ij} y_i(t) + u_j(t+1) \quad \ldots\ldots\ldots\ldots\ldots [2]$$

Here, $u_j(t+1)$ is a random component with zero mean. It is emphasized that the Markov process is still defined for the movement of individual particles. The state of the system is defined by the position of the particle whose movement is being considered.

We have no data on individual particle movements, but substitution of our estimates of $[y_i(t)]$, $[\gamma_i(t)]$, into equation [2], yields equation [3]. Equation [3] is the means by which we relate our aggregate data composed of particle distributions within the transition periods to the transition probability matrix, P, which defines the assumed Markov process.

$$\gamma_j(t+1) = \sum_{i=1}^{r} P_{ij} \gamma_i(t) + u_j(t+1) \quad \ldots\ldots\ldots\ldots\ldots [3]$$

For a nonstationary model, P_{ij} becomes a function of transition period t so that equation [3] is modified:

$$\gamma_j(t+1) = \sum_{i=1}^{r} P_{ij}(t)\,\gamma_i(t) + u_j(t+1) \quad \text{[4]}$$

If r is the number of states and T + 1 is the number of transition periods, the stationary model will have r^2 unknowns whereas the nonstationary model has r^2T unknowns. For each of T transition periods, r equations of the form given by equation [3] can be written. For each transition probability matrix, r equations can be written by requiring that the row sums equal 1. Thus, the stationary model is determinate if T + 1 = r and overdeterminate if T + 1 is greater than r. The nonstationary model has a total of 2rT equations. Unless certain assumptions are made, the nonstationary model is always indeterminate if r is greater than 2.

A typical approach is to assume that the variable transition probabilities are linearly dependent upon parameters which vary with the transition variable (Lee et al., 1977; Telser, 1963).

$$P_{ij}(t) = P_{ij} + \sum_{k=1}^{m} \delta_{ijk}\, Z_k(t) \quad \text{[5]}$$

Here, m is the number of so-called "external variables" to which the probabilities are related. Note that with m = 0, we have the stationary case. The P_{ij} are the entries of the stationary matrix, P, which has all the required properties of a transition probability matrix. That is, all its entries are nonnegative and the row sums equal 1. The δ_{ijk} are coefficients which are to be determined, and the $Z_k(t)$ are the external variables. An additional set of constraints on the δ_{ijk} requires that:

$$\sum_{j=1}^{r} \delta_{ijk} = 0$$

for all i and k. This ensures that the row sums of $P_{ij}(t)$ will always equal 1 regardless of the values of $Z_k(t)$. Note also, that for the range of values of $Z_k(t)$, the δ_{ijk} should be determined under the condition that for all t, the entries of P(t) are nonnegative. With the use of equation [5], the number of unknowns for the nonstationary model has been reduced to $(m+1)r^2$. Thus, with T+1 number of transition periods, it can be shown that the nonstationary model specified by equation [5] is determinate for T = (r-1)(m+1) and overdeterminate for T greater than (r-1)(m+1).

Note that in choosing the variables $Z_k(t)$, we are not concerned with variables that do not change with x-position (transition period t) such as slope and vibration rate. For the gravity table, some possible candidates for the variables $z_k(t)$ would include local deck geometry parameters, local superficial air velocity, settled bed depth, static air pressure, or even some parameter quantifying the distribution of particles in a particular transition period. It should be emphasized that it is extremely desirable to limit the number of parameters, m, to a few as possible (preferably 1 or 2). From a statistical standpoint, the estimates of the P_{ij} and δ_{ijk} become more reliable as the system becomes more overdetermined. Unfortunately, we are limited by the number of transition periods that are available, so m must be held small. In practice, we will be able to make better use of the number of transition periods we have by eliminating some of the unknowns. With a 4-state system, for example, if transitions are allowed only between adjacent states (a physically justifiable assumption), then the number of unknowns for a stationary model such as that which appears in Fig. 4 is reduced from 16 to 10 (P_{13}, P_{14}, P_{24}, P_{31}, P_{41}, and P_{42} equal zero).

Estimation of Transition Probabilities from Aggregate Data

It is more difficult to estimate transition probabilities from aggregate (macro) data than from individual particle data or "micro data." Micro data are records of individual particle positions as a function of time. The problem of estimating transition probabilities from aggregate data has most often been discussed with regard to management and market analysis applications. Telser (1963) used aggregate data to estimate transition probabilities for a Markov process which he postulated to govern the distribution of market shares for three brands of cigarettes. Ezzati (1974) used a similar approach to forecast market shares of home-heating units.

There are a number of methods for the estimation of the transition probabilities involved. The initial formulation of the aggregate data problem is always the same, however. For simplicity, consider the stationary case from which it is possible to generalize to the nonstationary model. The data are related to the transition probabilities using equation [3]. An unrestricted least squares estimator can be developed by using the method of Lagrange multipliers with the traditional error sum of squares as the objective function.

The minimization is subject to the constraints that the row sums of the transition probability matrix P must equal 1 (Lee et al., 1977; Lee et al., 1965). The primary difficulty with this approach is that is is not possible to include the nonnegativity constraints on the entries of matrix P. Thus, it is possible to obtain "infeasible" solutions—solutions for which some entries, P_{ij}, of matrix P (which are probabilities) may be negative or have absolute values greater than 1.

If the nonnegativity constraints are included, the objective function is still specified as the error sum of squares then the estimation of the P_{ij} becomes a classic quadratic programming problem. That is, the objective function is quadratic; and the constraints are all linear. See Boot (1964), Hadley (1964), or Sposito (1975) for discussions of the quadratic programming problem. Ezzati (1974) uses a maximum likelihood estimator proposed by Lee et al. (1965) which also results in a quadratic programming problem. Theil and Rey (1966) discuss the application of quadratic programming of Telser's data (1963). Judge and Takayama (1966) discuss different forms of the quadratic estimator and work with Telser's data as an example. Balascio (1985) discusses the Lagrange multiplier and quadratic programming approaches as they relate to this research.

A disadvantage of the quadratic programming method is that it is rather complicated. Use of the method to calculate estimates of transition probabilities requires a considerable amount of manipulation to arrange the data in a form which is compatible with that required by the available quadratic programming software.

There is no reason that an alternative objective function cannot be used, however. Lee et al. (1977) stated that there is no basis for preference of a least squares objective function over a minimum absolute deviation (MAD) function. The function, MAD, is defined as:

$$\text{MAD} = \sum_{t=2}^{T+1} \sum_{j=1}^{r} |\gamma_j(t) - \hat{\gamma}_j(t)| \qquad [6]$$

Let the estimate of proportion $\gamma_j(t+1)$ be given by:

$$\hat{\gamma}_j(t+1) = \sum_{i=1}^{r} P_{ij}\, \gamma_i(t)$$

We can now define $up_j(t) - um_j(t) = \gamma_j(t) - \hat{\gamma}_j(t)$ with $up_j(t)$ and $um_j(t)$ strictly nonnegative. We rewrite equation [6] as:

$$\text{MAD} = \sum_{t=2}^{T+1} \sum_{j=1}^{r} [up_j(t) + um_j(t)] \qquad [7]$$

We now formulate the optimization problem as follows: Minimize:

$$\text{MAD} = \sum_{t=2}^{T+1} \sum_{j=1}^{r} [up_j(t) + um_j(t)]$$

subject to:

$$up_j(t) - um_j(t) + \sum_{i=1}^{r} \gamma_i(t-1)P_{ij} = \gamma_j(t)$$

for all j and $t > 1$. We also have the row sums of P equal to 1 for all i:

$$\sum_{j=1}^{r} P_{ij} = 1.$$

Naturally, we require that P_{ij} be nonnegative for all i and j. Expressed in this manner, the estimation of P_{ij} is a linear programming problem. For the nonstationary problem, the objective function, MAD, remains the same, but the constraints are rewritten to include the parameters δ_{ijk} of equation [5] and the variable transition probability matrix P(t). We define $\delta_{ijk} = \delta p_{ijk} - \delta m_{ijk}$ and include the row sum constraints on the δ_{ijk}:

$$\sum_{j=1}^{r} (\delta p_{ijk} - \delta m_{ijk}) = 0$$

for all i and k. We use the equality constraints:

$$P_{ij}(t) - P_{ik} - \sum_{k=1}^{m} Z_k(t)\, (\delta p_{ijk} - \delta m_{ijk}) = 0$$

to define $P_{ij}(t)$. There are of course, the required nonnegativity constraints on the activities, $P_{ij}(t)$, P_{ij}, δp_{ijk}, δm_{ijk}, $up_j(t)$, and $um_j(t)$ for all i, j, k, and t.

The greatest disadvantage of this approach is the large number of parameters of "activities" which need to be estimated. For the nonstationary case, this number can reach the thousands quickly. Even so, the linear programming algorithms used to solve these problems are extremely efficient and computation costs are minimal even for nonstationary problems. Because of the simplicity of formulation, and the familiarity of literature concerning linear programming, the linear programming approach was chosen for this research. In addition, linear programming software was readily available and quite useable. The MPSX linear programming package available on Iowa State University's IBM computer was used for the computations. See Appendix A of Sposito (1975) for an explanation of the MPSX linear programming features and format. A Fortran computer program was written by Balascio (1985) to organize the data into proper input format for use by the MPSX software.

SUMMARY

Fan and Chang's time-transition random walk model (1929) was used as a starting point for the development of a time-transition Markov model to describe the steady state operation of a gravity separator. A number of serious problems were noted with the use of a time-transition model.

A distance-transition Markov model was proposed which overcomes these difficulties. The model is simplified considerably if longitudinal x-velocity is uniform across the width. It can then be assumed that a sample of particle distributions across the width for a particular transition period is an estimate of the true distribution of all particles which flow through that transition period. It should be emphasized that each particle type in the mixture has associated with it a transition probability matrix of its own. The differences among these transition probability matrices are responsible for the different movement of particles and the resultant separation phenomena.

Use of stationary and nonstationary Markov models was discussed, and methods of estimating the parameters in these models from the aggregate "macro" data were examined. It was decided that a linear programming approach is most suitable for the estimation probabilities associated with the Markov processes which describe the differential movement of various particle classes as they move through a gravity separator.

This is the first of two papers which examine the separation phenomenon in gravity table operation. In this paper, the theoretical model of particle movement has been developed and presented. In the second paper, the theoretical model is corroborated with experimentally obtained data for soybeans.

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