

Potential Function Explain of the Quick Algorithm of Synergetic Neural Network

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Abstract: We can determine the winner pattern of the synergetic neural network directly from order parameters and attention parameters when the attention parameters are equal or constant. In this paper, we explain that the basis of that quick algorithm is that the potential function of network and attractive domain of each attractor are fully determined for given attention parameters, and there is an analytic approximation for the division of attractive domains.

Key Words: Neurally-Based Network, Synergetics, Potential Function, Quick Haken Network, Neural Network

1. The Principle of Synergetic Neural Network

Synergetics is the science proposed by Haken in 1973 to explain the phase transition and self-organization in non-equilibrium system. It was introduced into computer science in late 1980s by Haken and brought about a new field: synergetic information processing, which includes synergetic neural network, synergetic information theory and synergetic computer, etc. [1]-[5]

The basic principle of synergetic neural network is that the pattern recognition procedure can be viewed as the competition progress of many order parameters [1] [2]. For a unrecognized pattern q we can construct a dynamic progress to make q evolving into one of the prototype pattern v_k which closest to $q(0)$ via some intermediate states $q(t)$. This progress can be described as $q(0) \rightarrow q(t) \rightarrow v_k$.

A dynamic equation can be given for a unrecognized pattern q [3]:

$$\dot{q} = \sum_{k=1}^M \lambda_k v_k (v_k^+ q) - B \sum_{k' \neq k} (v_k^+ q)^2 (v_k^+ q) v_k - C(q^+ q)q + F(t) \quad (1)$$

Where q is the **status vector** of input pattern with initial value q_0 , λ_k is **attention parameter**, v_k is **prototype pattern** vector, v_k^+ is the **adjoint vector** of v_k that satisfies

$$(v_k^+, v_{k'}^T) = v_k^+ \cdot v_{k'}^T = \delta_{kk'}. \quad (2)$$

Order parameter is $\xi_k = v_k^+ q$

Corresponding dynamic equation of order parameters is

$$\dot{\xi}_k = \lambda_k \xi_k - B \sum_{k' \neq k} \xi_{k'}^2 \xi_k - C \sum_{k'=1}^M \xi_{k'}^2 \xi_k \quad (3)$$

The strongest order parameter will win by competition and desired pattern will be recognized.

2. Quick algorithm of Haken Network

The classical Haken model has some weakness in that the iteration progress will cost a great deal of space and time resource in resolving high dimensional nonlinear equation sets when we have large volume of pattern[7]. For example, figure 1 shows the evolution of 100 order parameters with initial value sampled randomly from (0,1), iterative step length $\gamma = 0.1/D$ [8], and all attention parameters are set to 1. The network converges in around 750 steps.

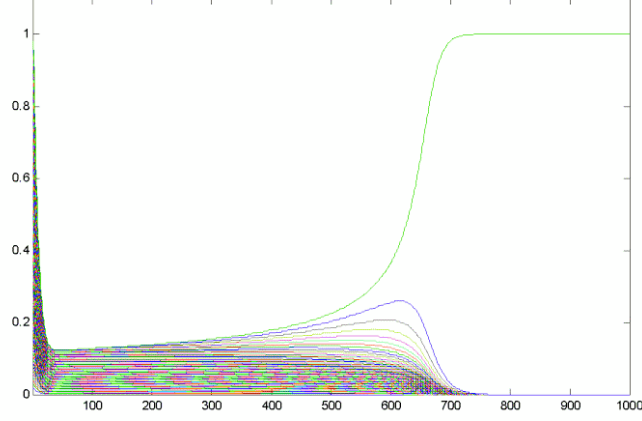


Fig1: the evolution of classical Haken network: 100 patterns

The time and space cost (omit all assistant blocks) during this iteration is

Space cost:

Order parameter	100
Global coupling D	100
Attention parameter	100

Time cost:

Time of adding	$(100+1)*750$
Time of multiplying	$(100-1)^2*750$

In practice, we may encounter competition between more samples. For example, when retrieving an image database, we usually need to find a specific sample out of tens of thousand of or even more images. Apparently, classical Haken network will have bad performance.

There are two ways to solve this problem:

1) Classify the patterns at matching layer via hierarchical competition. Competition occurs first between sub classes and then inside the recognized sub class.

2) Improve the competition layer; simplify the competition progress and the space / time cost during this progress. It can be further divided into two sub methods:

2.1) Reduce the time cost by judging the competition result form the initial condition of equation.

2.2) Reduce the space cost by transforming the differential equation sets into matrix differential equation expressed by sparse matrix using the fact that the similarities between most of patterns are low.

In this paper we will only discuss about the method 2.2, that's, determining the survivability of order parameter directly by the initial conditions of the equation. By the discuss about the attention parameter, we can find that the system status can be directly determined by order parameters, attention parameters and the property of other parameters. In the flowing experiments we prove that the winner pattern can be distinguished out directly by the order parameter and attention parameter when the attention parameters are equal or constant.

Literature [6] proves that:

- If all λ_k are equal, when $\xi_i < \xi_j$, we have $\xi_i < \xi_j$. So the evolutionary loci of all order parameters have no intersection, the largest initial order parameter will win and the network will then converge.
- For any $i, 1 \leq i \leq M$, if there exists $1 \leq j \leq M$ that satisfies

$$\begin{aligned} \lambda_j - \lambda_i &> (1/\tau) \ln(\xi_i(0) / \xi_j(0)) \\ \Leftrightarrow \xi_j(0)e^{\lambda_j\tau} &> \xi_i(0)e^{\lambda_i\tau} \Leftrightarrow \xi_j(\tau) > \xi_i(\tau) \end{aligned} \quad (4)$$

Then the order parameter corresponding to pattern i will be attenuated.

According to this theorem, we can choose winner order parameter and corresponding prototype vector based on attention parameter and initial value of order parameter. It can be used as a quick

algorithm of synergetic neural network.

3. Potential Function Explain of the Quick Algorithm

Now we will try to give explain of the quick Haken network algorithm from the viewpoint of potential function. The core of quick Haken network is that after the attention parameter is determined the potential function and all attractors and attractive domains are determined too. The evolutionary direction of certain initial status is also decided by initial value of order parameters and attention parameter.

Before the discuss we give following theorem [6]

Theorem suppose

$$\begin{cases} B > 0, C > 0 \\ (B + C)\lambda_i - C\lambda_j > 0 \quad 1 \leq i \neq j \leq M \end{cases} \quad (5)$$

The static points in corresponding synergetic neural network can be classified into

(1) Zero point $\xi = 0$ is a unstable fixed point;

(2) $2M$ stable points at $\xi = (0, \dots, \pm \sqrt{\frac{\lambda_k}{C}}, \dots, 0)^T$ with potential function value of

$$V_k = -\frac{\lambda_k^2}{4C}$$

(3) $3^M - 2M - 1$ saddle point

For lucidness, we only discuss the case of $M=2$. The evolutionary equation of order parameter of two patterns is:

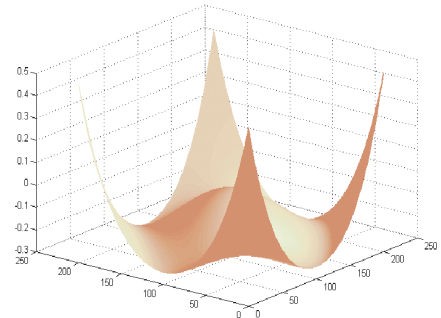
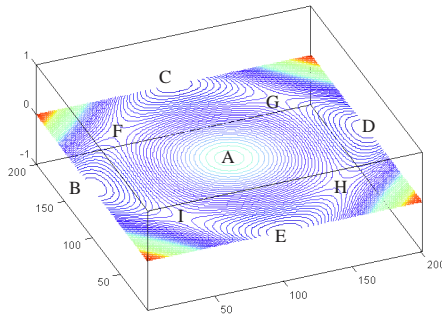
$$\begin{aligned} \dot{\xi}_1 &= \lambda_1 \xi_1 - \xi_2^2 \xi_1 - (\xi_1^2 + \xi_2^2) \xi_1 \\ \dot{\xi}_2 &= \lambda_2 \xi_2 - \xi_1^2 \xi_2 - (\xi_1^2 + \xi_2^2) \xi_2 \end{aligned} \quad (6)$$

The potential function is

$$V = -\frac{1}{2} \lambda_1 \xi_1^2 - \frac{1}{2} \lambda_2 \xi_2^2 + \frac{1}{2} \xi_1^2 \xi_2^2 + \frac{1}{4} (\xi_1^2 + \xi_2^2)^2 \quad (7)$$

3.1 Balanced attention parameter

Firstly let's observe the potential function under balanced attention parameter case when $\lambda_1 = \lambda_2 = 1.0$:



Contour of potential function, x axis corresponds to ξ_1 and y axis corresponds to ξ_2 , both in the range of (-1, +1)

Please note that there are 4 attractors with the value of -0.25 in the figure (B,C,D,E); 1 unstable fixed point (A), 4 saddle point (F,G,H,I). It fit the theory result exactly.

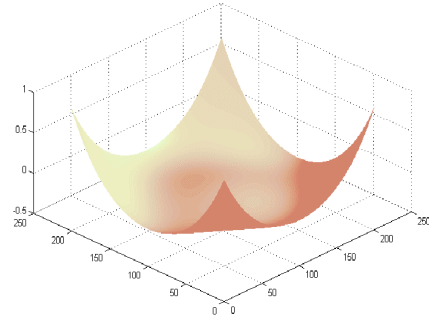
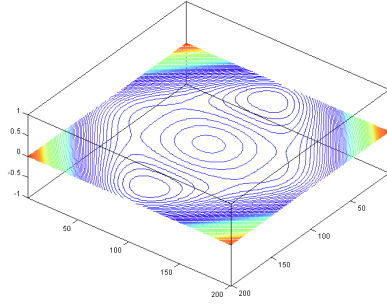
The identity of the two attractor's shape represents the meaning of "balanced" attention parameter

Potential function. There are 4 apparent attractors in this figure that are axial symmetrical respectively. The hill at the original point is an unstable equilibrium point. Along the diagonal direction we can find 4 saddle points.

Fig2: the potential function with balanced parameter

3.2 Unbalanced attention parameter 0.6 / 0.4

The potential function is shown as following when the attention parameter is unbalanced with $\lambda_1 = 0.6, \lambda_2 = 0.4$:



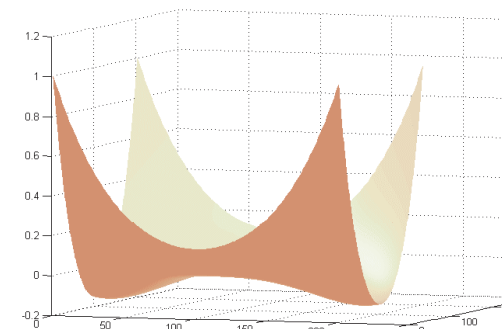
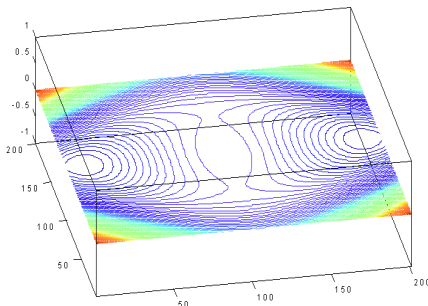
The attractive domain of pattern 1 is enlarged and the attractive domain of pattern 2 is reduced. It illuminates that attention parameter has great influence on the attractive domain.

Potential function. Note that both the width and the depth of the attractive domain of pattern 2 now is greatly decreased compared to that of balanced attention parameter.

Fig3: Potential function with unbalanced attention parameter 0.6 / 0.4

3.3 unbalanced attention parameter 0.8 / 0.2

The potential function is shown as following when the attention parameter is unbalanced with $\lambda_1 = 0.8, \lambda_2 = 0.2$



Note that the attractive domain of pattern 2 now is very small because the current difference between two attention parameters is quit large

Potential function. Please note that the well depth distance between two attractors now is quite large

Fig 4: Potantial function with unbalanced attention parameter 0.8 / 0.2

3.4 Attractive domain of attractors

Now we will compare the contour of potential function under these three cases to further observe attractive domain

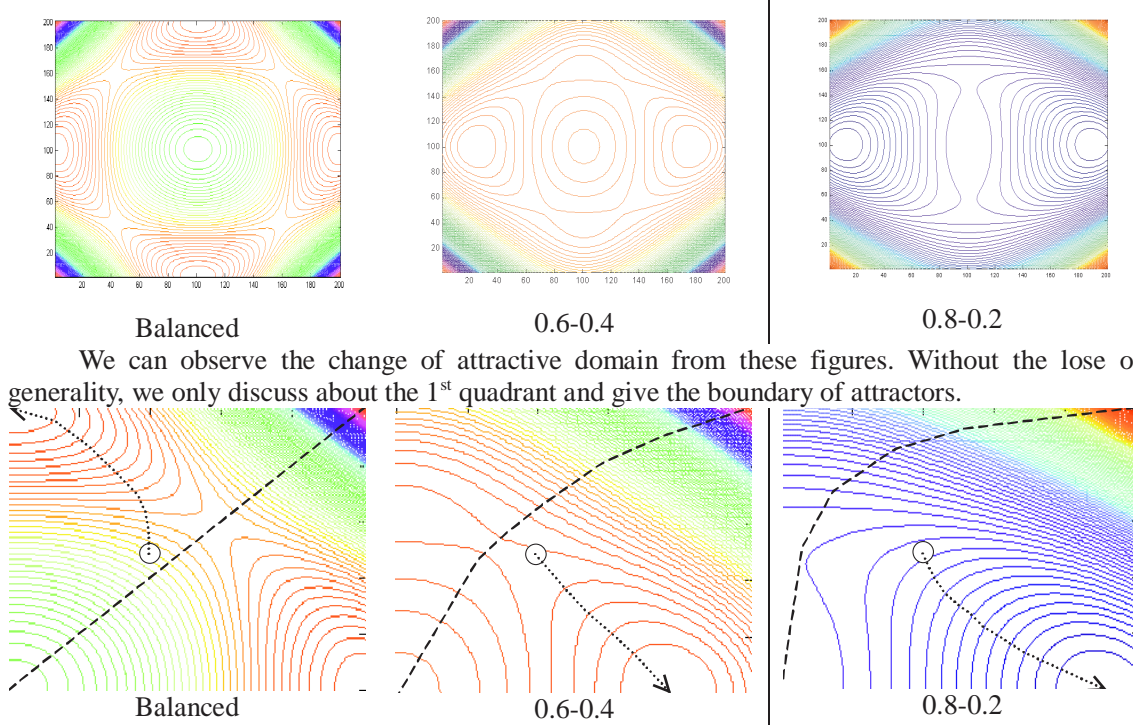


Fig5: the contour and attactive domain of the potantial function

The attractive domain of v_1 is increasing when λ_1 is increasing, and some patterns which will be attracted by v_2 under balanced parameter case, can be attracted by v_1 under the last two cases. Evolutionary locus of initial status (0.4, 0.5) is given for those three kinds of conditions.

4. Conclusions

These examples enable us to understand the principle of Quick Haken Network much better. The potential function of network and attractive domain of each attractor are fully determined for given attention parameters, and there is an analytic approximation for the division of attractive domains.

For balanced attention parameters, since attractive domains are entirely axial symmetry, pattern will finally “roll” to the attractive domain corresponding to the maximal order parameter.

For unbalanced attention parameters, although the attractive domains aren’t entirely axial symmetry, they can also be entirely determined and controlled, and the evolution path of pattern with given initial value is determined too.

The top-down nature of the potential function of synergetic neural network makes the distribution of attractors and attractive domains is very regular and easy to observe. The boundary of attractive domain in synergetic network is smooth. So we can easily determine the competition result based on the system initial status.

However, we have very strong requirement of that the attention parameter should be much greater than the initial value of order parameter in the unbalanced attention parameter case. The key

issue is how to depict the boundary of attractive domains precisely and analytically, which is possible for synergetic neural network. That will be our future work.

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