FRACTURE MECHANICS OF FIBER-REINFORCED COMPOSITES

E. M. Wu
Lawrence Livermore Laboratory
Livermore, California 94550

ABSTRACT

Quantitative understanding of the parameters which control composite fracture is imperative to the implementation of fail safe design and inspection of critical load bearing structures. For isotropic materials, fracture is essentially controlled by a single parameter, e.g., the fracture toughness or the stress-intensity factor. This one dimensional nature lends itself to experimental quantification. However, for anisotropic composites there are at least seven primary controlling parameters: 1) crack length; 2) crack orientation with respect to material axis of anisotropy; 3) nature of applied combined stresses; 4) lamination geometry; 5) deformational and strength responses of the constituent lamina; 6) three kinematically admissible modes of crack extension and 7) crack trajectory. Because of this large number of parameters, experimental quantification by systematic permutation of the parameters must be realistically viewed as intractable. This paper presents an analytical method of reducing these parameters from seven to two and furnishes experimental observations which lend support to the theoretical model. An experimental program is conducted on fiberglass reinforced epoxy where a centrally notched-crack is subjected to combined loading. Several lamination geometries are tested and by varying the external combined loading, different crack trajectories are predicted by the theoretical model. These predicted trajectories agree well with the experimental observed fracture mode. Such agreement suggests that with further refinement, the general condition of laminated fracture can be characterized.

Introduction

In contrast to isotropic metals and polymers, fracture of anisotropic composites is a multi-parameter problem because 1) the near-field stress distribution depends not only on the crack geometry but also on the relation of the crack to the material orientation, and 2) the material resistance to crack propagation is a strong function of the material orientation, e.g., it is more difficult to break fibers than to separate them. As a consequence of the multi-parameter characteristics, the prediction of crack initiation under biaxial loading and the prediction of crack trajectory become relevant to the fundamental understanding of fracture and to the rational analysis of composite engineering structures.

In the trend of current research practices, characterization of the strength of anisotropic multiphase composites is usually separated into two broad categories: 1) the composite strength in the absence of macroscopic flaws, and 2) the composite strength in the presence of macroscopic flaws (and stress-risers). These two categories are usually referred to as anisotropic failure criterion characterization and fracture mechanics respectively, and they are treated as separate physical phenomena. Clearly, such arbitrary categorizing is a consequence of attempting to identify the critical paths of composite strength characterization through association with those experiences gained from isotropic solids. The one-parameter nature of isotropic fracture follows directly from the physical observation that isotropic crack extension is always perpendicular to the direction of maximum tension and the dissipation always occurs via a crack opening mode. Thus, the similarity between the mathematical model and physical observation is easily maintained. In contrast, composites, particularly in the laminated form, exhibit a large range of instability conditions involving various amounts of slow crack growth. First of all, the modes of energy dissipation are not limited to the opening mode but also include forward sliding and out-of-plane shear; the crack trajectories seldom follow the maximum tensile stress direction and often lead to non-self-similar crack extension including complex branching. The effects of the external loads (symmetric and stress-symmetric to the crack) as well as combined loading on crack instability also need to be documented. Finally, the size effect of flaws is far more dominant in composites than in homogeneous isotropic materials.

Whereas the one dimensional nature of isotropic fracture lends itself to experimental quantification in the form of a single critical stress-intensity factor or fracture toughness parameter, the multiple-parameter nature of crack extension in composites precludes empirical permutation of the parameters. For anisotropic composite laminates, there are at least seven primary parameters controlling the fracture characterisitics. These are:

1. Deformational and strength responses of the constituent lamina
2. Lamination geometry
3. Crack orientation with respect to the material axis of anisotropy
4. Crack length
5. Nature of applied stresses
6. Energy dissipation associated with the three kinematically admissible modes of crack extension
7. Crack trajectory

Theoretical Model

Because of this large number of parameters, experimental quantification by systematic permutation of the parameters must be realistically viewed as intractable. This paper presents an

analytical model which reduces the above parameter list from seven to two and furnishes experimental observations which lend support to the theoretical model.

The theoretical model is based on the hypothesis that, in the case of quasi-static crack extension, the necessary and sufficient condition for failure of a volume element around a macroscopic crack tip is similar to that condition for failure of a volume element in the absence of a macroscopic crack. Since the presence of a macroscopic crack gives rise to a combined complex state of stress in the neighborhood of the crack tip, it is necessary to know the condition of failure of the composite under complex loading, which is commonly referred to as the failure envelope or failure criterion. Thus, the major ingredients required in the implementation of the theoretical model are a) a mathematically operational anisotropic failure criterion for the composite lamina, and b) a suitable stress analysis technique through which the stress distribution in the neighborhood of the crack tip in a laminate can be computed. In recent years, numerous failure criterion have been proposed. Examination of their formulations, Ref. 1, reveals that they can be cast and compared in terms of tensor polynomials and that the majority of them are mathematically awkward; some even lack consistency of conversion between stress and strain. It was found that the tensor polynomial failure criterion, Ref. 2, encompasses maximum flexibility without redundancy and further, that it lends itself to the design of critical experiments, Ref. 3. The tensor polynomial failure criterion is used here, although it should be emphasized that other experimentally verified criteria may be substituted. The tensor polynomial failure criterion when expressed in terms of stress takes the form:

$$f(\sigma_i) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k$$
$$+ \ldots = 1, \qquad i = 1,2,\ldots 6 \tag{1}$$

where in Eq. (1) contracted notation is used. For a typical engineering composite (graphite epoxy), the linear and quadratic terms in Eq. (1) provide sufficient correlation of the experimental data as shown in Fig. 1. These experimental data were obtained from tubular samples tested under combined stress conditions along radial loading paths by an axial-rotary-internal pressure mechanical testing machine controlled by an on-line digital computer. The experimental details have been reported in Ref. 4. The data actually populate a three-dimensional space in $\sigma_1\sigma_2\sigma_6$, but they have been convoluted (or projected) onto the $\sigma_1\sigma_2$ plane for easy comparison. In Fig. 1, the same set of experimental data are convoluted onto the $\sigma_1\sigma_2$ plane by three different failure criteria. Better correlation by the tensor polynomial criterion exhibited both visually as well as by the lowest RMS (the root mean square) deviation of experiments from theory.
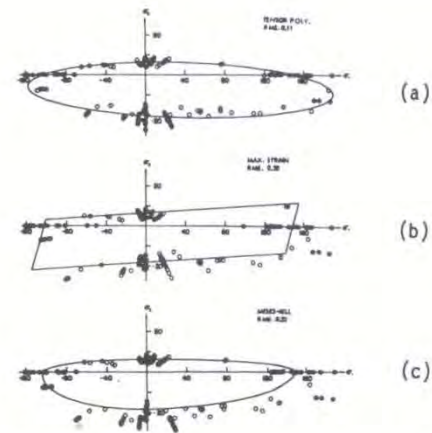

Figure 1. Failure data of graphite epoxy lamina convoluted onto $\sigma_1\sigma_2$ plane, stresses in {ksi}. (a) By tensor polynomial failure criterion; (b) by maximum strain failure criterion; (c) by modified Mises-Hill failure criterion.

The physical interpretation of the failure envelope requires some attention, since the composite is assumed to be homogeneous, anisotropic, and contains a population of randomly distributed microscopic flaws $C_1, C_2 \ldots C_i$. While the flaws are small compared to the characteristic dimension D of the body as depicted in Fig. 2a, continuum analysis discloses that under arbitrary loads P_i the state of stress is unbounded at the location of the geometric singularities $C_1, C_2 \ldots C_i$, and thus would lead to immediate failure even for extremely small P_i. This is contrary to physical observations and the stresses appearing in Eq. (1) should be interpreted as the average stress acting on a small but finite characteristic volume (specified by a dimension r_c, Fig. 2a) which fully encapsulates one microscopic flaw. Thus, although the stress is singular inside this characteristic volume r_c, the average stresses external to r_c are bounded and may be used to characterize the failure of this volume through a failure criterion of the form

$$\mathcal{A} < \mathcal{F} \tag{2}$$

where $\mathcal{A}$ is the average stress vector acting external to the characteristic volume defined in terms of the unit vector e_i in the stress space of Fig. 2b as

$$\mathcal{A} = \sigma_i \vec{e}_i \qquad i = 1,2,\ldots 6 \tag{3}$$

and $\mathcal{F}$ is the strength vector to the failure surface $f(\sigma_i)$ as determined by Eq. (1) and illustrated in Fig. 2b. Under an arbitrary loading P_i, the stress vector $\mathcal{A}$ at any location of the body can be determined through continuum analysis or numerical techniques. It follows that when criterion $f(\sigma_i)$ is known, then the location of a prevalent failure condition can be determined. Hence,

the problem of analyzing the crack initiation is possible with the availability of the second ingredient, i.e., the stress analysis of the crack.
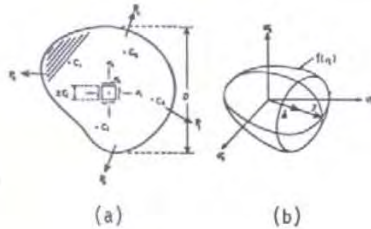

(a)   (b)

Figure 2. (a) Homogeneous anistropic body with randomly distributed microscopic flaws. (b) Criticality of stress vector $\mathscr{A}$ acting on characteristic volume r_c, the failure surface $f(\sigma_i)$ the strength vector $\mathscr{F}$.

It has been found, Ref. 5, that for composite lamina the problems of biaxially loaded crack initiation and crack trajectory can be examined by introducing the criterion of comparing the stress vector $\mathscr{A}$ around the crack tip to the strength vector $\mathscr{F}$ of the parent material. The essentials of this concept are shown in Fig. 3, where 3b depicts a crack subjected to a system of biaxial loads, P_i. Through anisotropic stress analysis of the crack, the near-field stress σ_i around the crack tip can be computed as a function of material orientation as specified by compliance coefficients S'_{ij}, Ref. 6.

$$\sigma_i = g_i(S'_{ij}, 2a, r) \qquad i = 1, 2, 6 \qquad (4)$$

Here 2a and r are respectively the crack length and the distance from the crack tip. The crack tip stress is singular when r approaches zero. However, in accordance with our model, we only need to compute the average stress exterior to the critical volume which encapsulates the crack tip. Thus, Eq. (4) can be computed in terms of the critical volume r_c, and the stress vector $\mathscr{A}$ can be expressed in the stress space $\sigma_1 \sigma_2 \sigma_6$ from Eq. (3). From Fig. 3a it can be seen that whether or not the material will fail under the influence of the stress vector $\mathscr{A}$ (determined from Eq. (3)) can be determined by comparing the stress vector $\mathscr{A}$ to the strength vector $\mathscr{F}$ which is defined by the failure surface, i.e., Eq. (2). In Fig. 3a $\mathscr{F} = \sigma_i^* e_i$ and σ_i^* are the roots of the lamina failure criterion (Eq. 1) in the direction of $\mathscr{A}$. Thus, the coincidence of the stress and strength vectors determine both the initiation of failure and the trajectory of crack extension. This is graphically illustrated in Fig. 3b where the polar contours of the stress vector $\mathscr{A}$ and the strength vector $\mathscr{F}$ are plotted. It can be seen that the location of the maximum stress vector $\mathscr{A}$, i.e.,

θ_i, in Fig. 3b, is not a sufficient condition for crack initiation; whereas the coincidence of $\mathscr{A}$ and $\mathscr{F}$ at θ_c defines initiation and crack extension along the θ_c direction. Extensive experiments on composite lamina (glass-epoxy) subjected to combined loading verified this concept (Fig. 4) where the characteristic dimension r_c was computed to be 0.076 inch for fracture under tension and 0.077 inch for fracture under shear. Futhermore a single characteristic $r_c = 0.077$ correlates all combined stress fracture results completely suggesting that r_c need not be an adjustable empirical constant. The net effect is that a formerly multi-dimensional fracture under complex stress problem can be completely characterized by two parameters, i.e., the characteristic volume r_c and one critical stress intensity factor.
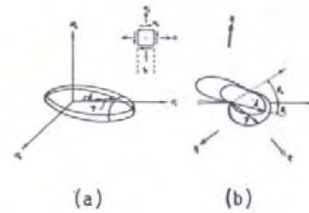

(a)   (b)

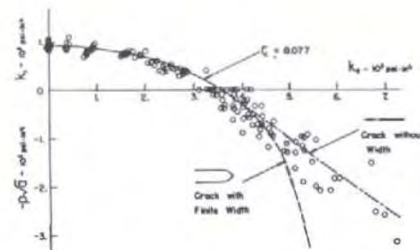Figure 3. Relation between failure of characteristic volume r_c (a), and fracture of crack (b).

Figure 4. Crack extension of unidirectional composite (Scotch-ply 1002) under combined loading; k_1 - symmetric stress intensity factor (tension-compression), k_2, skew symmetric stress intensity factor (shear). Experimental results relatable by a single critical volume r_c.

It can be readily seen in the case where the anisotropic composite is a laminate that additional dimensions are introduced in the form of lamination angles and stacking sequences. If we are permitted to use classical linear laminated plate theory which appears to be a reasonable compromise, we can proceed to extend the above parameter reduction concept to the fracture of laminates by using

Eqs. (1,2,3,4) respectively. The basic equations for these computations are described in the following section.

Calculation of Stress and Strength Vectors For Laminate Composites

The following calculation of the stress and strength vector for a laminate is based on the assumptions that linear laminated plate theory is applicable and that the deformational and strength properties of the lamina are known, i.e.,

$$S_{ij} \qquad \text{known} \qquad (5)$$

$$f(\sigma_i) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \text{ known} \qquad (6)$$

The determination of S_{ij} and F_i, F_{ij} has been discussed in Refs. 7 and 1. With Q_{ij} for the lamina known, the stiffness matrix of the laminate can be computed from:

$$A_{ij} = \int_{-h/2}^{h/2} Q_{ij} dZ = \sum_{k=1}^{n} Q_{ij}^{(k)} (h_{k+1}-h_k) , \qquad (7)$$

where $Q_{ij}^{(k)}$ is the plane stress stiffness coefficient computed from S_{ij}^{-1} and transformed to the orientation of the kth lamina. For the case of symmetrical stacking sequence the stress-strain relation for the laminate is

$$N_i = A_{ij}\epsilon_j^0 \qquad (8a)$$

and

$$\epsilon_i^0 = A_{ij}^* N_i \qquad (8b)$$

where ϵ_i^0 is the average strain through the thickness.

The general plane problem of a crack in the laminate then requires solution of the Airy's stress function χ in the form, Ref. 6,

$$\frac{\partial^4\chi}{\partial x_1^4} - \frac{2A_{26}^*}{A_{22}^*} \frac{\partial^4\chi}{\partial x_1^3 \partial x_2} + \left(\frac{2A_{12}^*+A_{66}^*}{A_{22}^*}\right) \frac{\partial^4\chi}{\partial x_1^2 \partial x_2^2}$$

$$- \frac{2A_{16}^*}{A_{22}^*} \frac{\partial^4\chi}{\partial x_1 \partial x_2^3} + \frac{A_{11}^*}{A_{22}^*} \frac{\partial^4\chi}{\partial x_2^4} = 0 \qquad (9)$$

For a crack oriented in an arbitrary direction in such a laminate (Fig. 5), the A_{ij}^* coefficients can be transformed from the principal direction to the direction of the crack. These coefficients can then be used in Eq. (9) to obtain the near field stress distribution around the crack tip. The solution is

$$\sigma_1 = 2R_e[S_1^2\phi_1'(Z_1)+S_2^2\phi_2'(Z_2)]$$

$$\sigma_2 = 2R_e[\phi_1'(Z_1)+\phi_2'(Z_2)]$$

$$\sigma_6 = -2R_e[S_1\phi_1'(Z_1)+S_2\phi_2'(Z_2)] \qquad (10)$$

$$Z_1 = x_1+S_1x_2$$

$$Z_2 = x_1+S_2x_2$$

where for uniform stress σ^∞ and τ^∞ the stress functions are, Ref. 6,

$$\lim_{Z_1 \to a} \phi_1'(Z_1) = -\frac{(\tau^\infty\sqrt{a} + S_2\sigma^\infty\sqrt{a})}{2(S_1-S_2)+\{2r(\cos\theta+S_1\sin\theta)}$$

$$(11)$$

$$\lim_{Z_2 \to a} \phi_2'(Z_2) = -\frac{\tau\beta\sqrt{a} + S_1\sigma^\infty\sqrt{a}}{2(S_1-S_2)+\{2r(\cos\theta+S_2\sin\theta)\}}$$

and $S_1 = \alpha_1 +_i\beta_2$, $S_2 = \alpha_2 +_i\beta_2$ are the roots of the characteristic equation of Eq. (9). Thus, Eq. (10) is the laminate analogue of Eq. (4). From Eq. (10) for a given direction θ from the crack tip, the direction cosines of the stress vector can be obtained and the laminate analogue of Eq. (3) can be computed in the stress space as a function of distance r from the crack tip.

$$\vec{\sigma} = \frac{N_1}{h} \vec{e}_1 + \frac{N_2}{h} \vec{e}_2 + \frac{N_6}{h} \vec{e}_6 \qquad (12)$$
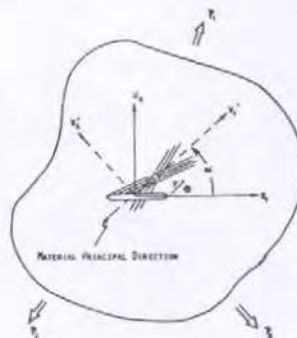

Figure 5. Definition of coordinates of a crack oriented in an arbitrary direction with respect to the principle direction of the laminate.

Now in order to compute the strength vector $\vec{\mathscr{F}}$, the laminate analogue of a failure criterion is needed. This required laminate failure criterion can also be computed from the lamina failure criterion (Eq. (1)) through use of the linear laminated plate theory using the definition that 1st ply failure constitutes total failure of the laminate. This computation can be most efficiently performed by noting that every point in the lamina failure surface has a corresponding failure point in the laminate failure surface. We may then systematically choose a stress ratio

$$\frac{\sigma_1}{\sigma_2}, \; \frac{\sigma_1}{\sigma_6}, \; \sigma_1$$

in the lamina and obtain the failure condition $\sigma_1^*, \sigma_2^*, \sigma_6^*$ by Eq. (1). Substituting these stresses σ_i^* into

$$\epsilon_i^* = S_{ij}^{(k)} \sigma_j^*,$$

we obtain the failure strain condition. Assigning this failure strain condition to be the average strain of the laminate

$$\epsilon_{io}^* = \epsilon_i^*,$$

we can then compute the failure condition for the laminate by

$$N_i^* = A_{ij} \epsilon_j^*.$$

This procedure is summarized in the sequential solution of the following equations:

for kth layer subjected to

$$F_i^{(k)} \sigma_i^* + F_{ij}^* \sigma_i^* \sigma_j^* = 1, \tag{13}$$

$$\epsilon_i^* = S_{ij}^{(k)} \sigma_j^*, \tag{14}$$

$$\epsilon_{io}^* = \epsilon_i^*, \tag{15}$$

$$N_i^* = A_{ij} \epsilon_{io}^* \tag{16}$$

where * denotes failure conditions.

Systematic solution of Eqs. (13) to (16) for different ratios

$$\frac{\sigma_1}{\sigma_2}, \; \frac{\sigma_1}{\sigma_6}, \; \sigma_1$$

results in a laminate failure surface when discrete points are fitted with a third-order polynomial in the form, Ref. 8,

$$F_i \frac{N_i}{h} + \overline{F}_{ij} \frac{N_i N_j}{h^2} + \overline{F}_{ijk} \frac{N_i N_j N_k}{h^3} = 1 \tag{17}$$

where

$$\overline{F}_i, \; \overline{F}_{ij}, \; \overline{F}_{ijk}$$

are failure tensors for the laminate. A sample of the least square fit is shown in Fig. 6.
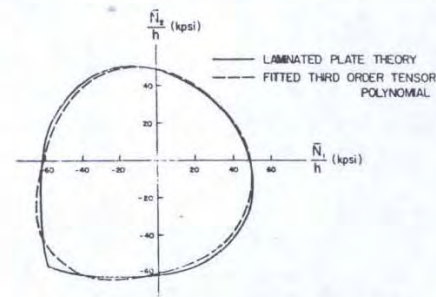

Figure 6. Third order tensor polynomial failure surface for 0/90/0/90/90/0/90/0 laminate.

In essence, Eq. (17) is the laminate correspondence of Eq. (1). Hence, for every stress vector determined by Eq. (12) a corresponding laminate strength vector can be computed from Eq. (17) by

$$\overline{F} = \frac{N_i^*}{h} \vec{e}_i \qquad i = 1,2,6 \tag{18}$$

The above computation has the following meaning: For a given loading condition (Fig. 3) along each polar direction from the crack tip there exists a stress vector $\vec{\mathscr{d}}$ which is representative of the driving force in that direction. For such a stress vector the material resistance to rupture is represented by the strength vector $\vec{\mathscr{F}}$. If the driving force is less than the resistance, no rupture can take place; hence, the stability for the crack is defined by

$$\vec{\mathscr{d}}_c = \vec{\mathscr{F}}_c \tag{19}$$

where the subscript c refers to the neighborhood of the crack tip.

Experimental Observations

The purpose of the experimental program is to examine the theoretical model to see if the coincidence of the stress vector $\mathscr{A}_c$ and the strength vector $\mathscr{F}_c$ defines the magnitude and direction of crack initiation trajectory.

The samples are fabricated from 3M glass-epoxy pre-preg. Four variations of the lamination sequences are examined. The fracture samples are rectangular, 4.75" wide and 8" between grips. A centrally located initial crack is sawed in the sample by a jeweler's saw producing a crack width around .008". The specimens are tested in tension in a standard testing machine under displacement control. A motor driven 35 mm camera is triggered by the experimentalist and the event of each picture taken is also marked on the load-deformation recordings, thus providing a recording of crack length corresponding to different load levels.

We note that by varying the crack orientation with respect to the direction of tension, we can effectively apply combined tension and shear loading to the crack and, consequently, vary the contour of the stress vector $\mathscr{A}$. Furthermore, by varying the lamination geometry, we can vary the strength of the composite and, hence, the contour of the strength vector $\mathscr{F}$. In accordance with the method discussed in the previous section, the polar contour stress vector and strength vector are computed for four lamination geometries with different crack orientations as shown in Figs. 7 to 10. In Fig. 7a the crack is oriented perpendicular to the tensile load in an $0°/45°/90°/-45°$ laminate. The stress vector $\mathscr{A}$ and the strength vector indicate two critical orientations, θ_c, where crack initiation is predicted by the theoretical model. In Fig. 7b this predicted trajectory is clearly confirmed by experimental observation. In Fig. 8a, the crack is oriented at 60° to the tensile load in a 30°/60° laminate. The $\mathscr{A}$ and $\mathscr{F}$ contours indicate a single critical orientation — $\theta_c = 44°$ from original crack direction. Figure 8b shows that this predicted trajectory is substantiated by experimental observation. In Fig. 9a the crack is oriented at 45° to the tensile load in a 90°/45° laminate where the $\mathscr{A}$ and $\mathscr{F}$ contours predict a co-linear crack extension. Again, this is clearly substantial by the photograph shown in Fig. 10b.
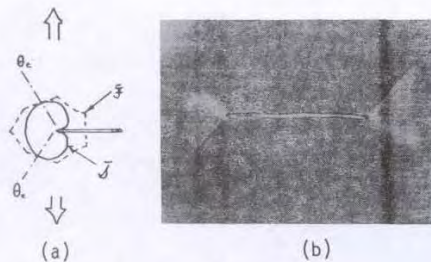

(a)   (b)

Figure 8. (a) Computed stress vector $\overline{\mathscr{A}}$ and strength vector $\mathscr{F}$ contour for 60/30/60/30/60/ 30/60 laminate. (b)Experimentally observed crack initiation.

(a)   (b)

Figure 9. (a) Computed stress vector $\overline{\mathscr{A}}$ and strength vector $\mathscr{F}$ contour for 45/45/45/0/45/45/45 laminate. (b) Experimentally observed crack initiation.

(a)   (b)

Figure 7. (a) Computed stress vector $\overline{\mathscr{A}}$ and strength vector $\mathscr{F}$ contour for 90/45/0/-45/-45/0/ 45/90 laminate. (b) Experimentally observed crack initiation.
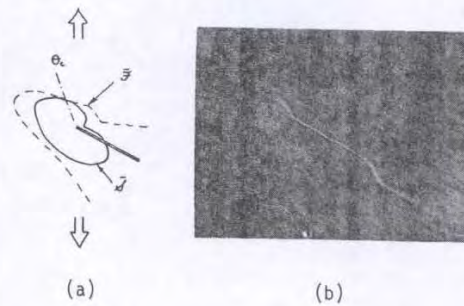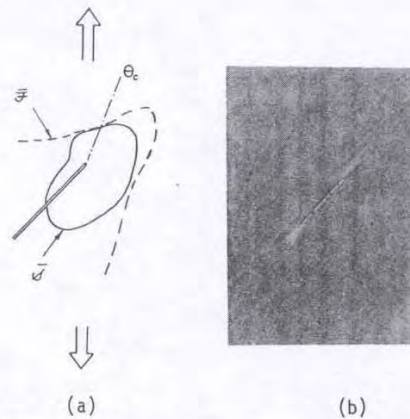

(a)   (b)
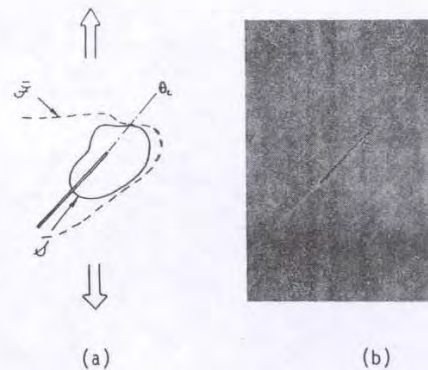
Figure 10. (a) Computed stress vector $\overline{\mathscr{A}}$ and strength vector $\mathscr{F}$ contour for 45/45/90/ 45/45/90/45/45 laminate; (b) experimentally observed crack initiation.

It is important to note that for a given laminate, both the contours of the $\mathscr{J}$ and $\mathscr{F}$ vectors are a function of external loading. Naturally, for different laminate configurations, the differences in these contours will be even more drastic. Furthermore, in the computation of the local stress around the crack tip, Eq. (10) is only applicable for a straight crack. If the crack extension is not self-similar, i.e., not co-linear with the parent crack then the mapping function changes in form and Eq. (10) is no longer applicable. Thus, for the cases of crack deflection, the above computation is only applicable for the point of initiation.

Finally, Eq. (19) checks the criticality of the stress and strength vector as modified in the neighborhood of the crack tip. It should be recognized that the criticality of the $\mathscr{J}$ and $\mathscr{F}$ vectors has to be checked for the far field or global condition where the stress distribution is not under the influence of the crack, i.e., whether the

$$\bar{\mathscr{J}}_g < \bar{\mathscr{F}}_g \qquad (20)$$

here the subscripts g refer to the far field global stress.

It is immediately apparent that if $\mathscr{J}_g \ll \mathscr{J}_c$, then crack extension will be confined completely to the crack tip. However, if $\mathscr{J}_g \leq \mathscr{J}_c$, global damage may occur in addition to the crack extension.

Conclusion

The characterization of the fracture responses of laminated composites is of great engineering importance not only for the prediction of the criticality of macroscopic flaws but also for the design of crack-arresting, fail-safe structures. Recent investigations of laminated composite fracture have utilized characterization methods established for isotropic solids and, hence, are revelant only to the particular lamination configuration tested. These results typically do not address the condition of combined loadings and the non-selfsimilar crack trajectory. Thus, these findings cannot be generalized to arbitrary lamination geometry and loading conditions for efficient structural design. A theoretical model has been presented herein to combine the lamina failure criterion with stress analysis of the crack. With this theoretical model, seven major parameters which control laminate fracture can be reduced to two, i.e., the lamina failure criterion and a critical volume characteristic to the composite. The reduction of controlling parameters to two makes quantification of the fracture of laminates a tractable task. Several lamination geometries are tested. By varying the external loading, different crack trajectories are predicted by the theoretical model. These predicted trajectories agree well with the experimentally observed modes. Such agreement suggest that, with refinement, the general condition of laminate fracture can be characterized within useful engineering accuracy.

Acknowledgment

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