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STUDY OF DAMPING POWER IN INTERCONNECTED POWER SYSTEMS

by

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I. INTRODUCTION

The analysis of steady and transient stability with automatic control devices constitutes the most important tool for judging the system performance in the present day. Oscillations of power flow between synchronous machines have been known to be present not only in the transient state, but also in the steady state. Theoretically no such oscillations exist in the steady state, however power systems in the steady state cannot be adjusted continuously to meet new conditions as there are disturbances of small or large size.

The study of power system stability under transient conditions is a tedious task because the differential equations of motion describing even the simplest system are nonlinear. The general approach has been to obtain a time solution and to observe if the system is stable or not by inspecting the first and subsequent swings. The amplitude of these swings, whether increasing or decreasing, determines the magnitude of the sustained oscillation and the ability of the system to be stable when operating in the dynamic region.

It is the purpose of this dissertation to analyze the characteristics of the following factors under transient conditions:

1. Excitation system response
2. Synchronous machine and connected system characteristics
3. Governor response

These components are interrelated and the interrelation is quite complex. The effects of each component will be discussed independently, and then the combined results will be evaluated.

For the purpose of this study, the system shown in Fig. 1 is taken to be representative of the problem. It consists of a synchronous machine (S) driven by a turbine (T) which is controlled by a governor (G) to respond for any speed variation $\dot{\delta}$, and the generator is electrically connected to a load system. The differential equation defining the behavior of the synchronous generator subjected to a small power pulsation due to a sudden change in electrical output power of the machine is

$$M \frac{d^2\delta}{dt^2} + D \frac{d\delta}{dt} = P_{in}(\delta) - P_{out}(\delta) = P_a(\delta) \qquad (1)$$

where

M is the inertia of the generator which is assumed to be constant

D is the damping power coefficient of the system, described on page 18 and 23

P_{in} is the mechanical input power

P_{out} is the electrical output power

P_a is the accelerating power

δ is the angular rotor position

In the above equation it is assumed that the oscillations are very small so that $P_{out}(\delta)$ may be assumed to be a linear function of δ and not sin δ. In Fig. 1 it should be assumed that the controller feedback loop is open so that $P_{in}(\delta)$ is constant.

The solution of Equation 1 can be written as

$$\delta(t) = A\ e^{-\frac{DT}{2M} \pm \left[\frac{D^2}{4M^2} - \frac{P_a}{M}\right]^{\frac{1}{2}} t} \qquad (2)$$

where A is a constant depending on the initial conditions.

Since $\frac{P_a}{M}$ is greater than $\frac{D^2}{4M^2}$ due to the fact that the damping coefficient is usually less than the accelerating coefficient, then

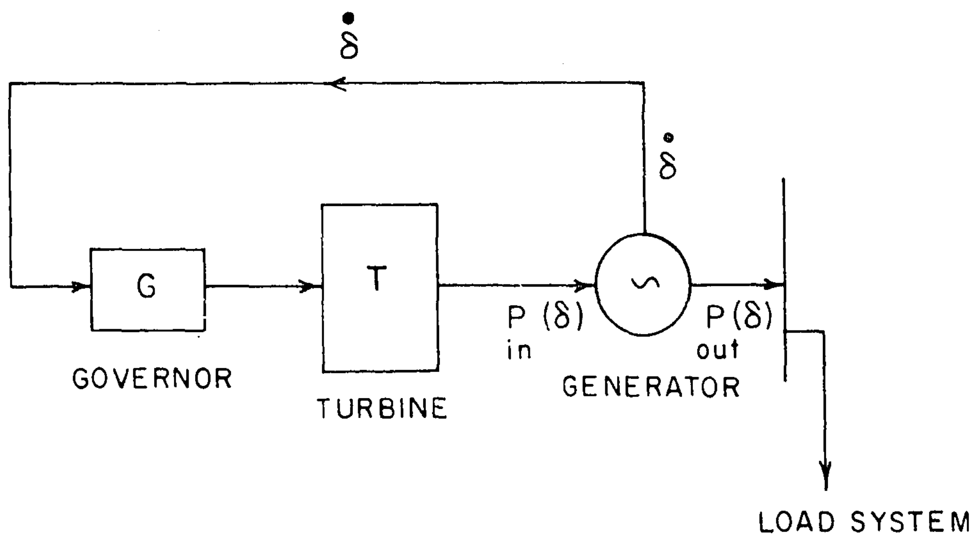

Fig. 1. System-line diagram

Equation 2 can be written as

$$\delta(t) = \delta_o \, e^{-\frac{D}{2M}t} \cos \left[\frac{P_a}{M} - \frac{D^2}{4M^2}\right]t \tag{3}$$

Equation 3 describes the motion of the system following a disturbance. It is seen that if D is equal to zero, the angular oscillation would simply continue at its initial amplitude, and also if D is negative, the amplitude of oscillation would build up with time, but in practical cases the damping coefficient is positive which will help in decaying the oscillation rapidly.

II. REVIEW OF LITERATURE

Stability oscillation in interconnected power systems has become more evident during the last fifty years, and during this period power systems have grown to such dimensions that special measures have had to be taken and special designs introduced in order to ensure a maximum of reliability. Very large blocks of powers are now transmitted over long distances, so it is becomming more likely for disturbances to happen and oscillations will become more significant. However, in order to prevent instability during a reasonable disturbance, a line capacity must be provided to permit sufficient transfer of synchronizing power over the tie-line. This immediately implies that ties between systems should be designed, not only for the economic normal interchange power, but also to handle the required synchronizing power. In other words, the question of stability limit is the major key in judging the system performance. Kimbark (1, 2), Crary (3,4), and Dahl (5) give excellent discussion of the various aspects of transient and steady state stability limits.

Since the problem of power system stability is to determine whether or not the various synchronous machines on the system will remain in synchronism with one another, the characteristics of these machines obviously play an important part in the problem, and many notable papers have been written on this subject.

In the period from 1923 to 1928 the solution for a three-phase short circuit of a cylindrical-rotor machine was accomplished by Bakku, Biermanns, Dreyfus, and Lyon (6) under these assumptions: (1) The total armature self inductance is constant, (2) the rotor has only one circuit,

(3) the saturation is negligible. Their work was the first step in evaluating the armature reactance of synchronous machines.

Doherty, R. E. and Nickle, C. A. (7) published a paper in 1928 for the solution of single phase short circuits for salient and non salient-pole machine. In their paper they assumed the self inductance is variable with respect to the rotor position, and the author believes that their brilliant work was a major advance in analyzing the machine characteristics during the transient period.

In 1929, Park, R. A. (8) published a paper presenting a generalization and extension of the work of Blondel, Dreyfus, Doherty and Nickle (7). He established a new and general method of calculating current and torque of a salient-pole machine in transient state. In 1933 (9), he published a paper for evaluating the damping torque coefficient of a rotor in transient state condition, he concluded in his paper that this damping torque is similar to the torque produced by an induction motor due to the difference in slip.

Wagner, C. E. (10) published a paper in 1931 for the damper winding in water wheel generators. He discussed the effects of damper windings upon both real and reactive components of the negative sequence impedance.

In 1937 (11) Concordia, C. published a paper for the analysis of non salient-pole machine in terms of direct and quachature quantities in the case when any balanced impedance is connected to the armature terminals, in 1960, (12) he published a paper which he considered as expressing the results of the previous paper in more convenient form. He concluded that the effect of the air-gap and rotor surface curvature is very small, and a large solid-rotor machine develops a relatively large damping torque, and the resistance of the machine should be taken into account for calculating the proper value of subtransient impedance. In 1950 (13) Concordia, C. published a paper for the analysis of synchronous machine damping and synchronizing torques, and the object of his work was to present the results of some calculations of damping and synchronizing torque coefficients which were calculated by Park (9) with fixed excitation systems. He concluded in his paper that for any fixed value of armature resistance of the machine with armatessier winding, the damping coefficient never decreases, and usually increases with decreasing external line reactance in the range of system parameters studied.

A great deal has been written about the effects of excitation response on stability. Before modern quick-operating circuit breakers and relays were developed, the transient stability limits of a system was low as compared to the steady state limit. As a result, studies were made in those days which lead to improving the power limits by high speed excitation response.

Doherty, R. E. (14) published a paper in 1928 on excitation systems, and their influence on short circuits and maximum power. He discussed the advantage and disadvantage of quick response excitation upon short circuit current and internal voltage of a synchronous machine. He concluded that fast excitation systems may increase the transient stability limit by 10%.

Bothwell, F. E. (15) wrote a paper in 1950 on the stability of voltage regulators. He discussed the theory of Liapounoff on the stability of dynamical systems under small displacement from equilibrium together with Roath-Hurwitz stability criterion. A graphical method for locating the stability boundary for general control elements with imperical given characteristic were found.

Governor systems are now being used for water wheel and steam generators which have transient speed regulation up to 20%, which indicates, if there is no time lag in the governor, a range of equivalent damping coefficient of about 5 as mentioned by Crary (4). Such damping torque could have certain effects on the system stability.

Concordia, C. and Kirchmayer, L. C. (16,17) published a paper to represent the results of a study of the performance of two interconnected steam and hydroelectric power generating areas as affected by frequency and tie-line controllers. The object was to determine theoretically the best values of controller gains for best overall system performance. They concluded that the tie-line gain is more effective than the frequency gain in reducing the oscillation.

In 1957 Concordia, Kirchmayer, and Szymanski (18) wrote a paper to report the effects of speed governor dead-band on the supplementary controller performance. They concluded that speed governor dead-band tends to produce continuous oscillations of the tie-line power at the tie-line natural period of about 2 seconds.

III. EXCITATION SYSTEM RESPONSE

The ability of an automatic voltage regulator to increase the stability limit has been recognized. It is assumed that the flux linkage is constant; this is equivalent to considering the quadrature-axis component E''_{qo} and the direct-axis component E''_{do} of the internal voltage of the machine do not change suddenly, and they have the same values directly after a disturbance takes place as they had before. Hence the initial values of these components are used in the subtransient-calculations. However, subsequently the internal voltage will change, the rapidity of variation depending primarily upon the resistance of the field circuit, and the manner in which the voltage regulator and exciter respond.

1. Vector Diagram of a Salient-Pole Machine Connected to Infinite-Bus. The effect of saliency in the subtransient period is small, due to the fact that the damper winding is more effective than the field winding in keeping the stator flux out of the rotor circuit at the instant of a disturbance because the damper winding has lower reactance than the field winding, and is closer to the stator surface. Therefore it is assumed that the direct-axis subtransient reactance X''_d is equal to the quadrature-axis subtransient reactance X''_q which means that the excitation voltage E'' is equal to the subtransient internal voltage E''_i. The direct-axis components of the internal voltage E_d and E''_d appear suddenly in the subtransient period due to the effect of the quadrature-axis component of the armature reaction, and that is why the quadrature-axis armature current is considered in that period.

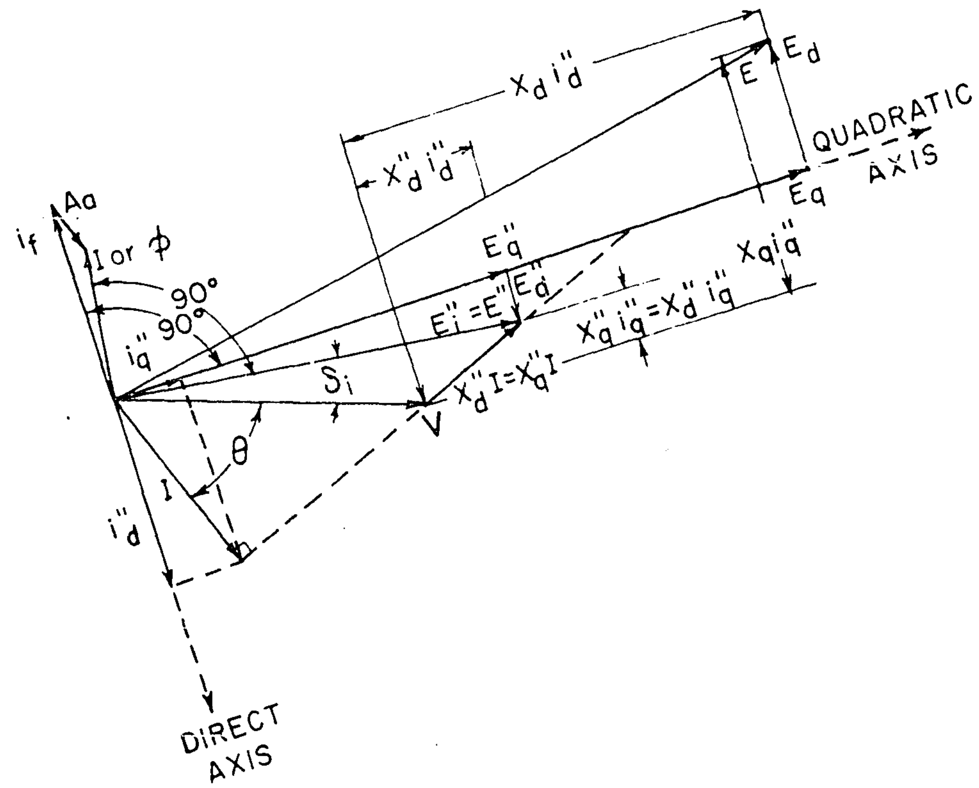

Fig. 2. Vector diagram of a salient-pole machine connected to infinite-bus in the subtransient period (resistance is neglected)

The differential equation expressing the rate of change of the quadrature-axis component of the internal voltage as given by Kimbark (2) and Crary (4) is

$$T''_{do} \frac{dE''_q}{dt} = E_{ex} - E_q \tag{4}$$

where T''_{do} is the direct-axis subtransient open circuit time constant, E_{ex} is the open circuit exciter armature voltage, and E_q is the voltage behind the direct-axis synchronous reactance which is proportional to the field current.

From Fig. 2

$$E_q = E''_q + (X_d - X''_d)\ i''_d \tag{5}$$

where X_d is the direct-axis synchronous reactance, i''_d the a-c subtransient direct-axis armature current.

The vector diagram shows the effect of the a-c component of the armature current on the internal voltage following a disturbance, but actually the d-c components of the armature current share the a-c components on this effect.

Therefore, Equation 5 can be reduced to

$$E_q = E''_q + (X_d - X''_d)\ I_{\epsilon_d} \tag{6}$$

where

$$I_{\epsilon_d} = [i_{a\text{-}c_d}{}^2 + i_{d\text{-}c_d}{}^2]^{\frac{1}{2}}$$

$$i_{d\text{-}c_d} = \sqrt{2}\ i''_{d\text{-}c_{do}}\ e^{-\frac{t}{T_a}} \tag{7}$$

T_a is the armature time constant, $i''_{a\text{-}c_{do}}$ is the initial value of subtransient a-c armature current.

From Equation 6, Equation 4 can be written as

$$T''_{do} \frac{dE''_q}{dt} + E''_q = E_{ex} - (X_d - X''_d) I_{\epsilon_d} \tag{8}$$

The initial value of E''_q is

$$E''_{q_o} = E_{qS} - (X_d - X''_d)\ I_{dS} \tag{9}$$

where E_{qS} is the steady state voltage behind synchronous reactance, I_{dS} is the steady state direct-axis component of the armature current.

Similarly, the rate of change of direct-axis component of internal voltage can be deduced from Equation 8, except there is no excitation on the quadrature-axis, then

$$T''_{q_o} \frac{dE''_q}{dt} - E''_d = -\ (X_q - X''_q)\ I_{t_q} \tag{10}$$

The initial value of E''_d is

$$E''_{do} = -\ (X_q - X''_q)\ I_{qS}$$

where

$$I_{\epsilon_q} = [i_{a\text{-}c_q}{}^2 + i_{d\text{-}c_q}{}^2]^{\frac{1}{2}}$$

$$i_{d\text{-}c_q} = \sqrt{2}\ i''_{a\text{-}c_{qo}}\ \epsilon^{-\frac{\epsilon}{T_a}} \tag{11}$$

I_{qS} is the steady state quadrature-axis component of the armature current, $i''_{a\text{-}c_{qo}}$ is the initial value of subtransient a-c quadrature-axis component of the armature current, T''_{qo} is the quadrature-axis open circuit time constant and X_q is the quadrature-axis synchronous reactance

2. Two-Machine System.

Consider the system network shown in Fig. 3, from the theorem of

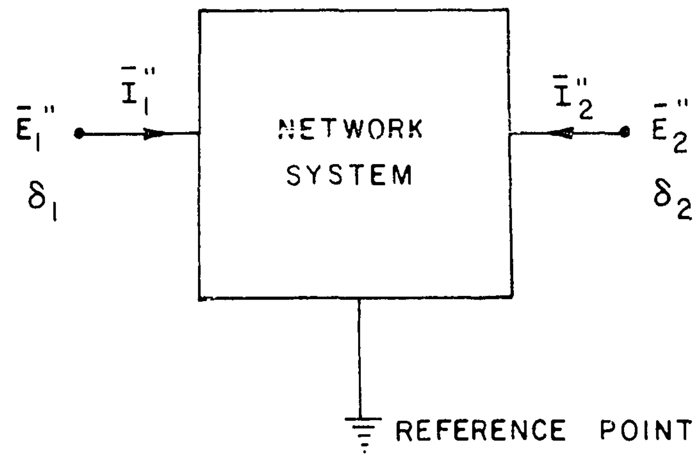

Fig. 3. The 2-part system network showing the direction of $\bar{I}''$ and $\bar{E}''$

Superposition as given by Crary (3).

$$\bar{I}_1'' = \frac{\bar{E}_1''}{\bar{Z}_{11}} - \frac{\bar{E}_2''}{\bar{Z}_{12}} = \frac{E_1'' \angle \delta_1}{Z_{11} \angle \theta_{11}} - \frac{E_2'' \angle \delta_2}{Z_{12} \angle \theta_{12}}$$

$$I_2'' = \frac{\bar{E}_2''}{\bar{Z}_{22}} - \frac{\bar{E}_1''}{\bar{Z}_{21}} = \frac{E_2'' \angle \delta_2}{Z_{22} \angle \theta_{21}} - \frac{E_1 \angle \delta_1}{Z_{21} \angle \theta_{21}} \tag{12}$$

The voltages $\bar{E}_1''$ and $\bar{E}_2''$ represent voltages behind subtransient reactances, in which case the subtransient reactances are assumed to be regarded as parts of the network. $\bar{Z}_{11}$, $\bar{Z}_{22}$ are the driving impedances of machines 1 and 2, and $\bar{Z}_{12}$ is the transfer impedance between the two machines.

The direct-axis subtransient component of the armature current is

$$i''_{a\text{-}c_{d_1}} = \frac{E_1''}{Z_{11}} \sin \theta_{11} - \frac{E_2''}{Z_{12}} \sin (\delta_1 - \delta_2 + \theta_{12})$$

$$i''_{a\text{-}c_{d_2}} = \frac{E_2''}{Z_{22}} \sin \theta_{22} - \frac{E_1}{Z_{21}} \sin (\delta_2 - \delta_1 + \theta_{21}) \tag{13}$$

Similarly the quadrature-axis subtransient component of the armature current is

$$i''_{a\text{-}c_{q_1}} = \frac{E_1''}{Z_{11}} \cos \theta_{11} - \frac{E_2''}{Z_{12}} \cos (\delta_1 - \delta_2 + \theta_{12})$$

$$i''_{a\text{-}c_{q_2}} = \frac{E_2''}{Z_{22}} \cos \theta_{22} - \frac{E_1''}{Z_{21}} \cos (\delta_2 - \delta_1 + \theta_{21}) \tag{14}$$

From Fig. 2

$$E'' = [E_q''^2 + E_d''^2]^{\frac{1}{2}} \tag{15}$$

Therefore from Equations 7, 8, 13, and 15, the rate of change of the quadrature-axis components of the internal voltages of the two machines is

$$T''_{do_1} \frac{dE''_{q_1}}{dt} + E''_{q_1} = E_{ex_1} - (X_d - X''_d)_1 \left\{ \left[\frac{(E''^2_{q_1} + E''^2_{d_1})^{\frac{1}{2}}}{Z_{11}} \sin \theta_{11} - \frac{(E''^2_{q_2} + E''^2_{d_2})^{\frac{1}{2}}}{Z_{12}} \sin (\delta_1 - \delta_2 + \theta_{12}) \right]^2 + [\sqrt{2}\, i''_{do_1} e^{-\frac{t}{T_{a_1}}}]^2 \right\}^{\frac{1}{2}} \tag{16}$$

$$T''_{do_2} \frac{dE''_{q_2}}{dt} + E''_{q_2} = E_{ex_2} - (X_d - X''_d)_2 \left\{ \left[\frac{(E''^2_{q_2} + E''^2_{d_2})^{\frac{1}{2}}}{Z_{22}} \sin \theta_{22} - \frac{E''^2_{q_1} + E''^2_{d_1})^{\frac{1}{2}}}{Z_{21}} \sin (\delta_2 - \delta_1 + \theta_{21}) \right]^2 + [\sqrt{2}\, i''_{d_{o2}} e^{-\frac{t}{T_{d2}}}]^2 \right\}^{\frac{1}{2}} \tag{17}$$

Similarly from Equations 10, 11, 14, and 15, the rate of change of the direct-axis components of the internal voltages is

$$T''_{q_{o1}} \frac{dE''_{d_1}}{dt} - Ed''_1 = - (X_q - X''_q)_1 \left\{ \left[\frac{(E''^2_{q_1} + E''^2_{d_1})^{\frac{1}{2}}}{Z_{11}} \cos \theta_{11} - \frac{(E''^2_{q_2} + E''^2_{d_2})^{\frac{1}{2}}}{Z_{12}} \cos (\delta_1 - \delta_2 + \theta_{12}) \right]^2 + [\sqrt{2}\, i''_{q_{o1}} e^{-\frac{t}{T_{a_1}}}]^2 \right\}^{\frac{1}{2}} \tag{18}$$

$$T''_{q_{o2}} \frac{dE''_{d_2}}{dt} - E''_{d_2} = - (X_q - X''_q)_2 \left\{ \left[\frac{(E''^2_{q_2} + E''^2_{d_2})^{\frac{1}{2}}}{Z_{22}} \cos \theta_{22} - \right.\right.$$

$$\frac{E''^2_{q_1} + E''^2_{d_1})^{\frac{1}{2}}}{Z_{21}} \cos (\delta_2 - \delta_1 + \theta_{21}) \Big]^2 + [\sqrt{2}\, i''_{q_{o2}} e^{-\frac{t}{T_{a_2}}}]^2 \Big\}^{\frac{1}{2}} \quad (19)$$

From Equation 1, the rate of change of the angular rotor positions of the two machines is

$$M_1 \frac{d^2\delta_1}{dt^2} + D_1 \frac{d\delta_1}{dt} + \frac{[E''^2_{q_1} + E''^2_{d_1}]}{Z_{11}} \cos \theta_{11} - \frac{[E''^2_{q_1} + E''^2_{d_1}]^{\frac{1}{2}}[E''^2_{q_2} + E''^2_{d_2}]^{\frac{1}{2}}}{Z_{12}}$$

$$\cos (\delta_1 - \delta_2 + \theta_{12}) = P_{in} (\delta_1, \delta_2) \quad (20)$$

$$M_2 \frac{d^2\delta_2}{dt^2} + D_2 \frac{d\delta_2}{dt} + \frac{[E''^2_{q_2} + E''_{d_2}]}{Z_{22}} \cos \theta_{22} - \frac{[E''^2_{q_1} + E''_{d_1}]^{\frac{1}{2}}[E''^2_{q_2} + E''^2_{d_2}]^{\frac{1}{2}}}{Z_{21}}$$

$$\cos (\delta_2 - \delta_1 + \theta_{21}) = P_{in} (\delta_2, \delta_1) \quad (21)$$

Therefore, the above six equations can be solved simulantaneously and time solutions of the angular rotor positions, and the internal voltage components can be obtained. This method can be extended for multi-machine systems with the same procedure used.

The effects of higher-than-normal armature currents during a disturbance are such as to increase saturation in the leakage paths as well as in the main paths. Increased saturation in the leakage paths decreases that portion of the total reactance due to the leakage flux. This can be equivalent to reactances lower than the unsaturated reactance, and as indicated by Concordia (H) the effective reactance appears as only from 0.6 to 0.8 of the unsaturated value.

IV. DAMPING POWER

A. Generator Damping

Since the generator rotor has two windings, namely the field and armotessier windings, each one develops an electrical power following any disturbance. In the following analysis each winding is considered separately in the absence of the other.

1. Field damping

During an oscillation the flux linkage of a machine is not constant, but it varies according to the machine characteristics, and the rate of change of the flux linkage produce an electrical damping power. In the derivation of this damping the following assumptions are considered:

1. No resistance in the armature circuit
2. Small slip
3. Damping action is caused by only one set of windings, i.e. the field winding.

From Equation 6

$$I_{\epsilon_d} = \frac{E_q - E''_q}{(X_q - X''_q)} \tag{22}$$

Similarily,

$$I_{\epsilon_q} = \frac{E_d + E''_d}{(X_q - X''_q)} \tag{23}$$

From Fig. 2, the electrical output power of a synchronous machine connected to an infinite-bus is

$$P_{output} = I_{\epsilon_d} V \sin \delta + I_{\epsilon_q} V \cos \delta \tag{24}$$

Substitute for I_{ϵ_d} and I_{ϵ_q} from Equation 22 and 23 in Equation 24, then

$$P_{output} = \frac{E_q - E''_q}{(X_d - X''_d)} V \sin \delta + \frac{E_d + E''_d}{(X_q - X''_q)} V \cos \delta . \qquad (25)$$

From Equations 4 and 10, then

$$E_q = E_{ex} - T''_{do} \frac{dE''_q}{dt} \qquad (26)$$

$$E_d = - T''_{qo} \frac{dE''_d}{dt} \quad , \text{[since } E''_d = (X_q - X''_q) I_{\epsilon_q} - E_d \text{ as shown in Fig. 2]} \qquad (27)$$

Therefore, from Equations 26 and 27, Equation 25 can be written as

$$P_{output} = E_{ex} - E''_q - T''_{do} \frac{dE''_q}{dt} \frac{V \sin \delta}{(X_d - X''_d)} + E''_d - T''_{qo} \frac{dE''_d}{dt} \frac{V \cos \delta}{(X_q - X''_q)} \qquad (28)$$

since $\delta = (\delta_o + \Delta\delta)$.

Then, $\sin \delta = \sin (\delta_o + \Delta\delta) = \sin \delta_o \cos \Delta S + \cos \delta_o \sin \Delta\delta$.

The oscillations are assumed very small, then $\cos \Delta\delta \simeq 1$, $\sin \Delta\delta \simeq \Delta\delta$.

Therefore

$$\sin \delta \simeq \sin \delta_o + \Delta\delta \cos \delta_o$$

$$\cos \delta \simeq \cos \delta_o - \Delta\delta \sin \delta_o \qquad (29)$$

From Equation 29, then P_{output} can be written as

$$P_{output} = \frac{V}{(X_d - X''_d)} \left[E_{ex} - E''_q - T''_{do} \frac{dE''_q}{dt} \right] [\sin \delta_o + \Delta\delta \cos \delta_o]$$

$$+ \frac{V}{(X_q - X''_q)} \left[E''_d - T''_{qo} \frac{dE''_d}{dt} \right] [\cos \delta_o - \Delta\delta \sin \delta_o]$$

$$= \frac{V \sin \delta_o}{(X_d - X''_d)} \left[E_{ex} - E''_q - T''_{do} \frac{dE''_q}{dt} \right] + \frac{V \Delta\delta \cos \delta_o}{(X_d - X''_d)} [E_{ex} - E''_q]$$

$$+ \frac{V \cos \delta_o}{(X_d - X''_d)} \quad [E''_d - T''_{do} \frac{dE''_d}{dt}] - \frac{V \Delta\delta \sin \delta_o}{(X_d - X''_d)} \quad E''_d$$

$$+ \frac{V \cos \delta_o}{(X_d - X''_d)} \left[- T''_{do} \frac{dE''_q}{dt} \right] \left(\frac{\Delta\delta}{\Delta t}\right) \Delta t - \frac{V \sin \delta_o}{(X_q - X''_q)} \left[- T''_{qo} \frac{dE''_d}{dt} \right] \left(\frac{\Delta\delta}{\Delta t}\right) \Delta t \quad (30)$$

By inspecting Equation 30, P_{output} can be reduced to

$$P_{output} = P(\delta_o) + P(\Delta\delta) + D_f \frac{\Delta\delta}{\Delta t} \quad (31)$$

As was mentioned before, the damping power is proportional to the slip $(\frac{\Delta\delta}{\Delta t})$ for small oscillations. Therefore D_f should be equal to the damping coefficient of the field winding, and by comparing Equation 30 with Equation 31, then

$$D_f = \frac{\Delta t \; T''_{qo}}{(X_q - X''_q)} \sin \delta_o \; V \; \dot{E}''_d - \frac{\Delta t \; T''_{do}}{(X_d - X''_d)} \cos \delta_o \; V \; \dot{E}''_q \quad (32)$$

It is clear from Equation 32, that D_f is flux linkage dependent which is effective in the first few cycles of disturbance due to the rapid decrease of the internal voltage.

2. Induction damping

Due to the rate of change of the rotor angle during an oscillation with respect to the armature flux, there will be induced current in the rotor. This induced current produces a damping power similar to the power developed by an induction motor. Park (9) derived this damping power under the following assumptions:

1. No resistance in the armature
2. No resistance in the field circuit
3. Small slip

4. Damping action is caused by damper winding only.

The induction damping coefficient as given by Park is

$$D_1 = V^2 \left[\frac{(X'_d - X''_d)}{X'^2_d} T''_{do} \sin^2 \delta + \frac{(X'_q - X''_q)}{X'^2_q} T''_{qo} \cos^2 \delta \right] \qquad (33)$$

where X'_d and X'_q are the direct and the quadrature-axis transient reactances.

Equations 33 and 34 are derived for one machine connected to an infinite-bus whose voltage is V.

For a two machine system, consider Fig. 4. In calculating the terminal voltage of machine 1, i.e. V_1, the other machine can be represented by its internal voltage E_2 in series with the reactance X_2, then

$$\bar{V}_1 = \bar{E}_2 \left[\frac{\bar{Z}_3}{\bar{Z}_3 + \bar{Z}_2 + JX_2} \right] + \left[\bar{Z}_1 + \frac{(\bar{Z}_2 + JX_2)}{(\bar{Z}_2 + \bar{Z}_3 + JX_2)} \right] \bar{i}_1 \qquad (34)$$

Similarly

$$\bar{V}_2 = \bar{E}_1 \left[\frac{\bar{Z}_3}{\bar{Z}_3 + \bar{Z}_1 + JX_1} \right] + \left[\bar{Z}_2 + \frac{(\bar{Z}_1 + JX_1)\, \bar{Z}_3}{(\bar{Z}_1 + \bar{Z}_3 + JX_1)} \right] \bar{i}_2 \qquad (35)$$

It is seen from the above two equations the the damping coefficient decreases by increasing the external reactances of the systems.

B. Load Damping

The input power to the system which is equal to the positive-sequence output power of the machine is the algebraic sum of the damping power and the synchronous power. Therefore from Equations 32 and 33 the input power to the system is

$$P_{\text{in system}} = P_S(\delta) + (D_f + D_I) \frac{d\delta}{dt} \qquad (36)$$

where $P_S(\delta)$ is the synchronizing power.

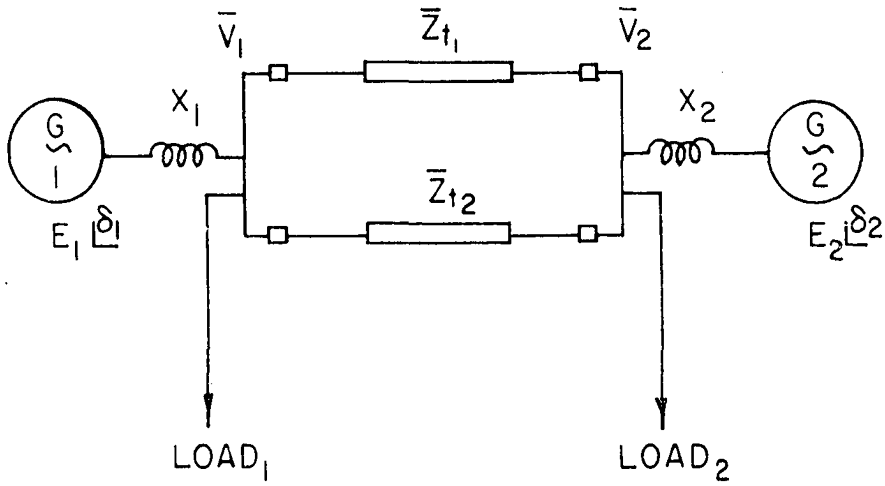

Fig. 4a. Two-machine system

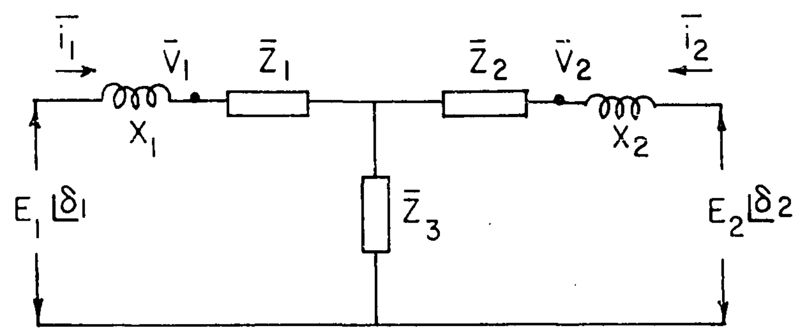

Fig. 4b. Equivalent circuit of two-machine system

Following any disturbance, especially a fault condition, the output power of synchronous generators decreases, and due to the discontinuity between the mechanical input and electrical output, then the generator rotor will speed up. Accordingly there is applied to the system a component of power having higher speed than normal due to the power generated by the generator which has increased its speed, and this is obviously clear from Equation 36. This component of power at slightly higher speed develops additional electrical power due to the characteristics of the connected system. This effect is reflected to the generator and produces damping in the generator.

Differentiating Equation 36 with respect to the slip $(\frac{d\delta}{dt})$, then

$$\frac{dP_{\text{in system}}}{dS} = D_f + D_I \tag{37}$$

Commercial loads in general are composite loads; they consist of lighting, synchronous motors, and induction motors, and each one has certain characteristic performance.

Therefore the rate of change of input power to the system with respect to the slip must be written as

$$\frac{dP_{\text{in system}}}{dS} = C\ (D_f + D_I) \tag{38}$$

where C is a constant depending on the load characteristic.

Therefore the additional electrical input power to the system which can be translated as the reflecting generator damping is

$$P_{D_{\text{Load}}} = C\ [D_f + D_I]\ \frac{d\delta}{dt} \tag{39}$$

V. GOVERNOR AND CONTROLLER

A. Tie-Line and Load Frequency Control

The main object of a tie-line and load frequency controller is to regulate the output of each area to absorb its own disturbances. To illustrate this point consider the system shown in Fig. 5.

The rate of change of the controller signal as given by Kirchmayer and Concordia (16) is

$$\frac{d}{dt}\,\Delta p_1' = K_{\epsilon_1}\,\Delta T_{12}\,(\delta_1 - \delta_2) + K_{f_1}\,\frac{d\delta_1}{dt} \tag{40}$$

where

$\Delta p_1'$ is power correction initiated by controller number 1

K_{ϵ_1} is the tie-line controller gain

K_{f_1} is the frequency controller gain

ΔT_{12} is the tie-line power deviation

Similarly for controller of area 2

$$\frac{d}{dt}\,\Delta p_2' = K_{t_2}\,\Delta T_{21}\,(\delta_2 - \delta_1) + K_{f_2}\,\frac{d\delta_2}{dt} \tag{41}$$

If $\Delta p'$ is constant, then $\frac{d}{dt}\,\Delta p'$ is equal to zero.

This means that the controller will not operate and reach steady state condition. Suppose there is a disturbance in area 1 due to sudden change in load 1, therefore $\frac{d}{dt}\,p_2'$ should be equal to zero, i.e. controller 2 will not operate, and the reverse is true if there is a disturbance in area 2.

For three-phase short circuits in the interconnected system, the two machines will be disturbed depending on the power flow, and the two controllers will operate.

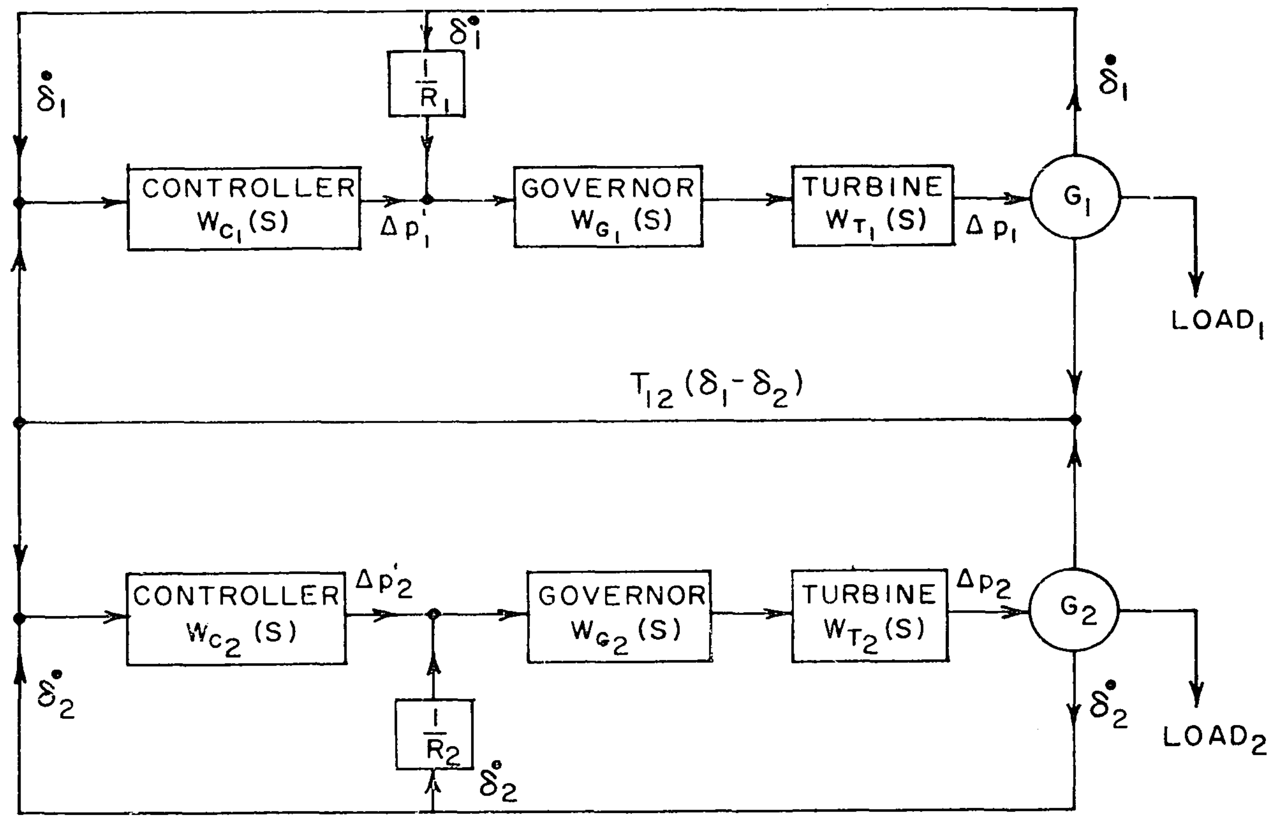

Fig. 5. Block diagram of interconnected system with governed supplementary controllers

The tie-line power deviation can be written as

$$\Delta T_{12} = \frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \sin (\Delta\delta_{12} - \alpha_{12})$$

where $\alpha_{12} = 90^{\circ} - \theta_{12}$

$$= \frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \left[\sin \Delta\delta_{12} \cos \alpha_{12} - \cos \Delta\delta_{12} \sin \alpha_{12} \right]$$

$$\simeq \left[\frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \cos \alpha_{12} \right] \Delta\delta_{12} - \frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \sin \alpha_{12} \qquad (42)$$

Therefore Equation 40 can be written as

$$\frac{d}{dt} \Delta p'_1 = K_{t_1} \left\{ \left[\frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \cos \alpha_{12} \right] \Delta\delta_{12} - \frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \sin \alpha_{12} \right\}$$

$$+ K f_1 \frac{d\delta_1}{dt} \qquad (43)$$

The output controller signal $\Delta p'_1$ is

$$\Delta p'_1 = \int_{t_{K_1}}^{t} K_{t_1} \left\{ \left[\frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \cos \alpha_{12} \right] \Delta\delta_{12} - \frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \sin \alpha_{12} \right\} dt$$

$$+ \int_{t_{K_1}}^{t} K_{f_1} \left(\frac{d\delta_1}{dt}\right) dt \qquad (44)$$

It is assumed that the oscillation is very small, i.e., the angle δ and the slip $\frac{d\delta}{dt}$ are slowing varying with time.

Therefore Equation 44 is reduced to

$$\Delta p'_1 = \left[K_{t_1} \frac{\Delta E''_1 \, \Delta E''_2}{Z_{12}} \cos \alpha_{12} \right] \Delta\delta_{12} \int_{t_{K_1}}^{t} dt - \frac{K_{t_1} \Delta E''_1 \, \Delta E''_2}{Z_{12}}$$

$$\sin \alpha_{12} \int_{t_{K_1}}^{t} dt + K_{f_1} \left(\frac{d\delta_1}{dt}\right) \int_{t_{K_1}}^{t} dt = K_{t_1}\left[\frac{\Delta E_1'' \Delta E_2''}{Z_{12}} \cos \alpha_{12}\right] \Delta\delta_{12} (t-$$

$$t_{K_1}) + K_{f_1} \left(\frac{d\delta_1}{dt}\right) (t - t_{K_1}) - K_{t_1} \frac{\Delta E_1'' \Delta E_2''}{Z_{12}} \sin \alpha_{12} \quad (t-t_{K_1}) \qquad (45)$$

where t_K is the controller time lag, and is equal to 0.26 second approximately. It is seen from the above equation that $\Delta p_1'$ is negative if t is less than t_K which means that the oscillation increases during the first few cycles of disturbance.

B. Governor Response

The input signal to the governor is

$$\Delta p_1 (t) = \frac{1}{R_1} \frac{d\delta_1}{dt} + \Delta p_1' (t) \qquad (46)$$

where R_1 is the steady state regulation of machine 1.

By convolution theorem, the output signal of the governor can be written as

$$\Delta p_{G_{o_1}} (t) = \int_{t_{d_1}}^{t} \Delta p_1 (\tau) W_{G_1} (t - \tau) d\tau \, . \qquad (47)$$

Substitute for $\Delta p_1 (\tau)$ from Equation 45 and 46 in the above equation, then

$$\Delta p_{G_{o_1}} (t) = \left[\frac{K_{t_1} \Delta E_1'' \Delta E_2''}{Z_{12}} \cos \alpha_{12}\right] \Delta\delta_{12} \int_{t_{d_1}}^{t} (\tau - t_{K_1}) W_{G_1}(t-\tau) d\tau$$

$$- \left[K_{t_1} \frac{\Delta E_1'' \Delta E_2''}{Z_{12}} \sin \alpha_{12}\right] \int_{t_{d_1}}^{\epsilon} (\tau - \epsilon_{K_1}) W_{G_1} (\epsilon-\tau) d\tau$$

$$+ \left[K_{f_1} \frac{d\delta_1}{dt} \right] \int_{\epsilon_{d_1}}^{t} (\tau - t_{K_1}) \, W_{G_1} \, (\epsilon - \tau) \, d\tau + \left[\frac{1}{R_1} \frac{d\delta_1}{dt} \right] \int_{t_{d_1}}^{t} W_{G_1}(\tau) \, d\tau. \tag{48}$$

where

$W_G(t)$ is the transfer function in the time-domain of the governor

t_d is the dead band time of the governor

If the time lag of the turbine is neglected compared to the governor and controller time lags, then the power output of the turbine is

$$P_{T_{o_1}} = P_{S_1} - \Delta p_{G_{o_1}} \tag{49}$$

where p_{S_1} is the steady state output power of the turbine 1

Δp_{G_o} is the output signal of the governor as given by Equation 48.

Differentiating Equation 49 with respect to the slip S_1 $\left(\frac{d\delta_1}{dt}\right)$

$$\frac{d}{dS_1} p_{T_{o1}} = - \frac{d}{dS_1} \Delta p_{G_o}$$

$$= - K_{f_1} \int_{t_{d_1}}^{t} (\tau - t_{K_1}) W_{G_1}(t-\tau) d\tau - \frac{1}{R_1} \int_{t_{d_1}}^{t} W_{G_1}(\tau) \, d\tau \tag{50}$$

Equation 50 gives the rate of change of the output signal of the turbine with respect to the slip, and due to the turbine characteristic the infinitesmal change in the output signal which can be expressed as the prime mover damping power coefficient is

$$D_{p_1} = - C_{p_1} \left[K_{f_1} \int_{t_{d_1}}^{t} (\tau - t_{K_1}) W_{G_1}(t-\tau) d\tau + \frac{1}{R_1} \int_{t_{d_1}}^{t} W_{G_1}(\tau) \, d\tau \right] \tag{51}$$

where C_{p_1} is the prime-mover characteristic constant.

It is seen that the damping power of the prime-mover is negative which means that the output power of the turbine decreases which improves the stability of the system.

Similarly

$$D_{p_2} = - C_{p_2} \left[K_{f_2} \int_{td_1}^{t} (\tau - t_{K_2}) W_{G_2}(t-\tau) d\tau + \frac{1}{R_2} \int_{t_{d_2}}^{t} W_{G_2}(\tau) d\tau \right] \quad (52)$$

By making the approximation that $\Delta\delta_{12} \simeq \Delta t \dot{\delta}_{12}$

and $\Delta E_1'' \simeq \Delta t \dot{E}_1''$ and $\Delta E_2'' \simeq \Delta t \dot{E}_2''$.

Then from Equations 32, 33, 39, 48, and 51, the rate of change of the angular rotor position of machine 1 is

$$M_1 \frac{d^2\delta_1}{dt^2} + (1 + c_1) \left[\frac{V_1 \dot{E}_{d_1}'' \sin \delta_{o_1}}{(X_q - X_q'')_1} \Delta t \, T_{q_{o_1}}'' - \frac{V_1 \dot{E}_{q_1}'' \cos \delta_{o_1}}{(X_d - X_d'')_1} \Delta t T_{do_1}'' \right.$$

$$\left. + \frac{V_1^2 (X_d' - X_d'')_1 \, T_{d_{o_1}}''}{X_{d_1}^{12}} \sin^2 \delta_1 + \frac{V_1^2 (X_q' - X_q'')_1}{X_{q_1}^{12}} T_{q_{o_1}}'' \cos^2 \delta_1 \right] \frac{d\delta_1}{dt}$$

$$+ C_{p_1} \left[K_{f_1} \int_{t_{d_1}}^{t} (\tau - t_{K_1}) W_{G_1}(t-\tau) d\tau + \frac{1}{R_1} \int_{t_{d_1}}^{t} W_{G_1}(\tau) d\tau \right] \frac{d\delta_1}{dt}$$

$$= p_{S_1} - \underset{\text{output}_1}{p} (\delta_1, \delta_2) - \left\{ \left[\frac{K_{t_1} \dot{E}_1'' \dot{E}_2''}{Z_{12}} (\Delta t)^3 \cos \alpha_{12} \right] \int_{t_{d_1}}^{t} (\tau - t_{K_1}) W_{G_1}(t- \right.$$

$$\left. \tau) \, d\tau \right\} \frac{d\delta_{12}}{dt} - (-) \left[\frac{K_{t_1} \dot{E}_1'' \dot{E}_2''}{Z_{12}} (\Delta t)^2 \sin \alpha_{12} \right] \int_{t_{d_1}}^{t} (\tau - t_{K_1}) W_{G_1}(t-\tau) d\tau$$

$$- \left[K_{f_1} \int_{t_{d_1}}^{t} (\tau - t_{K_1}) W_{G_1}(t-\tau) d\tau + \frac{1}{R_1} \int_{t_{d_1}}^{t} W_{G_1}(\tau)\, d\tau \right] \frac{d\delta_1}{dt} \qquad (53)$$

where p_{S_1} is the steady state mechanical input to the machine 1 $p(\delta_1, \delta_2)_{\text{output}_1}$ is the synchronizing power of machine 1, and as given by Crary (4) is equal to $\frac{E_1''^2}{Z_{11}} \sin \alpha_{11} + \frac{E_1'' E_2''}{Z_{12}} \sin (\delta_1 - \delta_2 - \alpha_{12})$.

Similarly

$$M_2 \frac{d^2\delta_2}{dt^2} + (1+c_2) \left[\frac{V_2 \dot{E}''_{d_2} \sin \delta_{o_2}}{(X_q - X_q'')_2} \Delta t\, T''_{q_{o_2}} - \frac{V_2 \dot{E}''_{q_2} \cos \delta_{o_2}}{(X_d - X_d'')_2} \Delta t\, T''_{d_{o_2}} \right.$$

$$\left. + \frac{V_2^2 (X_d' - X_d'')_2}{X^{12}_{d_2}} T''_{d_{o_2}} \sin^2 \delta_2 + \frac{V_2^2 (X_q' - X_q'') T''_{qo_2}}{X^{12}_{q_2}} \cos^2 \delta_2 \right] \frac{d\delta_2}{dt}$$

$$+ c_{p_2} \left[K_{f_2} \int_{t_{d_2}}^{t} (\tau - t_{K_2}) W_{G_2}(t-\tau) d\tau + \frac{1}{R_2} \int_{t_{d_2}}^{t} W_{G_2}(\tau)\, d\tau \right] \frac{d\delta_2}{dt}$$

$$= p_{S_2} - p(\delta_1, \delta_2)_{\text{output}_2} - \left\{ \left[\frac{-K_{t_2} \dot{E}_2'' \dot{E}_1''}{Z_{21}} (\Delta t)^3 \cos \alpha_{12} \right] \int_{t_{d_2}}^{t} (\tau - t_{K_2}) W_{G_2}(t-\tau) d\tau \right\} \frac{d\delta_{21}}{dt}$$

$$-(-) \left[\frac{K_{t_2} \dot{E}_2'' \dot{E}_1''}{Z_{21}} (\Delta t)^2 \sin \alpha_{12} \right] \int_{t_{d_2}}^{t} (\tau - t_{K_2}) W_{G_2}(t-\tau) d\tau$$

$$- \left[K_{f_2} \int_{t_{d_2}}^{t} (\tau - t_{K_2}) W_{G_2}(t-\tau) d\tau + \frac{1}{R_2} \int_{t_{d_2}}^{t} W_{G_2}(\tau)\, d\tau \right] \frac{d\delta_1}{dt} \qquad (54)$$

where p_{s_2} is the steady state mechanical input to the machine 2 $p(\delta_1,\delta_2)_{\text{output}_2}$ is the synchronizing power of machine 2, and as given by Crary (4) is equal to $\frac{E''^2}{Z_{21}} \sin \alpha_{22} + \frac{E''_2 E''_1}{Z_{21}} \sin (\delta_2 - \delta_1 + \alpha_{21})$.

VI. ILLUSTRATIVE PROBLEM

The following data are from Westinghouse.

1. 3938 K.V.A., 480 volt, three-phase, 60 cycles, 240 rpm, 80% power factor, salient-pole machine has the following values:

$X_d = 131\%$	$X'_d = 31\%$	$X''_d = 24\%$
$X_q = 71.5\%$	$X'_q = 71.5\%$	$X''_q = 21\%$
$T'_{d_o} = 2.5$ secs	$T'_d = 0.446$ secs	$T_a = 0.0314$ secs
$T''_{d_o} = 0.0045$ secs	$T''_d = 0.035$ secs	

$M = 2.5 \times 10^{-4}$ unit power sec^2 per electrical degree.

2. 40 KW, 125 volt, 1150 r.p.m., separate excited exciter has the following values:

Four main poles, two commutating poles

475 turns/pole (main pole)

Field poles all in series

Total field resistance 3.94 ohms at 75 C^o

Field leakage inductance 0.06 hennies

Nominal field inductance 0.440 hennies

Armature resistance 0.0091 ohm at 75 C^o

Armature inductance 0.000638 hennies

Pilot exciter voltage 62 volt (battery)

The exciter has 126 bars, one turn per single coil, four circuits, three single coils per coil, 6 conductors per slot and 42 slots

Steady state exciter current 8.4 amp.

Steady state exciter voltage 3.15 volt

The system shown in Fig. 4a illustrates the problem.

The generators G_1 and G_2 have the same rating as indicated above. Load in area 1 takes half of the generating power from G_1, so the power flows over the double transmission lines from G_1. Load in area 2 takes one and half of the total generated power of G_1 and G_2 minus the losses in the transmission lines.

$Z_{Ld_1} = 2 \angle 36.9^\circ$ p.u, and $Z_{Ld_2} = 0.601 \angle 30.1^\circ$ p.u which are approximated to be constant, due to the fact that most power systems have feeder voltage regulators which hold the voltages at the individual loads nearly constant.

$$Z_{t_1} = Z_{t_2} = 0.08 + J0.60 \quad \text{p.u}$$

$$K_{f_1} = K_{f_2} = 0.002 \text{ p.u per second}$$

$$K_{t_1} = K_{t_2} = 0.02 \text{ p.u per second}$$

$$R_1 = R_2 = 0.17 \text{ p.u}$$

$$t_{d_1} = t_{d_2} = 0.028 \text{ second}; \quad t_{K_1} = t_{K_2} \simeq 0.26 \text{ second}$$

$$W_{G_1}(s) = W_{G_2}(s) = \frac{1}{TS + 1}$$

where T is the time lag of the governor $\simeq$ 0.50 second.

Initial conditions:

$$E_{qd_1} = v_1 + Jx_{q_1} I_1$$

$$= 1 \angle 0 + 0.715 \angle 90^\circ \times 1 \angle -36.9^\circ$$

$$= 1.54 \angle 22.8^\circ \quad \text{p.u}$$

$$\delta_{1_0} = 22.8^\circ$$

$$E_{qd_2} = V_2 + JX_{q2}I_2$$
$$= 0.903 \angle{-6.8^\circ} + 0.715 \angle{90^\circ} \times 1 \angle{-36.9^\circ}$$
$$= 1.400 \angle{18.7^\circ} \quad \text{p.u}$$
$$\delta_{20} = 18.7^\circ$$

$$I_{dS_1} = I_1 \sin(\delta_{10} + \theta_1) = 1 \sin(22.8^\circ + 36.9^\circ)$$
$$= 0.855 \text{ p.u}$$

$$I_{qS_1} = I_1 \cos(\delta_{10} + \theta_1) = 0.517 \text{ p.u}$$
$$E_{qS_1} = E_{qd_1} + (X_{d1} - X_{q1}) I_{dS_1}$$
$$= 1.54 + 0.855 (1.31-0.715) = 2.04 \text{ p.u}$$

$$I_{dS_2} = I_2 \sin(\delta_{20} + \theta_2)$$
$$= 1 \sin(18.7^\circ + 36.9^\circ) = 0.833 \text{ p.u}$$

$$I_{qS_2} = I_2 \cos(\delta_{20} + \theta_2) = 0.555 \text{ p.u}$$
$$E_{qS_2} = E_{qd_2} + (X_{d_2} - X_{q_2}) I_{dS_2}$$
$$= 1.400 + 0.833 (1.31-0.715) = 1.895 \text{ p.u}$$

$$E''_{i_1} = V_1 + JX''_{d_1} I_1$$
$$= 1.0 + 0.24 \angle{90^\circ} \times 1 \angle{-36.9^\circ}$$
$$= 1.15 \angle{9.8^\circ} \text{ p.u} \; ; \quad \delta_{i_{10}} = 9.8^\circ$$

$$E''_{q_{10}} = E_{qS_1} - (X_{d_1} - X''_{d_1}) I_{dS_1}$$
$$= 2.04 - 0.855 (1.31-0.24) = 1.11 \text{ p.u}$$

$$E''_{d_{10}} = -0.517 (0.715-0.24) = -0.245 \text{ p.u}$$

$$E''_{i_2} = V_2 + J X''_{d_2} I_2$$
$$= 0.903 \angle{-6.8^\circ} + 0.24 \angle{90^\circ} \times 1 \angle{-36.9^\circ}$$
$$= 1.04 \angle{7.5^\circ} \text{ p.u}; \quad \delta_{i_{20}} = 7.5^\circ$$

$E''_{q_{20}} = 1.895 - 0.833\ (1.31-0.24) = 1.000$ p.u

$E''_{d_{20}} = -0.553\ (0.715-0.24) = -0.265$ p.u

$\bar{Z}_{11} = 0.437\ \angle 83.3^\circ$, $\bar{Z}_{22} = 0.410\ \angle 76^\circ$, and $\bar{Z}_{12} = \bar{Z}_{21} = 3.31\ \angle 160^\circ$ p.u

It is assumed that a 3-ϕ short circuit occurs at the middle of one of the transmission lines in Fig. 4a. Subtransient period is assumed to be 0.001 second.

$\bar{Z}_{11} = 0.485\ \angle 84.9^\circ$, $\bar{Z}_{22} = 0.450\ \angle 76.3^\circ$, and $\bar{Z}_{12} = \bar{Z}_{21} = 3.90\angle 113^\circ$ p.u

in the transient period.

It is assumed that the fault is cleared at 0.15 second, following disconnection of the faulted line.

$\bar{Z}_{11} = 0.862\angle 72.5^\circ$, $\bar{Z}_{12} = 0.607\angle 60^\circ$, $\bar{Z}_{12} = \bar{Z}_{21} = 1.55\angle 100$ p.u

Exciter circuit:

The differential equation expressing the field current of separately excited exciters

$$L\frac{di}{dt} + Ri = E \qquad \text{(a)}$$

Pilot exciter voltage E is constant

The solution of Equation a is

$$i(t) = (i_S - \frac{E}{R})\ e^{-\frac{R}{L}t} + \frac{E}{R} \qquad \text{(b)}$$

The differential equation of voltage around the field circuit is

$$c\frac{de}{dt} + Ri\ (t) = E \qquad \text{(c)}$$

By substituting for i(t) in Equation c, then

$$e(t) = \frac{L}{c}\ (\frac{E}{R} - i_s)\ (1 - e^{-\frac{R}{L}t}) + e_o \qquad \text{(d)}$$

where

$$c = \frac{\sigma N}{K}$$

σ = constant coefficient of dispersion and it is equal to 1.15

$$K = \frac{Z\eta P}{60a} = \frac{6 \times 42 \times 1150 \times 4}{60 \times 4} = 4830$$

$$c = \frac{\sigma N}{K} = \frac{1.15 \times 475}{4830} = 0.1127 \text{ second}$$

Then from Equation d

$$e_1(t) = E_{ex_1}(t) = \frac{0.440}{0.1127} \left(\frac{62}{3.94} - 8.4\right)\left(1 - e^{-\frac{3.94}{0.440} t}\right) + 31.5$$

$$= 1.97\ (1 - e^{-8.89t}) + 2.04 \quad \text{p.u}$$

$$e_2\ (t) = E_{ex_2}(t) = 1.804\ (1-e^{-889t}) + 1.895 \quad \text{p.u}$$

A computer program which utilizes the Runge-Kutta method was used to obtain the time solution of Equations 16, 17, 18, 19, 53, and 54. The time solution is obtained in each period by using different parameters in the specified period. The results are tabulated in Tables 1 and 2. Additional run is made by setting K_f and K_t equal to zero, and t equal to 0.15 second to see if it is possible to neglect the controller signal. The additional results are tabulated in Table 3.

Table 1. Time solution of slip, angular rotor position, and internal voltage with damping power and governor responses

Time second	$\dot{\delta}_1$ %	$\dot{\delta}_2$ %	$\dot{\delta}_1-\dot{\delta}_2$ %	δ_1 Elec deg	δ_2 Elec deg	$\delta_1-\delta_2$ Elec deg	E''_{q_1}, E'_{q_1} p.u	E''_{q_2}, E'_{q_2} p.u	E''_{d_1} p.u	E''_{d_2} p.u
0	0	0	0	9.8000	7.500	2.300	1.1100	1.000	-0.2450	-0.2650
0.0001	1058×10^{-6}	1150×10^{-6}	-92×10^{-6}	9.8020	7.504	2.298	1.0637	0.9444	-0.2591	-0.2845
0.0002	2272×10^{-6}	1826×10^{-6}	$+391\times10^{-6}$	9.8072	7.5060	2.3012	1.01834	0.89133	-0.2730	-0.3043
0.0003	3427×10^{-6}	2226×10^{-6}	$+120\times10^{-5}$	9.8100	7.507	2.3030	0.9784	0.84055	-0.28831	-0.3245
0.0004	4692×10^{-6}	2530×10^{-6}	$+216\times10^{-5}$	9.8160	7.510	2.3060	0.9301	0.7919	-0.3033	-0.34508
0.0005	598×10^{-5}	280×10^{-5}	$+317\times10^{-5}$	9.8350	7.519	2.3140	0.8878	0.7452	-0.3187	-0.3660
0.0006	736×10^{-5}	308×10^{-5}	$+428\times10^{-5}$	9.8500	7.428	2.3220	0.8453	0.70031	-0.3438	-0.3874
0.0007	874×10^{-5}	335×10^{-5}	$+539\times10^{-5}$	9.8710	7.540	2.3310	0.8041	0.6570	-0.3504	-0.4092
0.0008	1071×10^{-5}	364×10^{-5}	$+707\times10^{-5}$	9.893	7.549	2.344	0.7636	0.6153	-0.3667	-0.4134
0.0009	1177×10^{-5}	386×10^{-5}	$+791\times10^{-5}$	9.907	7.559	2.348	0.7238	0.5750	-0.3835	-0.4541
0.001	1334×10^{-5}	409×10^{-5}	$+925\times10^{-5}$	10.001	7.602	2.399	0.6847	0.5399	-0.4006	-0.4773
0.005	70.8×10^{-3}	57.5×10^{-3}	$+13.3\times10^{-3}$	10.240	7.640	2.600	0.6815	0.5359	-	-
0.01	128.8×10^{-3}	110.4×10^{-3}	$+18.4\times10^{-3}$	10.370	7.710	2.550	0.6780	0.5300	-	-
0.015	186.3×10^{-3}	165.6×10^{-3}	$+20.7\times10^{-3}$	10.659	7.895	2.764	0.6750	0.5254	-	-
0.02	243.8×10^{-3}	220.8×10^{-3}	$+23\times10^{-3}$	11.012	8.181	2.831	0.6714	0.5220	-	-
0.025	295×10^{-3}	270×10^{-3}	$+25\times10^{-3}$	11.710	8.700	3.010	0.6686	0.5195	-	-
0.03	348.8×10^{-3}	321.2×10^{-3}	$+27.6\times10^{-3}$	12.101	8.860	3.241	0.6658	0.5153	-	-
0.035	396×10^{-3}	366.0×10^{-3}	$+30.0\times10^{-3}$	12.802	9.200	3.602	0.6623	0.5119	-	-
0.040	473.8×10^{-3}	441.6×10^{-3}	$+32.2\times10^{-3}$	13.529	9.697	3.832	0.6595	0.5089	-	-

Table 1 (Continued)

Time second	$\dot{\delta}_1$ %	$\dot{\delta}_2$ %	$\dot{\delta}_1-\dot{\delta}_2$ %	δ_1 Elec deg	δ_2 Elec deg	$\delta_1-\delta_2$ Elec deg	E''_{q_1}, E'_{q_1} p.u	E''_{q_2}, E'_{q_2} p.u	E''_{d_1} p.u	E''_{d_2} p.u
0.045	529×10^{-3}	493×10^{-3}	$+35.2\times10^{-3}$	19.121	9.995	4.126	0.6565	0.5051	-	-
0.050	612.8×10^{-3}	572.5×10^{-3}	$+40.3\times10^{-3}$	17.363	10.315	7.048	0.6533	0.5027	-	-
0.100	107.3×10^{-2}	100.2×10^{-2}	$+71\times10^{-2}$	35.152	24.959	10.193	0.6201	0.4810	-	-
0.150	166.2×10^{-2}	158×10^{-2}	$+82\times10^{-2}$	71.899	58.709	13.190	0.5897	0.4500	-	-
0.350	2.4374	2.3348	+0.1025	201.33	185.31	16.02	0.7329	0.5707	-	-
0.550	1.0560	1.0130	+0.043	280.13	262.20	17.93	0.8595	0.7001	-	-
0.750	-0.5880	-0.5018	-0.0862	290.9	277.0	13.90	0.9009	0.7599	-	-
0.950	-2.184	-2.233	+0.049	228.8	213.9	14.90	0.9476	0.7978	-	-
1.15	-4.021	-4.000	-0.0210	119.99	106.60	13.39	0.9989	0.8309	-	-
1.35	-3.624	-3.638	+0.0140	89.90	77.50	12.40	1.0132	0.8597	-	-
1.55	-2.238	-2.228	-0.0100	47.32	36.20	11.30	1.0597	0.8923	-	-
1.75	-1.5202	-1.5122	-0.0080	31.78	21.58	10.20	1.0838	0.9246	-	-

Table 2. Time solution of slip, angular rotor position, and internal voltage without damping power and govenor responses

Time Second	$\dot{\delta}_1$ %	$\dot{\delta}_2$ %	$\dot{\delta}_1 - \dot{\delta}_2$ %	δ_1 Elec deg	δ_2 Elec deg	$\delta_1 - \delta_2$ Elec deg	E''_{q_1}, E'_{q_1} p.u	E''_{q_2}, E'_{q_2} p.u	E''_{d_1} p.u	E''_{d_2} p.u
0	0	0	0	9.800	7.500	2.300	1.1100	1.000	-0.2450	-0.2650
0.0001	$1072\text{x}10^{-6}$	$1162\text{x}10^{-6}$	$-90\text{x}10^{-6}$	9.802	7.505	2.297	1.0635	0.9440	-0.2591	-0.2843
0.0002	$2725\text{x}10^{-6}$	$1992\text{x}10^{-6}$	$+733\text{x}10^{-6}$	9.8099	7.5070	2.3029	1.01830	0.89127	-0.2728	-0.3041
0.0003	$3639\text{x}10^{-6}$	$2425.4\text{x}10^{-6}$	$+1213.6\text{x}10^{-6}$	9.8216	7.5110	2.3106	0.9779	0.84034	-0.28825	-0.3242
0.0004	$4899\text{x}10^{-6}$	$2620\text{x}10^{-6}$	$+217.9\text{x}10^{-5}$	9.8510	7.5163	2.3147	0.9300	0.7914	-0.3030	-0.3450
0.0005	$613.7\text{x}10^{-5}$	$290.7\text{x}10^{-5}$	$+323\text{x}10^{-5}$	9.8417	7.5201	2.3216	0.8871	0.7450	-0.3188	-0.3660
0.0006	$772\text{x}10^{-5}$	$341\text{x}10^{-5}$	$+431\text{x}10^{-5}$	9.8600	7.5300	2.3300	0.8453	0.7003	-0.3435	-0.4035
0.0007	$892\text{x}10^{-5}$	$362\text{x}10^{-5}$	$+530\text{x}10^{-5}$	9.8822	7.5510	2.3312	0.8042	0.5571	-0.3503	-0.4091
0.0008	$1175\text{x}10^{-5}$	$391\text{x}10^{-5}$	$+728.4\text{x}10^{-5}$	9.8945	7.5600	2.3345	0.7633	0.6153	-0.3669	-0.4136
0.0009	$1209\text{x}10^{-5}$	$421.7\text{x}10^{-5}$	$+777.3\text{x}10^{-5}$	9.9210	7.5672	2.3538	0.7237	0.5750	-0.3835	-0.4540
0.001	$1397.9\text{x}10^{-5}$	$442.6\text{x}10^{-6}$	$+955.3\text{x}10^{-5}$	10.1095	7.6730	2.4363	0.6842	0.5390	-0.4000	-0.4771
0.005	$79.6\text{x}10^{-3}$	$63.9\text{x}10^{-3}$	$+15.7\text{x}10^{-3}$	10.3490	7.6950	2.6540	0.6812	0.5355	-	-
0.010	$140.8\text{x}10^{-3}$	$123.7\text{x}10^{-3}$	$+17.1\text{x}10^{-3}$	10.499	7.7950	2.7040	0.6780	0.5301	-	-
0.015	$187.9\text{x}10^{-3}$	$167.5\text{x}10^{-3}$	$+20.4\text{x}10^{-3}$	11.690	7.809	3.881	0.6751	0.5250	-	-
0.020	$244.8\text{x}10^{-3}$	$220\text{x}10^{-3}$	$+24.8\text{x}10^{-3}$	12.121	8.000	4.121	0.6712	0.5221	-	-
0.025	$300.1\text{x}10^{-3}$	$271\text{x}10^{-3}$	$+29.1\text{x}10^{-3}$	12.789	8.500	4.289	0.6689	0.5192	-	-
0.030	$358.7\text{x}10^{-3}$	$321\text{x}10^{-3}$	$+37.7\text{x}10^{-3}$	13.112	8.710	4.402	0.6655	0.5150	-	-
0.035	$400\text{x}10^{-3}$	$361\text{x}10^{-3}$	$+39\text{x}10^{-3}$	13.995	8.929	5.066	0.6620	0.5117	-	-
0.040	$475\text{x}10^{-3}$	$431\text{x}10^{-3}$	$+44\text{x}10^{-3}$	14.720	9.400	5.320	0.6580	0.5083	-	-

Table 2 (Continued)

Time Second	$\dot{\delta}_1$ %	$\dot{\delta}_2$ %	$\dot{\delta}_1-\dot{\delta}_2$ %	δ_1 Elec deg	δ_2 Elec deg	$\delta_1-\delta_2$ Elec deg	E''_{q_1}, E'_{q_1} p.u	E''_{q_2}, E'_{q_2} p.u	E''_{d_1} p.u	E''_{d_2} p.u
0.045	529.7×10^{-3}	473.2×10^{-3}	$+56.5 \times 10^{-3}$	16.012	10.001	6.011	0.6560	0.5047	-	-
0.050	601.9×10^{-3}	500.1×10^{-3}	+0.1018	18.817	10.775	8.042	0.6526	0.5020	-	-
0.100	1.149	1.022	+0.127	40.321	26.00	14.321	0.6200	0.4807	-	-
0.150	1.750	1.590	+0.162	78.152	61.972	16.180	0.5872	0.4501	-	-
0.35	4.7000	4.7801	-0.081	260.32	240.25	20.07	0.7324	0.5702	-	-
0.55	4.766	4.906	-0.140	377.70	363.50	14.20	0.8592	0.7000	-	-
0.75	3.952	3.820	+0.132	480.60	461.00	19.60	0.8996	0.7593	-	-
0.95	1.404	1.524	-0.120	500.40	587.0	13.40	0.9429	0.7973	-	-
1.15	-2.250	-2.320	+0.130	589.23	570.33	18.90	0.9889	0.8300	-	-
1.35	-7.513	-7.413	-0.100	440.80	427.50	13.30	1.0112	0.8723	-	-
1.55	-9.184	-9.296	+0.112	209.52	190.95	18.57	1.0569	0.8992	-	-
1.75	-11.012	-10.918	-0.094	59.30	46.60	12.70	1.0717	0.9239	-	-
1.95	-12.998	-13.078	+0.08	-8.0	-25.1	17.1	1.0996	0.9598	-	-

Table 3. Time solution of slip, angular rotor position, and internal voltage with damping power and governor responses and neglecting controller signal

Time Second	$\dot{\delta}_1$ %	$\dot{\delta}_2$ %	$\dot{\delta}_1-\dot{\delta}_2$ %	δ_1 Elec deg	δ_2 Elec deg	$\delta_1-\delta_2$ Elec deg	E''_{q_1}, E'_{q_1} p.u	E''_{q_2}, E'_{q_2} p.u	E''_{d_1} p.u	E''_{d_2} p.u
0.15	$166.2\text{x}10^{-2}$	$158\text{x}10^{-2}$	$+82\text{x}10^{-2}$	71.899	58.709	13.190	0.5897	0.4500	-	-
0.35	2.8720	2.7260	+0.1460	221.92	205.49	16.43	0.7330	0.5708	-	-
0.55	1.3882	1.3362	+0.520	310.75	292.50	18.25	0.8595	0.7006	-	-
0.75	-0.7980	-0.6912	+0.1068	331.30	317.10	14.2	0.9000	0.7594	-	-
0.95	-2.482	-2.535	+.053	279.00	264.00	15.0	0.9477	0.7978	-	-
1.15	-4.323	-4.301	-0.0220	170.10	156.70	13.4	0.9989	0.8309	-	-

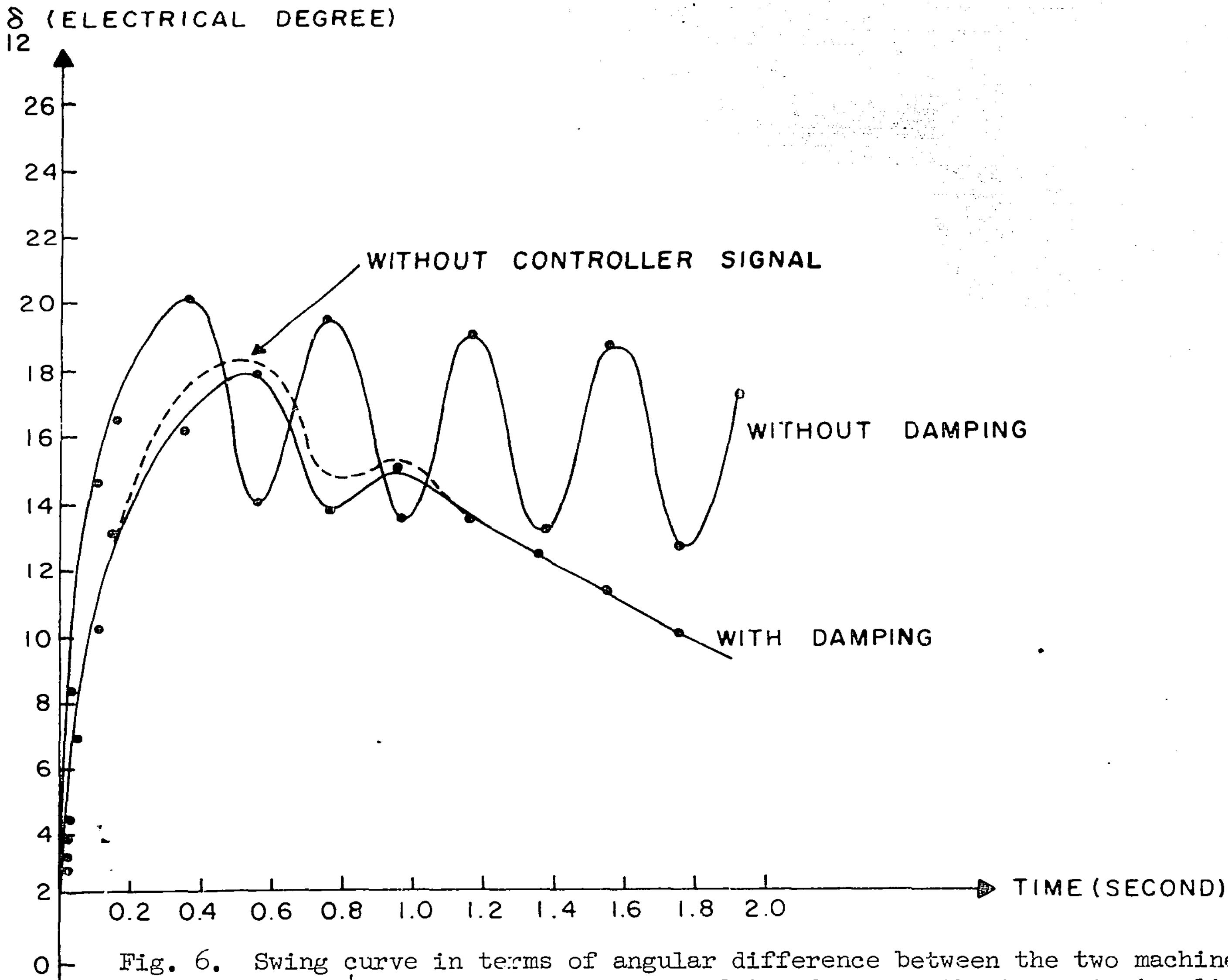

Fig. 6. Swing curve in terms of angular difference between the two machines for 3-ϕ short circuit at the middle of one of the transmission lines (fault cleared at 0.15 sec)

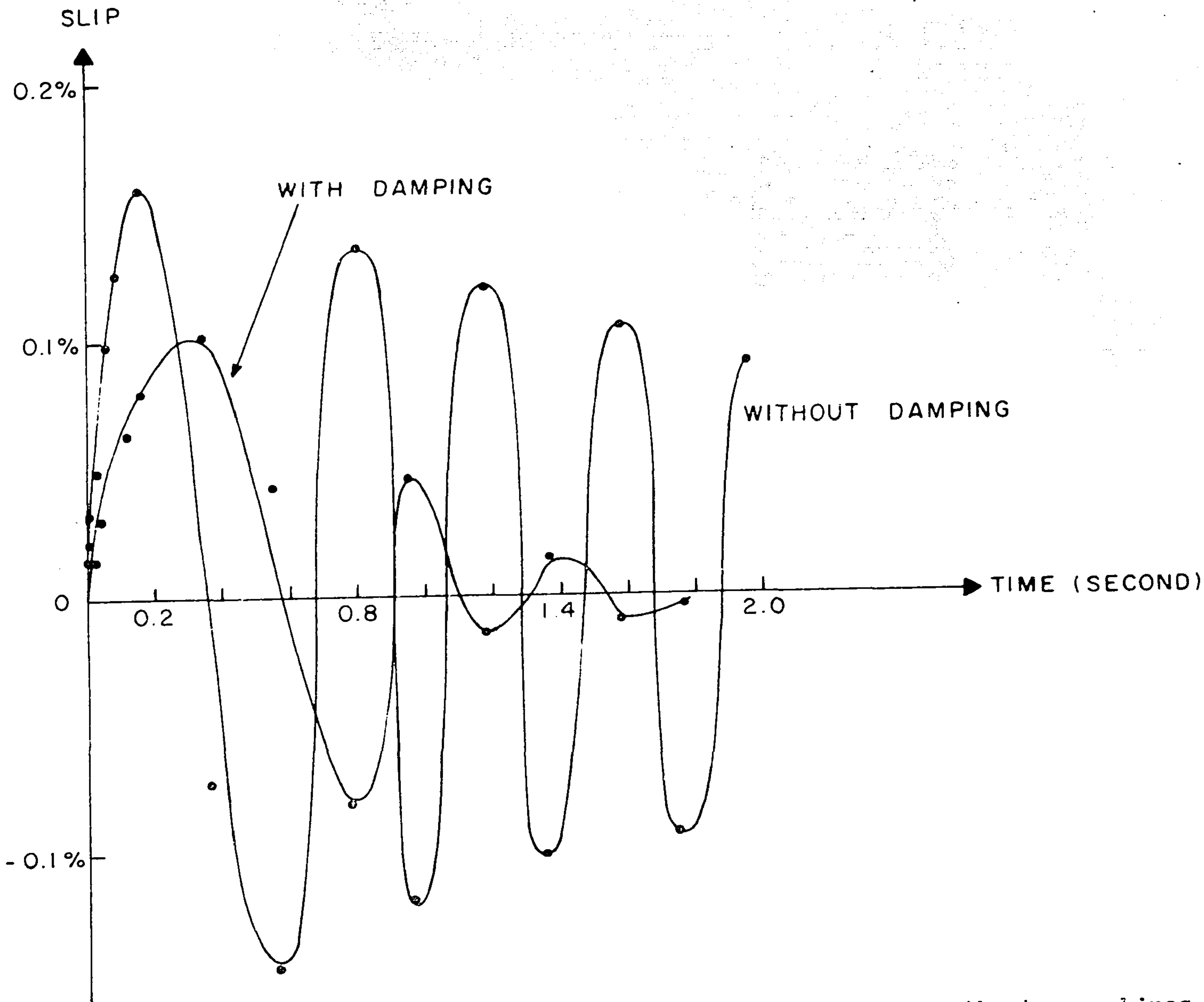

Fig. 7. Swing curve in terms of slip difference between the two machines

VII. CONCLUSIONS

General equations describing the behavior of synchronous machines following a disturbance in a system network have been developed. These equations illustrate how the damping power is flux linkage and angular rotor position dependent. The field damping is very effective at the instant of short circuit due to the rapid decrease of the internal voltage components in the first few cycles of disturbance. This immediately implies the transient stability limit of the system to be more likely to increase due to the slow increase in δ_{12} as shown in Fig. 6. and the synchronous machines will have more chance for relief from the sudden disturbances. The governor responses are more effective during the subsequent swings according to the time lags of the governor and controller. This is clear from Fig. 6, 7, since the amplitudes of δ_{12} and the slip δ'_{12} decrease rapidly in magnitude till equilibrium is reached. However the amplitudes in the subsequent swings without taking damping into account are decreasing very slowly due to the sustained oscillations and the system is not known to be stable until two seconds and the transient stability limit will be lower and lower. Therefore the damping power and governor responses play an important rule in increasing and judging the stability of the system and they should be taken into consideration.

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