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STATISTICAL ANALYSIS OF RESOURCE PRODUCTIVITY
IN SELECTED FARMING REGIONS OF TAIWAN

by

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I. INTRODUCTION

Increasing attention has been focused upon the resources productivities and adjustment problems of agriculture during recent years in Taiwan. The growth rate of agricultural output has been declining, while population has continued to increase at rapid pace. More food has to be produced each year in order to feed the increasing population. In addition to satisfying the increasing needs of the population for food, agricultural production must also provide sufficient raw material for industrial development. Furthermore, for economic stabilization purposes, the relative price of agricultural products in Taiwan have been kept stable and low in order to reduce the cost of production for industries and to enable consumers to get cheap food. The relative low price in agriculture results in low farm income. It increases, however, the income effect for city workers and produces an expansionary effect on industrial production, which is favorable to both the accumulation of industrial capital and the economic development as a whole. Although the relatively low price of agricultural products may reduce the cost of industrial production and thus contribute to industrial development, it cuts down farmers' income at the same time and thus weakens their purchasing power for industrial products, with the result that the industries will be hard to expand since the domestic market for industrial goods is limited. It is a dual task for future agricultural development, therefore, to raise farm productivity so that enough food and raw materials can be supplied to consumers and industries at a reasonable price level on the one hand, and to keep farm income on a sufficiently high level so as to increase the income effect for farmers and enlarge their effective demand for industrial products to promote industrial development on the other hand. Many solutions have been suggested for solving the "dual task" of agriculture in Taiwan such as, in rather broad categories, the expanding foreign market for industrial goods, reducing the agricultural labor force through further industrial development, increasing resource productivity and efficiency in agriculture, etc. This study deals with some aspects of the last category of these general solutions. From the standpoint of whole economic program and policy, this study indicates the differentials in resource productivity between agricultural regions and certain causes of differentials in resource productivity. From the standpoint of the individual farmer, this study provides the information of resource productivity when different quantities and combinations of resources are used at a particular agricultural region.

A. Objectives

The general objective of this study in production economics is designed to measure and examine some aspects of efficiency in selected agricultural regions in Taiwan. One of the major problems falls in the field of agricultural economics is to determine the nature of resource productivity in agriculture. However, the economist is not expected to establish the technical physical production relationships which are possible for attending a production target from a given set of resources in agriculture; this is the work of technical agriculturists. But the agricultural economist is usually interested to establish, on the basis of (1) production techniques in use by farmers or (2) techniques established by physical scientists as technically feasible, the economically optimum method of attaining given levels of production. This study concerns solely the productivity of resources as used by farmers, and deals only with tangible measures of economic efficiency and resource productivity; it does not deal with intangible and subjective aspects of farming such as the values which farmers may attach to a certain type of farming *per se*. While many of these factors are important, they are not subject to easy measurement.

The major objective of the investigation is to measure the value productivity of resources and their services used in different agricultural regions and to predict, within the limitations of the data and method, the effect of varying combinations and quantities of resources used on the value of the product produced.

More specific objectives are:

1. To examine the average income or returns to be expected from different types of farming;
2. To analyze the effects of factors affecting the income variation between agricultural region;
3. To provide information on farm income and costs of production in different agricultural regions;
4. To estimate "average" production functions for groups of farmers in different agricultural regions;
5. To examine different methods of estimation of production functions;
6. To measure the marginal productivities of resources and corresponding confidence intervals when different quantities and combinations of resources are used in different agricultural regions;
7. To derive isoquants and isoclines and their confidence intervals in different regions;
8. To determine the least cost combination of resources for attaining given income levels in different regions.

This study shows the extent of differentials in resource productivity between agricultural regions and the possible income that can be expected from different types of farming. It also examines the implications and limitations of current statistical analyses applied to cross-sectional farm survey data for estimating production functions on Taiwan's small farms.

B. Source of Data

The main statistics of this study are based on a random sample and survey farms in the Shihmen Reservoir area in northern Taiwan. The area of investigation covers eight townships with a total land area of 46,578 hectares, of which 30,510 hectares are under cultivation. Based on information provided by the township and village offices, six farming regions were classified within the area. A total of 400 farms were then drawn at random in 1962 from these six rather homogeneous agricultural regions. All information obtained in the farm survey related to the year of 1961; specifically, the data of investigation are for the farm business year from December 1960 to November 1961, and cover the yearly crop period of the area. In the following text, data cited are from this source unless specially noted otherwise.

C. Method of Analysis

Both tabular and production function analyses were employed for the estimation of productivity or returns for resources used in the different agricultural regions. Previously, farm economic surveys in Taiwan have been mainly analyzed through tabular procedures. The great limitation of this approach is the assumption of constant productivity coefficients through the whole observed range of resources used. When average returns are computed for a group of farms and used for recommendations, it is implicitly assumed that returns for all units of resources are the same as the "computed" average productivity figures, regardless of the quantity or proportions of resources used. However, the tabular procedures may allow more flexibility in the form of relationships expressed in the data. On the other hand, the approach of the statistical analysis of production functions, in general, will yield more refined marginal productivity estimates. Whether the estimated parameters of functions are meaningful, however, depends on (1) the accuracy of data, (2) the appropriateness of the model used and (3) the appropriateness of the statistical estimation procedures.

Each approach has advantages and disadvantages. What approach to use should be judged in terms of the information we are seeking and should be determined by the data we have on hand. One of the objectives of this study is to compare the different inferences which can be made from productivity estimates based on alternative empirical procedures of estimation.

D. Definitions

Some of the important terminologies and definitions used in this study are as follows:

1. Agricultural region is a larger land area consisting of a number of homogeneous "land sections" (villages) that differ from other regions in respect to the land type, crop distribution and irrigation features. It is sometimes referred to as a "type" of farming area in the following text.
2. Man-equivalent is a measure of the farm labor force. The computation of man-equivalent depends not only on how many days family laborers actually work on the farm but also depends on the number of days which family laborers are available on farm. It represents the stock of the farm labor force rather than the flows of farm labor. An adult male, age 16 to 60, working full time on a farm during the year

is considered to be a standard unit of man-equivalent. The following conversion factors were used in the computation of man-equivalent on a farm in this study:

One full time man, age 16 to 60, is considered as one unit of man-equivalent.

One full time woman, age 16 to 60, is considered to be 0.8 unit of man-equivalent.

One full time man or woman, under 16 or above 60 years old to be 0.5 unit of man-equivalent.

The farm operator is always accounted as one unit of man-equivalent. For those family workers who have temporary off-farm jobs, the off-farm working days were excluded in the computation of man-equivalent.

3. Productive man-work day, or abbreviated as man-day, is a 10-hour day of productive farm work by an adult male of average skill, age 16 to 60, under average working conditions. This measure only takes into account directly productive work for crops and livestock production on the farm. The total productive work days contributed by various kinds of farm labor were all converted into the standard unit of man-equivalent.

4. Farm area is the total land area consisting of cultivated land and non-cultivated land. Cultivated land area will represent as the size of farm business and it includes both self-owned and rented land for the production of various crops. Non-cultivated land includes forest land, water pond, farmstead and others.
5. Crop land area represents the total acreage of various individual crops planted during the investigation period. This measure takes account of the crop planting area as well as the frequencies of land use during the year. Some parts of the farm may produce 3 or 4 crops in the year, other parts may produce only one and still other areas less than one crop, depending on the length of production period of the individual crop.
6. Total value of fixed capital is the sum of the present values of land (excluding rented land), buildings, farm equipment and machinery, working animals, livestock, and trees on the farm.
7. Total value of production is defined as the sum of the value of crops, livestock, and fruit or trees in production on the farm during the investigation period. In other words, it consists of all production values of various crops and different kinds of

livestock on the farm, except for those minor by-products, such as the rice straw and livestock manure, which are used directly on the farm rather than sold and whose values are hard to evaluate.

8. Gross farm receipts represent the total farm cash receipts from crop and livestock, adjusted for the changes in inventory during the investigation period, plus the total value of farm privileges.

9. Farm privileges are the estimated value of crop and livestock products which are produced on the farm but are consumed by the farm family. They do not include an estimate of house rent.

10. Off-farm income consists mainly of wage receipts by family members from off-farm jobs and other miscellaneous income which is not directly related to the farm production.

11. Gross family receipts consists of gross farm receipts and off-farm income. It represents the total receipts from both farm business and off-farm jobs during the year. The difference between this measure and the total value of farm production is that the value of farm products used as inputs for farm production, such as self-provided seeds or home produced feeds, is not counted as a source of farm re-

ceipts but is included in the computation of the total value of farm production. Of course, off-farm income is not included as a source of total value of farm production.

12. Total cost of production consists of three rather broad categories: (1) total operating cost of crop production; (2) total operating cost of livestock production, and (3) total fixed cost of production for both crop and livestock. Both cash purchased and self-provided input resources are included in the items of cost of production.
13. Cash operating cost consists of only the cash expenditures for operating cost items, including both for crop and livestock production. It does not include the self-provided cost items and unpaid family labor or animal labor.
14. Farm expenditure is defined as the sum of cash operating cost and fixed cost of production.
15. Farm income is the difference between farm cash receipts and farm expenditures. It represents the amount of cash income left over from farming, available for family living, investment or saving.
16. Farm earnings are farm income plus farm privileges.
17. Farm family income and farm family earnings are the corresponding "farm" measures plus off-farm income.

II. CHARACTERISTICS OF AGRICULTURAL REGIONS AND SAMPLE FARMS

A. The General Situation of Investigation Area

The Shihman Reservoir area, the investigation area, is located in the northern part of Taiwan. This reservoir is a newly multi-purpose project designed and planned to fully exploit the potentialities of water resources in that area. It was designed to serve the purposes of irrigation, flood control, power generation and supply of water for urban use. This area covers a total land area of 46,578 hectares, of which about 65%, the equivalent of 30,510 hectares is under cultivation, with an addition 6,437 hectares in forest. Approximately 20,000 hectares of the cultivated land are under irrigation at the present time. The paddy fields occupy 71 percent of the total cultivated land area and the up-land fields constitute the other 29 percent. According to the 1961 Census of Agriculture, there were 20,469 farm families in this area, with an average in cultivated land size of 1.5 hectares per farm and 9 persons of family size. The major crops produced in this area consist of paddy rice, sweet potatoes and different kinds of vegetables. Rice and sweet potatoes are dominant over the whole area, while vegetables extend within its favorable marketing location. Tea is the principal product along the mountain district, and its production constitutes the major portion of total tea production in the whole island of Taiwan. Besides tea production, there are also some kinds of fruit and tree products produced in the mountain area. Livestock production is made up mainly of hogs and poultry, which is widespread in the whole area. Almost all farms engage in the breeding of fattening of hogs, however, the size of enterprise is not necessarily large. Cattle is the main draft power on the farm. Although the number of power tillers is gradually increasing during the recent years, it is still nonsignificant compared with working cattle. The number of milk cows has also been gradually establishing and expanding in this area. Dairy production, however, is still a minor part compared with the hog production.

Based on the information provided by the township and village offices, the whole area had been classified into six agricultural regions according to the land type, crop distribution and cropping system, and the irrigation feature. Sample farms were then selected from these regions. These regions are known as: (1) Rice region, (2) Sweet potato region, (3) Tea region, (4) Rice-Sweet potato region, (5) Rice-Tea region, and (6) Mixed region. Table 1 shows the main characteristics of these six regions, and their locations are shown in Figure 1.

Table 1. Characteristics of agricultural regions[a]

Items	Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
	Agricultural regions					
Total land (ha.)	13,950	1,937	6,674	4,570	13,271	6,176
Cultivated land (ha.)	10,091	1,098	4,176	2,943	7,983	4,219
Paddy land	9,314	689	980	2,647	4,893	3,176
Up-land	777	409	3,196	296	3,090	1,043
Percentage of cultivated land under irrigation (%)	83	43	23	89	61	72
Total crop land (ha.)	19,458	2,380	7,477	6,030	14,568	8,093
Distribution of main crops (% of total crop land):						
Paddy rice	88.00	46.60	26.02	65.10	63.10	76.40
Sweet potatoes	5.17	36.05	9.65	16.67	6.90	11.19
Vegetables	4.10	11.30	1.70	6.30	3.30	6.00
Tea	1.80	0.08	43.30	1.70	22.60	4.00
Total number of farm families	7,244	1,099	2,168	1,911	4,886	3,164

[a]Based on the information provided by township and village offices in the investigation area.

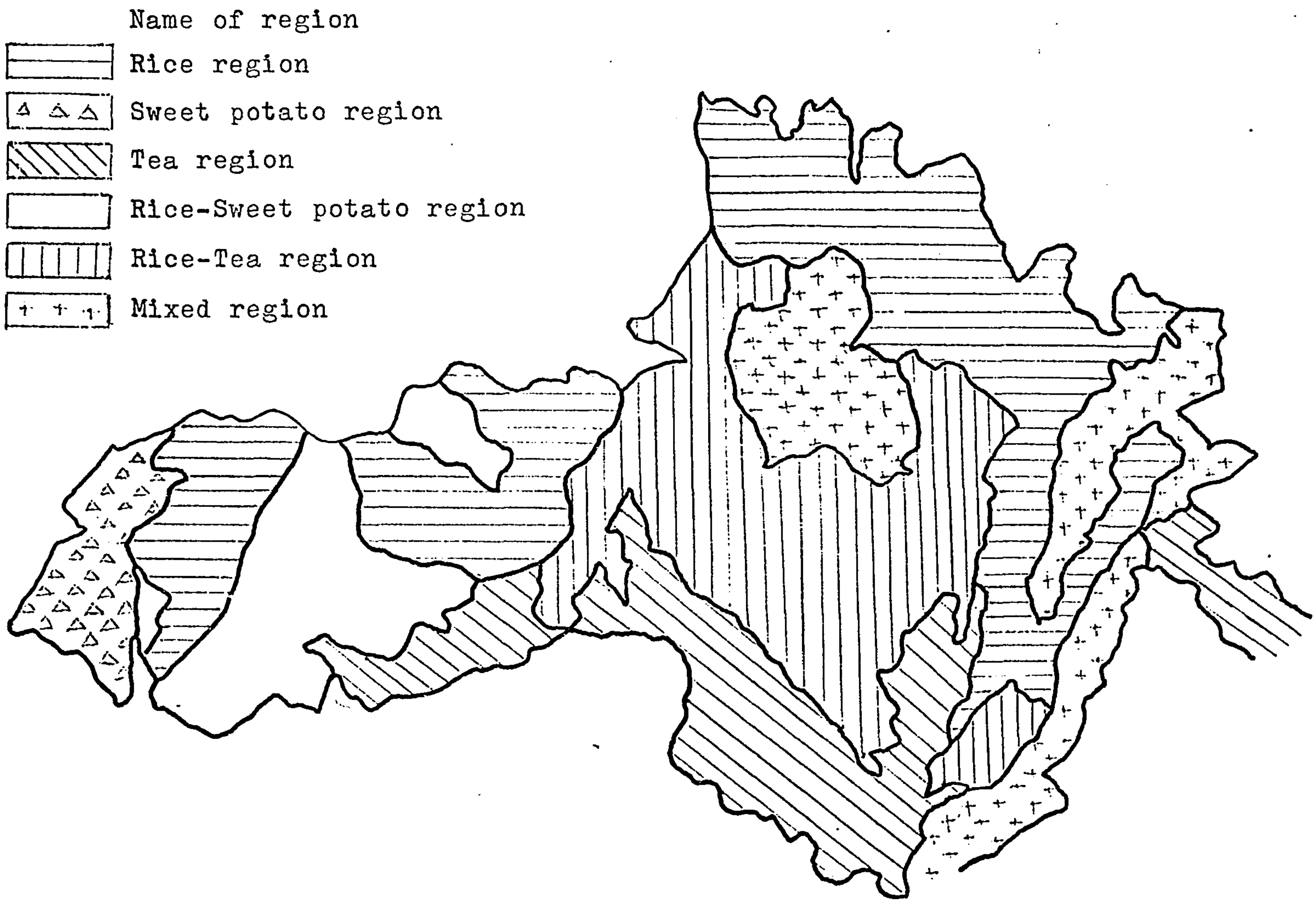

Figure 1. Map of investigation regions

The main region is the Rice region with around 14,000 hectares of land area, of which about 10,000 hectares are cultivated. The Rice-Tea region covers about 13,300 hectares of total land with about 8,000 hectares under cultivation. Then follows the Mixed and Tea regions. Both of them cover about the same size of cultivated land area. Although the Tea region occupies more than 1,000 hectares of cultivated land compared with that in the Rice-Sweet potato region, the paddy land area in the latter region is much larger than that in the Tea region. The smallest region is the Sweet potato region in which only about 1,000 hectares of land are under cultivation.

Paddy rice, sweet potatoes and tea are the principal crops in this area. With the exception of the Tea and Sweet potato regions, more than 65 percent of the total crop area is used for rice production. Even in these two exception regions, there are still 26 percent and 47 percent of total crop area used for rice production respectively.

In the Tea region, the tea crop area constitutes about 43 percent of the total crop area. The distribution of various crops in each region is shown in the lower part of Table 1. In general, paddy land with enough water supply in the proper seasons is mainly used for rice, while for those land with shortage of water is usually used for planting sweet potatoes. Up-land fields can be used either for sweet potatoes or tea production, depending on situations of water supply and the preference of farmers. Vegetables are usually found in the sub-urban areas with suitable marketing facilities.

B. Sample Farms

A total of 400 farm families were drawn at random from these six regions. Approximately, a 5 percent of sample fraction was used to determine the number of sample farms in each region. The distribution of sample farms in each region with the size groups is shown in Table 2.

In this section, the general results of the farm economy survey based on these 400 sample farms are to be described and summarized, and some points in input-output relationship are to be emphasized for further study in the following chapters.

1. Land resources and its utilization

The total farm land area of all sample farms varied from 1.20 hectares to 2.56 hectares with an average of 1.95 hectares per farm, of which 1.67 hectares were cultivated land consisting of paddy land and up-land. The total cultivated land area per farm varied from 0.78 hectares in the Sweet

Table 2. Distribution of sample farms

Agricultural regions	Total number of sample farms	Cultivated land size groups				
		Less than 0.5 ha.	0.5-1.0 ha.	1.0-2.0 ha.	2.0-3.0 ha.	3.0 or more
Rice	138	12	29	54	27	16
Sweet potato	18	12	2	3	0	1
Tea	48	8	8	12	7	13
Rice-Sweet potato	38	6	10	19	3	0
Rice-Tea	100	11	22	41	9	17
Mixed	58	12	8	22	10	6
Whole area	400	61	79	151	56	53

potato region to 2.19 hectares in the Tea region. In the Rice region, the average cultivated land area was 1.87 hectares of which 1.70 hectares were paddy fields, while only 0.17 hectares were up-land fields. On the other hand, the Tea region had a larger portion of up-land fields than the paddy land on the average.

In general, most of the paddy land is used for raising two paddy rice crops during the year or to plant various kinds of vegetables. Since rice in Taiwan is used as the main food and can be also used as means to pay taxes or to trade for fertilizers, farmers usually devote their paddy land to rice production. It might be said that if the farm is lacking in paddy fields, there will be little chance to develop the farm business in Taiwan. In this survey, paddy rice occupied about 74 percent of total crop land, on the average. In the Rice region, this percentage was much higher; up to 87 percent as shown in Table 3. Vegetable production usually needs a lot of fertilizer and labor inputs, and is facing larger uncertainty of price and yield variations. Consequently, farmers usually devote only a small portion of their land to planting different kinds of vegetables. Sweet potatoes can be produced either in the up-land fields or in the paddy land during drough season. Farmers in the Sweet potato region tend to use their paddy field for planting sweet potatoes instead of a second paddy rice crop. In the

Table 3. Land resources and the distribution of crops per farm

Items	Agricultural regions						
	Whole area	Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Total farm land (ha.)	1.95	2.13	1.20	2.56	1.25	2.18	1.69
Cultivated land (ha.)							
Paddy land	1.22	1.70	0.64	0.82	1.09	1.22	1.20
Up-land	0.45	0.17	0.14	1.37	0.02	0.67	0.26
Total	1.67	1.87	0.78	2.19	1.11	1.89	1.46
Others	0.28	0.26	0.42	0.37	0.14	0.29	0.23
Crop area per 100 ha. of cultivated land:							
1st paddy rice	40	46	42	25	46	36	42
2nd paddy rice	34	41	28	21	36	30	37
Sweet potatoes	8	7	13	8	15	9	8
Vegetables	3	3	5	2	2	3	6
Tea	13	2	0	41	...[a]	20	3
Fruit	1	...	11	3	...	1	...
Others	2	1	1	...	1	1	4
Total	100	100	100	100	100	100	100
Total crop land (ha.)	2.91	3.45	1.50	3.17	2.22	3.13	2.69
Multiple cropping index	174	184	193	145	200	165	184

[a] '...' stands less than 1 percent.

case of the Tea region, farmers on the average devoted more than 40 percent of their crop area for tea production. Table 3 shows the composition of land resources and the distribution of main crops in each region. Roughly speaking, farmers with small size of cultivated land but a greater portion in paddy fields than in up-land fields, will tend to use their land resources more intensively and in turn cause an increase of farm income.

2. Labor resources

The labor resource of farms consists of operator labor, family labor and hired labor. The labor available for farm production is denoted by the number of man-equivalent units as described previously, and the utilization of labor on the farm is discussed in terms of total productive man-work days, or alternatively, as total amount of man-day or man-hour per year on farm.

In this farm economy survey, the average size of the farm family in the whole area was 9 persons, with an average of 4.4 man-equivalents. The number of man-equivalent was not different greatly from region to region, except for the Rice-sweet potato region. The group of farms in the Sweet potato region had, on the average, the largest number of man-equivalents while farms in the Rice-Sweet potato region had had only 3.6 man-equivalents on the average. The total amount of farm labor available, or the number of man-equivalents, depends largely on (1) family size, (2) age distribution of family member, and (3) the opportunity of off-farm work. On the other side, the total amount of labor used on the farm depends mainly on (1) the size of farm, (2) the intensity of resource use and also (3) the degree of diversity of farming or the choice of cropping system. Generally speaking, much more labor is needed for the production of tea, vegetables and paddy rice than for sweet potatoes production. On the average, a total of 4,200 man-hours of labor were used per farm during the year for the whole group of 400 farms. It varied from 2,080 man-hours per farm in the Sweet potato region up to 4,640 man-hours in the Tea region.

Of the total amount of man-hours used per farm, about 83 percent, or 3,500 man-hours, was contributed by family labor (including operator's labor). Hired labor had supplied 17 percent of the total. The size of farm family and the utilization of labor resources in each region are shown in detail in Table 4.

3. Capital resources

Total employed by farmers consists of value of land used, farm buildings and equipments, trees, working cattle and livestock. All of these capital items were evaluated at the beginning of the investigation period and again at the

Table 4. Size of farm family and labor resources per farm

Items	Whole area	Agricultural regions					
		Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Total number of persons:							
Male	4.5	4.6	4.9	4.6	4.2	4.5	4.3
Female	4.5	4.8	4.7	3.9	3.5	4.5	4.6
Total	9.0	9.4	9.6	8.5	7.7	9.0	8.9
Total number of man-equivalents	4.4	4.5	4.9	4.3	3.6	4.2	4.5
Total number of man-hour used	4,170	4,450	2,080	4,640	3,280	4,280	4,190
Family labor	3,470	3,580	1,960	3,660	3,060	3,560	3,660
Hired labor	700	870	120	980	220	720	530

end of the period, then the average figure was taken to indicate the average amount of capital used by farmers.

The total capital per farm in the whole area averaged N.T.$110,668[1], of which about 82 percent or N.T.$90,285 represents the value of land and buildings. The working capital such as equipment, working cattle and livestock amounted to only about N.T.$11,000, or 10 percent of the total capital investment per farm as shown in Table 5. The low ratio of working capital is a rather common situation in Taiwan. However, it is a hasty conclusion to say that farming in this studied area is operated primitively with unequipped labor. With a relatively small size of cultivated land and without a significant number of productive animals, limit of capital intensive will be found on the farms. In recent years, mechanization of Taiwan's farming has been gradually developed. However, it does not lead to a large size of farm and hence any significant saving of labor. Machinery has been increased to intensify farming rather than to save labor.

Farmers in the Rice, Tea, Rice-Sweet potato and Rice-Tea regions had on the average invested about the same amount of capital, varying from N.T.$110,000 to N.T.$130,000. Much

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[1]N.T.$ stands for New Taiwan dollar currency. N.T.$40 is in equivalent to U.S. $1.

lower capital investment was found in the Sweet potato region: an average of N.T.$56,150 per farm. Table 5 shows the total capital investment for the different groups of farmers in each region, stratified by size of cultivated land area.

4. Gross farm return

The main categories of resources used by farmers in different regions has been described briefly in previous sections. This section includes an analysis of farm returns from crop and livestock production. The cost of production will be discussed in the following section.

a. Total value of production The average total value of farm production was estimated to be about N.T.$41,000 per farm for the whole group of farms in this area. The value of crop production constituted about 70 percent of the total value, while the residual of 30 percent was made up by livestock production.

The group of farmers in the Rice region produced an average of about N.T.$48,000 of total value of production per farm during the year, while the farmers in the Sweet potato region had had only about N.T.$20,000 which was only about 40 percent of total value of production per farm in the Rice region. Farmers in the Rice-Tea and Mixed regions were at the average level while the Tea and Rice-Sweet potato

Table 5. Total capital investment per farm stratified by size of farm (Unit: N.T.$)

Items	Whole area	Cultivated land size groups				
		Less than 0.5 ha.	0.5-1.0 ha.	1.0-2.0 ha.	2.0-3.0 ha.	3.0 ha. or more
Agricultural regions:						
Rice	112,600	36,935	49,960	83,957	165,435	290.565
Sweet potato	56,150	32,308	85,763	133,581	...	50,730
Tea	126,982	19,201	51,946	65,865	99,888	310,489
Rice-Sweet potato	109,654	65,719	79,605	151,876	30,278	...
Rice-Tea	130,112	28,457	53,772	96,866	145,218	366,806
Mixed	76,580	31,338	51,414	103,152	78,430	100,106
Capital items per farm:						
Land	72,344	18,822	34,527	63,881	91,630	194,052
Buildings	17,941	9,645	11,487	18,000	21,413	33,072
Equipment	2,012	349	1,322	2,200	2,193	4,425
Trees	9,225	1,015	2,688	4,872	4,028	46,310
Working animals	3,450	508	2,140	3,513	5,041	6,926
Livestock	5,696	3,561	3,865	5,887	6,928	9,035
Total	110,668	33,900	56,029	98,353	131,233	293,820

regions were below the average as shown in Table 6. The proportion between crop and livestock production did not vary greatly from region to region except for those farmers in the Rice-Sweet potato region. Livestock production contributed approximately 40 percent of the total value of farm production in the Rice-Sweet potato region, while in all other regions, it constituted around 30 percent of the total.

b. Gross farm receipts   This measure includes farm cash receipts and farm privileges. Specifically, farm receipts consist of total cash receipts from crops and livestock, adjusted for the changes in inventory during the investigation period. In addition to the crop sale, the value of paddy rice used for the payment of land taxes and the barter for fertilizers were also included in the computation of farm receipts. However, the intermediate farm products such as sweet potatoes used as feed, or self-produced seeds and seedlings were excluded in this measure.

The total gross farm receipts for the whole area were about N.T.$37,500 per farm, of which 68 percent were farm receipts and about 32 percent were consumed by the farm family. The ratio of farm receipts to farm privileges indicates roughly the degree of commercialization. The higher ratio, the larger the portion of farm product which is put into the channel of marketing. More than 70 percent of farm products

Table 6. Total value of farm production and gross farm receipts (Unit: N.T.$)

Items	Whole area	Agricultural regions					
		Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Total value of farm production:							
Crop production	28,991	34,826	14,053	25,412	20,288	28,861	28,635
Livestock	12,146	12,770	5,690	9,668	13,745	13,356	11,584
Total	41,137	47,596	19,743	35,080	34,033	42,217	40,219
Gross farm receipts:							
Farm cash receipts	25,511	31,017	8,302	23,088	20,455	25,732	22,691
Crop	15,993	20,701	3,830	15,209	9,166	15,009	15,251
Livestock	8,478	9,170	4,459	5,641	10,441	9,043	8,167
Inventory change	+1,060	+1,146	+ 13	+2,238	+ 848	+1,680	- 727
Farm privileges	11,983	12,912	8,521	9,154	10,512	12,631	13,033
Total	37,494	43,929	16,823	32,242	30,967	38,363	35,724
Off-farm income	5,719	4,281	4,928	7,764	1,337	6,963	8,420
Gross family receipts	43,213	48,210	21,751	40,006	32,304	45,326	44,144

produced in the Tea and Rice regions were sold, while in the region of Sweet potato, farmers consumed more than half of their products by themselves.

c. Off-farm income and gross family receipts Gross family receipts is defined as the sum of gross farm receipts and off-farm income. Off-farm income represents income from off-farm activities rather than from farm production. As pointed out previously, with relatively large families and small farm size, farmers in Taiwan are usually seeking for off-farm jobs during off-seasons. The amount of off-farm income depends on the opportunities of work both on farm and off-farm.

On the average, the gross family receipts per farm for the whole group of farmers in this area were estimated to be about N.T.$43,000, of which nearly 87 percent were contributed by gross farm receipts and the off-farm income merely constituted about 13 percent, or N.T.$5,719 per farm. Farmers in the Sweet potato region had had about 23 percent of gross family receipts from off-farm income, which was the largest portion among all regions investigated, while the Tea and Mixed regions had had around 19 percent, and the smallest portion of off-farm income was found in the Rice-Sweet potato region; only about 4 percent of gross family receipts were made up by off-farm income. Table 6 shows the various measures of farm returns per farm in each region. The figures of different returns measures for each region are of the same order as shown in the table.

5. Production cost

This section describes the average cost of production for those groups of farms in different regions in this study. Production cost will vary from farm to farm and between regions, particularly when different methods of production are used or different cropping systems are adopted. However, in this section the cost of production were analyzed only for the farms between regions, no attempts were made for the cost variation from farm to farm within region. The production cost described here was computed based on whole farm business rather than a single enterprise production.

For the calculation which follows, total cost of production per farm was divided into operating costs and fixed costs. Operating costs are those items which vary with the size and economy of operation. Fixed costs are those which are largely determined in advance of the year's operation and subject to little or no control by the farmer. Operating costs which vary with the amount of products produced on the farm include those for seed or seedling, fertilizer, feed, pesticides, medicine and livestock insurance, labor and others which vary with the number of livestock raised and the yield level attained. Fixed costs which usually are constant for a given size of farm organization, include depreciation on buildings and farm equipment, payment of land taxes, rent of land, payment of irrigation water fee and others which are not directly related to the level of production. In general, operating costs or variable costs are usually termed as direct costs and fixed costs as indirect costs in Taiwan. The production cost per farm for the whole area as well as for each region are shown in Table 7. Those figures in parentheses in the table represent the percentage distribution under each subcategory.

The total cost of production, consisting of both operating and fixed costs, was estimated at about N.T.$28,500 per farm for the whole group of 400 farms in the area, of which about N.T.$24,000 were classified as operating costs while the fixed costs were only about N.T.$4,600 per farm. The total production cost per farm varied considerably between regions. The average total cost of production in the Rice region, the highest cost region, was about N.T.$32,000 against only about N.T.$14,000 in the Sweet potato region, the lowest of all regions. The latter was merely 44 percent as much as the former. The relative proportion of operating and fixed costs per farm, however, was not greatly different between regions; it was about 84 to 16 as shown in Table 7.

Table 7. Production cost per farm[a] (Unit: N.T.$)

Items	Whole area	Agricultural regions					
		Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Total cost per farm	28,538	31,999	14,318	27,062	20,530	28,430	31,357
	(100)	(100)	(100)	(100)	(100)	(100)	(100)
Total operating cost	23,922	26,389	12,600	22,936	17,540	23,958	26,505
	(83.8)	(82.5)	(88.0)	(84.8)	(85.4)	(84.3)	(84.5)
Total fixed cost	4,616	5,610	1,718	4,126	2,990	4,472	4,852
	(16.2)	(17.5)	(12.0)	(15.2)	(14.6)	(15.7)	(15.5)
Total operating cost:							
Seed or seedling	682	810	295	500	576	648	773
	(2.9)	(3.1)	(2.3)	(2.2)	(3.3)	(2.7)	(2.9)
Feeder pig	1,669	1,727	941	1,613	1,765	1,635	1,801
	(7.0)	(6.5)	(7.5)	(7.0)	(10.1)	(6.8)	(6.8)
Fertilizer	4,831	6,470	2,257	2,859	3,287	4,132	5,583
	(20.2)	(24.5)	(17.9)	(12.5)	(18.7)	(17.2)	(21.1)
Feed	4,411	5,035	2,826	3,280	2,302	4,330	5,874
	(18.4)	(19.1)	(22.4)	(14.3)	(13.1)	(18.1)	(22.2)
Man-labor	10,872	10,468	5,451	13,231	8,676	11,760	11,387
	(45.4)	(39.7)	(43.3)	(57.7)	(49.5)	(49.1)	(43.0)
Animal-labor	1,161	1,582	617	859	692	1,258	716
	(4.8)	(6.0)	(4.9)	(3.7)	(3.9)	(0.7)	(0.9)
Pesticides & medicine	273	294	119	535	239	179	238
	(1.1)	(1.1)	(0.9)	(2.3)	(1.4)	(0.7)	(0.9)

[a]Figures in parentheses represent the percentage distribution under each subcategory.

Table 7 (Continued).

Items	Whole area	Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
		Agricultural regions					
Miscellaneous	23	3	94	59	3	16	33
	(0.2)	(...)	(0.8)	(0.3)	(...)	(0.1)	(0.4)
Total	23,922	26,389	12,600	22,936	17,540	23,958	26,505
	(100)	(100)	(100)	(100)	(100)	(100)	(100)
Total fixed cost:							
Depreciation	1,047	1,078	483	772	661	1,293	1,185
	(22.7)	(19.2)	(28.2)	(18.7)	(22.1)	(28.9)	(24.4)
Land tax	940	1,139	388	934	529	922	945
	(20.4)	(20.3)	(22.6)	(22.6)	(17.7)	(20.6)	(19.5)
Rent	1,397	1,835	199	1,067	1,514	1,154	1,337
	(30.3)	(32.7)	(11.6)	(25.9)	(50.6)	(25.8)	(27.6)
Repairs expense	824	865	349	1,239	202	804	974
	(17.8)	(15.4)	(20.2)	(30.0)	(6.8)	(18.0)	(20.1)
Water fee	250	533	222	2	0	80	249
	(5.4)	(9.7)	(12.9)	(...)	(0.0)	(1.8)	(5.1)
Miscellaneous	158	160	77	112	84	219	162
	(3.4)	(2.7)	(4.5)	(2.8)	(2.8)	(4.9)	(3.3)
Total	4,616	5,610	1,718	4,126	2,990	4,472	4,852
	(100)	(100)	(100)	(100)	(100)	(100)	(100)

Out of the total of N.T.$23,922 of operating cost of production per farm, man-labor expense accounted for about 45 percent. Fertilizer was second in rank, with a percentage of about 20 percent. Feed took about 18 percent of the total. Thus, these three cost items claimed more than 80 percent of the total operating cost of production. Feeder pig and animal labor costs made up 7 and 5 percent respectively. Other items including pesticides, medicines and miscellaneous expenses were insignificant. In the computation of total operating cost, all the items consisted of both payments in cash for purchased productive goods and services and estimated value of self-provided resources such as the value of natural fertilizer, self-provided seed and feed, operator's and unpaid family laborers.

With the total fixed cost of N.T.$4,616 per farm, rent of land, depreciations on farm buildings and equipment, payments of land taxes and repair expenses were the important items which on the average made up more than 90 percent of the total fixed cost. Depreciation was computed by the straight-line method in this study. On the average, approximately 5 percent of the total present value of buildings and equipment was charged as the annual depreciation expenses.

The variation of production costs from region to region is mainly due to the size difference of farm business and the intensity of resource use in each region. Figure 2 shows roughly a high relationship between cost of production and the acreage of crop area per farm among regions. Each dot in this figure represents average cost per farm in each region.

6. <u>Total cash operating cost of production</u>

The operating cost of production has been presented in the last section. In the computation of production costs, both purchased productive goods and services as well as self-provided productive goods and services were all included, in order to show the physical relationship between output and resource inputs. The self-provided goods and services included such items as self-produced seed or seedling, natural fertilizer produced on farm, feed provided by family, unpaid operator's and family laborers and some other operating items which were not purchased from outside of the farm. However, the total amount of self-provided or non-cash costs may be considered as an opportunity for farmers to fully use their resources and hence to increase family income. This is true particularly for those productive goods or services which do not have or hardly find a market for sale. Therefore, farmers in Taiwan usually concern only the cash expenses as an important portion of production cost. Table 8 shows the cash

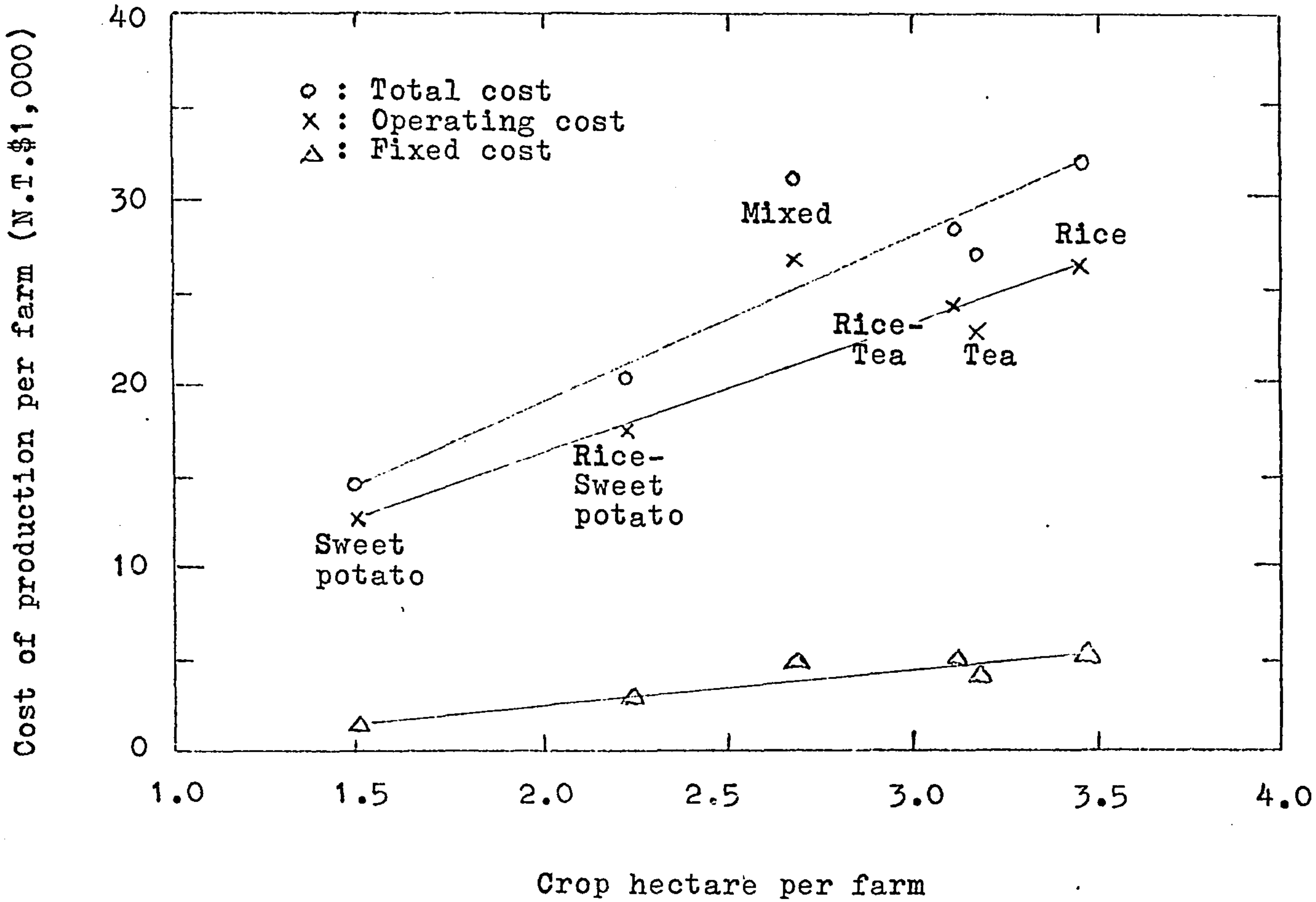

Figure 2. Relationship between cost of production and crop area per farm

Table 8. Cash operating cost of production per farm (Unit:N.T.$)

Items	Whole area	Agricultural regions					
		Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Cash operating cost:							
Seed or seedling	50	46	23	47	36	62	58
Livestock	911	1,072	746	693	893	676	1,178
Fertilizer	4,831	6,470	2,257	2,859	3,287	4,132	5,583
Feed	2,214	2,787	993	1,815	952	2,021	2,717
Hired labor	3,240	4,030	585	4,304	1,027	3,395	2,481
Pesticides & medicine	273	294	119	535	239	179	238
Miscellaneous	23	3	94	59	3	23	33
Total	11,542	14,695	4,817	10,313	6,437	10,488	12,288
Percentage of total operating cost	48%	56%	38%	45%	37%	44%	46%
Total farm expense	16,158	20,305	6,535	14,439	9,427	14,960	17,140

expenses out of total operating cost of production which were computed for different regions in this study. The average cash operating cost of production was estimated to be about N.T.$11,542 as compared with the total operating cost of N.T.$23,922. More than one-half of the total operating cost was self-produced on the farm or provided by the farm family on the average. The proportion of cash operating cost to total operating cost varied from 37 percent to 56 percent among regions. The groups of farmers in the Sweet potato and Rice-Sweet potato regions had the lowest percentage of cash operating cost, while those farmers in the Rice region had the highest percentage.

Total farm expenses shown in the last row of Table 8 represented the sum of cash operating cost and total fixed cost of production.

As pointed out previously, the most important item of production cost was labor. It accounted for about one-half of the total operating cost, or about one-fourth of total production cost on the average. Therefore, the total labor cost was further decomposed into unpaid operator's and family laborers and hired labor as shown in Table 9. On the average about two-thirds of the total labor used was contributed by the operator and members of his family. For those farms which produced a large amount of sweet potatoes, the operator and members of his family provided approximately 90 percent

Table 9. Unpaid family labor and hired labor cost (Unit:N.T.$)

Items	Whole area	Agricultural regions					
		Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Total man-labor cost:							
Unpaid family labor	7,799	6,580	4,877	9,227	7,680	8,634	9,068
Hired labor	3,073	3,888	574	4,004	996	3,119	2,419
Total	10,872	10,468	5,451	13,231	8,676	11,753	11,487
Total animal labor cost:							
Unpaid self-provided	994	1,440	606	559	661	982	654
Hired labor	167	142	11	300	31	276	62
Total	1,161	1,582	617	859	692	1,258	716
Total hired labor cost	3,240	4,030	585	4,304	1,027	3,395	2,481
Total labor cost	12,033	12,050	6,068	14,090	9,368	13,011	12,203
Hired labor/total labor ratio	27%	33%	10%	31%	11%	26%	20%

of the total amount of labor used. In Taiwan, sweet potatoes can be planted either in the spring or fall seasons, or even as a kind of inter-crop, depending on the availability of family labor and the adaptation of the cropping system. In other words, production of sweet potatoes may provide an opportunity to fully utilize family labor during the year in Taiwan. But for rice or tea, the situation is different. The harvesting and planting of rice are usually limited in time. Considerable amount of hired labor must be employed to supplement the family labor during the peak seasons. As for tea production, the tender leaf must be picked within a shorter period and thus hired labor is usually needed to supplement family labor during the peak of tea production.

C. Summary of Farm Business

Through previous sections, resource use, cost of production and farm returns has been discussed one by one for groups of farmers in different regions. A summary of the whole farm business for those average or representative farms of each region will be described in this section. Some points in the input-output relationships will be also emphasized for further study in the following chapters.

1. Farm returns measures

By taking account of both farm return and production cost simultaneously, the results of farm business or profitability of farming can be thus calculated. With different concepts of returns and costs, alternative measures of farm returns can be presented to provide different kinds of information on the outcome and feasibility of given practices of farming. Four such measures are to be defined in this study.

The first returns measure concerned is farm profit. It represents the difference between total value of farm production and total cost of production. All sales, increase in inventories, and the value of farm privileges are included in the credit side, while the total cost includes all values of purchased and self-provided factors of production. Consequently, the resulting figure shown as farm profit, is the net return from the operation of the whole farm business.

Farm income, the second measure concerned, in the strict sense of definition, is obtained by substracting the farm expenditure, i.e., the sum of cash operating cost and total fixed cost, from the sum of cash sales of farm products adjusted by the changes in inventories. However, the value of farm products being consumed by the farm family are not included as a source of income. Therefore, this measure indicates the amount of cash returns which is available for family living expenses and reinvestment in the farm business.

Farm earnings differ from farm income only by the amount of farm-raised products consumed in the household. Since the former includes the value of farm-raised products which are consumed in the household, the magnitude of farm income will be generally smaller than farm earnings. In those regions which are of high commercialization of farm product, i.e., a large portion of farm products are produced for and sold in the market, the difference between these two measures is usually smaller than those regions where a large portion of the farm products are consumed by the farm family. Therefore, the measure of farm earnings can be considered as an useful measure of successfulness of farm operation for those high subsistance or semi-subsistance farms. This measure, by the definition employed in this study, represents the total value of cash returns and farm products consumed in the household from the business operation which could be withdrawn without reducing the future scale of the farm business. In other words, it represents the cash returns and total value of farm products consumed by the family from the use of the farmer's own resources, such as his own capital, land, labor by himself and the members of his family as well as his management.

Family earnings are derived from gross farm receipts less farm expenditures plus off-farm income. It also can be obtained directly from farm earnings plus off-farm income. It includes not only the earnings from the farm business but also the proceeds of family labor from off-farm jobs.

These four measures of farm returns figures for the different groups of farms in each region for the year investigated in this study are shown in Table 10. In general, the group of farmers in the Rice region had the highest returns measured in farm profit and farm earnings, while the group of farmers in the Rice-Sweet potato and Rice-Tea regions had the highest farm income and family earnings respectively. Farmers in the Sweet potato region always had the lowest returns in all the returns measures. This was mainly due to the small scale of farm operation in this region.

The difference in returns figures among regions results from the efficiency of farm organization and technical operation. Some of the important measures of performance or efficiency factors had been derived to reflect the general efficiency of farm organization and operation in different regions. Those efficiency factors involved scale of operation, crop and labor efficiency factors and some cost ratios as shown in Table 10.

Table 10. Farm returns and efficiency factors of farm performance

Items	Whole area	Agricultural regions: Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Farm returns measures:							
1.Total value of farm production (N.T.$)	41,137	47,596	19,743	35,033	34,033	42,217	40,219
2.Gross farm receipts (N.T.$)	37,494	43,929	16,823	32,242	30,967	38,363	35,724
3.Farm profit (N.T.$)	12,599	15,597	5,425	8,018	13,508	13,787	8,862
4.Farm income (N.T.$)	9,353	10,704	2,387	8,650	11,332	10,772	5,551
5.Farm earnings(N.T.$)	21,336	23,616	10,908	17,804	21,844	23,403	18,584
6.Family " "	27,055	27,904	15,217	25,568	22,877	30,366	27,004
Measures of performance (efficiency factors):							
1.Scale of operations:							
Total cultivated ha.	1.67	1.87	0.78	2.17	1.11	1.89	1.46
Total crop ha.	2.91	3.45	1.50	3.17	2.22	3.13	2.69
Total capital(N.T.$)	110,668	112,620	56,150	126,982	109,654	130,112	76,580
Total man-work hours	4,170	4,450	2,080	4,640	3,280	4,280	4,190
2.Crop efficiency:							
Gross crop value per hectare (N.T.$)	17,360	18,624	18,017	11,604	18,278	15,270	19,613
Main crop yields(kg/ha.)							
Paddy rice	2,639	2,705	2,070	2,369	2,501	2,726	2,824
Sweet potatoes	13,849	15,373	13,724	10,141	8,627	14,843	15,036
3.Labor efficiency:							
Crop area per man-equivalent (ha.)	0.6613	0.7666	0.3061	0.7372	0.6166	0.7452	0.5977
Livestock production per man-equivalent (N.T.$)	2,761	2,838	1,161	2,248	3,818	3,180	2,574
Total value of production (N.T.$)	9,349	10,577	4,029	8,158	9,454	10,052	8,938
4.Cost ratios (per N.T.$100 of total value of production):							
Total operating cost ratio	58.15	55.44	63.82	65.38	51.53	56.74	65.90
Cash expense cost ratio	28.05	30.87	24.39	29.39	18.91	24.84	30.55
Farm expenditure ratio	39.27	42.66	33.10	41.16	27.69	35.43	42.61
Total cost per ha.	17,089	17,112	18,356	12,357	18,495	15,042	21,477
Total cost ratio	69.37	67.23	72.52	77.14	60.32	67.34	77.96

2. Scale of operation

The scale of operation or size of business is commonly considered as an important factor affecting farm returns. The first measure of size likely to occur to farmers in Taiwan is the acreage of the farm, either of total area of cultivated land or total crop area. The total cultivated land area is a satisfactory indication for comparing a given type of land and a given type of farming. However, this measure fails to indicate the intensive utilization of land resource. Therefore, total crop area can be used to supplement cultivated land area. Another measure of scale of operation is the total capital investment in the farm. This measure is most significant when all the farms compared produce essentially the same products. It may have little meaning for comparison if the farms have different specialities. Total productive man-work hours are also usually considered as a measure of business size of farm. As with most measures of efficiency, total productive man-work hours do not always give very exact expressions of size as between farms. Some products require the use of much more labor than others and different types of machinery and different cultural methods under various conditions of climate, soil, or topography. In this study, the size of business or scale of operation has been expressed in terms of resource inputs such as land, capital and labor.

3. Efficiency in the cropping system

Only two aspects of cropping system were concerned in this study. The first was its aggregate size. In our studied area, farms were dominated by crop production. The gross value of crops per hectare was thus used as an indicator of the aggregate size and composition of cropping system in each region. This term was computed by dividing the total value of all crops produced during the year by the cultivated acreage. Therefore, the gross crop value per hectare depended on the kind and combination of crops grown, the prices received, and the yields per hectare. Of course, it did not consider production costs.

The second aspect of crop efficiency concerned was the main crops yield. As stated previously, paddy rice and sweet potatoes were the dominant crops in the whole area studied. Even in the Tea region, farmers devoted more than one-half of their crop land for the production of rice and sweet potatoes. Therefore, yields of rice and sweet potatoes was used as measures of crop efficiency to supplement gross crop value.

4. Labor efficiency

Three ratios were employed in this study to reflect the average productivity of labor. However, no single ratio was entirely satisfactory when farming was well diversified and of a different type. Crop area per man-equivalent indicated how many crop hectares were operated by one man-equivalent. This measure did not serve satisfactorily if the kinds of crops varied greatly from region to region or if some farms grew mainly crops while others had a smaller crop acreage but a larger production of livestock. Therefore, total value of livestock production per man-equivalent was also computed to supplement the crop hectares per man. Together the two gave a better indication of labor productivity than either one alone. A better common indicator of labor productivity is the gross value of production per man. It was computed by dividing total value of production by the number of man-equivalents.

5. Cost ratios

Most of the ratios or efficiency factors discussed so far were designed to indicate strong or weak points in the organization or operation of the farm business and to call attention to the specific phase or areas of the business where greater managerial attention was needed. In addition, there are other ratios that are often used in more general analysis. Among the different aspects reflected by these general ratios, the most important is the cost ratios which are used to determine whether costs are high or low related to specific type of farming. The success of farming is determined as much on the income side as on the cost side. Cost ratios are averages, and their magnitude reflects physical production efficiency, selection of enterprises, prices received for commodities, and the expense for the production elements. As averages for the entire business they do not relate the added cost to the added revenue of an increment of production.

The first measure of cost ratios employed in this study was the total operating cost ratio. It is the percentage which total operating cost absorbs out of the total value of production. However, the cash operating cost ratio indicated only the percentage of cash operating cost rather than total operating cost out of total value of production. It showed the total value of production used in (1) hiring labor, (2) buying small animals, feeds, seeds, fertilizers, and other production supplies during the year, excluding the imputed value of family labor and self-provided cost items. The third measure of cost ratio was farm expenditure ratio which represented the percentage of both cash operating and fixed costs out of total value of production. Both total operating and fixed cost ratios and cost per hectare were also computed by dividing the total cost (total operating and fixed) by the total value of production and the number of cultivated land hectare respectively in this study.

Through the above information, the measure of farm earnings is chosen as the indicator of farm returns for the group of farmers in each region. The variation of farm earnings among regions is mainly due to the scale of farm business. Figure 3 shows the average relationship between farm return and crop acreage per farm for each region. Each dot represents the average farm earnings per farm against the total crop hectare per farm for the region as given in the figure. Average farm earnings and total crop hectare per farm for all 400 farms in the whole area are also shown as the horizontal and vertical lines. On the average, the groups of farmers in the Sweet potato and Mixed regions had both below average farm earnings and total crop hectare, while the groups of farmers in the Rice-Tea and Rice regions had both above average farm earnings and total crop hectare. The group of farmers in the Rice-Sweet potato region had about average farm earnings but considerably below average of total crop hectare per farm. On the other side, the group of farmers in the Tea region had above average farm earnings but below average total crop hectare.

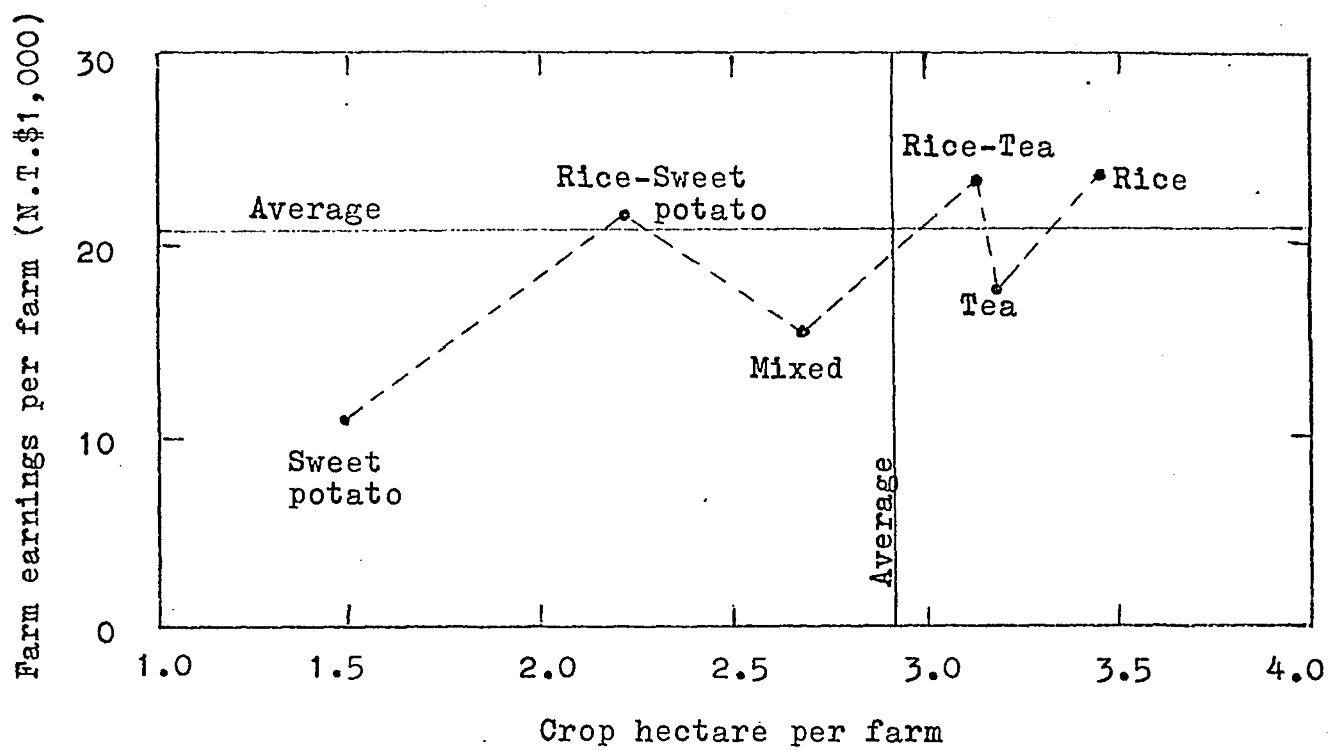

Figure 3. Relationship between farm earnings and crop area per farm

It is also known that farmers in the Rice-Sweet potato, Rice-Tea, and Rice regions have higher farm earnings than those farmers in the Sweet potato, Mixed and Tea regions. These two groups of farmers, in general, will have higher farm earnings if the total crop hectare per farm increases. Therefore, the primary observation may provide the conclusions that the farm earnings per farm will be higher as the crop acreage is expanded, and farmers who raise more paddy rice will have higher farm earnings than the farmers who are in the Sweet potato, Tea or in the Mixed regions.

The relative high position of farm earnings for those farmers in the Rice-Sweet potato region compared with farmers in the Sweet potato and Mixed regions was mainly due to the relative high efficiency of labor, particularly, the highest livestock production per man-equivalent, which is generally not effected by the size of crop area. On the other hand, the relative low farm earnings in the Tea region was largely due to the low efficiency in crop production. This low efficiency in crop production was largely due to the low price level of tea and the low productivity of land which can be mainly explained by the low capital-land ratio and low labor-land ratio in the Tea region.

III. ESTIMATION OF PRODUCTION FUNCTIONS

In the previous chapter, various important factors, such as the scale of farm, crop and labor efficiency, cost ratios, important to successfulness of farm operation, have been pointed out. These factors are relevant to farm organization in the various agricultural regions. Tabular methods were employed extensively in the analysis of the last chapter. In this chapter, we are now interested in the estimation of production functions based on cross-sectional survey data for each agricultural region.

A. Basic Concept of Production Function

A production function may be broadly defined as the transformation of resource inputs into product outputs. Technical possibilities are open to firms and define the particular manner in which the resource inputs can be converted into product outputs (Hicks 1953). In other words, the production function may be considered as a technical or engineering relation between inputs and outputs. As long as the natural laws of technology remain unchanged, the production function is also unchanged. Based on the cross-sectional data, we may reasonably assume that the technical possibilities are open to all individual firms, and thus the production function can be the same for each firm.

There are, in general, two different kinds of production functions which can be estimated from a group of firms; i.e. cross-sectional data; namely, aggregate production functions and average production functions. The former includes estimate of the total output of the whole group, given the total quantity of various productive factor inputs used by the firms. On the other hand, the average production function is one representative of the individual firm in the group. This study is concerned with average production functions for different group of farm-firms.

It has been argued by Soper that, under the assumptions of "ideal conditions" and profit maximization, the identification of the parameters of average production function from cross-sectional survey data by ordinary least-squares method is impossible (Soper 1958). However, the assumptions imposed by Soper, that all individual production functions are "identical" and have the same form as that of the average function, that producers maximize their profits exactly, and that resources are used in proportionate fixity, are too strong to apply to real-world data. As pointed out by Konijn, if these assumptions do not hold true, it is still possible to derive a meaningful average production function estimate (Konijn 1959 and Soper 1959).

B. Economic Model

In the study of production function analysis, a first step in estimation is the determination of the algebraic form of the production function which appears or is known to be consistent with the phenomena of the functional relationship between inputs and output under investigation. Appropriate algebraic form of production function may be selected by the information from previous investigations and the theories of the sciences involved in the study (Heady and Dillon 1961). The Cobb-Douglas production function was employed in this study for the different agricultural regions. Some of the important characteristics of this type function should be carefully examined in order to know whether this function is appropriate and consistent with the factual situation under investigation. The well known form of the Cobb-Douglas production function can be generalized as shown in Equation 3.1 (Cobb and Douglas 1928).

$$Y = AX_1^{\beta_1} X_2^{\beta_2} \ldots\ldots X_m^{\beta_m}$$

$$\text{or } Y = A \prod_{i=1}^{m} X_i^{\beta_i} \qquad (3.1)$$

where Y is the physical output, X_i (i=1,2,.....,m) are the relevant physical inputs in the production procession, A and

β_i are constants. The properties of this function are summarized as follows:

1. The β_i's are the production elasticities with respect to productive input, X_i, respectively.
2. The function is homogeneous of degree $\sum_{i=1}^{m} \beta_i$, the sum of all production elasticities. If the sum of production elasticities is greater than, equal to, or less than one, then there is, correspondingly, increasing returns to scale, constant returns to scale, or decreasing returns to scale.
3. Marginal physical productivity of the ith input, X_i, declines if $\beta_i<1$ as the quantity of ith input is increased; or specifically, $\frac{\partial^2 Y}{\partial X_i^2} = \beta_i(\beta_i-1)\frac{Y}{X_i^2}$ is negative if $\beta_i< 1$.
4. The marginal rate of substitution between any inputs is $\frac{\partial X_i}{\partial X_j} = \frac{\beta_j X_i}{\beta_i X_j}$, a linear function passed through the original, and so the elasticity of substitution is equal to one (Allen 1953, p. 343).
5. The expansion line is a straight line through the origin, i.e., it is a scale line, and so it assumes the combination of resources which gives the least

cost for one level of output must also give the least cost for all other levels of output.

It should be noted that the Cobb-Douglas function indicates only either one of the three types of returns to scale for a particular production phenomenon; it does not allow a production function embracing two or more different returns to scale simultaneously.

Based on the Cobb-Douglas production function, the marginal productivity of any one particular resource, holding others as being given, is represented by Equation 3.2,

$$\frac{\partial Y}{\partial X_i} = B_i A X_1^{\beta_1} X_2^{\beta_2} \ldots . X_i^{\beta_i - 1} \ldots . X_m^{\beta_m}$$

$$\text{or} \quad \frac{\partial Y}{\partial X_i} = \beta_i \frac{Y}{X_i}, \quad i = 1,2,\ldots..m \tag{3.2}$$

and the profit maximization by firms can be expressed in Equation 3.3.

$$\beta_i \frac{Y}{X_i} = \frac{P_i}{P_y} \tag{3.3}$$

or in alternative form as

$$\beta_i \frac{YP_y}{X_i P_i} = 1 \quad , \quad i = 1,2,\ldots..m \tag{3.3'}$$

where P_y and P_i represent the price of output Y and the price of the ith input respectively, and assumed to be the same constant to all individual firms under competition. Equation

3.3 is the usual marginal condition for profit maximization which states that the marginal product of a given input, while other inputs are all constant, be equal to the input-output price ratio, or in other words, the value of the marginal product must be equal to the price of factor input. Equation 3.3' merely expresses that the profit maximization implies that one dollar marginal product of each input should be equal one dollar worth of the input used.

Equation 3.3, or its alternative Equation 3.3', involves implicitly the assumption of decreasing returns to scale or increasing marginal cost situation. Since decreasing returns to scale is the second order condition for a maximization.

Furthermore, it is apparent that Equation 3.3, or 3.3' implies that the individual firm is exactly at the equilibrium point of maximum profit. The production scale will be expanded as far as marginal revenue is greater than marginal cost. However, some factors such as risk and uncertainty, individual expectations and preferences, institutional restrictions, time lags, capital or other resource limitations, or even mistakes, etc. could affect the decision behavior of any individual firm. Therefore, a more general statement of the firm's decision process can be presented as shown in Equation 3.4 instead of Equation 3.3'.

$$\beta_i \frac{YP_y}{X_i P_i} = R_i \quad , \quad i = 1,2,\ldots m \tag{3.4}$$

where R_i is some constant value not necessarily equal to unity. Equation 3.4 states that firm's equilibrium position has been restricted by some factors mentioned above and the restricted maximum profit is attained while the firm sets the value of marginal product equal to the price of the factor input times some constant. If the value of R_i is one, then the restricted profit maximization becomes the same as unrestricted profit maximization as shown in Equation 3.3'.

C. Statistical Model

The economic model consisting of Equations 3.1 and 3.3 or 3.4 in more general expression, can be transformed into a statistical framework by introducing random disturbances, U and V, into each equation and can be also transformed with ease into a linear function by converting all variables to logarithms.

The statistical equation system corresponding to the economic model of Equations 3.1 and 3.4 is shown in Equations 3.5 and 3.6. While the Equations 3.7 and 3.8 represent the transformation of all variables in the system into logarithmic form.

$$Y = A\prod_{i=1}^{m} X_i^{\beta_i} U \qquad (3.5)$$

$$X_i = \beta_i \frac{YP_y}{R_i P_i} V_i \quad , \quad i = 1,2,\ldots,m \qquad (3.6)$$

$$y = a + \sum_{i=1}^{m} \beta_i x_i + u \qquad (3.7)$$

$$x_i = k_i + y + v_i \quad , \quad i = 1,2,\ldots,m \qquad (3.8)$$

Where $y=\log Y$, $a=\log A$, $x_i=\log X_i$, $u=\log U$,

$v_i=\log V_i$, $k_i=\log (\beta_i P_y/R_i P_i)$

The logrithmic form of production function is a linear form as shown in Equation 3.7. A linear function, in a statistical sense, is linear in parameters, but not necessarily in the variables. One of the advantages of using a Cobb-Douglas function is it is economical in the use of degrees of freedom, or parameters, and yet gives us a nonlinear relationship in original form as shown in Equation 3.5.

There are generally three possible, though not mutually exclusive, components in the random disturbance terms. The first kind of error component is due to the unpredictable element of randomness in human behavior which can be only adequately characterized by the inclusion of a random variable term. Another kind of error is so called errors of observation. Since all the observation of variables in our model were obtained from personal interviews conducted by a farm survey, this source is of inaccurate. We have, however, no norm with which to evaluate them and their significance should not be underestimated. In addition to errors of random factors and observation, there is so called error of specification. This error is caused by the omission of variables and by the approximation or over-simplified nature of the model structure. Such kind of error is never absent in actual economic analysis: the only model of an economy that would be perfect in this aspect is the economy itself.

D. Methods of Estimation

1. Single equation estimation

The single equation least-squares estimation has been traditionally used for estimating the parameters of a linear production function such as Equation 3.7. In employing the method of least-squares for the estimation of coefficients in a single equation implies the assumptions that the explanetory variables, i.e., x_i's in Equation 3.7, are statistically independent of the random disturbance term u. Hence, according to the Gauss-Markoff theorem, given the values of the independent variables x_i, the least-squares method is the best linear unbiased estimate of the coefficients in Equation 3.7 under the assumptions of the random disturbance independently distributed with zero mean and constant variance (David and Neyman 1938, Mood and Graybill 1963). However, in the analysis of the production function, the "statistically independent" assumption in the explanatory variables may not be always true. Output, land used, labor and capital inputs are all endogeneous variables subject to simultaneous entreprenenrial decision as shown in the system of Equations 3.7 and 3.8. It is apparent, as shown in Equation 3.8, that the x's are correlated with the u's, indirectly through Equation 3.7. This contradicts the assumptions underlying single least-squares equations since the presumed independent variables are in fact correlated with the distrubance term in the production function equation. If we made the crucial assumption that the disturbance in the production equation affects only the output and is not transmitted to the other variables in the system, then there is not so called "simultaneous equation bias", and single least-squares equation estimates are appropriate (Hoch 1958 and 1962). This can be done by arguing that firms do not maximize profit by differentiating current output with respect to input but rather differentiate "expected" output with respect to input. Expected output can be defined as $A\prod_{i=1}^{m} X_i^{\beta_i}$ and expressed as E(Y). Therefore, the marginal condition Equation 3.6 can be replaced by Equation 3.9 and thus the observed values of X_i's are not correlated with the production disturbance. Hence the assumption of statistical independence of single equation estimation is not contradicted.

$$P_y \frac{\partial E(Y)}{\partial X_i} \cong P_y \frac{\partial \left(A \prod_{i=1}^{m} X_i^{\beta_i} \right)}{\partial X_i} = \frac{P_i R_i}{V_i} \tag{3.9}$$

* In the case of agriculture, the disturbance U probably includes the effects of weather variability, and it can be argued that these effects do not affect the level of inputs used, X, by farmers. The farmers' decision process may be looked at as selecting inputs for an expected or anticipated level of output with input levels unaffected by "good" or "bad" weather. This argument leads some support to use single equation least-squares estimation of the production function.

2. Klein's estimation

Based on the decision function of Equation 3.3, i.e., the assumption of unrestricted profit maximization, Klein developed a straightforward method to estimate the parameters of the production function under the assumption that the logarithm of the disturbance is distributed independently among firms, with mean equal to zero (Klein 1953, pp. 193-196).

From Equation 3.3', we may introduce an "economic" disturbance term, V, and make it into a statistical equation as shown in Equation 3.10.

$$\beta_i = \frac{X_{ij}P_i}{Y_jP_y} \cdot V_{ij} \quad , \quad \begin{matrix} i = 1,2,\ldots\ldots,m \\ j = 1,2,\ldots\ldots,n \end{matrix} \tag{3.10}$$

or expressing in logarithmic form:

$$\log \beta_i = \log \frac{X_{ij}P_i}{Y_jP_y} + \log V_{ij} \tag{3.11}$$

Assuming the logarithms of disturbance V_{ij} to be normally distributed about a zero mean value, then the estimates of β_i are:

$$\text{est} \log \beta_i = \frac{1}{n} \sum_{j=1}^{n} \log \left(\frac{X_{ij}P_i}{Y_jP_y}\right) + \frac{1}{n} \sum_{j=1}^{n} \log V_{ij}, \quad i = 1,2,\ldots\ldots,m \tag{3.12}$$

where n is the total number of firms observed. The last term on the right-hand side in Equation 3.12 is equal to zero by assumption, and thus the logarithm of the elasticity coefficient of production, β_i, is estimated as the arithmetic mean of logarithms of input factor shares. Alternatively, we may view the estimate coefficients to be geometric means as shown in Equation 3.13.

$$\text{est } \beta_i = \left(\prod_{j=1}^{n} \frac{X_{ij}P_i}{Y_jP_y}\right)^{\frac{1}{n}} \quad , \quad i = 1,2,\ldots\ldots,m \tag{3.13}$$

Klein has pointed out that if we assume the disturbance has the logarithmic normal distribution, then Equation 3.13

is the best linear unbiased estimate of parameter coefficients of the production function.

Klein's estimates are based on the assumption that the average firm is on the exact optimal position. Therefore, if the average firm is not optimal or when one of the inputs is predetermined, Equation 3.13 leads to biased and inconsistent estimates. However, the bias appears only with respect to the coefficient corresponding to the predetermined input.

3. Hoch's estimation

It was pointed out previously in the economic model that the conditions of profit maximization and the technical production function fully determine the equilibrium position of a firm that operates under condition of perfect competition in the product market, obtains its inputs at fixed prices, and experiences decreasing returns to scale. If all the relationships hold exactly, then all the firms in the industry will be producing identical quantities of output and will be employing identical quantities of inputs, providing the inputs are freely variable and substitutable. Variations from firm to firm will exist if one or more of the inputs are fixed; and hence the profit maximizing quantities of output and inputs will depend on the amount of fixed input or inputs in each firm. However, if the production function as well as the profit-maximizing decision functions contain disturbance terms, differences in actual outputs and inputs of firms will appear even in the case of absence of fixed factors of production. In this case a solution of the system of equations, Equation 3.5 and 3.6 in our case, shows that the quantities of output and inputs of any firm is a function of all disturbances in the system. Hence, the inputs are not independent of the disturbance in the production function, and single equation least-squares estimates of the production function parameters based on cross-sectional data will be, in general, biased and inconsistent. This situation was first pointed out in a classical article by Marschak and Andrews in 1944 (Marschak and Andrews 1944). Alternative methods of estimation have since been proposed. One of these proposals is developed recently by Hoch in 1958, and examined the small sample properties by a Monte Carlo study by Kmenta and Joseph in 1963 (Kmenta and Joseph 1963). A summary of Hoch's estimation is briefly presented as follows.

Consider the equation system shown in Equations 3.7 and 3.8 based on Equations 3.5 and 3.6. Equations 3.7 and 3.8 can be referred to as our starting structural equations. The solution of this structural system is shown in Equations 3.14 and 3.15.

$$y^* = K_y + \frac{1}{D} \left(\sum_{i=1}^{m} \beta_i v_i + u \right) \tag{3.14}$$

$$x_i^* = K_i + \frac{1}{D}\left(\sum_{i=1}^{m} \beta_i v_i + u\right) + v_i \quad , \ i = 1,2,\ldots\ldots,m \qquad (3.15)$$

where $D = 1 - \sum_{i=1}^{m} \beta_i$, K_y and K_i are constants and are the equilibrium values of y and x_i. y^* and x_i^* are presumably the observed values of each firm by the investigator. Since the disturbance u appears in the observed values of the "independent" variables as shown in Equation 3.15, therefore, single equation least-squares estimates contradict the basic assumption of statistically independent input variables and thus leads to bias.

If we assume that the disturbance in the production function is not correlated with the disturbances in the marginal decision equation, and the disturbance in a given marginal decision equation is not correlated with the disturbance in any other marginal decision equation, i.e., $E(uv_i) = 0$, $i=1,2,\ldots\ldots,m$, and $E(v_i v_j) = 0$, $i \neq j$, $i,j = 1,2,\ldots\ldots,m$ where assuming $E(u)=E(v_i)=0$, then Hoch's estimates of production parameters, $\hat{\hat{\beta}}_i$, are given by Equation 3.16.

$$\hat{\hat{\beta}}_i = \hat{\beta}_i \left[1 + \hat{\hat{\sigma}}_u^2 \sum_{i=1}^{m} \frac{1}{\hat{\sigma}_{v_i}^2}\right] - \frac{\hat{\hat{\sigma}}_u^2}{\hat{\sigma}_{vi}^2} \qquad (3.16)$$

where $\hat{\beta}_i$ is the ordinary least-squares estimate of β_i, and $\hat{\hat{\sigma}}_u^2$ is the Hoch's estimate of the disturbance variance in production function, i.e. $E(u^2)$, which can be obtained by Equation 3.17.

$$\hat{\hat{\sigma}}_u^2 = \frac{\hat{\sigma}_u^2}{1 - \hat{\sigma}_u^2 \Sigma \frac{1}{\hat{\sigma}_{vi}^2}} \qquad (3.17)$$

In Equation 3.17, $\hat{\sigma}_u^2$ and $\hat{\sigma}_{vi}^2$ are the ordinary least-squares estimates of disturbance variances and can be obtained from observable sample moments as shown in Equations 3.18 and 3.19.

$$\hat{\sigma}_u^2 = C_{yy} - \sum_{i=1}^{n} \beta_i C_{yi} \qquad (3.18)$$

$$\hat{\sigma}_{vi}^2 = C_{yy} + C_{ii} - 2C_{yi} \quad , \quad i=1,2,\ldots,m \qquad (3.19)$$

where C_{yy} is the sample variance of y, C_{ii} is the sample variance of x_i, and C_{yi} is the sample covariance of y and x_i.

Hoch's also points out a similar result if some inputs are predetermined. If we assume $x_1, x_2, \ldots, x_R$ to be endogenous, and let $x_{R+1}, \ldots\ldots, x_m$ be predetermined, without an economic disturbance in the marginal decision equation and taken as exogenously determined, then Hoch's estimates are obtained as shown in Equations 3.20 and 3.21 for those production coefficients corresponding to random inputs and fixed inputs respectively.

$$\hat{\hat{\beta}}_r = \hat{\beta}_r \left[1 + \hat{\hat{\sigma}}_u^2 \sum_{r=1}^{R} \frac{1}{\hat{\sigma}_{v_r}^2} \right] - \frac{\hat{\hat{\sigma}}_u^2}{\hat{\sigma}_{v_r}^2} \quad ; \; r = 1,2,\ldots,R \qquad (3.20)$$

$$\hat{\hat{\beta}}_f = \hat{\beta}_f \left[1 + \hat{\hat{\sigma}}_u^2 \sum_{r=1}^{R} \frac{1}{\hat{\sigma}_{v_r}^2} \right] \qquad (3.21)$$

$$\hat{\hat{\sigma}}_u^2 = \frac{\hat{\sigma}_u^2}{1 - \hat{\sigma}_u^2 \sum_{r=1}^{R} \frac{1}{\hat{\sigma}_{v_r}^2}} \qquad (3.22)$$

IV. EMPIRICAL PRODUCTION FUNCTIONS

The model used for empirical analysis in this study has been discussed in the previous chapter. The parameters of an "average" or "representative" production function of the Cobb-Douglas form were estimated for the group of random farms in each agricultural region.

Single equation least-squares method has been employed for estimating the parameter coefficients of production functions for those six agricultural regions. However, the Klein's and Hoch's estimations are also used as well as least-squares method for the Rice region in order to make a tentative comparison among these three methods of estimation.

A. Statistical Estimation and Procedures

The data used in this study for the production function analysis were the same as presented in earlier chapters, with minor modification in order to fit our purposes. The form of the production function is assumed to be the same for all agricultural regions in the Cobb-Douglas type, as shown again in Equation 4.1.

$$Y = A\, X_1^{\beta_1}\, X_2^{\beta_2}\, X_3^{\beta_3} \,\ldots\ldots X_m^{\beta_m} \qquad (4.1)$$

After deciding the form of the production function and the variables to be included in the function, we come to the problem of statistical estimation. We may first transform the above function into a statistical equation and consider that the variable Y depends upon these m explanatory factors X_1, X_2, and X_3, etc., such that

$$Y_j = A\, X_{1j}^{\beta_1}\, X_{2j}^{\beta_2}\, X_{3j}^{\beta_3} \cdots X_{mj}^{\beta_m}\, U_j \qquad (4.2)$$

$$j = 1,2,3,\ldots n$$

where U_j denotes a disturbance term for the jth farm, reflecting the stochastic nature of the relationship. This relationship can be easily transformed into one which is linear in the parameters, but not necessarily in the variables, by taking logarithms for each variable in Equation 4.2 as represented in Equation 4.3.

$$y_j = \alpha + \beta_1 x_{1j} + \beta_2 x_{2j} + \beta_3 x_{3j} + \cdots + \beta_m x_{mj} + u_j \qquad (4.3)$$

$$j = 1,2,3,\ldots,n$$

where y,x's and u stand for log Y, log X's and log U respectively.

If we now apply the least-squares method to Equation 4.3 as it stands, we can obtain the estimates of α and β_i, i=1,2, 3,....m, which are such that

$$\sum_{j=1}^{n} (y_j - \alpha - \beta_1 x_{1j} - \beta_2 x_{2j} - \beta_3 x_{3j} - \cdots - \beta_m x_{mj})^2$$

is minimized. Traditional F and student-t tests are then available to make the tests of significancies of the multiple correlation coefficients and the production coefficients respectively (Anderson and Bancroft 1952). The strict validity of such tests depends upon the following assumptions:

1. The disturbance term is a random variable distributed normally with zero mean and constant variance for all farms. The later property is usually referred to as "homoscedasticity" and it implies that the probability of a discrepancy of a given size occurring is independent of the level of output and is constant all along the production function.
2. The disturbances are serially independent. For time series data on a given farm this implies that the production discrepancy in a given period is independent of that of other periods. In the case of this study based on cross-sectional data, it can be explained that the production discrepancy of a particular farm is independent of other farms. If this condition is not fulfilled, we then have an autocorrelated disturbance situation.
3. The disturbance term is distributed independently of the explanatory variables x_i, $i = 1,2,3,\ldots,m$.

Based on these assumptions, the least-squares estimates of production coefficients are best linear unbiased, and can be expressed in matrix notation as shown in Equation 4.4.

$$\hat{\beta} = Y'X(X'X)^{-1} \qquad (4.4)$$

where $\hat{\beta}$ is the estimate of the parameter vector of production coefficients, Y and X are the observed "dependent" variable vector and "independent" variables matrix in terms of logarithmic form respectively. In general, $\hat{\beta}$ is a (m+1)-element column vector, Y is a n-element column vector and X is a n by (m+1) order matrix, with 1 in the first column.

The variance of $\hat{\beta}$ is given by

$$\mathrm{var}(\hat{\beta}) = \sigma^2 (X'X)^{-1} \qquad (4.5)$$

where σ^2 is the variance of u_j for all j. The unbiased estimator of σ^2 is given by Equation 4.6.

$$\text{est } \sigma^2 = \frac{Y'Y - \hat{\beta}' X'Y}{n - m - 1} \qquad (4.6)$$

where n is the total number of observations and (m+1) is the total number of parameters involved in the production function. In our case, n stands the total number of sample farms in each agricultural region, and m is the number of resource categories used in production function.

The hypotheses that in the population the multiple correlation coefficient is zero can be tested by using Snedcor's F-test as:

$$F = \frac{R^2 / m}{(1 - R^2) / (n - m - 1)} \tag{4.7}$$

with (m, n-m-1) degrees of freedom. The production coefficients can be tested by Student's "t" as:

$$t = \frac{\hat{\beta}_i - \beta_i}{(\text{var } \hat{\beta}_i)^{1/2}} \tag{4.8}$$

with (n-m-1) degrees of freedom. If the hypotheses is that the population coefficient is zero, then the test is simply as:

$$t = \frac{\hat{\beta}_i}{(\text{var } \hat{\beta}_i)^{1/2}} \tag{4.9}$$

B. Alternative Grouping of Resources and Estimation

Three different sets of production functions based on different groupings of resource inputs were estimated from the sample data for those six agricultural regions. Of these three sets of production functions, the third set is logically and statistically most acceptable. In outline form, the functions are as follows:

First set: $Y = A\, X_1^{\beta_1}\, X_2^{\beta_2}\, X_3^{\beta_3}\, X_4^{\beta_4} U$

Variables involved in the function are:

Y is the value of gross product or the total value of production for each farm and measured in money terms by N.T. dollar. Gross product includes the total value of crops and livestock produced on farm during the investigation period. X_1 is the quantity of land used for the production of crops on farm and usually referred as the "cultivated land area". It is measured by "are" which is 1/100 hectare (1 hectare is equal to 2.471 acres).

X_2 is the quantity of labor used on crops and livestock production. It consists of operator's and family labor as well as the temporary hired labor and is measured in man-work days. Ten hours of productive work on the farm were considered as one man-work day.

X_3 is the amount of expenses on crops and livestock production measured in N.T. dollar. It consists of the items such as seeds, fertilizers, pesticides, feeds and other miscellaneous crop and livestock outlays. Generally speaking, operating expenses are the variable expenditure which are more or less connected with the total product of crops and livestock; it does not, however, include labor inputs.

X_4 is the total amount of depreciation and repairs of farm buildings and farm equipment, the depreciation cost on working cattle, rent for land, taxes, water fee and other miscellaneous outlays.

Second set: $Y = A X_1^{\beta_1} X_2^{\beta_2} X_3^{\beta_3} X_4^{\beta_4} U$

where all the variables, Y, X_1, X_2 and X_3 are exactly the same as indicated in first set. However, the variable X_4 in second set includes only depreciation, repairs and miscellaneous outlays. Rent for land, land tax and water fee which were presented in first set are not included in X_4 of this set.

Third set: $Y = A X_1^{\beta_1} X_2^{\beta_2} X_3^{\beta_3} U$

where variables, Y, X_1, and X_2 are still exactly the same as presented in first set or in second set. But the variable X_3 represents the sum of variables X_3 and X_4 presented in second set, and denoted as capital input measured in N.T.$.

The empirical production functions or regression equations estimated for these three sets of alternative groupings of resource inputs in each region are as follows:

First set:

Rice region $\hat{Y} = 47.98\ X_1^{0.2204}\ X_2^{0.3969}\ X_3^{0.3189}\ X_4^{0.0318}$

Sweet potato region $\hat{Y} = 160.15\ X_1^{0.1405}\ X_2^{-0.0113}\ X_3^{0.2459}\ X_4^{0.2776}$

Tea region $\hat{Y} = 240.95\ X_1^{0.5155}\ X_2^{-0.1276} X_3^{0.2493} X_4^{0.0902}$

Rice-Sweet potato region $\hat{Y} = 51.64\ X_1^{-0.0311}\ X_2^{0.5812}\ X_3^{0.3625}\ X_4^{0.0021}$

Rice-Tea region $\hat{Y} = 22.35\ X_1^{0.1142}\ X_2^{0.3334}\ X_3^{0.4879}\ X_4^{0.0448}$

Mixed region $\hat{Y} = 8.62\ X_1^{0.1462}\ X_2^{0.4211}\ X_3^{0.4535}\ X_4^{0.0941}$

Second set:

Rice region $\hat{Y} = 47.40\ X_1^{0.2230}\ X_2^{0.4162}\ X_3^{0.3183}\ X_4^{0.0230}$

Sweet potato region $\hat{Y} = 127.14\ X_1^{0.3520}\ X_2^{-0.0446}X_3^{0.1826}\ X_4^{0.3224}$

Tea region $\hat{Y} = 203.13\ X_1^{0.5143}\ X_2^{-0.1106}X_3^{0.2530}\ X_4^{0.1068}$

Rice-Sweet potato region $\hat{Y} = 48.04\ X_1^{-0.0400}\ X_2^{0.6281}\ X_3^{0.3911}\ X_4^{-0.0585}$

Rice-Tea region $\hat{Y} = 25.89\ X_1^{0.1233}\ X_2^{0.3681}\ X_3^{0.5227}\ X_4^{-0.0448}$

Mixed region $\hat{Y} = 10.55\ X_1^{0.1898}\ X_2^{0.4735}\ X_3^{0.4808}\ X_4^{-0.0206}$

Third set:

Rice region $\hat{Y} = 41.88\ X_1^{0.2213}\ X_2^{0.4341}\ X_3^{0.3339}$

Sweet potato region $\hat{Y} = 138.67\ X_1^{0.2489}\ X_2^{0.0636}\ X_3^{0.4014}$

Tea region $\hat{Y} = 46.41\ X_1^{0.3948}\ X_2^{-0.0017}\ X_3^{0.4872}$

Rice-Sweet potato region $\hat{Y} = 48.74\ X_1^{-0.0169}\ X_2^{0.5440}\ X_3^{0.3825}$

Rice-Tea region $\hat{Y} = 15.12\ X_1^{0.1041}\ X_2^{0.3984}\ X_3^{0.5233}$

Mixed region $\hat{Y} = 7.70\ X_1^{0.1841}\ X_2^{0.4816}\ X_3^{0.4866}$

All these equations were estimated by the least-squares method. Table 11 presents the elasticity or regression co-

Table 11. Statistics for first estimate of production functions

Items	Agricultural regions: Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Number of sample farms (n)	138	18	48	38	100	58
Value of constant (log A)	1.6810	2.2045	2.3819	1.7130	1.3493	0.9353
Value of production coefficients:						
Land	0.2204	0.1405	0.5155	-0.0311	0.1142	0.1462
Labor	0.3969	-0.0113	-0.1276	0.5812	0.3334	0.4211
Operating expense	0.3189	0.2459	0.2493	0.3625	0.4879	0.4535
Fixed capital	0.0318	0.2776	0.0902	0.0021	0.0448	0.0941
Sum of production coefficients	0.9680	0.6527	0.7274	0.9147	0.9803	1.1149
Value of standard error:						
Land	0.0455	0.2849	0.0873	0.1670	0.0411	0.0892
Labor	0.0704	0.2872	0.1468	0.2275	0.0754	0.1606
Operating expense	0.0464	0.1540	0.0545	0.0799	0.0458	0.0829
Fixed capital	0.0274	0.1856	0.0748	0.0511	0.0356	0.0680
t-value:						
Land	4.84	0.49	5.90	-0.19	2.78	1.64
Labor	5.64	-0.04	-0.87	2.55	4.42	2.62
Operating expense	6.88	1.59	4.58	4.53	10.65	5.47
Fixed capital	1.16	1.49	1.21	0.04	1.26	1.38
Value of R^2	0.9140	0.7799	0.7618	0.8098	0.9117	0.9044

efficients, along with other statistics of interest in this analysis for the first set of equations in each region. Because of large error of estimate, the low value of t and a relative high correlation between land and fixed capital variables in some regions, some of items which originally were included in the variable X_4 (fixed capital) were dropped from the second set of production functions. These items include rent for land, land taxes, water fees and other expenses having a close relation with land area.

Table 12 shows the statistics for the second set of equations. The result was deemed not entirely satisfactory. Only 13 out of 24 production coefficients are significant at the probability level of 5-percent, and numerous coefficients of production are negative in value. Those negative coefficients or elasticities of production are hardly conceivable. Also, the elasticities of "fixed capital" in every region are not significantly different from zero at 10-percent significant level. This indicates that the fixed capital factor is not an important input affecting farm production within the observation of data in this study. In the current situation of farming in Taiwan, particularly in the short run, it is quite possible that farmers carry out their production plan without taking account of how much quantity they have in the form of "fixed capital". The total amounts

Table 12. Statistics for second estimate of production functions

Items	Agricultural regions					
	Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Number of sample farms (n)	138	18	48	38	100	58
Value of constant (log A)	1.6758	2.1042	2.3078	1.6816	1.4131	1.0232
Value of production coefficients:						
Land	0.2230	0.3520	0.5143	-0.0400	0.1233	0.1898
Labor	0.4162	-0.0446	-0.1106	0.6281	0.3681	0.4735
Operating expense	0.3183	0.1826	0.2530	0.3911	0.5227	0.4808
Fixed capital	0.0230	0.3224	0.1068	-0.0585	-0.0448	-0.0206
Sum of production coefficients	0.9805	0.9016	0.9847	0.9207	0.9693	1.1235
Value of standard error:						
Land	0.0457	0.2866	0.0889	0.1600	0.0415	0.0864
Labor	0.0680	0.2917	0.1458	0.2430	0.0728	0.1597
Operating expense	0.0469	0.1755	0.0548	0.0962	0.0462	0.0840
Fixed capital	0.0296	0.2101	0.1113	0.1108	0.0416	0.0592
t-value:						
Land	4.88	1.23	5.78	-0.25	2.97	2.19
Labor	6.12	-0.15	-0.76	2.58	5.06	2.96
Operating expense	6.78	1.04	4.69	4.07	11.31	5.72
Fixed capital	0.78	1.53	0.96	-0.53	-1.08	-0.35
Value of R^2	0.9142	0.7817	0.7590	0.8113	0.9112	0.9012

of farm buildings, farm equipment and working cattle historically exist; they have played no important role of production process in the short run. Another reason of non-significancy of fixed capital might be due to large observed error. Since farm buildings may have lasted for several decades, it is difficult to evaluate accurately the depreciation expenses by the method of farm survey within a short time of face-to-face interview. However, the logic of production suggests no basis for dropping all of these expenses. Therefore, a third set of equations were thus tried by combining the variables X_3 and X_4 presented in the second set and treated as a single bundle of resource service. Hence, the function presented in the third set includes only land, labor and capital services as resource inputs. Statistics for this third set of production functions are presented in Table 13. All of the production coefficients in the logarithmic form are significant at the 5-percent probability level, excepting two negative coefficients. However, these negative coefficients are no significantly different from zero even at the 50-percent level of significance. They could arise with a probability of more than one-half chance, even if the true population elasticity is zero.

In order to make a tentative comparison among different methods of estimation, Klein's and Hoch's estimation methods were also applied for the Rice region under the second set

Table 13. Statistics for third estimate of production functions

Items	Agricultural regions Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Number of sample farms (n)	138	18	48	38	100	58
Value of constant (log A)	1.6220	2.1421	1.6667	1.6879	1.1795	0.8866
Value of production coefficients:						
Land	0.2213*	0.2489	0.3948*	-0.0169	0.1041**	0.1841**
Labor	0.4341*	0.0636	-0.0017	0.5440**	0.3984*	0.4816*
Capital	0.3339*	0.4014**	0.4872*	0.3825*	0.5233*	0.4866*
Sum of production coefficients	0.9893	0.7139	0.8803	0.9096	1.0258	1.1523
Value of standard error:						
Land	0.0455	0.2800	0.0965	0.1570	0.0448	0.0883
Labor	0.0675	0.2842	0.1408	0.2252	0.0783	0.1633
Capital	0.0487	0.1513	0.1016	0.0841	0.0525	0.0915
t-value:						
Land	4.86	0.89	4.09	-0.11	2.32	2.09
Labor	6.43	0.22	-0.01	2.42	5.09	2.95
Capital	6.86	2.65	4.79	4.55	9.98	5.32
Value of R^2	0.9127*	0.7525*	0.7435*	0.8079*	0.8941*	0.8946*

*Significant at probability level of 1%.

**Significant at probability level of 5%.

of groupings. The empirical results of these two estimates as well as the least-squares estimates are presented in Table 14. The procedures of Klein's and Hoch's estimations were described in the last chapter.

It is apparent that Hoch's estimates are not quite different from ordinary least-squares estimates based on our observations. However, Klein's estimates show substantial deviation from the results of the two other methods as shown in Table 14. Under the assumption that the average farm is optimal, Klein's estimates are probably reliable.

It is hardly conceivable that comparison among these three alternative methods of estimation can be made from the empirical results of this study. Direct comparison and evaluation among these methods of estimation are interesting but not quite definite. As far as it concerns only the least-squares and Hoch's methods, both provide the same reasonable estimates of the parameters. This evaluation, however, is made entirely upon the basis of what is expected from the observation as to the reasonable character of the estimate. No quantitative test of the better method is available.

Kmenta and Joseph have carried out a Monte Carlo experiment to examine the small sample properties of these three alternative estimates of the parameters of the Cobb-Douglas

Table 14. Alternative estimates of regression coefficients of production function for Rice region[a]

Production coefficients	Ordinary least-squares estimates	Klein's estimates[d]	Hoch's estimates: Case I[b] (all X's are variables)	Hoch's estimates: Case II[c] (X_1 and X_2 are fixed)
Constant term ($\hat{A}$)	47.40	144.90	47.56	47.70
Land ($\hat{\beta}_1$)	0.2230	0.2290	0.2235	0.2246
Labor ($\hat{\beta}_2$)	0.4162	0.2336	0.4158	0.4147
Operating exp. ($\hat{\beta}_3$)	0.3183	0.2988	0.3184	0.3176
Fixed capital ($\hat{\beta}_4$)	0.0230	0.0395	0.0224	0.0232
Sum of coeff.	0.9805	0.8009	0.9801	0.9801

[a]Based on the second set of production functions.

[b]Estimated by Equation 3.16.

[c]Estimated by Equations 3.20 and 3.21.

[d]Estimated by Equation 3.13.

production function (Kmenta and Joseph 1963). In general, the ordinary least-squares estimates are inconsistent under certain conditions. Hoch's estimates are inconsistent only under the situation when incorrect assumptions are made about the nature of interrelationship of the disturbances. Klein's estimates are consistent except in the case where the average firm is not optimal or when estimated coefficients attach to fixed inputs. The conclusion of the study by Kmenta and Joseph is:

> In general, our results indicate that no single estimation procedure is satisfactory in all circumstances. Ordinary least-squares estimates tend to have an upward bias; Klein's estimates, though highly efficient, are biased in the absence of effective profit maximization; and Hoch's, and to some extent indirect least-squares estimates, can be highly unstable in small samples. A choice of an estimation method has to depend on the specific field of application and on the knowledge of the technical and economic characteristics of the industry.

C. Elasticity Coefficients and Scale Returns

Through the discussion in the previous section, the production functions of the third set given in Table 13 are accepted in the following text. Three categories of resource inputs were used in the production function for each region--namely land services, labor services and capital services.

All discussion which follows is based on these equations of the third set unless specifically noted otherwise.

As mentioned previously, the production coefficients are the elasticities of production which show approximately the average percentage change in total value of output which would result if the input of any one resource is increased by one percent, _ceteris paribus_. For example, an increase of one percent in the quantity of labor used in the Rice region would increase the total value of production by 0.4341 percent, _ceteris paribus_. As shown in Table 13, all the elasticities of production in every region are less than one, which can be interpreted as indicating diminishing returns for the individual resource services.

Out of the total 18 production coefficients estimated, 10 coefficients are significant at 1-percent level of probability, and 4 are significant at 5-percent. However, the coefficients of land and labor in the Sweet potato region are not significant even at the 30-percent level of probability. This is due to the large error in this region. The negative elasticities or production coefficients of labor in the Tea region and that of land in the Rice-Sweet potato region are hardly conceivable that the total value of production would decrease if more of these two inputs were employed in these regions. However, these negative coefficients are not significantly different from zero even at the 50-percent level of significance. They could arise with a probability of more than one-half even if the true population elasticity is zero.

The multiple correlation coefficient, R, in all regions are significant at the 1-percent level of probability. The square of the multiple correlation coefficient, R^2, indicating the percentage of the variation in the sample observed values of total production per farm which can be explained by the fitted regression production function are also shown in Table 13 for each region. When the number of observations is small, the computed value of R^2 tends to overestimate the true value of the population. To take account of this, the adjusted R^2 may be used as the following formula:

$$\text{Adjusted } R^2 = 1 - (1-R^2)(n-1/n-m)$$

The sum of the elasticities for each region is also shown in Table 13 which indicates the returns to scale. In this study, all the agricultural regions show decreasing returns to scale except the Rice-Tea and Mixed regions which possess increasing returns to scale. However, all these values would probably test not significantly different from one and thus can be interpreted as approximately constant returns to scale in all regions.

V. MARGINAL PRODUCTIVITIES

After the production function has been estimated as shown in the previous chapter, the next step in the analysis is derivation of the marginal productivity of resources. The magnitude of the marginal productivity of resources depends on the quantity of other resources with which it is used. As stated previously, marginal productivity is a measure indicating the quantity by which the value of output (per farm in this study) is predicted to increase when one more unit of the particular resource input is to be employed with (1) inputs of the specific resource at stated levels and (2) inputs of other resources held constant or increased by a stated amount.

A. Equations of Marginal Productivity

Equations of marginal product returns of resources were derived from the third set of production functions presented previously. With other resources held constant at their geometric means, the equations of marginal product returns of land, labor and capital for each region are as follows:

1. Marginal productivity of land (MP_1)

Rice region $\quad MP_1 = 2,928\ X_1^{-0.7787}$

Sweet potato region	$MP_1 = 1{,}478\ X_1^{-0.7511}$
Tea region	$MP_1 = 1{,}423\ X_1^{-0.6052}$
Rice-Sweet potato region	$MP_1 = -584\ X_1^{-1.0169}$
Rice-tea region	$MP_1 = 2{,}034\ X_1^{-0.8959}$
Mixed region	$MP_1 = 2{,}526\ X_1^{-0.8159}$

2. Marginal productivity of labor (MP_2)

Rice region	$MP_2 = 1{,}288\ X_2^{-0.5659}$
Sweet potato region	$MP_2 = 721\ X_2^{-0.9364}$
Tea region	$MP_2 = -44\ X_2^{-1.0017}$
Rice-Sweet potato region	$MP_2 = 770\ X_2^{-0.4560}$
Rice-Tea region	$MP_2 = 1{,}278\ X_2^{-0.6016}$
Mixed region	$MP_2 = 902\ X_2^{-0.5184}$

3. Marginal productivity of capital (MP_3)

Rice region	$MP_3 = 545\ X_3^{-0.6661}$
Sweet potato region	$MP_3 = 200\ X_3^{-0.5986}$
Tea region	$MP_3 = 160\ X_3^{-0.5128}$
Rice-Sweet potato region	$MP_3 = 389\ X_3^{-0.6175}$

Rice-Tea region $MP_3 = 136\ X_3^{-0.4767}$

Mixed region $MP_3 = 152\ X_3^{-0.5134}$

The numerical values of the marginal product or return from any particular resource can be computed directly from these equations, while inputs of other resources are held constant at their geometric means. By inspecting these equations, it can be seen that increases in returns become smaller and smaller as resources employed is increased. This is due to the fact that all the elasticities of production are less than one, and hence diminishing returns for individual resource inputs are prevailing.

Table 15 shows the returns which might be expected, as an average for the farm sample in each region, or for the "average" or "representative" farm in the sense of a normal distribution in each region, if one more unit of resource is to be used on a farm while inputs of all other resources are given at their geometric means. The geometric mean quantity of resource services and the mean value of production are also presented in the table. Also included are the "average" product or returns of resources, which is obtained by dividing the total value of production (predicted at the geometric means again) by the geometric mean quantity of each resource. The "average" resulting includes the product returns of all resources, and not simply the product returns attributable

Table 15. Average production and resource inputs and marginal and average productivity of resources at geometric means

Items	Agricultural regions: Rice	Sweet potato	Tea	Rice-Sweet potato	Rice-Tea	Mixed
Geometric mean:						
Product (N.T.$)	39,100	15,530	25,690	31,940	33,130	32,300
Land (are)	134	48	144	96	128	105
Labor (man-day)	381	124	384	307	351	369
Capital (N.T.$)	13,530	5,297	7,740	8,182	10,640	13,780
Marginal product or returns:[a]						
Land (N.T.$/are)	64.57	80.53	70.43	-5.62	26.94	56.63
Labor (N.T.$/man-day)	44.55	7.97	-0.11	56.60	37.60	42.15
Capital (N.T.$/N.T.$)	0.96	1.18	1.62	1.49	1.63	1.14
Average product or returns:[b]						
Land (N.T.$/are)	291.79	323.54	178.40	332.71	258.83	307.62
Labor (N.T.$/man-day)	102.62	125.24	66.90	104.04	94.39	87.53
Capital (N.T.$/N.T.$)	2.89	2.93	3.32	3.90	3.11	2.34

[a]Marginal product or returns are based on the predicted product returns from the production function with inputs at their geometric means rather than based on the total product returns as the geometric mean of sample farms.

[b]The average product or returns are computed from the geometric mean product of the sample farms dividing by mean quantity of each resource. The "average" resulting includes the product returns of all input resources, and not simply the product returns attributable to the single resource.

to the single resource.

The marginal product returns derived at means for the group of farms in each region as shown in Table 15 represent only one marginal quantity from a large number of possible marginal quantities. The marginal product figures indicate the quantity of total value of production which will be added as one more unit of the particular resource is used on a farm, with its input and that of other resources at the mean of the quantities shown in the top of the table. An increase in land services in the Rice region will add to total value of production at the rate of N.T.$64.57 for one added unit of land. The marginal product return per man-work day of labor in the same region is N.T.$44.55 while a N.T.$1 input in capital services return only N.T.$0.96.

The negative marginal productivity indicates that the total value of production will decrease if one more unit of the particular resource is used. In the Rice-Sweet potato region, if one more unit of land is used with all resources at the mean quantities, it will cause the total value of production to decrease by N.T.$5.62, rather than increasing its value.

B. Test for Departure of Marginal Productivities from Resource Prices

With the analysis of marginal productivities above, one thing should be remembered. All the marginal product returns were measured with resource services at the mean quantities. Because of the diminishing returns nature of the productivity coefficients, marginal returns will be greater than the indicated figures when the amount of resource used is smaller than the mean quantities as shown in the top of Table 15.

Another thing to be concerned with is the equilibrium test for difference between marginal product returns and prices of the resources. As stated in chapter III, Klein's estimates are based on the assumption that the average farm is in the exactly optimal position. Under this optimal assumption, the production coefficients can be estimated by the equation of marginal productivity-price condition. In other words, while probability tests suggest that the elasticity coefficients differ significantly from zero, we may inquire whether they differ significantly from a level necessary to give marginal productivities equal to the market price or cost of each of the resources.

Table 16 shows the test of departure between marginal resource returns and market prices of resources in each

Table 16. Test of departure between marginal resource returns and market prices of resources

Agricultural regions	Value of production elasticity to give marginal product returns equal to market price of resource and its t-value[a]					
	Land		Labor		Capital	
	β*	t	β*	t	β*	t
Rice	0.2286	0.16[b]	0.2339	2.97[e]	0.3806	0.96[c]
Sweet potato	0	0.89[c]	0	0.22[b]	0.3752	0.17[b]
Tea	0.1754	2.27[e]	0	0.01[b]	0.3314	1.53[d]
Rice-Sweet potato	0	0.11[b]	0.2499	1.31[c]	0.2818	1.20[c]
Rice-Tea	0.1781	1.65[d]	0.2860	1.44[d]	0.3533	3.24[e]
Mixed	0.2168	0.37[b]	0.2970	1.13[c]	0.4693	0.19[b]

[a]Only those elasticities which are significantly at the level of 5-percent of probability are computed in this table. The elasticity, β*, necessary to give a marginal product returns for mean quantity of resources equal to the market price of the resources has been computed as

$$\beta^* = P \cdot \frac{\overline{X}}{\overline{Y}}$$

where P is the market price for the particular resource, $\overline{X}$ is the geometric mean quantity of the resource and $\overline{Y}$ is the geometric mean value of production. The value of t has been derived as

$$t = \frac{\hat{\beta} - \beta^*}{s_{\hat{\beta}}}$$

where $\hat{\beta}$ is the elasticity or regression coefficient and $s_{\hat{\beta}}$ is the standard error of estimate as shown in Table 13.

[b]Probability level < 50%.

[c]Probability level < 20%.

[d]Probability level < 10%.

[e]Probability level > 5%.

region. For those production elasticities with a small t-value, and being not significant at the 20-percent level of probability, we may say that the corresponding marginal returns, as on the average of farms in the sample, do not differ significantly from the market price of the resource. For instance, we cannot say that the marginal product return of N.T.$64.57 per are of land in the Rice region, on the average of sample farms, differs significantly from the market rent of N.T.$66.70 per are of land, or the marginal return of N.T.$0.96 differed significantly from N.T.$1.10 (N.T.$1 principal plus 10 percent interest) per N.T.$1 input of capital on the farm. Therefore, we may say that farmers in the Rice region were, on the average maximizing returns in the use of land and capital under the particular prices and yields of the year; efficiency in production has been attained in the sense that the cost of resources approximatety approached the added returns for more of these resources used beyond the per farm mean. However, the same conclusion cannot be made for the use of labor services in the Rice region, since it differs significantly from the equilibrium position at the 5-percent probability level.

Several points can be drawn from an observation of Table 16. Land services have been used efficiently in both the Rice and Mixed regions in the sense that the marginal product return of land approached its market price in each region. The elasticity of land production in the Tea region differs in a highly significant manner from the efficient condition. The marginal return per are of land in the Tea region was much greater than its price. Farm size in this region will probably be enlarged under the diminishing returns nature of land. On the other hand, the size of farms in the Rice-Tea region should be reduced in order to improve the land used toward its optimal situation. The productivities of land services in both the Sweet potato and the Rice-Sweet potato regions were not significantly different from zero.

Farmers in most of the regions did not use very efficiently their labor services in the acceptable probability level. The amount of labor used on farms seems much smaller than the optimal quantity under the particular price levels and production techniques. However, since the analysis does not give full consideration to functional relationships between products and resources which fall in the complementary and supplementary phases, a more detailed analysis is needed for the determination of efficient utilization of labor services on farms.

The production elasticity of capital service is not significantly different from the optimal position at the level of 20-percent of probability except in the Tea and Rice-Tea regions. Farmers were, on the average, maximizing the return from the use of capital under the given conditions. This result seems reasonable in view of the fact that farmers in Taiwan usually have used their capital resource more carefully than their labor resource.

Among these three categories of resources, the movability of capital service is greater than that of land and labor resources among regions. The differences in marginal productivity of land do not cause concern about the allocation of this resource between different producing regions. It is an immobile resource between different producing regions.

Marginal labor return is greatest in the Rice-Sweet potato region which can be explained mainly by the high return from livestock production in this region. Also, since the elasticity of land is negative in this region, farms with small acreage of land input and thus a low land-labor ratio may have the effect of "pulling up" labor productivity. While the low marginal labor return in the Rice-Tea region may be largely due to the relative low capital-labor ratio on the average, compared with that in the Rice and Mixed regions. Although the small farm and a smaller quantity of capital per worker in general provide the major explanation for a lower marginal labor productivity, there is not such evidence in this analysis.

The lowest marginal capital productivity in the Rice region can be best explained in the form of capital. This group of farmers generally have better buildings on their farms relative to the farm size and production, and thus subject to commit more depreciation expense. However, added more investment in farm buildings alone, as on the average of all farms, would likely add less to value of annual production than the annual depreciation cost on buildings. Farmers in the Rice region have pushed building investment to a relative high level to add to the living satisfaction of the family.

The marginal capital returns vary from N.T.$1.14 to N.T.$1.63 for each N.T.$1 input among regions. This situation might be due to the fact that farmers in Taiwan generally cannot borrow or hesitate to borrow more capital because of equity and uncertainty considerations. Hence, a large gap is found between the return from capital used and its cost or price in the form of interest.

C. Marginal Productivity Curve and Its Reliability

In evaluating the levels of marginal productivity as shown in the last section, they were all computed at the mean quantity of the sample farms for each region. The magnitude of the marginal return of any particular resource depends not only on its input level but also levels of other resources. The marginal product return will be larger for a given level of a resource input if larger amounts were used of other resources, under the condition of positive elasticities of production. On the other hand, because of the diminishing returns nature, the marginal return of a resource will be decreasing as the amount of the resource used increases, while other resources are held at a constant level.

As mentioned previously, the marginal productivity equations shown in section A of this chapter were derived for each resource when all other resources were held constant at their geometric means for the sample farms. From these marginal productivity equations, a curve can be then derived for each resource by inserting different quantities of the resource in the equation. Such curves of marginal productivity for each resource in different regions are shown as the solid curve in Figures 4 to 17. Figures 4 to 7 are the marginal return curves for land service, while Figures 8 to 11 are the marginal labor return curves and Figures 12 to 17 represent the marginal return curves for the capital resource. In deriving these marginal productivity curves, only resources with a significant elasticity of production were concerned.

As shown in Figures 4 to 7, the marginal productivity of land in the Rice and Tea regions is greater than that in the Rice-Tea and Mixed regions for different quantities of land resource within the observed data and with other resources held at their mean quantities of the sample farms. By the comparison between the Rice and Tea regions, the marginal productivity of land is greater in the Rice region than that in the Tea region for small farms. But as the farm size increases, marginal productivity of land in the Rice region will be decreased at a greater rate than that in the Tea region. In other words, farms in the Rice region, on the average, have higher land productivity with a greater diminishing return rate than farms in the Tea region. As shown in Figures 4 and 5, for farm size less than 80 ares, the marginal return of land in the Rice region is greater than that in the Tea region, while it will become smaller and smaller as farm size grows beyond 80 ares. This situation may explain why the farm size in the Tea region is usually larger than Rice farms in Taiwan.

The marginal productivity of labor resource at different levels in the Rice-Sweet potato region is found to be the highest among regions as shown in Figures 8 to 11. For instance, at the labor input of 400 man-work days, the marginal return of labor in the Rice-Sweet potato region is N.T.$50, compared with N.T.$43, N.T.$40 and N.T.$34 in the Rice, Mixed and Rice-Tea regions respectively. It will be decreased to N.T.$41 per man-work day in the Rice-Sweet potato region when labor resource used is increased to 600 man-work days and can be compared with N.T.$34, N.T.$33, and N.T.$27 in the other regions respectively. We must remember again, in evaluating such comparisons, all other resources are held unchanged at their mean quantities. The rate of diminishing marginal return to labor is not greatly different among regions, it varies from 0.60 in the Tea region to 0.46 in the Rice-Sweet potato region.

Figures 12 to 17 show the marginal productivity curves of capital service for each region. Among these regions, the highest marginal return of capital is the Rice-Tea region, while the lowest is the Sweet potato region. When the capital investment in a farm amounts to N.T.$10,000, the marginal capital return per N.T.$1 still remained at a rate greater than N.T.$1 in most of the regions--except in the Rice and Sweet potato regions. However, marginal returns to capital are all less than N.T.$1 as capital investment increases to the amount of N.T.$20,000, with the exception of the farms in the Rice-Tea region. This situation may indicate that farmers in Taiwan have not much opportunity to invest a large amount of capital in their farms, under the current farm size, crop and livestock system, and the given techniques of

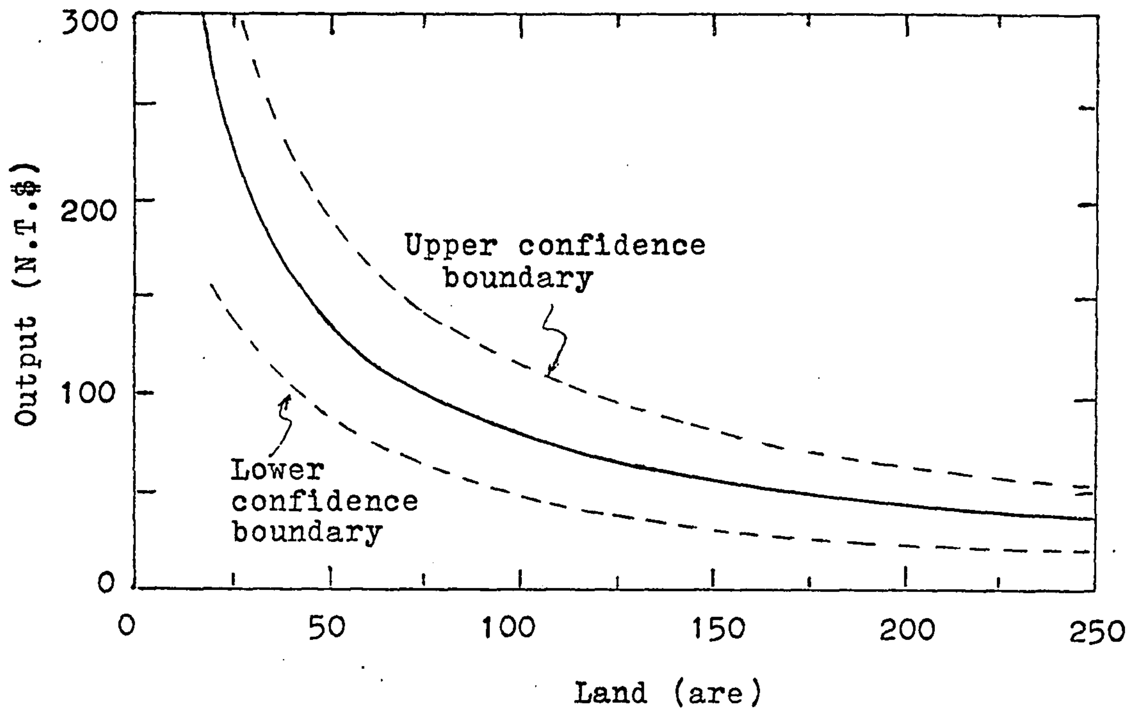

Figure 4. Marginal productivity of land resource and the 95 percent confidence interval in Rice region

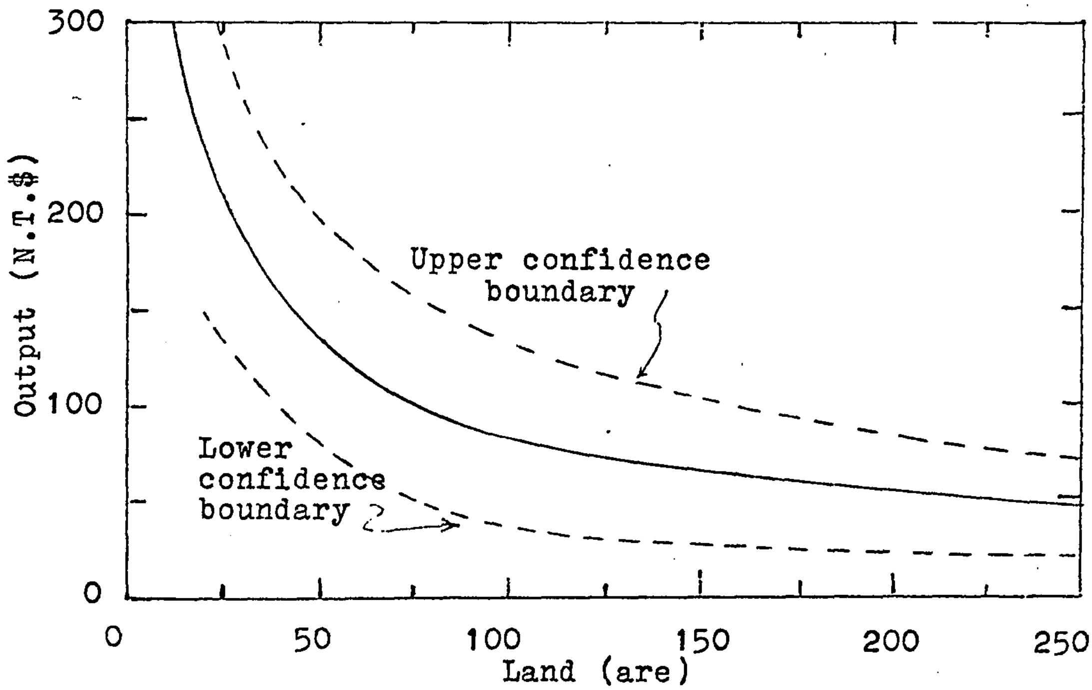

Figure 5. Marginal productivity of land resource and the 95 percent confidence interval in Tea region

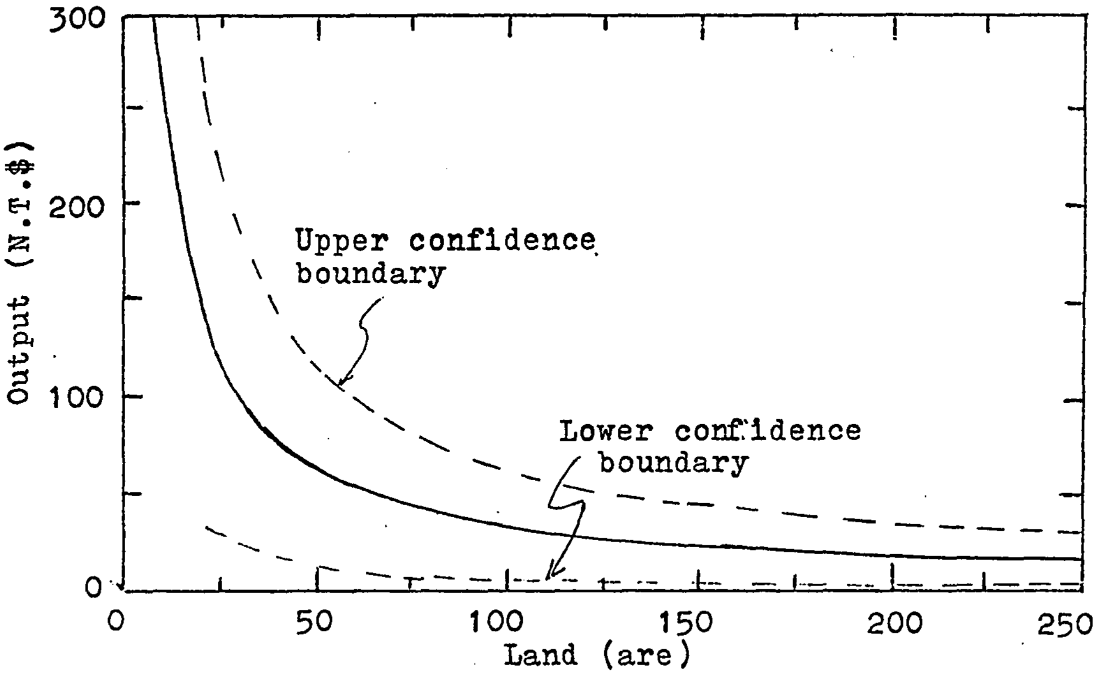

Figure 6. Marginal productivity of land resource and the 95 percent confidence interval in Rice-Tea region

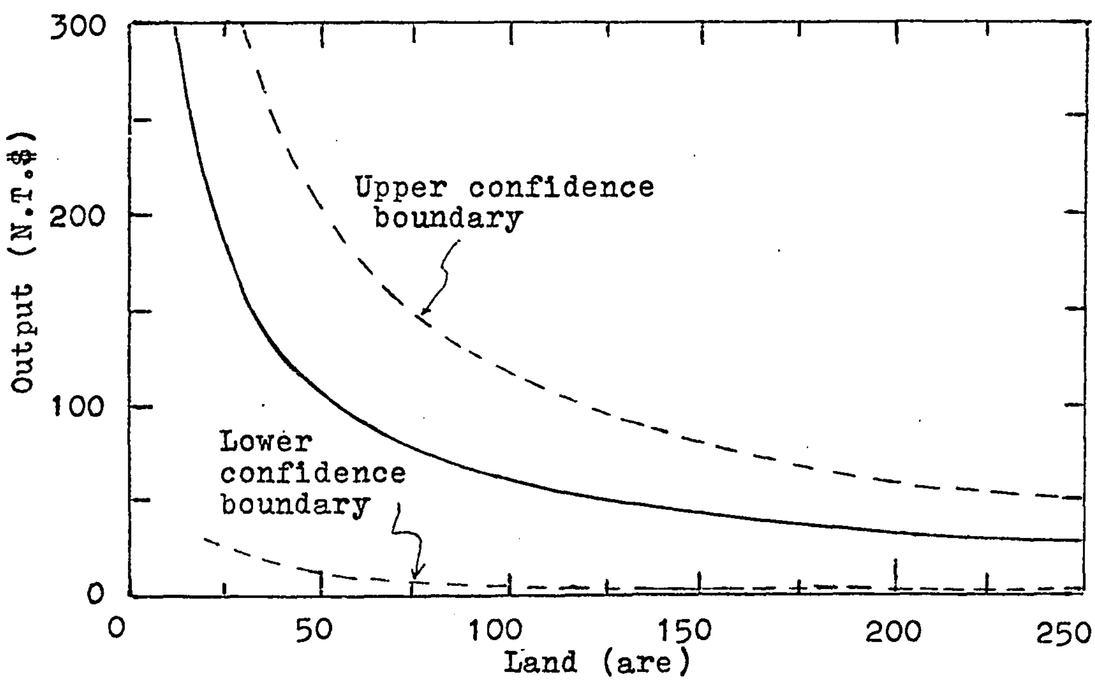

Figure 7. Marginal productivity of land resource and the 95 percent confidence interval in Mixed region

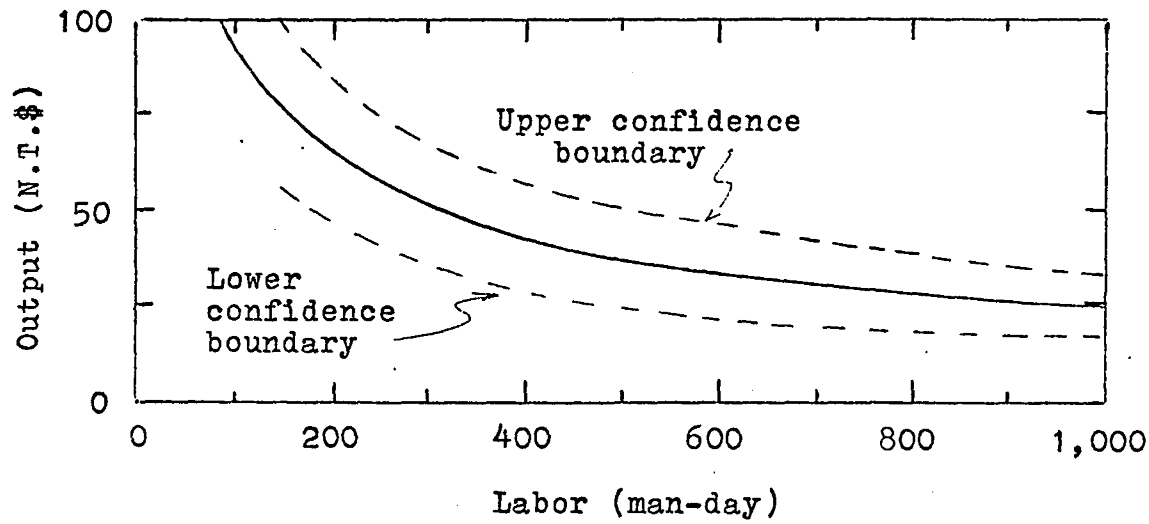

Figure 8. Marginal productivity of labor resource and the 95 percent confidence interval in Rice region

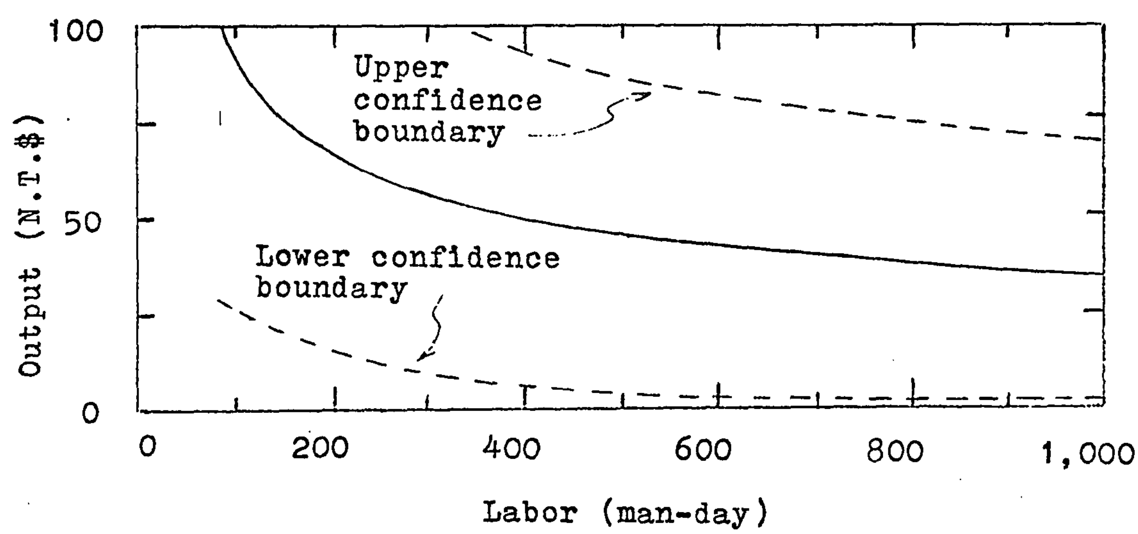

Figure 9. Marginal productivity of labor resource and the 95 percent confidence interval in Rice-Sweet potato region

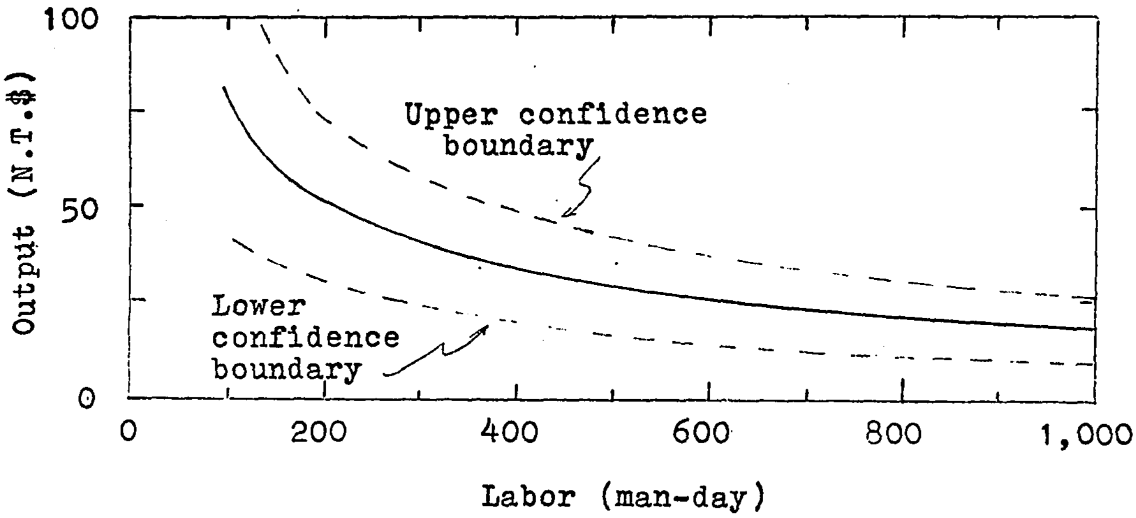

Figure 10. Marginal productivity of labor resource and the 95 percent confidence interval in Rice-Tea region

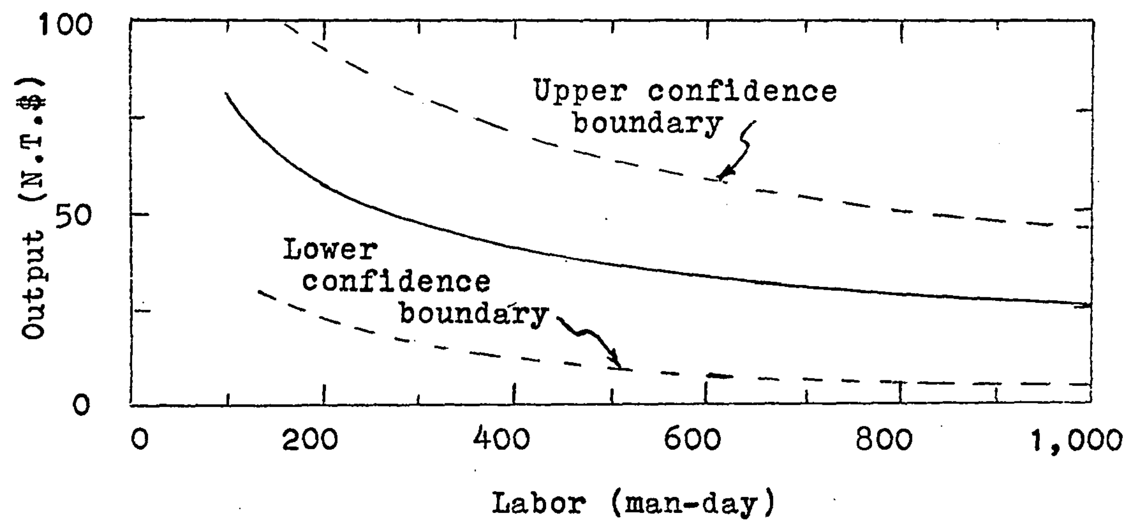

Figure 11. Marginal productivity of labor resource and the 95 percent confidence interval in Mixed region

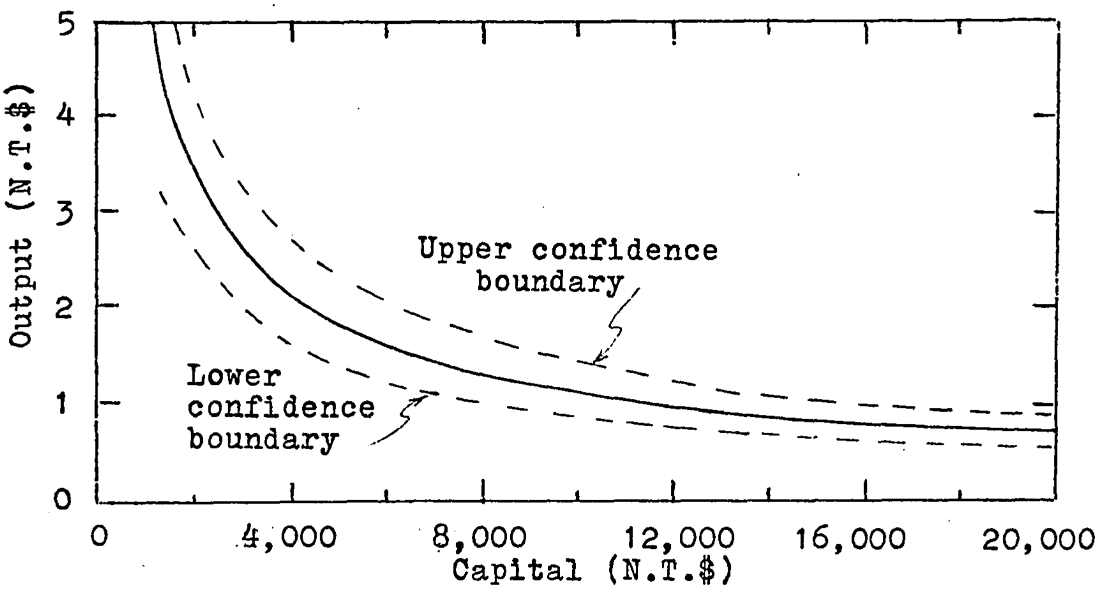

Figure 12. Marginal productivity of capital resource and the 95 percent confidence interval in Rice region

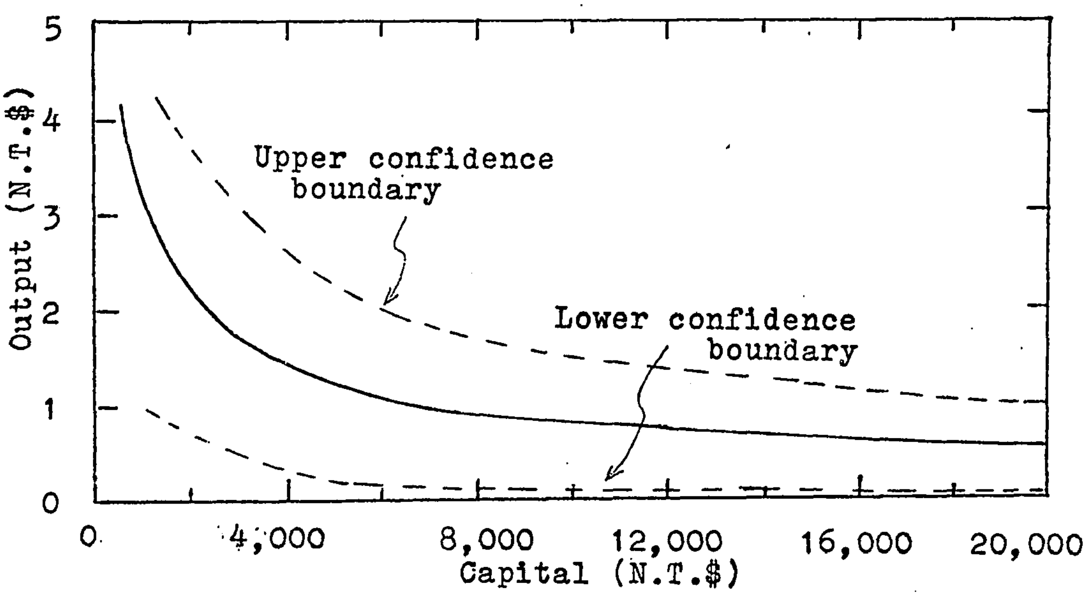

Figure 13. Marginal productivity of capital resource and the 95 percent confidence interval in Sweet potato region

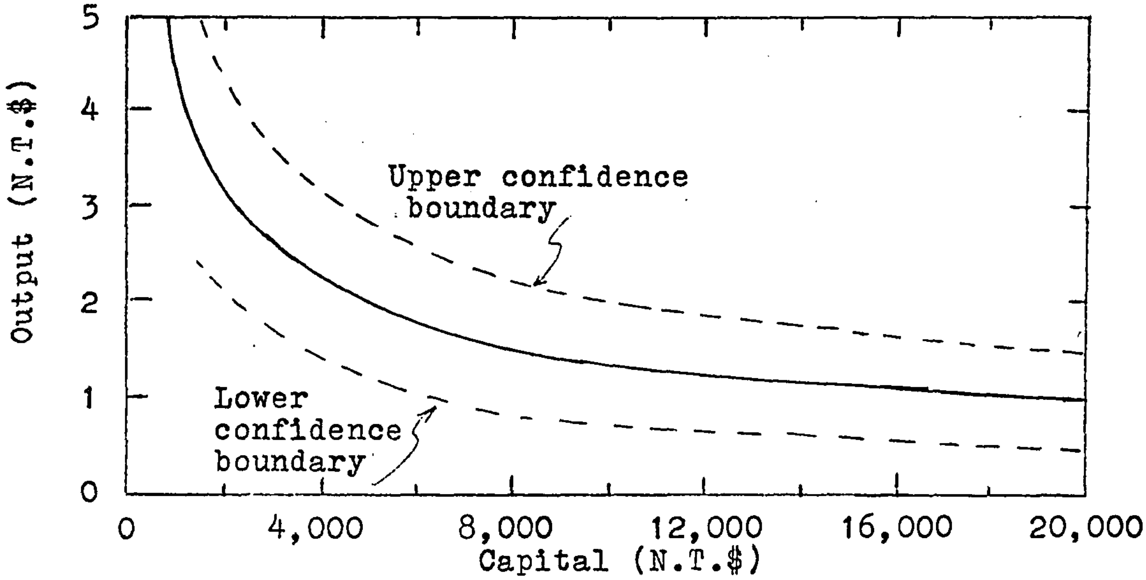

Figure 14. Marginal productivity of capital resource and the 95 percent confidence interval in Tea region

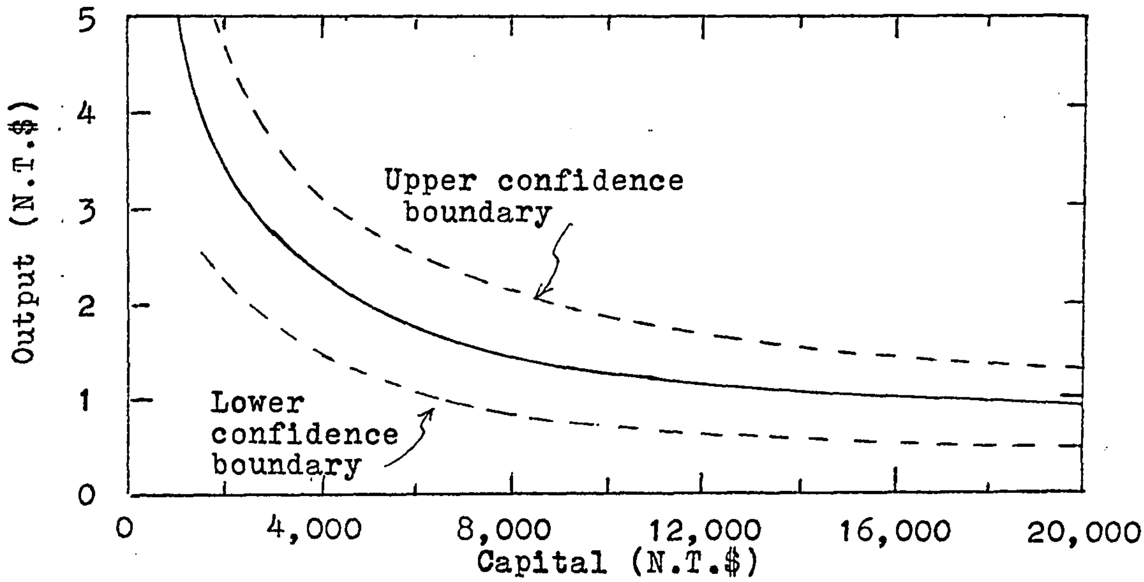

Figure 15. Marginal productivity of capital resource and the 95 percent confidence interval in Rice-Sweet potato region

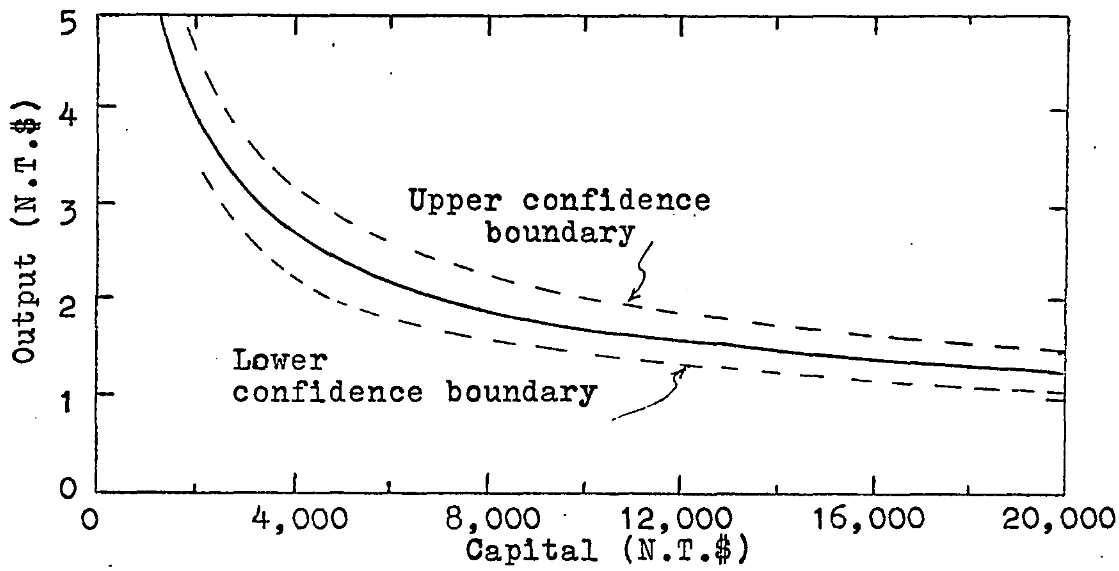

Figure 16. Marginal productivity of capital resource and the 95 percent confidence interval in Rice-Tea region

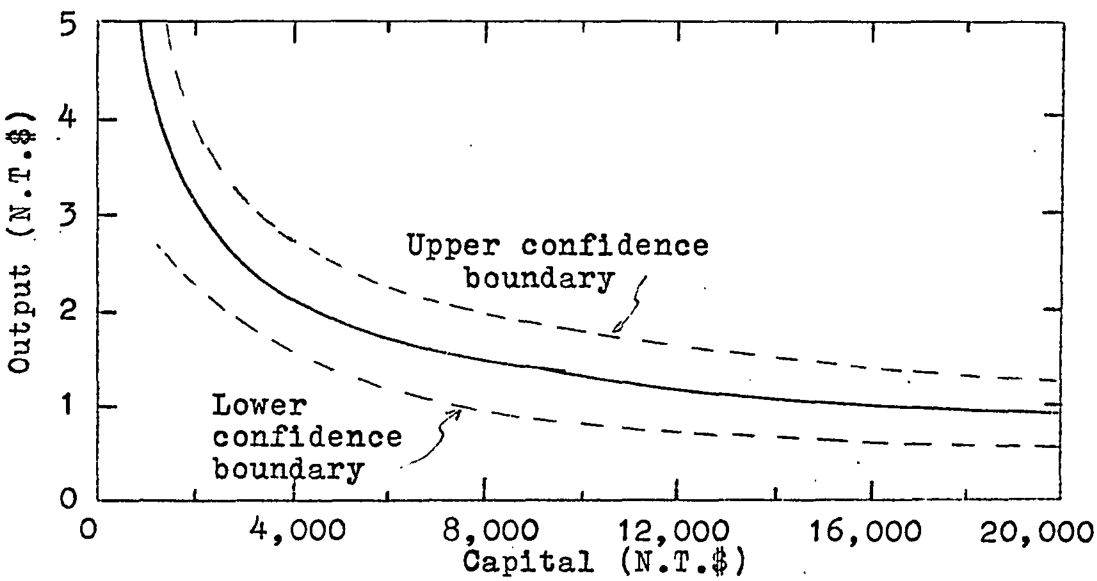

Figure 17. Marginal productivity of capital resource and the 95 percent confidence interval in Mixed region

production. Therefore, reorganization of farm business and adoptation of new technology are the necessary conditions for farmers to employ profitably a larger amount of capital on farms.

Given the estimates of marginal productivity as above, the next step in the analysis is to calculate their variance. Historically, the variance of the marginal productivity has generally been derived by assuming the values of predicted output and resource inputs to be constants. The variance of marginal productivity, i.e., $MP_i = \frac{\partial \hat{Y}}{\partial X_i} = \hat{\beta}_i \frac{\hat{Y}}{X_i}$, is then estimated by Equation (5.1).

$$\mathrm{var}\left(\hat{\beta}_i \frac{\hat{Y}}{X_i}\right) = \left(\frac{\hat{Y}}{X_i}\right)^2 \mathrm{var}(\hat{\beta}_i) \qquad (5.1)$$

where $\mathrm{var}(\hat{\beta}_i)$ are variances of the estimated production coefficients as shown in Table 13 and $\hat{Y}$ is the predicted value of output at the given value of X_i, with other resources used held constant at their geometric means. Equation (5.1) assumes that predicted Y value is constant. However, such an assumption is unrealistic because the value of the predicted Y will vary over alternative samples and it is estimated based on the $\hat{\beta}_i$ values which are only estimates of the true parameters. For estimates of marginal productivity with all resources at their geometric means, Equation (5.1) leads to negligible errors in the variance estimate. These errors, however, increase rapidly as marginal productivity estimates are made further away from the geometric mean levels. The implication of nonconsistancy of predicted Y value has been discussed by Carter and Hartley (Carter and Hartley 1958). They derived a more accurate expression of the variance of the marginal productivity estimates. This formula is given in Equation (5.2)

$$\text{var}\left(\hat{\beta}_i \frac{\hat{Y}}{X_i}\right) = \text{var}(\hat{Y})\left(\frac{\hat{Y}}{X_i}\right)^2 \left(\frac{\hat{\beta}_i^2}{n} + \mu C \mu'\right) \qquad (5.2)$$

where var $(\hat{Y})$ is the estimated variance of σ^2 or predicted Y based on a regression equation as shown in Equation 4.6, n is the total number of sample farms, μ represents the vector

$$\hat{\beta}_i \left(\log X_m - \overline{\log X_m}\right) \qquad \text{for } i \neq j$$

$$\hat{\beta}_i \left(\log X_m - \overline{\log X_m}\right) + 1 \qquad \text{for } i = m$$

and $C=(X'X)^{-1}$ where the element in the rth row and cth column of the matrix $(X'X)$ is $\Sigma(\log X_r - \overline{\log X_r})(\log X_c - \overline{\log X_c})$. In Equation 5.2, it is assumed that the logarithmic transformation used in the least-squares estimation is to the base e. For using a transformation to the base 10, the term $(\hat{\beta}_i^2/n)$ in Equation 5.2 must be multiplied by the value of $(2.3026)^2$ (Heady and Dillon 1961, p. 232).

After the variance of marginal productivity has been calculated by Equation 5.2, then it is well known that the confidence interval at the α-level for a particular point of marginal productivity can be computed by Equation 5.3 where $t_{(1-\alpha)}$ has (n-m-1) degrees of freedom.

$$\hat{\beta}_i \frac{\hat{Y}}{X_i} \pm t_{(1-\alpha)} \sqrt{\text{var} \left(\hat{\beta}_i \frac{\hat{Y}}{X_i}\right)} \qquad (5.3)$$

Following the general procedure stated above, the 95-percent confidence intervals for the marginal productivity of resources have been computed. The broken lines shown in Figures 4 to 17 indicate the magnitude and position of these confidence intervals for the marginal productivity of the respective resources.

As shown in Figure 4, the confidence interval for marginal land productivity in the Rice region is quite narrow, implying that the marginal productivity equation of land in this region provides a reasonably reliable estimate of land productivity, with other resources held constant at their geometric means of the sample, farms in the region. For example, the marginal return per unit of land resource is estimated to be N.T.$64.57 at 134 ares of land input, while the variance at this point is N.T.$13.34 estimated by Equation 5.2. Therefore, the 95-percent confidence interval at this mean point is N.T.$64.57 ± N.T.$26.41, extending from N.T.$90.98 to N.T.$38.16. On the other hand, the confidence interval is much wider in the Mixed region, indicating that the marginal return of land resource in this region cannot be ascertained with certainty on the basis of the available data compared with that in other regions in this study.

As for labor productivity, confidence intervals in the Rice and Rice-Tea regions are much more narrow than that in the Rice-Sweet potato and Mixed regions as shown in Figures 8 to 11. Figures 12 to 17 show the size and position of the 95-percent confidence interval for capital in each region. More specific predictions can be made by lessening the confidence intervals simultaneously with reducing the residual variance of farm output Y and the variances and covariances for the regression coefficients in the production function. Reduction in the residual variance, or unexplained variance in farm output can be accomplished by: (1) increasing the number of farm observation; (2) the inclusion of additional explanatory variables in the model, and (3) reducing the errors in observation or measurement by more refined sampling survey techniques.

VI. ISOQUANT, ISOCLINE AND OPTIMAL RESOURCE COMBINATION

The marginal productivity curves of resources and their confidence intervals have been discussed above. This chapter will deal with the nature of output isoquants and isoclines for the average farms in the different regions. Farmers in Taiwan are more or less faced with a given size of farm. The acreage of cultivated land of a farm is usually difficult to change rapidly in a short run period. Farmers can easily adjust, however, the amounts of labor and capital to combine with the given size of land resource in farming. Therefore, in the following analysis, farm size is considered to be fixed at the mean value of the farm sample. Isoquants and isoclines are then derived to show the various combinations of labor and capital resources on an "average size" of farm in different agricultural regions. As stated previously, since the regression coefficients of labor resource in the Sweet potato and Tea regions were not significant at the acceptable probability level, the isoquants and isoclines were only derived for those other regions with significant coefficients both in labor and capital services. Furthermore, in order to overcome the sampling variability inherent in sample survey data, confidence intervals were also computed for the isoquants and isoclines, indicating a range of values within which the expected or average value of the estimate may lie, given a probability level. The methods employed to derive the confidence interval are to be described in the text below.

A. Nature of the Output Isoquants

Output isoquants can be derived directly from production function. Given our production function as Equation 6.1, then it becomes as Equation 6.2 after the value of X_1 (land resource in our case) was set constant at a particular level. The isoquant equation can be then derived to express that X_3 (capital) is a function of X_2 (labor) for the given level of output as shown in Equation 6.3.

$$\hat{Y} = \hat{A} X_1^{\hat{\beta}_1} X_2^{\hat{\beta}_2} X_3^{\hat{\beta}_3} \qquad (6.1)$$

$$\hat{Y} = (\hat{A}\, \bar{X}_1^{\hat{\beta}_1})\, X_2^{\hat{\beta}_2} X_3^{\hat{\beta}_3} \qquad (6.2)$$

$$X_3 = \left(\hat{A}\, \bar{X}_1^{\hat{\beta}_1}\right)^{-\frac{1}{\hat{\beta}_3}} \hat{Y}^{\frac{1}{\hat{\beta}_3}} X_2^{-\frac{\hat{\beta}_2}{\hat{\beta}_3}} \qquad (6.3)$$

Equation 6.3 indicates the various combination of capital and labor resources that are required to produce the particular output level, setting variable X_1 equal to $\bar{X}_1$. In this study, X_1 was set at the geometric mean of sample farms in each region.

The procedures illustrated by Fuller was employed in this study for computing the confidence interval for the isoquant (Fuller 1962). The confidence limits for a given isoquant at the α-level are given by Equation 6.4, where

$$y_{ci} = \hat{y}(x_1,x_2,x_3) \pm \sqrt{var(\hat{Y})} \cdot t_{(1-\alpha)}\sqrt{\frac{1}{n} + \xi C^{-1}\xi'} \qquad (6.4)$$

y_{ci} is the log value of the confidence limit of $\hat{Y}$ predicted at the point $(\bar{x}_1, x_2, x_3)$, and $\hat{y}$ denotes the estimated log value of Y at the same point, ξ is the row vector of deviations of resource variables from their respective geometric means at the same point on the isoquant curve. C is the matrix of sums of squares and products of the resource variables in terms of logs.

As Equation 6.4 is difficult to evaluate directly, the following procedure of approximation was used in this study. Choose a point, say $(\bar{x}_1, x_2^*, x_3)$, on the isoquant, and evaluate first

$$V = t_{(1-\alpha)}\sqrt{\left(\frac{1}{n} + \xi C^{-1}\xi'\right) var(\hat{Y})}$$

Then fixing x_2 at the value of x_2^* compute the values of x_3 given by Equations 6.5 and 6.6.

$$y_{ci} - V = \hat{y}\ (\bar{x}_1, x_2^*, x_3) \qquad (6.5)$$

$$y_{ci} + V = \hat{y}\ (\bar{x}_1, x_2^*, x_3) \qquad (6.6)$$

The output isoquant equations derived from the estimated production functions given in Chapter IV are shown as follows, with Y and X_1 set at their respective geometric means of sample farms.

Rice region: $X_3 = 30,590,000\ X_2^{-1.30008}$

for Y = N.T.$39,940 and X_1 = 134 ares

Rice-Sweet potato region: $X_3 = 28,230,000\ X_2^{-1.42222}$

for Y = N.T.$31,940 and X_1 = 96 ares

Rice-Tea region: $X_3 = 922,200\ X_2^{-0.76132}$

for Y = N.T.$33,130 and X_1 = 128 ares

Mixed region: $X_3 = 4,796,000\ X_2^{-0.98972}$

for Y = N.T.$32,300 and X_1 = 105 ares

These isoquant equations are used to graph the mean-output isoquant curves for each region as shown by the solid lines in Figures 18 to 21. The approximation of the size and shape of a 95-percent confidence interval for the mean output isoquant is also shown as broken lines in these Figures. To improve the approximation, a method of successive approximation suggested by Fuller can be done (Fuller 1962).

The confidence intervals for a mean-output isoquant in the Rice and Rice-Tea regions are much more narrow than that

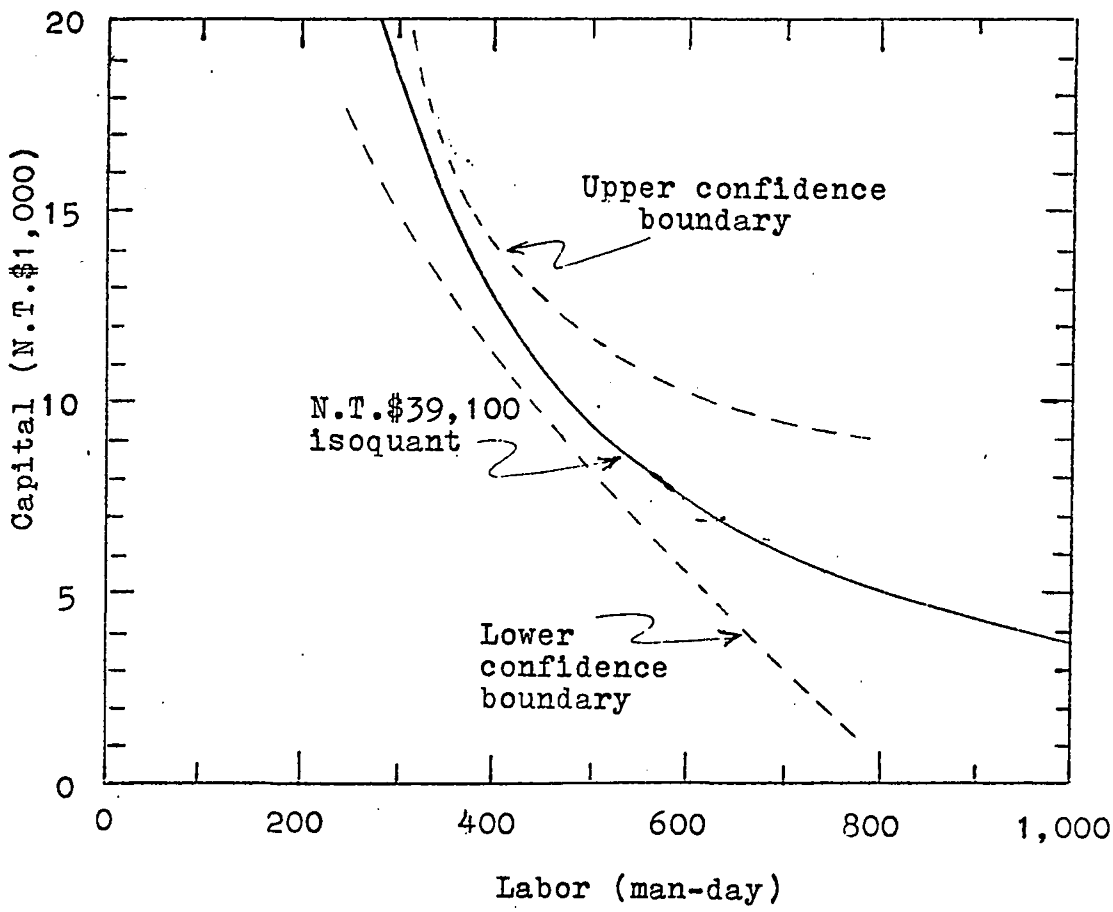

Figure 18. The 95 percent confidence interval for the mean-output isoquant in Rice region

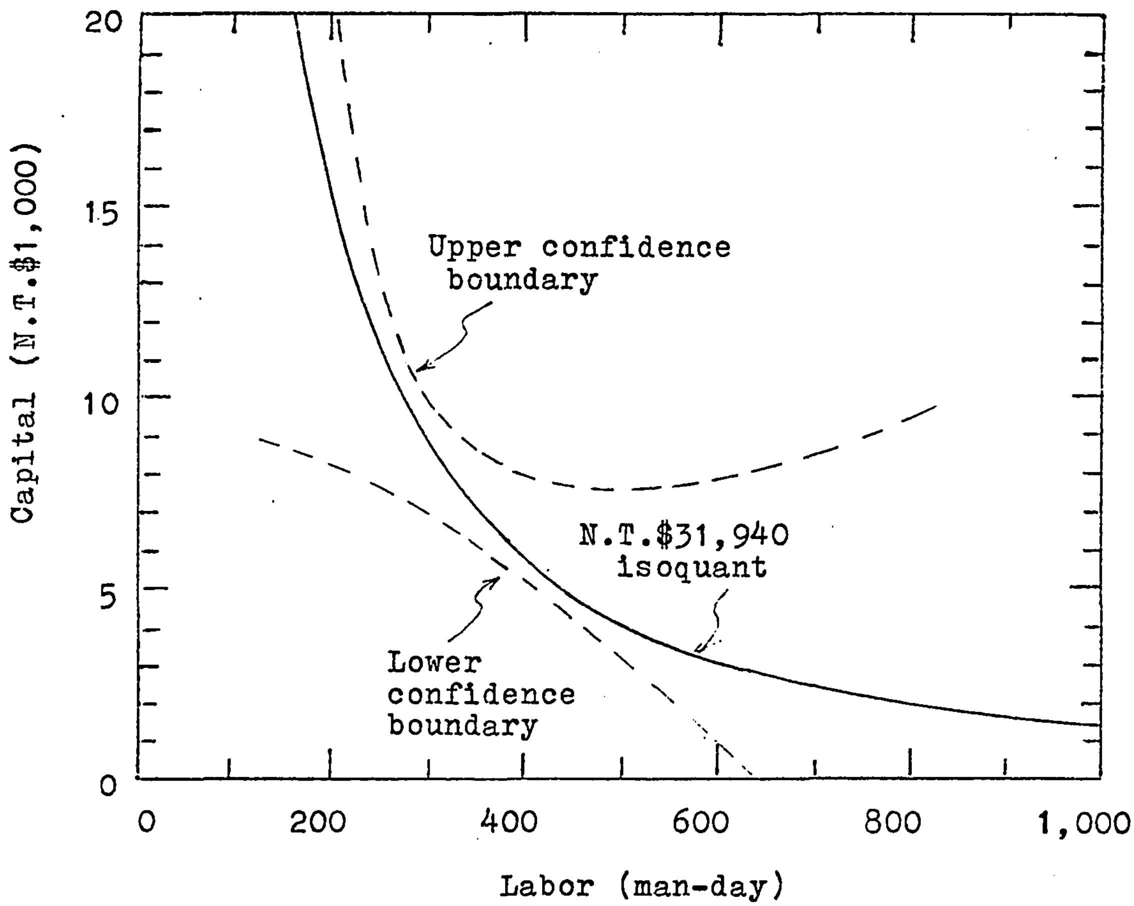

Figure 19. The 95 percent confidence interval for the mean-output isoquant in Rice-Sweet potato region

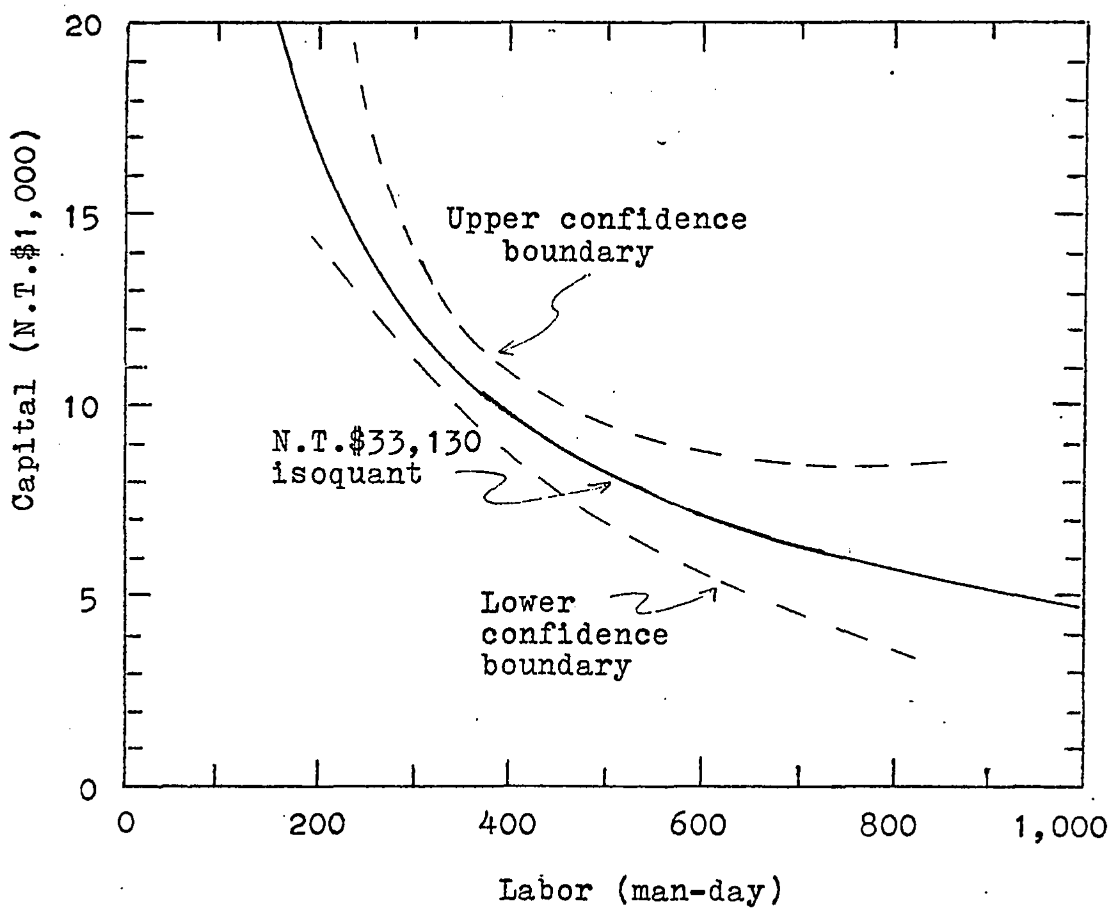

Figure 20. The 95 percent confidence interval for the mean-output isoquant in Rice-Tea region

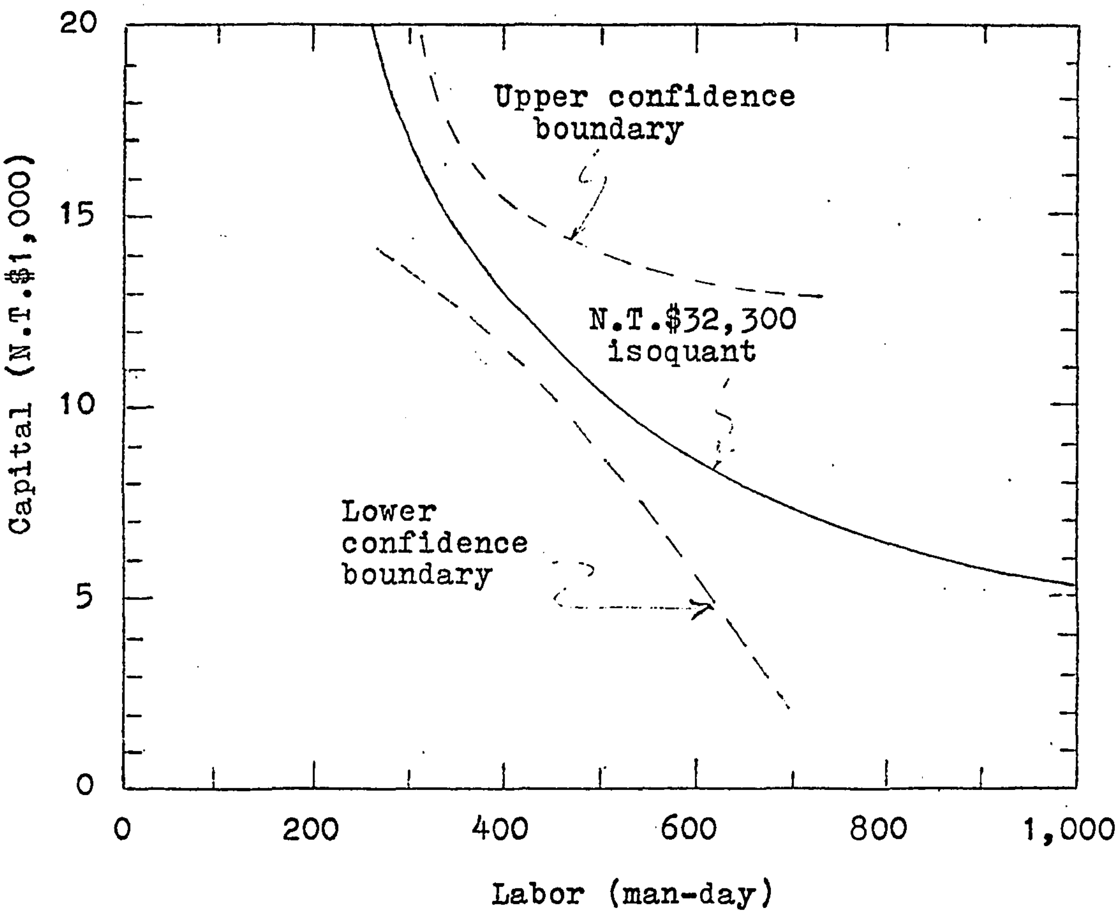

Figure 21. The 95 percent confidence interval for the mean-output isoquant in Mixed region

in the regions of Rice-Sweet potato and Mixed. The narrowness of a confidence interval implies that the estimated production functions provide a reasonably reliable estimate of capital quantity with a fixed amount of labor and land for a given level of production output. It was shown before that in the Rice-Sweet potato and Mixed regions the confidence intervals for the marginal productivity of labor were much wider than those in the Rice and Rice-Tea regions, while the confidence intervals for marginal capital productivity in these four regions were not greatly different. Therefore, the confidence intervals for the isoquants were expected to be narrower in the Rice and Rice-Tea regions. As expected, each confidence interval is narrowest near the point corresponding to the mean value of the observed amounts of capital and labor used on the farm.

B. Nature of the Output Isoclines

Output isoclines are directly related to isoquants. A particular isocline intersects all isoquants at points where the isoquants have the same given slope. Isoclines, like isoquants, are derived from the estimated production functions given in the previous chapter. The general form of the isocline equation in our study is shown in Equation 6.7.

$$X_3 = k\frac{\beta_3}{\beta_2} X_2 \qquad (6.7)$$

where k is any constant. Let $k = P_2 P_3$, be the price ratio of labor and capital resources in our case, then the isocline equation becomes as:

$$X_3 = \frac{P_2}{P_3} \cdot \frac{\hat{\beta}_3}{\hat{\beta}_2} X_2 \tag{6.8}$$

Equation 6.8 represents the least cost expansion path, indicating changes in the combination of resources necessary to give maximum profits as higher output is attained under the given price ratio of resources. Along a particular isocline, the marginal rate of substitution of resources, $\frac{\partial X_3}{\partial X_2}$, equals the price ratio of X_2 (labor) to X_3 (capital).

A confidence interval also can be derived for isoclines. Based on the isocline equation, we may consider that for a given level of X_2, what the level of X_3 should be in order to give the marginal rate of substitution of X_2 for X_3 equal to k. Two different approaches are used here to establish the confidence intervals for isoclines.

The first approach is deriving directly the variance of X_3 from Equation 6.7 as shown in Equation 6.9.

$$\text{var}\ (X_3) = (kX_2)^2 \cdot \text{var}\ (\hat{\beta}_3/\hat{\beta}_2) \tag{6.9}$$

The variance of ratio R = $\hat{\beta}_3/\hat{\beta}_2$ is then estimated approximately by the Equation 6.10 (Cochran 1953),

$$\text{var}\left(\frac{\hat{\beta}_3}{\hat{\beta}_2}\right) = \left(\frac{\hat{\beta}_3}{\hat{\beta}_2}\right)^2 \left(\frac{\hat{\sigma}_3^2}{\hat{\beta}_3^2} + \frac{\hat{\sigma}_2^2}{\hat{\beta}_2^2} - \frac{2\hat{\sigma}_{23}}{\hat{\beta}_2\hat{\beta}_3}\right) \tag{6.10}$$

where $\hat{\sigma}_2^2$, $\hat{\sigma}_3^2$ and $\hat{\sigma}_{23}$ denote the estimated variances and covariance of $\hat{\beta}_2$ and $\hat{\beta}_3$, which can be obtained directly from C^{-1} (var $\hat{Y}$). Substituting the value of var $(\hat{\beta}_3/\hat{\beta}_2)$ into Equation 6.9, var (X_3) is then obtained. Based on the assumption of normality, confidence limits at the α-level are given by Equation 6.11.

$$X_{3ci} = k\, X_2 \frac{\hat{\beta}_3}{\hat{\beta}_2} \pm t_{(1-\alpha)} (\text{var } (X_3))^{1/2} \qquad (6.11)$$

where var (X_3) is estimated by Equation 6.9.

An alternative method of computing confidence intervals was originally established by Fieller (Fieller 1932). This approach requires fewer basic assumptions and takes some account of the skewness of the distribution of $\frac{\hat{\beta}_3}{\hat{\beta}_2}$ (Cochran, 1953). Let R denote the limits of the ratio $\frac{\hat{\beta}_3}{\hat{\beta}_2}$, and assume that $\hat{\beta}_2$ and $\hat{\beta}_3$ follow a bivariate normal distribution. It follows that Equation 6.12 is following a Student's t-distribution.

$$\frac{(\hat{\beta}_3 - \hat{\beta}_2 R)}{\left(\hat{\sigma}_2^2 - 2R\hat{\sigma}_{23} + R^2\hat{\sigma}_3^2\right)^{1/2}} \qquad (6.12)$$

Confidence intervals for R are then found by setting Equation 6.12 equal to the specific t-value and solving the resulting quadratic equation for R. Then the confidence intervals can be immediately obtained for the isocline, k $(\hat{\beta}_3/\hat{\beta}_2)$ X_2. After some manipulation, the two roots of the confidence limits of R may be expected as:

$$R = \frac{-B \pm (B^2 - 4AC)^{1/2}}{2A} \qquad (6.13)$$

where $A = \hat{\beta}_2^2 - t^2 \hat{\sigma}_3^2$

$B = 2(t^2 \hat{\sigma}_{23} - \hat{\beta}_2 \hat{\beta}_3)$

$C = \hat{\beta}_3^2 - t^2 \hat{\sigma}_2^2$

It is possible for both roots to be imaginary. However, Cochran points out that imaginary roots are unlikely to occur when the coefficients of variation for $\hat{\beta}$ are less than 0.3.

The output isoclines derived from empirical production functions in this study are as follows:

Rice region:	$X_3 = 0.76918\ k\ X_2$
Rice-Sweet potato region:	$X_3 = 0.70313\ k\ X_2$
Rice-Tea region:	$X_3 = 1.31350\ k\ X_2$
Mixed region:	$X_3 = 1.01038\ k\ X_2$

where k is a constant. These isoclines indicate the quantity of X_3 (capital) necessary to provide a rate of substitution of k magnitude when X_2 (labor) is given various values. These are linear equations, indicating that the isoclines are straight lines passing through the origin. In other words, the least cost ratio of labor to capital remains the same regardless of the level of output when k equals the price ratio of labor to capital.

In Figures 22 to 25, the isoclines denoted by solid lines, were drawn by letting k equal to the market price ratio of labor to capital in each region. The market price of labor (P_2) varies from region to region but capital price (P_3) is considered to be the same in each region as N.T.$1.10 (N.T.$1 in principal plus 10 percent interest) for each N.T.$1 of capital investment. The size and position of a 95-percent confidence interval for the market price isoclines is also shown in Figures 22 to 25. As outlined above, two different approaches in the derivation of confidence intervals were employed for the isoclines in each region. These confidence boundaries are shown as dashed lines in the figures. They all turned out to be straight lines passing through the origin. It is obvious that the confidence intervals estimated by the second approach from Equation 6.12 are always in the linear form. However, if the first approach was employed for computing var (X_3) as shown in Equation 6.9, the confidence boundaries may be a curved line. In this study, however, the confidence boundaries estimated by these two methods are all straight lines in all regions.

As it can be seen in Figures 22 to 25, the confidence intervals estimated from Equation 6.11, the first method, are not different greatly from those estimated by the second

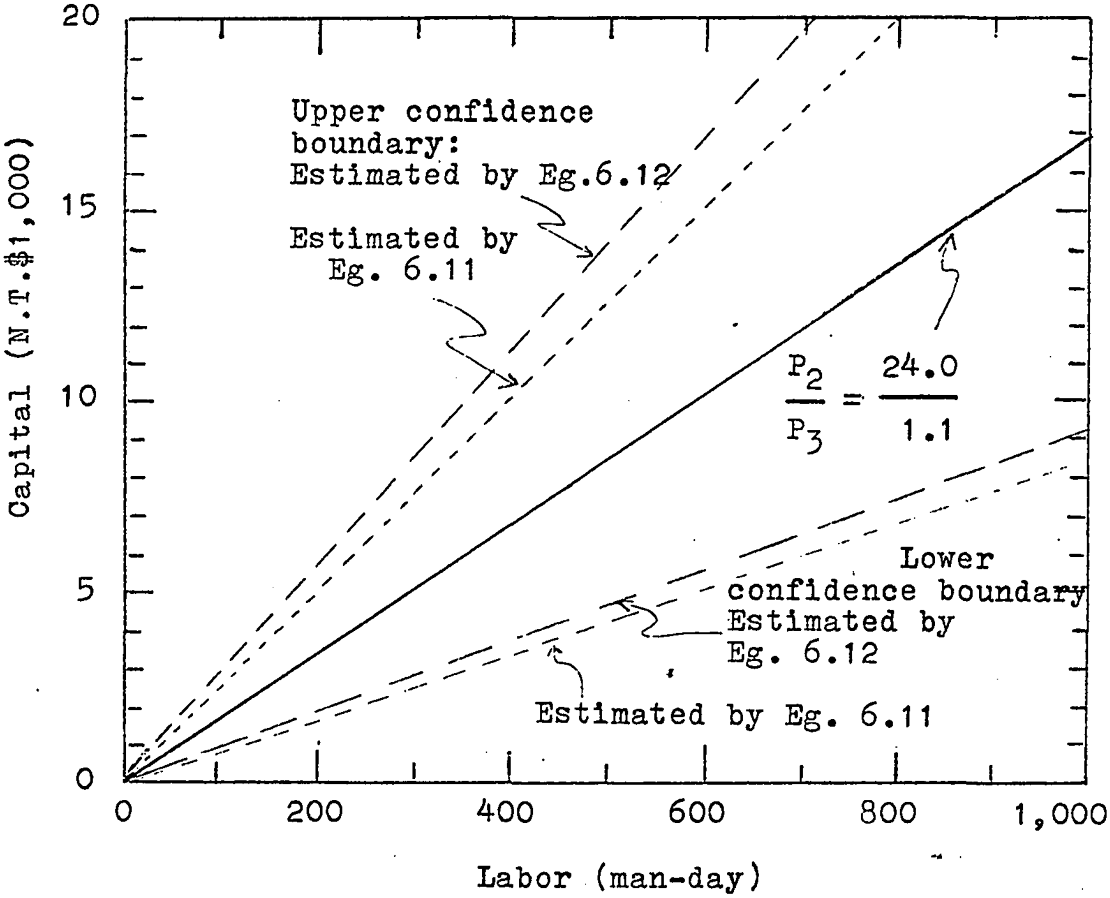

Figure 22. Market prices isocline and the 95 percent confidence intervals in Rice region

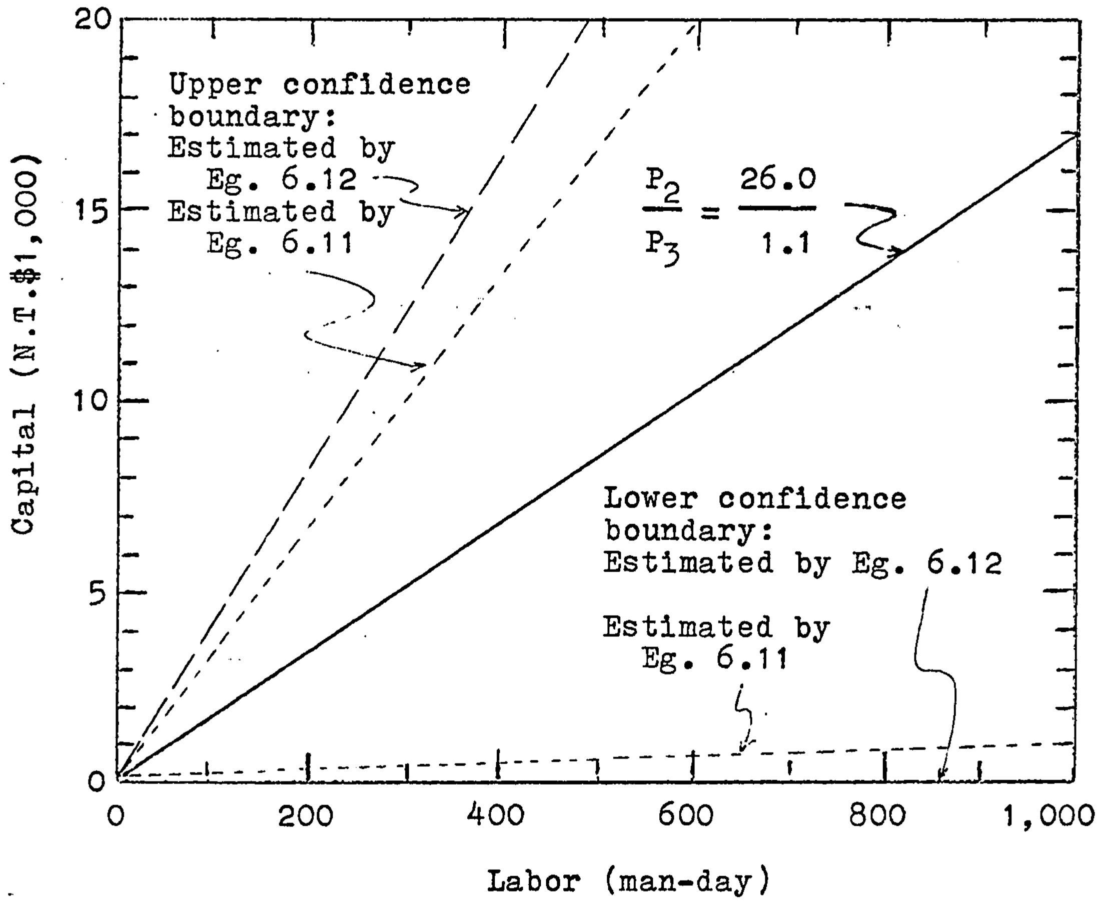

Figure 23. Market prices isocline and the 95 percent confidence intervals in Rice-Sweet potato region

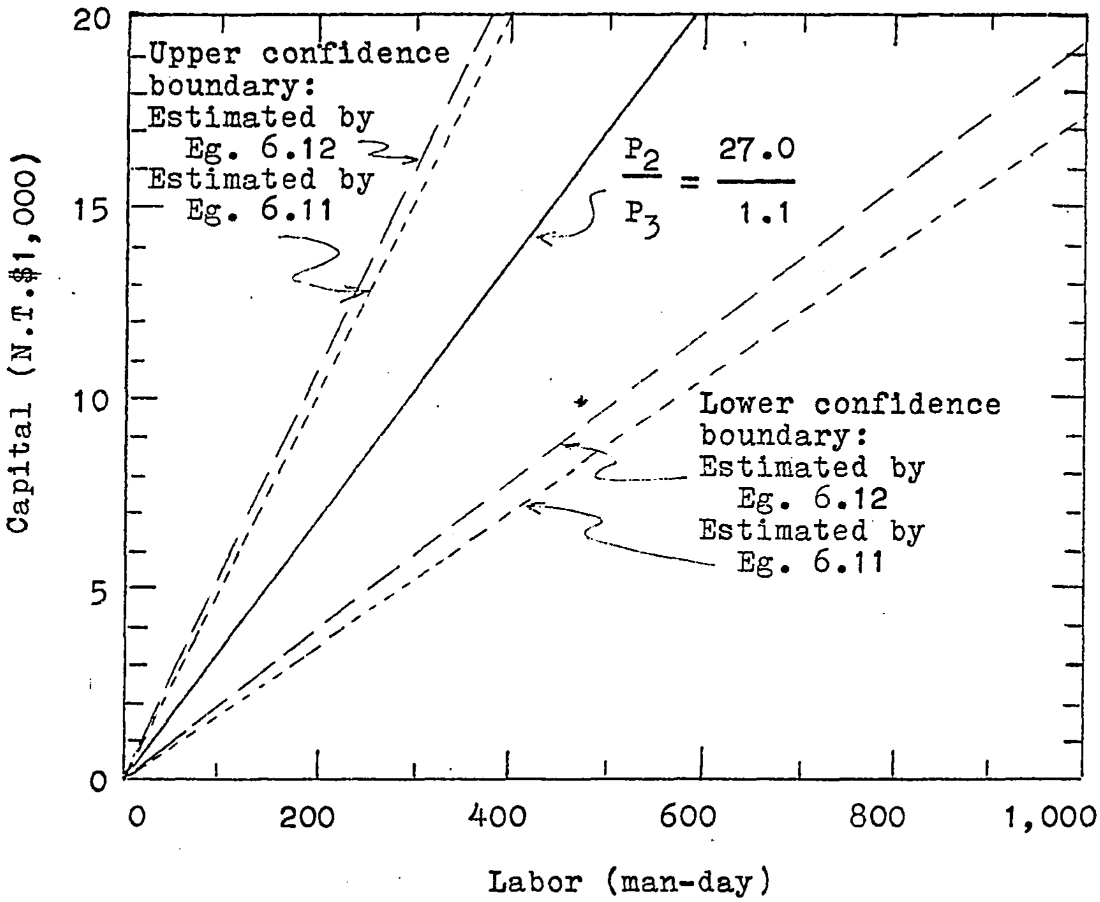

Figure 24. Market prices isocline and the 95 percent confidence intervals in Rice-Tea region

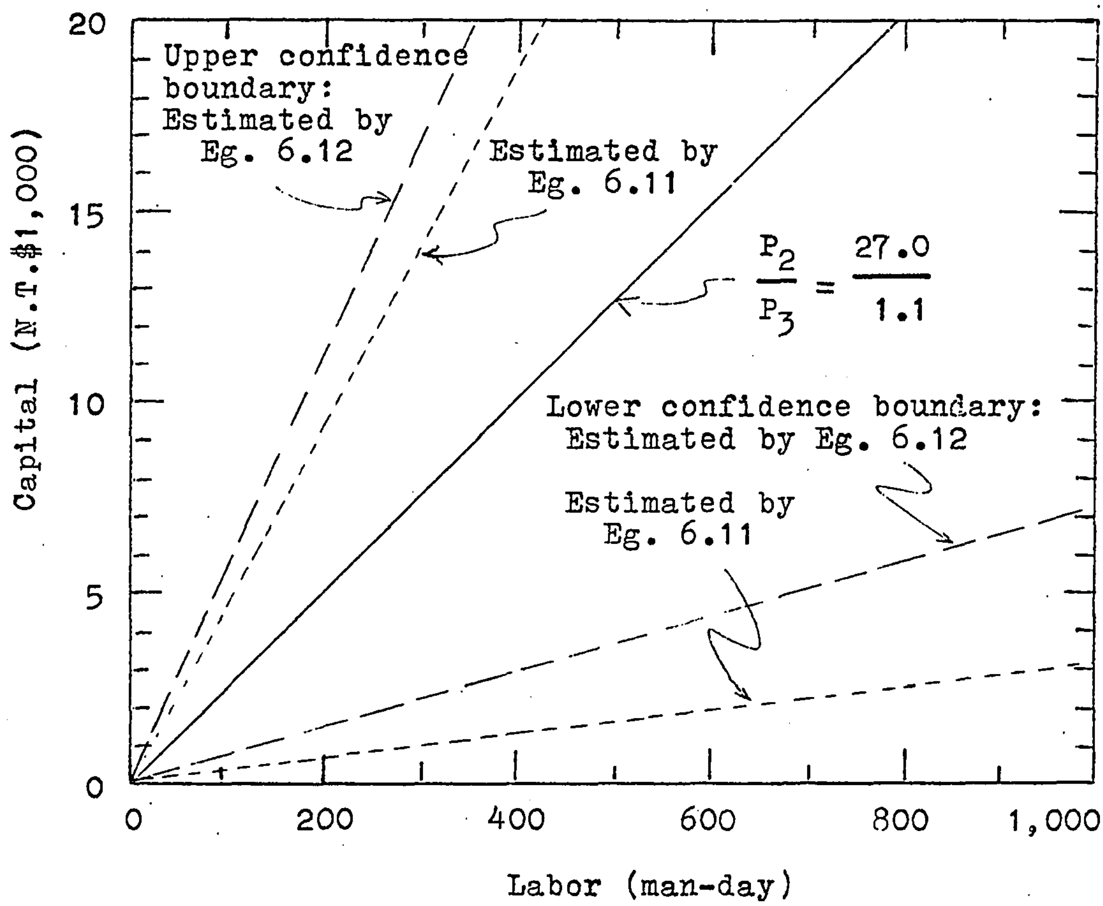

Figure 25. Market prices isocline and the 95 percent confidence intervals in Mixed region

method from Equation 6.12, particularly in the Rice and Rice-Tea regions. As expected, the width of each confidence interval increases as the amount of labor and capital increases along the isocline. In other words, for higher level of outputs, it cannot be ascertained with certainty for the least cost combination of labor and capital on the basis of the available data.

C. Least Cost Combination of Resources

Isoclines presented previously provide the optimum combination of labor and capital for any given level of output. The point of intersection of the appropriate isocline with a specified isoquant gives the optimum combination of labor and capital for the given yield output. For current market price conditions, the optimum combination of labor and capital for the geometric mean output are shown as point L_1 in Figures 26 to 29. If the market price of labor in each region decreases to N.T.$20 per man-day, the optimum combination of labor and capital for attaining the same mean-value of output will shift to the point L_2. The optimum amount of labor is increased under the relative low labor price. Point M in the figures denotes the observed geometric means of output and the three basic resources in our production functions in each region as shown in Table 15.

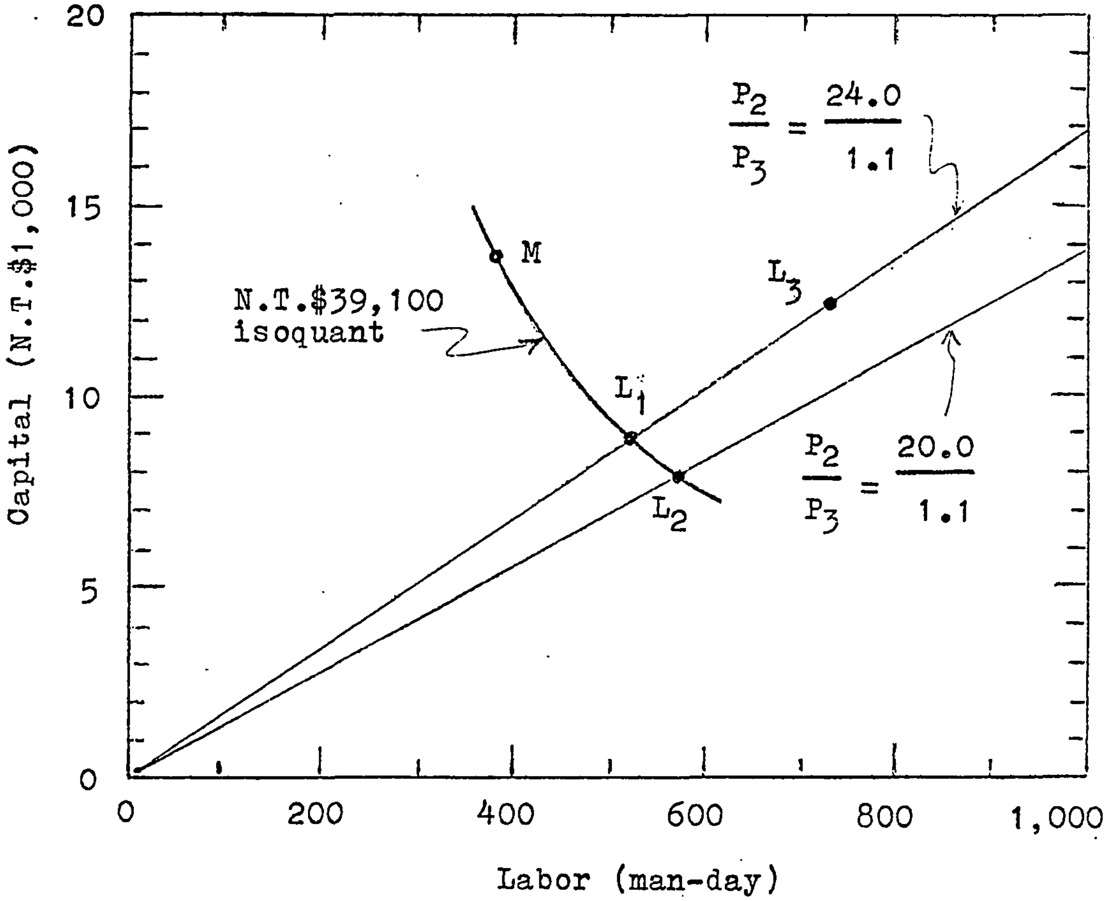

Figure 26. Least cost combinations of resources in Rice region

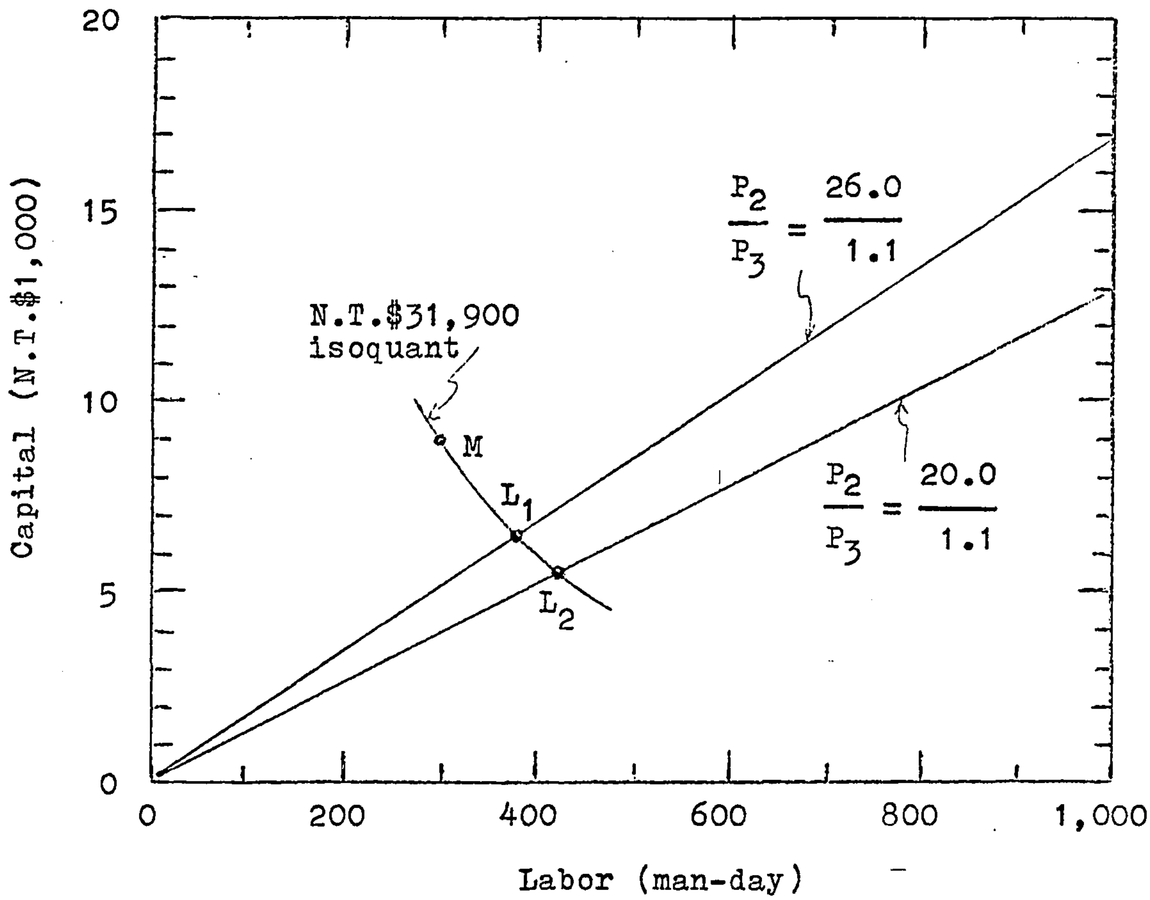

Figure 27. Least cost combinations of resources in Rice-Sweet potato region

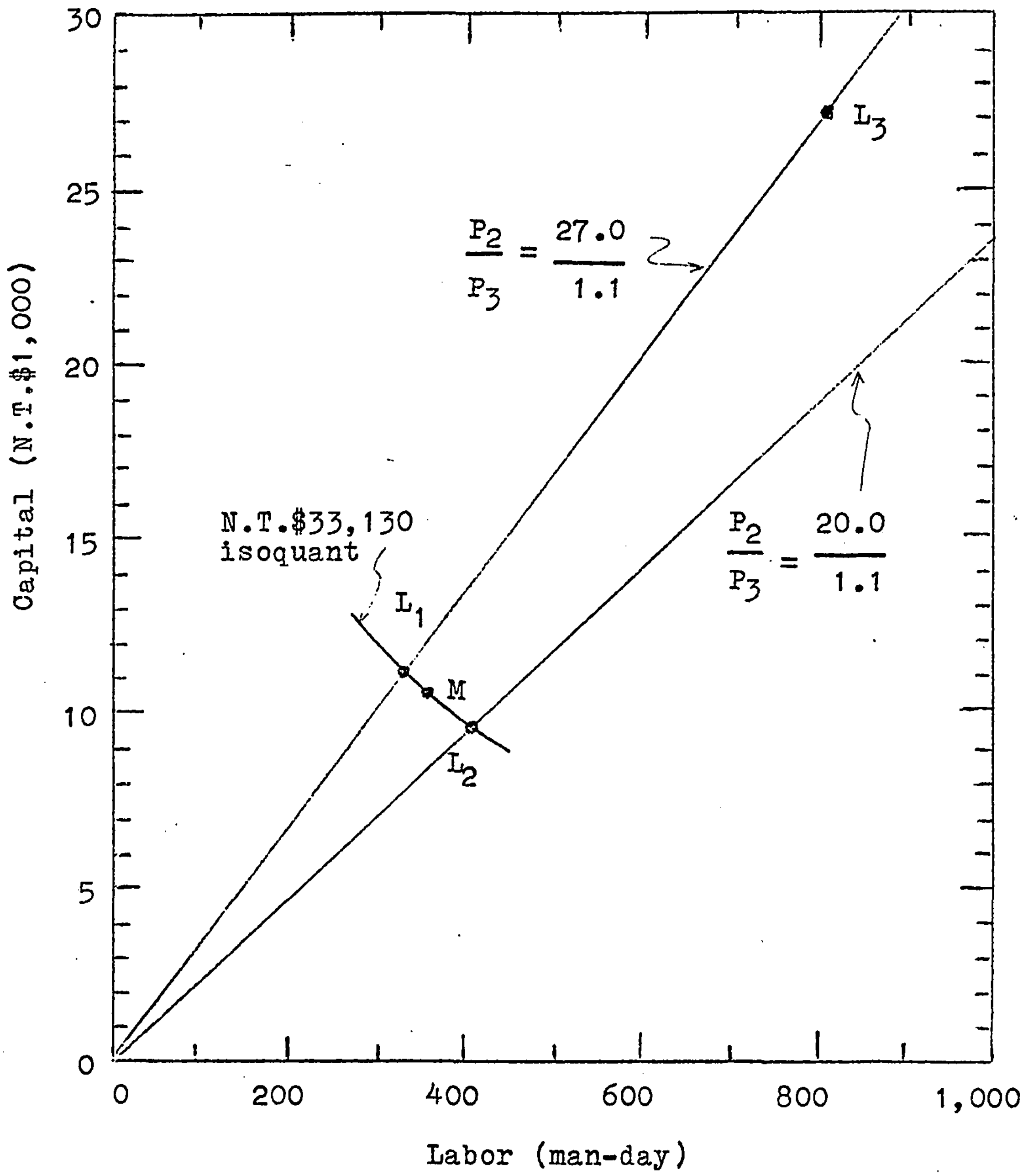

Figure 28. Least cost combinations of resources in Rice-Tea region

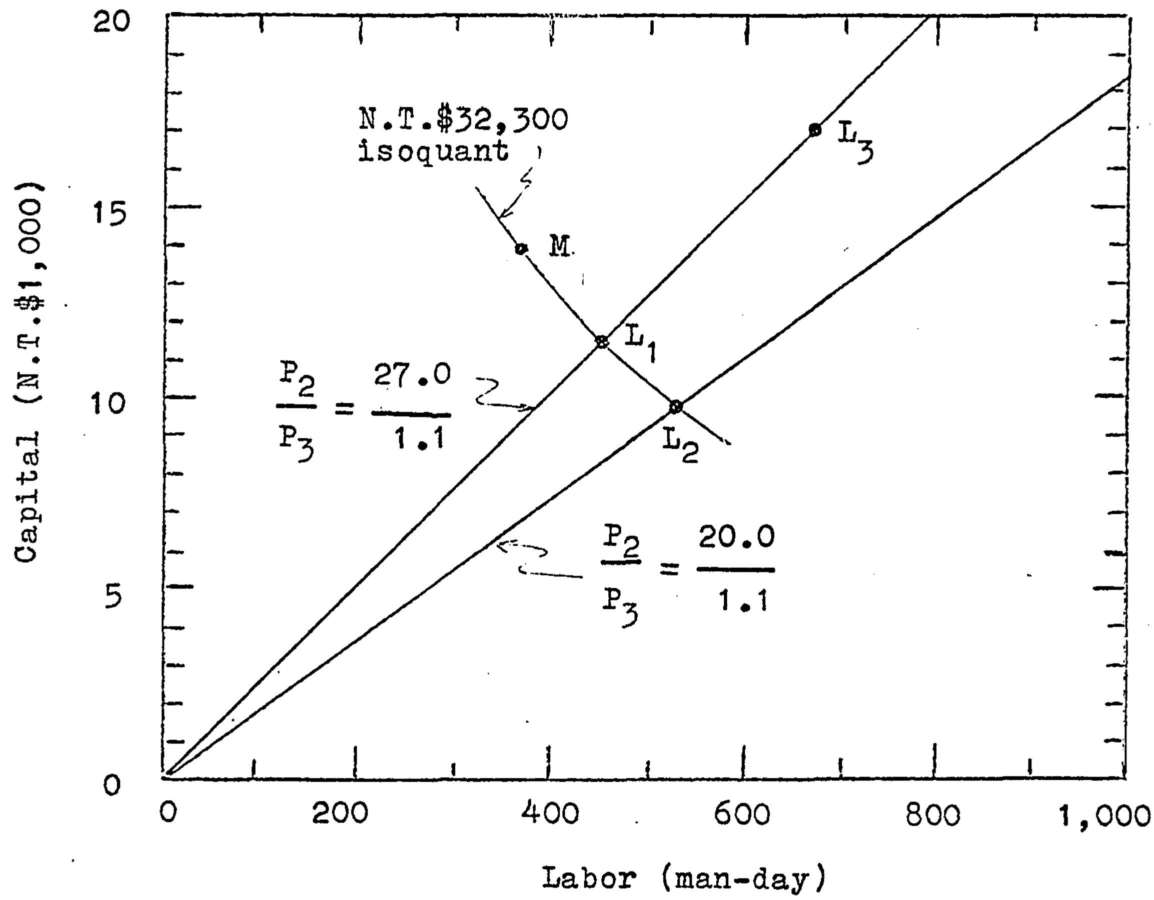

Figure 29. Least cost combinations of resources in Mixed region

By the comparison of points M and L_1, it is obvious that farmers in the Rice, Rice-Sweet potato and Mixed regions have had too much capital investment related to the labor resource under the current market price situation. While in the Rice-Tea region, farmers should spend more for capital on their farms for the least cost combination.

The least cost or optimum combination of resources discussed so far is concerned with labor and capital resources only. It is, however, possible to find the least cost combination for all three resource inputs. Such a least cost combination is obtained when the marginal productivities of land, labor and capital are all equal to their market prices respectively. The condition for the least cost combination can be thus expressed by Equation 6.14 in a general form.

$$\frac{X_i}{X_1} = \frac{\hat{\beta}_i}{\hat{\beta}_1} \cdot \frac{P_1}{P_i} = \quad ; \quad i = 2 \text{ and } 3 \qquad (6.14)$$

In the Rice region, for example, under current price conditions, Equation 6.14 is as follows:

$$\frac{X_2}{X_1} = \frac{0.4341}{0.2213} \cdot \frac{66.70}{24.00} = 5.45$$

$$\frac{X_3}{X_1} = \frac{0.3339}{0.2213} \cdot \frac{66.70}{1.10} = 91.49$$

The least cost combination of land, labor, and capital are then as the proportions of 1:5.45:91.49. These proportions remain the same for any given output level. With farm land fixed at the geometric mean of the sample farms in the region, the least cost combinations of these three resources in different regions are shown in Table 17, and they are also indicated as point L_3 in Figures 26, 28 and 29. In the calculation of least cost combination of these three resources, the Rice-Sweet potato region was excluded because of the negative elasticity coefficient of land in this region. It can be seen that points L_1 and L_3 are on the same isocline. This indicates that these two points are both least cost combinations for different levels of output under the same market price situations. Point L_3, however, represents the least cost combination of all three resources when land is fixed at the geometric mean quantity of the sample farms, while the point L_1 indicates only the least cost combination of labor and capital, for the particular mean-value of output. Although the point L_1 is an optimum point for the combination of labor and capital, it is not an optimum point for using land resource. As shown in Figure 26, farmers in the Rice region on the average can use their resources more efficiently by employing larger amount of labor and less capital on their farms to attain a higher level of output. This is obvious by comparing the points M and L_3. As for the Rice-Tea and Mixed regions, both labor and capital must be increased in

Table 17. Least cost combination of resources under current market price

Resources combination in different regions		Geometric mean of sample farms	Least cost combination for land at mean value
Rice region:			
Land	(are)	134	134
Labor	(man-day)	381	730
Capital	(N.T.$)	13,530	12,258
Expected output	(N.T.$)	39,100	50,220
Rice-Tea region:			
Land	(are)	128	128
Labor	(man-day)	351	836
Capital	(N.T.$)	10,640	26,970
Expected output	(N.T.$)	33,130	76,160
Mixed region:			
Land	(are)	105	105
Labor	(man-day)	369	678
Capital	(N.T.$)	13,780	16,827
Expected output	(N.T.$)	32,300	46,665

order to utilize efficiently farm resources under the average farm land size and current price situations.

D. Profit Maximizing Quantities of Resources

Quantities such as those derived in the previous sections provide the basis for specifying (1) the least cost combination of resources for any yield output level and (2) profit maximizing quantities of resource combinations. This section deals with these quantities under given market price ratio for a farmer who might have unlimited capital.

In general, the profit is maximized from a given farm when the marginal products of resources are simultaneously equal to their respective prices divided by the price of the product. In this study, output was measured in dollars, therefore, the maximum profit condition may be expressed as when the marginal product returns of resources are simultaneously equal to their respective prices as shown in the following:

$$\frac{\partial Y}{\partial X_1} = \beta_1 \frac{Y}{X_1} = P_1 \qquad (6.15)$$

$$\frac{\partial Y}{\partial X_2} = \beta_2 \frac{Y}{X_2} = P_2 \qquad (6.16)$$

$$\frac{\partial Y}{\partial X_3} = \beta_3 \frac{Y}{X_3} = P_3 \qquad (6.17)$$

As discussed above, if farm size was predetermined at the geometric mean of the sample farms in each region, the production functions then become as Equation 6.2, and the corresponding profit maximizing quantities of labor and capital can be obtained by solving Equations 6.16 and 6.17 simultaneously. Such optimum quantities of labor and capital were calculated for the groups of farmers in the Rice, Rice-Sweet potato, Rice-Tea, and Mixed regions as shown in Table 18. It is obvious that the optimum combination gives very large quantities of labor and capital, far beyond the range of observed data, except in the Rice region. Such a strange combination of these two resources is due to the large value of the production coefficients of labor and capital as shown in Table 18. The sum of the production coefficients of these two resources is larger than 0.92 in the Rice-Sweet potato, Rice-Tea and Mixed regions. It indicates that a 1% joint increase in the use of labor and capital gives more than 0.92% increase in the output. Therefore, the maximum profit calls for large quantities of labor and capital in these regions.

As outlined above, the profit maximizing quantities of all three resources can be obtained by solving Equations 6.15, 6.16 and 6.17 simultaneously. However, as the sum of the three production coefficients is greater than 1 in some

Table 18. Profit maximizing quantities of labor and capital with land predetermined at the geometric mean

Items	Rice region	Rice-Sweet potato region	Rice-Tea region	Mixed region
Profit maximizing quantities:				
Labor (man-day)	1,871	1,028,000	36,450	827,100
Capital (N.T.$)	31,400	17,100,000	1,175,000	20,510,000
Sum of production coefficients of labor and capital	0.7680	0.9265	0.9217	0.9687

regions, there is no combination of the three resources which gives maximum profit. For example, in the Mixed region, a 1% joint increase in the use of all three resources gives 1.1522% increase in the yield output and thus adds to the profit. When one of the resources has a fixed value, it is possible, because the sum of the other two coefficients is less than 1, to find the optimum combination of the other two resources as shown above in Table 18.

The maximum profit combination of all three resources has been calculated for the Rice region, which has significant coefficients for all resources and the sum of these three coefficients is less than 1. The resulting quantities which give the maximum profit for the particular farm production function under analysis are very large, far beyond the range of the data in the sample. Consider the profit function =W-E, where $E=P_1X_1+P_2X_2+P_3X_3$ is a cost function and W is the total value of output. Since E is a homogenous function of the first degree, and W in our example is a homogeneous of degree β, $\beta = \beta_1 + \beta_2 + \beta_3 = 0.9893$, we can write $E_m = E_o$ and $W_m = {}^{\beta}W_o$ for any constant and resource combination $L_o = (X_{1o}, X_{2o}, X_{3o})$ and $L_m = (X_{1m}, X_{2m}, X_{3m})$ respectively, such that $X_{im} = X_{io}$, i = 1, 2, 3. Correspondingly, we have $_o = W_o - E_o$ for the first resource combination, L_o and $_m = W_m -$

E_m, or alternative $\pi_m = \lambda^{\beta} W_o - \lambda E_o$, for the second resource combination. It is now possible to find the profit rate of change, π_m / π_o, for any chosen value of λ.

For our specific sample in the Rice region let us choose the point L_o as the least cost combination of all three resources when X_1 equals to its geometric mean as shown at the point L_3 in Figure 26, giving us by computation, W_o = N.T.\$50,220, E_o = N.T.\$38,716 and π_o = N.T.\$11,504. If we can contract all resource inputs by one half, i.e., choosing $\lambda = 0.5$, we obtain $W_m = \lambda^{\beta} W_o = (0.5)^{0.9893}$ (50,220) = 25,296, $E_m = \lambda E_o$ = (0.5) (38,716) = 19,358, and π_m = 5,938. The profit rate, π_m / π_o, is now found to be 0.5162. If all three resources employed are doubled in quantity, i.e., choosing $\lambda = 2$, then the profit rate, π_m / π_o = 22,505/11,504, is found to be 1.9389. It is also possible to find the necessary value of λ which gives the maximum profit resource combination. This optimum value of λ can be obtained by the following steps:

$$\pi_m = \lambda^{\beta} W_o - \lambda E_o \tag{6.18}$$

$$\frac{\partial \pi_m}{\partial \lambda} = \beta \lambda^{\beta - 1} W_o - E_o \tag{6.19}$$

$$\lambda = \left(\frac{E_o}{\beta W_o} \right)^{\frac{1}{\beta - 1}} \tag{6.20}$$

Given the profit function Equation 6.18, the maximum profit value of λ can be thus obtained as shown in Equation 6.20.

In our example, the optimum value of λ for the group of farmers in the Rice region is very large and turned out to be 1,323 x 10^7, far beyond the range of the data in our sample.

VII. SUMMARY

The main purpose of this study is to analyze the resource productivity in different agricultural regions of Taiwan. One of the problems of production economics which confronts farmers is that of determining the proper combination of resources to use in production. This study shows the extent of differentials in resources productivities between agricultural regions and the possible income that can be expected from different types of farming areas. It also intends to examine the implications and limitations of current statistical analyses applied to the cross-sectional farm survey data for estimating production functions on Taiwan's small farms.

Six agricultural regions, namely the Rice region, Sweet potato region, Tea region, Rice-Sweet potato region, Rice-Tea region and Mixed region, in the northern part of Taiwan were selected as the survey area for this study. A total of 400 farms was then drawn at random in 1962 from these six regions. All information obtained in this study related to crop year of 1961.

Extensive tabular analysis is first employed to examine the situation of the farm economy in each region. From farm sample in each region, production functions and marginal resource productivities are then derived. Three different methods of estimation, the least squares estimate, Hoch's estimate and Klein's estimate have been examined in this study. Other quantities, such as the estimation of isoquants and isoclines, and their confidence intervals have also been derived in this study.

The production functions derived were as follows:

Rice region:

$$\hat{Y} = 41.88\ X_1^{0.2213}\ X_2^{0.4341}\ X_3^{0.3339}$$

Sweet potato region:

$$\hat{Y} = 138.67\ X_1^{0.2489}\ X_2^{0.0636}\ X_3^{0.4014}$$

Tea region:

$$\hat{Y} = 46.41\ X_1^{0.3948}\ X_2^{-0.0017}\ X_3^{0.4872}$$

Rice-Sweet potato region:

$$\hat{Y} = 48.74\ X_1^{-0.0169}\ X_2^{0.5440}\ X_3^{0.3825}$$

Rice-Tea region:

$$\hat{Y} = 15.12\ X_1^{0.1041}\ X_2^{0.3984}\ X_3^{0.5233}$$

Mixed region:

$$\hat{Y} = 7.70\ X_1^{0.1841}\ X_2^{0.4816}\ X_3^{0.4866}$$

For those functions, Y refers to output in dollars, X_1 refers to land in ares (i.e., 0.01 hectare), X_2 refers to labor in man-days and X_3 refers to capital in dollars.

Marginal resource productivities differ greatly among the six regions. The marginal product returns per are of land, with resources at the geometric mean, vary from N.T. $80.53 in the Sweet potato region to N.T.$-5.62 in the Rice-Sweet potato region. Marginal product returns per man-day of labor vary from N.T.$56.60 in the Rice-Sweet potato region to N.T.$0.11 in the Tea region. Marginal productivity of capital, per N.T.$1 input varies from N.T.$1.63 in the Rice-Tea region to N.T.$0.96 in the Rice region.

In terms of the estimates of this study, the efficiency of resources has been tested. Land service has been used efficiently in both the Rice and Mixed regions in the sense that the marginal productivity of land approached the market price in these two regions. Marginal product return to land in the Tea region was statistically much greater than the market price. Farm size in this region should be enlarged under the diminishing returns nature of land. On the other hand, the size of farms in the Rice-Tea region should be reduced, on the average, in order to improve the land utilization toward its optimal situation. The productivity of land in both the Sweet potato and Rice-Sweet potato regions were not significantly different from zero.

Farmers in most of the regions did not use efficiently their labor service in the acceptable probability level. The amount of labor used on farms seems much smaller than the optimal quantity under the particular price level and production techniques. Generally speaking, farmers were maximizing returns from the use of capital under the given condition. This result seems reasonable in view of the fact that farmers in Taiwan usually have used their capital resource more carefully than labor resource.

Marginal productivity curves of resources and the 95 percent confidence intervals have also been derived in different regions.

The geometric mean-output isoquants were derived from production functions for four regions. These isoquants show the necessary combination of labor and capital to attain the mean-output level of sample farms in each region, with land input held constant at the geometric mean value. The approximate 95 percent confidence intervals were also computed. The confidence intervals in the Rice and Rice-Tea regions were much more narrow than in the regions of Rice-Sweet potato and Mixed. The narrowness of the confidence interval implies that the estimated production function provides a reasonably reliable estimate of capital quantity with a fixed amount of labor and land for a given level of output.

Isoclines have also been derived from the production functions for the same four regions. The 95 percent confidence intervals are also narrower in the Rice and Rice-Tea regions.

The least cost combination of labor and capital for a given level of output can be determined from the isoclines under a given price ratio. On the average, farmers in the Rice, Rice-Sweet potato and Mixed regions have had too much capital investment related to the labor resource under current production techniques. While in the Rice-Tea region, farmers should spend more capital on their farms to approach the least cost combination. However, since the analysis did not give full consideration to functional relationships between products and resources which fall in the complementary and supplementary phases, a more detailed analysis is needed for the determination of efficient utilization of labor and capital resources.

The least cost combination of all three resource inputs was also derived for the Rice, Rice-Tea and Mixed regions. Given current prices and production technique situations, the least cost combination should be 134 ares of land, 730 man-days of labor and N.T.$12,258 of capital for farmers in the Rice region. In the Rice-Tea region, the least cost combination was 128 ares of land, 836 man-days of labor and N.T. $26,970 of capital, while in the Mixed region, it was 105 ares of land, 678 man-days of labor and N.T.$16,827 of capital. The proportions of the least cost combination of resources remains constant for any level of output.

The limitation of this study is the estimation of productivity coefficients based on the particular algebraic production functions. Agriculture involves a highly complex production process and it is doubtful that any single algebraic function can accurately predict all of the relevant productivity coefficients. A function may allow estimates with small error over some range of the data, or it may involve a larger error over some other range of the data. This study probably provides some information for those who are interested in the total returns expected and resource productivities in different agricultural regions. However, more detailed studies are still needed to specify the optimum combination of all resources and the techniques of production to be employed for any individual farm.

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