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INCOME OPPORTUNITIES AND OPTIMUM FARM PLANS IN SOUTHERN MAYENNE, FRANCE

by

André François Blanchard

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In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

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INTRODUCTION

In a developing economy the agricultural sector takes a decreasing importance, in relative terms. Producing outputs whose price and income elasticities of demand are smaller than one, this sector takes on a progressively lesser importance in the economy when progress alters the traditional input-output relationships, the price situation and the per capita level of income. Within the farm sector, capital is substituted for labor and land through modern equipment and techniques of production. Farms become larger, capital is applied in higher quantity as more farm inputs are produced by the industry and the excess of farm labor is transferred to the developing sectors of the economy. At the same time the risks involved in farming increase.

Under such conditions it becomes important to know, even for an area as small as the "Bocage Angevin" region, what are the main economic forces in action and their relative strength in order to guide political leaders and people. The main problems, among others, are related to:

- level of income opportunities in farming and other jobs
- degree of risk involved in agriculture due to price and technological progress uncertainties
- optimum farm plans related to various levels of capital, land and labor resources at disposal. This knowledge helps to build a program for developing the existing commercial structure of the area and to set up an adequate extension program.
- rate of resource accumulation.

In this study, after having reported briefly the main characteristics of the area we have worked on, we define the main economic problems which have to be solved and the economic model we have built in order to help people in their task of defining adequate policies and development programs. In Part II we discuss, on theoretical grounds, various ways of setting up a set of adequate constraints for our linear programming model. Finally, after having reported in Part III the main input-output relationships and coefficients which have been used in the model, we give our results in Part IV. They are related to the preceding problems.

PART I. PROBLEMS AREA AND RESEARCH OBJECTIVES

CHAPTER 1. THE "MAYENNE DEPARTEMENT" AND ITS PROBLEMS

Any economic activity is determined by the characteristics of its near and far off environment. Before defining our research purpose we will therefore present briefly the "Departement de la Mayenne" and one of its natural regions called "Bocage Angevin" for which this study has been undertaken.

A. The "Departement de la Mayenne" (33, pp. 1-43)

It is located between latitude 47° 44' and 48° 31' north and between longitude 2° 23' and 3° 35' west of the Paris meridian. Its chief town, Laval, is

- 285 kilometers distant from Paris
- 136 kilometers distant from Nantes
- 142 kilometers distant from Caen

1. Geology

The northern part is constituted of granite and slate. The central part is formed of various rocks: sandstone, slate and limestone.

The southern part, or the Bocage Angevin region, is mainly constituted of pre-cambrian schist and of few silurian slate. In the former case, this bedrock has generated fertile clayed soils when they are deep and well drained. They occupy the major part of the total area.

2. Relief and climate

The highest point is 417 meters above the sea level but the most frequent altitude is from 200 to 100 meters with a general inclination from north to south.

Distant 80 kilometers from the sea, the climate is very mild, humid with a small range of extreme temperature during the year since, in average, it has been recorded over the last 10 years, 4^{o} centigrade in January and 19.1^{o}. In July in Laval, for a total of about 0.758 meter of water, it rains 160 days a year (or about 4.5 days out of 10). Under such a climate a large number of plants can be cultivated.

3. Demography

In spite of a high birth rate (1.86%) and of an excess of birth over death of 6.3 per 1000 habitants, the "Departement de la Mayenne" is continuously losing its population. The "average" marginal losses are, in number of people per year and for the following periods:

1876-1936: 1670 persons

1931-1936: 535 persons

1946-1954: 2225 persons

1954-1962: 1872 persons

This emigration benefits equally to the Parisian region and to the bordering "departements". However, for the first time since many years "the departement" maintained its population in 1963 (247,000 habitants). This result is mainly due to a resolute and concerted action whose purpose consists of creating new jobs. There is no doubt for anybody that this equilibrium is very weak for two reasons:

- This "departement", as the major part of the west of France, has remained apart from the great industrialization movement of the nineteenth century. Since then, the small manufacturers which have been attracted here will never play a leading role and the Common Market won't reverse the situation.

- The agricultural sector which has lost about half of its workers from 1892 to 1962 and about 2,000 per year from 1954 to 1962 still employed one person (man or woman) for eight hectares at this last date. The technological progress which is continuously taking place will release labor from agriculture. If the farm youth don't find a job locally the total population of this "departement" will decrease steadily. The relative importance of the agricultural sector (Table 1) in the economy of this "departement" makes this problem still more acute.

Table 1. Composition of the working population (1962)

Occupation	Number of workers	%
Agriculture and forestry	58,351	51.6
Building	6,969	6.2
Transport	1,723	1.5
Other industries	17,327	15.3
Trade, banking and insurance	1,723 10,803	1.5 9.6
Services	17,908	15.8
Total	113,100	100.0

4. Characteristics of the agricultural sector

Since over 50% of the working population is engaged in agriculture we won't describe here the other sectors of activity.

- Farm size: the average size of farm is equal to 17.85 hectares and varies from 15.10 to 20.10 from one natural region to another. Table 2 gives, in relative terms, the distribution of the number of farms by size of acreage group for the "Bocage Angevin" region and the whole "Mayenne" area.

Table 2. Number of farms by size of acreage group (cumulative percentage)

Farm size (hectares)	Mayenne	Bocage Angevin
< 10	25.9%	30.1%
< 20	55.5%	61.0%
< 50	97.1%	97.4%
Total number of farms	25,818	5,773
Average size	17.85	20.61

- Land use and farm output: Over half of the total land is presently occupied by permanent pastures. On the tilled land is grown about 45% of cereals, 10% of row crops and 42% of forage crops (temporary pastures, red clover, alfalfa). Since cereals, except wheat, are fed to livestock on farms, animal products made up about 89% of the total farm output in 1966 (61, p. 8).
- Age of the farm managers: Farmers get control over farms and expand the size of their holding, when possible, between 25 and

50 years of age. They keep their holding until they are about 65 years old and from there on they start to give up a part, or all of it to a younger manager. Today, farmers of less than 50 years of age control 56% of the total land acreage as shown in the following table.

Table 3. Age and control over farm land

Age (years)	Cumulative % of the total acreage
30	2.814
40	25.69
50	56.38
60	79.32
70	96.32

B. The "Bccage Angevin" (33)

1. Demography

Being essentially a rural area, this region has 45 inhabitants/square kilometer and only 34 inhabitants/square kilometer in the rural district. Of a total of 58,200 people, 44.7% make up the total labor force. Sixty and fifty-five one hundredths percent of the working population is engaged in agriculture, 15.04% in manufacturing and 24.41% in the "tertiary" sector of activity. The "Bocage Angevin" population has decreased at an average rate of about 0.27% per year over the last century. Stable, for a certain lapse of time after the wars (1870, 1914, 1940), the total population decreases steadily between them. In

1962, youngsters of less than 15 years of age made up 30% of the total population (Mayenne, 28.2%; France, 24.8%) while people over 65 years of age constituted 10.5% of it (France, 12.6%).

2. The farm managers

Almost half of the farmers are 50 years old or over and they hold more than one third of the total farm land. The largest and the smallest farms are held more frequently by elderly people. The land/labor ratio is equal, in average, to 7.7 hectares or 13.6 hectares per man, if we consider that women are not working full time on farms and if we omit them.

3. Educational status of the farm population

One of the greatest difficulties faced by the rural population is its level of education (Table 4). It slows down the rate at which technological progress can be applied in agricultural production and the adjustment process of the present farm labor force. Under these conditions the jobs to which the farm youth has access are not very rewarding. But unfortunately this situation is not particular to our "departement". In 1964, out of 100 French college students only six had been raised on a farm and 7.6 belonged to the working class even though they were originating from sectors which made up respectively 20.1 and 36.7% of the total labor force (70, p. 36).

C. The Economic Problems of this Area

A permanent migration rate, an important proportion of youngsters

Table 4. Educational status of the farm population over 15 years of age in percent

Diploma	Men	Women	Total
Primary school	22.49	25.74	24.15
High school	1.78	2.80	2.30
College	0.44	0.12	0.28
Vocational training	9.92	5.00	7.41
None	65.36	66.34	65.86
Total	99.99	100.00	100.00

among its inhabitants, a still small land/labor ratio even in the presence of new technological progress especially in dairy and forage production, will force the people of this region to face again two very important and crucial problems in the near future.

1. The problem of those who will start and/or keep on farming

For them the important problems to solve are related to optimum production plans, risks, rate of capital accumulation, degree of specialization, and for a small minority, the economic advantages and welfare that might be provided by joint farming. However, the most crucial question for many young farmers consists of accumulating and getting the control over a minimum amount of resources within an acceptable range of privation. This minimum has still to be defined.

2. The problem of those who can't get control over a farm; land being the most scarce resource

Acquiring a farm is expensive (8,000-10,000 F/ha) and getting a farm lease is almost impossible, the lease being more frequently renewed to a member of each tenant family. The main obstacle to get a rewarding job lies in the lack of education of people who are forced to quit agriculture. Therefore, for the "departement" as a whole the problem consists of:

a. Attracting enough trades and manufacturing within its largest towns In the absence of such a program, the economic activity of this region will decline in the long run.

b. Increasing the level of knowledge of young people whatever the location where they will later work A large proportion of those who will get to college probably won't find a job locally. But it is urgent to enlarge the opportunities which are presently faced by the youth.

Finally, those who are lucky enough to have the opportunity of choosing between farming and other jobs will have to compare the relative advantages of the two situations. The comparison has to be done both in money terms and in function of the related standards of living (the final choice being, of course, a function of peoples' own preferences).

CHAPTER 2. RESEARCH OBJECTIVES AND THE ECONOMIC MODEL

Taking mainly into account the two above-mentioned problems of this area, we will define our research objectives and give a brief outline of our programming model.

A. Research Objectives

The general objectives of this study are to determine:

Problem 1: Optimum production plans (and their related levels of income) under different levels of management in forage and milk production (output per hectare or cow).

Problem 2: The influence on income of specialization in milk, steers and cereal production. Young farmers aspire for the simplification of their work. They want to bring it up-to-date, but such a transformation requires, in most cases, large initial building investments. They think these investments can become profitable if they are spread over a larger number of units.

Problem 3: The set of optimum combinations of two limiting resources when the others can be bought at an unlimited level. This research has been undertaken for the following resources:

- land-capital
- land-labor
- labor-capital

Labor is not, strictly speaking, a limiting resource since it is always possible to hire extra farm workers. But, in fact, we know that it is not always feasible to hire them on a part-time basis. Therefore, in order to study its influence on income, it is considered as a limiting resource. Furthermore, it is assumed that modern dairy facilities are either available or not.

These results will allow us to infer on the economic advantages of joint farming. However, this problem won't be fully studied here: It would require the building of a specific model to evaluate the additional income due to an increasing degree of resource pooling in agriculture.

Problem 4: The degree of stability of the optimum plans and related levels of income under situations of variable prices especially those of milk, beef, cereals and grass seed.

Problem 5: The optimum investment in building facilities.

The results of the preceding work will help mainly those who will keep on farming. But, however, from these results it is possible to infer few important consequences for those who will quit farming and for the political leaders of the area. It will be mainly tried:

a. To determine the minimum level of resources allowing farmers to get an income equal to different categories of urban people wages. This knowledge will help the vocational guidance service to set up its program.

b. To estimate the amount of disguised unemployment in agriculture and the potential decrease of rural population.

c. To set up a program for the extension service after having compared our results with the present production of this area.

d. To examine the government price program which would help to solve, in this area, the problem of surplus in milk production.

B. The Programming Model

In order to solve the preceding problems we have set up a unique linear programming model whose schematic representation is shown in Table 5. The following symbols - A, B, b - represent submatrices whose coefficients are different from zero. Other submatrices have coefficients equal to zero. The following computation procedures have been used:

- Bounding of variables. Problems 1 and 3.
- Parametric linear programming. The coefficients of the A matrix, the objective function, cost coefficients and the constants of the right hand side of the equations have been varied to solve respectively problems 1, 4, 2 and 3.
- Integer linear programming. It has been used to solve problem 5 since the investment cost functions are of the form: $Y = a + bx$.

A correct setting up of the linear programming model constraints requires a careful analysis of the variables and of the corresponding production possibility set. It is particularly the case for:

- capital
- crop rotations
- feeding programs
- investment functions and mutually exclusive set of variables

These related problems will be fully discussed in the following chapters.

C. Form of Results

Most of our results are given under the form of linear and non-linear equations. The method of least-square regression has been used

Table 5. Schematic representation of the programming model

List of constraints	Crops (forage, cereals)	Grassland manage-ment	Livestock	Buying	Selling	Transfer & miscel-laneous	Right-hand side
	List of activities						
Objective function Max. f(x)	C, - C	- C	C, - C	- C	C	C, 0, - C	
Tractor hour requirements[a]	B	B	B				
Land and crop rotation	B B, - A						≤ b
Accounting constraints on							
Grass seeding	- A	B					
Fodder	- A	- A	B	- A	B		
Cereals and seeds	- A		B		B		
Animals and livestock products			- A, B	- A	B		
Labor							
Crops	B	B	0	- A		B	
Livestock	0	0	B	- A		- A	≤ b
Capital							
Working	B, - A	B	B, - A	B	- A	- A	
Investment	B	B	B	- A	0	B	
Buildings			B	- A		- A	≤ b

[a]The equation "tractor hour requirement" is included to estimate the total annual number of tractor hours which are required by the related optimum production plans.

Table 5 (Continued)

List of constraints	List of activities						Right-hand side
	Crops (forage, cereals)	Grassland manage-ment	Livestock	Buying	Selling	Transfer & miscel-laneous	
Initial fixed costs				B		(δ=1 or 0)	

to derive all of them. However, the original data come from two different sources. The first set of observations has been collected from unplanned experiments; they are real world observations. The second set of data, in a sense, comes from a controlled economic experiment, if we admit that a parametric linear programming can be viewed as such.

Consequently, two resulting assumptions are made:

- The unobservable random variables are normally and independently distributed with mean zero and variance σ^2. In this case, the classical statistical inferences are derived. This assumption is made in Chapters 11 and 13 (first part) since the observations come from unplanned experiments.
- No specific assumptions are made on the error term of the equations to be regressed. We use a descriptive linear regression model in order to show, in a more convenient fashion, a very large set of results. This procedure allows us, in particular, to derive a series of iso-revenue and iso-product curves. Making no particular assumptions on the disturbance term of the equations, we can't make any statistical inference and probability statements about the regression results. Consequently, the coefficients of determination (R^2) are used as a measure of goodness of fit. Descriptive linear regression has been used to estimate the elasticity of demand (60) or the elasticity of supply (47) from step functions originating from a parametric linear programming model. Assuming that the midpoints of the vertical portions of the steps are most stable with respect to price change, these points were used as observations for fitting the corresponding

equations. Trying to improve this procedure, Burt (15) proposed to minimize the integral of squared distances between the fitted curve and the original step function. Working, in most cases, into a two or three dimensional space (price or input spaces), we use the standard descriptive linear least-square regression model and, in most cases, take observations at equally spaced magnitudes of the independent variables. Such a procedure is adopted since a linear programming model whose size is large requires many iterations when matrix coefficients and vectors are continuously varied in a certain range.

PART II. BUILDING UP AN ADEQUATE SET OF LINEAR PROGRAMMING CONSTRAINTS: A THEORETICAL INVESTIGATION

CHAPTER 3. CROP ROTATION CONSTRAINTS

When setting up a linear programming problem we can insert into the model either a set of crop rotations or a set of linear constraints which will bound all feasible crop sequences. One will be preferred to the other according to the specific assumptions which underlie each particular problem and the corresponding model size.

When crops can't be dissociated from particular sequences of crops and recombined into other ones without violating the additivity assumption of linear programming model, a pre-established crop rotation should be included within models. It will always be the case when soil conservation problems arise. In all other cases, even if each particular crop can be fertilized at n different levels or be cultivated in m different ways, we can split off rotation activities into their components and link them with a set of adequate constraints.

A. Setting up crop rotation constraints

1. The relevant requirements

Adequate crop rotation constraints should satisfy to several conditions.

a. Sequence rules

(1) A crop can follow another one if

(a) the preceding crop has been harvested

(b) agronomic laws allow it

(2) Certain crops can be cultivated on the same soil only if a certain number of years has elapsed since they have been plowed in (alfalfa and rapes are such examples).

b. Isolation rules

Plants grown for seed have frequently to be Z miles apart from other specific crops. It is a conditional mutually exclusive type of constraint. Location rules cannot be expressed in percentage of soil occupation or even as a sequence constraint. All relevant variables are not even under the control of a manager, he has to consider his neighbor's decisions.

What can only be done is the determination, a priori, of a maximum surface which can be assigned to those crops. From particular solutions it will be decided if they are feasible or not.

2. Converting sequence rules into an adequate set of linear constraints.

The first step consists of building an oriented graph or its associated matrix, which shows all possible circuits and oriented chains.

a. The problem to be solved

Given a graph $\Gamma = (G,E,E)$ and its associated matrix A where

j = origin

i = destination

$a_{ij} = 1$, if i and j are connected by an oriented arc; $a_{ij} \varepsilon G$

$a_{ij} = 0$, otherwise; $a_{ij} \varepsilon G$,

we have to find a system of inequations such that all possible solutions of the linear programming model will satisfy two conditions.

Condition 1: From any solution it is always possible to form one or several circuits.

Condition 2: Each crop i can't give up more surface than the area x it occupied (Table 6).

This condition can be stated as follows (Table 6):

Table 6. Constraints on the matrix associated to the graph $\Gamma = (G,E,E)$

Destinations (following crops i)	Origins (preceding crop j) 1	2 ... n	Total
1	x_{11}	$x_{12} \ldots x_{1n}$	$x_{1.} = \sum_j x_{ij}$
2	x_{21}	$x_{22} \ldots x_{2n}$	$x_{2.} = \sum_j x_{ij}$
	...	...	...
m	x_{m1}	$x_{m2} \ldots x_{mn}$	$x_{m.}$
Total	$x_{.1}$	$x_{.2} \ldots x_{.n}$	$\sum_j x_{.j} = \sum_i x_{i.} = x_{..}$
	or		and
	$\sum_i x_{ij}$		$x_{.j} = x_{i.}, \forall (i=j)$

b. Systems of inequations

The first condition, which states that there exists a way from i to j and from j to i can be expressed as a ($\leq$) relation since:

$i \leq j \Rightarrow j$ precedes i

$i \leq j \leq k \Rightarrow i \leq k$ and k precedes i

The second condition states that all particular roads within a circuit will be, in all points, large enough.

Theorem 1

The relation ($\leq$) guarantees, for every possible solution $X = \{x_1, x_2 .. x_j .. x_n\} \neq \emptyset$, that we can always form with it one or several circuits which can be either connected or disjointed.

Proof

Suppose that $X \neq \emptyset$ doesn't constitute one or several circuits but an oriented tree, we have for at least one variable j:

$$x_j \leq 0$$

since trees have n nodes and (n - 1) arcs (25, p. 354).

Furthermore, $x_j \geq 0$ for all variables in a linear programming model solution. Hence,

$$x_j = 0$$

and the system of inequation becomes:

$$x_j \leq 0$$

$$x_{j+1} \leq 0$$

$$x_m \leq x_{m-1}$$

which implies that $X = \emptyset$.

First adequate set of linear constraint -

The first necessary condition stated in Table 6:

$$\sum_j x_{.j} = \sum_i x_{i.} \qquad (1)$$

will always be satisfied if the second one

$$x_{.j} = x_{i.} \ ; \ \forall (i = j) \qquad (2)$$

is fulfilled.

Equation 2 is a sufficient condition for a circuit being a feasible crop rotation. By Theorem 1, equation 2 becomes

$$x_{.j} \geq x_{i.}, \ \forall(i = j). \qquad (3)$$

We define each activity as crop j following crop i. Since crops are considered successively as origins and destinations, then i = j in equations 2 and 3. The corresponding matrix takes the form shown in Table 7. Its size is:

$$(m = n) \times (\sum_i \sum_j a_{ij})$$

where a_{ij} = elements of the matrix associated with the graph $\Gamma = (G,E,E)$.

Table 7. Constraints on the oriented arcs of the graph $\Gamma = (G,E,E)$

Crops i (i = j)	x_{11}	$x_{21} \ldots x_{m1}$	$x_{12} \ldots x_{m2}$	$x_{1n} \ldots x_{mn}$	Right hand side
1	0	- 1 ... - 1	+ 1 ...	+ 1 ...	≤ 0
2		+ 1 ...	- 1 ... - 1	...	≤ 0
...		...	...	...	...
m = n		... + 1	... + 1	- 1 ... 0	≤ 0

Remarks:

1. Crops for which $x_{ij} = 1$, i = j, are not bounded by any crop rotation constraint. By themselves, they constitute a circuit or a rotation.

2. Table 7 shows a matrix for which the column vectors are in

fact the oriented arcs of the graph $\Gamma = (G,E,E)$. If x_i and U_j are respectively its corresponding nodes and oriented arcs then the matrix shown in Table 7 has a_{ij} values which are equal to:

. $+1$ if U_j originates from x_i

. -1 if U_j ends in x_i

. 0 otherwise

3. The sum of each column vector elements in Table 7 is equal to zero.

Second adequate set of linear constraints -

The preceding constraint set, although correct, can be found too large since we have as many rows as nodes and the number of columns equals to the number of arcs. Instead of considering arcs and nodes we can take only into account nodes (or crops). Given the associated matrix A of the graph $\Gamma = (G,E,E)$ we can write

$$IE \leq AE \quad (4)$$

where $E = \{x_1 x_2 \ldots x_n\}'$ and I = an identity matrix. Equation 4 becomes

$$IE - AE \leq 0$$

$$CE = (1 - A)\ E \leq 0 \quad (5)$$

and states that

$$\underset{i=j}{c_{ij}x_j} \leq \sum_{\substack{j \\ i \neq j}} c_{ij}x_j \ ; \ c_{ij} = 1 \quad (6)$$

<u>Theorem 2</u>

If $\sum_i C_{ij} = 0$, $\forall(j)$, where C_{ij} are elements of the C matrix (equation 5) then each linear programming solution: $\{x_1, x_2 \ldots x_n\} = X \neq \emptyset$, constitutes one or several disconnected crop rotation, and all variables x_j which belong to a particular circuit are equal.

Proof:

Since $X \neq \emptyset$, the corresponding solution contains at least one circuit (by Theorem 1). To guarantee that a circuit is also a crop rotation we must satisfy, for all j, inequation 3. Then $\sum_i c_{ij} = 0 \Rightarrow$

$$[x_{.j} = x_{j.}]_{(j = i)} \leq \sum_{\substack{j \\ j \neq i}} x_{ij},$$

since any nodes precede only another one.

If several crop rotation were connected, then we would have at least for one node, several following crops. What violates our assumption: $\sum_i c_{ij} = 0$. Therefore, all variables are equal for a given crop rotation since our set of linear inequation states:

$x_i \leq x_{i-1}$

$x_{i+1} \leq x_i$

....

$x_{i+n} \leq x_{i+(n-1)}$

$x_{i-1} \leq x_{i+n}$

what implies that

$$x_{i-1} \geq x_i \geq x_{i+1} --- \geq x_{i+(n-1)} \geq x_{i+n} \geq x_{i-1}$$

can only be satisfied if and only if

$$x_{i-1} = x_i = x_{i+1} ---- x_{i+(n-1)} = x_{i+n} = x_{i-1}.$$

Theorem 3

If $\sum_i c_{ij} \leq -1$ for at least one variable j, in equation 5, then each linear programming solution : $\{x_1 ----- x_n\} = X \neq \emptyset$; $x_j \in X$, might not constitute a feasible crop rotation, although it will form a circuit.

Proof:

For node k the incoming arcs are constrained to take the following value according to equation 6:

$$\underset{(i=k)}{c_{ik}x_k} \leq \sum_{\substack{j \\ j\neq i,k}} c_{ij}x_j \ .$$

If the outgoing arcs of node k, as stated by Theorem 3, are at least equal to $2x_k$, $(-\sum_{\substack{i \\ i\neq k}} c_{ik} \geq 2)$ then we can write the corresponding inequation:

$$(\text{at least } 2x_k) \leq \underset{i=k}{c_{ik}x_k} \leq \sum_{\substack{j \\ j\neq i,k}} c_{ij}x_j$$

or

$$(\text{at least } 2x_k) \leq x_k \leq \sum_{\substack{j \\ j\neq i,k}} c_{ij}x_j$$

since $c_{ik}=1$.

Therefore, we are violating the existence condition shown in Table 6 which states: Sum of origins = sum of destination and a feasible crop rotation might not be found although the solution constitutes, by Theorem 1, at least one circuit.

If the excess of destination right is not used and goes to disposal then the solution can be a feasible crop rotation, but such a case is not likely. Let's show, under Theorem 3 assumption that all $x_j \in X$ might not be equal, as they would be under Theorem 2 hypothesis. Let's study the following graph (Figure 1):

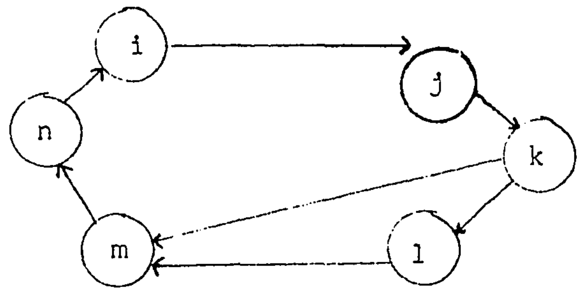


Figure 1. Graph of two connected crop rotations

In such a case, equation 5 is written:

$$
\begin{aligned}
&x_j \leq x_i \\
&x_k \leq x_j \\
&x_l \leq \quad\quad x_k \\
&x_m \leq x_l + x_k \\
&x_n \leq x_m \\
&x_i \leq x_n
\end{aligned}
\tag{7}
$$

Hence,

$$x_i \leq x_n \leq x_m [\leq(x_l + x_k) \geq x_l] \leq x_k \leq x_j \leq x_i \tag{8}$$

then

$$x_m >< x_k.$$

Furthermore, the preceding set of inequation can give rise to unfeasible crop rotations.

If we partition by rows the matrices A and 1 in equation 5, A_{i1}, i = 1...m we can see that 1 forms a set of disjoint subsets of positive coefficients since they are equal to zero when $i \neq j$.

A constitutes, when $\sum_i c_{ij} < -1$ (equation 5) a set P of overlapping subsets P_j of positive coefficients p_i; i = 1...k; $p_i \epsilon P_i \epsilon P$. Some nodes are allowed to give up more capacity than they own.

Since we are considering, in this method, nodes and not arcs as variables we can't guarantee directly the sufficient condition stated by equation 2.

<u>Theorem 4</u>

Given a set P of overlapping subset P_i such that $P = \{P_i \subseteq P_2 \subseteq \ldots P_k\}$, then it is sufficient, to guarantee equation 2 fulfillment, that we <u>substitute</u> to the corresponding equations in system 5, the following ones:

$$x_1 \leq P_1$$

$$x_2 + x_1 \leq P_2$$

$$x_k + x_{k-1} + \ldots + x_2 + x_1 \leq P_k.$$

Proof:

Equation 2 states that, through a given node, the incoming flow is equal to the outgoing one.

Equation 5 states that each following node flow is smaller or equal to the preceding one(s). But, by Theorem 4 assumption, some nodes precede several ones. Therefore, it is sufficient, to satisfy equation 2, to write that each destination right will not be used up several times.

By equation 5 we have

$$x_1 \leq P_1$$

$$x_2 \leq P_2$$

$$x_k \leq P_k \qquad (9)$$

but since $P_1 \subseteq P_2 \ldots P_k$, we are overestimating the value which can be taken by x_j providing that some $x_i \neq 0$; $i < j$. To rectify the system of equation 8 we can write the following one:

$$x_1 \leq P_1$$

$$x_2 \leq P_2 - x_1$$

$$x_k \leq P_k - \sum_{i=1}^{i=k-1} x_i \qquad (10)$$

since each P_j has to be corrected for the destination rights which have been used by the other crops.

Theorem 5

Given of set P of overlapping subject P_i such that

$$P = \begin{cases} P_i \not\subset P_j \\ P_i \cap P_j \neq \emptyset; \ \forall(i,j), \ i = 1 \ldots s \end{cases}$$ then it is sufficient, to guarantee equation 2 fulfillment, that we add to the equations of system 5, the following ones:

$$\bigcup_i x_i \leq \bigcup_i P_i; \text{ some } i, \ \forall\left(\bigcup_{i=C_2^s}^{i=C_s^s} P_i \in P\right).$$

Proof:

We know that (1) $x_i \geq 0$, $\forall(i)$

(2) Each preceding node supplies destination rights to each element of the same set of variables, as stated above. Therefore, to guarantee that each node outgoing flow can't exceed its capacity we have to write

$$\sum_{i=1}^{s} x_i \leq \bigcup_{i=1}^{s} P_i \qquad (11)$$

but if $x_i = 0$, one i, then we overestimate the right hand side of the preceding equation by:

$$[P_i] - [P_i \cap (\underset{\substack{j=1 \\ i \neq j}}{\overset{s}{\cup}} P_j)]. \qquad (12)$$

Consequently, to rectify equation 10 we write in addition of it:

$$E_m = [\sum_{i=1}^{s-1} x_i \leq \underset{i=1}{\overset{s-1}{\cup}} P_i], \qquad (13)$$

But the set $L = \{x_1 \ldots x_s\}$ has to be partitioned into a pair of subsets containing s - 1 and 1 elements respectively. What can be done in C^s_{s-1} ways. Therefore 12 is a system of C^s_{s-1} equations.

If $x_i = 0$, two i (i = i, 1'), we overestimate the right hand side of the equation 12 by

$$[\{P_i + P_{i'}\} - \{(P_i + P_{i'}) \cap (\underset{\substack{j=i \\ j \neq i, i'}}{\overset{s}{\cup}} P_j)\}].$$

Therefore, in addition to the system of equation 12, we write the following ones:

$$E_n = [\sum_{i=1}^{s-2} x_i \leq \overset{s-2}{\cup} P_i], \; n=1 \ldots C^{s-2}_s. \qquad (14)$$

The subset of $x_i = 0$ for which this reasoning applies is formed of 1,2...(s-2) elements since the relevant equations for i = (s-1) are included in 5. Consequently we will add to equation 5: $(\sum_{i=2}^{s} C^s_i) = (2^s - s - 1)$ supplementary equations.

Remarks:

1. To reduce the number of combinations, which can be quite large, particular activities can be aggregated by pair. For example, x_k is transformed into the following one, $(x_i + x_j)$, as shown in Table 8. The overlapping of subsets P_1 and P_2 has been removed.

Table 8. Aggregation of activities and reduction of the number of complex constraints

i	(a) Original simple constraints 1	2	3	4	(b) Constraints after aggregation 1	2	3	4	3+2
1	1	-1	-1		1	-1	-1		-1
2	0	1	-1	-1	0	1		-1	0
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...

2. When $P_i \not\subset P_j$

$$P_i \cap P_j \neq \emptyset;\ \forall(j),\ i \neq j.$$

Then the system of equation 5 will be sufficient to guarantee equation 2, since all crop rotations or linear combinations of them are allowed, provided that any crop can't form a circuit by itself. This last possibility is ruled out by equation 5.

Hovelaque (41, p. 62) states that we should add to simple constraints (the equivalent of system 5) a set of complex ones, when several crops compete for occupying the soil liberated by a preceding crop. They are found by the enumeration of all combinations of m equations taken r at a time, r = 2...m. Before him, Mazoyer wrote (57, p. 531): "To a given set of preceding crops corresponds one and only one set of following ones". But none of them proved it. We can do it as follows:

Theorem 6

Given C, the set of equation 5, and C_1 and C_2 such that

$C_1, C_2 \varepsilon C$

$C_1 \cap C_2 = \emptyset$

where $C_1 = \{P_i \cap P_j \neq \emptyset;\ i \neq j,\ i = 1 \ldots . s\}$

$C_2 = \{P_i \cap P_j = \emptyset;\ i \neq j,\ i = 1 \ldots . (m-s)\}$

then it is sufficient to guarantee equations 1 and 2 to write for C, in addition to equation 6, the following set of complex constraints

$$D_i = \left\{ \bigcup_i (c_{ij} x_j)_{i=j} \leq \bigcup_i (\sum_{j,\ i \neq j} c_{ij} x_j) \right\} \quad i = 2 \ldots 2^s$$

Proof:

When we write $C_{ij} x_j{}_{i=j} \leq \sum_{j,\ i \neq j} c_{ij} x_j$, $c_{ij} = 1$, this constraint is effective if $c_{ij} x_j{}_{i=j} \neq 0$, whatever the value of the right hand side of the inequation. Therefore, for each activity $X_j = 0$, there exists a corresponding inequation i which becomes ineffective. The number of possible inactive constraint sets is equal to $\sum_{i=o}^{m} C_i^m = 2^m$. Since set C_2 assumption guarantees equation 2 fulfillment, it is unnecessary to add any supplementary constraint to the system 5. However, the set of constraints C_1 doesn't guarantee that equation 2 will be fulfilled unless we add it to equation 5 and $x_j \neq o$ or $x_j = o$, $\forall (j)$. Since each x_j can supply destination rights to any $P_i \varepsilon C$, and we ignore the specific inequations which will be ineffective, we combine them 2 by 2, 3 by 3, ..., s by s, in all possible ways. Therefore, any node, although linked to several succeeding ones, will never supply more than its capacity, and equation 2 will be fulfilled. We have added to equations 5: $2^s - s - 1$ equations.

Row dominance within crop rotation constraints

Heady (36, p. 154) writes: "if two resources have the same supply... then for any activity the resource with the larger input requirement will limit production before the resource with the smaller requirement". And he takes an example with labor constraints while Hovelaque (41, p. 62) shows that two crop rotation constraints become unnecessary. We can summarize the necessary conditions for ruling out a set of crop rotation constraints. They are already standardized since their coefficients are composed of -1, 1 or zero.

Let's define:

$S_i = \{-x_{j,j} = 1....n\}$ = Supply of destination rights

$D_i = \{x_{j,j} = 1....n\}$ = Demand of destination rights.

For each particular row i = 1....m, we write $D_i \leq S_i$

(1) if $D_i = D_k$ and $S_i \subseteq S_k$; $i \neq k \Rightarrow D_i \leq S_i$

since it is the only constraint which limits production, the other one being either dominated or redundant.

(2) if $D_i \subseteq D_k$ and $S_i = S_k$; $i \neq k \Rightarrow D_k \leq S_k$

The other constraint is dominated or redundant.

Within the set of crop rotation constraints, defined in this paragraph, a large number of them are either dominated or redundant (they might be identical to another one or become empty, after simplification, on one or both sides of the inequation).

Crop frequency constraints

Up to now we have only taken into account the first sequence rule (p. 19). To guarantee that one or several particular crops alternate within the rotation sequence, with a set of other ones we can express the corresponding constraints in two ways:

(1) $(\alpha + \beta)\ X_j \leq S$ (15)

where α = X_j crop land requirement

β = set of other crops land requirement

X_j = quantity of crop j subject to a frequency constraint

S = land supply.

Even if we add to this equation a correction term for crops which last longer than β years, this formulation won't be correct unless we get a solution containing only one circuit. Two circuits, either connected or disjoint, would allow equation 15 to overestimate the value which should be taken by X_j.

(2) Aggregate crops to form a set of oriented chains. We find the set of all possible oriented chains which can be formed with X_j. All sequences of crops will last, at least, $(\alpha + \beta)$ years on the same soil. This set, added to the set of simple crops, will transform our problem into ordinary sequence constraints, which have been defined in the preceding paragraphs. We can run into large sets of chains, especially when β becomes large, but the problem is correctly stated.

(3) Introducing preestablished crop rotations into models. Instead of transforming the sequence rules into an adequate set of linear programming constraints we can use them to find the set of all possible crop rotations. This set can be introduced within the model either alone[1] or in addition to the set of corresponding crops, these two sets being

[1]Such procedure has been used by many researchers: Heady (36), Lefort and Sebillote (48), Hildreth and Reiteir (40) and many others.

Table 9. Constraints linking simple crop with crop rotation activities

Crops i	Crop rotations ABC	CBH	C	...	ABCH	Crops A	B	C	...	H	Right hand side
A	-1				-1	1			...		≤ 0
B	-1	-1		...	-1		1		...		≤ 0
C	-1	-1	-1	...	-1			1	...		≤ 0
...				...					...		≤ 0
H		-1		...	-1				...	1	≤ 0

linked by a system of linear inequations as follows (Table 9).

This system is especially used when each crop can be grown in n different ways without altering any crop rotation. But when the graph $\Gamma = (G,E,E)$ contains many nodes and/or quite a few arcs, we have to use an efficient method to enumerate them.

(a) Matrix method.

A circuit is an oriented chain connecting a node X_i to itself. A directed arc a_{ij} represents an allowable precedence between i and j $(i \rightarrow j)$. If $i \rightarrow j$, $j \rightarrow k$, then $i \rightarrow k$. Or $a_{ij} + a_{jk} = a_{ik}$, the corresponding sequence of nodes being: (X_i, X_j, X_k). A sequence which doesn't contain the same node twice, is said elementary. A sequence with p arcs has (p + 1) nodes. Given two oriented chains, S_1 and S_2:

$$S_1 = (X_1, X_2 \ldots X_p)$$

$$S_2 = (X_p, X_{p+1} \ldots X_n)$$

we can link them to form S_3:

$$S_3 = (X_1, X_2, \ldots X_p, X_{p+1} \ldots X_n) = S_1 * S_2^1$$

where $S'_2 = S_2$ whose first node has been dropped. Likewise, given two oriented chains:

$$S_4 = (X_1, X_2 \ldots . X_p)$$

$$S_5 = (X_p, X_{p+1} \ldots . X_1)$$

we get a circuit S_6 if we link S_4 and S_5 as follows:

$$S_6 = S'_4 * S'_5 = (X_2 \ldots . X_p,\ X_{p+1}, \ldots . X_1).$$

Kaufmann and Malgrange (45) after having made these preliminary remarks develop a method to find all elementary oriented chains without omission and redundancy. Circuits are found by the same method. The following are drawn from their work:

$S'_1 * S'_2 = S'_1 S'_2$ if the sequence is elementary

$S'_1 * S'_2 = \emptyset$ if the sequence is not elementary

$\emptyset * S'_2 = \emptyset$

$S_1 * \emptyset = \emptyset$

$\emptyset * \emptyset = \emptyset$

C^p_{ij} = the set of all possible p arcs linking i to j and forming as many elementary oriented chains. $C^p_{ij} \otimes C^q_{jk}$ = the product of C^p_{ij} by C^q_{jk} such that it represents all possible oriented chains connecting i with k.

Therefore, $C^p_{ij} \otimes C^q_{jk} = C^p_{ik} \times C'^q_{kj}$ for oriented chains

$$= C^{p+q}_{ij}$$

$$C^{p+q}_{ij} = \bigcup_{k=1}^{n} C^p_{ik} \otimes C^q_{kj} = \bigcup_{k=1}^{n} C^p_{ik} \times C'^q_{kj}$$

M^p = A matrix whose element C_{ij} is the set of all p arcs elementary oriented chains connecting node i with node j and forming as many elementary oriented chains. M'^p is obtained from M_p by dropping the first node in all elementary chains.

The method:

The following computations are carried out:

(1) $M^{p+q} = M^{p} \times M'^{q}$

(2) $M''^{p+q} = M'^{p} \times M'^{q} = C''^{p+q}_{ij} = \underset{k}{\cup} C'^{p}_{ik} \times C'^{q}_{kj}$.

To get all possible circuit with p + q arcs it is sufficient to calculate C''^{p+q}_{jj}, the diagonal elements of the matrix M''^{p+q}.

When $p + q = n$, then any c''_{jj} will contain the enumeration of all circuits, since they cross each node. Furthermore, the usual properties of exponent is still valid.

$$M^{p} \otimes M^{q} = M^{p+q}$$

$$(M^{p})^{q} = M^{pq} \;.$$

Therefore, we can reduce the computation burden.

Example: suppose we want to find all possible circuits of graph C (Figure 2). We compute first M^1 and M^2.

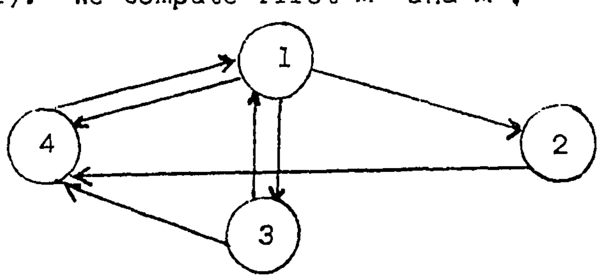


Figure 2. Graph C

$$M' = \begin{bmatrix} \emptyset & \emptyset & 13 & 14 \\ 21 & \emptyset & \emptyset & \emptyset \\ 31 & \emptyset & \emptyset & \emptyset \\ 41 & 42 & 43 & \emptyset \end{bmatrix}$$

$$M^2 = M^1 \times M'^1 = \begin{bmatrix} \emptyset & \emptyset & 13 & 14 \\ 21 & \emptyset & \emptyset & \emptyset \\ 31 & \emptyset & \emptyset & \emptyset \\ 31 & 42 & 43 & \emptyset \end{bmatrix} \times \begin{bmatrix} \emptyset & \emptyset & 3 & 4 \\ 1 & \emptyset & \emptyset & \emptyset \\ 1 & \emptyset & \emptyset & \emptyset \\ 1 & 2 & 3 & \emptyset \end{bmatrix} = \begin{bmatrix} \emptyset & 142 & 143 & \emptyset \\ \emptyset & \emptyset & \emptyset & 204 \\ \emptyset & \emptyset & \emptyset & 314 \\ 421 & \emptyset & 413 & \emptyset \\ 431 & & & \end{bmatrix}$$

then $M''^1 = \emptyset$ since $a_{ii} = \emptyset$, $\forall (i)$ in M'

$$M''^2 = M'^1 \times M'^1 = \begin{bmatrix} \substack{31 \\ 41} & & & \\ & \emptyset & & \\ & & 13 & \\ & & & 14 \end{bmatrix}$$

$$M''^3 = M'^1 \times M'^2 = \begin{bmatrix} \substack{421 \\ 431} & & & \\ & 142 & & \\ & & 143 & \\ & & & \substack{214 \\ 314} \end{bmatrix}$$

$$M''^4 = M^{12} \times M^{12} = \emptyset$$

There exist, therefore, four circuits: 31, 41, 421, and 431.

(b) Method limits to solve our problem.

This method is not perfectly appropriated to our needs since:

(1) It doesn't avoid redundancy in circuit enumeration. A circuit with n nodes appearing n times within our list.

(2) The set of crop rotations is a subset of the set of all elementary circuits. When several ones link the same set of nodes, one of them might be more productive than the others and dominates them. However, all of them are enumerated. Furthermore, two elementary circuits can connect the same set of nodes than a longer one, without being more efficient. The latter is, therefore, redundant as a crop rotation. On practical examples, this last case is frequent.

CHAPTER 4. LIVESTOCK FEEDING PROGRAMS

Improved knowledge in livestock nutrition, nutritional value of feeds and economic model building should be significant to the farm industry. For most livestock activities, feed cost is a major part of their total cost. Not only should adequate livestock activities be chosen but they also should be associated to corresponding least-cost feeding programs.

A. Simple Blending Problem

Dantzig states: "The problem is to give a recipe showing how much of each commodity should be purchased and blended with the rest so that the characteristics of the mixture lie within specified bounds and the total purchase cost is minimized". (26, p. 42).

Mathematically it can be stated as follows:

$$\text{Min: } \sum_j C_j X_j \qquad (16)$$

$$\text{Subj. to: } a_{ij} x_j \begin{matrix} \geq \\ = \\ < \end{matrix} b_i; \; i = 1 \ldots m$$

$$x_j \geq o; \; j = 1 \ldots n$$

where x_j = number of units of the j feedstuff

a_{ij} = i nutrient content per unit of j

C_j = cost by unit of j

b_i = i nutrient requirement.

The overall objective can be either to minimize the cost of producing a certain output weight or a given daily animal diet. Additional constraint(s) would be added accordingly.

Linear programming technique has been widely used to solve this problem either for feed manufacturers or cattle feeders. However least-cost feed-mix problems can be solved only if three sets of data are available: nutrient requirements, nutrient contents, and primary feed costs.

Many feedstuffs have no market value. They can't be sold without being transformed into animal products. Produced and used up on farms, these resources have, however, non-zero shadow prices. Such values can't even be used to solve a particular least cost feed-mix problem. They are a by-product of the overall farm income maximization problem.

The latter includes both production and feed-mix problems since each one interacts with the other. Consequently, they have to be solved at once. Corresponding shadow prices can't even be used to solve any subsequent feed-mix problem.

They vary with price systems, production possibilities, initial amount of scarce resources and therefore from one farm to another. In such a case, the least-cost feed determination has to be made in a profit-maximizing framework. This procedure has been suggested by Heady (36, p. 146) and Becker (4, p. 226).

B. Combining Feed-Mix and Overall Profit Maximization Problems

Trying to find optimum livestock diet we have to add to our simple blending problem a few equations. They will express:

- intake limits on fodder subsets
- fodder complementary or antagonism.

Besides fodder production activities, livestock activities have to be introduced within our general model. Furthermore, certain feedstuffs are available either all the year around or only part of it, and animals have not, over time, constant needs. Our feed-mix subproblem becomes:

$$\text{Max: } \sum_{k}\sum_{t}\sum_{j} C_{jtk}X_{jtk} + \sum_{k} C_k X_k$$

$$\text{Subj. to: } \quad a_{ijtk}X_{jtk} \gtreqless b_{itk}X_{tk} \qquad (17)$$

$$X_{jtk}, X_{tk} \geq 0$$

where t = 1...t time subperiod

k = 1...k livestock activities

i = 1...m nutrient

j = 1...n feedstuffs

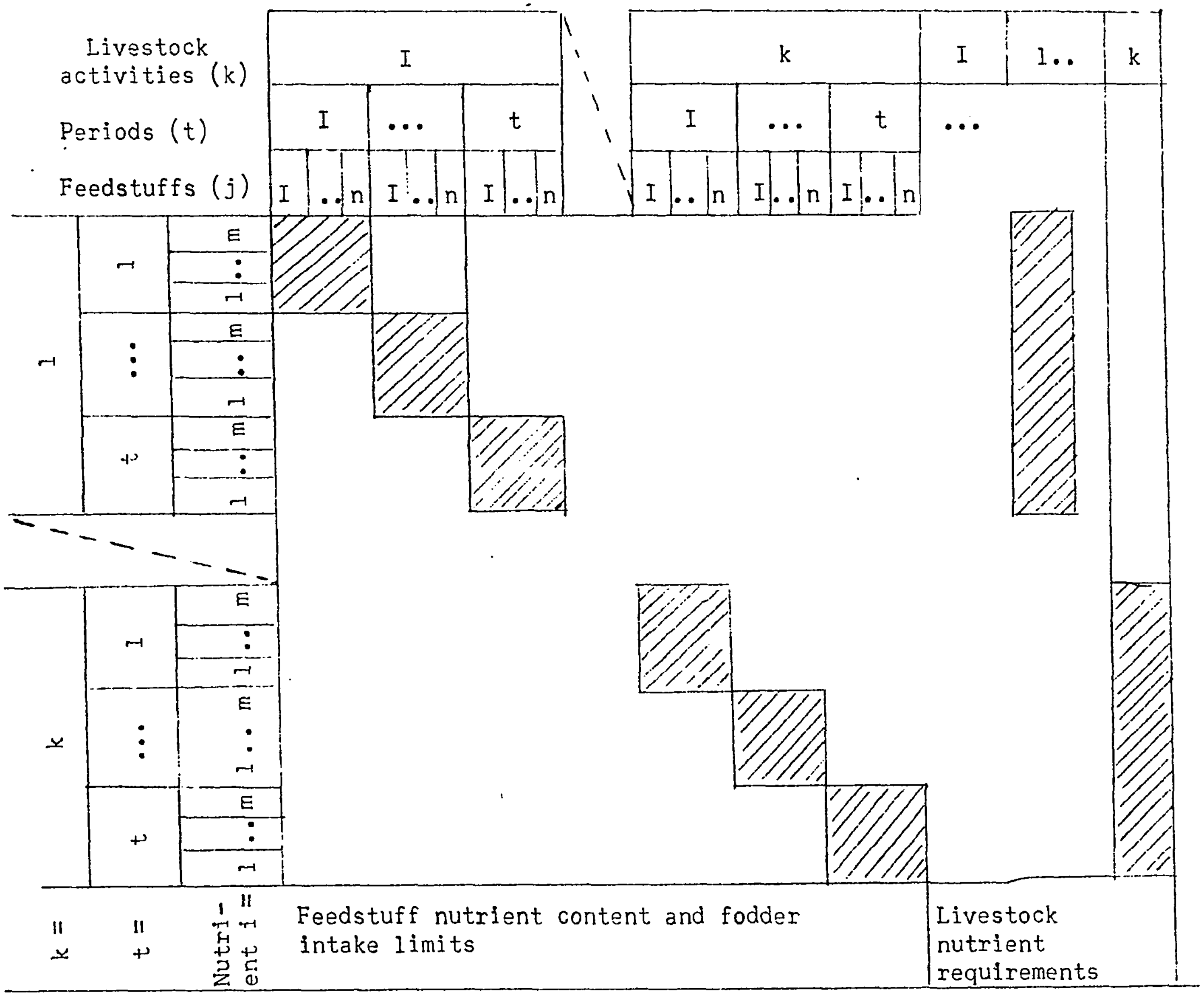
With such a model we run very quickly into dimensional problems although our matrix is quite empty, as shown below (Figure 3). Our feed-mix submatrix dimension is:

$\left[\sum_{k}\sum_{t}\sum_{i}(itk)\right] \times \left[\sum_{k}\sum_{t}\sum_{j}(jtk) + k\right]$. If i, j, k, t were constant, then its size would be: (itk) x (jtk + k). Jullian and Tirel (44) have built such a model. Although they have included in it only three livestock activities (dairy herd, three years heifer and steer), and nine periods their blending submatrix is a 371 x 546 one. That is quite large for only three animal activities.

Nutrient requirements and contents, intake limits and palatability, marginal rate of substitution of one feedstuff to another and of one source of a specific nutrient to another should be known accurately. Our state of knowledge hasn't yet reached, according to nutritionists, such an achievement, especially beyond certain specific values.

Linear programming allows only constant marginal rate of substitution which results from its basic assumption. It is not proved that

Figure 3. Feed-mix submatrix


Livestock activities (k)
Periods (t)
Feedstuffs (j)
I
k
I
1..
k
I
...
t
I
...
t
...
I .. n
k =
t =
Nutrient i =
1
...
t
1 .. m
Feedstuff nutrient content and fodder intake limits
Livestock nutrient requirements

we can express the basic biological relationships of animal nutrition in its framework, particularly for large input variations.

If livestock nutrient requirements could always be supplied in fixed proportion the preceding model would be adequate (independent of its limits). But it is well known that most factors can be substituted to others, within certain limits. Energy can be substituted by proteins, for example.

To determine least-cost feeding programs, using this opportunity, our previous model should be modified. Livestock production functions 18 and corresponding isoquant maps 19 have to be known with accuracy.

$$Y = f(X_1, X_2 | \ldots X_n) \qquad (18)$$

$$X_1 = f(Y^o, X_2 | \ldots X_n) \qquad (19)$$

when Y = output

X = input

To get a least-cost feeding program for producing a given output Y^o we have to minimize the following equation:

$$P_1X_1 + P_2X_2 = C \qquad (20)$$

which would be written, in the general case framework

$$\text{Max:} \quad R = P_yY^o - C = \text{Constant-C} \qquad (21)$$

where P = price

R = revenue

A series of least-cost feed mix problems corresponding to an isoquant set should be solved to maximize equation 21, assuming that feeds costs are known. Dent has followed this procedure to find least-cost bacon pig rations (28). When such market prices don't exist we have to substitute the system of equation 17 to the system 16.

Large range of technical substitution between factors and high input cost differences will result in substantial feed cost saving. Inversely this long and costly study shouldn't have been undertaken when small range variations were observed.

C. Pre-established Livestock Rations

When the preceding procedure can't be used, especially to get over dimensional problems, alternative pre-determined rations are introduced into a general maximizing model. Among them, the least-cost ration is chosen. But this simplifying procedure might mask the true optimum of the objective function and reduce it by a substantial amount. Pre-established rations are largely arbitrary and through a chain of linked constraints they can narrow the income opportunity set that we wanted to explore. This difficulty arises each time we aggregate activities instead of separating them. The best means of overcoming it consists of multiplicating the number of aggregate activities in such a way that the complete production possibility set is entirely included into our model. But alternative activities can be so numerous that we are again running into dimensional problems.

1. Extreme aggregate activities

It is well known that a convex set S is completely defined by its extreme points U. The set of all convex combinations of sets of points U will generate S. Making a large use of this property we can overcome our dimensional difficulties without altering the original problem. Heady has pointed out this principle. He writes, "aggregate activities only into their extreme relationships" (36, p. 217).

Practically it is convenient to mix, in a given ruminant ration, a maximum of three bulky feeds. In this case a minimum of eight vertices (extreme ration) will be sufficient to take into account the set of all possible rations which can be formulated with these three fodders.

Extreme rations are formed with all feasible extreme proportions of bulky feeds, concentrates being added in variable quantity to make up the difference with total animal nutrient requirements. We get more than eight extreme rations when we have to make linear approximations of non-linear extreme combinations of feed. As a consequence of this formulation, all possible combinations of bulky and concentrated feed are allowed (as it was with the blending problem).

Although our problem has been considerably narrowed without altering it, we however end up with a large number of activities. If hay should be incorporated into all rations to satisfy dry matter minimum requirements, then, the minimum number of activities to be defined, by period and animal, is equal to N, where $N = 2^n$ and n = number of bulky feed, including hay.

It still would be possible to reduce N if we knew in which subset of rations the optimum choice is always made.

2. Economic dominance and extreme aggregate activities

When such a dominance exists it becomes useless to write the dominated subset of rations in a model. If we knew for example that the optimum subset X of feeding programs could be defined as follows:

$$\text{optimum } X = \left\{ \begin{matrix} \text{minimum weight of concentrates and minimum quantity} \\ \text{of hay)} \end{matrix} \right\}$$

then, if n = 3, our extreme rations would be reduced from 8 to 3. It would be sufficient to use with the corresponding minimum quantity of hay, the maximum of one or both other fodder to minimize concentrated feed requirements. The number of activities and constraints would, therefore, be reduced in a large extent.

D. Specific Difficulties in Optimum Dairy Herd Feeding Program Determination

The main complications arise from two causes:

- dairy herd characteristics
- practical constraints

1. Dairy herd characteristics

We always observe a large variation of daily milk production from cow to cow within different herds. To the distribution of daily milk production per cow corresponds a similar distribution of daily nutrient requirements per animal. These variations are due to different

- calving dates
- cow production potentials
- lactation numbers...etc.

Furthermore, the arithmetic mean of these distributions are not constant over time. The production of milk is a decreasing function of time within each lactation.

2. Practical constraints

As it wouldn't be realistic to calculate, in a model, an optimum ration for each cow, it wouldn't be practical to distribute a different menu to each individual of a medium size herd. However, sub-herd with similar characteristics could be constituted.

In short, we try to find a single least-cost ration for a herd which is constituted of heterogeneous individuals. We must satisfy, not a sum of nutrient requirements but a distribution of them. Individuals can be overfed, if profitable, but underfeeding is ruled out. The optimum level of the common ration should be found.

If bulky feeds are cheaper than concentrated ones, then we must balance the cost of overfeeding few cows with bulky feeds over concentrate savings in more nutritive cow rations. This problem could be solved easily with a mixed integer code since we have to write a set of mutually exclusive constraints to express it correctly (26, p. 538). Among a set of basic rations, we have to choose the one which, distributed to each sub-group of cows, minimizes total feed cost. The corresponding sub-matrix can be written as follows (for a given period):

$$\text{Max:} \quad ..\sum_j C_j X_j + \sum_p \sum_k C_{kp} X_{kp}$$

$$\text{Subj:} \quad ...A_{ij} X_j - A_{ikp} X_{kp} \leq 0 \qquad (22)$$

$$...X_j, X_{kp} \geq 0$$

...mutually exclusive subsets of constraints i

where i = basic ration level = 1...n

j = herd type = 1...m

k = basic ration = 1...m'

p = herd sub-group = 1...p'

Our alternative problem becomes:

$$\text{Max:} ..\sum_j C_j X_j + \sum_k \sum_p C_{kp} X_{kp}$$

$$\text{Subj: } + A_{ij}X_j - A_{ikp}X_{kp} + L_i\theta_i \leq 0 \quad (23)$$
$$.\sum_i \theta_i \leq 1$$
$$...\theta_i = 0 \text{ or } 1$$

where L_i = lower limit of constraint i.

Besides computation difficulties we run again into dimensional problems since the latter constraint extends our initial models.

The possibility of cow profitable overfeeding and the particular shape of the overtime milk production function generates accuracy problems in feed cost determination. These difficulties arise with linear combination of aggregate activities.

a. Overfeeding and approximation problems If A and B are two feeding activities satisfying exactly to all animal nutrient requirements (i = 1...n), then activity C will have the same characteristics, where

$$C = \alpha A + (1-\alpha)B.$$

If, however, at least one of the two activities exceed one or several animal nutrient requirements, the quantity of feed inputs will be overestimated for all linear combinations C of A and B,

where $A = \begin{Bmatrix} a_1 \\ a_2 + d \end{Bmatrix}$, $B = \begin{Bmatrix} b_1 \\ b_2 \end{Bmatrix}$

$a_i = b_i$ = minimum nutrient requirement

d = nutrient excess

$$C = \begin{Bmatrix} \alpha a_1 + (1-\alpha)b_1 \\ \alpha(a_2+d) + (1-\alpha)b_2 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 + \alpha d \end{Bmatrix}$$

To overcome this problem overfeeding can be excluded, but we restrict the production possibilities to a particular subset without being sure that it contains the optimum one. Hovelaque (41) has, nevertheless, chosen this alternative. Doing so, certain basic rations, especially those formed with nutritive fodders, are limited to a level of 6.5 kilograms of daily milk production, although they could satisfy to the energy requirement of 11.5 kilograms. If the production of these fodders is profitable, energy is an expensive nutrient and daily milk output per cow reaches high level (for the herd or sub-group of it), then found solutions are not true optimum. When feeding aggregate activities are introduced within linear programming model a choice has to be made between overfeeding or low basic ration level. If such a choice can't be made, then optimum feeding programs will be studied as blending problems which overpass these difficulties.

b. Decreasing milk production function and accuracy problems

Cow nutrient requirements vary as their milk production function. Their needs decrease over time from a calving date to the following one. Basic rations supply a certain proportion of total nutritive needs. The remainder is brought by concentrates. To satisfy specific requirements, added concentrate quantities vary with basic ration levels. The linear combination of certain aggregate activities results in accurate concentrates cost estimates, others lead to erroneous estimations.

Adequate linear combinations - Without loss of generality we can restrict our demonstration to a single nutritive element such as energy, for example (Figure 4), where

R_a,R_b: level of disposable energy supplied by basic rations A and B

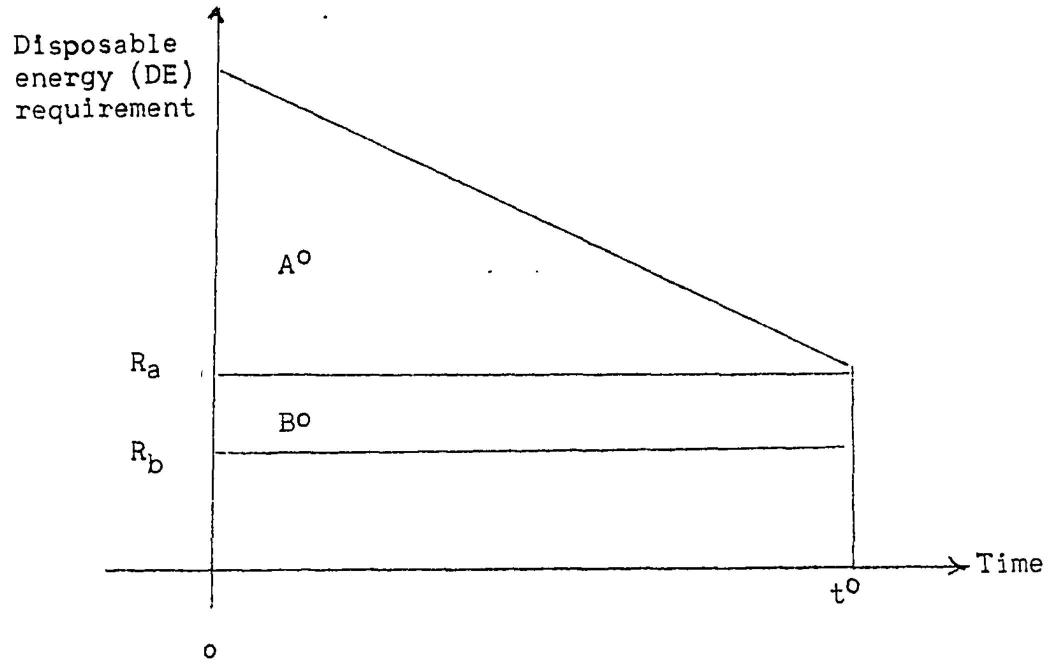


Figure 4. Supply of concentrate in addition of two alternative basic rations (example one)

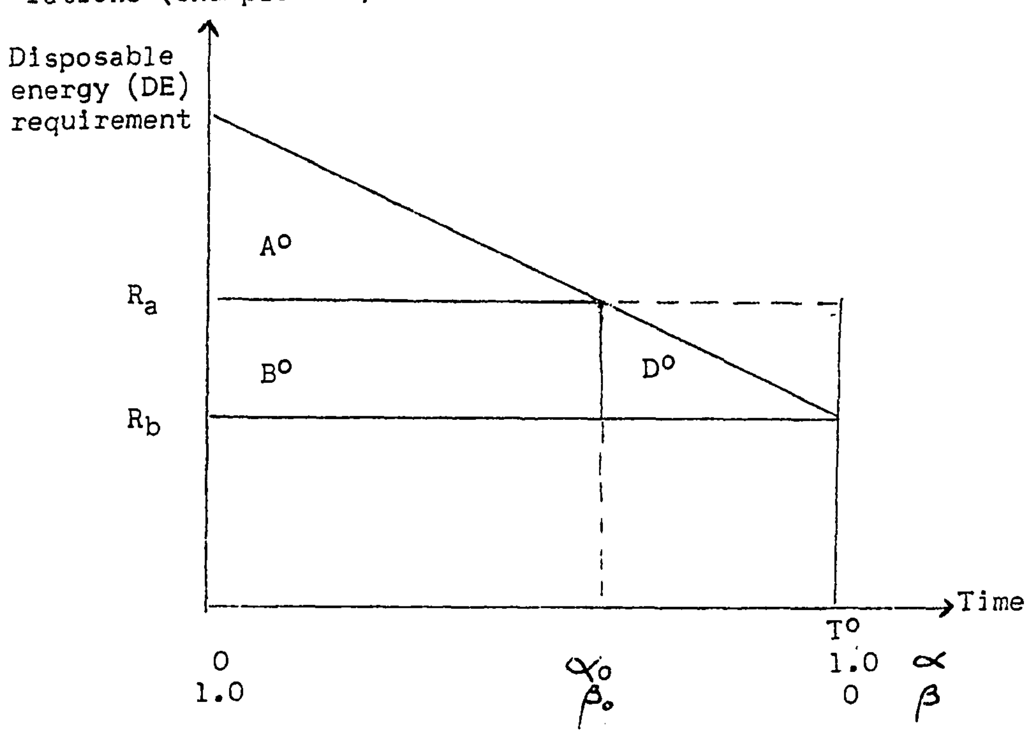


Figure 5. Supply of concentrate in addition of two alternative basic rations (example 2)

$A^o, A^o + B^o$: total quantity of concentrates added respectively to basic rations R_a and R_b, within the t_o period.

Any linear combination of these two rations will give a correct estimation of concentrated feed consumption. We get:

$$C^o = \alpha A^o + \beta(A^o + B^o); \ \beta = 1 - \alpha$$

$$C^o = A^o + \beta B^o.$$

The value of C^o is accurate since

- the first term A^o is constant for all values of β
- B^o input consumption is directly proportional to β (time)

Inaccurate linear combinations - When R_a and R_b take higher values as shown in Figure 5, the estimation of C^o won't longer be adequate, where

$A^o, (A^o + B^o + D^o)$: total quantity of concentrates added respectively to basic rations R_a and R_b, within the T^o period.

We get:

$$C^o = \alpha A^o + \beta[A^o + B^o + D^o]; \ \alpha = 1 - \beta$$

$$C^o = A^o + \beta[B^o + D^o]$$

But in fact:

for $\alpha \geq \alpha_o$; $B^o = 0$

$\alpha < \alpha_o$; D^o should equal D^o

Furthermore, even if we write the preceding combination as follows:

$$C^o = A^o + \beta D^o; \ \beta \leq \beta_o$$

$$C^o = A^o + D^o + \beta B^o; \ \beta \geq \beta_o$$

we would not get a correct estimation of C^o since the consumption of concentrated feed is not directly proportional to time. This case, in fact, violates the proportionality assumption of linear programming model. To overcome this difficulty (if we suppose it is not negligible) we can only divide our time axis into more subperiods, in order to make a linear approximation of nutrient requirement curves.

E. Conclusion

In model building we can choose either one of three alternatives:

- the blending framework
- the set of extreme pre-established ration
- the ration subset which presents known economic advantages over other subsets.

Particular choice will be a function of the problem to be studied and the amount of available economic results. When optimum feeding programs constitute the main research objective, then the blending framework should be chosen. Otherwise, other procedures are more convenient.

CHAPTER 5. LABOR CONSTRAINTS

Each production process requires a definite amount of labor inputs. Due to the particular pattern of farm production and the high cost of those resources, it is particularly important to avoid both under-estimation and over-estimation of production opportunity surface. What can easily be done when labor constraints have not been set up very rigorously.

A. The Problem

Any livestock and crop activity can be characterized by a sequence of discrete jobs. This sequence is an ordered collection of field and tending animal operations.

1. Job definition

A job is completely defined by:

(a) Its period of completion. The "when should we execute it" question will determine its place on the time axis. First of all, jobs are divided into two subsets according to the possibility of postponing them or not.

Postponable jobs, such as fencing or machinery maintenance, don't generate a set of strenuous constraints. They are omitted.

Non-postponable jobs, on the contrary, limit really and sometimes strongly, production plans. They are included in models. These jobs can only be executed efficiently within certain periods, say $(t - t_o) = B_k$. Timeliness of each operation is subject to certain environmental conditions such as the weather and/or biological laws. But generally, within any given time period, only a certain proportion of days or hours can be devoted to the corresponding job completion.

Let's define:

j_{kj} = job k of activity j

B_{kj} = time period of job k in activity j

β_{kj} = environmental conditions required by j_{kj} in B_{kj}

b_{kj} = number of effective hours which can be allocated to job k in activity j

A_{kj} = man labor requirement for job k in activity j.

If B_{kj} can be viewed as constants or variables with small over-time variances, we observe that b_{kj} varies greatly from year to year: b_{kj} being a function of β_{kj}.

Since a linear programming assumption states that coefficients are fixed we have to choose a single value for b_{kj}. Should we take the mean, the mode or extreme values of the distribution function? The chosen value will depend upon farmer's willingness to accept risks of non-completion of his field work during bad years or in order to avoid them, to use any exceptional means. To determine such a value we need a "risk criteria". This concept has been used by Link (51) and Reboul (65).

(b) Its man labor and machine time requirements. After having defined all different ways which could be chosen to execute each particular job, labor and machine input requirements are calculated.

2. <u>Setting up adequate labor constraints</u>

Labor inputs are "flow resources" and therefore the marginal rate of substitution (M.R.S.) of one period input to the subsequent one is frequently equal to zero. However, within certain subperiods it is equal to 1.0. A correct setting-up of constraints requires that

(a) Within each labor constraint the M.R.S. of labor from one job to another is equal to 1.0.

(b) The supply of labor $\geq$ its demand.

We assume that the number of workers is constant over time although the number of work hours per day can vary. If a single activity j ($j \neq 0$) were chosen as the best solution it would be limited by the smallest ratio θ_j:

$$\theta = \min_k \frac{b_{kj}}{A_{kj}} \tag{24}$$

A corresponding constraint should be included to limit activity j accordingly (except if $k \epsilon i$, case 1 below).

When several activities j differ from zero we have to add our second requirement stated above:

Definition:

$$\dot{I} = \left\{ (t_1 - t_0) = T \text{ such that: } B_{kj} \cap B_{kj'} = \emptyset;\ j \neq j',\ \forall (k,j) \right\}$$

$\dot{I}$ = time period

i = time subperiod such that $i \epsilon \dot{I}$

The constraints:

Different sets of i, $i \epsilon \dot{I}$, will be defined as follows according to corresponding assumptions.

Case 1: If within $\dot{I}$:

(a) $B_{kj} = B_{kj'}$; $j \neq j', \forall(j)$

(b) $b_{kj} = b_{kj'}$; $j \neq j', \forall(j)$

(c) $\beta_{kj} = \beta_{kj'}$; $j \neq j', \forall(j)$

Then, it is sufficient to write a single labor constraint for the corresponding time periods. Labor, in this case, can be viewed as a stock resource since its M. R. S. within the time period $\dot{I}$ is equal to 1.0. The corresponding constraint is:

$$\sum_j (\sum_k A_{kj}) X_j \leq \sum_j \sum_k b_{kj} \tag{25}$$

Case 2: If within $\dot{I}$:

(a) $B_{kj} = B_{kj'}$; $j \neq j', \forall(j)$

(b) $b_{kj} \gtrless b_{kj'}$; $j \neq j', \forall(j)$

(c) $\beta_{kj} \cap \beta_{kj'} = \emptyset$; $j \neq j', \forall(j)$

then, although all jobs can be performed within exactly the same period, they are, however, a set of mutually exclusive events due to weather conditions as for example:

- if it rains we transplant fodder beet
- if it doesn't we make hay.

The corresponding constraints are:

$$A_{kj} X_j \leq b_{kj};\ j = 1 \ldots n \tag{26}$$

but we retain only those k which correspond to $\theta = \underset{k}{\text{Min}} \frac{b_{kj}}{A_{kj}}$. The others k are dominated by the constraint which corresponds to θ.

Case 3: If within $\dot{I}$:

(a) $B_{kj} = B_{kj'}$; $j \neq j, \forall(j)$

(b) $b_{kj} \leq b_{k,j+1} \leq b_{k,j+2} \cdots \leq b_{k,j+n}$

(c) $\beta_{kj} \subseteq \beta_{k,j+1} \subseteq \beta_{k,j+2} \cdots \subseteq \beta_{k,j+n}$

In this case, where the subset j is included within the subset j + 1, for all j. An adequate set of i = n + 1 constraints will be written as follows:

$$\underset{j=l+1}{U} A_{kj}X_j \leq \underset{j=l+1}{U} b_{kj};\ l = 0,1\ldots n\ . \tag{27}$$

Case 4: If within $\dot{I}$:

(a) $B_{kj} \cap B_{kj'} \neq \emptyset;\ j \neq j', \forall(j)$

$B_{kj} \not\subset B_{kj'}\quad ;\ j \neq j', \forall(j)$

or

$B_{kj} \subseteq B_{kj'}\quad ;\ j \neq j', \forall(j)$

(b) $b_{kj} >< b_{kj'};\ j \neq j'$

(c) $\beta_{kj} \cap \beta_{kj'} \neq \emptyset;\ j \neq j'$

$\beta_{kj} \not\subset \beta_{kj'};\ j \neq j'$

To take into account this set of overlapping subsets of constraints we have to define, in addition to constraints specified by the θ ratio, the following ones:

$$\underset{j=C_r^n}{U} A_{kj}X_j \leq \underset{j=C_r^n}{U} b_{kj};\ r = 2\ldots n \tag{28}$$

This set of constraints has $\sum_{r=2}^{n} C_r^n = (2^n - n - 1)$ elements.

Case 5: If within $\dot{I}$:

(a) $B_{kj} \cap B_{kj'} \neq \emptyset;\ j \neq j'$

$B_{kj} \not\subset B_{kj'}$

(b) $\beta_{kj} = \beta_{kj'};\ j = j'$.

An adequate set of constraints will be formulated by equation 28. We are led to define $(2^n - n - 1)$ subperiods i, $i \in \dot{I}$.

Case 6: If within $\dot{I}$:

(a) $B_{kj} \cap B_{kj'} \neq \emptyset;\ i \neq j'$

$B_{kj} \subset B_{kj'}; \; j \neq j', \; \forall(j)$

(b) $\beta_{kj} = \beta_{kj'}; \; j \neq j'$

In this case equation 27 will constitute a sufficient set of constraints.

B. Conclusion

We have not proved the validity of the preceding five equations. Proofs would lie on the same arguments that we have developed to prove theorems 4, 5 and 6 relative to crop rotations. They are therefore omitted. Those equations are necessary to guarantee that the supply of labor in any period is not smaller than the effective requirements of labor inputs which arise from any possible solution of the linear programming model. However, in practical problems, we have to make use of good judgment to select among this large number of constraints, those which are more likely to be effective. Otherwise, the model would frequently be very large.

CHAPTER 6. CAPITAL

Any production plan purpose is the transformation of resources into final products such as wheat, corn, milk. Each process can be a direct one (wheat) or the result of a succession of intermediate processes ending into a final one. Heifer and fodder, for example, have to be produced before we can get milk or steer output.

Service of resources could be classified into three main groups: (37, p. 23) stock, flow or stock-flow. They are used up in the production process and as such are a part of the resulting output. Since, in every case, the combination of inputs has to precede any output obtainment, one can't start producing if he has not accumulated a minimum amount of scarce resources. At one time, farmers had to save seeds and enough food from the preceding harvest if he wanted to get the following one. Nowadays, larger and larger amount of inputs are produced out of farms. Farmers have to buy them on the market and to pay the corresponding bill within a certain delay which is usually shorter than the production period. Therefore, modern farmers, like our ancestors, have to own some particular inputs (or their money equivalent) before they can undertake something. To start farming one should own a certain amount of capital which could be invested into resources which embody either stock or flow services. In the former case the value of those inputs will be entirely recovered at the output sale time (otherwise tne corresponding activity wouldn't be selected from the vector list by the computer). In the latter case, we will get back only the annual amortized payment of the assets and the annual expense associated with owning and using it (since this type of input lasts longer than the chosen lapse of time). In particular, capital output from a production period is an input for the next one. Therefore, at each production cycle, if all processes start and finish within it and even if he doesn't get any external contribution of liquid assets, the manager will be able to choose the same production plan so long as he doesn't consume more than he earns. On the contrary he will enhance its production possibilities and consequently its future income expectation by relaxing its capital constraints (if he has any). In any model the adequate expression of outgoings and incomings will be a rewarding effort since capital flows are counterparts of almost every decision which might be made in the farm-firm. It is not so easy to do so. Capital has several dimensions through its role of expressing any outgoing and incoming inputs and outputs in money terms. The main ones are time and quantity.

Capital and its time dimension - Inputs and outputs are sold and bought at certain dates which are different for each activity production cycle. Furthermore, we have to cut the time axis since our linear programming model assumes a finite number of activities. Practically, it is important to construct a model which does represent reality as closely as possible and includes the minimum number of variables and constraints. Once the time period is chosen we will be able to classify inputs and production processes into two main groups:

- variables inputs and one period activities.

  Within the time period length certain inputs are fully "used up" and the production cycle of a subset of activities ends. Typical cases are fertilizer inputs and barley.

- Durable inputs and multi-period activities.

Some inputs and production processes do not end with the usual time period. They last longer. Examples are buildings or apple production. To take them fully into account we have two solutions. We could lengthen our time period in order to include the longest production cycle or the inputs wearing out time. Another way would be to use artificial means. Durable inputs are amortized and multi-period activities are combined into single period ones. To do so, we can divide into parts the entire production cycle such as we introduce at each period an amount of a starting process equal to the one which has been ended. For example, if apple trees are pulled up after 15 years we will define the corresponding orchard activity in the following way:

- total planted acreage: 15 hectares
- acreage in full production: 10 hectares
- acreage pulled out each year: 1 hectare
- new planting: 1 hectare

Capital and its quantity dimension - We have to consider two important classes of problems: divisibility and scarcity.

- Capital can be a scarce or a plentiful resource. In the latter case it will never be an efficient constraint on the production plan.
- Although capital can be, by nature, divided *ad infinitum*, the physical inputs which are bought with it might be indivisible, and so the production processes which are associated with them.

A. Multistage Linear Programming Versus Mono-Periodic Programming

Within a multistage linear model the capital flow is entirely described since the chosen programmed period includes the longest production cycle. We answer, not only the question "what should we produce", but also "when should we start each production process"?

A mono-periodic model can take into account capital flows which arise from a production plan already in cruising speed. To the initial investment we have to add working (or operating) capital. From this model objective function the durable input annual depreciation is subtracted. Doing so, we will fail to solve the "when" of investment decision, except in some particular cases.

To know which one we should use, in a particular problem, we have to consider several cases.

1. Project benefits are constant-over-time

a. Unlimited amount of capital When value of goods and services provided by a set of investment alternatives doesn't change over time it would be necessary "only to compare the present value of the benefits of each proposed project with its construction outlay. If the former exceeded the latter, the project would be constructed at once; otherwise, it would be rejected" (55, p. 11). It is therefore sufficient, in every case, to build a mono-periodic model since we only have to decide whether a project should be undertaken. It is, however, necessary to evaluate correctly the required amount of capital in order to assign to the specified project the corresponding opportunity or real interest charge.

b. Limited amount of capital Even though capital is a limiting resource, the necessity of building multistage models will be decided upon the absence or the presence of joint characteristic assumptions. In a first approach we will consider that every process yields all its output within a single period. We rule out every multi-period activity and intertemporal constraint other than capital.

Divisible and variable inputs - Although capital is limited, a classical linear programming model will allow us to specify what and when each production plan should be attempted: each year plan being different from the preceding one by a certain amount of capital. The set of all solutions being completely defined by the following equations:

$$Y = f(K) = f(K + \theta K) \tag{29}$$

$$X = f(K) = f(K + \theta K) \tag{30}$$

where Y = objective function value

X = solution vector of a linear programming model

K = initial amount of capital

θ = a variation parameter (> 0).

Or the solution of equations 29 and 30 can easily be found in an inexpensive way by using a parametric procedure on an initial linear programming solution. Such a model has been described by Candler (16).

Divisible and durable inputs - The amortization rate of durable inputs can always be chosen so that it coincides with the repayment rate of a bank loan. The corresponding depreciation rate will be linear since most banks require lump sum annual installments. Durable inputs being divisible, their corresponding costs will be directly proportional.

When such inputs can be sold on a market at a price near their residual value we can always, at any time, transform a durable input set into another one. Capital being perfectly adaptable, equations 29 and 30 still hold and the corresponding procedure will be valid. In each subsequent year additional durable inputs will be bought at the maximum permissible amount allowed by capital availabilities. If, at certain firm growth level, the transformation of the basic equipment is required, the corresponding solution will still be feasible. We must observe, however, that the notions of flexibility and adaptability are different. Flexible plant can be used for producing one product out of n possible ones, according to specified substitution rates. Perfect adaptability of capital allows productive bundle of durable inputs to be transformed, at zero cost, into another one.

However, capital input is not always so malleable. Secondhand inputs can have a very low salvage value or even no market at all, for different reasons, a typical one being location obsolescence. Under these assumptions it will be necessary, in most cases, to build a multi-stage model. Continuous variation of capital constraints would possibly give a sequence of solutions requiring a steady change from one durable input set to another. Since these inputs have no salvage value their amortization rate should vary with their effective use span. These two variables are negatively related. Their effective life span being unknown we have to consider time as an additional variable in order to take their real cost and their potential use into account. A multi-stage linear programming model is required unless we should get from the parametric solutions of the mono-periodic program one of the two cases:

- $X_{i-1} \subseteq X_i$; $i = 2 \ldots n$ (X_i = durable input set required by capital level i)
- the growth rate is such that we get an exact correspondence between the capital accumulation rate and the life span of the successive bundles of durable inputs. If the first case can be frequent (e.g. field tilling), the second one is quite improbable.

Indivisible and variable inputs - It is difficult to think of an essential real life example when the corresponding period of time lasts one year. However, under such assumptions, a multistage model wouldn't be an essential one. Time doesn't need to be taken into account since inputs are destroyed each year by the production process. At the end of each period their value is found in the marketed products. Capital being, through time, fully adaptable we can still use the two preceding procedures. However, the main difficulty would be to find a good mixed integer algorithm capable of finding the optimal solution within a suitable time. Furthermore, none of these codes can perform a parametric procedure allowing the finding of equation 29 and equation 30. We would have to find, if the main difficulty can be overcome, a set of solutions for the corresponding amount of liquid assets.

Indivisible and durable inputs - Indivisibility requires the use of an integer or mixed integer algorithm. Durable inputs require a multistage model when they are such that they rule out any form of adaptability. We must weigh savings from economies of scale relative to the opportunity costs of temporary excess capacity. On the contrary, when they are perfectly adaptable a mono-periodic model is adequate.

<u>Multi-period activities</u> - We reintroduce here multi-period activities into production plans excluding other assumptions, especially

- indivisible inputs
- inadaptability of capital.

Multi-period productions can present four different characteristics:

(1) Over-time constant stream of income

(2) Over-time variable stream of income

(3) Over-time variable input requirements

(4) Intermediate products can be marketed

Over-time constant stream of income and outlay - When we get in each period the same total amount of outgoings and incomings and when they appear, within each subsequent period, at the same date we can still choose a mono-periodic model. Each year is similar to the preceding one. We may have to amortize some initial costs over the total production period. One example can be provided by alfalfa.

Over-time variable stream of income and outlay - Outputs of quite a few productions are located within the last periods of their production cycle. When we combine all subsequent periods into a single one, costs and incomes of the production process are aggregated. Continuous variation of capital constraints will give a set of solutions which won't be feasible. Capital is overestimated as long as we have not reached the corresponding cruising speed. Popular plantations whose yield is marketed after more than 15 years give us an extreme example of this situation.

Over-time variable input requirements - Although activities are independent from each other in the sense that their benefits and costs do not depend on whether other projects in the program are undertaken, they are, however, interdependent through their competing use of limiting resources. A project whose input requirements vary over its life allows other projects to be associated with it at time-varied levels. When such a process enters into the solution of a multi-stage model we get a sequence of optimum plans, each one differing from the other partially by the level of interrelated projects. Stable solutions will appear when capital constraints will cease being effective and the maturation time of multi-period projects will be ended. Under that assumption, a mono-periodic model can't be used. Orchard investment is a good example of a project which requires over-time variable inputs (especially labor) and provides an irregular stream of net income.

Existence of a market for intermediate outputs - We define the producing units arising from the maturation time of the multi-period activity as intermediate outputs. In some cases they can be acquired in a market place. A dairy herd can be formed with bought heifers. Although they represent a typical multi-period process it has been split into a single period one by the market. Each subsequent intermediate output becomes a final product with its real cost, its opportunity cost and its market value. This last characteristic rules out the need of a multi-period model, a mono-periodic one being sufficiently accurate.

Other inter-temporal constraints - We can imagine a few situations which cannot be correctly expressed within a mono-period linear model. They have in common the property of generating constraints in such a way that decisions in one period modify next year's possibility set and impose a series of new constraints upon it. Year t crops generate liquid assets, which extend next year's income opportunity curve, and at once restrict the number of crops which can be cultivated to a new subset.

When capital is unlimited we need to find optimal crop rotations (circuits on a graph). Capital constraints impose the finding of an optimal crop sequence (an oriented tree or chain ending into a circuit when capital ceases to be scarce). The introduction of crop rotations into a mono-periodic (or a multistage model) can lead to:

- non-optimum
- and infeasible solutions.

In the first case the optimum solution can be a time sequence of crops rather than a sequence of crop rotations. The latter can impose crops in the first stages of a firm growth, although they could really be undertaken more economically in later stages. When the present crop is meadow (year 0) it can be followed by corn (year 1) and oats (year 2) rather than cultivating an equal amount of each one in every successive year.

Infeasible solutions can be obtained when there exists no way of linking correctly the found crop rotation sequence.

Crop rotation rules are frequently such that it is impossible to cultivate any crop after any other one. This assumption leads to the preceding remarks. However, in particular cases, we can find subset of crops for which the preceding assumption doesn't apply and mono-periodic models would be accurate enough. It is the case when the expansion path is not determined by crop rotation constraints.

2. Over-time increasing project benefits

Even in the absence of capital constraints a positive present value of total benefits may lead to non-optimal decisions when the demand for a particular output increases over time. Projects can be undertaken too soon. "The optimal construction date is t_0, at which time the value of the project's output catches up to the interest cost. Until this time the project would be losing money each minute of its existence" (55, p. 23). Under this assumption when projects are interdependent through the competitive use of resources, we are led to build a multi-stage model. If capital is scarce we come to the same conclusion.

3. Conclusion

To maximize their income ($\dot{i}$), farmers should act upon such choice variables as the level of inputs (X) and/or outputs (Y). In other words they should find the solution of the following equation (56, p. 83):

$$\max_{Y} \max_{X \epsilon f(Y)} R(Y) - C(X) = \max (\dot{i}) \tag{31}$$

where X,Y are vectors

C = costs

R = receipts

To maximize it over-time we can use a parametric mono-periodic linear model even under our first two simplifying assumptions:

- capital is the only varying inter-temporal constraint
- there exist no multi-period activities in the model.

This procedure is valid when capital is perfectly adaptable through the market, as for durable inputs, or through product sales, as for variable resources. If, furthermore, a few inputs are indivisible, we will solve the problem with a mixed-integer linear model, instead of using a classical one. In all other cases, when capital is inadaptable, classical or mixed integer multistage linear programming will be required.

Under our two second simplifying assumptions:

- multi-period activities are present among choice variables
- crop rotation rules constitute a subset of over-time constraints.

We can still maximize equation 31 using a parametric mono-periodic linear model. This procedure is only valid when:

- multi-period activities generate constant stream of income or marketable intermediate outputs
- crop rotation rules are such that any crop can be followed by any other one.

In all other cases, classical or mixed integer multistage linear programming will be needed, especially if multi-period activities require varying amounts of inputs through time or generate fluctuating streams of income and if crop rotation rules are severe.

If capital scarcity is a necessary condition to the need of building multistage linear model, it is not a sufficient condition. Other ones have to be associated with it.

B. Capital as a Constraint

Many farmers mention capital rationing as a constraint. It can be imposed by external agents such as merchants, bankers...or by farmers on themselves (risk bearing motives...).

A few authors neglect the corresponding restrictions for different reasons. Tyler (73, p. 9) thinks that operating capital shouldn't be a constraint so long as projects are profitable and loans can be repaid within a reasonable lapse of time. du Boullay (13) distinguishes two cases. When capital is an effective constraint we should integrate it to a multistage model in order to maximize the growth rate of the firm. He mentions that multi-period activities and durable inputs rule out the possibility of expressing correctly the corresponding money flux within a mono-periodic model (13, p. 2). When capital is plentiful we should exclude from mono-periodic model both investment and working capital constraints (13, p. 8).

However, many authors integrate capital constraints into their models (36, p. 206). This practice is even done when they recognize "the capital problem" (14, pp. 72-77).

Even though capital is not an active constraint we may desire to:

- estimate the required amount
- take full account of the loan interest cost
- charge the project with a certain interest opportunity cost.

When capital is an effective constraint we have shown that, in a few cases, it can be integrated correctly within a mono-periodic model.

For this subset of problems it is, therefore, necessary to be able to express the corresponding capital constraints correctly within our mono-periodic models.

1. Working capital constraints

a. General hypothesis

- future is known with certainty
- family consumption plus loan repayments don't exceed their total income
- the model assumptions are such as is adequate to write a mono-period linear program
- all bills are paid within a certain time limit (for example three months) after delivery

b. Definitions

J = activity set

T = period set

X = level of activity

a^{+}_{tj} = positive balance; $t \varepsilon T$, $j \varepsilon J$

a = (receipts-payments)

K = initial amount of capital

θ = capital transfer from period (t) to period (t + 1)

a^{-}_{tj} = negative balance; $t \varepsilon T$, $j \varepsilon J$

c. Model ≠ 1

We assume:

(1) a^{-}_{tj}; $\forall (j, t < t_o)$

$a^{+}_{tj};\ \forall(j, t \geq t_o)$

(2) $a^{+}_{tj} \cap a^{-}_{ti} = \emptyset;\ i \neq j, i, j \in J,\ \forall(j,t)$

(3) $U(a_{tj}) = \emptyset;$ some $t \in T,\ \forall(j)$

In other words the time axis can always be cut in such a way that every activity production cycle can be included within it.

- negative balances always precede positive ones.
- any activity i incoming cannot pay out for activity j outgoing.

Under these conditions a unique capital constraint will be sufficient

$$\sum_t \sum_j a^{-}_{tj} X_j \leq K \tag{32}$$

d. <u>Model ≠ 2</u>

In model 1 we substitute assumption number 1 by the following hypothesis to obtain model 2 assumptions

(1) $a^{-}_{tj};\ \forall(t \leq t_o;\ t_1 < t_2)$

$a^{+}_{tj};\ \forall(t_o < t \leq t_1;\ t_2 < t \leq t_n)$

In this case we will be able to pay period $(t_2 - t_1)$ bills with periods $(t_1 - t_0)$ net receipts. Total capital needs would be less than the total sum of negative balances. The corresponding capital constraints could be written as follows:

$$\sum_t^{t_o} \sum_j a^{-}_{tj} X_j \qquad + 1.0 \leq K \tag{33}$$

$$\sum_{t_1}^{t_2} \sum_j a^{-}_{tj} X_j - \sum_{t_0}^{t_1} \sum_j a^{+}_{jt} - 1.0 \leq 0$$

The first equation guarantees that initial outlay will be paid on the disposable stock of capital. The second one allows $(t_2 - t_1)$ payments to be made on $(t_1 - t_0)$ net receipts and through a capital transfer from the initial stock of capital, if needed.

e. <u>Model $\neq$ 3</u>

We assume:

(1) $a^{+}_{tj} \cap a^{-}_{ti} \neq \emptyset$; $i \neq j, ij\ \dot{J}, \forall(t)$

(2) $U(a_{tj}) = \emptyset$, some $t \in T, \forall(j)$

All activity production cycles can take place within the defined time period t_o, but some activities possess a capital profit enabling them to pay out for other activity expenses. When capital is scarce optimum solutions will make use of this opportunity.

Equation 34 takes the corresponding constraints into account. a_{tj} submatrix coefficients will be negative or positive when they represent respectively the following balances: a^{+}_{tj} and a^{-}_{tj}.

$$\begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1k} & +1 & & & \\ a_{21} & \cdots & a_{2j} & \cdots & a_{2k} & -1 & +1 & & \\ \vdots & & \vdots & & \vdots & & & & \\ a_{i1} & & a_{ij} & \cdots & a_{ik} & \cdots & -1 & +1 & \\ \vdots & & \vdots & & & & & & \\ a_{t1} & \cdots & a_{tj} & \cdots & a_{tk} & \cdots & \cdots & \cdots & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_j \\ \vdots \\ X_k \\ \theta_{k+1} \\ \theta_{k+2} \\ \vdots \\ \theta_n \end{bmatrix} \leq \begin{bmatrix} K_o \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{34}$$

or in matrix notation

$$\underset{(txn)}{A} \underset{(nx1)}{X} \leq \underset{(tx1)}{B}$$

f. Model ≠ 4

We assume:

(1) a^-_{tj}; $\forall(j, t < t_o)$

(2) $a^+_{tj} \cap a^-_{ti} = \emptyset$; $i \neq j, ij \epsilon J, \forall(t_o \leq t \leq t_1)$

(3) $a^+_{tj} \cap a^-_{ti} \neq \emptyset$; $i \neq j, ij \epsilon J$, some $(t \geq t_1)$

(4) $U(a_{tj}) \neq \emptyset$; some $t \epsilon T, \forall(j)$

Here we have combined model number 1 with model number 3 hypothesis. Constraints will be easily taken into account by equation 35.

$$\begin{bmatrix} a^-_{11} & \cdots & a^-_{1j} & \cdots & a^-_{1k} & +1 & & \\ a_{21} & \cdots & a_{2j} & \cdots & a_{2k} & -1 & +1 & \\ a_{31} & \cdots & a_{3j} & \cdots & a_{3k} & & -1 & \\ a_{t1} & \cdots & a_{tj} & \cdots & a_{tk} & & & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_k \\ \theta_{k+1} \\ \cdot \\ \theta_n \end{bmatrix} \leq \begin{bmatrix} K_o \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \qquad (35)$$

or in matrix form: $\underset{(txn)}{A} \underset{(nx1)}{X} \leq \underset{(nx1)}{B}$

equation 35 is equal to equation 34 to which we have added equation 32 after two modifications: We have substituted zero for K_o in the B vector (equation 34), and allowed capital transfer from period t_o to period t.

g. Model ≠ 5

We assume:

(1) $a^+_{tj} \cap a^-_{ti} \neq \emptyset$; $i \neq j, ij \epsilon J$, some t

(2) $U(a_{tj}) \neq \emptyset$, $\forall(t \epsilon T; j \epsilon J)$

It is impossible to divide the time axis into periods in such a way that every period includes an entire production cycle. But if we define T_i periods such that the longest production cycle can be included within it, we have the following relationship:

$$b_j \left[\sum_{t=i}^{m} (a^{+}_{tj} - a^{-}_{tj})\right] = b_j C_j; \ \forall (j \epsilon \dot{J}, t \epsilon T_i) \qquad (36)$$

where $b \geq 1.0$ = number of cycles within T_i

C_j = j activity net return, above fixed costs.

$1 \leq T_0 \leq m; m + 1 \leq T_1 \leq 2m; \ldots$

Assumption 2 allows capital constraints to be useless. When the production activities are profitable, then

$$\sum_j b_j C_j X_j > 0$$

$$\sum_j b_j C_j X_j = 0 \text{ otherwise.} \qquad (37)$$

Since we are maximizing equation 31, within each period T_i, a few a^{-}_{tj} will yield their corresponding output in t_{i+1} period, as a few a^{+}_{tj} result from T_{i-1} expenses. It is, therefore, possible that capital becomes an unconstrained resource when quite a few activities yield their output within T_i first subperiods and when production has reached its cruising speed. Nevertheless, capital is an effective constraint within the first period in which production plans are starting. We must consider, not a single period as previously, but two periods. The former represents the starting process, the latter describes a stable capital profile. Beyond it, T_{i-1} capital output will be sufficient to satisfy T_i capital input requirements.

Therefore, we have to choose a date at which the first project will be undertaken. Equation 35 will express correctly the corresponding capital constraints with A being now a (2t x n) matrix.

2. Investment capital constraints

Initial outlays which represent investment expenditures (equipment, livestock) can be added to the operating capital requirements of the first subperiod T_1 in any preceding system of equations 32, 33, 34, and 35. However, when borrowing activities have different characteristics (interest rate, upper bound...) according to their subsequent use we should distinguish both types of capital. A first equation represents investment outlay, the second one being the T_1 subperiod working capital constraint. A capital transfer activity links them as follows:

$$\begin{aligned} &\sum_j a_{0j} X_j - X_{j+1} \leq 0 \\ &\sum_j a_{1j} X_j + X_{j+1} \leq K \end{aligned} \qquad (38)$$

This last equation shows that we should not artificially separate operating and investment capital since a plant cannot be operated if we are unable to finance both its construction and its working expenditures.

C. Conclusion

Under the preceding assumptions, our models ensure us that the found solution is optimum since scarce capital is used in the most efficient way. Furthermore, a continuous variation of initial capital supply will determine an optimum and feasible sequence of production plans.

When capital flows can't be correctly expressed within a mono-period model, a multistage model is the only known substitute. But we are running into dimensional, residual value appraisal, uncertainty and indivisibility problems.

CHAPTER 7. INVESTMENT AND MUTUALLY EXCLUSIVE SET OF VARIABLES

To maximize their income (1), farmers should adjust not only their production plan (Υ) but also the set of fixed and variable inputs (X) which are necessarily associated with it. If we want to make a choice of the best production plan, we have to include, within our model, the set of all possible production possibility surfaces. In other words, we have to maximize the following equation:

$$\underset{\Upsilon}{\text{Max}} \; \underset{X \varepsilon \phi(\Upsilon)}{\text{Max}} \; R(\Upsilon) - C(X) \quad = \text{Max}(\dot{I})$$

R = sales

C = cost

Furthermore, the resulting choice should be practically feasible.

To be fully taken into account, the two preceding considerations require that our classical linear programming model be transformed into a mixed integer problem. However, we are running into the problem of finding an efficient code. But at the start of our study we have thought that, by the end of it, new codes might have been developed and made available to potential users. Work is being constantly done on this subject (28, 24, 2).

Main problems to be solved with mixed integer models.

Among problems we have met and which are easily formulated within the framework of a mixed integer linear programming model we can enumerate:

- the fixed charge problem
- mutually exclusive activities or set of activities.

A. The Fixed Charge Problem

It is frequently found mainly associated with investment decisions. Building cost functions, for example, are generally of the form

$$Y = a + bx \tag{39}$$

where $X = 0 \Rightarrow a = 0$

$x > 0 \Rightarrow Y = a + bx$

when they are extended in only one direction. Following Dantzig (26, p. 545) we can transform 39 as follows:

$$Y = a\delta + bx \tag{40}$$

$$x \leq U\delta$$

$$\delta = 0 \text{ or } 1.0$$

inequation $X \leq U$ linked with the condition $\delta = 0$ or 1 guarantee that $Y = 0$) when $X = 0$ (U = upper limit of X).

B. Mutually Exclusive Activities or Set of Activities

It can be required, for example, that a building space could be transformed into adequate facilities for either one of the following livestock activities: dairy, hogs, beef, heifer or yearling bull.

Another practical problem is also encountered. Farmers expect to ask about the type of dairy breed they should raise, but they reject, in some cases, the divisibility and convexity assumptions of linear programming. Their problem could be stated as follows: I expect to raise zero cows, or more than 10, but they should belong to the same breed and the total number of head cannot exceed 50.

Mathematically, these two problems can be expressed by the following set of equations.

Problem 1:

$0 \leq x_1 \leq a$ or $0 \leq x_2 \leq b$ or $0 \leq x_3 \leq c$ or $0 \leq x_4 \leq d$

Problem 2:

Without last of generality assume we have two breeds, then

$$\begin{matrix} X_1 \leq 0 \\ X_2 \leq 0 \end{matrix} \quad \text{or} \quad \begin{matrix} X_1 \geq 10 \\ X_2 \leq 0 \end{matrix} \quad \text{or} \quad \begin{matrix} X_2 \geq 10 \\ X_1 \leq 0 \end{matrix} \qquad (41)$$

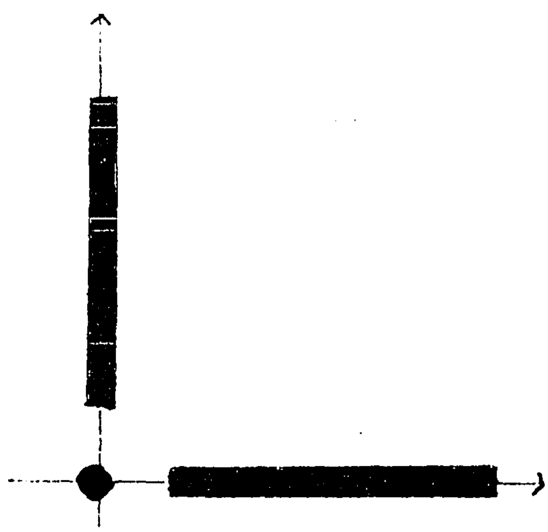


Figure 6. Domain of acceptable solutions

On Figure 6, the double lines depict the domain of feasible solutions. As for the fixed charge problem we can transform the set of equations 42 and 43 into the equivalent equations, using 0,1 variables.

Problem 1:

$$\begin{aligned} X_1 - a(1 - \delta_1) &\leq 0 \\ X_2 - b(1 - \delta_2) &\leq 0 \\ X_3 - c(1 - \delta_3) &\leq 0 \\ X_4 - d(1 - \delta_4) &\leq 0 \end{aligned} \qquad (42)$$

$$\sum_{j=1}^{4} \delta_j = 3.0$$

$$0 \leq \delta_j \leq 1.0$$

$$\delta_j = \text{integer}$$

Problem 2:

$$-X_1 + 50(1 - \delta_1) \geq 0 \tag{43}$$

$$-X_2 + 50(1 - \delta_1) \geq 0$$

$$X_1 - 10 + 10(1 - \delta_2) \geq 0$$

$$-X_2 + 50\ (1 - \delta_2) \geq 0$$

$$X_2 - 10 + 10(1 - \delta_3) \geq 0$$

$$-X_1 + 50(1 - \delta_3) \geq 0$$

$$\sum_{j=1}^{3} \delta_j = 1.0$$

$$0 \leq \delta_j \leq 1.0$$

$$\delta_j = \text{integer}$$

Problem 1: A particular case

If we want to choose n from N variables when all of them take specific values, then it is sufficient to write the following constraints:

$$\sum_{j}^{N} a_{ij}\delta_j = n \tag{44}$$

$$\delta_j = 0 \text{ or } 1.0$$

where

$$a_{ij} = 1.0$$

This procedure is used when we have to make a choice among N equipment for example.

D. Conclusion

Providing we can find an efficient mixed integer code we will be using the preceding means to solve some of our sub-problems. Doing so, we will avoid the computation burden involved in testing 2^k possibilities arising from the fact that k variables can take only one of two specific values (X_j = 0 or 1.0). When we explore the solutions allowed by different subsets of investment, in livestock facilities for example, we will give up the mixed integer algorithm and use the classical one, after having set up adequate bounds on the corresponding subset of variables.

PART III. SETTING UP THE LINEAR PROGRAMMING MODEL

CHAPTER 8. GENERAL ASSUMPTIONS AND MATRIX COEFFICIENTS ORIGIN

After having discussed, on theoretical grounds, the setting up of the programming model we will now specify our general assumptions and the origin of the matrix coefficients.

A. General Assumptions

Beside the four assumptions which underlie any linear programming model (linear objective function, additivity, proportionality, non-negativity) we have made a few other ones, related to particular input-output relationships.

(1) The programming model has been set up for the "Bocage Angevin" area whose soils are deep (tillable soil $\geq$ 0.20 meter) and well drained. Are excluded, from this definition, shallow soils as well as those whose bedrock is formed of sand and gravel.

(2) The chosen input-output relationships to be obtained with regularity from year to year are such that they require a good level of technical management from the farmer. However, when the techniques of production are more difficult to master or when a production has been recently brought into the region, several levels of management are defined (milk yield per cow, fodder corn).

(3) Crop and livestock enterprises in the programming model are those common to the area, plus a few other ones. They have been included because they truly belong to the production possibility set of this region (yearling bull for example).

(4) The economies of scale in labor and machinery input coefficients are ruled out. In order to satisfy this hypothesis it has been assumed

that small farms as well as greater ones are able to hire or own heavy machinery and that the corresponding costs are equal in all cases. This assumption is valid when there exists a certain competition between field work contractors and co-operative societies. It has also been assumed that mutual-aid was always possible between small farmers. So, they are able to be as efficient as any other farmer who has the possibility of organizing the most desirable team of workers to achieve any particular job efficiently.

Besides this set of general hypothesis, the particular ones which are specific to each chapter of results are stated in Part IV.

B. Origin of Matrix Coefficients

Input-output coefficients, prices, initial stock of resources were obtained from many sources and particularly from the different specialists of the extension service and other agencies of this region. The set of data, which was gathered for the purpose of this study and some other ones, are published (18). Consequently we won't give a full report of it here. Table 10 refers to this publication and indicates the corresponding chapters in which the coefficients of each submatrix are found. However, in the following pages we will:

- indicate the further assumptions which are made (e.g. annual price distribution and relative level of prices or yield)
- justify the reasons of our choice when specific constraints can be set up in several ways (e.g. livestock feeding programs)
- define more particularly few linear programming constraints (e.g. crop rotations)

- present a few research results which were obtained. They were necessary either for setting up a more adequate set of activities (e.g. livestock rations) or for transforming the original data into a more suitable form (e.g. livestock facilities). Our research on grass yield was made to provide the necessary forage input-output coefficients which were badly lacking.

In the following pages, information will be given on the following subject matter:

- crop rotations
- livestock rations
- seasonal variations of grass output
- working and investment capital
- livestock facilities
- prices
- crop and livestock yields
- labor constraints

Table 10. Origin of the matrix coefficients

Constraints code		List of activities: Crops (forage, cereals)	Grass-land manage-ment	Live-stock	Buying	Selling	Transfer and miscel-laneous	Right-hand side
00	Objective function	Chp. 5	Chap. 5	Chp. 11 Chp. 13	Chp. 5 Chp. 11-15, 14	Chp. 5 Chp. 11		
29/34	Tractor hour requirements	Chp. 4	Chp. 4	Chp. 10				
01/15	Land and crop rotation	Chp. 3						
	Accounting constraints on:							
16/17	Grass seeding	Chp. 6	Chp. 6					
18/24	Fodder	Chp. 6	Chp. 6	Chp. 9	Chp. 5	Chp. 5		
25/28	Cereals and seeds	Chp. 5		Chp. 9		Chp. 5		
56/82	Animals and livestock products			Chp. 11	Chp. 11	Chp. 11		
35/55	Labor - crops - livestock	Chp. 4	Chp. 4	Chp. 10				Chp. 4
110,83/87	Capital - working - investment	Chp. 7	Chp. 7	Chp. 12	Chp. 5, 11, 14, 15	Chp. 5, 11		
88/93	Buildings			Chp. 10	Chp. 14			Chp. 14
94/98,111	Initial fixed costs				Chp. 14, 15			

CHAPTER 9. CROP ROTATION CONSTRAINTS

In the "Bocage Angevin" region, 14 crops can be grown. They are:

Subset 1

1. Corn
2. Winter wheat
3. Spring barley
4. Winter oats
5. Spring oats
6. Rape
7. Seed production of tall fescue
8. Seed production of Italian Rye-Grass
9. Temporary pasture

Subset 2

10. Winter barley
11. Fodder corn
12. Fodder beet
13. Potato
14. Intercrop fodder kale (after winter barley or Rye-Grass pasture)

Besides this set of crops, few other ones are excluded. They are dominated by other crops, their yield and/or their price per kilogram being too low. We can enumerate: seed production of red-clover, alfalfa, timothy, meadow fescue and cocks-foot, hemp.

A. Setting up Crop Rotation Constraints

From present production plans adopted by farmers and linear programming results (41, annex 1; 34, pp. 45 and 64; 5, pp. 49 and 66) we can observe that crops 10 to 14 are always chosen at a very low level relative to farm total acreage. Furthermore, to avoid crop rotations requiring a large number of small tracts of land, farmers have a root crop rotation more or less disconnected from the main one. They isolate these fodder crops on a tract of land, not far from the farm buildings, when possible. We followed this practice when setting up our model and defined a pre-established root-crop rotation. It should have the following characteristics:

- livestock activities shouldn't be limited, in any case, by the corresponding crop rotation
- the defined fodder crops should be chosen at will, in any proportion relative to one another.

Consequently, we inserted the following rotation in our model:

First and second year: (and/or)

Intercrop fodder kale (after Italian Rye-Grass)

Winter barley + intercrop fodder kale

Fodder kale

Fodder beet

Fodder corn

Potato

Third year: (and/or)

Winter wheat

Spring barley

Spring oats

Fourth year:

Italian Rye-Grass

Root crops and/or potato can only be followed by three cereals if we want to avoid a continuous rotation of root crops. Finally, Italian Rye-Grass was included to remake initial soil structure which could be damaged by harvesting roots with heavy machinery. Intercrop fodder kale was associated with winter barley, since they are two supplementary enterprises.

The main crop rotation includes at most 10 crops. Winter wheat was differentiated into two distinct activities according to its gross margin level which vary from the first subset of preceding crops to the second one (additivity assumption). Winter barley, being dominated by winter oats (18, chp. 5), was excluded. Table 11 summarizes the corresponding crop constraints. However, they had to be transformed into activity rotation constraints. To decrease the number of complex ones, as defined in Chapter 3, we aggregated the following crops:

- (spring barley + tall fescue) and (rape or corn)
- (Italian Rye-Grass) and (rape or corn)

Furthermore, due to labor time constraints, we did it also for:

- Corn + winter wheat type 1
- Temporary pastures and their two alternative nurse crops: spring barley or oats. Temporary grassland can be sown either in springtime with a companion crop or in September as a main crop. When several crops require the same set of preceding ones the

Table 11. Crop rotation constraints (matrix associated with the graph: $\Gamma = (G, x, x)$

Crop code	Following crops	Preceding crops: Corn	Rape	Winter oats	Winter wheat type 1	Winter wheat type 2	Spring oats	Spring barley	Tall fescue (for seed)	Italian Rye-Grass (for seed)	Temporary pasture
0	Corn	1	1	1	1	1	1	1	1	1	1
1	Rape			1	1	1	1	1	1	1	1
2	Winter oats				1	1		1			1
3	Winter wheat type 1	1	1								1
4	Winter wheat type 2			1	1		1				
5	Spring oats	1	1	1	1	1		1			1
6	Spring barley	1	1	1	1	1	1				1
	Seed production -										
7	Tall fescue							1			
8	Italian Rye-Grass		1	1	1	1	1	1			
9	Temporary pasture		1	1	1	1	1	1			

Table 12. The simple constraints of rotation of the crop activities

Simple constraint code	Following activities[a]	Preceding activities[b] Corn (1)	Rape (2)	Winter oats (3)	Winter wheat type 1 (4)	Winter wheat type 2 (5)	Spring oats (6)	Spring barley (7)
A	3				1	1		1
B	5			1	1		1	
C	2			1	1	1	1	1
D	4		1					
E	9+11+12		1	1	1	1	1	1
F	5+14	1	1	1	1	1		1
G	7+8+10+13	1	1	1	1	1	1	
H	1+15	1	1	1	1	1	1	1

[a]Activities are numbered rather than named.

[b]Numbers in parentheses are activity code numbers.

Table 12 (Continued)

	Preceding activities[b]							
						Temporary pasture + its nurse crop		
Simple constraint code	Spring barley + tall fescue + rape (8)	Italian Rye-Grass + rape (9)	Spring barley + tall fescue + corn (10)	Italian Rye-Grass + corn (11)	Temporary pasture (12)	Spring barley (13)	Spring oats (14)	Corn + winter wheat type 1 (15)
A					1	1	1	1
B								1
C					1	1	1	1
D	1	1			1	1	1	
E	1	1						1
F	1	1	1	1	1	1	1	1
G	1	1	1	1	1	1	1	1
H	1	1	1	1	1	1	1	1

corresponding constraints were also aggregated, according to rule 2, page 21 and Theorem 4, page 28 (Chapter 3). To the first seven simple constraints A to G in Table 12 we added a subset of complex ones (Theorem 4 and 5, Chapter 3, pages 28 and 29). Constraint H is redundant since, after simplifying it, we get: $-\sum_j X_j \leq 0$. The subset of complex constraints was found from the following union of simple ones, as shown in Table 13, all others being either dominated or redundant.

Table 13. Valid complex constraints

Complex constraint code	Origin of the complex constraint
1	A U B U C
2	A U D
3	A U D U F
4	B U D U G
5	A U B U C U D U E
6	B U E

Moreover, some complex constraints dominate simple ones. They are:

1 and C

3 and F

4 and G

6 and E

The subset of valid constraints contains nine rows. They are shown in matrix format in Table 14.

To eliminate unacceptable crop rotations we added to the preceding constraints, two supplementary ones:

Table 14. Necessary and sufficient crop rotation constraints

Constraint code	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Right-hand side
A			1	-1	-1		-1					-1	-1	-1	-1	≤ 0
B			-1	-1	1	-1									-1	≤ 0
D		-1		1				-1	-1			-1	-1	-1		≤ 0
1		1		-1		-1	-1					-1	-1	-1	-1	≤ 0
2		-1	1		-1		-1	-1	-1			-1	-1	-1	-1	≤ 0
3	-1	-1			-1	1	-1	-1	-1	-1	-1	-1	-1		-1	≤ 0
4	-1	-1	-1			-1	1		-1		-1	-1		-1	-1	≤ 0
5						-1	-1	-1			1		-1	-1	-1	≤ 0
6		-1	-1	-1		-1	-1	-1			1	1			-1	≤ 0

$$\sum_j X_j \leq .66S$$

where j = any cereal activity

$$\sum_j X_j \leq .33(S - \sum_k a_{ik} X_k)$$

where j = rape activity

S = total land resource

k = activity of more than three years' duration

$a_{ik} > 0$

a_{ik} = (activity k duration in years - three)

They fix a limit on the maximum quantity of rape and cereals which can be grown on a farm (57, p. 531 and 35, p. 81). In fact we did not write strict frequency constraints but "average" ones. As expected these constraints are not frequently efficient. This procedure simplified our model.

B. Enumerating Elementary Crop Rotations

Using the "matrix method" we enumerated all possible circuits. After the elimination of

- any rotation which contains too many cereals
- any rotation which doesn't satisfy to rape frequency constraint

we ended with 144 elementary crop rotations. The submatrix shown in Table 14 is obviously more compact than the one which would be associated with those aggregate crop rotations. They are enumerated in Table 15 which can be read as follows: to form the set of crops, which can be linked by, at least, one circuit, combine for each line of the table, the subset s_1 with one element of the subset s_2 when

Table 15. Enumeration of elementary crop rotations

Subset of cereals and plants grown for seed[a] (s_1)	Subset of rotation leading crop(s)[b] (s_2)				
	0	1	9	01	19
0	x				
2			x		x
3	x		x		x
5	x		x		x
6	x		x		x
19					x
23			x	x	x
26			x	x	x
42			x		x
43			x	x	x
45			x	x	x
52			x		x
53			x	x	x
63			x	x	x
65			x		x
76	x	x		x	x
82					x
83	x			x	x
85	x			x	x
86	x			x	x
423			x		x
453			x		x
523			x		x
542			x		x
623			x		x
642			x		x
643			x		x
645			x		x
652			x		x
653			x		x
763	x	x			
765	x	x			
823	x	x			x
826	x	x			x
843	x	x			x
845	x	x			x

[a]The crop code is given in Table 11.

[b]x indicates that the crop is included in the rotation.

Table 15 (Continued)

Subset of cereals and plants grown for seed[a] (s_1)	Subset of rotation leading crop(s)[b] (s_2) 0	1	9	01	19
853	x	x			x
863	x	x			x
865	x	x			x
876				x	
825					x
824					x
5 423			x		
6 423			x		
6 523			x		
6 542			x		
6 543			x		
7 623	x	x			
7 643	x	x			
7 645	x	x			
7 653	x	x			
8 245	x	x			
8 265	x	x			
8 423	x	x			
8 426	x	x			x
8 523	x	x			
8 543	x	x			
8 623	x	x			
8 643	x	x			
8 645	x	x			
8 653	x	x			
64 523					x
76 423	x	x			
76 523	x	x			
76 543	x	x			
76 245	x	x			
765 243	x	x			

allowed. For example in line two, we have two elementary crop rotations: 29 and 219 or (winter oats + temporary pasture) and (winter oats + rape + temporary pasture). When several circuits connect the same set of crops they are viewed as equivalent since their economic contributions are identical, due to the additivity assumption.

C. Conclusion

We inserted into our model the root-crop rotation previously defined and the set of crop rotation constraints shown in Table 14. We think that this last method requires less desk work than the other one.

CHAPTER 10. LIVESTOCK RATIONS

In Chapter 4 we have studied three alternatives of writing within a linear programming model, all possible rations which could be fed to livestock. Of these three alternatives we chose the last one, which includes, within the model, the only dominating extreme aggregate activities. We decided to choose this last procedure in order to

- use available and valuable information
- decrease our model size, especially in view of the large number of selected livestock activities.

A. Available Information on Dominated Fodder Inputs

A careful study of Jullian and Tirel's results (44, pp. 100-145), which were obtained from a model combining feed-mix and profit maximization problems, shows that:

- hay is introduced into rations for milk cows at its minimum level
- concentrated feeds are never substituted for bulky ones. They are introduced into rations to adjust them to the minimum required amount of nutrient elements.

1. Minimum weight of hay per animal

In Jullian's model, when annual labor is equal to 1.5 units, hay is always at its minimum level (nine subperiods). When labor is limited to 1.0 unit, then hay is minimum in six out of nine subperiods.

But, for these three exceptions, the additional amount of hay, above the minimum level, is equal respectively to 38 and 33% of the total weight allowed above minimum. Heifers and steers get hay and/or straw above minimum level in nine periods out of 19 with 1.5 units of labor and in four periods out of 19 in the second model (1.0 unit of labor). These last results can easily be explained since the production of fodder associated with hay is limited by labor constraints.

Hovelaque's results (41, pp. 116-117 and Appendix A) confirm the preceding ones. Hay has a very high marginal value and is, therefore, substituted by other fodders. Since he was working with extreme aggregate rations the chosen ones are those which require the minimum weight of hay per head.

2. Highest basic rations

From Jullian's results (44, pp. 79-80 and 104) we can draw the following table.

Table 16. Percent of nutritive elements brought by concentrate into total dairy cow rations

Hypothesis	Nutrient requirements Fodder units (U.F.)	Digestible protein
1.5 labor unit	1.84%	5.95%
1.0 labor unit	0.48%	0.26%
Total requirement	3,082 U.F.	336 Kg

We note that concentrate feeds are reduced to their minimum levels. They are not substituted for bulky fodders, they are complementary to them. Otherwise, they would be removed from daily rations.

B. Extrapolation of these Results to the Region we are Working for

The preceding results were found out for two regions: "Le Bassin de Rennes" and "le Pays d'Ouche" respectively 50 and 100 miles apart from the "Bocage Angevin" region. Although small differences in soil fertility and climate exist between these regions we accepted these results and assumed that their extrapolation to the "Bocage Angevin" was valid. We will come back to this problem later on. In order to try extending these results and specifying a few basic rules which would allow us to classify extreme aggregate rations into two main groups:

- the dominated subset
- the dominating subset

we built a linear model which emphasizes the feeding program problem (7). This model (125 x 370) was set up for a 25 hectares farm. Eight livestock activities were selected:

- two dairy herds differentiated by their most frequent calving dates, February and October
- two steer activities for each calving date
- one heifer activity for each calving date.

Furthermore, the year was divided into eight feeding subperiods according to fodder availability and animal nutrient requirements. On the whole, 296 pre-established extreme rations were included. They were chosen according to the preceding rules:

- minimum weight of hay
- minimum amount of concentrate feeds.

Hay was associated to one or two of the following fodders:

- fodder beet
- fodder kale
- corn silage
- grass silage

Four types of hay were differentiated according to their contents of digestible proteins and energy. They are:

- first cut alfalfa (second choice)
- second and third cut alfalfa (first choice)
- rye-grass
- mixed grass

1. Results

Table 17 shows, for each 20 cow-herd and subperiod, the different rations shadow prices. Each one is an average of four results corresponding to the four types of hay associated with the same quantity of other fodders. An increasing cost order is set up for the first herd.

We observe that, for October calving, the chosen rations are mainly those which consist of three fodders. Grass silage + hay rations are never chosen for both herds. They have the highest average shadow prices in every case.

For February calving, chosen ration rank order differs from the October calving one, especially for subperiods 1 and 2.

The February calving herd being dried off at these periods, its minimum requirements are very low. It was assumed that farmers would

Table 17. Shadow prices of various ration types

Subperiod	Fodder with hay	Kg of hay	Oct. calving[a] $\overline{c}$	Oct. calving[a] s_c	Feb. calving[a] $\overline{c}$	Feb. calving[a] s_c
I (5/11-1/12)	Kale - beet	5	94.75	122	112.00	25.8
	Kale	9	357.75	86.7	39.75	46.7
	Beet	10-11	360.00	175	147.75	84.2
	Grass silage	5	831.00	78.9	426.25	26.0
II (1/12-1/01)	Kale + corn silage	4	39.25	39.3	163.00	25.8
	Beet + corn silage	4	112.75	102	170.75	25.5
	Kale + beet	5	126.25	14.3	291.50	146
	Corn silage	4	190.00	95.4	21.75	25.5
	Kale	9	373.25	71.2	226.25	57.7
	Beet	11	378.00	258	360.75	104
	Grass silage	5	987.75	174.5	706.25	267
III (1/01-10/03)	Beet + corn silage	4	221.75	242	147.75	126
	Corn silage	4	230.25	194	124.50	133
	Beet	11	843.00	445	574.50	352
	Grass silage	5	2,030.00	189	1,849.50	104

[a]$\overline{c}$ is the mean of the shadow prices; s_c is the standard error

not feed several different basic rations each day, even if their herds are partly dried off and partly in full lactation.

Consequently, since basic rations were set up such that they contain the highest possible level of energy, cows are more or less overfed with different alternative rations. This bias introduces a disturbance factor into the comparison and explains some differences we will see below.

To conclude we can say that:

(1) Grass silage + hay rations are always dominated.

(2) Other rations are ranked differently from one period to another especially when the herd is dried off.

(3) Rations for the October calving herd can be ranked as follows:

three fodder basic rations (including hay)

two fodder basic rations.

Among them, hay + fodder beet rations are those which have the most fluctuating shadow prices around the mean.

2. Setting up a choice rule

The model results, as many others (41, 44) show that land shadow price is very high. Furthermore, the yield of different fodders vary widely, in certain cases from 1.0 to 2.5. It is natural to think that land requirements associated with alternative subperiod herd rations, could explain a fraction of the ration shadow price variances. On the other hand, varying weights of concentrated feeds are added to different basic rations. The corresponding cost is, in some cases, quite high. It is hypothesized that these two variables are the most significant ones. Let's define:

X_2 = Land requirement for producing subperiod 20-cow herd rations (hectares)

X_1 = Ration concentrated feed cost (francs)

C = Shadow price

To verify this assumption we ran the following regression:

$$C = a + b_1X_1 + b_2X_2. \qquad (45)$$

To be sure that this relation was adequate we also calculated the following ones:

$$C = a + b_1X_1 + b_2X_2 + b_3X_1X_2 \qquad (46)$$

$$C = a + b_1X_1 + b_2X_2 + b_xX_2^2 + b_4X_1X_2 \qquad (47)$$

$$C = a + b_1X_1 + b_2X_2 + b_3X_2^{\frac{1}{2}} + b_4(X_1X_2)^{\frac{1}{2}} \qquad (48)$$

In each case the coefficient of determination R^2 is high, except for period 1, February calving as shown in Table 18.

Table 18. Range of coefficients of determination for equations 45 to 48

Subperiods	October calving	February calving
1	$.792 \pm .037$	$.0696 \pm .0072$
2	$.828 \pm .009$	$.515 \pm .005$
3	$.948 \pm .006$	$.898 \pm .013$

Because non-linear equations and interactions between X_1 and X_2 do not explain a significant additional amount of total variance we chose equation 45. The corresponding results are shown in Table 19.

Table 19. Value of R^2 and regression coefficients b_i of equation 45 for various subperiod cow rations

Subperiod cow-ration	Equation number	b_0	b_1	b_2	R^2
October calving					
Subperiod 1	49	-424.66	1.284	2.681	.753
Subperiod 2	50	-1369.85	1.231	16.715	.819
Subperiod 3	51	-3226.37	.845	21.506	.941
February calving					
Subperiod 1	52	-654.85	0	12.5	.068
Subperiod 2	53	-1092.78	0	17.75	.511
Subperiod 3	54	-4178.88	.978	22.785	.883

Table 19 shows that equations 49 to 54 present a good fit of shadow prices except for equation 52. In fact, for this period cows are overfed with basic rations and the rate of overfeeding is higher with high yield fodder than with others. This disturbance factor narrows the range of X_2 variation to a great extent and explains why the corresponding equation is not significant. In the subsequent periods, the same disturbance factor is still present but rations with corn silage are allowed, they require less surface than others and cows are less overfed. The coefficient of determination of equation 53 is much higher, although not very good.

(a) The choice criteria:

Since in every case,

$$\frac{\partial C}{\partial X_i} > 0, \ i = 1,2. \tag{55}$$

Then we can minimize C by decreasing both X_1 and X_2 values. In other words we should choose pre-established extreme rations which satisfy one of the following criteria or both:

- the required amount of land to produce them, is as low as possible
- basic rations should be such that they minimize the needed quantity of concentrated feeds to be added.

The marginal rate of substitution of land for concentrated feeds for each feeding subperiod (October calving herd) is equal to:

$$\frac{dx_1}{dx_2} = -2.681/1.284 = -2.08 \quad (56)$$

$$" = -16.715/1.231 = -13.57 \quad (57)$$

$$" = -21.506/.845 = -25.45 \quad (58)$$

We can, therefore, substitute 1350 F of concentrates to 1 hectare of land in period 2 and 2545 F in period 3.

Figure 7 and Figure 8 show that rations are composed of one or two of the following fodders, besides hay: kale, corn silage, fodder beet, are those from which we can expect to get the lowest shadow prices. How can we explain why the result is thus, as well as, the value taken by equation 55? Table 20 gives us the answer.

Basic rations requiring the lowest quantity of concentrated feed are those which allow the largest intake of highly nutritive dry matter when animal needs are big. Columns A and B of Table 20 show that they are composed of a minimum quantity of hay associated with the fodders mentioned above. They are, also, those which supply the highest quantity of fodder units per hectare. Furthermore, in every case, variable costs

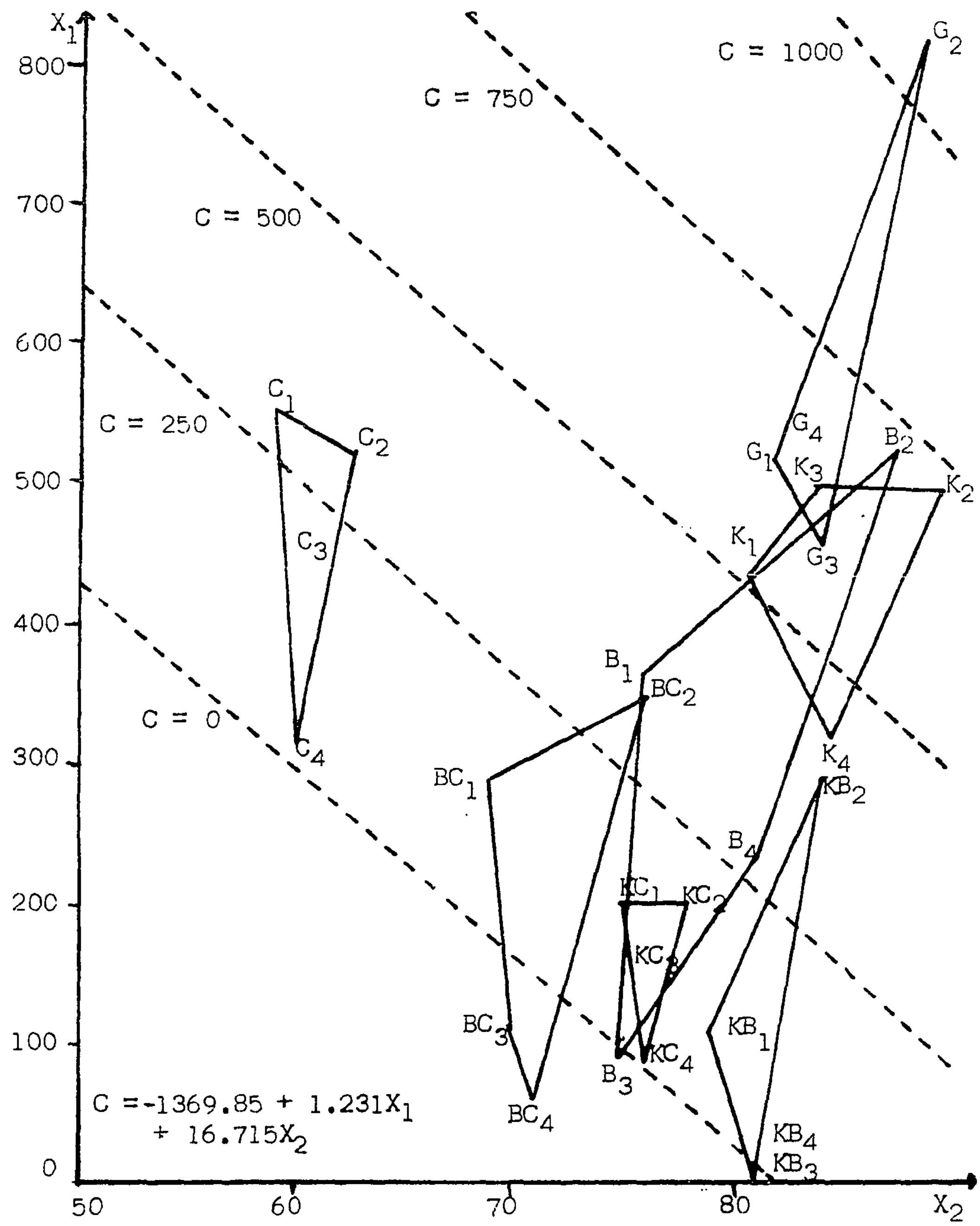


Figure 7. Iso-shadow prices; land and concentrated feed required by October calving alternative rations in Period 2

Symbols for observed data	1:	Mixed hay	G:	Grass silage
	2:	Rye-grass hay	C:	Corn silage
	3:	1st choice alfalfa hay	B:	Fodder beet
	4:	2nd choice alfalfa hay	K:	Kale

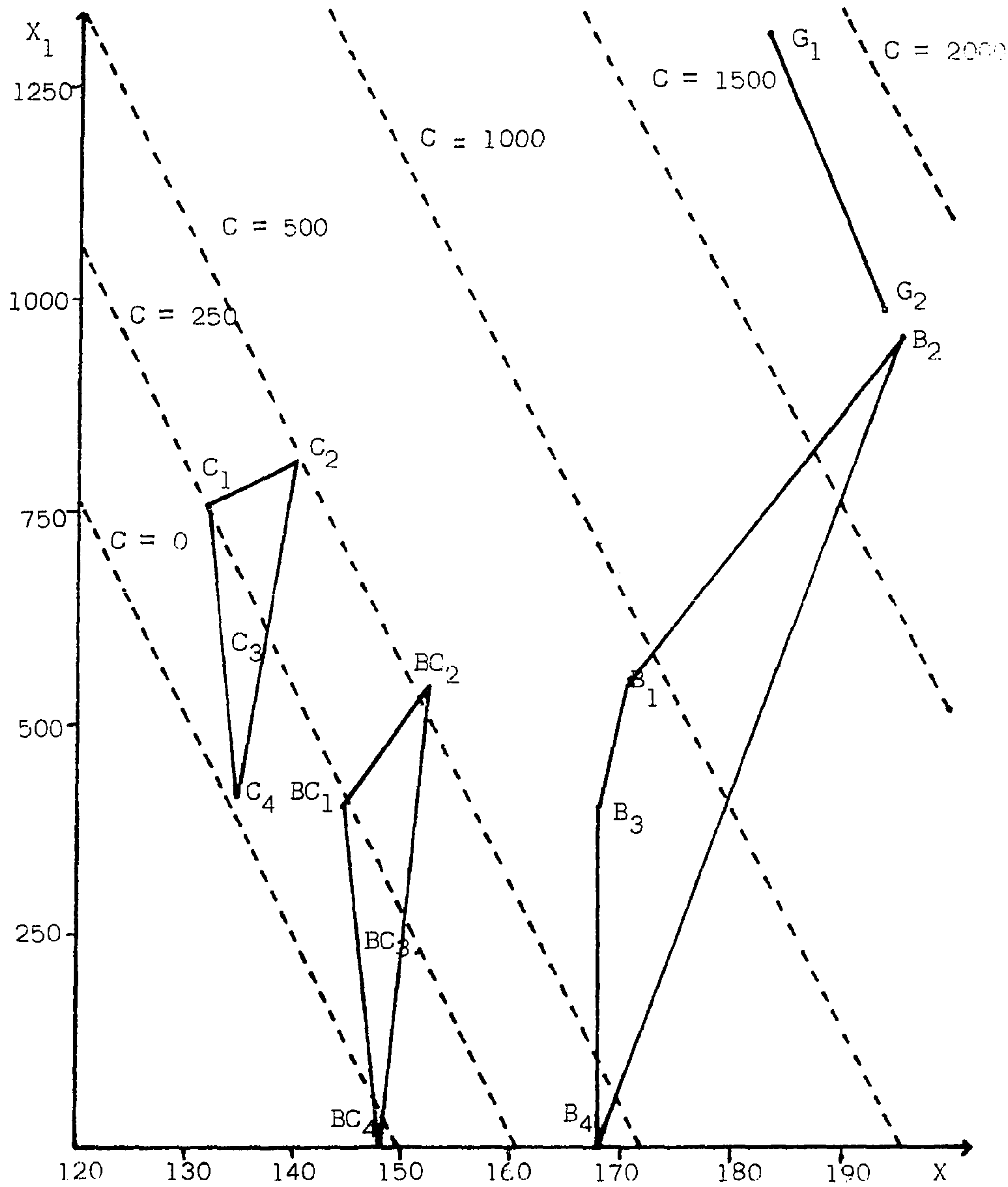


Figure 8. Iso-shadow prices, land and concentrated feed required by October calving. Alternative basic rations in Period 3

Table 20. Characteristics of fodder and related variable costs

Fodder type	Order of animal appetency for fodder A	Fodder unit per Kg of dried matter[b] B	Fodder unit yield (thousand ha)[c] C	Variable cost[a] per fodder unit (f)[c] D
Fodder beet	1	1.0	10.0-13.0	.095-.073
Fodder beet	1	1.0	9.0-11.0	.037-.032
Corn silage	2	.75-.80	6.4-8.0	.175-.135
Kale	3	.85	6.0	.67
Mixed grass silage	4	.50-.55	4.2	.198
Hay	5	.45-.50	4.2	.095

[a]Including harvest custom work.

[b]Ergan, M., Maison de l'Agriculture, Laval, 53. Fodder nutrient content. Private communication. 1968.

[c]Source 18, chapter 5.

per fodder unit, including or excluding harvest custom work expenditures, are also lower for these fodders. All economic forces converge toward the same fodders and favor them.

(b) Validity test of this choice rule:

It may be asked if this choice rule is valid in all cases. We did not test it to enumerate its conditions of validity. However, the study of two particular cases (34 and 5) which were undertaken in this region, independently from our work, came as a confirmation of our rule. The results of Berson's study show that grass silage was never chosen; rations were composed of kale, fodder beet, corn silage and hay (5, pp. 51 and 67). Gaultier remarks that "hay is always inserted (into rations) at its minimum level when kale and fodder beet are introduced at their maximum" (34, p. 42).

C. Extreme Aggregate Rations Inserted into our Model

Figures 7 and 8, beyond illustrating our choice rule, show that different types of hay are a factor of variation of ration shadow prices.

In general, for any ration type (characterized by fodder(s) associated with hay), rye-grass and mixed grass hays are dominated by alfalfa hay which contribute to decrease the corresponding ration shadow prices. This is due particularly to the high digestible protein content and the high yield level of alfalfa; two factors which allow to reduce, for any rations, the corresponding concentrated feed cost and land requirements. Consequently, we eliminated grass hay from our final model. Only average choice alfalfa hay was included.

Although we defined the dominating livestock rations, we could choose only one of them to be inserted in our model. Other constraints, such as labor, could limit farmers' income. As harvesting dates of each dominating fodder is different from one another, we retained all of them to keep the maximum of flexibility within our model. Therefore, our main basic rations are the following:

- alfalfa hay + fodder beet
- alfalfa hay + corn silage
- alfalfa hay + kale
- alfalfa hay + fodder beet + corn silage

- alfalfa hay + fodder beet + kale
- alfalfa hay + corn silage + kale

Rations composed of three fodders can only be fed to animals with high level nutrient requirements such as dairy cows or feeder steers. They allow, generally, to decrease the required minimum of hay and added concentrated feeds. The rest of the year, livestock is fed on grass with a complement of silage, if necessary, during late summer.

After we had started our computation work, livestock experimental results were published (54). They state that yearling-bulls can be raised with a lower quantity of hay than it was generally admitted previously. Corresponding yearling-bull activities were included in subsequent computations and hay minimum requirements reduced from 1200 Kg to 300 Kg. Our first results show that this substitution is surely profitable.

D. Conclusion

The determination of our choice rule for selecting extreme aggregate basic rations, besides its practical interest especially for this region's extension service, allowed us to narrow our model to a great extent. We estimate that the number of extreme aggregate rations was divided by, at least, 10.

CHAPTER 11. TOTAL ANNUAL PASTURE OUTPUT AND ITS SEASONAL VARIATION

In this region, climatic conditions necessitate full hand feeding in fall and winter but livestock obtain the bulk of their feed by grazing in other periods. Since it has never been considered, up to now, to suppress green fodder from livestock spring and summer feeding programs and since stocking rates are unknown for each separate livestock activity, we can't avoid taking into account pasture outputs. Even average herd stocking rates are little known and their variance is large. Furthermore, on each farm, different types of pasture and other grazing crops are fed to a set of different animals producing few products such as heifers, feeder calves, steers, milk and so on. Under these conditions, it is difficult to use these stocking rates since our objective includes the research of the most rewarding livestock activities. We cannot, therefore, avoid estimating total pasture output per year as well as its breakdown by subperiods, grazing output being not constant over time. These estimations are usually made by two different methods.

Experimental design results: In any design, split-plot or others, forage is cut with a motor mower at different stages of growth and at different intervals of time according to the "type" of forage utilization which is being referred to. In most cases, yield is estimated in terms of weight of dry matter per hectare. Researchers need a precise measure of yield since they are studying the influence of factors such as variety, fertilizers, and dates of cutting. Many studies of this type have been published (53, 62, 42). Even though feeding value of herbage from temporary pastures at the grazing stage is starting to be well known (27) we hesitated to use these data, mainly for two reasons:

- To transform this crude forage yield into effective fodder intake we need to know the transformation coefficients which would take into account the unavoidable wasting of forage by livestock and yield differences between small and large plots. But these coefficients are little known and difficult to determine.
- Many experiments are often located far from the region we are studying. Even the nearest ones were made under slightly different climate and soil conditions than ours. Response surfaces should integrate soil and climate indexes in order to extrapolate these costly findings. To our knowledge, no such production functions have been published. They would be very useful since we have problems when we want to use this set of data apart from experimental station soils and climate conditions.

Forage evaluation through livestock: Since we need to evaluate animal products which can be obtained per hectare from a set of forage crops and since crude yield results are insufficient to define the number of animals which can be fed by surface unit, it is natural to estimate pasture grazing through the amount of nutrients required for producing a given observed livestock output. This method has been described by Faike and Geith and reported by Kohnlein (46, p. 13), Jarrige and Journet (43, p. 698). In spite of its inaccuracy, this method has been used extensively due to its inexpensiveness. This method has even been simplified: effective forage intake being estimated with the three following variables:

(1) number of grazing days per hectare

(2) theoretical feeding value of herbage at various grazing stage

(3) theoretical intake of herbage by different animals when fed *ad libitum*.

More than others, the last variable might be a source of errors in measurement. The *ad libitum* feeding assumption is not always valid in real world situations, especially during summertime. In spite of the lack of precision in this last simplified method, extension people have been estimating temporary and natural pasture yields for three years (31) when we started our study in 1966. We decided to use these data since they had the advantage of having been elaborated in the region we were studying. This original set of data was collected but no one, up to this date, has interpreted it. Our needs being particular and deviating from these people's objectives we undertook this work although we knew these data were not very accurate. But in any case we had not much choice.

A. Forage Crops Output

1. Total estimated grazing intake per year and consumable yield after conservation of other fodder crops

Total forage crop output is seldom transformed into hay. Generally part of it is grazed, the rest is cut and dried.

To take into account this fact and add grazing to hay yield, the latter was estimated as the number of fodder units which would be necessary to produce, without any loss of dry matter, the corresponding

Table 21. Yield of different forage crops per year

Fodder type		Number of obser- vations	Average yield (fodder units)[a]	Standard error s	Data reference
Alfalfa	M[b]	28	5,600*	1,627	
Rye-grass	M	20	4,980*	1,346	
Rye-grass	P[b]	6	4,875*	1,235	
Mixed grass	M	23	5,470*	1,440	
Mixed grass	P	28	4,640*	1,350	(18)
Natural pasture	M	14	4,170*	1,400	Chapter 6
Natural pasture	P	31	3,030*	1,290	
Corn		17	6,000	1,250	
Fodder beet					
Transplanted		46	9,700	2,400	
Drilled		25	12,200	3,280	
Kale			6,000		

[a]*Over time.

[b]M indicates meadow; P indicates pasture.

quantity of hay. In other words, extension people estimated that dry matter losses are equivalent in both cases. Table 21 shows that the fodder output is slightly smaller when forage is grazed rather than dried in field. In view of the large variation of yield in temporary pastures or meadows we decided to calculate the influence of such a yield variation on farmers' revenue. According to extension people, yield differences are due mainly to the farmer's range of knowledge. Since the probability of having an important area of pastures into our model solutions was high, we decided to define three levels of pasture management differentiated by input requirements and output levels. They correspond to the required managerial ability for producing, <u>in average</u>:

- 3,000 fodder units per hectare
- 4,000 fodder units per hectare
- 5,000 fodder units per hectare

A 6,000 fodder unit output level has been left out. It was thought to be difficult to obtain, especially in bad years.

2. Total forage output breakdown by subperiods

To evaluate such data, two similar methods are available. The first one consists of computing a forage cumulative production function, the second one, its derivative.

a. Forage cumulative production function Let's define the following variables:

Y_T = cumulative forage output at time T

T = time (decade number)

F = fertilizer inputs

S = soil fertility index

E = type of pasture management

S_p = forage specie

V = variety

The cumulative production function would be:

$$Y_T = f(T, F \mid S, E, S_p, V). \quad (59)$$

But the original set of data excluded to compute such a function for three reasons:

- F was not always known and, even if it had been, we would not be able to take into account initial soil nutrient contents, soil tests being lacking.

- E was not homogeneous. Forage was cut and/or grazed. The number of observations was relatively small in each sub-group.
- Finally even when total forage output was grazed, grazing intervals were not equally spaced. They were varying from two to eight weeks, between and even within different observations. It is well known, however, that grazing intervals are correlated, within a certain range, with total output (63, pp. 13-28; 53, p. 21). This last remark incited us to use the following method.

b. Daily output of forage crops When grazing intervals are defined, forage cumulative production functions are composed of a series of linked segments: each one could be viewed as either:

- the average daily output of forage within the corresponding sub-period
- the "average" marginal output of our cumulative production function within the corresponding subperiod.

Our objective consists of finding the slope of this series of segments. We can express it as follows:

$$Y_t = f(F,\ Y^o,\ TY^o,\ G\ E) \qquad (60)$$

where

Y_T = "average" marginal grazing output, time t

Y^o = total forage crop output (maximum of Y_t in hundreds of fodder units)

G = grazing interval.

However, the first grazing cannot be regressed on equation 59 since G is unknown: the first interval being not defined. Consequently equation 61 was substituted for equation 60.

$$Y_{to} = f(T,\ Y^o,\ TY^o \mid E) \qquad (61)$$

where

Y_{to} = total first grazing output

Forms of grazing production functions - The interaction factor TY^o has been included upon logical considerations. Whatever the total forage yield, forage growth stops by the end of October. For any cumulative production function we get

$$\partial Y_T/\partial T = 0, \text{ for } T = T^*$$

Daily forage growth reaches, through the year, two peaks: the highest one in May, the smallest one in September (74, p. 63). If the former is always observed, the latter can be completely erased in drought years. Our cumulative production function, which is a function of time, has a double S shape. To take into account the non-linear effect of time upon Y_t, non-linear equations were calculated such as cubic, quadratic, square root and linear equations (with a dummy time variable t). They are:

$$Y_t = a + b_1 t + b_2 T + b_3 Y^o + b_4 G \qquad (62)$$

$$Y_t = a + b_1 t + b_2 T + b_3 Y^o + b_4 G + b_5 TY^o \qquad (63)$$

$$Y_t = a + b_2 T + b_3 Y^o + b_4 G \qquad (64)$$

$$Y_t = a + b_2 T + b_3 Y^o + b_4 G + b_5 TY^o \qquad (65)$$

$$Y_t = a + b_2 T + b_3 Y^o + b_4 G + b_5 TY^o + b_6 T^2 \qquad (66)$$

$$Y_t = a + b_2 T + b_3 Y^o + b_4 G + b_5 (TY^o)^{.5} + b_6 T^{.5} \qquad (67)$$

$$Y_t = a + b_2T + b_3Y^o + b_4G + b_5TY^o + b_6T^2 + b_7T^3 \quad (68)$$

$$Y_{to} = a + b_1T + b_2Y^o \quad (69)$$

$$Y_{to} = a + b_1T + b_2Y^o + b_3TY^o \quad (70)$$

$$Y_{to} = a + b_1T + b_2Y^o + b_3TY^o + b_4(Y^o)^2 \quad (71)$$

$$Y_{to} = a + b_1T + b_2Y^o + b_3(TY^o)^{.5} \quad (72)$$

$$Y_{to} = a + b_1T + b_2Y^o + b_3(TY^o)^{.5} + b_4(Y^o)^{.5} \quad (73)$$

B. Computation Results and Choice of Coefficients

Y^o was differentiated into two sub-groups. The first one is composed of results with both hay and grazing output. The second one contains only grazed forage yield. These sub-groups were distinguished to take into account the disturbances which could come up from the variable E.

In view of the above set of multiple correlation coefficients (Table 22), the utility of one or two independent variables may be questioned and their omission proposed. In the first series (equations 62 to 68) these variables are: t, TY^o, T^2, T^3. They condition, if significant, both the cumulative production function shape relative to time, and the date at which this function becomes maximum.

The following F test is made:

$$F_R^{R-K} = \frac{SS_R - SS_{R-K}}{R-K} \Big/ \frac{SS_E}{R}$$

where

SS = sum of squares

R = the set of independent variables

K = the first K independent variables

Table 22. Level of significance (LS) of multiple correlation coefficients (R) for equations 63 to 68 and 69 to 73

	Pasture						Meadow					
Equation number	Temporary pasture R	LS[a]	Natural pasture R	LS[a]	Rye-Grass R	LS[a]	Temporary meadow R	LS[a]	Rye-Grass R	LS[a]	Alfalfa R	LS[a]
21	.763	**	.545	**			.543	*	.745	**	.602	*
22	.776	**	.548	**			.545	*	.778	**	.687	**
23	.763	**	.542	**	.720	**	.543	**	.711	**	.561	*
24	.775	**	.548	**	.798	**	.545	**	.764	**	.655	**
25	.783	**	.557	**	.798	**	.587	*	.320	**	.673	*
26	.794	**	.576	**	.770	**	.592	**	.791	**	.686	**
27	.797	*	.601	**	.800	**	.628	**	.821	**	.673	*
31	.813	**	.688	**								
32	.374	**	.725	**								
33	.878	**	.729	**								
34	.861	**	.689	**								
35	.862	**	.713	**								

[a]For *, LS = 5%; for **, LS = 1%.

R-K = variables to be deleted

E = error.

It takes the following values (Table 23).

According to Table 23, the independent variable (TY^o) makes, in most cases, a significant contribution to regression. As mentioned above, this result stands on theoretical grounds. The variables t, T^2, T^3 can be deleted. It would seem that Y_t is linear relative to time, and therefore that Y_T is a quadratic function. Two reasons can be invoked to justify this finding:

- Equation Y_{to} takes into account the entire increasing phase of equation Y_t. Consequently the first peak mentioned above cannot appear in equation Y_T.
- The output estimation procedure is not precise enough and the second peak is erased: animals playing the role of a buffer.

In the second series of equations, 69 to 73, the interaction factor (TY^o) makes again a significant contribution to regression. However, the independent variables $(Y^o)^2$ and $(Y^o)^{.5}$ can be deleted. As previously, the significance of the interaction factor can be justified on theoretical ground. The first grazing output increases at an increasing rate through time when total grazing output becomes larger.

In view of these results we chose equations 65 and 70 to evaluate respectively Y_t and Y_{to}. These equations are given below.

1. Forage output

a. Grazing output Temporary pasture

$$Y_t = 10.794 - 0.011\, T + 1.016\, Y^o - .219\, G - .347\, TY^o$$

Table 23. F values for different comparisons of equations

Pair of equations being compared	Pasture						Meadow					
	Temporary pasture		Natural pasture		Rye-Grass		Temporary meadow		Rye-Grass		Alfalfa	
	F[a]	R-K/R	F[a]	R-K/R	F[a]	R-K/R	F[a]	R-K/R	F[a]	R-K/R	F[a]	R-K/R
21,22	3.99*	1/80	0.36	1/61			0.11	1/34	2.16	1/17	4.77*	1/23
21,23	0.008	1/81	0.25	1/62			0.006	1/35	1.98	1/18	1.84	1/24
22,24	0.091	1/80	0.07	1/61			0.01	1/34	0.93	1/17	1.90	1/23
23,24	3.99*	1/81	0.54	1/62	11.12**	1/34	0.10	1/35	3.35°	1/18	4.82*	1/24
24,25	2.45	1/80	0.96	1/61	0.07	1/33	2.51	1/34	4.61*	1/17	1.01	1/23
24,27	3.66	2/79	2.84	2/60	0.25	2/32	2.67	2/33	2.24	2/16	0.48	2/22
31,32	16.88**	1/26	4.05**	1/39								
31,33	6.02**	2/25	2.39	2/38								
32,33	0.78	1/25	0.49	1/38								
34,35	0.157	1/25	2.67	1/38								

[a]For °, LS = 10%; for *, LS = 5%; for **, LS = 1%.

$$Y_{to} = 1321.37 - 105.135\ T - 43.667\ Y^o + 6.310\ TY^o$$

- Natural pasture

$$Y_t = 49.615 - 1.793\ T - .141\ Y^o - .254\ G + .0208\ TY^o$$

$$Y_{to} = 799.3 - 102.276\ T - 22.988\ Y^o + 5.653\ TY^o$$

- Rye-Grass pasture

$$Y_t = 10.803 + 2.296\ T + 1.371\ Y^o - 287\ G - .081\ TY^o$$

$$T \leq 20$$

- Temporary meadow

$$Y_t = 13.792 - .332\ T + .568\ Y^o - .286\ G - .0092\ TY^o$$

- Rye-Grass meadow

$$Y_t = -36.346 + 1.662\ T + 1.297\ Y^o - .089\ G - .046\ TY^o$$

- Alfalfa meadow

$$Y_t = -33.372 + 1.792\ T + 1.153\ Y^o - .127\ G - .042\ TY^o$$

Contrary to what is expected, the coefficient of the variable G takes a negative value. Plant physiology theory contradicts this result. However, it proves that the estimation method used by extension people to measure the output of pastures and meadows, is not very accurate. An acceptable explanation of this negative coefficient can be found. When the stocking rate is too high for a given period, pasture output grazing intervals become smaller and smaller. Livestock are underfed and pasture output overestimated, as it is expressed by the sign of the G variable.

Furthermore, in the natural pasture equation, Y_t behaves incorrectly. Since $\frac{\partial Y_t}{\partial Y^o} = -.141 + .0208\ T$, its slope decreases as Y^o increases. Here too, plant physiology contradicts this result.

Is this result due to the fact that natural pastures are very heterogeneous in respect to soil fertility and moisture content or is it due to summer and fall overgrazing which gives rise to an overestimation of the corresponding period pasture output? A definite answer has not been given to this question, but we can hypothesize that the two preceding causes are effective. Usually, natural pastures are found on poor soils and it is observed that they are more often damaged by overgrazing than the more productive temporary pastures.

b. Hay yield It is generally admitted that alfalfa produces about 50% of its annual hay output at the first cutting, the remainder being harvested at the second and third cuttings (42, 1964 report, p. 61 and 1965 report, p. 54). On the contrary, grasses which are associated with alfalfa produce about 70% of their total output by July 1 (62, p. 45). Grasses have a tendency to predominate over alfalfa, especially with a nitrogen fertilization. A first cutting of alfalfa and grasses yields 65% of their total annual output.

c. Grassland output inserted into our model From the preceding result we built a series of activities which differ from one another on total output level and grassland utilization. Although these results are based on somewhat poor data, they are not in contradiction with published experimental results (62, p. 45) which show that spring output (by July 1) represents from 64 to 75% of the annual output. Moreover, these results were submitted to extension agents[1] for agreement. Finally, the following set of data was inserted into our model (Table 24).

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[1]Ergan, M., Vignier, D. and Houdan, M., Maison de l'Agriculture, Laval, 53. Input-output relationships. Private communication. 1967.

Table 24. Annual grassland output

Output level (fodder unit/ha)	Type of forage	Grassland utilization[a]	Spring grazing (by July 1) fodder units/ha	Summer and early fall grazing fodder units/ha	Hay Metric quintal/ha	Silage Metric quintal/ha
5,000	Alfalfa + grasses	HHH	0	0	110.0	0
		HGGG	770	1,600	71.5	0
		GHGG	1,600	1,920	53.0	0
	White clover + grasses	GGGG	3,100	1,900	0	0
		SGGG	1,730	820	0	164.0
	Natural pasture	GGGG	2,100	1,400	0	0
		HGG	0	1,400	35.0	0
4,000	Alfalfa + grasses	HHH	0	0	90.0	0
		HGGG	600	1,320	58.5	0
		GHGG	1,360	1,530	42.0	0
	White clover + grasses	GGGG	2,480	1,520	0	0
		SGGG	670	1,430	0	128.0
	Natural pasture	GGGG	1,650	1,150	0	0
3,000	Alfalfa + grasses	HHH	0	0	70.0	0
		HGGG	420	1,060	45.5	0
		GHGG	1,220	1,220	29.5	0
	White clover + grasses	GGGG	1,860	1,140	0	0
		SGGG	520	1,120	0	100.0
	Natural pasture	GGGG	1,k60	840	0	0

[a]H indicates hay cut; G indicates grazing.

Permanent pastures occupy, at least, 1/10 of the total farm acreage. This amount is generally admitted[1] as the minimum acreage which is usually found on most farms.

2. Fertilizer inputs

In spite of the lack of known response surfaces in fertilizer use (N, P_2O_5, K) which would be valid for the soil and rainfall characteristics of this region, we had to vary the level of fertilization with the level of forage output. The level of P_2O_5 and K was indicated by extension agents[1]. The rates of "average" marginal productivity of N which were used are based on Bougle's experimental results (10, 11, 12). The corresponding amount of fertilizer inputs are given in Table 25.

C. Conclusion

In a region where forage crops occupy about 66% of the total area (8, p. 33) precise and reliable forage input data are still lacking for each different soil and rainfall condition. In this domain a large amount of experimental research is needed, as well as, the insertion of soil and climatic indexes into production functions which could be derived from the set of results found in the past, by all experimental stations located in the western part of France.

In our case, for the type of soil and rainfall characteristics which are those of the "Bocage Angevin" region, the preceding data,

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[1] *Ibid*.

Table 25. Rate of fertilization of different pastures and meadows

Type of pasture or meadow	Level of output (100 fodder units)	Fertilization rate N	P_2O_5	K
Natural pastures	20.0	0	70	70
	27.5	50	80	80
	35.0	120	100	100
Temporary pastures	30.0	0	80	80
	40.0	80	100	100
	50.0	170	120	120
Rye-Grass	40.0	80	80	80
	50.0	150	100	100
	60.0	240	120	120
Alfalfa + grasses	42.0	0	80	80
	54.0	40	100	100
	66.0	100	120	120

derived from real observations, seem to constitute a first good approximation of reality. Obviously they will have to be confirmed or corrected, when new experimental results are available. A new series of experiments is actually taking place to reach these objectives.

CHAPTER 12. CAPITAL

In Chapter 6, we have discussed the necessary and sufficient conditions for building multistage models. Our model being a mono-periodic one we will define the conditions under which we are working, as well as the corresponding capital constraints.

A. Model Assumptions

At some stage of our study, capital will be viewed as a scarce resource since we want to study the effect upon farmers' income of different combination of land, capital and labor inputs. To this necessary, but not sufficient, assumption to build a multistage model we add the following ones:

(1) Multiperiodic activities such as steers, heifers and alfalfa belong to the production possibilities of this region. However, there exists a market for almost every intermediate animal output, such as one-year and two-year steers and heifers. Although alfalfa and temporary pastures are multi-periodic activities they can be seeded in each late summer and be ready to start full production next spring. Furthermore, they can follow a large set of crops and, in this respect, are not limited by crop rotation constraints. Installment costs being very low, if optimum solutions require a shorter than usual duration of these activities, only a negligible error will be made. Under these conditions a mono-periodic model is valid.

(2) Durable inputs such as livestock facilities are partly divisible and partly indivisible. In the case of dairy facilities, the barn can be extended beyond a certain minimum of square meters each year.

However, such a possibility is ruled out with milking parlor, especially when the size of the herd necessitates a shift from a tandem parlor to a herringbone one.

When capital is scarce we know that cash cropping will be selected rather than forage crops which have to be transformed by animals. The average productivity of capital, according to our estimation, is always greater than 2.15 with cereals (barley, 3.25; wheat, 3.40) and smaller than 1.7 with steer and dairy production (dairy production, 1.0; 26-month steers, 1.65).

Under these conditions, an increasing supply of capital will generate production plans whose proportion of forage crops will correlatively increase, as well as, the related amount of livestock facilities, if profitable. A mono-periodic model will be able to take fully into account the cost of the divisible and durable inputs, but it will underestimate the cost of the undivisible and durable ones, when they are not perfectly adaptable.

It will also overestimate the rate of growth of the firm if capital can only be accumulated very slowly; in practice, farmers will never extend their barn a few square meters each year.

Although a multistage model would take into account more accurately milking parlor costs, we chose the mono-periodic model, the extra computation costs involved being too high. Furthermore, if the dairy herd increases very rapidly when capital becomes less scarce, it is always possible, in practice, to budget the relative advantages of building ahead of present needs and to choose flexible and extensible equipment.

(3) Crop rotations are not such that we can cultivate one crop after any other. However, since the expansion path is negatively correlated with the quantity of cereal crops, and since temporary pasture can be cultivated after any one of those except corn, we can expect that the set of found solutions will be feasible.

B. Capital Constraints

Since, in spite of certain approximations, a mono-periodic model is adequate, we include within it a set of capital constraints.

The working capital profile of the activity set is such that it is impossible to include all expenditures and receipts of every production cycle within the same period (a year). This situation corresponds to our model 5 described in Chapter 6, page 75. The chosen starting date is the third quarter (September - October) period at which time winter crops are sown.

The time at which incoming and outgoing is taken into account, is not defined as the date of the movement of goods. Farmers run accounts and doing so, they vary their supply of credit by varying the time lapse before payment. It is assumed that farmers are able to get three months' credit without losing good standing. This delay has been reported as also admitted in England and New Zealand by Taplin (71, p. 63).

Six constraints were defined:

- investment capital
- year T^o net balance
- year T' net balance

- third quarter
- fourth quarter
- first quarter
- second quarter

The investment capital constraint takes into account building and livestock initial outlay. In the case of multi-periodic activities such as three-year steers, initial outlays are equal to the market value of two young steers (one and two years old). This number of young animals of different age has to be such that each year a steer will be sold and a calf will be raised. When the market values were not well defined we estimated the cost of those animals by extrapolation. It was admitted that their value increases proportionally to their gain of weight.

Year T^{o} net balances represent a subperiod in which balances are negative within each quarter for every activity except dairy, hog, fattening calf and broiler activities. These last are positive in each quarter.

Year T' net balances represent subperiods in which there exist a subset of positive balances and a subset of negative ones. In each quarter, the elements of each subset are different.

C. Initial Amount of Capital

The purpose of our study, when varying the amount of available capital, is to study the influence of different combinations of scarce resources on farmer's income. Since farmers possess different amounts of credit (equity), save different proportions of their income, are able to delay payment of debts for a variable span of time, an initial amount of capital has not been defined. A borrowing activity supplies the necessary capital at a cost of five percent, the present interest rate discounted by banks. The level of this activity has to be unlimited or varied continuously from zero to its unlimited level.

D. Conclusion

Although capital is an effective constraint which is measured in money terms it is not easy to quantify precisely its components. Furthermore, it is even difficult to estimate the real requirements and the present supply of capital in particular cases. We could even say, with Taplin (71), that the maximum amount of capital which could be used by a particular farmer would be defined "when his bank and stock firm would extend no further accommodation, merchants refused further credit, and family living expenses could be reduced no more. However, virtually no one reaches this limit. As it is approached, the farmer faces increasing degrees of unpleasantness and embarrassment (71, p. 64). Consequently, we feel that each farmer has to define, for himself, the amount of capital available for farming. Our purpose is only to determine how much is required to run, efficiently, particular production plans.

CHAPTER 13. LIVESTOCK ACTIVITIES

From one farm to another, buildings differ in many ways:

- construction period
- degree of flexibility and specialization
- general organization and layout
- space supply

However, when setting up a representative linear model, we have to define precisely initial livestock facilities at hand. Otherwise, we would either overestimate or underestimate attainable income. It can be hypothesized that building space supply varies with farm size. This relationship, if valid, would allow us to increase, continuously, both land and building space and to study the influence of these variables on farmers' incomes.

A. Farm Size and Building Space Relationship

Pigsties and hen roosts were excluded from our estimations: they are more and more left idle by lack of adaptation to modern farming. We only took into account large buildings such as barns, granaries, cow houses which can be used, after small transformation into hog, sheep, dairy cow, calf and steer facilities. Having access to two sources of data we made two series of estimations.

1. Extension service data[1]

We had 12 observations for farms whose size varies from 11.50 to

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[1]Service d'Utilité et de Developpement Agricole. Plans d'exploitations agricoles représentatives du Bocage Angevin. Projet n^{o} 10, 3, rue de l'Ancien Evêché, Laval. Private communication. March 1967.

42.0 hectares. The following linear relationship

$$Y = a + bx$$

where

Y = building space in square meter (m^2)

x = farm size (ha)

was computed for different buildings. We got the following results:

$Y_0 = -6.12 + 4.97\ X$, $^{**}F^{1}_{10} = 45.55$, $r = .90$

$Y_1 = 36.47 + 5.11\ X$, $^{**}F^{1}_{10} = 20.54$, $r = .81$

$Y_2 = 68.23 + 5.27\ X$, $^{**}F^{1}_{10} = 19.81$, $r = .81$

$Z_1 = 4.67 + 13.06\ X$, $^{**}F^{1}_{9} = 13.88$, $r = .77$

where

Y_0 = cattle barn

Y_1 = cattle barn + stable

Y_2 = cattle barn + stable + fodder storage room

Z_1 = hayloft (cubic meter = m^3)

r = coefficient of correlation

* and ** = 5% on 1% significance level

F^n_m = statistical F ratio with n and m degree of freedom

Furthermore, we had 40 observations relative to field barns ($5 \leq x \leq 50$). We got the following relationship:

$$Z_2 = 47.01 + 4.55\ x\ ;\ ^{**}F^{1}_{38} = 10.2\ ,\ r = .462$$

All these relationships being significant, we did go on with our investigations.

2. Fire insurance company data[1]

The "Mutuelle Agricole du Maine" had collected for each fire insurance contract the corresponding building drafts. A random sample of 94 observations was drawn from their "Bocage Angevin" policy subset. Three types of equations were calculated. They are:

$Y = a + bx$

$Y = a + bx + cx^2$

$Y = a + bx + cx^{.5}$

The linear one was found as good as the other two. The latter are therefore omitted here. The corresponding relationships are:

$S_1 = 112.715 + 5.232x; \; r^{**} = .505$

$S_5 = 169.839 + 6.765x; \; r^{**} = .414$

$S_6 = 185.964 + 9.318x; \; r^{**} = .507$

where

S_1 = large buildings (≥ 40 m^2), mainly used for cattle housing in square meter (m^2)

S_5 = large buildings + sheds which can be transformed and used as cattle barn (m^2)

S_6 = total farm buildings; m^2 (excluding small out-buildings)

3. Choice of one relationship

Although the definition of S_1 and Y_2 variables is not rigorously similar they are almost identical. Furthermore Y_2' and S_1' are practically identical. We chose S_1 for three reasons:

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[1]Tardieu, M., Mutuelle Agricole du Maine, le Mans, 72. Fire insurance company data. Private communication. 1967.

- $N_{S_1} \simeq 8N_{Y_2}$; N = number of observations
- S_1 represents present livestock housing facilities. If we had used S_5 we would have probably underestimated the amount of building space required by modern farming: sheds, although transformable, are likely to have a specific use in most production plans.
- Hay lofts and field barns are supposed to be sufficient for hay and straw storage. The increase in the number of cattle raised on a farm corresponds to a decrease in hay and straw consumption per animal (modern feeding programs and building facilities).

B. Building Investment Functions

When building space is too scarce and limits drastically the feasible production plans, it might be profitable to add new buildings and equipment to the old ones, such as:

- milking parlor
- loose housing stable
- hog house
- sow house
- chicken house
- laying-hen house

The corresponding investment functions are more frequently of the form:

$Y = a + bx$

where

Y = total investment

x = number of animal facilities

This relationship is particularly true when only the lengthening of buildings is undertaken to provide extra space. The derived amortization functions have the same form and theoretically require to be inserted into a mixed integer program instead of a straight linear one. Amortization length, interest rate[1] and maximum number of animal facilities are summarized in Table 26.

Table 26. Equipment maximum size and loan characteristics

Equipment type	Maximum number of animal facilities	Interest rate	Amortization length (years)
Milking parlor	70	5%	12
Loose housing stable	70	5%	12
Hog house	180	5%	7
Sow house	50	5%	7
Chicken house	10,000	5%	5
Laying house	5,000	5%	5

Investment and amortization cost function coefficients were calculated from different estimates of quantities and costs. Our results are summarized in Table 27. Milking parlor investment step functions were not inserted, as such, into our model, but transformed into a continuously increasing function. We assumed milking parlor divisibility beyond initial fixed costs. For each equipment set, such as tandem or herringbone milking parlors, we supposed that the corresponding cost supplied a fixed number (mean range) instead of a range of milking

[1]Auphan, M., Caisse Régionale de Crédit Agricole, Laval, 53. Interest rate. Private communication. 1967.

Table 27. Total livestock facilities investment and amortization cost functions[a]

Livestock facility	Total cost (F)	Y or Z = a + bx: Fixed initial cost	Variable cost	r	Source of data
Milking parlor	Y	7,802	570.23 X	.98	
Loose housing stable	Y	1,021	750.80 X	.99	
					(22)
Dairy cow facilities	Y	8,823	1,320.00 X		
	Z	995	149.00 X		
Hog house	Y	6,180	162.50 X	.99	
	Z	1,068	28.00 X		(20)
					(21)
Sow house	Y	1,045	881.80 X	.98	(24)
	Z	180	152.40 X		
Chicken house	Y	7,375	5.03 X	.98	
	Z	1,703	1.15 X		
Laying house type 1	Y	1,112	10.50 X	.98	(24)
	Z	257	2.42 X		
Laying house type 2	Y	4,550	8.38 X	.98	
	Z	1,074	1.93 X		

[a]The cost functions have the form Y or Z = a + bx where Y is investment cost and Z is amortization cost.

facilities. This assumption was made to reduce the number of 0,1 variables associated with linear non-proportional functions.

CHAPTER 14. PRICES

Farm input and output prices have a tremendous effect on income. A careful choice has to be made of different farm production product prices. However, the number of variables which influence price determination are so numerous that we cannot, in planning, assume perfect knowledge. Price expectations are valid within a certain range: the knowledge of the future is so imperfect that enterpreneurs usually formulate multi-valued expectations. Since we were working with a classical linear programming model and not with a quadratic one, we were not able to take directly into account price variation. However, we studied the hehavior of our solutions in varying certain price coefficients.

A. Determination of the Present Price Level

1. Price for milk

The average milk prices, at the farm level, which had been paid by the dairy industry in the "Bocage Angevin" region are given in Table 28. These values were computed from unpublished data gathered by the farmers' union (32).

We observe (Table 28) that the annual mean price increased from 1964 to 1965 while its variance decreased. The difference between winter and summer milk prices narrowed in 1965 and was expected to be kept at this level (32). In consequence, we decided to insert into our model 1965 milk price as the basic price.

Table 28. Mean price and standard deviation for milk

Year	Annual mean price $\overline{X}$ (francs/liter)	S
1964	.459	.026
1965	.458	.038
1965	.473	.020

2. <u>Price for beef</u>

The meat processing industry paid the farmers, the following prices (Table 29) as recorded by the main marketing association[1]. Generally, 1965 prices were higher than 1966 ones. Their variance varied in the same direction. We averaged these two year prices and inserted them into our model. They are considered as "present prices".

Table 29. Standard deviation and annual mean prices for beef carcass[a]

Beef or cow, carcass grade	Annual mean price (francs/kilogram) 1965	1966	Standard deviation 1965	1966
V	6.39	6.45	.180	.116
M	6.02	6.00	.145	.166
Z	5.93	5.85	.185	.143
F	5.86	5.71	.210	.103
D	5.40	5.34	.198	.168

[a]Source: SICAVEM, EVRON - 53.

[1]de Parcevaux, M., Directeur de SICAVEM, Evron, 53. Carcass price data. Private communication. 1967.

3. Price for cereals

According to the "ONIC staff"[1] prices are expected to vary within the ranges given in Table 30.

Table 30. Price of cereals and other crops

Type of crops	Crops	Floor price (francs/metric quintal)	Range of price variation
Cereals	Winter wheat	42.0	42.0-46.0
	Winter and spring barley	38.0	38.0-42.0
	Winter and spring oats	36.0	36.0-39.5
	Corn	39.00	39.0-43.0
Seeds	Rye-grass	160.00	160.0-120.0
	Tall fescue	330.00	
Others	Potatoes	17.00	
	Rape	80.00	

Present prices are equal to 107% of the floor price. They were inserted into our model.

B. Range of Price Variation

The preceding chosen prices are viewed as "present prices". Beside their role as a starting point, they provide a price distribution through the year which was assumed stable even though the annual mean price increases. In order to study the influence of the price level on income, we varied, within a certain range, the annual mean price of

[1]Direction de l'Office National Interprofessionnel des Cereals, Laval, 53. Expected price for cereals. Private communication. 1967.

milk, meat, seeds and cereals. In no case whatever, did we change the relative prices of products which belong to the same group.

1. Variation of milk price

The intervention price has been fixed at .46 F/liter (37% butterfat) at the milk factory gate. As the shipping cost can be presently estimated about .04 F/liter, the expected minimum milk price at the farm gate is about .42 F/liter (49). The corresponding range of price variation would be equal to .42 - .47 F/liter. However, we decided to vary it from .40 to .50 F/liter (4.0% butterfat).

2. Variation of meat price

Target prices of first grade steers have been fixed at (58)

- 335.72 F/100 kilograms liveweight from April 1968 forward
- 345.59 F/100 kilograms liveweight from April 1968 forward

As the intervention price is equal to 90 - 93% of the target price we can expect that meat price will never decrease below:

$\simeq$ 5.65 F/kilogram for a Z beef carcass. At present the corresponding price is equal to about 5.89 F/kg (first grade steer $\simeq$ Z carcass grade).

The range of meat price variation was chosen equal to:

5.65 - 6.85 F/kilogram for carcass (corresponding to a first grade steer).

3. Price variation of cereals

Although prices were expected to vary from 100 to 110% of the floor prices (Table 30), we decided to allow a larger price variation: from 80 to 120%. We did it to guarantee the detection of the range of prices which generate stable farmer's income and identical crop rotation, if any. We wanted to check the hypothesis that cereal activities were both economically dominated by livestock production and a set of supplementary enterprises.

4. Other input and output prices

They were considered as constant except rye-grass seeds.

C. Conclusion

Even though we were not able to take price variation directly into account, as we would have with quadratic programming, we can foresee the degree of stability of farm production plans with respect to their main producing target.

CHAPTER 15. YIELD LEVEL OF CROPS AND LIVESTOCK

The variances of the distribution functions for total forage output, milk production per cow, daily gain of steers, and crop yields are large enough. To specify our linear programming activities a choice had to be made on particular fixed values. However, in every case, we assumed that the efficiency level reached by good farmers cannot be inferior to the mean of any specific yield distribution function.

A. Average Milk Production Per Cow

From published cow testing data (1) for this region (Mayenne) we calculated the following characteristics (Table 31).

Table 31. Milk production for different breeds

Breed	Average yield (liter/lactation) 1966	1967	Standard error S 1966	1967	Average number of observations
Frisian (black and white)	3,645	3,575	1,341	1,321	4,420
Normande	3,151	3,106	1,210	1,204	19,600
Maine-Anjou	2,540	2,382	1,037	1,093	2,200

Since the Maine-Anjou breed possesses the highest fattening ability we decided to maintain it as an activity. The average milk output per cow was fixed at 2,850 liters per year. Instead of defining dairy activities according to breed type we did it in function of the level of average milk production per cow and per year. We are referring indifferently to double purpose breeds, such as Normande, or specialized breeds, such as black and white Frisian. The corresponding two dairy activities supply respectively 3,000 and 3,800 liters of milk per cow per year. Furthermore, two calving dates were chosen for three reasons:

- milk prices vary from month to month; winter prices being higher than summer ones
- requirements of various bulky fodders are function, to a certain extent, of calving dates
- farmers, keeping in mind the two above variables, ask for the most profitable calving date.

Consequently, five dairy activities were inserted into our model, they are:

- Maine-Anjou
- 3,000 liters dairy activity - February calving
- 3,000 liters dairy activity - October calving
- 3,800 liters dairy activity - February calving
- 3,800 liters dairy activity - October calving

Although we knew which subset of dairy activities dominates the other ones, all of them were inserted to compute the effect of different management level on farmers' income.

B. Steer Weight Gain Per Day

Here we take the case of fall born animals. Their liveweight gains vary according to the season and more particularly to the corresponding feeding programs which are traditionally chosen by farmers in this region.

Table 32. Average daily liveweight gain and standard deviations for fall-born animals

Period	Animal age (months)	Steers			Heifers			Steers and heifers	
		n[a]	Average gain (grams)	Standard deviation	n[a]	Average gain (grams)	Standard deviation	n[a]	Average gain (grams)
Spring and summer (pasture feeding)	6/8-13/14	37	566	129	46	569	125		
	16/17-23/25	26	648	186	34	611	125		
Fall and winter	10/12-15/18							48	275
	22/25-27/30[b]							21	$\simeq 0$
	22/25-27/30[c]							X	1,000

[a]n is the number of animals per group.

[b]Maintenance ration.

[c]Fattening ration.

Most steer liveweight gains which were inserted into our model are based on Barbier's results (3, p. 25). They confirm the traditional practice which makes heavy use of the compensatory growth phenomenon. Very low winter growths are correlated with daily spring liveweight gain. Coleou estimated this relationship for "Charolais" steers (23, p. 41). He found:

$Y = 100 - .4\ X$

where

$b = .4 \pm .1$

x = winter weight gain

Y = spring weight gain

Traditionally, it has been admitted that pasture feeding programs are both cheap and labor saving. Consequently, steer growth curves are typically in the form shown in Table 32. However, we can question the validity of such a traditional belief. It is even doubtful that the corresponding beef production processes are still the most profitable ones, when tremendous technological progress is continuous.

We can imagine, in addition to the preceding steer growth curve, few others which would have the following shape:

(1) A strictly linear curve, ruling out all possibility of compensatory growth.

(2) A step function, such as the traditional growth curve, characterized by alternate high and low daily liveweight gains into subsequent subperiods. Contrary to the traditional curve, high daily growth would take place in winter and the lowest in summer. The

hypothesized profitableness of such beef activities can be justified on the following grounds:

- Considerable technological progress has been made in fodder, beet, and corn silage production (yield and cultivation methods...) two feeds which enter, in large quantities, into fall and winter rations.
- Possibility of controlling fall and winter feeding programs and consequently, the level of daily gains. They can be high even when concentrated feeds are excluded from feeding programs, basic rations composed of fodder beet and corn silage being very nutritive.
- High winter daily gains would largely compensate for the lowering of summer ones. At this period they are usually fed 450 grams per day as compared to 950 grams in spring and winter (3, p. 28). Summer is not a period which favors steer growth. Nutrient content of grass has decreased (27, p. 29) and temperature may have a negative effect on daily gains (38, p. 434). Our first results (Chapter 18) led us to formulate the shape of such a steer growth curve and ask a nutritionist[1] to define it more precisely.

(3) An intermediate growth curve. When small and high daily liveweight gains alternate, it would be interesting to determine the optimum level of low daily growth associated with higher gain levels of the other periods. The resulting steer growth curve would be a

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[1]Ergan, M., Directeur de la Maison de l'Elevage de la Mayenne, Laval, 53. Hypothesized steer growth curve. Private communication. 1968.

compromise between the linear and the step functions described above. This research implies that we have a pretty accurate quantitative knowledge of the compensatory growth phenomenon. Thinking that it is not yet the case, we limited our inquiry to the two preceding problems.

Consequently, we inserted into our model:

- linear growth curves (yearling bull)
- traditional growth curves (beef activities)
- modified step functions (first choice steers characterized by high daily liveweight gains from October to June)

C. Crop Yield

Crop yields (18, chp. 5) are summarized in Table 33. These data represent the results which can be obtained by good farmers; the above average yields being higher than the mean yields of the whole population.

D. Conclusion

We consider that two farm enterprises, which are more critical and difficult to manage, should be differentiated with respect to their output level. They are pasture-meadow and dairy activities. The others are supposed to supply a given fixed output under an average level of management.

Table 33. Various crop yields

Crop type	Average yield (metric quintal/hectare)	Range of variation
Wheat (on pasture, corn, rape...)	45	38-52
Wheat (on beet, potato...)	42	35-50
Wheat (on cereals...)	40	30-50
Winter barley	35	30-45
Spring barley	40	35-45
Winter oats	38	30-45
Spring oats	33	30-45
Corn	54	45-60
Rape	20	15-25
Potato	320	200-400
Rye-grass seeds	14	12-18

CHAPTER 16. LABOR

Most of the labor coefficients used in this study were derived from data obtained by personal interview with good farmers (18, chp. 2). The corresponding constraints were set up according to the rules stated in Part II (Chapter 5). Since few of them could explain some results, we will define the labor constraints very briefly.

A. Labor Constraints on Crop Activities

Let us recall the definition of the following variables (Chapter 5, page 53).

- $\dot{J}_{kj}$ = job k of activity j
- B_{kj} = time period of job k in activity j
- b_{kj} = number of effective hours which can be allocated to job k in activity j per farm worker.

Table 34 gives the specific jobs which have to be undertaken within each time period and subperiod. When a subperiod or a period includes other subperiods, it is necessary to include the specific jobs of these other subperiods. We find this situation in the following periods and subperiods.

21 + 22 + 24 C 23 C 20

31 C 30

41 C 40

61 + 62 C 63; 63 + 64 + 65 C 60.

Table 34. Labor constraints and corresponding jobs

Period and subperiod code	B_{kj}	b_{kj}	J_{kj}
L10	15/3-1/4	125	Tilling for and seeding of spring barley and tall fescue. Nitrogen broadcasting on grassland. Rape spraying. Tall fescue second cultivation and weeding.
L20	16/4-8/6	315	Rye-Grass ensiling.
L21	10/5-20/5	72	Mixed grass ensiling - nitrogen broadcasting on grassland.
L22	21/5-31/5	50	Alfalfa (HHH) First cutting.
L23	10/5-8/6	215	Potato earthing-up - fodder beet thinning.
L24	1/6-8/6	40	Alfalfa (GHGG) First cutting.
L30	9/6-10/7	245	Fodder beet transplanting. Plowing and tilling for kale. Potato spraying. Kale transplanting. Alfalfa second cutting. Nitrogen broadcasting on grassland. Rape and tall fescue harvesting.
L40	11/7-20/8	250	Potato spraying. Kale and fodder beet second cultivation. Catch crop kale transplanting. Nitrogen broadcasting on grassland. Cereal harvesting.
L50	21/8-13/9	180	Alfalfa third cutting. Pasture seeding. Plowing for winter barley.
L60	14/9-15/12	460	
L61	14/9-12/10	180	Tall fescue first cultivation. Corn ensiling. Tilling for and seeding of winter and spring oats.
L62	13/10-10/11	115	Fodder beet harvesting.
L63	14/9-10/11	355	Plowing and fertilizer (P_2O_5 + K_2O) broadcasting for winter wheat.

Table 34 (Continued)

Period and subperiod code	B_{kj}	b_{kj}	$\dot{J}_{kj}$
L64	13/10-30/11	230	Corn harvesting. Fertilizer (P_2O_5 + K_2O) broadcasting on grassland.
L65	13/10-15/12	250	Tilling for and seeding of winter wheat. Plowing + tilling for and seeding of winter wheat following corn and beet.

B. Labor Constraints on Livestock Activities

Since tending of livestock can usually be done when it is not always possible to work in fields, three separate livestock labor constraints were defined.

Table 35. Livestock labor constraints

Period code	B_{kj}	$\dot{J}_{kj}$ (feeding program of ruminants)
A1	15/4-10/7	Animals on pasture
A2	10/7-15/10	Pasture feeding (with additional fodder, hay or silage)
A3	15/10-1/1	Indoor feeding

In order to make our programming model more realistic three transfer activities were set up. Labor can, therefore, be allocated more efficiently between field and farm-yard jobs since the initial amount of labor assigned to crops is maximum. This procedure decreases also the number of the matrix labor coefficients.

C. Conclusion

The risk criteria was chosen such that there exists a high probability of performing any job, 7 years out of 10, without difficulties, even though the initial labor stock is exhausted in the programming solution. This choice was made according to the evaluation of farmers' risk-bearing reported by Reboul (65, p. 60). In practice, we realize that an error of evaluation of this risk criteria would cause a larger variation of the objective function than an approximation of any labor input requirement.

PART IV. RESULTS

CHAPTER 17. INFLUENCE OF INVESTMENT IN BUILDING FACILITIES ON FARMERS' INCOME

The input requirements of various farm products differ greatly. We can distinguish three classes of output according to the needs of land and building space:

- The land consuming output. Cereals and certain slaughter steers belong to this group.
- The building consuming output. Hog, weaned pig, broiler, slaughter calf production are good examples of the elements of this class.
- The land and building consuming output. In this group we find most of the livestock activities (sheep, dairying, steers).

The level of income[1] is, therefore, a function of both farm size and building space at farmers' disposal, when these inputs are scarce.

Having stated our particular hypothesis, we study two initial solutions in which investment in dairy facilities are either excluded or allowed. These two solutions were found with a standard linear programming code. The subsequent results were obtained with a mixed integer one[2].

In the second part, we show the influence of building investments for hogs, pigs and dairy cows on income (five discrete variables). In the third part, to the above opportunities, we have added the following ones: broiler and egg production (eight discrete variables).

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[1]After deduction of the fixed costs FC; where FC = -1249.75 + 436.66 S, S = farm acreage in hectare (18, chap. 12).

[2]I.B.M. computing center, Paris.

A. Particular Assumptions

The following particular assumptions were made:

1. Standard linear model
   a. All production, even the second group defined above, are allowed as long as they utilize only the initial stock of buildings (old ones). Consequently broiler and egg production are excluded. However it is possible to build for young cattle.
   b. Present prices
   c. 30 hectares farm and unlimited amount of capital
2. Mixed integer model
   a. Few activities were excluded on various ground (risk-market situations). They are purchase of hay and feeder steers; production of potatoes and grass seed. Furthermore, it was assumed that nobody would hire more than three men on a 30 hectares farm (risk bearing).
   b. "Present" prices
   c. 30 hectares farm and unlimited amount of capital.

B. Results

1. Standard linear model
   a. Without investment in dairy facilities. The solution is characterized by the absence of dairy cows and farm-born steers. The main production is cereals, grass for seed, slaughter calves and steers. Two beef fattening activities were chosen: the 36-month steers which are bought in

fall and sold in spring and the 36.5 month ones which are fed on pasture from March to October. The purchase of feeder steers costs 156,400 francs. This plan requires a large amount of capital and furthermore, is very unstable. When the price of feeder steers raises by 5% their number decreases by 65.6%. A price increase of 8.93% causes their disappearance from the optimum solution. Cereals are substituted for pastures and row crops. At the same time income decreases rapidly since we have the following relationship:

$$\dot{I} = 48{,}270.75 - 1564.24\ X + 97.713\ X^2;\ R^2 = .994$$

where

I = income

X = increasing cost (%) of fodder steers.

The corresponding farm plans are summarized in Table 36.

b. With investment in dairy facilities. This opportunity could have caused the substitution of cereals by dairy cows, but in fact we get about the same solution as above. This solution is also very unstable since slaughter calves disappear from the optimum solution when their price decreases very slightly (- .063 F/kg of carcass). They are substituted by hogs. Income is almost constant due to a large decrease of labor inputs (-5,600 F). The production of hogs requires a smaller amount of labor input per head than the production of slaughter calves. The corresponding farm plans are given below (Table 37).

Table 36. Optimum farm plans without investment in dairy facilities

Activities	Change in price of feeder steers + 0%	+ 8.93%
Income (francs)	48,420	42,127
Hired labor (man per year)	.44	.21
Potatoes (ha)	.75	1.09
Cereals (ha)	9.913	10.412
Pasture (ha)	11.936	4.561
Fodder row crop (ha)	3.061	.718
Grass for seed (ha)	4.334	4.206
Slaughter calves (head)	388.5	329.0
Dairy cows + yearling bulls (head)	< 3.0	< 3.0
36.5 months steers (head)	43.1	0
36.0 months steers (head)	50.9	0

Table 37. Optimum farm plans with investment in dairy facilities

Activities	Price of slaughter calves 640 francs	634 francs
Income (francs)	47,027	45,842
Hired labor (man per year)	.56	.0
Potatoes (ha)	1.034	.945
Cereals (ha)	9.793	9.738
Pastures (ha)	11.332	11.564
Fodder row crops (ha)	2.766	3.059

Table 37 (Continued)

Activities	Price of slaughter calves 640 francs	634 francs
Grass for seed (ha)	5.063	4.688
Slaughter calves (head)	388.5	0
Hogs (head)	0	462.5
Dairy cows + yearling bulls (head)	< 3.0	< 6.0
36.5 month steers (head)	40.30	40.56
36.0 month steers (head)	47.83	47.92

2. Mixed integer linear model

a. With five discrete (0,1) variables. The following discrete variables are taken into consideration:

- labor hiring
  - one man
  - two men
- investment in
  - dairy facilities
  - hog house
  - weaned pig facilities

These three solutions are very close with respect to the level of income. But, however, the production plans corresponding to the three discrete solutions are somewhat different. When an investment is made in dairy facilities (solution 2 versus solution 1) cereals are substituted by fodder crops, yearling bulls by dairy cows and the number of hogs produced per year increases. At the optimum, two men are hired against one in the two first discrete solutions. A further substitution

Table 38. Optimum farm plans with five discrete variables

Items	Continuous	Solution number 1	2	3
Income (francs)	52,148	49,004	49,466	50,932
Discrete variables				
hired labor (men)	1.78	1.0	1.0	2.0
dairy facilities	.179	.0	1.0	1.0
hog facilities	1.0	1.0	1.0	1.0
sow facilities	1.0	1.0	1.0	1.0
Production plan				
cereals	16.040	18.726	11.744	8.893
pasture	10.580	9.062	14.717	15.912
fodder row crops	3.375	2.204	3.502	5.190
sows	50.0	50.0	48.6	50.0
hogs	450.0	802.0	916.0	450.0
slaughter calves	390.0	10.0	.0	390.0
dairy cows	12.51	10.0	23.8	24.7
yearling bulls	24.41	20.1	0.	14.7

of cereals by fodder crops is made, hogs are partly substituted by slaughter calves and yearling bulls belong to the optimum solution.

b. With eight discrete variables. To the preceding five discrete variables we have added the following ones:

- investment in hen-house type 1
- investment in hen-house type 2
- investment in chicken-house

The solution differs from the preceding ones in the following respect:

- the level of income is much higher
- cereals constitute the main crop
- labor is mainly allocated to the production of eggs, slaughter calves and hogs

- sheep and cows belong to the programming solution in order to transform the intermediate production of the permanent pastures whose acreage is minimum.

The corresponding farm plan is shown in Table 39.

Table 39. Optimum farm plans with eight discrete variables

Item	Solution value
Income (francs)	82,088.0
Discrete variables	
hog facilities	1.0
hen-house type 2 facilities	1.0
hen-house type 1	1.0
hired labor	1.0
Cereals (ha)	23.789
Pastures (ha)	5.466
Fodder row crops (ha)	.737
Hogs (head)	660.0
Slaughter calves (head)	165.40
Dairy cows (head)	5.40
Yearling bulls (head)	3.19
Sheep (head)	46.84
Hens (head)	10,000.0

C. Are the Preceding Farm Plans Too Risky?

Most of the preceding farm plans are characterized by the combination of activities which belong to the two following classes of production:

- the land consuming output
- the building consuming output

When to these corresponding farm plans, dairy cows are added (a building and land consuming output) the level of income increases only a slight amount, while the total number of produced hogs and calves is almost constant. This type of farm plans are frequently rejected by farmers who consider them as too risky, especially when the number of head of calves and hogs produced per year is large. This remark is valid for all farm productions whose farm input requirements are mainly constituted of building space and labor. As we will show later, the land/labor ratio is too small in agriculture. Consequently, farmers develop those activities whose land requirement per output unit is very small. Furthermore, large producing units are created under the influence of integrators, but the price squeeze reduces the corresponding benefits. These productions being undertaken on a large scale, the corresponding risk is high: the variance of the results which are presently observed is large.

Let us define the following gross benefit as follows:

G.B. = S - VC

where

G.B. = gross benefit per head

S = sales

VC = variable cost excluding building and labor costs

Then we get the following results:

slaughter calf: G.B. = 55 $\mp$ 35 F.[1]

broiler: = .50 $\pm$ .25 F.[2]

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[1]de Parcevaux, M., Directeur de SICAVEM, Evrn, 53. Risky production plans. Private communication. 1968.

[2]Paillard, M., Directeur de la CAC, Craon, 53. Risky production plans. Private communication. 1968.

hog = 30 ± 20 F.[1]

hen = 7 ± 3 F.[1]

The value of the objective functions is consequently very unstable. If, for example, the average price of eggs decreases .01 F per unit, income (Table 39) decreases 22,000 francs.

Presently, integrators supply, if needed, the required amount of capital inputs. In spite of the risk involved, these activities are consequently very attractive for those farmers whose capital is scarce.

However, all these considerations led us not to pursue the preceding investigation. These activities belong to the optimum solutions either at their maximum level or at their minimum one ($X_j = 0$). Given the risk bearing ability of farmers, the optimum solutions are certainly formed of more diversified farm plans characterized by the combination of risky and very profitable activities with very secure ones.

We think that a quadratic model which takes risk into account would be more realistic than the mixed integer model which has been used. Furthermore, the fixed charge problem does not seem to influence largely the value of the objective function. Consequently, in a first approach of risk study, the fixed costs could be neglected.

D. Conclusion

In the subsequent chapters we will omit the building consuming output in our model. The corresponding value of income will decrease

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[1] *Ibid.*

by a large amount, to about 28,650 F. It would, therefore, be rewarding to integrate these productions into farm plans at a level which would be secure enough. On a 30 hectares farm, average income opportunities vary from 50,000 to 80,000 francs when large risks are taken and equal 28,000 F when a more secure plan is chosen.

CHAPTER 18. INFLUENCE OF TECHNICAL MANAGEMENT ON INCOME LEVELS

Four variables are considered to be greatly affected by farmers' level of technical management and knowledge. They are:

Forage yield per hectare of the following crops:

- temporary pasture
- corn for ensilaging

Milk yield per cow

Acreage of permanent pasture on farms. In this case an important acreage of permanent pasture can also be due to particular soil conditions. But they don't fall under the general hypothesis we are working.

The productivity of land, when allocated to forage production, is mainly a function of two factors: its forage yield and the rate of transformation of forage into animal products (for a given set of prices). Therefore, we will study the influence of the forage yield level associated with different levels of milk yield per cow. Since the substitution of animal products by cereals can occur at certain low levels of technical management, the sale of cereals was excluded in a series of solutions and reinserted into the subsequent computations.

A. Particular Assumptions

The following assumptions were made:

- "present" prices
- farm acreage: 30 hectares
- unlimited supply of capital
- some activities were excluded, they are:

- weaned pig, hog, grass for seed, potato and slaughter calf production[1] as well as hay making on permanent pastures[2]
- purchase of feeder steer, hay and dairy cow facilities[2]

- The replacement of cull cows by purchased three-year heifers is only possible with Normande or Frisian herd, producing an average of 3,000 liters of milk per cow.
- The maximum number of cows is fixed at 20; the purchase of modern dairy facilities being excluded from the activity set.

Besides the above assumptions, the following activities had already been added when we studied the influence of the total acreage of permanent pasture on farms[2].

- Modified feeding programs for yearling bulls; they are
  - corn silage + minimum of hay
  - the above ration in which 500 kg of cereals are substituted for 2.77 metric tons of corn silage.
- First choice steer with high daily liveweight gain from October to June. Their ration being mainly constituted of fodder beet and corn silage in winter.

Furthermore, the sheep flock is excluded from this last model.

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[1]Pig, hog, seed, slaughter calf productions and feeder steer purchase are considered as risky enterprises or limited in extent. We therefore omit them in this part of the study.

[2]The addition of these activities to our model is due to the results of the following chapter.

B. Results

1. Yield level of corn for ensilage

Being recently introduced in this area, hybrid corn for ensilage, yields a variable quantity of forage per hectare. Furthermore, it is a substitute to, and a complement of fodder beets within livestock feeding programs. To study the influence of corn yield on income and on the optimum livestock ration set, the yield level was varied from 37 metric tons per hectare to 62 tons of silage. Let us define:

$\dot{I}_c$ = income in francs

Y_1 = yield level of hybrid corn (metric ton of corn silage).

Then we have for:

$$Y_1 = 37.0 + X;\ 0 \leq X \leq 25, \text{ the following results:}$$

$$\dot{I}_c = 28478.47 + 109.90\ X + .5966\ X^2;\ R^2 = .9972$$

$$\dot{I}'_c = 109.90 + 1.1932\ X.$$

The marginal revenue of X being quite high within its range of variation, ($109.9 \leq \dot{I}'_c \leq 139.73$) it is therefore worthwhile to popularize the best methods of corn cultivation.

When X increases, then fodder beet are substituted by corn silage in dairy cow rations at the following corn yield level:

winter ration: 40.86 tons/hectare

fall ration: 46.61 tons/hectare

When Y_1 = 46.71, then fodder kale is partly substituted to fodder beet and when Y_1 = 58.67, we substitute entirely, corn silage for kale and beet in the heifer rations. However, in these solutions yearling bulls are always fed with beet, a minimum of hay not yet being introduced within corn rations in this model.

The series of solutions are quite alike. In general we have:

- winter wheat: 8.5 ha
- spring barley: 1.5 ha
- corn (grain): 3.8 ha
- pasture: 10.5 ha
- fodder cow crops: 5.0 ha
- hired labor: 0.0
- capital: about 8C,000 F

However, as X increases, the fodder beet acreage decreases from 2 ha to 0.7 ha to the benefit of corn for ensilage which goes from 1 ha to 2.5 ha. At the same time the number of yearling bulls increases. The livestock activities are:

- 20 dairy cows (the maximum)
- 5 heifers (the minimum)
- 4 to 10 yearling bulls.

In conclusion we can say that an increasing productivity of fodder corn doesn't change much the optimum farm plan even though its influence on farmers' income is relatively high in comparison with the acreage concerned. It is probably easy enough to get an extra production of 10 tons of corn silage per ha, but it is rewarding to try since, in this case, $\Delta \dot{I}_c \simeq 1{,}150$ F.

2. Acreage of permanent pasture

Let's define the following variables:

X = total acreage of permanent pasture

where

$3 \leq X \leq 25$

$\dot{I}_p$ = income

a. The solutions

Income

$I_p = 29{,}980.59 + 95.459\ X - 42.45\ X^2$, $R^2 = .995$. When X = 3, then there is the same acreage of permanent pasture as in the preceding solutions (paragraph a). The difference ($\dot{I}_p$ - $\dot{I}_c$) = 1406 F, being due to the introduction of new rations for yearling bulls in the model. The corresponding solution differs from the preceding one since we have:

winter wheat:	6.75 ha
corn (grain):	.65 ha
spring barley:	2.08 ha
corn (silage)	5.62 ha
yearling bulls:	23.0 heads
hired labor:	.10 units
capital:	91,600 F

The substitution of fodder corn for cereals is profitable when it is transformed by yearling bulls.

If I_p is quite high when x = 0, it decreases rapidly when X increases. The marginal revenue of X being negative and equal to:

$$I'_p = 95.459 - 84.90\ X.$$

Decreasing at the constant rate of 84.90 F per hectare, I'_p takes, rapidly, large negative values as X increases. In fact I_p decreases to 10,000 F when X goes from 0 to 16.587.

Sale of products (in francs)

- The sale of cereals (S_c) decreases rapidly at a rate of 618.962 F/hectare of X and becomes equal to zero when X = 25.889 since we get the following relationship:

$$S_c = 16{,}024.888 - 618.963\ X;\ r^2 = .999$$

- The sale of slaughter cows and steers (S_m) decreases rapidly ($S''_m = 110.252$), but never takes a zero value within the range of investigation since $S_m = 5{,}701.157$ when X = 25.0.

$$S_m = 44{,}032.939 + 1{,}223.028\ X - 110.252\ X^2;\ R^2 = .9976.$$

- The sale of milk is almost constant, however, and equal to: 32,543.6 F with s* = 1,042.78[1]. Milk sales are even slightly higher when $X \geq 15$: some milk is sold instead of being consumed by calves.

Livestock activities

- Dairy cows. They are almost always equal to the permissible maximum since the average number of cows is equal to: 19.62, s* = .71 and starts to decrease when $X \geq 23$.
- Yearling bulls, (Y_b). They constitute the main source of meat production. Being equal to 23 head when X = 3, they are still 20 when X = 13.8, but they disappear very rapidly afterward from the programming solutions ($Y_b = 0$ when X = 15.025) since we have:

$$Y_b = 24.60 - .33333\ X - 1.30393\ X_d;\ R^2 = .998$$

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[1] $s^* = \left[\sum_j (x_j - \bar{x})^2/n\right]^{.5}$.

$25 \geq X_d > 13.8$; 0 otherwise

- Other activities, when the yearling bull activity starts to decrease very rapidly, first choice steers are introduced into the programming solutions. But they are never numerous since seven steers are produced when X = 19.2. Afterward they disappear at the benefit of steer at livery: tilled land being used to produce the necessary fodders (beet, corn...) for dairy cows.

Crop activities

As X increases, the acreage of tilled land decreases the same amount. Among crops, the acreage of winter wheat (W.W.), temporary pastures (T.P.) and corn for ensilage decrease the most. Corn for grain (C.G.) disappears when X = 18.2. We get the following relationship (in hectares).

$$T.P. = 16.753 + .3444\ X - 4.654\sqrt{X};\quad R^2 = .9958$$

$$W.W. = 7.174 - .2787\ X;\quad r^2 = .9814$$

$$C.G. = .685 - .0129\ X - .0278\ X_d;\quad r^2 = .998$$

$$C = 9.72 - .3976\ X;\quad r^2 = .9958$$

where

C = total acreage of cereals

$25 \geq X_d > 13.8$; 0 otherwise.

Since W.W.' > C' the acreage of other cereals decreases also, but, however, winter wheat is the main cereal crop.

Input requirements

- Labor. Equal to .10 unit when X = 3, hired labor becomes practically negligible when X = 8.4. Such a farm is run by the farmer and his wife.

- Capital (K) as expected from the solution activity set, capital requirement decreases very rapidly when steers are left out, as shown below:

$$K = 91264.63 - 96.28\ X - 934.24\ X_d;\ R^2 = .9892$$

where

$$25 \geq X_d \geq 19.2;\ 0 \text{ otherwise}$$

b. Conclusion When the acreage of permanent pasture increases, income decreases more rapidly than can be expected at first. The value of the objective function decreases 50% when X = 23.35. The dairy activity dominates all traditional steer ones, although all these animals are mostly fed with grass and hay. Besides yearling bulls, only the non-traditional first choice steers and steer at livery are complementary to the dairy activity. The production of milk is, therefore, the most profitable activity which can transform the forage of permanent pasture. However, in the real world, farmers frequently substitute feeder steers for dairy cows to a large extent.

3. Yield level of temporary pastures and level of milk production per cow

a. Level of income Let's define the following variables:

Z = total annual forage yield in hundreds of fodder units

X = (Z - 30)

$\dot{I}_{ij}$ = income

where

i = 0 when cereals sale is excluded

i = 1 when cereals sale is allowed

j = herd type:

MA = Maine-Anjou: 2,850 liters of milk

N30 = Normande or Frisian: 3,000 liters of milk

N38 = Normande or Frisian: 3,800 liters of milk

Sale of cereals is excluded.

The following results were found:

$$\dot{I}_{0,MA} = 4,622.08 + 482.19\ X - 3.36\ X^2,\ R^2 = .998$$

$$\dot{I}_{0,N30} = 9,871.91 + 619.69\ X - 11.35\ X^2,\ R^2 = .998$$

$$\dot{I}_{0,N38} = 12,672.87 + 633.24\ X - 10.40\ X^2,\ R^2 = .998$$

We can observe that the difference of income is equal to 8,050 F between $\dot{I}_{0,MA}$ and $\dot{I}_{0,N38}$, when X = 0. Beyond this last value the difference increases slightly and becomes equal to 8,256 F when X = 20.0. Similarly an increase of forage production per acreage unit provides a substantial increase of income even though the cow herd type is kept identical. When X varies from .0 to 20.0 the corresponding increase of income is equal to:

M.A. = 8,300 F

N30 = 7,854 F

N38 = 8,504 F

It seems possible to increase farmers' income to about 16,550 F when they are helped to increase both the productiveness of their grassland (X = 0 to X = 20) and the production of their dairy herd (MA to N38).

If the value of $\dot{I}_{0,N30}$ is closer to $\dot{I}_{0,N38}$ than to $\dot{I}_{0,MA}$, it is partly due to our particular hypothesis relative to the purchase of the replacement stock. The production of three-year heifers has relatively high shadow prices. In the $I_{0,N30}$ related solution the smallest ones are equal to 400 F $\pm$ 30 F, according to different feeding programs. The corresponding shadow prices are almost twice this last value in the $\dot{I}_{0,N38}$ solution.

If we assume that we can extrapolate beyond X = 20.0 and that $\dot{I}_{ij}$ declines at a constant rate (quadratic function) the optimum levels of grassland production are equal to:

MA = Z = 101.75

N30 = Z = 57.29

N38 = Z = 60.44

If, on the contrary, we assume that $\dot{I}_{ij}$ declines at a diminishing rate (square root function) the related optima are equal to:

N30 = Z = 63.35

N38 = Z = 70.11

For MA we get such a high value that we think the extrapolation improper; the stage of large diminishing returns being not reached at $.0 \leq X \leq 20.0$.

Sale of cereals is allowed.

The following results were found:

$\dot{I}_{1,MA} = 17{,}243.63 + 319.45\ X - 4.819\ X^2;\ R^2 = .998$

$\dot{I}_{1,N30} = 20{,}592.56 + 395.04\ X - 5.640\ X^2;\ R^2 = .998$

$\dot{I}_{1,N38} = 22{,}203.06 + 441.69\ X - 6.030\ X^2;\ R^2 = .998$

When X = 0, the difference of income between $\dot{I}_{1,MA}$ and $\dot{I}_{N38}$ is equal to 4,960 F. This difference increases with X and is equal to 6,920 F when X = 20. For a given herd the difference of income increases also when X varies from 0 to 20.0. The income increment being equal to:

MA = 4,461 F

N30 = 5,645 F

N38 = 6,421 F

The difference in income between the two following situations:

(MA; X = 0)

and

(N38; X = 20.0)

is equal to 11,380.00 F. Therefore a substantial reward can be expected for those who are able to improve, at the same time, the productiveness of grasslands and dairy cows.

The optimum productions of temporary pasture are equal to:

- quadratic function	- square root function
MA = X = 63.14	X = 76.72
N30 = X = 65.03	X = 81.73
N38 = X = 66.62	X = 86.50

These results, as those of the preceding section, show that the optimum level of temporary pasture production lies between 5,500 and 8,500 fodder units per hectare. A set of experiments should be undertaken within this range of production in order to find the exact input-output relationship and to measure the corresponding risk increase, if any. However, it can be expected to be higher with high levels of forage production per hectare, especially when bad weather conditions occur. If that is the case, the overtime variability of yield increases with the yield level.

The two preceding sets of solutions ($\dot{I}_{0j}$ and $\dot{I}_{1j}$) differ mainly in four respects:

- The income differences are smaller for $\dot{I}_{1j}$ than for $\dot{I}_{0j}$, when the productiveness of grassland and dairy cows increase. Furthermore, $\Delta I_{1j}/\Delta I_{0j}$ ratios vary from about 50 to 70%.

- $\dot{I}'_{1j} \leq \dot{I}'_{0j}$; $0 \leq X \leq 49.67$.

The marginal income due to temporary pasture intermediate output is higher when cereal sales are excluded from the possibility set.

- $I''_{1j} \not\leq I''_{0j}$, except for $I''_{i,MA}$.

- The optimum level of grassland production is always higher when cereal sales are allowed.

b. <u>Optimum farm plans</u> Without cereals sale. For a given herd, when the production of forage per hectare is increased, the solutions differ by the value of certain variables, few are almost stable, the others increase. We will not give here in detail all solutions or calculate a series of equations. The mean and the root mean square of these variables will characterize, precisely enough, the influence of an increasing forage production per hectare on the related optimum farm plans.

From Table 40 we observe that the variation of X value has mainly an effect upon the following variables:

- income

- capital
- labor
- yearling bull
- ewe
- sale of steer

The other ones are almost constant, especially:

- fodder crop acreage
- dairy cow
- sale of milk
- steer at livery

The number of Normande or Frisian cows is almost maximum in every solution. However, the Maine-Anjou ones are economically dominated by other animals, the few cows[1] which are present in the solutions are necessary to supply milk to yearling bulls and to avoid a waste of forage in pasture. When X increases, the extra production of forage is allocated to yearling bulls and to the sheep flock. From the original solutions it can even be seen that the number of ewes starts to increase when the number of yearling bulls has reached its maximum.

To the more productive dairy herds are associated:

- a higher income
- a smaller requirement of capital and labor inputs
- a smaller production of yearling bulls and lambs

With cereals sale

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[1]In fact $\dot{I}_{i,MA}$ should be called $\dot{I}_{i,ewe}$ since the number of sheep is much higher than the number of dairy cows.

Table 40. Mean and root mean square of the main variables when X varies from 0 to 20.0, herd type held constant[a]

		Income (francs)	Capital (francs)	Hired labor (year)	Acreage of fodder crops (hectares)	Dairy cow (head)	Yearling bull (head)	Ewe (head)	Steer at livery (head)	Sale of milk (francs)	Sale of steer (francs)	Sale of cereals (francs)
$\dot{I}_{0,MA}$	$\bar{X}$	9,109	119,453	.99	19.66	8.02	34.60	54.86	5.86	4,579	61,605	.0
	S_x^*	3,415	5,856	.14	.124	.96	1.11	37.10	4.47	1,317	1,682	.0
$\dot{I}_{0,N30}$	$\bar{X}$	14,177	126,997	.75	20.92	20.0	32.38	21.53	.0	23,106	62,197	.0
	S_x^*	3,244	9,975	.09	.61	.0	3.8	20.65	.0	629.4	6,301	.0
$\dot{I}_{0,N38}$	$\bar{X}$	17,272	98,787	.53	21.26	19.63	19.79	7.04	.0	31,970	41,256	.0
	S_x^*	3,503	17,381	.23	.064	.51	6.7	9.96	.0	868.6	10,997	.0

[a] $S_x^* = \left[\sum_j^n (X_j - \bar{X})^2/n\right]^{.5}$.

The insertion of a cereal marketing activity into the programming possibility set has considerably modified the initial solution.

Similar to the preceding one, capital requirement as well as income increases with X. However, the two solutions differ mainly from one another in the following respect:

- The three following variables equal zero in the second solution: hired labor, ewe number and steer at livery
- The number of cows increase with X
- The production of cereals for sale goes up at the expense of the fodder crop production which was initially transformed by steer at livery, the sheep flock, cows and yearling bulls. This last activity is considerably reduced, but the number of cows is kept constant for $Z \geq 40.0$.
- Almost constant in every farm plan of the first solution, the acreage of fodder crops increases with the productiveness of the dairy herds, in the second solution. At the same time the acreage of cereals decreases, due to per hectare increasing returns of fodder crops.
- The second solution (Table 41) provides a higher income than the first one (Table 40) with significantly lower capital and labor requirements (from about 1/2 to 2/3 of the initial ones). The second plan is characterized by a more efficient allocation of resources.

However, if the two solutions differ in many respects, they have at least a common feature: none of the feeder and slaughter steers which are traditionally produced in this area are selected from

Table 41. Mean and root mean square of the main variables when X varies from .0 to 20.0, herd type held constant

		Income (francs)	Capital (francs)	Hired labor (man year)	Acreage of fodder crops (hectares)	Dairy (head)	Yearling bull (head)	Ewe (head)	Steer at livery (head)	Sale of milk (francs)	Sale of steer (francs)	Sale of cereals (francs)
$\dot{I}_{1,MA}$	$\bar{X}$	19,635	59,631	.0	10.94	8.84	13.73	.0	.0	9,668	27,126	35,334
	S_X^*	1,835	4,415	.0	.41	.82	1.39	.0	.0	856	2,720	634
$\dot{I}_{1,N30}$	$\bar{X}$	23,603	61,990	.0	13.57	18.04	11.90	.0	.0	23,731	27,200	30,310
	S_X^*	2,316	5,047	.0	.56	2.77	.90	.0	.0	3,929	1,991	1,141
$\dot{I}_{1,N38}$	$\bar{X}$	25,615	68,226	.0	16.32	18.14	4.11	.0	.0	31,615	14,250	25,746
	S_X^*	2,630	8,762	.0	.85	2.62	1.23	.0	.0	4,794	1,538	1,622

production possibility set and included within the optimum farm plans. These activity shadow prices are relatively high in almost every solution. They range from 200 to 780 F with N38 cow herd type and from 100 to 550 F with the Maine-Anjou herd, even though the sale of cereals is allowed and Z takes any value within the range 30.0 - 50.0.

c. Iso-revenue and necessary level of forage production to make up for a difference in dairy herd productivity Although it is impossible to get the same income with different herds when the level of forage production per hectare is optimum, it is however feasible, within a certain range, to make up for a lack of cow productivity with an increasing production of fodder units per hectare of temporary pasture.

Let us define:

- Z_0 as the level of grass production associated with the most productive herd
- Z_1 as the level of grass production associated with the less productive herd
- $\dot{I}_{ij} = a + bZ - cZ^2$

If we equalize $\dot{I}_{ij}$ with $\dot{I}_{ik}$; $k \neq j$, then we get $Z_1 = F(Z_0)$. This particular equation takes the following form:

$$Z_1 = \frac{-b_1 \pm [b_1^2 + 4a_1(a_0 Z_0^2 - b_0 Z_0 - c_0 + c_1)]^{.5}}{-2a_1}$$

However, in order to express $Z_1 = F(Z_0)$ in a more convenient fashion, successive values of Z_1 were computed in function of Z_0. The corresponding results were regressed on the following functions:

$$Z_1 = a + bZ_0 + c(Z)^5 \text{ or } Z_1 = a + b_{Z_0}^2 ,$$

whose first and second derivative are positive in the range with which we are concerned. There exists, however, one exception which occurs when we compare $\dot{I}_{0,MA}$ and $\dot{I}_{0,N30}$. But it has been stated above that the extrapolation beyond Z = 50.0 was improper for MA.

Excluded sale of cereals

To get $(\dot{I}_{0,N38} - \dot{I}_{0,MA}) = 0$ within the range of observations of $\dot{I}_{0,MA}$, it is necessary that $Z_1 = 49.288$ when $Z_0 = 30.0$. That is quite a large difference of forage productivity. However, the following iso-revenue $(\dot{I}_{0,N30} - \dot{I}_{0,MA}) = 0$ requires a smaller difference of forage productivity to make up for a lack of per head livestock production, the following Z values being associated:

$Z_0 = 30.0$ and $Z_1 = 41.868$

$Z_0 = 36.0$ and $Z_1 = 50.751$

The iso-revenue $(\dot{I}_{0,N38} - \dot{I}_{0,N30}) = 0$ requires a much smaller difference between Z_0 and Z_1 since

$$Z_1 = 255.899 + 9.4039\, Z_0 - 91.832\, (Z_0)^5;\ 30 \leq Z_0 \leq 40.876;$$

$$R^2 = .9988$$

This function increases at an increasing rate equal to $22.958\, Z_0^{-1.5}$ and takes the following extreme values we are concerned with:

$Z_0 = 30.0$, $Z_1 = 35.05$

$Z_0 = 40.876$, $Z_1 = 57.16$

Allowed sale of cereals

For $(\dot{I}_{1,N38} - \dot{I}_{1,MA}) = 0$ it is almost necessary that Z_0 and Z_1 take their permissible extreme values.

If $Z_0 = 30.0$ then $Z_1 = 54.811$

If $Z_0 = 30.76$ then $Z_1 = 62.764$

Even the comparison of N30 with MA leads to almost the same conclusion.

For $(\dot{I}_{1,N30} - \dot{I}_{1,MA}) = 0$, then:

$Z_1 = 624.545 + 24.507\ Z_0 - 240.479\ (Z_0)^{.5}$; $30.0 \leq Z_0 \leq 35.327$,

$R^2 = .9818$.

This function increases at a rate equal to: $60.119\ Z_0^{-1.5}$ and takes the following permissible extreme values:

$Z_1 = 42.332$ when $Z_0 = 30.0$

$Z_1 = 64.562$ when $Z_0 = 35.327$

The iso-revenue $(\dot{I}_{1,N38} - \dot{I}_{1,N30}) = 0$ shows that $(Z_1 - Z_0)$ takes quite small values when Z_0 is small but increases rapidly afterward. The following function:

$Z_1 = 14.3071 + .0219\ Z_0^2$; $30.0 \leq Z_0 \leq 45.14$, $r^2 = .9956$,

increases rapidly within the range of Z_0 since $Z_1'' = .0438$.

Extension workers know very well that it is easier and faster to increase the productivity of temporary or permanent grasslands than to improve the productiveness of livestock. But even so, it is rewarding to intensify any effort directed toward livestock and feeding program improvement. The preceding results show that it is fallacious to believe that an increasing productivity of fodder crops can make up for a lack of productiveness of the livestock herd.

d. <u>Farmers' income opportunities related to cow and pasture output level</u> . Large variations of income result from a variation of forage yield and animal output (Table 42). A lack of knowledge and technical management can reduce the level of income, since the smallest income is equal to 59.3% of the highest one.

Table 42. Farmers' incomes in francs and yield levels when sales of cereals are allowed

Dairy herd type	Pasture output (100 fodder units/hectare) 30	50
Maine-Anjou (MA)	16,590	21,050
Normande 3800 liters (N38)	21,560	27,960

C. Conclusion

Three important results arise from the preceding paragraphs.

(1) The most efficient use of resources call for somewhat diversified production plans with, at least, two main activities: dairy and cereals. The number of yearling bulls is small in solutions $\dot{I}_{ij}$ (various levels of cow and pasture output with sale of cereals allowed). However, they are more numerous in the programming solutions when rations with small hay requirements are allowed. The relative advantages of, and the obstacles to, specialization will be studied in the next chapter.

(2) The traditional slaughter and feeder steers which are commonly produced in this area are never present in the different optimum solutions. Yearling bulls are substituted instead.

(3) Farmers' income can be reduced to about 50%, when the acreage of permanent pasture occupy 2/3 of a 30 hectares farm, or when the level

of farmers' technical management is low in respect to forage and dairy production.

CHAPTER 19. DIVERSIFIED OR SPECIALIZED FARM OUTPUT?

Although the results of the preceding chapter show that the optimum farm plans are somewhat diversified, it is worthwhile to study the relative advantages of, and the obstacles to, specialization. Such a study will enable us to discover the relationship in the model as well as the degree of competition and complementarity between different activities. These results lead to the formulation of few research hypothesis.

A. Method and Particular Assumptions

1. Method

One or several products can be produced with the limited stock of resources available to the farmer. In the former case we get a specialized farm, in the latter one the farm output is diversified. In our model, extra building space can be bought, harvesting machines rented and farm labor hired, if profitable. The specialization path is therefore open. In order to study the influence of the degree of specialization in meat, milk and cereal production, these outputs are varied within the following range:

$$0 \leq X_i \leq b_i$$

where

X_i = sale of product i

b_i = upper limit of variation

2. Particular assumptions

The particular assumptions are those of the preceding chapter with two exceptions:

- yearling bull rations with minimum of hay and first choice steers are not included in the model

- the purchase of extra building space for cows and milking parlor are either allowed (specialization in dairy production) or excluded (diversification or specialization in cereals and meat production). In the latter case the permissible maximum number of cows is equal to 20.0.

B. Results

Let's define the following variables:

$\dot{I}_j$ = income related to the j specialization (in francs)

K_j = capital (in francs)

L_j = hired labor (number of men/year)

C_j = number of dairy cows

Y_j = number of yearling bulls

S_j = number of sheep

$(TC)_j$ = total acreage of cereals (hectares)

$(TP)_j$ = total acreage of temporary pasture (hectares)

$(RC)_j$ = total acreage of fodder row crop (hectares)

X_j = amount of sale of product j (1,000 francs)

where

j = b = specialization in beef production

= d = specialization in dairy production with purchase of dairy facilities

d_{20} = specialization in dairy production with no purchase of dairy facilities

= c = specialization in cereal production

1. <u>Main characteristics of specialization</u>

a. <u>In meat production</u> When X_b varies from 0 to 120 the sale of meat can be constituted of slaughter steers and/or yearling bulls and/or cull cows.

When X_b increases we get the following relationship:

Income:

$$\dot{I}_b = 19{,}831.35 - 400.90\ X_b + 3775.214\ (X_b)^{.5};\ R^2 = .988.$$

Within the range of observation, the income function takes its maximum when $X_b = 22.169$ (Figure 9). The corresponding marginal revenue is equal to

$$R_b' = -400.90 + \frac{1{,}887.607}{(X_b)^{.5}}\ .$$

Capital:

$$K_b = 60{,}642.79 + 557.554\ X_b + 6.949\ X_b^2;\ R^2 = .992.$$

This increasing function increases at a constant rate within the range of observation. A higher degree of specialization calls for a reduction in complementary production, especially in working capital requirements. Furthermore, steer activities have a positive need of investment capital. Specialization in beef production requires a large amount of capital: when X_b varies from 0 to 120 when K_b increases over three times.

Hired labor:

$$r_b = \begin{cases} -.2327 + .0173\ X_b;\ r^2 = .9958 \\ 0 \text{ when } X_b \leq 15 \end{cases}$$

Almost zero at the maximum value of R_b, L_b increases rapidly afterward.

Number of cows:

$$C_b = \begin{cases} 12.2; & X_b \leq 5.0 \\ 20.0; & 5.0 < X_b \leq 60.0 \\ -453.63 + 24.2529\ X_b - .45485\ X_b^2 + .00369\ X_b^3 - .00001\ X_b^4; & 60 \leq X_b \leq 120.0,\ R^2 = .974 \end{cases}$$

When X_b increases, cull cows are sold, instead of being left out, in the disposal activity. The profitableness of dairying increases as well as the value of C_b which reaches its upper limit in the programming solution. When $X_b = 120.0$, then the most productive cows are substituted by less productive ones. The latter do not require the replacement stock be raised on the farm (by assumption). Within the range: $60 < X_b < 120$, $\overline{C}_b = 17.69$ and $S_{C_b} = 1.432$, which is a small variation.

Yearling bulls:

$$Y_b = -4.788 + .5997\ X_b;\ r^2 = .9994.$$

One yearling bull is sold 1668 F. When X_b increases the additional meat sold consists mostly of yearling bull since (Y_b') (1,668) = 1,000.29 F. However, when $X_b = 110$, 28-month slaughter steers start to appear in the solution at an insignificant level ($\leq$ three heads), as well as, steers at livery.

Number of sheep:

$$S_b = \begin{cases} 61.5; & X_b \leq 5.0 \\ 0\ ; & X_b > 10.0 \end{cases}$$

When the profitableness of dairying increases with the value of X_b, then sheep are replaced by dairy cows.

Acreage of cereals:

$$(TC)_b = \begin{cases} 6.74 \text{ when } X_b \geq 110 \\ 16.14 - .0232\ X_b - .610\ (X_b)^{.5};\ R^2 = .9662. \end{cases}$$

An increasing production of yearling bull is undertaken at the expense of cereal production, since $(TC)_b'$ is negative. This substitution has been taking place up to its feasible upper limit. Furthermore, we observe, from the original set of data, that the total cereal acreage is minimum for $X_b \geq 110$. Winter wheat and spring barley are still in the programming solutions to satisfy the crop rotation requirements.

Acreage of temporary pastures:

$$(TP)_b = 9.143 + .0476\ X_b;\ r^2 = .962.$$

The total acreage of temporary pasture is directly related to X_b, while the acreage of fodder row crops increases less rapidly with X_b.

Fodder row crops acreage:

$$(RC)_b = 2.476 + .0652\ X_b - .00036\ X_b^2;\ R^2 = .937.$$

In short, we can say that an increasing degree of specialization in beef production requires an additional amount of resources, especially in capital and labor. We can distinguish three stages within the range of variation of X_b:

(a) dairy cows are substituted for sheep

(b) cereal production is replaced by yearling bulls

(c) Dairy cows and heifers are replaced by yearling bulls, slaughter steers, and steers at livery. We have not fully explored this stage; beyond a certain point it would not be realistic to do so.

b. <u>In dairy production (with allowed investment in building facilities</u> When X_d is varied from 0 to 70 we get the following results:

Level of income:

$$\dot{I}_d = 20{,}554.296 + 315.707\ X_d + 15{,}687.538\ X_{dd} - 4.9123\ X_d^2 - 242.9909\ X_{dd}^2;\ R^2 = .998$$

where

$$X_{dd} = X_d \text{ when } X_d \geq 65$$
$$= 0, \text{ otherwise.}$$

Within the above range of observation this function increases very slowly up to $X_d = 32.13$[1] value, at which point it has its maximum. Beyond this point it decreases very slowly up to $X_d = 55$, and very rapidly afterwards. We can note that the value of income is almost stable within a large range of variation of X_d.

Capital:

$$K_d = 72{,}557.31 + 26.867\ X_d^2;\ R^2 = .988.$$

Dairying is also a specialization which requires a large amount of capital. When X_d increases from 0 to 60, then K_d takes exactly three times its initial value. The reasons are similar to those enumerated for K_b.

Hired labor:

$$L_d = \begin{cases} -.5006 + .0135\ X_d + .0678\ X_{dd};\ R^2 = .998 \\ \quad \text{where } 65 \leq X_{dd} \leq 70;\ 0 \text{ otherwise} \\ 0 \text{ for } X_d \leq 45 \end{cases}$$

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[1]This function is very flat in a certain range (Figure 9). For the original data, the maximum is reached when $X_d = 41.181$.

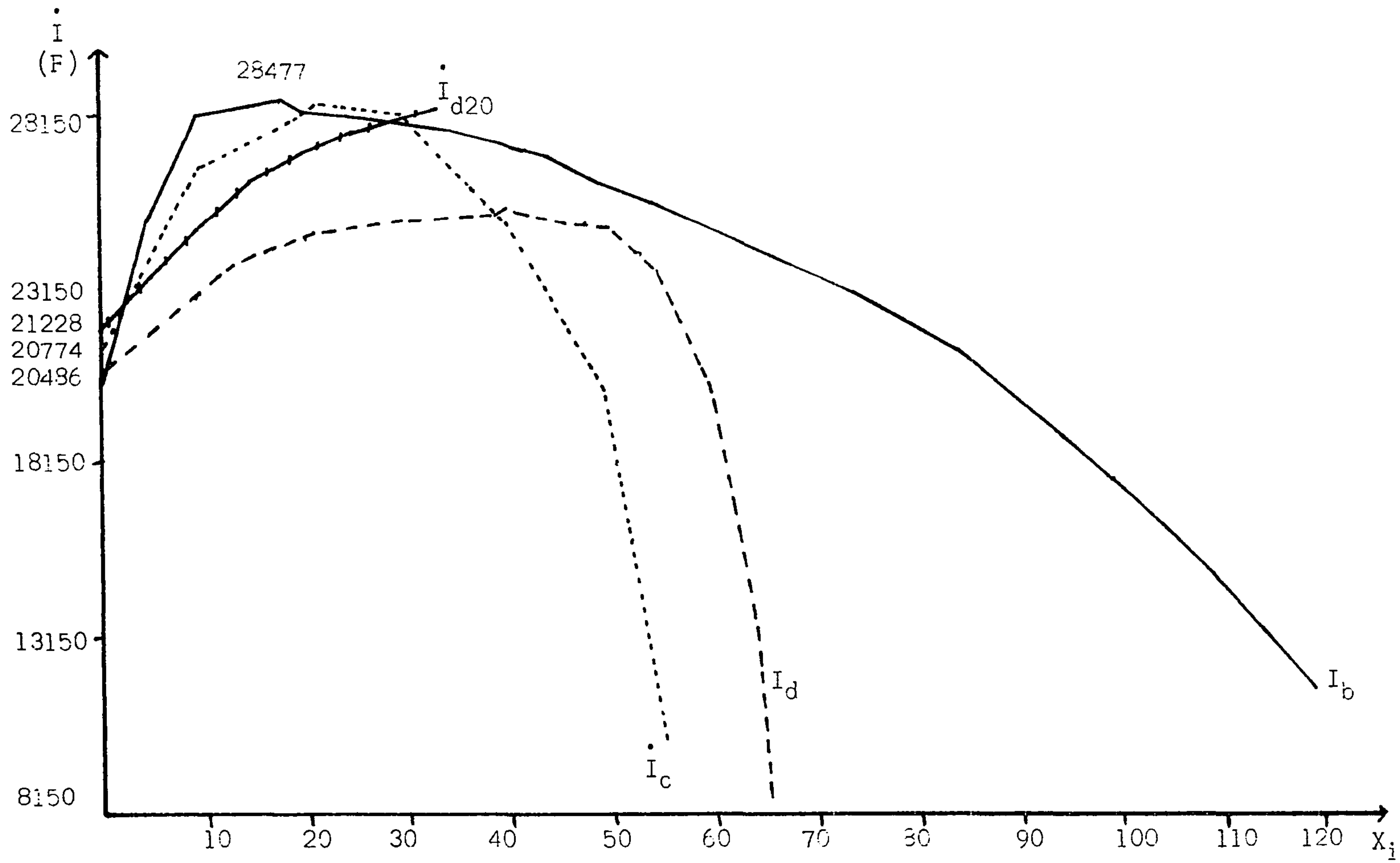

Figure 9. Specialization and income levels (original data)

Almost insignificant for $X_d \leq 65$, the labor requirements increase suddenly afterward, forcing I_d to become negative.

Number of dairy cows:

$$C_d = 3.46 + .279 X_d + .0052 X_d^2; R^2 = .986.$$

Since the total number of cows increases at a constant rate when X_d varies, we have to observe a distinction between less and more productive dairy cows. The dairy herd consists of the following types of cows: Normande or Frisian with 3,800 liters of milk per year (N38) and Normande or Frisian with 3,000 liters of milk (N30). C_d can be broken into the two following relationships:

$$C_{d,N38} = 2.041 + .515 X_d - .459 X_{dd}; R^2 = .998$$

$$55 < X_{dd} \leq 60; 0 \text{ otherwise}$$

$$0 \leq X_d \leq 60$$

$$C_{d,N30} = -1654.28 + 51.749 X_d - .393 X_d^2; R^2 = .994.$$

When $X_d > 55.0$, then $C_{d,N38}$ starts to decrease and $C_{d,N30}$ becomes positive when $X_d > 54.811$, In fact, the substitution of N30 for N38 is complete when $X_d = 65$.

Number of yearling bulls and sheep:

$$Y_d = 18.669 - .0973 X_d - .00799 X_d^2; R^2 = .972.$$

The number of yearling bulls decreases very rapidly and becomes almost equal to zero when $X_d = 41.18$ value at which $\dot{I}_d$ is maximum.

$$S_d = \begin{cases} 47.034 - 1.948 X_d; r^2 = .990 \\ 0 \text{ when } X_d > 10 \end{cases}$$

When an increasing number of cows is allowed in the programming solution, then the number of sheep decreases very rapidly.

Cereals acreage

$$(TC)_d = \begin{cases} 20.447 - .0909\ X_d - .00297\ X_d^2;\ R^2 = .996 \\ \quad \text{where } X_d < 55 \\ 6.749 \text{ when } 55 \leq X_d \leq 70 \end{cases}$$

The expansion of the dairy herd is made at the expense of the production of cereals. $(TC)_d$ becomes constant and minimum when the crop rotation constraints rule out any further substitution between dairy and cereal production. At the same time the acreage of temporary pastures and fodder row crops increase steadily as shown below.

Fodder row crops and pasture acreage:

$$(TP)_d = 4.326 + .1529\ X_d;\ r^2 = .895$$

$$(RC)_d = .9752 + .09148\ X_d - .0366\ X_{dd};\ R^2 = .857$$

$$\text{where } X_{dd} > 65.0.$$

In short, an increasing degree of specialization requires large amounts of capital, but however, labor requirements are kept low and minimum within a large range of variation of X_d. Within this same range the income function is very flat. The substitution of dairy production for other production is taking place in three stages:

(a) When $0 \leq X_d \leq 40$, the sheep flock and then the yearling bull herd are replaced by dairy cows. At the same time, the total acreage of cereals decreases.

(b) When $40 \leq X_d \leq 55$, dairy expansion is made at the expense of cereal production.

(c) When $55 < X_d \leq 70$ the preceding substitution becomes infeasible. The dairy herd is modified, and less productive cows, which do not require the replacement stock to be raised on the farm, are substituted for productive ones. At this stage income decreases very rapidly.

c. In dairy production (investment in dairy facilities is ruled out The maximum number of dairy cows is fixed at 20, by assumption.

We find the same type of relationship as in the preceding problem. What has been said about the first stage could be repeated here; we therefore omit it. We will give, however, four interesting relationships which will be used later, as we make some comparisons. They are:

$$\dot{I}_{d20} = 21,932.34 + 370.827 X_{d20} - 5.294 X^2_{d20};\ R^2 = .996$$

$$K_{d20} = 58,450.40 + 12.9 X^2_{d20};\ r^2 = .942$$

$$C_{d20} = 2.117 + .5212 X_{d20};\ r^2 = .996$$

$$\text{where } 0 \leq X_{d20} \leq 34.54$$

$$Y_{d20} = \begin{cases} 18.696 - .288 X_{d20};\ r^2 = .982 \\ 26.509 - .599 X_{d20};\ r^2 = .992 \end{cases}$$

d. In cereal production When X_c varies from 0 to 60.0 the cereals sold consist of winter-wheat, corn and barley. The following results are found:

Level of income:

$$\dot{I}_c = 21,755.456 + 420.8175 X_c - 2.66504 X_c^2 - .11876 X_c^3;$$
$$R^2 = .984$$

This function, whose maximum is located at $X_c = 25.54$, increases and decreases very rapidly before and beyond this point.

Capital:

$$K_c = 123,368.83 - 10,393.889 (X_c)^{.5};\ r^2 = .929.$$

An increasing degree of specialization in cereal production goes with a decreasing capital requirement, contrary to specialization in animal products.

Hired labor:

$$L_c = .750 - .10463\ X_c + .00928\ X_c^2 - .00026\ X_c^3;\ R^2 = .976.$$

Equal to .75 man when $X_c = 0$, this function decreases very rapidly afterward. After having reached a minimum at $X_c = 9.03$ and a maximum at $X_c = 14.76$, it becomes equal to zero when $X_c = 23.906$.

Number of dairy cows:

$$C_c = \begin{cases} 20 & ;\ 0 \leq X_c \leq 22.85 \\ 32.424 - .554\ X_c;\ r^2 = .996 & \text{where } 22.85 < X_c \leq 60.0. \end{cases}$$

Kept up to its feasible upper limit at first, the number of dairy cows decreases stadily beyond the point where the objective function takes its maximum.

Number of yearling bulls:

$$Y_c = 27.96 - .8695\ X_c + .3391\ X_{cd};\ R^2 = .931$$

X_{cd} is a dummy variable: $X_{cd} > 22.858$.

The number of yearling bulls decreases steadily from $X_c = 0$ to $X_c = 22.858$ (true maximum value of the observed data). It diminishes slightly afterward.

Number of sheep:

$$S_c = \begin{cases} 12.0 - 1.510\ X_c;\ r^2 = 1.0 \\ 0 \text{ when } X_c > 5.0 \end{cases}$$

As in the preceding solutions, the sheep flock disappears rapidly from the series of farm plans.

Acreage of cereals:

$$(TC)_c = 9.128 + .1190\ X_c + .00306\ X_c^2;\ R^2 = .992.$$

The total acreage of cereals increases at a constant rate. A close examination of the original set of data shows that the winter and spring barley acreage decreases steadily over the range of variation of X_c while the winter wheat and then the corn acreage increase throughout. As expected, cereals are substituted for temporary pastures and fodder row crops. We have:

$$(TP)_c = 11.422 - .00333\ X_c^2;\ r^2 = .972$$

$$(RC)_c = 6.371 - .109\ X_c;\ r^2 = .980$$

In short, we can say that an increasing degree of specialization in cereal production requires, for a given farm acreage, a decreasing requirement of capital and labor inputs. When X_c varies, the substitution of cereals for other products takes place in two stages:

(a) When X_c increases from 0 to 25.54, the point at which the income function takes on its maximum value, we observe that

- less hired labor is needed and becomes unnecessary at the optimum
- cereals are substituted for temporary pastures and fodder row crops. At the same time the number of yearling bulls decreases and the sheep flock disappears from the series of solutions. The winter wheat acreage increases first and then the acreage of corn.

(b) Beyond the optimum point, the same substitution is taking place within crops, In addition, the number of dairy cows starts to decrease, while the number of yearling bulls increases slightly at first, and then becomes equal to zero when $X_c > 40$.

Finally, for the first time, we observe that every labor constraint has a zero shadow price when $X_c > 40$. The most effective labor constraints are subperiods 22 and 24 (hay making). Specialization in animal products makes these constraints very effective.

2. Scarce resources and economic dominance among activities

In this model, the most limiting resources are land and labor whose scarcity is especially effective at the hay-making time (sub-periods 22 and 24) and to a lesser extent in the spring and fall seeding periods. The fall seeding coincides with the harvest of fodder row crops (fodder beet and corn). Due to these constraints, hay may have, in some circumstances, a very high shadow price. In order to understand what is happening within the model when we specialize the farm plans, we will try to classify (in original order) the value of the productivities of scarce resources with respect to each type of output.

a. Productivity of hay   Hay is always produced at its maximum level, except in cereal specialization. When labor is fixed at a certain level, the quantity of available hay is constant. We will show here that the productivity of hay is higher when it is used by cows and lower when it is consumed by yearling bulls.

First example: specialization in dairying. When $0 \leq X_d \leq 45$, 36.32 metric tons of hay are produced. But, after the disappearance of the sheep flock from the programming solution, yearling bulls are replaced by dairy cows at the following rate:

$\frac{-\Delta Y_d}{\Delta C_d} = \frac{.666}{.50} = 1.332$. The corresponding relationships are:

$Y_d = 27.798 - .666\ X_d;\ r^2 = .998$

$C_d = 2.580 + .5X_d;\ r^2 = 1.0$

$15 \leq X_d \leq 40$

At the same time the value of the income function increases slowly: $\Delta\dot{I}_d = 1{,}138.262$.

The same relationships are found when the investment in dairy facilities are excluded. We have:

$Y_{d20} = 26.50 - .5997\ X_{d20};\ r^2 = .996$

$C_{d20} = 2.117 + .5212\ X_{d20};\ r^2 = .995$

where $15 \leq X_{d20} \leq 34.537$

The corresponding value of the income function increases steadily since $\Delta\dot{I}_{d20} = 2{,}120.86$ F.

Second example: specialization in cereal production. When X_c increases from 0 to 22.85, the total production of hay decreases with the quantity of hired labor. However, at the same time, the number of dairy cows is constant while the number of yearling bulls decreases rapidly. The scarce hay input is allocated to cows in priority.

b. <u>Productivity of land</u> Specialization in dairy and beef production generates labor peaks and requires an additional amount of labor. However, the number of family workers can be higher on a farm than in our model and sufficient to supply the needed labor inputs. In this case, land becomes the most limiting resource since the purchase of building facilities is allowed. Under such an assumption, we will show that the productivity of land with respect to its use can be classified as follows:

Yearling bulls > dairy cows > cereals.

The productivity of land is higher with yearling bulls than with dairy cows.

When $X_d = 55$ and $X_b = 110$, then $(TC)_b = (TC)_d = 6.74$ hectares. The number of yearling bulls is, respectively, equal to 0 and 61.17 and the corresponding number of cows is equal to 31.0 and 16.0. The following equation:

$$(\dot{I}_b + 10{,}000\ L_b) - (\dot{I}_d + 10{,}000\ L_d) = \Delta\dot{I} = 6{,}559.88$$

gives the difference in value between the two income functions if, family labor were sufficient to satisfy the corresponding requirements. When yearling bulls are substituted for dairy cows the productivity of land increases since $\Delta\dot{I}$ is positive (10,000 francs is the per year cost of hired labor).

The productivity of land is higher with dairy cows than with cereals.

When X_c varies from 22.8 to 40, Y_c increases slightly (three yearling bulls, at most, in the observations) but C_c decreases rapidly. In the same interval cows are substituted for cereals.

$$C_c = 32.424 - .554\ X_c;\ r^2 = .996 \text{ and } L_c = 0.$$

Since the productivity of yearling bulls is higher than the productivity of cows, the replacement of 2.6 cows by three yearling bulls has a positive effect upon the value of the income function. But, however, the net result shows that dairy cows are more productive than cereals since $\dot{I}_c$ is negative in the corresponding interval.

We have ranked the different production in the following order:

Yearling bulls > dairy cows

Dairy cows > cereals

We should get, by the law of transitivity, the following relationship:

Yearling bulls > cereals.

The productivity of land is higher with yearling bulls than with cereals.

The following relationship:

$$(\dot{I}_b + 10{,}000\ L_b) = 29{,}344.35 - 227.90\ X_b + 3{,}775.214\ (X_b)^{.5}$$

has a maximum value when $X_b = 68.60$ and decreases slightly afterward. It can be recalled that $\dot{I}_b$ is maximum when $X_b = 22.169$. In the interval $5 \leq X_b \leq 60.0$ the number of dairy cows is constant and there exists only a substitution of yearling bulls for cereals. Beyond $X_b = 60$ the transformation of the production plan is more complex: the number of cows decreases, less productive ones are substituted for the more productive, the manual harvesting of fodder beet disappears, cereals for feed and bedding straw are bought. In the first stage ($X_b < 68.6$), when the substitution of yearling bulls for cereals is taking place, $(\dot{I}_b + 10{,}000\ L_b)$ increases as a result of the higher productivity of land when used for yearling bull raising. Beyond this point we reach a stage of complementarity between productions and the objective function decreases.

Complementarity of dairy and yearling bull productions.

If we exclude specialization in cereals, the maximum of hay is always produced. The shadow prices of the labor constraints in subperiods 22 and 24 are different from zero. But this maximum is reached if, and only if, some acres of grassland are first grazed, then cut for hay and finally grazed. The first grazing delays June hay cutting. When the maximum quantity of hay is produced, dairy cows (or sheep) have to be associated with yearling bulls since grass wastage is here unprofitable, and yearling bulls cannot be fed grass. However, the reverse is not true.

c. The action of the main economic forces within the programming model Although the productivity of land is higher when used for yearling bull production, a small acreage of land is allocated to it. With a limited amount of hay, too many acres of cereals (lowest productivity) would be associated to yearling bull production, if dairying were excluded from the programming solution. Out of one acre of land allocated to yearling bull production, about 2/3 can supply the required amount of hay (three cuttings). Since dairying is the second best activity is respect to land productivity, and is the best one with respect to hay input, it always belongs to the programming solution. Out of one acre of land allocated to dairying, about 1/5 provides cows with hay. Hired labor is one of the most costly inputs. Furthermore, labor is a flow resource. In this respect it is a tremendous force which acts upon the yearly allocation of work, flattening out, to a certain extent, the labor peaks. The production of fodder beet is not yet limited (the lowest shadow price per fodder unit), and therefore, can be allocated to yearling bull production. Consequently, the optimum solution is found on the following basis: the maximum of hay is produced and allocated to the two most profitable activities, dairying and yearling bull production. The former one, being able to transform a large set of fodder inputs into animal products, allows a good enough yearly allocation of work. On the contrary, the latter one, being only able to transform three fodder inputs, generates one tremendous labor peak at hay making time. A larger acreage of land will, therefore, be allocated to dairying, and a smaller one to yearling bull production. Cereals and temporary pastures are two joint products since the seeding of the latter one has to be done in or after a cereal. They are, therefore, two complementary outputs. But beyond this point, cereals constitute a set of supplementary enterprises since they are never competitive with animal and fodder crop productions, in respect to labor inputs. However, they are competitive with fodder crops, and economically dominated by animal products in respect to land input. Consequently, the production of cereals is always associated with the optimum farm plans. In order to flatten out the yearly allocation of work, they enter the optimum solution beyond their range of complementarity, in most cases. Their supplementary relationship with fodder crops is used.

3. Hypothesis for further research in beef and dairy production

The above results lead us to formulate two important and fruitful hypotheses:

a. The hay requirement of the livestock feeding program should still be decreased   Several ways are possible:

- Straw and corn silage are substituted for hay in some rations. In most of the subsequent computations a large quantity of hay has been substituted by corn silage (see Chapter 10, Part 3) in the yearling bull rations. But further research has to be pursued on the minimum of hay which can be associated with other fodders, without risks.

- Less costly, and more efficient, means of hay harvesting are found and used in order to reduce the labor requirements per ton of harvested hay.
- Finally, the hay making time is spread over a longer period using several types of pasture management, different varieties and/or species of grass and legumes and different methods of harvest such as artificial drying. Our model makes use of these first two opportunities, but in order to make more use of it, further research has to be done.

b. Different feed shadow prices are not identical When the value of the income function is maximum (I_{d20}, $\dot{I}_c$, $\dot{I}_b$), we get the following shadow prices per fodder unit:

hay:	.734 F
kale:	.233 F
fodder beet:	.233 F
corn silage:	.361 F
grass silage:	.538 F
pasture (first period):	.319 F
pasture (second period):	.361 F

In winter, the traditional feeder steers receive only a minimum quantity of hay. Consequently, their daily liveweight gain is very low. The preceding results lead us to formulate a new slaughter steer activity. To the poor wintering rations, we add a certain amount of fodder beets and substitute some corn silage for hay in order to get a very high daily liveweight gain whose level is maintained in spring but not in summer. These first choice steers are sold in spring (March or April), at which time beef price is the highest. This activity has been defined more precisely in Chapter 15, part 2, and will be included within the model in most of our subsequent computations.

4. Future and limits of specialization

In the industrial sector, the division of labor has led to the specialization of workers. But even if a firm is specialized in the production of some product, the stream of output is often continuous. In almost every case labor requirements are constant over time. On the contrary, in agriculture the firm size is such that the division of labor is impossible. Furthermore, firm specialization in reducing the number of products generates some important labor peaks even with more use of machinery. Unless farmers have the opportunity of hiring seasonal labor or getting an off-farm job, specialization reduces their level of income. Moreover, too high a degree of specialization in animal products requires large amounts of capital. The addition of cereal production to the farm plan decreases the requirement of capital and improves income. In addition to these advantages, a diversified farm plan is less risky.

If it is profitable to diversify the output of a farm, it is probably difficult for a farmer to be very efficient in every farm sector, especially, when his level of technological knowledge is not very high. He can have some difficulty in getting the input-output relationships previously defined when the farm plans are too diversified. In this case, the association of two main productions: cereals + dairy or yearling bulls + cereals would be the best choice. If it is possible to buy a certain quantity of hay on the market each year, then the specialization in milk production is feasible and provides a relatively high income. Unfortunately, it requires a too large amount of capital equity for many farmers. In this respect it is interesting to compare the farm plans in Table 43. Specialization in dairying is obviously less

Table 43. Variation of income and capital requirements in three farm plans

Variable name	With investment	Without investment	
X_{d20}		15	34.5
X_d	55		
K	153,267.0	61,627.0	76,059.0
L	.31	.0	.05
C	31.0	10.0	20.0
Y	0.0	18.0	6.3
(TC)	6.74	18.8	13.3
I	23,038.0	26,491.0	28,477.0

profitable and requires over two times the capital of the plans without investment.

Specialization in cereal or beef production reduces income considerably and is only feasible with larger farm size or part-time farming. But such opportunities are very limited in this area.

C. Conclusion

Diversified farm plans are undoubtedly the best ones in every respect. The association of cereals with one or two animal activities is the most profitable one. For a given farm size, the improvement in income can only be reached if it is possible to find a means of making profitable a further replacement of cereals by livestock. Such a substitution would bring about a shift of the income functions I_j, $[I_j = f(X_j)]$. But to make this substitution profitable more research has to be undertaken. Decreasing the labor requirements of forage crops and hay making, improving the input-output relationships of fodder crops, reducing the livestock minimum requirements or finding a good substitution for hay, diminishing the cost per head of building facilities are the main directions of research which are the most promising. Furthermore when the ratio, family labor/farm acreage, increases, the corresponding farm plan will be mainly oriented toward the production of milk and meat.

CHAPTER 20. EFFECT OF PRICE VARIATION ON FARMERS' INCOME AND CORRESPONDING OPTIMUM FARM PLANS

The level of farmers' income varies with many factors. We have already studied the influence of a few of them. But, among all these factors price variation is one of the most important. The price level determines income and optimum farm plans. When prices vary to a large extent, optimum farm plans are very unstable and the degree of uncertainty which is generated complicates the decision-making. In this chapter we will study the influence of price variation on income level and farm plans. In the first part we state the particular assumptions. The second part will be devoted to the influence of seed grass price variation, the third part to beef and milk price change, the fourth part to beef, milk and cereal price variation, and the fifth part to milk price and forage yield variation.

A. Particular Assumptions

In addition to the general assumptions, we make the following specific assumptions:

- Some activities are excluded from the activity set of the programming model. They are:
    - potato
    - slaughter calf production
    - egg and broiler production
    - hog and weaned pig production
    - purchase of feeder steers
    - sale of straw.

Furthermore, in parts three and four, first choice steer and yearling bull rations with a minimum of hay will belong to the activity set. The purchase of hay, although profitable[1], is excluded, its market being limited. In part four, when the selling price of cereal is varied, the purchase of grain for feed is excluded.

- Capital is available in unlimited amounts, as well as labor.
- The farm size is 30 hectares. However, when we vary the price of rye-grass seed, the solution is valid for farms whose acreage ranges from 15 to 60 hectares.

B. Influence of Grass Seed Price Variation

The production of grass seed and especially of rye-grass seed is traditional in this area. But new production methods were recently adopted. At the same time the production of tall fescue seed was introduced in certain farm plans.

1. Tall fescue seed

At a price of 3.8 F per kilogram of seed, about five hectares are allocated to tall fescue (Table 44). As the price of seed decreases we observe the following changes:

- Tall fescue and cereals for feed are replaced by winter wheat and temporary pastures.
- The number of sheep increases
- Farmers' income decreases.

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[1] $I = 30{,}488.703 - 416.610\ X + 18.218\ X^2$; $R^2 = .968$ where X = (price of hay - 18 F/100 kg), I = income.

Table 44. Present price of tall fescue seed and optimum farm plans

Items	Without investment	With investment in dairy facilities
Income (F)	31,973	29,425
Capital	106,166.0	141,962.0
Fodder row crops (ha)	3.108	3.204
Cereals (ha)	9.793	8.356
Pastures (ha)	12.248	13.677
Tall fescue (ha)	4.842	4.752
Dairy cows (head)	20.0	26.08
Sheep (head)	23.10	.0
Yearling bulls (head)	22.64	15.95

These effects upon the optimum solutions are similar under both assumptions relative to investment in dairy facilities.

Define the following variables:

$\dot{I}_j$ = income

F_j = acreage of tall fescue

S_j = number of sheep

X_j = decreasing price (%) of seed

where j = d when dairy facilities are allowed

= blank otherwise

Then we get the following relationships:

$$\dot{I}_d = 29{,}391.92 - 108.01\ X_d + 2.69\ X_d^2;\ R^2 = .954$$

$$I = 31{,}996.26 - 116.54\ X + 2.24\ X^2;\ R^2 = .996$$

Within the range of observations:

$0 \leq X_d \leq 31.0$ and $0 \leq X \leq 27.68$ the total variation of $\dot{I}_j$ is small since we have:

$$\Delta \dot{I}_d = 758\ F$$

and ΔI = 1,510 F

Furthermore, the solution is unstable: F_j becomes equal to zero for a small price variation of seed since

$$F_d = 0 \text{ when } X_d = 17.765$$

$$F = 0 \text{ when } X = 28.10$$

The corresponding relationships are equal to:

$$F_d = 4.734 - .015\ X_d^2;\ r^2 = .998$$

$$F = 4.901 - .17777\ X - .0012\ X^2;\ R^2 = .994$$

At the same time the number of sheep increases rapidly enough as shown below:

$$S_d = -252.68 - 11.051\ X_d + 115.985\ (X_d)^{.5};\ R^2 = .996$$

Conclusion: Although this production belongs to the original farm plan, its influence on the level of income is almost insignificant. A price decrease of 0.6 to 1.0 F per kilogram brings about its disappearance from farm plans. But price uncertainty is not the most troublesome one. Yield variability is large since the yield level varies from 200 to 1,000 kilograms per hectare. Under these conditions it seems wise enough to withdraw this production from optimum farm plans. Furthermore, its substitutes are easily found. The production of rye-grass seed and winter wheat being among the most important.

2. Rye-grass seed

The price of seed has been varied from 1.60 F to 1.12 F per kilogram, which corresponds to a price decrease of 30%. Before setting forth our results let's define the following variables:

I = income (F)

S = farm size (ha)

G = decreasing price in percent of rye grass seed

(RG) = acreage of rye grass

(WB) = acreage of winter wheat and spring barley

(OC) = acreage of spring oats and corn

(TP) = acreage of temporary pasture.

We found the following results:

a. <u>Income</u> The following function gives the level of income for different prices of seed and farm size:

$$\dot{I} = 6547.644 + 31.141\ G + 1265.485\ S + .505\ G^2 - 6.536\ S^2 - 4.450\ GS;$$

$$R^2 = .990$$

When the price of seed decreases, income decreases, whatever the size of the farm. But the corresponding loss becomes more important when the farm size increases. The following function decreases more rapidly when S is large since $\partial I/\partial G = 31.141 + 1.01\ G - 4.45\ S$. The extreme losses ($\Delta I$) are equal to:

ΔI = 6,622 F when S = 60 hectares

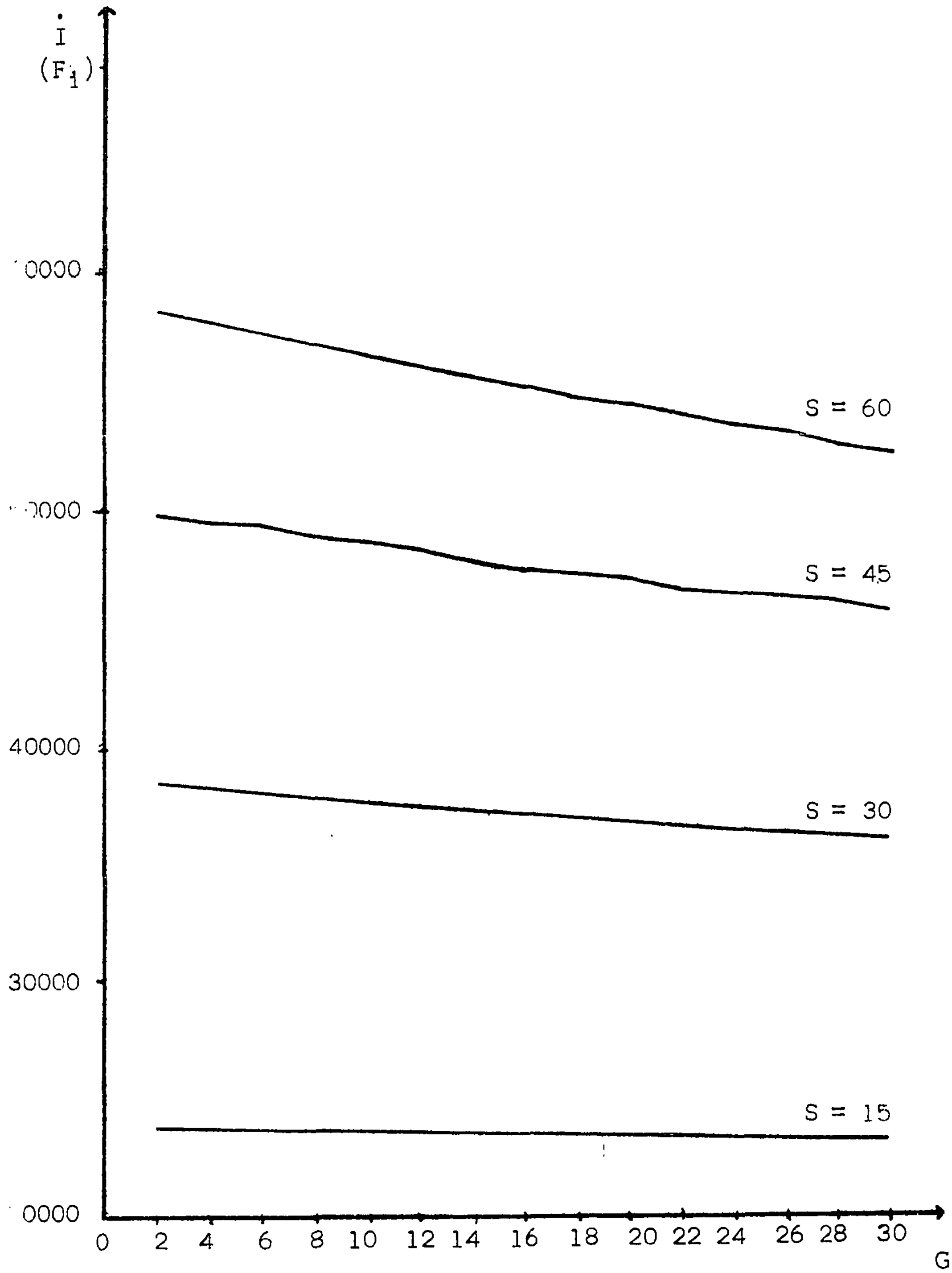
and ΔI = 4,629 F when S = 45 hectares

= 2,617 F when S = 30 hectares

= 614 F when S = 15 hectares

The influence of farm size on income is very important. It is an increasing function of S (Figure 10). The following function is always positive, whatever the values of S and G within the range of observations:

$$\partial I/\partial S = 1265.485 - 13.072\ S - 4.45\ G.$$



'igure 10. Level of income in function of farm size and decreasing price of seed

But, however, $\partial^2 \dot{I}/\partial S^2$ being negative, the function I is concave downward and increases at a constant rate. When G = 0 the corresponding incomes are about equal to:

58,900 F when S = 60.0

50,600 F when S = 45.0

35,900 F when S = 30.0

25,300 F when S = 15.0

b. Acreage of rye-grass Although the acreage allocated to rye-grass for seed decreases when the price of seed goes down, this production never disappears from the optimum farm plans, at least within the range of our observations.

$$(RG) = -2.720 + .00348\ G + .27782\ S + .00036\ G^2 - .00085 S^2 - .00158\ GS$$

$$R^2 = .994$$

The larger is the farm, the larger the decrease in rye-grass acreage when the price of seed falls.

$\partial(RG)/\partial G$ takes a more negative value when S increases.

$$\partial(RG)/\partial G = .00348 + .00072\ G - .00158\ S$$

Therefore, for each farm size, the corresponding functions decrease at a constant rate and are concave upward. When G varies from 0 to 30, then:

Δ (RG) = -2.41 ha when S = 60.0 and

Δ (RG) = -.28 ha when S = 15.0

S determines, to a large extent, the acreage allocated to the production of seed. RG is a function of S and is concave downward. When G = 0, then RG takes the following values:

10.88 ha when S = 60

8.05 ha when S = 45

4.84 ha when S = 30

1.25 ha when S = 15

c. <u>Crops substituted for rye-grass</u> When the acreage allocated to the production of seed decreases we observe three main substitutions:

- The acreage of temporary pastures increases

$$TP = 8.891 - .03871\,G - .302615 + .00078\,G^2 + .004855^2 + .00225\,GS$$

$$R^2 = .774$$

This function has a positive slope with respect to the variable G, except in a very small area where G and S take their minimum values. The partial derivative $\partial(TP)/\partial G$ is negative when G = 0 and S = 15; but becomes positive when G = 3.11 and S = 15 or G = 0 and S = 17.20. That is a very insignificant area in our range of investigation.

The larger the farm, the greater the increase in temporary pastures acreage when the price of seed decreases. The value of $\partial(TP)/\partial G$ is larger when S is higher.

$$\partial(TP)/\partial G = -.03871 + .00156\,G + .00225\,S.$$

The acreage of winter wheat and spring barley increases in most of the area of investigation.

$$(WB) = .1339 + .03082G + .251238\,S - .00437\,G^2 - .00155S^2 + .00557GS$$

$$R^2 = .968$$

This function has always a positive slope, $\partial(WB)/\partial S$, when G is considered as a constant, but the partial derivative $\partial(WB)/\partial G$ is not positive in the whole area of investigation.

$\partial(WB)/\partial G = .03082 - .00874\ G + .00557\ S.$

$\partial^2(WB)/\partial G^2$ being negative, when the price of seed decreases (WB) increases, reaches a maximum and then decreases in every farm whose size is smaller than 41.53 hectares. The maximum of this function being located on the following boundary:

$S = -5.533 + 1.569\ G$

The acreage of oats and corn decreases

$$(OC) = -3.374 + .01752\ G + .31010\ S + .00269\ G^2 + .00065\ S^2 - .00638\ GS$$

$$R^2 = .964$$

As (WB), this function always increases with S when G is considered as a constant. The partial derivative $\partial(OC)/\partial G$ is not negative in the whole area of investigation. $\partial^2(OC)/\partial G^2$ being positive, OC reaches a minimum for every farm size smaller than 28.03 hectares. This minimum is located on the following boundary:

$S = 2.746 + .8432\ G.$

However, in a very large part of the area investigated, $\partial(OC)/\partial G$ is negative and (OC) decreases more rapidly when the farm is large in size.

Having stated the most important feature of these results, we now try to explain the stability of these production plans and especially the presence of rye-grass, whatever the price of seed within the range of investigation. The seed yield is high and its value lies between 2240 F and 1680 F (1980 and 1845 F for winter wheat). To seed production, it is necessary to add a certain number of fodder units whose per unit shadow price is equal to:

-.25 F in period one

-.45 F in period two.

The corresponding production per hectare of rye-grass is 2000 and 1100 fodder units in periods one and two, respectively. The value of this production is not affected by G, but it would be by the price of milk and beef. This additional forage production partially explains the stability of rye-grass and the instability of tall fescue in the optimum farm plans when G varies, the forage production of the latter being almost null. A second factor of stability is the range of supplementarity of this production with the livestock activities with respect to labor requirements.

The crop substitutions which have been observed can be explained easily. In the preceding chapter we have shown that certain livestock activities dominate the production of cereals whose acreage is always minimal in the programming solutions. Here, we observe that the production of milk and yearling bulls is almost constant. A decreasing acreage of rye-grass reduces the production of forage. To compensate for it, an increasing acreage of temporary pasture and spring barley, its companion crop, is grown. Spring oats is replaced by spring barley and corn. This last crop is also associated with rye-grass for weeding problems; it also decreases with it. Winter wheat, being one of the most profitable crops among cereals, is partly substituted for rye-grass and corn. Since the production of cereals increases rapidly with farm size, the above substitutions are made on a larger scale when the farm is larger.

The production of rye-grass seed is profitable and resists price reduction well. The influence of price on income increases rapidly with farm size. The corresponding necessary adjustments can be made easily and rapidly. It is a production to insert in many farm plans when the production contract can be obtained. Unfortunately, their number is somewhat limited. The profitableness of this production explains its rapid extension in this region.

C. Influence of Beef and Milk Price Variation

The corresponding prices are varied within the following ranges:

First grade steer: 5.65 - 6.85 F per kilogram

Milk: .40 - .50 F per liter

Such a large variation has been chosen since beef and milk are the two most important farm outputs of this area. When their price is varied we get the following results (assuming the present level of price of cereals).

1. Results

Before reporting our results, let us define the following variables:

I = income (francs)

M = price of milk (francs)

B = price of beef (francs)

C = number of cows

Y = number of yearling bulls

S = number of steers

E = number of ewes

L = labor (man year)

K = capital (francs)

SC = sale of cereals (francs)

We now define a few variables which are functions of M and B. The corresponding equations define boundaries within the price map and delimit the areas in which the corresponding activities are present or absent in the optimum farm plans. They therefore define the zero level of the corresponding activities.

OE = zero level of the ewe activity

OS_{30} = zero level of the 30-month steer activity

OS_{28} = zero level of the 28-month steer activity

OY_c = zero level: yearling bull fed with corn silage

OY_b = zero level: yearling bull fed with fodder beet

OY_{cc} = zero level: yearling bull fed with corn silage + cereals

OS_m = zero level: milk selling activity

OS_c = zero level: cereals selling activity

a. <u>$\dot{\text{I}}$ncome</u> The following equation gives the level of income:

$$\dot{I} = 956{,}568.958 + 224{,}393.145\ B + 635{,}886.417\ M - 827{,}938.453\ B^{.5} - 325{,}679.358\ (BM)^{.5};\ R^2 = .994$$

Expressing M as a function of B we get a family of iso-revenue curves. They are given in Figure 11. A careful analysis of this figure shows that:

- A certain income increment ΔI requires a smaller variation of B when the price of beef is higher and the price of milk at its present value.

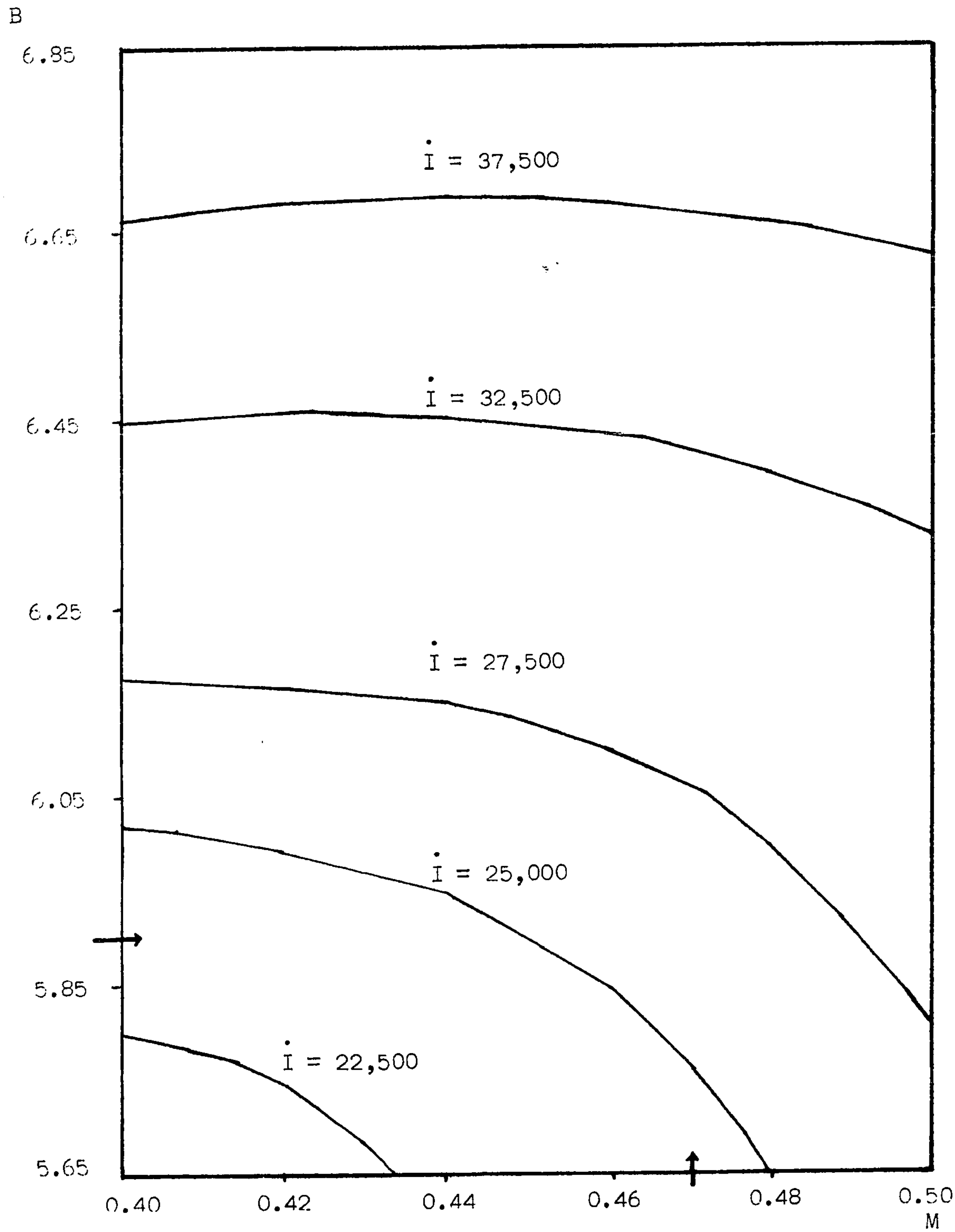


Figure 11. Income under different price situations
(↑ : Present price)

- The level of income is almost independent of the price of milk when the price of beef is high enough and is about equal to 6.30 F.
- Given the present price situation, it is necessary to have a large enough variation of price to increase income by a certain amount, say 2,500 F. It is likely that the price situation will never move toward a very high price of beef, the ambitious individual, who wants to improve his income by, say, 10,000 F, has to look for other means. Probably one of the best is to obtain more resources.

b. Price situation and corresponding activities When the price situation varies we observe certain modifications in the optimum farm plans. Few activities move in and out of the optimum solutions. The boundaries on which these changes occur are defined below and shown on Figure 12.

Dairy cows are always present whatever the price of beef and milk. In a certain area the sale of milk becomes equal to zero and cows are associated with the production of yearling bulls; they only supply the milk input to calves. Everywhere else, milk is sold.

$OS_m = -291.947 - 37.65769\ B + 228.365 \sqrt{B}$; B, $R^2 = .996$

where B means: $X_j > 0$ below the curve.

Yearling bulls are nearly always included in the production plans. They are excluded from them in a very small area (Figure 12). For different price situations different feeding programs are chosen. Yearling bulls with a maximum of corn silage are present between the two following curves:

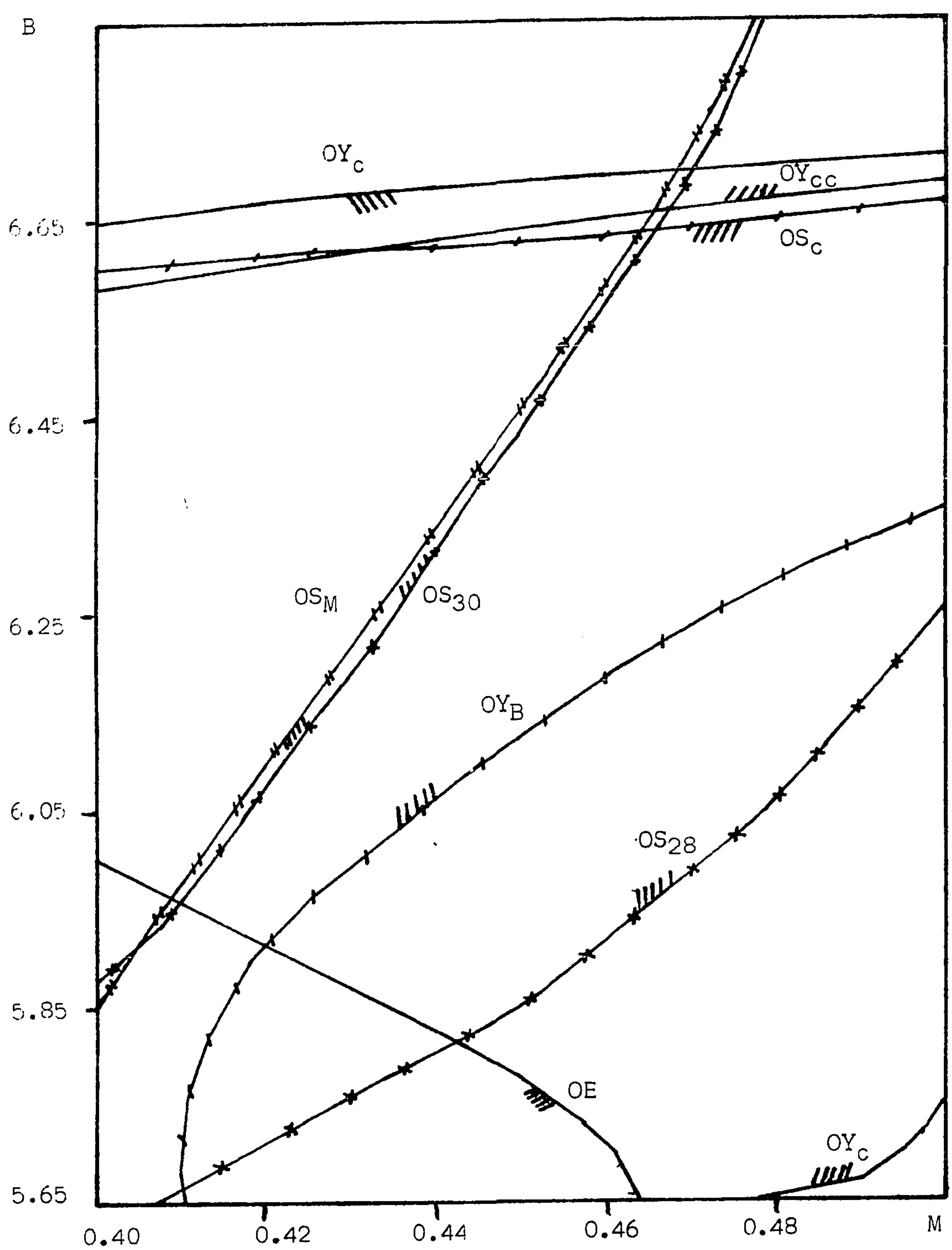


Figure 12. Present price situation and profitable activities (____ area in which the activity is present)

$$OY_c = 1{,}310.11 + 86.492\ B + 300.147 \sqrt{B};\ B,\ R^2 = .946$$

$$OY_c = -1482.54 - 225.44\ B + 1180.177\ B^{.5};\ A,\ R^2 = .996$$

where A means: $X_j > 0$ above the curve.

When, for a high price of beef, yearling bulls fed with the maximum of corn silage disappear from the optimum solutions they are replaced by others whose rations contain a lower quantity of corn silage and more cereals. This substitution takes place on the following boundary:

$$OY_{cc} = -791.078 + 125.984\ B;\ A,\ r^2 = .514$$

Yearling bulls fed with fodder beet are also present in the solutions, in a large area.

$$OY_b = 3{,}327 + 574.48\ B - 2{,}749.025 \sqrt{B};\ A,\ R^2 = .996$$

Once more, these results show that it is profitable to have a minimum of hay and cereals within rations in the present price situation.

First choice steers are present in a large area. The 28-month ones appear at a lower price of beef than the 30-month steers.

$$OS_{28} = -1639.11 - 254.17\ B + 1311.42\ B^{.5};\ A,\ R^2 = .998$$

$$OS_{30} = -524.99 - 74.25\ B + 413.17\ B^{.5};\ A,\ R^2 = .994$$

Ewes belong to the optimum solutions only when the prices of milk and beef are low.

$$OE = -1{,}445.37 - 300.723\ B + 1{,}342.95 \sqrt{B};\ B,\ R^2 = .964$$

The sale of cereals disappears when the price of beef is high.

$$OS_c = -1187.15 + 125.4\ B + 155.49\ B^{.5};\ A,\ R^2 = .749.$$

In this region, the production plans are very stable. Whatever the price situation, milk and beef outputs are in the optimum solutions. If the production of yearling bulls is excluded from the possibility set, they will probably be replaced by first choice steers. The optimum farm plans differ, however, in the proportion of steers and cows when the prices vary.

c. Level of activities Dairy cows and yearling bulls are two activities which are affected the most by price changes. The others, particularly crop activities, are adjusted to the level of these two main productions.

Yearling bulls. The number of yearling bulls increases very rapidly with B; the partial derivative, with respect to the price of beef, of the following function being always positive. When $B > 6.02$, Y starts to increase at an increasing rate.

$$Y = -14999.668 + 7696.271\ B - 1700.339\ M - 1291.78\ B^2 + 71.496\ B^3 + 251.487\ BM;\ R^2 = .970$$

The iso-product curves (Figure 13) show that:

- the interaction effect becomes less important when B increases. As for income, the price of milk has almost no effect upon the number of yearling bulls when $B = 6.30$.
- The price of milk determines the number of yearling bulls when B is low. But to change this number by 10, in the present price situation, the price of milk has to vary more than is expected in the near future.

Dairy cows

$$C = -954.655 + 138.279\ B + 2888.888\ M^{.5} - 1094.983\ (BM)^{.5};\ R^2 = .902$$

The number of cows increases as the price of milk increases and price of beef decreases. Even with a small variation in the present

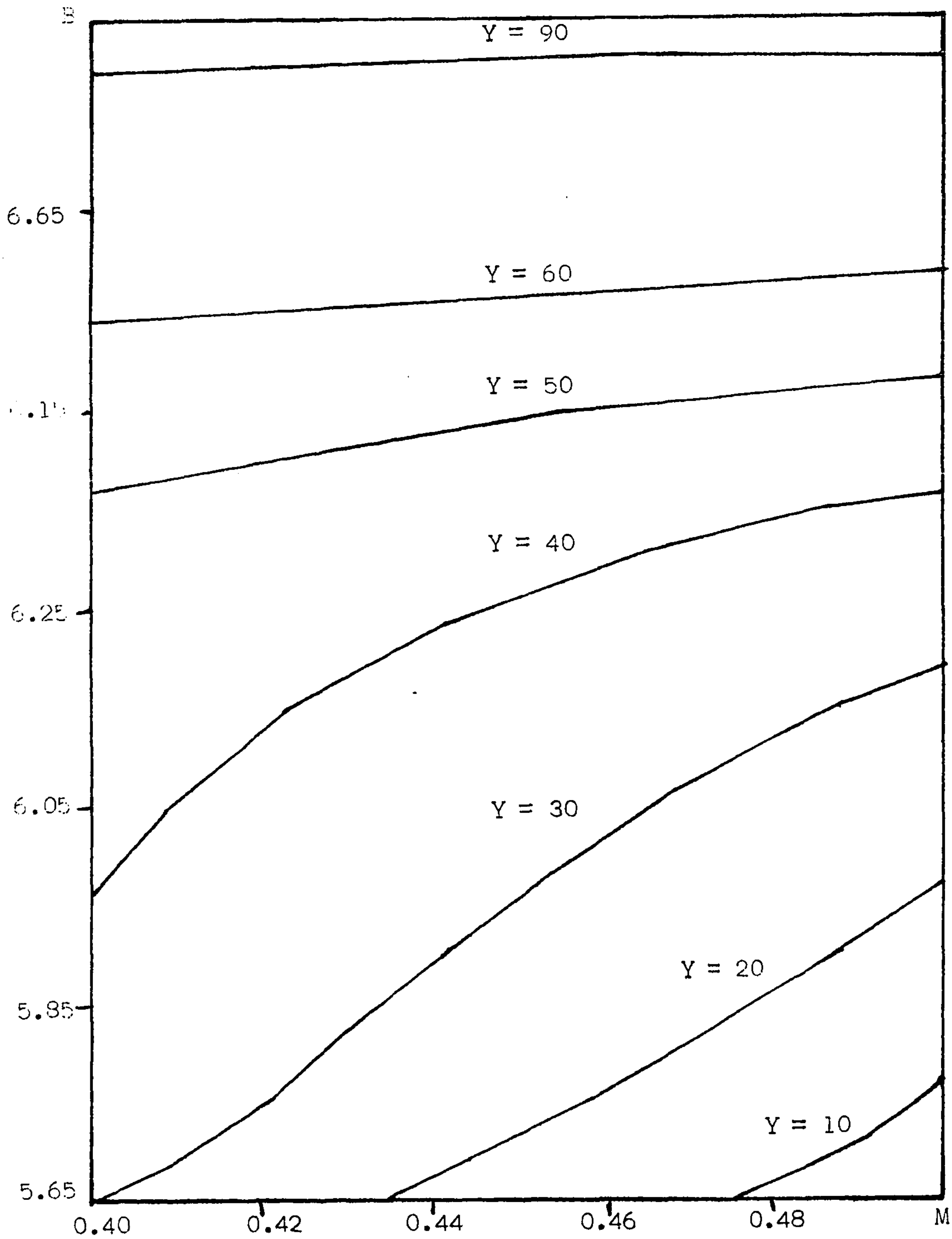


Figure 13. Price situation and number of yearling bulls

prices, the number of cows and yearling bulls is respectively equal to about 18 and 25 (Figures 13 and 14).

First choice steers

$$S = -1922.015 - 357.961\ B - 805.49\ M + 1544.436\ B^{.5} + 396.784\ (BM)^{.5};$$
$$R^2 = .705$$

The number of steers decreases as M increases, except in a small area located above the curve: $B = 16.484\ M$ (upper left corner of the figure). This number increases and then decreases as B increases. The total number of first choice steers is limited, however, and varies from 0 to 7.0.

Ewes

$$E = 22029.308 - 1659.035\ B + 28334.943\ M - 66430.866\ \sqrt{M} +$$
$$12012.427\ \sqrt{BM};\ R^2 = .876$$

The number of ewes decreases very rapidly when M and/or B increase. The maximum number of ewes being equal to 33.76 when M and B are minimum.

Sale of cereals

$$S_c = 644{,}189.719 - 99344.91153\ B - 1{,}530{,}919.3912\ M^{.5} + 604{,}599.7926$$
$$(BM)^{.5};\ R^2 = .896$$

The sale of cereals always decreases as B increases. A larger quantity of grain is fed to the increasing number of yearling bulls and steers and the acreage allocated to the production of grain diminishes. On the other hand, $\partial(S_c)/\partial M$ is negative when $B < 6.411$ and positive when $B > 6.411$. But at this price the sale of cereals is not far from being equal to zero (Figure 12). The total amount of sale varies from 23,575 F to zero.

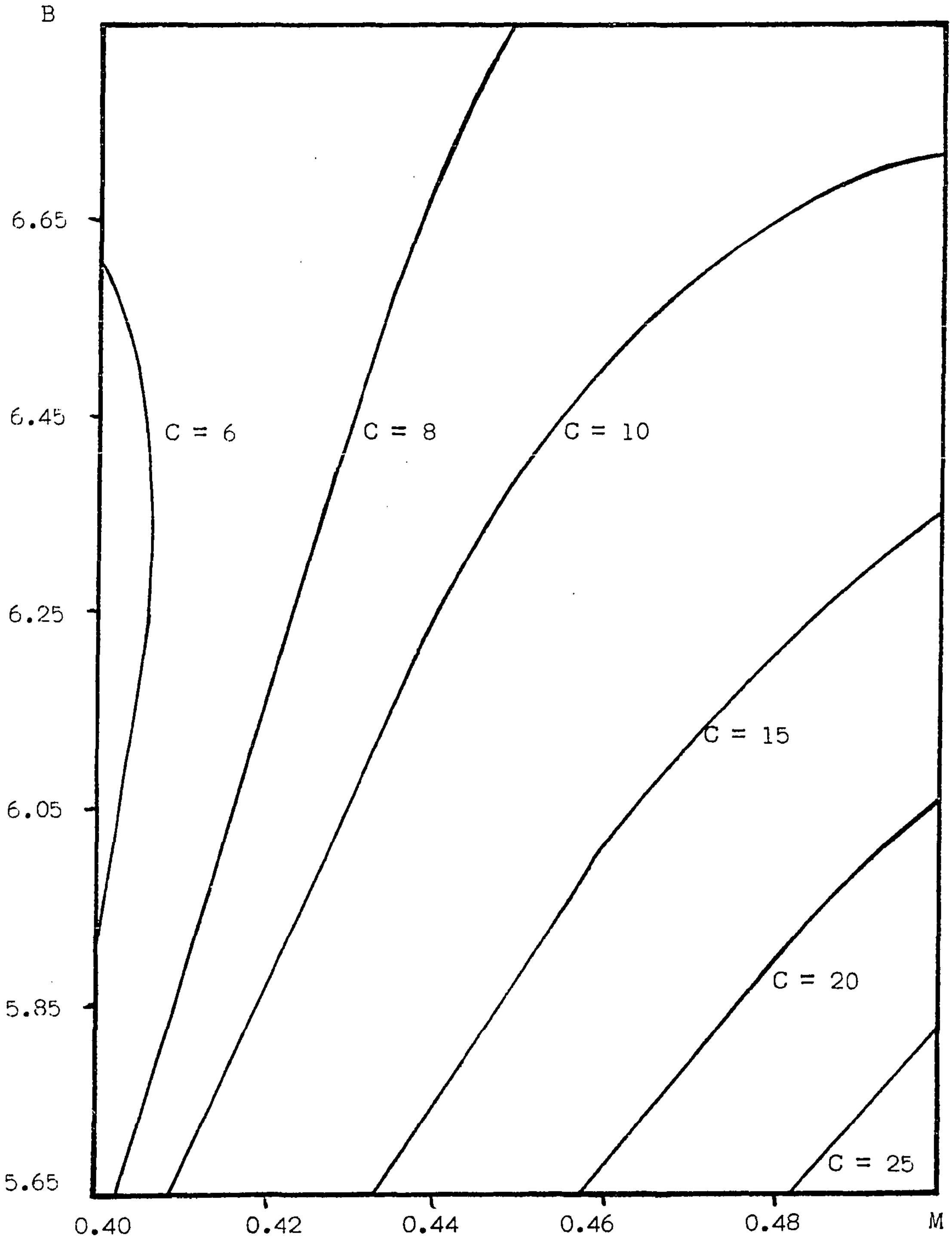


Figure 14. Price situation and number of dairy cows

d. Level of resources   Labor

$$L = 99.106 + 18.517\,B - 86.023\sqrt{B} + 15.791\sqrt{M} - 6.138\sqrt{BM};\ R^2 = .935$$

Being equal to zero in the lower left corner of the price map, L takes its minimum value when $\sqrt{B} = 2.3228 + .1657\sqrt{M}$. Above this curve located at the bottom of the map, L increases at an increasing rate with the value of B. Equal to about .2 when B = 6.45, L becomes equal to .62 when B = 6.85.

Capital

Capital requirement is a function of the beef price, the price of milk having an insignificant influence.

$$K = -55{,}543{,}572.731 + 27{,}766{,}729.907\,B - 4{,}620{,}366.634\,B^2 + 256{,}514.772\,B^3;\ R^2 = .952$$

This function increases within the range of B and has an inflection point when B = 6.004 F. Increasing at $K'' = -9{,}240{,}733.268 + 1{,}539{,}088.632\,B$ the requirement of capital rapidly becomes very large and takes the following values:

B = 5.65, K = 110,346 F

B = 6.05, K = 132,119 F

B = 6.45, K = 165,212 F

B = 6.85, K = 308,127 F

When the price of beef goes from 6.45 to 6.85, then the requirement for capital increase by about 90%.

2. Conclusion

Three important conclusions can be drawn from the preceding results.

a. When the price of beef becomes greater than 6.45 F, then we get into an area of the price map in which:

- The sale of cereals vanish and the purchase of grain for feed becomes profitable.
- The number of yearling bulls increases very rapidly and the number of first choice steers decreases slightly.
- The substitution of grain for corn silage is made in yearling bull rations.
- The requirements for capital and labor inputs increase very rapidly.
- The level of income, as a result, goes up at an increasing rate.

At the present price of cereals and as long as the price of beef is smaller than about 6.60 F:

- The production of yearling bulls has to be undertaken with a minimum of grain in the rations.
- This region will export a certain quantity of cereals unless feed input requirements for hogs and poultry reverse the balance.

b. If, in order to solve the milk surplus problem, the government raises the support price of beef, then income will increase, as well as, capital requirements. While they accumulate this capital, it is not evident that farmers will be better off. Furthermore, if the price of milk is kept constant then, as B increases, the number of dairy cows decreases and the number of yearling bulls increases. On the whole, we cannot raise more than one calf per cow per year. Calves from double purpose cows, whose production of milk is high, are selected to produce yearling bulls. Under these conditions it seems unlikely that a moderate beef price change can solve the milk surplus problem.

c. Given the moderate price changes which can be expected in the near future, the optimum production plans are very stable. Only small adjustments in the allocation of resources to beef and milk production will be made, one activity being slightly developed at the expense of the other. The level of income varies accordingly, by a small amount. To increase income by a large amount it is necessary to have control of a larger bundle of resources. This problem will be examined in the next chapter. Having studied the influence of milk and beef price changes on income, keeping the price of cereals at their present level, we now relax this assumption.

D. Influence of Milk, Beef and Cereal Price Variation

The price of cereals is now varied within the range 80 - 120% of the present price situation. This variation corresponds to a wheat price of 33.6 to 50.40 F per hundred kilograms. Only the selling price of cereals is varied; the purchase of grain for feed is excluded by assumption. A complete investigation of a three-dimensional price situation being quite laborious and expensive we have varied only the price of cereals for eight given milk and beef extreme price situations.

1. Results

Let us again define a few variables.

ΔR = A separating hyperplane locating the area of increasing income under the influence of a higher C value. Below ΔR, income is constant in the area of investigation.

MR = The value of the corresponding income on the ΔR hyperplane.

I = Total income above the ΔR hyperplane.

Y = Total number of yearling bulls above the ΔR hyperplane.

ΔC = A separating hyperplane locating the area of increasing acreage allocated to cereal production. This hyperplane is defined as the acreage of cereals when C = 80.0 plus .25 hectares. ΔC is expressed in terms of C.

AC = Total acreage of cereals.

C = Price index of cereals (80 - 120% of the present price).

B = Price of beef.

M = Price of milk.

a. Income

ΔR is expressed in terms of C

$$\Delta R = 3{,}454.58579 + 599.48129\, B - 2844.96025/B; \quad R^2 = .998$$

Within our range of observations the separating hyperplane ΔR is equally valid for the following variables:

- labor
- number of yearling bulls
- total acreage of cereals

This function increases at an increasing rate when B varies (Figure 15). The level of income is a function of C, but ΔR expressed in terms of C is a function of B.

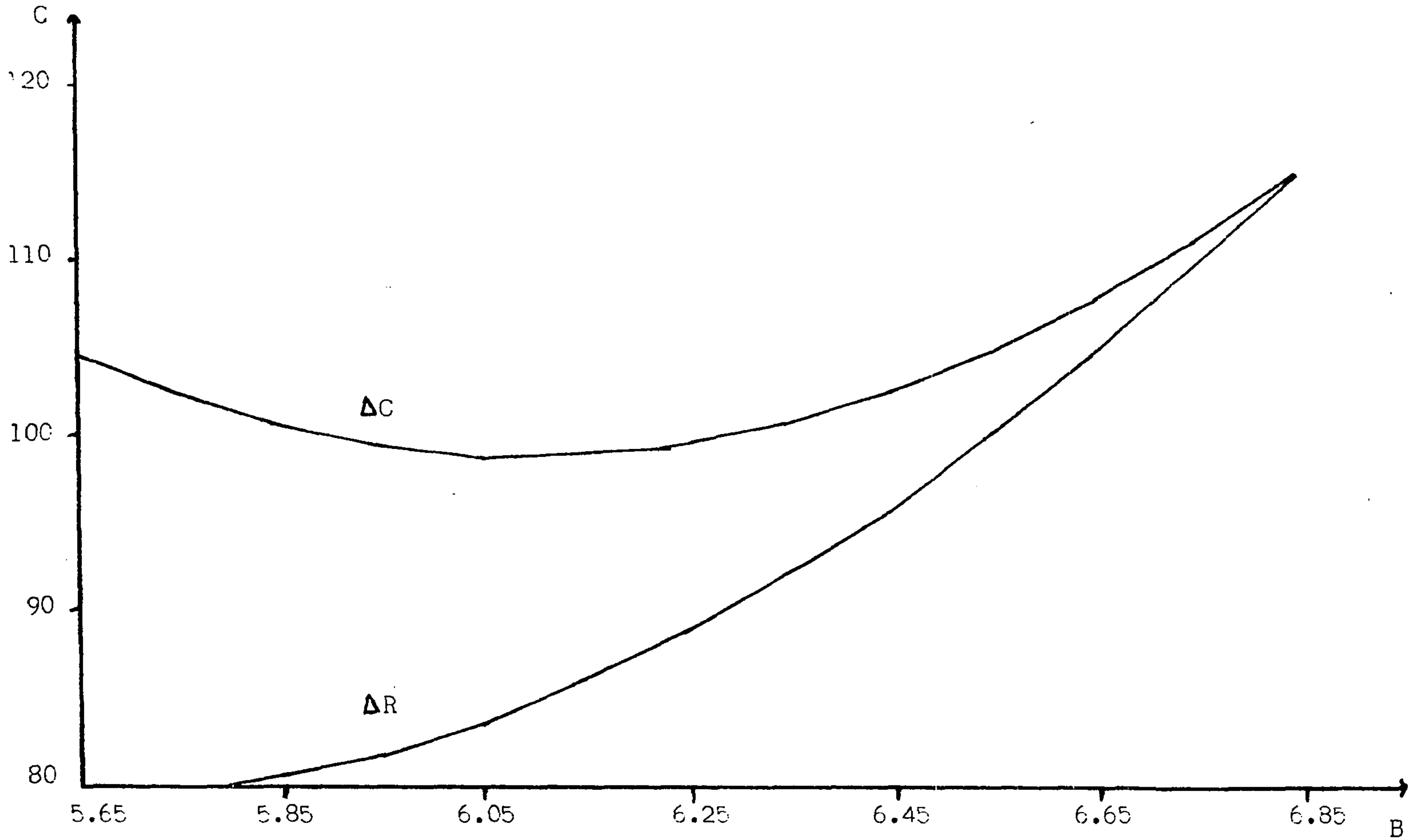


Figure 15. Separating hyperplanes

$MR = -12{,}314.3107 + 449.5352\ C;\ R^2 = .992$

Due to the small number of observations, the interaction effect of the prices of beef and milk on the level of income is not significant as in Figure 11. The following equation is more adequate:

$$I = -45799.36 + 377.279\ C + 393062.37\ M^2 + 4071.73\ B^2 + 2.581\ C^2 - 51743.113\ BM - 130.172\ BC;\ R^2 = .996$$

This function indicates that income increases more rapidly when C is large and B is small if the price of cereals increases. The partial derivative:

$$\partial I/\partial C = 377.279 + 5.162\ C - 130.172\ B$$

is positive for all values of C and B above ΔR, C being a function of B.

An increasing price of beef has a greater effect on income when M and C are cheap. $\partial I/\partial B$ is positive for all values of M and C.

The higher the price of beef, the smaller the effect on income from an increase in milk price. The partial derivative $\partial I/\partial M$ is positive for all values of B except above the following curve:

$$B > 15.192\ M$$

This area is located in the upper left corner of the price map and corresponds roughly to situations in which the whole production of milk is fed to calves.

The total variation of I is given in Table 45. The variance of income attributed to a variation of milk and beef price would be much higher than the variance attributed to a variation of the selling price of cereals, other prices being constant. In the present price situation of beef and milk, a variation of the price of cereals from 90 to 110%, which seems realistic enough, would change the income level from 24,318 F

Table 45. Mean income and its variation at given milk and beef price situation when the price of cereals varies from 80 to 120%

Mean	$S_i^o = [\sum_j (x_j-\bar{x})^2/n]^{.5}$	Milk price (francs)	Beef price (francs)
24131	1647	.43	5.87
26048	1767	.47	5.87
26758	2051	.50	5.65
41657	44	.50	6.85
35357	479	.47	6.55
29783	983	.45	6.25
35112	598	.43	6.55
41316	194	.40	6.85

to 27,274 F, a difference of about 3,000 F.

b. Level of activities

Number of cows

This activity is very stable when C varies. The number of cows is essentially a function of the beef and milk price ratio. Table 46 is quite significative in this respect. For the present price situation of beef and milk, the total number of cows is almost constant. When the price of cereals increases more than 110% the number of cows decreases only slightly. Between 90 and 105%, the number of cows is stable.

Number of yearling bulls and steers

The number of first choice steers being quite small, we have added

Table 46. Average number of cows and its variation for various price situations

Milk price (francs)	Beef price (francs)	C = 90-100 $\bar{X}$	S_i^o	C = 80-120 $\bar{X}$	S_i^o
.43	5.87	10.8	1.6	10.44	1.26
.47	5.87	19.6	.8	19.0	1.24
.50	5.65	27.2	1.6	25.22	2.57
.50	6.85	10.0	0	10.0	0
.47	6.55	10.0	0	9.66	.67
.45	6.25	10.4	1.2	10.0	1.41
.43	6.55	6.04	.54	5.86	.67
.40	6.85	6.6	0	6.42	.50

Table 47. Average number of yearling bulls and its variation for various price situations

Milk price (francs)	Beef price (francs)	C = 90-110 $\bar{X}$	S_i^o	C = 80-120 $\bar{X}$	S_i^o
.43	5.87	37.4	3.2	36.55	8.73
.47	5.87	22.4	1.35	23.22	2.82
.50	5.65	0	0	4.22	8.02
.50	6.85	64.0	0	63.55	1.28
.47	6.55	64.6	3.2	62.0	6.66
.45	6.25	50.2	5.94	51.55	6.09
.43	6.55	65.6	5.42	63.88	6.59
.40	6.85	71.0	0	69.33	4.71

them to yearling bulls. Their number is negatively correlated to the number of dairy cows as shown in Tables 46 and 47. For the present price situation of beef and milk, their number is very stable even for the largest variation of C. The smallest variation of the number of yearling bulls occurs when M and B are maximum. This number, above the ΔR hyperplane is given by the following equation:

$$Y = 12541.6839 - 5833.259\ B - 1328.987\ M - 5.375\ C + 947.901\ B^2 - 50.853\ B^3 + 10.882\ MC; \quad R^2 = .931$$

The preceding equation shows that:

- The number of yearling bulls increases with B, for all values of M and C, $\partial T/\partial B$ being always positive within the range of investigation. Increasing at first at an increasing rate, the corresponding equation increases at a decreasing rate when $B > 6.213$. We have the reverse situation in Figure 13. But here we are not investigating the same beef and milk price map for the value of C is not constant.
- The number of yearling bulls decreases when the price of milk increases, but less rapidly however when C is large.
- Y is a decreasing function of C, but when M is large, the number of yearling bulls decreases more slowly.

Total acreage of cereals

Table 48 shows that the total acreage allocated to the production of cereals varies to about the same extent, either with C for a given ratio B/M or with B/M for a given C. Moreover, the total acreage of cereals decreases when B goes up. For a range of variation in C of

Table 48. Average acreage allocated to the production of cereals and its variation for various price situations

Milk price (francs)	Beef price (francs)	C = 90-110 $\overline{X}$	S_i^o	C = 80-120 $\overline{X}$	S_i^o
.43	5.87	9.11	.92	10.75	3.38
.47	5.87	9.25	1.14	9.62	1.45
.50	5.65	9.19	1.35	9.75	1.67
.50	6.85	6.23	0	6.50	.77
.47	6.55	7.77	.45	8.48	1.49
.45	6.25	9.20	1.05	9.74	1.58
.43	6.55	7.94	1.23	8.47	1.64
.40	6.85	6.09	0	6.56	1.35

90-110, the acreage of cereals varies only a few hectares for all ratios B/M. This stability is particularly shown in Figure 15, the curve ΔC being located above the curve ΔR. The difference (ΔC-ΔR) being larger for small values of B.

$\Delta C = 4457.329 + 716.099\ B - 3533.314 \sqrt{B};\ R^2 = .998$

Above this separating hyperplane, the total acreage of cereals is equal to:

$AC = 10.200 + 1.6753\ C - 28.0611\ B - 3.072\ CM + 50.8579\ MB;\ R^2 = .853$

and varies as follows:

- AC increases with the price of cereals, but at a smaller rate when

M is high. The partial derivative $\partial AC/\partial C$ is positive for all values of M (within the area of investigation).

- AC decreases when M increases. The slope of the partial derivative is smaller when C is high and B is small. Above the hyperplane ΔC, $\partial AC/\partial M$ is always negative, however.
- AC decreases when B increases but at a smaller rate when the milk price is high. The partial derivative $\partial AC/\partial M$ is always negative for all values of M.

The acreage allocated to cereals is mainly used for the production of winter wheat and spring barley (nurse crop of temporary pastures). When C is high ($C \geq 105$), then corn for grain belongs to the optimum solution.

2. <u>Conclusion</u>

From the preceding results we can draw two important conclusions:

(1) The number of cows is almost completely determined by the beef/milk price ratio. The number of yearling bulls is negatively correlated to the number of cows and is therefore a direct function of B/M. But the price of cereals affects the number of yearling bulls more than the number of cows. When C varies from 90 to 110, the resulting variation of the total acreage of cereals is small. When the beef/milk price ratio varies the average acreage of cereals ranges from 6.00 to 9.20 hectares. Unless we get an extreme beef price ratio, the production of meat, milk and cereals belong to the optimum plans. For a given price situation, the resulting adjustments due to price change are very small when the beef/milk price ratio is stable. Otherwise, the main change

consists of a re-allocation of the resources which were primarily allocated to the production of meat and milk. The former being substituted for the latter or vice versa.

If we relax the assumption of no purchase of cereals for feed, are our conclusions still valid? The answer is yes when $B \leq 5.87$ and probably above this value. In this range of prices, the production of cereals is partly sold on the market and partly fed to animals. But when $B \geq 6.25$ we have underestimated income and overestimated the acreage of cereals below the curve ΔR, since it would be profitable to buy the required grain for feed. Above ΔR, our results are valid. Given the present price situation the preceding conclusions would be unchanged even though we relaxed this assumption.

(2) On a farm of 30 hectares, a good farmer can make an income of a minimum of 20,000 F when C and the beef/milk price ratio is small. When C increases, he gets additional revenue of 5,000 F, but if the beef price ratio were propitious he would increase his income by 100%.

E. Influence of Milk Price and Forage Yield Variation

Given the present level of forage yield and particularly of pasture output which is obtained in this area, we can question the effectiveness of a milk price reduction to solve the milk surplus problem. In order to throw some light on the possible reactions of the farm sector as a whole, we have varied the milk price and the level of forage yield.

Define these variables:

P = decreasing percentage of the milk price where $0 \leq P \leq 30$

F = forage yield in hundreds of fodder units

I = income for a 30 hectares farm

QB = quantity of meat equivalent (kilograms) sold

QM = quantity of milk (hundreds of liters) sold

1. Sale of cereals excluded

The corresponding level of income is equal to

$$I = -71918.28 - 106.2080\ P - 1423.4352\ F - 582.8547\ P^{.5} + 23185.4095\ F^{.5} - 34.2333\ (PF)^{.5};\ R^2 = .996$$

As expected the level of income increases at a decreasing rate with F and decreases with P. The level of income decreases by about 6,500 F when P goes from zero to 30.0, whatever the value of F. Since it is more realistic to allow the sale of cereals we will study this case in the following paragraph.

2. Sale of cereals allowed

The corresponding level of income is given below:

$$\dot{I} = -9814.914 - 61.2259\ P - 537.9377\ F + 651.7535\ P^{.5} + 10768.4776\ F^{.5} - 221.4651\ (PF)^{.5};\ R^2 = .996$$

This equation has the same characteristics as the preceding ones shown in Figure 16. Even if the price of milk goes down, farmers will be able to maintain their income levels by improving forage yields (Figure 16). The lower the present income, the easier the obtainment of a stable income since the production of forage is far from its optimum level. When the present production of forage is greater or equal to 4,000 fodder units per hectare it becomes relatively difficult to maintain

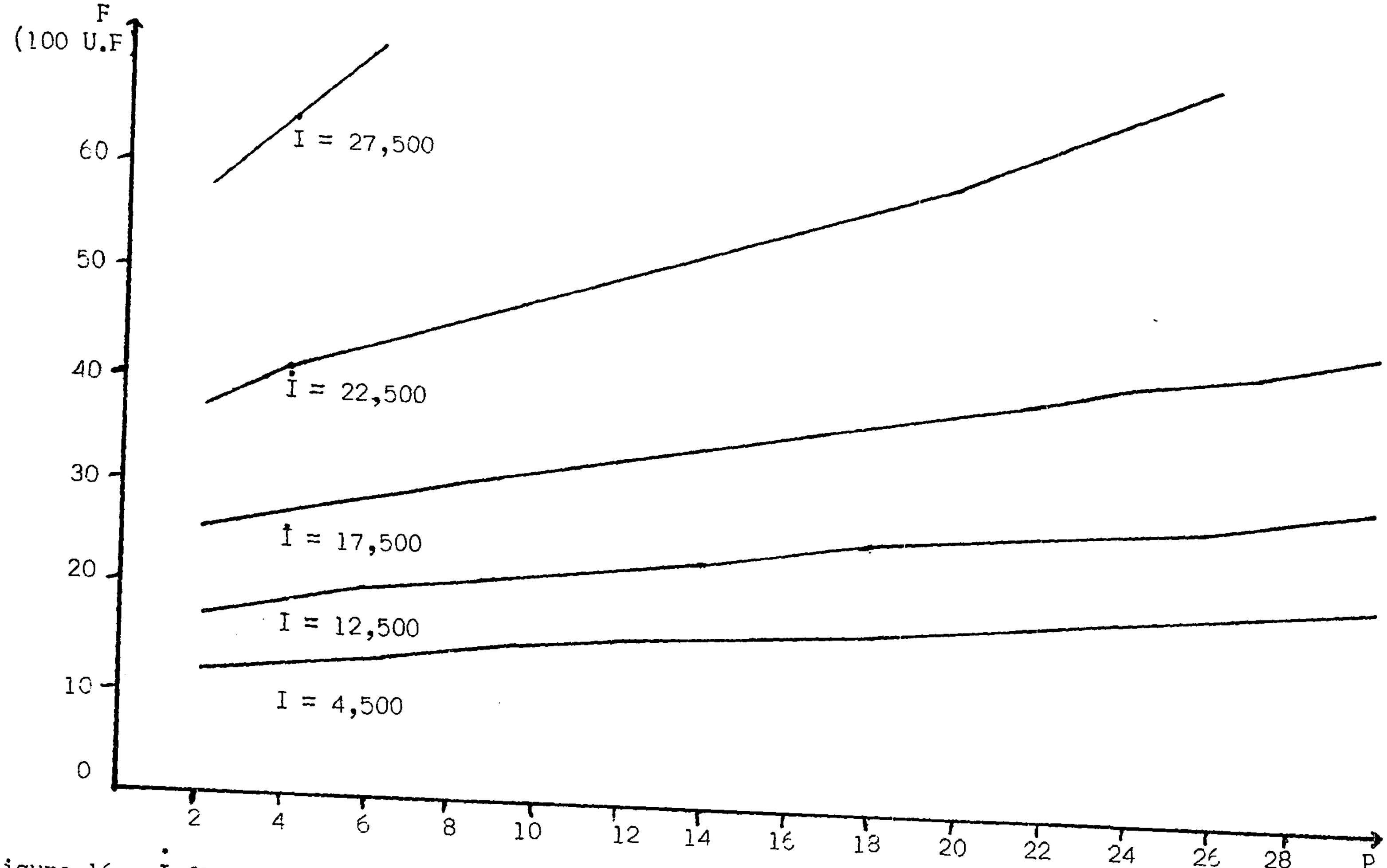


Figure 16. Influence of forage yield and milk price level on income

a stable income. The level of present forage yield of the area being relatively low, farmers possess the means of reacting effectively to a milk price decrease in order to keep constant or even improve their present level of income. At the same time the production of milk increases as indicated by the following relationship:

$$QM = \frac{-1994.6245 - 48.322\ F + 674.5736/F - 7.1297/FP}{.47\ (1 - P/100)};\ R^2 = .982$$

When the price of milk decreases, the production of milk decreases at an increasing rate but when higher forage yields are obtained, milk output increases. If, other things being equal, the production of milk decreases with a milk price reduction, this relationship does not hold when farmers react to a milk price decrease by improving forage yields. As a result they are urged to increase the production of milk to maintain their present level of income. (Compare the slope of the isoquant QM and the iso-revenue 1, respectively, in Figures 16 and 17).

At the same time, the production of meat (QB) increases as shown in Figure 17. The corresponding isoquants are drawn from the following equation:

$$QB = 2419.48 + 161.175\ P - 7.7612\ P^2 + 3.2810PF;\ R^2 = .940$$

QB increases and then decreases with the value of P, as indicated by the following relationship:

$$\partial QB/\partial P = 161.175 - 15.5224\ P + 3.281\ F$$

The particular shape of QB can be explained as follows: When P goes up, the production of yearling bulls increases. They are first substituted for dairy cows and then, when they reach a maximum, sheep are inserted in the programming solutions at the expense of dairy cows. At

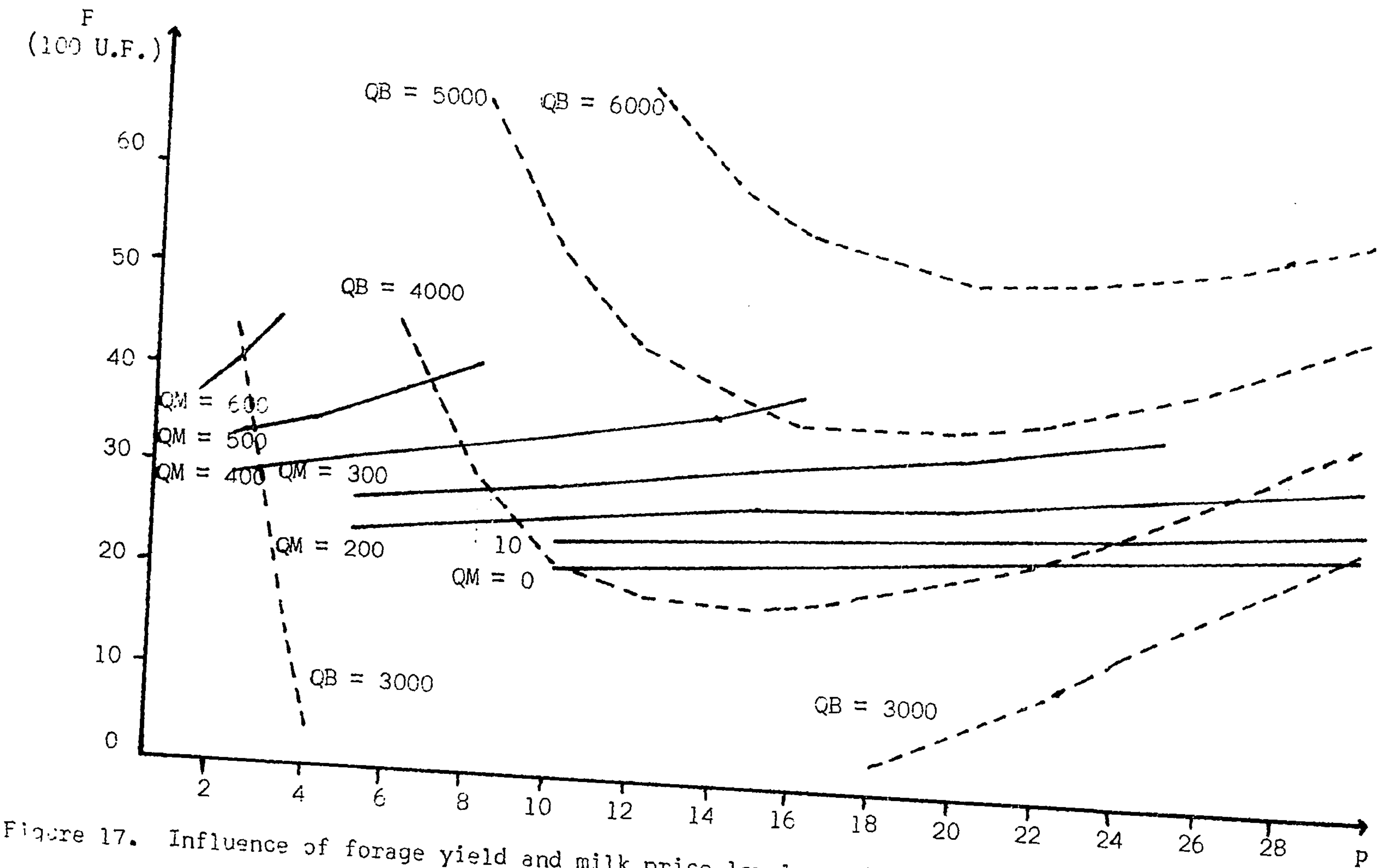


Figure 17. Influence of forage yield and milk price level on milk and beef output

the same time the production of cull cows decreases and provokes a decrease in the function QB.

F. Conclusion

The results of this chapter are very encouraging. They should urge young farmers to start farming with a certain confidence in the future. Although the variation of farm product prices have a non-negligible influence on the corresponding levels of income, the farm production plans seem very stable, at least in a certain area of the price maps. And even if the price situation moves far away from the present one, farmers still have the possibility of adjusting production plans, at a low cost. The main and most costly shift is the re-allocation of resources from milk to beef production or vice versa. But the corresponding inputs are not so specialized that it would be costly to move from the early optimum plan to the new one. Buildings, silo and harvesting machine requirements are similar for both productions. Only the milking parlor would be left idle. Such a conclusion would not be drawn if a small cereal price variation, relative to the milk/beef price ratio, withdrew large bundles of resources from animal production to cereals or vice versa. In such a case, the long and short-run opportunity curves differ greatly. This conclusion is also valid for the production of rye-grass seed. Seed price variation generates only a small variation within the crop rotation but does not significantly affect the production of livestock. The production of tall fescue seed should not be encouraged since this production is profitable only within a small price range.

CHAPTER 21. ACCUMULATION OF RESOURCES AND INCOME LEVEL

Having studied the influence of price variation on income, we now measure the effect of an accumulation of resources on income, assuming the present price situation.

From the results of the preceding chapters we are already able to foresee the importance of farm size, capital, and labor input and their influence on income level. The affect of these variables is now measured thoroughly. In the first part we define a few variables. In the second part, land and capital inputs are considered as the only scarce resources, investments in dairy facilities being either allowed or excluded. In the third part we assume that land and labor are the only two scarce resources. The same assumption is made relative to building investments. Finally, in the fourth part, we examine the possibility of substituting capital for labor.

A. Definition of Variables

1. Dependent variables

The following variables will be used throughout this chapter:

I = income (francs)

C = number of cows (head)

Y = number of yearling bulls (head)

S = number of sheep (head)

SL = steers at livery (head)

RC = acreage of fodder row crops (hectare)

TP = acreage of temporary pastures (hectare)

AC = acreage of cereals, rape and corn for grain (hectare)

Subscripts are added to distinguish equations according to the particular assumptions which have been made. The subscripts i and j are used, where:

i = k when capital and land are scarce

= l when labor and land are scarce

= w when labor and capital are scarce

j = b when investment in dairy facilities are allowed

= blank otherwise.

2. Independent variables

The scarce resources are defined as follows:

L = hired labor (man year)

S = farm size (hectare)

K = capital (1,000 F)

B. Influence on Income of the Scarcity of Land and Capital Inputs

1. Investment in dairy facilities excluded

a. Border lines of the resource map The maximum amount of capital which can be profitably associated with a given amount of land is given by the following ridge line:

$K = 9.3749 + 2.6608\ S;\ r^2 = .996$

When S increases, the maximum amount of income can only be obtained if, and only if, 2,660.83 francs of capital are associated with each additional hectare of land.

On the other hand, the maximum acreage of land which can be cultivated with a given amount of capital is given below.

$K = -7.239 + 1.185\ S;\ r^2 = .992$

If capital were the only scarce resource, 1,185 F would be associated with each additional hectare of land.

Furthermore, $S \leq 60.0$ by assumption.

b. Level of income The level of income increases rapidly with the value of S and K as shown below:

$$\dot{I}_k = -11342.605 - 592.164\ K - 974.425\ S + 3092.329\sqrt{K} + 2995.011\sqrt{S} + 1489.974\sqrt{KS};\ R^2 = .998$$

But, however, the corresponding iso-revenue curves (Figure 18) show that income increases at a decreasing rate when a larger bundle of capital and land is inserted in the production process. Total income varies from 10,000 F on small farms to 43,000 F on larger ones, when capital is available in unlimited amounts. But when this resource is scarce the corresponding income decreases by 5,000 F for small holdings and by 10,000 F for large ones. At a certain degree of scarcity, the iso-revenue curves become nearly flat and a large acreage is required to compensate for a very small amount of capital. The best strategy for a young farmer is, therefore, to accumulate capital as rapidly as possible even though he has to reduce consumption at its acceptable lower bound. We will come back to this problem later since it is related to intertemporal preferences and to firm-household conflict.

c. Effective constraints and labor requirement The profitable land-capital combinations are shown in Figure 18. The quantity of labor associated with them are given by the following equation:

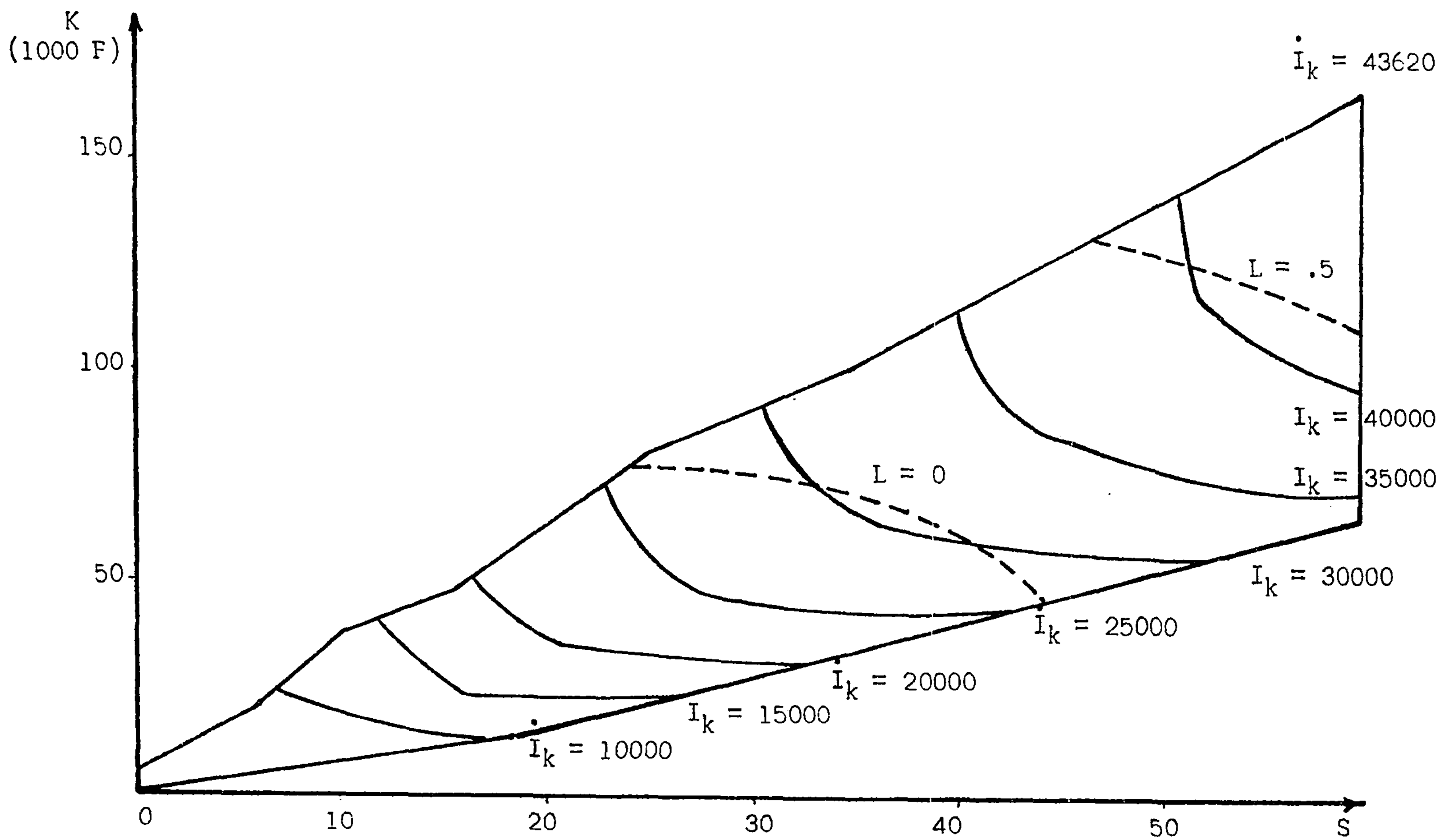


Figure 18. Iso-revenue and combination of land, labor and capital inputs

$$L_k = -.403 + .0139\,K + .0510\,S + .07\sqrt{K} - .2263\sqrt{S} - .0327\sqrt{KS};$$
$$R^2 = .986$$

Contrary to the preceding equation this curve increases at an increasing rate. The corresponding isoquants are shown in Figure 18. According to the amount of capital at their disposal, a farmer and his wife can run a farm whose size varies from 25 to 47.5 hectares. As K and L become less scarce, they will hire a part-time farm worker, 85% of his total work time being the maximum requirement. We give now the main limiting constraints of the model and the boundary on which they become effective. The subscripts of K indicate the name of the constraints we are referring to. They have been defined in Tables 34 and 35, Chapter 16.

Livestock labor constraints are rapidly effective when the level of capital increases. They are located in the same area of the resource map and very close to the lower ridge line (Figure 19).

$$K_{A_1} = 86.576 + 2.8201\,S - 23.3768\sqrt{S};\ A^1,\ R^2 = .998$$
$$K_{A_2} = 82.830 + 1.9947\,S - 16.6301\sqrt{S};\ A^1,\ R^2 = .956$$
$$K_{A_3} = 97.173 + 2.3253\,S - 21.0043\sqrt{S};\ A^1,\ R^2 = .970$$

The labor constraints on crop activities which become rapidly effective are related to the production of forage, hay and fodder row crops.

$$K_{L_{22}} = 71.069 + 2.4805\,S - 17.865\sqrt{S};\ A^1,\ R^2 = .998$$
$$K_{L_{24}} = 17.621 + .6706\,S;\ A^1,\ r^2 = .990$$

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[1]A = the K_{ij} constraint is effective in the area located above the curve.

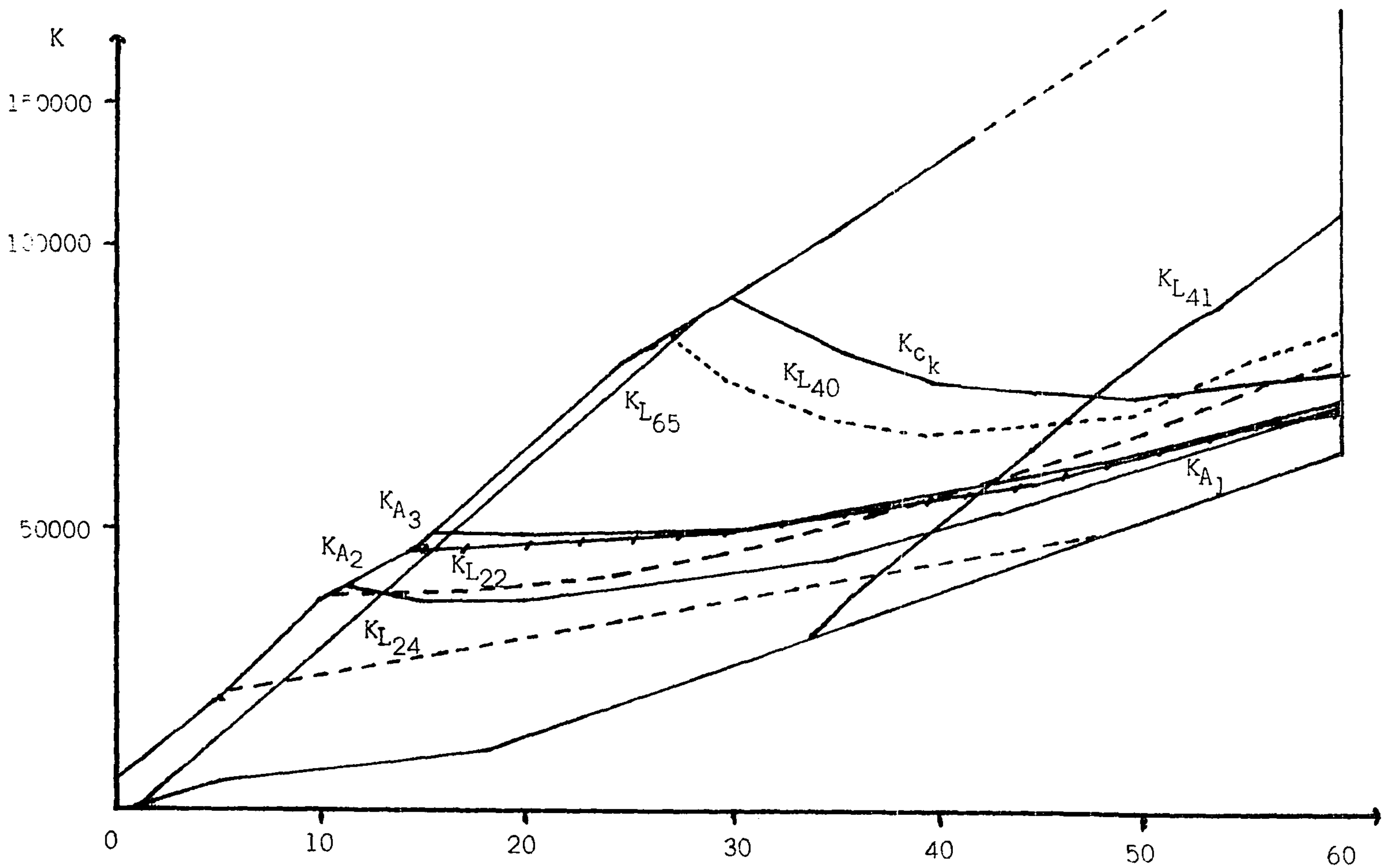

Figure 19. Boundaries of the effective constraints

These two constraints on hay making have been found very effective throughout our results.

$K_{L_{41}} = -196.918 + 39.298 \sqrt{S}$; B^1, $r^2 = .992$

The cereal harvesting constraint is effective on farms whose size is higher than 30 hectares and for which capital is very scarce. When, instead of considering this subperiod, we take into account period 40, labor becomes scarce when capital is available in a larger amount.

$K_{L_{40}} = 528.263 + 11.202\ S - 143.651 \sqrt{S}$; $R^2 = .992$

Finally, labor is scarce in period 65 (wheat seeding and row crop harvesting) for almost the whole resource map.

$K_{L_{65}} = -.3148 + 3.1098\ S$; B^1, $r^2 = .998$

All these boundaries are shown in Figure 19. They indicate, once more, that any research or farm budgeting program cannot underestimate the importance of livestock production and related problems for this area as a whole.

d. Main features of farm production plans   Number of cows.

In this solution they are limited to 20 by assumption. This maximum is reached on the following boundary line; above it, the number of cows is constant and maximum.

$K_{C_k} = 468.318 + 8.250\ S - 113.976 \sqrt{S}$; $R^2 = .960$

The total number of cows is given by the following equation:

$$C_k = -4.4590 - .7883\ K - 1.2845\ S + 3.4930 \sqrt{K} - 2.9433 \sqrt{S} + 2.2481 \sqrt{KS}; \quad R^2 = .992$$

---

[1]B: the K_{ij} constraint is effective below the corresponding curve.

The herd size increases at a decreasing rate with the value of K. However, for a given amount of capital, the number of cows decreases when S increases, capital being more profitably allocated to other production (Figure 20).

Number of yearling bulls

We are reporting here a result, disregarding the specific feeding programs of yearling bulls. More are fed with corn silage than with fodder beet (four against one when S = 60 and K = 150). If the size of farm is small ($S \leq 30$) and capital limited, only rations constituted with fodder beet belong to the optimum solutions.

$$Y_k = -.2533 + .4646\ K - .8245\ S + .0034\ KS;\ R^2 = .946$$

The total number of yearling bulls increases with the value of K. The corresponding rate of increase being higher when S is larger. Yearling bulls are less numerous when the size of farm increases and capital is kept constant. Their maximum number is 59 when K and S are maximum (Figure 21).

Number of sheep

The sheep flock appears in the optimum solution only when K and S are available in very large amounts. It is composed of 62 head when these two resources are maximum.

$$S_k = 18.733 - 3.715\ K + 8.291\ S + .0371\ K^2 - .0864\ KS;\ R^2 = .980$$

As for the preceding ones, the partial derivation of this function takes the following values when $S_k > 0$:

$\partial S_k / \partial S < 0$ and $\partial S_k / \partial K > 0$

Number of steers at livery

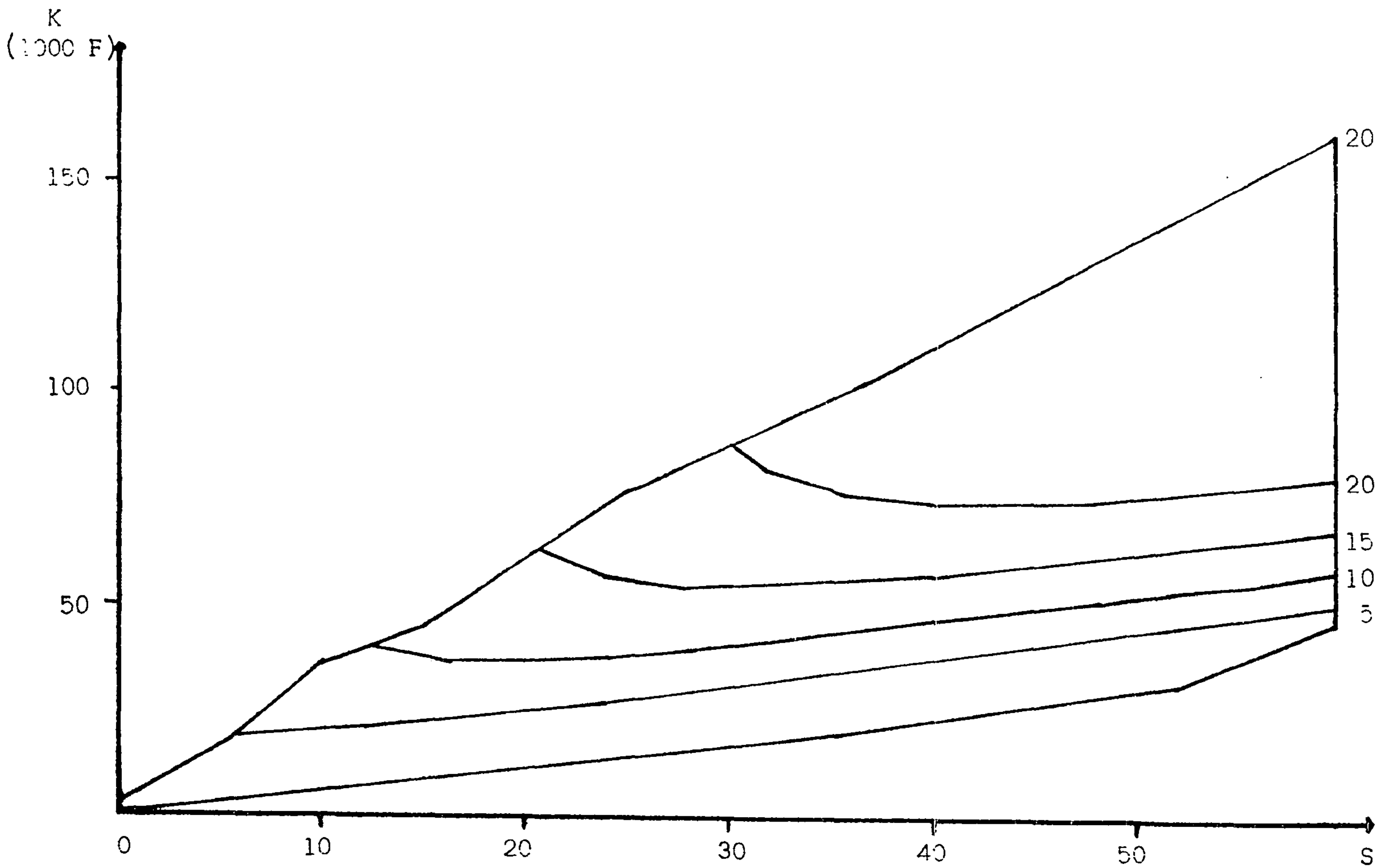

Figure 20. Number of dairy cows (C_k)

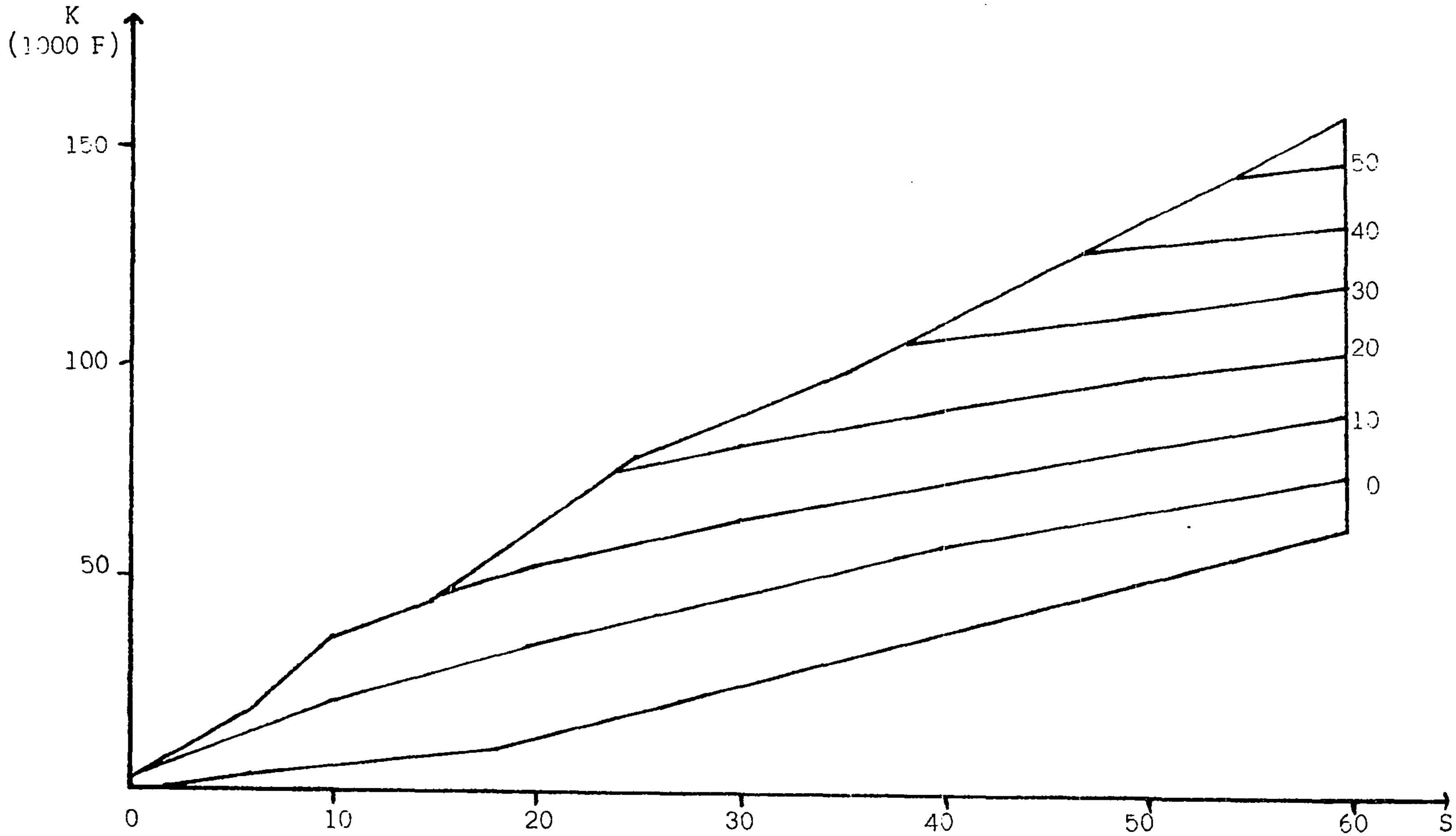

Figure 21. Number of yearling bulls (Y_k)

When capital is very scarce, few steers at livery appear in the optimum solution.

$$S_{L_k} = 2.065 + .0129\ K^2 + .0191\ S^2 - .0323\ KS;\ R^2 = .978$$

But, as K increases they disappear very rapidly. They are present only on the lower border line.

Acreage of fodder row crops

$$RC_k = -2.0289 + .10273\ K + .00069\ K^2 - .00177\ KS;\ R^2 = .921$$

The number of cows and yearling bulls increases with the value of K. It is therefore expected that these intermediate products which are feed inputs for animals, increase also with K. As expected $\partial RC_k/\partial K > 0$ and $\partial RC_k/\partial S < 0$. At most 16.60 hectares of those crops are cultivated (Figure 22).

Acreage of temporary pastures

As for the preceding variable and for the same reasons, it is expected here that the acreage of temporary pasture increases with K.

$$TP_k = -.41456 + .18320647\ K - .070882\ S - .000\ 51886\ K^2;\ R^2 = .940$$

Here again, $\partial TP_k/\partial K > 0$ and $\partial TP_k/\partial S < 0$.

When K and S are maximum, 12 hectares of temporary pastures are cultivated (Figure 23).

Acreage of cereals, rape and corn for grain

The total acreage allocated to these crops is equal to:

$$AC_k = .228 - .29971\ K + 1.09461\ S + .50499\sqrt{K} - 1.33469\sqrt{S} + .14328\sqrt{KS};\ R^2 = .992$$

It decreases when K increases, and increases with the farm size. Within the range of investigation the partial derivatives take the following values: $\partial AC_k/\partial K < 0$ and $\partial AC_k/\partial S > 0$. At most, we have 26.50 hectares of

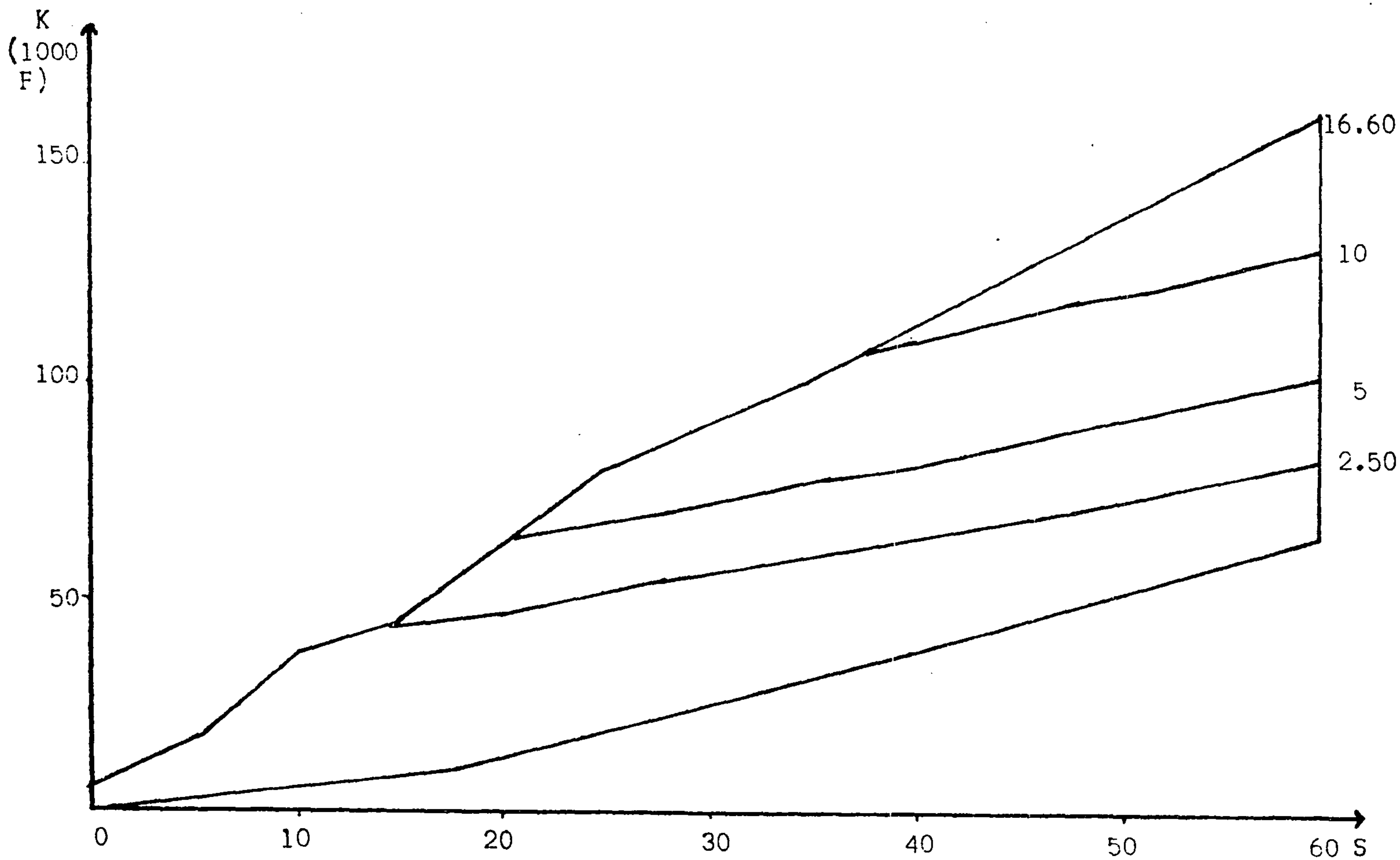

Figure 22. Acreage of fodder row crops (RC_k)

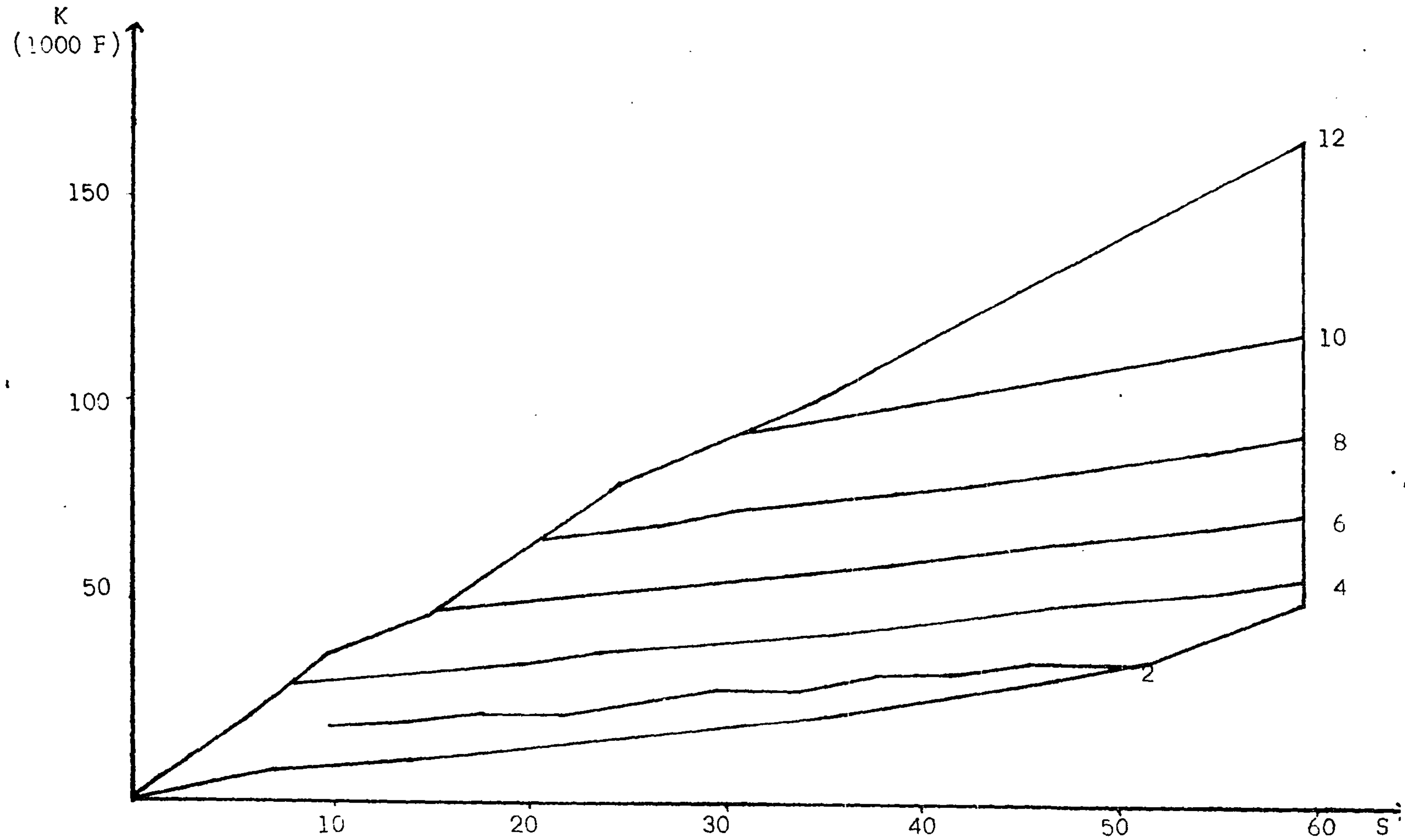

Figure 23. Acreage of temporary pasture (TP_k)

cereals when S and K are maximum. That corresponds to about half of the total acreage of plough-land (Figure 24).

In short, we can say that an increasing level of income can be obtained when it is possible to have control over a larger bundle of resources. When capital is kept constant, and additional hectares are rented, then the number of dairy cows and yearling bulls decreases very slightly, as does the production of forage. The additional land is allocated to the production of cereals and corn. On the contrary, when for a given acreage, capital becomes less scarce, cereals are replaced by fodder crops and transformed by animals. The number of cows starts to increase before the number of yearling bulls. The latter are not limited in number, they become many when K and S are maximum. An upper bound has been defined for the former. The effects of relaxing this assumption on the results are shown below.

2. Investment in dairy facilities allowed

a. Ridge lines of the resource map As before, $S \leq 60$. The border lines are given below:

$K_b = 17.4347 + 3.7516\ S;\ r^2 = .994$

$K_b = 28.5192 + 1.6543\ S - 9.4753 \sqrt{S};\ R^2 = .996$

When capital is available in unlimited amount the corresponding requirement is equal to 3,751 F per additional hectare of land, a value much higher than in the preceding case (2,660 F). Now the maximum value of K is 242,530 F compared with 169,020 F previously.

The minimum amount of capital which can be combined with each supplementary hectare of land is equal to:

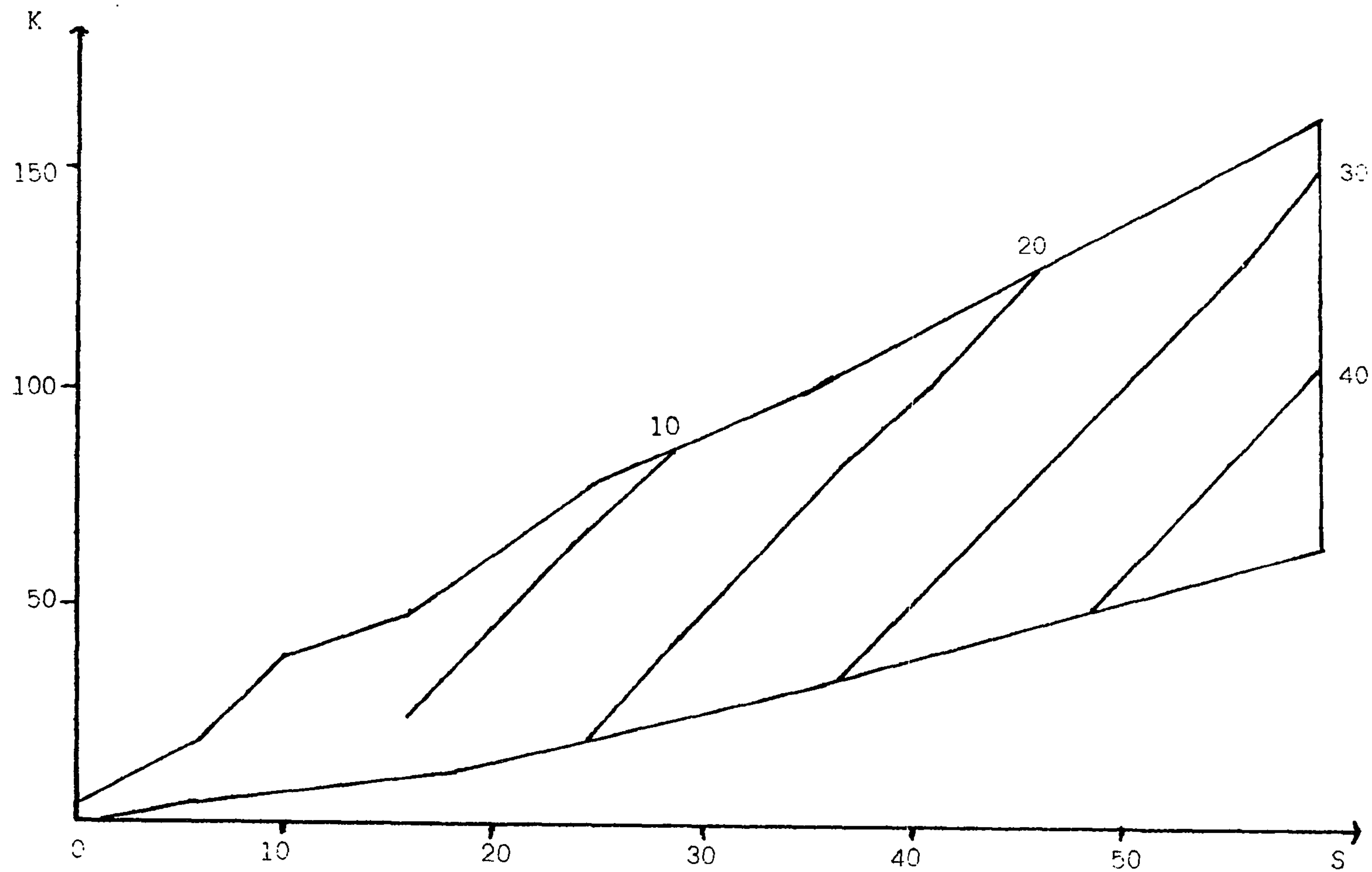

Figure 24. Acreage of cereals; rape and corn (AC_k)

$dK_b/dS = 1{,}654 - 4737.65\ S^{-.5}$ francs.

b. Level of income   The maximum level of income given below is not much different from the one which can be obtained when investment in dairy facilities are excluded by assumption. The comparison of

$$\dot{I}_{kb} = -14615.05319 - 466.5487\ S - 278.99343\ K + 2065.23706\sqrt{S} + 2848.93626\sqrt{K} + 768.93722\sqrt{KS};\quad R^2 = .998$$

the iso-revenues of Figure 18 and Figure 25 are significant in this respect. When K and S are maximum, then $\dot{I}_{kb}$ = 42,854 francs against 43,621 F for $\dot{I}_k$. Investment in dairy facilities is not profitable when K is scarce, however. The reasons will be discussed later when this solution is completely reported.

c. Hired labor requirement

$$L_{kb} = .46504 + .01651\ K + .0802\ S + .04063\sqrt{K} - .49222\sqrt{S} - .0409\sqrt{KS};\quad R^2 = .990$$

Labor requirement increases with S and K, in most parts of the resource map. $\partial^2 L_{kb}/\partial K^2$ and $\partial^2 L_{kb}/\partial S^2$ are positive, this curve is therefore concave upward. At most, L_{kb} = 1.17 against .85 for Lk. An investment in dairy facilities increases labor requirements, at least at certain levels. We omit here the equations of the boundaries on which the main constraints become effective. They differ little from the preceding ones, when investments in dairy facilities are excluded.

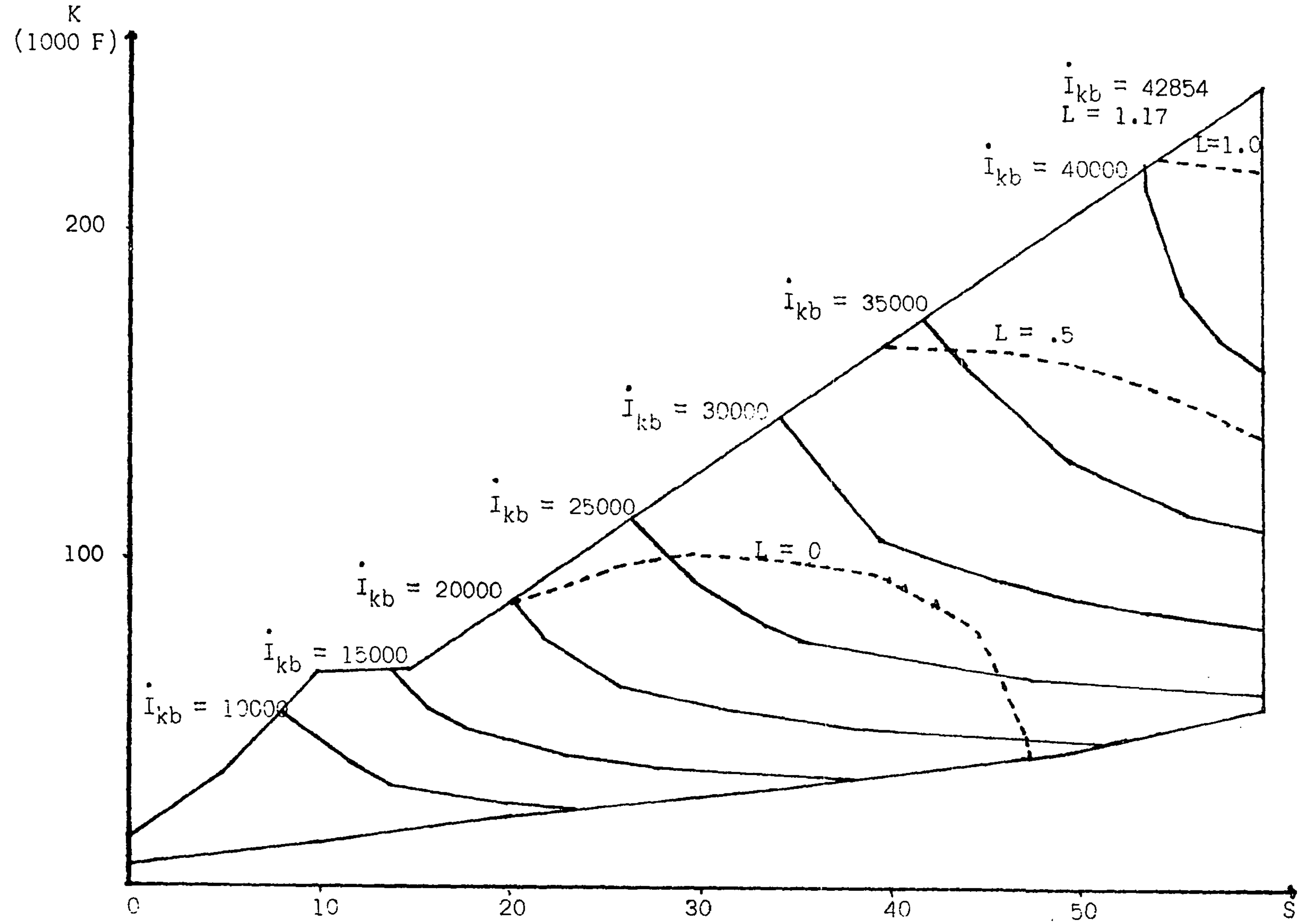

Figure 25. Iso-revenue and combination of resources

d. Main characteristics of the production plans   Number of cows.

As in the preceding hypothesis, dairy cows are present in practically the whole resource map. Their number varies from very few to at most 42 (Figure 26).

$$C_{kb} = .17338 + .09718\ K - .40919\ S - .61032\sqrt{K} + .44799\sqrt{S} + .40113\sqrt{KS};\ R^2 = .984$$

The number of cows increases very rapidly with the value of K but at a decreasing rate. The influence of S on the number of cows is quite small when K is fixed at a given level.

Number of yearling bulls

If the number of dairy cows increases by 100% when investments in dairying are allowed instead of being excluded, the maximum number of yearling bulls decreases by 35%. When K increases, Y_{kb} increases at a decreasing rate.

$$Y_{kb} = -6.3869 + .38310\ K - .38752\ S - .00339\ K^2 - .02690\ S^2 + .01863\ KS;\ R^2 = .982$$

The maximum number of yearling bulls is equal to 39 (Figure 27).

Steers at livery and sheep

As for the preceding hypothesis, steers at livery belong to the optimum solutions when K is very scarce and the farm size is large enough. In the whole resource map sheep are excluded from the production plans.

$$SL_{kb} = 3.05604 - .22859\ K + .31379\ S + .00848\ K^2 + .01521\ S^2 - .02399\ KS;\ R^2 = .998$$

Acreage of fodder row crops

The acreage allocated to the production of fodder row crops is more

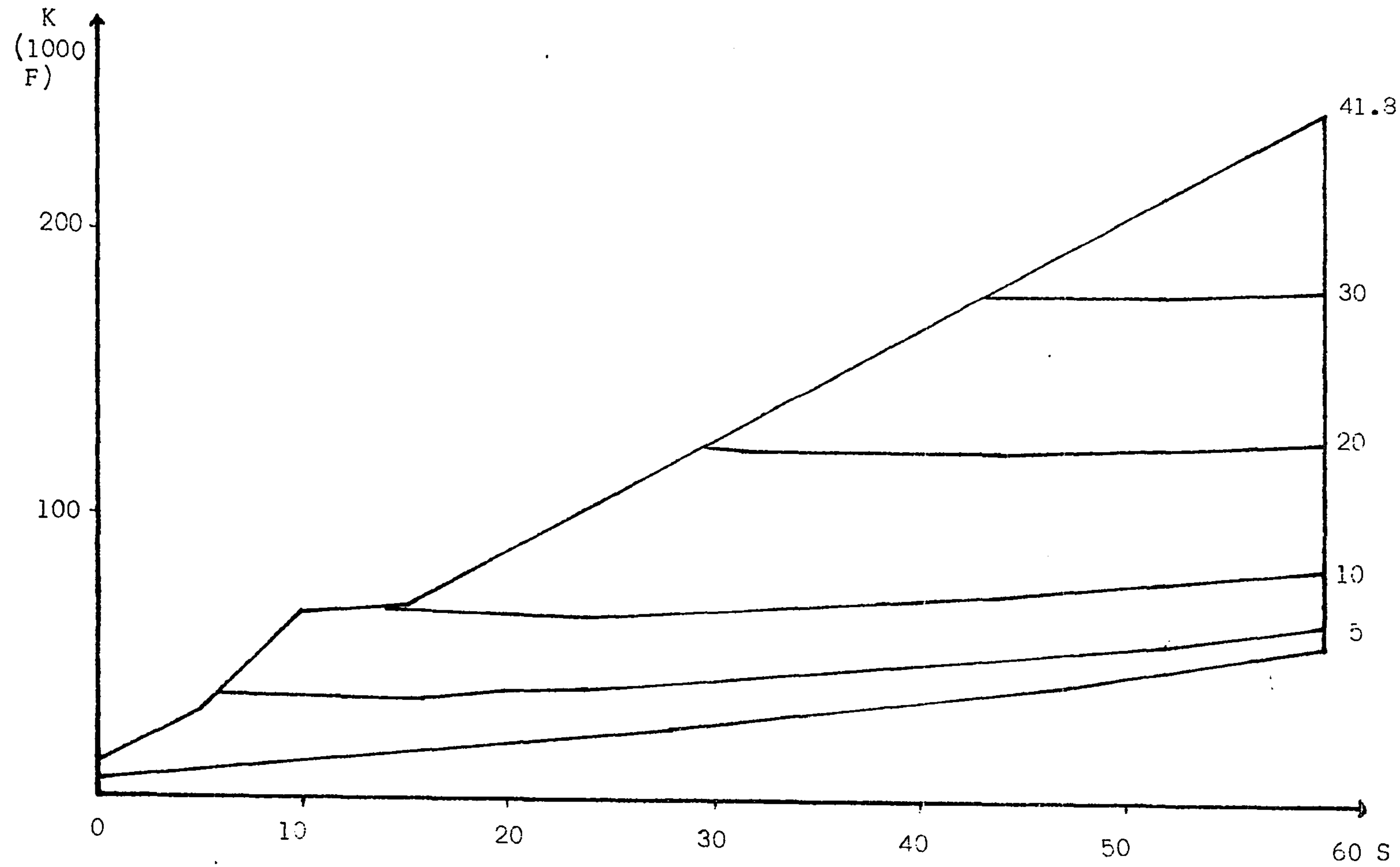

Figure 26. Number of dairy cows (C_{kb})

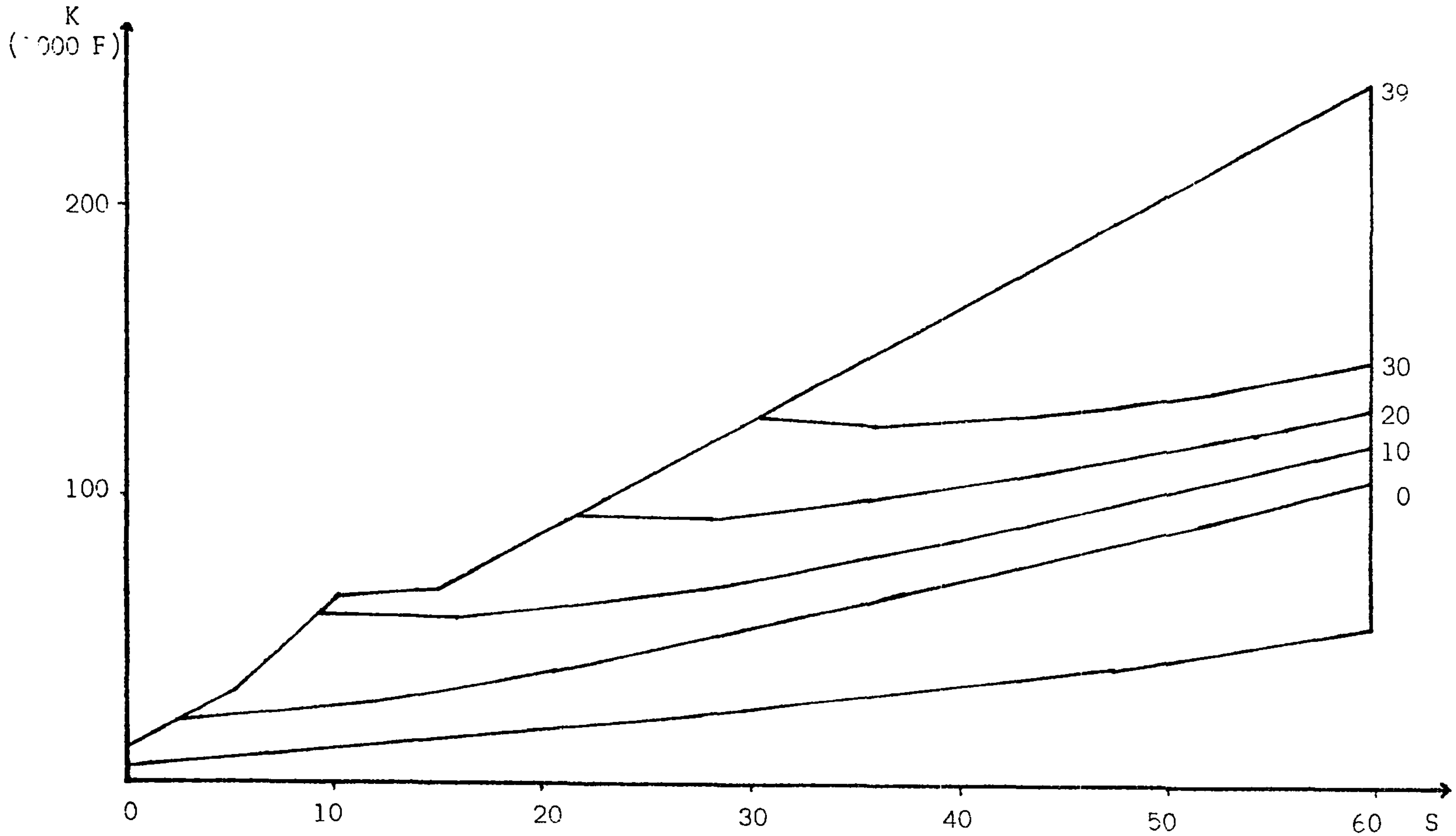

Figure 27. Number of yearling bulls (Y_{kb})

directly a function of K. The interaction effect KS is not significant.

$$RC_{kb} = -2.724 + .09201\ K - .05867\ S;\ R^2 = .972$$

Furthermore, when K is not limited: $RC_k - 16$ (Figures 28 and 22)

Acreage of temporary pastures

When K and S are maximum the acreage of temporary pastures is equal to 20.50 hectares. Due to a larger cow herd than in the preceding solution, TP_{kb} is higher than TP_k since the latter is equal to 19 hectares, at most (Figures 23 and 29).

$$TP_{kb} = .34732 + .09653\ K - .18686\ S - .34583\ \sqrt{K} + .50319\ \sqrt{S} + .07678\ \sqrt{KS};\ R^2 = .988$$

Acreage of cereals, rape and corn

$$AC_{kb} = -3.5069 - .24964\ K + 1.41782\ S + 2.53595\ \sqrt{K} - 2.82651\ \sqrt{S} - .17801\ \sqrt{KS};\ R^2 = .996$$

At most, $AC_{kb} = 16.80$ when K and S are maximum. A value much smaller than the value of AC_k. The total acreage of cereals increases at an increasing rate with S, and decreases at a decreasing rate when K becomes less scarce (Figure 30).

To summarize the influence of the scarcity of land and capital on the corresponding production plans we can say that:

- The level of income increases rapidly with the availability of capital until it reaches an upper bound. This upper bound is directly correlated with S, the farm size.
- The number of dairy cows increases first and next the number of yearling bulls. Roughly, when K is unlimited we get a proportion of one bull and one cow.
- The forage requirements increase with the importance of the

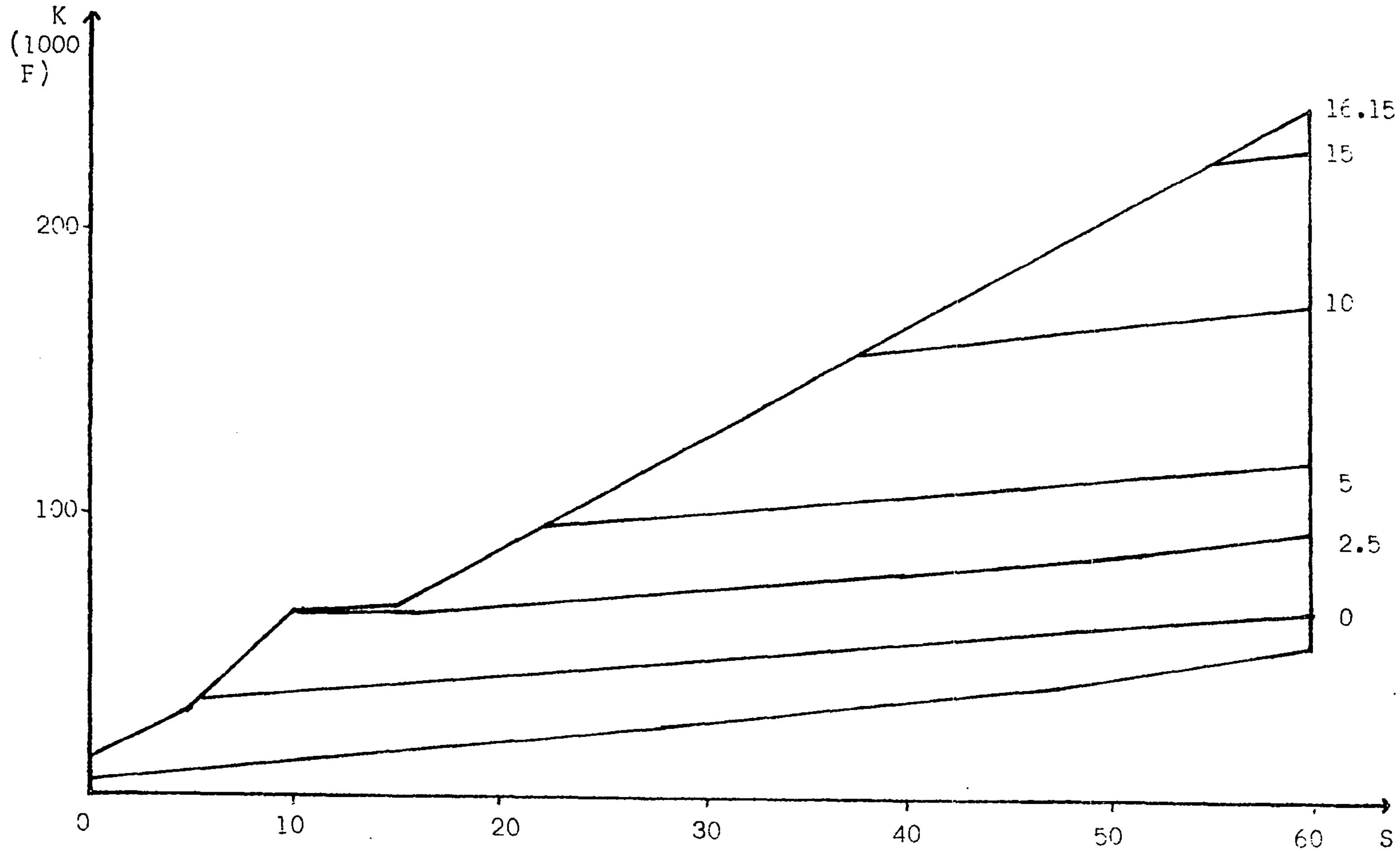

Figure 28. Acreage of fodder row crops (RC_{kb})

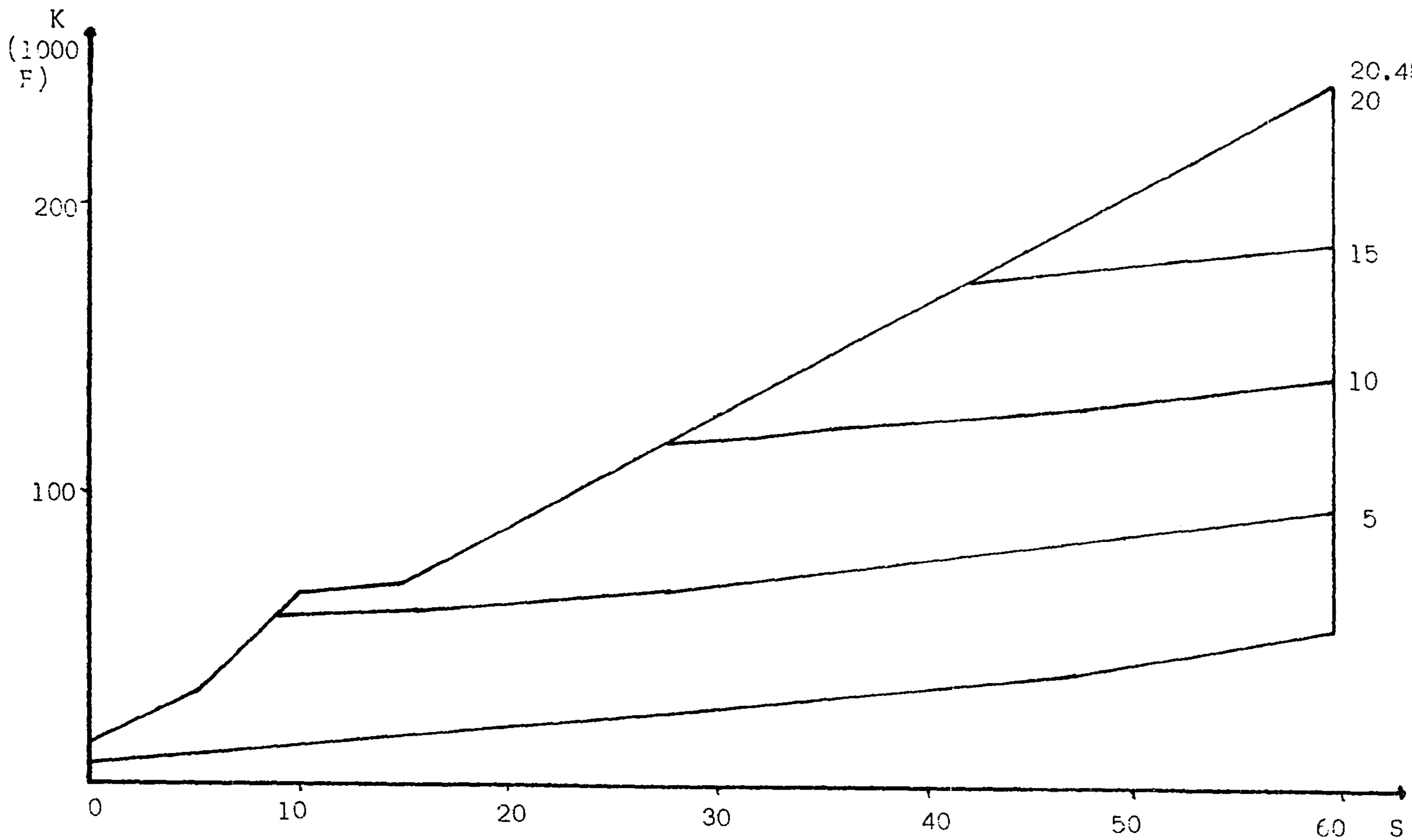

Figure 29. Acreage of temporary pastures (TP_{kb})

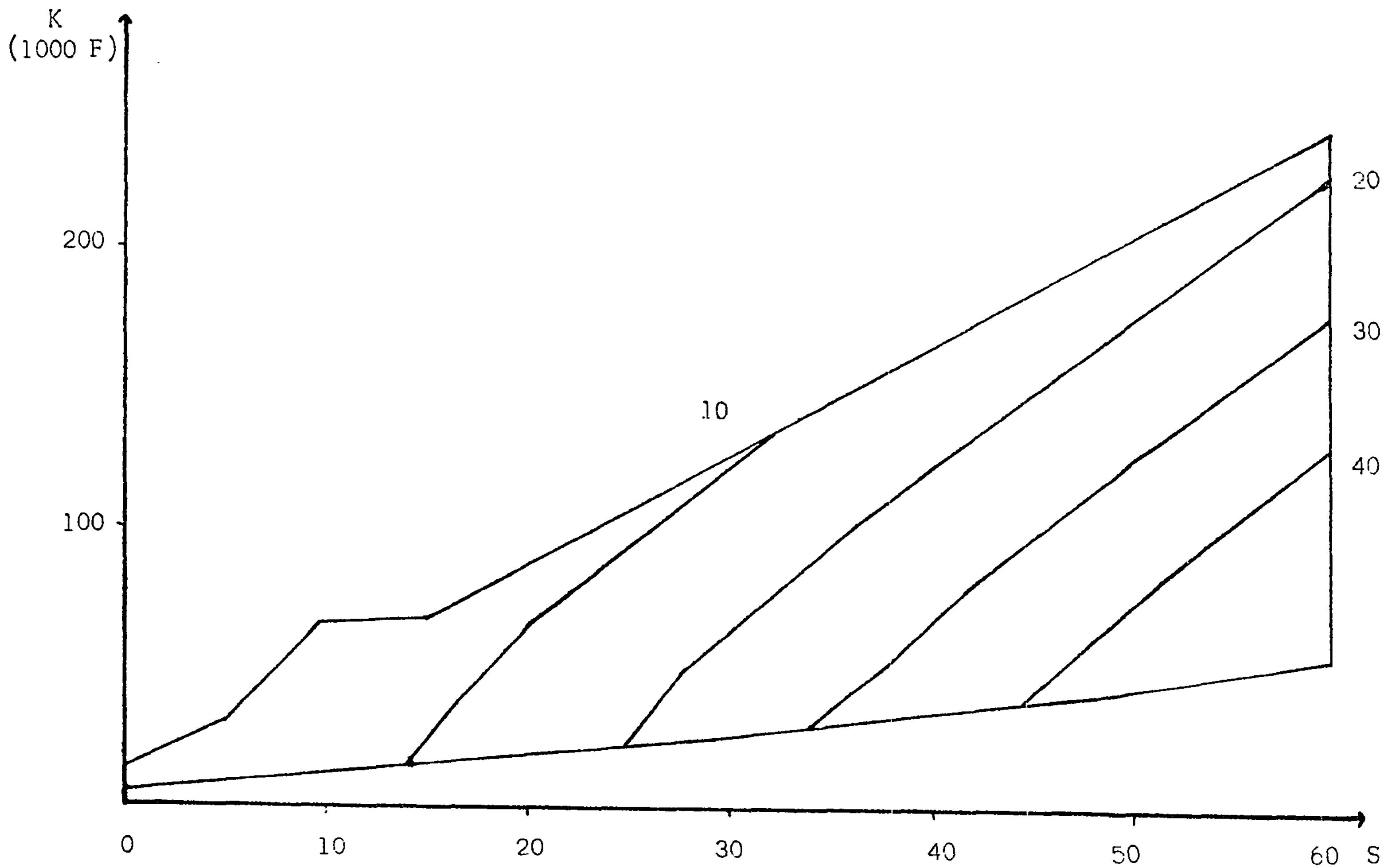

Figure 30. Total acreage of cereals (AC_{kb})

livestock production. The acreage of fodder row crops and temporary pastures increases at the expense of cereals and corn production.

3. Why investment in dairying is not profitable

If it is practically indifferent to invest or not in dairy facilities when K and S are maximum, it becomes profitable to avoid such an expenditure when K is scarce or the farm size limited. The level of income decreases, and several reasons can be given to explain these results.

- Investment functions have the form $Y = a + bX$, where X is the space unit required per animal. The value of a is much larger when we invest in cows than in yearling bulls; the corresponding fixed costs are practically equal to zero for the latter. The larger the herd size, the smaller the building amortization cost.
- The replacement of cereals and yearling bulls by dairy cows, beyond the boundary $C_k = 20$ requires additional labor. Furthermore, if the replacement of cereals by dairy cows increases the productivity of land, the reverse is true when yearling bulls are replaced by dairy cows (see Chapter 19). This complex replacement of bulls and cereals by cows, when all additional labor costs are paid, is not profitable.
- When investment in dairying is excluded it is possible, however, to invest in yearling bulls. For a given supply of labor, the maximum of livestock production can be undertaken and cereals

are included in the production plans for two reasons: they have the smallest capital requirements per hectare and they allow a better yearly allocation of labor.

- Finally, these production plans combine activities which have very different working capital profiles. Cows require large initial capital investments in buildings and animals but supply a continuous stream of income. On the other hand, cereals and yearling bulls have a low investment cost but a long maturity period (8-16 months). The association of these activities decreases the requirements of working capital.

If investment in dairying is not profitable in most cases, it becomes indifferent for those who have a larger farm and are not limited in capital. For these people, and for those who have a high preference for good working conditions, investment in dairying can be undertaken. For others, it is rewarding to accumulate first, a minimum amount of capital and equity.

C. Influence on Income of the Scarcity of Land and Labor Inputs

Capital is frequently a scarce resource which limits the level of income. Labor is not a scarce input when capital is available in large enough quantity. Labor can be hired, if it is profitable to do so. But this input does not always satisfy the assumption of divisibility. Before discussing this problem in Part IV, we estimate labor requirements for different farm sizes. Since it is difficult to study this problem in three dimensional space, capital is assumed to be available in unlimited amounts.

1. Investment in building excluded

a. Ridge line of the resource map   This resource map is bounded as follows:

$S \leq 60$ hectares

$L \geq 0$

$S = 25.40 + 44.40\ L;\ r^2 = .998$

Furthermore, after fixed costs are deducted from the objective function, two very small areas are excluded from the resource map (shaded area of Figure 31).

b. Level of income   As expected, the level of income increases with the value of S and L within the resource map defined above, Figure 31.

$$\dot{I}_1 = -37566.0 - 1343.1112\ S + 2881.74332\ L + 19681.32451\sqrt{S} - 28201.124216\sqrt{L} + 4685.48897\sqrt{LS};\ R^2 = .998$$

Once more, we observe that $\dot{I}_1$ increases at a decreasing rate. When L and S are maximum, then $\dot{I}_1 = 43{,}915$ F.

c. Capital requirement   It is equal to:

$$K_1 = -34.1907 - .7369\ S + 71.6931\ L + 25.2609\sqrt{S} + 2.2264\sqrt{L} - 1.7563\sqrt{SL};\ R^2 = .990$$

K_1 increases with S and L. K_1 is negative in an insignificantly small area of the resource map. This function increases at an increasing rate with respect to L and at a decreasing rate with respect to S. The maximum capital requirement is 166,000 F.

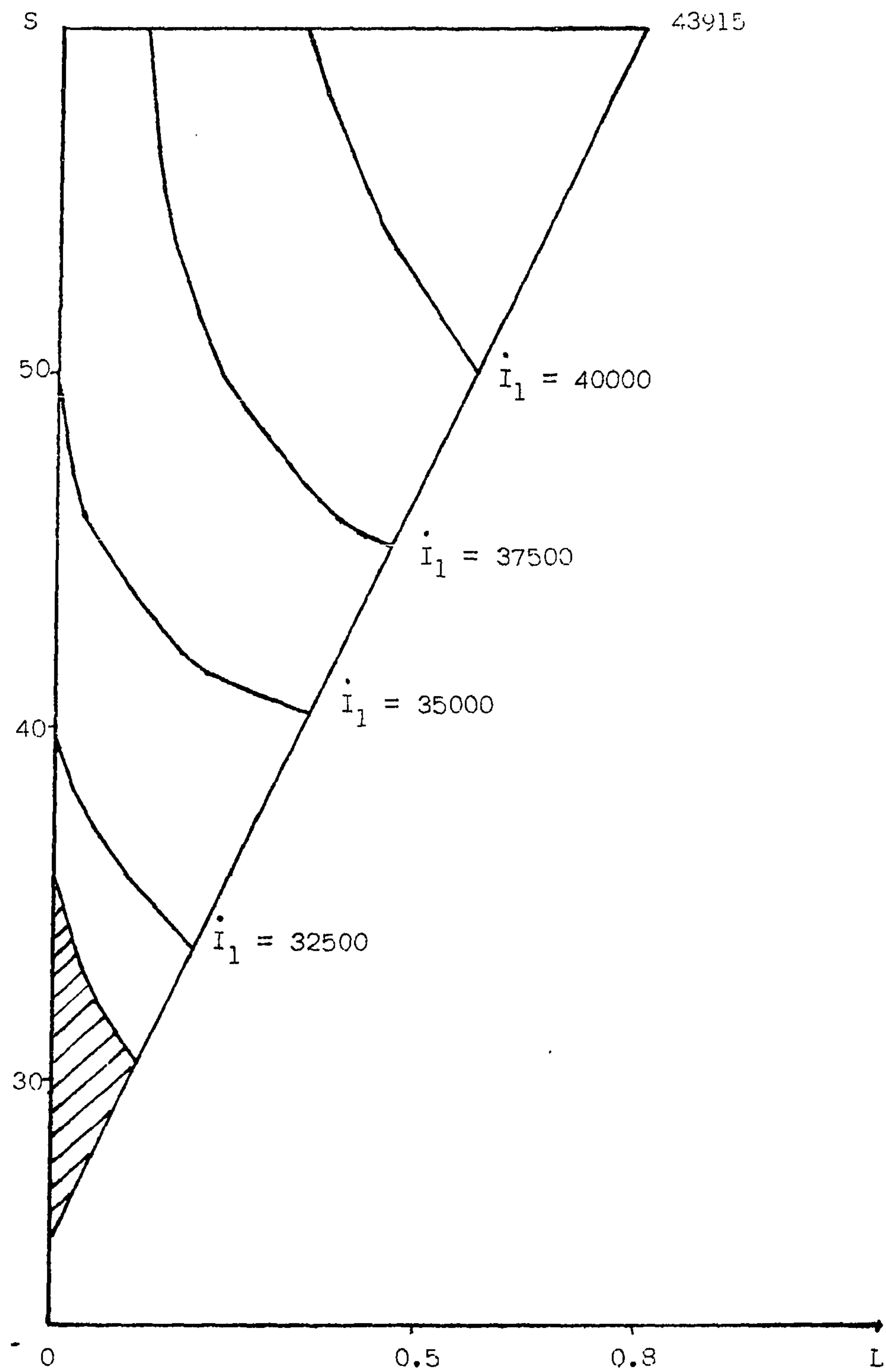

Figure 31. Iso-revenue and optimum combination of land and labor inputs

d. Main features of the production plans

Number of cows

Limited by assumption to 20, the number of cows:

- increases and then decreases with the value of S. When $L = 0$, $\partial C_1/\partial S$ becomes negative for all $S \geq 24.94$

$$C_1 = -27.02576 - 1.73016\ S + 15.8663\ L + 17.28215\sqrt{S} - 1.74374\sqrt{L} + .6719\sqrt{LS};\ R^2 = .940$$

- increases at a decreasing rate with the value of L.

Number of yearling bulls and first choice steers

First choice steers belong to the optimum solutions when S is large ($S \geq 55$) and $L = 0$ but, however, in limited quantity (≤ 10 heads). Yearling bulls are present in all solutions and are equal to 54 at most, S and L being maximum.

$$Y_1 = -21.754 + 2.3039\ S - 77.7730\ L - .0298\ S^2 - 45.7599\ L^2 + 2.4654\ SL;\ R^2 = .978$$

Number of sheep

Sheep are present in all solutions of the resource map as long as $S \geq 30$. They are also present in the upper side of the capital-land resource map when capital becomes less scarce. Their number increases and then decreases with S (when first choice steers are substituted for them). They increase with the value of L also.

$$S_1 = -244.697 + 11.75514\ S - 223.6631\ L - 12183\ S^2 + 4.4104\ SL;\ R^2 = .855.$$

Livestock activities explain the rate of increase of the function K_1, since their level increases with L and decreases with S.

Acreage of cereals, rape and corn

$$AC_1 = 19.619 + 1.8631\ S - 17.7813\ L - 11.6838 \sqrt{S} + 9.3886 \sqrt{L} - 1.188 \sqrt{SL};\ R^2 = .984$$

The acreage of cereals increases at an increasing rate with the value of S and at a decreasing rate with the value of L. At most, $AC_1 = 41$ hectares when S = 60.0 and L = 0. When L increases and equals .8, S being kept constant, 14.2 hectares of those crops are replaced by forage crops.

Fodder row crops

They are equal to:

$$RC_1 = -7.218 - .2038\ S + 5.5571\ L + 3.656 \sqrt{S} + .8450 \sqrt{L} + .0571 \sqrt{LS};\ R^2 = .964$$

The acreage of fodder row crops increases at a decreasing rate with the value of L and S. Equal to 8.87 hectares when S = 60.0 and L = 0, they occupy 14.50 hectares of land when S = 60 and L = .8.

Temporary pastures

$$TP_1 = -9.521 - .6432\ S + 12.727\ L + 6.720 \sqrt{S} + 2.3242 \sqrt{L} - 5390 \sqrt{SL};\ R^2 = .984$$

The acreage of temporary pasture decreases at a decreasing rate with S and increases at an increasing rate with L. This function is the image of the grass requirement for cows, sheep and steers, yearling bulls being fed almost exclusively with corn silage.

$$TP_1 = 3.90 \text{ when } L = 0 \text{ and } S = 60.0$$

$$= 12.58 \text{ when } L = .8 \text{ and } S = 60.0$$

This land-labor resource map has the following characteristics. An increasing level of income can be obtained with larger amounts of inputs.

However, the iso-revenue curves are pretty sloped (Figure 31). The marginal rate of substitution of labor for land is quite high and significant. But such a substitution generates more production of animal products which is undertaken at the expense of cereals output. A larger acreage of fodder crops has to be cultivated to feed the corresponding livestock. The same conclusion is reached when we withdraw the assumption of no investment in dairying. The various solutions differ somewhat. Consequently, we report them very briefly below.

2. <u>Allowed investment in dairy facilities</u>

a. <u>Border lines of the resource map</u> The corresponding resource map is bounded by the following curves:

$S \leq 60$ and $L \geq 0$

$S = 27.389 + 27.758\ L,\ r^2 = .998$

L and S reach respectively their maximum at 1.17 and 60.0.

b. <u>Level of income</u> Here again, as for the case of the capital-land resource map, there is no benefit from investing in building facilities for dairying. Indifferent when S and L are maximum, this investment becomes unprofitable when L is scarce. The comparison of Figure 31 with Figure 32 is significant in this respect. The corresponding iso-revenue are drawn from the following equation:

$$\dot{I}_{1b} = -23172.76 - 925.306\ S + 11552.088\ L + 14057.220\sqrt{S} + 29989.552\sqrt{L} - 3959.359\sqrt{SL};\ R^2 = .994$$

Within the corresponding resource map this function increases at a decreasing rate with the values of L and S. At most

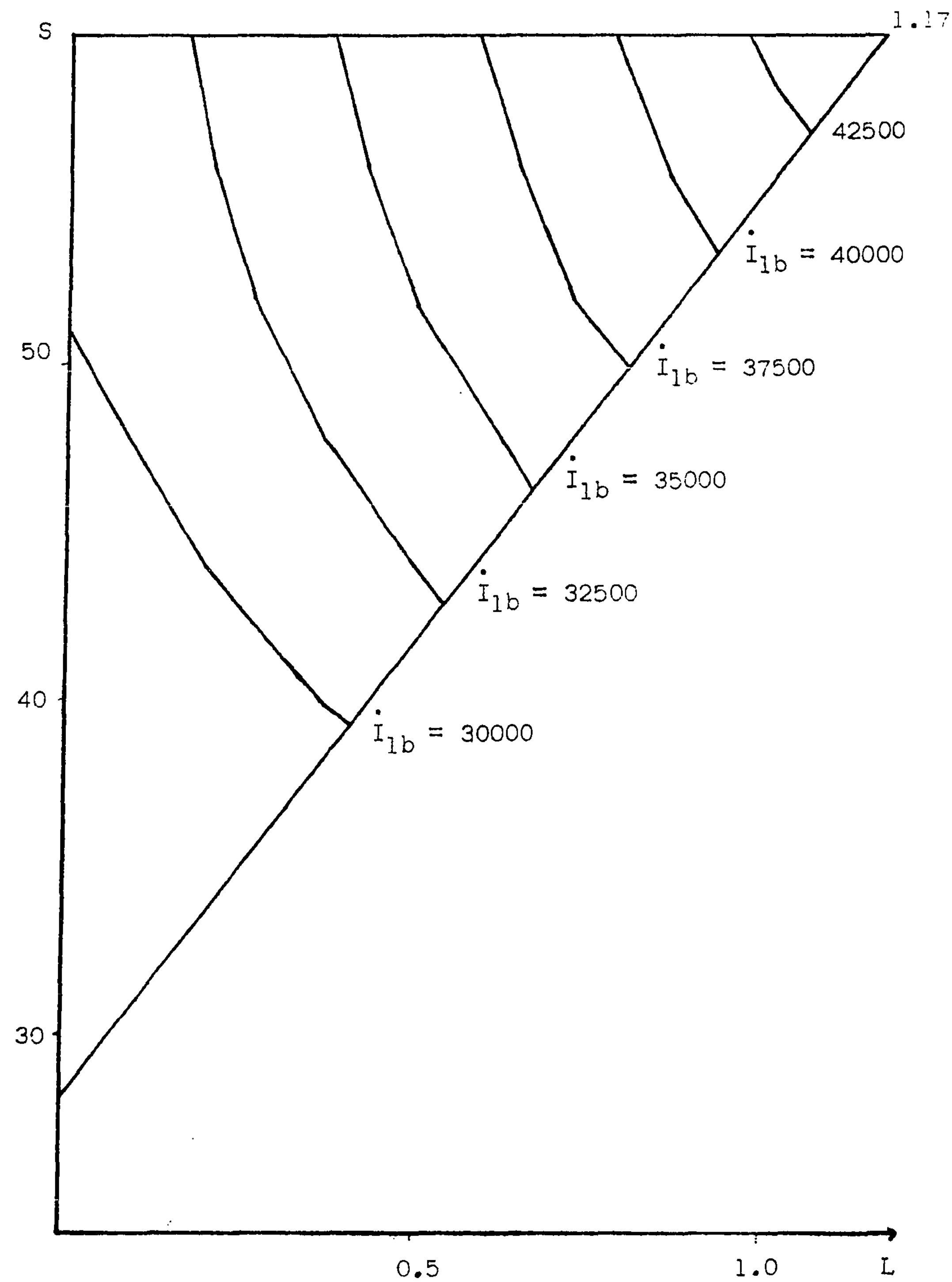

Figure 32. Iso-revenue and optimum combination of land and labor inputs

$\dot{I}_{1b}$ = 43090 F (Figure 32).

c. Capital requirement It is equal to:

K_{1b} = -73.790 - 4.1917 S + 1519671 L + 55.734 $\sqrt{S}$ + 42.6805 $\sqrt{L}$ - 10.3634 $\sqrt{SL}$; R^2 = .990

The maximum requirement is equal to 244,900 francs when S = 60 and L = 1.17. A higher value than the corresponding maximum of K_1.

d. Main characteristics of the production plans

(1) Number of cows

When L increases, this activity increases beyond the previous assumed upper limit. At most there are 43 cows in the solution (when L and S are maximum). The number of cows is given below:

C_{1b} = -18.693 - 1.369 S + 60.843 L + 12.514 $\sqrt{S}$ + 80.885 $\sqrt{L}$ - 14.964 $\sqrt{LS}$; R^2 = .964

C_{1b} increases and then decreases at a decreasing rate with S. When L = 0, C_{1b} is maximum for S = 20.88. The number of cows increases with the value of L.

Number of yearling bulls

Cows are substituted for yearling bulls. At most, 38 bulls are present when S and L are maximum, as compared to 55 for Y_1.

Y_{1b} = -1.060 + 1.114 S + 42.729 L - .01467 S^2 - 18.2234 L^2; R^2 = .927

$\partial Y_{1b}/\partial S$ is equal to zero when S = 37.968, the point at which we have a maximum, and $\partial Y_{1b}/\partial L$ is positive for all relevant values of L.

Number of sheep

They are present in a very small area of the resource map, when L is scarce ($L \leq .5$) and $35 \leq S \leq 50$. At most, we have 60 head. They are replaced by dairy cows elsewhere.

Acreage of crops

As for the preceding problem, cereals are replaced by forage crops when the number of cows increases and additional land inputs are allocated to the production of cereals. The reverse is true when the number of cows decreases.

We obtained the following relationships:

$$AC_{1b} = 22.316 + 2.146\ S - 57.315\ L - 13.543\ \sqrt{S} - 177.683\ \sqrt{L} + 27.702\ \sqrt{LS};\ R^2 = .986$$

$$TP_{1b} = -13.924 - .890\ S + 4.294\ L + 8.987\ \sqrt{S} - 63.229\ \sqrt{L} + 9.808\ \sqrt{LS};\ R^2 = .970$$

$$RC_{1b} = -10.015 - .449\ S + 28.413\ L + 5.364\ \sqrt{S} + 93.960\ \sqrt{L} - 14.839\ \sqrt{LS};\ R^2 = .950$$

Their main characteristics being already known, we shall not repeat them here.

D. Range of Feasible Capital-Labor Substitution

As new progress in applied mechanics and engineering are made, a further substitution of capital for labor becomes feasible. This possibility is excluded, to a certain extent, in our model. It is assumed that daily jobs are performed with a given set of equipment. When expensive harvesting equipment is required the corresponding jobs are carried out by a contractor. Beside this type of substitution there exists also the possibility of choosing different production plans which allow the substitution of capital for labor.

1. Without investment in dairying

The border lines are shown in Figure 33. The hatched part of the resource map represents various combinations of K and L for which S is smaller than 60 hectares. Few values of S are given in this part of the graph. They show that the associated acreage of land decreases very rapidly when L and K become scarce. Our analysis is limited to the case where S = 60 hectares (upper side of the graph).

a. Level of income Equal to:

$$\dot{I}_w = -50584.67 - 758.89\ K - 4534.485\ L + 15967.859\ \sqrt{K} - 7815.174\ \sqrt{L} + 2156.290\ \sqrt{KL};\quad R^2 = .998$$

This function increases with the value of K and L as shown by the corresponding iso-revenue curves of Figure 33. The maximum level of income which can be obtained is equal to 43,268 F when K = 166 and L = .8.

b. Optimum production plans The substitution of capital by labor is possible through an adequate change in the combination of activities. The following modifications are made.

Number of cows

It is equal to:

$$C_w = -97.235 - 1.61\ K + 7.6027\ L + 26.7266\ \sqrt{K} - 79.3632\ \sqrt{L} + 9.0409\ \sqrt{KL};\quad R^2 = .944$$

The number of cows decreases at a decreasing rate when K increases, but becomes larger when L increases. As capital is kept constant and

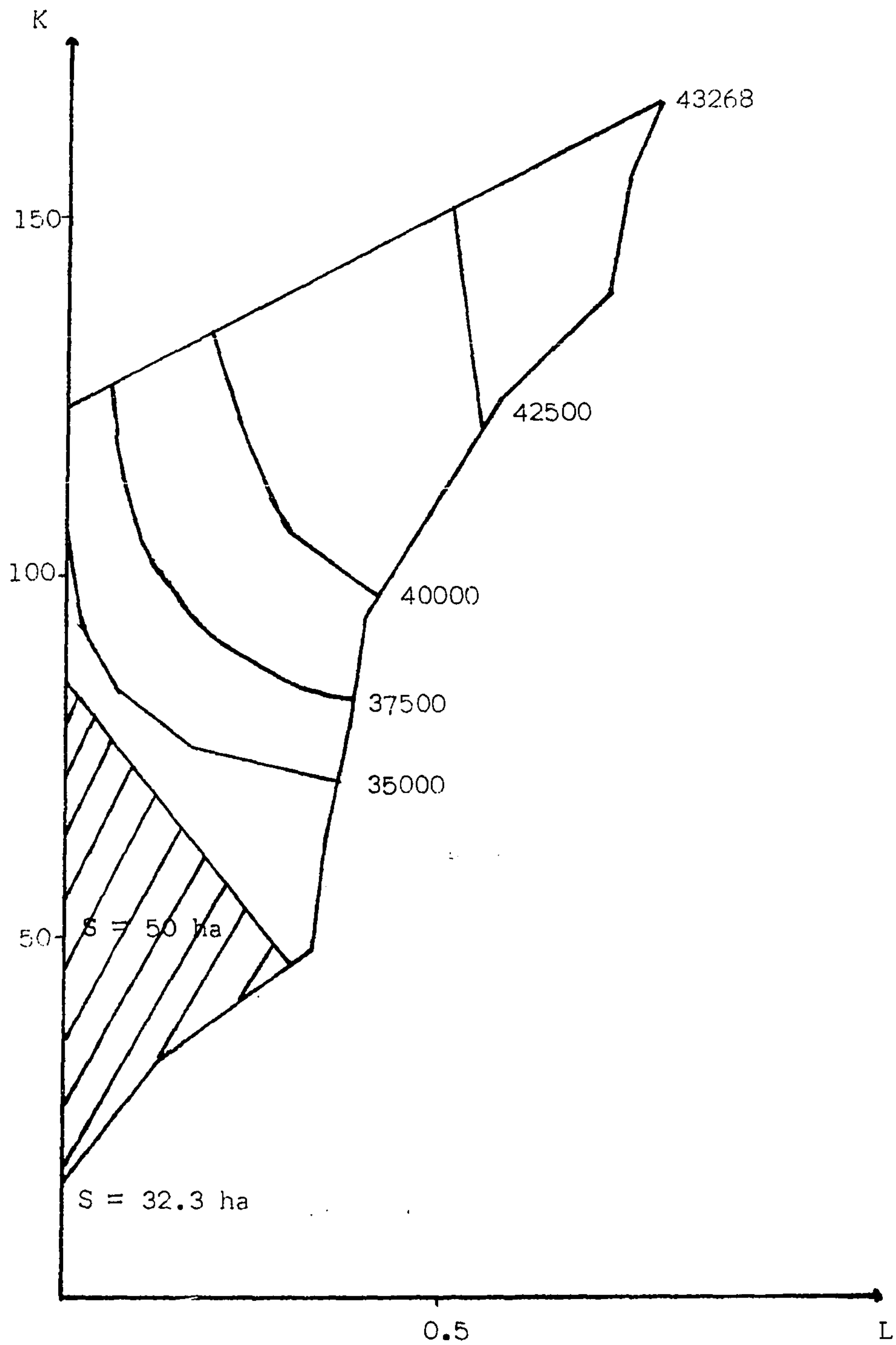

Figure 33. Iso-revenue and optimum combination of capital and labor inputs

the amount of labor increases, the dairy herd becomes larger.

Number of yearling bulls

Equal to:

$$Y_w = -157.94 - .77\ K - 127.777\ L + 25.419\ \sqrt{K} - 143.664\ \sqrt{L} + 20.899\ \sqrt{KL};\ R^2 = .968$$

The number of yearling bulls decreases when L increases and goes up when K is less scarce. We have an inverse relationship of the preceding one.

Number of sheep

It decreases with the value of L,

$$S_w = -2298.05 - 23.29\ K - 881.355\ L + 466.50\ \sqrt{K} - 1693.723\ \sqrt{L} + 210.119\ \sqrt{KL};\ R^2 = .958$$

but increases and then decreases with the value of K. $\partial S_w/\partial K$ is equal to zero when $K = [(466.50 + 210.119\ \sqrt{L})/46.58]^2$

Acreage of cereals

$$AC_w = 12.92 - .24\ K + 29.507\ L + 5.386\ \sqrt{K} + 65.986\ \sqrt{L} - 8.6349\ \sqrt{KL};\ R^2 = .962$$

$\partial AC_w/\partial K$ is negative in almost the whole resource map, but the acreage of cereals decreases and then increases when L increases. This relationship is a function of the total forage requirements for livestock.

Acreage of fodder row crops

$$RC_w = 13.289 + .35\ K - 10.464\ L - 4.2384\ \sqrt{K} - 1.2425\ \sqrt{L} + .6239\ \sqrt{KL};\ R^2 = .994$$

The acreage of fodder row crops increases when K increases and decreases in almost the whole resource map when L increases. This function has the same shape as Y_w. The acreage of fodder row crops is determined to a large extent by the number of yearling bulls.

Acreage of temporary pastures

The acreage of temporary pastures varies inversely with cereals acreage. It increases, reaches a maximum and then decreases when L goes up. On the other hand, $\partial TP_w / \partial K$ is positive within the whole resource map.

$$TP_w = -6.239 + .01\,K - 15.686\,L + 1.0792\,\sqrt{K} - 12.527\,\sqrt{L} + 2.4197\,\sqrt{KL};\quad R^2 = .855.$$

In short, for a given amount of land input, the replacement of capital by labor generates:

- A substitution of dairy cows for yearling bulls and sheep
- A substitution of cereals for forage crops

We can observe that dairy cows generate a flow of net returns while yearling bulls, cereals and sheep give their output at one time. Cereals have the lowest capital requirement. The combination of a dairy activity with cereal production can reduce the needs for capital, but as an offset, requires a larger amount of labor. These relationships explain a few of the production plans which are traditionally found in this region. Dairying is associated with cereals and hog production in many farms. When building space can't be used indifferently by all farm animals, as is the case when we assume that dairy cows will be sheltered only in new buildings, the range of capital-labor substitution is narrowed to a large extent. The substitution described above makes available a certain number of square meters of building but, according to this assumption, they are idle.

E. Conclusion

In this chapter it has been shown that limited amounts of capital or labor reduce the level of income, unless holdings can be enlarged. This possibility is presently ruled out, in most cases. These results are very important and their economic implications will be examined in the next chapter.

When capital is scarce, it is possible to replace capital by labor; but this possibility is somewhat limited. It is even ruled out in small holdings, an excess of labor being already available there. The largest difficulty comes from the fact that in the most profitable production plans a large proportion of resources is allocated to the production of livestock products.

These activities require large amounts of capital and labor inputs. For given farm sizes, the highest levels of income can therefore be obtained if the corresponding production plans are mainly devoted to the production of livestock outputs and, consequently, more intensive applications of labor and capital are made on the fixed area of land.

CHAPTER 22. FURTHER RESULTS IMPLICATIONS

In the preceding chapters we have accumulated a large quantity of results. By themselves, they answer many questions. But we can use them more intensively in order to throw some light on present and crucial problems such as farm labor migration, efficiency of farm resources, progress and agricultural policy problems.

A. Causes of Discrepancies Between Present Farm Plans and Programming Results

When we consider our best results and the present farm plans of this region, we are struck by the large discrepancies between them. Only a small number of farmers get the levels of income shown in this study, for the present price situation and state of knowledge. In some cases when they obtain it, they insert into their production plans additional activities such as hog and broiler productions. These activities are excluded from the farm plans to which we refer. When allowed in our solutions, these activities were undertaken on a much larger scale than is usually found in this area. Roughly, we can say that three farmers out of four who are members of the accounting agency[1] do not obtain the level of income we found. The variance of results is quite large since they obtain on a 30-40 hectares farm, about 600 F ± 600 F of income per hectare with continuous variation between the extreme values (17, p. 28).

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[1]Centre de Comptabilité et d'Economie Rurale - 6, rue de l'Ancien Evêché - Laval - 53 - France.

These discrepancies can be explained very easily. The main differences between our results and the existing farm plans are observed in the following variables.

1. Level of forage yield and stocking rate

In this area the stocking rate varies from 0.8 to 1.8 livestock units per hectare of fodder crops with a mode at 1.25 (17, p. 28). Most of our results show a stocking rate equal to 2.0 except when we force into the solution an increasing acreage of permanent pasture. Such a difference is due to various causes:

(a) The level of forage yield is lower than 5,000 transformed forage units per hectare of temporary pasture on many farms. To give an example, the alfalfa hay yield is estimated at six tons by the French Department of Agriculture (59, pp. 207-212), although the potential yield is much higher. Six tons is considered as a lower bound by scientists.

(b) A much higher acreage of permanent pasture is presently kept on farms than in our models, in which it is always reduced to its minimum level (10%). In present farm situations this percentage varies from 5 to 80% with a mode of 35% (17, p. 32).

(c) A larger acreage is allocated to the production of fodder row crops in our models than in present farm situations. Our models presume new methods of fodder beet harvesting and the possibility of cultivating corn for silage. These productions and methods are starting to be adopted by farmers but their adoption is not yet widespread.

In the optimum farm plans, a more important acreage is allocated to the production of fodder row crops than in the present farm plans. Furthermore, the summer green forage deficit is supplemented with corn silage, while presently farmers underfeed their animals or distribute an additional hay ration in drought periods. This practice contributes to increasing the stocking rates since the corn yield is higher than the hay output per hectare.

2. Milk output per cow

The more productive dairy herds are chosen in the optimum farm plans, but actually Frisian or the productive Normande herds are not adopted by all farmers. Three thousand liters of milk per cow per year is more frequent than 3,800 liters.

3. Production of slaughter and feeder steers

Only first choice steers, the activity we had defined after we analyzed our first results, and yearling bulls belong to the optimum solutions. The shadow prices of the traditional slaughter and feeder steer activities are high and range from 200 to 780 F per animal when they are associated with productive dairy herds and from 100 to 550 F per head when they are associated with unproductive dairy cows. If yearling bull activities are excluded from the possibility set, then income decreases by about 4,500 F or more, when the sale of cereals, the production of sheep and yearling bulls fed almost exclusively with corn silage are forced to zero by assumption. The following table is significant in this respect.

Table 49. Influence on income of the exclusion of yearling bulls from the production possibility set[a]

Item	Normande or Frisian 3,000 liters	3,800 liters	Maine-Anjou 2,750 liters
Without yearling bulls			
Income (francs)	12,686	16,969	7,032
Labor (man year)	.33	.27	.23
Capital (francs)	97,590	93,930	94,010
Slaughter steers			
28-month	9.0	5.07	8.8
26-month	11.2	7.45	0.0
Steer at livery	0.0	0.0	7.61
With yearling bulls			
Income (francs)	17,405	21,201	17,934
Labor (man year)	.96	.77	.96
Capital (francs)	153,210	130,840	144,730
Yearling bulls	45.20	32.85	49.44

[a]Forage yield, 5,000 fodder units.

This area has been traditionally oriented toward the production of feeder and slaughter steers. Of 245,300 head of livestock on "Mayenne" farms on the first of October, 1964, 22.34% were destined to be sold as replacement stock, 52.30% sold as feeder cattle, and 24.29% to be slaughtered.

The input-output relationship has not been altered for years, however. It is significant that the steer activities which belong to the programming solutions are not traditional for this area. They differ from the classical steer activities by:

a. The date of slaughtering Beef prices are higher in spring and lower in fall. Most of the slaughter steers which are produced in this region are sold in October or November. Those in our solutions are slaughtered from February to May, the period of higher prices.

b. The feeding programs The bulk of traditional feeding rations consists of grass. Fattening animals are fed on pasture while the programming solutions show that the optimum feeding programs contain a larger proportion of corn silage and fodder beet and a smaller quantity of hay. During the fattening period animals get rations consisting mainly of beet and corn silage. In a few cases they are fed with spring grass (28-month slaughter steers).

c. Age at slaughter The more profitable activities are characterized by a lower slaughter age. In no case are 36-month steers profitable. They even have the highest shadow price per kilogram of carcass. There is an exception, however. These activities are profitable for the beef fattener when these animals are bought as feeder steers at a market and fed with corn silage and fodder beet. But this assertion implies that someone has probably produced this animal at a loss, depending on his forage production possibilities which could be limited on certain soils. This case is not frequent in this region, however.

In short, the most profitable steer activities are those which include the fattening period, utilize a large proportion of productive and highly nutritious fodder crops, require a lower quantity of energy per kilogram liveweight gain and are sold during the period of higher prices. Our results show that the beef/milk price ratio is presently such that a larger quantity of resources is allocated to the production of milk and a lesser one to the production of meat. Unless the beef/milk price ratio changes significantly, the only way to make the production of steers competitive is to favorably alter their input-output relationships. If we knew with precision the compensatory growth relationships, then it would be worthwhile to integrate within a linear farm model a series of activities in such a way that the programming solution determines simultaneously the optimum steer growth curve and the cheapest feeding program. This is a subject for further research.

4. Livestock feeding programs

We determined (Chapter 10) a few choice rules for selecting the subset of the more profitable feeding rations. But in practice, such rules are not being followed.

The discrepancies which exist between present farm plans and our most profitable ones are narrowed when we choose, as a point of comparison, the results reported in Chapter 18.

B. Means of Improving Income

To improve income we rule out various possibilities such as:

- acting upon the price level through a government policy
- subsidizing a research program
- making available larger bundles of resources
- decreasing uncertainty.

We are more concerned with the task of reducing the discrepancies between optimum farm plans and present ones. To determine the causes of such discrepancies and indicate one means of reducing them, we take a specific example: the improvement of forage production. It has been shown profitable (Chapter 18) but is far from being used by farmers.

1. Some obstacles to the adoption of progressive forage production methods in the "Bocage Angevin" region

Although grasslands occupy 66% of the agricultural area (50% and 7% respectively for natural and sown swards) in the Mayenne departement, the ability (or the decision) to adopt the methods that could increase forage production has been lacking. Per hectare grassland yield estimates of the French Department of Agriculture (59) are far behind the potential yield levels since, in 1966 for example, they are equal to six tons of alfalfa hay while, according to local extension agents[1], it is possible to produce 10 tons of hay.

This general under-intensification of forage in the area is probably due to many factors. Among the main ones we can enumerate:

(a) The lack of productivity of livestock in general, as shown by different statistics (1, 3). This situation is due partly to environmental conditions such as unadapted rations and insufficient selective breeding programs. This is probably one of the main obstacles which hinders the adoption of modern techniques of forage production within the present price and yield set. (See the Results of Chapter 18 and particularly the influence on income of milk yield per cow.)

(b) The difficulties of integrating modern techniques of forage

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[1]Houdan, M. and Vignier, D., Maison de l'Agriculture, Laval, 53. Hay output. Private communication. 1967.

production in a harmonious manner into all the activities already present on the farm. The techniques which have been rapidly adopted, or refused, have few common features. They improve the working conditions, yield an income which can be readily disposed of, and contribute to the improvement of the standards of living. Now when forage production is being intensified, there exists in the first stage, rather unpleasant prospects such as the need to care for a larger herd, to maintain more accurate accounting records, and to accept momentary financial sacrifices. The harmonious integration of modern techniques of forage production into a farm plan is not easy and requires farmers to accept new ideas readily (8).

(c) The extension service programs over the past ten years. Historically, in Mayenne, the extension service has set up programs which can be summarized in the two following models (8):

Model one:

$$Y_j = F[x_1 \mid x_2 \ldots x_n]$$

where

x_i = factor of production

Y_j = output of product j

Here, extension agents develop the use of only one input whose marginal revenue is high enough to get a rapid and widespread adoption of the recent technological progress.

Model two:

$$Y_j = F[x_1, x_2 \ldots x_m \mid X_{m+1}, \ldots X_n]$$

However, when extension agents have won farmers' confidence they try to act upon several relevant variables of a production function: seed quality, tilling efficiency, fertilization and so on.

Although these two preceding models are necessary to provide the farmers with up-to-date technological knowledge relative to every type of product, they are insufficient to generate a significant trend of progress within the agricultural sector. These models have generated only slow and gradual progress, although they require no deep thought. A careful imitation of what has been done by a neighbor is almost sufficient for success in most cases. But farmers usually realize that new practices cause a chain reaction in the farming system. When they are reluctant to set up all at once, a good new farm plan which incorporates efficiently the new techniques, they keep the status quo or decide on making insignificant shifts each year. A new model is therefore badly needed and should be applied without further delay.

2. <u>A more ambitious model for the extension service</u>

Let's define this model as follows:

$R = f[z_j \mid x_i]$

where

R = income

z_j = activity j, j = 1...n

x_i = environment conditions; i = 1...n

Generally, when it is attempted to maximize R for each farm of the district, farm reorganization is called for. Here, innovation is a discontinuous phenomenon rather than a continuous one as in model two. The adoption of progress has to be done at once and therefore has the appearance of successive mutations with the continuous incorporation of the recent techniques which are consistent with the previous reorganization. Presently this type of model seems to be developed in different parts of the world. It is called "systems basis" in the United States (39, p. 110). In France, if it has not yet received a formal name and a wide application, people realize that much work has to be done in this direction, in order to make any extension work more efficient. A. Leveque, in his paper "Psychological obstacles to the adoption of progress among stock-breeders" (50), shows that it is necessary to provide the farmer with the means of convincing himself, when new solutions are offered to him, to apply global perspectives for the farm and to adopt the information to the level of progress actually reached by the farmer. In the same issue Rouch, Bonnefous and Prugniaud (66) argue that extension services should provide farmers with "ready to use" farm-plan models. From all this, it is clear that economics has to be of more importance within the present extension service organization. In the models of farm development it is necessary to take into account the managerial abilities of farmers through their own input-output coefficients, their repayment capacity and level of equity and their preferences. But one of the most important decision elements of decision is related to the risk and uncertainty problem. These elements should be integrated in decision models (parametric or quadratic programming) or evaluated subjectively. But in any case, they affect deeply the process of decision making. More than ever, it is necessary to place economic principles and models at farmers' disposal through an efficient extension service or accounting office. Integrators and other people try to develop such and such production in the area, in order to develop their own business. As a result farmers, solicited by many canvassers, risk altering their present production plans in a rambling and unprofitable manner.

C. Resource Pooling and Economic Efficiency

It is well known that the allocation of resources within the farm sector is far from optimum. This fact is recognized by many authors and in particular by Heady (37, p. 272). Suppose that two farmers have the following control over labor resources. The first has a very small farm and cannot get an off-farm job; the second has a large farm and cannot hire farm workers on a part-time basis. We get the following income function:

$$\pi_j = P_y Y(L_i) - P_i L_i$$

where

π_j = income of farmer j, j = 1,2

P_i = price of input i; i = 1 = labor

P_y = price of output Y

When farmer one maximizes his revenue, he allocates labor in such a way that:

$$\frac{\partial \pi_1}{\partial L_1} = P_y \frac{\partial Y}{\partial L_1} - P_1 = 0$$

$$\text{or } \frac{\partial Y}{\partial L_1} = \frac{P_1}{P_y}$$

But $P_1 = 0$, since labor is a fixed resource available in a large amount on a small holding. Consequently, farmer one works as long as he gets a positive marginal return for his labor.

Farmer two, however, has a limited amount of labor. He acts in such a way that the following relationship is true:

$$\frac{\partial \Pi_2}{\partial L_2} = P_Y \frac{\partial Y}{\partial L_2} - P_1 = 0$$

$$\text{or } \frac{\partial Y}{\partial L_2} = \frac{P_1}{P_y}$$

In this case, if the corresponding value of L_2 is not an integer, then the true optimum is equal to either one of the following values:

$$L_2' = L_2 + \theta'$$

$$L_2'' = L_2 - \theta''$$

where L_2' and L_2'' are integer

The optimum allocation of resources is characterized by a relationship such as:

$$\frac{\partial \Pi_2}{\partial L_2} = P_Y \frac{\partial Y}{\partial L_2} - P_1 - \lambda P_1 = 0$$

$$\text{or } \frac{\partial Y}{\partial L_2} = \frac{P_1}{P_y}(1 + \lambda)$$

where λ = lagrangian multiplier.

The marginal productivity of labor is equal, in the first case to zero, in the second case to a positive value whose magnitude increases with the value of λ. It is obvious that resource pooling would be profitable to both farmers.

We can enumerate two sources of economic advantages and benefits in favor of resource pooling:

- A better allocation of scarce resources between farms

- A size of operation, such that it becomes feasible to make certain economies of scale (on machinery, labor inputs...). But, there also exists, certain diseconomies of scale which can be crucial and particularly when decisions are made corporately by people whose management abilities are very different.

Four resources can be pooled by farmers. They are: management, land, capital and labor. Management is pooled when people discuss their own expectations and decisions with others. This resource can even be bought from a management office or employment office. Since land is one of the most scarce resources whose shadow price is very high (1,300 F per hectare), the demand for pooling would be higher than the existing supply. We therefore discuss only capital and labor pooling.

1. Capital

Figures 18 and 25 show that the influence of capital scarcity on income level is very strong.

There are many ways of financing a farm firm. Before discussing them we summarize the ground rules for loans. They are:

- The discounted marginal revenue of capital has to be equal to its discounted cost.
- The loan should never exceed the repayment capacity of the firm. In this respect we can distinguish two types of assets. Those which are paid from gross income (feed, fertilizers...) and those which are paid from net income. The first ones rarely

give repayment problems. The second ones are more troublesome, especially when the difference between the repayment amounts and the stream of income they generate is large.

- The loan is proportional to the risk-bearing ability of the firm and household.

For farmers, one of the biggest problems consists of generating equity and risk bearing abilities rapidly enough within the farm-firm. The "rapidly enough" is important since the opportunity cost of capital is very high. For a 30 hectares farm we have:

$$d\dot{I}_K/dK^1 = -592.164 + 5626.607\ K^{-.5}$$

Farmers should, therefore, save as much as they can in the early stages of their life. The propensity to save, however, is a function of the level of income and of the intertemporal preferences of the family. Through time, the objectives of farmers should follow the following pattern:

- To save the maximum when their children are still young. The household requirements for consumption are then at their minimum level and the opportunity cost of capital is very high in the farm-firm. This strategy allows the level of income and the equity position of the firm to increase rapidly.
- To transfer a larger proportion of income and even some accumulated capital from the firm to the household when children get to secondary school and college, if needed.
- And finally, to transfer income and capital from the firm to the household for retirement purposes.

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[1] $\dot{I}_K$ and K are defined in Chapter 21, pp. 251-252.

Under such a pattern, however, lies the following assumption and value judgment. An important fraction of farm youths has to get an off-farm job. They are rejected by the agricultural sector. But they have the right to be prepared to make such a transfer.

When personal saving is not sufficient, farmers can use equity and risk bearing generated outside of the farm firm by others. The main sources are:

(a) Traditional crop and livestock share leasing. Although the traditional share leases are established on a 50%-50% basis, a 30%-70% share lease might be better for a young farmer with a very small amount of capital, allowing him to take a larger farm.

(b) Sharing arrangements between operators and merchants. Vertical integration is being developed in the production of broilers, hogs, slaughter calves and steers. Trying to obtain an expanded market for his product and/or services, integrators are therefore interested in the timing, quantity, efficiency, quality aspects of production. To retain control, they retail credit to farmers using only the flock as security. But such loans are usually granted for financing nondurable goods. Most of the risk is therefore left to farmers.

(c) Closely held corporation

Presently, stocks have not been sold to raise equity capital. Loans are still received from the same sources and operators are still personally liable for farm debts. Under such conditions, a corporation is not a means of generating equity since the risk ability of the firm is equal to the sum of the initial personal equity of the stock holders.

Family-held corporations will probably be developed in the future due to their intergeneration transfer advantages and to the possibility of interesting all children in the family business.

Leaving the area of joint account farming enterprises, there are other ways of increasing the risk-bearing and financing ability of the farm firm.

(d) Bail-bond. It is a common practice for the cooperative credit associations to ask for a bail-bond. In case of no loan repayment the bailsman has to pay for the bailee. This is therefore an efficient and cheap way of transferring equity.

(e) Resource pooling between farms. It is frequently used to get the benefit of returns to scale when modern equipment is used on a large scale. On the other hand, total investment costs are much smaller for each entrepreneur.

(f) Leasing. Cash land lease and machinery leasing are other means of decreasing the total capital requirement. In this domain, the most difficult problems of management are generated by acquiring ownership.

(g) Making the future less uncertain. Crop and life insurance as well as technological devices increase the risk-bearing ability of farmers while reducing uncertainty.

Although there exists many ways of financing the farm, we feel that the problem of capital accumulation will be, along with management, the most difficult task of tomorrow's farmers. The minimum level of capital which is required to start farming increases with farm size (Figures 18 and 25) and technological progress.

2. Labor

Although labor does not constitute a scarce resource by itself, it can be considered as a limiting input when it can be hired in only discrete amounts. If it were easy enough to hire day-farm workers in the past, this opportunity is almost ruled out today. When farm size is not a multiple of 27 hectares (Figure 30) or is larger than 25 hectares (Figure 31), then the allocation of labor inputs is not optimum. The farm manager has to choose between hiring too much or too little labor relative to the optimum solution which results from the assumption of divisibility. We are taking as an example the case of scarce land and labor when investment in dairy facilities is excluded (Figure 31). The level of income is given by the following equation:

$$\dot{I}_1 = -37{,}566.0 - 1343.1112\ S + 2881.74332\ L + 19681.32451\ \sqrt{S} - 28201.124216\ \sqrt{L} + 4685.48897\ \sqrt{LS};\quad R^2 = .998$$

The corresponding border line is equal to:

$$S = 25.399 + 44.40\ L$$

or

$$L = -.571166 + .0225\ S;\quad r^2 = .998$$

If $L = 0$ or $L = -.571166 + .0225\ S$, then we get the corresponding revenue curves R_a and R_b (Figure 34). When $L = 1.0$, then the corresponding revenue curve (R_c in Figure 34) represents the lower bound which can be reached under such an assumption, since here we consider that the labor surplus is left idle. In our computations L was considered as a variable input, while now it is viewed as a fixed cost. The difference: $\Delta R = [R_b - (\text{Max:}\ R_a, R_c)]$ shows the maximum opportunity cost

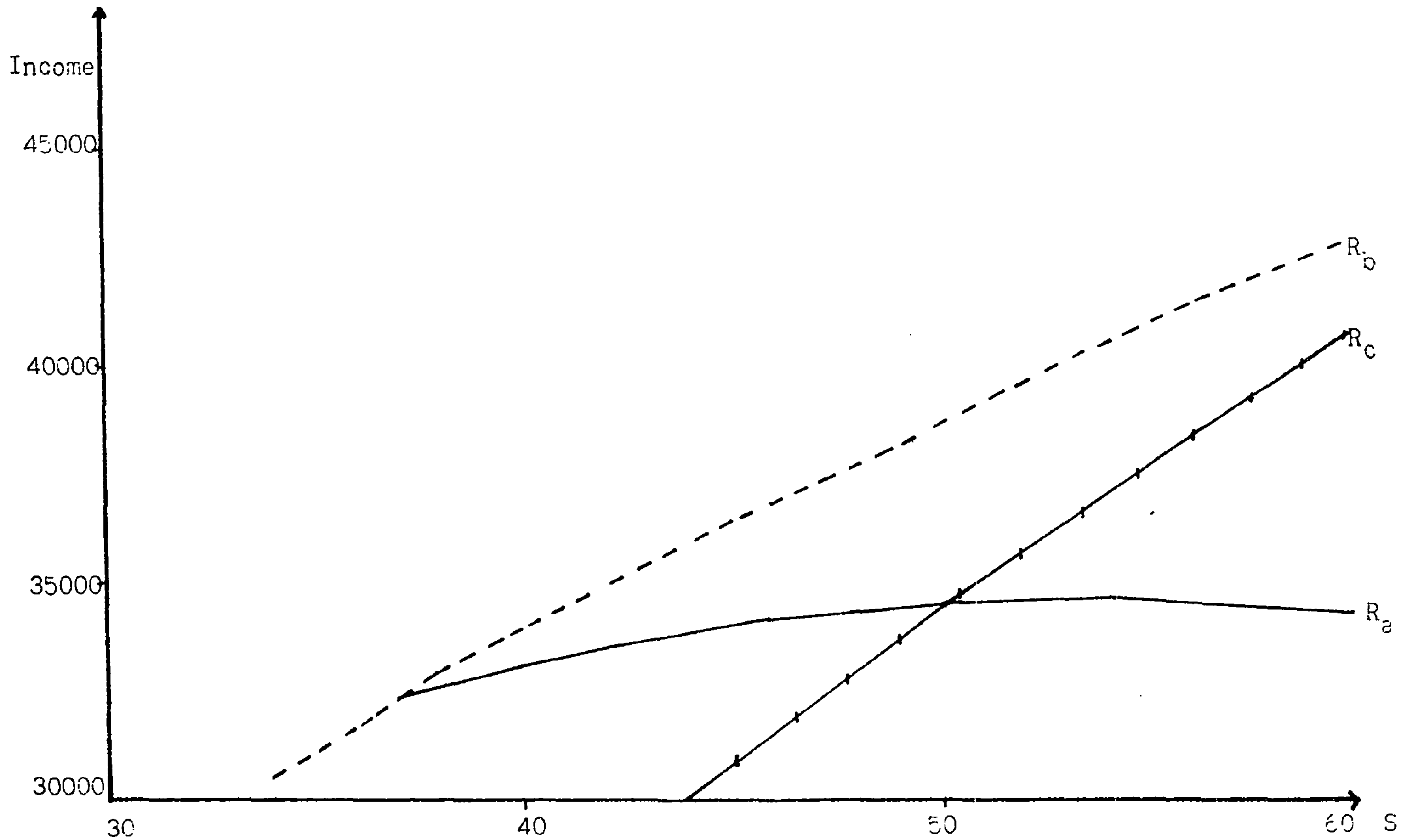

Figure 34. Opportunity cost due to labor indivisibility

due to the indivisibility of labor (or no labor-input pooling). Labor pooling would therefore be highly profitable. It could be done according to the following relationship (among others):

$L = L_1 + L_2 = 1.0$

$L = L_1 + L_2 + L_3 = 1.0$ or 2.0

L = total hired labor

where

L_i = hired labor used by farm i; i = 1,2,3

S_i = size of farm iji = 1,2,3

To get optimum returns from resource pooling, it would be necessary to pool hired labor for farms whose size is defined by the following relationship:

$S_1 + S_2 = 95.214$ hectares; $L = 1.0$

$S_1 + S_2 + S_3 = 120.60$ hectares; $L = 1.0$

$S_1 + S_2 + S_3 = 165.04$ hectares; $L = 2.0$

The corresponding curves are shown in Figure 35. We have limited resource pooling to three partners but we could conceive a more general model although there are many optimum associations among farms of various sizes. It can be difficult to organize labor resource pooling on a small scale. Farm workers would be transformed into day workers and they refuse such a status. They have several employers who save for them the harder tasks. It would probably be more realistic and more successful to create modern enterprises which would hire farm workers, buy modern farm equipment and perform farm jobs on a contract basis. Presently there are few organizations whose purpose is resource pooling: machinery cooperatives, work banks and contract threshers. They have a limited

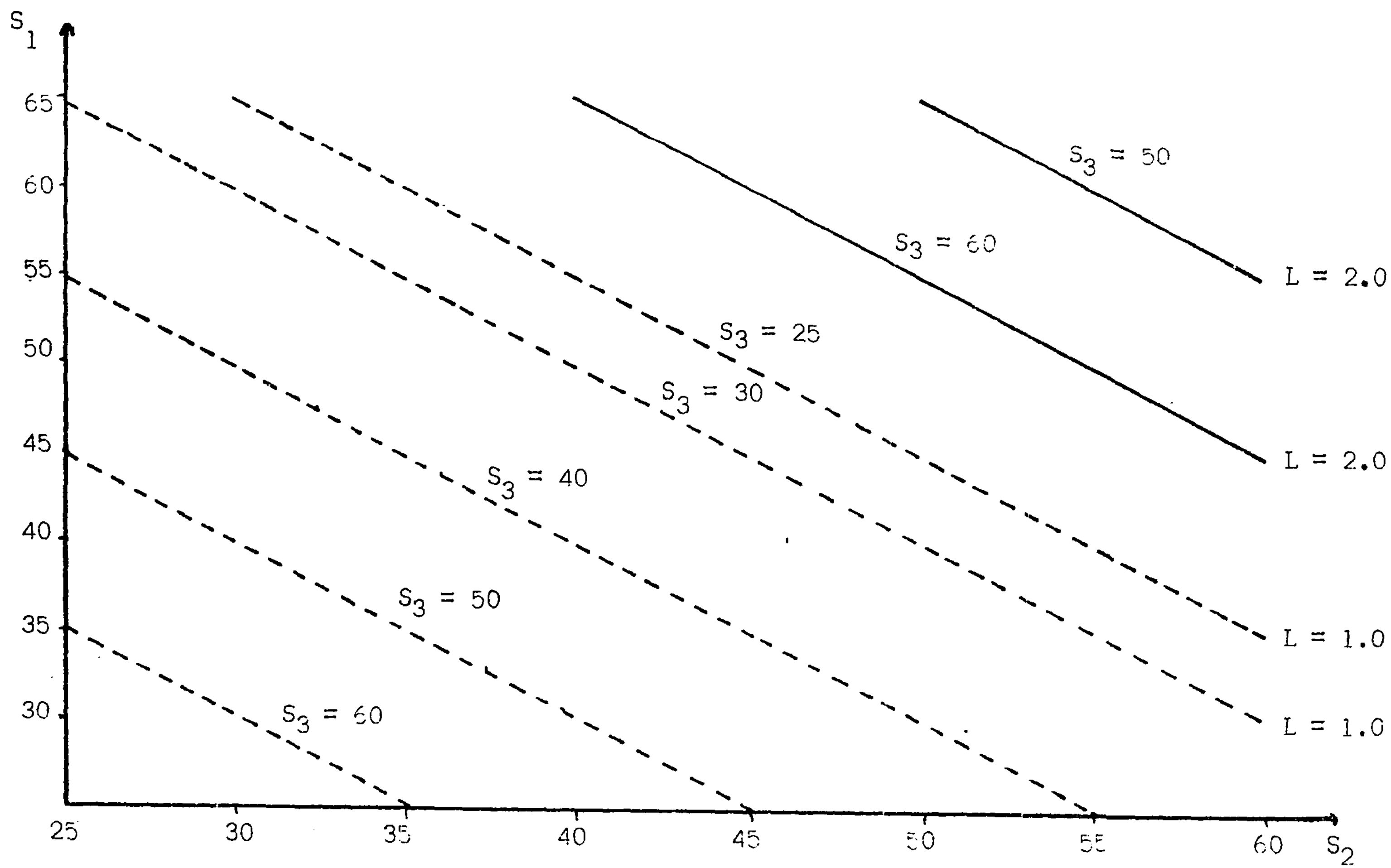

Figure 35. Labor pooling and optimum combination of farms of various size

development. They are too specialized. The enterprises we are thinking of should be able to undertake any farm job and to make farm labor as well as equipment divisible. Farmers whose farm size is smaller than 25 hectares could even work as part-time workers for these enterprises. There has been some experience with closely held corporations. A few have already been broken.

Modern enterprises of farm jobs could incorporate most of the economic advantages of closely held corporations, since labor and equipment resource pooling constitute the main sources of benefits which are frequently enumerated and estimated. Such an enterprise would be more flexible and potentially larger than the corporation system, which is rejected by people who do not want to lose too much of their freedom.

D. Elements of Choice Between Farm and Nonfarm Employment

It is not possible to provide a general answer to such a problem. Too many variables and particular cases are involved. Furthermore, many elements of choice are related to taste, preferences and family values. The income differentials are only one element of choice. The final decision can be made only by individual families.

1. Equivalent level of living

Some adjustments should be made to correct income functions for differences in the cost of living between farms and towns. But this adjustment is very difficult. Among the main factors responsible for a cost of living differential, those that favor rural people (72, pp. 26-28) are:

- The importance of home-grown food. Only 2/3 of the total consumed food is bought.
- The cost of housing and transportation. Farm dwellings are usually provided as part of the farm business.

The cost of education (boarding school fees) works to the disadvantage of rural people.

Urban people usually have to save to buy a home. Farmers, although dwelling is almost always supplied with the farm, have to save in order to set a profitable business. Since we cannot fully take all of these considerations into account, we will compare only unadjusted income opportunities.

2. Nonfarm income opportunities

Nonfarm income opportunities are especially known for wage workers. Wage income in manufacturing industries, trade and services are shown in Table 50. These results, obtained from an analysis of income tax

Table 50. Average net wage opportunities in France in 1966, F/year[a]

	All categories	Top executives	Medium executives	Employees	Workers[b]
Men, average wage	12,600	41,476	20,689	11,585	9,976
Women, average wage	8,079	25,714	14,317	8,873	6,299

[a]Source: (9, p. 12).

[b]Including foremen and apprentices.

Table 51. Average wage opportunities for unskilled workers in Mayenne in 1967 in Francs per year[a]

	Processing industry	Manufacturing	Building
Starting wage	6,000	4,200	6,000
Full wage	8,400	6,000	8,400

[a]Source: Mr. Hebert, service d'orientation professionnelle, Laval, 53. Wage opportunities. Private communication. 1968.

returns, are relative to the general French situation. Local employment and income opportunities are given above for unskilled workers. Those who have vocational school training make higher incomes, especially in manufacturing industries.

3. Farm income opportunities

Figures 18 and 25, Chapter 21, pages 254 and 267, show that it is possible to get the level of income of:

- a top executive on a 50-60 ha farm
- a medium executive on a 20-30 ha farm
- manual worker on a 10-20 ha farm

depending upon the level of available capital. But in order to get such a level of income in farming, it is necessary to be able to

- be an efficient manager
- control a farm whose nontillable land is less than or equal to 10% of the total farm acreage.

When such abilities are lacking or when the farm is of only medium soil quality, the corresponding income opportunities decrease very rapidly. On a 30 hectares farm, a top manager will obtain an income of 28,600 francs, an average one: 23,000 francs (3,000 liters of milk per cow and 4,000 fodder units per hectare) and a below-average manager, 17,200 francs (see Chapter 18, Table 42). If, on the other hand, the acreage of nontillable land increases, the corresponding income decreases by 50% very rapidly. Although the variance is high, farm income opportunities are still such that farming is a lucrative job, especially when it is possible to control a certain quantity of capital and land resources. A further price-cost squeeze can be expected in the future. Consequently, to start farming on a farm larger than the minimum size required for a certain level of income is surely a wise precaution.

Farm income opportunities have been computed for cash tenants. When, after a few years, operators are forced to buy their farms (when they cannot or refuse to quit farming) the rate of saving they have to impose on their families can be very high and at the limit, almost unbearable. With the price-cost squeeze it is the highest risk facing young operators.

E. Aggregate Implications of Results

1. Labor transfer from agriculture

Table 2, page 7, shows that 30.1% of the "Bocage Angevin" farms are smaller than 10 hectares (including retirement holdings), 31% range from 10 to 20 hectares and 36.4% from 20 to 50 hectares.

Furthermore, the land labor ratio is 7.7 hectares (men and women) and 13.6 hectares (men only). Our results show that a 25-30 hectares farm can be run by a couple of young operators and a 60 hectares farm with two men. Moreover, the land shadow prices vary from 900 to 1,500 francs while the per hectare cash rent is worth 300 francs. On the other hand, land is currently sold for 10,000 francs per hectare in small track and 8,000 francs in larger ones. From these figures we can easily deduce that, in the future, the number of farms and the number of persons employed in agriculture will decrease while the price of land will probably go up. The level of farm income opportunities and the degree of uncertainty tied up with nonfarm jobs are such that the transfer of labor from the farm sector to other jobs will be the result of migration of a group of people who are either rejected from agriculture or who prefer off-farm jobs. The main obstacles to labor transfer are the lack of preparation for nonfarm professions, general knowledge, guidance, and efficient employment services. Employment services should be able to inform people of nonfarm employment opportunities, job openings, personal adjustments required for new occupations and new living environments. These services should not only inform people but also help them to adapt to their new environments. One of the main objectives of public organizations and of the rural community should be to give more opportunity for education to farm youth who will be rejected by agriculture. Table 4, page 10, shows that opportunities for education are low in rural communities. This aspect is, however, one of the most important and crucial choices which have to be made and which determines the professional future of many individuals. A recent survey among unemployed young people (22 years of age or less) shows (48) that 42.6% of them have no diploma while 50.6% obtained a primary school certificate, 6.4% a junior high school certificate and 0.4% are graduates of a secondary school. Furthermore, 70.9% have no technical school diploma and 20.1% have an apprenticeship certificate. Unemployed young people are characterized by their lack of general education and technical training. Rural communities and public organizations should, therefore, guide and give higher opportunities for general education and technical training to the farm youth who will be rejected by the farm sector.

2. Milk surplus problem

This problem is very complex. In Europe, there is a deficit of meat production, but milk surpluses are a main concern (6, p. 21). Our results show that the most profitable activities are milk, yearling bulls and slaughter calf production. The higher the milk yield per cow, the more profitable the dairy activities become. Yearling bulls are produced out of dairy or double purpose herd calves. Furthermore, milk and beef outputs are joint products through the production of calves. Under such conditions we doubt that a policy which tries to develop total beef output from a breed specialized in meat production will be efficient without a very expensive price support program, the price of milk being kept constant (Table 49, page 293). One way of helping to solve this problem is to produce the largest quantity of meat out of the given number of dairy calves which represent the most scarce resource for the beef industry. Actually our results show that such a policy would be feasible. In France in 1953, 53% of calves born were slaughtered (6, p. 24). If a higher number of calves were used for beef production, a smaller proportion of farm resources would be allocated to the production of milk. If it is necessary to increase the number of cows and/or to decrease the number of slaughtered calves for developing the production of beef, it is not necessary to increase the production of beef to increase the production of milk.

A milk/beef price ratio which favors the production of milk, as is now the case, increases the number of specialized dairy farms. Since in France out of 9,500,000 cows, 7,500,000 are dairy cows (6, p. 24) the main problem is one of finding adequate ways for increasing the quantity of meat produced per calf. We see three directions for further research:

- encouraging the production of heavier carcasses
- discouraging the consumption of veal. This production contributes to decreasing the milk surplus
- developing an efficient extension program to encourage the production of steers by new producing methods.

One of the most efficient ways would be to separate the production of beef from the production of milk. Such a procedure is impossible since calves are still produced from cows. But, if it were possible to get artificially a larger proportion of twin calves, the solution of the problem would be easier to find.

F. Conclusion

If farming can be a lucrative job for those who have control over a certain quantity of resources, it is also an attractive occupation for those who can get a small farm and are not prepared for an off-farm job. But in order to succeed in farming it is necessary to take economic advantage of technological progress, otherwise the level of income which can be expected decreases greatly. It is also necessary to pool labor and equipment when farms are a certain size or to find ways of using excess labor efficiently. It is, therefore, urgent to give farm youth real opportunities in being prepared for off-farm jobs, especially for those who will be rejected from agriculture. Otherwise, 30 years from now, we will have to face a tremendous surplus problem and income disparities between sectors due to the adoption of progress and an excess supply of labor in agriculture.

CHAPTER 23. GENERAL CONCLUSION

This study was designed to determine:

- Optimum farm plans and related levels of income under:
  - (a) different levels of management in forage and milk production
  - (b) different combinations of limiting resources such as capital, labor and land
- The influence on income of:
  - (a) specialization in milk, steers and cereals production
  - (b) seed, milk, beef and cereals price variation
  - (c) large investment in building facilities

In addition to these results, this study was also designed to draw, from the preceding results, further implications which are particularly relevant to the problems of this area, especially the comparison of farm and nonfarm incomes, the estimation of disguised unemployment and the setting up of an adequate extension program.

In order to throw some light on the preceding problems a mixed integer linear programming was set up and used to solve the problems of investment while a standard linear model was used to study the other ones. In order to avoid narrowing the true production opportunity set through aggregation of activities such as crop rotation, livestock rations or aggregation of labor constraints, we discussed various ways of setting up a linear programming model.

Crop rotation constraints are taken into account in three ways:

(a) Pre-established rotations were defined in an exhaustive manner through a series of matrix multiplications in order to find the corresponding circuits.

(b) A transportation model type of constraints.

(c) A series of sufficient constraints on a set of crop activities. This last solution was described and chosen.

Labor constraints are defined in such a way that the marginal rate of substitution between different labor inputs equals one. But, as the corresponding labor requirements or environmental conditions frequently constitute a set of overlapping subsets, a sequence of set unions was defined to avoid the sum of the parts exceeding the value of the whole. This principle was used also for the crop rotation constraints.

Livestock rations can either be calculated within the maximization framework or pre-established either at random or in such a way that they constitute the set of extreme points in the convex set of feasible solutions. If, furthermore, the set of feasible feeding programs can be dissociated into two subsets and if the first dominates the second economically, the problem becomes simpler. The extreme aggregate rations are therefore defined according to simple choice rules. In order to define them, a preliminary linear programming model was solved. A descriptive linear regression was run on the shadow prices of alternative feeding programs. The results show that the least-cost rations are found when the cost of concentrated feeds and the acreage of harvested fodder are minimized.

Besides measurement problems, capital is a resource that gives some difficulties in model building. If its scarcity is a necessary condition for building a multistage model, it is not a sufficient one. In addition it has to be non-perfectly adaptable from one programming period to the subsequent one. On the other hand, indivisibility of capital input (equipment for example) calls for a mixed integer model. The problems to be solved were such that a one-stage model was considered adequate. However, we had to restrict few variables to an integer value when building investments were taken into account.

The results have some encouraging implications as well as a few alarming ones. The implications are encouraging for those who have or will have control over a large enough bundle of resources.

For those people who will go on farming, future prospects are good for the following reasons:

(a) The reserve of unused progress is still large. If it is correctly exploited it can bring additional income to farmers. On a 30 hectares farm:

- production of an extra 10 tons of corn silage per hectare increases total income by 1,150 francs.
- decreasing the acreage of permanent pastures from 16.5 hectares to 0.0 hectare raises total income by 10,000 francs. Productive temporary pastures (5,000 fodder units) are substituted for permanent ones.
- improving the productivity of temporary pastures from 3,000 to 5,000 fodder units per hectare and substituting good dairy cows for less productive ones, increase total income by about 10,000 francs.

(b) The production plans are very stable when the price of cereals varies from 80 to 120% of the present price, the beef/milk price ratio being held constant. The number of cows is almost stable and the number of yearling bulls varies slightly ($S_Y = 6.0$). At the same time the acreage allocated to the production of cereals changes a small amount ($S_{AC} \simeq 1.5$ ha). However, the stability of farm production plans is deeply disturbed when the beef/milk price ratio varies. When this ratio goes from 6.85/.4 to 5.65/.5, the number of dairy cows increases from about 6 to 25, while the production of yearling bulls is considerably reduced (from 70 to 5 animals). At the same time, the associated income decreases from 41,300 F to 26,800 F. These results should urge young farmers to start farming with a certain confidence in the future. Although their expected income cannot be stable, production plans can easily be adapted to new price situations. Even if the beef/milk price ratio changes much, the corresponding adjustment will not be too costly since building, silo and harvesting machine requirements are similar for beef and milk production. Such a conclusion would not be drawn if a new price situation could shift farm resources from animal to cereal production.

(c) Specialization as such is not profitable when important labor peaks are generated at certain periods of the years. This occurs particularly when we rule out, to a certain extent, the production of cereals. These productions behave as complementary enterprises in the first stage (they are companion crop) and as supplementary enterprises in the second stage, when the total cost of labor has to be minimized. Diversification is therefore the best strategy since it allows, at the same time, maximum profits and spreading risks over several activities (eggs are not all in the same basket). If, however, farmers want to simplify their production plans, the best association of two activities is formed by milk and cereal production. On a 30 hectares farm, they will have 6.7 ha of cereals, 31 dairy cows, capital of 153,267 F and get an income of 23,000 F. However, if they accept a more diversified farm plan, they will invest 76,000 F, raise 20 cows and 6 yearling bulls, grow 13.3 hectares of cereals and receive an income of 28,500 F. Many farmers are inclined to reduce their dairy herds for convenience reasons. In this case, with 10 dairy cows, 18 yearling bulls and 19 hectares of cereals, they invest 61,600 F and still get an income of 26,500 F.

(d) Labor indivisibility can reduce income considerably for certain farm sizes (up to 4,150 F). But through labor and equipment resource pooling, farmers can obtain most of the economic advantages of closely held corporations or large farms. Practically, they can obtain it through sharing arrangements or creation of modern enterprises which would perform farm jobs on a contract basis.

(e) Investment in building facilities, especially that which allows important production of hogs, slaughter calves, eggs and broilers is highly profitable. When they are undertaken, income rises from about 28,600 F to 50,000 F or more. But, while the amortization cost of buildings represents a small percentage of total costs, total investment is high. Considering the large fluctuations of the corresponding output prices and the increasing degree of competition in these sectors of production, farmers are reluctant to build on a large scale. They do not want to take the risk of minimizing losses in the near future.

(f) When an individual, with a good level of technological knowledge and management, can get control over the required bundle of resources, his income can be as high as urban wages. The following income equivalence between farming and off-farm jobs are found:

- top executive $\simeq$ 50-60 hectares farm (40,000 F)
- medium executive $\simeq$ 20-30 hectares farm (20,000 F)
- manual worker $\simeq$ 10-20 hectares farm (10,000 F)

Given a certain farm size, the scarcity of capital inputs reduces income by about 10,000 F or more while the scarcity of labor reduces it by 2,500 F to 10,000 F, depending on the size of the farm. The most profitable farm plans are mostly oriented toward the production of livestock outputs which requires higher levels of capital and labor than the production of cereal. These results, in addition of those found when the output prices are varied, prove that the economic development of the agricultural sector of this area is through the development and the improvement of animal production.

Although their future prospects are encouraging, these people will surely have some difficulties and problems. The biggest ones are:

- The accumulation of capital at a rapid enough rate. There are many ways of financing a farm-firm with various types of loans. But to get them, it is necessary to satisfy certain equity ratios.
- The acquisition of knowledge and know-how. The price-cost squeeze and the higher degree of competition in agriculture forces farmers to use the most efficient producing processes, as well as the best farm plans. A new generation of farmers, characterized by

their capability for running the farm as a modern business, is slowly emerging.

- The existence of a large discrepancy between the rate of return from leased land and the opportunity cost of the corresponding capital. Aside from the economic venture and the risk of money depreciation, land is considered a secure investment. But, its low rate of return (1-2%), in comparison with other investments (5-8%), reduces considerably the number of potential landlords and pull capital out of the Agricultural Sector. One of the most crucial risks faced by beginning farmers lies in the contingency of being forced to buy their farm at a certain stage of their professional life. Discrepancies between annual loan repayments and the stream of income generated by this investment are such that it becomes unbearable for a family to save the required money.

For those who will not be able to control a bundle of resources large enough, and/or will not have a sufficient level of technological knowledge and management, or even a certain readiness of mind to adopt progress, the future prospects are not so good. Since presently, the land/labor ratio is 13.6 hectares while we find about 25-30 hectares when capital is unlimited, rural poverty will still be a problem. Most of these people are either too old to quit farming or they do not have adequate training for finding good off-farm jobs. In this case it will be necessary to assist people who belong to the poverty sector of agriculture in two different ways. In particular, it is necessary:

- to help them terminate a decent life when they are a certain age and the transfer from agriculture is infeasible
- to give a good opportunity for education or training to the farm youth and to those who are still able to make a successful transfer from agriculture. We have seen that this last principle is far from being applied.

The extension service can help the farmers to increase their present level of income. Its program should emphasize the economic aspects of production and optimum farm plans as well as the benefits resulting from the adoption of progress and modern techniques of production. The main income discrepancies are presently due to a lack of forage and livestock production, inadequate feeding programs and the presence of traditional feeder and slaughter steers in today's farm plans. The techniques of production and the input-output relationships of these last activities have been kept almost constant for decades. It should also emphasize every technical and economic aspect of livestock production since, as resources become less scarce, the increase in animal output is made at the expense of cereals.

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APPENDIX A. THE LINEAR PROGRAMMING MODEL

CROP ACTIVITIES (CEREALS AND FODDERS)

	P.DE.T	BLE.S.B	ORG.P	AV.P	CHX.S.BG	CHX.ORGE	CHX	BET.TRAD	BET.SM	PTT.M	MAIS.R	AS.F	MAIS.ENS	MAIS.G	COLZA	AVOINE.H	BLE.1	BLE.2	AVOINE.FT	ORGE.F	D.FLT.CZ	BGG.CZ	BGGLZ.SP	D.FET.MO	BGG.MG	BGGMG.SP	PT3.SN	PT2.MI	ORGE.PT
OC FT	3873.00	-612.40	-462.80	-392.45	-721.30	-835.20	-425.60	-335.10	-667.10	-957.00	-1081.50	-602.55	-1241.50	-976.50	-871.85	-382.30	-542.25	-607.15	-392.45	-457.84	-3716.10	-1720.45	-120.45	-4071.35	-2025.10	-2025.10	-189.50	-189.50	-610.[illegible]
02 FAILLE												4.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	5.00	2.00	2.00	1.00	2.00	2.00	3.00	2.00	-X
03 COLZA												4.00				1.51	1.51	1.51	1.51	1.51	1.51			1.51					1.51
04 ASS.FS	1.00				1.00	1.00	1.00	1.00	1.00	1.00	1.00	-2.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		2.00	2.00	-3.00	2.00	2.00	3.00	2.00	3.00
05 ASS.FS		1.00	1.00	1.00								-1.00																	

CROP ACTIVITIES

	CALE.PT2	AV.PT3	AV.PT2	MG.BLE
ZZ ZZ	-912.15	-534.60	-534.60	-1522.65
01 TERRE	1.00	4.00	3.00	2.00
02 PAILLE	1.51	1.51	1.51	1.51
03 COLZA	3.00	3.00	3.00	2.00
06 R.COLZ	1.00	1.00	1.00	1.00
07 R.AV.H	1.00	1.00	1.00	1.00
08 R.AV.M	1.00	1.00	1.00	1.00
09 R.BLE1	1.00	1.00	1.00	
10 R.BLE2				1.00
11 R.AV.P	1.00			1.00
12 R.ORGE		1.00	1.00	1.00
13 R.MG.P	1.00	1.00	1.00	1.00
14 R.MG.M				1.00
16 PT	2.00	1.00	2.00	
17 GR.[illegible]Z	2.00			
24 PATURE	500.00	500.00	500.00	
26 GRAINS	1520.00	1368.00	1368.00	3990.00
27 [illegible]	-40.00	35.00	35.00	
29 T1	11.25			
30 T2				10.50
32 T4	9.00	7.50	7.50	4.00
34 T6				18.00
35 L10	13.25	2.75	2.75	
36 L20				12.25
42 L40	18.16	15.16	15.16	7.25
43 L41	18.16	15.16	15.16	7.25
45 L60				19.50
49 L64				9.00
50 L65				19.50
51 PAILLE				10.50
83 [illegible]	244	72	72	670
84 [illegible]	-1391	-1239	-1239	-1602
85 [illegible]				-1534
86 [illegible]	237	334	334	99
[illegible] KL	1050	975	975	1057

GRASSLAND MANAGEMENT

	L21.FFF	L21.FFPP	L21.FFFF	T1.FFFF	PT1.EPPP	PN1.FFPP	PN1.FFF
ZZ ZZ	-335.70	-411.20	-392.40	-414.20	-574.90	317.00	-362.00
01 TERRE						1.00	1.00
15 PN						1.00	1.00
16 PT	1.00	1.00	1.00	1.00	1.00		
17 GR.[illegible]Z	1.00	1.00	1.00				
21 FOIN	-110.00	71.50	33.00				-35.00
22 E.HERB					-154.00		
23 PATURE		-770.00	-1600.00	3120.00	-420.00	-2100.00	
24 PATURE		-1470.00	-1925.00	1900.00	-1750.00	-1400.00	-1400.00
29 T1			1.00	1.00		1.00	1.00
30 T2	10.50	11.40	9.30	1.30	8.50	1.00	7.00
31 T3	6.75	1.00	1.00	1.00	1.00		
33 T5	5.75	1.00	1.00	1.00	1.00	1.00	1.00
34 T6	1.50	1.50	1.50	1.50	1.50	1.50	1.50
35 L10			1.00	1.00		1.00	1.00
36 L20	21.75	22.75	18.35	1.00	19.00	1.00	13.25
37 L21					18.00		
38 L22	21.75	21.75					
39 L23	21.75	22.75	18.35	1.00	19.00		13.25
40 L30	12.75	1.00	1.00	1.00	1.00		1.00
41 L31	12.75	1.00		1.00	1.00		
43 L41						1.20	
44 L50	9.25	1.00	1.00	1.00	1.00		1.00
45 L60	1.50	1.50	1.50	1.50	1.50	1.50	1.50
49 L64	1.50	1.50	1.50	1.50	1.50	1.50	1.50
50 L65	1.50	1.50	1.50	1.50	1.50	1.50	1.50
52 L24			18.35				13.25
83 [illegible]	114	190	172	125	[illegible]	94	141
84 [illegible]	57	56	56	42	42	42	42
85 [illegible]	164	164	164	176	164	149	137
86 [illegible]				70	73	42	42
[illegible] KL	278	355	336	372	55[illegible]	275	325

	L22.FFF	L22.FFPP	L22.FFPP	PT2.PFPP	PT2.FFPP	PN2.FFPP
ZZ ZZ	294.50	-238.[illegible]	266.00	-261.00	407.50	-188.60
01 TERRE						1.00
15 PN						1.00
16 PT	1.00	1.00	1.00	1.00	1.00	
17 GR.[illegible]Z	1.00	1.00	1.00			
21 FOIN	90.00	38.50	42.00			
22 E.HERB					-128.00	
23 PATURE		-900.00	-1360.00	-2480.00	670.00	-1650.00
24 PATURE		-1320.00	-1550.00	-1520.00	1130.00	-1150.00
29 T1			1.00	1.00		1.00
30 T2	9.75	10.75	7.55	1.00	7.50	1.00
31 T3	5.06	1.00	1.00	1.00	1.00	
33 T5	5.50	1.00	1.00	1.00	1.00	1.00
34 T6	1.52	1.50	1.50	1.50	1.50	1.50
35 L10			1.00	1.00		1.00
36 L20	19.75	20.75	14.65	1.00	16.00	1.00
37 L21					15.00	
38 L22	19.75	19.75				
39 L23	19.75	23.75	14.65	1.00	16.00	
40 L30	10.75	1.00	1.00	1.00	1.00	
41 L31	10.75	1.00		1.00	1.00	
43 L41						1.00
44 L50	9.00	1.00	1.00	1.00	1.00	
45 L60	1.50	1.50	1.50	1.50	1.50	1.50
49 L64	1.50	1.50	1.50	1.50	1.50	1.50
50 L65	1.50	1.50	1.50	1.50	1.50	1.50
52 L24			14.65			
83 [illegible]	95	131	129	42	200	42
84 [illegible]	49			28	28	
85 [illegible]	137					
86 [illegible]		137	137	149	137	118
[illegible] KL	215	288	266	233	379	184

	L23.FFF	L23.FFPP	L23.FFPP	PT3.PPFP	PT3.EPPP	PN3.PPPP
ZZ ZZ	237.40	189.40	-169.50	121.60	-266.00	-101.90
02 PAILLE						1.00
15 PN						1.00
16 PT	1.00	1.00	1.00	1.00	1.00	
17 GR.[illegible]Z	1.00	1.00	1.00			
21 FOIN	70.00	45.50	29.50			
22 E.HERB					-100.00	
23 PATURE		-420.00	-1220.00	-1860.00	-320.00	-1160.00
24 PATURE		-1060.00	-1220.00	-1140.00	-1120.00	-840.00
29 T1			1.00	1.00		1.00
30 T2	9.16	10.10	6.75	1.00	6.50	1.00
31 T3	5.90	1.00	1.00	1.00	1.00	
33 T5	5.50	1.00	1.00	1.00	1.00	1.00
34 T6	1.50	1.50	1.50	1.50	1.50	1.50
35 L10			1.00	1.00		1.00
36 L20	17.75	18.75	12.25	1.00	14.00	1.00
37 L21					13.00	
38 L22	17.75	17.75				
39 L23	17.75	18.75	12.75	1.00	14.00	
40 L30	10.25	1.00	1.00	1.00	1.00	
41 L31	10.25	1.00		1.00	1.00	
43 L41						1.00
44 L50	9.00	1.00	1.00	1.00	1.00	
45 L60	1.50	1.50	1.50	1.50	1.50	1.50
49 L64	1.50	1.50	1.50	1.50	1.50	1.50
50 L65	1.50	1.50	1.50	1.50	1.50	1.50
52 L24			12.75			
83 [illegible]	85	79	59		136	
84 [illegible]	42					
86 [illegible]	109	109	109	121	109	101
[illegible] KL	194	189	169	121	266	101

LIVESTOCK ACTIVITIES

	V.PAIL	MOUTON	IRP.VT.B	IRP.VT.M	IRP.M	IRP.P
ZZ ZZ		697.00	0.28	0.06	-2806.39	2766.[illegible]
20 E.MAIS		8.60	0.24			217.00
21 FOIN				0.26	248.00	
22 E.HERB		10.24	0.04	0.04	31.00	68.20
23 PATURE		539.00			20020.00	20020.00
24 PATURE		346.00			18190.00	18190.00
25 ENGL		1.90			3.72	
28 GRAINE	-10.00				180.00	180.00
33 T5	1.25					
43 L41	2.50					
52 L24	2.50					
53 T.A1		2.36			41.00	43.00
54 T.A2		2.33			48.50	48.50
55 T.A3		1.70				
56 A.VL.T					1670.00	1670.00
57 R.MA.E					5.00	5.00
58 G.MA					-8.00	8.00
59 A.MA					3.00	3.00
61 R.NA18			1.00	1.00	1180.00	1180.00
75 V.MEAT					-26413	-26413
76 V.MORE					-9541	-9541
78 E.[illegible].N					-3760	-3760
84 [illegible]		225			-11702	-11702
85 [illegible]		-840			-7065	-7065
86 [illegible]		-100			-1724	-1726
[illegible] KL		23	0.08	0.06	-17445	-17485
[illegible]		248	0.08	0.06	-30371	-30371
[illegible]		232			50000	50000
[illegible]		80				
90 [illegible]					20.00	20.00
91 [illegible]					30.00	30.00
92 [illegible]					25.00	2500

LIVESTOCK ACTIVITIES

	1RP.BM	1R.VE.B	1R.VE.M	1EG.C.B	1EG.C.M	1EG.B	1EG.M	1RA.VE.C	1RA.VE.B	1RA.VE.M	2RP.M	2RP.B	2RP.BM	2RA.C	2RA.B	2RA.M	2RA.CB	2RA.CM	2RA.BM	2RA.VE.C	2RA.VE.B	2RA.VE.M	2EG.C.B	2EG.C.M	2EG.B	2EG.M	3RP.M	3RP.B	3RP.BM
00 FF	-2922.42	-7.50	-5.00	-352.13	-347.96	-357.93	-349.50	-0.08	-0.08	-0.06	-3199.28	-2852.67	-3336.66	-106.72	-361.53	-494.79	-343.29	-348.10	-666.66	-5.48	-7.00	-16.44	-301.72	-282.85	-285.55	-281.07	-2267.95	-2202.39	-2368.58
01 TERRE																													
02 PAILLE																													
03 COLZA																													
04 ASS.PS																													
05 ASS.PS																													
06 R.COLZ																													
07 R.AV.M																													
08 R.AV.H																													
09 R.BLE1																													
10 R.BLE2																													
11 R.AV.P																													
12 R.ORGE																													
13 R.RC.M																													
14 R.RC.H																													
15 FM																													
16 FT																													
17 CO.LCE																													
18 CHOU				60.83	60.83			0.39						340.00			212.50	212.50		24.40			34.65	34.65					
19 BETTE	167.40	19.84		38.74		78.78			0.24			540.00	540.00		255.00		255.00		255.00		17.09		25.20		46.76			208.50	165.60
20 E.MAIS	134.40		24.80		36.63		75.13			0.26	630.00		270.00			297.50		204.00	127.50			21.35		27.00		50.10	252.00		108.00
21 FOIN	37.20	6.20	3.10	23.70	22.51	23.70	20.98	0.06	0.06	0.04	72.00	180.00	90.00	68.00	85.00	34.00	42.50	34.00	42.50	4.88	5.69	2.44	16.36	14.36	16.36	13.02	28.80	64.80	35.00
22 E.HERB																													
23 PATURE	20020.00			1046.00	1046.00	1046.00	1046.00				18200.00	18200.00	18200.00										1410.00	1410.00	1410.00	1410.00	21702.00	21702.00	21702.00
24 PATURE	18190.00			1230.00	1230.00	1230.00	1230.00				18060.00	18080.00	18080.00										1493.00	1493.00	1493.00	1493.00	19260.00	19260.00	19260.00
25 ORGE				3.27	3.36	2.50	2.97				28.75	21.60	8.38	23.53	8.20	11.81	3.50	12.53	1.62	0.61		0.05	1.05	1.05	1.05	1.05	4.46	4.31	3.69
26 GRAINS																													
27 LIT	180.00			16.20	16.20	16.20	16.20				203.00	203.00	203.00										15.30	15.30	15.30	15.30	180.00	180.00	180.00
28 GRAINE																													
29 T1																													
30 T2																													
31 T3																													
32 T4																													
33 T5																													
34 T6		0.65	0.50	3.70	3.70	3.97	2.83	0.02	0.02	0.02				21.86	27.34	3.16	33.08	23.73	30.16	1.54	1.74	1.30	2.14	2.14	2.21	1.64			
35 L10																													
36 L20																													
37 L21																													
38 L22																													
39 L23																													
40 L30																													
41 L31																													
42 L40																													
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45 L60																													
46 L61																													
47 L62																													
48 L63																													
49 L64																													
50 L65																													
51 PAILLE																													
52 L26																													
53 T.A1	43.00			12.04	12.04	12.04	12.04				43.00	43.00	43.00										5.16	5.16	5.16	5.16	48.01	48.01	48.01
54 T.A2	48.50			8.73	8.73	8.73	8.73				51.62	51.62	51.62										7.32	7.32	7.32	7.32	51.34	51.34	51.34
55 T.A3		1.30	1.00	5.80	5.80	6.07	4.65	0.03	0.03	0.02				31.31	36.24	7.22	43.06	32.45	39.46	2.05	2.32	1.74	11.02	11.02	11.10	10.40			
56 R.VL.T	1670.00							-1.00	-1.00	-1.00	818.00	818.00	818.00														693.00	693.00	693.00
57 R.MA.E	3.00	-1.00	-1.00																										
58 G.MA	-8.00			1.00	1.00	1.00	1.00																						
59 A.MA	5.00			-1.00	-1.00	-1.00	-1.00																						
60 R.VL.T	1180.00																										1328.00	1328.00	1328.00
61 R.MA38											1.00	1.00	1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00										
62 R.MA.E											5.00	5.00	5.00							-1.00	-1.00	-1.00							
63 C.MA38											-9.00	-9.00	-9.00										1.00	1.00	1.00	-1.00			
64 A.MA38											5.00	5.00	5.00										1.00	-1.00	-1.00	-1.00			
65 R.RP38																											1.00	1.00	1.00
66 C.RP38																											-9.00	-9.00	-9.00
67 A.RP38																											5.00	5.00	5.00
68 R.MA30																													
69 R.RP30																													
70 R.MA25																													
71 R.RP36																													
72 R.RP26																													
73 A.MERE																													
74 V.LAIT	-26413			142	142	142	142				-36541	-36541	-36541										124	124	124	124	-36226	-36226	-36226
75 V.MEAT	-9541										-7528	-7528	-7528														-8141	-8141	-8141
76 V.MERE																													
77 E.V.MA	-3760																												
78 E.V.M											-3450	-3450	-3450														-3180	-3180	-3180
79 E.C.AM																													
80 CX.MA																													
81 CX.M36																													
82 CX.M26																													
83 TR2	-11202			174	174	174	174				-10436	-10436	-10436										147	147	147	147	-22127	-22127	-22127
84 TR3	-7065		3								-5366	-5366	-5366														-11691	-11691	-11691
85 TR4	-1724	3	3	59	57	64	60	0.08	0.08	0.06	-16902	-16902	-16902	106	361	494	543	348	666	5	7	16	35	34	35	33	-3144	-3144	-3144
86 TR1	-17329	3		200	198	201	196				-12083	-12430	-11746										179	178	179	177	-4791	-4857	-4691
110 KC	-30371	7	3	375	372	375	371	0.08	0.08	0.06	-39442	-39442	-39442	106	361	494	543	348	666	5	7	16	362	360	362	358	-34064	-34129	-33963
87 K	50000			3285	3285	3285	3285				46000	46000	46000										2900	2900	2900	2900	44000	44000	44000
88 V.EAT																													
89 ET.VL	20.00										20.00	20.00	20.00														20.00	20.00	20.00
90 ET.JB	30.00			14.00	14.00	14.00	14.00				30.00	30.00	30.00										12.00	12	12	12			
91 AUGE	25.00			2.75	2.75	2.75	2.75				25.00	25.00	25.00										2.00	2	2	2	20.00	20.00	[illegible]
92 PORC																													
93 TRUIE																													
94 F.ST.L																													
95 F.PORC																													
96 F.TR																													
97 F.P.CH																													
98 F.E.F																													
111 F.P.T																													

LIVESTOCK ACTIVITIES

	3RA.C	3RA.E	3RA.P	3RA.CE	3RA.CM	3RA.EM	3EG.C.B	3EG.C.M	3EG.B	3EG.M	4RP.M	4RP.B	4RP.EM	4RA.C	4RA.E	4RA.P	4RA.CE	4RA.CM	4RA.EM	5RP.M	5RP.E	5RA.C	5RA.E	5RA.P	5RA.CE	5RA.CM	5RA.EM	[illegible]	[illegible]
00 FE	- 121.13	- 231.72	[illegible]	[illegible]	- 238.35	[illegible]	- 323.73	- 321.00	- 329.46	- 321.20	- 2822.56	- 2537.31	- 3111.09	- 25.42	- 236.57	[illegible]	- 353.43	- 225.06	- 558.45	- 2052.20	[illegible]	- 205.59	- 162.95	- 186.22	- 257.13	- 112.31	- 295.01	[illegible]	321.20
01 TERRE																													
02 PAILLE																													
03 COLZA																													
04 ASS.PS																													
05 ASS.PS																													
06 B.COLZ																													
07 R.AV.N																													
08 R.AV.M																													
09 R.BLE1																													
10 R.BLE2																													
11 R.AV.P																													
12 R.ORGE																													
13 R.BG.M																													
14 R.BG.M																													
15 PM																													
16 PT																													
17 GR.LUZ																													
18 CHOU	338.80			211.75	[illegible]		50.00	50.00						288.80			160.50	160.50				327.20			122.70	204.50		50.00	50.00
19 BETTE		254.10		254.10		254.10	24.34		37.41			340.00	340.00		215.60		215.60		215.50		215.00		237.22		245.40		168.14	24.54	
20 E.MAIS			296.45		201.28	127.05		22.72		55.48	630.00		270.00			252.70		173.28	108.30	252.00				286.30		195.32	122.70		22.72
21 FOIN	67.76	84.70	33.58	42.35	33.58	42.35	22.90	22.25	22.90	20.74	72.00	180.00	90.00	37.76	72.20	28.85	35.10	28.58	31.10	28.80	50.40	55.44	73.62	32.72	40.90	32.72	40.90	22.90	22.28
22 E.HERB																													
23 PATURE							1019.00	1019.00	1019.00	1019.00	16380.00	16380.00	16380.00							19682.00	19682.00							1019.00	1019.00
24 PATURE							1198.00	1198.00	1198.00	1198.00	15820.00	15820.00	15820.00							18190.00	18190.00							1198.00	1198.00
25 ORGE	17.11	3.39	6.57	1.90	7.23	1.90	3.72	3.81	3.34	3.49	15.83	10.26	1.24	16.77	3.38	5.51	1.05	7.23	0.46	1.24	2.01	6.05	1.29	1.29	1.29	1.29	1.29	3.72	3.41
26 GRAINS																													
27 LIT							16.20	16.20	16.20	15.20	203.00	203.00	203.00							180.00	180.00							16.20	16.20
28 GRAINE																													
29 T1																													
30 T2																													
31 T3																													
32 T4																													
33 T5																													
34 T6	21.78	28.24	3.36	32.94	23.65	30.06	3.21	3.21	3.46	2.71				18.57	23.22	2.68	28.09	20.15	25.62			21.04	24.92	3.04	28.09	22.84	24.67	3.21	3.21
35 L10																													
36 L20																													
37 L21																													
38 L22																													
39 L23																													
40 L30																													
41 L31																													
42 L40																													
43 L41																													
44 L50																													
45 L60																													
46 L61																													
47 L62																													
48 L63																													
49 L64																													
50 L65																													
51 PAILLE																													
52 L24																													
53 T.A1							12.04	12.04	12.04	12.04	43.00	43.00	43.00							45.81	45.81							12.04	12.04
54 T.A2							8.73	8.73	8.73	8.73	51.50	51.50	51.50							49.04	49.04							8.73	8.73
55 T.A3	30.19	35.02	6.12	42.22	31.17	39.02	3.18	3.18	3.44	4.54				26.02	29.99	5.44	36.04	26.80	33.21			22.39	31.50	4.49	35.74	28.83	31.59	5.18	5.18
56 B.VL.T											946.00	946.00	946.00							722.00	722.00								
57 B.MA.E																													
58 C.MA																													
59 A.MA																													
60 B.VL.T																				1328.00	1328.00								
61 B.MA38																													
62 B.MA.E											5.00	5.00	5.00																
63 G.MA38																													
64 A.MA38																													
65 B.HP38	- 1.00	- 1.00	- 1.00	- 1.00	- 1.00	- 1.00																							
66 G.HP38							1.00	1.00	1.00	1.00																			
67 A.HP38							- 1.00	- 1.00	- 1.00	- 1.00																			
68 B.MA30											1.00	1.00	1.00	- 1.00	- 1.00	- 1.00	- 1.00	- 1.00	- 1.00										
69 B.HP30																				1.00	1.00	- 1.00	- 1.00	- 1.00	- 1.00	- 1.00	- 1.00		
70 B.MA25																													
71 B.HP26																													
72 B.HP26																													
73 A.HCRE																													
74 V.LAIT							127	127	127	127	- 28801	- 28801	- 28801							- 28505	- 28505							127	127
75 V.MEAT											- 7526	- 7528	- 7528							- 8141	- 8141								
76 V.MCRE																													
77 E.V.MA																													
78 E.V.N											- 4500	- 4500	- 4500							- 4230	- 4230							250	250
79 E.G.AN											10500	10500	10500							10000	10000							- 2000	- 2000
80 CX.MA																													
81 CX.M36																													
82 CX.M26																													
83 TR2											- 7687	- 7687	- 7687							- 20699	- 20699								
94 TR3											- 3935	- 3935	- 3935							- 9241	- 9241								
84 TR4	121	251	412	383	238	530	222	221	228	224	- 6351	- 6351	- 6351	75	236	383	383	225	508	- 5417	- 5417	105	142	186	257	112	295	222	221
85 TB1							176	174	176	173	- 9981	- 10257	- 9693							6105	6180							- 1573	- 1572
110 KC	121	251	417	383	238	530	176	174	176	173	- 24020	- 24306	- 23732	75	236	383	383	225	508	- 20012	- 19932	105	142	186	257	112	295	176	174
87 K							2810	2810	2810	2810	42000	42000	42000							40000	40000							2460	2460
88 V.BAT																													
89 ET.VL											20.00	20.00	20.00							20.00	20.00								
90 ET.JB							14.00	14.00	14.00	14.00	30.00	30.00	30.00															14.00	14.00
91 AVGE							2.75	2.75	2.75	2.75	25.00	25.00	25.00							20.00	20.00							2.75	2.75
92 PORC																													
93 TRUIE																													
94 F.ST..																													
95 F.T.VC																													
96 F.JB																													
97 F.T.CN																													
98 F.B.F																													
111 F.P.F																													

	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6E25B	6E25M	6E25C.M	6E25C.B	6F25B	6F25M	[illegible]	[illegible]	[illegible]
00 TE	[illegible]	[illegible]	295.26 -	282.85 -	285.55-	281.02 -	305.71-	304.23-	299.00 -	299.81-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
01 TERRE															
02 FAILLE															
03 COLZA															
04 ASS.PS															
05 ASS.PS															
06 P.DOLE															
07 B.AV.E															
08 B.AV.H															
09 B.BLE															
10 B.BLE															
11 B.AV.P															
12 B.ORGE															
13 B.BG.M															
14 B.BG.M															
15 PN															
16 PT															
17 [illegible]															
18 CHOL			34.63	34.63					3.85	3.85			50.10		55.43
19 BETTE	53.91		25.20		46.76		13.95			7.20	50.10		41.75	33.40	
20 E.MAIS		11.49		27.00		50.10		11.69	6.30			58.45		35.74	
21 FOIN	24.40	20.74	16.36	14.56	16.36	13.02	11.48	11.48	11.48	11.48	13.16	6.68	8.35	5.58	11.59
22 E.HERB															
23 PATURE	1019.00	1019.00	1410.00	1410.00	1410.00	1410.00	546.00	546.00	546.00	546.00					
24 PATURE	1195.00	1195.00	1493.00	1493.00	1493.00	1493.00	642.00	642.00	642.00	642.00					
25 ORGE	3.34	3.49	1.05	1.05	1.05	1.05	2.95	2.95	3.64	3.64	1.53	1.83		2.33	1.67
26 GRAINS															
27 LIT	16.23	16.20	15.30	15.30	15.30	15.30	3.60	3.60	3.60	3.60	5.40	5.40	5.40	5.40	5.40
28 GRAINE															
29 T1															
30 T2															
31 T3															
32 T4															
33 T5															
34 T6	3.46	2.73	2.14	2.14	2.21	1.64	0.74	0.36	0.44	0.44	1.06	0.78	1.31	1.07	0.86
35 L10															
36 L20															
37 L21															
38 L22															
39 L23															
40 L30															
41 L31															
42 L40															
43 L41															
44 L50															
45 L60															
46 L61															
47 L62															
48 L63															
49 L64															
50 L65															
51 PAILLE															
52 L16															
53 T.A1	12.04	12.04	5.16	5.16	5.16	5.16	10.12	10.32	10.32	10.32					
54 T.A2	8.73	8.73	7.32	7.32	7.32	7.32	6.79	6.79	6.79	6.70					
55 T.A3	5.44	4.54	11.02	11.02	11.10	10.40	1.45	1.22	1.19	1.19	3.26	2.53	3.83	3.18	2.40
56 B.VL.T															
57 B.MA.E															
58 G.MA															
59 A.MA															
60 B.VL.T															
61 B.MA38															
62 B.MA.E															
63 C.MA38															
64 A.MA38															
65 B.NP38															
66 G.NP38															
67 A.NP38															
68 B.MA30															
69 B.NP30															
70 B.MA25							- 1.00	- 1.00	- 1.00	- 1.00	1.00	1.00	1.00	1.00	1.00
71 B.NP36															
72 B.NP26															
73 A.MGRE															
74 V.LAIT	127	127	124	124	124	124	159	159	159	159					
75 V.MEAT											- 2174	- 2174	- 2174	- 2174	- 2174
76 V.MGRE															
77 E.V.MA							320	320	320	320					
78 E.V.F	250	250	250	250	250	250									
79 E.G.AN	- 2000	- 2000	- 2100	- 2100	- 2100	- 2100									
80 CX.MA											- 77	- 77	90	- 77	90
81 CX.N36															
82 CX.N26															
83 TB2			147	147	147	147	165	165	165	165	- 2174	- 2174	- 2174	- 2174	- 2174
84 TB3							8	8	8	8					
85 TB4	228	224	- 1814	- 1815	- 1814	- 1816	40	39	34	35	11	9	14	12	10
86 TB1	- 1573	- 1576	179	178	179	177	527	527	526	527	9	5	11	9	7
1K KC	175	173	362	360	362	358	165	165	165	165					
87 F	2460	2460	2600	2600	2600	2600	1600	1600	1600	1600					
88 V.BAT															
89 ET.VL															
90 ET.JB	14.00	14.00	12.00	12.00	12.00	12.00	4.00	4.00	4.00	4.00	5.00	5.00	5.00	5.00	5.00
91 AUGE	2.75	2.75	2.00	2.00	2.00	2.00	0.75	0.75	0.75	0.75	1.00	1.00	1.00	1.00	1.00
92 PORC															
93 TRUIE															
94 F.ST.L															
95 F.PORC															
96 F.TR															
97 F.P.CH															
98 F.P.F															
111 F.P.P															

	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
00 TE		- 311.32-	312.61-	319.15-	314.06-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
01 TERRE														
02 FAILLE														
03 COLZA														
04 ASS.PS														
05 ASS.PS														
06 P.DOLE														
07 B.AV.E														
08 B.AV.H														
09 B.BLE														
10 B.BLE														
11 B.AV.P														
12 B.ORGE														
13 B.BG.M														
14 B.BG.M														
15 PN														
16 PT														
17 [illegible]														
18 CHOL		33.11	33.11				31.57	31.57					31.57	31.57
19 BETTE		21.35		46.00			23.16		41.25		6.82		27.00	
20 E.MAIS			16.69		40.25			18.13		38.89		6.20		26.50
21 FOIN		16.92	16.92	16.92	16.92		16.92	16.92	16.92	16.92	1.24	1.24	18.16	18.16
22 E.HERB														
23 PATURE		1510.00	1510.00	1510.00	1510.00	1010.00	500.00	500.00	500.00	500.00	664.00	664.00	1165.00	1165.00
24 PATURE		388.00	388.00	388.00	388.00		388.00	388.00	388.00	388.00	281.00	281.00	1370.00	1370.00
25 ORGE		3.76	3.76	3.30	3.30	1.00	2.76	2.76	2.30	2.30			2.76	2.76
26 GRAINS														
27 LIT		9.00	9.00	9.00	9.00		9.00	9.00	9.00	9.00			9.00	9.00
28 GRAINE														
29 T1														
30 T2														
31 T3														
32 T4														
33 T5														
34 T6		2.06	2.06	2.42	1.69		2.01	2.01	2.32	1.67			2.01	2.01
35 L10														
36 L20														
37 L21														
38 L22														
39 L23														
40 L30														
41 L31														
42 L40														
43 L41														
44 L50														
45 L60														
46 L61														
47 L62														
48 L63														
49 L64														
50 L65														
51 PAILLE														
52 L16														
53 T.A1		11.84	11.84	11.84	11.84	1.57	10.32	10.32	10.32	10.32	1.72	1.72	12.04	12.04
54 T.A2		6.79	6.79	6.79	6.79		6.79	6.79	6.79	6.79	1.94	1.94	8.73	8.73
55 T.A3		3.38	3.38	3.38	2.85		3.15	3.15	3.54	2.73			3.15	3.15
56 B.VL.T														
57 B.MA.E														
58 G.MA														
59 A.MA														
60 B.VL.T														
61 B.MA38														
62 B.MA.E														
63 C.MA38														
64 A.MA38														
65 B.NP38														
66 G.NP38														
67 A.NP38														
68 B.MA30														
69 B.NP30														
70 B.MA25	- 1.00													
71 B.NP36														
72 B.NP26														
73 A.MGRE	1600					1850					1750	1750		
74 V.LAIT		159	159	159	159		159	159	159	159			159	159
75 V.MEAT		- 2168	- 2168	- 2168	- 2168	- 2168					- 2172	- 2172	- 2172	- 2172
76 V.MGRE							- 1750	- 1750	- 1750	- 1750				
77 E.V.MA		320	320	320	320		320	320	320	320			320	320
78 E.V.F														
79 E.G.AN														
80 CX.MA														
81 CX.N36														
82 CX.N26														
83 TB2		165	165	165	165		165	165	165	165			165	165
84 TB3		- 2159	- 2159	- 2159	- 2159	- 2168	8	8	8	8			8	8
85 TB4	1600	63	42	48	46	1	63	41	48	46	- 2169	- 2169	- 2127	- 2131
86 TB1		532	531	533	530	1850	- 1217	- 1218	-1217	-1219	1752	1751	531	524
1K KC	- 550	749	747	755	750	1850	697	696	695	695	1752	1751	743	715
87 F		1880	1880	1880	1880		1340	1340	1340	1340			2210	2210
88 V.BAT														
89 ET.VL														
90 ET.JB		9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00			9.00	9.00
91 AUGE		1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75			1.75	1.75
92 PORC														
93 TRUIE														
94 F.ST.L														
95 F.PORC														
96 F.TR														
97 F.P.CH														
98 F.P.F														
111 F.P.P														

LIVESTOCK ACTIVITIES

	6F315P	6F315M	6F10	6F15P	6F15M	6F15M..	PF25..	PF30..	TF27B	TF27M	TF27..P	TF27..M	TF29B	TF29M	TF29C.B	TF29C.M	TF365B	TF365M	TF365C.B	TF365C.M	TF36'B	TF365M	TF33B	TF33M	TF33C.B	TF33C.P	TF33C.B	TF33C.M	TF33M
0? TE	- 324.14	- 320.86	- 198.94	- 133.62	- 25.55	- 37.94	- 29.55	- 38.08	- 25.70	224.20	- 221.44	223.01	- 251.19	247.05	- 251.26	249.15	- 17.66	16.97	- 260.26	238.05	- 262.73	259.95	- 269.46	- 263.44	264.26	265.54	- 26.34	- 24.45	23.10
1 TERRE																													
02 PAILLE																													
03 COLZA																													
04 ASS.PS																													
05 ASS.PS																													
06 P.COLZ																													
07 R.AV.H																													
08 R.AV.H																													
09 R.BLE1																													
10 R.BLE2																													
11 R.AV.P																													
12 R.ORGE																													
13 R.BG.H																													
14 R.BG.H																													
15 PN																													
16 PT																													
17 CR.LUZ																													
18 CNCP											13.55	13.85			36.96	36.96			36.96	36.96					40.36	40.36	24.64	24.64	
19 BETTE	50.10		15.00	32.50			64.23	35.00	14.96		6.49		-2.85		20.32		5.49		26.41		46.74		60.12		32.40		14.93		
20 E.MAIS		45.09			21.30	724.00	35.42	39.60		15.32		7.28		-5.88		22.01		5.27		27.28		51.15		61.79		33.30		15.00	33.40
21 FOIN	15.16	18.16	10.00	11.56	11.66	1.39	13.80	14.70	8.44	7.08	8.44	7.85	13.12	12.09	15.12	13.63	1.24	1.24	16.36	14.87	16.36	13.33	16.36	13.02	16.36	14.56	6.68	5.78	5.01
22 E.HERB																													
23 PATURE	1165.00	1165.00					1475.00	792.00	318.00	318.00	314.00	318.00	846.00	846.00	946.00	946.00	637.00	637.00	1483.00	1483.00	1493.00	1483.00	1692.00	1692.00	1692.00	1692.00	791.00	791.00	791.00
24 PATURE	1370.00	1370.00					564.00	1128.00	374.00	374.00	374.00	374.00	495.00	495.00	995.00	995.00	749.00	749.00	1744.00	1744.00	1744.00	1744.00	909.00	909.00	909.00	909.00			
25 ORGE	2.30	2.30	4.50	5.30	9.66	2.20	1.95	2.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45			0.45	0.45	0.45	0.45	1.95	1.95	2.41	2.41			
26 GRAINS																													
27 LIT	9.00	9.00	1.20	1.60	1.50	3.60	9.30	15.30	7.20	7.20	7.20	7.20	14.10	14.10	14.10	14.10			15.30	15.30	15.30	15.30	15.30	15.30	15.30	15.30	7.20	7.20	7.20
28 GRAINE																													
29 T1																													
30 T2																													
31 T3																													
32 T4																													
33 T5																													
34 T6	2.32	1.67	0.21	1.12	0.62	0.62	3.38	10.50	0.92	0.69	0.91	0.91	2.26	1.68	2.25	2.25			2.25	2.25	2.26	1.68	2.66	1.92	2.37	2.37	1.40	1.40	1.02
35 L10																													
36 L20																													
37 L21																													
38 L22																													
39 L23																													
40 L30																													
41 L31																													
42 L40																													
43 L41																													
44 L50																													
45 L60																													
46 L61																													
47 L62																													
48 L63																													
49 L64																													
50 L65																													
51 PAILLE																													
52 L24																													
53 T.A1	12.04	12.04	1.00	3.05	3.05	3.05	11.84	3.44	1.72	1.72	1.72	1.72	3.44	3.44	3.44	3.44	1.72	1.72	5.16	5.16	5.16	5.16	4.96	4.96	4.96	4.96	1.52	1.52	1.52
54 T.A2	8.73	8.73	9.70	4.85	4.85	4.85	6.79	5.38	3.44	3.44	3.44	3.44	5.38	5.38	5.38	5.38	1.94	1.94	7.32	7.32	7.32	7.32	5.38	5.38	5.38	5.38			
55 T.A3	3.54	2.73	3.31	2.05	2.05	2.05	4.01	12.79	9.17	8.89	9.16	9.16	11.17	10.45	11.16	11.16			11.16	11.16	11.17	10.45	11.66	10.71	11.30	11.30	2.00	2.00	1.53
56 R.VL.T																													
57 R.MA.E																													
58 G.MA																													
59 A.MA																													
60 R.VL.T																													
61 R.MA38																													
62 R.MA.E																													
63 G.MA38																													
64 A.MA38																													
65 R.NP38																													
66 G.NP38																													
67 A.NP38																													
68 R.MA30																													
69 R.NP30																													
70 R.MA25																													
71 R.NP36																													
72 R.NP26																													
73 A.MGRE																	1500	1500									1600	1600	1600
74 V.LAIT	159	159	178	235	235	235	159	159	124	124	124	124	124	124	124	124			124	124	124	124	124	124	124	124			
75 V.MEAT	- 2172	- 2172		- 1806	- 1806	- 1806	- 2742	- 2900									- 2074	- 2074	- 2074	- 2074	- 2074	- 2074	- 2114	- 2114	- 2114	- 2114	- 2114	- 2114	- 2114
76 V.MGRE			-1150						- 1000	- 1000	- 1000	- 1000	- 1500	- 1500	- 1500	-1500													
77 E.V.MA	320	320	320	320	320	320	320	320																					
78 E.V.N									250	250	250	250	250	250	250	250			250	250	250	250	250	250	250	750			
79 E.G.AM																													
80 CX.MA																													
81 CX.N36																													
82 CX.N26																													
83 TR2	165	165	43	- 1634	- 1534	- 1534	157	- 2900																					
84 TR3	8	8	41	42	50	50	- 2742																- 2114	- 2116	2116	- 2114	- 2114	- 2114	- 2114
85 TR4	- 2121	- 2124	271	82	81	81	101	423	426	427	428	427	435	433	435	434	- 2069	- 2069	- 1639	- 1637	- 1635	- 1638	440	437	439	438	1608	1607	1606
86 TP1	534	531	- 830	644	642	643	587	256	- 848	- 849	- 849	- 849	- 1343	- 1345	- 1343	-1344	1501	1500	157	155	157	154	158	155	158	156	5	5	4
110 FC	749	744	363	171	171	171	651		151	150	151	150	156	154	156	155	1501	1500	595	592	595	590	599	593	598	594	1614	1613	1612
87 K	2210	2210	320	622	620	620	2290	2530	510	510	510	510	1520	1520	1520	1520			2410	2410	2410	2410	1850	1850	1850	1850			
88 V.BAT																													
89 ET.VL																													
90 ET.JB	9.00	9.00	3.00	4.00	4.00	4.00	9.00	12.00	9.00	9.00	9.00	9.00	12.00	12.00	12.00	12.00			12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	5.00	5.00	5.00
91 AIGE	1.75	1.75		0.75	0.75	0.75	1.75	2.00	1.00	1.00	1.00	1.00	2.00	2.00	2.00	2.00			2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	1.00	1.00	1.00
92 PORC																													
93 TRUIE																													
94 T.ST.L																													
95 T.PORC																													
96 [illegible]																													
97 [illegible]																													
98 [illegible]																													
111 [illegible]																													

	7F33B	7F31B	7F31M	7F31C.B	7F31CBM	7F31C.B	7F31CBM	7F31M	7F31P	8P25B	8P25M	8F25C.B	[illegible]	[illegible]	[illegible]	8P325P	8P325M	8F325B	8F325M	8E3?B	8E36M	8E36.B	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
PI	25.32	274.21	265.02	274.11	258.38	42.27	38.39	35.33	42.47	277.39	273.8	271.8	270.15	279.5	281.78	267.2	283.23	33.97	31.3	258.02	253.22	252.5	259.2	18.4	22.5	23.	17.	[illegible]	
18 ?OX				31.57	31.57	16.17	16.17					23.12	23.10	23.10	23.10							30.03	30.31	43.20	31.32				
19 BETTE	33.40	85.82		65.80	15.40	51.40	15.40		59.10	31.30		14.??			20.11	35.57		5.27		44.81		24.30			21.60	31.32		21.60	
20 C.MAIS			87.49		31.30		36.00	59.10			27.07		12.76	17.42			31.72		4.65		39.76		21.60				37.80	21.60	
21 ?OIN	6.68	21.50	13.92	20.73	16.23	11.05	7.45	5.91	11.82	15.73	15.73	15.?	15.73	16.97	16.97	16.97	16.97	1.24	1.24	16.97	16.97	16.97	16.97	7.56	6.46	8.44	3.24	4.32	
23 PATURE	791.00	900.00	900.00	900.00	900.00					455.00	455.00	455.00	455.00	1065.00	1065.00	1065.00	1065.00	610.00	610.00	1055.00	1055.00	1055.00	1055.00						
24 PATURE		909.00	909.00	909.00	909.00					535.00	535.00	535.00	535.00	1252.00	1252.00	1252.00	1252.00	717.00	717.00	1241.00	1241.00	1241.00	1241.00						
25 ?NGE		1.95	3.13	2.41	3.13		0.72	1.18		3.35	3.50	3.73	3.82	3.22	3.73	3.35	3.50			3.44	3.50	3.73	3.82	1.83		3.54	1.09		
27 ?LT	7.20	15.30	15.30	15.30	15.30	7.20	7.20	7.20	7.20	9.00	9.00	9.00	9.00	9.90	9.90	9.90	9.90			9.90	9.90	9.90	9.90	4.80	4.80	4.80	4.80	4.80	
34 T6	1.44	3.39	2.33	3.23	3.23	2.25	2.25	1.43	2.17	1.74	1.25	1.54	1.54	1.54	1.34	1.74	1.25			2.04	1.45	1.86	1.86	0.86	1.18	1.00	0.79	1.03	
53 T.A1	1.32	3.44	3.44	3.44	3.44					10.32	10.32	10.32	10.32	12.04	12.04	12.04	12.04	1.72	1.72	12.04	12.04	12.04	12.04						
54 T.A2		5.38	5.38	5.38	5.38					6.79	6.79	6.79	6.79	8.73	8.73	8.73	8.73	1.94	1.94	8.73	8.73	8.73	8.73						
55 T.A3	2.06	12.57	11.39	12.40	12.40	3.07	3.07	2.11	2.97	2.96	2.38	2.77	2.77	2.77	2.77	2.98	2.38			3.35	2.63	3.18	3.15	2.71	3.58	3.12	2.49	3.16	
71																				- 1.00	- 1.00	- 1.00	1.00	1.00	1.00	1.00	1.00	- 1.00	- 1.30
73 A.MCAL	1500					1600	1600	1600	1600									1300	1300										1300
74		124	124	124	124					127	127	127	127	127	127	127	127			127	127	127	127						
75 V.MEA	- 2114	- 2242	- 2242	- 2242	- 2242	- 2242	- 2242	- 2242	- 2242					- 1843	- 1843	- 1843	- 1843	- 1843	- 1843					- 2311	- 2311	- 2311	- 2311	- 2311	
76										- 1300	- 1300	- 1300	- 1300																
78		250	250	250	250					220	220	220	220	220	220	220	220			220	220	220	220						
81																								31	31 -	77 -	77 -	77	
83		- 2242	- 2242	- 2242	- 2242	- 2242	- 2242	- 2242	- 2242	147	147	147	147	147	147	147	147			147	147	147	147						
84	- 2114																												
85	1608	443	435	442	439	1611	1610	1609	1612	49	47	44	53	- 1795	- 1797	- 1792	- 1794	- 1840	- 1840	52	50	47	46	8	10	9	6	14	1800
86	5	153	158	163	150	10	4	6	10	- 901	- 903	- 901	- 902	397	395	399	397	1301	1301	400	396	400	399	- 2305	- 2304	- 2302	- 2306	- 2303	
87	1514					- 618	- 622	- 625	- 615	545	544	545	544	585	591	597	593	1301	1301	548	545	547	545	5	6	5	4	7	- 505
88		2120	2120	2120	2120					1080	1080	[illegible]	[illegible]	[illegible]	[illegible]	1750	1749			2430	2430	2430	2430						
90	5.30	12.00	12.00	12.00	12.00	5.00	5.00	5.00	5.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00			9.00	9.00	9.00	9.00	5.00	5.00	5.00	5.30	5.30	
91	1.50	2.00	2.00	2.00	2.00	1.00	1.00	1.00	1.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00			2.00	2.00	2.00	2.00	1.00	1.00	1.00	1.00	1.00	

LIVESTOCK ACTIVITIES

	6E26B	6E26M	6E26C.B	6E26C.H	6F26CB	6F26B	6F26M	6F26M.B	6F26C	6F16B	6F16M	6F16M.UP	6F16M.AN
00 FL	-289.31	-286.37	-281.51	-283.19	-22.58	-65.22	-16.8	-17.7	-15.07	-436.8	-433.24	-430.24	-430.24
01 TERRE													
02 PAILLE													
03 COLZA													
04 ASS.PE													
05 ASS.PS													
06 B.COLZ													
07 B.AV.H													
08 B.AV.H													
09 B.BLE1													
10 B.BLE2													
11 B.AV.P													
12 B.ORGE													
13 B.EG.H													
14 B.EG.H													
15 PV													
16 PT													
17 CE.LUZ													
18 CHOU			4.46	4.46	41.73				38.45				
19 BETTE	11.03		6.30		33.40	50.10		33.40		38.00			
20 E.MAIS		9.54		5.40			33.44	21.71			21.30	75.28	45.13
21 FOIN	10.29	10.29	10.29	10.29	6.68	8.35	3.34	6.68	10.02	11.78	11.78	3.00	4.00
22 E.HERB													
23 PATURE	482.00	482.00	482.00	482.00									
24 PATURE	567.00	567.00	567.00	567.00									
25 ORGE	4.12	4.10	4.52	4.52	1.50	1.67	2.17	1.67	3.84	6.78	9.02	3.00	8.01
26 GRAINS													
27 LIT	4.50	4.50	4.50	4.50	5.40	5.40	5.40	5.40	5.40	4.50	4.50	4.30	4.50
28 GRAINE													
29 T1													
30 T2													
31 T3													
32 T4													
33 T5													
34 T6	0.38	0.43	0.41	0.41	1.06	0.95	0.66	0.90	0.75	1.12	0.59	0.39	0.39
35 L10													
36 L20													
37 L21													
38 L22													
39 L23													
40 L30													
41 L31													
42 L40													
43 L41													
44 L50													
45 L60													
46 L61													
47 L62													
48 L63													
49 L64													
50 L65													
51 PAILLE													
52 L24													
53 T.A1	10.32	10.32	10.32	10.32						4.30	4.30	4.30	4.30
54 T.A2	6.79	6.79	6.79	6.79						4.85	4.85	4.85	4.85
55 T.A3	1.22	1.09	1.09	1.09	3.33	3.16	2.34	2.91	2.67	2.07	1.34	1.34	1.34
56 B.VL.T													
57 B.BA.E													
58 G.BA													
59 A.BA													
60 B.VL.T													
61 B.BA30													
62 B.BA.E													
63 G.BA36													
64 A.BA36													
65 B.BF36													
66 G.BF36													
67 A.BF36													
68 B.BA30													
69 B.BF30													
70 B.BA23													
71 B.BF36													
72 B.BF26	-1.00	-1.00	-1.00	-1.00	1.00	1.00	1.00	1.00	1.00				
73 A.VEAU													
74 V.LAIT	186	186	186	186						166	166	166	166
75 V.BBAT					-2007	-2007	-2007	-2007	-2007	-1668	-1668	-1668	-1668
76 V.BEEE													
77 E.V.BA													
78 E.V.B	230	230	220	220						220	220	220	220
79 E.C.AN													
80 CE.BA													
81 CE.X36													
82 CE.V26					97	-77	-77	-77	90				
83 TB2	147	147	147	147	-2007	-2007	-2007	-2007	-2007	-1502	-1502	-1502	-1502
84 TB3										67	64	64	64
85 TB4	35	33	30	30	11	30	28	9	8	89	87	87	87
86 TB1	433	433	433	433	9	29	26	6	6	480	478	478	478
110 EC	147	147	147	147						165	163	163	163
87 E	1480	1480	1480	1480						440	440	440	440
88 V.BAT													
89 ST.VL													
90 ST.J8	6.00	6.00	6.00	6.00	5.00	5.00	5.00	5.00	5.00	6.00	6.00	6.00	6.00
91 AUGE	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
92 PORC													
93 TRUIE													
94 F.ST.L													
95 F.PORC													
96 F.TR													
97 F.P.CH													
98 F.F.P													
111 F.P.P													

LIVESTOCK ACTIVITIES (continued)

	OEUFS	POULET	PORCELET	PORC	EL.PORC	VEAU.B	POULES
00 FL	3.60	1.09	2375.00	75.00	4983.00	192.52	4.00
53 T.A1	0.08	0.05	56.00	1.01	96.12	3.77	0.08
54 T.A2	0.09	0.09	113.76	1.14	133.60	6.30	0.09
55 T.A3	0.07	0.04	29.42	0.81	53.45	5.18	0.07
83 TB2	10	0.11	-37		-37	-35	10
84 TB3	-4	0.21	-2181	-30	-3521	-35	6
85 TB4	-3	0.21	1244	-30	1006	-55	5
86 TB1	-4	0.21	-1321	-15	-2431	-27	5
110 EC	10	0.63	-112	-56	-1462	-144	10
87 E	10	5	2500		2500	150	8
88 V.BAT						2.50	
92 PORC				1.00	35.00		
93 TRUIE			5.00		5.00		
97 F.P.CH		1.00					
98 F.F.P	1.00						
111 F.P.P							1.00

SELLING ACTIVITIES

	1VG	2VG	3VG	V.GRAINS	VBETTE	VE.MAIS	VFOIN	VPAILLE	V.LAIT
00 FL				107.20			14.00	5.00	100.00
19 BETTE					1.00				
20 E.MAIS						1.00			
21 FOIN							1.00		
26 GRAINS				100.00					
27 LIT								1.00	
59 A.BA	1.00								
64 A.BA36		1.00							
67 A.BF36			1.00						
74 V.LAIT									100
77 E.V.BA	400	300	300						
78 E.V.B	-2500	-2300	+2200						
84 TB3							-14	-3	
85 TB4		-2000							
86 TB1	-2100		+1900						

SELLING ACTIVITIES / BUYING ACTIVITIES

	9VIANDE	9MAIGRE	9VEAU.MA	9VEAU.N	9GJU	9PENSION	9GRAINES	VP.LAIT	VP.MEAT	VP.GRAIN	P.LAIT	P.VIANDE	P.GRAINS	SG.GRAIN	SG.LAIT	SG.MEAT	SG.MORE	OGRAINS	OFOIN	OPAILLE	OVACHE	OVEAU.MA	OVEAU.N	OGTE	OORGE	OTERRE	OV.ST.VI	OST.JB	OMAIS
00 FE	150.00	170.00	100.00	100.00	100.00	240.00	100.00	0.50	6.85	120.00	0.40	3.65	100.00	107.00	100.00	100.00	100.00	-	- 18.00	- 7.00	- 100.00	- 100.00	- 100.00	- 100.00	- 41.00	- 320.00	- 82.97	- 3.[illegible]	- 15.00
01 TERRE																										- 1.00			
02 PAILLE																										- 1.30			
03 COLZA																													
04 ASS.PS																													
05 ASS.PS																													
06 R.COLZ																													
07 R.AV.N																													
08 R.AV.N																													
09 R.B.EI																													
10 R.BLEI																													
11 R.AV.P																													
12 R.ORGE																													
13 R.RG.N																													
14 R.RG.N																													
15 [illegible]																										- 0.10			
16 PT																													
17 GR.LUZ																													
18 CHOU																													
19 BETTE																													
20 E.MAIS																													
21 FOIN																			- 1.00										
22 E.HERB																													
23 PATURE						328.00																							
24 PATURE						620.00																							
25 ORGE																		- 1.00							1.00				
26 GRAINS										100.00			100.00	100.30				38.00											
27 LIT																				- 1.00									
28 GRAINE							100.00																						
29 T1																													
30 T2																													
31 T3																													
32 T4																													
33 T5																													
34 T6																													
35 L10																													
36 L20																													
37 L21																													
38 L22																													
39 L23																													
40 L30																													
41 L31																													
42 L40																													
43 L41																													
44 L50																													
45 L60																													
46 L61																													
47 L62																													
48 L63																													
49 L64																													
50 L65																													
51 PAILLE																													
52 L74																													
53 T.A1																											7.52		
54 T.A2																											8.49		
55 T.A3																											6.74		
56 R.VL.T																													
57 R.MA.E																													
58 G.MA																													
59 A.MA																													
60 B.VL.T																													
61 R.MA38																													
62 R.MA.E																													
63 G.MA38																													
64 A.MA38																													
65 R.NP38																													
66 G.NP38																													
67 A.NP38																													
68 R.MA30																													
69 R.NP30																													
70 R.MA25																													
71 R.NP26																													
72 R.NP25																													
73 A.MORE																					- 100								
74 V.LAIT								0.47			0.47				100														
75 V.MEAT	100								5			5				100													
76 V.MORE		100															100												
77 E.V.MA			100																			- 100							
78 E.V.N				100																			- 100						
79 E.V.AM					100																			- 100					
80 CE.MA																													
81 CE.N38																													
82 CE.N25																													
83 TR2																										160			
84 TR3						- 40												35	18	7					41				
85 TR4						- 80																				160	148	6	27
86 TR5						- 60																							
112 KC																		38	19	7					41	160			
87 K																											1310	60	240
88 V.BAT																										- 5.21			
89 ET.VL																											- 1.00		
90 ET.JB																												- 1.00	
91 AUGE																													- 1.00
92 PORC																													
93 TR.TI																													
94 F.ST..																													
95 F.PORC																											1.00		
96 F.TR																													
97 F.F.CH																													
98 F.F.P																													
111 F.F.F																													

Row	BUYING ACTIVITIES	TRANSFER AND MISCELLANEOUS	CONST
00 PE	- 19.97 - 108.33 - 5.00 - 10000.00 - 450.00 - 55.37 - 759.15 - 128.39 - 133.68 - 201.34 - 451.50	173.3	
01 TERRE			15.00
02 PAILLE			15.00
15 PN			1.50
21 POIS		1.00	
22 E.HERB		1.00 200.00	
23 PATURE		- 2780.00	
24 PATURE		- 2.20 - 11.00 - 18.00	
30 T2		- 14.00	
35 L10	- 12500		125.00
36 L20	- 31500	35.20 - 22.00	315.00
37 L21	- 7200	8.00 - 22.00	72.00
38 L22	- 5000		50.00
39 L23	- 21500	24.00 - 22.00	215.00
40 L30	- 24500	27.00	245.00
41 L31	- 11000		110.00
42 L40	- 25000	28.05	250.00
43 L41	- 15000		150.00
44 L50	- 18000	19.95	180.00
45 L60	- 46000	17.83 37.45	480.00
46 L61	- 18000	20.00	180.00
47 L62	- 11500		115.00
48 L63	- 35500	21.54 20.10	355.00
49 L64	- 23000	28.70	230.00
50 L65	- 26000	32.40	260.00
52 L24	- 4000		40.00
53 T.A1	- 8600 156.52	18.42 - 86.00	258.00
54 T.A2	- 9500 176.54	18.32 - 95.00 0.25 0.15 0.15	289.00
55 T.A3	- 13500 190.10	18.30 - 27.00	308.00
83 TR2	2500 3300 425	- 100 - 180	
84 TR3	2500 900 425		
85 TR4	28 152 2500 3300 595 1048 190 425	- 100 100	
86 TR1	2500 900 475 256 1074	- 100 100 - 100 100	
110 KC	1500 5500 1277	- 100 100	
87 K	183 582 - 120 8820 5180 1065 7373 1112 4650	100	
88 V.BAT		6.00 1.00 1.50 8.00	200.00
89 ET.VL		- 1.00	
90 ET.JB		- 1.00	
91 AVGE		1.00	
92 PORC	- 1.00	- 1.00	
93 TRUIE	- 1.00	- 1.00	
94 F.ST.L	- 70.92		
95 F.PORC	1.00 - 190.00		
96 C.TR	1.00 - 50.00		
97 F.P.CH	- 10000.00		
98 F.P.P	- 5000.00		
111 F.P.P	- 5000.00		

APPENDIX B. LIST OF ACTIVITIES AND CONSTRAINTS

Activities

Code	Units	Definition
P.DE.T	1 Ha	Potato
BLE.S.B	1 Ha	Winter wheat
ORG.P	1 Ha	Spring barley
AV.P	1 Ha	Spring oats
CHX.S.RG	1 Ha	Fodder kale (after rye grass)
CHX.ORGE	1 Ha	Fodder kale + winter barley
CHX	1 Ha	Fodder kale
BET.TRAD	1 Ha	Transplanted fodder beet: manual harvesting
BET.SM	1 Ha	Transplanted fodder beet: machine made harvest
BET.M	1 Ha	Seeded fodder beet: machine made harvest
MAIS.E	1 Ha	Corn for ensilage
AS.F	4 years	Fodder crop rotation
MAIS.ENS	1 Ha	Corn for ensilage
MAIS.G	1 Ha	Corn
COLZA	1 Ha	Rape
AVOINE.H	1 Ha	Winter oats
BLE.1	1 Ha	Winter wheat (higher yield level)
BLE.2	1 Ha	Winter wheat (lower yield level)
AVOINE.P	1 Ha	Spring oats
ORGE.P	1 Ha	Spring barley
O.FET.CZ	5 Ha	Spring barley + tall fescue for seed + rape
RGG.CZ	2 Ha	Rye grass for seed + rape
RGGCZ.SP	2 Ha	Rye grass for seed + rape (with straw manuring)
O.FET.MG	5 Ha	Spring barley + tall fescue for seed + corn
RGG.MG	2 Ha	Rye grass for seed + corn
RGGMG.SP	2 Ha	Rye grass for seed + corn (with straw manuring)
PT3.NU	1 Ha	September seeding of three years temporary pasture
PT2.NU	1 Ha	September seeding of two years temporary pasture
ORGE.PT3	1 Ha	Spring barley + seeding of three years temporary pasture
ORGE.PT2	1 Ha	Spring barley + seeding of two years temporary pasture
AV.PT3	1 Ha	Spring oats + seeding of three years temporary pasture

Code	Units	Definition
AV.PT2	1 Ha	Spring oats + seeding of two years temporary pasture
MG.BLE	2 Ha	Corn + winter wheat
		Pasture Management
		(a) Yield level: 5,000 fodder units/Ha
LZ1.FFF	1 Ha	Grass + alfalfa: three hay cuttings
LZ1.FPPP	1 Ha	Grass + alfalfa: one hay cutting + three grazings
LZ1.PFPP	1 Ha	Grass + alfalfa: one hay cutting + three grazings
PT1.PPPP	1 Ha	Temporary pasture: four grazings
PT1.EPPP	1 Ha	Temporary pasture: one ensilaging + three grazings
PN1.PPPP	1 Ha	Natural pasture: four grazings
PN1.FPP	1 Ha	Natural pasture: one hay cutting + two grazings
		(b) Yield level: 4,000 fodder units/Ha
LZ2.FFF	1 Ha	Grass + alfalfa: three hay cuttings
LZ2.FPPP	1 Ha	Grass + alfalfa: one hay cutting + three grazings
LZ2.PFPP	1 Ha	Grass + alfalfa: one hay cutting + three grazings
PT2.PPPP	1 Ha	Temporary pasture: four grazings
PT2.EPPP	1 Ha	Temporary pasture: one ensilaging + three grazings
PN2.PPPP	1 Ha	Natural pasture: four grazings
		(c) Yield level: 3,000 fodder units/Ha
LZ3.FFF	1 Ha	Grass + alfalfa: three hay cuttings
LZ3.FPPP	1 Ha	Grass + alfalfa: one hay cutting + three grazings
LZ3.PFPP	1 Ha	Grass + alfalfa: one hay cutting + three grazings
PT3.PPPP	1 Ha	Temporary pasture: four grazings
PT3.EPPP	1 Ha	Temporary pasture: one ensilaging + three grazings
PN3.PPPP	1 Ha	Natural pasture: four grazings
R.PAIL	1 metric ton	Straw harvesting
MOUTON	4 ewes	Sheep flock

Livestock Feeding Programs

Code	Units	Livestock activities	Basic fodder in winter	Period
1RP.VT.B	daily ration	Dry cow	Beet	1/1-1/4
1RP.VT.M	daily ration	Dry cow	Corn[1]	1/1-1/4
1RA.VT.C	daily ration	One dry cow	Kale	15/10-1/1
1RA.VT.B	daily ration	One dry cow	Beet	15/10-1/1
1RA.VT.M	daily ration	One dry cow	Corn	15/10-1/1
		Maine-Anjou Breed		
1RP.M	20 rations	20 dairy cows	Corn[1]	1/3-31/3
1RP.B	20 rations	20 dairy cows	Beet	1/3-31/3
1RP.BM	20 rations	20 dairy cows	Beet + corn	1/3-31/3
1R.VE.B	1 ration	One feeder cow	Beet	15/10-1/4
1R.VE.M	1 ration	One feeder cow	Corn	15/10-1/4
1EG.C.B	1 ration	One heifer (from 0 to 3 years)	Kale + beet	15/10-1/4
1EG.C.M	1 ration	One heifer (from 0 to 3 years)	Kale + corn	15/10-1/4
1EG.B	1 ration	One heifer (from 0 to 3 years)	Beet	15/10-1/4
1EG.M	1 ration	One heifer (from 0 to 3 years)	Corn	15/10-1/4
		Normande and Frisian Breed Cows: October calving, 3,800 liters of milk/year		
2RP.M		20 dairy cows	Corn	1/1-1/4
2RP.B	20 rations	20 dairy cows	Beet	1/1-1/4
2RP.BM	20 rations	20 dairy cows	Beet + corn	1/1-1/4
2RA.C	20 rations	20 dairy cows	Kale	15/10-1/1
2RA.B	20 rations	20 dairy cows	Beet	15/10-1/1
2RA.M	20 rations	20 dairy cows	Corn	15/10-1/1
2RA.CB	20 rations	20 dairy cows	Kale + beet	15/10-1/1
2RA.CM	20 rations	20 dairy cows	Kale + corn	15/10-1/1
2RA.BM	20 rations	20 dairy cows	Beet + corn	15/10-1/1
2RA.VE.C	1 ration	One feeder cow	Kale	1/11-1/1
2RA.VE.B	1 ration	One feeder cow	Beet	1/11-1/1
2RA.VE.M	1 ration	One feeder cow	Corn	1/11-1/1
2EG.C.B	1 ration	One heifer (from 0 to 3 years)	Kale + beet	15/10-1/4
2EG.C.M	1 ration	One heifer (from 0 to 3 years)	Kale + corn	15/10-1/4

[1]Corn = corn silage.

Code	Units	Livestock Activities	Basic fodder in winter	Period
2EG.B	1 ration	One heifer (from 0 to 3 years)	Beet	15/10-1/4
2EG.M	1 ration	One heifer (from 0 to 3 years)	Corn	15/10-1/4

Normande and Frisian Breed
Cows: February calving
3,800 liters of milk/year

3RP.M	20 rations	20 dairy cows	Corn	1/1-1/4
3RP.B	20 rations	20 dairy cows	Beet	1/1-1/4
3RP.BM	20 rations	20 dairy cows	Corn + beet	1/1-1/4
3RA.C	20 rations	20 dairy cows	Kale	15/10-1/1
3RA.B	20 rations	20 dairy cows	Beet	15/10-1/1
3RA.M	20 rations	20 dairy cows	Corn	15/10-1/1
3RA.CB	20 rations	20 dairy cows	Kale + beet	15/10-1/1
3RA.CM	20 rations	20 dairy cows	Kale + corn	15/10-1/1
3RA.BM	20 rations	20 dairy cows	Beet + corn	15/10-1/1
3EG.C.B	1 ration	One heifer (from 0 to 3 years)	Kale + beet	15/10-1/4
3EG.C.M	1 ration	One heifer (from 0 to 3 years)	Kale + corn	15/10-1/4
3EG.B	1 ration	One heifer (from 0 to 3 years)	Beet	15/10-1/4
3EG.M	1 ration	One heifer (from 0 to 3 years)	Corn	15/10-1/4

Normande and Frisian Breed
Cows: October calving,
3,000 liters of milk/year

4RP.M	20 rations	20 dairy cows	Corn	1/1-1/4
4RP.B	20 rations	20 dairy cows	Beet	1/1-1/4
4RP.BM	20 rations	20 dairy cows	Beet + corn	1/1-1/4
4RA.C	20 rations	20 dairy cows	Kale	15/10-1/1
4RA.B	20 rations	20 dairy cows	Beet	15/10-1/1
4RA.M	20 rations	20 dairy cows	Corn	15/10-1/1
4RA.CB	20 rations	20 dairy cows	Kale + beet	15/10-1/1
4RA.CM	20 rations	20 dairy cows	Kale + corn	15/10-1/1
4RA.BM	20 rations	20 dairy cows	Beet + corn	15/10-1/1
4EG.CB	1 ration	One heifer (0 to 3 years)	Kale + beet	15/10-1/4
4EG.CM	1 ration	One heifer (0 to 3 years)	Kale + corn	15/10-1/4
4EG.B	1 ration	One heifer (0 to 3 years)	Beet	15/10-1/4
4EG.M	1 ration	One heifer (0 to 3 years)	Corn	15/10-1/4

Code	Units	Livestock Activities	Basic fodder in winter	Period
		Normande and Frisian Breed Cows: February calving, 3,000 liters of milk/year		
5RP.M	20 rations	20 dairy cows	Corn	1/1-1/4
5RP.B	20 rations	20 dairy cows	Beet	1/1-1/4
5RA.C	20 rations	20 dairy cows	Kale	1/1-1/4
5RA.B	20 rations	20 dairy cows	Beet	15/10-1/1
5RA.M	20 rations	20 dairy cows	Corn	15/10-1/1
5RA.CB	20 rations	20 dairy cows	Kale + beet	15/10-1/1
5RA.CM	20 rations	20 dairy cows	Kale + corn	15/10-1/1
5RA.BM	20 rations	20 dairy cows	Beet + corn	15/10-1/1
5EG.CB	1 ration	One heifer (0 to 3 years)	Kale + beet	15/10-1/4
5EG.C.M	1 ration	One heifer (0 to 3 years)	Kale + corn	15/10-1/4
5EG.B	1 ration	One heifer (0 to 3 years)	Beet	15/10-1/4
5EG.M	1 ration	One heifer (0 to 3 years)	Corn	15/10-1/4
		Steer Production (1) Maine-Anjou breed (a) 25 months slaughter steer		
6E25B	1 ration	Feeder steer production	Beet	15/10-1/4
6E25M	1 ration	Feeder steer production	Corn	15/10-1/4
6E25C.M	1 ration	Feeder steer production	Kale + corn	15/10-1/4
6E25C.B	1 ration	Feeder steer production	Kale + beet	15/10-1/4
6F25B	1 ration	Steer fattening	Beet	15/10-1/4
6F25M	1 ration	Steer fattening	Corn	15/10-1/4
6F25CB	1 ration	Steer fattening	Kale + beet	15/10-1/4
6F25BM	1 ration	Steer fattening	Beet + corn	15/10-1/4
6F25C	1 ration	Steer fattening	Kale	15/10-1/4
6M.T.F25	1 ration	Transfer of feeder steer		
		(b) 28 months slaughter steer		
6P28C.B	1 ration	0 to 28 months steer production	Kale + beet	15/10-1/4
6P28C.M	1 ration	0 to 28 months steer production	Kale + corn	15/10-1/4
6P28B	1 ration	0 to 28 months steer production	Beet	15/10-1/4

Code	Units	Livestock activities	Basic fodder in winter	Period
6P28M	1 ration	0 to 28 months steer production	Corn	15/10-1/4
6F28HERB	1 ration	Fattening on pasture of steer bought in March	None	-
		(c) 24 months feeder steer		
6P24C.B	1 ration	0 to 24 months steer production	Kale + beet	15/10-1/4
6P24C.M	1 ration	0 to 24 months steer production	Kale + corn	15/10-1/4
6P24B	1 ration	0 to 24 months steer production	Beet	15/10-1/4
6P24M	1 ration	0 to 24 months steer production	Corn	15/10-1/4
		(d) 31.5 months slaughter steer		
6F315B	1 ration	Fattening on pasture of steer bought in March	Beet	15/10-1/4
6F315M	1 ration	Fattening on pasture of steer bought in March	Corn	15/10-1/4
6P315C.B	1 ration	0 to 31.5 months steer production	Kale + beet	15/10-1/4
6P315C.M	1 ration	0 to 31.5 months steer production	Kale + corn	15/10-1/4
6P315B	1 ration	0 to 31.5 months steer production	Beet	15/10-1/4
6P315M	1 ration	0 to 31.5 months steer production	Corn	15/10-1/4
6P10	1 ration	(e) 10 months feeder steer	Beet	15/10-1/4
		Frisian or Normande Breed (Fall Born Animals)		
		(a) Yearling bull		
6P15B	1 ration	0 to 15 months bull production	Beet	15/10-1/4
6P15M	1 ration	0 to 15 months bull production	Corn with large quantities of hay	15/10-1/4
6P15M.UP	1 ration	0 to 15 months bull production	Corn with hay minimum	15/10-1/4

Code	Units	Livestock activities	Basic fodder in winter	Period
		(b) Prime slaughter steer		
6P28LBEL	1 ration	0 to 28 months steer production	Beet + corn	15/10-1/4
6P30LBEL	1 ration	0 to 30 months steer production	Beet + corn	15/10-1/4
		(c) 17 months feeder steer		
7P17B	1 ration	0 to 17 months steer production	Beet	15/10-1/4
7P17M	1 ration	0 to 17 months steer production	Corn	15/10-1/4
7P17C.B	1 ration	0 to 17 months steer production	Kale + beet	15/10-1/4
7P17C.M	1 ration	0 to 17 months steer production	Kale + corn	15/10-1/4
		(d) 29 months feeder steer		
7FP29B	1 ration	0 to 29 months steer production	Beet	15/10-1/4
7P29M	1 ration	0 to 29 months steer production	Corn	15/10-1/4
7P29C.B	1 ration	0 to 29 months steer production	Kale + beet	15/10-1/4
7P29C.M	1 ration	0 to 29 months steer production	Kale + corn	15/10-1/4
		(e) 36.5 months slaughter steer		
7F365B	1 ration	Fattening on pasture of steer bought in March	Beet	15/10-1/4
7F365M	1 ration	Fattening on pasture of steer bought in March	Corn	15/10-1/4
7P365C.B	1 ration	0 to 36.5 months steer production	Kale + beet	15/10-1/4
7P365C.M	1 ration	0 to 36.5 months steer production	Kale + corn	15/10-1/4
7P365B	1 ration	0 to 36.5 months steer production	Beet	15/10-1/4
7P365M	1 ration	0 to 36.5 months steer production	Corn	15/10-1/4
		(f) 33 months slaughter steer		

Code	Units	Livestock activities	Basic fodder in winter	Period
7P33B	1 ration	0 to 33 months steer production	Beet	15/10-1/4
7P33M	1 ration	0 to 33 months steer production	Corn	15/10-1/4
7P33C.B	1 ration	0 to 33 months steer production	Kale + beet	15/10-1/4
7P33C.M	1 ration	0 to 33 months steer production	Kale + corn	15/10-1/4
7F33C.B	1 ration	Fattening of steer bought in November	Kale + beet	15/10-1/4
7F33C.M	1 ration	Fattening of steer bought in November	Kale + corn	15/10-1/4
7F33M	1 ration	Fattening of steer bought in November	Corn	15/10-1/4
7F33B	1 ration	Fattening of steer bought in November	Beet	15/10-1/4
		(g) 31 months slaughter steer		
7P31B	1 ration	0 to 31 months steer production	Beet	15/10-1/4
7P31M	1 ration	0 to 31 months steer production	Corn	15/10-1/4
7P31C.B	1 ration	0 to 31 months steer production	Kale + beet	15/10-1/4
7P31C.BM	1 ration	0 to 31 months steer production	Kale + corn + beet	15/10-1/4
7F31C.B	1 ration	Fattening of steer bought in November	Kale + beet	15/10-1/4
7F31C.BM	1 ration	Fattening of steer bought in November	Kale + corn + beet	15/10-1/4
7F31M	1 ration	Fattening of steer bought in November	Corn	15/10-1/4
7F31B	1 ration	Fattening of steer bought in November	Beet	15/10-1/4
		Frisian or Normande Breed (Spring Born Animals)		
8P25B	1 ration	(a) 25 months feeder steer	Beet	15/10-1/4
8P25M	1 ration	25 months feeder steer	Corn	15/10-1/4
8P25C.B	1 ration	25 months feeder steer	Kale + beet	15/10-1/4

Code	Units	Livestock Activities	Basic fodder in winter	Period
8P25C.M	1 ration	25 months feeder steer	Kale + corn	15/10-1/4
		(b) 32.5 months slaughter steer		
8P325C.M	1 ration	0 to 32.5 months steer production	Kale + corn	15/10-1/4
8P325C.B	1 ration	0 to 32.5 months steer production	Kale + beet	15/10-1/4
8P325B	1 ration	0 to 32.5 months steer production	Beet	15/10-1/4
8P325M	1 ration	0 to 32.5 months steer production	Corn	15/10-1/4
8F325B	1 ration	Fattening on pasture of steer bought in March	Beet	15/10-1/4
8F325M	1 ration	Fattening on pasture of steer bought in March	Corn	15/10-1/4
		(c) 36 months slaughter steer		
8E36B	1 ration	Feeder steer production	Beet	15/10-1/4
8E36M	1 ration	Feeder steer production	Corn	15/10-1/4
8E36C.B	1 ration	Feeder steer production	Kale + beet	15/10-1/4
8E36C.M	1 ration	Feeder steer production	Kale + corn	15/10-1/4
8F36C	1 ration	Steer fattening	Kale	15/10-1/2
8F36CB	1 ration	Steer fattening	Kale + beet	15/10-1/2
8F36B	1 ration	Steer fattening	Beet	15/10-1/2
8F36M	1 ration	Steer fattening	Corn	15/10-1/2
8F36MB	1 ration	Steer fattening	Corn + beet	15/10-1/2
8M.T.F36	1 ration	Transfer of feeder steer		
		(d) 26 months slaughter steer		
8E26B	1 ration	Feeder steer production	Beet	15/10-1/4
8E26M	1 ration	Feeder steer production	Corn	15/10-1/4
8E26C.B	1 ration	Feeder steer production	Kale + beet	15/10-1/4
8E26C.M	1 ration	Feeder steer production	Kale + corn	15/10-1/4

Code	Units	Livestock Activities	Basic fodder in winter	Period
8F26CB	1 ration	Steer fattening	Kale + beet	15/10-1/4
8F26B	1 ration	Steer fattening	Beet	15/10-1/4
8F26M	1 ration	Steer fattening	Corn	15/10-1/4
8F26M.B	1 ration	Steer fattening	Corn + beet	15/10-1/4
8F26C	1 ration	Steer fattening	Kale	15/10-1/4

(e) Yearling bulls production

8P16B	1 ration	0 to 16 months bull production	Beet	15/10-1/4
8P16M	1 ration	0 to 16 months bull production	Corn + large quantities of hay	15/10-1/4
8P16M.UP	1 ration	0 to 16 months bull production	Maximum of corn	15/10-1/4
8P16M.AV	1 ration	0 to 16 months bull production	Corn partly substituted by 500 Kg of concentrates	15/10-1/4

Code	Units	Definition
OEUFS	1 hen	Egg production. Type I hen-house
POULET	4 broilers	Broiler production
PORCELET	5 sows	Weaned pig production
PORC	2.5 hogs	Hog production
EE.PORC	10 litters	Weaned pig + hog production
VEAU.B	3.5 calves	Slaughter calf
POULES.V	1 hen	Egg production. Type II hen-house
1VG	1 heifer	Sale of 1 Maine-Anjou heifer
2VG	1 heifer	Sale of 1 Normande or Frisian heifer born in October (from herd producing 3,800 liters of milk)
3VG	1 heifer	Sale of 1 Normande or Frisian heifer born in spring (from herd producing 3,800 liters of milk)
9GRAINS	100 F	Sale of cereals (present price)
9BETTE	1 qx	Sale of fodder beet
9E.MAIS	1 qx	Sale of corn silage
9FOIN	1 qx	Sale of hay
9PAILLE	1 qx	Sale of straw
9LAIT	100 F	Sale of milk (present price)
9VIANDE	100 F	Sale of slaughter steers or cows (present price)
9MAIGRE	100 F	Sale of feeder steers (present price)
9VEAU.MA	100 F	Sale of Maine-Anjou calves
9VEAU.N	100 F	Sale of Normande or Frisian calves
9G36	100 F	Sale of heifer (3,000 liters of milk)
9PENSION	1 animal	Steer at livery
9GRAINES	100 F	Sale of grass seeds
UP.LAIT	0.5 F	Sale of milk (maximum price)
UP.MEAT	6.85 F	Sale of slaughter steer or cow (maximum price)
UP.GRAIN	120.00 F	Sale of cereals (maximum price)
P.LAIT	0.40 F	Sale of milk (minimum price)
P.VIANDE	5.65 F	Sale of slaughter steers of cows (minimum price)
P.GRAINS	80.00 F	Sale of grains (minimum price)
SG.GRAIN	107.00 F	Sale of cereals (present price)
SG.LAIT	100.00 F	Sale of milk (present price)
SG.MEAT	100.00 F	Sale of slaughter steer or cow (present price)
SG.MGRE	100.00 F	Sale of feeder steer (present price)
OGRAINS	1 qx	Transfer of cereals to feeding programs
OFOIN	1 qx	Purchase of hay
OPAILLE	1 qx	Purchase of straw
OMAIGRE	100 F	Purchase of feeder steer
OVEAU.MA	100 F	Purchase of Maine-Anjou calf
OVEAU.N	100 F	Purchase of Normande or Frisian calf
OG36	100 F	Purchase of heifer
OORGE	1 qx	Purchase of barley

Code	Units	Definition
OTERRE	Ha	Land renting
OV.ST.VL	Number	Purchase of dairy cow facilities
OST.JB	Number	Purchase of steer facilities
OAUGE	Number	Purchase of feeding rack
OVP.GRAS	Number	Purchase of hay facilities
OV.TRUIE	Number	Purchase of sow facilities
OK	100 F	Capital borrowing
OTRAVAIL	Number	Labor hiring
FIXE	1 F	Fixed farm expenditures and minimum private expenditure requirements
OF.ST.VL	1	Purchase of fixed dairy cow facilities
OFP.GRAS	1	Purchase of fixed hog cow facilities
OF.TRUIE	1	Purchase of fixed sow facilities
OP.CHAIR	1	Purchase of fixed broiler facilities
OP.OEUF	1	Purchase of fixed type 1 hen house facilities
POULES.F	1	Purchase of fixed type 2 hen house facilities
VB.T.VL	Number	Transfer of building space to dairy activities
VB.T.JB	M^2	Transfer of building space to steer activities
VB.T.PE	Number	Transfer of building space to hog activities
VB.T.T.	Number	Transfer of building space to sow activities
TV.T.A1	1 hour/day	Transfer of labor from crop to livestock activities period A_1
TV.T.TA2	1 hour/day	Transfer of labor from crop to livestock activities period A_2
TV.T.TA3	1 hour/day	Transfer of labor from crop to livestock activities period A_3
F.T.ETE	1 qx	Transfer of hay to summer feeding programs
EH.T.ETE	1 qx	Transfer of grass silage feeding programs
EM.T.ETE	1 qx	Transfer of corn silage feeding programs
K.T.KC	100 F	Transfer of capital to the initial period
KC.T.TR3	100 F	Transfer of working capital: initial period to third quarter
TR3.T.T4	100 F	Transfer of working capital: third quarter to fourth quarter
TR4.T.T1	100 F	Transfer of working capital: fourth quarter to first quarter
TR1.T.T2	100 F	Transfer of working capital: first quarter to second quarter
EH.T.23P	200 qx	First period rye grass grazing
01RHS.1	200 qx	Vector of constraints

Constraints

Code	Constraints	Units	Definition
00 FE	N	F	Objective function
01 TERRE	≤ 15	Ha	Land
02 PAILLE	≤ 15	Ha	Maximum of cereals in crop rotations
03 COLZA	≤ 0	Ha	Maximum of rape in crop rotations
04 ASS.PS	≤ 0	Ha	Preestablished fodder crop rotation
05 ASS.PS	≤ 0	Ha	Preestablished fodder crop rotation
			Main Crop Rotation
06 R.COLZ	≤ 0	Ha	Rape constraint
07 R.AV.H	≤ 0	Ha	Winter oats constraint
08 R.AV.H	≤ 0	Ha	Winter oats constraint
09 R.BLE1	≤ 0	Ha	Type I winter wheat constraint
10 R.BLE2	≤ 0	Ha	Type II winter wheat constraint
11 R.AV.P	≤ 0	Ha	Spring oats constraint
12 R.ORGE	≤ 0	Ha	Spring barley constraint
13 R.RG.M	≤ 0	Unit	Rye-grass + corn constraint
14 R.RG.M	≤ 0	Ha	Rye-grass + corn constraint
15 PN	≥ 1.5	Ha	Permanent pasture
16 PT	≤ 0	Ha	Temporary pasture use
17 GR.LUZ	≤ 0	Ha	Grass + alfalfa pasture use
18 CHOU	≤ 0	qx	Kale for feed
19 BETTE	≤ 0	qx	Fodder beet for feed
20 E.MAIS	≤ 0	qx	Corn silage for feed
21 FOIN	≤ 0	qx	Hay for feed
22 E.HERB	≤ 0	qx	Grass silage for feed (20% dry matter)
23 PATURE	≤ 0	Fodder unit	Pasture equivalent (from April to July 1)
24 PATURE	≤ 0	Fodder unit	Pasture equivalent (from July 2 to October 15)
25 ORGE	≤ 0	qx	Barley for feed
26 GRAINS	≤ 0	I F	Cereals equivalent
27 LIT	≤ 0	qx	Straw for litter
28 GRAINE	≤ 0	I F	Grass seeds equivalent
		Hour/period	Requirement of Tractor Hours
29 T1	N	Hour/period	Period: 15/3-15/4
30 T2	N	Hour/period	Period: 16/4-8/6
31 T3	N	Hour/period	Period: 9/6-10/7
32 T4	N	Hour/period	Period: 11/7-20/8
33 T5	N	Hour/period	Period: 21/8-13/9
34 T6	N	Hour/period	Period: 14/9-15/12

Code	Constraints	Units	Definition
			Labor Constraints on Crop Rotation
35 L10	≤ 125.0	Hour/period	Period: 15/3-15/4
36 L20	≤ 315.0	Hour/period	Period: 16/4-8/6
37 L21	≤ 72.0	Hour/period	Period: 10/5-20/5
38 L22	≤ 50.0	Hour/period	Period: 21/5-31/5
39 L23	≤ 215.0	Hour/period	Period: 10/5-8/6
40 L30	≤ 245.0	Hour/period	Period: 9/6-10/7
41 L31	≤ 110.0	Hour/period	Period: 20/6-10/7
42 L40	≤ 250.0	Hour/period	Period: 11/7-20/8
43 L41	≤ 150.0	Hour/period	Period: 11/7-20/8
44 L50	≤ 180.0	Hour/period	Period: 21/8-13/9
45 L60	≤ 460.0	Hour/period	Period: 14/9-15/12
46 L61	≤ 180.0	Hour/period	Period: 14/9-12/10
47 L62	≤ 115.0	Hour/period	Period: 13/10-10/11
48 L63	≤ 355.0	Hour/period	Period: 14/9-10/11
49 L64	≤ 230.0	Hour/period	Period: 13/10-30/11
50 L65	≤ 260.0	Hour/period	Period: 13/10-15/12
51 PAILLE	≤ 0	Hour	Straw harvesting
52 L24	≤ 40.0	Hour/period	Period: 1/6-8/6
53 T.A1	≤ 258.0	Hour/period	Labor constraints on live-stock production period: 15/4-10/7
54 T.A2	≤ 289.0	Hour/period	Period: 11/7-15/10
55 T.A3	≤ 308.0	Hour/period	Period: 16/10-1/1
56 R.VL.T	≤ 0	One day ration	Dry cow ration. Period: 15/10-1/1
57 R.MA.E	≤ 0	One period ration	Fattening ration for Maine-Anjou cow
58 G.MA	≤ 0	Number	Maine-Anjou heifer calf
59 A.MA	≤ 0	Number	Maine-Anjou three years heifer
60 R.VL.T	≤ 0	A day ration	Dry cow ration. Period 1/1-1/4
			Normande or Frisian - 3,800 liters of milk (a) October calving
61 R.NA38	≤ 0	One period ration	Dairy cow feeding program (15/10-1/1)
62 R.NA.E	≤ 0	One period ration	Feeder cow ration (15/10-1/1)
63 G.NA38	≤ 0	Number	Heifer calf
64 A.NA38	≤ 0	Number	3 years heifer

Code	Constraints	Units	Definition
			(b) February calving
65 R.NP38	≤ 0	One period ration	Dairy cow feeding program (15/10-1/1)
66 G.NP38	≤ 0	Number	Heifer calf
67 A.NP38	≤ 0	Number	3 years heifer
			Normande or Frisian - 3,000 liters of milk
			(a) October calving
68 R.NA30	≤ 0	One period ration	Dairy cow feeding program (15/10-1/1)
			(b) February calving
69 R.NP30	≤ 0	One period ration	Dairy cow feeding program (15/10-1/1)
			Beef Feeding Constraints
70 R.MA25	≤ 0	One period ration	Maine-Anjou 25 months slaughter steer
			Normande or Frisian Born in February
71 R.NP36	≤ 0	One period ration	(a) 36 months slaughter steer
72 R.NP26	≤ 0	One period ration	(b) 26 months slaughter steer
73 A.MGRE	≤ 0	I F	Feeder steer purchase and use
74 V.LAIT	≤ 0	I F	Milk equivalent
75 V.MEAT	≤ 0	I F	Slaughter steer equivalent
76 V.MGRE	≤ 0	I F	Production and sale of feeder steer
77 E.V.MA	≤ 0	I F	Maine-Anjou calf equivalent
78 E.V.N	≤ 0	I F	Normande and Frisian calf equivalent
79 E.G.AM	≤ 0	I F	3 years heifer equivalent
80 CX.MA	≤ 0	%	Maximum of kale in 25 months Normande or Frisian steer feeding program

Code	Constraints	Units	Definition
81 CX.N36	≤ 0	%	Maximum of kale in 36 months Normande or Frisian steer feeding program
82 CX.N26	≤ 0	%	Maximum of kale in 26 months Normande or Frisian steer feeding program
83 TR2	≤ 0	I F	Second quarter working capital
84 TR3	≤ 0	I F	Third quarter working capital
85 TR4	≤ 0	I F	Fourth quarter working capital
87 TR1	≤ 0	I F	First quarter working capital
110 KC	≤ 0	I F	Working capital: starting period
87 K	≤ 0	I F	Investment capital
88 V.BAT	≤ 200	M^2	Present building facilities
89 ET.VL	≤ 0	Number	Dairy cow facilities
90 ET.JB	≤ 0	Number	Steer facilities
91 AUGE	≤ 0	Number	Feeding rack
92 PORC	≤ 0	Number	Hog facilities
93 TRUIE	≤ 0	Number	Saw facilities
94 F.ST.L	≤ 0	Number	Fixed cost of purchased dairy cow facilities
95 F.PORC	≤ 0	Number	Fixed cost of purchased hog facilities
96 F.TR	≤ 0	Number	Fixed cost of purchased saw facilities
97 F.P.CH	≤ 0	Number	Fixed cost of purchased broiler facilities
98 F.P.P	≤ 0	Number	Fixed cost of purchased hen facilities (type I)
111 F.P.P	≤ 0	Number	Fixed cost of purchased hen facilities (type II)
99 M.K	≤ 0	I F	Maximum of capital borrowing
100 M.MO	≤ 0	One man year	Labor hiring
101 M.SAU	≤ 45	Ha	Land renting maximum
			Specialization Constraints
102 L.L	≤ 0	I F	(a) Milk production
103 G.L	≥ 0	I F	
104 L.V	≤ 0	I F	(b) Slaughter steer production
105 G.V	≥ 0	I F	
106 L.G	≤ 0	I F	(c) Cereals production
107 G.G	≥ 0	I F	

Code	Constraints	Units	Definition
108 L.AM	≤ 0	I F	(d) Feeder steer production
109 G.AM	≥ 0	I F	

APPENDIX C. MATHEMATICAL SYMBOLS

X,Y,S - Set

$\subseteq$ - Strict inclusion

$\cap$ - Intersection

$\forall$ - For all

$\emptyset$ - Empty set

a^+_{tj} - Positive balance

a^-_{tj} - Negative balance

a - (receipts-payments)

K - Initial amount of K

θ - Capital transfer from period (t) to period (t+1)

R - Multiple correlation coefficient

r - Simple correlation coefficient

0 - 10% level of significance

* - 5% level of significance

** - 1% level of significance

*** - .1% level of significance

R^2 - Coefficient of multiple determination