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APPLICATION OF SEVERAL ECONOMETRIC TECHNIQUES TO A THEORY OF DEMAND
WITH VARIABLE TASTES

by

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SUMMARY

Existing theory of consumer demand does not contain a body of theorems purporting to explain the consumption behavior of individuals when their preferences are changed, either autonomously or by advertising and other forms of selling effort. In view of the importance of advertising in the modern economy, it is desirable that such a body of theorems be worked out. The objective of this study is the formulation of a theory of consumer demand with variable preferences; the governing criterion is that the theory be applicable in econometric demand analysis.

The assumption that the individual consumer has only one ordinal utility function is replaced by the assumption that he has a whole family of ordinal utility functions; advertising expenditures by sellers of commodities are assumed to determine which one of these ordinal utility functions is maximized. From these assumptions are derived a number of a priori restraints on measurements defining advertising elasticities of demand. It is shown that advertising elasticities are weighted averages of elasticities of substitution between goods in consumption, the weights being measurements defining advertising elasticities of marginal utilities.

Estimates of empirical advertising elasticities of demand for tobacco are presented along with estimates of the elasticity of marginal utility of tobacco with respect to advertising expenditure. The computation of these estimates serves as an example of the application of the theory to problems of econometric demand analysis.

INTRODUCTION

The modern theory of consumer demand as formulated by Edgeworth,¹

¹F. Y. Edgeworth. *Mathematical Psychics*. London. Kegan Paul. 1881.

Antonelli,² and Pareto,³ and worked out by Slutsky,⁴ Hicks and Allen,⁵

²G. B. Antonelli. *Sulla teoria matematica della economia pura*. Pisa. Folchetto. 1886. Reprinted in *Giornale degli Economisti*, N. S., Vol. 10, 1951, pp. 233-263.

³V. Pareto. *Manuel d'Économie Politique*. 2d ed. Paris. Giard. 1927.

⁴E. Slutsky, "Sulla teoria del bilancio del consumatore," *Giornale degli Economisti*, Vol. 51, 1915, pp. 1-26.

⁵J. R. Hicks and R. G. D. Allen, "A reconsideration of the theory of value," *Economica*, N. S., Vol. 1, 1934, pp. 52-73, 196-219.

and Hicks,⁶ is based on the assumption that the individual consumer

⁶J. R. Hicks. *Value and Capital*. Oxford. Clarendon Press. 1939.

allocates expenditures on commodities as if he had a fixed, ordered set of preferences described by an indifference map or by an ordinal utility function which he maximizes subject to restraints imposed by the money income he receives and the prices he must pay. From the point of view of the econometrician, this theory has served well by indicating criteria to govern the construction of aggregate commodities and consumer price indexes,⁷ by providing certain plausible a priori

⁷Cf. E. von Hofsten. *Price Indexes and Quality Changes*. London. Allen and Unwin. 1952. passim.

constraints to be imposed on estimates of the parameters of empirical demand functions,^{8,9} and by suggesting several hypotheses to be subjected

⁸Cf. H. Wold. *Demand Analysis: A Study in Econometrics*. New York. John Wiley. 1953. p. 47.

⁹Cf. T. C. Koopmans and W. C. Hood, "The estimation of simultaneous linear economic relationships," in W. C. Hood and T. C. Koopmans, eds. *Studies in Econometric Method*. New York. John Wiley. 1953. pp. 120-122.

to statistical test. However, consumer demand theory, not having taken variable preferences explicitly into account until very recently, has not asserted any hypothetical laws governing relations among the shifts in demand functions which a change in preference orderings should cause. It is desirable that such hypothetical laws be derived, and that econometricians bring them under empirical test; the logical consequence of the assumption of fixed preferences differs markedly from experience in the modern economy, for in the latter it is commonly observed that the consumption behavior of real individuals and households is changed more or less systematically by advertising and other forms of selling effort, and by changes in social and technological factors exogenous with respect to the consumer economy; in econometric demand analysis, the introduction of time as an independent trend variable to "explain" the effects of changes in taste is at best an expedient it would be better to have done with, if possible, since

trend parameters are not capable of causal or legal interpretation.

In recent years two economists have cleared the way for the development of a theory of consumer demand with variable preferences. In two short papers growing out of the Lange-Robertson-Hicks debate over the nature of related goods,¹⁰ Ichimura¹¹ and Tintner¹² defined

¹⁰Cf. O. Lange, "Complementarity and interrelations of shifts in demand," *Rev. Econ. Studies*, Vol. 8, 1940, pp. 58-63 and D. H. Robertson, J. R. Hicks, and O. Lange, "The interrelations of shifts in demand," *Rev. Econ. Studies*, Vol. 12, 1944-45, pp. 71-78.

¹¹S. Ichimura, "A critical note on the definition of related goods," *Rev. Econ. Studies*, Vol. 18, 1950-51, pp. 179-183.

¹²C. Tintner, "Complementarity and shifts in demand," *Metroeconomica*, Vol. 4, 1952, pp. 1-4.

a change in preferences by a change in the form of the ordinal utility function or indifference map, and derived for shifts in demand algebraic expressions which are linear combinations of Slutsky-Hicks substitution terms¹³ which play a central role in existing consumer demand theory.

¹³Cf. J. R. Hicks. *Value and Capital*. 2d ed. Oxford. Clarendon Press. 1946. pp. 309-311.

Straightforward linear transformations of the expressions derived by Tintner for shifts in demand can be shown to follow the same mathematical rules as the corresponding linear transformations of substitution terms. Economic interpretations of the latter transformations and the rules which govern them, constitute all but one of the major analytical laws

of existing consumer demand theory; thus, in general, the same mathematical rules that govern the reactions of consumers' expenditures and consumption to income-compensated price changes, govern the reactions of consumers' expenditures and consumption to changes in the variables that determine the form of their preference orderings.

In this essay the laws of consumer demand with variable preferences are worked out and interpreted. Throughout the theoretical analysis, it is assumed that the form of a representative individual's utility function depends on the expenditures which sellers make for advertising goods and services. The analysis is entirely in terms of comparative statics; it provides theoretical explanations of the way an hypothetical consumer, who always maximizes his utility function, reallocates a fixed total expenditure to purchases of goods and services when his preference structure (ordinal utility function) is changed by advertising in certain defined ways. No attempt has been made to extend the theory of consumer demand with variable preferences in the direction towards a union with Keynesian theories of the marginal propensity to consume and the liquidity-preference schedule, though such a union would be highly desirable; the absence of any general postulate from which existing consumer demand theory on the one hand, and the theories of the marginal propensity to consume and the liquidity-preference schedule on the other can be derived as special instances makes the success of such an attempt seem very unlikely at this time. From the point of view of the econometrician engaged in empirical demand analysis, however, existing consumer demand theory has shown itself to be essentially

adequate for the analysis of effects of prices and income on demand, and it seems plausible that the theory of consumer demand with variable preferences will be equally useful in the empirical analysis of the effects of advertising on demand functions.

A THEORY OF DEMAND WITH VARIABLE CONSUMER PREFERENCES

Definitions and Assumptions

The point of view adopted in this essay is that advertising costs are incurred by a seller in an effort to secure a favorable change in consumers' subjective evaluations of the commodity he offers for sale; in terms of economic theory, the seller seeks by advertising his goods and services to increase their marginal utilities to consumers with respect to the marginal utilities of the commodities offered by other sellers. This point of view conforms to that expressed in the institutional studies of advertising by Borden¹⁴ and Lever.¹⁵

¹⁴N. H. Borden. *The Economic Effects of Advertising*. Chicago. Richard D. Irwin. 1947. pp. 162-163.

¹⁵E. A. Lever. *Advertising and Economic Theory*. London. Oxford Univ. Press. 1947. p. 28.

In order to express this concept of advertising in terms of ordinal utility theory, it is first assumed that individual preferences can be represented by the utility function, $u(x_1, \dots, x_n; \theta_1, \dots, \theta_n)$, where the x_i denote the quantities of distinct goods and services the individual consumes during a given period of time, during which he receives money income, M , and pays prices, p_1, \dots, p_n , which he cannot influence. The θ_i denote parameters which describe the form of the ordinal utility function; the θ_i are assumed to depend on the variables a_j , where a_j denotes the money expenditure by the seller of X_j in

advertising that commodity to the consumer; it is also assumed that the form of the utility function is uniquely determined by the advertising expenditures, i. e.,

$$(2.1) \quad \theta_j = \theta_j(a_1, \dots, a_n/x_1, \dots, x_n), \quad j = 1, \dots, n,$$

are single-valued functions. The utility function is presumed to possess first-order partial derivatives, u_1, \dots, u_n , with respect to the x_1 , and these denote the marginal utilities of the commodities, x_1, \dots, x_n ; the marginal utilities are presumed to possess in turn the first-order partial derivatives, u_{1j} , and u_{j1} ; it is assumed that the inverse of transformation (2.1) exists, so the second-order partial derivatives, u_{ia_j} and $u_{j\theta_k}$, $i, j, k = 1, \dots, n$, are related according to

$$(2.2) \quad \frac{\partial(u_1, \dots, u_n)}{\partial(\theta_1, \dots, \theta_n)} \cdot \frac{\partial(\theta_1, \dots, \theta_n)}{\partial(a_1, \dots, a_n)} = \frac{\partial(u_1, \dots, u_n)}{\partial(a_1, \dots, a_n)}.$$

A final a priori restraint on the $u_{j\theta_j}$ and u_{ia_j} is justified by the concept of ordinal utility. Given a fixed set of parameters, θ_j , two different budgets, (x_1, \dots, x_n) and (x'_1, \dots, x'_n) can be compared according to the ordinal relation

$$(2.3) \quad u(x'_1, \dots, x'_n; \theta_1, \dots, \theta_n) \begin{matrix} > \\ < \end{matrix} u(x_1, \dots, x_n; \theta_1, \dots, \theta_n),$$

which is invariant against the substitution of the equally valid ordinal utility function $\rho(u)$, where $\rho'(u) > 0$, for u . On the other hand, a comparison of the same budget before and after a change in preferences is essentially meaningless, and the same is true of a comparison of one

budget after a change of preferences with another budget before the change; that is, the relations

$$(2.4) \quad u(x'_1, \dots, x'_n; \theta'_1, \dots, \theta'_n) \begin{matrix} \geq \\ \leq \end{matrix} u(x_1, \dots, x_n; \theta'_1, \dots, \theta'_n),$$

where at least one $\theta'_j \neq \theta_j$, make no meaningful assertions about the consumer's preferences. In the context of ordinal utility theory, it makes no sense to assert that a change in preferences, with or without a change in the quantities consumed, increases, decreases, or leaves the individual consumer's welfare unchanged. It follows that the sign and magnitude of the partial derivative u_{θ_j} are meaningless, and the same is true of the sign and magnitude of u_{x_j} , for all $j = 1, \dots, n$. Finally, none of the demand function parameters, e. g., shifts in demand with respect to changes in preference, depend on the u_{θ_j} or the u_{x_j} ; this follows from the proof that shifts in demand are invariant against the transformation, $\phi(u)$.¹⁶ Consequently, nothing is lost, and simplicity

¹⁶Cf. R. L. Basmann, "A note on an invariant property of shifts in demand," *Metroeconomica*, Vol. 6, 1954, pp. 69-71.

of exposition is gained, if the arbitrary restrictions

$$(2.5) \quad u_{\theta_j} = 0,$$

$$(2.6) \quad u_{x_j} = 0,$$

are imposed on the ordinal utility function.

Under the restrictions (2.5) and (2.6) the partial derivatives u_{θ_j}

and u_{ia_j} are invariant in sign. Let $u_{ia_j} = b_{ij}u_i$, where b_{ij} denotes the rate of proportionate increase (or decrease) in the marginal utility of X_i with respect to expenditure on advertising X_j . The b_{ij} are invariant against the substitution of $\phi(u)$, where $\phi'(u) > 0$, for \bar{u} , and are single-valued functions of the advertising expenditures, a_j , and quantities, X_k . Let \underline{a} denote the advertising expenditure on a group of commodities, e. g., X_1, \dots, X_m , where $m \leq n$; i. e.,

$$(2.7) \quad \underline{a} = \sum_{i=1}^m a_i;$$

let b_h denote the rate of proportionate increase in the marginal utility of X_h with respect to the aggregate advertising expenditure, \underline{a} ; then

$$(2.8) \quad b_h = \sum_{i=1}^n b_{hi}.$$

Finally, it is assumed that an increase in advertising expenditure, a_i , never decreases the marginal utility of X_i ; i. e.,

$$(2.9) \quad b_{ii} \geq 0, \quad i = 1, \dots, n.$$

Derivation of the Tinbergen-Ichimura Relation

The derivation of expressions for shifts in demand functions with respect to small changes in advertising expenditures begins here with the assumption that the usual first and second order conditions for individual consumer equilibrium are fulfilled; i. e.,

$$(2.10) \quad \sum_{i=1}^n p_i x_i = M;$$

$$(2.11) \quad -\lambda p_i \neq u_i = 0, \quad i = 1, \dots, n;$$

and the matrix

$$(2.12) \quad U = \begin{bmatrix} 0 & u_j \\ u_1 & u_{1j} \end{bmatrix}, \quad i, j = 1, \dots, n,$$

is negative definite. The Lagrangean multiplier, λ , denotes the marginal utility of money income, M , and is positive. Income, M , and all prices, p_i , are assumed to remain constant.

An important invariant derived from the equilibrium conditions is the Slutsky-Hicks Substitution Term.¹⁷ It is obtained by differentiat-

¹⁷J. R. Hicks. *Value and Capital*. 2d ed. p. 309.

ing the first order conditions (2.11), along with the side condition

$$(2.13) \quad u(x_1, \dots, x_n) \theta_1, \dots, \theta_n = \text{constant},$$

partially with respect to p_j and solving the resulting system of equations for

$$(2.14) \quad s_{1j} = \lambda \frac{u_{1j}}{u}, \quad i, j = 1, \dots, n;$$

where u_{1j} is the cofactor of u_{1j} in the matrix U . The Substitution Term, s_{1j} , denotes the rate of change in demand for x_1 with respect to a small change in the price of x_j , all other prices remaining constant, but with income adjusted to maintain a constant level of utility. The mathematical laws governing relations among Substitution Terms play a central role in the theory of consumer demand with variable preferences, and so are summarized here, though without complete proofs being furnished.

Complete proofs are available elsewhere.¹⁸ The four major theorems are:

¹⁸ J. L. Mosak. General Equilibrium Theory in International Trade. Bloomington, Ind. Principia Press. 1944. pp. 9-30.

$$(2.15) \quad s_{ii} < 0;$$

$$(2.16) \quad s_{ij} = s_{ji}, \quad i, j = 1, \dots, n;$$

$$(2.17) \quad \sum_{j=1}^n p_j s_{ij} = 0;$$

$$(2.18) \quad \sum_{i=1}^m \sum_{j=1}^n s_{ij} z_i z_j < 0, \quad m < n,$$

where not all $z_i = 0$.

Expressions for shifts in demand with respect to advertising are obtained from the equilibrium conditions (2.10) and (2.11) by differentiating them partially with respect to a_j and solving the resulting system of equations. Denote the shift in demand for X_i by x_{iaj} , where $x_{iaj} = \frac{\partial x_i}{\partial a_j}$; then

$$(2.19) \quad x_{iaj} = - \sum_{k=1}^n \frac{w_{ka}}{\lambda} s_{ki}, \quad i, j = 1, \dots, n.$$

The relation (2.19) was first derived by Tintner;¹⁹ a special case of

¹⁹G. Tintner, "Complementarity and shifts in demand," Metroeconomica, Vol. 4, 1952, pp. 1-4.

(2.19) was derived earlier by Ichimura.²⁰ Ichimura's assumption was

²⁰S. Ichimura, "A critical note on the definition of related goods," Rev. Econ. Studies, Vol. 18, 1950-51, pp. 179-183.

that there is a solitary increase in the marginal rate of substitution of (say) X_1 for one of the other commodities, all other marginal rates of substitution remaining constant; thus, the expression he derived is equivalent to (2.19) when $u_{ja_j} > 0$, and $u_{ia_j} = 0$, for all $i \neq j$.²¹

²¹Cf. R. L. Basmann, "A note on Mr. Ichimura's definition of related goods," Rev. Econ. Studies, Vol. 22, 1954-55, pp. 67-69.

The expression (2.19), which is called the Tintner-Ichimura Relation, satisfies the criterion of invariance independently of the restriction (2.6); for if the equally valid utility function, $v = \phi(u)$, where $\phi'(u) > 0$, is substituted for u , $v_h = \phi'(u)u_h$, $v_{ha_j} = \phi'(u)u_{ha_j} + \phi''(u)u_h u_{a_j}$, and the marginal utility of money income, M , is $\phi'(u)\lambda$; Slutsky-Hicks Substitution Terms are invariant against this transformation,²² and the Tintner-Ichimura Relation, after elementary simpli-

²²Cf. J. R. Hicks, op. cit., pp. 306-307.

fication

$$(2.20) \quad x_{ia_j} = -\sum_{h=1}^n \frac{u_{ha_j}}{\lambda} s_{hi} - \phi''(u)u_{a_j} \sum_{h=1}^n \frac{u_h}{\lambda} s_{hi};$$

$u_h/\lambda = p_h$, according to the first order conditions (2.11), so it follows from the linear dependence relation (2.17) that the second

right hand term of (2.20) is equal to zero independently of the restriction that $u_{aj} = 0$, for all $j = 1, \dots, n$. This completes the proof, outlined earlier in this essay, that the arbitrary restrictions (2.5) and (2.6) do not affect any of the empirically meaningful propositions of ordinal utility theory.

It follows from the initial assumption that the individual consumer spends his entire money income, M , that the expressions for shifts in demand are related according to the linear dependence

$$(2.21) \quad \sum_{i=1}^n p_i x_{iaj} = 0.$$

That the Tintner-Ichimura Relations satisfy this linear dependence can easily be verified by straightforward substitution of (2.19) into (2.21), having regard to the linear dependence rule for Slutsky-Hicks Substitution Terms.

The Tintner-Ichimura Market Relation is obtained by summing over all individuals in the market; that is,

$$(2.22) \quad X_{iaj} = \sum x_{iaj}.$$

(The notation, X_i , will denote market demand in the rest of this essay.) Since each individual is presumed to be charged the same prices for his purchases of commodities, it follows from (2.21) that the Tintner-Ichimura Market Relations (2.22) satisfy the linear dependence

$$(2.23) \quad \sum_{i=1}^n p_i X_{iaj} = 0.$$

Interrelations of Elasticities of Demand

The transformations

$$(2.24) \quad e_{ij} = \frac{a_j}{x_i} x_{iaj}, \quad i, j = 1, \dots, n;$$

$$(2.25) \quad E_{ij} = \frac{a_j}{X_j} X_{iaj},$$

define the empirically useful concepts of individual and market advertising elasticities of demand.²³ e_{ij} is a dimensionless number

²³Cf. R. Dorfman and P. O. Steiner, "Optimal advertising and optimal quality," Amer. Econ. Rev., Vol. 44, 1954, pp. 826-836.

representing the percentage change in demand for X_i with respect to a one per cent change in advertising expenditure, a_j ; E_{ij} represents the corresponding concept for market demand functions. These concepts are closely related to the concept of elasticity of substitution, sometimes employed in existing consumer demand theory. The elasticity of substitution between two commodities, X_i and X_j , is defined as

$$(2.26) \quad s_{ij} = - \frac{s_{ij} p_j}{x_i}, \quad i, j = 1, \dots, n,$$

and represents the percentage change in the consumer's demand for X_i with respect to a one percent change in the price of X_j . If b_{nj} is substituted for u_{nj} in (2.19), then, after simplification, the expression for e_{ij} becomes

$$(2.27) \quad e_{ij} = \sum_{h=1}^n a_j b_{hj} \left\{ - \frac{s_{ih} p_h}{x_i} \right\}$$

$$= \sum_{h=1}^n w_{hj} s_{1h}$$

where $w_{hj} = a_j b_{hj}$ is the elasticity of marginal utility of X_h with respect to the advertising expenditure, a_j , and represents the percentage change in the marginal utility with respect to a one per cent change in expenditure on advertising X_j . Formula (2.27) shows that advertising elasticities of demand are linear combinations of substitution elasticities with advertising elasticities of the marginal utilities acting as weights. Since, in empirical demand analysis, it is sometimes possible to obtain independent estimates of substitution elasticities and advertising elasticities of demand, it is also possible to obtain estimates of the advertising elasticities of marginal utilities; empirical verification of several hypotheses concerning the w_{hj} might throw some light on problems encountered in the theory of selling costs.²⁴ Some possibilities will be mentioned

²⁴Cf. N. S. Buchanan, "Advertising expenditures: a suggested treatment," Jour. Pol. Econ., Vol. 20, 1942, pp. 537-557.

below.

Using relations (2.21) and (2.23), one can easily verify that advertising elasticities of demand are linearly dependent according to

$$(2.28) \quad \sum_{i=1}^n (p_i x_i) e_{1j} = 0,$$

for individual consumers, and

$$(2.29) \quad \sum_{i=1}^n (p_i x_i) E_{ij} = 0, \quad j = 1, \dots, n,$$

for the market demand functions.

The linear dependence relations impose several restrictions on the signs and magnitudes of advertising elasticities of demand. Since the coefficients, $p_i x_i$ and $p_i X_i$, are non-negative, it follows from (2.28) and (2.29) that the elasticity of demand with respect to a_j must be positive for at least one i , and negative for at least one other, except in the logically (but not empirically) trivial case where all advertising elasticities are zero.

Another restriction imposed by the linear dependence relations limits the number of elasticities, e_{ij} and E_{ij} which can be constant; not all the e_{ij} can be constant unless all are (trivially) zero, and the same holds for the E_{ij} . For if all the e_{ij} were constant, then expenditures on the several commodities would be determined according to the system of equations

$$(2.30) \quad [e_{ji}] n = 0, \quad j, i = 1, \dots, n$$

$$n = \text{col. } (m_1, \dots, m_n),$$

where $m_i = p_i x_i$, and the rank of the matrix $[e_{ji}]$ is $n-1$. It follows from (2.30) that the ratios of expenditures, m_i/m_n are constant; hence

$$(2.31) \quad \frac{\partial}{\partial a_j} \frac{m_i}{m_n} = 0,$$

which implies

$$(2.32) \quad \frac{a_j}{m_1} \frac{\partial m_1}{\partial a_j} = \frac{a_j}{m_n} \frac{\partial m_n}{\partial a_j},$$

1. e.,

$$(2.33) \quad e_{1j} = e_{nj}, \quad j = 1, \dots, n-1.$$

Thus, if all the elasticities of demand with respect to a_j are constant, and if this is true for all $j = 1, \dots, n$, then the elasticities of all the demand functions with respect to each a_j are equal, but the only magnitude of the e_{1j} which is consistent with this result is zero.

Some interesting results are obtained when it is assumed that advertising expenditure, a_1 , affects only the marginal utility of x_1 , leaving all other marginal utilities unchanged.²⁵ Let $u_{1a_1} = b_1 u_1 > 0$,

²⁵ Cf. G. Tintner, op. cit., p. 3.

$u_{ja_1} = 0$, for all $j \neq 1$. In this case the Tintner-Ichimura Relation becomes

$$(2.34) \quad x_{ja_1} = -b_{11} \frac{u_1}{\lambda} g_{j1} \\ = -b_{11} g_{j1} p_1$$

and

$$(2.35) \quad e_{j1} = w_{11} g_{j1}, \quad j = 1, \dots, n.$$

From (2.34) and (2.35) it follows that $x_{1a_1} > 0$, and from (2.22) that $x_{1a_1} > 0$, assuming that $u_{1a_1} > 0$ and $u_{ja_1} = 0$ for all consumers.

According to the definition of related goods given by Hicks,²⁶ X_1 is

²⁶J. R. Hicks, op. cit., pp. 309-311. Also see S. Ichimura, op. cit., p. 181 and R. L. Basmann, op. cit., p. 69.

a substitute for X_j if $s_{1j} > 0$; X_1 is a complement of X_j if $s_{1j} < 0$; and X_1 and X_j are independent commodities if $s_{1j} = 0$. Accordingly, if X_1 is a substitute for X_j , then by (2.34) $x_{ja_1} < 0$; if X_1 is a complement of X_j , then $x_{ja_1} > 0$; that is, if there is an increase in advertising expenditure, a_1 , there results an increase in the demand for X_1 , a decrease in the demand for commodities which are substitutes for X_1 , an increase in the demand for commodities which are complements of X_1 , and no change in the demand for commodities which are independent of X_1 . From the definition of elasticity of substitution between X_1 and X_j , i. e., from formula (2.26), the algebraic sign of s_{j1} is opposite to that of s_{j1} ; the elasticity, w_{11} , of the marginal utility of X_1 with respect to advertising expenditure, a_1 , is a positive number; it follows from the definition of related goods, that the advertising elasticity of demand for X_1 with respect to a_1 is greater than zero, that the advertising elasticity of demand for X_j with respect to a_1 is less than zero if X_1 is a substitute for X_j and greater than zero if X_1 is a complement of X_j , and zero if X_1 is independent of X_j .

In this case where the advertising expenditure, a_1 , affects only the marginal utility of X_1 , the advertising elasticities of demand are directly proportional to the elasticities of substitution; indeed, the

the advertising elasticity of demand for X_j with respect to a_1 is simply the product of the elasticity of marginal utility of X_1 with respect to a_1 and the elasticity of substitution of X_1 for X_j .

If the marginal utility of X_1 is relatively inelastic, i. e., if $w_{11} < 1$, then formula (2.35) indicates that the consumer's demand for X_j responds relatively less to a one per cent increase in advertising expenditure, a_1 , than to a one per cent income-compensated decrease in the price of X_1 ; if the marginal utility of X_1 is relatively elastic, i. e., if $w_{11} > 1$, then (2.35) indicates that the consumer's demand for X_j responds relatively more to a one per cent increase in advertising expenditure, a_1 , than to a one per cent income-compensated decrease in the price of X_1 . This points to the possible convenience of classifying commodities as "necessities" or "luxuries" according as the elasticities, w_{11} , are less than or equal to, and greater than unity; possibly such a classification would agree closely with a corresponding classification based on the magnitudes of price elasticities.²⁷

²⁷Cf. H. Wold, op. cit., pp. 114-115.

Some Theorems on Composite Commodities

From the point of view of empirical demand analysis, one of the most important propositions of existing consumer demand theory is the well-known Leontief-Hicks theorem,²⁸ which asserts that if the prices

²⁸Cf. W. Leontief, "Composite commodities and the problem of

index numbers," *Econometrica*, Vol. 4, 1936, pp. 39-59. J. R. Hicks, *op. cit.*, pp. 312-313. H. Wold, *op. cit.*, pp. 108-109.

of a group of goods all change in equal proportion, then that group of goods can be logically treated as if it were but a single commodity. It is frequently the experience with market data that the prices of individual goods in a general class of commodity undergo approximately uniformly proportionate changes. By using the Leontief-Hicks theorem, one is then able to define the concepts of related commodities as applied to composite goods in exactly the same way as one defines complementarity, substitutability, and independence of single, well-defined goods, and to define the concepts of price elasticity, income elasticity, and substitution elasticity of demand for composite commodities, and to verify that these measures satisfy exactly the same laws as do the corresponding measures defined for single goods.

There is an analogous theorem in the theory of consumer demand with variable preferences, but here it is the marginal utilities of a group of goods which are assumed to change in equal proportion. If X_1, \dots, X_m , are all sub-commodities of a general class, it is plausible to assume that expenditure devoted to advertising the class of commodity will affect the marginal utilities of the sub-commodities more or less uniformly. The theorem asserts that if the marginal utilities are all increased in the same proportion, then the group of goods can be logically treated as a single commodity; specifically, an aggregate quantity, called the composite demand for the group,

which is a weighted sum of the demands for individual commodities within the group, can be defined such that this quantity behaves exactly as do the x_{ja_1} in formula (2.34); elasticities of composite demand can be defined such that they behave exactly as the e_{j1} in formula (2.35). The proof of the theorem is essentially equivalent to one of the steps in the proof of the Leontief-Hicks theorem.

Before the proof of this theorem is undertaken, it is necessary to define the concept of related composite commodities. Suppose that the prices of X_1, \dots, X_m , all increase in the same proportion, and denote this aggregate by Z_1 ; suppose that all other prices remain constant, but partition the remaining commodities into two aggregates, X_{m+1}, \dots, X_p , denoted by Z_2 , and X_{p+1}, \dots, X_n . According to the Leontief-Hicks theorem, each of these groups of goods, Z_1 , Z_2 , and Z_3 , may be treated as if they were single commodities. In addition, suppose that the equal proportionate changes in the prices, p_1, \dots, p_m , are income-compensated. Thus, the change in expenditure on X_j , $j = 1, \dots, n$, due to the proportionate change in the price of X_i , $i = 1, \dots, m$, is equal to $s_{ji}p_jp_i$; if X_j is a substitute for X_i , then it follows from the definition of related goods that $s_{ji}p_jp_i > 0$; if X_j is a complement of X_i , it follows from the definition that $s_{ji}p_jp_i < 0$; in particular

$$(2.36) \quad s_{11}p_1^2 < 0.$$

Summing over the commodities making up the aggregate, Z_1 , one obtains the result that the change in aggregate expenditure on Z_1 with respect to the proportionate increase or decrease in prices, p_1, \dots, p_m ,

obeys the same law of sign as $s_{11}p_1^2$; for, according to the theorem (2.18), it follows that

$$(2.37) \quad \sum_{h=1}^m \sum_{i=1}^m s_{hi} p_h p_i < 0$$

where the left-hand member is the rate of change in expenditure on the group Z_1 with respect to the equal proportionate change in prices.

$$(2.38) \quad R_2^1 = \sum_{j=p_1+1}^p \sum_{i=1}^m s_{ji} p_j p_i$$

is the rate of change in expenditure on the aggregate Z_2 , and

$$(2.39) \quad R_3^1 = \sum_{k=p_1+1}^n \sum_{i=1}^m s_{ki} p_k p_i$$

is the rate of change in expenditure on the aggregate Z_3 with respect to the proportionate change in prices, p_1, \dots, p_m . If $R_2^1 > 0$, the commodities in the aggregate Z_2 are said to be predominantly substitutable for the commodities in the aggregate Z_1 ; if $R_2^1 < 0$, the commodities in the aggregate Z_2 are said to be predominantly complementary with the commodities in Z_1 . It follows from the linear dependence rule (2.17) that, if $R_2^1 < 0$, that $R_3^1 > 0$; if Z_2 is predominantly complementary with Z_1 , then Z_3 is predominantly substitutable for Z_1 . Ultimately this last result is derived from the assumption that the consumer spends his entire income, M .

Suppose that there is advertising which increases all the marginal utilities, u_1, \dots, u_m , in equal proportion, and denote the proportional increase by \underline{b} . Denote by \underline{a} the total advertising on Z_1 ; denote by r_1 the total expenditure on Z_1 , r_2 the total expenditure

on Z_2 , and r_3 the total expenditure on Z_3 ; i. e.,

$$(2.40) \quad r_1 = \sum_{h=1}^m p_h x_h,$$

$$(2.41) \quad r_2 = \sum_{j=m+1}^p p_j x_j,$$

$$(2.42) \quad r_3 = \sum_{k=p+1}^n p_k x_k.$$

From (2.19) the Tintner-Ichimura Relation for the single good, X_j , is given by

$$(2.43) \quad x_{ja} = -b \sum_{i=1}^m s_{ji} p_i, \quad j = 1, \dots, n.$$

The rates of change of expenditures, r_1 , r_2 , and r_3 , are given by

$$(2.44) \quad \frac{\partial r_1}{\partial a} = -b \sum_{h=1}^m \sum_{i=1}^m s_{hi} p_h p_i,$$

$$(2.45) \quad \frac{\partial r_2}{\partial a} = -b \sum_{j=m+1}^p \sum_{i=1}^m s_{ji} p_j p_i,$$

$$(2.46) \quad \frac{\partial r_3}{\partial a} = -b \sum_{k=p+1}^n \sum_{i=1}^m s_{ki} p_k p_i.$$

It follows once more from (2.18) that $\partial r_1 / \partial a > 0$; that $\partial r_2 / \partial a < 0$, if Z_2 is predominantly substitutable for Z_1 , and that $\partial r_2 / \partial a > 0$, if Z_2 is predominantly complementary with Z_1 , follow from the definitions of related aggregate commodities given earlier in this section. Comparing (2.34) with (2.44), one sees that $\partial r_1 / \partial a$ obeys the same law of sign as $p_1 x_{1a}$; with (2.45), that $\partial r_2 / \partial a$ and $\partial r_3 / \partial a$ obey the same

law of sign as $p_j x_{ja_1}$, where $j \neq 1$.

From the linear dependence rule (2.21) it follows that

$$(2.46) \quad \frac{\partial r_1}{\partial a} + \frac{\partial r_2}{\partial a} + \frac{\partial r_3}{\partial a} = 0,$$

from which it follows that

$$(2.47) \quad \frac{a}{r_1} \frac{\partial r_1}{\partial a} r_1 + \frac{a}{r_2} \frac{\partial r_2}{\partial a} r_2 + \frac{a}{r_3} \frac{\partial r_3}{\partial a} r_3 = 0,$$

which is the linear dependence rule for advertising elasticities of composite commodities, i. e.,

$$(2.48) \quad r_1 e_{Z_1 a} + r_2 e_{Z_2 a} + r_3 e_{Z_3 a} = 0,$$

where $e_{Z_1 a}$ is the advertising elasticity of demand for the composite commodity, Z_1 , $e_{Z_2 a}$, the elasticity of demand for the composite commodity, Z_2 , and $e_{Z_3 a}$, the elasticity of demand for the composite commodity, Z_3 , all with respect to the aggregate advertising expenditure on Z_1 .

(2.48) is seen to have exactly the same form as (2.28); this completes the proof of the theorem that a group of goods, all of whose marginal utilities increase or decrease in equal proportion, can be treated as a single commodity.

Further Interrelations of Elasticities of Demand: Composite Goods

In formulas (2.27) and (2.35) the connections between advertising elasticities of demand and substitution elasticities were pointed out. Similar connections subsist between advertising elasticities of demand for composite goods and the advertising elasticities of demand for the

sub-commodities that make up the aggregates, which elasticities are in turn related to elasticities of substitution between the sub-commodities. In order to show these relations, the concept of elasticity of substitution between two aggregate commodities is first defined. This is done in a fashion similar to the fashion in which the concept of related composite commodities was defined above: from formula (2.26) defining the elasticity of substitution of X_j for X_i , one has

$$(2.49) \quad \sigma_{ji} = - \left\{ \frac{\sum_{j=1}^n p_j p_i}{(p_j x_j)} \right\};$$

analogously, in terms of the composite commodities, Z_1 and Z_2 , one defines the elasticity of substitution

$$(2.50) \quad \sigma_{21} = - \frac{1}{\epsilon_2} \sum_{j=1}^n \sum_{i=1}^m \sigma_{ji} p_i p_j,$$

and similarly for σ_{31} . σ_{11} is analogously defined.

It follows from the definitions of elasticities, σ_{1a} , σ_{2a} , and σ_{3a} ; and from the formulas (2.44), (2.45), and (2.46); that

$$(2.51) \quad \sigma_{1a} = w_{Z_1} \sigma_{11},$$

$$(2.52) \quad \sigma_{2a} = w_{Z_1} \sigma_{21},$$

$$(2.53) \quad \sigma_{3a} = w_{Z_1} \sigma_{31};$$

where $w_{Z_1} = \sigma_{aZ_1}$ defines the elasticity of marginal utility of the aggregate Z_1 with respect to aggregate advertising expenditure, a .

That relations (2.51), (2.52), and (2.53) have the same form as (2.35)

shows that the same relations subsist between aggregate elasticities of demand with respect to advertising expenditure and elasticities of substitution of one aggregate commodity for another, as subsist between elasticities of demand with respect to advertising which affects the marginal utility of only one commodity and the elasticities of substitution of that one commodity for itself and other commodities.

The aggregate advertising elasticities of demand for Z_1 , Z_2 , and Z_3 are also weighted averages of the individual elasticities of demand for X_1 , where $i = 1, \dots, m$; for X_j , where $j = m/1, \dots, p$; and for X_k , where $k = p/1, \dots, n$; all with respect to the advertising expenditure, a . According to the Tintner-Ichimura Relation (2.43), and the definition of advertising elasticity of demand,

$$(2.54) \quad \epsilon_{ha} = w_{Z1} \sum_{i=1}^m \epsilon_{hi}, \quad h = 1, \dots, n.$$

Then

$$\begin{aligned} (2.55) \quad \epsilon_{Z_1 a} &= \frac{w_{Z1}}{r_1} \sum_{h=1}^m \sum_{i=1}^m \epsilon_{hi} p_i p_h \\ &= \sum_{h=1}^m (p_h x_h) w_{Z1} \frac{1}{r_1} \sum_{i=1}^m \epsilon_{hi} \\ &= \sum_{h=1}^m \frac{(p_h x_h)}{r_1} \epsilon_{ha}, \end{aligned}$$

similarly,

$$(2.56) \quad \epsilon_{Z_2 a} = \sum_{j=m/1}^p \frac{(p_j x_j)}{r_2} \epsilon_{ja},$$

and

$$(2.57) \quad e_{z_3^a} = \sum_{k=p+1}^n \frac{(p_k x_k)}{r_3} e_{kz}.$$

Conclusion

By the device of summation over all individual consumers in a given market, or within a market stratum, market formulas equivalent to the formulas from (2.35) to (2.57) can easily be derived and shown to have the same algebraic form. In empirical demand analysis it will be frequently convenient to postulate that aggregate market behavior of individuals can be represented as the market behavior of a conceptual average individual; the formulas then purport to describe the consumption behavior of this statistical individual.

The linear dependence rules (2.28) and (2.48) may be employed in empirical demand analysis in essentially two different ways. Firstly, the linear dependence may be imposed a priori, i. e., as side conditions which estimated advertising elasticities of demand are forced to satisfy. One might do this when, for example, one is chiefly interested in testing the hypothesis that an increase in expenditure on advertising a commodity, X_1 , affects only the marginal utility of that commodity. Secondly, elasticities estimated unconditionally may be "tested" by substitution in the appropriate linear dependence relation.

If independent estimates of elasticities of substitution and advertising elasticities can be obtained, it is possible, using the

the relations (2.27), for one to estimate advertising elasticities of marginal utilities and to test hypotheses such as those represented by formula (2.35). For example, from the point of view of a firm which sells and advertises several goods, say, X_1 and X_2 , knowledge of the probable effect of an increase in advertising expenditure, a_1 , on the marginal utility of X_2 is relevant to the problem of selecting an optimal advertising budget, i. e., a profit-maximizing budget. From another point of view, knowledge of the parameters of a utility function is interesting in its own right, if only to satisfy intellectual curiosity. It also seems likely that experienced consumption economists will be able to formulate, on the basis of the relationships shown here, a number of significant hypotheses to be put to test.

SOME ESTIMATES OF ADVERTISING ELASTICITIES OF DEMAND FOR TOBACCO

Introduction

The purpose of working out a theory of demand with variable consumer preferences was to formulate definitions and derive analytical propositions useful as a priori restraints or tentative empirical hypotheses to be tested against market and budget data. In this chapter an exploratory empirical analysis of market elasticities of demand for tobacco in the United States, 1926-1945 is described; empirical demand elasticities with respect to price, income, tobacco advertising, and aggregate advertising on all other goods are given; in addition, the elasticity of the marginal utility of tobacco with respect to tobacco advertising is computed by the use of estimated price and advertising elasticities of demand.

Available advertising data were found to be classified according to general type of commodity, e. g., according to definitions of aggregate commodities such as food and drink, textiles, home furnishings, smoking materials, etc. It was found that the prices of individual tobacco commodities, cigarettes, cigars, smoking tobacco, and chewing tobacco remained approximately proportional one to the other for a considerable part of the time between 1926 and 1945, and after; what deviations there were from constant proportions seem to be negligible. Thus, empirical conditions essentially equivalent to the premises of the Leontief-Hicks theorem given above were exhibited by prices of tobacco commodities. Moreover, a composite retail price of tobacco

products is regularly computed and published by the Bureau of Agricultural Economics — the source is cited in the next section — and the construction of this composite price series apparently satisfies the criteria required by the Leontief-Hicks theorem.

The prices of individual goods in no other general class of commodity for which advertising expenditures were available appeared to satisfy the criteria for composite commodities as well as tobacco. Moreover, the percentage of consumers' disposable income spent on tobacco is relatively small compared with the percentage expenditure on other commodity groups. From the point of view of statistical estimation of simultaneous economic relationships, this warrants the specification of disposable income as an exogenous variable, a specification which would be grossly imprecise for most of the other general classes of commodity, e. g., food and drink.²⁹ Budget restrictions limited statistical

²⁹M. A. Girshick and T. Haavelmo, "Statistical analysis of the demand for food: examples of simultaneous estimation of structural equations," *Econometrica*, Vol. 15, 1947, pp. 79-110.

procedures to the simplest and least expensive feasible.

The purpose of this empirical analysis is only to take the first step in connecting the theoretical relations worked out earlier with observations of consumers' consumption behavior in real markets. This first step is that of observing whether computed measurements of advertising elasticities of demand have the signs they are asserted to

have by the logical relations. Except in one instance, estimates of advertising elasticities of demand for any commodity³⁰ do not appear

³⁰Cf. A. R. Prest, "Some experiments in demand analysis," Rev. Econ. Stat., Vol. 31, 1949, pp. 33-49.

to exist in the literature.

Approximation of the Demand Function

According to the conditions (2.10) - (2.12) for consumer equilibrium, the demand function for tobacco, X_1 , is functionally represented by

$$(3.1) \quad x_1 = f_1(p_1, p_2, M, a_1, a_2)$$

where x_1 denotes the consumption of tobacco, p_1 denotes the composite price of tobacco products, p_2 denotes the composite price of all other commodities, M denotes money income, a_1 denotes per capita expenditure by sellers on advertising tobacco, and a_2 denotes aggregate per capita expenditure on advertising all other products. The demand function is homogeneous of degree zero in the prices and income.

A point needs to be made here about the advertising expenditures, a_1 and a_2 . These are presumed to be measured in real terms; i. e., if a_{1t} is total money expenditure on advertising X_1 divided by the number of consumers during the time period t , and P_t is the consumer price index for time period t , then

$$(3.2) \quad a_{1t} = \frac{a_{1t}^i}{P_t}, \quad t = 1, 2.$$

It is also assumed that

$$(3.3) \quad p_{2t} = P_t p_{20},$$

where p_{20} is the average composite price of the aggregate, X_2 , in the price index base period. The real price of tobacco and the real income in time period, t , are defined by

$$(3.4) \quad z_{1t} = \frac{P_{1t}}{P_t},$$

and

$$(3.5) \quad Q_t = \frac{M_t}{P_t}.$$

By the homogeneity property

$$(3.6) \quad x_{1t} = f_1(z_{1t}, Q_t, a_{1t}, a_{2t}).$$

Since the exact form of the demand function is not of primary interest in this study, the function (3.6) may conveniently be replaced by a linear approximation according to Taylor's Formula. The primary interest being focused on the elasticities, however, make it more convenient to approximate $\log f_1(z_{1t}, Q_t, a_{1t}, a_{2t})$ by the linear relation

$$(3.7) \quad \log x_{1t} = b_{10} + b_{11} \log z_{1t} + b_{1Q} \log Q_t + b_{1a_1} \log a_{1t} \\ + b_{1a_2} \log a_{2t}.$$

From (3.7), and by Taylor's Theorem, it is seen that $-b_{11}$ is approximately the price elasticity of demand for tobacco, b_{1Q} the income elasticity, b_{1a_1} and b_{1a_2} the advertising elasticities of demand for tobacco.³¹ This form of approximation is widely used in

³¹R. G. D. Allen. *Mathematical Analysis for Economists*. London. Macmillan. 1950. pp. 456-459.

econometric demand analysis.³²

³²Cf. J. A. Nordin, G. G. Judge, and O. Wahby. *Application of Econometric Procedures to the Demands for Agricultural Products*. Iowa Agr. Exp. Sta. Res. Bul. 410. 1954. R. Stone, "The analysis of market demand," *Jour. Roy. Stat. Soc.*, Vol. 108, Series A, 1945, pp. 286-382. H. Wold, *op. cit.*, pp. 231-244.

A second equation is introduced to complete the notion of structure as applied to the tobacco retail market. It is assumed that tobacco consumption and tobacco advertising are jointly determined endogenous variables; retail tobacco prices, disposable income, and advertising expenditures on all other goods are considered to be exogenous with respect to tobacco consumption and tobacco advertising. The assumption that advertising is the second endogenous variable rather than price is considered plausible on the ground that tobacco prices have been more or less institutional, while advertising has been chief means of competition within the industry.³³ It is assumed

³³Cf. W. H. Nicholls. *Price Policies in the Cigarette Industry*.

Nashville. Vanderbilt Univ. Press. 1951. pp. 67-70, 177-191.

that current advertising expenditure on tobacco depends on current consumption of tobacco, current price, current disposable income, and current advertising expenditure on all other goods; but in addition to these variables, current advertising expenditure on tobacco depends on the tax-rate imposed by State and Federal governments. The functional form of this equation is approximated by the methods outlined above in connection with the demand function; the approximation is

$$(3.8) \quad \log a_{1t} = b_{20} + b_{2x_1} \log x_{1t} + b_{21} \log z_{1t} + b_{2Q} \log Q_t \\ + b_{2a_2} \log a_{2t} + b_{22} \log z_{2t},$$

where z_{2t} denotes taxes per pound in real terms.

Statistical Estimation Procedures

Equations (3.7) and (3.8) provide mathematical approximations to the economic relationships jointly determining tobacco consumption and tobacco advertising. The notion of the approximate empirical structure of the tobacco retail sector of the economy is expressed by

$$(3.9) \quad \log x_{1t} = b_{10} + b_{11} \log z_{1t} + b_{1Q} \log Q_t + b_{1a_1} \log a_{1t} \\ + b_{1a_2} \log a_{2t} + y_{1t}$$

$$(3.10) \quad \log a_{1t} = b_{20} + b_{2x_1} \log x_{1t} + b_{21} \log z_{1t} + b_{2Q} \log Q_t \\ + b_{2a_2} \log a_{2t} + b_{22} \log z_{2t} + y_{2t}$$

where:

- x_{1t} = Per capita consumption of tobacco, pounds. (U. S. Bureau of Agricultural Economics. Agricultural Statistics. 1953. Table 157. p. 117.)
- a_{1t} = Composite retail price of tobacco products, cents/pound, deflated by B. L. S. Cost-of-Living Index. (U. S. Bureau of Agricultural Economics. Agricultural Statistics. 1953. Table 155. p. 116.) (U. S. Department of Commerce. Statistical Abstract of the United States. 1953. Table 339. p. 307.)
- Q_t = Per capita disposable income, dollars, deflated by B. L. S. Cost-of-Living Index. (U. S. Department of Commerce. Statistical Abstract of the United States. 1954. Table 333. p. 302.)
- a_{1t} = Per capita advertising expenditure on tobacco, cents, deflated by B. L. S. Cost-of-Living Index. (Compiled from U. S. Department of Commerce. Survey of Current Business. 1925-1947, and from U. S. Department of Commerce. Statistical Abstract of the United States. 1951. Table 1043. p. 907.)
- a_{2t} = Per capita advertising expenditure on all other goods, dollars, deflated by B. L. S. Cost-of-Living Index. (Compiled from same sources as a_{1t} .)
- a_{2t} = Federal and State taxes on tobacco products, composite, cents/pound, deflated by B. L. S. Cost-of-Living Index. (U. S. Bureau of Agricultural Economics. Agricultural Statistics. 1953. Table 155. p. 116.)

None of the variables has been cleared of trend, nor in the demand function (3.9) has time been included as a regressor as an equivalent to clearing of trends.³⁴ The motive behind clearing of

³⁴R. Frisch and F. V. Waugh, "Partial time regressions as compared with individual trends," *Econometrica*, Vol. 1, 1933, pp. 387-401. G. Tintner. *Econometrics*. New York. John Wiley. 1952. pp. 301-302.

trend would be to avoid spurious correlation and spurious regression of consumption on price, income, and advertising expenditures due to other variables not explicitly introduced as regressors. On the other

hand, trends in price, income, and advertising expenditures are supposed to influence per capita consumption through the theoretical relation specified by the demand function (3.9); so trend removal before fitting might reduce the amount of information on the demand relation. It was considered plausible that the regressors in (3.9) constituted a fairly complete specification of factors influencing per capita consumption of tobacco, several other sources of possible trend, e. g., from changes in age composition of the population, and changes in the distribution of incomes, having first been investigated and found to be negligible; so time was not initially introduced as a regressor, though it still would have been easy to do so later on, if necessary. As things turned out, however, it was not necessary.

In the demand function (3.9) y_{1t} denotes the error due to influencing factors not taken into account; in (3.10) y_{2t} denotes the same kind of error. It is assumed that the errors in the equations are normally distributed with zero means and constant variances and constant covariance between them; they are assumed to be not autocorrelated. The normality of the distributions is not essential to the selection of methods of estimation.

For estimating the coefficients of equation (3.9) two different methods are employed: the method of least squares³⁵ and the method of simultaneous equations.³⁶ Under the assumption that (3.9) constitutes a

³⁵G. Tintner, op. cit., pp. 83-92.

³⁶Ibid., pp. 154-184.

a single-equation model, i. e., when it is not assumed that x_{1t} and a_{1t} are jointly determined endogenous variables, then it is a well-known result that direct application of the method of least squares yields best (in the sense of having minimum variance), unbiased estimates of the parameters, b_{10} , b_{11} , etc.³⁷ Under the further

³⁷H. Wold, op. cit., pp. 208-209.

assumption that the errors in the equations, y_{1t} , are normally distributed, the least squares estimates are equivalent to maximum likelihood estimates.³⁸

³⁸G. Tintner, op. cit., pp. 26-28.

By the method of least squares, the following estimate of the demand function (3.9) was obtained:

$$\begin{aligned}
 (3.11) \quad \widehat{\log x_{1t}} = & .245 - .218 \log x_{1t} + .403 \log Q_t + .051 \log a_{1t} \\
 & (t = \quad 6.91 \quad \quad \quad 6.01^{**} \quad \quad 1.39 \\
 & \quad \quad \quad - .023 \log a_{2t}, \\
 & \quad \quad \quad 1.48)
 \end{aligned}$$

$$R_1 = .976.$$

$\widehat{\log x_{1t}}$ denotes the predicted value of $\log x_{1t}$ for time period t .

Directly beneath each elasticity coefficient is shown the corresponding magnitude of Student's t -criterion. Where the double asterisk appears it denotes that the criterion is significant at the 0.99 level.

Given that b_{11} , the price elasticity of demand, is really zero, the probability of a value of Student's t as large or larger than 0.91 is less than 0.40; given that $b_{1Q} = 0$, the probability that $|t| > 6.01$ is something less than 0.01; given that $b_{1a_1} = 0$, the probability that $|t| > 1.39$ is less than 0.20; given that $b_{1a_2} = 0$, the probability that $|t| > 1.48$ is less than 0.20. It appears plausible that none of the coefficients is zero. As concerns the general fit of the equation to the data, the multiple correlation coefficient, $R_1 = 0.976$ is highly significant for four variables and twenty observations.

From the estimated demand function (3.11) it appears that per capita consumption of tobacco products is relatively inelastic with respect to all the variables included. Of especial interest are the advertising elasticities of demand. According to the estimated elasticity of demand for tobacco with respect to tobacco advertising, a one per cent increase in advertising expenditure leads to an increase in consumption of about one-twentieth of a per cent. An increase of one per cent in advertising expenditure on all other goods leads to a decrease of about one-fiftieth of a per cent in the per capita consumption of tobacco. It thus appears that tobacco consumption is relatively more elastic with respect to price than with respect to advertising, a one per cent decrease in the price of tobacco leading to an increase in consumption of about one-fifth of one per cent.

As a crude check on the non-autocorrelation assumption about the errors in the equation (3.9), the residuals

$$(3.12) \quad \hat{u}_{1t} = \log x_{1t} - \log x_{1t}, \quad t = 1, 2, \dots, 20,$$

were computed and tested for autocorrelation according to the von Neumann ratio test,³⁹

³⁹Ibid., pp. 252-254.

$$(3.13) \quad \frac{s^2}{s^2} = \frac{N}{N-1} \frac{\sum_{t=1}^{N-1} (u_{1,t+1} - u_{1t})^2}{\sum_{t=1}^N (u_{1t} - \bar{u}_1)^2}.$$

$$= 2.70.$$

For $N = 20$,

$$(3.14) \quad E \frac{s^2}{s^2} = 2.05.$$

According to the tables provided by Hart,⁴⁰ the probability of a devla-

⁴⁰B. I. Hart, "Tabulation of the probabilities for the ratio of the mean square successive difference to the variance," *Annals of Math. Stat.*, Vol. 13, 1942, pp. 207-214.

tion as large as, or larger than 0.65 is 0.09. This appears to cast doubt on the assumption that the errors in the equation are not autocorrelated. Strictly speaking, however, the von Neumann ratio test is not valid for residuals.⁴¹

⁴¹G. Tintner, op. cit., p. 253.

If it is assumed that x_{1t} and a_{1t} are jointly determined by (3.9) and (3.10), the computation of maximum likelihood estimates calls for the application of the method known as simultaneous equations. Since a_{2t} , the tax rate per pound of composite tobacco products, is absent from the demand equation (3.9) and present in (3.10) it follows that (3.9) is hypothetically just-identified.⁴² The estimated coefficients⁴³

⁴²Ibid., p. 157.

⁴³Ibid., p. 167.

obtained by elimination of $\log a_{2t}$ between the reduced form equations are given by

$$(3.15) \quad \log x_{1t} = .704 - .390 \log a_{1t} + .374 \log Q_t + .085 \log a_{1t} - .036 \log a_{2t}.$$

Only the von Neumann test was computed for this estimated equation. The computed ratio was 2.695, substantially equivalent to the value for (3.9) when estimated by least squares; consequently the same remarks apply to the residuals in (3.15).

According to (3.15) the elasticity of demand for tobacco products with respect to tobacco advertising is .085. Given a ceteris paribus increase of one per cent in tobacco advertising, this estimate asserts

that there will be an increase of about one-twelfth of one per cent in the consumption of tobacco products. In (3.15) the elasticity of demand with respect to advertising expenditure on all other goods is $-.036$. Given a *ceteris paribus* increase in advertising expenditure on all other goods, this estimate asserts that there will be an increase of slightly more than one-third of one per cent in the consumption of tobacco products. According to (3.15) tobacco consumption is more elastic with respect to price and advertising expenditures than is indicated by the coefficients in (3.11) obtained by the method of least squares, whereas the least squares estimate of income elasticity is greater than the estimate obtained by simultaneous equations methods.

According to the theory of consumer demand with variable preferences it is a sufficient condition for $b_{1s_1} > 0$ and $b_{1s_2} < 0$ that each of the separate tobacco products be predominantly substitutable for the composite commodity representing all other goods, provided that increasing advertising expenditure on tobacco does not decrease the marginal utility of any tobacco product. Since, however, this is not a necessary condition, the fact that the elasticity estimates exhibit these inequalities is only very weak evidence for concluding that aggregate advertising expenditure does not decrease the marginal utility of any tobacco product.

It is interesting to make a rough estimate of a sort of average elasticity of marginal utility of tobacco with respect to tobacco advertising, that is, to compute w_{z_1} in equation (2.51). According to

the Fundamental Equation in Value Theory,⁴⁴ the elasticity of substitution

⁴⁴J. L. Mosak, "On the interpretation of the Fundamental Equation in Value Theory," in O. Lange, ed. *Studies in Mathematical Economics and Econometrics*. Chicago. Univ. of Chicago Press. 1942. pp. 69-74.

Z_{11} is related to price elasticity and income elasticity by the formula

$$(3.16) \quad Z_{11,t} = -b_{11} + \frac{P_{1t}x_{1t}}{M} b_{1Q}.$$

During the years 1926-1945 the proportion of income spent on tobacco remained at about two per cent. It follows by formula (2.51) that, using the least squares estimates of b_{11} and b_{1a_1} with b_{1Q} in (3.16), one obtains

$$(3.17) \quad w_{Z_1}^1 = \frac{.051}{.210} = .242;$$

and by using the simultaneous equations estimates of b_{11} , b_{1a_1} , and of b_{1Q} , one obtains

$$(3.18) \quad w_{Z_1}^s = \frac{.085}{.383} = .222.$$

It appears that a one per cent ceteris paribus increase in tobacco advertising increases the marginal utility of tobacco by about one-fifth of one per cent. The marginal utility of tobacco appears to be inelastic, but since these estimates represent a sort of average elasticity for all tobacco products, one should expect that the marginal utility of one or more individual tobacco products would be significantly more elastic than these estimates indicate.

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APPENDIX

Appendix. Data used for computations.

Year	x_{1t}	x_{2t}	Q_t	s_{1t}	s_{2t}	n_{2t}
1926	7.09	166	494	36
1927	7.04	167	498	38
1928	7.03	164	511	40
1929	7.18	158	552	4.63	1.44	42
1930	6.91	157	500	6.08	1.50	43
1931	6.62	168	408	9.06	1.41	47
1932	6.01	188	392	9.71	1.17	53
1933	6.14	174	389	6.41	1.02	59
1934	6.54	183	426	7.34	1.22	58
1935	6.65	176	463	7.95	1.26	59
1936	7.16	173	521	9.14	1.69	60
1937	7.22	173	536	11.50	1.66	59
1938	7.17	177	500	13.87	1.48	62
1939	7.31	180	539	14.84	1.65	67
1940	7.49	183	572	17.80	1.82	73
1941	8.09	177	656	17.95	1.86	73
1942	8.63	165	743	16.15	1.70	73
1943	8.68	164	782	17.53	2.09	76
1944	8.25	165	845	17.28	2.49	72
1945	9.06	168	840	14.43	2.62	76