NUMERICAL SIMULATION OF FLAW DETECTION WITH A CAPACITIVE
ARRAY SENSOR USING FINITE AND INFINITE ELEMENTS

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INTRODUCTION

Capacitive array sensors are one among many electromagnetic techniques that can be used to detect flaws or other irregularities at or near the surface of materials. Capacitive sensors have an advantage over inductive sensors in that insulating materials may be interrogated as well as conducting materials. These sensors have seen some application in nondestructive evaluation, including flaw detection, the monitoring of porosity and thickness of thermal barrier coatings, dielectric cure monitoring, and robotic proximity sensing [1, 2]. Only surface features can be examined on metallic plates because the accumulation of surface charges blind the capacitive probe to interior features. In dielectric materials, both surface and subsurface features can be examined.

The basic element of the capacitive probe is a parallel plate capacitor with the electrodes unfolded such that they lie in the same plane. The electric field generated by this configuration can be used to scan the specimen and obtain some relative response that is a function of the flaw geometry, specimen constitution, and probe location relative to the flaw. This response is caused by the change in current flow between the electrodes of the probe that results from the influence of the specimen. This type of probe is versatile because its behavior may be changed by altering the size, shape, number, and spacing of the probe fingers. Hence the probe design may be optimized to suit a particular application.

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Other models have been developed for capacitive array sensors using finite difference and finite element techniques [3, 4], but both of these approaches approximated the infinite space surrounding the test region by a large but finite domain. Although good qualitative agreement was found between these numerical studies and existing experimental data, the numerical model should take into account the effects of an infinite domain. The purpose of this paper is to combine finite and infinite elements to accurately model the response of a capacitive array sensor in the presence of flawed dielectric and metallic slabs. The probe region is modeled using finite elements and the infinite domain surrounding the probe region is modeled using infinite elements. Although numerous probe configurations could be considered, the analysis in this paper will be restricted to a three-finger probe geometry. This configuration consists of three conducting strips spaced at equal intervals on a dielectric substrate. The geometry and nomenclature of the probe and a sample specimen are shown in Figure 1. Although the probe and the specimen are three-dimensional, we assume that they are infinitely long in the direction of the fingers. Hence, a two-dimensional idealization is assumed for this study.

Governing Equations

In modeling the capacitive array sensor, the assumption is made that the fields are electrostatic. For a two-dimensional region Ω with no free charge, the governing equation may be written as

$$-\frac{\partial}{\partial x} \left(\epsilon_x \frac{\partial \Phi}{\partial x} \right) - \frac{\partial}{\partial y} \left(\epsilon_y \frac{\partial \Phi}{\partial y} \right) = 0 \quad \text{in } \Omega \quad (1)$$

Here Φ is the electrostatic potential and ϵ_x and ϵ_y are the dielectric constants in the two coordinate directions x and y . The corresponding essential and natural boundary conditions are given by

$$\begin{aligned} \Phi &= \hat{\Phi} \\ &\text{on } \Gamma \end{aligned} \quad (2)$$

$$\epsilon_x \frac{\partial \Phi}{\partial x} n_x + \epsilon_y \frac{\partial \Phi}{\partial y} n_y = \hat{t}_n ,$$

where the circumflex denotes a known quantity, Γ represents the boundary of the domain Ω , and n_x and n_y are the x and y components of the unit normal vector.

Fig. 1 Geometry of probe and specimen

To obtain the variational or weak form of Eq. (1) over an element, we multiply this equation by a sufficiently differentiable test function, taken here as the first variation of Φ , and integrate over the domain of a typical finite element Ω_e . Integrating by parts and applying Green's theorem yields

$$\int_{\Omega_e} \left[\epsilon_x \frac{\partial \delta \Phi}{\partial x} \frac{\partial \Phi}{\partial x} + \epsilon_y \frac{\partial \delta \Phi}{\partial y} \frac{\partial \Phi}{\partial y} \right] dx dy - \oint_{\Gamma_e} \delta \Phi \left[\epsilon_x \frac{\partial \Phi}{\partial x} n_x + \epsilon_y \frac{\partial \Phi}{\partial y} n_y \right] ds, \quad (3)$$

where Γ_e represents the boundary of the element domain Ω_e and ds is the element of arc along the path Γ_e . Equation (3) forms the basis of the finite element model of Eq. (1).

Finite and Infinite Element Model

Decay function infinite elements [5,6] are formed by multiplying the original finite element shape function $N_i(\xi, \eta)$ by a decay function $f_i(\xi, \eta)$ which will model the behavior of the element at infinity

$$\psi_i(\xi, \eta) = N_i(\xi, \eta) f_i(\xi, \eta). \quad (4)$$

The decay function is chosen such that the rate of decay qualitatively matches that of the problem. Two major requirements of the infinite element shape functions $\psi_i(\xi, \eta)$ are that they tend to the far field value at infinity and that they equal unity at their own node. Since the latter requirement is already met for the conventional shape functions, this implies that the decay function is equal to unity at its own node. Although the infinite element shape functions are used to describe the behavior of the unknowns of the problem, the conventional shape functions $N_i(\xi, \eta)$ are used to define the coordinates of the infinite element. This implies that the mapping of the element through the Jacobian is restricted to the terms traditionally used in conventional finite element analysis.

We next assume that the value of Φ within an element may be approximated by the expression

$$\Phi(x, y) = \sum_{j=1}^n \Phi_j \psi_j(x, y) \quad (5)$$

where Φ_j are the nodal values of the scalar potential, ψ_j are the appropriate shape functions for the given element, and n is the number of nodes for a given element. The shape function ψ_j can be reduced to a conventional finite element shape function by setting $f_i(\xi, \eta)$ equal to unity. Substituting Eq. (5) into Eq. (3) yields the final form of the element equation

$$[K]\{\Phi\} = \{F\} \quad (6)$$

where $[K]$ is the element coefficient matrix, $\{F\}$ is the element force vector, and $\{\Phi\}$ is the vector of unknown nodal potentials. The elements of $[K]$ and $\{F\}$ are given by

$$K_{ij} = \int_{\Omega_e} \left[\epsilon_x \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \epsilon_y \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right] dx dy \quad (7)$$

$$F_i = \oint_{\Gamma_e} \psi_i \left[\epsilon_x \frac{\partial \Phi}{\partial x} n_x + \epsilon_y \frac{\partial \Phi}{\partial y} n_y \right] ds. \quad (8)$$

The assembly and solution of the global system of equations follows the standard procedures used in finite element analysis. For the capacitive array sensor, the force vector $\{F\}$ will in general be null, and the forcing function will be in the form of the nonhomogeneous potentials prescribed at the sensor locations.

From the variational form given in Eq. (3), we note that the order of approximation for $\Phi(x,y)$ must be at least bilinear. The surface term in Eq. (3) also indicates that Φ must only be C^0 continuous across element boundaries. Linear Lagrange quadrilaterals were used exclusively in the numerical examples that follow.

The choice of the decay function used in Eq. (7) must meet a number of criteria. One requirement places restrictions on ψ_i such that Green's theorem will hold (see Bettess [6]). The infinite element shape function must allow the field variable to tend to the far field value at infinity, and the value of ψ_i must be equal to unity at its own node. Although a number of decay functions meet these requirements, we chose exponential decay functions to model the behavior of the capacitive array sensor. The form of the decay function used in this study was originally proposed by Bettess [6] and is given by

$$f_i(\xi, \eta) = \exp [(\xi_i - \xi)/L] , \quad (9)$$

which implies that the decay is only in the positive ξ direction. Here L is a decay parameter which determines the relative rate of decay. There are several advantages of using the form of f_i given in Eq. (9), including its rapid decay and ease of mathematical manipulation. It is possible to construct an element with decay in both the ξ and η directions. For the domain considered in this study of the capacitive array sensor, one row of infinite elements is used to surround a mesh of conventional finite elements. Each of the infinite elements are oriented such that the element sides lie along a radius from the center of decay, which was taken to be the geometric center of the middle probe finger. Hence the functions f_i only need to decay in the ξ direction.

Evaluating the terms of the infinite element stiffness matrix involves computing the value of an integral of the form

$$G(\xi) = \int_{-1}^{\infty} g(\xi) \exp (-2\xi/L) d\xi \quad (10)$$

Integration in the η direction is performed numerically using Gauss-Legendre integration. Performing a simple mapping on the variables in Eq. (10) allows the integral $G(\xi)$ to be written as (See Bettess [6])

$$G(\xi) = \int_0^{\infty} g\left(\frac{L}{2}s - 1\right) \exp\left(\frac{2}{L}\right) \exp(-s) ds . \quad (11)$$

This integral is of the form

$$\int_0^{\infty} f(x) e^{-x} dx , \quad (12)$$

which is evaluated using Gauss-Laguerre integration [7]. This integration is exact if $f(x)$ is a polynomial and the appropriate number of terms in the integration formula are taken.

The major difficulty in using infinite elements with exponential decay as in Eq. (9) lies in determining the correct value of the decay length L to be used in formulating the infinite element matrices. It is

possible to match the decay length to known forms of decay by evaluating the decay function at several points of the infinite element [6]. It is also possible to match the global behavior of the decay function to the true behavior of the field variable beginning at the edge of the infinite element. Both of these approaches require expressions for the true rate of decay. This information is not always available, particularly for problems with complex geometries and nonuniform source potentials.

The procedure we used to determine L for the capacitive array sensor involves several simplifying assumptions. The domain of the probe and specimen are discretized into a square mesh of finite elements with the mesh center coinciding with the center of the middle probe finger. This point is assumed to represent the center of decay. A single row of infinite elements is placed around the outer boundary of the conventional element mesh. Each of the infinite element sides is parallel to a radius from the center of decay. It is further assumed that the decay length L is a constant for all of the infinite elements. This avoids the tedium of optimizing the decay length for each element and ensures continuity of potentials between adjacent infinite elements. The decay length for a specimen with a given dielectric constant is determined by averaging the two solutions obtained by specifying homogeneous potential and flux at the outer boundaries of the finite element mesh for an unflawed sample. This results in an underestimate and an overestimate of the true potential [8], respectively. The decay length used in subsequent studies is the value that yields the most consistent results to this average solution. Although the presence of the flaw will slightly change the rate of decay at certain locations around the mesh, this effect will be neglected in this study. The advantage of this approach is that once the decay length has been computed, the same value may be used for each probe location.

Numerical Modeling of Capacitive Array Sensor

The major objective of this paper is to apply the preceeding formulation to model flaw detection in dielectric and metallic slabs using a capacitive array sensor. The domain surrounding the probe and the specimen is assumed to be filled with air ($\epsilon = 1.0$) and is assumed to extend to infinity. The domain and geometry of the basic capacitive probe are shown in Fig. 1. The spacing of the three probe fingers, denoted by the letter B , will be used as a characteristic length throughout the remainder of this paper. The liftoff distance between the unflawed specimen surface and the bottom of the probe fingers is taken as $0.3B$ unless otherwise noted. The probe fingers have dimensions $0.2B \times 0.6B$ and are perfect conductors. The fingers are mounted on a dielectric material of thickness $1.5B$ with a dielectric constant $\epsilon = 3.9$. All voids are assumed to be filled with air, $\epsilon = 1.0$. The dielectric constant of the specimen is denoted by ϵ_s and will be noted for each particular case.

The change in the probe admittance caused by a flaw in a specimen in the vicinity of a probe can be written, as derived from the Lorentz reciprocity theorem [1], as

$$\Delta Y_{21} = \frac{j\omega\epsilon_0}{V^2} \int_A \left[\Phi_1 \frac{\partial \Phi_2'}{\partial y} - \Phi_2' \frac{\partial \Phi_1}{\partial y} \right] dx dy . \quad (13)$$

Here, $j = \sqrt{-1}$, ω is the angular frequency, ϵ_0 is the permittivity of free space, V is the excitation voltage applied to the probe, Φ is the electrostatic potential, and A is the specimen surface area. The unprimed quantities refer to the electric potential on an unflawed specimen, and primed quantities represent potential values on a flawed specimen. Subscripts 1 and 2 denote whether the excitation voltage is

applied to the source or receiver terminals. This condition follows from the use of reciprocity in the derivation of Eq. (13). For the probe configuration shown in Fig. 1, the center electrode is the source and the outer electrodes are differential receivers. For the nomenclature defined in Fig. 1, these conditions imply that $\Phi_a = \Phi_c = 0$, $\Phi_b = 1.0$ for case 1 and $\Phi_a = -0.5$, $\Phi_b = 0$ and $\Phi_c = 0.5$ for case 2. Potentials for case 2 correspond to connecting the probe's receiving terminals to an ideal transformer with a grounded center tap on the input side and a single-ended terminal connected to an ideal operational amplifier on the output side.

The quantity of interest to model the probe's response is the relative change in admittance caused by changes in the specimen's dielectric constant, flaw geometry, or probe location. The quantity that was actually calculated in this study was therefore slightly modified from Eq. (13) and is given by

$$\Delta Y_{21} = \int_L \left(\Phi_1 \frac{\partial \Phi_2'}{\partial y} - \Phi_2' \frac{\partial \Phi_1}{\partial y} \right) dx, \quad (14)$$

where L represents the horizontal line surface of the unflawed specimen and all other quantities are kept as defined above. The magnitude of ΔY_{21} is therefore the relative change in admittance per unit thickness of the specimen. This quantity can be computed for any flaw as long as the flaw geometry falls within the constraints of the finite element mesh. To model the probe signal obtained from scanning the probe over a flaw, the probe is kept in the center of the mesh and the specimen was marched across the domain width from left to right. This minimizes any possible edge effects that may arise from positioning the probe fingers too close to the boundary.

As indicated by Eq. (14), two analyses are required for the computation of ΔY_{21} for each position of the flaw in a scan. Because of the rapid decay of the potential to a non-zero value for case 1, conventional elements are used (specifying $\epsilon_x \partial \Phi / \partial x n_x + \epsilon_y \partial \Phi / \partial y n_y = 0$ at the boundaries) to model this case. Infinite elements are used (with $\Phi = 0$ at the far field) to model case 2. The decay length for case 2 is determined by matching the averaged solution obtained from imposing the two types of boundary conditions for the case of an unflawed specimen. This procedure assumes that the presence of the flaw will not have a significant contribution to the far field rate of decay. These two analyses, which correspond to the computation of Φ_1 and Φ_2' in Eq. (14), are repeated for n flaw positions. The final plot of the change in admittance as a function of the flaw position relative to the probe are thus obtained.

Two different flaw geometries were examined using the capacitive array sensor. The first is a step type of flaw with the lower plane of the step to the right of the upper plane. The second type of flaw is a narrow groove, which has a height and depth equal to one probe finger width. To model these flaw geometries, the probe fingers were kept fixed at the center of the modeled domain. The flaw is marched across the width of the domain from left to right, with the relative change in admittance being computed for each flaw location using Eq. (14).

The response curves for the step geometry are given in Figs. 2 and 3. The three curves in Fig. 2 represent the response for a dielectric constant of 2.0 with the three different types of boundary conditions: homogeneous flux, homogeneous potential, and infinite elements. As was the case for the potential and the flux, the relative change in admittance is underestimated when $\Phi_1=0$ is specified at the far field and

Fig. 2 Relative change in admittance for step geometry ($\epsilon=2.0$)

Fig. 3 Effect of dielectric constant on relative change in admittance for step geometry

overestimated when $\partial\Phi/\partial n = 0$ is specified. The response curve obtained using infinite elements falls between these two curves. The effect of dielectric constant for the step geometry is shown in Fig. 3.

The response curves for the groove geometry exhibit the same type of behavior for the three types of boundary conditions. For this example, only half of the response curve is shown in Fig. 4 (for $\epsilon=2.0$), since the results are antisymmetric about the probe centerline. As before, the response obtained using infinite elements lies between the response curves obtained using the extreme boundary conditions. The effect of dielectric constant on a flaw signal is represented by the curves in Fig. 5.

Fig. 4 Relative change in admittance for groove geometry ($\epsilon=2.0$)

Fig. 5 Effect of dielectric constant on relative change in admittance for groove geometry

Conclusions

Infinite elements provide a means for obtaining more accurate representations of field variables and their derivatives for open boundary problems. If conventional finite elements are used to compute variables which tend to zero at infinity, the only boundary conditions that may be specified relate to the homogeneous potential and flux. These conditions result in underestimates and overestimates of the true potential, respectively. By using infinite elements with a physically meaningful decay rate, fairly accurate approximations of the field variables may be obtained. The main advantage of infinite elements is their ease of implementation.

Infinite elements with exponential decay were used to model a three-fingered capacitive array sensor interrogating dielectric slabs. Flaw geometries modeled were a step and a square groove. The influence of boundary condition and dielectric constant were examined for these two geometries. The response of the probe was measured using a line integral that is a function of the electrostatic potential and its normal derivative along the surface of the tested sample. By using infinite elements to model the infinite region around the probe, more accurate values of the change in admittance were obtained.

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