Debt- Versus Equity-Financing in Auction Designs

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Abstract

A social planner wishes to launch a project but the contenders capable of running the project are cash-constrained and may default. To signal their capabilities, the contenders may finance their bids through debt or equity, depending on the mechanism chosen by the social planner. When moral hazard is absent, it is established as theorems that the ex post efficient social choice function cannot be achieved by any mechanism using only debt financing and can be achieved by a mechanism using equity financing. When moral hazard is present, however, it is illustrated heuristically that equity share discourages effort and exacerbates default more than risky debt does.

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1 Introduction

Social planners hoping to launch high-tech or economy-rescuing projects often find themselves in the following predicament. The contenders that are technically capable of carrying out the project are financially constrained. In the case of high-tech projects such as nuclear power plants or high-speed rails, the investment necessary to start a project is so large that it dwarfs the liquid assets of the contenders.[1] In the case of economy-rescuing projects such as the ill-fated toxic asset auction plan of the United States Treasury Department [5], the contenders are themselves financially constrained. Financially constrained, a contender may be unable to signal, through bids or initial investments, the social surplus it could produce should it run the project. Then the social planner would have trouble finding the most capable contender to run it. To cope with this problem, social planners often offer the contenders some financial subsidies such as loan or equity guarantees. However, some of such financial subsidies may give the winner an incentive to default, thereby failing to complete the project even when the social benefit from completing it outweighs its cost. The question is How to choose a contender to run the project and, if necessary, how to finance the chosen contender? Is equity financing better than debt financing? Or are they equivalent?

The above situation may be mapped into a theoretical problem of designing a (socially) efficient mechanism to select among several contenders a winner to run a project, given that the contenders are more or less financially constrained. That leads to the agenda of efficient auction design given budget-constrained bidders. The question is Can efficiency be achieved if the contenders may finance payments exceeding their liquid assets?[2]

Che and Gale [2] have developed an auction model where the winner may finance its bid at a cost. They used the model to study the relationship between budget constraints and revenues. To be applied to our issue of the possibility of social efficiency, however, the model needs to incorporate the possibility that a winner defaults. Had there been no default risk, a firm could have borrowed any amount it needs with a promise to return fully so budget

[1] For example, according to a policy brief issued by the Nuclear Energy Institute [4], to set up new nuclear power plants, the electric power industry needs to invest between \$1.5 trillion and \$2 trillion in new generating capacity and infrastructure, while the largest of the U.S. electronic companies has a value of approximately \$36 billion and most of the rest is much smaller.

[2] The case where contenders' payments can never exceed their budget constraints have been considered by Maskin [3] and Pai and Vohra [6]. Pai and Vohra use that model to consider revenue-maximizing mechanisms.

constraint would be a non-issue. Furthermore, various means to finance bids may change the likelihood of default. Default also causes external costs such as delaying the project that policy makers are anxious to launch.

The effect of default has been considered by Waehrer [8] in a model where bidders have no budget constraint. With a model combining the features of Che and Gale and Waehrer, Zheng [9] demonstrates that, depending on whether loans available to bidders are sufficiently cheap or not, a first-price auction is won either by the poorest bidder who defaults most likely or by a rich bidder who defaults least likely. In that paper, bidders are assumed to have the same valuation of the item being auctioned. Removing this assumption, Board [1] considers a model where bidders are different in both valuations and wealth and he analyzes effects of default risks on first- and second-price auctions. Incorporating a capital market into an auction model similar in spirit to those of Board and Zheng, Rhodes-Kropf and Viswanathan [7] investigate the relationship between a first-price auction and various financing methods. They demonstrate that both debt and equity financing, as well as a more general state-contingent financing, can implement the efficient allocation with the proviso that the terms of the financing are determined based on the winner's bid and wealth.[3]

The efficiency result of Rhodes-Kropf and Viswanathan, while interesting, has its limitations. In their model, the external cost of default is never counted, hence default merely means that the winner transfers the realized return of the project to others without interrupting the project at all. Thus, their efficient mechanism is not really efficient when we take into account the possibility that default generates an external cost such as disrupting the project.[4] In terms of the possibility of implementing the efficient allocation, their efficiency result does not distinguish between equity and debt financing. In addition, they have made some restrictive assumptions.[5]

[3] In addition, Rhodes-Kropf and Viswanathan also consider the case of "pre-auction financing," where the terms of the financing are independent of the winner's bid. Those considerations are based on the assumption that the winner's valuation is known to the capital market.

[4] In their model, Rhodes-Kropf and Viswanathan assume that the value of the project is always positive. Thus, for their mechanism to be efficient, either default does not disrupt the project at all, or default should never occur (while default does occur in their equilibrium).

[5] Rhodes-Kropf and Viswanathan consider almost only the case where every bidder-type must bid above their wealth, an assumption that they needed to carry out the analysis with calculus. (See Zheng [9] for the subtleties when a bidder need not bid above its budget.) The only exception is their Proposition 9, which deals with the case where some bidder-types do not need to borrow, but its proof is unclear to me.

In this note, I shall consider the efficient mechanism design problem where bidders are different in their valuations, wealth, and limited liabilities. Default means the complete uselessness of the project, so default is socially desirable only if the the realized net return of the project is negative. Theorem 1 says that it is impossible to implement full efficiency via debt financing and Theorem 2 sayst that full efficiency can always be implemented via a modified Vickrey auction that offers a certain equity financing to the winner.

The idea of my efficiency result is that equity financing shrinks the range of a bidder's valuation down to the level that the bidder can afford, thereby removing its budget constraint. This idea cannot be applied to a usual auction environment where the object being auctioned is indivisible and cannot be shared. But in the context of a firm running a project, its ownership is typically divisible. In fact, this efficiency result sheds a new light on the prevalence of equity corporate ownership and the frequent phenomena that economies heavily in debt are prone to default crises.

In §6, the model is extended to incorporate the moral hazard problem of the winning contender. Then it is impossible to achieve full efficiency (Corollary 1). Like risky-debt financing, equity financing undermines the winning contender's incentive to exert efforts that could have improved the actual value of the project. With effort inefficiently exerted, default occurs more often.

2 The Primitives

A project is to be run by one of n contenders. At the beginning, each contender i privately knows a signal $\theta_i \in \Theta_i$, its wealth $w_i \in \mathbb{R}_+$, and its limited liability $\lambda_i \in \mathbb{R}_{++}$, so a bidder-type is $(\theta_i, w_i, \lambda_i)$. The value of the project depends on who runs it and a random variable $\mathbf{z}$. If the realized value of $\mathbf{z}$ is z and contender i completes the project, the ex post value of the project is equal to a real number $v_i(\theta_i, z)$. If the project is incomplete, its value is zero. Let $\underline{w}_i$ and $\underline{\lambda}_i$ denote the infimums of the supports for w_i and λ_i, respectively.

For example, in calculating the present value of a bidder's payment in the case where the bidder does not default, the authors did not include the bidder's payment in the event where the project results in low value (line -7, page 810). The argument for the case $c_i + L$ is less than the cash bid is also unclear.

To establish their efficiency result, Rhodes-Kropf and Viswanathan also assume that a bidder cannot lie about its available cash. They need this assumption to reduce the bidder-type into a one-dimensional variable so as to calculate the equilibrium.

Before any contender launches the project, the variable $\mathbf{z}$ remains uncertain and each contender i may receive a loan $\delta_i \in \mathbb{R}_+$ and may deliver an *interim payment* p_i. The interim payment needs to satisfy its budget constraint

$$p_i \leq w_i + \delta_i. \tag{1}$$

If a contender i promises an interim payment that violates its budget constraint, the payment cannot be delivered and i gets zero payoff for the entire game.

If a contender i receives a loan, i.e., $\delta_i > 0$, i promises to pay Δ later or pays a *default penalty* ψ_i if i defaults on the debt. Including interest payment, Δ needs to satisfy

$$\Delta_i \geq \delta_i. \tag{2}$$

The default penalty needs to satisfy the limited liability constraint

$$\psi_i \leq \lambda_i. \tag{3}$$

Any contender i who borrows a loan is subject to an audit so that if it misrepresents its limited liability as a higher number than λ_i then it receives no loan at all. Hence (3) can always be satisfied.

If a contender i is selected as the winner, i signs a contract specified by an *equity share* $\alpha_i \in (0, 1]$ which results from the selection and negotiation process. If it completes the project, the winner receives a fraction α_i of the ex post value of the project.

The winner i launches the project and discovers the realized value z. Then i chooses whether to default or not. If it does not default, i completes the project, receives its share $\alpha_i v_i(\theta_i, z)$ of the value of the project, and pays off its debt Δ_i. If i defaults, the project is incomplete (hence of zero value to the society), i pays the default penalty ψ_i and makes no other payment, thereby walking away from its debt Δ_i.

A *social choice function* (SCF) is a mapping that associates to each bidder-type profile $(\theta_i, w_i, \lambda_i)_{i=1}^n$ two objects: (i) a lottery that selects a winner out of the contenders and (ii) a default decision rule that maps any possible realized value z of the random variable to either default or not default.

An SCF is *efficient* if and only if, for any bidder-type profile $(\theta_i, w_i, \lambda_i)_{i=1}^n$,

a. the project is won by the contender i_* that maximizes the expected (social) value

$$\mathbb{E}\left[\max\{v_i(\theta_i, \mathbf{z}), 0\}\right]$$

of the project across all contenders i with random variable $\mathbf{z}$, and

b. if contender i wins, for any realized value z of $\mathbf{z}$, default occurs if and only if $v_i(\theta_i, z) < 0$.

3 Mechanisms

Clearly the revelation principle holds in this environment. Let us therefore restrict attention to direct revelation mechanisms. Thus, a mechanism is an SCF, defined previously, coupled with a payment and financing scheme. To each bidder-type profile $(\theta_j, w_j, \lambda_j)_{j=1}^n$, this scheme associates a lottery that determines every contender i's interim payment p_i, debt and its payment scheme (δ_i, Δ_i), default penalty ψ_i, and, in the event that i is the winner, i's share α_i of the value of the project. For the scheme to be feasible, every realized vector $(p_i, \delta_i, \Delta_i, \psi_i)$ needs to satisfy the financial constraints (1)–(3).

Let us arbitrarily pick any mechanism and focus our attention to it. For any bidder-type profile $(\theta_j, w_j, \lambda_j)_{j=1}^n$ and any realized z, denote the following aspects of the mechanism:

- $q_i((\theta_j, w_j, \lambda_j)_{j=1}^n) :=$ the probability with which contender i wins
- $\alpha_i((\theta_j, w_j, \lambda_j)_{j=1}^n) := i$'s share of the value of the project if i is the winner
- $p_i((\theta_j, w_j, \lambda_j)_{j=1}^n) := i$'s interim payment
- $\delta_i((\theta_j, w_j, \lambda_j)_{j=1}^n) :=$ the loan provided to i
- $\Delta_i((\theta_j, w_j, \lambda_j)_{j=1}^n) :=$ the principal and interest payments by i if i does not default
- $\psi_i((\theta_j, w_j, \lambda_j)_{j=1}^n, z) :=$ the default penalty for i when the realized value of $\mathbf{z}$ is z.

The financial constraints (1)–(3) mean that for every $(\theta_j, w_j, \lambda_j)_{j=1}^n$

$$p_i((\theta_j, w_j, \lambda_j)_{j=1}^n) \leq w_i + \delta_i((\theta_j, w_j, \lambda_j)_{j=1}^n), \tag{4}$$

$$\delta_i((\theta_j, w_j, \lambda_j)_{j=1}^n) \leq \Delta_i((\theta_j, w_j, \lambda_j)_{j=1}^n), \tag{5}$$

$$\psi_i((\theta_j, w_j, \lambda_j)_{j=1}^n, z) \leq \lambda_i. \tag{6}$$

If $\alpha_i((\theta_j, w_j, \lambda_j)_{j=1}^n) = 1$ for any contender i and any bidder-type profile $(\theta_j, w_j, \lambda_j)_{j=1}^n$, the mechanism is using only *debt-financing*. If $\Delta_i((\theta_j, w_j, \lambda_j)_{j=1}^n) > \psi_i((\theta_j, w_j, \lambda_j)_{j=1}^n, z)$

for some contender i and a positive-measure set of bidder-type profiles $(\theta_j, w_j, \lambda_j)_{j=1}^n$, the mechanism is engaged in *risky-debt financing*, with

$$\Delta_i((\theta_j, w_j, \lambda_j)_{j=1}^n) - \psi_i((\theta_j, w_j, \lambda_j)_{j=1}^n, z)$$

being the quantity of the risky debt, as a borrower i may want to default on a debt which exceeds the default penalty. If $\Delta_i((\theta_j, w_j, \lambda_j)_{j=1}^n) = \psi_i((\theta_j, w_j, \lambda_j)_{j=1}^n, z)$ for any i, any $(\theta_j, w_j, \lambda_j)_{j=1}^n$ and any z, the mechanism is using *equity-financing*, with the debt risk-free.

Let $(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)$ denote the type reported by contender i, who expects others to be truthful. If i is the winner and if the realized value of $\mathbf{z}$ is z, i's default decision is clearly to default if and only if

$$\alpha_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_{-i}, w_{-i}, \lambda_{-i})v_i(\theta_i, z) - \Delta_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_{-i}, w_{-i}, \lambda_{-i}) < -\psi_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_{-i}, w_{-i}, \lambda_{-i}; z). \quad (7)$$

Thus, if a contender i is the winner, its ex post payoff is equal to

$$\begin{aligned} & u_i^W(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i; \theta_i; \theta_{-i}, w_{-i}, \lambda_{-i}; z) \\ := \ & \max \left\{ \begin{array}{c} \alpha_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_{-i}, w_{-i}, \lambda_{-i})v_i(\theta_i, z) - \Delta_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_{-i}, w_{-i}, \lambda_{-i}), \\ -\psi_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_{-i}, w_{-i}, \lambda_{-i}; z) \end{array} \right\}. \end{aligned} \quad (8)$$

Thus, during the selection process, for any contender i with type $(\theta_i, w_i, \lambda_i)$, i's expected payoff $u_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i; \theta_i, w_i, \lambda_i)$ from reporting $(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)$ is equal to

$$\begin{aligned} & u_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i; \theta_i, w_i, \lambda_i) \\ = \ & \mathbb{E}\left[q_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_{-i}, w_{-i}, \lambda_{-i})u_i^W(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i; \theta_i; \theta_{-i}, w_{-i}, \lambda_{-i}; \mathbf{z}) - p_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_{-i}, w_{-i}, \lambda_{-i})\right] \end{aligned}$$

if the budget and liability constraints (4)–(6) are satisfied and $u_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i; \theta_i, w_i, \lambda_i) = 0$ if (4)–(6) are not satisfied. The incentive compatibility condition is that for each contender i and for any type $(\theta_i, w_i, \lambda_i)$,

$$u_i(\theta_i, w_i, \lambda_i; \theta_i, w_i, \lambda_i) = \max_{(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)} u_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_i, w_i, \lambda_i).$$

Coupled with the participation constraint

$$u_i(\theta_i, w_i, \lambda_i; \theta_i, w_i, \lambda_i) \geq 0, \quad (9)$$

the incentive compatibility condition becomes

$$u_i(\theta_i, w_i, \lambda_i; \theta_i, w_i, \lambda_i) = \max_{(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)} u_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \theta_i, w_i, \lambda_i) \quad (10)$$

$$\text{s.t.} \quad \left\{ \begin{array}{l} \hat{w}_i \leq w_i \\ \hat{\lambda}_i \leq \lambda_i \end{array} \right. \quad (11)$$

That is because over-reporting one's wealth or liability can never make the contender better-off than reporting them truthfully, as by our assumption a contender gets zero payoff if it violates its budget constraint or if it is found over-reporting its liability.

A mechanism is (ex post) efficient if and only if it implements the efficient SCF. In other words, an efficient mechanism satisfies the budget constraint (4), limited liability constraint (6), incentive compatibility (10) and participation constraint (9), and the SCF of the mechanism is efficient. By condition (b) of efficient SCF, in any efficient mechanism, for any contender i who becomes the winner, (7) holds if and only if $v_i(\theta_i, z) < 0$.

As we are discussing in the context of financially constraints, it is reasonable to assume that the social planner is itself subject to some budget constraint so that it cannot be overly generous in subsidizing the contenders. Specifically, we consider only those mechanisms that give zero surplus to any bidder-type which at the efficient SCF generates zero social surplus. I.e., with $U_i(\theta_i, w_i, \lambda_i)$ denoting the right-hand side of (10), for any contender i and any bidder-type $(\theta_i, w_i, \lambda_i)$,

$$\mathbb{E}\max\{v_i(\theta_i, \mathbf{z}), 0\} < \max_{j \neq i} \mathbb{E}\max\{v_j(\theta_j, \mathbf{z}), 0\} \text{ a.e. } \theta_{-i} \Longrightarrow U_i(\theta_i, w_i, \lambda_i) = 0. \tag{12}$$

The "if" clause of (12) means that a bidder with type $(\theta_i, w_i, \lambda_i)$ would never win the project if the efficient SCF is implemented. If the "then" clause of (12) is not satisfied, then such a bidder-type gets a positive surplus (nonnegative due to the participation constraint) even though it has no chance to contribute any social surplus. This positive surplus has to be subsidized by the social planner, because we assume that a contender has no other source of income. Clearly it is wasteful for the social planner to subsidize such a bidder-type at all. We may call (12) *no free lunch* constraint.

4 The Inefficiency of Debt Financing

A purely debt-financing mechanism means the share $\alpha_i((\theta_j, w_j, \lambda_j)_{j=1}^n) = 1$ for any contender i in case that i wins. With any winner owning the entire return of the project, debt is the only channel for a contender to make up the difference between its wealth and payment.

It is impossible for a purely debt-financing mechanism to achieve efficiency fully. That is because a debt larger than a winning contender's liability would induce the winner to default more often than the efficiency level, and a debt less than a contender's liability may constrain its ability to win even when it is the most capable contender to run the project.

To state the impossibility theorem, we define two notations:

$$\begin{aligned} \nu_i &:= \mathbb{E}\max\{v_i(\theta_i,\mathbf{z}),0\} \\ (\underline{\nu}_i,\overline{\nu}_i) &:= \left(\inf_{\theta_i\in\Theta_i}\mathbb{E}\max\{v_i(\theta_i,\mathbf{z}),0\}, \sup_{\theta_i\in\Theta_i}\mathbb{E}\max\{v_i(\theta_i,\mathbf{z}),0\}\right). \end{aligned} \tag{13}$$

As long as the default decision is aligned with efficiency, ν_i plays the role of contender i's private value before the winner launches the project. The theorem uses an assumption, (14) in the following, which says that the range of every contender's possible private values is sufficiently large compared to the contender's lowest possible wealth and liability.

Theorem 1 *Assume: for any contender i,*

$$\mathbb{E}\left[\left(\max_{j\neq i}\nu_j\right)\mathbf{1}_{\max_{j\neq i}\nu_j\leq\overline{\nu}_i}\right] > \underline{w}_i + \underline{\lambda}_i. \tag{14}$$

For any mechanism $(q_i,\alpha_i,p_i,\delta_i,\Delta_i,\psi_i)_{i=1}^n$, if $\alpha_i((\theta_j,w_j,\lambda_j)_{j=1}^n)=1$ for any contender i and any bidder-type profile $(\theta_j,w_j,\lambda_j)_{j=1}^n$, then the mechanism is not efficient.

Proof By definition of efficiency, we need only to show that there exist a positive-measure set of bidder-type profiles and a positive-measure set of realized values of $\mathbf{z}$ at which the equilibrium outcome of the mechanism is not efficient.

As efficiency requires that the default decision of any winner i be efficient, (7) holds if and only if $v_i(\theta_i,z)<0$. Since in this mechanism $\alpha_i((\theta_j,w_j,\lambda_j)_{j=1}^n)=1$ for any i and any bidder-type profile, the condition of efficient default becomes, for any $((\theta_j,w_j,\lambda_j)_{j=1}^n),z)$,

$$v_i(\theta_i,z)-\Delta_i((\theta_j,w_j,\lambda_j)_{j=1}^n))<\psi_i((\theta_j,w_j,\lambda_j)_{j=1}^n),z)\Leftrightarrow v_i(\theta_i,z)<0,$$

hence

$$\Delta_i((\theta_j,w_j,\lambda_j)_{j=1}^n))=\psi_i((\theta_j,w_j,\lambda_j)_{j=1}^n),z)\overset{(6)}{\leq}\lambda_i$$

for any contender i and any bidder-type profile $(\theta_j,w_j,\lambda_j)_{j=1}^n)$.

It follows from the budget constraint (4) and debt constraint (5) of the mechanism that every contender i's interim payment has to be always bounded from above by $w_i+\lambda_i$.

Now pick a contender i such that $\underline{\nu}_i\leq\underline{\nu}_j$ for all contenders j. If the mechanism achieves the efficient SCF, then the project is always won by the contender with the highest ν_{j_*} among all contenders j. Fix contender i's wealth and liability at $(\underline{w}_i,\underline{\lambda}_i)$ and follow the standard "envelope theorem" routine along the dimension of ν_i. The incentive compatibility of the efficient SCF implies that, when i's type is such that $\nu_i = \overline{\nu}_i$, the expected amount of i's interim payment is equal to

$$\mathbb{E}\left[\left(\max_{j\neq i}\nu_j\right)\mathbf{1}_{\max_{j\neq i}\nu_j\leq\overline{\nu}_i}\right] - U_i(\underline{\theta}_i, \underline{w}_i, \underline{\lambda}_i),$$

where $\underline{\theta}_i$ is an arbitrarily chosen element of $\{\theta_i \in \Theta_i : \mathbb{E}v_i(\theta_i, \mathbf{z}) = \underline{\nu}_i\}$. By the budget constraint derived above, we have

$$\mathbb{E}\left[\left(\max_{j\neq i}\nu_j\right)\mathbf{1}_{\max_{j\neq i}\nu_j\leq\overline{\nu}_i}\right] - U_i(\underline{\theta}_i, \underline{w}_i, \underline{\lambda}_i) \leq \underline{w}_i + \underline{\lambda}_i.$$

By the choice of i and $\underline{\theta}_i$, $\nu_i < \max_{j\neq i}\nu_j$ for sure, hence condition (12) implies that $U_i(\underline{\theta}_i, \underline{w}_i, \underline{\lambda}_i) = 0$. Thus, $\mathbb{E}\left[(\max_{j\neq i}\nu_j)\,\mathbf{1}_{\max_{j\neq i}\nu_j\leq\overline{\nu}_i}\right] \leq w_i + \lambda_i$, which contradicts (14). Hence the incentive compatibility condition for contender i is violated when its type is $(\theta_i, \underline{w}_i, \underline{\lambda}_i)$ such that $\nu_i = \overline{\nu}_i$. By continuity of the surplus function (which would follow from incentive compatibility), this violation holds for a positive-measure set of i's types. ■

5 Implementing Efficiency through Equity Financing

By contrast, efficiency can be implemented by a mechanism with equity financing. The idea is to choose the winner based on who can bring the highest expected value to the project and to give the winner a sufficiently small equity share thereby shrinking the range of a contender's possible values down to what its budget constraint allows. A contender's incentive of being truthful about its valuation of the project is not affected by the equity share, as the share is designed to be independent of the report of its valuation. And a contender has no incentive to under-report its wealth and liability, because the lower the sum of the two, the smaller its equity share in the event that the contender wins.

Theorem 2 *Assume for every contender i, $\overline{\nu}_i < \infty$, $\underline{\nu}_i$ is a constant identical across all i, and the prior distribution has zero mass at $\underline{\nu}_i$. There exists a mechanism, using equity financing, that implements the efficient SCF. In this mechanism, given the profile $(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)_{i=1}^n$ of reported bidder-types,*

a. the winner belongs to $\arg\max_{i=1,\ldots,n}\mathbb{E}v_i(\hat{\theta}_i, \mathbf{z})$,

b. if contender i is the winner then i shares

$$\alpha_i(\hat{w}_i, \hat{\lambda}_i) := \frac{\hat{w}_i + \hat{\lambda}_i}{\overline{\nu}_i} \tag{15}$$

of the ex post value of the project if it is completed, and

c. *if contender i is not the winner, i's interim payment is zero; if i is the winner, i's interim payment is equal to*

$$p_i\left(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \hat{\theta}_{-i}, \hat{w}_{-i}, \hat{\lambda}_{-i}\right) := \alpha_i(\hat{w}_i, \hat{\lambda}_i) \max_{j \neq i} \mathbb{E} v_j(\hat{\theta}_j, \mathbf{z}), \tag{16}$$

with default penalty, for any possible realized value z,

$$\begin{aligned} \psi_i((\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)_{i=1}^n, z) &:= \Delta_i((\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)_{i=1}^n) \\ &:= \delta_i((\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)_{i=1}^n) \\ &:= \max\{p_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)_{i=1}^n) - \hat{w}_i, 0\}. \end{aligned} \tag{17}$$

Proof If the mechanism described above is incentive compatible, then any winner's default decision is aligned with the efficient SCF. That is because, by (17), the winner i's optimal decision is to default if and only if (7) holds. With the share $\alpha_i(w_i, \lambda_i)$ positive, this condition becomes $\alpha_i(w_i, \lambda_i)v_i(\theta_i, z) < 0$, i.e., the efficient default condition $v_i(\theta_i, z) < 0$.

Next, we verify that the mechanism satisfies the budget and limited liabilities constraints. By (16) and provision (a) of the theorem, for any contender i,

$$p_i\left(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \hat{\theta}_{-i}, \hat{w}_{-i}, \hat{\lambda}_{-i}\right) = \left(\alpha_i(\hat{w}_i, \hat{\lambda}_i) \max_{j \neq i} \hat{\nu}_j\right)\left(\mathbf{1}_{\max_{j\neq i} \hat{\nu}_j < \hat{\nu}_i}\right) \leq \frac{\hat{w}_i + \hat{\lambda}_i}{\overline{\nu}_i}\overline{\nu}_i = \hat{w}_i + \hat{\lambda}_i. \tag{18}$$

If $p_i\left(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i, \hat{\theta}_{-i}, \hat{w}_{-i}, \hat{\lambda}_{-i}\right) > \hat{w}_i$, then by (17) we have

$$\psi_i((\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)_{i=1}^n, z) = \delta_i((\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)_{i=1}^n) = p_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)_{i=1}^n) - \hat{w}_i \overset{(18)}{\leq} \hat{\lambda}_i,$$

where the second equality implies the budget constraint (4) and the last inequality implies the liability constraint (6).

Note that the mechanism satisfies the participation constraint (9), as every contender i can report its value to be the infimum $\underline{\nu}_i$ of the support thereby securing zero payoff.

Finally, we verify that incentive compatibility. Recall ν_i defined by (13). Define

$$\begin{aligned} q_i(\nu_i) &:= \text{Prob}\left\{\theta_{-i} : \max_{j \neq i} \nu_j < \nu_i\right\} \\ P_i(\nu_i) &:= \mathbb{E}\left[\frac{p_i(\theta_i, w_i, \lambda_i, \theta_{-i}, w_{-i}, \lambda_{-i})}{\alpha_i(w_i, \lambda_i)}\right] \\ &\overset{(16)}{=} \mathbb{E}\left[\left(\max_{j \neq i} \nu_j\right)\left(\mathbf{1}_{\max_{j\neq i} \nu_j < \nu_i}\right)\right]. \end{aligned}$$

Clearly q_i is a weakly increasing function and, through integration by parts,

$$q_i(\nu_i)\nu_i - P_i(\nu_i) = \int_{\underline{\nu}_i}^{\nu_i} q_i(\nu_i')d\nu_i'. \tag{19}$$

For any contender i and any bidder-type $(\theta_i, w_i, \lambda_i)$, with $\nu_i = \mathbb{E}\max\{v_i(\theta_i, \mathbf{z}), 0\}$, consider the deviation of misrepresenting i's type as another bidder-type $(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)$, with $\hat{\nu}_i = \mathbb{E}\max\{v_i(\hat{\theta}_i, \mathbf{z}), 0\}$. Without loss of generality, assume that the expected payoff from such misrepresentation is nonnegative (otherwise i could have reported $\underline{\nu}_i$), i.e.,

$$q_i(\hat{\nu}_i)\nu_i - P(\hat{\nu}_i) \geq 0. \tag{20}$$

Let us calculate the difference between contender i's expected payoff from truthtelling and that from reporting $(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)$, suppressing the subscript i:

$$\begin{aligned}
& \alpha(w,\lambda)\left(q(\nu)\nu - P(\nu)\right) - \alpha(\hat{w},\hat{\lambda})\left(q(\hat{\nu})\nu - P(\hat{\nu})\right) \\
\overset{(15)}{=} \quad & \frac{w+\lambda}{\overline{\nu}}\left(q(\nu)\nu - P(\nu)\right) - \frac{\hat{w}+\hat{\lambda}}{\overline{\nu}}\left(q(\hat{\nu})\nu - P(\hat{\nu})\right) \\
\overset{(11),(20)}{\geq} \quad & \frac{\hat{w}+\hat{\lambda}}{\overline{\nu}}\left(\left(q(\nu)\nu - P(\nu)\right) - \left(q(\hat{\nu})\nu - P(\hat{\nu})\right)\right) \\
= \quad & \frac{\hat{w}+\hat{\lambda}}{\overline{\nu}}\left(\left(q(\nu)\nu - P(\nu)\right) - \left(q(\hat{\nu})\hat{\nu} - P(\hat{\nu})\right) + q(\hat{\nu})\hat{\nu} - q(\hat{\nu})\nu\right) \\
\overset{(19)}{=} \quad & \frac{\hat{w}+\hat{\lambda}}{\overline{\nu}}\left(\int_{\hat{\nu}}^{\nu} q(\nu')d\nu' + q(\hat{\nu})(\hat{\nu} - \nu)\right) \\
= \quad & \frac{\hat{w}+\hat{\lambda}}{\overline{\nu}}\left(\int_{\hat{\nu}}^{\nu} (q(\nu') - q(\hat{\nu}))d\nu'\right) \\
\geq \quad & 0 \quad \text{by monotonicity of } q.
\end{aligned}$$

It follows that the contender cannot profit from misrepresenting its type. ■

The mechanism designed in Theorem 2 has an intuitive interpretation: Every contender i reports its wealth w_i and liability λ_i to the financial sector and then obtains—

i. an equity guarantee which promises, in the event that contender i wins the project, to pay $1 - (w_i + \lambda_i)/\overline{\nu}_i$ of the price for the license to run the project, in return for owning $1 - (w_i + \lambda_i)/\overline{\nu}_i$ of the actual profit of the project unless default occurs, and

ii. loan guarantee which promises, in the event that contender i wins the project, to lend contender i the amount by which i's interim payment $((w_i + \lambda_i)/\overline{\nu}_i$ of the price for the license of the project) exceeds i's wealth, with the amount of this lending being the default penalty in the event that i defaults on the loan. (A winner's interim payment is shrunk by the equity share $(w_i + \lambda_i)/\overline{\nu}_i$ down below $w_i + \lambda_i$, so this debt is risk-free: it never exceeds the winner's liability.)

Then the contenders compete for the right to run the project via a Vickrey auction.

Even without the loan guarantee provision, the efficient SCF can still be implemented. We just need to modify the share function (15) into $\alpha_i(w_i, \lambda_i) := w_i/\overline{\nu}_i$, completely not taking advantage of a contender's liability λ_i. By contrast, the equity guarantee provision is indispensable, as Theorem 1 has demonstrated.

6 The Case with Moral Hazard

A possible drawback of equity financing is the moral hazard problem in the case where the ex post value of the project depends upon the hidden effort of the winner who runs it. Let us extend our model to incorporate that case. In the sequel, assume that the ex post value of the project, if completed, is equal to

$$v_i(\theta_i, e, z)$$

if contender i with type θ_i runs it with an effort e, which is an element of an interval $E \subseteq \mathbb{R}$, and realized state z. Assume that the cost bourn by contender i due to effort e is equal to $c_i(e)$, also unobservable to others.

In the sequel, we make the following assumptions for any contender i.

a. For any $\theta_i \in \Theta_i$ and any effort level $e \in E$, $v_i(\theta_i, e, \cdot)$ is continuously differentiable with derivative $D_3 v_i(\theta_i, e, \cdot) > 0$ over the range of z, and $v_i(\theta_i, e, \underline{z}) < 0 < v_i(\theta_i, e, \overline{z})$.

b. c_i is continuously differentiable and is convex on $\mathbb{R}_{++}$.

c. For any $\theta_i \in \Theta_i$, and any realized state z, $v_i(\theta_i, \cdot, z)$ is continuously differentiable with derivative $D_2 v_i(\theta_i, \cdot, z) > 0$ overall the domain of e.

Let us focus on an arbitrarily chosen incentive feasible mechanism $(\alpha_i, q_i, p_i, \delta_i, \Delta_i, \psi_i)_{i=1}^n$.

First, consider a contender i's default decision in the event that it has won the project with a financing package $(\alpha_i, \delta_i, \Delta_i, \psi_i)$ (which may depend on the messages submitted by the contenders) and has exerted effort level e. Given any realized state z, i's payoff from not defaulting is equal to $\alpha_i v_i(\theta_i, e, z) - \Delta_i$ and its payoff from default is equal to $-\psi_i$. Thus, i's optimal default decision is to default if and only if the realized state z is below a cutoff z_i^*, where z_i^* is the solution for the equation

$$v_i(\theta_i, e, z_i^*) = \frac{\Delta_i - \psi_i}{\alpha_i}. \tag{21}$$

The existence and uniqueness of z_i^* are guaranteed by the above assumption (a) and the incentive feasibility of the mechanism under consideration.

Lemma 1 *The default cutoff z_i^* given by Eq. (21) is an increasing function of $\frac{\Delta_i - \psi_i}{\alpha_i}$. Furthermore, z_i^* is a differentiable function of α_i and*

$$\gamma_i := \Delta_i - \psi_i, \tag{22}$$

and

$$\frac{\partial}{\partial \alpha_i} z_i^* = -\frac{\gamma_i}{\alpha_i^2 D_3 v_i(\theta_i, e, z_i^*)} \tag{23}$$

$$\frac{\partial}{\partial \gamma_i} z_i^* = \frac{1}{\alpha_i D_3 v_i(\theta_i, e, z_i^*)}, \tag{24}$$

where D_3 denotes the partial derivative operator for the third argument of a function.

Proof The monotonicity claim follows directly from Assumption (a) of the function $v_i(\theta_i, e, \cdot)$. That assumption also says that the partial derivative of v_i with respect to z_i^* is always nonzero. Hence the implicit function theorem implies that z_i^* is locally a differentiable function of α_i and γ_i. Differentiating Eq. (21) with respect to z_i^* and α_i, we get (23); differentiating (21) with respect to z_i^* and γ_i, we obtain (24). ■

The amount $\frac{\gamma_i}{\alpha_i}$ is the quantity of risky debt per equity share. Lemma 1 implies that a larger per-equity-share risky debt results in a higher probability for the winner to default. Call this the *default-exacerbating effect*.

Second, consider contender i's decision on the effort level after it has won the project, given the financing package $(\alpha_i, \delta_i, \Delta_i, \psi_i)$, and before the state $\mathbf{z}$ is revealed.

$$\begin{aligned}
& \max_{e \in \mathbb{R}_{++}} \left\{-c_i(e) + \mathbb{E}\left[\max\left\{\alpha_i v_i(\theta_i, e, \mathbf{z}) - \Delta_i, -\psi_i\right\}\right]\right\} \\
\overset{(22)}{=} \quad & -\psi_i + \max_{e \in \mathbb{R}_{++}} \left\{-c_i(e) + \alpha_i \mathbb{E}\left[\max\left\{v_i(\theta_i, e, \mathbf{z}) - \frac{\gamma_i}{\alpha_i}, 0\right\}\right]\right\} \\
= \quad & -\psi_i + \max_{e \in \mathbb{R}_{++}} \left\{-c_i(e) + \alpha_i \int_{z_i^*}^{\overline{z}} \left(v_i(\theta_i, e, z) - \frac{\gamma_i}{\alpha_i}\right) d\mu(z)\right\}.
\end{aligned} \tag{25}$$

The first-order condition for problem (25) is

$$\frac{c_i'(e)}{\alpha_i} = \int_{z_i^*}^{\bar{z}} D_2 v_i(\theta_i, e, z) d\mu(z), \tag{26}$$

where Eq. (21) has been used. Assume that c_i'' is sufficiently larger than $\frac{\partial^2}{\partial e^2} v_i$ for the second-order condition to be satisfied. Solving the simultaneous equations (21) and (26), we obtain the unique optimal effort level e_i^* and optimal default cutoff z_i^* for contender i.

Plugging the solutions for z_i^* and e_i^* into (25), we derive the expression for contender i's optimal expected payoff from running the project given financing package (α_i, d_i, ψ_i):

$$-\psi_i \underbrace{-c_i(e_i^*) + \alpha_i \int_{z_i^*}^{\bar{z}} \left(v_i(\theta_i, e_i^*, z) - \frac{\gamma_i}{\alpha_i} \right) d\mu(z)}_{=: V_i(\theta_i, \alpha_i, \gamma_i, \psi_i)} = -\psi_i + V_i(\theta_i, \alpha_i, \gamma_i, \psi_i). \tag{27}$$

Thus, a contender i's expected payoff in the mechanism is equal to

$$u_i(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i; \theta_i) = \mathbb{E}_{\theta_{-i}, w_{-i}, \lambda_{-i}} \left[q_i V_i(\theta_i, \alpha_i, \gamma_i, \psi_i) - (q_i \psi_i + \min\{p_i, \hat{w}_i\}) \right], \tag{28}$$

where $q_i, \alpha_i, \gamma_i, \psi_i, p_i$ depend *only* on i's message $(\hat{\theta}_i, \hat{w}_i, \hat{\lambda}_i)$ and the other contenders' presumably truthful messages $(\theta_{-i}, w_{-i}, \lambda_{-i})$. The incentive feasibility condition for the mechanism is that $(\theta_i, w_i, \lambda_i)$ maximizes $u_i(\cdot, \cdot, \cdot, \theta_i)$ subject to the constraints that $\hat{w}_i \le w_i$ and $\hat{\lambda}_i \le \lambda_i$.

The expected payoff function in (28) differs from the usual quasilinear form in auction theory in that the part for payment, $q_i \psi_i + \min\{p_i, \hat{w}_i\}$, is bounded from above by $\lambda_i + w_i$ due to the budget and liability constraints (4)–(6). That is an obstacle to finding the contender with the highest V_i to be the winner of the project.

The expected value of the social welfare generated by the mechanism is equal to

$$\mathbb{E} \left[\sum_{i=1}^{n} q_i(\theta, w, \lambda) \int_{z_i^*}^{\bar{z}} v_i(\theta_i, e_i^*, z) d\mu(z) \right], \tag{29}$$

where z_i^* and e_i^* are given by Eqs. (21) and (26).

Achieving full efficiency, i.e., maximizing (29) without binding the constraints of incentive, budget and liability, would require achieving not only default efficiency and winner-selection efficiency, defined in §2, but also effort efficiency, i.e., any winner i's effort level e_i^{**} solves problem (25) with the restriction that $\alpha_i = 1$ and $\gamma_i = 0$ always.

Corollary 1 *In the extended model with moral hazard, denote*

$$\tilde{\nu}_i \ := \ \mathbb{E} \max\{v_i(\theta_i, e_i^{**}, \mathbf{z}), 0\} - c_i(e_i^{**})$$

and $\overline{\tilde{\nu}}_i$ for the supremum of the range of the random variable $\tilde{\nu}_i$. Assume: for any contender i,

$$\mathbb{E}\left[\left(\max_{j\neq i}\tilde{\nu}_j\right)\mathbf{1}_{\max_{j\neq i}\tilde{\nu}_j\leq\overline{\tilde{\nu}}_i}\right] > \underline{w}_i + \underline{\lambda}_i. \tag{30}$$

Then there does not exist a feasible mechanism that implements efficiency almost surely.

Proof As in the proof of Theorem 1, default efficiency implies that every contender i is subject to the hard budget constraint $w_i + \lambda_i$. By the requirement of effort efficiency, every i's equity share $\alpha_i = 1$ always. Then the same envelope theorem routine used in the proof of Theorem 1, with (14) replaced by (30) here, implies the impossibility claim. ■

The dilemma for the social planner is: on one hand, selection efficiency would require that the contenders' budget constraints be sufficiently relaxed via equity or risky debt financing; on the other hand, each of these instruments may compromise default or effort efficiency. Now that they are unavoidable to achieve selection efficiency, which financing instrument is less detrimental to default, effort, and winner-selection efficiency? Presented below are some heuristic comparative statics potentially useful for further analysis.

First we compare the effects of the two instruments in relaxing contenders' budget constraints. Inspecting (27)–(28), we see that both financing instruments are to shrink the range a contender's expected payoff $V_i(\theta_i, \alpha_i, d_i, \psi_i)$ from winning. With V_i shrunk, the interim payment a contender i needs to deliver in order to signal the value of V_i also shrinks thereby partially relaxing the budget constraint. To calculate the budget-relaxing effect, let us calculate the partial derivative $\frac{\partial}{\partial\theta_i}V_i$ by applying the envelope theorem to the maximization problem (25) and the definition of V_i in (27):

$$\frac{\partial}{\partial\theta_i}V_i(\theta_i, \alpha_i, \gamma_i, \psi_i) \quad = \quad \alpha_i \int_{z_i^*(\theta_i)}^{\overline{z}} D_1 v_i(\theta_i, e_i^*(\theta_i), z)d\mu(z),$$

where D_1 denotes the derivative operator of the function $v_i(\cdot, e_i, z)$ and $(z_i^*(\theta_i), e_i^*(\theta_i))$ signifies the dependence of (z_i^*, e_i^*) on θ_i via Eqs. (21) and (26). With the envelope theorem applied to the incentive compatibility of a contender whose expected payoff function is given by (28), the expected interim payment made by a type-$(\theta_i, w_i, \lambda_i)$ contender i is equal to

$$\alpha_i \int_{\underline{\theta}_i}^{\theta_i} \mathbb{E}_{\theta_{-i},w_{-i},\lambda_{-i}}\left[q_i(\theta_i', w_i, \lambda_i; \theta_{-i}, w_{-i}, \lambda_{-i})\int_{z_i^*(\theta_i')}^{\overline{z}} D_1 v_i(\theta_i', e_i^*(\theta_i'), z)d\mu(z)\right]d\theta_i'$$
$$-U_i(\underline{\theta}_i, w_i, \lambda_i). \tag{31}$$

With the assumption that v_i is strictly increasing in θ_i, $D_1 v_i(\theta_i', e_i^*(\theta_i'), z) > 0$ always, hence Lemma 1 implies that a global increase of $\frac{\gamma_i}{\alpha_i}$ (for all θ_i') reduces

$$\int_{z_i^*(\theta_i')}^{\overline{z}} D_1 v_i(\theta_i', e_i^*(\theta_i'), z) d\mu(z)$$

thereby reducing the expected interim payment (31) and hence partially relaxing i's budget constraint of $w_i + \lambda_i$.

Thus, both equity financing and risky-debt financing have the effect of partially relaxing the budget constraint through enlarging the quantity $\frac{\gamma_i}{\alpha_i}$ globally. However, equity financing has an additional effect on relaxing budget constraints because α_i is a coefficient of the entire integral in the expression (31) thereby scaling down the expected payment. Moreover, this effect of α_i does not require α_i to scale down globally.

Second, we compare the effects of the two financing instruments in default and effort. For the maximization problem (25), a second-order condition is

$$\frac{c_i''(e)}{\alpha_i} > \int_{z_i^*}^{\overline{z}_i} D_2^2 v_i(\theta_i, e, z) d\mu(z). \tag{32}$$

As long as (32) holds, the first-order condition (26) is sufficient and necessary, the winner i's optimal effort level e_i^* is the intersection between its marginal benefit $\int_{z_i^*}^{\overline{z}_i} D_2 v_i(\theta_i, e, z) d\mu(z)$ and its per-share marginal cost $c_i'(e)/\alpha_i$.

Lemma 2 *If (32) always holds, then any winner i's optimal effort e_i^* is a differentiable function of the equity share α_i and risky debt quantity γ_i, and*

$$\alpha_i \cdot \frac{\partial}{\partial \alpha_i} e_i^* + \gamma_i \cdot \frac{\partial}{\partial \gamma_i} e_i^* = \frac{c_i'(e_i^*)}{c_i''(e_i^*) - \alpha_i \int_{z_i^*}^{\overline{z}_i} D_2^2 v_i(\theta_i, e_i^*, z) d\mu(z)}. \tag{33}$$

Proof As (32) always holds, the first-order condition for the maximization problem (25) is sufficient and necessary, hence a winner i's optimal effort level e_i^* is equal to the solution for e in Eq. (26). By the implicit function theorem (whose nonsingularity condition follows from (32), e_i^* is locally a differentiable function of (α_i, γ_i), and we can obtain the partial derivatives by totally differentiating Eq. (26) and plugging in Eqs. (23)–(24).

$$\begin{aligned} \frac{\partial}{\partial \alpha_i} e_i^* &= \frac{(\gamma_i/\alpha_i)\mu'(z_i^*) D_2 v_i(\theta_i, e_i^*, z_i^*) \frac{\partial}{\partial \gamma_i} z_i^* + c_i'(e_i^*)/\alpha_i^2}{c_i''(e_i^*)/\alpha_i - \int_{z_i^*}^{\overline{z}_i} D_2^2 v_i(\theta_i, e_i^*, z) d\mu(z)}, & (34) \\ \frac{\partial}{\partial \gamma_i} e_i^* &= -\frac{\mu'(z_i^*) D_2 v_i(\theta_i, e_i^*, z_i^*) \frac{\partial}{\partial \gamma_i} z_i^*}{c_i''(e_i^*)/\alpha_i - \int_{z_i^*}^{\overline{z}_i} D_2^2 v_i(\theta_i, e_i^*, z) d\mu(z)}. & (35) \end{aligned}$$

Plugging (35) into (34), we obtain

$$\frac{\partial}{\partial \alpha_i} e_i^* = -\frac{\gamma_i}{\alpha_i} \cdot \frac{\partial}{\partial \gamma_i} e_i^* + \frac{c_i'(e_i^*)/\alpha_i}{c_i''(e_i^*) - \alpha_i \int_{z_i^*}^{\overline{z}_i} D_2^2 v_i(\theta_i, e_i^*, z) d\mu(z)}, \tag{36}$$

which simplifies to (36). ■

By (35), $\frac{\partial}{\partial \gamma_i} e_i^* < 0$. Thus, Eq. (36) implies that

$$\left|\frac{\partial}{\partial \alpha_i} e_i^*\right| = \frac{\gamma_i}{\alpha_i} \left|\frac{\partial}{\partial \gamma_i} e_i^*\right| + \frac{c_i'(e_i^*)/\alpha_i}{c_i''(e_i^*) - \alpha_i \int_{z_i^*}^{\overline{z}_i} D_2^2 v_i(\theta_i, e_i^*, z) d\mu(z)}.$$

Hence $\left|\frac{\partial}{\partial \alpha_i} e_i^*\right| > \left|\frac{\partial}{\partial \gamma_i} e_i^*\right|$ if $\gamma_i/\alpha_i \geq 1$.

Lemma 2 can be illustrated heuristically with Figure 1. In that figure, the curve MC depicts the left-hand side, and the curve MB the right-hand side, of Eq. (26) as a function of e.[6] The optimal effort level e_i^* is the intersection between the two curves. For a higher $\frac{\gamma_i}{\alpha_i}$, Lemma 1 implies that z_i^* is higher and then the right-hand side of Eq. (26) is smaller, meaning the curve MB shifts downward to the dashed curve MB'. For a smaller α_i, the left-hand side of Eq. (26) is scaled up, meaning that the curve MC pivots upward to the dashed MC'.

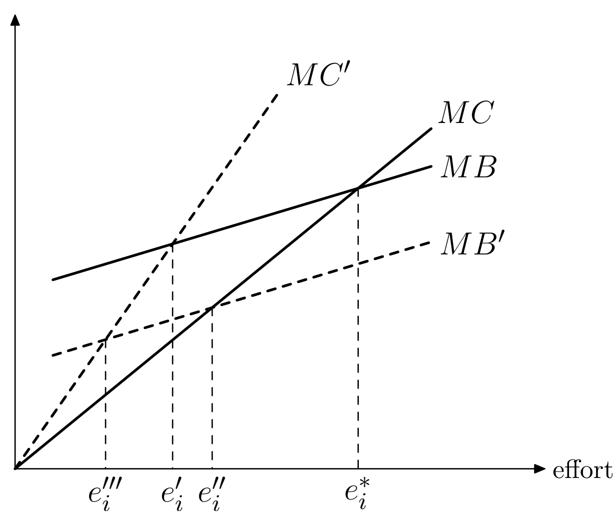

Figure 1: A larger quantity of risky debt shifts curve MB down to MB'. A smaller equity share shifts down MB and pivots curve MC up to MC'.

Thus, if the risky debt γ_i is larger while equity share α_i is unchanged, curve MB

[6] Note that the marginal benefit is not necessarily decreasing in the effort level unless the $D_2^2 v_i(\theta_i, e, z)$ is sufficiently negative. This observation uses the fact that z_i^* is decreasing in e and the assumption that $v_i(\theta_i, e, z)$ is increasing in e.

shifts down ward and the optimal effort decreases from e_i^* to e_i'' in the figure. Such default exacerbation is the only effect of risky debt financing on the winner's effort.

The effect of equity share, by contrast, is two-fold. First, it has the default-exacerbating effect as risky debt does, as a smaller α_i amounts to a bigger $\frac{\gamma_i}{\alpha_i}$ and hence curve MB shifts downward to MB'. Second, equity share has another effect that risky debt does not have. Scaling down α_i amounts to raising the per-share marginal cost $c_i'(e)/\alpha_i$ of one's effort, which means the curve MC pivots upward to MC'. Thus, a smaller equity share reduces the winner's optimal effort level from e_i^* to e_i''', more detrimental than increasing risky debt.

In sum, equity and risky-debt financing have similar effects on relaxing contenders' budget constraints, exacerbating default, and discouraging efforts. However, the local analysis presented above indicate that the effects of equity financing might be stronger than those of risky-debt financing, be it the positive effect or negative effect. If they are both applied globally for all types, equity financing relaxes budget constraints more than risky debt does. If the second-order condition (32) holds, equity financing exacerbates default and discourages efforts more than risky-debt financing does.

7 Discussion

This note is a beginning attempt to introduce finance into the theory of auction design. From a social planner's viewpoint, the role of auctions is essential in making financing an issue. If every contender is assigned a project to manage, then it is not an issue at all to select the most capable contender to run the project, so there is no need for them to signal their capabilities through bids, so it would be unnecessary to finance them.

The theorems reported in this paper demonstrate that, in the case where moral hazard is a non-issue for the winner of the project, equity and debt financing are distinct in their roles of achieving efficiency. While no mechanism relying only on debt financing can achieve efficiency fully, there exists a mechanism with equity financing that achieves efficiency fully. The default-prone nature of risky debt financing is the driving force of this distinction.

In the case where the winner of the project has a moral hazard problem, it is impossible to achieve efficiency fully. In this paper, we observe that, while equity and risky-debt financing both lead to less effort and more default, the two instruments differ in the magnitudes of their effects, with those of equity financing stronger. It remains an open question how the socially optimal mechanism looks like and whether there is an efficiency-ranking between debt and equity financing.

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Index