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DEVELOPMENT OF A COMPUTER-ASSISTED INSTRUCTION UNIT IN PROBABILITY

by

Donald Henry McClain

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I. INTRODUCTION

Americans are realizing today, as never before, the importance of education, both in its qualitative and in its quantitative aspects. This prominence is noticeable at all levels of the educational system, from kindergarten through the university, and is accompanied by a desire for equality of opportunity, allowing each individual to develop his potential to the maximum. As a consequence of this, our total educational effort is faced with a number of major challenges. Some of these, if properly realized, may have far-reaching implications for changing the character and nature of education and, in turn, society.

Radio Corporation of America (RCA) Instructional System personnel (64, p. 1) summarize some of these challenges as:

> Overcrowded classrooms . . . overloaded teachers . . . subject matter that grows in volume and complexity every year . . . the vitally important challenge of the disadvantaged, the under-motivated, the under-achiever.
>
> Disproportionate student/teacher ratios that force teachers to concentrate on class norms--yet, as every educator knows, the more individual teaching attention each student can be given, the closer he'll come to achieving his maximum learning potential.

As a consequence of this new sense of importance of education, E. N. Adams (1, p. 12) of the International Business Machines (IBM) Watson Research Center states:

> The educational profession finds itself under unprecedented pressure to broaden its responsibility and increase its productivity. The profession is being

> asked to train the mass of our young people to higher standards than ever before. It is being pressed to bring an end to conditions which presently permit a great many underachievers to drop out of the process of conventional education. It is being called on to devise and develop institutions and techniques for the re-education of adults whose education is either inadequate or obsolete.

Richard Lanham and his colleagues of System Development Corporation (SDC) (48, 83, 85) substantiate these views by discussing the demand for American education to meet an unprecedented variety of needs for an ever-growing number of people. Some of these needs are: to impart more knowledge in the same amount of time, to accommodate instruction to the widely varying learning rates and skills of individual students, to fulfill the long-overdue promise of equal education for the racially and economically disadvantaged, to expand adult-education programs--not only for recreational but for continuing- and re-educational purposes, and to rectify the inadequacies of our present educational system which result in the alienation and underachievement of many students.

The preceding problems and challenges for education are complex with many interrelationships. They are not meant to comprise an exhaustive list but are merely illustrations of shortcomings of our present educational system. They all, however, have common demands for excellence in the quality of education and for the preservation of the American ideal of education for the masses.

In this time of social crisis in our country, the

existence of such problems places a big burden upon education and educators for educational reforms aimed at realizing this excellence. There seems a note of urgency in these demands, and thus, some educators are looking to technology as one source of techniques and methods for improving the quality of educational programs. Technology is by no means a remedy for all the ills that afflict the educational process, but it does represent a significant advancement toward the solution of some of these problems.

The technological development which some educators, such as Suppes (77, 78, 81), Adams (1), Mitzel (52), and the personnel at SDC (43, 83) feel holds much promise for alleviating some of these educational challenges is the computer. The call for computer innovations in education is emphasized by John I. Goodlad (27, pp. 49-50), Dean, Graduate School of Education, University of California, Los Angeles, when he says:

> The era that is in full bloom and is about to fade is human-to-human instruction. The prime exhibit of this era is the human-based school--a school almost without machines. Here, we like to believe, children and youth are inducted into the culture, their individual potentialities are discovered and developed, they take on a sense of identity and ultimately transcend themselves, and they are inculcated in those values that make for the ideal adult. Increasingly, however, we have become aware that school is not accomplishing these things with a large segment of our population. Indeed, present-day education appears to increase the gap between the haves and the have-nots.
>
> Nonetheless, we are in an inventive period, and old ways of doing things are tumbling before our drive to increase the effectiveness of the school.

> We have not yet eliminated the track systems, with their self-fulfilling prophecies, nor have we broken down the grade barriers, with their nefarious adjustment mechanism of nonpromotion, nor have we learned to teach inductively, with the child learning for himself the skill of inquiry. But we have caught the spirit of these things.
>
> The challenge now is much less one of inventing than of implementing the several powerful and viable innovations that have appeared during recent years. . . . The era of instruction that will supercede the era of human-based instruction is to be one of man-machine interaction--and the machine is the computer.

It seems logical that computer technology, which has contributed so heavily to the knowledge explosion, the advent of automation, and the demand for new skills which has complicated the educator's task, should be used to help satisfy the needs of education.

A. Rationale for the Problem

Computer technology is having far-reaching effects in government and business, on the one hand, and in science and engineering, on the other; and it is beginning to have implications in education. Dr. Patrick Suppes (78) of Stanford University's Institute for Mathematical Studies predicts that if the potentials of the computer for education are properly realized, the character and nature of education during the course of our lifetimes will be radically changed.

Dr. Wayne H. Holtzman (40), Dean, College of Education, University of Texas at Austin, has outlined the following six uses which computers find in the field of education:

1. as an administrative tool for payroll, inventory, auditing, report generating, student-record keeping, student registration, class scheduling, etc.,
2. as a research tool for analyzing data and solving mathematical problems,
3. as a subject of research itself, such as software and hardware development,
4. as a subject of instruction in the computer science department,
5. as an instructional tool through computer-assisted instruction, simulation, problem solving, and a reference system,
6. as an information-storage and retrieval system in library use.

All such activities serve very necessary functions. However, possibly the real contribution of computers to education is in the instructional process, Item 5 in the above list. Early investigations into the instructional potential of the computer led Dr. Patrick Suppes (81, p. 207) to state:

> One can predict that in a few more years millions of schoolchildren will have access to what Philip of Macedon's son Alexander enjoyed as royal prerogative: the personal services of a tutor as well-informed and responsive as Aristotle.

Suppes (78, p. 420) clarifies his prediction when he says:

> Perhaps the most important aspect of computerized instructional devices is that the kind of individualized instruction once possible only for a few members of the aristocracy can be made available to

all students at all levels of abilities.

If these statements are correct, the dream for individualized instruction which has been prominent in American education for almost 50 years is closer to realization.

Using the computer as a new instructional tool, commonly called computer-assisted instruction (CAI) and involving an on-line instructional process in which the computer and student engage in discourse designed to promote student learning, is widely discussed and probably contains the greatest possibilities for individualizing instruction. The qualities of the computer which emphasize these possibilities include the fact it can store large quantities of information in its memory system, it can make elementary yes-no decisions guided by a previously written program, and it can perform operations with astonishing speed. Although mainly in the experimental stages, CAI's potential is so great that education can ill afford not to give it serious consideration, and one of the most-important tasks of present-day educators is to explore the means and methods of using the computer as an instructional tool.

B. Statement of the Problem

The problem of this study is to investigate the use of the computer as an instructional tool by developing a program for individualized instruction on the basic concepts of probability for a graduate educational statistics class.

This probability unit is subdivided into six concept blocks, each including related practice problems. These concept blocks are as follows: multiplication principle, permutations, combinations, binomial theorem, basic probability, and probabilities of combinations of events.

C. Purposes of Study

The purposes of this study are two-fold. The first is the development of a programmed unit in probability to be presented by the computer, and the second is the development of the necessary CAI software to permit the computer to be used as an instructional tool.

The objectives of the probability unit are:

1. to develop course objectives,
2. to develop the order of the content sequences according to the objectives,
3. to develop the CAI instructional strategy,
4. to write the course materials using features of the author language,
5. to develop individualized problem sets as a test of terminal behavior.

The objectives of the CAI software are:

1. to design the features of the author language,
2. to design an author language not requiring previous computer knowledge,
3. to implement the author language in the existing

time-sharing system, CPS, of the 360/65 computer,

4. to write and flow chart the necessary computer programs.

D. Outline of Chapters

The materials presented in this report are organized into seven chapters. The first chapter includes a brief introduction to the need for developmental and experimental research in educational technology, particularly in computer-assisted instruction. It also contains a statement of the problem of this study and its purposes. The second chapter consists of a review of the literature relating to the application of the computer to the instructional process. A general description of the computer-assisted instructional system, including the computer hardware, computer software, and educational software, is found in Chapter III. Chapter IV elaborates on the computer software by presenting a detailed explanation of the computer programs and their organizations. A further discussion of the educational software and a listing of the course objectives for the probability unit are contained in Chapter V. The sixth chapter is devoted to discussion, observations, and recommendations of this study, and a summary of the project is presented in Chapter VII. Computer programs, flow charts, and listings of the probability concept blocks are included in the appendixes.

II. REVIEW OF LITERATURE

One of the biggest stumbling blocks in the educational process has been our failure in the past to adapt the instructional process to individual differences of learners. An educational psychologist, John DeCecco (17), stated that in the attempt to provide for individual differences in the classroom, teachers have sought ways to organize and instruct students using a variety of grouping patterns which have included team teaching, flexible scheduling, nongraded instruction, and continuous-progress models. Educators have also teamed with psychologists and scientists to develop new instructional materials, particularly in the subject-matter areas of modern mathematics and science. In addition, teaching kits, programmed workbooks, closed-circuit television, and teaching machines have been additional innovations aimed at meeting individual educational needs. Even the collection of uniform boxes (rooms) of the conventional schoolhouse has not been safe from innovation. New schools are designed to enclose large, open spaces. By utilizing movable partitions, these open spaces can be divided into smaller areas more responsive to the particular instructional needs.

All of these innovations are attempts to add flexibility to the educational process so that it can be more responsive to the large differences found among students in any given grade level. Although all of these practices add degrees of

flexibility to the educational process, none of these, nor any combination of them, has yet demonstrated that it can fulfill the educator's dream of individualized instruction. However, Dr. Patrick Suppes (80), a leader in the field of CAI, feels the present day, general-purpose computers have great potential for making the goal of individualized instruction a closer reality. Suppes (78, p. 423) further declares:

> Just as books freed serious students from the tyranny of overly simple methods of oral recitation, so computers can free students from the drudgery of doing exactly similar tasks unadjusted and untailored to their individual needs.

The following sections of this chapter describe some of the possibilities and realities that the computer holds for individualizing the instructional process.

A. Potentials of CAI

Computer-assisted instruction is viewed by Robert Glaser (25) of the University of Pittsburgh as a type of learning experience in which the computer places the student in direct contact with the subject matter. The computer allows the student to explore, make mistakes, and find his own unique path to learning within a highly structured, well-prepared environment. It guides the student through a set of specified behaviors that are designed and sequenced to make it more probable he will react in a given, desired manner in the future.

Robert Bundy (11) of Syracuse University describes CAI as

a technology which seeks to provide an individualized learning environment for each student. Instructional programs are stored in the computer, and the student interacts with these programs by means of electronic interface devices. The instructional programs may include a broad range of subject-matter content and extend across a spectrum, from simple drill-and-practice routines to complex tutorial-type learning experiences.

Adams (1, p. 12) lists the following services a CAI system, when properly programmed, can provide for a student as:

1. Engage in two-way communication with a student by means of natural language messages.

2. Guide the student through a program of tasks, helping him where he has difficulty, and accelerating his progress where he finds little challenge.

3. Observe and record significant details of the student's behavior, including steps undertaken in performing tasks, time taken for particular steps, and values of varying physiological or environmental quantities.

4. Simulate the operation of a physical, mathematical, or social process responding to variations in parameters.

5. Analyse and summarize performance records and other behavioral records of individual students and also groups of students.

SDC personnel (85, p. 3) represent the potential of CAI as follows:

> CAI is ideally adapted to using programmed instruction to maximum advantage. The teaching sequence can proceed in carefully preplanned steps that the student can easily grasp; active response can be

> required of the student at every key step; the student can know at all times how well he is doing; if he is not doing well he can be given remedial material according to his individual needs; and the student who is able to progress faster than his peers can be allowed to pass over the amount of detail permitted by his grasp of the subject and can move ahead to more advanced material.
>
> CAI also has the ability to ease administrative burdens by having the computer assume the task of record-keeping. The student can be graded even as he is taking a lesson; the results can be compared to past scores almost instantaneously; future performance of a student can be predicted by comparison with data derived from the histories of former students; and records of varying completeness, according to need, can be made available at a remote printer.

Furthermore, CAI can process student responses and provide immediate feedback to correct erroneous answers. Also, due to the computer's great speed, CAI can interact with hundreds of students at once while giving each student the illusion that he alone is communicating with it. This (1) creates a feeling of personal privacy for the student, (2) reduces the student's inhibitions toward what his peers think of his performance, and (3) eliminates the student's fear of having his ignorance revealed if he chooses an incorrect response.

Another capability of CAI is to obtain vast information of students' performances for teachers, administrators, and educational researchers. Suppes (81) views this subject by saying that the power of the computer to assemble and provide data will perhaps be the most powerful impetus to development of educational theory yet to appear. Using this day-by-day, concept-by-concept data, researchers can study the nature of

competent performance and the process of learning itself.

The preceding descriptions of several of the potential features of the computer for instruction may seem to imply that it has a built-in instructional methodology or philosophy. Although the computer is sophisticated and intricate, in reality, it remains a physical, electronic device termed hardware. Thus, it can perform none of the services described above on its own initiative but must be directed in the most explicit terms by technical personnel and/or specially trained educators. The complete sequence of instructions, explicit orders telling the computer exactly what to do, is called software. It is the computer software which manipulates the instructional process and carefully controls the learning sequence. The computer software, together with the stored course materials, are the keys to a CAI system and are dependent upon the creativity, educational expertise, and know-how of the designer.

Even with present-day computers, the hardware capability far exceeds our present knowledge of the process of learning and instruction. According to Suppes (81), the real obstacles to CAI lie in the areas of curriculum design and present educational methods, which are based upon group needs rather than individual needs. At this time, there is not a purely scientific method for individualizing instruction. Harold Mitzel of The Pennsylvania State University (52, p. 323) alludes to

this, by saying:

> Unfortunately, psychologists are hard put to know which characteristics of the learner (immediate response history, pattern of aptitudes, response latencies, etc.) are optimum for branching decisions, and until we discover the instructional experiences which are optimal for a student with a particular learning history, the full power of CAI for individualizing instruction will not be realized.

Thus, the extent to which the potential of CAI is realized will depend upon the ingenuity of the instructional programmer who must design software according to teaching models and theories of learning and upon the ability of the researcher to provide answers to the complex questions concerning the systematic design of instruction.

DeCecco of San Francisco State College (17) asserts that, in the meantime, American industry has entered the educational scene to hasten the development of educational technology, particularly for the purpose of providing the hardware. Companies are designing the hardware to meet what they believe will be the demands of present and future educational markets. However, in its acquisition of many major textbook publishing companies, the business community is also showing interest in educational software. Now that the business world is invading their classrooms, educators have to make a choice. Either they let big business dictate the future direction of educational technology or they become better informed so as to share in shaping the future. Furthermore, before CAI will be accepted into the schools, educators must properly emphasize

the role of computers in education and establish educational programs for teachers. Don Kraft of Science Research Associates (47, p. 10) sums this up as he says:

> The computer's role in instruction today is still largely experimental. The challenge for educators is to develop a workable solution for orchestrating the capabilities of the computer and the teacher. To prepare for this challenge, educators who have not already done so might begin by learning about computers, their capabilities and limitations, and in particular what is being done in education with them. With this understanding, the educator will be better prepared to give direction to this promising new media.

1. Means of communication

To be informed about CAI, educators must understand the means of communicating with the computer. Briefly, instructional programs are stored in the computer, and the student interacts with these programs by means of an electric device called an interface. In CAI terminology, interface refers to the input and output device through which the subject matter is presented to the student and through which he makes responses. This device is commonly called a terminal. Suppes (78) describes the simplest and most-common terminal as a device similar to an electric typewriter or teletype in which messages are typed out by the computer directly to the student's individual terminal; and the student, in turn, types a reply, question, or other response on the terminal keyboard. Kenneth Wodtke (89) lists the capabilities of an ideal student-subject matter interface as being:

1. Keyboard for response input,
2. Cathode-ray tube (CRT) display with light pen response capability,
3. Video tape or closed circuit television capability built into the CRT unit,
4. Random access image projector with positive address (may be unnecessary if the system provides video tape capability),
5. Random access audio communication device with positive address.

With such an interface, the student can receive information from audio messages, typewritten messages, or visual displays that may range from graphics to complex photographs. In turn, the student responds to the system and gives his own answers by using the keyboard on the typewriter or the electronic light pen. The CAI interface, then, must provide maximum flexibility in adapting display and response modes to differences in students and variations in subject matter.

The interface between the student and the computer is a critical factor in the instructional use of the computer. Kenneth Wodtke (89) of The Pennsylvania State's CAI Laboratory elaborates on the two major dimensions of the CAI terminal: (1) its stimulus-display capabilities and (2) its response-processing capabilities. Stimulus display capability refers to the means by which the interface can display subject matter to the learner (i.e., written, audio, pictorial, graphics, etc.). Response processing capability refers to the variety of student responses which can be detected by the interface

and processed by a computer (i.e., typewritten characters, light sensing, spoken words, etc.). Wodtke (89) also discusses the educational variables which determine the characteristics of an effective student-subject matter interface. The educational variables can be divided into four categories: characteristics of the subject matter, characteristics of the learner, nature of the instructional process, and the objectives of the instruction. Robert Glaser (25) has likewise considered the difficulties encountered in matching the properties of console displays and controls to the properties of the subject matter and learner characteristics in order to design relevant conditions for learning. The plasma display device of the University of Illinois (8) seems to hold great potential as a viable interface. The plasma display panel stores and displays digital information on an array of gas discharge cells, each of which is isolated from the exciting electrodes. This device combines the properties of memory, display, and high brightness in a simple structure of potentially inexpensive fabrication. In contrast to the commonly used cathode-ray tube display, on which images must be continually regenerated, the plasma display retains its own images and responds directly to the digital signals from the computer. The plasma display is discussed in detail in two listed references (7, 8).

The number of terminals which can be attached to the

central computer is largely dependent upon the capacity of the computer and the efficiency of its programming system. A small instructional system, such as the IBM 1500, can simultaneously operate a maximum of 32 terminals. A description of the 1500 system can be found in materials by Mitzel (54), Hansen et al. (34), and Inman (42). A large computer, for example, the RCA Spectra 70, can simultaneously operate as many as 192 individual student terminals. A description of the Instructional 70 system can be found in RCA manuals, references 66, 67, 69. Therefore, it is possible for a computer with 192 terminals to present instructional programs to as many as 6,000 students daily, with each student using the terminal approximately 15 minutes. In all likelihood, these numbers will, in the near future, increase to 4,000 terminals and more than 100,000 students. In fact, such a system is in the developmental stages at present. Though the number of terminals appears large, the computer can deal with many users seemingly at once by giving only a fraction of a second to each program or task. Even with all terminals operating, the time elapse between the end of a student's response and the start of the computer's next message must be short so that no delay is detectable. If it is too great (i.e., exceeding five seconds), the user will detect the delay and become bored or frustrated.

Terminals can be remotely located, which means that

responsibility to attempt the challenging task of helping students who are not proceeding successfully with the tutorial program and who need special attention. CAI does not replace the teacher but extends his/her capabilities by enriching the learning situation to ensure the student's utmost success. This second level of CAI is the area in which most current research and development is being concentrated. Researchers are developing curriculum materials and identifying learner characteristics which, hopefully, will give insight into the learning process itself. Examples of tutorial projects are the materials of The Pennsylvania State University (52), University of Illinois (4), Florida State University (34), and this study.

At the third and most-complex level of CAI instruction are the dialogue systems. These systems are aimed at permitting the student to conduct a genuine dialogue with the computer. Rosenbaum and Bennik (71) of SDC believe this is a necessary capability of an ideal CAI system. Since there are a number of difficult, technical problems to solve, the dialogue systems currently exist primarily at the conceptual rather than the operational phase. One major problem is to teach the computer how to communicate in the language of the student. Pattern recognition is very difficult to implement on a computer, and the use of man's natural language requires pattern-recognition capabilities. Another problem is the

difficulty of writing programs which will recognize and provide answers to questions that are broad and complex. Examples of dialogue systems are ELIZA, created by Weizenbaum (88) and PLATO Inquiry Mode, described by Bitzer et al. (6).

Yet, the situation is not hopeless. Dr. Patrick Suppes (81), a pioneer in CAI, predicts that within the next decade many children will use individualized drill-and-practice systems in elementary school; and by the time these children reach high-school age, tutorial systems will be available on a broad basis. Their children may use dialogue systems throughout their school experience. If these predictions are even approximately correct, they have far-reaching implications for education and society.

In addition to providing individualized instruction through CAI systems, the computer can be used for independent study or research by either students or faculty. Tedious problems which perhaps involved months or even years of study can now be solved by a computer in seconds. Furthermore, a computer can simulate sophisticated problems, situations, or conditions. In this simulation mode, the student interacts with a model, in the form of a previously stored program, of a physical, social, or economic system or phenomenon in order to discover its behavior and/or properties. Dr. Karl Zinn (95) of the University of Michigan feels that the computer's greatest potential for instruction lies in the areas of calculation

and simulation rather than in tutorial CAI.

B. The Role of Industry in CAI

How far has CAI progressed toward fulfilling the promise of meeting the specified educational requirements? A discussion of the state of the art should give some indication.

CAI has a very short history. The first experiments were carried out in an IBM research laboratory in 1958 when a 650 computer was connected to a special typewriter and programmed to teach binary arithmetic to engineers (37, 47). Since its entry into CAI, IBM has expended a great amount of money and effort in developing new systems and curriculum materials. Company personnel have devised courses in German (2, 55), statistics (28, 38), reading (60), and arithmetic drill-and-practice exercises (61, 62). The pilot studies and field tests seem to indicate that CAI test groups achieved equally as well as, if not superior to, control test groups in German (2, 55), statistics (28, 29, 30, 31, 38), and arithmetic (61, 62).

Along a similar trend, IBM has produced the IBM 1500 Instructional System, which is fully dedicated to CAI. This system uses an IBM 1130 as a central computer and has thirty-two 1510 cathode-ray tube displays as student terminals. The Company has produced approximately 24 such systems and has placed them in various educational institutions. This system is also used by the IBM Field Engineering Division in training

IBM customer engineers in the use of new equipment. Thus, CAI not only has potential for formal classroom education but for industrial training as well.

To complement its hardware, IBM has developed a special author language called Coursewriter and the necessary computer programs for it. Hartman (37) characterizes Coursewriter as a high-level language in that each mnemonic symbol can call into play a complex sequence of computer instructions. The author merely labels each unit of course material with a mnemonic Coursewriter code and enters both the unit and code at the instructional station. Response processing includes exact match, key-word match, and partial-answer match. To assist the exact-match function, the system monitor automatically edits the student's response, eliminating any undesired characters. Coursewriter has undergone constant revisions and the third version is now in operation.

Science Research Associates (SRA), an IBM subsidiary engaged in publishing, is working in all phases of CAI. However, emphasis at SRA (47) is mainly in the development and evaluation of experimental CAI materials. One of their projects is the research of materials and programs for science and mathematics students in grades 11-14, using problem-solving and simulation techniques. Another project is that of diagnosis and practice in elementary arithmetic. Some of these materials being developed are now undergoing initial student

testing in several high schools and colleges throughout the country.

In 1967, the Radio Corporation of America (RCA) entered the CAI field commercially with the RCA Instructional System (RCA/IS) (64). With Dr. Patrick Suppes, Dr. Richard Atkinson of Stanford University, and Dr. Duncan Hansen of Florida State University as technical consultants, they have developed a computer-based instructional system. This project is the first endeavor to make CAI successful on a commercial basis. The RCA/IS (66, 67) is a multi-purpose CAI system designed to provide instruction for students during classroom hours and also to perform special after-hour functions. These after-hour functions are comprised of special instructional tasks, such as adult education, a broad variety of educational services, and the administrative data-processing requirements of a large, city school district, multi-campus university, or other major educational institution. This instructional system utilizes an RCA Spectra 70/45 computer to drive a maximum of 192, remotely located, RCA 733 student typewriter terminals simultaneously.

The curriculum materials for use with the RCA/IS are offered directly to system users by leading textbook publishers and are primarily drill-and-practice systems. L. W. Singer Company provides mathematics drill-and-practice programs consisting of drill-and-review lessons for students

in grades two through six and for remedial students above grade six (64, 66). The mathematics program is intended to supplement teaching in the classroom by providing drill-and-practice exercises for students on concepts selected by their classroom teacher. The instructional strategy and some of the materials were developed by Dr. Suppes and other curriculum experts at Stanford University.

Elementary English and remedial-reading lessons for elementary grades four through six are available from Harcourt, Brace and World (36). Elementary English is a program of individualized practice in language skills and can be used in conjunction with most published textbooks. The reading materials are prepared to be used with upper-grade elementary students or older students and perhaps even adults whose reading skills are below their desired levels.

Simplicity is the keynote to the RCA/IS, and it is designed to be used by professional educators, not computer specialists. Besides making use of commercially available curriculum materials, the curriculum author can prepare his own course materials. Teaching strategies are contained in sets of instructions called procedure programs; and the procedure programs are written in a special language called Instructional Language-1, which is designed to allow individuals with little or no previous computer experience to communicate with the computer. For greater details of the

language or the operating system, the reader is referred to User's Guide to Instructional Language-1 (69) and Operating System Description (67).

Three RCA CAI systems are in operation. The prototype installation was completed at RCA's CAI Laboratory, Palo Alto, California (64), in the spring of 1967. A second instructional system, which began operation in the New York City Public School system in March, 1968, provides individualized daily instruction to approximately 6,000 students during normal classroom hours. This project was made possible through a three-year grant of $2.5 million from the U.S. Office of Education under provisions of Title III of the Elementary and Secondary Education Act of 1965. Few educational innovations have been watched as closely and as hopefully as this system. An impartial evaluation of it was prepared by City University of New York and indicates initial favorable results, both in achievement and in acceptance (58). The third system, a 32-terminal system, is operating in Waterford, Michigan (64).

In 1958, a small group of System Development Corporation (SDC) psychologists (83) began to explore automated teaching and, later, the computer's role in education. This led to the development of a computer-based educational laboratory for automated school systems, called CLASS. CLASS allowed extensive experimentation in identifying important variables in materials for programmed instruction to ensure the best

quality and organization of the subject matter. Experimentation was concerned mainly with presentation, evaluation, and revision cycles in the development of courses. More recent studies have investigated the effects of other variables; for example, the amount of student control over the sequence of instruction, different methods of branching, and varying degrees of teacher involvement. Now, experiments and trials have moved from the laboratory setting into the field as SDC (48) is providing a variety of services to the educational community, including the military and other federal agencies, state departments of public instruction, school districts, universities, and regional educational laboratories.

One of SDC's contributions to CAI is in the area of author languages. An author language is a special syntax which allows the course author to communicate with the computer. PLANIT (Programming Language for Interactive Teaching) (22, 84) is a computer language designed to enable teachers, counselors, administrators, and students to communicate easily with a computer, even though they may have no knowledge of computers or computer programming. Originally intended for the design and execution of programmed-mathematics instruction, PLANIT (48, 83) has been used for the development of a wide range of materials, ranging from first-grade reading to college-level statistics. Other applications include its adaptation for automated counseling interviews and for school-

administrative planning.

Feingold (22) characterizes PLANIT as a general-purpose teaching system, developed for time-shared use. Interacting with PLANIT, the user can build and edit lessons, execute them, and perform calculations. By using the CALC mode, PLANIT can be employed as a calculator for defining and evaluating mathematical functions. PLANIT also provides service functions for evaluating student answers that depart from the expected response by making phonetic comparisons, key-word matches, and equivalent formula or algebraic matches. The language includes statistical functions and routines which are very useful in statistics instruction. SDC's staff contains specialists in linguistics, education, engineering, psychology, and computer science (85).

Realizing this educational potential, many computer companies are turning their computer systems toward this market. UNIVAC, a division of Sperry Rand, already has developed the computer hardware and is now producing the necessary software for an educational system. COPI (Computer Oriented Programmed Instruction) (87) is UNIVAC's computer programming language for education and is designed to provide the educational community with versatile and flexible teaching capabilities. COPI has undergone constant revision and updating, and the third version is in the research and development stages. Operation of COPI programs does not require that the computer

system be dedicated solely to education. Instead, other data processing activities can proceed simultaneously with the execution of COPI lessons to allow for greater computer efficiency. The UNISCOPE 300 Visual Communication Terminal provides the interface device for the system. Moreover, by developing curriculum materials in data processing for the Company's various educational centers, UNIVAC is showing an interest in educational software.

Another company, Measurement Research Center (MRC) (51), a division of Westinghouse Learning Corporation, has a different approach to educational computer use, which is referred to as Computer-Managed Instruction (CMI). In CMI, the computer is used as a means of assisting the teacher in managing the individualized educational process. The computer system stores and accesses information on each student's performance, suggests how he should proceed in his learning experience, keeps records up to date, grades and records the results of tests, and uses the knowledge of educational experts in recommending which lesson the student should work on next. The teacher has access to a remotely located terminal and can query the computer system at any time. Importantly, however, the teacher is responsible for making final decisions.

The student uses conventional classroom resources, either commercially or teacher prepared, for obtaining the proficiency required by the lessons. The lessons are a series of

printed study guides called Teaching-Learning Units (TLU). These TLU's were prepared by classroom teachers and professional educators in the attempt to suit individual differences and to take advantage of a wide range of available learning aids and techniques. They were developed under project PLAN (Program for Learning in Accordance with Needs) by the American Institutes for Research (91) under funding by government grants and Westinghouse Learning Corporation.

MRC (51) asserts that in 1969 there were approximately 9,000 students in PLAN classrooms, and by September, 1970, an additional 30,000 students are expected to participate. As this project grows, CAI can be added as an additional feature.

This section has not contained an all-inclusive listing of companies with ventures in the area of applying computers to education. However, these projects do give some indication as to the extent to which the business community has entered the field of CAI. The primary emphasis of current research in educationally oriented computer industries is on the development of computer hardware and software.

C. The Role of the University in CAI

In 1963, the Institute for Mathematical Studies in the Social Sciences at Stanford University initiated a program in research and development of computer-based instruction. Since then, Stanford has become one of the research leaders in computer-assisted instruction, and its personnel have

published a great quantity of material in this field. Dr. Patrick Suppes, Dr. Richard Atkinson, and their colleagues at Stanford (74) have developed curriculum materials and instructional strategies for drill-and-practice lessons in arithmetic, reading, and English for elementary grades. These curriculum materials are now available through commercial publishing companies. Much of the initial research on these programs was completed at Brentwood Elementary School in Palo Alto, California.

Other research studies at Brentwood have emphasized the great diversity in the rate of learning of individuals. In an accelerated mathematics program for gifted students, Stanford researchers (78, 81) found strikingly large differences, even within relatively homogeneous groups of students. The fastest students completed 400 percent more problems than the slowest, and speed was not strongly correlated with IQ. A variety in the rate of progress was also found in reading classes. After one year, the span between the fastest and the slowest students was found to be 6,250 completed problems, with an interquartile range of 1,875 problems. Examples of significant individual differences in learning ability are in no sense restricted to mathematics and reading but are found in all disciplines. These extremes emphasize the need for (1) further research in identifying factors influencing the learning rate and (2) an instructional aid capable of being

sensitive to the diversities in learning rates.

Dr. Suppes (78, 80, 82) has also done much research in the area of how students learn mathematics and in the application of mathematical models of learning in education. He believes elementary mathematics affords an ideal testing ground for mathematical models of learning or performance. Other current work at Stanford University includes courses in logic and algebra, spelling, and dial-a-drill program at the elementary-school level, and elementary Russian at the college level. These courses are described in detail in the Stanford Progress Report (74).

The University of Illinois is another of the pioneers in the CAI field. The University's Computer-based Education (CBE) Research Laboratory has been experimenting with a CBE system called PLATO (Programmed Logic for Automatic Teaching Operations) for the past eight years. The PLATO system has evolved from a single terminal connected to the University-developed ILLIAC computer to 20 graphic-pictorial terminals linked to a Control Data Corporation (CDC) 1604 computer (4, 5, 6). Alpert and Bitzer (4) state that the PLATO program has developed in two distinct phases. The first phase was the investigation of the computer's potential role in the instructional process. The second phase involved the design of an economically and educationally viable system incorporating the most valuable approaches of teaching and learning developed in

Phase I.

In Phase I of the program, three successive and increasingly versatile systems (PLATO I, II, and III) were designed and constructed to explore the educational possibilities of the computer. Along similar lines, exploratory experiments and studies have been conducted in many disciplines and at all levels of formal education. Some of these are electrical engineering, geometry, biology, nursing, library science, pharmacology, chemistry, algebra, mathematics drill, computer programming, and foreign languages (49). Lesson materials have been prepared, edited, and modified in an author language called TUTOR. According to Alpert and Bitzer (4), these materials have been presented through the use of a variety of teaching strategies, ranging from drill-and-practice to tutorial to student-directed inquiry. In the inquiry mode, the student is presented with a problem statement which cannot be dealt with by a simple or multiple-choice answer. The problem may call for a sequential analysis or a constructed response. The student is provided with reference material of his choice, which may include definitions, theorems, helpful relationships, tables, experiments, data, and textual information. Then, by selecting the information needed, the student solves the problem. The inquiry mode is an example of a dialogue system.

Phase II of the PLATO program has addressed itself to the problem of the high cost of existing technology for CBE. This

portion of the program has been concerned with the design of an economically viable, large-scale CBE system. The guidelines for the development of the software and hardware for this system, called PLATO IV, are listed in The Design of an Economically Viable Large-scale CBE System (7). The key to the economic feasibility of PLATO IV is a newly invented graphics display device, as several existing computers already meet some of the requirements. Under the trade name DIGIVUE, the plasma display panel is capable of storing on its viewing surface either computer-generated or student-generated information without the need for auxiliary storage devices. The plasma display panel is presently in the commercial prototype-development stage and offers promise of reduction in student terminal costs. When produced in quantity, it will sell for approximately $1,500 as opposed to the $5,000 expenditure for present terminals. The plasma display panel's technical aspects are viewed in greater detail by Bitzer and Slottow (8). While the total operational cost per student-contact hour for the PLATO III system is $1.90 to $2.90, PLATO IV proposals set the cost at $0.31 to $0.68.

In April, 1964, a CAI laboratory was begun at The Pennsylvania State University with the funding of a project to conduct a CAI feasibility study for four college courses (52). In the last six years, under the direction of Dr. Keith Hall and Dean Harold Mitzel, the Penn State Laboratory has

become one of the leaders in CAI. The Laboratory personnel view CAI as an aid to conventional teaching and see CAI's role as a tutorial one. The major purposes of the Penn State CAI work are educational research and the development of curriculum materials. Much emphasis is on the best methods of writing CAI courses. The courses are constructed in terms of student responses and course objectives and are written in the IBM author language, Coursewriter. The reader is referred to The Development and Presentation of Four College Courses by Computer Teleprocessing, principal investigator being Harold Mitzel (52), for information on Coursewriter and sample courses.

The Penn State computer system consists of an IBM 1500 Instructional System, which is solely dedicated to CAI. At present, the Laboratory has approximately 15 IBM student terminals. These terminals consist of a cathode-ray tube screen for computer-generated materials and a typewriter keyboard and light pen for student responses. Some of the terminals are equipped with computer-accessed image projectors and audio play or record devices, which allow for a widely diversified range of course materials and student characteristics. An example of a course applying all three modes of presentation is the instrumental music (clarinet) course designed by Deihl (18). This system is frequently used by course authors in the development of programmed materials. The system allows four dictionaries of 128 characters each, which gives the course

author much flexibility. The Penn State system has served as a functioning prototype of a CAI system for many visitors.

Many of the Penn State CAI Laboratory staff are faculty members of the College of Education, whose job it is to train teachers. Thus, many of the programs are geared to in-service training of teachers or teacher education. Courses developed include accounting, audiology, engineering economics, modern mathematics for elementary teachers, recognition of malaria parasites (for the U.S. Naval Medical School), spelling, algebra, general mathematics, music, and the identification and diagnosis of handicapping conditions (18, 32, 52, 53, 54).

The Laboratory is also responsible for two additional IBM 1500 Instructional Systems being used in the school districts of Pittsburgh and Philadelphia. Programs in general mathematics and algebra are currently in use, and in the fall of 1970, a mobile 1500 system will be in service. This system will be moved from school to school and will provide in-service training for elementary school teachers. The system will be contained in a specially designed and built truck-trailer, and if successful, could play an important role in future in-service training.

Since 1966, the CAI Laboratory has conducted many field studies to test its curriculum courses and to determine student attitude. A questionnaire has been developed to provide information on student attitude toward CAI (52), and informal

interviews with students have also been conducted. Student reaction, generally, appears to be favorable. Short-term studies (52) by the Laboratory have indicated that students seem to learn at least as well with CAI as with conventional classroom instruction. However, more complete studies are needed before long-term effects can be determined.

Many other universities are doing research and development in the rapidly expanding area of CAI. It is impossible to summarize the work of them all in this study, but the following are contributing to this area: Florida State University, Harvard University, University of Wisconsin, University of Michigan, University of Texas, University of California, University of Pittsburgh, the United States Naval Academy, and the University of Alberta, Edmonton, Alberta, Canada. Efforts in CAI do not constitute a well-defined and coordinated program. Rather, they cover a wide range of educational objectives and interests and an even greater diversity in available computer software and hardware. Research emphasis in the universities has included varied and effective instructional strategies, greater accommodation of individual differences, and better evaluation of the instructional process.

D. The Role of the Teacher in CAI

A prominent issue in most discussions of CAI is the teacher's role in a CAI system, and it is fitting to consider

this aspect. The teacher will not be replaced by the computer, however, his role in the educational process will be changed. Goodlad (27, p. 50) says:

> I do not see the computer as the teacher's competitor. Not at all! I see it rather as replacing the teacher for certain instructional tasks that I believe it can and will do better than any human teacher can perform them.

The teacher will be freed from much of the routine that presently occupies a good part of the day. The amount of paper work required of instructors is substantial and is increasing. The computer seems to offer the possibility of reducing time spent in these administrative tasks.

According to Hansen and Harvey (33), as CAI absorbs more of the information presentation and correction functions, the educator will be able to devote more time to other activities, such as individual tutoring, group discussions, enrichment materials, social skills, and guidance. The teacher can become more involved in managing the instructional process and designing instructional strategies. The role of the teacher is likely to revolve around human relations, construction of learning materials, and research into the learning process itself. Even the best CAI system cannot possibly foresee all student trouble spots, and Suppes (80, 81) believes it will be the teacher's duty to identify these troublesome concepts and provide additional remedial instruction.

E. Cost of CAI

Another issue which sometimes creates lively debate is the cost of a CAI system, both hardware and software. At the present time, CAI is very expensive when compared with traditionally administered instruction (TAI). Kopstein and Seidel (46) did cost analyses of CAI and TAI in 1968 and found TAI considerably more economical. This finding was especially true at the elementary and high school levels. After complex studies using a 32-terminal system, they arrived at a cost of $0.36 per student hour for TAI in contrast to $3.73 per student hour for CAI. However, in evaluating these figures it must be remembered that CAI is, for the most part, still in the research and exploratory stages. Therefore, cost is not a criterion of effectiveness as pointed out by Seidel (72). It appears, furthermore, as if the cost of computer technology is in the grip of a sharply downward trend.

Bitzer and Skaperdas (7) see three factors which reduce the expense of CAI. These are: (1) more and larger systems, (2) compatibility of curriculum materials, and (3) 24-hour utilization of the computer. In discussing the first point it can be stated that large systems, supporting many terminals, perhaps thousands, can greatly lower the operating cost per terminal. These systems can be shared by many schools, thus lessening the expenditure for each school, while allowing all the capabilities of CAI. The more of these large systems in

operation, the cheaper the computer software. Also, the educational software, the curriculum materials, can be shared by many users, consequently reducing production expenditures.

To elaborate on Bitzer's second point, if technology discovers a way to easily adapt programs and curriculum materials designed for one computer to other computers, the cost of constructing a course will be lower. If this happens, economic rewards should encourage authors to produce CAI courses rather than textbooks.

Finally, 24-hour utilization of the computer would help to reduce hardware operating costs. According to Kraft (47), a typical day for the computer might be: CAI presentations for students from 8:00 a.m. to 5:00 p.m.; adult-education uses between 5:00 p.m. and 10:00 p.m.; and school data processing functions during remaining hours. RCA personnel feel that the administrative data processing capability alone might justify computer costs. If so, the instructional benefit would be a bonus. However, even if the expenses are high, education must be prepared to make a significant investment. It cannot afford to forfeit any of the potential rewards of CAI research.

F. Summary

The complete computer-assisted school has yet to arrive on the educational scene, but the tools and techniques for applying the computer to the educational process are being developed. Since CAI's simple beginning in 1958, computer

industries have been investing large sums of money and energies of capable personnel in the research and development of computer hardware and software. Colleges and universities of all sizes are establishing computer-learning centers for research into the process of learning and the development of CAI curriculum materials. To date, CAI projects have expended tremendous energy in the development of such studies.

Commercial publishing companies are also producing educational materials to be used for computer presentations or in conjunction with computer-presented lessons. Elementary schools, high schools, and colleges are becoming involved in trial studies, data collection, and evaluation of materials. Full realization of the computer's potential, however, is in the future but must be preceded by a thorough analysis and evaluation of hardware requirements, a study of the characteristics of learners and the learning process, and a painstaking development of curriculum materials. These steps should be taken through a meaningful collaboration of educators and technologists.

In summary, CAI offers a significant potential for providing individualized instruction and opportunities for carefully manipulated presentations of verbal and visual stimuli rarely possible in previous research or instruction. However, more extensive research and development is imperative at all levels of educational development.

III. DESCRIPTION OF THE CAI SYSTEM

This research project, an introductory venture into computer-assisted instruction at Iowa State University, was entirely developmental in nature. The completed unit and system have been tested on a graduate educational statistics class for the purposes of obtaining (1) constructive criticism regarding the curriculum material and (2) an estimate of student contact time required at the terminal. This chapter focuses on the general description of the CAI system by discussing the three primary parts: (1) the hardware component, (2) the software component, and (3) the educational component.

A. The CAI Hardware Component

The hardware component, comprised of the computer and the necessary peripheral equipment, was dictated by the existing facilities of the Computation Center at Iowa State University, Ames, Iowa. These facilities included an IBM 360/65 time-sharing system and peripheral equipment featuring the following: magnetic tape units, disk and drum storages, bulk core storage, card readers and punches, line printers, and a control console. The IBM 360/65 operating system (OS/360) provided on-line batch processing of University assignments, a part of which contained student problem-solving and research programs. The OS/360 software allowed these assignments to be programmed in high-level computer languages, such as Fortran,

PL/1 (Programming Language One), Algol, Cobol, etc. The operating system was also designed to provide a time-sharing conversational mode.

Supplying time sharing, Conversational Programming System (CPS) runs in a partition of OS/360 and is described in Conversational Programming System Manual (43). It supports a conversational language which is a dialect of PL/1 and includes a modified subset of the BASIC language of IBM CALL/360. According to the CPS manual (43), these languages are designed for interactive, conversational programming. Providing a powerful file-manipulation capability from a terminal, they also lend themselves readily to the writing and debugging of programs interpretively and to the interactive execution of these programs. These features hold potential for the instruction of computer science courses and for the solution of mathematical problems. Comparison of the cost of CPS with that of other languages is difficult to make, but it appears that CPS is relatively expensive. Consequently, programming jobs not specifically requiring terminal interaction or conversational mode should be submitted for batch processing computer runs.

CPS (43) supports communication through IBM 2741 and 1050 typewriter terminals and ASR Model 33 and 35 teletype terminals. Due to the availability of 2741 terminals at Iowa State University, one of these was used as the interface for this

study and was provided for the testing of the probability unit by Dr. Clair Maple, Director of the Computation Center.

Equipped with a dataphone, this terminal was connected to the central computer by means of a voice-grade telephone line. This line transmitted information between the computer and the terminal, allowing the latter to be remotely located. The user was provided access to the computer by following a procedure similar to that of operating a dial telephone. The telephone line was on a roll-over basis and switched automatically to any available line whenever the correct number was dialed. If the user received no answer, the computer system was not operational due to regularly scheduled maintenance or a system malfunction. If a busy signal was received, all the lines were occupied.

E. N. Adams (1) states that one of the most-promising applications of a time-sharing system is CAI; and the recent rapid advancement of CAI can be attributed, in a large part, to the development of computer time-sharing techniques. The general-purpose computer seems to hold potential for providing many of the educational functions described by Dr. Holtzman (40) and particularly as an instructional tool. The following section describes one method of adapting a conversational time-sharing system for CAI.

B. The CAI Software Component

The second component of the CAI system, the computer software, consists of an author language and associated computer programs to provide for its implementation. The author language allows a course author to design and write a course in his subject-matter area for computer presentation without having previous knowledge of computers or formal computer languages. However, the author is required to learn a minimum of convention in order to develop instructional materials for selected teaching strategies.

Dr. Karl Zinn (95) states that an author language is distinguished from scientific and business languages by: (1) its convenience for displaying textual material, such as questions, problems, assignments, and factual information; (2) its acceptance and classification of student responses and a comparison of these responses with pre-stored, anticipated answers, either correct or incorrect; (3) its automatic recording of performance data; and (4) its implicit branching determined by the categorization of an answer or the contents of a counter which is part of the response history of the individual student. According to SDC (85), in addition to its versatility, an author language's most attractive feature should be its simplicity, which should require the user to consult only a summary sheet, briefly describing the system, its simple conventions, and its special commands. The

language should not be bound by or committed to any particular teaching methodology and may have a calculation mode. Particularly oriented to mathematical subject matter, the calculation mode allows the student to use the computational ability of the computer. This ability, one of the computer's greatest assets, assists the student in rapidly computing problems that, under conventional classroom conditions, might take hours or days. Moreover, the student can construct his own program or use system-provided functions.

Karl Zinn (92, 95) established various criteria for author languages and divided instructional programming languages into the following four categories: (1) presentation of successive frames or items, (2) conversation within a limited context, (3) presentation of a curriculum file by a procedure program, and (4) interactive programming language for student use. The author languages for tutorial CAI belong in the first three categories, with most in the first classification. However, it should be noted that the above categories are not mutually exclusive, and some languages, depending upon their applications, may fit into more than one area. The following are examples of author languages: Coursewriter, PLANIT, ISL-1, TUTOR, COPI, ELIZA, and MENTOR. Details of each are described by Hickey (39) and Zinn (95).

The ideal CAI language would be the student's and author's native spoken language. Prior to the advent of CAI,

people were attempting to develop man-computer languages which simulated natural languages; but because of the complex semantics, large-scale ventures have never been successful. However, research is currently being conducted which should some day make natural-language communication with the computer possible.

One of the strengths of a CAI language is the range of its answer-matching capability. Because of the characteristics of different subject areas and the variety of teaching strategies, several types of answer-matching routines are necessary. Frye (24) lists several types of these and examples of languages which use them. Included in Frye's list are: exact match, key-word match, selected character string match, percentage match, partial match, phonetic match, algebraic match, numeric match (within specified limits), and calculated numeric match. All of the features of these matches are not found in any presently existing language, and, therefore, a course author must consider many variables before choosing a particular author language.

The CPS language for the IBM 360/65 system at the Iowa State University Computation Center is an interactive programming language and is primarily a conversational problem and calculating language. It is not designed for text processing as is a tutorial author language. So, for this CAI research study, the problem arose whether to: (1) adapt the CPS

language to provide for tutorial CAI capabilities or (2) implement one of the existing CAI languages to the 360/65 system. Because this is a developmental research study, it was decided to design and implement an author language within the structure of CPS. This research language is a hybrid of ideas from various languages, especially Coursewriter and ISL-1. The basic unit in this language, a statement, consists of three components: the label, the operation code, and the argument.

The first of these components, the label, is comprised of any four alphanumeric characters. It, a unique identifying symbol, allows the computer to reference the location of the statement (i.e., a mailing address). Further, the label enables the author to inform the computer which statement to process next if the sequence changes.

The second component, a four-character mnemonic symbol called the operation code, indicates to the computer program what type of information is stored in the argument. Labeling material with a particular operation code informs the system that, whenever encountering this code in the course sequence, the computer should respond accordingly. Each operation code executes a different section of the author language's associated computer program. Also, each operation code gives the author a different capability for preparing the course materials and the teaching strategy. The mnemonic symbols represent codes similar to the names of commands in the normal

instructional presentation. The author language is made up of ten mnemonic symbols: TYPE, ANS1, ANS2, WANS, BANS, NANS, COMP, GOTO, BRAN, and STOP. An explanation of these is found later in this section.

The third component of the statement, the argument, contains curriculum materials, such as textual information, questions, and answers. It also stores control information to be used by the computer for branching or for changing the sequence of instructions. Unless the order is modified by a control statement, each statement in the curriculum materials is executed in the order of its physical appearance in the program. Control statements permit branching of the execution order to any labeled statement in the program. In a control statement, the argument stores the unique set of four characters to identify the specified label.

This and the following nine paragraphs contain explanations of each of the ten operation codes (op codes) designed for this study. The first op code, TYPE, informs the computer that the argument of the statement contains information to be output to the student through the student terminal. This op code is commonly used to display questions, problems, textual information, statements, assignments, etc., to a student. Since TYPE is probably the most-common op code, the computer recognizes a series of four blanks as an alternative for it. This feature can save a course author considerable writing

time.

ANS1 is the op code identifying the statement's argument as a correct answer to be stored in the computer's memory for comparison with the student's response. The answer can be a sequence of acceptable anticipated answers, each separated by the delimiter "|," the logical OR symbol. Upon encountering this op code, the computer control program automatically requests a student reply. This response is stored in the computer's memory and, after both have been UPCASED, is compared with items in the set of correct answers. UPCASE, a pre-programmed function in CPS that changes a string of alphabetic characters to capitals, allows the student to reply in any mixture of small and capital letters without requiring the author to anticipate all the possibilities. Numbers are not altered by this function. The two stored answers, the student's and the author's, are pattern compared, which means that for the two to be judged the same by the computer program they must be identical. Thus, it is necessary for the author to anticipate common misspellings and typographical errors and decide whether or not to interpret these as correct. After the comparison, the computer-control program randomly selects a feedback response for output to the student from the several possible stored responses. This feedback informs the student as to the correctness of his last reply, and if incorrect, the correct answer is supplied. Research studies described by

DeCecco (17) and Hickey (39) show that immediate feedback is an important part of learning. The author can add any additional material to this feedback that he feels is necessary to clarify ideas. Or, if the author does not want the randomly selected computer response, he can provide his own by using the next operation code, ANS2. The computer-control program increments student response counters, such as total number of problems and correct answers. Figure 1 shows a flow-chart diagram of the presentation logic just described.

Similar to ANS1, the third code, ANS2, invokes identical action except that no feedback response is provided to the student. Such feedback is delegated as the responsibility of the course author and is commonly used when the author does not want the student to observe the correct answer.

WANS, the fourth op code, is composed of anticipated wrong answers, each separated by the delimiter "|." The incorrect answers are compared with the student's previous reply. The student response counters are not incremented. However, if a student's reply matches a specified wrong answer, the course author provides information to help correct the mistaken reasoning.

The fifth op code, BANS, allows the course author to request an assessment from the student as to his comprehension of a particular concept. Upon receiving this assessment, a typed yes or no, the computer program branches the student

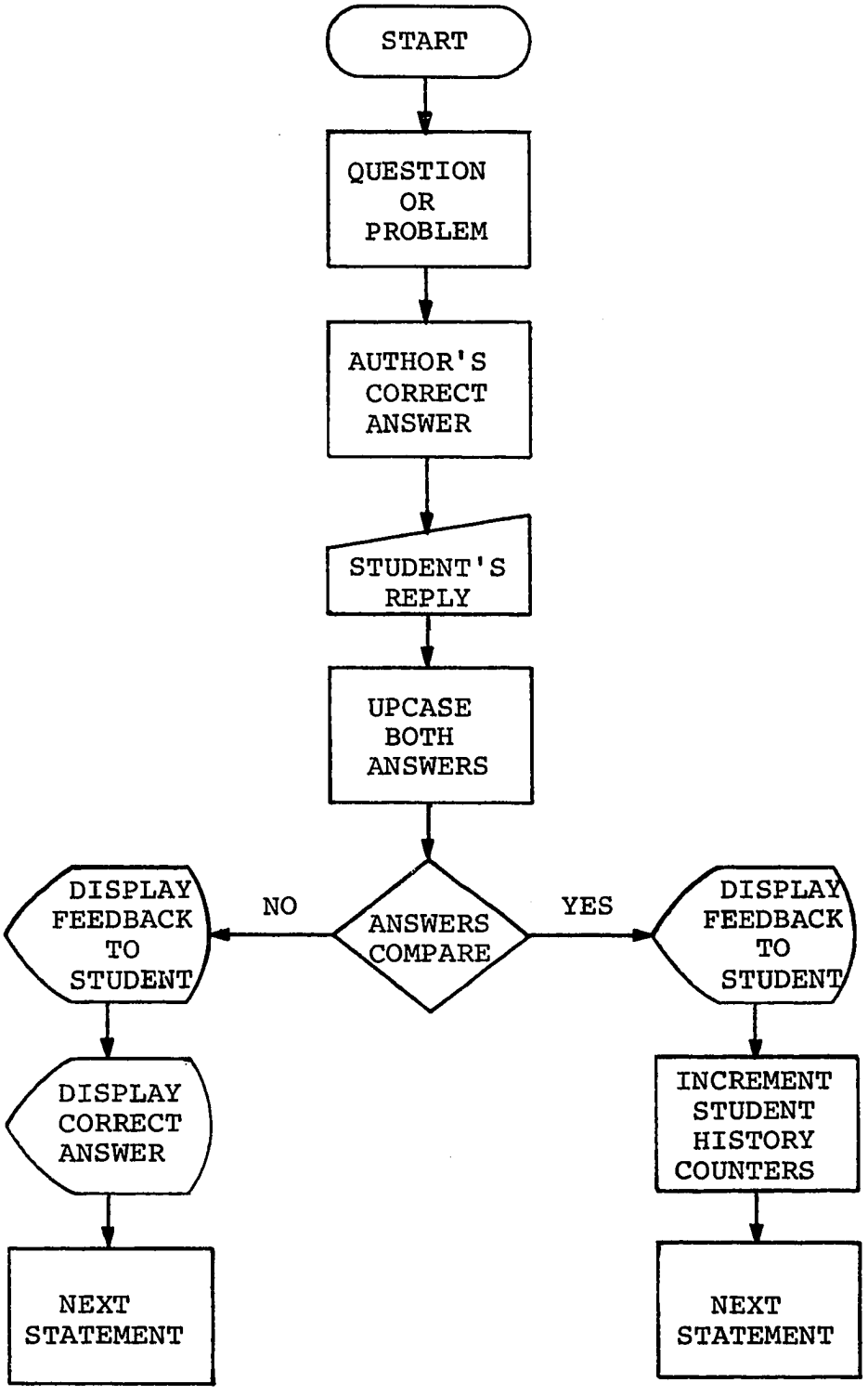

Figure 1. Flow chart of the presentation logic for ANS1

presentation to appropriate material. This feature gives the student the option to skip material in which he feels a degree of proficiency.

The sixth op code, NANS, employs the same logic as ANS1. However, NANS informs the computer-control program that the stored correct answer and the student's reply are numeric values and should be compared within a specified tolerance. In this way, to be considered correct, the absolute difference between the two values must be smaller than a specified number. The first four characters of the argument contain this tolerance which the course author establishes. If no tolerance is specified, a default of .01 automatically becomes the value used by the computer program. This provides an especially useful technique for the elimination of round off in problems requiring a numeric answer.

COMP, the op code for a general-control statement, allows change in the order of presentation of the lesson material. If the student's reply compares with the stored answer, the order of execution branches to a new statement whose label coincides with the first four characters of this statement's argument. If no comparison occurs, the order of execution proceeds to the succeeding physical statement.

GOTO, the op code for an unconditional-control statement, branches the presentation to a statement whose label coincides with the first four characters of the current statement's

argument.

The ninth op code, BRAN, is a conditional-control statement. In this type of statement, the sequence of presentation changes if the student's previous-response history meets a criterion set by the author. This criterion is a percentage stored as the fifth and sixth characters of the statement's argument. If the student's correct response percentage is equal to or greater than the author's set value, the first four characters contain the label of the statement to follow. If the criterion is not satisfied, the next physical statement is executed.

Used to indicate the end of the author's lesson, the tenth and final op code is STOP. The computer-control program calculates and displays the student's percentage of correct responses and terminates the presentation.

The author language is constructed to permit required new features to be added without interfering with the present ones. Some features are particularly designed for needs encountered in the development of the probability unit. If supplemental features to satisfy special requirements are desired, they can easily be implemented by additional op codes and sections of the pre-processor program. Therefore, the language can grow as further sophistication is demanded. Appendix A, Section A, exhibits a summary sheet, briefly describing the special operation codes for easy author use.

The student also has access to four keywords which allow him certain capacities. When the computer requests a reply, the student can input any one of these keywords: EXIT, TERM, SCORE, and CALC.

The keyword EXIT advises the computer program that the student wants to terminate the execution of the lesson without officially completing it; for example, in case of illness, previous commitments, etc. The computer program proceeds to inform the student of the last statement presented and then stops the lesson. This feature enables the student to return to the point where program termination occurred without responding to the material previously studied.

When wishing to proceed further with the lesson, the student merely types the keyword TERM whereupon the computer program asks for the number of the last statement executed before termination. The student replies with this numeric value, and the computer program starts the sequence of presentation.

SCORE requests the computer program to output the student's cumulative correct response percentage. This permits assessment of individual progress during a lesson and aids in making comparisons of the progress.

CALC, the final keyword, instructs the computer program to evaluate the following expression. This allows the student utility of the computational ability of the computer and the

mathematical functions available in CPS. The expressions are written to correspond with the format required under the rules of the CPS language (43). Use of this feature requires the student to have knowledge of a computer programming language, such as Fortran or PL/1.

In the author language just described, a statement consists of 80 characters, a value chosen because it represents the number of positions on an IBM punch card. In this form, the author's lesson materials can be punched on cards and read into the computer. The first four positions of each card are reserved for the label. For organization and ease of reading, the fifth position remains blank, and the sixth through ninth positions are reserved for the operation code. Similar to the fifth, the tenth position remains unpunched. Positions 11 through 80 (70 positions) are set aside for the argument. If the op code designates a control statement, the first four of these 70 positions, 11 through 14, contain the label of the next statement to be executed. Otherwise, these positions are not reserved. The 70 characters of the TYPE statement are not a limitation since the author can simply continue the material on additional TYPE statements. If any of the 70 positions in the argument are not used, they remain blank.

Using the author language op codes, the author designed the probability course and coded it on author-coding paper. Appendix A, Sections B and C, contains an example of the

coding paper and a format of each of the ten op codes. Punched on cards, these statements were processed by one of the computer programs and transferred to disk or tape storage. Accomplished in the batch job processing mode of the IBM 360/65 system, this procedure freed on-line terminal time for student instruction and lowered costs by operating in the less expensive OS/360 framework. Written in PL/1, the computer program is termed the pre-processor and is explained in greater detail in Chapter IV, page 74. The cards containing the course materials were returned to the author for storage and future use. Revisions to the course can easily be made by punching new or additional cards and re-processing the corrected deck.

The pre-processor creates two files of information to be stored externally. One file, called the prose file, contains curriculum material, such as questions, answers, and factual information. The other file, designated the object file, holds information to be used in determining the presentation order.

Another computer program, written in CPS, is titled the executor program. This program actually controls the sequence of presentation of the lesson according to the course author's strategy and is explained in greater detail in Chapter IV, page 84. It interacts with (1) the two files created by the pre-processor program and (2) the student in the presentation of the curriculum material.

The preceding author language, computer programs, and computer hardware illustrate the possible implementation of an instructional system for CAI on a general-purpose time-sharing computer system. It serves as a means for communicating the author's scheme in presenting his course materials to the computer.

C. The Educational Software Component

The third and final component of a CAI system is the educational software which consists of the curriculum material and the teaching strategies for its presentation. Because it is the actual instructional phase, the educational software is a very vital component of the CAI system. The student's success or failure depends upon the author's ability to provide a facility for direct contact with the subject matter and to allow for the redesigning of subject-matter elements that are relevant to particular individual differences. The author anticipates the decisions which are to be made by the computer during the course presentation and provides sufficient data for choosing among the pre-determined alternatives.

Developing and administrating a program are time-consuming activities. Bundy (11) approximates the time at 75 to 150 man hours for one student-contact hour, but this varies with teaching strategy, course content, number of branching choices, and complexity of models. Gloria and Leonard Silvern (73), having taught a course in methods of CAI at the University of

California, state that one student-contact hour of CAI requires approximately 1,000 to 2,000 punched cards. A 45-hour college course worth three units of credit might consume 45,000 to 90,000 cards, and a high school course of 55 minutes for a 20-week semester would take approximately 80,000 to 160,000 cards. Rogers (70) claims that to write CAI requires double the time needed to write programmed instruction which, in turn, takes twice as long to write as a textbook.

Mitzel (52) suggests that most subject matter can feasibly be taught via CAI, although some is especially well suited to this mode of instruction. Alpert and Bitzer (4) seem to confirm this statement in their description of the educational efforts with the PLATO system, which has involved exploratory teaching in at least 20 fields of study and over 100,000 student-contact hours (most for academic credit) in course work at the elementary, secondary, and college undergraduate and graduate levels.

The characteristics of programmed instruction, and particularly those of CAI, were kept in mind while designing this course. Some of these major characteristics are described by DeCecco (17) and Mitzel (52) as:

1. The material is broken down into small steps called frames.
2. Frequent response is required of the student. (Active participation by the student tends to produce

an effective learning experience.)

3. There is immediate feedback to the student. (Knowledge of one's progress contributes to effective learning.)
4. Each item can be viewed as a test question to determine whether or not the student has learned the material previously presented.
5. Appropriate corrective material can be selected if the student has not learned the concepts. (Continued failure by an individual at any level results in ineffective learning.)
6. CAI allows for privacy of responses and results, which reduces inhibitions caused by peer acceptance.
7. The content and sequence of the frames can be subjected to actual tryout with students, and revisions can be made if necessary. This builds a more effective learning device.
8. CAI permits adaptation to individual differences, such as differences in previous knowledge, rate of acquisition, and present performance.
9. CAI has the capability of evaluating student responses and storing information.

Since the present level of knowledge in educational psychology and technology does not permit a specific scientific guideline for designing a CAI course to be devised, the

outline suggested by DeCecco (17) was followed in this study. The development of the probability unit was divided into three major stages: (1) preparation, (2) writing, and (3) tryout and revision. Described by Hickey (39), a similar outline was used by the U.S. Navy in programmed instructional design and development. Conceptually, the two are very similar except for the breakdown of the major stages. The outline used by the Navy was divided into four stages as follows: (1) design, (2) fabrication, (3) validation and testing, and (4) revision.

The preparation stage for this study was subdivided into six categories: (1) selecting a topic, (2) determining instructional goals, (3) developing a content outline, (4) defining course objectives, (5) identifying prerequisite behavior, and (6) testing terminal behavior.

The selection of a topic was approached with the following ideas in mind: to become thoroughly familiar with the subject matter, to restrict the subject matter to a small area, and to choose a subject matter easy to program. Additional considerations were the subject matter-interface characteristics as described by Wodtke (89) and presented in Chapter II, page 16. Furthermore, the IBM 2741 terminal and the author language also limited the choice of subject matter. After careful study, statistics was selected as the broad subject-matter area, and the following four existing projects were reviewed.

The first project examined, by William Cooley (13) of the University of Pittsburgh, employed CAI in a numerical demonstration of statistical concepts and for statistical laboratory-exercise instruction. The project permitted student experience in data analysis by performing the specified statistical program on the appropriate selection of variables and samples. The student used a dataset (stored on disk) which contained data from a large educational survey, Project TALENT.

A second and similar project reviewed, called STATFLOW, was designed and implemented by David Starks (75) of the University of Michigan. It stored data and statistical routines and allowed for student interaction by selecting an appropriate sample and test hypothesis.

Third, IBM personnel have worked on two projects in this area. One, by Hartman and Schoonard (38), developed a CAI statistics problem laboratory. Essentially, this laboratory was a computer recitation of problems prepared by the student as homework. This facility provided limited tutoring and reviewing of homework problems from 13 units, one of which was probability. The second IBM project, by Grubb and Selfridge (31), formulated the first half of an ordinary one-semester college statistics course. This, an early venture into CAI, made the text the primary information device; the computer was used for testing and remedial material instruction. Later,

Grubb (29) modified the project into a learner-controlled statistics course. The student was provided with a series of subject-matter maps and could access various portions of interest to him.

The fourth and last project studied was by SDC (48). In conjunction with the Department of Psychology and the School of Education at UCLA, the Company developed a program for CAI in statistical inference. Difficult concepts of statistics were demonstrated by empirical sampling experiments and numerical examples. SDC also devised tutorial materials in probability and descriptive statistics. An important by-product of these developments was PLANIT, SDC's author language. Because of the built-in statistical routines, this language has great potential for probability and statistics instruction.

After reviewing these projects, a decision was made to (1) limit this study to the tutorial mode of presentation and (2) restrict the subject matter to an area that has implications affecting more-complex concepts. With the assistance of Drs. Trevor Howe and Anton Netusil, Department of Education, Iowa State University, Ames, Iowa, the area of probability was chosen. Dr. Huntsberger (41, p. 66) also notes the importance of probability when he says:

> The mathematical theory of probability provides a basis for the evaluation of the reliability of the conclusions we reach, or of the inferences we make, when we apply statistical techniques to the collection,

> analysis, and interpretation of quantitative data. Since probability plays this important role in the theory and applications of statistics we should acquire at least a nodding acquaintance with the subject.

Since, in statistics, a degree of proficiency in probability is desirable and the students had heterogeneous backgrounds, it was concluded that the graduate educational statistics class would be ideal for a program using the individualizing features of CAI. The author's familiarity with the concepts of probability resulted from teaching a unit in probability to a first-year college mathematics class and reviewing several textbooks (3, 20, 21, 41, 56, 57).

The second, third, and fourth categories of the preparation stage--determining instructional goals, developing a content outline, and defining course objectives--were considered after a topic was selected. However, discussion of these steps is delayed until Chapter V. The importance of these categories is the reason for devoting a full chapter to their descriptions.

The fifth category of the preparation stage involved the identification of prerequisite behavior. Designed for graduate behavioral science students with limited mathematical background and, on a limited basis, for undergraduate classroom use, the program assumed that the students possessed a high verbal ability. Consequently, this assumption limited the need for supplementary audio and visual presentations and

agreed with the existing capabilities of the 2741 interface. Because of the technical nature of the subject matter, the slow typing rate of the 2741 (slower than most college students' reading rates) was not considered a prevailing factor. No other restrictions were assumed concerning aptitude levels, previous relevant knowledge, or interests.

The sixth and final category of the preparation stage, the testing of terminal behavior, was centered upon student proficiency on a set of assigned problems at the completion of each concept block. These problems were designated according to the individual's performance, the percentage of correct responses, during the lesson. The problem difficulty was determined by the rating of each problem by a previous statistics class. This class was given the problems in mimeographed form and, after finding the solutions, every student was asked to rate each problem on a three-item scale: difficult, moderately difficult, and simple. In the process of designating a final rating, the individual student ratings were compared with the appropriate student's final statistics grade. After this step was completed, the problems were divided into the respective groups. Any additional tests of terminal behavior were determined by the class instructor.

The second stage of the development of the probability unit, that of writing, was divided into two parts--the determination of the instructional strategy and the actual writing

of the original draft. Prior to consideration of instructional strategies, a definition of learning was necessary. DeCecco (17, p. 243) defines it by saying:

> Learning is a relatively permanent change in a behavioral tendency and is the result of reinforced practice.

DeCecco (17) further states that three elements are essential for learning: a learner, a stimulus, and a response. In addition, teaching is described simply as a beneficial control and regulation of a learner's stimulus environment. In programmed learning, the program frame is the stimulus, the student's reply to this frame is the response, and the confirmation of the response is the reinforcement or feedback. The computer program, therefore, constitutes an operating model of instruction.

The rules governing the interaction between student and computer comprise an instructional or pedagogical strategy. Suppes (78, 82) denotes three levels of interaction--drill and practice, tutorial, and dialogue, and a description of each of these is contained in Chapter II, page 19. The level of interaction for the instructional strategy of this study was tutorial. In contrast to the learner-controlled mode in which the learner has major control of the learning event, the tutorial mode of this study is identified by Zinn (93) as author controlled. Hickey (39) further subdivides tutorial logic into linear, intrinsic, and adaptive categories.

Intrinsic and adaptive logic require successive computer-made decisions. These program decisions, choices among instructional alternatives as a function of the student's behavior, were used throughout this study. The two tutorial decisions differ on one basic principle: In intrinsic logic, the stimulus presentation is determined by the student's preceding response, whereas, in adaptive logic, the stimulus presentation is determined by a series of responses or behaviors.

The instructional strategy of the probability program was based on three basic assumptions of learning. First, effective learning requires active student participation. Second, learning cannot occur efficiently without the student obtaining immediate feedback on the quality of his efforts. Third, individuals exhibit wide ranges of differences, and effective learning requires recognition and adaptation for these differences.

The lesson frames were sequenced in the order of stimulus, response, and reinforcement or corrective feedback. The stimulus presentation is response dependent, or adaptive to student behavior. Learning was presumed to have taken place at the point of information dissemination, prior to the response. The student's response was used as a method of controlling the course of the programmed material.

The lesson material was organized into three types of instructional sequences. The first of these, the main-line

sequence, consisted of the minimum material for instructional objectives. The second, the remedial sequence, was designed for those students having difficulty with main-line concepts. The third type of sequence, that of enrichment, was given to those students demonstrating sufficient proficiency in the main-line sequence.

In order to further accommodate for the learning differences among the students in each of these instructional sequences, three types of branching were employed. The first type was based upon the correctness of the immediate response. If the response agreed with the stored answer, the student branched to the next frame. If the response did not agree, the student was provided corrective information. The second type of branching was based on the student's assessment of his concept proficiency. Usually, a check frame was provided to ensure correct assessment. The final type of branching was based upon the student's previous performance in the presentation.

Although the probability unit was designed as a tutorial system, the unit should be accompanied by a class-discussion period for every two periods at the terminal. This is important so as to permit deeper insight into the subject matter, to clarify misconceptions, and to allow student interchange of experiences. The sequential order of concepts was established to provide an optimal learning pattern for the

instructional objectives. Normally, this sequence was followed; however, if the student felt proficient in a particular concept, the option of skipping ahead was provided. Also, the student was given the alternative of receiving either the tutorial lesson or the problems at the beginning of each concept block.

The last part of the second stage of the development of the probability unit was the actual writing of the first draft of the tutorial program. During this writing, the course objectives and feasible student responses were constantly kept in mind. The frames were constructed utilizing the characteristics of programmed instruction and the outline of course content and order. The capabilities of the author language were both assets and limitations in designing and coding the presentation. For clarification, the techniques of hinting, prompting, and suggesting were employed as described by DeCecco (17) and Markle (50). Markle's text (50) on a grammar of frame writing was a good introduction to programmed instruction since preparing materials for CAI is different from preparing materials for a traditional lecture class. To conceive and develop a course which adapts to the abilities and interests of any learner requires skills which are not frequently found in teacher-education programs. Upon conclusion of the writing stage, the six concept blocks were written, coded, and punched on cards.

The third and final stage of the development of the probability unit, that of tryout and revision, was separated into two steps--edit and tryout-revision cycle. The probability unit was edited to comply with items in the check list developed by Markle (50). Two examples of these items are: (1) that frames be written concisely and in proper English and (2) that material be correct. The program was then stored in files by using the pre-processor computer program, a program designed to provide error diagnostic messages to the author. The second step of this final stage was the trial of the tryout-revision cycle. The total CAI system was first tested with graduate students, and their reactions and comments were employed to make the initial revisions. The CAI system was then tested on a graduate educational statistics class, which provided valuable data for the tryout-revision cycle. In addition to acquiring revision information, a time estimate was established for actual terminal time per concept block.

A CAI lesson is never completed, and constant use of the tryout-revision cycle is imperative for maintaining current material and for providing individualization.

IV. DESCRIPTION OF COMPUTER PROGRAMS

The author language described in Section B of Chapter III is a convenient notation which permits a course author to communicate instructions for the presentation of the curriculum material to the computer. These instructions are carried out by the computer under the direction of specially written computer programs. The computer programs are designed to (1) recognize each operation code defined in the author language and (2) include appropriate computer statements to execute each operation code.

This chapter contains a detailed description of these programs--file-creator, pre-processor, and executor. Since the programs perform independently, the remainder of this chapter is divided into three sections, one for a discussion of each.

A. File-Creator Program

The CPS manual (43) states that information stored on external disk files can be read internally by CPS programs and that this material can be written on the disk by other programs operating under OS/360. Basically, this is what has been done in this study. The executor program, the CPS program, reads the curriculum material which has been written on external disk files by the pre-processor program, the PL/1 program operating under OS/360. However, before it is

possible for the lessons to be written on the disk by the pre-processor program, the files must exist and appropriate space be allocated. The file-creator program accomplishes this step by creating two dummy files, identified with the same name as the two prospective curriculum files.

Written in CPS, the file-creator program is loaded into the terminal's assigned memory under the code name CREATOR. As the first stage of execution, the computer program requests the course author to reply by typing a unique four-letter code name that identifies the file. Usually this code name has some relationship to the curriculum material and identifies the object file which holds five records, each possessing 1,800 characters. Every record is a 100-element array, and each element of that array consists of three 6-digit numbers. Next, the three numbers of each of the 100 elements are initialized to zero. And finally, this initialized array is written on the disk five times to create the first dummy file, containing five records, all digit 0's. Thus, the process by which the file space is allocated and made available for the object-file information to be recorded by the pre-processor program is completed.

Furthermore, the file-creator program concatenates the four-letter file name given by the user with the letter P to form a five-character name for the second file. This file is comprised of seven 3,500-character records. Each record is a

50-element array, and every element has 70 characters. The file is initialized by writing the phrase PROSE FILE for each element of every record. At this point, the second file space is allocated and made available for the prose information to be recorded on it by the pre-processor program.

Following the creation of each file, the file-creator program displays a file list, making certain the necessary storage has been allocated. A listing and a flow chart of the file-creator program is contained in Appendix B, Section A.

B. Pre-processor Program

After the curriculum material has been coded and punched on cards, it is ready to be translated by the pre-processor program. Written in PL/1, this program pre-processes the curriculum material and formulates a prose file and an object file. The purpose of the prose file is to store the lesson material, such as questions, answers, and textual information; and the object file holds the location and type of each item in the prose file and the information needed for branching. Containing an element for each author statement, the object file provides the control information to the executor program, enabling the latter to make a selection from various educational alternatives. The lesson material and the control information are stored on disk as regional files rather than consecutive ones because in regional files the computer provides control of the physical placement of the records and

accesses them upon demand. On the other hand, in consecutive files, the records can be accessed only in sequential order. Placement of the regional records is accomplished by storing each record with a key. These records are then accessed by the same key, which optimizes the record access time if the records are read out of sequential order. In CAI, the order of curriculum materials differs with individuals, and file records must be readily accessible in any sequence. Regional files accomplish this important function.

In the first section of the pre-processor program, author statements, the curriculum material, are read individually by the program. The maximum number of author statements which can be read at one execution of the program is 500. Since this is an arbitrary figure, it can be increased; however, 500 statements in most cases is sufficient for a 30-minute CAI lesson. For later use, these author statements are internally stored in the program in an array of 500 eighty-character elements.

As the author statements are read, a label table composed of two arrays containing information concerning the location of the labels is created. A label table is necessary because the computer must be able to locate any specified label when processing a control statement later in the program. The contents of the label table's first array are the labels; and under the assumption that, at most, only 300 of the 500 author

statements would require a label, 300 is set as the maximum number of labels. The second array of the label table is comprised of pointers indicating the location of the label in the author statements. As these author statements are processed, each label is compared with previous ones to ensure uniqueness. If identified as a duplicate, the program outputs an error-diagnostic message to the author informing him of the mistake and increments an error counter accumulating the number of author errors. Upon encountering an error, the remainder of the author statements are processed, but the files are not created. For future use, both arrays are internally stored in the computer program.

Upon completing the reading of the author statements and building the label table, the pre-processor program prints a listing of the author statements and the label table via the line printer. A listing of the author statements for each concept block can be found in Appendix C. Table 1 is a segment of a label table taken from Concept Block One. The first column consists only of line numbers, the second column lists the labels which in this case are numeric, and the last column indicates the location of the labels in the author statements. By examining Item 10 in this table, it can be determined that the statement with 28 as its label is the eightieth author statement.

Table 1. Example of a label table

Line number	Label	Location
1	10	14
2	12	21
3	14	26
4	16	33
5	18	46
6	20	50
7	22	58
8	24	65
9	26	73
10	28	80
11	32	90
12	34	94
13	36	99
14	30	103
15	38	111

The second section of the pre-processor program employs the stored author statements and label table to build an object and a prose array. The object array contains 100 elements, each comprised of three 6-digit numbers, and the prose array has fifty 70-character elements. When either array is filled, the information stored within is written on an external-storage disk as a record of the particular file. These files are constructed by processing the stored author statements, one at a time, as follows: The op code of each author statement is compared with a pre-stored set of ten symbols. When a match is identified, the control branches to the subsection of the program which processes the particular

op code. If no match is recognized, an error-diagnostic message informs the author of an invalid operation code, the error counter is incremented by one, and the program proceeds to the next statement. The next several paragraphs describe in detail the processing steps of the ten op codes.

For the op code TYPE, the first number in the appropriate element of the object array is assigned the value 1 which identifies to the executor program that the argument of this statement should be processed as a TYPE statement. Since the object array requires a numeric value rather than an alphabetic letter, the digit 1 is associated to have an identical meaning as the op code TYPE. The remaining op codes are associated with numerals in a similar manner. Next, the program determines the length of the character string in the argument of the statement and stores this value as the second number in the proper element of the object array. Finally, the statement's argument is saved as the respective element of the prose array, and its location in the prose file is stored as the third number in the element of the object array.

The following example is included to add meaning to the description of the processing steps of op code TYPE. Specifically, this example is the eighth line in Concept Block One which states: TYPE THE LETTERS A AND B CAN BE ORDERED OR ARRANGED IN A ROW AS AB OR ___. The element of the object array for this statement is: 1 69 8. The first number in

the element, 1, identifies to the executor program that the lesson material is to be executed as a TYPE statement. The second number in the element, 69, indicates to the executor program that the curriculum material is 69 characters in length. The third number in the element, 8, informs the executor program that the curriculum material is stored as the eighth element in the prose file. The appropriate element of the prose array would read: THE LETTERS A AND B CAN BE ORDERED OR ARRANGED IN A ROW AS AB OR ___.

For the next op code ANS1, the program assigns to the first number in the appropriate element of the object array the digit 2. As for the first op code, the program determines the length of the character string and stores this value as the second number in the element of the object array. The statement's argument is saved as the present element of the prose array, and its location in the prose file is stored as the third number in the element of the object array.

The preceding process is clearly illustrated in the following example. This example, the fifty-fourth author statement of Concept Block One, reads: ANS1 3|THREE. The element of the object array for this statement is: 2 7 44. The first number in the element, 2, signifies to the executor program that the statement is an answer, and the second number in the element, 7, designates to the executor program that the answer is seven characters in length. The third number in the

element, 44, denotes to the executor program that the answer is stored as the forty-fourth element in the prose file. The appropriate element of the prose array is: 3|THREE.

A procedure identical to that of ANS1 is employed for the op codes ANS2, NANS, BANS, and WANS, with the exception that the digits 3, 4, 5, and 6, respectively, are assigned to be the first numbers in the appropriate element of the object array. Since examples of the processing of these op codes would be much the same as that illustrating ANS1, sample statements from Concept Block One are omitted.

For the seventh op code, COMP, the first number in the present element of the object array is assigned the identifying digit 7. Since it is a control statement, the program first reads the four-character label which is contained as the statement's argument and then searches the label table for a match. Upon matching, the location of the appropriate author statement is stored as the second number in the element of the object array. However, if this label is not matched with another in the label table, an error-diagnostic message informs the author of the mistake, the error counter is incremented by one, and the program proceeds to the next statement. In this control statement, the third number in the element of the object array is never used and retains the value of 0.

The thirty-ninth author statement of Concept Block One, used to illustrate the processing of op code COMP, is as

follows: COMP 18. The element of the object array for this statement is 7 46 0. The first number in the element notifies the executor program that the statement is to be executed as a COMP statement. If a decision is made by the executor program to change the order of presentation, the sequence is branched to the forty-sixth element of the object array, specified by the second number. As stated in the preceding paragraph, the third number in the op code COMP is not used and therefore is at all times 0.

Following the same pattern for the op codes GOTO and BRAN, the numerals 8 and 9, respectively, are assigned as the identifying values. Also for BRAN, the percentage criterion established by the author is stored as the third number in the element of the object array.

To clarify op code BRAN, author statement 293 in Concept Block One is used as an example. It reads as follows: BRAN 8660. The associated element of the object array is 9 311 60. The first number, 9, identifies this statement as the conditional branch statement BRAN. The third number, 60, is the percentage criterion. If the student's correct percentage is equal to or greater than 60, the order of presentation is branched to statement 311, the second number in the element of the object array. An example for the op code GOTO would be identical to that of op code COMP, except for the digit 8 as the first number in the element of the object

array. Therefore, a sample statement for GOTO is omitted.

For the final op code, STOP, the program stores the digit 10 as the first number in the proper element of the object array. Two additional numbers remain unused and retain the value 0. Thus, elements of the object array for the STOP statement are always designated as 10 0 0.

Upon completion of the processing of an author statement, counters for the number of elements in each array are incremented; and the two arrays, the object and the prose, are checked to determine if all the elements have been completed. If an array is not complete, the next statement is processed. Once an array is concluded and, if the error counter is 0, the array is written externally to the program on disk as a record of the specific file. The record, then, is stored on disk with a file-key, starting with the value 0 and incrementing by one for each additional record. If the error counter is greater than 0, the array is not written as a record in the file; and the author must make the necessary corrections for reprocessing. For the author's confirmation, the array is then printed via the line printer. Finally, the array counter is initialized, and the next statement is processed.

After the last author statement has been processed, any remaining, unused elements of each array must be completed. To complete the object array, the remaining elements are assigned the value of 11, and the remaining elements of the

prose array are stored as blanks. The arrays are then written on the disk as the last record for each respective file. Finally, the object and prose arrays are printed via the line printer. A listing and a flow chart of the pre-processor program is contained in Appendix B, Section B.

Table 2 is an example of sections of the first and last records in the object file for Concept Block One. The first column is comprised of the digits 1 through 10 which identify the particular op code. The second column contains one of the following: the length of the statement's argument for op code values 1 through 6, the location of the next statement in the object file to be executed for op code numerals 7 through 9,

Table 2. Example of an object file

First number	Second number	Third number
1	45	1
1	65	2
1	68	3
1	70	4
1	70	5
1	66	6
2	5	16
7	21	0
1	67	17
8	14	0
3	5	32
5	3	40
9	311	60
1	67	259
8	350	0
9	331	80
10	0	0

or a 0 for op code 10. The third column is composed of (1) the location of the statement's argument in the prose file for op codes 1 through 6, (2) a percentage criterion for op code 9, and (3) a 0 for op codes 7, 8, and 10.

Figure 2 is a segment of the first record in the prose file for Concept Block One. This section involves the argument of statements with op codes 1 through 6, the curriculum material.

C. Executor Program

The executor program, which is written in CPS, interacts with the student and the two curriculum files via the student's terminal. This program controls the sequence of the lesson presentation and processes the student's replies. It accesses one record at a time from the object and the prose files by calculating the appropriate key. The program also allows the student to use the capabilities of the four keywords as described in the author language.

The student initiates interaction with this program by completing the following steps, called the sign-on procedure. First, the student turns on the 2741 terminal and dials the computer, using the attached dataphone and appropriate telephone number. The student then follows the customary CPS login procedure, which identifies the master account and the sub-account for accounting purposes. Next, CPS requests a password from the student, and upon replying with the proper

CONCEPT BLOCK ONE: MULTIPLICATION PRINCIPLE.
FOR GENERATIONS, PEOPLE HAVE BEEN INTRIGUED BY PROBLEMS REQUIRING THEM TO FIND THE NUMBER OF WAYS OF ARRANGING A SET OF OBJECTS. SUCH COUNTING QUESTIONS ARE FASCINATING AND CHALLENGING IN THEIR OWN RIGHT. HOWEVER, WE SHALL CONSIDER THEM FOR AN ADDITIONAL REASON. THIS REASON IS THAT STUDENTS OFTEN NEED ANSWERS TO SUCH QUESTIONS IN THE STUDY OF PROBABILITY. LET'S CONSIDER SOME EXAMPLES.
THE LETTERS A AND B CAN BE ORDERED OR ARRANGED IN A ROW AS AB OR ___.
(IF YOU ARE NOT SURE OF THE ANSWER, GUESS AND SEE WHAT HAPPENS.)
BA
ORDER IS THE ESSENCE OF SUCH ARRANGEMENTS; A CHANGE IN ORDER YIELDS A DIFFERENT ARRANGEMENT. SO AB IS CONSIDERED DIFFERENT FROM BA.
EACH DIFFERENT ARRANGEMENT OF A AND B (AB OR BA) IS CALLED A PERMUTATION. A PERMUTATION OF A NUMBER OF OBJECTS IS ANY ARRANGEMENT OF THESE OBJECTS IN A DEFINITE ___.
ORDER
TO PERMUTE OBJECTS IS TO ARRANGE THEM IN A ROW IN A DEFINITE ORDER.
ABC AND ACB ARE TWO POSSIBLE ___ OF A, B, AND C, TAKEN TOGETHER.
PERMUTATION|PERMUTATIONS|ARRANGEMENT|ARRANGEMENTS
NOTICE THE DIFFERENCE IN THE TWO ARRANGEMENTS. THEY ARE PERMUTATIONS OF A, B, AND C.
ANOTHER PERMUTATION OF A, B, AND C, TAKEN TOGETHER, IS ___.
BCA|BAC|CBA|CAB
TO FORM A PERMUTATION, SELECT ANY ONE OF THE LETTERS FOR THE FIRST ELEMENT. THEN, SELECT EITHER OF THE REMAINING TWO LETTERS FOR THE SECOND ELEMENT, LEAVING ONE LETTER FOR THE LAST ELEMENT.

Figure 2. Example of a prose file

word, the student is connected to the computer. The executor program is loaded into the terminal's memory using the code name CAICON; and when the student types XEQ or EXECUTE on the terminal's keyboard, the executor program initiates execution. The executor program first displays a message at the terminal that requests the student to indicate which lesson he wishes to study, and the student replies by typing a four-letter code word from a table of contents which is provided. The executor program presents the lesson and processes the student's replies. At the completion of the lesson, the student signs out and turns off the terminal. An example of the sign-on procedure comprises Appendix D.

The following paragraphs contain a detailed description of the structure of the executor program. As previously stated, the program queries the student as to which lesson the student wishes to study. The student's reply, a four-letter code word, identifies the name of the object file for the particular lesson requested and concatenated with the letter P forms the name of the prose file. Next, the first records of the prose and object files are read into a prose array and an object array, respectively. These two arrays are defined with the same dimensions as stated for the pre-processor program. The first number in each element of the object array identifies the particular op code, and the program branches to the specific section responsible for execution of the command.

Following is a description of each of these sections.

In the section representing the op code TYPE, the second and third numbers of the appropriate element of the object array inform the program of the curriculum material's location in the prose file and its length, respectively. By interpreting this information, the program is able to locate and display the curriculum material to the student via his terminal. Information received relating to length of the material is used by the program to save time by not displaying excess blanks.

In the section representing the op codes ANS1, ANS2, NANS, BANS, and WANS, the second and third numbers of the proper element of the object array tell the program the location of the answer in the prose file and its length, respectively. By using this information, the program stores the element of the prose array as the correct answer. The program then requests a reply from the student. Employing the CPS function UPCASE, both the stored answer and the student's reply are converted to capital letters. The student's reply is first checked to see whether or not it compares with one of the four keywords. If no match can be made, the student's response is then compared with the correct answer through a pattern match, using the CPS function INDEX. This function permits the answer to be a sequence of acceptable correct answers by searching a character string, the stored correct answer, for a specified

substring pattern, the student's reply. If the correct pattern is found, an integer representing the starting location of the first occurrence of the substring, within the character string, is returned as the value of the function. If this value is other than the digit 0, the student's reply is evaluated as correct and a correctness counter is assigned the value of 1. However, if the student's reply is judged to be erroneous, the correctness counter is assigned the numeral 0.

When encountering the op codes BANS and WANS, the program proceeds to the next element of the object array. Rather than using the pattern comparison previously described, the op code NANS compares the stored answer and the student's reply through application of a numeric comparison. In this process, both answers are changed to floating-point decimal numbers by the CPS function FLOAT. The program determines whether or not the author specifies a tolerance level, denoted by the first four characters of the element of the prose array. If no tolerance is specified, the program assumes a default of .01. The absolute value of the difference between the stored answer and the student's reply is calculated using the CPS function ABS, and the value of this function is compared with the tolerance level. If less than it, the program judges the student's reply to be correct, and the correctness counter is assigned the value 1. On the other hand, if the value of the CPS function ABS is greater than the tolerance level, the

student's reply is not acceptable and the correctness counter is assigned the digit 0.

In executing the op code ANS2, the program increments a score counter by one if the correctness counter is 1. This process accumulates the number of correct replies, and the program proceeds to the succeeding element of the object array. If the correctness counter is 0, the score counter is not incremented.

A process identical to that previously described for ANS2 is executed for op codes ANS1 and NANS. However, rather than continuing next to the following element of the object array the program branches in accordance with the contents of the correctness counter. If the value of the correctness counter is 1, the program randomly chooses a confirmation message to be displayed to the student. This selection is accomplished through application of the CPS function RANDOM, which calculates a random number between 0 and 1. Each of the confirmation messages is associated with a range of numbers, and the message selected to be displayed to the student is chosen according to the range within which the random number lies. Finally, the program proceeds to the next element of the object array. If, on the other hand, the correctness counter is 0, the program randomly chooses a corrective message, through a process identical to that of selecting a confirmation message, which is displayed to the student informing him

of his error and providing the correct answer. If the correct response can be any one in a sequence of possible answers, only the first of these is displayed to the student. After displaying an answer, the program continues to the succeeding element of the object array.

The section of the computer program which executes the control statements functions as follows. If the correctness counter is 0, COMP directs the executor program to execute the next element of the object array. However, if the correctness counter value is 1, the order of presentation is changed by branching to a new element of the object array, which is identified by the second number in the appropriate element. The program then proceeds to execute the new element of the object array. This section of the executor program also carries out identical instructions for changing the order of presentation for the op code GOTO.

The third and final op code which executes control statements is BRAN. The third number in the element of the object array is compared with the student's percentage of correct responses; and, if this percentage equals or exceeds the author's criterion, the order of presentation is changed through the process described in the preceding paragraph. If the student's percentage is less than the author's criterion, the succeeding element in the object array is performed.

Whenever a new element in either the object or the prose

array is to be executed, the program checks to determine whether or not it is within the limits of the record stored in each respective array. If so, the program executes the element. If not, the key for the necessary record is calculated, and this record is then read into the respective array. Finally, the new element is carried out.

Following is a description of the section of the executor program which permits the student to use features of four keywords, CALC, SCORE, EXIT, and TERM. When the student types the keyword CALC on the terminal keyboard, the executor program evaluates the expression following this keyword by using the CPS function EVAL, which calculates the expression and returns a numeric value for it. Then the program requests another reply from the student, which can be (1) any one of the keywords or (2) the student's reply to the previous question. If the student types the keyword SCORE, the internal LET function PERCEN calculates the student's percentage of correct responses. The executor program displays this value to the student and then requests another reply. After processing the keywords CALC and SCORE, the control of the program returns to the section previously in process.

If the student types the keyword EXIT, the number of the last completed element of the object file is displayed, and the executor program terminates operation. Whenever the student wishes to resume the lesson, he completes the sign-on

procedure and then types the keyword TERM at the first opportunity to reply. The executor program next requests the statement number supplied at the previous termination, and, using this number, the program branches the presentation to the appropriate element of the object file to continue the lesson in progress.

The last section of the executor program carries out the op code STOP by displaying the student's percentage of correct responses and then terminating the program. A listing and a flow chart of the executor program is contained in Appendix B, Section C.

V. DESCRIPTION OF PROBABILITY CONCEPT BLOCKS

As stated in Chapter III, Section C, the first step in the development of the probability unit was the preparation stage. This stage was subdivided into the following six categories: (1) selecting a topic, (2) determining instructional goals, (3) developing a content outline, (4) defining course objectives, (5) identifying prerequisite behavior, and (6) testing terminal behavior.

While a discussion of Categories 1, 5, and 6 is contained in Chapter III, this chapter will expound upon Categories 2, 3, and 4 of the preparation stage. Section A is devoted to Categories 2 and 3, whereas the remaining category, defining the course objectives, is discussed in Section B.

A. General Objectives

After deciding upon probability as the subject matter of the curriculum material, the next step was to specify what should actually be included in the tutorial presentation. To facilitate the decision-making process, the question of how the probability unit should relate to other statistical concepts was pondered. Through this analysis, two instructional goals were identified. Defined in general terms, an instructional goal is the broad statement or general objective of the curriculum material. It can also be considered a task description.

The first of the two instructional goals determined for the probability unit is to provide the student with an understanding of the theory of binomial expansion, permitting the binomial distribution to be employed as:

1. a mathematical model for the description of a variety of real-life phenomena,
2. an introduction to normal distribution and, later, to additional theoretical distributions.

The second instructional goal is to provide the student with an understanding of basic principles of probability frequently employed in modern statistics, such as:

1. that of representing probability as an area under the normal curve,
2. that of calculating the probability that a random variate X will fall between two given values of a continuous distribution,
3. that of applying probability in tests of hypothesis (statistical inference), such as critical regions, confidence intervals, and levels of significance,
4. that of applying probability in estimation,
5. that of calculating the probability of repeated binomial trials.

The two instructional goals not only describe but also assist in analyzing the tasks that the student is to perform. The goals were analyzed by defining the prerequisite behaviors

necessary for the student to attain each goal. This procedure is called a task analysis and identifies the sub-tasks which must be learned in the process of learning the original tasks or goals. The task analysis generated a first-level hierarchy of sub-tasks or prerequisite skills and knowledge.

The identification of the sub-tasks was aided by the careful examination of a number of textbooks and reference sources. Also, professors experienced in teaching the concepts of probability assisted this process by providing additional insights and illustrations.

These sub-tasks designated the major subject-matter areas for the probability unit and supplied an initial ordering of the presentation sequence. Using this material, a content outline, Category 3 of the preparation stage, was developed in which the probability unit was divided into six concept blocks as follows: multiplication principle, permutations, combinations, binomial theorem, basic probability, and probabilities of combinations of events. The identification of these concept blocks completed formulation of the content outline, and expansion of these concept blocks assisted in defining the course objectives.

B. Objectives of Each Concept Block

A more extensive task analysis of each of the six concept blocks identified additional sub-tasks or concepts. These concepts determined the actual structure and order of the

curriculum material to be taught and were employed to construct the specific course objectives. These instructional objectives were written in behavioral terms and were used as a guideline in composing the particular frames. The following paragraphs contain descriptions and instructional objectives of each of the six concept blocks.

Concept Block One, the multiplication principle, is an initial introduction to permutations and to a method of counting them. The first problem, that of finding the number of permutations of three letters, is solved by applying a counting technique called the multiplication principle. This principle is then employed to determine the number of permutations in two additional problems and also a third which, although not a permutation problem, is solved using the same technique.

Upon completion of this concept block, the student should be able to:

1. write a sample permutation given a set of letters or objects,
2. calculate the number of permutations of N objects using the multiplication principle,
3. state that the number of alternatives for each successive element in a permutation is one less than that of the element immediately preceding it,
4. calculate the number of permutations of N objects,

taken R at a time, using the multiplication principle,

5. calculate, through application of the multiplication principle, a counting problem which allows items to be repeated.

Concept Block Two continues with the principles of permutations by presenting some simplifying notations and formulas. First, the number of permutations of N objects, taken R at a time, is denoted as PER(N,R). Next, factorials are introduced as a convenient method of representing the factors derived from the multiplication principle in problems such as PER(10,10). Finally, for students achieving at least 50 percent correct, the following two formulas are developed: (1) PER(N,R) = N! / (N-R)! and (2) PER(N,N) = N!. The remaining students are branched to the practice problems.

The following are the objectives for the second concept block, and upon completion, the student should be able to:

1. denote the number of permutations of N items, taken R at a time, as PER(N,R),
2. evaluate a factorial, such as 6!,
3. write PER(5,5) as 5! and evaluate it,
4. solve a permutation problem by either applying the multiplication principle or a formula,
5. state and evaluate the two formulas, PER(N,R) = N! / (N-R)! and PER(N,N) = N! (for students achieving 50 percent accuracy or better).

In the third concept block, combinations are distinguished from permutations, and the notation C(N,R) is used to denote the number of combinations. The formula C(N,R) = PER(N,R) / R! is developed by comparing the number of combinations with the number of permutations in a numeric problem.

For those students achieving at least 60 percent accuracy, the formula C(N,R) = N! / (N-R)!R! is derived from two preceding formulas: C(N,R) = PER(N,R) / R! and PER(N,R) = N! / (N-R)!. Using the examples C(8,2) and C(8,6), the student is also shown that C(N,R) equals C(N,N-R). Students achieving under 60 percent correct are presented a method of solving the number of combinations involving use of a modified multiplication principle.

An enrichment section pertaining to permutations of like items is provided to students achieving at least 75 percent accuracy. This section develops a formula for calculating the number of permutations of N items of which N1 and N2, respectively, are alike.

The student, after completing Concept Block Three, should be able to:

1. distinguish between combinations and permutations,
2. denote the number of combinations of N items, taken R at a time, as C(N,R),
3. express C(N,R) as PER(N,R) / R!,
4. solve a combination problem by either applying the

formula C(N,R) = PER(N,R) / R! or the formula C(N,R) = N! / (N-R)!R!,

5. evaluate the number of permutations of N items of which N1 and N2, respectively, are alike (for students achieving 75 percent accuracy or better).

The background for the theory of binomial expansion is provided in Concept Block Four by defining such terms as binomials, exponents, coefficients, and the degree of the expansion. The expansion is first illustrated by multiplying two binomial factors. After analyzing this expansion, the same binomials are expanded by applying the binomial theorem. The factors of the expansion are written by inspection of the original expression, and the coefficients are written as the number of possible combinations. The preceding method is then applied to the expansion of a binomial to a power greater than two.

Upon completion of this concept block, the student should be able to:

1. select the exponent of a term and write factors as powers,
2. write the degree of a given term,
3. determine the number of unique terms in an expansion of a binomial,
4. expand a binomial of power two by direct multiplication of the binomial factors,

5. write the terms in an expansion of a binomial and calculate the corresponding coefficients,
6. state that in the expansion of (A+B)**4 the exponent of the A factor decreases by one for each term in the expansion and the exponent of the B factor increases by one.

Basic probability is introduced in the fifth concept block by examining a simple experiment, that of flipping a coin. Rather than striving for comprehension of the various theories concerning probability, only an operational understanding of probability is sought in this concept block. The terms outcome, event, and sample space are defined. The symbol P(E) is used to denote the probability of event E. The probability of an event is calculated as the number of outcomes in the sample space. Various types of experiments are described to provide practice in determining probabilities. The lesson concludes by deriving the fact that the probability of an event is between zero and one, inclusive.

After completing Concept Block Five, the student should be able to:

1. state that the probability of event E is ¼ as P(E) = ¼,
2. describe a typical outcome of the sample space in an experiment,
3. list the three steps in a probability model,

4. state that the minimum value which can be assigned a probability is zero,
5. state that the maximum value which can be assigned a probability is one,
6. calculate the probability of a given experiment.

The final concept block considers combinations of events and how to determine their probabilities. Mathematical set notation is employed to represent both the sample space and the event. The symbols N(S) and N(E) are introduced to denote the number of outcomes in the set that designate the respective sample space and the event. Thus, the formula for probability becomes P(E) = N(E) / N(S).

The two set operations, union and intersection, are two ways of combining events and lead to the following formulas. Given two sets, A and B, the probability of A union B is expressed as P(A OR B) and is evaluated as N(A OR B) / N(S). The probability of A intersection B is denoted as P(A AND B) and is calculated by N(A AND B) / N(S). After examples to illustrate these concepts, the student is provided with several practice problems.

In addition, two special set relationships which influence the above set operations are described. The first, mutually exclusive, gives the formula P(A OR B) = P(A) + P(B) which leads to the general formula P(A OR B) = P(A) + P(B) - P(A AND B). The second, independent, permits use of the

formula P(A AND B) = P(A) x P(B). In concluding Concept Block Six, students achieving at least 75 percent accuracy are given a brief encounter with conditional probability.

Upon completion of this concept block, the student should be able to:

1. calculate the probability of event E using the formula P(E) = N(E) / N(S),
2. determine the number of outcomes in the union of two events,
3. determine the number of outcomes in the intersection of two events,
4. calculate the probability of the union of two events,
5. calculate the probability of the intersection of two events,
6. calculate the probability of the union of two mutually exclusive events by applying the formula P(A OR B) = P(A) + P(B),
7. list the condition that holds for two events to be mathematically independent,
8. determine the probability of two independent events using the formula P(A AND B) = P(A) x P(B),
9. calculate the conditional probability of event A given event B using the formula P(A|B) = P(A AND B) / P(B) (for students achieving 75 percent accuracy or better).

This, then, concludes a brief description of the six concept blocks, as well as an itemization of the objectives of each. A listing of each concept block is contained in Appendix C.

VI. DISCUSSION

The CAI unit on probability described in the preceding chapter was presented to a group of 17 students enrolled in a graduate educational statistics class. The students appeared to be very heterogeneous in previous behavioral patterns and were similar only with respect to majoring in the behavioral sciences, primarily education. So, with these differences, the class seemed to be an excellent one for a trial testing of the unit. The students were scheduled for approximately one concept block every two days. However, the option was provided for self-pacing, which allowed for the completion of the concept blocks according to a student's schedule or motivation. The students were able to ask questions of the instructor and refer to the class-required text or any additional supplementary material. Upon completion of each concept block, a printout of each student's CAI presentation was analyzed and student-provided notations were studied. Furthermore, pertinent comments made by students during informal discussions were evaluated.

Some of these ideas, in addition to other observations made during the course of the study, are discussed in the following section of this chapter. Also, several of the limitations of the study are considered. Recommendations for future development of the CAI unit on probability are reflected upon in Section B of this chapter.

A. Observations

Student reaction to the CAI unit on probability seemed to be polarized: either it was extremely favorable or very unfavorable. However, the majority of the students reacted approvingly, with many intrigued by this new instructional method. After completing the unit, numerous students expressed the hope that additional units would be available for use.

Nearly all of the students appeared highly motivated by the new instructional method and were eager to complete the six concept blocks. Some of the students were so enthused that they selected the self-pacing option and completed the probability unit within two days. In fact, one student with a mathematical background finished the unit using only one session at the terminal. This, however, was not recommended although it emphasized the importance of adapting instruction to individual differences found in students. On the other hand, another student stopped work after the first concept block and never completed the unit. These two extremes illustrated the polarity: the enthusiasm for CAI or the lack of it.

A few of the students worked through several of the concept blocks more than once. This was done for review purposes, and conscientious students used it as a means of measuring progress. An accurate method of recording the number of times a student repeated a concept block was not employed, and the students usually submitted only the best lesson. The more

capable students seemed to especially enjoy CAI because it allowed self-pacing, which is lacking in most conventional classes. Feedback was also viewed as a positive feature, and students requested that it be added in situations where it had been intentionally omitted.

Student enthusiasm again was demonstrated when the computer encountered system problems, and the system was not operational for a full day. Students did not lose interest and continually inquired as to whether or not the system was functional. However, because of this breakdown it was necessary to set up a new terminal-assignment schedule.

The students provided valuable information by indicating terms or sections of the program that were not clearly stated or could be easily misunderstood and by making suggestions for improving these areas. Through the analysis of student responses, additional acceptable correct answers and clarifying statements were added.

The situation that permitted the probability unit to be trial tested evolved so suddenly that the students were deprived of a well-prepared introduction. It was necessary to introduce the unit without a terminal available as a display example, and the students' first encounter with a terminal and with a CAI lesson was during the presentation of the first concept block. Many of the initial errors made by the students were related to unfamiliarity with the terminal's

keyboard. Typing errors also accounted for a share of the initial errors.

In addition, the lack of a meaningful introduction resulted in many students not using the presentation as it was intended, especially while working through the first concept block. By answering only when the correct answer was known, the students carried over a conventional classroom inhibition. This resulted in slow responses and limited use of the built-in individualizing features of the lesson. At first, too, the students considered the CAI presentation a test rather than an instructional method, and this belief was reinforced by the executor program which randomly provided the student with a percentage of correct responses. Rather than using the information as a progress report, the students thought of it as a grading process. This function of the executor program was eliminated, and the students were instructed as to the proper use of the CAI presentation.

At the end of two terminal sessions, nearly all of the students were more relaxed and functioning well within the new instructional environment; and in most instances, the final correct percentage increased with each succeeding concept block.

The amount of time required to complete a concept block ranged from 19 to 53 minutes, with an average range of 25 to 30 minutes. The higher amounts of time were required for the

first concept block and by individuals lacking a strong verbal command of the English language. Scores ranged from 43 to 100 percent correct, with the lowest average scores on Concept Block One. The average score for all concept blocks was approximately 79 percent correct. Cost estimates were $6.90 per terminal hour or $4.08 per concept block. However, these estimates were probably high since they reflected only one student session per concept block, whereas students were free to use the terminal whenever it was available. Also, several of the students brought friends to view and experiment with an operational CAI presentation. Furthermore, demonstrations were given for members of the faculty of the Iowa State University College of Education.

Limitations discussed in the remainder of this section were encountered during the development and trial testing of the CAI unit. One of the major restrictions was the inability of the response processing section to distinguish between legitimate errors and what should have been evaluated as correct answers but, because they were entered in a wrong form, were regarded as incorrect. These included typing and spacing errors. One of the problems causing this was the inflexibility of the exact-answer match which required student replies to be identical to one of several author-designated answers. Students often became frustrated when they correctly answered a question but were branched to remedial sequences

because their answer was not recognized as being acceptable by the program.

Comprised of 52 alphabetic letters (26 small letters and 26 capital letters), ten numerals, and 23 special characters, the typewriter symbol dictionary was a limitation in the lesson construction as it restricted the usage of standard mathematical notation, especially subscripts and superscripts. Moreover, mathematical set symbols were not available, and this necessitated the invention of new symbols and notations, particularly for permutations, combinations, and the set operations. Fortran and PL/1 symbolisms were employed for the mathematical operations. Since curriculum material was input on cards, the alphabetic characters were limited to capital letters which hindered the emphasis of certain letters, words, and phrases. The use of quotation marks around such material was considered but was vetoed because it seemed to complicate rather than clarify sentences. In several sections of the concept blocks, the information displayed to the student had to be read very carefully to distinguish special usage. Examples of such usage are words being defined, letters used to represent sets, and letters representing elements of permutations or outcomes of events.

Other limitations encountered in this study include: the restricted storage area in CPS, the necessity for students to have prior computer knowledge to use the calculation mode, the

sometimes inadequate response and display facilities of the 2741, the inadequate response processing functions of the executor program, and, finally, the considerable time required, approximately eight months, to program the relatively small amount of student contact time at the terminal.

B. Recommendations

Computer-assisted instruction requires further research and development at all stages, especially research in learning theory, development of curriculum materials, and evaluation of long-term effects. However, the remainder of this section will consider only the recommendations resulting from this study.

The first recommendation is that, prior to future use of the CAI system, a well-planned introduction should be devised and available for student use. This introduction should include a short preliminary lesson via the terminal to assist students in becoming familiar with the terminal's keyboard, the sign-on procedure, and the calculation mode.

Second, it is believed that additional testing of the CAI probability unit should be conducted with subsequent educational statistics classes. Since an effective CAI program requires periodic content updating and expansion of individualizing features, the curriculum material should be continually revised. Analysis of the results of the further trial testing, as well as student comments, can aid this process.

A third recommendation which has resulted from this study is that the executor program be expanded to provide for additional response processing. A very crucial addition would be a partial-answer compare which would automatically eliminate blanks and trivial characters. This feature would allow the student to make errors in typing or spelling and yet receive whole or partial credit. In addition, the executor program should be designed to include greater data collection capabilities.

Fourth, to increase the instructional potential of the CAI system, it is suggested that additional operation codes such as a counter branch, add counters, and a help feature be incorporated into the author language. The author-controlled counters would increase the author's capability for changing the order of the curriculum-material presentation and would assist in the partial-answer compare which was proposed in the preceding paragraph. The help feature would enable the author to provide supplementary material to the student, who would receive it simply by asking for assistance.

A fifth recommendation of this study is the addition of a feature which would save the author considerable time in making revisions by providing the capability for changing materials directly in the files rather than having to preprocess the entire deck of author statement cards. Specifically, the design and implementation of an edit mode is

suggested.

And finally, as CAI has good potential for becoming a useful vehicle in obtaining information concerning curriculum development, it is recommended that research in adapting the CAI system to other curriculum materials be conducted. If CAI is to be employed in various other subject-matter areas, use of the calculation mode should be expanded and greater allowance made for student control in selecting the order of presentation.

VII. SUMMARY

This study attempted to develop a computer-assisted instruction unit designed to teach the basic concepts of probability. The CAI probability unit was devised to provide tutorial instruction and related practice problems to university graduate students enrolled in an educational statistics class. Because the emphasis of the study was on the development of an operational CAI system, no attempt was made to compare the effectiveness of the CAI probability unit with that of conventional instructional methods.

The CAI system was considered and discussed in three primary parts: (1) the hardware component, (2) the software component, and (3) the educational component.

The hardware component, utilizing the facilities of the Computation Center, Iowa State University, Ames, Iowa, included an IBM 360/65 operating system. This system contained a conversational programming mode which was provided by the IBM Conversational Programming System (CPS). CPS supports communication through IBM 2741 terminals, and such a terminal was used as the interface for this study.

Since no language at the Computation Center provided tutorial CAI capabilities, the second part of the CAI system, the software component, involved the design and implementation of an author language and its associated computer programs. The author language which was devised permitted the curriculum

material to be easily programmed for presentation and was comprised of the following ten operation codes: TYPE, ANS1, ANS2, NANS, BANS, WANS, COMP, GOTO, BRAN, and STOP. When recognized by the associated computer programs, the operation codes (op codes) initiated a complex sequence of computer instructions to provide the specific, desired educational features.

To perform the features of the author language, two computer programs were designed and written which (1) recognize each op code defined in the author language and (2) include the appropriate statements to process or execute each op code. One of these programs, written in PL/1, was constructed to preprocess the probability lesson material and to form two files on external disk storage. The first file, called the prose file, contained the curriculum material to be displayed to the student via a terminal and the author-provided correct answers to be compared with the student responses. The second file, titled an object file, held the location and type of each item in the prose file and also stored the control information required for the branching decisions. Written in CPS, the other computer program permitted the student to interact with the files under its supervision. To accomplish this interaction, the program was devised to direct the sequence of lesson presentation and to process the student replies.

The third and final part of the CAI system was the

educational software component which consisted of the curriculum material and the teaching strategies for its presentation. The development of the probability unit involved three principal steps. The first step was to define the objectives of the course and to determine the subject matter to be included. The objectives were written in behavioral terms, and the lesson material was divided into six concept blocks. Each consisting of a tutorial presentation and applicable practice problems, the six concept blocks were designated as follows: multiplication principle, permutations, combinations, binomial theorem, basic probability, and probabilities of combinations of events. The second step in the development of the probability unit was to write the probability lessons according to the pre-determined instructional strategy and in compliance with the structure of the author language.

The final step in the development of the probability unit involved editing, trial testing, and revising the curriculum material. The trial test on a group of 17 graduate students permitted the author to acquire (1) estimates of terminal time per concept block and (2) data utilized in course revision. The data for making revisions was obtained by analyzing printouts of each student's CAI presentation, studying student-provided notations, and evaluating suggestions made by the students during informal discussions.

Generally speaking, student reaction to computer assisted

instruction was favorable, with several individuals appearing highly motivated by this type of presentation. Some students selected the self-pacing option and finished the concept blocks ahead of schedule. Upon completing the material, numerous students said they hoped additional lessons would be made available.

Major recommendations resulting from this study include the following:

1. Prior to future use of the CAI system, a well-planned introduction should be devised and available for student use.
2. Additional testing of the CAI probability unit should be conducted with subsequent educational statistics classes.
3. The executor program should be expanded to provide for additional response processing.
4. Research in adapting the CAI system to other curriculum materials should be conducted.

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X. APPENDIX A: AUTHOR LANGUAGE

A. Summary of Operation Codes

Op Codes	Description
TYPE	Designates material, such as questions, problems, and textual information, for display to the student.
ANS1	Designates acceptable answers to the preceding question or problem, each one separated by "|," the logical OR symbol. Evokes the computer program to request a reply from the student, to compare the reply with the correct answers, and to provide feedback to the student.
ANS2	Identical to ANS1 except that the choice of whether or not to provide feedback is delegated to the course author. Employed if the author doesn't want the student to observe the correct answer.
WANS	Designates incorrect answers which, in turn, can be used to provide students with material to correct mistakes pre-diagnosed by the author.
BANS	Designates answers which permit the student to control the order of presentation.
NANS	Designates that a numeric answer, which is not an exact integer, and the student's reply be compared and judged the same if the values are within a specified tolerance.
COMP	Designates where the order of presentation is to be branched if the student's reply is correct.
GOTO	Designates a control statement which automatically changes the order of presentation.
BRAN	Designates that if the student's progress is equal to or greater than a given percentage the order of presentation is changed.
STOP	Designates end of lesson.

B. Coding Form

80 COLUMN DATA SHEET

PROGRAM	JOB NO.	BY	DATE

LABEL 1	OP CODE	0	ARGUMENT 20	30	40	50	60	70	80

C. Example of Format of Operation Codes

80 COLUMN DATA SHEET

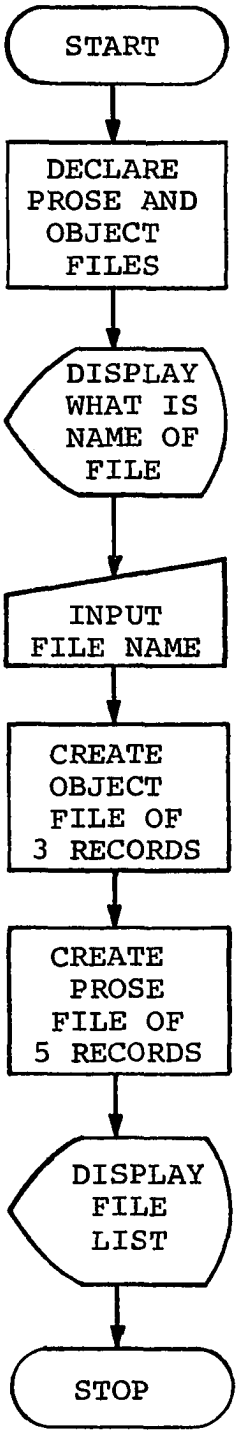
PROGRAM	JOB NO.	BY	DATE

LABEL	OP CODE	ARGUMENT (0 / 20 / 30 / 40 / 50 / 60 / 70 / 80)
134	TYPE	THE PROBABILITY OF A HEAD ON THE TOSS OF A FAIR COIN IS
	ANS1	ONE-HALF|1/2|.5|0.5|HALF
	ANS2	SIX|6
	WANS	FIVE|5
	BANS	YES
	NANS	.001 156.2578
	COMP	134
	GOTO	1
	BRAN	BA 80
	STOP	END OF LESSON

XI. APPENDIX B: COMPUTER PROGRAMS

A. File-Creator Program

1. Flow chart

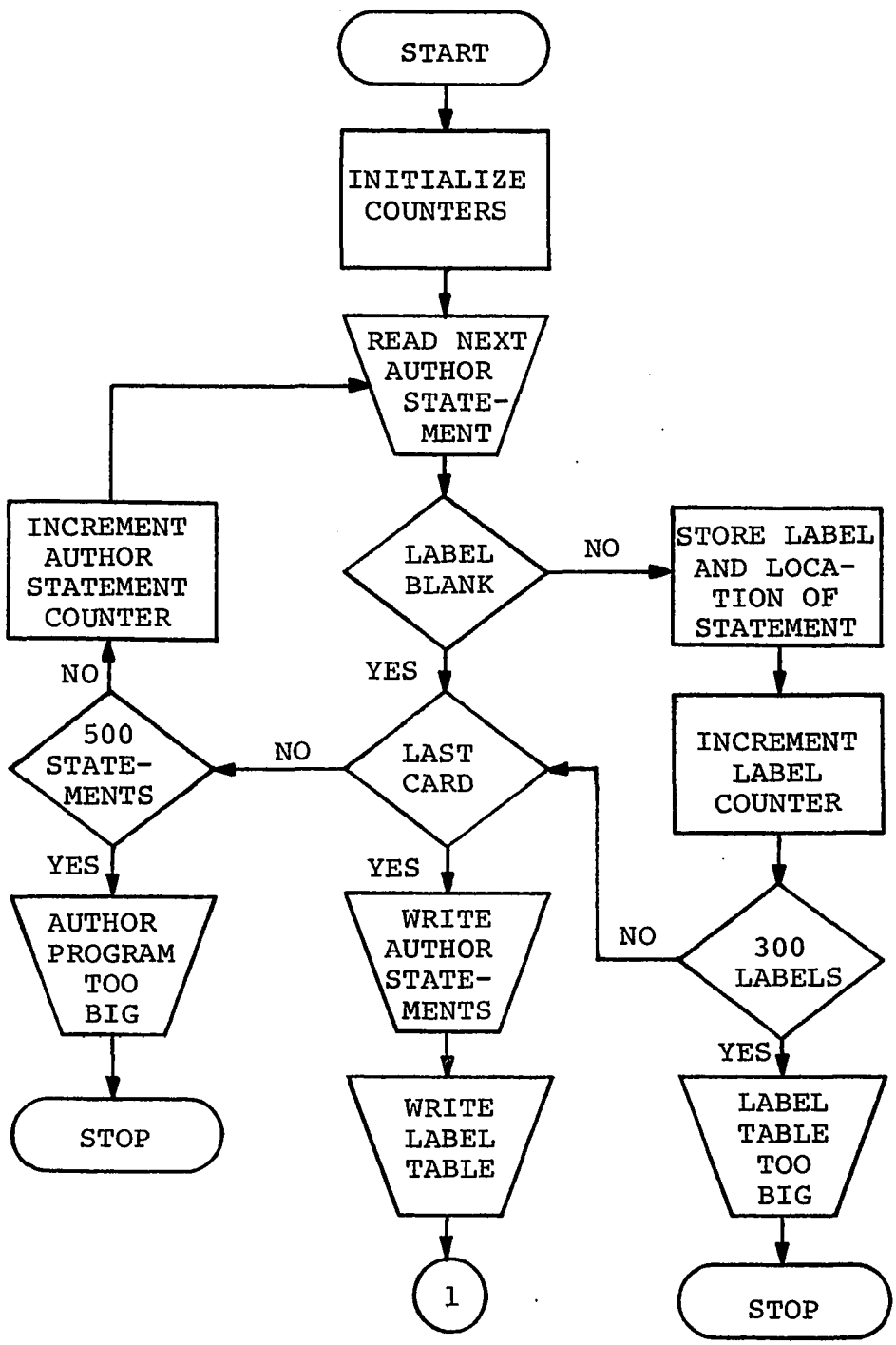

START
DECLARE PROSE AND OBJECT FILES
DISPLAY WHAT IS NAME OF FILE
INPUT FILE NAME
CREATE OBJECT FILE OF 3 RECORDS
CREATE PROSE FILE OF 5 RECORDS
DISPLAY FILE LIST
STOP

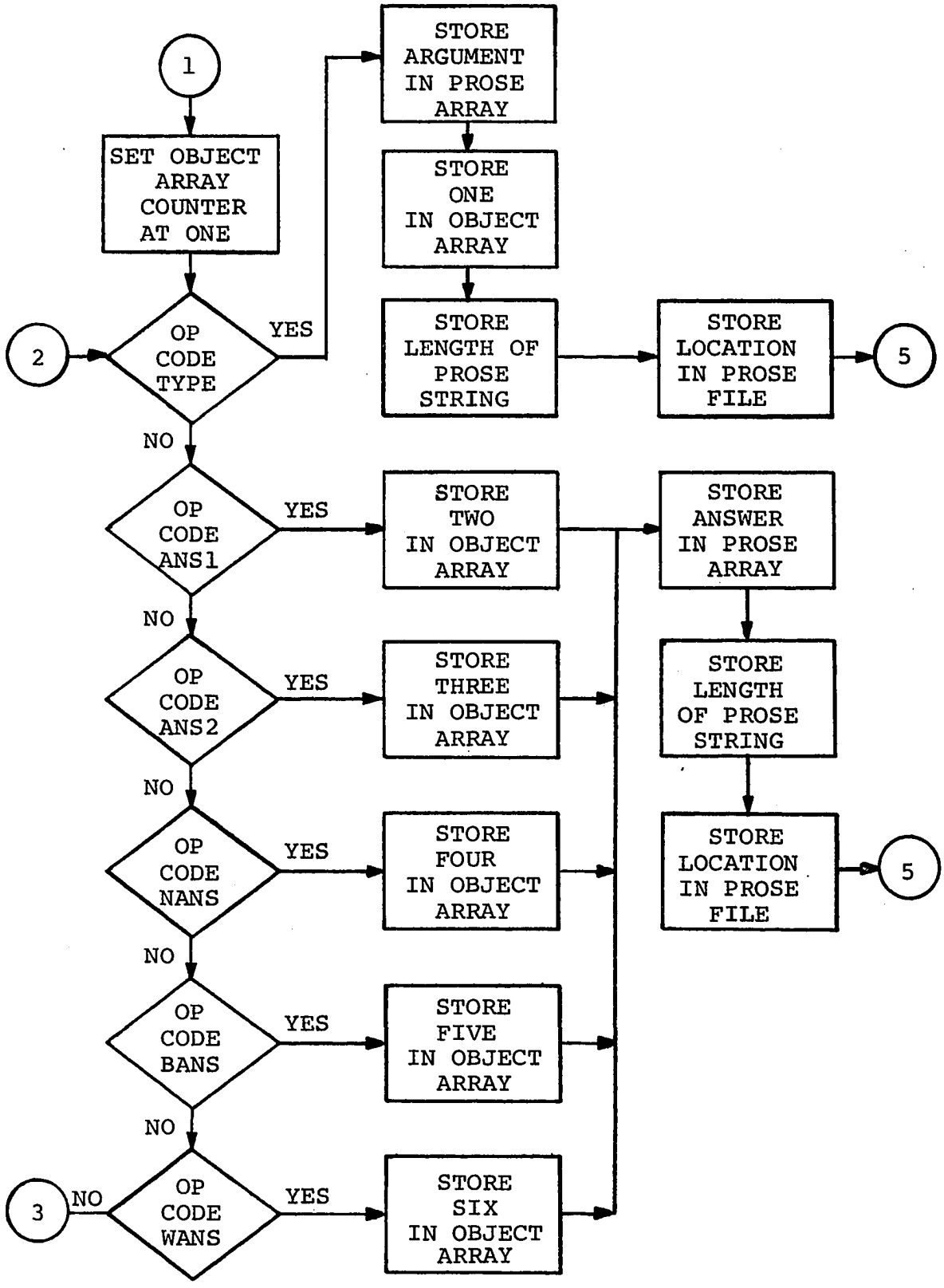
2. Listing of program

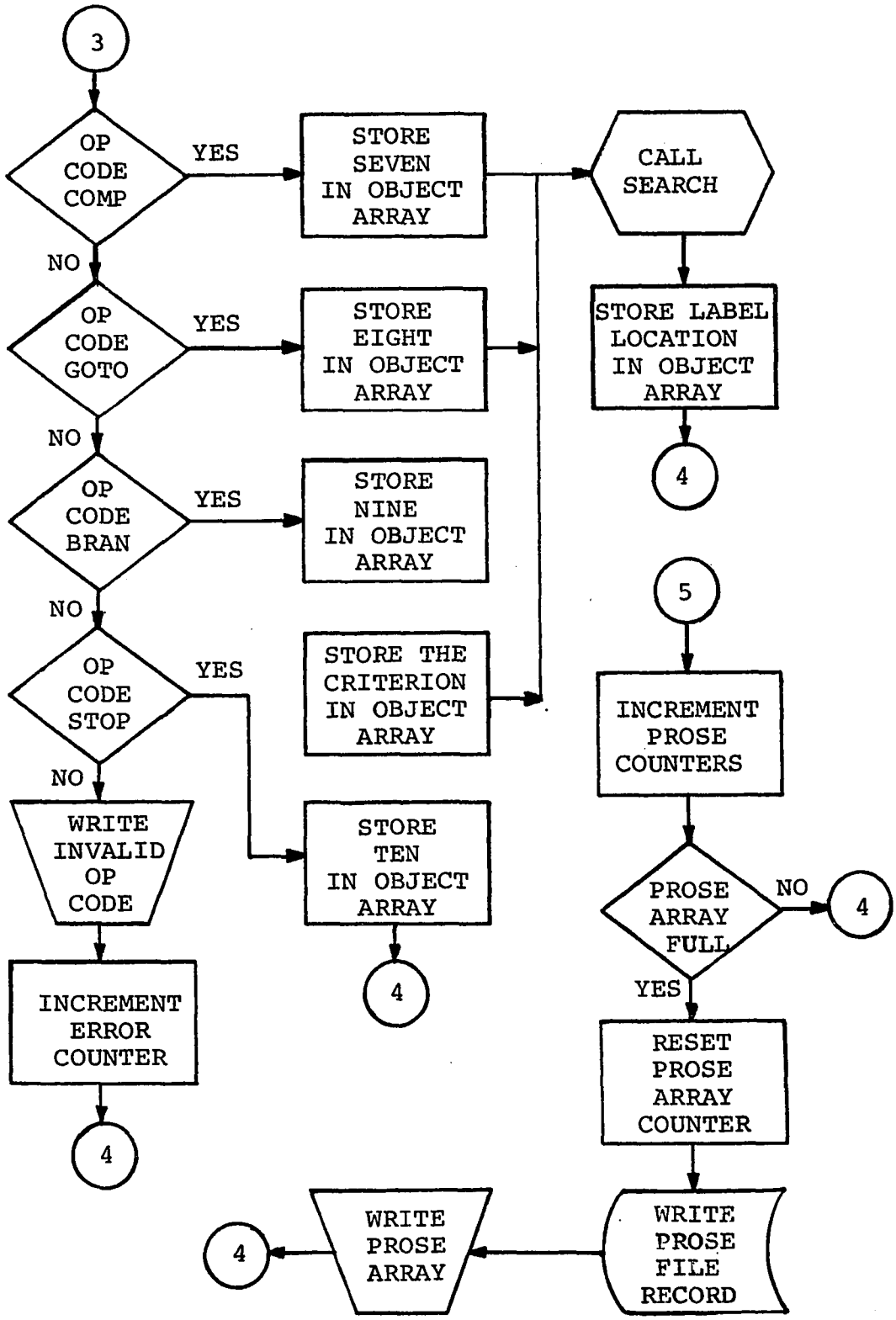
```
 1    /*CAI:  FILE CREATOR */;
 2    /* DON MCCLAIN */;
 3    /* DECLARATIONS */;
 4    DECLARE PROB FILE ENV( REGIONAL(1) F(1200) SPACE(TRK,2));
 5    DECLARE PROBP FILE ENV( REGIONAL(1) F(3500) SPACE(TRK,3));
 6    DECLARE PROB CHAR(4), PROBP CHAR(5);
 7    /* CREATION OF OBJECT FILE */;
 8    PUT LIST('WHAT IS THE NAME OF YOUR FILE');
 9    READ INTO (PROB);
10    DECLARE OBJ(100,3) DECIMAL(6);
11    OBJ=0;
12    OPEN FILE (PROB) OUTPUT;
13    DO J=1 TO 5;
14    WRITE FILE (PROB) FROM (OBJ);
15    END;
16    /* LIST FILE TO CHECK IF CREATED */;
17    FILE LIST PROB;
18    /* CREATION OF PROSE FILE */;
19    PROBP=PROB||'P';
20    DECLARE PROSE(50) CHAR(70);
21    PROSE='PROSE FILE';
22    OPEN FILE (PROBP) OUTPUT;
23    DO J=1 TO 7;
24    WRITE FILE (PROBP) FROM (PROSE);
25    END;
26    FILE LIST PROBP;
27    STOP;
```

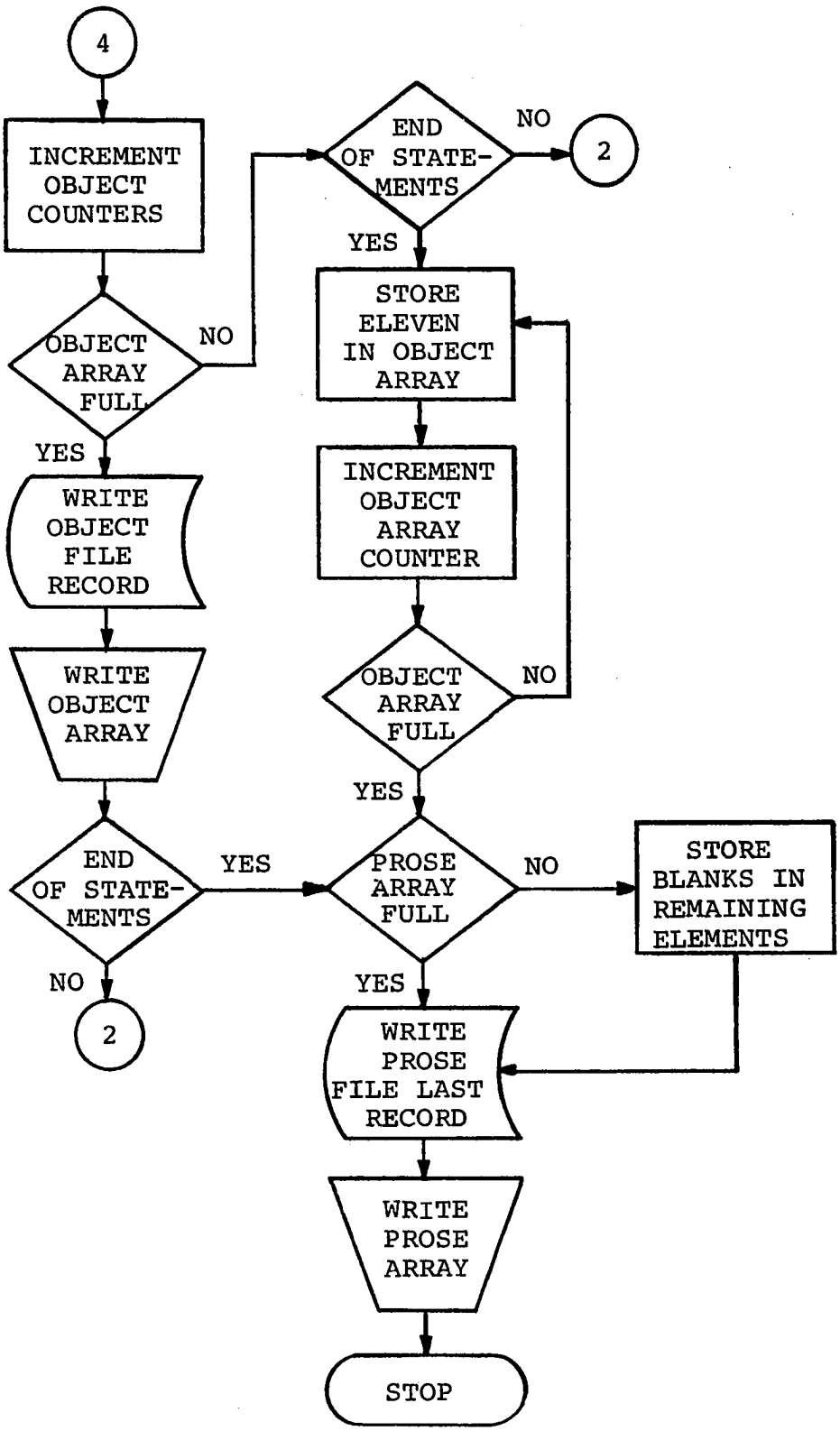
B. Pre-processor Program

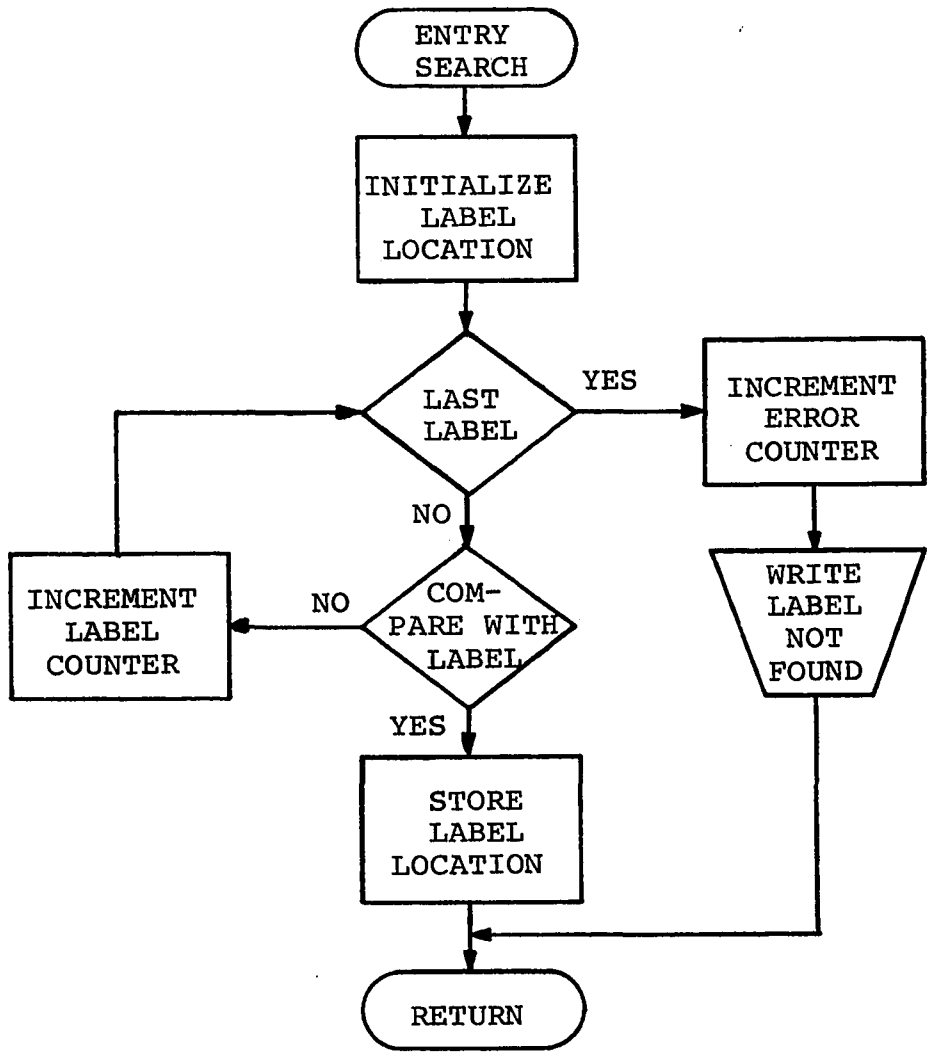
1. Flow chart


START
INITIALIZE COUNTERS
READ NEXT AUTHOR STATE-MENT
LABEL BLANK
NO
STORE LABEL AND LOCA-TION OF STATEMENT
YES
INCREMENT AUTHOR STATEMENT COUNTER
NO
500 STATE-MENTS
NO
LAST CARD
INCREMENT LABEL COUNTER
YES
YES
NO
300 LABELS
AUTHOR PROGRAM TOO BIG
WRITE AUTHOR STATE-MENTS
YES
STOP
WRITE LABEL TABLE
LABEL TABLE TOO BIG
1
STOP


1
SET OBJECT ARRAY COUNTER AT ONE
2
OP CODE TYPE
YES
STORE ARGUMENT IN PROSE ARRAY
STORE ONE IN OBJECT ARRAY
STORE LENGTH OF PROSE STRING
STORE LOCATION IN PROSE FILE
5
NO
OP CODE ANS1
YES
STORE TWO IN OBJECT ARRAY
STORE ANSWER IN PROSE ARRAY
NO
OP CODE ANS2
YES
STORE THREE IN OBJECT ARRAY
STORE LENGTH OF PROSE STRING
NO
OP CODE NANS
YES
STORE FOUR IN OBJECT ARRAY
STORE LOCATION IN PROSE FILE
5
NO
OP CODE BANS
YES
STORE FIVE IN OBJECT ARRAY
NO
3
NO
OP CODE WANS
YES
STORE SIX IN OBJECT ARRAY


3
OP CODE COMP
YES
STORE SEVEN IN OBJECT ARRAY
CALL SEARCH
NO
OP CODE GOTO
YES
STORE EIGHT IN OBJECT ARRAY
STORE LABEL LOCATION IN OBJECT ARRAY
4
NO
OP CODE BRAN
YES
STORE NINE IN OBJECT ARRAY
5
NO
OP CODE STOP
YES
STORE THE CRITERION IN OBJECT ARRAY
INCREMENT PROSE COUNTERS
NO
WRITE INVALID OP CODE
STORE TEN IN OBJECT ARRAY
PROSE ARRAY FULL
NO
4
YES
INCREMENT ERROR COUNTER
4
4
RESET PROSE ARRAY COUNTER
4
WRITE PROSE ARRAY
WRITE PROSE FILE RECORD


4
INCREMENT OBJECT COUNTERS
OBJECT ARRAY FULL
NO
YES
WRITE OBJECT FILE RECORD
WRITE OBJECT ARRAY
END OF STATE-MENTS
YES
NO
2
END OF STATE-MENTS
NO
2
YES
STORE ELEVEN IN OBJECT ARRAY
INCREMENT OBJECT ARRAY COUNTER
OBJECT ARRAY FULL
NO
YES
PROSE ARRAY FULL
NO
STORE BLANKS IN REMAINING ELEMENTS
YES
WRITE PROSE FILE LAST RECORD
WRITE PROSE ARRAY
STOP


ENTRY SEARCH
INITIALIZE LABEL LOCATION
LAST LABEL
YES
INCREMENT ERROR COUNTER
NO
INCREMENT LABEL COUNTER
NO
COM-PARE WITH LABEL
WRITE LABEL NOT FOUND
YES
STORE LABEL LOCATION
RETURN

2. Listing of program

```
/* PL/1 PRE-PROCESSOR FOR CAI          */
/* D. MCCLAIN 11-4-69                  */
/*                                     */
/*************************************/
EJN: PROCEDURE OPTIONS(MAIN);
/*        */
/*        */
/* DECLARATIONS AND INITIALIZATIONS */
/*        */
DECLARE AR(500) CHAR(80), ATHR CHAR(80),
                          LBL CHAR(4) DEF ATHR,
                            OP CHAR(4) DEF ATHR POS(6),
                            OPA CHAR(4) DEF ATHR POS(11),
                            OPB CHAR(70) DEF ATHR POS(11),
                            PR(70) CHAR(1) DEF ATHR POS(11);
DECLARE LBTBL(300) CHAR(4);
DECLARE OBJ(100,3),LBPT(300);
DECLARE OPS(15) CHAR(4);
DECLARE PROSE(50) CHAR(70);
DECLARE OPLBL(15) LABEL;
DECLARE OBFILE UPDATE DIRECT KEYED ENVIRONMENT(REGIONAL(1) F(1200));
DECLARE PRFILE UPDATE DIRECT KEYED ENVIRONMENT(REGIONAL(1) F(3500));
/*        */
/*        */
OPS(1)='    ';/*TYPE*/
OPS(2)='TYPE';
OPS(3)='ANS1';
OPS(4)='ANS2';
OPS(5)='NANS';
OPS(6)='BANS';
OPS(7)='WANS';
OPS(8)='COMP';
OPS(9)='GOTO';
OPS(10)='BRAN';
OPS(11)='STOP';
/*        */
OPLBL(1)=TYPE;
```

```
OPLBL(2)=TYPE;
OPLBL(3)=ANS1;
OPLBL(4)=ANS2;
OPLBL(5)=NANS;
OPLBL(6)=BANS;
OPLBL(7)=WANS;
OPLBL(8)=COMP;
OPLBL(9)=GOTO;
OPLBL(10)=BRAN;
OPLBL(11)=STOP;
/*       */
/*       */
/* READS AUTHOR STATEMENTS AND BUILDS LABEL TABLE */
/* RESTRICTED TO 500 AUTHOR STATEMENTS AND 300 LABELS */
/*       */
/* M LABEL COUNTER, AND KATHR AUTHOR STATEMENT COUNTER */
/* LBTBL LABELS, AND LBPT LOCATION */
/* AR AUTHOR STATEMENT ARRAY */
/* ERR ERROR COUNTER */
/*       */
ERR=0;
KOBJ,KPR=0 ;
M=1;
ON ENDFILE(SYSIN) GO TO DONEIN;
STRT: DO K=1 TO 500;
GET EDIT(AR(K))(A(80));
KATHR=K;
  ATHR=AR(K);
  IF LBL='    ' THEN GO TO NEXT;
  DO L=1 TO M-1;
     IF LBL=LBTBL(L) THEN DO;
                          PUT EDIT('***ERROR*** AT INSTRUCTION',K)
                                   (COL(5),A,F(5));
                 PUT EDIT(LBL,'IS A DUPLICATE ') (COL(2),A(4),A);
                          ERR=ERR+1;
                          END;
     END;
```

```
  LBTBL(M)=LBL;
  LBPT(M)=K;
  M=M+1;
  IF M<= 300 THEN GO TO NEXT;
PUT EDIT('LABEL TABLE TOO LARGE.')(SKIP,A(80),SKIP);
  M=M-1;
  CALL LBOUT;
  STOP;
  NEXT: END STRT;
M=M-1;
PUT EDIT('AUTHOR PROGRAM EXCEEDS 500 STATEMENTS.')(SKIP,A(80),SKIP);
CALL ATHOUT;
STOP;
DONEIN: M=M-1;
CALL ATHOUT;
CALL LBOUT;
/* THIS ENDS I/O AND LABEL TABLE BUILDING */
/*       */
/*       */
/* CREATES OBJECT AND PROSE FILE */
/* L AUTHOR STATEMENT COUNTER */
/* LL OBJECT RECORD COUNTER */
/* LN PROSE RECORD COUNTER */
/* N PROSE FILE COUNTER */
/*       */
L,N=1;
LN=1;
/* RESET LL*/
KLL:LL=1;
THREE:ATHR=AR(L);
OBJ(LL,1),OBJ(LL,2),OBJ(LL,3)=0;
TESTOP: DO KK=1 TO 11;
  IF OP=OPS(KK) THEN GO TO OPLBL(KK);
/* OP AUTHOR OP CODE */
/* OPS STORED ARRAY OF OP CODES */
/* BRANCH TO OPLBL */
  END TESTOP;
```

```
/* INVALID OP CODE */
PUT EDIT(L,ATHR)(F(5),COLUMN(15),A(80));
PUT EDIT(OP,'IS AN INVALID OPERATOR') (COL(2),A(4),COL(10),A);
ERR=ERR+1;
GO TO TWNTY;
/*       */
/*       */
/* OP CODE TYPE */
/*       */
TYPE: PROSE(LN)=OPB;
OBJ(LL,1)=1. ;
DO LEN=70 TO 1 BY -1 WHILE (PR(LEN)=' ');    END;
/* LEN LENGTH OF LESSON MATERIAL */
OBJ(LL,2)=LEN;
OBJ(LL,3)=N;
/* N PROSE FILE LOCATION */
/*       */
/* PROSE FILE RECORD COMPLETE? */
FIVE: N=N+1; LN=LN+1;
IF LN<=50 THEN GO TO TWNTY;
LN=1; /* RESET LN */
PUT EDIT ('PROSE FILE')(PAGE,A(80));
/* WILL WRITE PROSE FILE HERE */
IF ERR=0 THEN DO;
     REWRITE FILE(PRFILE) FROM(PROSE) KEY(KPR);
/* KPR PROSE FILE RECORD KEY */
     KPR=KPR+1;
     END;
PUT SKIP;
PUT EDIT((KK,PROSE(KK) DO KK=1 TO 50)) (COL(2),F(5),COL(15),A(80));
GO TO TWNTY;
/*       */
/*       */
/* OP CODES ANS1, ANS2, NANS, BANS, AND WANS */
/*       */
ANS1:XNEW=2.;
GO TO TEAM;
```

```
/*       */
ANS2:XNEW=3.;
GO TO TEAM;
/*       */
NANS:XNEW=4.;
GO TO TEAM;
/*       */
BANS:XNEW=5.;
GO TO TEAM;
/*       */
WANS:XNEW=6.;
/*       */
/*       */
TEAM: PROSE(LN)=OPB;
OBJ(LL,1)=XNEW;
DO LEN=70 TO 1 BY -1 WHILE (PR(LEN)=' ');  END;
/* LEN LENGTH OF ANSWER */
OBJ(LL,2)=LEN;
OBJ(LL,3)=N;
/* N LOCATION IN PROSE FILE */
GO TO FIVE;
/*       */
/*       */
/* OP COMP */
/*       */
COMP: OBJ(LL,1)=7.;
LAB19:  ;
CALL SEARCH;
OBJ(LL,2)=NN;
/* NN LOCATION OF LABEL */
GO TO TWNTY;
/*       */
/*       */
/* OP CODE GOTO */
/*       */
GOTO: OBJ(LL,1)=8.;
GO TO LAB19;
```

```
/*        */
/*        */
/* OP CODE BRAN */
/*        */
BRAN: OBJ(LL,1)=9.;
OBJ(LL,3)=SUBSTR(ATHR,15,2);
/* PERCENTAGE CRITERION */
GO TO LAB19;
/*        */
/*        */
/* OP CODE STOP */
/*        */
STOP: OBJ(LL,1)=10.;
GO TO TWNTY;
/*        */
/* OBJECT FILE RECORD COMPLETE? */
TWNTY: L=L+1; LL=LL+1;
IF LL<= 100 THEN GO TO CHK;
/* WILL WRITE OB FILE HERE */
IF ERR=0 THEN DO;
     REWRITE FILE(OBFILE) FROM(OBJ) KEY(KOBJ);
/* KOBJ OBJECT FILE RECORD KEY */
     KOBJ=KOBJ+1;
     END;
PUT PAGE;
PUT EDIT('OBJECT FILE')(SKIP,A(80),SKIP);
PUT SKIP;
DO KL=1 TO 100;
PUT EDIT(OBJ(KL,1),OBJ(KL,2),OBJ(KL,3))(COLUMN(2),3 F(10));
  END;
IF L>KATHR THEN GO TO DONE;
GO TO KLL;
/*        */
/* ALL STATEMENTS PROCESSED? */
CHK: IF L<=KATHR THEN GO TO THREE;
/* COMPLETES REMAINING ELEMENTS WITH 11 */
DO KL=LL TO 100;
```

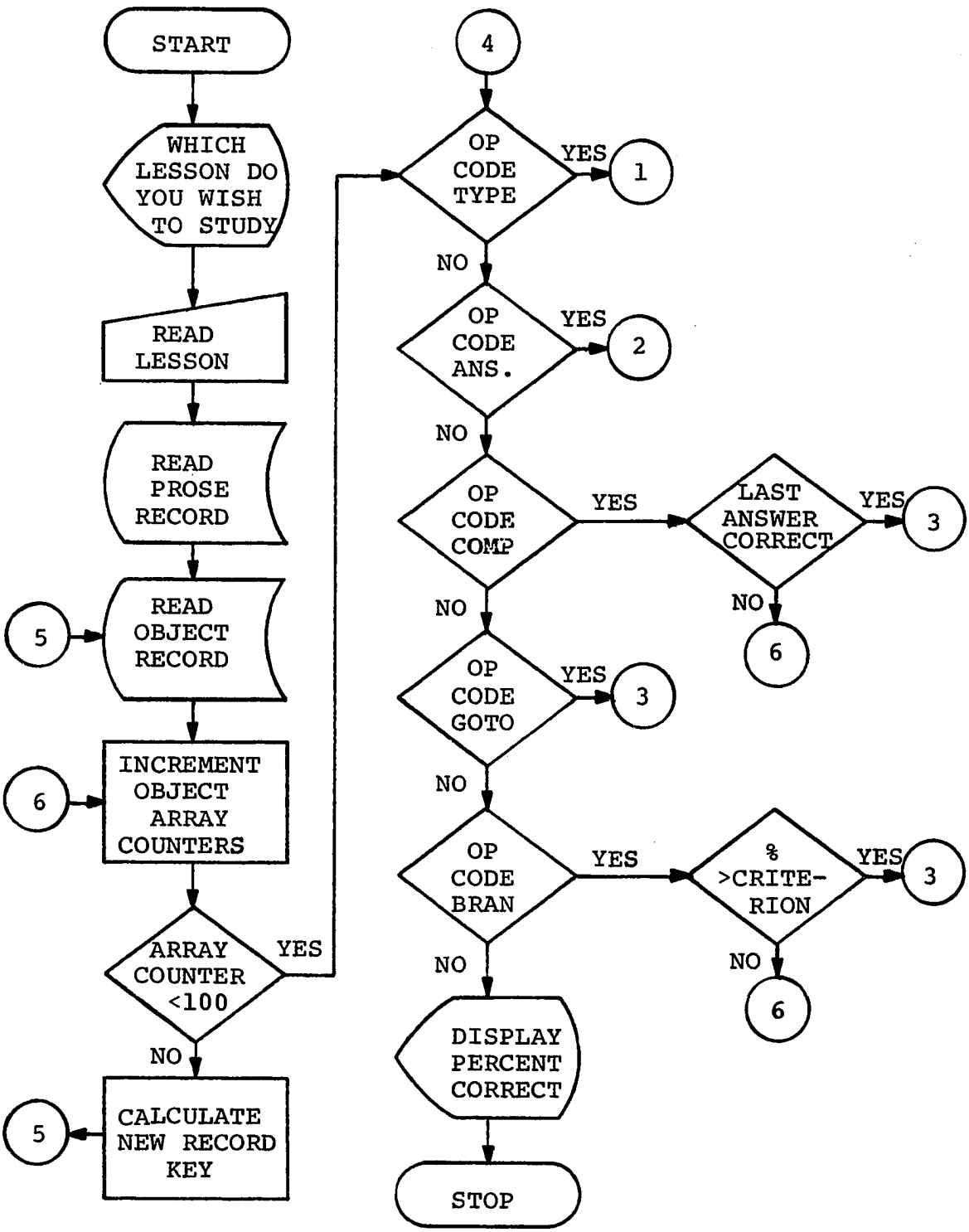
```
  OBJ(KL,1),OBJ(KL,2),OBJ(KL,3)=11.;
  END;
/* WRITES OBJECT FILE */
PUT PAGE;
IF  ERR=0 THEN REWRITE FILE(OBFILE) FROM(OBJ) KEY(KOBJ);
PUT LIST('KOBJ IS',KOBJ);
PUT EDIT('OBJECT FILE')(COLUMN(2),A(80));
DO KL=1 TO LL;
PUT EDIT(OBJ(KL,1),OBJ(KL,2),OBJ(KL,3))(COLUMN(2),3 F(10));
END;
DONE: IF LN=1 THEN GO TO FINIS;
/* COMPLETES REMAINING ELEMENTS WITH BLANKS */
DO KL=LN TO 50;
   PROSE(KL)=' ';
  END;
/* WRITES PROSE FILE */
IF ERR=0 THEN REWRITE FILE(PRFILE) FROM(PROSE) KEY(KPR);
PUT EDIT('PROSE FILE')(PAGE,COL(2),A,SKIP);
PUT SKIP;
DO KL=1 TO 50;
PUT EDIT(KL,PROSE(KL))(COLUMN(2),F(5),COLUMN(15),A(80));
  END;
/*       */
/*       */
/* END OF PROGRAM */
/* CLOSES FILES */
/*       */
FINIS:   ;
PUT EDIT('NUMBER OF AUTHOR ERRORS =',ERR)(PAGE,COL(2),A,F(5));
PUT EDIT('PROGRAM COMPLETE')(SKIP,A(80),SKIP);
CLOSE FILE(OBFILE);
CLOSE FILE(PRFILE);
STOP;
/*       */
/*       */
/* SEARCHES LABEL TABLE -- RETURNS LABEL LOCATION */
/*       */
```

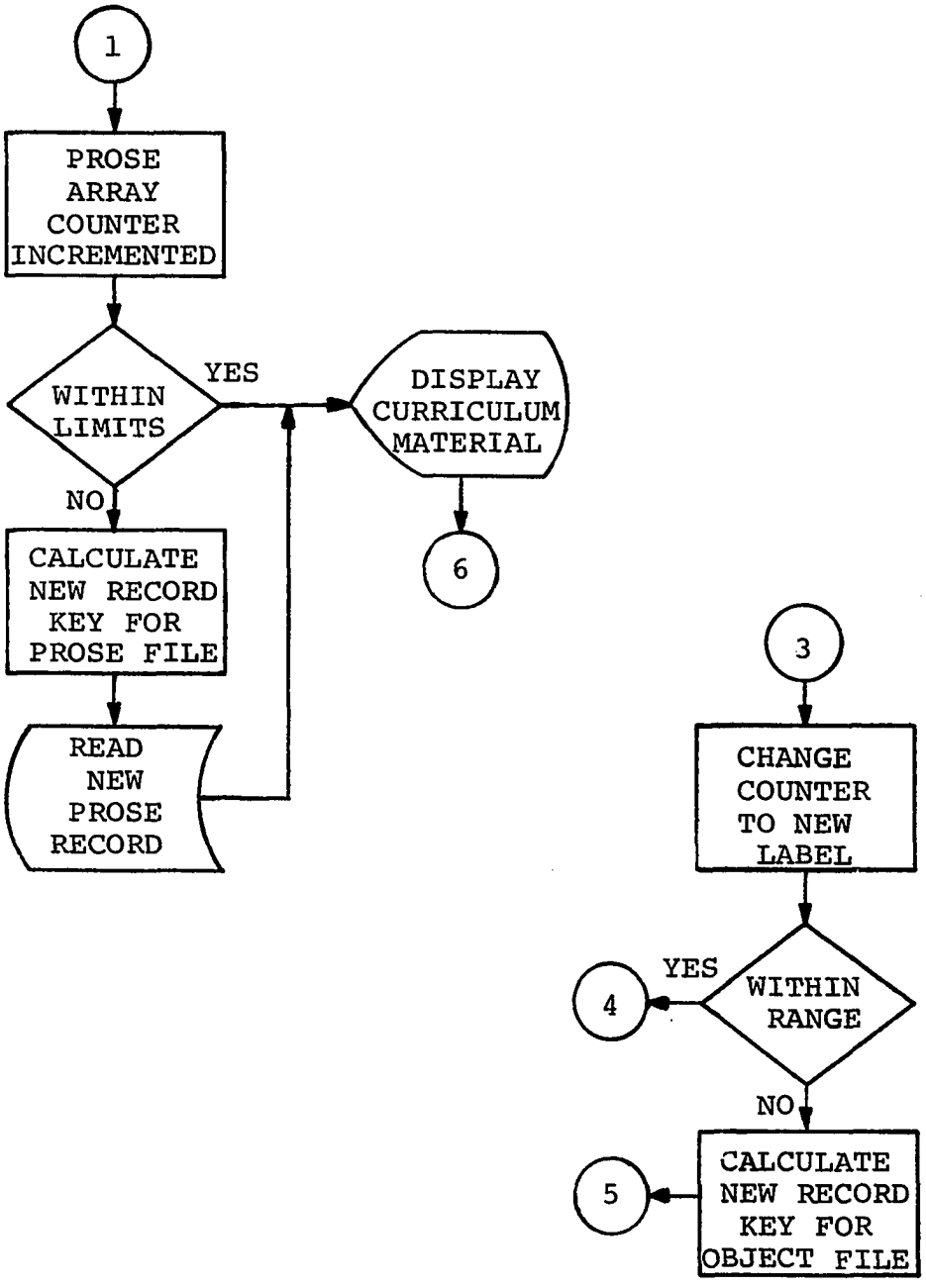
```
SEARCH: PROCEDURE;
NN=0;
DO KKK=1 TO M;
  IF OPA=LBTBL(KKK) THEN NN=LBPT(KKK);
  END;
IF NN¬=0 THEN RETURN;
ERR=ERR+1;
/* ERROR DIAGNOSTICS IF LABEL NOT FOUND */
PUT EDIT(OPA,'NOT FOUND IN LABEL TABLE.')
(COLUMN(2),A(4),COLUMN(10),A(80));
PUT EDIT('----')(COLUMN(2),A(4));
RETURN;
END SEARCH;
/*       */
/*       */
/* OUTPUTS AUTHOR SOURCE PROGRAM */
/*       */
ATHOUT: PROCEDURE;
PUT PAGE;
PUT EDIT('AUTHOR SOURCE PROGRAM')(SKIP,A(80),SKIP);
PUT SKIP;
DO KKK=1 TO KATHR;
PUT EDIT(KKK,  AR(KKK))(COLUMN(2),F(5),COLUMN(15),A(80));
  END;
PUT EDIT('END AUTHOR SOURCE')(SKIP,A(80),SKIP);
RETURN;
END ATHOUT;
/*       */
/*       */
/* OUTPUTS LABEL TABLE */
/*       */
LBOUT: PROCEDURE;
PUT PAGE;
PUT EDIT('LABEL TABLE')(SKIP,A(80),SKIP);
PUT SKIP;
DO KKK=1 TO M;
PUT EDIT(KKK,LBTBL(KKK),LBPT(KKK))
```

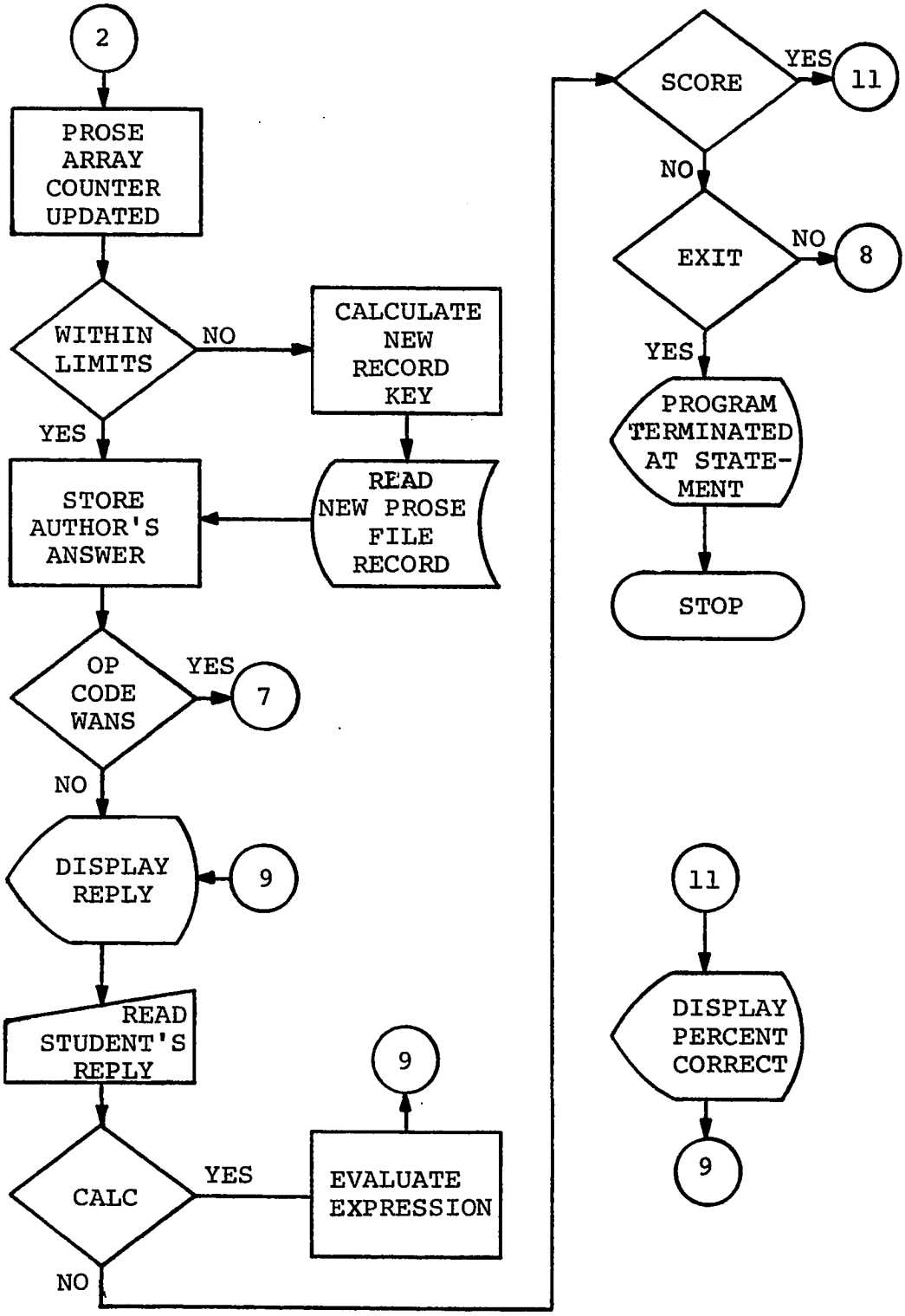
```
(COLUMN(2),F(5),COLUMN(10),A(4),F(10));
  END;
PUT EDIT('END LABEL TABLE')(SKIP,A(80),SKIP);
RETURN;
END LBOUT;
/*       */
/*       */
END EJN;
```

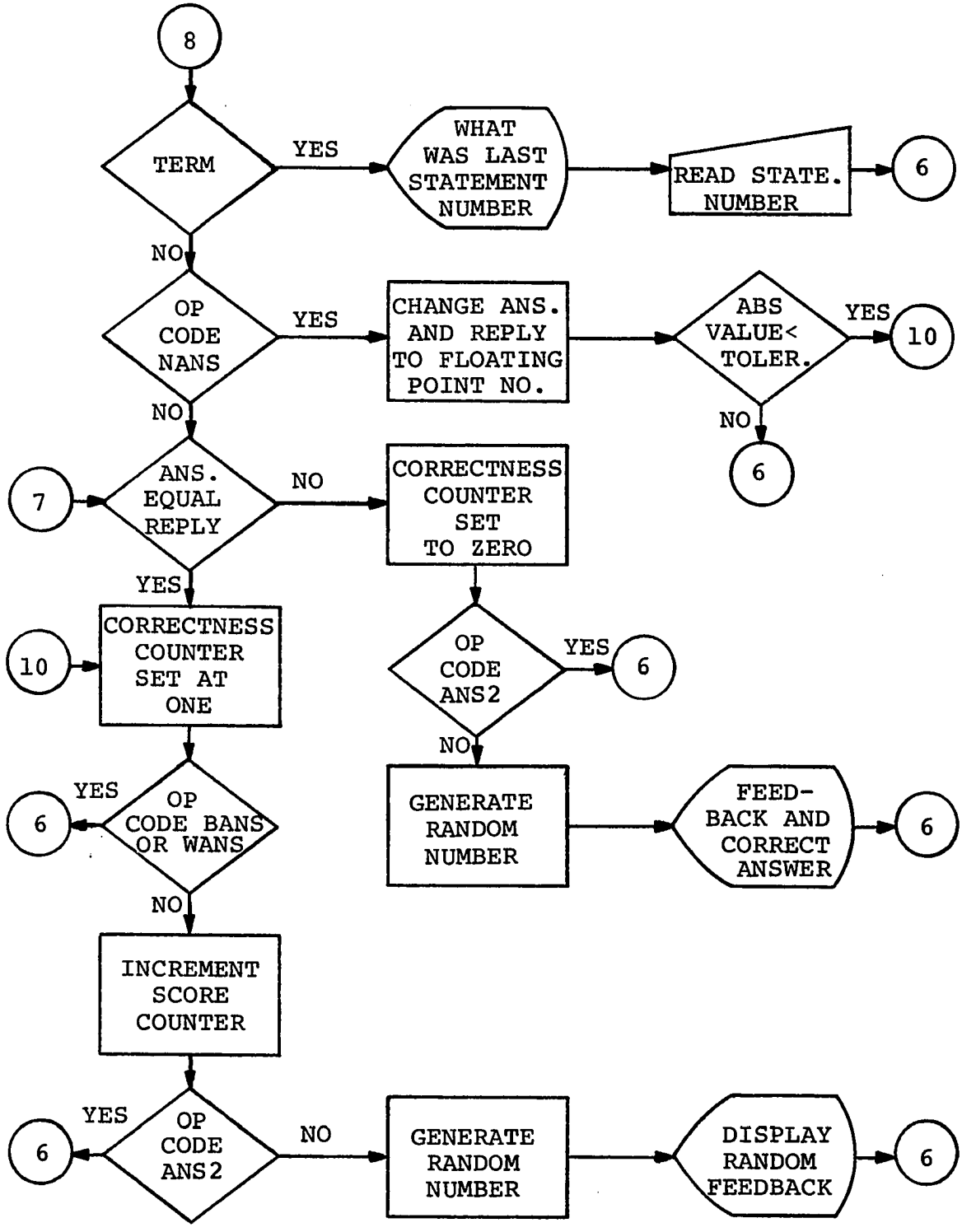
C. Executor Program

1. Flow chart


START
WHICH LESSON DO YOU WISH TO STUDY
READ LESSON
READ PROSE RECORD
5
READ OBJECT RECORD
6
INCREMENT OBJECT ARRAY COUNTERS
ARRAY COUNTER <100
YES
NO
5
CALCULATE NEW RECORD KEY
4
OP CODE TYPE
YES
1
NO
OP CODE ANS.
YES
2
NO
OP CODE COMP
YES
LAST ANSWER CORRECT
YES
3
NO
6
NO
OP CODE GOTO
YES
3
NO
OP CODE BRAN
YES
% >CRITE-RION
YES
3
NO
6
NO
DISPLAY PERCENT CORRECT
STOP


1
PROSE ARRAY COUNTER INCREMENTED
WITHIN LIMITS
YES
DISPLAY CURRICULUM MATERIAL
NO
6
CALCULATE NEW RECORD KEY FOR PROSE FILE
READ NEW PROSE RECORD
3
CHANGE COUNTER TO NEW LABEL
WITHIN RANGE
YES
4
NO
CALCULATE NEW RECORD KEY FOR OBJECT FILE
5


2
PROSE ARRAY COUNTER UPDATED
WITHIN LIMITS
NO
CALCULATE NEW RECORD KEY
YES
READ NEW PROSE FILE RECORD
STORE AUTHOR'S ANSWER
OP CODE WANS
YES
7
NO
DISPLAY REPLY
9
READ STUDENT'S REPLY
9
CALC
YES
EVALUATE EXPRESSION
NO
SCORE
YES
11
NO
EXIT
NO
8
YES
PROGRAM TERMINATED AT STATE-MENT
STOP
11
DISPLAY PERCENT CORRECT
9


8
TERM
YES
WHAT WAS LAST STATEMENT NUMBER
READ STATE. NUMBER
6
NO
OP CODE NANS
YES
CHANGE ANS. AND REPLY TO FLOATING POINT NO.
ABS VALUE< TOLER.
YES
10
NO
6
NO
7
ANS. EQUAL REPLY
NO
CORRECTNESS COUNTER SET TO ZERO
YES
10
CORRECTNESS COUNTER SET AT ONE
OP CODE ANS2
YES
6
NO
6
YES
OP CODE BANS OR WANS
GENERATE RANDOM NUMBER
FEED-BACK AND CORRECT ANSWER
6
NO
INCREMENT SCORE COUNTER
6
YES
OP CODE ANS2
NO
GENERATE RANDOM NUMBER
DISPLAY RANDOM FEEDBACK
6

2. Listing of program

```
 5.            /*CAI:CONTROL PROGRAM.MCCLAIN.292-5248*/;
10.            DECLARE A(50) CHAR(70), LES# CHAR(10) VAR;
15.            DECLARE OBJ(100,3) DEC(6), ANS CHAR(70) VAR, RETURN LABEL;
20.            DECLARE REPLY CHAR(50) VAR, FILNAM CHAR(25) VAR;
25.            DECLARE FIL FILE ENV( REGIONAL(1) F(1200) );
30.            DECLARE FILP FILE ENV( REGIONAL(1) F(3500) );
35.            DECLARE LBL(10) LABEL;
40.            LBL(1)=TYPE;
45.            LBL(2),LBL(3),LBL(4),LBL(5),LBL(6)=ANS1;
50.            LBL(7)=COMP;
55.            LBL(8)=GOTO;
60.            LBL(9)=BRAN;
65.            LBL(10)=STOP;
67.            ON ATTENTION GO TO BEGIN;
70.            SCORE,PT,J,JKEY,I,IKEY=0;
75.            PUT LIST('WHICH LESSON DO YOU WISH TO STUDY?');
80.            READ INTO(LES#);
85.            FILMNAM='CPS06.I3203.'||LES#;
90.            OPEN FILE(FILP) INPUT DIRECT TITLE(FILNAM||'P');
95.            READ FILE(FILP) INTO(A) KEY(IKEY);
100.           OPEN FILE(FIL) INPUT DIRECT TITLE(FILNAM);
105.   START:  READ FILE(FIL) INTO(OBJ) KEY(JKEY);
110.   CHECK:  J=J+1;
115.           IF J<=100 THEN GO TO BEGIN;
120.           J=J-101;
125.           JKEY=JKEY+1;
130.           GO TO START;
135.   OBCK:   J=OBJ(J,2);
140.           IF J-JKEY*100>100|J-JKEY*100<=0 THEN GO TO CHANGE;
145.           J=J-JKEY*100;
150.           GO TO BEGIN;
155.   CHANGE: JKEY=TRUNC((J-1)/100);
160.           J=J-JKEY*100;
162.           J=J-1;
165.           GO TO START;
170.   BEGIN;  GO TO LBL(OBJ(J,1));
175.   TYPE:   I=OBJ(J,3)-IKEY*50;
```

```
180.            IF I<=50&I>=1 THEN GO TO TYPE1;
185.            RETURN=TYPE1;
190.            GO TO PROSE;
195.    TYPE1:  PUT LIST(SUBSTR(A(I),1,OBJ(J,2)));
200.            GO TO CHECK;
215.    ANS1    I=OBJ(J,3)-IKEY*50;
220.            IF I<=50&I>=1 THEN GO TO TRY;
225.            RETURN=TRY;
230.            GO TO PROSE;
235.    TRY:    ANS=SUBSTR(A(I),1,OBJ(J,2));
240.            IF OBJ(J,1)=6 THEN GO TO NEXT;
245.            PUT LIST(' ');
250.            PUT LIST('REPLY');
255.    RPLY:   READ INTO(REPLY);
260.            PUT LIST(' ');
262.            REPLY=UPCASE(REPLY);
265.            IF SUBSTR(REPLY,1,4)='CALC' THEN GO TO CALC;
270.            IF SUBSTR(REPLY,1,4)='EXIT' THEN GO TO EXIT;
275.            IF SUBSTR(REPLY,1,5)='SCORE' THEN GO TO SCR;
276.            IF SUBSTR(REPLY,1,4)='TERM' THEN GO TO TERM;
277.            PT=PT+1;
278.            IF OBJ(J,1)=5 THEN PT=PT-1;
280.            IF OBJ(J,1)=4 THEN GO TO NANS;
285.    NEXT:   ANS=UPCASE(ANS);
292.            ANS='|'||ANS||'|';
293.            REPLY='|'||REPLY||'|';
295.            IF INDEX(ANS,REPLY)=0 THEN GO TO NO;
300.    YES:    IOK=1;
305.            IF OBJ(J,1)=5|OBJ(J,1)=6 THEN GO TO CHECK;
310.            SCORE=SCORE+1;
315.            IF OBJ(J,1)=3 THEN GO TO CHECK;
320.            Y=RANDOM;
325.            IF Y<=.4 THEN PUT LIST('YES, THAT IS CORRECT');
330.            IF Y>.6&Y<=.9 THEN PUT LIST('GOOD, TRY ANOTHER QUESTION');
335.            IF Y>.9 THEN PUT LIST('VERY GOOD');
340;            GO TO CHECK;
345.    NO:     IOK=0;
```

```
350.            IF OBJ(J,1)¬=2 THEN GO TO CHECK;
355.            Y=RANDOM;
360.            IF Y<=.4 THEN PUT LIST('NO, THE ANSWER IS ',ANS);
365.            IF Y>.4&Y<=.8 THEN PUT LIST('THE CORRECT ANSWER IS ',ANS);
370.            IF Y>.8&Y<=.9 THEN PUT LIST('INCORRECT ',ANS,'IS RIGHT');
375.            IF Y>.9 THEN PUT LIST('WRONG, THE ANSWER IS ',ANS);
385.            GO TO CHECK;
390.    NANS:   T=FLOAT(SUBSTR(ANS,1,4));
392.            IF T=0 THEN T=.01;
395.            Z=FLOAT(SUBSTR(ANS,5));
400             ZZ=FLOAT(REPLY);
405.            IF ABS(Z-ZZ)<=T THEN GO TO YES;
410.            GO TO NO;
415.    COMP:   IF IOK=0 THEN GO TO CHECK;
420.    GOTO:   GO TO OBCK;
425.    BRAN:   IF PERCEN(PT,SCORE)>=OBJ(J,3) THEN GO TO GOTO;
430.            GO TO CHECK;
435.    PROSE:  IF I>100|I<1 THEN GO TO FORWAR;
440.            I=I-50;
445.            IKEY=IKEY+1;
450.    READ:   READ FILE(FILP) INTO(A) KEY(IKEY);
455.            GO TO RETURN;
460.    FORWAR: IKEY=TRUNC((OBJ(J,3)-1)/50);
465.            I=OBJ(J,3)-IKEY*50;
470.            GO TO READ;
475.    CALC:   PUT LIST(EVAL(SUBSTR(REPLY,2)));
480.            PUT LIST('NOW REPLY');
485.            GO TO RPLY;
490.    EXIT:   PUT LIST('PROGRAM TERMINATED');
492.            PUT LIST(J+JKEY*100);
495.            GO TO LAST;
500.    SCR:    PUT LIST(PERCEN(PT,SCORE),'% CORRECT OUT OF ',PT);
505.            PUT LIST('NOW REPLY');
510.            GO TO RPLY;
511     TERM:   PUT LIST('WHICH LINE NUMBER DID YOU STOP AT?');
512.            READ INTO(J);
513.            J=J-1;
```

```
514.            GO TO CHECK;
515.    STOP:   PUT LIST('END OF PROGRAM');
520.            PUT LIST('YOU HAVE ANSWERED ', SCORE,'QUESTIONS CORRECTLY');
521.            PUT LIST('OUT OF ',PT,'FOR ',PERCEN(PT,SCORE),'%');
525.            LET PERCEN(PT,SCORE)=TRUNC(SCORE*100/PT);
530.    LAST:   STOP;
```

XII. APPENDIX C: PROBABILITY UNIT

A. Concept Block One

```
   TYPE CONCEPT BLOCK ONE:  MULTIPLICATION PRINCIPLE.
   TYPE FOR GENERATIONS, PEOPLE HAVE BEEN INTRIGUED BY PROBLEMS REQUIRING
        THEM TO FIND THE NUMBER OF WAYS OF ARRANGING A SET OF OBJECTS.  SUCH
        COUNTING QUESTIONS ARE FASCINATING AND CHALLENGING IN THEIR OWN RIGHT.
        HOWEVER, WE SHALL CONSIDER THEM FOR AN ADDITIONAL REASON.  THIS REASON
        IS THAT STUDENTS OFTEN NEED ANSWERS TO SUCH QUESTIONS IN THE STUDY
        OF PROBABILITY.  LET'S CONSIDER SOME EXAMPLES.
   TYPE THE LETTERS A AND B CAN BE ORDERED OR ARRANGED IN A ROW AS AB OR ___.
        (IF YOU ARE NOT SURE OF THE ANSWER, GUESS AND SEE WHAT HAPPENS.)
   ANS1 BA
   COMP   10
   TYPE ORDER IS THE ESSENCE OF SUCH ARRANGEMENTS; A CHANGE IN ORDER YIELDS
        A DIFFERENT ARRANGEMENT.  SO AB IS CONSIDERED DIFFERENT FROM BA.
10 TYPE EACH DIFFERENT ARRANGEMENT OF A AND B (AB OR BA) IS CALLED
        A PERMUTATION.  A PERMUTATION OF A NUMBER OF OBJECTS IS ANY ARRANGE-
        MENT OF THESE OBJECTS IN A DEFINITE ___.
   ANS1 ORDER
   COMP   12
   TYPE TO PERMUTE OBJECTS IS TO ARRANGE THEM IN A ROW IN A DEFINITE ORDER.
   GOTO   10
12 TYPE ABC AND ACB ARE TWO POSSIBLE ___ OF A, B, AND C, TAKEN TOGETHER.
   ANS1 PERMUTATION|PERMUTATIONS|ARRANGEMENT|ARRANGEMENTS
   COMP   14
   TYPE NOTICE THE DIFFERENCE IN THE TWO ARRANGEMENTS.  THEY ARE PERMUTATIONS
        OF A, B, AND C.
14 TYPE ANOTHER PERMUTATION OF A, B, AND C, TAKEN TOGETHER, IS ___.
   ANS2 BCA|BAC|CBA|CAB
   COMP   16
   TYPE TO FORM A PERMUTATION, SELECT ANY ONE OF THE LETTERS FOR THE FIRST
        ELEMENT.  THEN, SELECT EITHER OF THE REMAINING TWO LETTERS FOR THE
        SECOND ELEMENT, LEAVING ONE LETTER FOR THE LAST ELEMENT.
   GOTO   14
16 TYPE ANY ONE OF THE FOLLOWING PERMUTATIONS IS CORRECT:  BCA, BAC, CBA, AND
        CAB.
   TYPE TAKING THE SAME THREE LETTERS, THERE ARE HOW MANY POSSIBLE
        PERMUTATIONS?  (REPETITION OF A LETTER IS NOT ALLOWED, SUCH AS
        AAA, ABA.)
```

```
   ANS2 6|SIX
   COMP   18
        NO, YOUR ANSWER IS INCORRECT.
   TYPE YOU COULD HAVE ARRIVED AT THE CORRECT ANSWER BY COUNTING ALL OF THE
        POSSIBLE PERMUTATIONS, BUT IN SOME PROBLEMS THIS IS TOO TIME
        CONSUMING.  THEREFORE, LET'S SEE IF WE CAN DEVELOP A BETTER TECHNIQUE
        FOR COUNTING THE NUMBER OF PERMUTATIONS.
   GOTO   20
18 TYPE DID YOU ARRIVE AT THIS ANSWER BY USING ANY TECHNIQUE OTHER THAN THAT
        OF LISTING AND COUNTING ALL OF THE POSSIBLE PERMUTATIONS?  (YES OR NO)
   BANS YES
   COMP   42
20 TYPE CONSIDER THE PERMUTATION AS BEING CONSTRUCTED WITH THREE POSITIONS,
        EACH POSITION CONTAINING AN ELEMENT FROM THE SET OF LETTERS A, B,
        AND C.  IN HOW MANY DIFFERENT WAYS CAN THE FIRST POSITION OF THE
        PERMUTATION BE CHOSEN?
   ANS1 3|THREE
   COMP   22
        THE FIRST ELEMENT OF THE PERMUTATION CAN BE FILLED WITH A, B, OR C.
        THUS, THERE ARE THREE CHOICES FOR FILLING THE FIRST POSITION.
22 TYPE HAVING DECIDED UPON ONE LETTER TO FILL THE FIRST POSITION, HOW MANY
        CHOICES ARE LEFT FOR THE SECOND ELEMENT IN THE PERMUTATION?
   ANS1 2|TWO
   COMP   24
   TYPE REMEMBER, ONE LETTER HAS ALREADY BEEN SELECTED FOR THE FIRST POSITION;
        FOR EXAMPLE, THE LETTER C.  THIS LEAVES ONLY A OR B AS ALTERNATIVES
        FOR THE SECOND ELEMENT OF THE PERMUTATION.
24 TYPE SIMILARLY, THE LAST ELEMENT CAN BE SELECTED IN HOW MANY WAYS?
   ANS1 1|ONE
   COMP   26
   TYPE IF THE LETTER C HAS BEEN CHOSEN FOR THE FIRST ELEMENT AND THE LETTER A
        HAS BEEN CHOSEN FOR THE SECOND ELEMENT, THEN THERE IS ONLY ONE LETTER
        LEFT FOR THE LAST ELEMENT, THE LETTER B.  SO ONCE THE FIRST TWO
        ELEMENTS ARE CHOSEN, THERE IS ONLY ONE CHOICE FOR THE REMAINING
        ELEMENT.
26 TYPE HOW MANY LESS IS THE NUMBER OF ALTERNATIVES FOR THE SECOND POSITION
        OF THE PERMUTATION THAN FOR THE FIRST POSITION?
```

```
   ANS1 1|ONE
   COMP   28
   TYPE THIS IS DETERMINED BY SUBTRACTING TWO (THE NUMBER OF WAYS OF FILLING
        THE SECOND POSITION) FROM THREE (THE NUMBER OF WAYS OF FILLING THE
        FIRST POSITION).
28 TYPE SINCE THERE ARE THREE LETTERS WHICH CAN BE PLACED IN THE FIRST
        POSITION AND WITH EACH OF THESE THERE ARE TWO LETTERS FROM WHICH A
        SELECTION CAN BE MADE FOR THE SECOND POSITION, THE TOTAL NUMBER OF
        WAYS OF FILLING THE FIRST TWO POSITIONS IS ___.
   ANS1 6|SIX
   COMP   30
   TYPE IF THE PERMUTATION BEGINS WITH THE LETTER A, THEN THE NEXT LETTER CAN
        BE EITHER THE LETTER B OR ___.
   ANS1 C
   COMP   32
32 TYPE HOW MANY PERMUTATIONS BEGIN WITH THE LETTER A?
   ANS1 2|TWO
   COMP   34
   TYPE THESE ARE AB AND AC.  (DON'T WORRY ABOUT THE THIRD ELEMENT NOW.)
34 TYPE THEN, HOW MANY PERMUTATIONS BEGIN WITH THE LETTERS B AND C,
        RESPECTIVELY?
   ANS1 2|TWO
   COMP   36
   TYPE THESE ARE BA AND BC FOR THE LETTER B AND CA AND CB FOR THE LETTER C.
36 TYPE SINCE EACH LETTER IS THE BEGINNING OF TWO PERMUTATIONS, THE TOTAL
        NUMBER OF POSSIBLE TWO-LETTER PERMUTATIONS IS HOW MANY?
   ANS1 6|SIX
   COMP   30
30 TYPE THIS CAN BE DETERMINED BY ADDING 2+2+2 OR BY MULTIPLYING 3X2.  THE
        LATTER METHOD IS MORE CONVENIENT BECAUSE IT IS THE NUMBER OF
        ALTERNATIVES FOR THE FIRST POSITION MULTIPLIED BY THE NUMBER OF
        ALTERNATIVES FOR THE SECOND POSITION.
   TYPE THE NUMBER OF CHOICES FOR THE THIRD POSITION IS HOW MANY LESS THAN
        THE NUMBER OF CHOICES FOR THE SECOND POSITION?
   ANS1 1|ONE
   COMP   38
38 TYPE FOR EACH OF THE SIX POSSIBILITIES FOR FILLING THE FIRST TWO POSITIONS,
```

```
      THERE IS ONE CHOICE FOR THE LAST POSITION.  SO THE TOTAL NUMBER OF
      PERMUTATIONS FOR THE THREE LETTERS IS ___.
   ANS1 6|SIX
   COMP   40
40 TYPE THE ANSWER IS COMPUTED BY TAKING SIX (THE NUMBER OF WAYS FOR THE FIRST
      TWO POSITIONS) TIMES ONE (THE NUMBER OF WAYS FOR THE LAST POSITION) OR
      3X2X1, WHICH EQUALS SIX.  TO DETERMINE THE TOTAL NUMBER OF
      PERMUTATIONS, SIMPLY MULTIPLY TOGETHER THE NUMBER OF WAYS OF CHOOSING
      A LETTER FOR EACH ELEMENT.
   TYPE IN SUMMARY, THE NUMBER OF ALTERNATIVES FOR EACH SUCCESSIVE ELEMENT IS
      ONE LESS THAN THAT OF THE ELEMENT IMMEDIATELY PRECEDING IT.  THIS
      HOLDS TRUE FOR ALL PERMUTATIONS BECAUSE REPETITION OF A LETTER IS NOT
      ALLOWED.
      IN ADDITION, THE TOTAL NUMBER OF PERMUTATIONS CAN BE CALCULATED BY
      MULTIPLYING TOGETHER THE NUMBER OF WAYS THAT EACH ELEMENT CAN BE
      CHOSEN.  IN PART, THIS IS AN EXAMPLE OF WHAT IS CALLED THE
      MULTIPLICATION PRINCIPLE.
      TRY THIS PROBLEM.
   TYPE HOW MANY DIFFERENT WAYS CAN FOUR BOOKS BE ARRANGED ON A SHELF?
   ANS2 24|TWENTY-FOUR|TWENTY FOUR|TWENTYFOUR
   COMP   42
      NO, THAT IS NOT COMPLETELY CORRECT.
      SINCE ORDER IS IMPORTANT, THE QUESTION REALLY ASKS HOW MANY
      PERMUTATIONS CAN BE FORMED BY USING THE FOUR BOOKS.
      THE ANSWER IS ___.
   ANS2 24|TWENTY-FOUR|TWENTY FOUR|TWENTYFOUR
   COMP   42
   TYPE NO, LET'S BREAK THE QUESTION DOWN INTO PARTS.
   TYPE HOW MANY POSITIONS ARE NEEDED IN THE PERMUTATION TO REPRESENT THE
      PLACES ON THE SHELF?
   ANS1 4|FOUR
   COMP   44
44 TYPE HOW MANY ALTERNATIVES ARE THERE FOR THE FIRST POSITION?
   ANS1 4|FOUR
   COMP   46
   TYPE REMEMBER, THE FIRST PLACE CAN BE ASSUMED BY ANY ONE OF THE BOOKS.
46 TYPE THE SECOND PLACE HAS HOW MANY ALTERNATIVES?
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   ANS1 3|THREE
   COMP   48
   TYPE WHENEVER REPETITION IS NOT ALLOWED, EACH SUCCESSIVE CHOICE HAS ONE
        LESS ALTERNATIVE THAN THE ONE IMMEDIATELY PRECEDING IT.
48 TYPE THE THIRD POSITION CAN BE FILLED USING HOW MANY ALTERNATIVES?
   ANS1 2|TWO
   COMP   50
   TYPE THE LAST POSITION HAS ONLY ONE POSSIBLE CHOICE.  THE POSITIONS HAVE
        BEEN SELECTED IN 4, 3, 2, AND 1 WAYS, RESPECTIVELY.
50 TYPE WE HAVE ALREADY FOUND IN THE PRECEDING PROBLEM THAT THE LAST THREE
        ELEMENTS CAN BE FILLED IN 3X2X1 OR ___ WAYS.
   ANS1 6|SIX
   COMP   52
   TYPE THE TOTAL NUMBER OF WAYS THAT THE LAST THREE POSITIONS CAN BE CHOSEN
        IS CALCULATED BY MULTIPLYING TOGETHER THE NUMBER OF ALTERNATIVES FOR
        EACH INDIVIDUAL POSITION, WHICH IS 3X2X1 OR 6.
52 TYPE THUS, FOR EACH ALTERNATIVE FOR THE FIRST POSITION, THE REMAINING THREE
        POSITIONS CAN BE CHOSEN IN SIX WAYS.  THE TOTAL NUMBER OF PERMUTATIONS
        IS DETERMINED BY MULTIPLYING ___ X 6.
   ANS1 4|FOUR
   COMP   54
54 TYPE FOUR BOOKS CAN BE ARRANGED ON A SHELF ___ WAYS.
   ANS1 24|TWENTY-FOUR|TWENTY FOUR|TWENTYFOUR
   COMP   42
        THE FOUR POSITIONS ARE FILLED IN 4, 3, 2, AND 1 WAYS, RESPECTIVELY;
        AND THE TOTAL NUMBER OF ARRANGEMENTS IS 4X3X2X1 OR 4X6, WHICH IS
        EQUAL TO 24.
42 TYPE HOW MANY DIFFERENT WAYS CAN SIX LETTERS BE ARRANGED, TAKEN TWO LETTERS
        AT A TIME?  (IF THE LETTERS ARE THE FIRST SIX IN OUR ALPHABET, EXAMPLE
        ARRANGEMENTS ARE:  AB, DF, FB, CE, AND EA -- REPETITION NOT ALLOWED.)
   ANS2 30|THIRTY
   COMP   56
   TYPE HOW MANY POSITIONS ARE NEEDED TO REPRESENT THE PERMUTATION OF
        SIX LETTERS, TAKEN TWO AT A TIME?
   ANS1 2|TWO
   COMP   58
   TYPE EACH PERMUTATION OR ARRANGEMENT CONSISTS OF ONLY TWO LETTERS;
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        HENCE, WE NEED ONLY TWO POSITIONS.
58 TYPE THE NUMBER OF ALTERNATIVES FOR THE FIRST ELEMENT IS ___.
   ANS1 6|SIX
   COMP   60
   TYPE THE FIRST ELEMENT CAN BE SELECTED FROM ANY ONE OF THE SIX LETTERS.
        THEREFORE, THERE ARE SIX POSSIBLE ALTERNATIVES FOR IT.
60 TYPE ARE YOU NOW ABLE TO ANSWER THE ORIGINAL PROBLEM:  HOW MANY DIFFERENT
        WAYS CAN SIX LETTERS BE ARRANGED, TAKEN TWO AT A TIME?  (YES OR NO)
   BANS YES
   COMP   62
68 TYPE THERE ARE ___ CHOICES FOR THE SECOND ELEMENT.
   ANS1 5|FIVE
   COMP   64
   TYPE ONE LETTER HAS BEEN USED TO FILL THE FIRST POSITION, LEAVING ONE LESS
        OR FIVE ALTERNATIVES FOR THE SECOND POSITION.
64 TYPE THE TOTAL NUMBER OF DIFFERENT WAYS OF SELECTING THE TWO POSITIONS IS
        ___.
   ANS1 30|THIRTY
   COMP   66
        TO CALCULATE THE NUMBER OF PERMUTATIONS, SIMPLY MULTIPLY THE NUMBER OF
        ALTERNATIVES FOR THE FIRST POSITION (SIX) BY THE NUMBER OF
        ALTERNATIVES FOR THE SECOND POSITION (FIVE) FOR AN ANSWER OF THIRTY.
   GOTO   66
62 TYPE WHAT DO YOU NOW CALCULATE THE ANSWER TO BE?
   ANS2 30|THIRTY
   COMP   56
   GOTO   68
66 TYPE SO, THE NUMBER OF DIFFERENT PERMUTATIONS OF SIX THINGS, TAKEN TWO
        AT A TIME, IS THIRTY.
56 TYPE TRY THIS PROBLEM, WHICH IS NOT A PERMUTATION BUT CAN BE SOLVED USING
        THE SAME TECHNIQUE.
        HOW MANY LICENSE PLATES CAN BE MADE USING TWO LETTERS FOLLOWED BY A
        THREE-DIGIT NUMBER?  (DO NOT DO ALL THE MULTIPLICATION.  SIMPLY REPLY
        WITH THE NUMBERS THAT WOULD BE MULTIPLIED TOGETHER.  ANSWER IN THIS
        FORM, _X_X_X_X_, WHERE X IS A MULTIPLICATION SYMBOL AND _ IS THE
        NUMERIC ANSWER.)
   TYPE REMEMBER, THIS IS NOT A PERMUTATION PROBLEM BUT CAN BE SOLVED USING
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      THE SAME TECHNIQUE.
   ANS2 26X26X9X10X10|26 X 26 X 9 X 10 X 10|608,400
   COMP   84
   TYPE THE FIRST LETTER CAN BE SELECTED FROM HOW MANY ALTERNATIVES?
   ANS1 26|TWENTY-SIX|TWENTYSIX|TWENTY SIX
   COMP   72
   TYPE ALL THE LETTERS OF THE ALPHABET ARE POSSIBLE CHOICES.
72 TYPE PERMITTING REPETITION OF LETTERS, THE SECOND LETTER CAN BE DETERMINED
      IN ANY ONE OF ___ WAYS.
   ANS1 26|TWENTY-SIX|TWENTY SIX|TWENTYSIX
   COMP   74
   TYPE REPETITION OF LETTERS IS ONE OF THE REASONS THIS IS NOT A PERMUTATION
      PROBLEM, BUT IT CAN BE SOLVED USING THE SAME TECHNIQUE.
74 TYPE THE FIRST TWO SPACES IN EACH LICENSE-PLATE NUMBER CAN BE FORMED
      IN ___X___ WAYS.  (REPLY IN THE FORM _X_.)
   ANS1 26X26|26 X 26|TWENTY-SIXXTWENTY-SIX|TWENTY-SIX X TWENTY-SIX
   COMP   76
76 TYPE NOW, THE REMAINING POSITIONS IN EACH LICENSE-PLATE NUMBER (OR THE
      THREE-DIGIT NUMBER) CAN BE SELECTED IN HOW MANY WAYS?  (AGAIN USE
      THE FORM _X_X_.)
   ANS2 9X10X10|NINEXTENXTEN|NINE X TEN X TEN
   COMP   84
   WANS 10X10X10|TENXTENXTEN|TEN X TEN X TEN
   COMP   78
   TYPE IF A ZERO IS NOT ALLOWED BECAUSE A THREE-DIGIT NUMBER IS DESIRED,
      THE FIRST NUMERAL CAN BE CHOSEN FROM ANY ONE OF ___ DIGITS.
   ANS1 9|NINE
   COMP   80
   TYPE THERE ARE NINE POSSIBLE DIGITS (1, 2, 3, 4, 5, 6, 7, 8, 9).
80 TYPE IF BOTH ZERO AND REPETITION OF A DIGIT ARE ALLOWED, THE REMAINING
      TWO NUMERALS OF THE LICENSE PLATE CAN EACH BE SELECTED IN ___ WAYS.
   ANS1 10|TEN
   COMP   82
      IN MATHEMATICS, REPETITION OF A DIGIT, SUCH AS 99, IS PERMISSIBLE,
      AS IS THE USE OF ZERO, SUCH AS 100.  EACH OF THE LAST TWO NUMERALS
      CAN BE SELECTED FROM ANY ONE OF TEN DIGITS.
82 TYPE THE FIRST TWO LETTERS CAN BE FORMED IN 26X26 WAYS, AND THE THREE
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      DIGITS CAN BE FORMED IN ___X___X___ WAYS.  (AGAIN SIMPLY SUPPLY THE
      NUMBERS.  USE THE NUMBERS IN THE PREVIOUS THREE QUESTIONS.)
   ANS1 9X10X10|9 10 10|91010|900
   COMP   84
84 TYPE THE TOTAL NUMBER OF LICENSE PLATES IS CALCULATED BY MULTIPLYING
      25X26X9X10X10, WHICH EQUALS 608,400 WAYS.
   GOTO   70
78 TYPE NO, TO FORM A THREE-DIGIT NUMBER THE FIRST DIGIT CANNOT BE ZERO.
   TYPE SO, THERE ARE ONLY NINE POSSIBLE WAYS OF FORMING THE FIRST DIGIT.
   GOTO   84
70 TYPE THE MAIN CONCEPT OF THIS LESSON HAS BEEN TO INTRODUCE PERMUTATIONS AND
      A WAY OF COUNTING THEM.  THE COUNTING TECHNIQUE THAT HAS BEEN
      EMPLOYED IS CALLED THE MULTIPLICATION PRINCIPLE.  SIMPLY STATED, IT
      SAYS:  IF AN OPERATION CAN BE PERFORMED IN N WAYS AND, AFTER IT HAS
      BEEN PERFORMED IN ANY ONE OF THESE WAYS, A SECOND OPERATION CAN BE
      PERFORMED IN M WAYS AND, AFTER IT HAS BEEN PERFORMED IN ANY ONE OF
      THESE WAYS, A THIRD OPERATION CAN BE PERFORMED IN P WAYS, THEN THE
      TOTAL OPERATION CAN BE PERFORMED IN NXMXP WAYS.  THIS TECHNIQUE CAN BE
      GENERALIZED TO INCLUDE ANY NUMBER OF OPERATIONS, EVEN THOUGH ONLY
      THREE HAVE BEEN LISTED.  AN EXAMPLE OF THIS IS THE LICENSE-PLATE
      PROBLEM, WHICH HAS FIVE OPERATIONS.
   TYPE LET'S TRY SOME HOMEWORK PROBLEMS.
   BRAN   8660
      1.  IF EIGHT PERSONS APPLY FOR THREE JOBS, IN HOW MANY WAYS CAN THE
          JOBS BE FILLED?
      2.  AN AUTOMOBILE BUYER CAN CHOOSE FROM THREE BODY STYLES, TWO TYPES
          OF ENGINES AND SIX COLORS.  IN HOW MANY WAYS CAN HE CHOOSE A CAR?
      3.  A TEST CONSISTS OF FIVE MATCHING QUESTIONS AND FIVE ANSWERS.
          (ANSWERS CAN ONLY BE USED ONCE.)  IF A STUDENT GUESSES AT RANDOM
          ON EACH QUESTION, HOW MANY DIFFERENT WAYS CAN HE ANSWER THE TEST?
      4.  HOW MANY FIVE-DIGIT NUMBERS CAN BE FORMED FROM THE INTEGERS 1, 2,
          4, 6, 7, AND 8 IF NO INTEGER CAN BE USED MORE THAN ONCE?
      5.  IN HOW MANY WAYS CAN EIGHT PEOPLE LINE UP AT A TICKET WINDOW?
      6.  HOW MANY DIFFERENT PARTIES OF THREE CAN BE FORMED FROM NINE
          PEOPLE?
      7.  GIVEN THE DIGITS 1, 2, 3, 4, AND 5, FIND HOW MANY FOUR-DIGIT
          NUMBERS CAN BE FORMED FROM THEM
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            A.  IF NO DIGIT MAY BE REPEATED?
            B.  IF REPETITIONS OF A DIGIT ARE ALLOWED?
   GOTO   90
86 BRAN   8880
        1.  IF FIVE FAIR COINS ARE TOSSED, IN HOW MANY WAYS CAN THEY FALL?
        2.  HOW MANY FOUR-DIGIT NUMBERS CAN BE FORMED?
        3.  A TEST CONSISTS OF FIVE MATCHING QUESTIONS AND FIVE ANSWERS.
            (ANSWERS CAN ONLY BE USED ONCE.)  IF A STUDENT GUESSES AT RANDOM
            ON EACH QUESTION, HOW MANY DIFFERENT WAYS CAN HE ANSWER THE TEST?
        4.  IF THERE ARE THREE ROUTES FROM A TO B, FOUR ROUTES FROM B TO C,
            AND TWO ROUTES FROM C TO A, IN HOW MANY WAYS CAN A ROUND TRIP FROM
            A TO B TO C TO A BE MADE WITHOUT GOING BACK THROUGH B?
        5.  A TEST CONSISTS OF FIVE MULTIPLE CHOICE QUESTIONS.  THERE ARE
            THREE CHOICES FOR EACH QUESTION.  IF A STUDENT GUESSES AT RANDOM
            ON EACH QUESTION, HOW MANY DIFFERENT WAYS CAN HE ANSWER THIS TEST?
        6.  FIVE BOYS COMPETE IN A SWIMMING RACE.  IN HOW MANY WAYS CAN THE
            FIRST THREE PLACES BE TAKEN?
        7.  GIVEN THE DIGITS 1, 2, 3, 4, AND 5, FIND HOW MANY FOUR-DIGIT
            NUMBERS CAN BE FORMED FROM THEM
              A.  IF NO DIGIT MAY BE REPEATED?
              B.  IF REPETITIONS OF A DIGIT ARE ALLOWED?
   GOTO   90
88      1.  IN HOW MANY WAYS CAN SIX PERSONS LEAVE A BUILDING IF THERE ARE
            FOUR EXITS?
        2.  HOW MANY GROUPS OF FOUR OR LESS CAN BE FORMED FROM EIGHT PEOPLE?
        3.  A TEST CONSISTS OF FIVE MATCHING QUESTIONS AND FIVE ANSWERS.
            (ANSWERS CAN ONLY BE USED ONCE.)  IF A STUDENT GUESSES AT RANDOM
            ON EACH QUESTION, HOW MANY DIFFERENT WAYS CAN HE ANSWER THE TEST?
        4.  HOW MANY FIVE-DIGIT NUMBERS CAN BE FORMED FROM THE INTEGERS 1, 2,
            4, 6, 7, AND 8 IF NO INTEGER CAN BE USED MORE THAN ONCE?
            HOW MANY OF THESE NUMBERS WILL BE EVEN?
            HOW MANY OF THESE NUMBERS WILL BE ODD?
        5.  A TEST CONSISTS OF FIVE MULTIPLE CHOICE QUESTIONS.  THERE ARE
            THREE CHOICES FOR EACH QUESTION.  IF A STUDENT GUESSES AT RANDOM
            ON EACH QUESTION, HOW MANY DIFFERENT WAYS CAN HE ANSWER THIS TEST?
        6.  FIVE BOYS COMPETE IN A SWIMMING RACE.  IN HOW MANY WAYS CAN THE
            FIRST THREE PLACES BE TAKEN?
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7. THERE ARE SEVEN SEATS AVAILABLE IN A SEDAN. IN HOW MANY WAYS CAN SEVEN PERSONS BE SEATED FOR A JOURNEY IF ONLY THREE ARE ABLE TO DRIVE?

9C STOP STOP

B. Concept Block Two

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  TYPE CONCEPT BLOCK TWO:  PERMUTATIONS.
4 TYPE THE NUMBER OF POSSIBLE R-ELEMENT PERMUTATIONS FORMED FROM A SET OF
       N ELEMENTS IS DENOTED BY PER(N,R).  THE NUMBER OF THREE-ELEMENT
       PERMUTATIONS FORMED FROM A SET CONTAINING FIVE OBJECTS CAN BE
       INDICATED AS ___.
  ANS1 PER(5,3)|PER(FIVE,THREE)
  COMP    2
  TYPE IN THE NOTATION PER(N,R), THE FIRST ARGUMENT, N, DENOTES THE TOTAL
       NUMBER OF POSSIBLE OBJECTS AND THE SECOND ARGUMENT, R, DENOTES THE
       NUMBER OF ELEMENTS OR POSITIONS IN EACH PERMUTATION.  FOR EXAMPLE,
       PER(5,2) DENOTES THE NUMBER OF PERMUTATIONS OF FIVE DISTINCT OBJECTS,
       TAKEN ___ AT A TIME.
  ANS1 TWO|2
  COMP    2
  TYPE PER(6,3) REPRESENTS THE NUMBER OF PERMUTATIONS OF ___ DISTINCT ITEMS,
  ANS1 SIX|6
       TAKEN ___ AT A TIME.
  ANS1 THREE|3
  COMP    2
  GOTO    4
2      TO GIVE AN ILLUSTRATION, IF THE OBJECTS USED ARE THE FIRST FIVE
       LETTERS OF OUR ALPHABET, FOUR OF THE POSSIBLE PERMUTATIONS ARE:  ABE,
       CDA, EAD, AND DBE.  PER(5,3) IS A SYMBOL FOR REPRESENTING THE NUMBER
       OF PERMUTATIONS, AND THE NUMERIC VALUE OF THE NUMBER IS ___.
       (REMEMBER, THIS DENOTES THE NUMBER OF PERMUTATIONS OF FIVE THINGS,
       TAKEN THREE AT A TIME.  THE ANSWER CAN BE CALCULATED BY USING THE
       MULTIPLICATION PRINCIPLE.)
  ANS2 60|SIXTY
  COMP    6
  TYPE NO, THIS IS A DIRECT APPLICATION OF THE MULTIPLICATION PRINCIPLE.
       IN APPLYING IT, THERE ARE ___ POSITIONS IN EACH PERMUTATION WHICH
       CONTAIN DISTINCT OBJECTS FROM THE SET OF FIVE ITEMS.
  ANS1 THREE|3
  TYPE IN SELECTING ELEMENTS FOR THE THREE POSITIONS OF THE PERMUTATION,
       THE FIRST ELEMENT CAN BE CHOSEN IN ANY ONE OF ___ WAYS.
  ANS1 FIVE|5
  TYPE WITH EACH OF THESE, THE SECOND ELEMENT CAN BE SELECTED IN ___ WAYS.
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  ANS1 FOUR|4
  COMP    8
  TYPE IN CONCEPT ONE IT WAS STATED THAT THE NUMBER OF ALTERNATIVES FOR
       EACH SUCCESSIVE POSITION IS ONE LESS THAN THE NUMBER FOR THE
       POSITION IMMEDIATELY PRECEDING IT.
8 TYPE FINALLY, THE THIRD ELEMENT CAN BE CHOSEN IN ___ WAYS.
  ANS1 THREE|3
  TYPE TO CALCULATE THE NUMBER OF PERMUTATIONS, THE VALUES FIVE, FOUR, AND
       THREE ARE ___ TOGETHER.  (WHAT MATHEMATICAL OPERATION)
  ANS1 MULTIPLIED|MULTIPLY|TIMES|X
  COMP    9
  BRAN    950
  TYPE YOU HAVE ANSWERED THE LAST TEN QUESTIONS WITH LESS THAN FIFTY
       PERCENT ACCURACY.  IT IS SUGGESTED THAT YOU REVIEW THE MULTIPLICATION
       PRINCIPLE, CONCEPT BLOCK ONE, BEFORE CONTINUING FURTHER.  DO YOU FEEL
       YOU NEED MORE STUDY ON THE MULTIPLICATION PRINCIPLE?
       (YES OR NO)
  BANS NO
  COMP    7
  GOTO   35
9 TYPE THEREFORE, PER(5,3) IS EVALUATED AS FIVE X FOUR X THREE OR ___ WAYS.
  ANS1 60|SIXTY
  COMP    6
  TYPE FIVE X FOUR IS TWENTY, AND TWENTY X THREE IS SIXTY.
7 TYPE THE VALUE OF PER(5,3) IS CALCULATED BY MULTIPLYING TOGETHER FIVE,
       FOUR, AND THREE, WHICH GIVES A VALUE OF SIXTY.
  TYPE GIVEN A SET OF TEN LETTERS, THE NUMBER OF THREE-ELEMENT PERMUTATIONS
       WHICH CAN BE FORMED IS REPRESENTED BY ___.
  ANS1 PER(10,3)|PER(TEN,THREE)
  COMP    5
  TYPE THE NOTATION FOR THE NUMBER OF PERMUTATIONS OF N LETTERS, TAKEN R
       AT A TIME, IS PER(N,R).  N IS THE NUMBER OF POSSIBLE THINGS (10), AND
       R IS THE NUMBER OF ELEMENTS IN EACH PERMUTATION (3).  SUBSTITUTING
       THE CORRESPONDING VALUES FOR N AND R, PER(N,R) BECOMES PER(10,3).
5 TYPE THE VALUE OF R ALWAYS EQUALS THE NUMBER OF ELEMENTS OR POSITIONS IN
       EACH PERMUTATION.  THE THREE ELEMENTS OF THE PERMUTATION CAN BE CHOSEN
       IN TEN, NINE, AND EIGHT WAYS, RESPECTIVELY.  THE FIRST ELEMENT CAN BE
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        CHOSEN FROM ANY ONE OF THE TOTAL SET OR, IN OTHER WORDS, TEN WAYS.
        EACH SUCCESSIVE ELEMENT HAS ONE LESS CHOICE THAN DOES THE ELEMENT
        IMMEDIATELY PRECEDING IT.  THE NUMBER OF PERMUTATIONS IS CALCULATED BY
        MULTIPLYING THESE VALUES TOGETHER.  PER(10,3) IS EVALUATED AS 10X9X8
        OR 720.
 6 TYPE NOW, CONSIDER PER(N,2).  EACH PERMUTATION CONSISTS OF TWO ELEMENTS,
        AND THE FIRST ELEMENT CAN BE SELECTED FROM ANY ONE OF ___ ITEMS.
   ANS1 N
   TYPE THE SECOND ELEMENT CAN BE CHOSEN FROM ANY ONE OF THE REMAINING
        ___ ITEMS.
   ANS1 N-1
   TYPE APPLYING THE MULTIPLICATION PRINCIPLE, PER(N,2) IS EVALUATED AS
        NX(N-1).
   TYPE GENERALIZING, THE NUMBER OF PERMUTATIONS OF N THINGS, TAKEN R AT
        A TIME, CAN BE DENOTED BY PER(N,R), WHICH EQUALS NX(N-1)X(N-2)X...
        X(N-(R-1)).  (SINCE THE VALUE OF N IS NOT KNOWN, THE NOTATION OF THREE
        DOTS (...) IS USED TO INDICATE THAT NOT ALL THE TERMS ARE DENOTED.)
   TYPE NOTICE THE PATTERN.  EACH TERM IN THE PRODUCT IS ONE LESS THAN N MINUS
        THE POSITIONS OF THE TERM (NUMBERING FROM THE LEFT).
        THERE ARE R TERMS, SO THE LAST TERM IS N-(R-___).
   ANS1 ONE|1
   COMP    3
   TYPE THE THIRD TERM IS N-(3-1) OR N-2, WHICH IS THE VALUE OF THE THIRD
        TERM IN THE PRODUCT.
 3 TYPE THE LAST TERM IS N-(R-1).
   TYPE THIS CAN BE SIMPLIFIED AS N-R+1.  PER(N,R) EQUALS NX(N-1)X(N-2)X...
        X(N-R+1).  IN THIS FORMULA, N-R+1 DOESN'T NEED TO BE MEMORIZED
        BECAUSE IT CAN BE DETERMINED BY APPLYING THE MULTIPLICATION PRINCIPLE.
   TYPE USING THE NOTATION OF N-(R-1), THE TENTH TERM IS N-___.
   ANS1 NINE|9
   COMP   10
11 TYPE IF N IS EQUAL TO FIFTEEN AND R IS EQUAL TO FOUR, THE VALUE OF THE
        LAST TERM IN THE PRODUCT OF PER(15,4) IS ___.
   ANS1 TWELVE|12
   COMP   10
   TYPE THE TERM CAN BE CALCULATED USING N-R+1 OR 15-4+1, WHICH IS EVALUATED
        AS TWELVE.  IT CAN ALSO BE FOUND BY USING THE NOTATION OF CONCEPT
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        BLOCK ONE.  THE FIRST ELEMENT OF THE PERMUTATION CAN BE CHOSEN IN
        FIFTEEN WAYS, AND EACH SUCCESSIVE ELEMENT CAN BE CHOSEN IN ONE LESS
        WAY THAN CAN THE ELEMENT IMMEDIATELY PRECEDING IT.  THE REMAINING
        ALTERNATIVES FOR EACH ELEMENT OF THE PERMUTATION ARE FOURTEEN,
        THIRTEEN, AND TWELVE, RESPECTIVELY.  SO, THE LAST TERM HAS THE SAME
        NUMERIC VALUE AS DOES THE NUMBER OF WAYS OF CHOOSING THE LAST ELEMENT
        OF THE PERMUTATION.
    GOTO   11
10  TYPE  IN THE PROBLEM PER(50,47), HOW MANY POSITIONS OR ELEMENTS ARE NEEDED
        TO REPRESENT THE PERMUTATION?  (IN OTHER WORDS, THE PERMUTATION
        CONSISTS OF ___ ELEMENTS.)
    ANS1  47|FORTY-SEVEN|FORTY SEVEN|FORTYSEVEN
    TYPE  BY CONSIDERING THE PREVIOUS PROBLEM AND WITHOUT WRITING DOWN THE
        NUMERIC VALUE FOR THE WAYS OF CHOOSING THE FIRST FIVE ELEMENTS,
        DETERMINE THE NUMBER OF CHOICES FROM WHICH THE SIXTH ELEMENT CAN BE
        SELECTED.  (USE THE FORMULA N-R+1.)
    ANS1  45|FORTY-FIVE|FORTY FIVE|FORTYFIVE
    COMP   13
    TYPE  USING THE FORMULA N-R+1, N IS EQUAL TO FIFTY AND R IS EQUAL TO SIX.
        SUBSTITUTING INTO THE FORMULA, THE SOLUTION IS 50-6+1 OR 45.
13  TYPE  THE NUMBER OF PERMUTATIONS OF TEN THINGS, TAKEN TEN AT A TIME,
        IS EVALUATED AS 10X9X8X7X6X5X4X3X2X1.  SUCH A PRODUCT IS AWKWARD TO
        EXPRESS, AND A SYMBOL WHICH ALLOWS A MORE CONVENIENT NOTATION IS N!.
        THIS IS READ AS N FACTORIAL AND REPRESENTS THE PRODUCT OF ALL WHOLE
        NUMBERS FROM ONE THROUGH N.  FOR EXAMPLE, FOUR FACTORIAL IS THE
        PRODUCT OF 4X3X2X1 OR ___.  (ANSWER WITH A NUMERIC VALUE.)
    ANS1  24|TWENTY-FOUR|TWENTY FOUR|TWENTYFOUR|4!|FOUR!
    TYPE  THE NUMBER OF PERMUTATIONS OF A, B, C, AND D, TAKEN ALTOGETHER,
        IS DENOTED AS PER(4,4) AND IS EVALUATED AS ___!.
    ANS2  4|FOUR|4!|FOUR!|24|TWENTY-FOUR|TWENTY FOUR|TWENTYFOUR
    COMP   12
    TYPE  NO, LET'S ANALYZE THE PROBLEM FURTHER.
    TYPE  THE NUMBER OF POSITIONS OR ELEMENTS OF THE PERMUTATION IS FOUR.
        THE FIRST ELEMENT CAN BE CHOSEN IN FOUR WAYS.  THE REMAINING THREE
        ELEMENTS CAN BE CHOSEN IN THREE, TWO, AND ONE WAYS, RESPECTIVELY.
        THEREFORE, APPLYING THE MULTIPLICATION PRINCIPLE, PER(4,4) IS EQUAL
        TO 4X3X2X1 OR ___!.
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   ANS1 4|FOUR|4!|FOUR!|24|TWENTY-FOUR|TWENTY FOUR|TWENTYFOUR
12 TYPE THE NUMBER OF PERMUTATIONS OF FIVE DISTINCT OBJECTS, TAKEN ALTOGETHER,
        CAN BE REPRESENTED BY ___.
   ANS2 PER(5,5)|5!|FIVE!|FIVE FACTORIAL|120|ONE HUNDRED TWENTY
   COMP   15
   TYPE THE NOTATION FOR THE NUMBER OF PERMUTATIONS OF FIVE THINGS, TAKEN
        ALTOGETHER, IS PER(5,5).
   GOTO   14
15 WANS PER(5,5)
   COMP   14
   GOTO   17
14 TYPE PER(5,5) IS EQUAL TO ___!.
   ANS1 FIVE|5|5!|FIVE!|120|ONE HUNDRED TWENTY|ONE HUNDRED AND TWENTY
   TYPE THIS IS READ AS FIVE ___.
   ANS1 FACTORIAL|!
16 TYPE FIVE FACTORIAL IS THE PRODUCT OF ALL WHOLE NUMBERS FROM ONE
        THROUGH ___.
   ANS1 FIVE|5
17 TYPE PER(5,5) IS EQUAL TO FIVE FACTORIAL, WHICH IS EVALUATED AS 5X4X3X2X1
        OR ___.  (ANSWER WITH ONE NUMERIC VALUE.)
   ANS2 120|ONE HUNDRED TWENTY|ONE HUNDRED AND TWENTY
   COMP   18
   TYPE IF MULTIPLYING 5X4X3X2X1 SEEMS DIFFICULT, IT CAN BE SIMPLIFIED
        AS 5X(4X3X2X1) OR 5X(4!).  FOUR FACTORIAL IS TWENTY-FOUR.
        WITH THIS INFORMATION, TRY THE PROBLEM AGAIN.
   ANS1 120|ONE HUNDRED TWENTY|ONE HUNDRED AND TWENTY
   GOTO   20
18 TYPE PER(N,N) IS USED TO REPRESENT THE NUMBER OF PERMUTATIONS OF N
        DISTINCT OBJECTS, TAKEN ALTOGETHER, AND IS EQUAL TO ___!.
   ANS1 N|N!
   TYPE PER(N,N) IS THE PRODUCT OF ALL WHOLE NUMBERS FROM ONE THROUGH N,
        WHICH IS SIMPLY STATED AS N!.
20 BRAN   2250
   GOTO   34
22 TYPE THE MULTIPLICATION PRINCIPLE IS AN EFFICIENT METHOD OF COUNTING,
        BUT LET'S TRY TO REPRESENT THE NUMBER OF PERMUTATIONS IN A MORE
        CONCISE FORMULA.  IT WAS JUST STATED THAT PER(N,N) EQUALS N FACTORIAL,
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        WHICH CAN BE EXPRESSED AS NX(N-1)X(N-2)X...X(N-R+1)X(N-R)X...X2X1.
        THIS EXPRESSION DENOTES THE PRODUCT OF ALL WHOLE NUMBERS FROM ONE
        THROUGH N, WHERE R<=N.  THIS PRODUCT CAN BE SEPARATED INTO THE
        PRODUCT OF TWO EXPRESSIONS.  TAKING THE FIRST R TERMS, THE FIRST
        EXPRESSION IS NX(N-1)X(N-2)X...X(N-R+1).  USING PERMUTATION
        NOTATION, THIS EXPRESSION CAN BE DENOTED BY PER(___,___).
        WHAT IS THE ANSWER TO THE FIRST BLANK IN PER(___,___)?  (ONE ANSWER)
   ANS1 N
   COMP   23
   TYPE THIS CAN BE DETERMINED BY THE VALUE OF THE FIRST TERM IN THE
        EXPRESSION WHICH IS N.
23 TYPE WHAT IS THE ANSWER TO THE SECOND BLANK IN PER(___,___)?  (ONE ANSWER)
   ANS1 R
   COMP   24
   TYPE REMEMBER, PER(N,R) REPRESENTS THE NUMBER OF PERMUTATIONS OF N
        THINGS, TAKEN R AT A TIME.  R IS THE NUMBER OF ELEMENTS OR POSITIONS
        OF THE PERMUTATION, AND N IS THE NUMBER IN THE SET OF POSSIBLE
        CHOICES.
24 TYPE THE SECOND EXPRESSION, (N-R)X(N-R-1)X...X2X1, CAN BE WRITTEN IN
        FACTORIAL NOTATION AS ___!.
   ANS1 (N-R)|N-R|N - R|(N-R)!|N-R!
   COMP   26
   TYPE THE PRODUCT OF ALL WHOLE NUMBERS FROM ONE THROUGH N IS DEFINED
        AS N FACTORIAL.  THE SECOND EXPRESSION IS THE PRODUCT OF ALL WHOLE
        NUMBERS FROM ONE THROUGH (N-R) AND CAN BE DENOTED AS (N-R)!.
26 TYPE SUBSTITUTING THESE VALUES INTO THE EXPRESSION FOR PER(N,N), IT CAN
        BE WRITTEN AS PER(N,R)X(N-R)!.  SO, PER(N,N)=PER(N,R)X(N-R)!.
   TYPE BY DIVIDING BOTH SIDES OF THIS EQUATION BY THE QUANTITY (N-R)!, THE
        EQUATION CAN BE EXPRESSED AS PER(N,N)/(N-R)!=PER(N,R)X(N-R)!/(N-R)!.
   TYPE SIMPLIFYING, THE EQUATION CAN BE WRITTEN AS PER(N,N)/(N-R)! = ___.
   ANS1 PER(N,R)
   COMP   25
   TYPE (N-R)!/(N-R)! IS EVALUATED AS ONE, AND PER(N,R) X ONE IS SIMPLY
        PER(N,R).
25 TYPE BY DIVIDING BY (N-R)!, THE EQUATION CAN BE EXPRESSED IN TERMS OF
        PER(N,R).  THE FORMULA BECOMES PER(N,R)=PER(N,N)/(N-R)!.
29 TYPE FURTHER, PER(N,N) CAN BE EXPRESSED AS ___.
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   ANS2 N!|N FACTORIAL|NFACTORIAL
   COMP   27
   TYPE NO, THE NUMBER OF PERMUTATIONS OF N THINGS, TAKEN N AT A TIME, CAN BE
        EXPRESSED AS PER(N,N) OR NX(N-1)X...X2X1.  THIS IS N FACTORIAL.
   GOTO   29
27 TYPE SUBSTITUTING N! FOR PER(N,N), THE EQUATION BECOMES PER(N,R)=N!/(N-R)!.
        THIS EQUATION GIVES A CONCISE FORMULA FOR THE NUMBER OF PERMUTATIONS
        OF N ITEMS, TAKEN R AT A TIME.  CONSIDER THE EXAMPLE PER(5,2).
        THE VALUE OF N IS ___.
   ANS1 FIVE|5
        THE VALUE OF N-R IS ___.
   ANS1 THREE|3
   TYPE THEREFORE, PER(5,2) = ___/___.  (ANSWER IN THE FORM ___/___.  USE
        FACTORIALS TO EXPRESS VALUES.)
   ANS2 5!/3!|FIVE!/THREE!|FIVE FACTORIAL/THREE FACTORIAL
   COMP   31
   TYPE NO, THAT IS INCORRECT.
   TYPE APPLYING THE MULTIPLICATION PRINCIPLE, THE NUMBER OF PERMUTATIONS
        OF FIVE THINGS, TAKEN TWO AT A TIME, IS CALCULATED BY MULTIPLYING FIVE
        TIMES FOUR.  NOTICE THIS CAN BE WRITTEN AS (5X4)/1 WITHOUT CHANGING
        THE NUMERIC VALUE.  THE NUMERATOR 5X4 CAN BE EXPRESSED AS FIVE
        FACTORIAL IF IT CONTAINS THE FACTORS ___X___X___.  (ANSWER IN THE
        FORM ___X___X___X.)
   TYPE NOTICE THIS CAN BE WRITTEN AS (5X4)/1 WITHOUT CHANGING THE NUMERIC
        VALUE.  THE NUMERATOR 5X4 CAN BE EXPRESSED AS FIVE FACTORIAL IF IT
        CONTAINS THE FACTORS ___X___X___.  (ANSWER IN THE FORM ___X___X___.)
   ANS1 3X2X1|THREEXTWOXONE|THREE X TWO X ONE
   COMP   28
   TYPE THE DEFINITION OF N! IS THE PRODUCT OF ALL WHOLE NUMBERS FROM ONE
        THROUGH N.  APPLYING THIS DEFINITION, FIVE FACTORIAL IS THE PRODUCT
        OF ALL WHOLE NUMBERS FROM ONE THROUGH FIVE.  THE NUMBERS FIVE AND
        FOUR ARE GIVEN; AND TO COMPLETE THE FACTORIAL, THE NUMBERS THREE, TWO,
        AND ONE ARE NEEDED.
28 TYPE IN ORDER TO MAKE THE NUMERATOR FIVE FACTORIAL, BOTH NUMERATOR AND
        DENOMINATOR ARE MULTIPLIED BY 3X2X1.  THEREFORE, PER(5,2)=5!/3X2X1.
        THE DENOMINATOR CAN BE WRITTEN AS ___!.
   ANS1 THREE|3|3!|THREE!
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   TYPE PER(5,2)=5!/3!, WHICH IS EXACTLY WHAT THE FORMULA PER(N,R)=N!/(N-R)!
        GIVES BY SUBSTITUTING N=5 AND R=2.
31 TYPE THE FORMULA FOR PER(N,R) IS EASY TO REMEMBER WHEN WRITTEN IN
        FACTORIAL NOTATION, BUT EVALUATION BECOMES TEDIOUS IF MULTIPLICATION
        OF ALL THE TERMS IN THE NUMERATOR AND DENOMINATOR IS ACCOMPLISHED
        BEFORE SIMPLIFICATION.  WHENEVER POSSIBLE, SIMPLIFY BY DIVIDING
        BOTH NUMERATOR AND DENOMINATOR BY THEIR COMMON TERMS.
        FOR EXAMPLE, THE NUMBER OF PERMUTATIONS OF TEN THINGS, TAKEN TWO
        AT A TIME, CAN BE WRITTEN AS PER(10,2).  USING THIS FORMULA, PER(10,2)
        IS EQUAL TO 10!/___!.
   ANS1 EIGHT|8|8:|EIGHT!
   COMP   33
   TYPE THE DENOMINATOR IS (N-R)! OR (10-2)! OR 8!.
33 TYPE PER(10,2)=10X9X8X7X6X5X4X3X2X1/8X7X6X5X4X3X2X1.  DIVIDING BOTH
        NUMERATOR AND DENOMINATOR BY THE COMMON TERM OF 8X7X6X5X4X3X2X1,
        PER(10,2)=10X9 OR ___.
   ANS1 90|NINTY
   TYPE IN THE FORMULA PER(N,R)=N!/(N-R)!, THE FORMULA CAN BE REDUCED TO
        PER(N,N)=N!/___!, IF R=N.
   ANS2 ZERO|0|0!
   COMP   30
   TYPE IF N EQUALS FIVE AND R EQUALS FIVE, THEN N-R OR 5-5 EQUALS ___.
   ANS1 ZERO|0
   TYPE IF R EQUALS N, THEN (N-R) EQUALS ___.
   ANS1 ZERO|0
30 TYPE THE FORMULA PER(N,N) CAN BE EXPRESSED AS N!/0!.
        IN PREVIOUS PROBLEMS IT HAS BEEN CALCULATED THAT PER(N,N) = ___!.
   ANS1 N|N!
   TYPE IN ORDER TO MAKE THESE TWO EQUATIONS EQUAL, ZERO FACTORIAL IS DEFINED
        AS ___.
   ANS2 ONE|1
   COMP   32
   TYPE SO AS NOT TO CHANGE THE VALUE OF THE NUMERATOR AS N!, IT MUST BE
        DIVIDED BY ONE.  SO, ZERO FACTORIAL IS DEFINED AS ONE.
32 TYPE IN SUMMARY, THE MAIN CONCEPTS OF THIS LESSON HAVE BEEN THE FOLLOWING
        TWO FORMULAS:  PER(N,N)=N! AND PER(N,R)=N!/(N-R)!.
34 TYPE LET'S TRY SOME HOMEWORK PROBLEMS.
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    BRAN   3660
         1.  EVALUATE THE FOLLOWING:
               A.  50!/(2!48!)  B.  6!/(3!2!)  C.  7!/5!  D.  8!/10!
         2.  EVALUATE THE FOLLOWING:
               A.  PER(7,5)      B.  PER(10,3)      C.  PER(4,4)
               D.  PER(6,2)      E.  PER(9,2)       F.  PER(5,5)
         3.  THREE MEN ENTER A BUS THAT HAS TWENTY EMPTY SEATS.  IN HOW
             MANY DIFFERENT WAYS CAN THEY SEAT THEMSELVES?
         4.  A MAP OF FIVE STATES IS TO BE COLORED USING A DIFFERENT COLOR
             FOR EACH STATE.  HOW MANY DIFFERENT RESULTS ARE POSSIBLE IF
             THERE ARE NINE COLORS AVAILABLE?
         5.  HOW MANY WORDS CAN BE FORMED FROM THE LETTERS OF THE WORD
             'FRAGMENT'
               A.  TAKEN ALL AT A TIME?
               B.  TAKEN FOUR LETTERS AT A TIME?
             (A WORD SIMPLY MEANS AN ARRANGEMENT OF LETTERS -- THEY DO NOT
             HAVE TO MAKE SENSE.)
         6.  IN HOW MANY WAYS CAN A MAN CHOOSE THREE GIFTS FROM TEN DIFFERENT
             ARTICLES?
    GOTO   35
36  BRAN   3880
         1.  EVALUATE THE FOLLOWING:
               A.  100!/(2!98!)  B.  6!/(3!2!)  C.  7!/5!  D.  8!/(2!2!)
         2.  EVALUATE THE FOLLOWING:
               A.  PER(6,2)      B.  PER(9,2)       C.  PER(5,5)
         3.  A BOOKSHELF HAS SPACE FOR FIVE BOOKS.  IF THERE ARE SEVEN
             DIFFERENT BOOKS AVAILABLE, HOW MANY DIFFERENT ARRANGEMENTS CAN BE
             MADE ON THE SHELF?
         4.  A MAP OF FIVE STATES IS TO BE COLORED USING A DIFFERENT COLOR
             FOR EACH STATE.  HOW MANY DIFFERENT RESULTS ARE POSSIBLE IF
             THERE ARE NINE COLORS AVAILABLE?
         5.  HOW MANY WORDS CAN BE FORMED FROM THE LETTERS OF THE WORD
             'FRAGMENT'
               A.  TAKEN ALL AT A TIME?
               B.  TAKEN FOUR LETTERS AT A TIME?
             (A WORD SIMPLY MEANS AN ARRANGEMENT OF LETTERS -- THEY DO NOT
             HAVE TO MAKE SENSE.)
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        6.  FIND N IF PER(N,2)=3PER(N-1,2).
        7.  A CLUB HAS A MEMBERSHIP OF TEN.  A PRESIDENT, A VICE PRESIDENT,
            A SECRETARY, AND A TREASURER ARE TO BE ELECTED FROM THE
            MEMBERSHIP.  ONLY FOUR OF THE MEMBERS ARE ELIGIBLE FOR PRESIDENT
            AND VICE PRESIDENT.  HOW MANY SETS OF OFFICERS ARE POSSIBLE?
    GOTO  35
38      1.  EVALUATE THE FOLLOWING:
              A.  100!/(2!98!)  B.  6!/(3!2!)  C.  7!/5!  D.  8!/(2!2!)
        2.  THE FIRST FOUR LETTERS OF THE ALPHABET ARE TO BE ARRANGED TO FORM
            CODE WORDS.  (A CODE WORD MAY CONSIST OF ONE, TWO, THREE, OR FOUR
            LETTERS, RESPECTIVELY.)  HOW MANY SUCH WORDS CAN BE OBTAINED IF
            NO LETTER IS REPEATED IN ANY ARRANGEMENT?
        3.  IN HOW MANY WAYS CAN SEVEN STATISTICS BOOKS BE ARRANGED ON A SHELF
              A.  IF TWO SPECIFIED BOOKS MUST ALWAYS BE SIDE BY SIDE?
              B.  IF THESE TWO BOOKS MUST NOT BE SIDE BY SIDE?
        4.  HOW MANY DIFFERENT FOUR-FLAG SIGNALS CAN BE MADE BY DISPLAYING
            FOUR OF SEVEN AVAILABLE FLAGS IN A VERTICAL LINE?
        5.  FIND N IF PER(N,4)=4PER(N-1,3).
        6.  IN HOW MANY DIFFERENT ORDERS CAN SIX PEOPLE TAKE SEATS AT A
            ROUND TABLE?
        7.  A CLUB HAS A MEMBERSHIP OF TEN.  A PRESIDENT, A VICE PRESIDENT,
            A SECRETARY, AND A TREASURER ARE TO BE ELECTED FROM THE
            MEMBERSHIP.  ONLY FOUR OF THE MEMBERS ARE ELIGIBLE FOR PRESIDENT
            AND VICE PRESIDENT.  HOW MANY SETS OF OFFICERS ARE POSSIBLE?
35  STOP STOP
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C. Concept Block Three

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TYPE CONCEPT BLOCK THREE:  COMBINATIONS.
     SINCE SEQUENCE IS IMPORTANT, NONE OF THE PERMUTATIONS ABC, ACB, BAC,
     BCA, CAB, AND CBA ARE ALIKE.  THERE ARE TIMES, HOWEVER, WHEN THE
     ORDER OF THE ELEMENTS IS NOT IMPORTANT.  IN THE SELECTION OF THREE
     LETTERS FROM A, B, C, AND D, ABD AND BDA ARE THE SAME IF ___ DOES NOT
     MATTER.
ANS1 ORDER|SEQUENCE
TYPE A SELECTION OF THREE DIFFERENT LETTERS FROM THE LETTERS A, B, C, AND
     D, DISREGARDING ORDER, IS CALLED A COMBINATION OF THE FOUR LETTERS,
     TAKEN THREE AT A TIME.  A SELECTION OF OBJECTS WITHOUT REGARD TO
     THEIR ORDER IS CALLED A ___.
ANS1 COMBINATION
TYPE A SELECTION OF OBJECTS LISTED IN A SPECIFIC ORDER IS CALLED A ___.
ANS1 PERMUTATION
TYPE AB AND BA CONSTITUTE HOW MANY PERMUTATIONS?
ANS1 TWO|2
TYPE THEY CONSTITUTE HOW MANY COMBINATIONS?
ANS1 ONE|1
TYPE PER(N,R) DENOTES THE NUMBER OF PERMUTATIONS OF N OBJECTS, TAKEN ___ AT
     A TIME.
ANS1 R
TYPE THE SYMBOL C(N,R) DENOTES THE NUMBER OF COMBINATIONS OF N OBJECTS,
     TAKEN ___ AT A TIME.
ANS1 R
COMP   10
TYPE C(4,3) IS THE SYMBOL WHICH DENOTES THE NUMBER OF COMBINATIONS OF
     ___ OBJECTS,
ANS1 FOUR|4
TYPE TAKEN ___ AT A TIME.
ANS1 THREE|3
TYPE THE NUMBER OF COMBINATIONS OF FIVE OBJECTS, TAKEN TWO AT A TIME, IS
     DENOTED BY ___.
ANS1 C(5,2)
COMP   10
TYPE THE SYMBOL REPRESENTING THE NUMBER OF THREE-ELEMENT COMBINATIONS
     WHICH CAN BE FORMED FROM A SET OF SIX OBJECTS IS ___.
ANS1 C(6,3)
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   COMP   10
   TYPE REMEMBER, C(N,R) DENOTES THE NUMBER OF COMBINATIONS OF N OBJECTS,
        TAKEN R AT A TIME.
10 TYPE NOW, LET'S DEVELOP A FORMULA FOR C(N,R).
   TYPE A CLUE TO FINDING SUCH A FORMULA CAN BE DISCOVERED BY COMPARING THE
        NUMBER OF PERMUTATIONS AND COMBINATIONS FOR THE SAME SET OF LETTERS.
   TYPE THE COMBINATIONS OF A, B, C, AND D, TAKEN THREE AT A TIME, ARE ABC,
        ABD, ACD, AND BCD.  THE NUMBER OF THESE COMBINATIONS CAN BE
        REPRESENTED BY THE SYMBOL ___.
   ANS1 C(4,3)
   TYPE BY SIMPLY COUNTING, HOW MANY COMBINATIONS OF FOUR LETTERS, TAKEN
        THREE AT A TIME, CAN YOU FIND?
   ANS1 FOUR|4
   TYPE IN TURN, EACH OF THESE FOUR COMBINATIONS CAN FORM ___! PERMUTATIONS.
        IN OTHER WORDS, HOW MANY WAYS CAN THREE OBJECTS BE PERMUTED?
   ANS2 THREE|3|3!|THREE FACTORIAL|THREE!|6|SIX
   COMP   12
   TYPE WHAT THE QUESTION REALLY ASKS IS HOW MANY PERMUTATIONS CAN BE FORMED
        FROM THREE LETTERS, TAKEN ALL AT ONCE.
        REMEMBER, N DIFFERENT OBJECTS CAN BE PERMUTED IN N! WAYS.  SO, EACH
        COMBINATION, SUCH AS ACD, CAN BE PERMUTED IN ___! WAYS.
   ANS1 THREE|3|3!|THREE FACTORIAL|THREE!|6|SIX
12 TYPE THE TOTAL NUMBER OF COMBINATIONS OF FOUR LETTERS, TAKEN THREE AT A
        TIME, CAN BE DENOTED BY C(4,3); AND EACH OF THESE COMBINATIONS CAN BE
        PERMUTED IN THREE FACTORIAL WAYS.  THE TOTAL NUMBER OF PERMUTATIONS OF
        FOUR LETTERS, TAKEN THREE AT A TIME, IS EQUAL TO C(4,3)X ___ .
   ANS1 THREE|3|3!|THREE FACTORIAL|THREE!|6|SIX
   COMP   14
   TYPE IN THIS EXAMPLE, EACH OF THE FOUR DIFFERENT COMBINATIONS CAN BE
        PERMUTED IN THREE FACTORIAL WAYS.  SO, THE NUMBER OF PERMUTATIONS
        CAN BE FOUND BY MULTIPLYING FOUR, THE NUMBER OF COMBINATIONS, TIMES
        THREE FACTORIAL.  IN EVALUATING, PER(4,3)=4X3!, WHICH IS 4X6 OR 24.
        THIS IS THE SAME VALUE AS IS OBTAINED IN SOLVING PER(4,3) BY THE
        REGULAR METHOD.
14 TYPE IN GENERAL, THE NUMBER OF COMBINATIONS OF N OBJECTS, TAKEN R AT A
        TIME, IS DENOTED BY C(N,R).  EACH OF THESE COMBINATIONS CAN BE
        PERMUTED IN R FACTORIAL WAYS.  CONTINUING THIS REASONING, THE TOTAL
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       NUMBER OF PERMUTATIONS OF N THINGS, TAKEN R AT A TIME, IS DENOTED
       BY PER(N,R) AND IS EVALUATED AS C(N,R) TIMES R FACTORIAL, OR IN
       SYMBOLS AS PER(N,R)=C(N,R)XR!.  TO DEVELOP A FORMULA FOR C(N,R), EACH
       SIDE OF THE EQUATION PER(N,R)=C(N,R)XR! CAN BE DIVIDED BY ___.
   ANS1 R FACTORIAL|R!
   COMP   16
       DIVIDING BY R FACTORIAL IS NECESSARY TO ISOLATE THE TERM C(N,R).
16 TYPE THE FORMULA FOR THE NUMBER OF COMBINATIONS OF N OBJECTS, TAKEN R AT
       A TIME, IS C(N,R)=PER(N,R)/R!.  THE NUMBER OF COMBINATIONS IS FOUND
       BY DIVIDING THE NUMBER OF PERMUTATIONS BY R FACTORIAL.
   BRAN   1560
   GOTO   31
15 TYPE A FORMULA OBTAINED IN CONCEPT BLOCK TWO, PER(N,R)=N!/___!.
   ANS1 (N-R)|N-R|(N-R)!|N-R!
   COMP   19
       ARE YOU FAMILIAR WITH THIS FORMULA?  (YES OR NO)
   BANS NO
   COMP   31
19 TYPE SUBSTITUTING THIS EXPRESSION FOR PER(N,R), THE FORMULA BECOMES
       C(N,R)=N!/(R!X(N-R)!), OR N! DIVIDED BY R! TIMES (N-R)!.
   TYPE THE FORMULA, C(N,R)=PER(N,R)/R!, IS PROBABLY EASIER TO REMEMBER.
       HOWEVER, THE FORMULA C(N,R)=N!/(R!X(N-R)!) IS VERY HANDY, ESPECIALLY
       IF A TABLE OF FACTORIALS IS AVAILABLE.  USING THE LAST FORMULA, THE
       NUMBER OF COMBINATIONS OF SIX OBJECTS, TAKEN THREE AT A TIME, IS
       C(6,3)=N!/(R!X(N-R)!), WHERE N IS EQUAL TO ___.
   ANS2 SIX|6
   COMP   18
   TYPE THE TOTAL NUMBER OF OBJECTS IS SIX.
18 TYPE R IS EQUAL TO ___.
   ANS2 THREE|3
   COMP   20
   TYPE R IS THE NUMBER OF ELEMENTS OR POSITIONS IN EACH COMBINATION AND IS
       EQUAL TO THREE.
20 TYPE N-R IS EQUAL TO ___.
   ANS1 THREE|3
   TYPE SO, C(6,3)=6!/(3!X3!), AND THE FACTORIALS CAN BE EVALUATED.  HINT:
   TYPE IT SAVES A GREAT DEAL OF MULTIPLICATION AND THE NECESSITY OF WORKING
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      WITH LARGE NUMBERS IF TERMS OF THE DENOMINATOR ARE DIVIDED INTO TERMS
      OF THE NUMERATOR.  FOR EXAMPLE, C(6,3)=6X5X4X3X2X1/3X2X1X3X2X1.  BOTH
      THE NUMERATOR AND DENOMINATOR CONTAIN THE PRODUCT 3X2X1, AND DIVIDING
      BY THIS COMMON TERM GIVES C(6,3)=6X5X4/3X2X1.  SIMPLIFICATION IS NOT
      COMPLETE BECAUSE 6 DIVIDED BY 3 IS 2 AND 4 DIVIDED BY 2 IS 2.  WITH
      THESE REDUCTIONS, THE PROBLEM BECOMES C(6,3)=2X5X2 OR ___.
   ANS1 TWENTY|20
   TYPE SIMPLIFICATION CAN REDUCE THE NUMBER OF REQUIRED CALCULATIONS.
   TYPE THE NUMBER OF COMBINATIONS OF EIGHT OBJECTS, TAKEN TWO AT A TIME, IS
      ___.
   ANS2 TWENTY-EIGHT|28|C(8,2)|TWENTYEIGHT|TWENTY EIGHT
   COMP   22
   TYPE BY USING THE FORMULA C(N,R)=PER(N,R)/R!, C(8,2)=PER(8,2)/2!.
   TYPE THE VALUE OF PER(8,2) IS ___.
   ANS1 FIFTY-SIX|FIFTYSIX|FIFTY SIX|56
   COMP   24
   TYPE PER(8,2) IS EQUAL TO 8X7.  (THE FIRST POSITION CAN BE SELECTED IN
      EIGHT WAYS, AND THE SECOND POSITION CAN BE SELECTED IN SEVEN WAYS.)
24 TYPE WHAT IS THE VALUE OF R! IN THIS PROBLEM?
   ANS1 TWO|2|2!|TWO FACTORIAL
   TYPE R IS TWO, SO THE ANSWER IS TWO FACTORIAL, WHICH IS EVALUATED AS TWO.
   TYPE BY SUBSTITUTING, C(8,2) IS EQUAL TO 8X7/(2X1) OR 56/2 OR ___.
   ANS1 TWENTY-EIGHT|TWENTYEIGHT|TWENTY EIGHT|28
   TYPE USING THE FORMULA C(8,2)=N!/(R!X(N-R)!), C(8,2) IS EQUAL TO 8!/(2!X
      6!), WHICH SIMPLIFIES TO TWENTY-EIGHT.
22 TYPE WHAT IS THE VALUE OF N-R IN THIS PROBLEM?
   ANS1 SIX|6
   COMP   26
   TYPE N IS EQUAL TO EIGHT AND R IS EQUAL TO TWO, SO N-R IS EQUAL TO EIGHT
      MINUS TWO, OR SIX.
26 TYPE KEEPING N EQUAL TO EIGHT AND R EQUAL TO TWO, LET'S EVALUATE C(N,N-R).
      C(N,N-R) IS EQUAL TO C(8,6) WHICH HAS A VALUE OF ___.
   ANS2 TWENTY-EIGHT|TWENTYEIGHT|TWENTY EIGHT|28
   COMP   27
   TYPE USING THE FORMULA C(N,R)=N!/(R!X(N-R)!), C(8,6) IS EQUAL TO 8!/(6!X
      2!).  USING THE FORMULA C(N,R)=PER(N,R)/R!, C(8,6)=PER(8,6)/6! OR
      8X7X6X5X4X3/(6X5X4X3X2X1).  NOTICE THAT BOTH THE NUMERATOR AND
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      DENOMINATOR IN THE EXPRESSION CONTAIN TERMS OF 6X5X4X3.  DIVISION BY
      THESE TERMS SIMPLIFIES C(8,6) TO 8X7/2X1 OR ___.
   ANS1 TWENTY-EIGHT|TWENTY EIGHT|TWENTYEIGHT|28
27 TYPE THESE VALUES SEEM TO INDICATE THAT C(8,2) EQUALS C(8,6).  IN GENERAL,
      C(N,R) EQUALS C(N,N-R).
   BRAN   3075
   GOTO   38
31 TYPE TO CALCULATE THE NUMBER OF COMBINATIONS OF N OBJECTS, TAKEN R AT A
      TIME, SIMPLY EXPRESS THE PRODUCT OF THE WAYS OF SELECTING EACH ELEMENT
      IN THE PERMUTATION OF N OBJECTS, TAKEN R AT A TIME.  NEXT, DIVIDE EACH
      OF THESE NUMBERS IN THE PRODUCT BY A WHOLE NUMBER, STARTING WITH ONE
      AND INCREASING BY ONE EACH TIME UNTIL ALL THE TERMS IN THE NUMERATOR
      HAVE DIVISORS.  THE NUMERATOR IS PER(N,R), AND THE DENOMINATOR IS
      R FACTORIAL.  FOR EXAMPLE, HOW MANY ELEMENTS OR POSITIONS ARE NEEDED
      TO REPRESENT THE COMBINATION IN ORDER TO CALCULATE C(5,2)?
   ANS1 TWO|2
   COMP   32
   TYPE THE OBJECTS ARE TAKEN TWO AT A TIME (R=2), SO TWO ELEMENTS ARE NEEDED
      TO REPRESENT THE COMBINATION.
32 TYPE THIS TELLS THE NUMBER OF TERMS THAT ARE NEEDED IN BOTH THE NUMERATOR
      AND THE DENOMINATOR.  THE TWO TERMS OF THE NUMERATOR ARE THE NUMBER OF
      WAYS OF SELECTING EACH ELEMENT IN THE PERMUTATION OF FIVE ITEMS,
      TAKEN TWO AT A TIME.  THEY ARE ___ AND ___, RESPECTIVELY.  (ANSWER
      WITH A BLANK SPACE BETWEEN VALUES.)
   ANS2 5 4|FIVE FOUR| FIVE AND FOUR
   COMP   28
   TYPE PER(5,2) IS EQUAL TO FIVE TIMES FOUR.
28 TYPE THE TWO TERMS IN THE DENOMINATOR ARE ___ AND ___.  (ANSWER WITH A
      BLANK SPACE BETWEEN THE TWO VALUES.)
   ANS2 1 2|2 1|ONE TWO|TWO ONE|ONE AND TWO|TWO AND ONE
   COMP   34
   TYPE TWO TERMS ARE REQUIRED.  THE FIRST TERM IS ONE, AND THE SECOND IS TWO.
      THE FIRST TERM OF THE DENOMINATOR IS ALWAYS ONE, AND EACH ADDITIONAL
      TERM IS ONE GREATER THAN THE TERM IMMEDIATELY PRECEDING IT.
34 TYPE THUS, C(5,2)=PER(5,2)/(2X1) OR 5X4/(2X1) OR ___.
   ANS1 TEN|10
   TYPE C(8,3) IS EVALUATED AS ___.
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   ANS2 56|FIFTY-SIX|FIFTY SIX|FIFTYSIX
   COMP   36
   TYPE NO, LET'S SEPARATE THE PROBLEM INTO TWO PARTS.  FIRST, PER(8,3) IS
        EQUAL TO ___.  (ANSWER IN THE FORM _X_X..., SUPPLYING THE NECESSARY
        NUMBER OF VALUES AND X'S.)
   ANS2 8X7X6|336|EIGHTXSEVENXSIX|EIGHT X SEVEN X SIX
   COMP   35
   TYPE NO, THE ANSWER IS 8X7X6.
35 TYPE SINCE THERE ARE THREE ELEMENTS IN THE COMBINATION, THE THREE TERMS
        IN THE DENOMINATOR ARE ___.  (ANSWER IN THE FORM _X_X_.)
   ANS1 3X2X1|1X2X3|3!|6|SIX
   COMP   37
        THERE ARE THREE TERMS IN THE DENOMINATOR.  THE FIRST TERM HAS A VALUE
        OF ONE, AND EACH ADDITIONAL TERM IS ONE GREATER THAN THE TERM
        IMMEDIATELY PRECEDING IT.  SO, THE SECOND AND THIRD TERMS ARE TWO
        AND THREE, RESPECTIVELY.
37 TYPE DIVIDING THESE TWO VALUES, C(8,3)=8X7X6/(3X2X1).  SIMPLIFYING, THE
        DENOMINATOR CONTAINS THE VALUE SIX (3X2), AND THIS CAN BE DIVIDED
        INTO THE SIX IN THE NUMERATOR.  EVALUATING THE REMAINDER OF THE TERMS
        GIVES 8X7 OR ___.
   ANS1 56|FIFTY-SIX|FIFTY SIX|FIFTYSIX
36 GOTO   38
30 TYPE IN THE EVALUATIONS OF THE NUMBER OF PERMUTATIONS THUS FAR, ALL OF THE
        OBJECTS HAVE BEEN DISTINCT.  LET'S SEE WHAT HAPPENS WHEN NOT ALL THE
        OBJECTS ARE DIFFERENT.  FROM THE FORMULAS FOR PERMUTATIONS, THE NUMBER
        OF PERMUTATIONS OF FIVE DISTINCT LETTERS, TAKEN ALTOGETHER, IS
        PER(5,5)=___!.
   ANS1 FIVE|5|5!|FIVE!|FIVE FACTORIAL
   TYPE IN THE ARRANGEMENT AAABB, THE LETTER A IS REPEATED ___ TIMES.
   ANS1 THREE|3
   TYPE THE LETTER B IS REPEATED ___ TIMES.
   ANS1 TWO|2
42 TYPE DO YOU THINK THE NUMBER OF PERMUTATIONS OF THE LETTERS IN AAABB, TAKEN
        ALTOGETHER, IS ___ (MORE, LESS, EQUAL) THAN/TO PER(5,5)=5!=120?
   ANS2 LESS
   COMP   40
   TYPE THE NUMBER OF DISTINCT PERMUTATIONS THAT CAN BE CONSTRUCTED USING THE
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      THREE DIFFERENT LETTERS, A, B, AND C, TAKEN ALTOGETHER, IS ___.
   ANS1 SIX|6|3!|THREE!|3X2X1
   TYPE IN COMPARISON, THE THREE A'S OF THIS PROBLEM ARE INDISTINGUISHABLE
      (NO WAY OF TELLING THEM APART), AND THE NUMBER OF DISTINCT
      PERMUTATIONS THAT CAN BE CONSTRUCTED FROM THESE THREE LETTERS IS ___.
   ANS1 ONE|1
   COMP   44
   TYPE SINCE THE THREE LETTERS ARE NOT DISTINGUISHABLE, THERE IS ONLY ONE
      DISTINCT PERMUTATION, AAA, THAT CAN BE FORMED.  ANY OTHER ARRANGEMENT
      WOULD BE A REPEAT AND NOT DISTINCT.
44 TYPE KEEPING IN MIND WHAT HAS JUST BEEN STATED, TRY ANSWERING THE
      QUESTION AGAIN.
   GOTO   42
40 TYPE IF THE IDENTICAL LETTERS A IN AAABB CAN BE DISTINGUISHED, SUCH AS
      A1, A2, AND A3, THEY CAN BE PERMUTED IN HOW MANY WAYS WITHOUT
      ALTERING THE POSITIONS OF THE B'S?
   ANS2 3!|THREE!FACTORIAL|6|SIX|3X2X1|1X2X3
   COMP   46
   TYPE BY TEMPORARILY ADDING IDENTIFYING NUMBERS TO THE THREE A'S, THE
      ARRANGEMENT CAN BE DENOTED AS A1A2A3BB.  ANOTHER ARRANGEMENT IS
      A2A1A3BB, FORMED WITHOUT CHANGING THE POSITIONS OF THE B'S.  SO,
      THE QUESTION IS REALLY ASKING THAT IF THE A'S CAN BE DISTINGUISHED
      HOW MANY DIFFERENT PERMUTATIONS CAN BE FORMED FROM THESE THREE
      LETTERS, TAKEN ALTOGETHER.
   ANS2 SIX|3!|6|THREE FACTORIAL|3X2X1|1X2X3
   COMP   46
      DISREGARDING THE B'S, THIS BECOMES THE FAMILIAR PROBLEM OF FINDING
      THE NUMBER OF PERMUTATIONS OF THREE LETTERS, THE THREE A'S, TAKEN
      THREE AT A TIME.  THE ANSWER IS 3! OR 3X2X1 OR 6.
46 TYPE IF THE IDENTICAL LETTERS B IN AAABB ARE DISTINGUISHED AS B1 AND B2,
      THEY CAN BE PERMUTED IN ___ WAYS WITHOUT ALTERING THE POSITIONS OF THE
      A'S.
   ANS2 2!|TWO FACTORIAL|2|TWO|2X1
   COMP   45
      NOW, DISREGARD THE A'S.  THE TWO B'S, IF THEY ARE DISTINGUISHABLE,
      CAN BE ARRANGED IN 2! OR 2 WAYS.
45 TYPE IF THE LETTERS OF THE ARRANGEMENT AAABB ARE ALL DISTINGUISHABLE,
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      SUCH AS A1, A2, AND A3 FOR THE THREE A'S AND B1 AND B2 FOR THE
      TWO B'S, THEN THE NUMBER OF PERMUTATIONS FOR THESE FIVE DISTINCT
      LETTERS IS ___!.
   ANS1 FIVE|5|5!|FIVE!|FIVE FACTORIAL
   COMP   47
   TYPE IF THE FIVE LETTERS ARE DISTINCT, THEN THE PROBLEM IS TO FIND THE
      NUMBER OF PERMUTATIONS OF FIVE LETTERS, TAKEN ALTOGETHER, WHICH
      IS FIVE FACTORIAL OR 120.
47 TYPE BUT THE A'S AND THE B'S ARE INDISTINGUISHABLE.  TENTATIVELY, WE HAVE
      DECIDED THAT THE NUMBER OF PERMUTATIONS OF ELEMENTS WHICH ARE NOT
      DISTINCT IS LESS THAN THE NUMBER OF PERMUTATIONS OF EQUAL, DISTINCT
      LETTERS.  THE NUMBER OF PERMUTATIONS OF FIVE DISTINCT LETTERS IS 5!;
      AND SO, THE NUMBER OF PERMUTATIONS OF NON-DISTINCT LETTERS MUST BE
      LESS THAN 5!.  IN THE FIVE FACTORIAL ANSWER, THE NUMBER OF
      PERMUTATIONS IS DETERMINED AS IF THE A'S ARE DISTINGUISHABLE.  SINCE
      THEY ARE NOT, THEY HAVE BEEN COUNTED THREE FACTORIAL TOO MANY TIMES.
      LIKEWISE, THE B'S HAVE BEEN COUNTED TWO FACTORIAL TOO MANY TIMES.  SO,
      THE FIVE FACTORIAL MUST BE DIVIDED BY BOTH THREE FACTORIAL AND TWO
      FACTORIAL TO COMPENSATE FOR THIS OVER COUNTING.
48 TYPE THE NUMBER OF PERMUTATIONS OF FIVE LETTERS, THREE AND TWO,
      RESPECTIVELY, WHICH ARE ALIKE IS DENOTED AS PER(5|3,2).
   TYPE PER(5|3,2) DENOTES THE NUMBER OF PERMUTATIONS OF FIVE LETTERS, IN
      WHICH THREE AND TWO LETTERS, RESPECTIVELY, ARE INDISTINGUISHABLE.
      IN GENERAL, PER(N|N1,N2) IS CALCULATED BY DIVIDING THE NUMBER OF
      PERMUTATIONS OF N ITEMS, TAKEN ALTOGETHER, BY N1 FACTORIAL TIMES
      N2 FACTORIAL, WHERE N1 AND N2 ARE THE NUMBER OF INDISTINGUISHABLE
      ITEMS, RESPECTIVELY.  IN THE ABOVE EXAMPLE, PER(5|3,2)=PER(5,5)/
      (3!X2!) OR 5!/(3!X2!) OR 10.
38 TYPE IN REVIEW, TO COMPUTE THE NUMBER OF COMBINATIONS OF N ELEMENTS, TAKEN
      R AT A TIME, USE THE FORMULA C(N,R)=PER(N,R)/R!.
   TYPE LET'S TRY SOME ADDITIONAL PROBLEMS.
   BRAN   5060
      1.  EVALUATE THE FOLLOWING:
            A.  C(9,3)        B.  C(6,4)        C.  C(9,6)
            D.  C(12,9)       E.  C(5,2)        F.  C(5,5)
      2.  IN HOW MANY WAYS CAN A COMMITTEE OF FIVE BE CHOSEN FROM EIGHT
          PEOPLE?
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      3.  HOW MANY DIFFERENT SUMS CAN BE FORMED WITH A NICKEL, A DIME, A
          QUARTER, A HALF-DOLLAR, AND A SILVER DOLLAR IF ONLY THREE COINS
          CAN BE USED?
      4.  A CONTRACTOR NEEDS FOUR CARPENTERS, AND TEN APPLY FOR THE JOBS.
          IN HOW MANY WAYS CAN HE SELECT FOUR?
      5.  IN HOW MANY WAYS CAN A FOUR-MEMBER COMMITTEE BE SELECTED FROM A
          GROUP OF EIGHT MEN?
      6.  HOW MANY DIFFERENT SUMS OF MONEY CAN BE FORMED FROM A PENNY, A
          NICKEL, A DIME, A QUARTER, AND A HALF-DOLLAR IF EACH SUM IS
          DETERMINED BY
            A.  ONE COIN?
            B.  THREE COINS?
            C.  FIVE COINS?
GOTO   54
50 BRAN   5280
      1.  EVALUATE THE FOLLOWING:
            A.  C(9,3)     B.  C(6,4)     C.  C(9,6)     D.  C(5,5)
      2.  IN HOW MANY WAYS CAN A COMMITTEE OF THREE BE CHOSEN FROM FOUR
          MARRIED COUPLES
            A.  IF ALL ARE ELIGIBLE?
            B.  IF THE COMMITTEE MUST CONSIST OF TWO WOMEN AND ONE MAN?
      3.  HOW MANY DIFFERENT AMOUNTS OF POSTAGE CAN BE FORMED FROM A ONE-
          CENT STAMP, A THREE-CENT STAMP, A FIVE-CENT STAMP, AN EIGHT-CENT
          STAMP, AND A TEN-CENT STAMP USING
            A.  EXACTLY FOUR OF THE STAMPS?
            B.  AT LEAST TWO OF THE STAMPS?
      4.  IN HOW MANY WAYS CAN A FIVE-MEMBER COMMITTEE BE SELECTED FROM A
          GROUP OF NINE MEN?
      5.  IN HOW MANY WAYS CAN A COACH CHOOSE A TEAM OF FIVE FROM TEN BOYS
          IF TWO BOYS REFUSE TO PLAY TOGETHER?
      6.  HOW MANY DIFFERENT SUMS OF MONEY CAN BE FORMED FROM A PENNY, A
          NICKEL, A DIME, A QUARTER, AND A HALF-DOLLAR IF EACH SUM IS
          DETERMINED BY
            A.  ONE COIN?
            B.  THREE COINS?
            C.  FIVE COINS?
GOTO   54
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52      1.  IN HOW MANY WAYS CAN A COACH CHOOSE A TEAM OF FIVE FROM TEN BOYS?
        2.  IN HOW MANY WAYS CAN A COMMITTEE OF THREE BE CHOSEN FROM FOUR
            MARRIED COUPLES
              A.  IF ALL ARE ELIGIBLE?
              B.  IF THE COMMITTEE MUST CONSIST OF TWO WOMEN AND ONE MAN?
        3.  HOW MANY DIFFERENT AMOUNTS OF POSTAGE CAN BE FORMED FROM A ONE-
            CENT STAMP, A THREE-CENT STAMP, A FIVE-CENT STAMP, AN EIGHT-CENT
            STAMP, AND A TEN-CENT STAMP USING
              A.  EXACTLY FOUR OF THE STAMPS?
              B.  AT LEAST TWO OF THE STAMPS?
        4.  A COMPANY OF EIGHT MEN IS TO BE DIVIDED INTO TWO SECTIONS SO THERE
            ARE THREE MEN IN THE FIRST AND FIVE MEN IN THE SECOND.  IN HOW
            MANY WAYS CAN THIS BE DONE?
        5.  HOW MANY COMMITTEES OF EIGHT MEN CAN SELECTED FROM TWELVE MEN IF
            TWO MEN REFUSE TO SERVE TOGETHER?
        6.  FROM A COMPANY OF TWENTY SOLDIERS, A SQUAD OF THREE MEN IS CHOSEN
            EACH NIGHT.  FOR HOW MANY CONSECUTIVE NIGHTS COULD A SQUAD GO ON
            DUTY WITHOUT TWO OF THE SQUADS BEING IDENTICAL?
54 STOP STOP
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D. Concept Block Four

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    TYPE CONCEPT BLOCK FOUR:  BINOMIAL THEOREM.
    TYPE THE EXPANSION OF POWERS OF BINOMIALS IS IMPORTANT IN STATISTICS AND IS
         RELATED TO CONCEPTS ALREADY STUDIED.  SO, IN THIS LESSON WE ARE
         PRIMARILY INTERESTED IN LEARNING ABOUT BINOMIAL EXPANSION AND FINDING
         A TECHNIQUE BY WHICH SUCH EXPANSION CAN BE OBTAINED.  IN ORDER TO
         UNDERSTAND THIS DEVELOPMENT, ONE MUST HAVE A KNOWLEDGE OF WHAT IS
         MEANT BY THE TERMS BINOMIAL, EXPONENT, POWER, FACTOR, COEFFICIENT, AND
         BINOMIAL EXPANSION.  ARE THESE CONCEPTS FAMILIAR?  (YES OR NO)
    BANS YES
    COMP   10
14  TYPE A BINOMIAL IS THE SUM OF TWO TERMS; FOR EXAMPLE, (A+B) IS A BINOMIAL.
    TYPE (B+Y) IS A ___.
    ANS1 BINOMIAL
    COMP   12
    TYPE BOTH C AND 5 ARE TERMS AND THEIR SUM, (C+5), IS A BINOMIAL.  THE
         PARENTHESES CAN BE REMOVED WITHOUT EFFECTING THE BINOMIAL.  SO EITHER
         C+5 OR (C+5) CAN EXPRESS THE BINOMIAL.  (THE PREFIX BI MEANS TWO.)
    GOTO   14
12  TYPE (A+B)**2 IS THE BINOMIAL (A+B) RAISED TO THE SECOND POWER, WHERE ** IS
         THE SYMBOL TO DENOTE POWERS.  THE USUAL ALGEBRAIC NOTATION IS THE USE
         OF SUPERSCRIPTS, BUT DUE TO TERMINAL LIMITATIONS, THE ** NOTATION WILL
         BE USED THROUGHOUT THIS CONCEPT.  THE EXPONENT OR POWER OF (A+B)**2 IS
         THE NUMBER ___.
    ANS1 TWO|2
    COMP   16
    TYPE THE EXPONENT OR POWER TELLS THE NUMBER OF TIMES THAT THE BINOMIAL IS
         TO BE USED AS A FACTOR.  THE EXPONENT OF TWO INDICATES THAT (A+B) IS
         TO OCCUR AS A FACTOR TWICE.  (A+B)**2 CAN BE EXPRESSED AS
         (A+B) X (A+B).
    TYPE (A+B)**3 IS THE BINOMIAL (A+B) RAISED TO THE ___ POWER.
    ANS1 3RD|3RD.|THIRD|3|THREE
    TYPE THE EXPONENT OF (A+B)**3 IS THE NUMBER ___.
    ANS1 THREE|3
    COMP   16
    TYPE (A+B)**3 IS EQUAL TO (A+B) X (A+B) X (A+B).  THE BINOMIAL IS USED AS
         A FACTOR THREE TIMES.
16  TYPE (A+B)**2 IS EQUAL TO (A+B) ___ (A+B).
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   ANS1 TIMES|X|MULTIPLY|MULTIPLIED
   TYPE THE PROBLEM IS HOW TO EXPAND (MULTIPLY) THE SECOND POWER OF THIS
        BINOMIAL.  CAN YOU EXPAND (A+B)**2?  (YES OR NO)
   BANS YES
   COMP   20
30 TYPE THE FOLLOWING EXAMPLE ILLUSTRATES THE PROCEDURE FOR MULTIPLYING TWO
        BINOMIALS.
   GOTO   22
20 TYPE TO TEST YOUR KNOWLEDGE OF BINOMIAL EXPANSION, TRY ANSWERING THE
        FOLLOWING QUESTION.  THE TERM CONTAINING ONLY A'S IN THE EXPANSION IS
        RAISED TO WHAT POWER?
   ANS2 2|TWO|SECOND|2ND|2ND.
   COMP   24
   GOTO   30
24 TYPE THE COEFFICIENT OF THE AB TERM, AFTER COMBINING LIKE TERMS, IS ___.
   ANS2 TWO|2
   COMP   26
   GOTO   30
26 TYPE THE EQUATION TO REPRESENT THE EXPANSION OF THE SECOND POWER OF THE
        BINOMIAL (A+B) IS (A+B)**2 = A**2 + 2AB + B**2.
   GOTO   28
22 TYPE THE EXPANSION OF (A+B)**2 IS ACCOMPLISHED BY MULTIPLYING EACH LETTER
        OF ONE BINOMIAL FACTOR TIMES EACH LETTER OF THE OTHER BINOMIAL FACTOR.
        TO COMPLETE THE EXPANSION, THESE TERMS ARE SUMMED.  BY EXPANDING,
        (A+B)**2 = (A+B) X (A+B) = AX(A+B) + BX(A+B) = AXA + AXB + BXA + BXB.
        EACH TERM IN THE LAST EXPRESSION OF THE EQUALITY CONTAINS ___ FACTORS.
        (HOW MANY)
   ANS1 TWO|2
   COMP   32
   TYPE THE SECOND TERM OF THE EXPANSION AXB CONTAINS TWO FACTORS.  THE FIRST
        FACTOR IS THE A AND THE SECOND FACTOR IS THE B.
   TYPE THERE ARE TWO ___ IN EACH TERM OF THE EXPANSION OF A BINOMIAL TO THE
        SECOND POWER.
   ANS1 FACTORS|FACTOR
32 TYPE THE NUMBER OF FACTORS IN EACH TERM AND THE EXPONENT (OR POWER) OF THE
        BINOMIAL ARE IDENTICAL.  IN THIS PROBLEM, THEY BOTH EQUAL ___.
   ANS1 TWO|2
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      THE NUMBER OF FACTORS OF A AND B IN EACH TERM IS CALLED THE DEGREE
      OF THE TERM.
   TYPE EACH OF THE LETTERS IS FROM A DIFFERENT BINOMIAL FACTOR, WHICH MAY
      NOT BE OBVIOUS SINCE THE EXPANSION IS OF SIMILAR BINOMIALS.  LET'S
      INVESTIGATE THE EXPANSION OF TWO DISTINCT BINOMIALS, (A+B) X (C+D).
40 TYPE ONE TERM OF THE EXPANSION CONTAINS THE LETTER A FROM THE BINOMIAL
      (A+B) AND THE LETTER ___ FROM THE BINOMIAL (C+D).
   ANS2 C|D
   COMP   34
   TYPE THE OTHER FACTOR WITH A MUST BE FROM THE BINOMIAL (C+D) AND IS
      EITHER C OR D.
   GOTO   40
34 WANS D
   COMP   36
   TYPE EACH TERM OF THE EXPANSION IS OBTAINED BY MULTIPLYING ONE LETTER FROM
      EACH OF THE BINOMIAL FACTORS.  BY USING THE LETTER YOU SELECTED, THE
      APPROPRIATE TERM BECOMES AX___.
   ANS1 C|AXC|AX C|AC|A X C|A XC
   COMP   38
   TYPE THE LETTER A FROM (A+B) AND THE LETTER C FROM (C+D) ARE MULTIPLIED
      TOGETHER TO GIVE THE TERM AXC OR AC.  TO SIMPLIFY NOTATION, THE
      MULTIPLICATION SYMBOL CAN BE OMITTED BETWEEN TWO FACTORS.
38 TYPE ANOTHER TERM OF THE EXPANSION CONTAINS THE LETTER A FROM THE BINOMIAL
      (A+B) AND THE LETTER ___ FROM THE BINOMIAL (C+D).  (ANSWER WITH THE
      REMAINING LETTER IN THE BINOMIAL.)
   ANS1 D
      BY USING THIS LETTER, THE APPROPRIATE TERM BECOMES AX___.
   ANS1 D
   TYPE TWO TERMS OF THE EXPANSION ARE AXC AND AXD.
   GOTO   42
36 TYPE THE CORRESPONDING TERM IS AX___.
   ANS1 D
   COMP   44
   TYPE THE LETTER A FROM (A+B) AND THE LETTER D FROM (C+D) ARE MULTIPLIED
      TOGETHER TO GIVE THE TERM AXD OR AD.  TO SIMPLIFY NOTATION, THE
      MULTIPLICATION SYMBOL BETWEEN THE TWO FACTORS CAN BE OMITTED.
44 TYPE ANOTHER TERM OF THE EXPANSION CONTAINS THE LETTER A FROM (A+B) AND
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      THE LETTER ___ FROM (C+D).  (THE REMAINING ONE)
   ANS1 C
   TYPE THE APPROPRIATE TERM IS AX___.
   ANS1 C
   TYPE AXC AND AXD ARE TWO TERMS OF THE EXPANSION.
42 TYPE THE REMAINING TWO TERMS OF THE EXPANSION ARE DETERMINED BY MULTIPLYING
      THE LETTER B FROM THE BINOMIAL (A+B) TIMES C AND D, RESPECTIVELY.  THE
      RESULTING TERMS ARE BX___ AND BX___.  (ANSWER WITH A SPACE BETWEEN THE
      TWO ANSWERS.)
   ANS1 C D|D C| C D| D C|DC|CD
   COMP   46
   TYPE THE TWO TERMS ARE BXC AND BXD.
46 TYPE THE EXPANSION OF (A+B) X (C+D) IS COMPLETED BY ADDING THE FOUR TERMS
      TOGETHER.  (A+B) X (C+D) IS EQUAL TO AXC + AXD + BXC + BXD.  EACH OF
      THE ABOVE FOUR TERMS CONSISTS OF TWO FACTORS, ONE FACTOR FROM (A+B)
      AND ONE FACTOR FROM (C+D).
      CONSIDERING THE EXPANSION OF (A+B)**2 AGAIN, (A+B)**2 = AXA + AXB +
      BXA + BXB.  TO SIMPLIFY NOTATION, THE MULTIPLICATION SYMBOL IS OMITTED
      AND THE RESULT IS (A+B)**2 = AA + AB + BA + BB.  (WHENEVER TWO FACTORS
      ARE WRITTEN TOGETHER, MULTIPLICATION IS ASSUMED TO BE THE OPERATION.)
      THE COMMUTATIVE LAW OF MULTIPLICATION ALLOWS US TO CONSIDER THE TERM
      AB AND THE TERM ___ AS BEING EQUAL.
   ANS1 BA|BXA
   COMP   48
   TYPE THE COMMUTATIVE LAW OF MULTIPLICATION STATES THAT THE ORDER IN WHICH
      TWO VALUES ARE MULTIPLIED DOES NOT MAKE A DIFFERENCE IN THEIR PRODUCT.
      SO, AB AND BA ARE EQUAL.  TO ILLUSTRATE, SEVEN TIMES EIGHT AND EIGHT
      TIMES SEVEN ARE BOTH EVALUATED AS FIFTY-SIX.
48 TYPE THE TWO TERMS AB AND BA, BEING EQUAL, COMBINE AS ___AB.
   ANS1 TWO|2
   COMP   49
49 TYPE THE DIGIT 2 IS CALLED THE COEFFICIENT OF THE AB TERM.
      THE AA TERM CAN BE EXPRESSED IN EXPONENT NOTATION AS A**___.
   ANS1 TWO|2
   COMP   50
   TYPE THERE ARE TWO FACTORS OF A, AND USING EXPONENTS, THIS IS DENOTED AS
      A**2.  THE BB TERM IS EXPRESSED AS B**___.
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   ANS1 TWO|2
50 TYPE FINALLY, THE BINOMIAL (A+B) RAISED TO THE SECOND POWER IS
        A**2 + 2AB + B**2.
28 TYPE EXPANDING A BINOMIAL USING THE MULTIPLICATION PROCESS, AS WE JUST DID,
        SOON BECOMES LABORIOUS FOR POWERS HIGHER THAN TWO.
        LET'S ANALYZE (A+B)**2 IN GREATER DETAIL TO DEVELOP A TECHNIQUE THAT
        CAN BE APPLIED TO THE EXPANSION OF THESE HIGHER POWERS.  THE DEGREE OF
        EACH TERM IN THE PRODUCT EQUALS THE NUMBER OF FACTORS TO BE
        MULTIPLIED.  THE DEGREE OF EACH TERM IN THE EXPANSION OF (A+B)**2 IS
        ___.
   ANS1 TWO|2
   COMP   29
        THE DEGREE OF EACH TERM (THE NUMBER OF FACTORS) HAS THE SAME VALUE AS
        THE EXPONENT OF THE BINOMIAL.
29 TYPE THE NUMBER OF UNIQUE TERMS IN THE EXPANSION IS THE EXPONENT (POWER)
        PLUS ONE.  THE NUMBER OF UNIQUE TERMS IN THE EXPANSION OF (A+B)**2
        IS ___.
   ANS1 THREE|3
   COMP   31
   WANS FOUR|4
   COMP   33
   TYPE THE EXPONENT (POWER) IS TWO.  THE NUMBER OF UNIQUE TERMS IS TWO + ONE,
        OR THREE.  THIS AGREES WITH OUR EARLIER EXPANSION RESULTS.
   GOTO   31
33 TYPE USING THE MULTIPLICATION PROCESS, THERE ARE FOUR TERMS, BUT TWO OF
        THESE FOUR TERMS ARE IDENTICAL.  THE NUMBER OF UNIQUE TERMS IS THREE.
31 TYPE THE TERMS IN THE EXPANSION, WITHOUT THEIR COEFFICIENTS, CAN BE
        WRITTEN BY INSPECTION.  FOR EXAMPLE, THE EXPANSION OF (A+B)**2 HAS
        THREE TERMS OF DEGREE TWO; WITHOUT THEIR COEFFICIENTS, THE TERMS ARE
        A**2, AB, AND B**2.  NOTICE THAT THE EXPANSION IS TO THE SECOND POWER
        AND EACH TERM IS OF DEGREE TWO (HAS TWO FACTORS).
        NOW, THE QUESTION IS HOW DO WE DETERMINE THE COEFFICIENT OF THESE
        TERMS.  THE TERM A**2 REPRESENTS ___ FACTORS OF A.  (HOW MANY)
   ANS1 TWO|2
   COMP   52
   TYPE REMEMBER, A**2 IS EXPONENTIAL NOTATION FOR AXA.  SO, THE TERM A**2
        REPRESENTS TWO FACTORS OF A.
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52 TYPE THE COEFFICIENT OF THIS TERM IS THE NUMBER OF WAYS THAT THESE FACTORS
       CAN BE SELECTED.  TO OBTAIN A**2, WE CHOOSE AN A FROM THE FIRST
       BINOMIAL FACTOR AND AN A FROM THE SECOND.  EACH CHOICE CAN BE MADE
       IN ___ WAY/WAYS.  (HOW MANY)
   ANS1 ONE|1
   COMP   51
   TYPE BECAUSE THERE IS ONLY ONE LETTER A IN EACH OF THE BINOMIAL FACTORS,
       THE A'S CAN BE CHOSEN IN ONLY ONE WAY.
51 TYPE THE COEFFICIENT OF THE A**2 TERM IS ONE.  HOWEVER, A COEFFICIENT OF
       ONE IS USUALLY NOT WRITTEN.  ALSO, WHEN NO COEFFICIENT IS EXPRESSED,
       THE COEFFICIENT IS ASSUMED TO BE ONE.
       THE PROBLEM OF DETERMINING THE COEFFICIENTS FOR THE TERMS IN A
       BINOMIAL EXPANSION IS SIMPLY A COUNTING PROBLEM.  (HOW MANY WAYS CAN
       THE TWO FACTORS OF A BE SELECTED?)  YOU SHOULD RECOGNIZE THIS TYPE OF
       PROBLEM FROM CONCEPT BLOCKS TWO AND THREE.  SINCE THE ORDER OF
       SELECTING THE FACTORS OF A IS NOT IMPORTANT, WE WILL USE THE COUNTING
       TECHNIQUE OF DETERMINING THE NUMBER OF COMBINATIONS.  THE COEFFICIENT
       OF THE A**2 TERM IS THE NUMBER OF WAYS OF CHOOSING TWO FACTORS OF A
       FROM TWO POSSIBLE CHOICES.  THIS CAN BE REPRESENTED AS THE NUMBER OF
       COMBINATIONS OF TWO LETTERS, TAKEN TWO AT A TIME.  THE COEFFICIENT IS
       C(2,2).
       THE COEFFICIENT OF THE B**2 TERM CAN BE EXPRESSED AS ___.  (USE
       COMBINATION NOTATION.)
   ANS1 C(2,2)|C( 2,2 )|C(2, 2)|C( 2,2)|C(2 ,2)|ONE|1
   COMP   53
   TYPE THE COEFFICIENT OF THE B**2 TERM IS THE NUMBER OF WAYS OF SELECTING
       TWO FACTORS OF B FROM THE TWO POSSIBLE BINOMIAL FACTORS.  THIS CAN BE
       EXPRESSED AS THE NUMBER OF COMBINATIONS OF TWO LETTERS, TAKEN TWO
       AT A TIME, OR C(2,2).  C(2,2) IS EVALUATED AS ONE.
53 TYPE IN OBTAINING THE AB TERM, THE A FACTOR IS CHOSEN FROM EITHER OF THE
       TWO BINOMIAL FACTORS.  THERE ARE HOW MANY WAYS OF SELECTING IT?
       (GIVE A NUMERIC ANSWER.)
   ANS1 TWO|2
   COMP   54
   TYPE THE A FACTOR IN THE AB TERM CAN BE SELECTED FROM EITHER OF THE
       BINOMIAL FACTORS.  SO, THERE ARE TWO ALTERNATIVES FOR CHOOSING IT.
54 TYPE THE A FACTOR IS CHOSEN FROM ONE OF TWO POSSIBLE CHOICES, AND THE
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NUMBER OF WAYS OF SELECTING IT CAN BE EXPRESSED AS C(2,1). HAVING SELECTED THE A FACTOR FROM ONE OF THE TWO BINOMIAL FACTORS, THE B FACTOR IS CHOSEN FROM THE OTHER BINOMIAL FACTOR. REMEMBER, THE TWO FACTORS MUST BE SELECTED FROM DIFFERENT BINOMIALS. THE NUMBER OF WAYS OF SELECTING THE B FACTOR IS C(1,1) OR ___. (NUMERIC VALUE)

ANS1 ONE|1

COMP 55

TYPE ONCE THE A FACTOR HAS BEEN CHOSEN, THE B FACTOR IS CHOSEN FROM THE REMAINING BINOMIAL. THE B FACTOR CAN BE CHOSEN IN ONLY ONE WAY.

55 TYPE THE NUMBER OF WAYS OF SELECTING THE AB TERM IS C(2,1)XC(1,1) OR ___ WAYS. (NUMERIC ANSWER)

ANS1 TWO|2|C(2,1)

TYPE THE A FACTOR CAN BE SELECTED IN TWO WAYS AND THE B FACTOR CAN BE SELECTED IN ONE WAY. APPLYING THE MULTIPLICATION PRINCIPLE, THE AB TERM CAN BE SELECTED IN TWO X ONE OR ___ WAYS.

ANS1 TWO|2|C(2,1)

56 TYPE THE COEFFICIENT OF THE AB TERM IS C(2,1)XC(1,1) OR TWO. THE COEFFICIENTS OF THE A**2 AND THE B**2 TERMS ARE EACH C(2,2) OR ONE. SO, THE EXPANSION OF (A+B)**2 CAN BE EXPRESSED AS C(2,2)A**2 + C(2,1)X C(2,1)AB + C(2,2)B**2. THE COEFFICIENT OF EACH TERM IN THE EXPANSION IS THE NUMBER OF WAYS OF SELECTING THE A'S AND B'S FROM THE BINOMIAL FACTORS, WHERE EXACTLY ONE LETTER IS SELECTED FROM EACH BINOMIAL FACTOR.
WE HAVE NOW EXPANDED (A+B)**2 TWO WAYS: THE FIRST WAY IS BY MULTIPLYING THE BINOMIAL FACTORS TOGETHER, WHICH CAN BECOME COMPLICATED FOR HIGHER POWERS; THE SECOND WAY IS BY SELECTING COMBINATIONS FROM THE BINOMIAL FACTORS. THE SECOND METHOD IS MORE EFFICIENT FOR HIGHER POWERS, AND A SLIGHT CHANGE IN NOTATION RESULTS IN A VERY EASY PATTERN. EACH FACTOR IN THE AB TERM IS SELECTED FROM A DIFFERENT BINOMIAL FACTOR, SO ONCE THE NUMBER OF WAYS OF SELECTING THE FIRST FACTOR HAS BEEN DETERMINED, THE OTHER FACTOR CAN BE SELECTED IN ONLY ONE WAY. THUS, THE COEFFICIENT OF EACH TERM IN THE EXPANSION IS USUALLY WRITTEN AS THE NUMBER OF WAYS OF SELECTING ONLY ONE OF THE TWO FACTORS. THE CHOICE OF WHICH FACTOR (LETTER OF THE BINOMIAL) SHOULD BE USED IS IRRELEVANT, BUT ONCE DECIDED, IT SHOULD BE EMPLOYED THROUGHOUT THE EXPANSION. TO ILLUSTRATE, LET'S EXPAND (A+B)**2 AGAIN. LET THE B FACTOR BE USED TO REPRESENT THE NUMBER OF WAYS OF SELECTING

```
      EACH COEFFICIENT IN THE EXPANSION.  THE A**2 TERM CONTAINS ___ FACTORS
      OF B.  (HOW MANY)
   ANS1 NONE|NO|ZERO|0
      THE COEFFICIENT OF THIS TERM IS THE NUMBER OF WAYS OF CHOOSING ZERO
      B'S FROM THE TWO BINOMIALS.  THIS IS EXPRESSED IN COMBINATION NOTATION
      AS C(2,___).
   ANS1 ZERO|0
   COMP   58
      BEFORE BECOMING TOO DISCOURAGED IF THIS METHOD OF DETERMINING THE
      COEFFICIENTS DOESN'T SEEM TO MAKE SENSE, LOOK AT THE REMAINDER OF THIS
      CONCEPT.  HOPEFULLY, BY LOOKING AT THE ENTIRE PATTERN, YOU WILL
      UNDERSTAND WHAT PROCEDURE IS BEING FOLLOWED.
58 TYPE C(2,0) IS EVALUATED AS ONE, WHICH HAS ALREADY BEEN SHOWN TO BE THE
      COEFFICIENT OF THE A**2 TERM.
   TYPE THE AB TERM CONTAINS ___ FACTORS OF B.  (HOW MANY)
   ANS1 ONE|1
      THE COEFFICIENT OF THE AB TERM IS C(2,___).
   ANS1 ONE|1
   COMP   59
   TYPE THE COEFFICIENT OF THIS TERM IS THE NUMBER OF WAYS OF CHOOSING ONE
      FACTOR OF B FROM TWO POSSIBLE CHOICES.
59 TYPE THE COEFFICIENT OF THE B**2 TERM IS ___.  (ANSWER IN COMBINATION
      NOTATION.)
   ANS1 C(2,2)|C(2 ,2)|C(2, 2)|ONE|1
   COMP   60
   TYPE THE B**2 TERM CONTAINS ___ FACTORS OF B.  (HOW MANY)
   ANS1 TWO|2
   TYPE THE COEFFICIENT OF THE B**2 TERM IS THE NUMBER OF WAYS OF SELECTING
      TWO FACTORS OF B FROM TWO POSSIBLE CHOICES AND CAN BE EXPRESSED AS
      C(2,2).
60 TYPE COMBINING THESE TERMS, THE EXPANSION CAN BE EXPRESSED AS FOLLOWS:
      C(2,0)A**2 + C(2,1)AB + C(2,2)B**2.  BY EXAMINING THE COEFFICIENT, IT
      CAN BE SEEN THAT EACH ONE IS REALLY C(THE POWER OF THE EXPANSION, THE
      NUMBER OF FACTORS OF B).  ALSO, EACH TERM HAS ___ LESS FACTORS OF A
      THAN THE TERM IMMEDIATELY PRECEDING IT.  (HOW MANY)
   ANS1 ONE|1
   COMP   61
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      TYPE THE FIRST TERM HAS TWO FACTORS OF A, THE SECOND TERM HAS ONE FACTOR
           OF A, AND THE THIRD TERM HAS NO FACTORS OF A.  SO, EACH SUCCEEDING
           TERM HAS ONE LESS FACTOR OF A.  ON THE OTHER HAND, EACH TERM HAS ___
           MORE FACTORS OF B THAN THE TERM IMMEDIATELY PRECEDING IT.  (HOW MANY)
      ANS1 ONE|1
   61 TYPE THE SITUATION FOR THE FACTORS OF B IS THE REVERSE OF THAT FOR THE
           FACTORS OF A.  AS THE NUMBER OF FACTORS OF A DECREASES BY ONE, THE
           NUMBER OF FACTORS OF B INCREASES BY ONE.
           NOW, LET'S EXPAND THE BINOMIAL (A+B)**4 BY USING THE SAME PATTERN AS
           SUGGESTED BY THE ABOVE EXPANSION OF (A+B)**2; THAT IS, EXPANDING BY
           WRITING THE COEFFICIENTS OF EACH TERM AS COMBINATIONS.
           (A+B)**4 IS THE PRODUCT OF ___ FACTORS OF (A+B).  (HOW MANY)
      ANS1 FOUR|4
      COMP   62
      TYPE (A+B)**4 IS EQUAL TO (A+B)X(A+B)X(A+B)X(A+B).  THUS, (A+B) IS USED AS
           A BINOMIAL FACTOR ___ TIMES.  (HOW MANY)
      ANS1 FOUR|4
   62 TYPE THE NUMBER OF UNIQUE TERMS IN THE EXPANSION IS ___.  (HOW MANY)
      ANS1 FIVE|5
      COMP   64
           THE NUMBER OF UNIQUE TERMS IN THE EXPANSION IS THE EXPONENT (POWER) OF
           THE EXPANSION, PLUS ONE.  THE EXPANSION OF (A+B)**4 HAS ___ TERMS.
           (HOW MANY)
      ANS1 FIVE|5
   64 TYPE THE DEGREE OF EACH OF THE FIVE TERMS IN THE EXPANSION IS ___.
      ANS1 FOUR|4
      COMP   66
      TYPE THE DEGREE OF EACH TERM IS FOUR.  EACH TERM IN THE EXPANSION CONTAINS
           FOUR FACTORS.  THESE FOUR FACTORS ARE EACH SELECTED FROM A DIFFERENT
           ONE OF THE FOUR BINOMIALS.
   66 TYPE LET THE FIRST TERM OF THE EXPANSION BE THE TERM CONTAINING THE A
           FACTOR FROM EACH OF THE FOUR BINOMIALS.  EXPRESS THE COEFFICIENT OF
           EACH TERM AS THE NUMBER OF WAYS OF SELECTING THE B FACTORS.  THE
           FIRST TERM IS ___.  (ANSWER USING COMBINATIONS AND POWERS.  LEAVE NO
           BLANK SPACES IN YOUR ANSWER.)
      ANS2 C(4,0)A**4|C(4,0)XA**4|1XA**4|A**4|C(4,0)AAAA
      COMP   68
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    WANS C(4,4)A**4|C(4,4)XA**4|C(4,4)XAXAXAXA|C(4,4)AAAA
    COMP   69
    TYPE NO, THAT IS NOT COMPLETELY CORRECT.  LET'S CONSIDER THE QUESTION IN
         TWO PARTS.  FIRST, THE TERM, WITHOUT ITS COEFFICIENT, CAN BE EXPRESSED
         AS AXAXAXA OR A**___.
    ANS1 FOUR|4
    COMP   70
    TYPE THE DEGREE OF EACH TERM IS FOUR, WHICH MEANS THAT EACH TERM HAS FOUR
         FACTORS.  BY USING EXPONENTIAL NOTATION, FOUR FACTORS OF A CAN BE
         WRITTEN AS A**4.
70  TYPE THIS TERM IS OBTAINED BY SELECTING FOUR A'S AND ___ B'S.  (HOW MANY)
    ANS1 ZERO|0|NO|NONE
         THE COEFFICIENT OF THE A**4 TERM IS THE NUMBER OF WAYS OF CHOOSING
         ZERO B'S FROM FOUR POSSIBLE BINOMIAL FACTORS.  IT CAN BE EXPRESSED IN
         COMBINATION NOTATION AS ___.
    ANS1 C(4,0)|C(4 , 0)|C(FOUR,ZERO)|C(FOUR,0)|1|ONE|C(4, 0)|C(FOUR, ZERO)
   COMP   72
         THE NUMBER OF COMBINATIONS OF FOUR FACTORS, TAKEN NONE AT A TIME, IS
         C(4,0).  C(4,0) EQUALS ONE.
72  TYPE COMBINING THE TWO PARTS, THE TERM IS C(4,0)A**4.
    GOTO   68
69  TYPE IN VIEW OF WHAT THE QUESTION REALLY ASKS, YOUR REPLY IS NOT CORRECT.
         THE DESIRED RESPONSE IS C(4,0)A**4.  BOTH C(4,0) AND C(4,4) ARE EQUAL
         TO ONE.  HOWEVER, OF THE TWO, C(4,0) IS THE BEST ANSWER.
68  TYPE THE NEXT TERM OF THE EXPANSION CONTAINS ___ FACTORS OF A AND ONE
         FACTOR OF B.  (HOW MANY)
    ANS1 THREE|3
    COMP   74
    TYPE EACH SUCCESSIVE TERM OF THE EXPANSION CONTAINS ONE LESS FACTOR OF A
         AND ONE ADDITIONAL FACTOR OF B.  THE TOTAL NUMBER OF FACTORS IS
         ALWAYS THE DEGREE OF THE TERM, IN THIS CASE, FOUR.
74  TYPE THE SECOND TERM IS ___.  (ANSWER USING COMBINATIONS AND POWERS.  LEAVE
         NO BLANK SPACES.  USE THE FORM C(_,_)A**_B.)
    ANS2 C(4,1)A**3B|C(4,1)BA**3
    COMP   76
         NO, THE SECOND TERM IS C(4,1)A**3B.
76  TYPE THE THIRD TERM IN THE EXPANSION OF (A+B)**4, WITHOUT ITS COEFFICIENT,
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      IS A**2B**___.
ANS1  TWO|2
COMP    78
      EACH SUCCESSIVE TERM OF THE EXPANSION CONTAINS ONE LESS FACTOR OF A
      AND ONE ADDITIONAL FACTOR OF B.  THE TOTAL NUMBER OF FACTORS (THE SUM
      OF THE EXPONENTS) IS THE POWER OF THE EXPANSION.  SO, THE EXPONENT OF
      THE B FACTOR IS TWO.
78 TYPE THE COEFFICIENT OF THE A**2B**2 TERM IS ___.  (USE COMBINATIONS.)
ANS2  C(4,2)|C(FOUR,TWO)|C(4 , 2)|C( 4,2 )|6|SIX|C(4, 2)|C(FOUR, TWO)
COMP    80
TYPE  NO, THE COEFFICIENT IS THE NUMBER OF WAYS OF CHOOSING THESE TWO
      FACTORS OF B FROM THE FOUR POSSIBLE ALTERNATIVES.  THIS CAN BE
      REPRESENTED AS C(4,___).
ANS1  TWO|2|C(4,2)|SIX|6
COMP    80
      THE COEFFICIENT IS TO BE REPRESENTED IN COMBINATION NOTATION AS
      C(N,R).
TYPE  N IS THE POWER OF THE EXPANSION AND R IS THE NUMBER OF FACTORS OF B IN
      THE TERM.  N IS FOUR THROUGHOUT THIS EXPANSION OF (A+B)**4, AND R
      VARIES FROM ZERO TO FOUR, DEPENDING UPON THE TERM.  R START'S WITH
      ZERO AND INCREASES BY ONE IN EACH SUCCESSIVE TERM TO FOUR.
      WOULD YOU LIKE TO TRY THE ENTIRE EXPANSION OF (A+B)**4 AGAIN?  (ANSWER
      YES OR NO.)
BANS  NO
COMP    80
TYPE  LET'S TRY (A+B) AGAIN.
GOTO    66
80 TYPE THE FIRST THREE TERMS OF THE EXPANSION ARE C(4,0)A**4 + C(4,1)A**3B
      + C(4,2)A**2B**2.
BRAN    8280
TYPE  NOTICE THAT THE EXPONENT OF THE FACTOR OF A DECREASES BY ONE FOR EACH
      SUCCESSIVE TERM AND THE EXPONENT OF THE FACTOR OF B INCREASES BY ONE.
      IF NO EXPONENT IS WRITTEN, IT CAN BE ASSUMED TO BE ONE.  IF NO FACTOR
      OF B IS WRITTEN, IT CAN BE ASSUMED TO HAVE AN EXPONENT OF ZERO.  WHEN
      EXPRESSED AS C(N,R), THE EXPONENT OF THE FACTOR OF B IS ALSO THE
      VALUE OF R IN THE COEFFICIENT.
      USING THIS SCHEME, THE EXPONENT FOR THE FACTOR OF A IN THE FOURTH TERM
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      IS ___.
   ANS1 ONE|1
      THE EXPONENT FOR THE FACTOR OF B IN THIS TERM IS ___.
   ANS1 THREE|3
82 TYPE THE FOURTH TERM, WITHOUT ITS COEFFICIENT, IS AB**3.  THE EXPONENT FOR
      THE FACTOR OF A IS ONE AND CAN BE OMITTED.  THE COEFFICIENT OF THIS
      AB**3 TERM IS C(4,___).
   ANS1 THREE|3
   COMP   84
      THE EXPONENT OF THE FACTOR OF B IS THREE.  THE COEFFICIENT IS C(4,3).
84 TYPE THE FIFTH TERM, THE LAST ONE, IS C(4,4)B**4.  THE TERM CONTAINS NO
      FACTORS OF A.
   TYPE THE COMPLETED EXPANSION OF (A+B)**4 IS:
      C(4,0)A**4 + C(4,1)A**3B + C(4,2)A**2B**2 + C(4,3)AB**3 + C(4,4)B**4.
   BRAN   8680
      BY LOOKING AT THE COMPLETE EXPANSION, THE ENTIRE PATTERN CAN BE SEEN.
      EACH SUCCESSIVE TERM HAS ONE LESS FACTOR OF A AND ONE MORE FACTOR OF
      B.  THE TOTAL NUMBER OF FACTORS IN ANY TERM IS THE SAME AS THE POWER
      OF THE EXPANSION.  THE TOTAL NUMBER OF FACTORS IN EACH TERM (THE
      DEGREE OF THE TERM) IS ___.
   ANS1 FOUR|4
      THE COEFFICIENT OF EACH TERM IS THE NUMBER OF WAYS OF SELECTING THE
      FOUR FACTORS.  THIS CAN BE EXPRESSED AS THE NUMBER OF COMBINATIONS OF
      FOUR FACTORS.  IN C(N,R), N IS THE DEGREE OF EACH TERM AND R IS THE
      NUMBER OF FACTORS OF B.  (NOTE THAT R COULD HAVE BEEN THE NUMBER OF
      FACTORS OF A.  THE ONLY RESTRICTION IS THAT ONCE YOU HAVE DECIDED
      WHICH FACTOR TO USE, YOU MUST BE CONSISTENT.)
86 TYPE BY EVALUATING THE COEFFICIENTS, THE EXPANSION IS EXPRESSED AS:
      A**4 + 4A**3B + 6A**2B**2 + 4AB**3 + B**4.
   BRAN   8875
      SIMILARLY, LET'S EXPAND (A+B)**5, BUT FIRST ANSWER THE FOLLOWING TWO
      QUESTIONS.  FIRST, HOW MANY TERMS WILL THE EXPANSION CONTAIN?  (UNIQUE
      TERMS)
   ANS1 SIX|6
   COMP   90
      THE ANSWER IS DETERMINED BY ADDING ONE TO THE POWER OF THE EXPANSION;
      IN THIS CASE, FIVE PLUS ONE, OR SIX.
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90 TYPE SECOND, WHAT IS THE DEGREE OF EACH TERM?  (HOW MANY FACTORS IN EACH
       TERM?)
   ANS1 FIVE|5
   COMP   92
       THE DEGREE OF EACH OF THE SIX TERMS IS FIVE, THE POWER OF THE
       EXPANSION.
92 TYPE THE COMPLETED EXPANSION OF (A+B)**5 IS C(5,0)A**5 + C(5,1)A**4B +
       C(5,2)A**3B**2 + C(5,3)A**2B**3 + C(5,4)AB**4 + C(5,5)B**5.
       STUDY THE PATTERN OF THESE SIX TERMS OF DEGREE FIVE.  NOTICE THAT THE
       NUMBER OF FACTORS OF A FOR EACH SUCCESSIVE TERM DECREASES BY ONE WHILE
       THE NUMBER OF FACTORS OF B INCREASES BY ONE.  THE TOTAL NUMBER OF
       FACTORS IN EACH TERM IS FIVE.  STUDY THE COEFFICIENTS.  NOTICE THAT
       EACH IS OF THE FORM C(N,R) WHERE N IS FIVE AND R IS THE NUMBER OF
       FACTORS OF B.  THIS PATTERN SHOULD ENABLE YOU TO WRITE THE EXPANSION
       OF BINOMIALS VERY EASILY.
88 TYPE LET'S TRY SOME HOMEWORK PROBLEMS.
   BRAN   9460
       1.  HOW MANY TERMS ARE THERE IN THE EXPANSION OF (A+B)**50?  WRITE THE
           FIRST THREE TERMS OF THIS EXPANSION, WITHOUT THEIR COEFFICIENTS,
           IN ASCENDING POWERS OF B.
       2.  IN THE EXPANSION OF (X+Y)**6, WHAT IS THE DEGREE OF EACH TERM IN
           X AND Y?
       3.  EXPAND (X+Y)**2.  IF X=1/2 AND Y=1/2, FIND THE VALUES OF THE
           TERMS.
       4.  USE THE BINOMIAL FORMULA TO EXPAND THE FOLLOWING:
             A.  (X+Y)**5     B.  (1+B)**3     C.  (X-Y)**2     D.  (X+2Y)**2
   GOTO   98
   BRAN   9680
       1.  HOW MANY TERMS ARE THERE IN THE EXPANSION OF (A+B)**50?  WRITE THE
           FIRST THREE TERMS OF THIS EXPANSION, WITHOUT THEIR COEFFICIENTS,
           IN ASCENDING POWERS OF B.
       2.  IN THE EXPANSION OF (X+Y)**6, WHAT IS THE DEGREE OF EACH TERM IN
           X AND Y?
       3.  EXPAND (X+Y)**2.  IF X=1/2 AND Y=1/2, FIND THE VALUES OF THE
           TERMS.
       4.  USE THE BINOMIAL FORMULA TO EXPAND THE FOLLOWING:
             A.  (X+Y)**6     B.  (X-Y)**3     C.  (X+2Y)**2     D.  (C+D)**8
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      5.  EVALUATE 1.01**3 BY EXPANDING (1+.01)**3.
   GOTO   98
96    1.  HOW MANY TERMS ARE THERE IN THE EXPANSION OF (A+B)**50?  WRITE THE
          FIRST THREE TERMS OF THIS EXPANSION, WITHOUT THEIR COEFFICIENTS,
          IN ASCENDING POWERS OF B.
      2.  IN THE EXPANSION OF (X+Y)**6, WHAT IS THE DEGREE OF EACH TERM IN
          X AND Y?
      3.  EXPAND (R+S)**3.  IF R=1/2 AND S=1/2, FIND THE VALUES OF THE
          TERMS.
      4.  USE THE BINOMIAL FORMULA TO EXPAND THE FOLLOWING:
            A.  (C+D)**9    B.  (X-Y)**3    C.  (1-X)**3    D.  (X+2Y)**3
      5.  EVALUATE 1.01**3 BY EXPANDING (1+.01)**3.
98 STOP STOP
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E. Concept Block Five

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  TYPE CONCEPT BLOCK FIVE:  BASIC PROBABILITY.
  TYPE ALTHOUGH MAN'S INTEREST IN GAMES OF CHANCE UNDOUBTEDLY DATES BACK TO
       PREHISTORIC TIMES, IT WAS NOT UNTIL THE EARLY 1650'S THAT A TRULY
       MATHEMATICAL FOUNDATION WAS LAID FOR THE SOLUTION OF VARIOUS PROBLEMS
       SUGGESTED BY THESE GAMES.  AT ABOUT THIS TIME, A GAMBLING PROBLEM WAS
       BROUGHT TO THE ATTENTION OF TWO FRENCH MATHEMATICIANS, PIERRE FERMAT
       AND BLAISE PASCAL, WHO, IN APPROXIMATELY THREE MONTHS, NOT ONLY SOLVED
       THIS PROBLEM BUT ALSO LAID DOWN THE FUNDAMENTAL PRINCIPLES ON WHICH
       MODERN PROBABILITY IS BASED.  THIS THEORY, IN TURN, IS THE VERY BASIS
       OF ALL STATISTICAL ANALYSIS IN BUSINESS AND INSURANCE, AS WELL AS IN
       THE SOCIAL AND PHYSICAL SCIENCES.
       THE THEORY OF PROBABILITY IS CONCERNED WITH CHANCE EXPERIMENTS WHERE
       IT IS DESIRED TO KNOW THE CHANCES OF A FAVORABLE OR UNFAVORABLE
       OUTCOME.  AN EXPERIMENT WILL BE DEFINED AS A CHANCE PHENOMENA WHOSE
       OUTCOME CANNOT BE PREDICTED BEFOREHAND BUT WHICH, NEVERTHELESS, HAS
       CERTAIN FEATURES OF REGULARITY.  THAT IS TO SAY, IT IS POSSIBLE TO
       MAKE PROBABILITY STATEMENTS ABOUT HOW THE EXPERIMENT WILL TURN OUT.
       THE SIMPLEST EXPERIMENT TO BE STUDIED IS IN MANY WAYS ALSO THE MOST
       IMPORTANT ONE.  IT IS THE EXPERIMENT OF TOSSING A FAIR COIN AND
       OBSERVING THE OUTCOME.
       IF WE ASSUME THAT A TOSSED COIN CANNOT COME TO REST ON ITS EDGE,
       THERE ARE ___ (HOW MANY) POSSIBLE OUTCOMES FOR A TOSS.
  ANS1 TWO|2
  COMP    2
  TYPE THE TWO POSSIBLE OUTCOMES FOR A TOSS OF A COIN ARE HEADS AND TAILS.
2 TYPE WHEN A FAIR COIN IS TOSSED, IT IS REASONABLE TO ASSUME THAT IT
       IS JUST AS LIKELY TO FALL HEADS AS TO FALL TAILS.  THE TWO OUTCOMES
       ARE CONSIDERED TO BE EQUALLY LIKELY OR EQUALLY PROBABLE.
  TYPE THE SIX POSSIBLE OUTCOMES OF THE FAIR THROW OF A SYMMETRICAL DIE ARE
       CONSIDERED TO BE ___ LIKELY.
  ANS1 EQUALLY|EQUAL|=
  COMP    4
  TYPE IT IS ALSO REASONABLE TO ASSUME THAT WHEN THE DIE STOPS AFTER BEING
       THROWN, ONE FACE IS AS LIKELY TO BE ON TOP AS ANOTHER.  EACH OF THE
       SIX FACES OF THE DIE IS AN EQUALLY PROBABLE OUTCOME.
4 TYPE SINCE THE TWO POSSIBLE OUTCOMES OF THE TOSS OF A FAIR COIN ARE
       CONSIDERED TO BE EQUALLY LIKELY, WE COMMONLY SAY THAT CHANCES ARE
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      50-50 THAT IT COMES UP HEADS.  THIS MEANS THAT IF A COIN IS TOSSED
      100 TIMES, WE SHOULD EXPECT ABOUT ___ HEADS.
   ANS1 FIFTY|50|HALF|50%|50 %
   COMP    6
   TYPE ABOUT HALF OF THE TIME, THE COIN SHOULD LAND HEADS BECAUSE THE TWO
      POSSIBLE OUTCOMES ARE EQUALLY LIKELY.  ALSO, HALF OF THE TOSSES SHOULD
      RESULT IN TAILS.
   TYPE IF A FAIR COIN IS TOSSED 1,000 TIMES, YOU SHOULD EXPECT ABOUT ___
      TAILS.
   ANS1 FIVE HUNDRED|500|5 HUNDRED|FIVE HUNDRED|HALF|50%|50 %
   COMP    8
   TYPE YOU COULD ANTICIPATE APPROXIMATELY 500 HEADS AND 500 TAILS IN
      TOSSING A COIN 1,000 TIMES.  THE NUMBER OF HEADS SHOULD EQUAL THE
      NUMBER OF TAILS.
 8 TYPE IN TEN SETS OF 1,000 TOSSES OF A COIN, AN EXPERIMENTER FOUND THAT THE
      ACTUAL NUMBER OF HEADS WAS:  502, 511, 497, 529, 504, 476, 507, 528,
      504, AND 517.  YOU SHOULD NOT BE ALARMED BECAUSE NONE OF THESE NUMBERS
      IS EXACTLY 500.  THEY DO, HOWEVER, CLUSTER AROUND THE VALUE OF ___.
   ANS1 FIVE HUNDRED|500|5 HUNDRED|5HUNDRED|FIVEHUNDRED
      SINCE THE AVERAGE FOR MANY SETS OF TOSSES IS LIKELY TO BE CLOSE TO
      500, IT IS CALLED THE EXPECTED VALUE OF THE NUMBER OF HEADS.
   GOTO   10
 6 TYPE IN TEN SETS OF 100 TOSSES OF A COIN, AN EXPERIMENTER FOUND THAT THE
      ACTUAL NUMBER OF HEADS WAS:  52, 51, 49, 53, 49, 47, 51, 53, 48, AND
      52.  YOU SHOULD NOT BE ALARMED BECAUSE NONE OF THESE NUMBERS IS
      EXACTLY 50.  THEY DO, HOWEVER, CLUSTER AROUND THE VALUE OF ___.
   ANS1 FIFTY|50
      SINCE THE AVERAGE FOR MANY SETS OF TOSSES IS LIKELY TO APPROXIMATE
      50, IT IS CALLED THE EXPECTED VALUE OF THE NUMBER OF HEADS.
10 TYPE THIS MEANS THAT IN REPEATED TOSSES OF A COIN ONE-HALF CAN BE
      EXPECTED TO COME UP ___.
   ANS2 HEADS|TAILS|HEAD|TAIL
16 TYPE THE PROBABILITY OF ANY ONE OF THE OUTCOMES ASSOCIATED WITH A GIVEN
      EXPERIMENT IS A MEASURE OF THE LIKELIHOOD OF THE OCCURRENCE OF THAT
      OUTCOME.
   TYPE A TOSS OF A FAIR COIN CAN RESULT IN EITHER A HEAD OR A TAIL.  THE
      PROBABILITY OF A HEAD IS ___.
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    ANS2 ONE-HALF|1/2|.5|HALF|ONE HALF|ONEHALF
    COMP   12
    TYPE A 50-50 CHANCE FOR A HEAD MEANS THAT THE PROBABILITY OF THE OUTCOME
         BEING HEADS IS ONE-HALF.  A 50-50 CHANCE FOR A TAIL MEANS THAT THE
         PROBABILITY OF TAILS BEING THE OUTCOME IS ___.
    ANS1 ONE-HALF|1/2|.5|HALF|ONE HALF|ONEHALF
    TYPE WHEN A FAIR COIN IS TOSSED, THE ___ OF THE OUTCOME HEADS IS ONE-HALF
         (1/2 OR .5).
    ANS1 PROBABILITY|CHANCE
    COMP   14
    GOTO   16
 14 TYPE THE PROBABILITY OF EITHER HEADS OR TAILS IS THE PROBABILITY OF
         HEADS PLUS THE PROBABILITY OF TAILS OR 1/2 + 1/2 OR ___.
    ANS1 ONE|1
    COMP   12
    TYPE THE SUM OF THE PROBABILITIES OF TWO OUTCOMES IS ONE.
 12 TYPE THE PROBABILITY OF A HEAD IS 1/2, WHICH MEANS THAT THE VALUE OF 1/2
         HAS BEEN ASSIGNED TO THE LIKELIHOOD OF THAT OUTCOME OCCURRING.
         NOW CONSIDER THE TOSS OF A FAIR DIE.  THE PROBABILITY THAT THE FACE
         WITH SIX DOTS LANDS ON TOP IS ONE-SIXTH, WHICH CAN BE REPRESENTED IN
         FRACTIONAL NOTATION AS 1/6.  THE PROBABILITY ___ HAS BEEN ASSIGNED TO
         THE OUTCOME OF A SIX (THE TOTAL NUMBER OF DOTS ON THE TOP FACE) IN A
         TOSS OF THE DIE.
    ANS2 1/6|ONE-SIXTH|ONE SIXTH
    COMP   18
    TYPE THE DIE IS A SIX-SIDED CUBE, AND EACH OF THE SIX SIDES OR FACES IS AN
         EQUALLY LIKELY OUTCOME.  BY INTUITION, YOU CAN EXPECT EACH OF THE SIX
         POSSIBLE OUTCOMES TO OCCUR ___ OF THE TIME.  (HOW OFTEN)
    ANS2 1/6|ONE-SIXTH|ONE SIXTH
    COMP   17
    TYPE SINCE EACH FACE OF THE CUBE (DIE) HAS THE SAME CHANCE (EQUALLY LIKELY)
         OF LANDING ON TOP AFTER A TOSS, A PARTICULAR FACE OR SIDE OF THE
         CUBE SHOULD LAND ON TOP ONE TIME OUT OF SIX TOSSES OR 1/6 (ONE-SIXTH)
         OF THE TIME.
 17 TYPE IN THE TOSS OF A FAIR DIE, THE PROBABILITY THAT THE FACE WITH TWO
         DOTS LANDS ON TOP IS ___.
    ANS1 1/6|ONE-SIXTH|ONE SIXTH
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18 TYPE THE BASIC ELEMENT OF PROBABILITY THEORY IS THE OUTCOME, BUT IN SOME
       INSTANCES WE MAY WANT TO CONSIDER A COLLECTION OF OUTCOMES ASSOCIATED
       WITH A PARTICULAR EXPERIMENT.  THE TERM EVENT IS USED TO DESCRIBE THIS
       COLLECTION OF OUTCOMES OF THE EXPERIMENT.  AN EVENT MAY CONSIST OF A
       SINGLE OUTCOME OR IT MAY CONSIST OF SEVERAL OUTCOMES.  SINCE AN EVENT
       IS SIMPLY A SET OF OUTCOMES, IT CAN BE DESCRIBED BY LISTING ALL OF THE
       OUTCOMES OR BY WRITING SOME QUALITY SHARED BY ALL THE OUTCOMES.  AS
       AN ILLUSTRATION, LET'S CONSIDER THE EXPERIMENT OF DRAWING A NUMBERED
       BALL FROM A JAR, THE BALLS BEING NUMBERED FROM ONE THROUGH TEN AND
       BEING IDENTIFIED BY THEIR RESPECTIVE NUMBERS.  LET E DENOTE THE
       PARTICULAR EVENT THAT WE ARE INTERESTED IN HAVING HAPPEN.  EVENT E CAN
       BE DESCRIBED IN WORDS AS THE DRAWING OF A BALL WITH A NUMBER LESS THAN
       FIVE.  IT CAN ALSO BE DESCRIBED BY LISTING ALL OF THE OUTCOMES, SUCH
       AS 1, 2, 3, AND 4.
   TYPE TO SAY THAT THE PROBABILITY OF EVENT E IS 1/4 IS TO ASSIGN THE
       PROBABILITY ___ TO THAT EVENT.
   ANS1 1/4|ONE-FOURTH|.25|ONE FORTH|ONE FOURTH|ONEFOURTH|ONEFORTH
22 TYPE THE PROBABILITY OF EVENT A IS 3/4 CAN BE WRITTEN AS P(A)=3/4.  THE
       SYMBOL P( ) IS USED TO REPRESENT THE PROBABILITY OF THE EVENT.
       THE PROBABILITY OF EVENT B IS 1/36 CAN BE WRITTEN AS ___.
   ANS2 P(B)=1/36|P( B ) = 1/36|P(B) = 1/36|P(B)= 1/36|P(B) =1/36
   COMP   20
       NO, TRY AGAIN USING THIS ADDITIONAL INFORMATION.
       THE PROBABILITY OF EVENT B IS 1/36 CAN BE EXPRESSED AS ___=1/36.
   ANS1 P(B)|P( B)|P(B )|P( B )
   TYPE THE PROBABILITY OF EVENT D IS 1/10 IS WRITTEN IN SYMBOLS AS ___.
   ANS1 P(D)=1/10|P(D) = 1/10
   COMP   20
   GOTO   22
20 TYPE IN THE TOSS OF A COIN, THERE ARE ___ EQUALLY POSSIBLE OUTCOMES.
   ANS1 TWO|2
   COMP   24
   TYPE THEY CAN EASILY BE LISTED AS HEADS AND TAILS.
24 TYPE THIS SET OF ALL POSSIBLE OUTCOMES OF AN EXPERIMENT IS CALLED A
       SAMPLE SPACE.
   TYPE THERE ARE ___ (HOW MANY) OUTCOMES IN THE PARTICULAR SAMPLE SPACE FOR
       THE EXPERIMENT OF TOSSING A FAIR COIN.
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    ANS1 TWO|2
    TYPE IN A SIMPLE EXPERIMENT SUCH AS THIS ONE IT IS EASY TO LIST ALL THE
         POSSIBLE OUTCOMES AND COUNT THEM.  LATER, WE WILL USE OUR COUNTING
         TECHNIQUES TO SOLVE THIS PROBLEM.
    TYPE IF THE EVENT IS THE OBTAINING OF A HEAD, IT CAN HAPPEN IN ___
         WAY/WAYS.
    ANS1 ONE|1
    TYPE THEREFORE, P(H)= ___.  (H IS USED TO REPRESENT THE POSSIBILITY THAT
         THE COIN LANDS HEADS.)
    ANS1 1/2|ONE-HALF|.5|.50|ONE HALF|ONEHALF
    TYPE TO CALCULATE THE PROBABILITY OF AN EVENT, DIVIDE THE NUMBER OF
         WAYS THAT THE EVENT CAN HAPPEN BY THE NUMBER OF OUTCOMES IN THE
         SAMPLE SPACE.
    TYPE LET'S TRY ANOTHER PROBABILITY PROBLEM.
    TYPE IN A DECK OF 52-REGULAR PLAYING CARDS, THERE ARE 13 SPADES.  LET THE
         LETTER S REPRESENT THE EVENT OF DRAWING A SPADE.  APPLYING THE SAME
         PRINCIPLE, P(S)= ___.  (ANSWER IN EITHER FRACTIONAL OR DECIMAL FORM.)
    ANS2 13/52|1/4|.25|ONE-FOURTH|THIRTEEN-FIFTYSECONDS|ONE FOURTH|ONE FORTH
    COMP   26
    TYPE NO, LET'S TAKE A CLOSER LOOK AT THE PROBLEM.  IN DEVELOPING A
         PROBABILITY MODEL FOR THE EXPERIMENT, CONSIDER FOUR POINTS.
         FIRST, BE ABLE TO DESCRIBE A TYPICAL OUTCOME FOR THE EXPERIMENT.
         THIS CAN BE DONE MENTALLY BUT IS NECESSARY BECAUSE IT CAN GIVE MUCH
         INSIGHT INTO THE NEXT TWO POINTS.  SECOND, COUNT OR CALCULATE THE
         NUMBER OF OUTCOMES IN THE SAMPLE SPACE.  IN THIS PROBLEM, THERE ARE
         ___ POSSIBLE OUTCOMES IN THE SAMPLE SPACE.
    ANS1 52|FIFTY-TWO|FIFTY TWO|FIFTYTWO
    COMP   28
    TYPE IN DRAWING A CARD FROM A DECK OF 52 CARDS, THERE ARE 52 POSSIBLE
         OUTCOMES.  (EACH CARD IS AN EQUALLY LIKELY OUTCOME.)  SO, THE SAMPLE
         SPACE CONTAINS 52 ELEMENTS.
28  TYPE THIRD, COUNT OR CALCULATE THE NUMBER OF OUTCOMES IN THE EVENT.
         THERE ARE ___ OUTCOMES IN THE EVENT OF DRAWING A SPADE.
    ANS1 13|THIRTEEN
    COMP   30
         THERE ARE 13 SPADES IN THE DECK; THUS, THERE ARE 13 OUTCOMES IN THE
         EVENT OF DRAWING A SPADE.
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30 TYPE FINALLY, CALCULATE THE PROBABILITY OF THE EVENT.
31 TYPE THE PROBABILITY OF DRAWING A SPADE FROM A THOROUGHLY SHUFFLED DECK
       OF CARDS CAN BE DENOTED AS P(S) AND IS EQUAL TO ___.  (EXPRESS YOUR
       ANSWER IN EITHER FRACTIONAL OR DECIMAL FORM.)
   ANS1 13/52|1/4|.25|ONE-FOURTH|THIRTEEN-FIFTYSECONDS|ONE FOURTH|ONE FORTH
   COMP   32
   TYPE P(S)=13/52.  TO CALCULATE THE PROBABILITY OF THIS, DIVIDE THE NUMBER
       OF OUTCOMES IN THE EVENT (13) BY THE NUMBER OF OUTCOMES IN THE SAMPLE
       SPACE (52).
   GOTO   31
32 TYPE THERE ARE FOUR ACES IN A STANDARD DECK OF 52 CARDS.  THE PROBABILITY
       OF DRAWING AN ACE FROM A WELL-SHUFFLED DECK IS ___.
   ANS2 4/52|1/13|.08|FOUR FIFTY-SECONDS|FOUR FIFTY SECONDS|ONE-THIRTEENTH
   COMP   27
   TYPE NO, REMEMBER THE PROBABILITY MODEL DEVELOPED EARLIER.  (THE SAMPLE
       SPACE REMAINS THE 52 CARDS OF THE DECK.)  THE EVENT OF DRAWING AN
       ACE CONSISTS OF ___ (HOW MANY) POSSIBLE OUTCOMES.
   ANS1 FOUR|4
   TYPE THEREFORE, THE PROBABILITY OF DRAWING AN ACE IS THE NUMBER OF OUTCOMES
       IN THE EVENT DIVIDED BY THE NUMBER OF OUTCOMES IN THE SAMPLE SPACE
       OR ___.  (ANSWER IN FRACTIONAL OR DECIMAL FORM.)
   ANS1 4/52|1/13|.08|FOUR FIFTY-SECONDS|FOUR FIFTY SECONDS|ONE-THIRTEENTH
   COMP   27
   TYPE THE PROBABILITY OF DRAWING AN ACE IS 4/52 OR 1/13.
   GOTO   27
26 TYPE IN DEVELOPING A PROBABILITY MODEL FOR AN EXPERIMENT, CONSIDER FOUR
       POINTS.  FIRST, BE ABLE TO MENTALLY DESCRIBE A TYPICAL OUTCOME FOR THE
       EXPERIMENT.  SECOND, FIND THE NUMBER OF OUTCOMES IN THE PARTICULAR
       EVENT.  THIRD, FIND THE NUMBER OF OUTCOMES IN THE SAMPLE SPACE.
       FINALLY, CALCULATE THE PROBABILITY OF THE EVENT BY DIVIDING THE
       NUMBER OF OUTCOMES IN THE EVENT BY THE NUMBER OF OUTCOMES IN THE
       SAMPLE SPACE.
27 TYPE THE PROBABILITY OF OBTAINING A NUMBER LESS THAN FOUR WHEN TOSSING A
       FAIR DIE IS ___.  (THE NUMBER IS THE TOTAL OF THE DOTS ON THE FACE OF
       A DIE.  THE POSSIBLE NUMBERS ARE 1, 2, 3, 4, 5, AND 6.)
   ANS2 3/6|1/2|.5|.50|ONE-HALF|HALF|ONE HALF|THREE-SIXTHS|THREE SIXTHS
   COMP   34
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    TYPE NO, LET'S THINK IT THROUGH.
         THERE ARE ___ OUTCOMES IN THE SAMPLE SPACE.  (HOW MANY)
    ANS1 SIX|6
    TYPE THERE ARE ___ OUTCOMES IN THE EVENT OF OBTAINING A NUMBER LESS THAN
         FOUR WHEN TOSSING A DIE.
    ANS1 THREE|3
    COMP   36
    TYPE THE EVENT OF OBTAINING A NUMBER LESS THAN FOUR HAS THREE POSSIBLE
         OUTCOMES, AND THEY ARE THE NUMBERS 1, 2, AND 3.
36  TYPE THE PROBABILITY OF THIS EVENT IS ___.
    ANS1 3/6|1/2|.5|.50|ONE-HALF|HALF|ONE HALF|THREE-SIXTHS|THREE SIXTHS
    COMP   34
    TYPE THE PROBABILITY IS CALCULATED BY DIVIDING THE NUMBER OF OUTCOMES IN
         THE EVENT (3) BY THE NUMBER OF OUTCOMES IN THE SAMPLE SPACE (6), WHICH
         IS 3/6 OR 1/2.
34  TYPE THE MINIMUM VALUE THAT CAN BE ASSIGNED TO THE PROBABILITY OF AN EVENT
         IS ___.
    ANS2 ZERO|0
    COMP   38
    TYPE NO, LET'S EXAMINE AN EXAMPLE TO GAIN INSIGHT INTO THE PROBLEM.
    TYPE FOR A GIVEN EXPERIMENT, LET THE NUMBER OF OUTCOMES IN THE SAMPLE SPACE
         BE FIVE.  LET A BE THE EVENT.  WHAT IS THE SMALLEST NUMBER OF OUTCOMES
         IN EVENT A?
    ANS2 ZERO|0|NONE
    COMP   40
    TYPE NO, THE SMALLEST NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT IS ZERO.
40  TYPE SOMETIMES ONE ENCOUNTERS AN EVENT WHICH CONTAINS NO OUTCOMES OF THE
         EXPERIMENT.  SUCH AN EVENT IS CALLED THE IMPOSSIBLE EVENT.  FOR
         EXAMPLE, SUPPOSE THE EXPERIMENT IS TO TOSS A COIN FIVE TIMES, AND FOR
         SOME STRANGE REASON ONE REFERS TO THE EVENT THAT SEVEN HEADS ARE
         OBTAINED.  THIS IS, OF COURSE, IMPOSSIBLE; OR STATED MORE FORMALLY,
         THERE ARE NO OUTCOMES IN THE EVENT.
    TYPE THE PROBABILITY OF THE IMPOSSIBLE EVENT IS ___.
    ANS1 ZERO|0|0/5
    COMP   42
    TYPE THE PROBABILITY OF THE IMPOSSIBLE EVENT IS 0/5 OR 0.  SO, THE
         MINIMUM VALUE THAT CAN BE ASSIGNED TO THE PROBABILITY OF AN EVENT
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      IS ZERO.  THE PROBABILITY OF AN EVENT IS ALWAYS GREATER THAN OR EQUAL
      TO ZERO AND IS CALLED NON-NEGATIVE.
42 TYPE IF THE SMALLEST NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT IS ZERO, THEN
      THE LARGEST NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT IS ___.  (USE THE
      SAME EXPERIMENT AS BEFORE WHICH HAS A SAMPLE SPACE OF FIVE OUTCOMES.)
   ANS2 FIVE|5
   COMP   44
   TYPE NO, THE LARGEST NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT IS EQUAL TO
      THE NUMBER IN THE SAMPLE SPACE (ALL THE POSSIBLE OUTCOMES).  IN THIS
      CASE IT IS ___.
   ANS1 FIVE|5
44 TYPE THE MAXIMUM VALUE THAT CAN BE ASSIGNED TO THE PROBABILITY OF AN EVENT
      IS ___.
   ANS1 ONE|1|5/5
   COMP   46
   TYPE THE MAXIMUM PROBABILITY IS 5/5 OR 1 SINCE THE NUMBER OF OUTCOMES
      IN THE EVENT AND THE NUMBER OF OUTCOMES IN THE SAMPLE SPACE ARE EQUAL.
46 TYPE AN EVENT WHICH HAS AN ASSIGNED PROBABILITY OF ONE IS CALLED THE
      CERTAIN EVENT.
   TYPE THE PROBABILITY OF AN EVENT IS ALWAYS LESS THAN OR EQUAL TO ONE.
   TYPE COMBINING THESE TWO FACTS, THE PROBABILITY OF AN EVENT IS GREATER THAN
      OR EQUAL TO ZERO AND LESS THAN OR EQUAL TO ONE.
52 TYPE IN REVIEW, FOR ANY EVENT E, THE MINIMUM P(E)= ___.
   ANS1 ZERO|0
      THIS CAN BE STATED AS FOLLOWS:  ZERO IS LESS THAN OR EQUAL TO P(E) AND
      CAN BE DENOTED AS 0<=P(E).  (< IS A SYMBOL MEANING LESS THAN.)
   TYPE THE MAXIMUM P(E)= ___.
   ANS1 ONE|1
      THIS CAN BE STATED AS FOLLOWS:  P(E) IS LESS THAN OR EQUAL TO ONE AND
      CAN BE DENOTED AS P(E)<=1.
   TYPE THE INEQUALITY WHICH EXPRESSES THE VALUES THAT P(E) CAN ASSUME IS
      0<=P(E)<=1.
   GOTO   48
38 TYPE IF THE MINIMUM VALUE THAT CAN BE ASSIGNED TO THE PROBABILITY OF AN
      EVENT IS ZERO, WHAT IS THE MAXIMUM VALUE?
   ANS2 ONE|1
   COMP   50
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      NO, THE LARGEST NUMBER OF POSSIBLE OUTCOMES FOR AN EVENT IS EQUAL TO
      THE NUMBER IN THE SAMPLE SPACE.  SO, THE MAXIMUM PROBABILITY OF AN
      EVENT IS ___.
   ANS1 ONE!1
   COMP   50
   TYPE THE MAXIMUM PROBABILITY IS ONE SINCE BOTH THE NUMBER OF OUTCOMES
      IN THE EVENT AND THE NUMBER OF OUTCOMES IN THE SAMPLE SPACE ARE EQUAL.
50 TYPE AN EVENT WHICH HAS THE ASSIGNED PROBABILITY OF ONE IS CALLED THE
      CERTAIN EVENT.
   TYPE THE PROBABILITY OF AN EVENT IS ALWAYS GREATER THAN OR EQUAL TO ZERO
      AND LESS THAN OR EQUAL TO ONE.
   GOTO   52
48 TYPE IN SUMMARY, THERE HAVE BEEN TWO MAIN POINTS IN THIS CONCEPT BLOCK.
      FIRST, THE PROBABILITY OF AN EVENT IS ALWAYS ASSIGNED A VALUE BETWEEN
      ZERO AND ONE, INCLUSIVE.  SECOND, TO CALCULATE THE PROBABILITY OF AN
      EVENT, DIVIDE THE NUMBER OF OUTCOMES IN THE EVENT BY THE NUMBER OF
      OUTCOMES IN THE SAMPLE SPACE.
   TYPE LET'S TRY SOME HOMEWORK PROBLEMS.
   BRAN   5460
      1.  A MARBLE IS DRAWN AT RANDOM FROM A BAG WHICH CONTAINS FOUR WHITE,
          FIVE GREEN, AND TWO BLACK MARBLES.  WHAT IS THE PROBABILITY THAT
          THE MARBLE IS
            A.  WHITE?        B.  GREEN?        C.  BLACK?
      2.  A LETTER IS CHOSEN AT RANDOM FROM THE WORD 'EQUATION.'  WHAT IS
          THE PROBABILITY THAT THE LETTER IS
            A.  A VOWEL?    B.  A CONSONANT?    C.  THE LETTER M?
      3.  AN EXPERIMENT IS TO SELECT A STUDENT FROM A CLASS OF FORTY
          STUDENTS.  IF TWENTY-FIVE ARE GIRLS, WHAT IS THE PROBABILITY THAT
          A BOY WILL BE SELECTED?
      4.  A COMMITTEE OF FOUR PEOPLE IS TO BE SELECTED AT RANDOM FROM A
          GROUP OF FOUR MEN AND EIGHT WOMEN.  FIND THE PROBABILITY THAT THE
          COMMITTEE WILL CONSIST OF TWO MEN AND TWO WOMEN.
      5.  A BAG CONTAINS SIX WHITE, FIVE RED, AND FOUR BLACK BALLS.
            A.  IF TWO BALLS ARE DRAWN, FIND THE PROBABILITY THAT BOTH ARE
                WHITE.
            B.  IF THREE BALLS ARE DRAWN, FIND THE PROBABILITY THAT ALL
                ARE RED.
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   GOTO   58
54 BRAN   5680
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1. A THREE-DIGIT NUMBER IS FORMED BY RANDOMLY CHOOSING THREE OF THE DIGITS 1, 2, 3, 4, 5, WITHOUT REPETITION.
   A. WHAT IS THE PROBABILITY THAT THE NUMBER IS EVEN?
   B. WHAT IS THE PROBABILITY THAT THE NUMBER IS ODD?
2. FIVE BALLS, NUMBERED ONE TO FIVE, ARE PLACED IN A BAG, MIXED, AND DRAWN OUT ONE AT A TIME. WHAT IS THE PROBABILITY THAT THE BALLS ARE DRAWN IN ORDER 1, 2, 3, 4, AND 5?
3. IF THE LETTERS E, A, M, G, ARE ARRANGED IN A LINE AT RANDOM, WHAT IS THE PROBABILITY THAT THE ARRANGEMENT WILL SPELL THE WORD 'GAME'?
4. A COMMITTEE OF FOUR PEOPLE IS TO BE SELECTED AT RANDOM FROM A GROUP OF FOUR MEN AND EIGHT WOMEN. FIND THE PROBABILITY THAT THE COMMITTEE WILL CONSIST OF TWO MEN AND TWO WOMEN.
5. A BAG CONTAINS SIX WHITE, FIVE RED, AND FOUR BLACK BALLS.
   A. IF TWO BALLS ARE DRAWN, FIND THE PROBABILITY THAT BOTH ARE WHITE.
   B. IF THREE BALLS ARE DRAWN, FIND THE PROBABILITY THAT ALL ARE RED.

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   GOTO   58
56
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1. A THREE-DIGIT NUMBER IS FORMED BY RANDOMLY CHOOSING THREE OF THE DIGITS 1, 2, 3, 4, 5, WITHOUT REPETITION.
   A. WHAT IS THE PROBABILITY THAT THE NUMBER IS EVEN?
   B. WHAT IS THE PROBABILITY THAT THE NUMBER IS ODD?
   C. WHAT IS THE PROBABILITY THAT THE NUMBER IS A MULTIPLE OF 5?
2. A BOX OF EIGHT RAZOR BLADES CONTAINS TWO DEFECTIVE BLADES. IF TWO BLADES ARE DRAWN TOGETHER FROM THE BOX AT RANDOM, WHAT IS THE PROBABILITY THAT
   A. NEITHER OF THE TWO IS DEFECTIVE?
   B. BOTH ARE DEFECTIVE?
3. A BOY HAS IN HIS POCKET A PENNY, A NICKEL, A DIME, AND A QUARTER. HE TAKES TWO COINS OUT OF HIS POCKET, ONE AFTER THE OTHER.
   A. WHAT IS THE PROBABILITY THAT BOTH COINS ARE SILVER?
   B. WHAT IS THE PROBABILITY THAT THE VALUE OF THE COINS SELECTED IS LESS THAN 20 CENTS?
4. A COMMITTEE OF FOUR PEOPLE IS TO BE SELECTED AT RANDOM FROM A

GROUP OF FOUR MEN AND EIGHT WOMEN. FIND THE PROBABILITY THAT THE COMMITTEE WILL CONSIST OF TWO MEN AND TWO WOMEN.

5. ONE BAG CONTAINS FIVE WHITE AND EIGHT RED BALLS. ANOTHER BAG CONTAINS THREE WHITE AND TEN RED. FIND THE PROBABILITY OF DRAWING BOTH WHITE BALLS IF TWO BALLS ARE DRAWN FROM ONE OF THE BAGS TAKEN RANDOMLY.

58 STOP STOP

F. Concept Block Six

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   TYPE CONCEPT BLOCK SIX:  PROBABILITIES OF COMBINATIONS OF EVENTS.
        IN PRACTICE, MANY PROBABILITY PROBLEMS ARE CONCERNED WITH TWO OR MORE
        OF THE EVENTS WHICH OCCUR AS A RESULT OF A GIVEN EXPERIMENT.  A
        USEFUL AND VERSATILE WAY OF EXPRESSING THIS INFORMATION IS BY THE
        INTRODUCTION OF SET THEORY.  THE THEORY OF SETS HAS BECOME AN
        IMPORTANT AND SEPARATE MATHEMATICAL DISCIPLINE, BUT WE WILL NEED ONLY
        SOME OF THE BASIC NOTATION TO HELP CLARIFY SEVERAL OF THE CONCEPTS IN
        PROBABILITY.
   TYPE FOR OUR PURPOSES, LET'S THINK OF A SET AS ANY ASSEMBLAGE OR COLLECTION
        OF OBJECTS GROUPED TOGETHER SO AS TO FORM A SINGLE ENTITY.  THE
        FOLLOWING EXAMPLE WILL ILLUSTRATE SOME OF THE TERMINOLOGY THAT WE MAY
        ENCOUNTER.
        THE EXPERIMENT THAT WE WILL BE INTERESTED IN IS THAT OF TOSSING A COIN
        TWICE AND SEEING WHETHER THE COIN FALLS HEADS (H) OR TAILS (T).  THE
        POSSIBLE OUTCOMES OF THE EXPERIMENT MAY BE REPRESENTED BY THE SET HH,
        HT, TH, AND TT, WHERE HH MEANS THE OCCURRENCE OF A HEAD ON BOTH
        TOSSES.  WE CALL THE SET OF POSSIBLE OUTCOMES THE ___ ___.  (ANSWER
        REQUIRES TWO WORDS.)
   ANS1 SAMPLE SPACE|SAMPLESPACE|S
   COMP   10
   TYPE THE SET OF ALL POSSIBLE OUTCOMES OF AN EXPERIMENT IS THE SAMPLE SPACE.
10 TYPE IN SET NOTATION, THIS CAN BE DENOTED AS S=(HH, HT, TH, TT), WHERE S
        DENOTES THE SAMPLE SPACE.  (NOTICE THAT HT AND TH ARE DIFFERENT
        OUTCOMES.)
   TYPE LET N(S) DENOTE THE NUMBER OF OUTCOMES IN SET S; NOW N(S) EQUALS ___.
   ANS1 FOUR|4
   COMP   12
   TYPE EACH TOSS OF THE COIN HAS TWO POSSIBLE OUTCOMES.  SO, THE FIRST TOSS
        HAS TWO POSSIBLE OUTCOMES.  LIKEWISE, THE SECOND HAS TWO POSSIBLE
        OUTCOMES.  NOW, APPLYING THE MULTIPLICATION PRINCIPLE, THE NUMBER OF
        OUTCOMES FOR THE EXPERIMENT OF TOSSING A COIN TWICE IS 2X2 OR FOUR.
        THIS CAN ALSO BE ARRIVED AT BY SIMPLY COUNTING THE NUMBER OF OUTCOMES
        IN THE SAMPLE SPACE.  SIMPLE COUNTING IS USEFUL IN SMALL SAMPLE
        SPACES, BUT AS THE SAMPLE SPACE BECOMES LARGE, COUNTING TECHNIQUES ARE
        EMPLOYED.
12 TYPE LET E BE THE EVENT THAT THE FIRST COIN IS HEADS.  IN SET NOTATION
        THIS CAN BE DENOTED AS E=(HH, HT).  THE NUMBER OF OUTCOMES IN SET E,
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       N(E), EQUALS ___.
   ANS1 TWO|2
   COMP   14
   TYPE THE FIRST TOSS IS FIXED SINCE WE ARE ONLY INTERESTED IN IT BEING A
        HEAD.  THE SECOND TOSS CAN BE EITHER HEADS OR TAILS, SO N(E)=2.
14 TYPE SET E IS CALLED A SUBSET OF S.  EVERY OUTCOME IN THE EVENT MUST ALSO
        BE AN OUTCOME IN THE SAMPLE SPACE, SO EVERY EVENT IS A SUBSET OF THE
        SAMPLE SPACE.  (THE EVENT CAN BE THE ENTIRE SAMPLE SPACE.)
        WE CAN NOW WRITE A FORMULA FOR THE PROBABILITY OF AN EVENT.  IT IS
        P(E)=N(E)/N(S).  IN THE ABOVE EXPERIMENT, P(E) EQUALS ___.  (ANSWER
        IN DECIMAL OR FRACTIONAL FORM.)
   ANS1 2/4|1/2|TWO/FOUR|ONE/TWO|.5|0.5|.50|ONE-HALF|ONE HALF|HALF|TWO-FOURTHS
   COMP   16
   TYPE REMEMBER, TO CALCULATE THE PROBABILITY OF AN EVENT, DIVIDE THE NUMBER
        OF OUTCOMES IN THE EVENT, N(E), BY THE NUMBER OF OUTCOMES IN THE
        SAMPLE SPACE, N(S).  THEREFORE, P(E)=N(E)/N(S)=2/4 OR .5.
16 TYPE NOW CONSIDER TWO SETS, A AND B.  MANY TIMES WE ARE INTERESTED IN THE
        PROBABILITY OF THE OCCURRENCE OF EVENT A OR EVENT B OR BOTH.  THE SET
        OF ALL OUTCOMES BELONGING TO A OR TO B OR TO BOTH IS DENOTED AS A OR
        B.  IN SET THEORY, THIS IS CALLED THE UNION OF THE TWO SETS A AND B.
        LET A=(P, Q) AND B=(Q, R).  THEN N(A OR B) EQUALS ___.  (NUMBER IN
        SET)
   ANS2 THREE|3
   COMP   18
   WANS FOUR|4
   COMP   20
   TYPE REMEMBER, A OR B DENOTES THE SET OF ALL POINTS BELONGING TO A OR TO B
        OR TO BOTH.  N(A OR B) DENOTES THE NUMBER OF DISTINCT OUTCOMES IN THE
        SET A UNION B.  A UNION B IS THE SET (P, Q, R).  THEN N(A OR B) EQUALS
        ___.
   ANS1 THREE|3
   GOTO   22
20 TYPE NO, THE SET CONTAINS DISTINCT OUTCOMES, AND OUTCOME Q SHOULD NOT BE
        COUNTED TWICE.  THERE ARE ONLY THREE DISTINCT OUTCOMES IN THE SET A OR
        B.
18 TYPE A OR B=(P, Q, R).  N(A OR B)=THREE.
22      THE WORD 'OR' IN THE CONTEXT OF THIS MATERIAL IS CALLED THE INCLUSIVE
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      OR.  ITS USE INDICATES THE POSSIBILITY OF A OR B OR BOTH OF THESE
      EVENTS.  IF BOTH EVENTS CONTAIN THE SAME OUTCOME (I.E., Q), IT IS
      COUNTED ONLY ONCE IN N(A OR B).
      THE SET (Q) IS CALLED THE INTERSECTION OF THE TWO SETS A AND B AND IS
      DENOTED AS A AND B.  A INTERSECTION B REPRESENTS THE OUTCOMES WHICH
      BELONG TO BOTH SETS A AND B.  USING THE PREVIOUSLY DEFINED SETS A AND
      B, N(A AND B) IS ___.
   ANS1 ONE|1
   COMP   24
   TYPE THE SET A INTERSECTION B CONTAINS ONLY THE OUTCOMES FOUND IN BOTH
      SETS, SO N(A AND B) IS ___.
   ANS2 ONE|1
   COMP   24
   TYPE NO, THE SET A INTERSECTION B CONTAINS ONLY THE OUTCOME Q.  SO
      N(A AND B) IS ONE.
24 TYPE THE UNION OF SETS A AND B IS DENOTED AS A OR B, AND THE INTERSECTION
      IS DENOTED AS A AND B.  IN MOST MATHEMATICAL PROBABILITY TEXTBOOKS,
      THE UNION AND THE INTERSECTION ARE REPRESENTED BY CUP-SHAPED SYMBOLS
      SIMILAR TO THE LETTER U.  THE UNION SYMBOL HAS THE OPEN PART OF THE
      CUP FACING UP, AND THE INTERSECTION SYMBOL HAS THE OPEN PART OF THE
      CUP FACING DOWN.  THESE SYMBOLS ARE NOT AVAILABLE ON THE TERMINAL'S
      KEYBOARD, SO YOU MUST READ CAREFULLY TO DISTINGUISH THE USES OF OR
      AND OF AND.
   TYPE ASSUME SETS A AND B ARE SUBSETS OF THE SAMPLE SPACE S, WHICH CONSISTS
      OF THE OUTCOMES (P, Q, R, S, T).  N(S) IS FIVE.  WHAT IS THE
      PROBABILITY OF A INTERSECTION B?  P(A AND B) EQUALS ___.
      (LEAVE THE ANSWER IN FRACTIONAL FORM.)
   ANS1 1/5|ONE-FIFTH|ONE FIFTH|.20|0.2|.2|ONEFIFTH
   COMP   26
      REMEMBER, P(A AND B) = N(A AND B)/N(S) OR 1/5.  USING THE BASIC
      DEFINITION OF PROBABILITY, THE PROBABILITY OF AN EVENT OR COMBINATION
      OF EVENTS IS THE NUMBER OF OUTCOMES IN THE EVENT DIVIDED BY THE
      NUMBER OF OUTCOMES IN THE SAMPLE SPACE.
26 TYPE WHAT IS THE PROBABILITY OF A UNION B?  N(A OR B) EQUALS ___.
      (LEAVE THE ANSWER IN FRACTIONAL FORM.)
   ANS1 2/5|TWO-FIFTHS|TWO FIFTHS|.40|0.4|.4|TWOFIFTHS
   COMP   28
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   TYPE P(A OR B) EQUALS N(A OR B)/N(S) OR 2/5.
   TYPE DO YOU UNDERSTAND THESE CONCEPTS?  IF YOU WOULD LIKE TO REVIEW THIS
        MATERIAL, ANSWER YES.
   BANS YES
   COMP   16
28 TYPE THE PREVIOUS EXAMPLE WAS A BRIEF INTRODUCTION TO THE SET RELATIONSHIPS
        OF UNION AND INTERSECTION.  ARE THESE TWO CONCEPTS FAMILIAR TO YOU?
        IF THEY ARE, ANSWER YES.  IF YOU WOULD LIKE ADDITIONAL PRACTICE,
        ANSWER NO.
   BANS YES
   COMP   30
   TYPE FOR A GIVEN EXPERIMENT, IF S=(O1, O2, O3, O4, O5, O6, O7), A=(O1, O3,
        O6), AND B=(O2, O4, O5, O6) WHERE O IS AN OUTCOME AND THE NUMBER IS
        USED TO DISTINGUISH THE SEVEN POSSIBLE OUTCOMES IN THE SAMPLE SPACE,
        THEN THE INTERSECTION OF THE SETS A AND B IS (O___).  (ANSWER WITH A
        NUMBER.)
   ANS1 SIX|6
   COMP   32
   TYPE THE INTERSECTION OF TWO SETS IS THE SET OF ALL ELEMENTS IN COMMON IN
        BOTH SETS.  THE TWO SETS HAVE ONLY OUTCOME SIX, O6, IN COMMON.
        P(A AND B), THE PROBABILITY OF A INTERSECTION B, IS ___.  (LEAVE THE
        ANSWER IN FRACTIONAL FORM.)
   ANS1 1/7|ONE-SEVENTH|ONE SEVENTH|.143|ONESEVENTH
   COMP   34
   TYPE N(A AND B) EQUALS ___.
   ANS1 ONE|1
   TYPE N(S) EQUALS ___.
   ANS1 SEVEN|7
   TYPE THEREFORE, P(A AND B) EQUALS 1/7.
34 TYPE THE UNION OF THE SETS A AND B IS (___).  (LIST THE OUTCOMES IN
        NUMERICAL ORDER WITH A COMMA SEPARATING THEM.)
   ANS1 O1,O2,O3,O4,O5,O6|(O1,O2,O3,O4,O5,O6)|O1, O2, O3, O4, O5, O6
   COMP   35
   TYPE N(A OR B) EQUALS ___.
   ANS1 SIX|6
   COMP   36
   TYPE NO, A UNION B=(O1, O2, O3, O4, O5, O6).  THE ANSWER IS NOT N(A) + N(B)
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        BECAUSE THE TWO SETS HAVE ONE COMMON ELEMENT.  IN THIS PROBLEM,
        N(A OR B) EQUALS SIX.  THE UNION OF THE TWO SETS CONTAINS THE OUTCOMES
        IN SET A OR SET B OR BOTH.
    GOTO   35
36  TYPE A UNION B=(01, 02, 03, 04, 05, 06).
35  TYPE P(A OR B), THE PROBABILITY OF A UNION B, IS ___.
    ANS1 6/7|SIX-SEVENTHS|SIX SEVENTHS|.857|SIXSEVENTHS
    COMP   38
    TYPE N(A OR B)=6 AND N(S)=7; SO P(A OR B)=6/7.
38  TYPE LET'S TRY ONE MORE PROBLEM.
    TYPE THIS EXPERIMENT WILL BE THE FLIPPING OF A COIN AND THEN THE ROLLING
        OF A DIE.  EACH OUTCOME IN THE SAMPLE SPACE WILL CONSIST OF A PAIR.
        THE FIRST ONE OF THE PAIR IS EITHER A HEAD (H) OR TAIL (T), AND THE
        SECOND ONE IS THE NUMBER OF DOTS ON THE FACE OF THE DIE (1, 2, 3, 4,
        5, 6).
        THREE EXAMPLES OF THE OUTCOMES ARE H1, T2, AND H5.  N(S) EQUALS ___.
        (IF NECESSARY, LIST ALL THE OUTCOMES.)
    ANS1 TWELVE|12
    COMP   40
    TYPE S=(H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6); AND N(S) IS
        CALCULATED BY MULTIPLYING 2 (WAYS FOR A COIN TO FALL) TIMES 6 (NUMBER
        OF FACES ON A DIE).  THIS IS A DIRECT APPLICATION OF THE
        MULTIPLICATION PRINCIPLE IN CONCEPT BLOCK ONE.
40  TYPE IF A IS THE EVENT THAT THE COIN IS A HEAD AND THE DIE IS A NUMBER
        LESS THAN FIVE, THEN N(A) EQUALS ___.
    ANS1 FOUR|4
    COMP   42
    TYPE A=(H1, H2, H3, H4) AND N(A) IS FOUR.
42  TYPE IF B IS THE EVENT THAT THE COIN IS EITHER HEADS OR TAILS AND THE DIE
        IS EITHER THE NUMBER FOUR OR FIVE, THEN N(B) EQUALS ___.
    ANS1 FOUR|4
    COMP   44
        B=(H4, H5, T4, T5) AND N(B) EQUALS FOUR.
44  TYPE P(A AND B) EQUALS ___.  (LEAVE ANSWER AS A FRACTION.)
    ANS2 1/12|ONE-TWELFTH|ONE TWELFTH|ONETWELFTH
    COMP   46
    TYPE NO, THAT IS INCORRECT.  N(A AND B) EQUALS ONE.  A INTERSECTION B IS
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      ONLY THE OUTCOME H4.  N(S)=12 AND P(A AND B)=N(A AND B)/N(S)=1/12.
46 TYPE P(A OR B) EQUALS ___.
   ANS2 7/12|SEVEN-TWELFTHS|SEVEN TWELFTHS|SEVENTWELFTHS
   COMP   30
   TYPE NO, THAT IS WRONG.  N(A OR B)=7 AND A UNION B=(H1, H2, H3, H4, H5,
        T4, T5).  THEREFORE, P(A OR B)=N(A OR B)/N(S).
        IF YOU ARE STILL UNCERTAIN OF THE CONCEPTS OF INTERSECTION AND UNION,
        STOP THE LESSON BY TYPING EXIT IN PLACE OF TYPING A RESPONSE TO THE
        NEXT QUESTION.  THEN, CONSULT YOUR INSTRUCTOR FOR ADDITIONAL MATERIAL
        ON SETS.
30 TYPE SOME EVENTS HAVE SPECIAL RELATIONSHIPS, AND IF THEY DO, THIS CAN BE AN
        ASSET IN CALCULATING THEIR COMBINED PROBABILITIES.  ONE SPECIAL
        RELATIONSHIP IS CALLED MUTUALLY EXCLUSIVE.  THIS MEANS THAT THE
        INTERSECTION OF TWO SETS CONTAINS NO OUTCOMES AND IS THE EMPTY SET.
        WHEN TWO EVENTS HAVE NO OUTCOMES IN COMMON, THEY ARE CALLED MUTUALLY
        EXCLUSIVE OR DISJOINT.  MUTUALLY EXCLUSIVE IS THE MORE-COMMON
        TERMINOLOGY.  WHEN TWO EVENTS ARE MUTUALLY EXCLUSIVE, THE NUMBER OF
        OUTCOMES IN THE INTERSECTION IS ZERO.
        FOR EXAMPLE, CONSIDER THE SAMPLE SPACE S=(01, 02, 03, 04, 05, 06), THE
        EVENT A=(01, 05), AND THE EVENT B=(02, 04, 06).  THE INTERSECTION IS
        THE EMPTY SET, AND N(A AND B) EQUALS ZERO.  P(A AND B) EQUALS ___.
   ANS1 ZERO|0
   TYPE N(A OR B) EQUALS ___.
   ANS1 FIVE|5
   TYPE NOTE THAT THIS IS SIMPLY N(A)+N(B) OR 2+3=5.
   TYPE P(A OR B) EQUALS ___.  (LEAVE THE ANSWER IN FRACTIONAL FORM.)
   ANS1 5/6|FIVE-SIXTHS|FIVE SIXTHS|FIVE SIXTH|FIVE-SIXTH|FIVESIXTH
   TYPE NOTE THAT THIS IS SIMPLY P(A)+P(B) OR 2/6 + 3/6 = 5/6.
   TYPE IF TWO EVENTS ARE MUTUALLY EXCLUSIVE, P(A OR B) EQUALS P(A)+P(B).
   TYPE LET'S EXAMINE P(A OR B), AGAIN WHEN THE TWO EVENTS ARE NOT MUTUALLY
        EXCLUSIVE.  BY CLOSE EXAMINATION, WE SEE THAT N(A OR B) EQUALS N(A)+
        N(B)-N(A AND B).
        BOTH EVENTS A AND B CONTAIN OUTCOMES IN COMMON, AND BY SIMPLY ADDING
        N(A)+N(B), THESE OUTCOMES ARE COUNTED TWICE.  TO ADJUST FOR THIS, A
        NUMBER MUST BE SUBTRACTED, AND N(A AND B) IS THIS NUMBER.  TRY THIS
        WITH A SIMPLE EXAMPLE.  ASSUME S=(01, 02, 03, 04, 05, 06, 07, 08), THE
        EVENT A=(01, 03, 06, 08), AND THE EVENT B=(01, 05, 06, 07).  N(A)=4,
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        N(B)=4, AND N(A OR B)=6.  N(A AND B)=2; THEREFORE, N(A OR B)=N(A)+N(B)
        -N(A AND B)=4+4-2=6.
        IF TWO EVENTS ARE NOT MUTUALLY EXCLUSIVE, THEN P(A OR B)=P(A)+P(B)-
        P(A AND B).  IN FACT, THIS FORMULA APPLIES IN EITHER CASE BECAUSE THE
        MUTUALLY EXCLUSIVE ONE IS WHERE P(A AND B) IS EQUAL TO ZERO.
        P(A OR B)=P(A)+P(B)-P(A AND B) IS A GOOD FORMULA TO REMEMBER.
    TYPE LET'S USE THIS FORMULA TO SOLVE A PROBLEM.
        SUPPOSE TWO EVENTS ARE MUTUALLY EXCLUSIVE, AND P(A)=1/2 AND P(B)=1/4.
        THEN P(A OR B) EQUALS ___.
    ANS1 3/4|THREE-FOURTHS|THREE FOURTHS|.75|0.75|1/2+1/4|THREEFOURTHS
    COMP   48
    TYPE P(A OR B)=P(A)+P(B)=1/2 + 1/4 = 3/4.
48  TYPE ANOTHER WAY OF SAYING MUTUALLY EXCLUSIVE IS TO SAY N(A AND B)= ___.
    ANS1 ZERO|0
    COMP   50
    TYPE THE NUMBER OF OUTCOMES IN THE EMPTY SET IS ZERO.
50  TYPE ANOTHER SPECIAL RELATIONSHIP BETWEEN EVENTS IS CALLED INDEPENDENCE.
        THE TWO EVENTS A AND B ARE INDEPENDENT IF THE OCCURRENCE OF THE ONE
        HAS NO EFFECT ON THE OCCURRENCE OF THE OTHER.  COMMONLY, WE SAY THAT
        TWO EVENTS HAVE NOTHING TO DO WITH EACH OTHER, AND TECHNICALLY, THEY
        ARE CALLED INDEPENDENT EVENTS.
        WHEN TWO PENNIES ARE TOSSED, WE INTUITIVELY FEEL THAT THE FALL OF
        ONE PENNY IS INDEPENDENT OF THE FALL OF THE OTHER.  SIMILARLY, WHEN
        TWO DICE ARE TOSSED, THE FALL OF ONE DIE IS ___ OF THE FALL OF THE
        OTHER DIE.
    ANS1 INDEPENDENT
        THE TOSS OF A COIN TWICE IN SUCCESSION IS AN EXAMPLE OF INDEPENDENT
        EVENTS.
        CONSIDER THE EXPERIMENT OF TOSSING A COIN TWICE.  A SAMPLE SPACE FOR
        THIS EXPERIMENT IS S=(HH, TH, HT, TT).  THE PROBABILITY OF TWO HEADS
        IN SUCCESSION, P(HH), EQUALS ___.
    ANS1 1/4|ONE-FOURTH|ONE FOURTH|.25|0.25|ONEFOURTH
    COMP   52
    TYPE P(HH)=N(HH)/N(S)=1/4 OR .25.
52  TYPE THE PROBABILITY OF A HEAD ON THE FIRST TOSS IS ___.
    ANS1 1/2|ONE-HALF|ONE HALF|HALF|.5|0.5|.50|ONEHALF
    TYPE LIKEWISE, THE PROBABILITY OF A HEAD ON THE SECOND TOSS IS 1/2.
```

NOTE THAT P(HH)=1/2 X 1/2 = 1/4 OR .25.
WE ARE GIVEN THE FOLLOWING DEFINITION FOR INDEPENDENT EVENTS. EVENTS A AND B ARE INDEPENDENT IF AND ONLY IF P(A AND B)=P(A)XP(B). THE FOREGOING DEFINITION PROVIDES US WITH A CLEAN-CUT MEANING FOR INDEPENDENT EVENTS. IF TWO EVENTS ARE INDEPENDENT, THEN TO FIND P(A AND B) WE SIMPLY MULTIPLY P(A) TIMES P(B). IF WE CALCULATE P(A AND B) FROM N(A AND B)/N(S) AND IT IS EQUAL TO THE VALUE P(A) TIMES P(B), THEN WE CAN STATE THAT THE EVENTS ARE INDEPENDENT.
WARNING!
THERE IS A DANGER OF CONFUSING MUTUALLY EXCLUSIVE EVENTS WITH INDEPENDENT EVENTS. A SOURCE OF THIS CONFUSION IS THE COMMON EXPRESSION, 'HAVING NOTHING TO DO WITH EACH OTHER.' HOWEVER, MUTUALLY EXCLUSIVE SETS HAVE NO OUTCOMES IN COMMON, WHEREAS INDEPENDENT EVENTS IN A SAMPLE SPACE MUST HAVE A POINT IN COMMON IF BOTH EVENTS HAVE NON-ZERO PROBABILITIES. THIS CAN BE PROVEN MATHEMATICALLY.
ANALYZE THIS PROBLEM. P(A)=1/5, P(B)=1/5, AND P(A AND B)=1/25.
THEREFORE, A AND B ___ (ARE/ARE NOT) INDEPENDENT EVENTS.

ANS1 ARE
COMP 54

P(A AND B)=1/25, WHICH IS P(A)XP(B) OR 1/5 X 1/5.
IF TWO EVENTS ARE NOT INDEPENDENT, THE CONDITIONS IN THE DEFINITION ARE NOT MET. THEY ARE TERMED TO BE DEPENDENT.
ANALYZE THIS PROBLEM. P(A)=1/8, P(B)=1/8, AND P(A AND B)=1/40.
A AND B ARE ___ EVENTS.

ANS1 DEPENDENT
COMP 56

P(A AND B)=1/40, WHEREAS P(A)XP(B)=1/8 X 1/8=1/64. SO P(A AND B) DOES NOT EQUAL P(A)XP(B). THE CONDITIONS ARE NOT SATISFIED SO THE EVENTS ARE DEPENDENT.

56 TYPE SOLVE THIS PROBLEM. LET'S DRAW ONE CARD FROM EACH OF TWO WELL-SHUFFLED 52-CARD DECKS. IF WE ASSUME INDEPENDENT OUTCOMES, WHAT IS THE PROBABILITY THAT BOTH CARDS WILL BE SPADES? THE PROBABILITY OF DRAWING A SPADE FROM A DECK OF CARDS IS 13/52. THE PROBABILITY OF BOTH CARDS BEING SPADES IS ___X___. (ANSWER WITH TWO FRACTIONS SEPARATED BY X, THE MULTIPLICATION SYMBOL.)

ANS1 13/52X13/52|1/4X1/4|1/16|169/2704|13/52 X 13/52|1/4 X 1/4
COMP 58

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   TYPE ASSUMING INDEPENDENCE, THE PROBABILITY IS 13/52 X 13/52.
58 BRAN   6075
   GOTO   62
60 TYPE OFTEN WE DEAL WITH PROBABILITIES FOR PART RATHER THAN ALL OF A SAMPLE
       SPACE.  THE PROBABILITY OF AN EVENT IN THE SUBSET MAY DIFFER FROM THAT
       IN THE WHOLE SAMPLE SPACE.  THE SUB-SAMPLE SPACE IS DEFINED BY EXTRA
       CONDITIONS BEYOND THOSE FOR THE WHOLE SAMPLE SPACE, AND THE
       PROBABILITIES ASSOCIATED WITH EVENTS IN A SUB-SAMPLE SPACE ARE CALLED
       CONDITIONAL PROBABILITIES.  THE USUAL NOTATION FOR THE PROBABILITY OF
       AN EVENT A, GIVEN THAT AN EVENT B HAS ALREADY OCCURRED, IS P(A|B), THE
       VERTICAL BAR MEANING GIVEN.  THE CONDITIONAL PROBABILITY OF A, GIVEN
       B, IS DEFINED BY THE EQUATION P(A|B)=P(A AND B)/P(B) IF P(B) IS NOT
       EQUAL TO ZERO.
       FIND P(A|B) IF P(A)=6/36, P(B)=3/36, AND P(A AND B)=2/36.  P(A|B) IS
       EQUAL TO ___.  (NOTE THAT THE FRACTIONS WERE PURPOSELY NOT REDUCED.)
   ANS1 2/3|TWO-THIRDS|TWO THIRD|TWO THIRDS|TWOTHIRDS|.66|.666
   COMP   64
   TYPE P(A|B)=P(A AND B)/P(B)=2/36 / 3/36 = 2/3.  THE TWO 36'S DIVIDE OUT.
64 TYPE TRY THIS ONE.  TWO C'S AND TWO D'S ARE ARRANGED IN ORDER.  ALL
       ARRANGEMENTS ARE EQUALLY LIKELY.  GIVEN THAT THE LAST LETTER IN ORDER
       IS D, FIND THE PROBABILITY THAT THE TWO C'S ARE TOGETHER.  LET A BE
       THE EVENT OF TWO C'S TOGETHER AND B BE THE REDUCED SAMPLE SPACE WHOSE
       OUTCOMES HAVE D AS THE LAST LETTER.  P(A|B)=___.  (LEAVE THE ANSWER AS
       A FRACTION OR A TWO-PLACE DECIMAL.)
   ANS2 2/3|TWO-THIRDS|TWO THIRD|TWO THIRDS|TWOTHIRDS|.66|.666
   COMP   60
       NO, THAT IS NOT QUITE CORRECT.
       FIRST, DETERMINE THE SAMPLE SPACE FOR THE ORIGINAL EXPERIMENT.
       S=(CCDD, CDCD, CDDC, DCCD, DCDC, DDCC) AND N(S) EQUALS ___.
   ANS1 SIX|6
       CONSIDER THE REDUCED SAMPLE SPACE, B, WHOSE OUTCOMES HAVE D AS THE
       LAST LETTER.  B=(CCDD, CDCD, DCCD) AND N(B) EQUALS ___.
   ANS1 THREE|3
   TYPE CONSIDER EVENT A WHOSE OUTCOMES INCLUDE CC.  A=(CCDD, DCCD, DDCC) AND
       N(A) EQUALS ___.
   ANS1 THREE|3
       THEREFORE, A INTERSECTION B=(CCDD, DCCD) AND N(A AND B) EQUALS ___.
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   ANS1 TWO|2
   TYPE P(A|B)=P(A AND B)/P(B)=2/6 / 3/6= ___.
   ANS1 2/3|TWO-THIRDS|TWO THIRD|TWO THIRDS|TWOTHIRDS|.66|.666
62 TYPE IN SUMMARY, THERE HAVE BEEN TWO MAIN POINTS IN THIS CONCEPT, AND THEY
        CAN BE EXPRESSED BY TWO FORMULAS.  FIRST, P(A OR B)=P(A)+P(B)-
        P(A AND B).  SECOND, IF TWO EVENTS ARE INDEPENDENT, P(A AND B)=P(A)
        XP(B).
        LET'S TRY SOME HOMEWORK PROBLEMS.
   BRAN   6860
        1. A BAG CONTAINS FIFTEEN WHITE BALLS, OF WHICH FOUR HAVE A RED SPOT,
           AND TWENTY GREEN BALLS, OF WHICH TEN HAVE A RED SPOT.  A BALL WILL
           BE DRAWN AT RANDOM.  LET W, G, AND S REPRESENT THE EVENTS OF THE
           BALL BEING WHITE, GREEN, AND HAVING A RED SPOT, RESPECTIVELY.
           FIND THE FOLLOWING:
             A.  P(W)=          B.  P(G)=          C.  P(S)=
             D.  P(W AND S)=    E.  P(G AND S)=    F.  P(W OR S)=
             G.  P(G OR W)=     H.  P(G OR S)=
        2. ONE CARD IS DRAWN AT RANDOM FROM AN ORDINARY DECK OF FIFTY-TWO
           CARDS.  WHAT IS THE PROBABILITY OF DRAWING
             A.  AN ACE OR A KING?     B.  AN ACE OR A DIAMOND?
        3. IN A BASEBALL TOURNAMENT THE PROBABILITY THAT TEAM A WILL WIN IS
           1/7, AND THE PROBABILITY THAT TEAM B WILL WIN IS 1/9.  FIND THE
           PROBABILITY THAT ONE OF THESE TWO TEAMS WILL WIN.
        4. IF FOUR CARDS ARE DRAWN FROM A DECK OF FIFTY-TWO CARDS, FIND THE
           PROBABILITY THAT THEY ARE OF THE SAME SUIT?
        5. A MARBLE IS DRAWN AT RANDOM FROM A BAG WHICH CONTAINS FOUR WHITE
           MARBLES, FIVE GREEN MARBLES, AND TWO BLACK MARBLES.  WHAT IS THE
           PROBABILITY THAT THE MARBLE IS
             A.  WHITE OR GREEN?  B.  WHITE OR BLACK?  C.  GREEN OR BLACK?
   GOTO   72
68 BRAN   7080
        1. A BAG CONTAINS FIFTEEN WHITE BALLS, OF WHICH FOUR HAVE A RED SPOT,
           AND TWENTY GREEN BALLS, OF WHICH TEN HAVE A RED SPOT.  A BALL WILL
           BE DRAWN AT RANDOM.  LET W, G, AND S REPRESENT THE EVENTS OF THE
           BALL BEING WHITE, GREEN, AND HAVING A RED SPOT, RESPECTIVELY.
           FIND THE FOLLOWING:
             A.  P(W)=          B.  P(G)=          C.  P(S)=
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      D.  P(W AND S)=     E.  P(G AND S)=     F.  P(W OR S)=
      G.  P(G OR W)=      H.  P(G OR S)=
   2. ONE BAG CONTAINS FIVE BLACK AND TEN WHITE BALLS.  ANOTHER BAG
      CONTAINS EIGHT WHITE AND THREE BLACK BALLS.  IF TWO BALLS ARE
      DRAWN TOGETHER FROM EACH BAG, FIND THE PROBABILITY THAT ALL THE
      BALLS OBTAINED WILL BE WHITE.
   3. A MARBLE IS DRAWN AT RANDOM FROM A BAG WHICH CONTAINS FOUR WHITE
      MARBLES, FIVE GREEN MARBLES, AND TWO BLACK MARBLES.  WHAT IS THE
      PROBABILITY THAT THE MARBLE IS
        A.  WHITE OR GREEN?  B.  WHITE OR BLACK?  C.  GREEN OR BLACK?
   4. IF TWO CARDS ARE DRAWN FROM A STANDARD DECK OF PLAYING CARDS, WHAT
      IS THE PROBABILITY THAT BOTH ARE FACE CARDS OR BOTH SPADES?
   5. THE FOLLOWING QUESTIONS REFER TO THE EXPERIMENT OF ROLLING TWO
      DICE.  (ONE DIE IS RED AND THE OTHER IS BLACK.)  LET R DENOTE THE
      NUMBER OF SPOTS WHICH APPEAR ON THE RED DIE, AND LET B DENOTE THE
      NUMBER OF SPOTS WHICH APPEAR ON THE BLACK DIE.  FIND THE
      PROBABILITY THAT
        A.  R>5 AND B<2          B.  R>=5 OR B<=2
        C.  R+B=6                D.  R IS EVEN OR B IS ODD
GOTO  72
70 1. ONE CARD IS DRAWN AT RANDOM FROM AN ORDINARY DECK OF FIFTY-TWO
      CARDS.  WHAT IS THE PROBABILITY OF DRAWING
        A.  AN ACE OR A KING?              B.  A HEART OR A SPADE?
        C.  A RED CARD OR A BLACK CARD?    D.  AN ACE OR A DIAMOND?
   2. A COIN IS TOSSED AND THEN A DIE IS THROWN.  WHAT IS THE
      PROBABILITY OF A HEAD AND A FIVE?
   3. TWO DICE ARE TOSSED.  IF THE FIRST DIE SHOWS 5, WHAT IS THE
      PROBABILITY THAT THE SECOND DIE SHOWS EVEN?
   4. IF TWO CARDS ARE DRAWN FROM A STANDARD DECK OF PLAYING CARDS, WHAT
      IS THE PROBABILITY THAT BOTH ARE FACE CARDS OR BOTH SPADES?
   5. THE FOLLOWING QUESTIONS REFER TO THE EXPERIMENT OF ROLLING TWO
      DICE.  (ONE DIE IS RED AND THE OTHER IS BLACK.)  LET R DENOTE THE
      NUMBER OF SPOTS WHICH APPEAR ON THE RED DIE, AND LET B DENOTE THE
      NUMBER OF SPOTS WHICH APPEAR ON THE BLACK DIE.  FIND THE
      PROBABILITY THAT
        A.  R<3 OR B>=4          B.  R>=1 OR B=1          C.  R+B=5
        D.  R>2 AND B>3          E.  R+B=5 AND R+B=7
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72 STOP STOP
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XIII. APPENDIX D: SIGN-ON PROCEDURE

INSTRUCTIONS TO OPERATE THE 2741 TERMINAL FOR THE COMPUTER-ASSISTED INSTRUCTION (CAI) PROGRAM ON PROBABILITY

I. Dial-Up Procedure for CPS

A. Turn on the 2741 keyboard.
B. Pick up the appropriate telephone.
C. Dial the computer number 4-2630.
D. Wait for a high-pitched squeal.
E. Place the phone in the acoustic coupler.
F. Push the return key on the keyboard.
G. The computer will respond by nodding the typeball.

II. Login Procedure

A. Type the following information:
LOGIN(CPS10,E3100)
B. Correct any mistakes by backspacing and typing over.
C. Push the return key on the keyboard.
D. The computer will respond by typing:
PASSWORD
E. An underscore, __, will be typed indicating that control of the terminal is returned to you.
F. Type the following word:
STATIC
G. Push the return key on the keyboard.
H. The computer will respond by typing a line, such as:
XXXXX USER02: TIME 10:55:14 1/19/70
I. Type the following information:
LOAD(CAICON,DM)CPS06
J. Push the return key.
K. The computer will pause for a moment and then type an underscore.
L. Type the following information:
XEQ (or EXECUTE)
M. Push the return key.

III. CAI Program

A. The computer will respond by typing:
WHICH LESSON DO YOU WISH TO STUDY?
B. From the Table of Contents, decide which lesson is to be studied and type the keyword. (i.e., PROB)
C. Push the return key.

D. The computer will pause for a moment and then start the lesson material.
E. When the computer responds by typing:
REPLY
F. Type your answer to the question.
G. Push the return key.
H. At end of the lesson the computer will respond by typing:
530. STOP

IV. Logout Procedure

A. Type the following information:
LOGOUT(OFF)
B. Push the return key.
C. The computer will respond by typing a line, such as:
TIME 21:36:15 TIME USED: CPU 00:00:21
TERM 00:33:30 PAGE 01:38:53
D. Turn off the 2741 keyboard.
E. Tear off your lesson and keep for future reference.