

Further Evidence on the Asymmetric  
Behavior of Economic Time Series  
over the Business Cycle

by

Barry Falk  
Assistant Professor of Economics  
Iowa State University  
Ames, Iowa 50011

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### Abstract

Evidence has recently been put forward to support the hypothesis that recessions tend to be steeper than recoveries in economic activity. That evidence, however, was confined to the behavior of the unemployment rate. This paper looks at the behavior of real GNP, investment, and productivity in the United States since 1948 and concludes that these series' behavior do not seem to support the asymmetry hypothesis.

## I. Introduction

As economists continue to develop theoretical structures which can generate an economy which displays features thought to characterize observed business cycles, it is important that we clarify precisely what characteristics these cycles do, in fact, possess. Recently, Salih Neftci [1984] resurrected the issue of whether or not the business cycle displays symmetric behavior on both sides of the trough (or peak). Although other economists (such as Keynes [1936] and Hicks [1950]) have suggested that cycles are generally asymmetric, implicit in much of the recent theoretical and applied work is the notion that they are not.<sup>1/</sup> This latter point of view seems to be maintained primarily because of the simplicity it affords in both theoretical and applied work rather than on the basis of formal statistical tests whose results support the view.

Adapting a strategy developed by Heckman [1981] for use in panel data models, Neftci proposed a way to formally test at least one dimension of the proposition that economic time series exhibit asymmetric behavior over the course of a typical business cycle. The conventional view of the business cycle, among those who have argued that it is asymmetric, is that contractions in economic activity are more violent but tend to last for a shorter period of time than the subsequent expansions do.<sup>2/</sup> Neftci's notion of this asymmetry is that, if it is correct, "runs" of increasing values of a discrete economic time series characterized by a business cycle, should be more likely to persist than runs of decreasing values, if the series is "procyclical" (as would be the case, e.g., for real GNP, output per man-hour, etc.). Countercyclical variables, such as the unemployment rate, would be expected to show the reverse pattern.

Based on this idea, Neftci designed a test of the asymmetry proposition which utilizes the theory of finite Markov chains. He applies the test to quarterly data

on the unemployment rate of the United States for the 1948-1981 period and finds some support for asymmetry. The purpose of this paper is to see the extent to which Neftci's result pertains to other economic time series typically associated with the business cycle in the United States. The series that will be considered are: real GNP, output per worker-hour, and gross domestic private investment.

In the next section of the paper, Neftci's theory and test will be briefly summarized. Aside from providing a review, the discussion will serve to introduce the notation that will be used throughout the remainder of the paper. The series that were listed above share the characteristic of being procyclical with a significant positive trend. This combination turns out to bias the test toward rejecting the symmetry hypothesis and is one reason why Neftci focussed on the unemployment rate.<sup>3/</sup> Section III of the paper elaborates on this issue and describes what adjustments were made here to compensate for this problem. The main results obtained from this study are discussed in Section IV. The paper's conclusions are summarized in Section V.

## II. Neftci's Model and Test

Suppose that one has  $T+1$  sequential observations of an economic variable whose  $t$ -th observation is denoted  $X_t$ . Define a new sequence  $\{I_t\}$  such that  $I_t = +1$  if  $\Delta X_t > 0$  and  $I_t = -1$  if  $\Delta X_t \leq 0$ . Assume that the sequence  $\{I_t\}$  is a stationary stochastic process which is representable as a second-order Markov process.<sup>4/</sup> Let  $\lambda_{11}$  denote the probability that if  $I_{t-2}$  and  $I_{t-1}$  were both equal to  $+1$  then so would  $I_t$ . Let  $\lambda_{00}$  denote the probability that  $I_t$  will equal  $-1$  given that  $I_{t-2}$  and  $I_{t-1}$  did too. The idea is that if  $\{X_t\}$  is an economic time series characterized by a symmetric business cycle then  $\lambda_{00}$  and  $\lambda_{11}$  should be equal to one another.

The log-likelihood function corresponding to a given realization of  $\{I_t\}$  can be written

$$L(S_T, \lambda_{11}, \lambda_{00}, \lambda_{10}, \lambda_{01}, \pi_0) = \log \pi_0 + n_{11} \log \lambda_{11} + T_{11} \log(1 - \lambda_{11}) + n_{00} \log \lambda_{00} + T_{00} \log(1 - \lambda_{00}) + n_{10} \log \lambda_{10} + T_{10} \log(1 - \lambda_{10}) + n_{01} \log \lambda_{01} + T_{01} \log(1 - \lambda_{01}) \quad (1)$$

where  $S_T$  is the realization of  $\{I_t\}$ ,  $\lambda_{10} = P(I_k = 1 | I_{k-1} = 1, I_{k-2} = -1)$ ,  $\lambda_{01} = P(I_k = -1 | I_{k-1} = -1, I_{k-2} = 1)$ , and  $\pi_0$  is the probability of the initial state.<sup>5/</sup> The parameters  $n_{11}, \dots, T_{01}$  denote the number of occurrences of the various states implied by the associated transition probabilities.

The values of the four unknown parameters can then be estimated by maximizing the log-likelihood function. The form of the dependence of  $\pi_0$  on these parameters requires that an iterative search procedure be used. Once the  $\lambda$ 's have been estimated, a confidence ellipse for  $\lambda_{00}$  and  $\lambda_{11}$  can be constructed through the solution of the quadratic equation

$$(\lambda - \hat{\lambda})'(-H)(\lambda - \hat{\lambda}) = \chi^2_2(\alpha) \quad (2)$$

where  $\hat{\lambda} = [\hat{\lambda}_{00} \hat{\lambda}_{11}]'$ ,  $H$  is the  $2 \times 2$  matrix of second partial derivatives of  $L$  with respect to  $\lambda_{00}$  and  $\lambda_{11}$  evaluated at  $\hat{\lambda}$ , and  $\alpha$  is the selected confidence level. The null hypothesis of asymmetry, i.e.,  $\lambda_{11} > \lambda_{00}$  (for a procyclical variable) is rejected if any part of the ellipse falls on or below the 45-degree line (with  $\lambda_{00}$  measured on the horizontal axis). For the unemployment rate data, Neftci was unable to reject  $H_0$  at the 80 percent level.

### III. Problems with Procyclical, Increasing Series

We have already noted that Neftci's notion of asymmetric behavior of an economic time series is that runs of increasing values of the series are more or less likely to persist than are runs of decreasing values. A priori, one would expect that procyclical variables would show a greater tendency for persistent positive runs while countercyclical variables would show a greater tendency for persistent negative runs. In terms of the notation introduced in the previous section, we would expect  $\lambda_{11} > \lambda_{00}$  for procyclical series and  $\lambda_{00} > \lambda_{11}$  for countercyclical series (which is what Neftci found for the unemployment rate data).

A sticky problem arises, however, in applying the test to procyclical variables that are characterized by a positive trend generated by forces normally thought to be independent (or nearly so) of the forces generating the business cycle. Since a positive trend can be defined as a tendency for the series to show persistent positive runs, such a trend would tend to bias the test results toward acceptance of the asymmetry hypothesis offered above even though the business cycle itself might be symmetric.<sup>6/</sup> Further, the test described above would no longer be valid if the trend was reflected in the form of increasingly long runs of increases (or shorter runs of decreases) since the test requires that the index sequence  $\{I_t\}$  be stationary.

Thus, it would seem to be appropriate, in the case of a procyclical variable whose time series displays a positive trend, to remove the trend component from the series and define the business cycle in terms of deviations around trend.<sup>7/</sup> Of course in devising a method to extract the trend from the series, one must always be concerned with the possibility that part of what one removes from the data in the form of trend can actually have been an intrinsic part of the component of the data that one intends to focus on. Suppose, for example, that abstracting from the

effects of what we usually think of as trend forces (such as population growth and technological change), the business cycle is asymmetric in the following sense. It has a fairly regular length with the average period of a downswing being  $t_1$  periods and the average length of a upswing being  $t_2$  periods, where  $t_2 > t_1$ . Suppose further that the longer upswings, despite the fact that they may be less steep than the downswings, tend to generate an upward trend in the series. That is, suppose that an intrinsic property of the mechanism propogating business cycles is that successive peaks get higher and higher. In this case, removing the trend by conventional methods would remove more than was intended and would bias the test against asymmetry.

I know of no easy way out of this dilemma but will proceed anyway under the assumption that the business cycle itself does not generate a trend in the series. In this case it makes sense to attribute all of the observed trend to "other" forces, remove the observed trend from the series and proceed to analyze the behavior of the residual component. This procedure was followed here.

#### IV. Test Results

Quarterly data for the period 1948:I-1983:IV were collected for real GNP, real gross private domestic investment, and output per worker hour.<sup>8/</sup> The natural logarithm of each series was regressed on a linear trend and a constant. The regression summaries are reported in Table 1. The regression residuals formed the basis for further analysis.

Following Neftci, the sequence  $\{I_t\}$  was constructed for each of the three series (see Section II) and in each case it was assumed to be representable as a stationary, second-order Markov process whose log-likelihood function is given by equation (1). The values of the parameters  $n_{00}, \dots, T_{01}$  for each of the series are

displayed in Table 2, as are the initial states which determine the form of the function  $\pi_0(\lambda)$ .

Given this information, the log-likelihood function was maximized using a iterative search routine from the GQOPT package.<sup>9/</sup> The search was facilitated by the fact that the vector of first partial derivatives and the matrix of second partial derivatives of the log-likelihood function can be analytically derived. Various starting points were used including setting  $\pi_0$  equal to an arbitrary constant and solving the first order conditions for the  $\lambda$ 's. In all three of the cases these values turned out to come very close to the result generated by the search. Table 3 summarizes the numerical results obtained while the confidence ellipsoids for  $\lambda_{00}$  and  $\lambda_{11}$  are pictured in Figures 1-3.<sup>10/</sup>

The results that conform the most closely to Neftci's are those that were obtained for the real GNP series. In particular, the point estimates of  $\lambda_{00}$  and  $\lambda_{11}$  are consistent with the view that over the course of the business cycle GNP tends to remain in its upswing longer than it remains in its downswing. Further, when the roles of  $\lambda_{00}$  and  $\lambda_{11}$  are reversed, the point estimates and their standard errors are very close to the values Neftci obtained using the overall unemployment rate. However, the joint 80 percent confidence region for these two parameters indicate that the null hypothesis ( $\lambda_{11} > \lambda_{00}$ ) would be rejected at that level.

The evidence supporting asymmetry is even weaker in terms of the investment data. The point estimate of  $\lambda_{00}$  is slightly larger than the estimate of  $\lambda_{11}$ , though these two values (and their corresponding standard errors) are virtually identical. In addition, the 80 percent confidence region seems to have its interior nearly evenly divided on both sides of the symmetry line.



Perhaps the most interesting of the three cases is the output per worker-hour situation. Given the point estimates of  $\lambda_{00}$  and  $\lambda_{11}$ , and the nature of the 80 percent confidence region, it seems to be the most likely candidate for displaying an asymmetric cycle. However, the asymmetry suggested is one in which downturns in this procyclical variable tend to be more persistent than upturns.

Thus, taken individually or as a group, the results offer a different conclusion than what Neftci obtained using the unemployment data. Whereas he could not reject the asymmetry hypothesis at the 80 percent level, that hypothesis was rejected here for all three of the series considered. It was noted earlier that the removal of the trend from each series could, if part of the apparent trend is part of the business cycle, bias the estimate of  $\lambda_{11}$  downward. This could account for some of the discrepancy. However it seems as though the bias would have to be quite substantial to be the source of the rejections, especially in the investment and productivity cases.

## V. Summary

This research was motivated by Neftci's [1983] evidence supporting the view that cycles in the unemployment rate in the United States are asymmetric, with drops in that rate tending to last longer than the increases over the course of a business cycle. If this view is correct, and if it generalizes to other cyclical variables, then it could have rather severe (and inconvenient) implications for the theory and practice of macroeconometrics. To pursue this issue, I collected data for real GNP, real gross private domestic investment, and output per worker-hour covering about the same sample period and using the same sampling interval as Neftci's unemployment data. However, unlike Neftci, I detrended my data before

proceeding further (for reasons discussed in Section III). Applying Neftci's test procedure to these series, in all three cases I was able to reject the asymmetry hypothesis at the same level of confidence at which he was unable to reject it.

TABLE 1

## Regression Results - Detrending the Data

$$Y_t = a_0 + a_1 T_t + u_t$$

$Y_t =$	GNP	Investment	Output/Hour
$a_0 =$	8.513 (0.006)*	6.610 (0.0199)	-0.585 (0.0045)
$a_1$	0.00845 (0.000076)	0.00848 (0.000238)	0.00374 (0.000054)

\*: Standard Errors

TABLE 2Summary of the Index Sequence  $\{I_t\}$ 

	GNP	Investment	Output/Hour
$n_{00}$	26	24	27
$T_{00}$	16	16	17
$n_{11}$	36	24	15
$T_{11}$	15	17	18
$n_{10}$	15	18	18
$T_{10}$	9	12	14
$n_{01}$	16	16	17
$T_{01}$	8	14	15
Total	141	141	141
Initial State	+1,+1	+1,-1	+1,-1

Table 3

## Estimation Results

	$\hat{\lambda}_{00}$	$\hat{\lambda}_{11}$	$\partial L / \partial \lambda_{00}$	$\partial L / \partial \lambda_{11}$	$\partial^2 L / \partial \lambda_{00}^2$	$\partial^2 L / \partial \lambda_{11}^2$	$\partial^2 L / \partial \lambda_{00} \partial \lambda_{11}$
GNP	.6148 (.075)*	.7147 (.064)	3.7E-09	-1.4E-05	-179.96	-249.90	.985
Investment	.5959 (.077)	.5811 (.077)	-1.1E-06	-1.1E-06	-168.48	-170.87	.491
Output/Hour	.6094 (.073)	.4514 (.086)	-1.1E-05	7.0E-08	-187.54	-134.80	.336

\*: Standard errors

Figure 1

Real GNP

$$H_0: \lambda_{11} > \lambda_{00}$$

80% Confidence Region

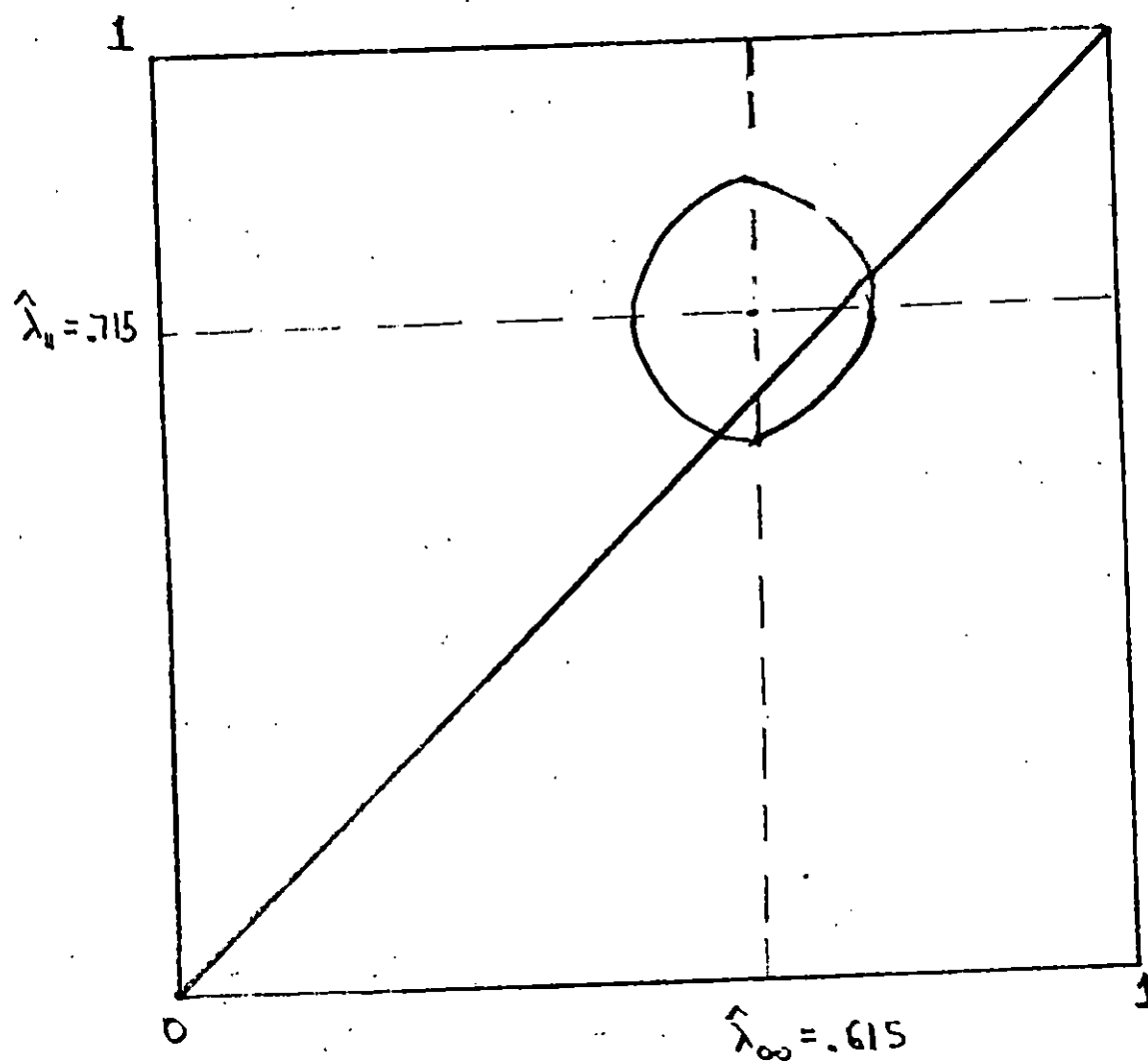


Figure 2

Real Gross Private Domestic Investment

$$H_0: \lambda_{11} > \lambda_{00}$$

80% Confidence Region

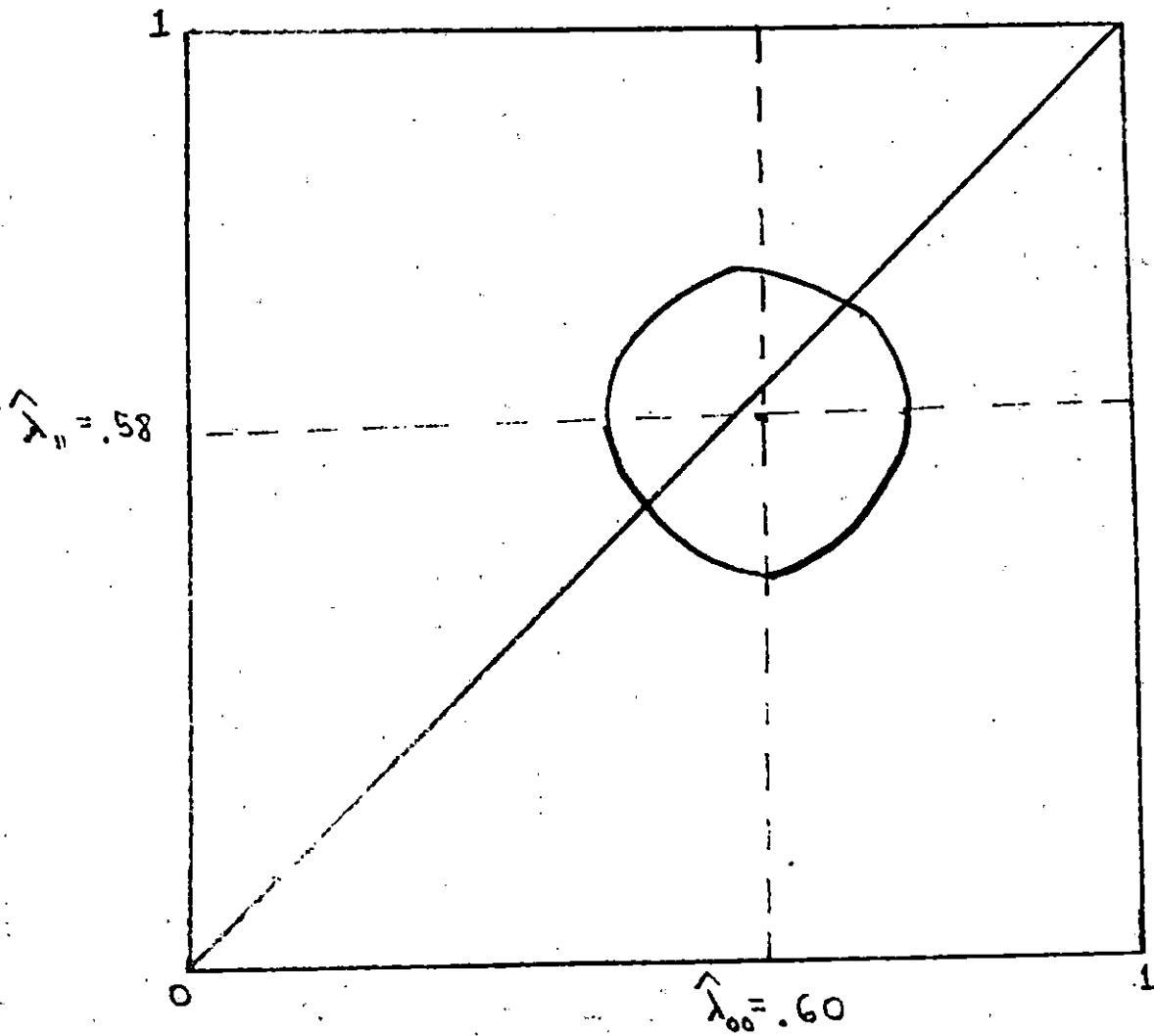
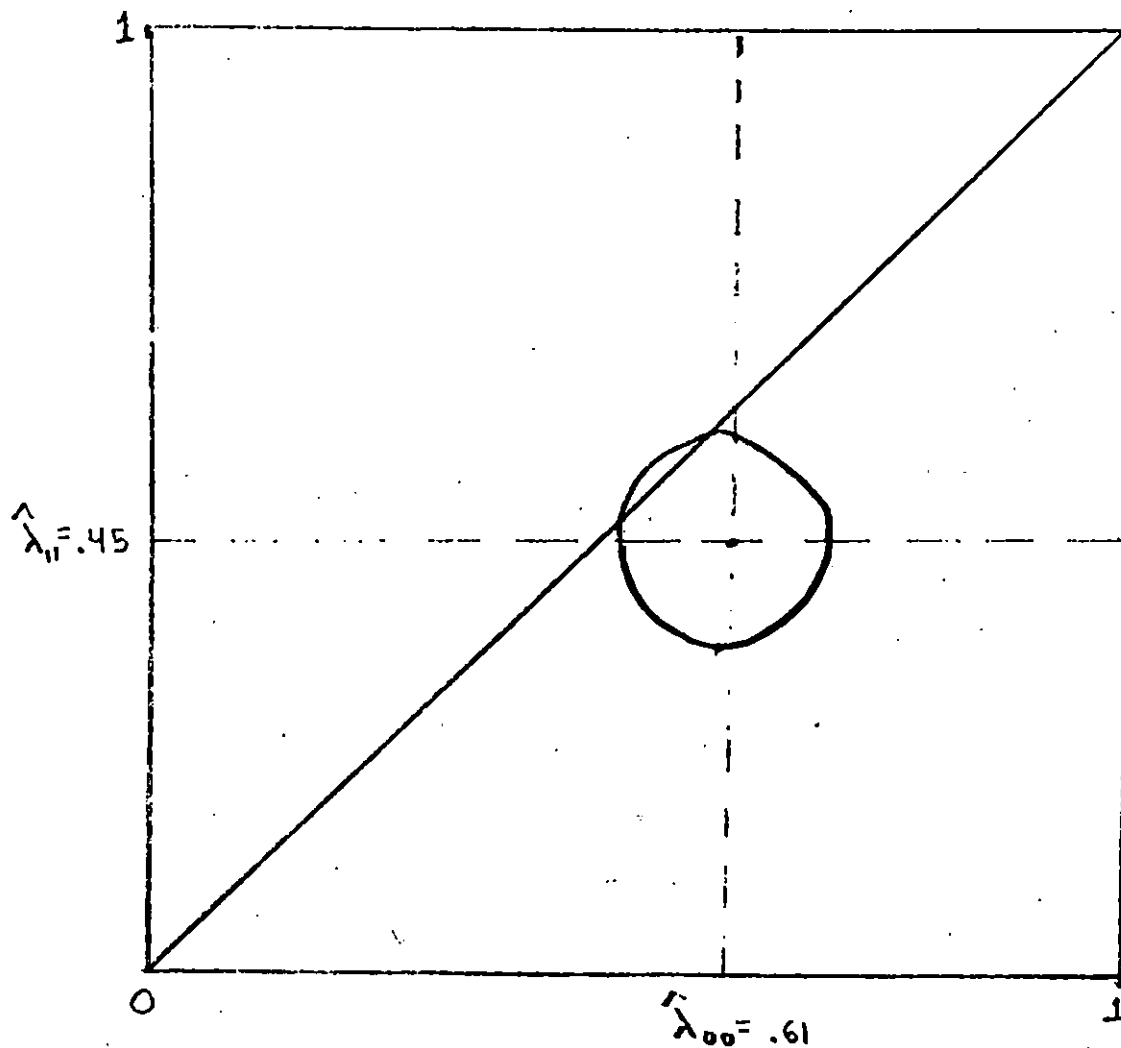


Figure 3

Output Per Employee Hour

$$H_0: \lambda_{11} > \lambda_{00}$$

80% Confidence Region





Notes

1/ See, for example, Neftci's (pp. 308-309) discussion of the way that the assumption of symmetry is reflected in rational expectations models such as those designed by Hansen and Sargent [1980].

2/ See Keynes [1936, p. 314].

3/ See Neftci (p. 316).

4/ In the case of quarterly data, the choice of a second-order process seems reasonable and coincides with Neftci's choice for the unemployment series.

5/ Neftci (pp. 326-327) provides the equations which can be used to solve for  $\pi_0$  in terms of the transition probabilities. I should add here that, in solving the steady state equations (p. 326), I obtain the same solutions given by Neftci (p. 327) for  $\pi_{00}$  and  $\pi_{11}$  but, what appear to be different solutions for  $\pi_{01}$  and  $\pi_{10}$ . I found the two to be equal to each other and equal to  $[(1-\lambda_{00})/\lambda_{01}]\pi_{00}$ .

6/ If the variable is countercyclical and has a positive trend, the trend biases the test toward symmetry since the presumption is that the asymmetry is in the form of  $\lambda_{00} > \lambda_{11}$ .

7/ This view of the cycle in an economic variable is fairly standard in recent theoretical and empirical analyses of it. Lucas [1973], for example, defined the cyclical component of output as its deviation from its normal level which, for annual data, was viewed as the value of output along a trend line. Also, see Gordon [1961, pp. 249-257].

8/ Data for real (1972\$) GNP and gross private domestic investment were obtained from the following U. S. Commerce Department publications: National Income and Product Accounts of the United States 1972-1976 (1948:I - 1950:VI); Business Statistics, 1982 edition (1951:I - 1982:IV); and recent issues of Survey of Current Business (1983:I - 1983:IV). Output per worker-hour was measure as real GNP divided by employee hours in nonagricultural establishments. The employee hours data were collected from the December 1983 edition of Business Conditions Digest (1948:I - 1981:IV) and the May 1984 edition (1982:I - 1983:IV). All data were quarterly, seasonally adjusted annual rates.

9/ The GQOPT package of search alogrithms is offered through the Economics Department at Princeton University. The particular alogrithm used was GRADX with user-supplied first and second partial derivatives.

10/ In Figure 1, the distance from  $(\hat{\lambda}_{00}, \hat{\lambda}_{11})$  along the major ("horizontal") axis is .1338 and .1135 along the minor ("vertical") axis. The slopes of these two axes are .014 and -.014, respectively. In Figure 2, the distance from  $(\hat{\lambda}_{00}, \hat{\lambda}_{11})$  along the major ("horizontal") axis is .1383 and .1372 along the minor ("vertical") axis. The slopes of these two axes are .2 and -5, respectively. Finally, in Figure 3, the distance from  $(\hat{\lambda}_{00}, \hat{\lambda}_{11})$  along the major ("vertical") axis is .1546 and .1310 along the minor ("horizontal") axis. The slopes of these axes are 156.26 and -.0063, respectively.

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