EXACT IMAGE RECONSTRUCTION IN CONE BEAM 3D CT

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INTRODUCTION

In cone beam CT, the measured x-ray data represent 1D line integrals through the 3D object. The object can be reconstructed via 3D inverse Radon transformation from the 2D planar integrals, or Radon transform, of the object if the latter can be constructed from the measured line integrals. However, because of the divergent nature of the cone beam x-rays, the 2D planar integrals cannot be constructed by simply summing up coplanar line integrals. This, in a nutshell, is the central problem in cone beam CT.

The algorithm developed by Feldkamp et al [1] is no doubt the most widely used reconstruction algorithm in cone beam CT. The algorithm is, however, an approximation which works well only at small cone beam angle. B.D. Smith [2] developed a method for computing from the cone beam data the 1-D convolution of the planar integrals with the Horn's kernel. Since the convolution mixes together the planar integrals on all the planes, the computation of one point of the convolved result requires all the data on the detector at one view angle. Thus the task is very computationally intensive. P. Grangeat [3] devised an algorithm to obtain the derivative of the planar integrals from the cone beam data. The computed data points, however, reside on a set of great circles on the Radon shell at each source position, which in general do not fall on the set of coaxial vertical planes in Figure 1a and hence are not suitable for input to inverse Radon transformation. It would require an extensive effort in three-dimensionalç interpolation to get the data on the vertical planes to be used in inverse Radon transformation, and furthermore interpolation would introduce errors to the data.

We have developed an exact algorithm to reconstruct a 3D object from its cone beam projection images. The image reconstruction proceeds in two steps: in the first step the cone beam projection data are converted to the Radon transform of the object, which is then inverse transformed in the second step to yield an image of the object via a fast $O(n^4)$ inverse Radon transformation algorithm. These 2 steps are illustrated in Figure 1.

3D INVERSE RADON TRANSFORMATION

The fast inverse Radon transformation (the second step) is made up of two 2D filtered backprojections operating in sequence, as illustrated in Figure 2. The first filtered backprojection takes as input the Radon transform of the object residing on a set of coaxial planes, and produces the 2D parallel beam projection images of the object on the same set of coaxial planes as output. The second filtered backprojection operates on the output images from the first to give the reconstructed image of the object on a set of planes parallel to the common axis of the coaxial planes.

Review of Progress in Quantitative Nondestructive Evaluation, Vol. 14
Edited by D.O. Thompson and D.E. Chimenti, Plenum Press, New York, 1995

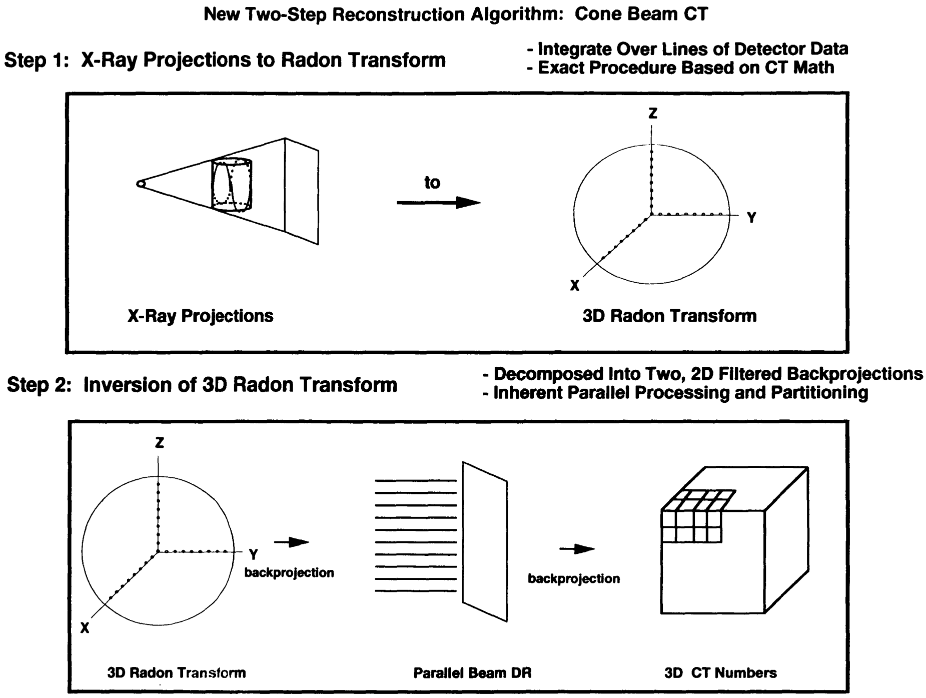

Figure 1. The two-step exact cone beam reconstruction algorithm.

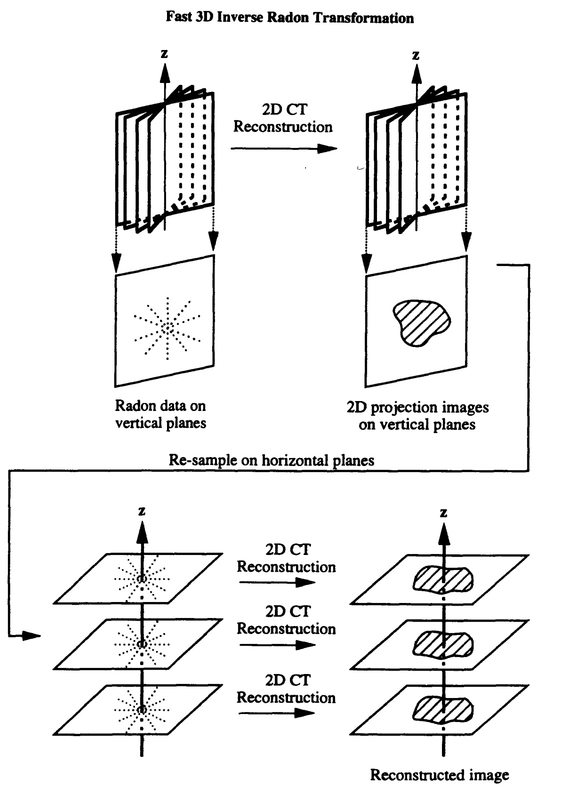

Figure 2. 3D Inverse radon transformation through 2D CT reconstructions.

Each 2D filtered backprojection involves N^3 computation. Thus the total amount of computation for all N planes (slices) is given by $N^3 \times N = N^4$.

RADON DATA FROM CONE BEAM DATA

In the first step of the reconstruction algorithm, the Radon transform data are generated directly on the set of coaxial planes required by the first filtered backprojection without going through 3D interpolation. The Radon data for any plane, $R(s,\widehat{n})$ (where in the object frame of reference, s is the distance of the plane Q from the origin and $\hat{n}$ is its normal), is given by:

$$R(s,\widehat{n}) = \iint f(r,\theta,s)\; r\, dr\, d\theta$$

where (r,θ,s) is a rotated cylindrical coordinate system with (r,θ) denoting the polar coordinates on the plane Q, and s denoting the distance of the plane from the origin in Radon space in the direction $\hat{n}$. The cone beam x-rays illuminating the object can be thought of as made up of an infinite set of planes of x-rays, with each of the planes

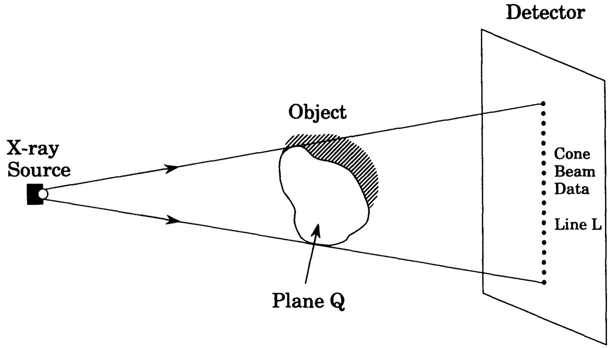

Figure 3. An integration plane intersecting the detector plane.

containing the x-ray source. Each of these planes intersects the detector plane in a line. The relevant geometry is illustrated in Figure 3. Drop a perpendicular from the source S to a line L where a plane Q intersects the detector plane, and let the 2 lines intersect at the point C. The datum X(t) on the line L, where t represents the displacement from the point C along L, is given by:

$$X(t) = \int f(r,\tan^{-1}\frac{t}{|SC|},s)\; dr \qquad (1)$$

Consider another plane Q' intersecting the source and the object, which is infinitesimally close to plane Q; the plane Q' is obtained by rotating the plane Q by a small angle $\delta\beta$ about an axis a' on the plane Q intersecting the source, as indicated in Figure 4. Define a weighted line integral J taken on the line L by the equation:

$$J = \iint \frac{f(r,\theta,s)\, dr\, d\theta}{\sin(\theta-\alpha)} \tag{2}$$

where α is the angle between SC and the a' axis. Using Equation (1) and coordination transformation from (r,θ) to t, one can express the desired integral J in the variable t as follows:

$$J = \int \frac{|SC'|\; X(t)}{(t-\Delta C)\sqrt{|SC|^2 + t^2}}\, dt \tag{3}$$

where C' is the point on the line L where the axis a' intersects, and ΔC is the displacement of C' from C. Let δJ be the change in the integral J in going from plane Q to plane Q'. By

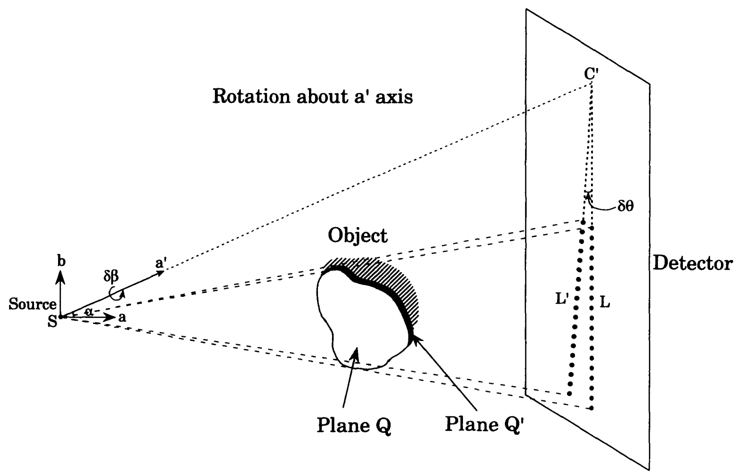

Figure 4. An infinitestimal rotation of the integration plane.

relating δJ to the displacement δr between origination and destination points on the 2 planes, it can be shown that the following equation holds:

$$\frac{dJ}{d\beta} = \lim_{\delta\beta \to 0} \frac{\delta J}{\delta\beta}$$

$$= \frac{\partial}{\partial s} \iint f(r,\theta,s)\, r\, dr\, d\theta$$

$$= \frac{\partial R(s,\hat{n})}{\partial s} \tag{4}$$

It can be shown that the rotation angle $\delta\beta$ is related to the angle $\delta\beta'$ between the lines L and L' on the detector through the equation:

$$\delta\beta = \cos\phi \frac{1+\tan^2\beta'}{1+\cos^2\phi\tan^2\beta'}\delta\beta' \tag{5}$$

where

β' = angle between the integration line L and the reference line M on the detector plane where the plane normal to SC' intersects

ϕ = angle between the normal plane and the detector plane

Using Equations (4) and (5) one can compute the radial derivative of the Radon data from the cone beam data, and the Radon data themselves can be obtained by integrating the result in the radial dimension. Consider a point P which resides on the Radon shell, the spherical shell with |SO| (O is the origin in Radon space) as diameter. Represent P as $s\hat{n}$ where s = |OP| and $\hat{n}$ = $\underline{OP}$/|OP|. To evaluate the Radon radial derivative at P, we do the following:

1. Determine the plane Q passing through the point P and orthogonal to the line OP.
2. Determine the line L where the plane Q intersects the detector plane.
3. Locate the point C on L such that the line SC is orthogonal to L.
4. Take any point C' on the line L, and rotate L through any small angle $\delta\theta$ about the point C', resulting in the line L'.
5. Compute the quantities J and J' on the lines L and L', respectively, using Equation (3).
6. Compute the angle of rotation $\delta\beta$ from $\delta\beta'$.
7. The Radon radial derivative at the point P is obtained from the quantities J, J', and $\delta\beta$ using the following equation:

$$\frac{\partial R(s,\hat{n})}{\partial s} = \frac{J' - J}{\delta\beta}$$

Using the above procedure we can obtain the Radon data for all the planes intersecting the object irradiated by the cone beam source.

COAXIAL PLANES IN RADON SPACE

Figure 5a illustrates the set of coaxial vertical planes Q_i containing the z axis as the common axis. It is desirable to generate Radon data directly on each vertical plane Q_i for input to Radon inversion, otherwise the Radon data on these vertical planes would have to be obtained by 3D interpolation from those generated anywhere in the Radon space, and thereby incurring errors in the interpolation process. Figure 5b shows a general cone beam scanning situation, which depicts the source S, the origin O, and the Radon shell generated at that source position; the combined geometry is shown in Figure 5c. Let the horizontal plane through the source S intersect the Radon shell in the circle G. It can be shown that each vertical plane Q_i intersects the Radon shell in a circle D_i, where D_i passes through the origin O, with the line segment OP_i as a diameter where P_i is the point on the circle G where D_i intersects. For illustration refer to Figure 6. Radon data can be generated on each circle D_i by the following operations:

1. Project the intersection point P_i on G onto the point C_i on the detector plane.

2. Construct lines on the detector plane at all orientations passing through the point C_i.

3. Compute the quantity J with the weighting function $1/\sin\theta$ on each of the lines.

4. Compute the derivative $dJ/d\beta$.

As the x-ray source moves on the scan path, the circle D_i varies in size and location throughout the plane Q_i, and consequently Radon data are generated throughout the plane. If the scan path is complete, each plane Q_i will be completely filled with Radon data. Thus Radon data can be generated directly on each plane Q_i, without the need for 3D interpolation.

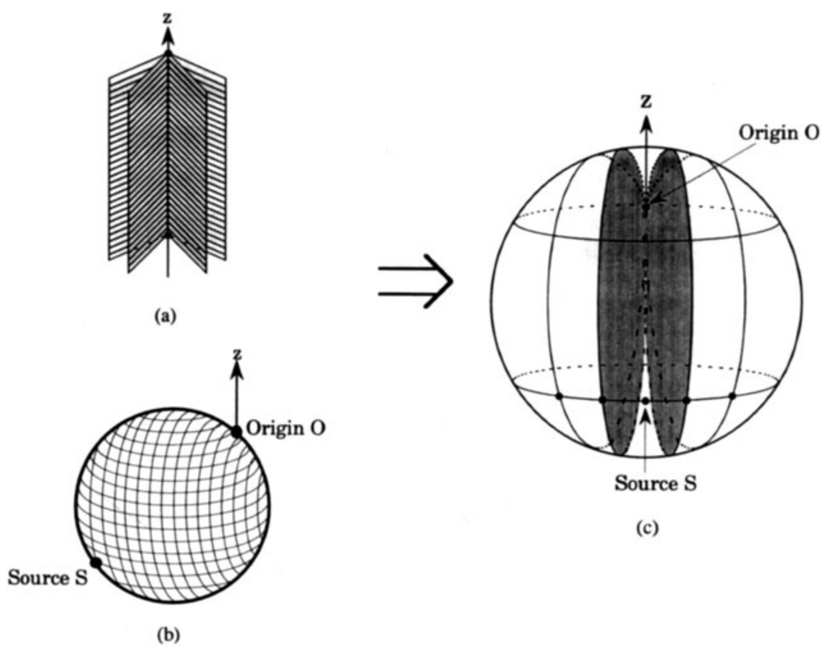

Figure 5. Locations of Radon data on the coaxial planes in Radon space.

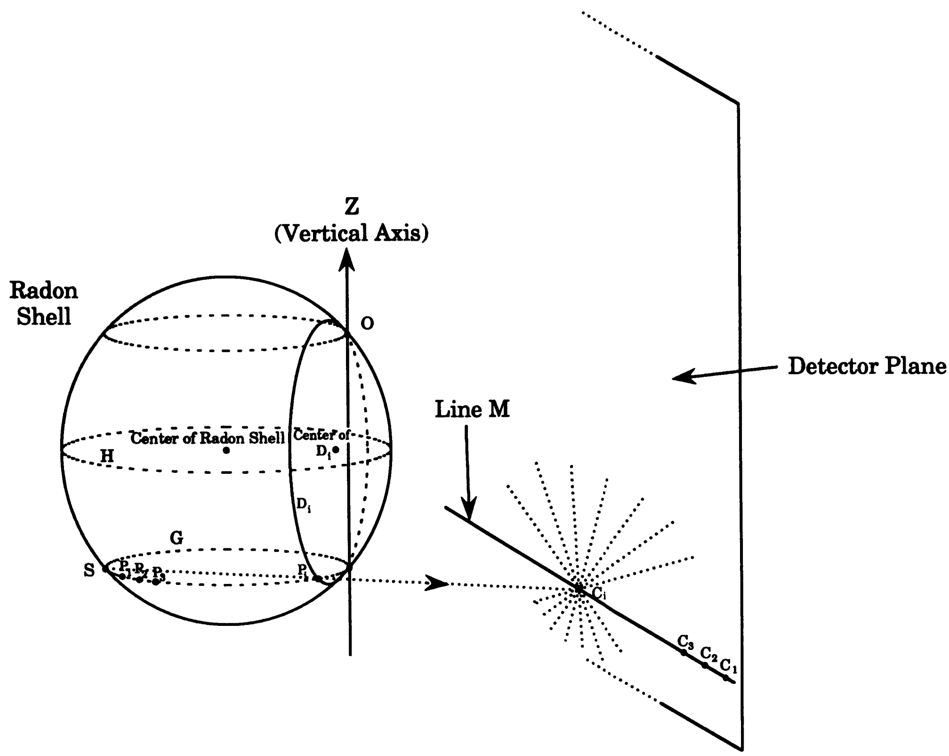

Figure 6. Generating Radon data on the coaxial planes in Radon space.

COMPUTATION ESTIMATION

At each view angle the generation of the datum at one point in the Radon space requires computations on a line of data on the detector plane, which contains ≈ N data points. Therefore to generate the data on a circle on the Radon shell requires ≈ $N \times N = N^2$ computations. The number of computations required to generate data on N circles covering the Radon shell is therefore equal to ≈ $N \times N^2 = N^3$. Finally, the total amount of computation at all N view angles is given by $N^3 \times N = N^4$.

A faster way to arrive at the same estimate is the following: as metioned above, to generate the datum at one point in the Radon space requires N computations. Since there are ≈ N^3 points in the Radon space, the total amount of computations is therefore equal to ≈ $N^3 \times N = N^4$.

CONCLUSION

We have developed an cone beam image reconstruction algorithm which is exact at all cone beam angles. Each step in the algorithm is well suited for parallel processing. Since all the cone beam data are synthesized and then re-distributed to each voxel through the Radon transformation and inverse transformation, the algorithm is not sensitive to ring artifact, which is a serious problem for filtered backprojection based algorithms. The algorithm has been validated on both simulated as well as experimental data, and the results are reported in the paper in this proceeding.

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