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USES OF
LINEAR PROGRAMMING TECHNIQUES IN THE FERTILIZER INDUSTRY

by

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TABLE OF CONTENTS

Page

INTRODUCTION

Much has been written during the past two hundred years relating population growth to the apparently inelastic supply of food. Since its postulation in 1789, the Malthusian theory has been reintroduced sporadically to outline the dangers associated with the growing or potential gap between food supply and population. The consensus of many recent writings is that certain underfed regions of the world will face disaster by 1975 unless large increases in agricultural production are forthcoming.

The magnitude of the problem is indicated by Ewell, for example, who estimated that world population will increase by 1.2 billion, or 35 percent, by 1980 (16).

Coleman has pointed out that future food needs will be greatest in areas least able to meet their present requirements (10). Generally, these areas also have high rates of population growth. How can they meet even larger food requirements in the future?

While birth control to achieve a better balance of food and people is the only long-range solution, other alternatives may offer more immediate opportunity for meeting minimum nutritional standards.

The traditional approach--bringing more land into cultivation--may be helpful in some areas. Coleman noted, however, that this approach has limited potential. No additional large acreages of highly productive land are available.

Another possibility is for countries with a highly developed agriculture to produce food for export to food-deficient nations. At best, this is a short-term answer. Exporting countries are limited as to the amount they can produce in excess of their own needs. Also, the food-deficient nations typically are severely restricted in their ability to generate the foreign exchange necessary for extended large-scale importation. Most, for example, are not sufficiently developed industrially to produce for export the quantities of high-value manufactured products which would be required to offset large food imports. Still another factor in the so-called less developed nations is the inadequacy of transportation and marketing systems for handling large tonnages of imported foods, particularly perishable products.

Thus, it appears that, if the rapidly expanding food needs of developing countries are to be met, much of the increased production must come from land now tilled in those countries.

The intensification of agriculture in many nations with consequent sizeable gains in production per unit area in recent years is closely related to greater use of fertilizers. Western Europe and Japan are leaders in this respect, applying as much as 450 pounds of plant nutrients per acre. The U.S. rate is much less, about 40 pounds per acre. This level, however, represents an almost fourfold increase since the end of World War II and is a major factor behind this Nation's agricultural abundance. In contrast, India averages only about two pounds of plant nutrients per acre (42).

In the United States, farmers have been educated to the advantages of increasing production by substituting fertilizer and other capital resources for land and labor. Prices for farm products generally have been sufficiently high to give a good return on capital invested in fertilizers. Also, unit prices of plant nutrients in recent years actually have declined while costs of other major inputs continued to rise. These changes have made substitution of fertilizers for other inputs increasingly profitable.

Among others, Carpentier has explored the possibility of increased fertilizer production in developing countries and in the economically well-developed countries (6). He concluded that greater use of fertilizers could, at least in the short run, reduce the threat of famine in underfed areas of the world.

One of the many obstacles to a rapid expansion in plant nutrient use in developing nations is the fact that returns to the fertilizer investment often are not so attractive as in most developed nations. While this may be due in part to government pricing policies in regard to farm products, it also is due in part to the high cost of fertilizer at the point of use. Thus, innovations which help reduce the cost of plant nutrients to farmers should encourage greater food production by food-deficient nations.

The cost of fertilizer at the farm is the sum of many costs--raw materials, manufacturing, transportation, distribution, profit margins at different points, etc. To minimize the final cost, the combination of the input costs must be minimized for given situations. With multiple alternatives for each input, and with variations in the cost of each from one situation to another, the number of choices in the total fertilizer production and marketing system is far too great to be reckoned with quickly and effectively on a "best judgment" basis.

On the other hand, great strides have been made in recent years in developing applications of mathematics which reduce such complex problems to more manageable proportions. This study explores the potential usefulness of linear programming techniques in analysis of ways to reduce costs in the fertilizer production and marketing system.

OBJECTIVES

The first objective is to review some of the important aspects of the world fertilizer industry as it has developed and as it exists today. This will be done separately for the nitrogen, phosphate, and potash industries. A basic knowledge of the industry should provide a better framework within which applications of mathematical approaches can be assessed.

The second objective is to analyze earlier applications of linear programming as it has been used to help develop least-cost formulations of mixed fertilizers.

The third objective is to develop a practical linear programming model for application to the operation of large fertilizer complexes. Such plants can produce many products from a few inputs. They face technological restrictions. Relative demand for the different products may fluctuate widely, either seasonally or over a period of a few years. Marginal income per ton differs greatly among products. Yet, within specified supply and demand requirements, the linear programming technique can be useful in determining plans which minimize costs or maximize profits.

The fourth objective is to investigate a linear programming model which can simulate the entire fertilizer industry of a nation. Such a model should be particularly useful to developing nations by suggesting how they can build at minimum cost the fertilizer industry--type of plants, location, product mix, etc.--which will meet their needs.

REVIEW OF LITERATURE

The simplex method of linear programming is a fairly recent development, and literature on its use by the fertilizer industry is limited.

The technique was developed in the mid-1950's. Economists quickly made adaptations of the tool to help solve various types of problems (8, 18, pp. 53-108). Peterson and Swanson at the University of Illinois made an early adaptation of the linear programming technique to fertilizer blending problems (30, 38, 39). Heady and Candler developed detailed instructions for general use of the simplex method of linear programming (18). Others have applied the method to problems of feed mixing, which are similar to fertilizer problems (24).

In 1958, the Tennessee Valley Authority and the University of Nebraska initiated a project to investigate the practical use of this mathematical tool within the fertilizer industry. The simplex method was used to analyze field operations of one fertilizer distribution firm. The system was developed to establish minimum cost formulations for both the grades and ratios which might be required by the business.

Results of the joint project showed that such an analysis can be quite useful in minimizing costs of raw materials to be used by fertilizer blenders. It also showed that such a system could be adapted to determine least-cost grades and ratios at the farm level as well as at the operating plant. Such information obviously would be of value to a dealer since he could use it to increase throughput of fixed operating facilities and thus reduce costs per ton.

The project resulted in an unpublished thesis, "Economic Analysis of Bulk-Blended Fertilizers," submitted by John I. Bucy to the University of

Nebraska in 1961. In addition, an experiment station bulletin and two semi-technical publications were written describing the study and its implications (11, 14, 17). These publications suggested that linear programming techniques could be used to help lower the cost of plant nutrients to farmers and at the same time provide information useful to management in establishing pricing policies and absolute price levels, especially on new multinutrient fertilizer materials.

THE FERTILIZER INDUSTRY

Nitrogen fertilizers

Of the three primary plant nutrients, nitrogen is used in the greatest quantity. U.S. consumption in 1965-66 was about 5.3 million tons of nitrogen (N), double the level seven years earlier and eight times as much as was used in 1946 (13, p. 32). World consumption totaled nearly 14 million short tons in 1963, compared with 7 million tons in 1955. World consumption was around 17 million short tons in 1965.

Use of nitrogen is increasing faster than that of other nutrients. There are three major reasons for this: the dramatic crop responses produced by nitrogen; development of more efficient production and marketing methods which help reduce the price at the farm; and the relatively unlimited natural resources for the production of nitrogen.

Crop response to nitrogen varies widely by crops and soils. Where soils have been reduced to a low fertility level, as in India, response of food grains (rice, wheat, millet, maize) to 20 to 40 pounds of nitrogen is 50 to 100 percent greater than to similar phosphate (P_2O_5) applications (41). The differential between responses to nitrogen and potash (K_2O) usually is even greater.

Even in the United States, where soils generally are maintained at higher fertility levels, response to nitrogen is dramatic. Demonstrations by educational agencies and the fertilizer industry continually produce increases in crop value several times greater than the cost of the fertilizer.

Before the advent of synthetic ammonia, sources of fertilizer nitrogen were few and relatively expensive. Major commercial materials were guano, Chilean nitrate of soda, and tobacco stems and other organic sources. These materials were relatively low in nitrogen content and often not found in or near major consuming areas. Thus, handling and transportation increased the cost to the point that the farmer had little incentive to use nitrogen. In fact, nitrogen for many years was the most expensive major nutrient.

With the introduction of low-cost synthetic ammonia, the picture was changed. Nitrogen suddenly became one of the lowest cost fertilizer nutrients and much more attractive in comparison with other inputs, such as land and labor.

Role of ammonia

Ammonia can now be produced economically almost anywhere in the world. Nitrogen for ammonia production is readily available from the air. Hydrogen to combine with nitrogen comes from a variety of feedstocks, including natural gas, crude oil, coal, lignite, naphtha, and even electrolytic hydrogen. Table 1 shows the relative importance of each source. Little use is made of refinery or coke oven gas, the major source of hydrogen 25 or 30 years ago.

Where large supplies of low-cost natural gas are available, as in the United States, this usually is the most economical hydrogen feedstock. Countries with large oil refineries have as a byproduct usable amounts of tail gas, which has low alternative value. Even lack of suitable local feedstocks may not preclude local ammonia production; recent technological developments have made it economical to use low-cost imported naphtha.

Coal and lignite still are used in some areas, but new facilities are based on other feedstocks in view of problems associated with use of these solids.

Table 1. Feedstocks used in the production of ammonia in 1964 (40).

Region	Hydrocarbon	Electrolytic	Coal	Other
		(% of total feedstocks)		
United States	95	4	1	-
Japan	56	10	33	1
Western Europe	41	6	51	2
Other free world	63	10	27	-
Total free world	57	6	36	1

The effect of improved ammonia technology is evidenced clearly in production and consumption data. In 1953, ammonia distribution in the United States totaled 2.3 million tons. About two-thirds was used for fertilizer. By 1963-64, production had reached 5.8 million tons of nitrogen (44, 45, 46). Agricultural consumption data indicate that ammonia accounted for 4.3 million tons of fertilizer nitrogen, or about 75 percent of all ammonia. Thus, not only has agriculture greatly enlarged its use of ammonia, but it also has enlarged its share of the total.

The availability of large amounts of anhydrous ammonia and nitrogen solutions also has helped change the pattern of application of nitrogen to the soil (Table 2). In 1950, approximately equal amounts of nitrogen were applied in mixed fertilizers and by direct application of nitrogenous materials. By 1964, direct application materials accounted for twice as much nitrogen as mixtures.

A major factor behind this change is the inherent cost advantage of using an intermediate product over one which incurs additional costs through additional processing. Development of application equipment suitable for use with various crops and the growth in custom application services by the industry also helped the trend.

Anhydrous ammonia now is the dominant source of nitrogen for direct application. The 1963 tonnage was nearly twice that for ammonium nitrate, the leading solid nitrogen fertilizer in the United States (Table 2). If recent trends continue, nitrogen applied as direct application ammonia will equal or exceed the total amount of nitrogen used in mixed fertilizers before 1970.

Ammonia also is used to make various liquid and solid nitrogen and multinutrient fertilizers.

Ammonium nitrate

Ammonium nitrate was introduced as a fertilizer during World War II, primarily as an emergency measure to boost food production. In 1943, two plants were producing solid ammonium nitrate fertilizer--the Hercules, California, plant of the Hercules Powder Company and TVA's Muscle Shoals, Alabama, plant. By the end of the war, the product had been accepted by many farmers. Demand grew rapidly, and ammonium nitrate soon became a leading nitrogen fertilizer.

Ammonium nitrate can be used as a solid fertilizer or as a solution for either direct application or manufacturing of mixed fertilizers. The

Table 2. Types of nitrogen fertilizers consumed in the United States (28, 31, 32, 33, 34, 35, 36, 37)

Year	Nitrogen used in mixtures	Nitrogen used as straight materials for direct application						
		Total[a]	Anhydrous ammonia	Aqua ammonia	Ammonium nitrate	Nitrogen solutions	Ammonium sulfate	Urea
	(Tons of nitrogen)							
1945	336,749	265,767	44,969	-	52,111	-	20,108	9,450
1950	467,009	488,630	70,123	5,540[b]	187,460	-	37,678	8,253
1955	770,945	1,126,402	287,922	73,520[b]	373,645	-	93,701	27,036
1960	983,414	1,702,158	581,436	76,776	412,295	194,740	106,908	61,762
1961	1,035,527	1,937,904	666,228	77,878	445,553	292,566	110,102	86,638
1962	1,107,603	2,195,742	766,668	91,834	467,564	371,378	111,893	120,592
1963	1,224,686	2,644,826	974,746	106,598	495,761	483,273	138,039	152,837
1964	1,348,279	2,993,698	N.A.	N.A.	N.A.	N.A.	N.A.	N.A.

[a]Includes sodium nitrate and other forms of material. Materials not based on ammonia production.

[b]Includes nitrogen solutions.

combined annual tonnage now exceeds 4 million tons of material (Table 3). The solid form still accounts for slightly more than half of the total, but use of solution is increasing more rapidly. During the 1959-64 period, solution use increased 168 percent while solid increased 90 percent. By far the largest share of ammonium nitrate solution is used in manufacturing--88 percent in 1963, according to U.S. Bureau of the Census data (50, 51, 52, 53).

Table 3. U.S. production of ammonium nitrate for use as solid and solutions (51, 52, 53)

Year	Solid	Solutions
	(Tons of material)	
1955	1,137,132	704,469
1956	1,178,310	715,262
1957	1,456,288	782,288
1958	1,521,132	847,829
1959	1,[illegible]5,463	1,109,408
1960	1,572,064	1,252,638
1961	1,653,430	1,286,995
1962	1,668,215	1,378,338
1963	1,926,843	1,647,475
1964	2,162,745	1,865,051

Ammonium sulfate

Ammonium sulfate is one of the older nitrogen fertilizers. It is made from coke oven byproduct ammonia or from synthetic ammonia. Its relatively low analysis, with consequent high distribution cost per unit of nitrogen, has kept ammonium sulfate from becoming a major source of nitrogen in the United States. Even so, substantial tonnages are used for direct application and for manufacturing. Large quantities also are exported. Ammonium sulfate is relatively more important in other parts of the world.

Urea

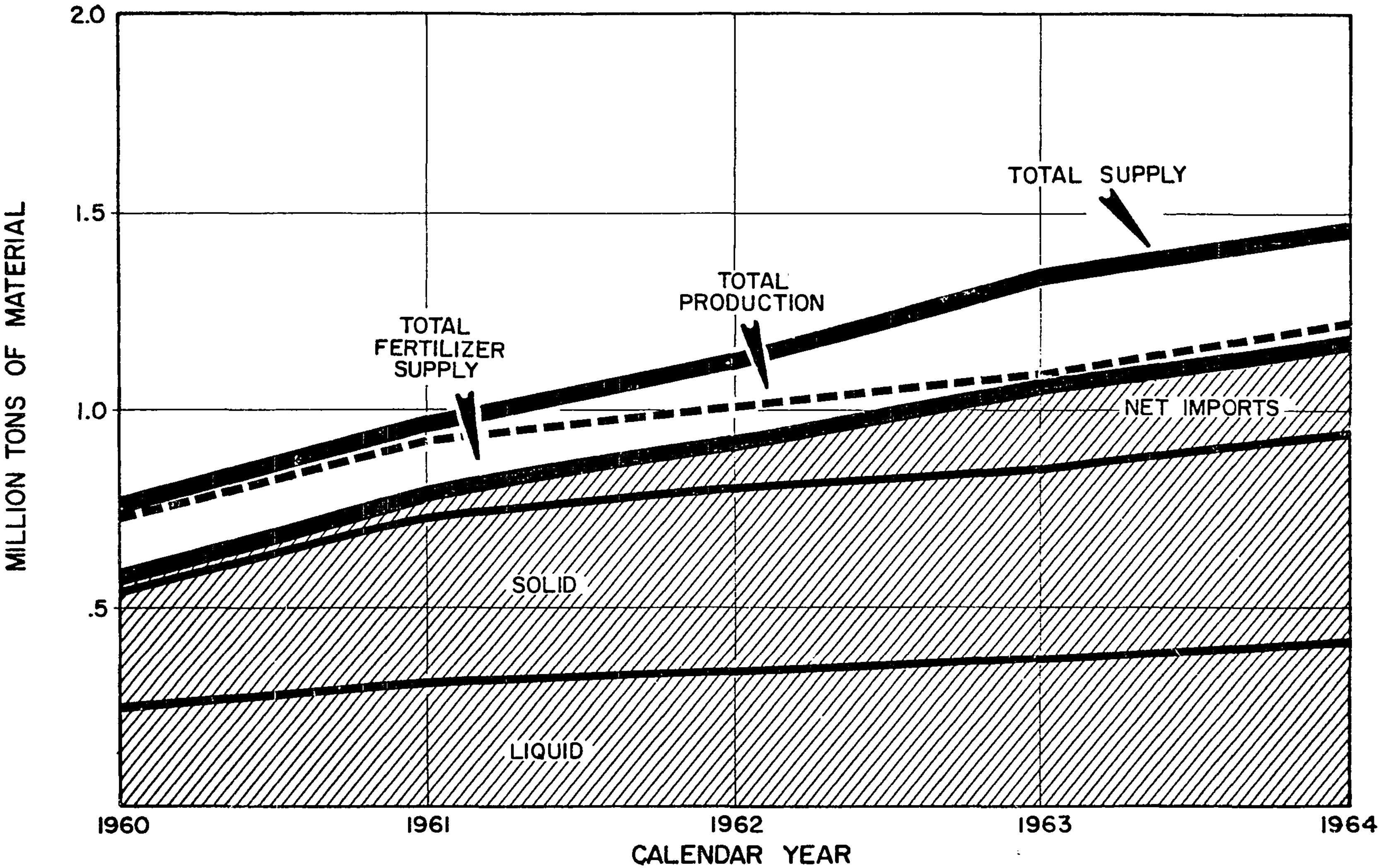
Although urea has long been known to be a good nitrogen fertilizer, its high production cost until recently severely limited its use by farmers. Breakthroughs in production technology during the late 1950's lowered this cost and at the same time resulted in a purer product. By the early 1960's, fertilizer urea was competitive in price with solid ammonium nitrate in the United States.

Use of urea has since grown rapidly (Figure 1). The total supply almost doubled from 1960 to 1964. Production climbed from 0.75 to 1.2 million tons of material, and net imports rose from a negligible amount to about 200,000 tons.

Nitrogen solutions

Prior to the mid-1950's, available nitrogen solutions were either high-pressure or low-analysis materials. Neither characteristic favored their use. With reductions in urea prices, however, it became possible to produce at attractive prices high-analysis straight nitrogen solutions of either low-pressure or nonpressure types. As a result, the tonnage used for

Figure 1. Urea supply situation (60, 61, 63)


TOTAL SUPPLY
TOTAL PRODUCTION
TOTAL FERTILIZER SUPPLY
NET IMPORTS
SOLID
LIQUID
2.0
1.5
1.0
.5
MILLION TONS OF MATERIAL
1960
1961
1962
1963
1964
CALENDAR YEAR

direct application soared from 109,000 in 1955 to 1,493,000 in 1963 (36). Average analysis was 31.5 percent nitrogen.

Multinutrient fertilizers

In addition to straight nitrogen fertilizers, the new sources of nitrogen which have come into common use in recent years also have been widely utilized in the manufacture of multinutrient fertilizers. Perhaps diammonium phosphate, in which ammonia is the nitrogen source, is the outstanding example. Since the early 1950's, use in the United States has increased from a few thousand to more than 3 million tons of material per year.

Diammonium phosphate (21-53-0) was first made with electric-furnace phosphoric acid. Engineering developments in the late 1950's by TVA opened the way to make a slightly lower analysis product (e.g., 16-48-0 or 18-46-0) from the more economical wet-process phosphoric acid. This, together with the realization by some acid producers that additional outlets were needed to maintain near-capacity operation of the many new plants coming on stream, provided the impetus for rapid market development. TVA studies indicate that diammonium phosphate capacity likely will continue to increase considerably faster than the total U.S. fertilizer market (13).

Although nitric phosphates are a major fertilizer in Europe, only a half dozen plants are operating in the United States. Finished products generally have been low in analysis and relatively costly to produce. However, recent trends to higher costs of sulfur and sulfuric acid have renewed interest in nitric phosphates, which require less sulfur than other phosphates.

Other products in various stages of development may be prominent nitrogen fertilizers in the years ahead. Potassium nitrate (13-0-44), for example, has only recently been produced commercially in the United States. Although relatively high priced, it may find fairly wide use where both potash and nitrogen are needed in large amounts and where the chloride ion is undesirable.

The introduction of solid ammonium polyphosphate (15-60-0) was initiated recently by TVA. Urea - ammonium phosphate, also under development by TVA, looks promising for use in this country and in other parts of the world, particularly with flooded rice. It can be manufactured in a variety of high-analysis grades and ratios.

World nitrogen fertilizer requirements

Numerous projections have been made of the amount of nitrogen which will be needed in the future. Most estimates have been based on the amount required to feed adequately the expanding population.

Carpentier predicted that developing countries alone would require 8 million tons of nitrogen by 1970 (3). Cascino suggested 1980 requirements for the major nutrients of 113 million tons (7). Parker cites a Food and Agriculture Organization estimate of 58 million short tons of plant nutrient by 1970 and 88 million by 1980, a major portion of which would be nitrogen (29). Ewell arrived at a combined total of 110 million tons needed by 1980 by extrapolating 1948-63 growth rate curves (15). He indicated that 55 million tons of this would be nitrogen. Ewell cautioned that pronounced acceleration in fertilizer use growth rates in developing countries could result in even higher totals.

U.S. nitrogen production capacity

Unpublished compilations by TVA indicate that by mid-1966 rated capacity of 92 U.S. anhydrous ammonia plants totaled 11.2 million tons of product. Completion on schedule of announced expansions or new plants would raise capacity to 18 million tons in 112 plants by the end of 1968.

The United States had 50 ammonium nitrate plants with a rated capacity exceeding 5.4 million tons of material by July 1966. Sixteen of the plants produce only solutions, while the others produce both solid and solution forms of ammonium nitrate. Announced new plant capacity would increase the total by less than 0.5 million tons through 1967.

Thirty-three urea plants with capacity of 2 million tons of material were operational in July 1966. Nine expansions or new plants were scheduled for operation before the end of the year, bringing capacity to 2.5 million tons, and three plants have been announced for completion during 1967 or 1968.

Ammonium sulfate production historically has been tied closely to the availability of byproduct ammonia from coke ovens or to spent sulfuric acid from such sources as oil refineries. Substantial quantities have been made in recent years from synthetic ammonia and new sulfur, but rising sulfur prices and the popularity of higher analysis nitrogen materials will tend to restrict further increases in ammonium sulfate production.

World nitrogen production capacity

Recent studies by TVA in conjunction with the United States Agency for International Development indicate present and anticipated world nitrogen productive capacity (Table 4). Capacity in July 1965 was 28.9 million short tons of nitrogen (N). Anticipated 1970 capacity is about 51 million tons, an increase of 77 percent.

Table 4. World nitrogen production, capacity, and anticipated production (27)

Region	Production 1963	Estimated capacity 1965	Anticipated capacity 1970
		(Thousand short tons)	
Western Europe	5,148	10,370	14,080
Eastern Europe	2,418	4,687	9,137
Africa	182	423	1,534
Asia	2,132	4,511	8,833
Oceania	29	86	179
Latin America	400	1,185	2,594
North America	4,116	7,680	14,919
Total	14,425	28,942	51,276

Most of the capacity, 78 percent in 1965 and 74 percent of that anticipated for 1970, is in Europe and North America. Thus, there is a great disparity between where nitrogen is produced and where food shortages are most critical.

Even within regions there are major imbalances. For example, most of Asia's nitrogen is produced in Japan, which exports half of its output, while India does not produce nearly enough for its needs.

World trade in nitrogen

Nitrogen exports increased from 2.5 million short tons of N in 1958 to more than 3.3 million in 1963. This represents about one-fourth of total production.

Table 5. Trade in nitrogen fertilizers 1958-1963 (42)

Region	1958	1959	1960	1961	1962	1963
	(Thousand short tons of nitrogen)					
	Exports					
Europe	1,630	1,698	1,994	2,047	2,255	2,146
North and Central America	421	418	384	441	450	418
South America	255	279	240	134	236	231
Asia	282	337	375	326	445	541
Africa	-	-	-	-	1	4
Oceania	-	3	4	3	2	2
	Imports					
Europe	747	884	974	920	936	1,102
North and Central America	535	539	542	531	639	634
South America	77	99	99	131	105	119
Asia	540	573	703	871	887	948
Africa	231	262	180	270	243	244
Oceania	17	19	10	12	19	25

Table 5 shows world trade during the 6-year period by regions. In 1963, Europe accounted for 64 percent of all nitrogen exports. Its tonnage increased by 500,000 tons from 1958 to 1963. Japan almost doubled its exports, from 282,000 to 541,000 tons.

It is interesting to note the increase in movement within regions, particularly Europe. Also, Asia increased its imports most, whether measured by tons or as a percentage.

Unpublished studies by the Distribution Economics Section of TVA indicate that substantial changes have been made in types of nitrogen products entering world trade. From 1958 to 1963, urea increased its share of the world market from 10 percent to 18 percent. Complex fertilizers increased from 6 percent to 10 percent. On the other hand, ammonium sulfate's share declined from 41 percent to 35 percent in the same period.

Phosphate fertilizers

Phosphate is the plant nutrient produced in second largest tonnage, both in the United States and worldwide. World consumption of phosphate has increased rapidly in recent years--from 7.5 million tons of P_2O_5 in 1955 to nearly 13.5 million tons in 1965.

The developed regions of the world are by far the heaviest users of phosphate. They accounted for over 12 million tons of P_2O_5, or 88 percent of the total, in 1965. During the 1955-65 period, developed regions increased consumption by nearly 5 million tons, while the increase in developing areas was much less (Table 6).

As with nitrogen, significant changes have occurred in types of phosphate fertilizers produced. One strong trend has been toward higher analysis materials. Another, particularly in the United States, has been a shift to use of multinutrient fertilizers.

Table 6. World P_2O_5 consumption and estimate of use in 1970 (9, 42)

Region	P_2O_5 consumption				Percentage change		
	Fiscal year						
	1955	1960	1965	1970	1955-1960	1960-1965	1965-1970
Western Europe	2,884	3,548	4,257	5,185	23	20	22
Eastern Europe	1,063	1,592	2,672	4,325	50	68	62
North America	2,283	2,560	3,425	4,492	12	34	31
Oceania	649	733	1,136	1,528	13	55	24
Africa	210	260	373	672	24	43	80
Asia	411	681	1,082	1,498	66	59	38
Latin America	162	224	537	682	38	40	27
World total	7,662	9,598	13,474	18,382	25	40	36

Normal superphosphate

For many years normal superphosphate was the most important phosphate fertilizer. Its relative importance has declined rapidly, however, as more concentrated materials gain in popularity. In the United States, for example, production of normal superphosphate declined from nearly 1.5 million tons of P_2O_5 in 1956 to about 1.1 million tons (Figure 2).

Normal superphosphate accounted for 54 percent of the world P_2O_5 production in 1958 but had declined to 47 percent in 1964 (Table 7).

Figure 2. Production of phosphate products (54, 55, 56, 57, 58, 59)

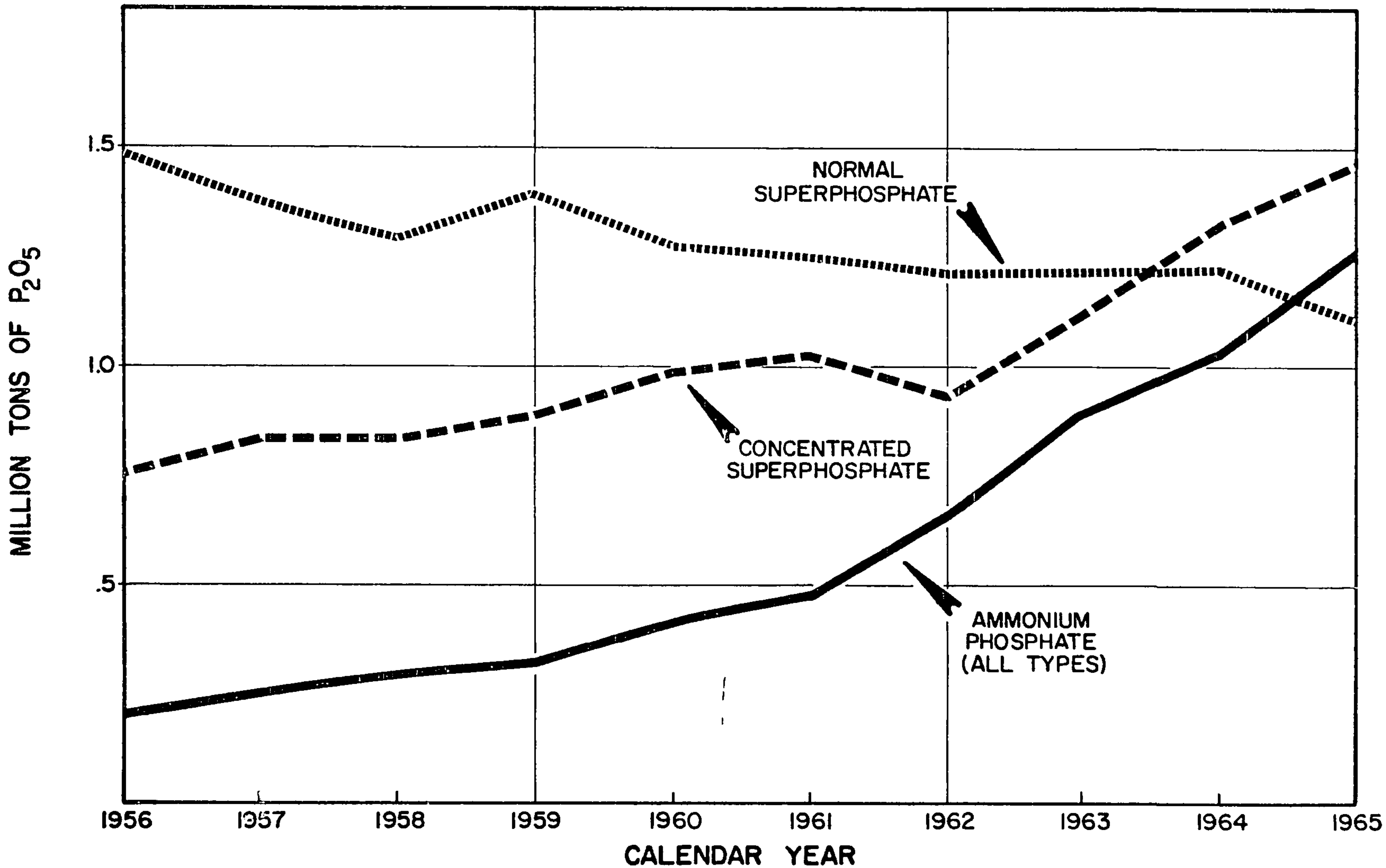

NORMAL SUPERPHOSPHATE
CONCENTRATED SUPERPHOSPHATE
AMMONIUM PHOSPHATE (ALL TYPES)
MILLION TONS OF P_2O_5
1.5
1.0
.5
1956
1957
1958
1959
1960
1961
1962
1963
1964
1965
CALENDAR YEAR

Table 7. Relative contribution of phosphate products to world production (42)

Fiscal year	Normal superphosphate	Concentrated superphosphate	Basic slag	Complex fertilizers[a]
		(Percent of total)		
1958	54	13	16	17
1959	53	14	15	18
1960	52	13	16	19
1961	50	14	15	21
1962	49	14	14	23
1963	48	13	14	25
1964	47	14	13	26

[a]Includes complex fertilizers, ammonium phosphates, other phosphatic fertilizers, and organics.

Concentrated superphosphate

Production of concentrated superphosphate in the United States almost doubled in the past decade. In 1956, production totaled about 0.75 million tons of P_2O_5, or about half as much P_2O_5 as was supplied in normal superphosphate. By 1965, production of concentrated superphosphate was approaching 1.5 million tons of P_2O_5, well in excess of that now supplied as normal superphosphate.

Worldwide, concentrated superphosphate has maintained its relative position, accounting for 14 percent of total output of P_2O_5.

Multinutrient products

The output of phosphate in multinutrient fertilizers has increased rapidly in recent years. In the United States this has been largely in the form of ammonium phosphates, production of which climbed from around 200,000 tons of P_2O_5 in 1956 to 1.2 million tons in 1965.

Complex fertilizers also have grown in importance throughout the world. In 1958 they accounted for 17 percent of the total phosphate production. By 1964 their share had increased to 26 percent. Much of these complex fertilizers is ammonium phosphate, but a substantial portion consists of nitric phosphates, which have become a relatively important product in Europe. They will gain in popularity elsewhere if prices of sulfur, which is used in phosphoric acid production, continue to rise. A technological breakthrough in production of nitric phosphates is needed, however, before they would have wide appeal at today's price levels.

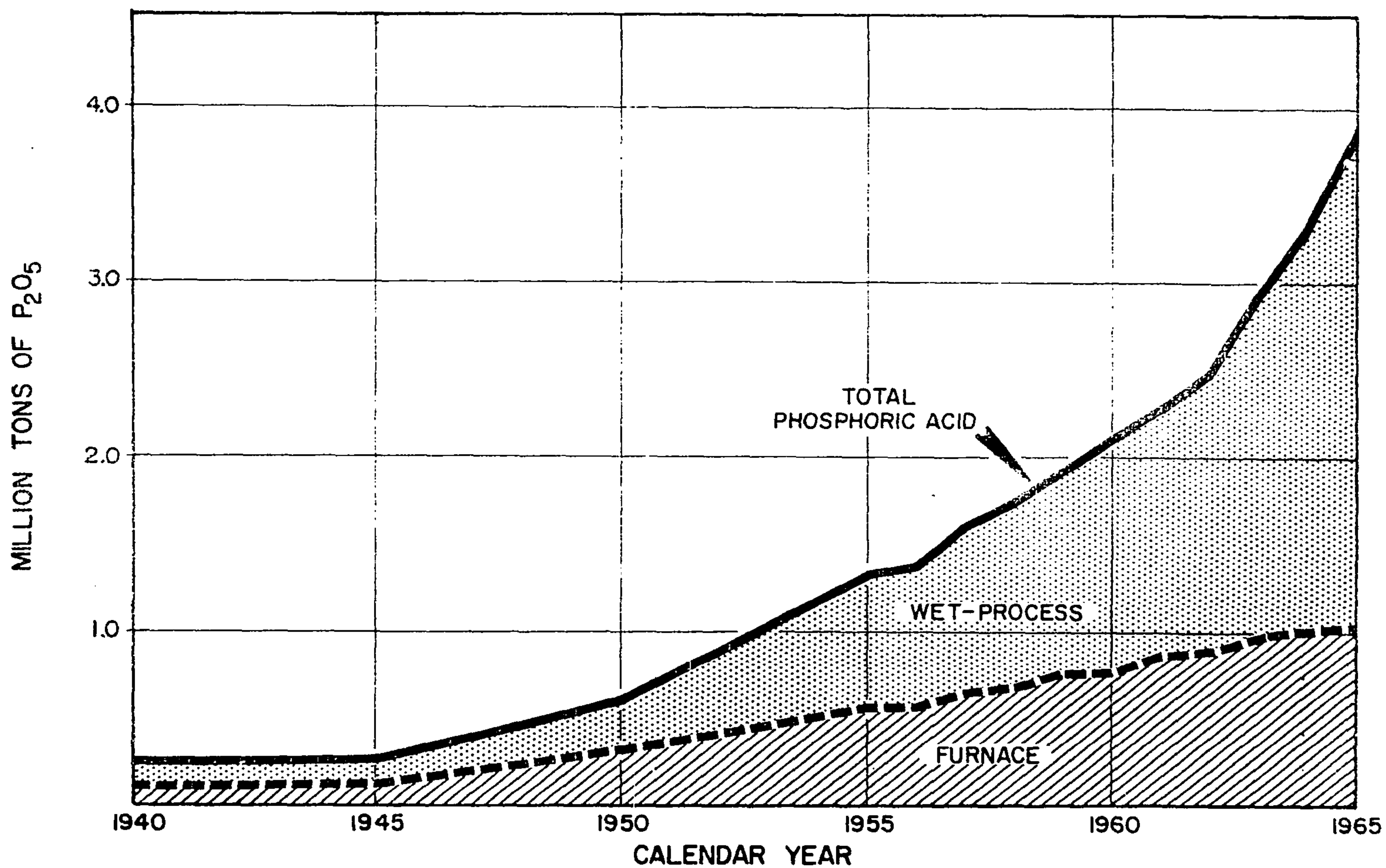
Basic slag

This industrial byproduct no longer is important as a fertilizer in the United States. However, its share of the world's phosphate market has declined only slightly in recent years, from 16 percent in 1958 to 13 percent in 1964. It remains an important fertilizer in Europe.

Phosphoric acid

With the introduction of triple superphosphate in 1940 and diammonium phosphate in the late 1950's and early 1960's, phosphoric acid has become the major intermediate product used by the U.S. phosphate fertilizer industry. Figure 3 shows the growth of phosphoric acid production in the United States. While the amount of electric-furnace acid used in fertilizers

Figure 3. Phosphoric acid production (47, 48, 49, 50, 51, 52, 53)



4.0
3.0
2.0
1.0
MILLION TONS OF P_2O_5
TOTAL PHOSPHORIC ACID
WET-PROCESS
FURNACE
1940
1945
1950
1955
1960
1965
CALENDAR YEAR

Table 8. Estimated world wet-process phosphoric acid capacity and announced future capacity by 1970 (27)

Region	Capacity 1965	Announced future capacity by 1970	Total planned capacity by 1970	No. concentrated superphosphate plants Present	No. concentrated superphosphate plants Future[a]	No. ammonium phosphate plants Present	No. ammonium phosphate plants Future[a]
	(Metric tons P_2O_5)						
Western Europe	1,443,000	1,412,000	2,855,000	30	2	27	7
Eastern Europe[b]	N.A.	380,000	380,000	2	2	-	2
Africa	270,000	531,000	801,000	4	7	1	2
Near East	17,000	114,000	131,000	1	2	-	-
Far East[c]	359,000	479,000	838,000	4	1	26	8
Oceania	96,000	57,000	153,000	2	2	-	1
Latin America	77,000	433,000	510,000	2	6	-	1
North America	3,180,000	2,518,000	5,698,000	19	10	38	33
Total	5,442,000	5,924,000	11,366,000	64	32	92	54

[a]Includes expansions.

[b]Does not include Russia.

[c]Does not include Mainland China.

has increased substantially, production of wet-process phosphoric acid has increased to a much greater extent. Total production has doubled in the last six years.

Table 8 summarizes world capacity to produce wet-process phosphoric acid. Capacity in 1965 exceeded 5.4 million metric tons of P_2O_5. By 1970 it is expected to approach 11.3 million tons, a gain of 108 percent. Production will continue to be centered in the developed regions; however, the developing countries will increase their share of the total P_2O_5 produced as wet-process phosphoric acid.

World phosphate requirements

Numerous individuals and organizations have estimated future phosphate "requirements," "needs," and "demands." The estimates vary widely, partly because of underlying assumptions. For example, some are based on the amount which would be needed to provide a minimum diet for an expanded world population, while others are based on what the estimators saw as a "realistic demand."

Among the estimates based on what would be needed to maintain a satisfactory diet, Carpentier predicted that developing countries alone will require about 4 million tons of P_2O_5 by 1970 (6). Ewell extrapolated the 1948-63 rate of increase in fertilizer use and estimated a 1980 worldwide requirement of 25 million tons of P_2O_5 (15).

Coleman, taking what he termed a "realistic demand" approach, estimated that consumption of phosphate fertilizers will increase 36 percent between 1965 and 1970, to more than 18 million tons of P_2O_5 (10). His estimate, however, did not include Albania, Mainland China, and other Communist countries in Asia for which statistics are not available.

Supply of phosphatic materials

World production of phosphate rock is increasing rapidly. The 1955-59 production was estimated at an average of 34 million metric tons per year. Production in 1964 was 59 million metric tons (25).

Table 9. World production of phosphate rock[a] (25)

Region	1960	1961	1962	1963	1964	1970 range
			Rock production			
			(Thousand metric tons)			
North America	17,938	19,038	19,854	20,331	23,465	35,000- 40,000
South America	1,073	852	800	509	511	-[b]
Russia	7,000	8,799	10,008	11,003	12,995	20,000- 25,000
Africa	11,216	11,967	12,395	14,045	15,694	20,000- 25,000
Near East	586	643	668	711	618	-[b]
Pacific Islands[c]	2,624	2,737	2,652	2,935	3,344	-[b]
Other areas	1,392	1,460	1,700	1,865	2,210	5,000- 10,000
Total	41,829	45,496	48,077	51,399	58,837	80,000-100,000

[a]1970 estimates are taken from unpublished records of the Distribution Economics Section of Tennessee Valley Authority, Muscle Shoals, Alabama.

[b]Included in other areas.

[c]Includes Christmas Island.

The United States supplied about 40 percent of the 1964 production; Africa, 25 percent; and Russia, between 16 and 22 percent. Table 9 shows regional production of phosphate rock for the past five years. It also shows estimated production for 1970 at 80,000 to 100,000 tons. Even at the maximum expected rate of increased demand, it appears that phosphate rock producers can readily meet the needs.

U.S. producers of phosphate rock in 1965 had capacity to produce 27 million metric tons. An additional 10 million tons of productive capacity is under development, plus at least one other major project for which no capacity figure has been announced.

North Africa has large reserves of phosphate rock. Production has increased steadily in recent years, and reported plans indicate further expansion, perhaps at an accelerated rate. Morocco, for example, expects to increase capacity by at least 1 million tons per year through 1970. Increases also have been announced by Israel, Jordan, Tunisia, and Senegal.

Russia's phosphate rock production is estimated at 13 million metric tons in 1964 and is expected to reach 25 million tons by 1970.

In addition to these three major sources of phosphate rock, increased production is likely in several other areas. Table 10 gives an indication of known phosphate rock reserves.

World phosphate production capacity

World capacity for fertilizer phosphate production at the end of 1965 was almost 18 million metric tons of P_2O_5 (27). Plans have been announced for expansions in capacity totaling about 9 million tons by 1970, excluding "targets" set by Russia and Mainland China. Including these targets results in a capacity of more than 31 million tons by 1970. Much of this capacity will be located far from the area in which the fertilizers will be used.

Table 10. World reserves of phosphate rock (4)

Region	Reported reserves[a]
	(Million metric tons)
North America	13,742
South America	1,531
Russia	7,689
Africa	47,004
Near East	919
Pacific Islands[b]	185
Other areas	681
Total	71,566

[a]Includes apatite and all grades of rock.

[b]Includes Christmas Island.

World trade in phosphates

In 1963 a total of 21 million metric tons of phosphate rock moved into world trade, with over 13 million tons imported by Europe. Asia received more than 3 million, while 2.5 million tons moved into Australia and New Zealand (3, 20, 21, 22, 23, 25).

Morocco is the leading exporter of phosphate rock, with more than 10 million metric tons in 1964. The United States was second, with 5.7 million tons, followed by Tunisia with 2.2 million tons. Several other countries export small quantities.

The United States also exports considerable tonnages of finished phosphate fertilizers. Normal superphosphate exports during the 1950's averaged 250,000 tons of material per year, but have recently declined to about 100,000 tons. On the other hand, exports of ammonium phosphates have climbed sharply, reaching 400,000 tons of material in 1964. Exports of concentrated superphosphate have increased consistently--from less than 34,000 tons in 1950 to 600,000 tons in 1964. The 1964 exports represent about 15 percent of U.S. production (60).

Europe also is a major exporter of finished phosphate fertilizers, moving 840,000 tons of P_2O_5 in 1963. However, it is estimated that 55 percent of the movement was within the continent. North America is the second largest exporting region, but accounts for only one-third as much as Europe. Nevertheless, the United States has overseas exports of nearly twice as much P_2O_5 as all of Western Europe; only 15 percent of the U.S. exports moves within the North American continent.

Table 11 shows the relative importance and trends of various phosphate products in world trade. Normal superphosphate and basic slag have declined in relative importance. The decline would be much more pronounced if data were based on "overseas shipment." On the other hand, high-analysis materials are becoming more important in world trade. Since 1958 concentrated superphosphate has increased its share from 26 percent to 30 percent of total world exports. The share held by complex fertilizers has climbed even more, from 16 percent to 28 percent. Almost all U.S. exports are high-analysis materials. This trend is also developing in other exporting countries.

Table 11. Relative importance of phosphate fertilizers in world trade (42)

Fiscal year	Normal superphosphate	Concentrated superphosphate	Basic slag	Complex fertilizers[a]
		(Percent of total)		
1958	20	26	38	16
1959	19	25	36	20
1960	20	22	37	21
1961	20	26	29	25
1962	18	29	28	25
1963	16	27	30	27
1964	14	30	28	28

[a]Includes ammonium phosphate, other complex fertilizers, organics, and other phosphatic materials.

Potash Fertilizers

Production and use of potash have increased consistently in recent years, though not nearly so rapidly as nitrogen and phosphates. Between 1956 and 1965, potash use increased from 6.8 million tons of K_2O to more than 11 million tons. World consumption has been estimated to exceed 14 million tons by 1970 (9).

Table 12. World consumption of potash and estimated consumption for 1970 (9, 42)

Year	Regions							Total
	Western Europe	Eastern Europe	North America	Oceania	Africa	Asia	Latin America	
			(Thousand metric tons K_2O)					
1956	2,708	1,609	1,807	45	65	467	126	6,827
1957	2,873	1,675	1,834	41	62	562	139	7,186
1958	3,020	1,830	1,832	66	72	425	145	7,390
1959	3,112	1,854	2,083	60	78	541	166	7,894
1960	3,277	1,919	2,038	70	85	663	165	8,217
1961	3,312	1,890	2,067	94	102	769	234	8,468
1962	3,679	1,652	2,166	130	104	682	195	8,608
1963	3,644	1,980	2,378	115	108	686	184	9,095
1964	4,112	2,225	2,568	132	132	874	336	10,379
1965	4,044	2,856	2,769	155	167	850	343	11,184
1970	5,056	3,400	3,757	201	227	1,155	564	14,360

Table 12 shows world use of potash by regions. Generally, trends in use of potash follow those of nitrogen and phosphate. Europe and North America remain the major consumers, with the United States using the major share of the North American total.

Potash products

Potassium chloride is by far the leading carrier of this important plant nutrient. It is used in dry blends and in the manufacture of two- and three-component mixed fertilizers.

Within the past two or three years, one plant has come on stream in the United States for production of potassium nitrate, a new type of fertilizer. Though more expensive than equivalent amounts of nitrogen and potassium from other sources, potassium nitrate has competed well as a specialty product. It is used primarily on crops which cannot tolerate chlorine, such as tobacco and some vegetables. To become a major fertilizer, however, potassium nitrate would have to be produced at lower cost.

Potassium metaphosphate is another possible new product. The economics of production and distribution, however, indicate that it could be economically manufactured only at points where both phosphate and potash deposits occur.

Another fertilizer possibility is potassium hydroxide--an excellent material, but generally too costly for extensive use as a fertilizer.

World potash capacity

World capacity to produce potash fertilizers in mid-1965 is estimated at more than 13.8 million tons K_2O. This is expected to increase to 22.8 million tons by 1970. Of this total, North America would account for 11.5 million tons; Western Europe, 5.4; and Eastern Europe, 4.6 (27, 42).

Potash has generally been discovered and produced only in the developed nations. The developing nations of the world have few known reserves. The summary of the known plans for production of potash in the developing nations indicates total productive capacity by 1970 of only 1.3 million tons (42).

Thus, the fertilizer industries of these developing nations will continue to rely primarily upon imported potash to supply their fertilizer requirements.

SOME APPLICATIONS OF LINEAR PROGRAMMING TECHNIQUES TO FERTILIZER DISTRIBUTION

Need for advanced techniques

In the early days of the fertilizer industry a mixer or dealer worked with a few simple, single-nutrient materials. A typical inventory might have included normal superphosphate (0-20-0), ammonium sulfate (20-0-0), nitrate of soda (16-0-0), and potash (0-0-60). Determining the most economical combinations of materials to produce a specified ratio or grade was very simple. First, he determined which of the two nitrogen sources was cheaper. Then, he calculated the amounts of the nitrogen, phosphate, and potash materials needed.

The advent of new and more complex materials since the end of World War II has made it much more difficult to compute least-cost formulations. In addition to the older materials, the dealer now must consider several high-analysis straight materials--such as ammonium nitrate (33.5-0-0), urea (45-0-0), and concentrated superphosphates (0-46-0 and 0-54-0). Also, a host of multinutrient materials are available--such as the ammonium phosphates (13-39-0, 11-48-0, 18-46-0, 21-53-0, and 30-10-0) and others.

For example, if a dealer wanted to make a 15-15-15 grade product from two ammonium phosphates--18-46-0 and 30-10-0--and potash (0-0-60), he essentially must solve the following simultaneous equations:

(1) $.30X + .18Y = 300$ (to ensure 15 units nitrogen)

(2) $.10X + .46Y = 300$ (to ensure 15 units phosphate)

(3) $.60Z = 300$ (to ensure 15 units potash)

(4) $X + Y + Z + W = 2{,}000$ (to ensure a ton weight)

Where:

X = amount of 30-10-0 grade ammonium phosphate

Y = amount of 18-46-0 grade ammonium phosphate

Z = amount of 0-0-60 grade potash

W - amount of 0-0-0 grade filler

The solution shows that he needs 500 pounds of 18-46-0, 700 pounds of 30-10-0, and 500 pounds of potash--plus 300 pounds of filler. However, all he knows at this point is that this is one of the many alternative ways to mix a 15-15-15 grade. It may or may not be the most economical mix. Only an extensive period of trial and error can provide this answer.

Basic linear programming application on least-cost mixes

A basic program has been developed for use on the IBM 704 (later converted to IBM 360) computer. It essentially is the simplest useful program possible. The program consists of five equations establishing "requirements" or "activity levels." Space is available for up to 30 real activities, together with the necessary slack vectors for each activity level to prevent both overfulfillment and underfulfillment.

The algebra of this system is as follows:

Restrictions

(1) $\sum n_i X_i \geq$ (to be inserted for each problem)

(2) $\sum p_i X_i \geq$ (to be inserted for each problem)

(3) $\sum k_i X_i \geq$ (to be inserted for each problem)

(4) $\sum w_i X_i \geq$ (to be inserted for each problem)

(5) $\sum z_i X_i \geq$ (to be inserted for each problem)

Subject to minimization function

(6)
$$\sum_{i=1}^{n} P_i X_i = \text{minimum}$$

In this algebraic system, X_i represents the many possible grades of raw materials available for any one location. The amount of nitrogen contained in each X_i is n_i; p_i represents the amount of phosphate; k_i, the amount of potash; w_i, the total weight requirements; and z_i, the amount of some secondary plant nutrient, such as sulfur. P_i designates the price per pound of each X_i.

Thus, to determine the least-cost mixture for a 15-15-15 grade of fertilizer, the equations might--after conversion to equalities--be:

Restrictions

(7) $.335X_1 + .21X_2 + .45X_3 + .30X_4 + .21X_5 + .16X_6 + .18X_7 + 14X_{12} =$ 300 (pounds nitrogen required in final mix)

(8) $.10X_4 + .53X_5 + .48X_6 + .46X_7 + .20X_8 + .46X_9 + .54X_{10} = 300$ (pounds phosphate required in final mix)

(9) $.60X_{11} + .44X_{12} = 300$ (pounds potash required in final mix)

(10)
$$\sum_{i=1}^{12} X_i = 2{,}000$$

Minimization function

(11)
$$\sum_{i=1}^{12} P_i X_i = \text{minimum}$$

Where:

P_i = price per ton of each X_i

X_1 = ammonium nitrate (33.5-0-0)

X_2 = ammonium sulfate (21-0-0)

X_3 = urea (45-0-0)

X_4 = ammonium phosphate nitrate (30-10-0)

X_5 = diammonium phosphate (21-53-0)

X_6 = ammonium phosphate (16-48-0)

X_7 = ammonium phosphate (18-46-0)

X_8 = normal superphosphate (0-20-0)

X_9 = concentrated superphosphate (0-46-0)

X_{10} = triple superphosphate (0-54-0)

X_{11} = potassium chloride (0-0-60)

X_{12} = potassium nitrate (14-0-44)

Various subroutines were designed for the machine operation to make it easier to evaluate results. These subroutines produce printout solutions which fit an 8-1/2- by 11-inch page.

A special program was designed for use for those cases wherein it is desired to arrive at the least-cost ratio of plant nutrients, such as a 1-1-1, without reference to grade. This program is essentially the same as that for least-cost grades except that the equation expressing the total weight limitation is eliminated and requirements are expressed in terms of units (i.e., 20 pounds) of plant nutrients. Also, a special subroutine is inserted to convert the final product mix to a ton and thus express the answer as a grade even though this was not the original objective. Thus, the following equation would be used to arrive at a 1-1-1 ratio of fertilizer.

Restrictive equations to arrive at least-cost mix for desired ratio

(12) $n_i X_i = 20$

(13) $p_i X_i = 20$

(14) $k_i X_i = 20$

Minimization function

Same as Equation 11

Subroutine equation to convert to a ton basis

(15) $$\frac{2000}{\sum_{i=1}^{n} X_i} = \text{jack-up factor to convert ratio to grade}$$

One example is presented to indicate the multiple practical uses for this application. Input data for this problem are shown in Table 13. Several materials are available commercially; others, however, are products under development. They are included in order to determine their "equilibrium" price in relation to existing materials. Also, many possible input materials were deleted prior to the computer run because previous runs had shown them to be completely out of the competitive price range.

The printout solution (Figure 4) indicates that at existing prices of raw materials

1. A 1-1-1 ratio can be most economically mixed using 642.32 pounds of ammonium nitrate, 768.50 pounds of 18-46-0 grade ammonium phosphate, and 589.18 pounds of potash.
2. Total cost of the raw materials will be $66.84.
3. The grade produced will be 17.68-17.68-17.68.

In addition, the column headed "Substitution Cost Per Ton" shows the extent to which each material not used in the least-cost mix was overpriced. For example, 30-10-0 was overpriced by $3.18 per ton for use at this location in this particular ratio.

Table 13. Input data used in least-cost example

Materials available	Grade	Delivered price/pound
1. Ammonium phosphate	30-10-0	$0.036875
2. Ammonium phosphate	29-14-0	.038750
3. Ammonium nitrate	33.5-0-0	.033500
4. Ammonium phosphate	18-46-0	.042300
5. Concentrated superphosphate	0-46-0	.029200
6. Urea - ammonium phosphate	33-20-0	.103350[a]
7. Urea - ammonium phosphate	25-35-0	.103350[a]
8. Ammonium polyphosphate	16-60-0	.103350[a]
9. Urea	45-0-0	.103350[a]
10. Nitric phosphates	20-20-0	.103350[a]
11. Combination fertilizers	17-17-17	.103350[a]
12. Combination fertilizers	15-15-15	.103350[a]
13. Diammonium phosphate	21-53-0	.103350[a]
14. Ammonium phosphate	25-25-0	.103350[a]
15. Potash	0-0-60	.021745

[a]Inserted as "dummy" prices in order to aid in making evaluation of the economic potential of these new or different products.

Figure 4. Example of IBM printout solution to least-cost mix problem

1-1-1 RATIO MACON, GEORGIA

OPTIMAL SOLUTION ON ROW 1 1

	BETA	
0	-0.18906775	1
0	0.00000000	2
6	1.81700195	3
7	2.17391303	4
19	1.66666666	5

X 6 = 642.32 LB	21.52	COST	33.5- 0. - 0.
X 7 = 768.50 LB	32.51	COST	18.0-46.0 - 0.
X 19 = 589.18 LB	12.81	COST	0. - 0. -60.0
X 0 = 0. LB	0.	COST	0. - 0. - 0.
X 00 = 0. LB	0.	COST	0. - 0. - 0.
2000.00 LB	66.84	TOTAL COST	

NITROGEN	PHOSPHORUS	POTASH
17.68	17.68	17.68

Z(J)-C(J),	(DELTA J)	SUBSTITUTION COST-PER TON	ANALYSIS
1	0.	0.	0. - 0. - 0.
2	0.	0.	0. - 0. - 0.
3	0.	0.	0. - 0. - 0.
4	0.001592	3.18	30.0-10.0- 0.
5	0.002354	4.71	29.0-14.0- 0.
6	-0.000000	-0.00	33.5- 0. - 0.
7	-0.000000	-0.00	18.0-46.0- 0.
8	0.004900	9.80	0. -46.0- 0.
9	0.059785	119.57	33.0-20.0- 0.
10	0.059861	119.72	25.0-35.0- 0.
11	0.055654	111.31	16.0-60.0- 0.
12	0.058350	116.70	45.0- 0. - 0.
13	0.072785	145.57	20.0-20.0- 0.
14	0.071208	142.42	17.0-17.0-17.0
15	0.074990	149.98	15.0-15.0-15.0
16	0.054352	108.70	21.0-53.0- 0.
17	0.065143	130.29	25.0-25.0- 0.
18	0.000000	0.00	0. - 0. -60.0

<u>The linear programming technique as a pricing tool</u>

A complicating factor in the realm of pricing is the different systems of pricing used for nitrogen, phosphate, and potash.

Phosphates are priced per unit of P_2O_5 f.o.b. Tampa, Florida, reflecting the fact that most phosphate fertilizer used in the United States is mined and processed, at least to some extent, in that area. For a similar reason, prices of potassic fertilizers are on a f.o.b. Carlsbad, New Mexico, basis.

Prices of nitrogen, however, have been based on an entirely different pattern for the past 10 years. Nitrogen fertilizer materials are produced at widely scattered points throughout the United States. As a result, instead of following a basing point pricing system similar to that used with phosphates and potash, producers have established a combination basing point-delivered price system. For instance, in 1965 the price per ton of ammonium nitrate was $65 f.o.b. nearest producing point or $67 per ton delivered to any location east of the Rockies. West of the Rockies the prices were based upon delivered prices to the various areas, regardless of where the material was produced.

Most multinutrient materials are chemical combinations of N and P_2O_5. In general, prices are based on the system for the nutrient present in largest quantity, or a combination of the nitrogen and phosphate pricing systems.

In this dilemma, linear programming can be used as a management tool for pricing new fertilizers. In using the linear programming system as an aid to analysis of pricing, the $z(j) - c(j)$ statistic evolving from the final iteration of machine operation (sometimes called the "shadow price") is analyzed. This shadow price indicates the price at which a certain grade of new fertilizer material would be competitive with the least-cost combination of all other materials used in the particular matrix.

To evaluate the pricing of a proposed new fertilizer material in relation to existing products on the market, the following procedure is followed:

1. Determine the end products in which the new material will probably be used, such as a 1-1-1 ratio.
2. Prepare least-cost matrices using as real activities all input factors which are currently available.
3. Insert the proposed new fertilizer material into the least-cost matrix at a specified artificially high cost--at some level so high as to ensure that it does not become a portion of the least-cost mix for the end product desired.
4. After solution of the linear programming problems, use the shadow price as the equilibrium price of the proposed new fertilizer in relation to the existing prices of other fertilizers already on the market and available at the various locations in which the new product may be useful.

The example chosen for analysis is an actual problem facing TVA in the summer of 1966.

As a result of the many least-cost programs which have been run, it is known that the most economical sources of commercially produced fertilizers consist of four multinutrient products, 30-10-0, 29-14-0, 18-46-0, and 16-48-0, and three single-nutrient sources, 33.5-0-0, 0-46-0, and 0-0-60. Therefore, the matrix can be shortened to include only these seven real materials plus those potential new products for which it is desired that pricing be evaluated.

There are nine theoretical products for pricing evaluation. Among these are three potential new products--34-17-0, 25-35-0, and 15-60-0. There are also two grades of nitric phosphates, 20-20-0 and 26-13-0, which are so new that pricing levels are still uncertain. In addition, there are four old products which are expected to remain as a part of the product mix for at least another year. On these it is desired to analyze the existing prices in order to determine whether they are at the proper level, i.e., not so high as to force them out of the market and not so low as to underprice all commercial products by a large amount.

In such an approach to pricing there must be an evaluation at different geographical locations. For this example, 13 locations have been used as evaluation points covering the United States east of the Rockies.

In practice it has been found that shadow prices on fertilizer materials may vary at one location, depending upon the end product desired. Thus, if the end product desired is a 1-4-4 ratio, a material containing high levels of P_2O_5, such as 15-16-0, may be more valuable than the same material if it is to be used in producing a 2-1-1 ratio. Therefore, in all such pricing evaluations an attempt is made to determine the highest N to P ratio and the highest P to N ratio which may be used generally. For purposes of the example, the following ratios were programmed at each location: 2-1-1 and 1-4-4.

Table 14. Estimated delivered prices/pound to retail dealers of fertilizer materials spring 1967

Grade	Macon, Ga.	Shef-field, Ala.	Murray, Ky.	St. Louis, Mo.	Grin-nell, Iowa	Minne-apolis, Minn.	Omaha, Nebr.
30-10-0	.037375	.037375	.037375	.037375	.037375	.037375	.037375
29-14-0	.040500	.040500	.040500	.040500	.040500	.040500	.040500
33.5-0-0	.030150	.030150	.030150	.030150	.030150	.030150	.030150
18-46-0	.043450	.044650	.045070	.045630	.046900	.047500	.047300
16-48-0	.043450	.044650	.045070	.045630	.046900	.047500	.047300
0-46-0	.032650	.033750	.034170	.034730	.036000	.036600	.036400
0-0-60	.022345	.021855	.021855	.020935	.020985	.020935	.020935
<u>Others</u> (each to be run at same price/pound for pricing evaluation)							
34-17-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950
25-35-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950
20-20-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950
26-13-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950
15-60-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950
30-10-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950
25-25-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950
21-53-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950
0-54-0	.103350	.100645	.101950	.103350	.104650	.105250	.104950

Table 14 (Continued)

Grade	El Paso, Tex.	Houston, Tex.	Lake Placid, N. Y.	Roanoke, Va.	Walnut Ridge, Ark.	Law-rence, Kan.
30-10-0	.037375	.037375	.037375	.037375	.037375	.037375
29-14-0	.040500	.040500	.040500	.040500	.040500	.040500
33.5-0-0	.030150	.030150	.030150	.030150	.030150	.030150
18-46-0	.048400	.045630	.047625	.044850	.045150	.046700
16-48-0	.048400	.045630	.047625	.044850	.045150	.046700
0-46-0	.037500	.034730	.036725	.033450	.034250	.035800
0-0-60	.016350	.018500	.023165	.023165	.020935	.020935
<u>Others</u> (each to be run at same price/pound for pricing evaluation)						
34-17-0	.106800	.104750	.106475	.104250	.102350	.104650
25-35-0	.106800	.104750	.106475	.104250	.102350	.104650
20-20-0	.106800	.104750	.106475	.104250	.102350	.104650
26-13-0	.106800	.104750	.106475	.104250	.102350	.104650
15-60-0	.106800	.104750	.106475	.104250	.102350	.104650
30-10-0	.106800	.104750	.106475	.104250	.102350	.104650
25-25-0	.106800	.104750	.106475	.104250	.102350	.104650
21-53-0	.106800	.104750	.106475	.104250	.102350	.104650
0-54-0	.106800	.104750	.106475	.104250	.102350	.104650

Table 14 gives the required input coefficients for use in the price evaluation program.

For each of the 13 different locations, two different linear programming solutions are required--one for each of the ratios.

Table 15 is a summary of the shadow or equilibrium prices of the nine different fertilizer materials at each of the 13 different locations.

At any single location a new material may, and usually does, have more than one shadow price. Its shadow or equilibrium price will vary depending upon other possible input factors and upon the final ratio required as an end product. When evaluating the pricing of a proposed new material, it is important to assess the material itself before deciding which of the equilibrium prices to accept. If the material is one which is high in nitrogen and low in phosphorus, such as 34-17-0, the equilibrium price of the 2-1-1 ratio is used in evaluation of pricing policy and levels, and vice versa.

As the program has been set up on this particular example, these equilibrium prices represent the price levels f.o.b. Sheffield, Alabama, which would result in equilibrium delivered prices to each specific location. It is equally possible to set up the program in such a way as to arrive at the proper level at delivered location. And, of course, from either the f.o.b. price or the delivered price the other can be computed.

As noted before, fertilizers are generally priced in one of three ways: (1) an f.o.b. price (basing point), (2) an area delivered price (same price to all locations in one area), or (3) some combination of the two. In case there are area delivered prices or f.o.b. prices for more than one area, it is normal to have the smallest possible number of areas--for administrative simplicity.

Table 15. Equilibrium prices f.o.b. Sheffield, Alabama, for various grades fertilizer for use at 13 different locations

Ratios of materials	Macon, Ga.	Shef-field, Ala.	Murray, Ky.	St. Louis, Mo.	Grin-nell, Iowa	Omaha, Nebr.	Lake Placid, N. Y.
34-17-0							
*2-1-1	$74.64	$80.94	$78.64	$76.25	$74.59	$74.29	$72.41
1-4-4	57.30	63.74	61.37	58.88	56.98	56.60	50.12
25-35-0							
*2-1-1	79.77	87.00	85.03	83.08	82.42	82.43	80.36
1-4-4	72.30	79.55	77.52	75.49	74.65	74.61	70.47
20-20-0							
*2-1-1	53.00	59.45	57.20	54.89	54.30	53.14	49.37
1-4-4	45.14	51.64	49.35	46.98	45.35	45.05	39.15
26-13-0							
*2-1-1	55.50	61.59	59.22	56.73	54.85	54.48	51.15
1-4-4	42.24	48.44	46.01	43.45	41.39	40.96	34.10
15-60-0							
2-1-1	91.39	99.93	98.41	97.07	97.79	98.23	96.28
*1-4-4	96.06	104.44	102.89	101.52	102.12	102.53	101.62
30-10-0							
*2-1-1	59.15	65.08	62.65	60.10	58.05	57.62	54.69
1-4-4	42.67	48.76	46.26	43.61	41.36	40.87	33.58
25-25-0							
*2-1-1	67.92	74.63	72.48	70.29	69.07	68.90	66.20
1-4-4	58.10	64.87	62.66	60.39	59.00	58.78	53.42
21-53-0							
2-1-1	93.89	102.07	100.43	98.92	99.24	99.57	98.06
*1-4-4	93.16	101.24	99.55	97.98	98.16	98.44	96.57
0-54-0							
2-1-1	57.28	65.51	63.88	62.40	62.78	63.12	58.55
*1-4-4	69.96	77.95	76.33	74.84	75.22	75.56	74.14

*Shadow price used for evaluation purposes.

Table 15 (Continued)

Ratios of materials	Roanoke, Va.	Walnut Ridge, Ark.	Minne-apolis, Minn.	Lawrence, Kan.	El Paso, Tex.	Houston, Tex.
			34-17-0			
*2-1-1	$73.88	$77.90	$72.83	$74.44	$71.40	$73.45
1-4-4	58.49	60.61	56.11	56.87	53.51	56.08
			25-35-0			
*2-1-1	80.10	84.35	82.13	82.11	80.40	80.28
1-4-4	73.48	76.83	74.29	74.38	72.43	72.69
			20-20-0			
*2-1-1	52.41	56.47	52.72	53.22	50.40	52.09
1-4-4	45.45	48.61	44.60	45.19	42.18	44.17
			26-13-0			
*2-1-1	54.49	58.46	53.99	54.74	51.40	53.93
1-4-4	42.73	45.24	40.44	41.30	37.72	40.65
			15-60-0			
2-1-1	93.24	97.82	98.15	97.27	97.40	94.27
*1-4-4	97.41	102.29	102.43	101.61	101.61	98.71
			30-10-0			
*2-1-1	57.96	61.89	57.11	57.96	54.40	57.30
1-4-4	43.34	45.48	40.32	41.30	37.47	40.81
			25-25-0			
*2-1-1	67.64	71.77	68.52	68.86	66.40	67.40
1-4-4	58.94	61.94	58.37	58.81	56.13	57.59
			21-53-0			
2-1-1	95.32	99.81	99.43	98.76	98.40	96.12
*1-4-4	94.69	98.93	98.27	97.72	97.15	95.18
			0-54-0			
2-1-1	58.77	63.27	62.99	62.31	62.00	59.60
*1-4-4	70.03	75.71	75.43	74.75	74.44	72.04

Table 16. Equilibrium delivered prices

Location	34-17-0	25-35-0	20-20-0	26-13-0	15-60-0	30-10-0	25-25-0	21-53-0	0-54-0
Macon, Ga.	$81.34	$86.47	$59.70	$62.20	$102.76	$65.85	$74.62	$ 99.86	$76.66
Sheffield, Ala.	82.23	88.29	60.74	62.88	105.73	66.37	75.92	102.53	79.24
Murray, Ky.	82.54	88.93	61.10	63.12	106.79	66.55	76.38	103.45	80.23
St. Louis, Mo.	82.95	89.78	61.59	63.43	108.22	66.80	76.99	104.68	81.54
Grinnell, Iowa	83.89	91.72	62.70	64.15	111.42	67.35	78.37	107.46	84.52
Omaha, Nebr.	84.19	92.33	63.04	64.38	112.43	67.52	78.80	108.34	85.46
Lake Placid, N. Y.	85.36	93.31	62.32	64.10	114.57	67.64	79.15	109.52	87.09
Roanoke, Va.	82.38	88.60	60.91	62.99	105.91	66.46	76.14	103.19	78.53
Walnut Ridge, Ark.	82.60	89.05	61.17	63.16	106.99	66.59	76.47	103.63	80.41
Minneapolis, Minn.	84.33	92.63	63.22	64.49	112.93	67.11	79.02	108.77	85.93
Lawrence, Kan.	83.74	91.41	62.52	64.04	110.91	67.26	78.15	107.02	84.05
El Paso, Tex.	85.00	94.00	64.00	65.00	115.21	68.00	80.00	110.75	88.04
Houston, Tex.	82.95	89.78	61.59	63.43	108.21	66.80	76.90	104.68	81.54

Table 15 is, as noted before, based upon prices per ton f.o.b. Sheffield, Alabama, which will result in equilibrium prices to each location being studied.

Table 16, equilibrium delivered prices, is computed from Table 15. The prices in Table 16 represent the equilibrium price per ton f.o.b. Sheffield, Alabama, plus the applicable rail freight from Sheffield to each delivery point.

From Tables 15 and 16, then, geographic pricing maps are constructed for each separate material (Maps 1 and 2). These maps indicate the equilibrium price levels for a 34-17-0 grade urea-phosphate fertilizer for use in broad-scale demonstration programs in early 1967.

Map 1 shows the equilibrium delivered price of 34-17-0. Map 2 shows the price f.o.b. Sheffield, Alabama, that will result in the equilibrium price delivered to each location.

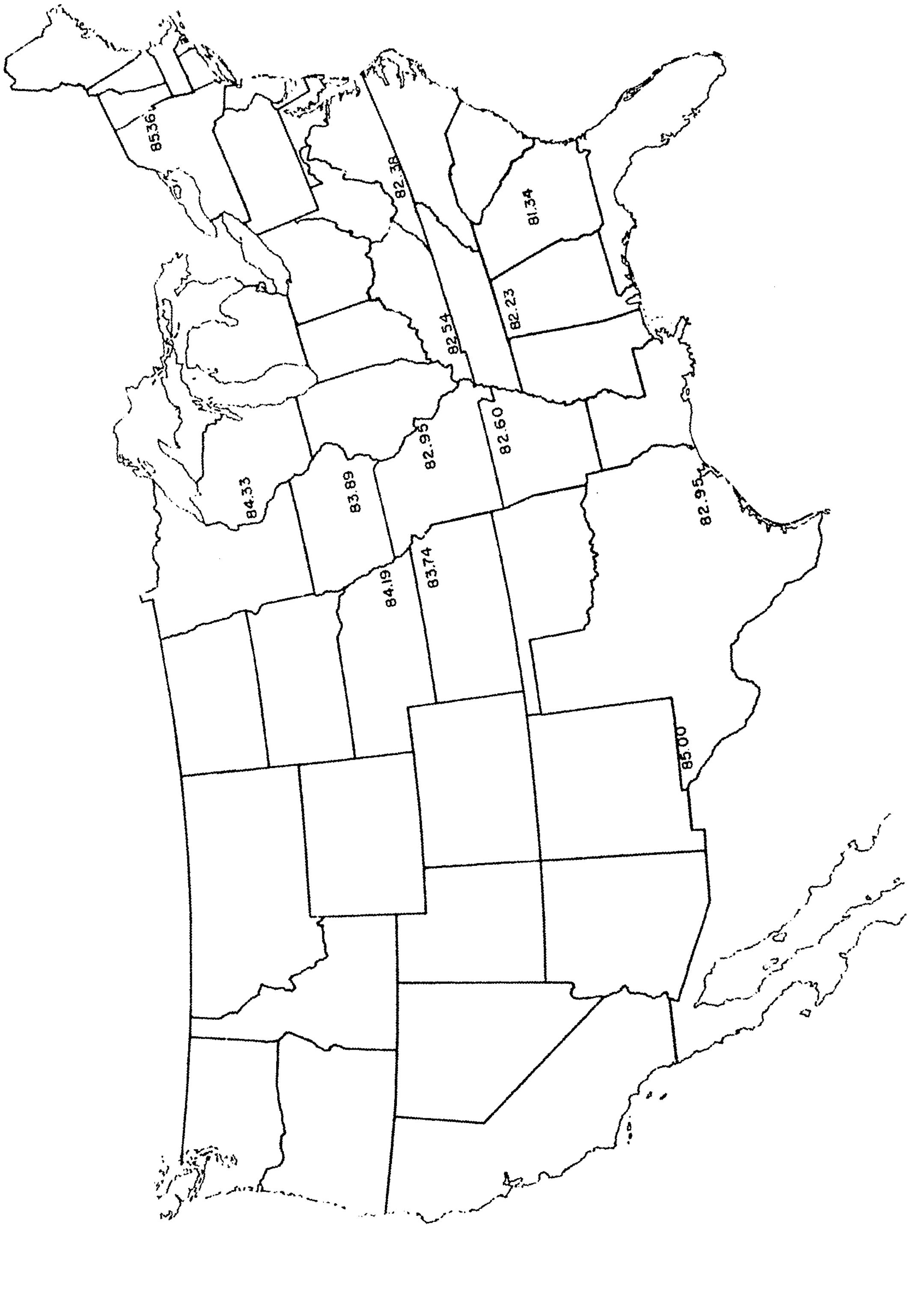
Visual analysis of these maps suggests two general conclusions. The 34-17-0 can be more easily priced on a delivered basis than on an f.o.b. Sheffield basis. The equilibrium delivered price at 13 widely separated geographic locations ranges from $81.34 to $85.36 per ton--a range of only $4.02. It is TVA policy to provide a price incentive ranging up to $5 per ton of material on a new fertilizer material such as this. Thus, one delivered price would suffice to give an incentive up to but not more than $5 per ton for all locations studied.

If priced f.o.b. Sheffield, however, and the same price incentive criteria are used, there obviously must be more than one price area. The range of equilibrium prices (Map 2) is from $71.40 f.o.b. Sheffield for delivery to El Paso, Texas, to $80.94 f.o.b. Sheffield for delivery to the immediate vicinity of Sheffield. If priced, then, on an f.o.b. basis, it would be necessary to have at least two and probably three different area prices.

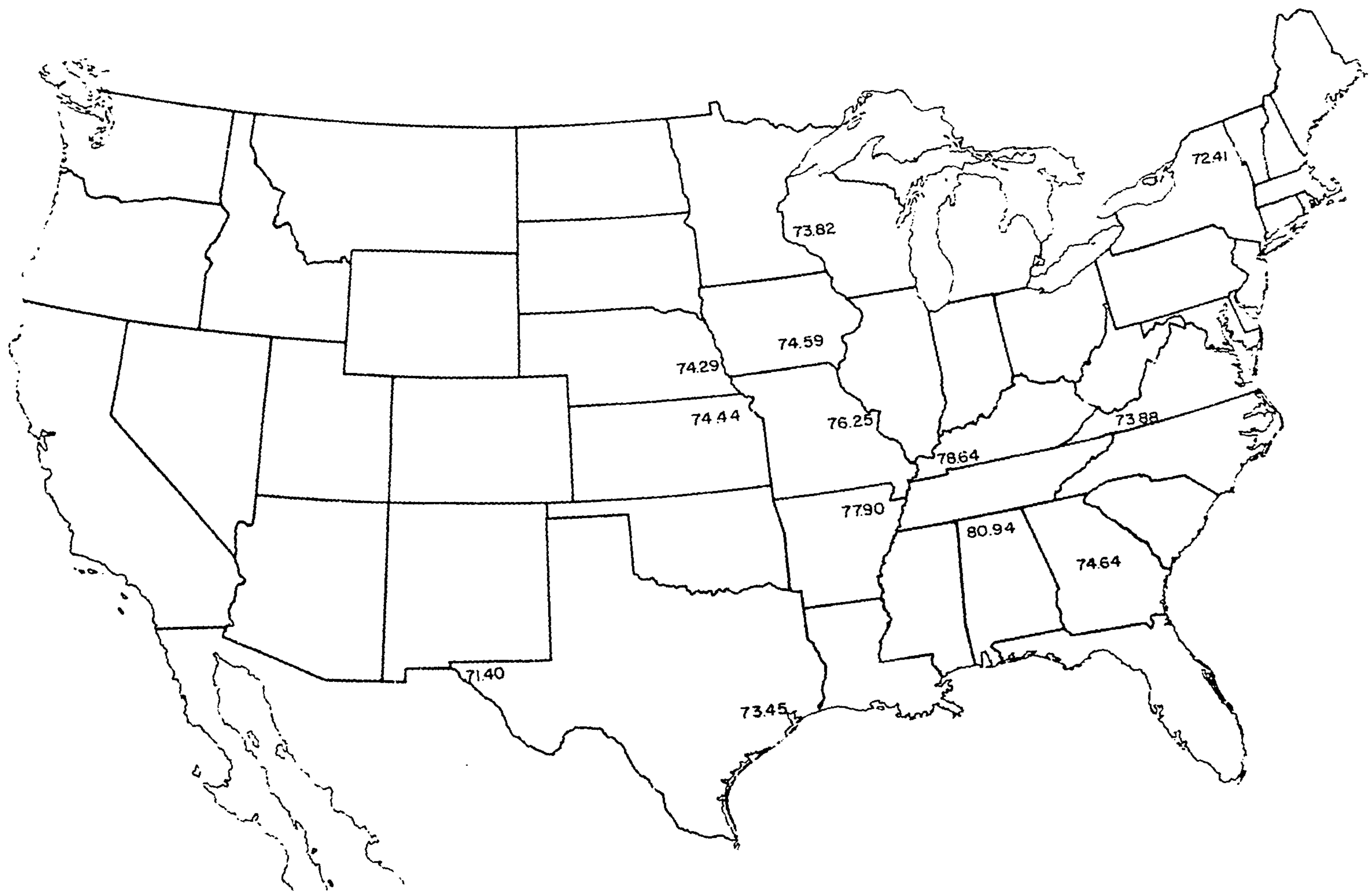
Analysis of the nine different potential fertilizers, using such a procedure, i.e., visual analysis of both the f.o.b. and the delivered price, indicates that all those with high nitrogen content probably can be priced more easily on a delivered basis. Those with higher phosphorus content probably can be priced best on an f.o.b. level, with a different price to each of two or three different areas.

Thus, using the linear programming technique as a pricing tool can be helpful to management in determining (1) whether pricing criteria can be better satisfied by a delivered price or an f.o.b. producing point concept and (2) the level at which a fertilizer should be priced to make it competitive on a price level with all other fertilizers on the market at that time.

Map 1. Computed equilibrium delivered prices, 34-17-0, spring 1967


85.36
82.36
81.34
82.54
82.23
82.60
82.95
84.33
83.89
82.95
83.74
84.19
85.00

Map 2. Computed equilibrium prices f.o.b. Sheffield, 34-17-0, spring 1967


72.41
73.82
74.59
74.29
74.44
76.25
78.64
73.88
77.90
80.94
74.64
71.40
73.45

SIMULATION OF SINGLE PLANT OPERATION

Developing need for linear programming techniques

As long as fertilizer production techniques and chemical technology encompassed production of only single-nutrient materials, such as normal superphosphate (0-20-0), it was relatively simple to decide how much of which product to produce. Normally, only one product was produced; it was produced at maximum capacity if there was a market demand for it.

With the advances of fertilizer technology, however, production scheduling problems arose. Huge chemical complexes were constructed for production of a large number of different products. A phosphate complex under the new technology might have capacity to produce 100,000 tons of P_2O_5 as phosphoric acid as an intermediate product and also have production facilities to use the entire amount of acid in any one of, or a combination of, two or more end products.

Management of a typical nitrogen complex faced even greater scheduling difficulties. The basic ammonia production facilities invariably acted as an overall bottleneck both for the total year's operation and for any one period of time. The complex plants produced many forms of nitrogen fertilizer. The ammonia in turn (1) could be sold, or (2) could be used to produce nitric acid which, in turn, would be sold or recombined with portions of the remaining ammonia to form any of many products for sale as a finished fertilizer material--either liquid or dry.

With technology advances to the stage of efficient economic production of multinutrient products--i.e., with both nitrogen and phosphoric acid being produced within the same industrial complex--the difficulties of production schedules were further multiplied. Under these circumstances, bottlenecks in the phosphoric acid facilities interacted with those within the nitrogen production facilities, and vice versa.

As long as the complex fertilizer facilities had sufficient capacity to produce the total amount of each product required, there was no major scheduling problem other than the normal one of preventing inventories from climbing too high at any one period. Under these circumstances it followed that one of two theses was in effect. Either (1) the total complex was exactly the proper-sized capacity for the existing market and each individual component part of the plant facility was sized perfectly or (2) at least some of the component parts were maintained at an excess capacity level. Usually the latter was true.

With a continually increasing and changing demand for fertilizers, however, most of the large complexes soon found themselves faced with demands for more total end product than they could produce. The question then facing their management became that of, With given production facilities and varying amounts of many different products, what product mix will lead to a maximization of revenue to the corporation?

General background of fertilizer complex

TVA was chosen as an example where an application of the general linear programming techniques might aid in policy decisions on product mix. Nitrogenous, phosphatic, and multinutrient materials are produced at its National Fertilizer Development Center at Muscle Shoals, Alabama. The facilities are used primarily to produce fertilizers, although they are available for munitions production in emergencies. The program is essentially a research and development program. Within program requirements, however, it is desirable to operate the facilities at maximum economy.

The nature of the program results in more products being produced in smaller quantities than in the average commercial production facility. Also, products change rapidly. As a new one is introduced to the fertilizer industry, an old product is deleted from the product mix.

In such a situation, linear programming can be useful to management by indicating in advance the financial results of increasing the emphasis on one product as compared with other possibilities.

In short, the fertilizer demonstration program objectives dictate the range of manufacturing operations for each specific fertilizer material. Linear programming techniques can be used--taking into account the technical limitations of facilities to produce primary, intermediate, and finished products--to determine the most economical product mix within the production parameters established by individual program requirements.

Alternative objective functions

Under the conditions described above, many objective functions could be investigated. It would be possible, for example, to attempt to maximize total gross revenue. Since a large percentage of the costs is composed of costs of raw materials, however, that attempt might well lead to a maximization of income with a corresponding maximization of costs.

It would be possible to set as an objective function the maximization of total net profit (book value). With the bookkeeping systems and allocations of plant fixed costs as they are at TVA, this approach does not necessarily lead to optimum economic results. In such a complex manufacturing facility, much of the manpower and other physical input factors are used in production of more than one product. The allocation of these costs often is accomplished on a highly subjective basis. Thus, "net returns" by specific products are not considered as adequate guides to optimizing total plant operations.

Over a period of years, however, it has been found that a modified version of marginal income analysis can be most helpful in analyzing the economic outcome of past, present, and future plant operations. Under this concept, as applied at TVA, marginal income on each product consists of gross income per ton of product less the out-of-pocket cash costs of raw materials, power, and other inputs directly attributable to that product.

Under normal conditions it would be expected that the cash costs of labor would also be deducted from income in order to arrive at marginal income. Under actual operational conditions at TVA it has been found that labor as an input is not infinitely divisible; in fact, the labor is a fixed cost over a relatively wide range of operational levels.

The physical plant facilities are such that in order to reduce labor by any appreciable amount some portion of the complex must be closed. For instance, in the phosphorus complex there are five electric furnaces. With five furnaces operational, there must be five full crews for actual furnace operations. The furnaces are supplied with raw materials from a single crew common to all. The phosphorus produced is handled by a single crew common to all furnaces. The five furnaces must operate full time for maximum efficiency, i.e., at a production level of approximately 44,000 tons elemental phosphorus per year. The furnace load can be reduced on one or all of the furnaces and total production reduced to approximately 38,000 tons.

Such a reduction in production of phosphorus does not result in a reduction of the labor force.

If a lower operational level is required, it becomes more profitable to close one furnace than to continue operation of all five. At the same time, to take this course of action leads to sharp reductions in overall economies enjoyed in comparison with the full capacity level. If one furnace is closed it is possible to reduce labor at only that one furnace. Labor and other inputs of all the flow of production processes prior to and subsequent to the actual furnace operation cannot be reduced when only one furnace is inoperative.

The "marginal income" concept, then, as applied in the present study, is useful in analyzing the economic consequences of changes in product mix within a certain level of operations. Higher production levels are not possible with current plant facilities; below the minimum levels, other cost and income data would be applicable.

But before these production (or supply) restrictions are described in detail, a mathematical simulation of the program requirements by type of product must be completed.

Demand restrictions and equations

In any such demonstration program as is carried on by TVA, there are generally no static levels of use of any single product. Instead, for any one product in any one time period, there are program requirements for a minimum level of each product. These minimum level requirements are generated in order to ensure continuity of programs, wide geographic coverage, and continuity of research and development activities.

On the other hand, there are generally certain maximum amounts of any one material which may be used for program requirements. These maximum levels are dictated by the necessity for such programs to preclude, insofar as possible, competition with commercial producers of materials with close similarity to the new and improved products of the demonstration program.

TVA now makes available 18 different products for use in agricultural programs. In addition, two of these products are also supplied to the military establishment for defense purposes.

The material supplied to the Department of the Army is priced differently from that being used in agricultural programs. The elemental phosphorus is priced higher than that going for agricultural uses since the Army specifications are much more restrictive, i.e., a higher quality phosphorus is required. The nitric acid being used by the Department of the Army for making munitions is sold at production cost--thus is lower priced than that used in agricultural programs.

Below is listed a set of 40 demand equations setting up the demand portion of the linear programming matrix for plant operation:

Demand equations

(16 & 17)	P4D $\geq$ 4,500 $\leq$ 8,000		Phosphorus to the Army
(18 & 19)	P4A $\geq$ 3,000 $\leq$ 4,000		Phosphorus to agriculture
(20 & 21)	HPO $\geq$ 7,000 $\leq$ 10,000		Phosphoric acid for agricultural use
(22 & 23)	CSP $\geq$ 15,000 $\leq$ 40,000		Concentrated superphosphate (0-54-0)
(24 & 25)	HAP $\geq$ 4,000 $\leq$ 10,000		Granulated CSP
(26 & 27)	LFA $\geq$ 50,000 $\leq$ 120,000		Liquid mixed fertilizer (11-37-0)
(28 & 29)	LFB $\geq$ 10,000 $\leq$ 20,000		Liquid mixed fertilizer (12-40-0)
(30 & 31)	NH3 $\geq$ 20,000 $\leq$ 40,000		Anhydrous ammonia (82-0-0)

(32 & 33)	NHA	$\geq$ 15,000 $\leq$	20,000	Nitric acid for agricultural use
(34 & 35)	NND	$\geq$ 2,000 $\leq$	5,000	Nitric acid for Department of Army
(36 & 37)	ANZ	$\geq$ 500 $\leq$	1,000	Ammonium nitrate with 8% zinc
(38 & 39)	ANS	$\geq$ 15,000 $\leq$	25,000	Ammonium nitrate with 5% sulfur
(40 & 41)	APN	$\geq$ 30,000 $\leq$	45,000	Ammonium phosphate nitrate (30-10-0)
(42 & 43)	PNA	$\geq$ 15,000 $\leq$	25,000	Ammonium phosphate nitrate (25-25-0)
(44 & 45)	APP	$\geq$ 3,000 $\leq$	10,000	Ammonium polyphosphate (15-60-0)
(46 & 47)	NPA	$\geq$ 6,000 $\leq$	15,000	Nitric phosphate (20-20-0)
(48 & 49)	DAP	$\geq$ 15,000 $\leq$	25,000	Diammonium phosphate (21-53-0)
(50 & 51)	CUB	$\geq$ 4,000 $\leq$	5,000	Custom blended fertilizers for special projects
(52 & 53)	NSL	$\geq$ 1,000 $\leq$	3,000	Ammonium nitrate solution
(54 & 55)	UNS	$\geq$ 1,000 $\leq$	3,000	Urea - ammonium nitrate solution

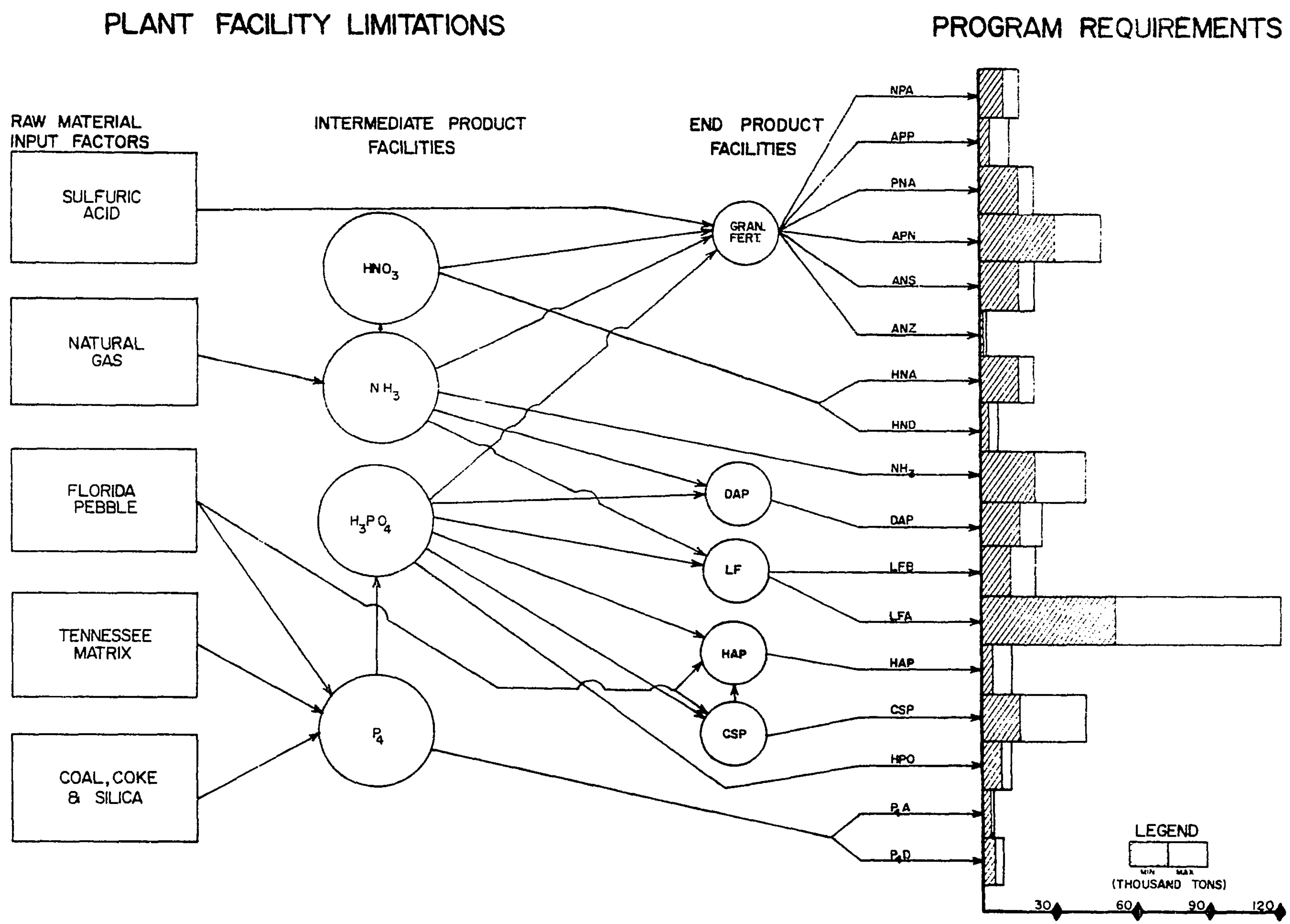
These equations assure that minimum program requirements will be met and, at the same time, assure that no product will be produced in excessive quantities. With these demand restrictions inserted into the matrix, it now is necessary to construct the production (or supply) restrictions.

Specific production restrictions

As mentioned above, the TVA plant as presently configured has the capacity to produce 18 different products in varying levels. Production facilities for the finished fertilizer products are generally large enough to produce more of any one single product than would normally be required in a demonstration program. There are production restrictions or bottlenecks in several of the intermediate facilities.

Figure 5 depicts the flow of materials through the plant. Though incomplete, it is useful in programming the operations since it gives some

Figure 5. Flow diagram TVA fertilizer facility



PLANT FACILITY LIMITATIONS
PROGRAM REQUIREMENTS
RAW MATERIAL INPUT FACTORS
INTERMEDIATE PRODUCT FACILITIES
END PRODUCT FACILITIES
SULFURIC ACID
NATURAL GAS
FLORIDA PEBBLE
TENNESSEE MATRIX
COAL, COKE & SILICA
HNO3
NH3
H3PO4
P4
GRAN. FERT.
DAP
LF
HAP
CSP
NPA
APP
PNA
APN
ANS
ANZ
HNA
HND
NH3
DAP
LFB
LFA
HAP
CSP
HPO
P4A
P4D
LEGEND
MIN
MAX
(THOUSAND TONS)
30
60
90
120

idea of the technical and physical bottlenecks and of the many interactions among the several bottlenecks as product mix is changed.

Seven major restrictions at one time or another come into play in the supply sector of the linear programming matrix. Some of these are policy decisions while others are purely technical in nature. The technical ones relate to the use of the outputs of certain intermediates, such as ammonia, nitric acid, phosphorus, or phosphoric acid. Table 17 indicates the production requirements per ton of finished product for each of the limited intermediate products.

Mathematical equations simulating these supply limitations, together with a short explanation of each, are shown below. Activity levels, i.e., X_i's, are coded in order to ensure easier interpretation and analysis.

(56) $$\sum_{i=1}^{20} X_i \geq 200{,}000$$

At any production level below a minimum of 200,000 tons annually the entire analysis would be faulty.

(57) $$\sum_{i=1}^{20} X_i \leq 310{,}000$$

TVA's board of directors has established a policy that, since the program is primarily one of demonstration, a production and distribution level of 310,000 tons is maximum. To produce and distribute larger quantities of total materials would risk interfering with industry sales of materials rather than helping introduce new and improved products.

(58) $LFA + LFB \leq 120{,}000$

(59) $LFA + LFB \geq 60{,}000$

Table 17. Production requirements per ton of product[a]

Products	Phosphoric acid requirement	Total phosphorus requirement	Nitric acid requirement	Total ammonia requirement
P4D		1.0000	-	-
P4A		1.0000	-	-
HPO	1.000	.3200	-	-
CSP	.576	.1840	-	-
HAP	.583	.1870	-	-
LFA	.514	.1640	-	.137
LFB	.555	.1780	-	.178
NH3	-	-	-	1.000
HNA	-	-	1.000	.300
HND	-	-	1.000	.300
ANZ	-	-	.800	.460
ANS	-	-	.680	.424
APN	.170	.0544	.645	.401
PNA	.395	.1260	.450	.337
APP	.837	.2680	-	.196
NPA	.160	.0512	.394	.261
DAP	.780	.2500	-	.275
CUB	.340	.1100	.600	.280
NSL	-	-	.835	.481
UNS	-	-	.230	.070

[a]Computed from unpublished TVA production records.

Two types of liquid mixed fertilizers are produced in one reactor type production facility. The upper limit represents maximum throughput of this facility, while the lower limit represents minimum throughput without unduly disrupting overall production scheduling.

(60) $\sum$ ANZ, ANS, APN, PNA, APP, NPA $\leq$ 110,000

(61) $\sum$ ANZ, ANS, APN, PNA, APP, NPA $\geq$ 70,000

These six different fertilizer materials are produced in one common production unit--a combination fertilizer unit. The two limits represent the maximum production capability and the lower limit for operations without unduly disrupting the overall production scheduling.

The remaining eight equations in the supply side of the linear programming matrix are those representing the technical limitations resulting from the production requirements per ton of product for each of the restricted intermediate products. The upper limit in each case is that of maximum annual productive capacity, while the lower limit represents the lower level of operational feasibility without making major changes in the entire operation of TVA's facilities. These equations are derived from Table 17.

Phosphorus production restrictions

$\sum$ P4D, P4A, .32 HPO, .184 CSP, .187 HAP, .164 LFA, .178 LFB, .0544 APN, .126 PNA, .268 APP, .0512 NPA, .250 DAP, .110 CUB

(62) $\geq$ 38,000

(63) $\leq$ 44,000

Phosphoric acid production restrictions

$\sum$ HPO, .576 CSP, .583 HAP, .514 LFA, .555 LFB, .170 APN, .395 PNA, .837 APP, .160 NPA, .780 DAP, .340 CUB

(64) $\geq$ 100,000

(65) $\leq$ 125,000

Ammonia production limitations

$\sum$ .137 LFA, .178 LFB, NH3, .300 HNA, .300 HND, .460 ANZ, .424 ANS, .401 APN, .337 PNA, .196 APP, .261 NPA, .275 DAP, .280 CUB, .481 NSL, .070 UNS

(66) $\geq$ 75,000

(67) $\leq$ 85,000

Nitric acid production limitations

$\sum$ HNA, HND, .800 ANZ, .680 ANS, .645 APN, .450 PNA, .394 NPA, .600 CUB, .835 NSL, .230 UNS

(68) $\geq$ 60,000

(69) $\leq$ 75,000

Objective function

As indicated earlier, the objective function used in solution of this problem is that of maximization of marginal income. Table 18 is a compilation of the data required for the cost factors in the maximization function.

It is evident that at least one of the products should be produced only for program reasons--the UNS with a negative marginal income. This negative marginal income results from the high cost of raw materials which must be purchased by TVA in order to produce the material. UNS is a proposed new suspension fertilizer material containing a large proportion of urea. TVA does not have production facilities for urea and thus purchases the material at market price. Yet, for program reasons, it is considered

Table 18. Cost and income factors for fertilizer plant products

Product	Estimated income per ton	Variable costs per ton	Marginal income per ton
P4D	$320.00	$149.85	$170.15
P4A	290.00	149.85	140.15
HPO	105.00	48.03	56.97
CSP	63.50	32.13	31.37
HAP	64.15	40.64	23.51
LFA	65.00	27.77	37.23
LFB	67.60	32.01	35.59
NH3	48.00	23.22	24.78
HNA	27.50	8.15	19.35
HND	25.00	8.15	16.85
ANZ	50.00	21.36	28.64
ANS	39.00	16.23	22.77
APN	56.50	21.42	35.08
PNA	63.00	29.16	33.84
APP	98.50	53.00	45.50
NPA	47.00	25.26	21.74
DAP	96.50	41.40	55.10
CUB	71.50	38.47	33.03
NSL	40.50	11.88	28.62
UNS	45.00	61.33	(- 16.33)

necessary to begin experiments on its demonstration and use.

The objective function of the linear programming problem becomes that of maximizing:

$$(70) \quad \sum_{i=1}^{20} X_i \, p_i$$

where X_i's are the individual products of the total product mix and the p_i's are the marginal incomes of each respective product.

Solution and interpretation

The IBM solution to this problem must be viewed within the overall framework of the inherent limitations of the problem as it has been established. It is a highly static approach, i.e., its results can only be expected to be of aid in management policy within an overall operational level approaching the maximum capacities. At levels significantly lower than this maximum capacity level, a new analysis would be required.

Its results are only as good as the estimates of total and marginal income per ton of product. If the prices of any one or more than one of the products change during the year, the total relationship may be changed, thus necessitating a complete new run of the program.

The same limitation applies if the costs of raw materials change during the period under study. If raw material costs change, then marginal incomes of individual products change.

If the demand for certain products changes, then the entire answer may change. For example, if minimum military demands for phosphorus increase or decrease, the program solution suggests that the change would be reflected first in the optimum limits of DAP. The solution does not give an answer as to what direction to take when once the lower or upper limit of DAP demand has been reached. Therefore, reducing the minimum demand for elemental phosphorus by as little as 1,500 tons would necessitate a new run of the program.

Table 19 summarizes the solution to the problem as set up in the foregoing sections. The solution optimizes marginal income at a level of $11,593,146.25.

This optimum solution is arrived at by satisfying minimum program demands of P4D, P4A, HPO, CSP, HAP, LFB, HNA, HND, PNA, APP, NPA, CUB, and, of course, UNS.

The production levels of the two liquid mixed fertilizers are limited by the upper capacity limit of the single dual facility for their production. Maximum amounts of LFA (11-37-0) are produced, consistent with the minimum amount of LFB (12-40-0) required for program purposes.

Maximum demand for ANZ, APN, and NSL will be a part of the optimum product mix, thereby using a portion of the total nitrogen in a finished form.

When the above requirements are met, the remainder of the scarce resources such as ammonia, phosphorus, and nitric acid will be used to produce DAP (21-53-0) and ANS (30-0-0-5S). The DAP will use the remainder of the phosphorus capacity and a portion of the ammonia capacity. The ANS will use the remainder of the ammonia and nitric acid capacities.

Under this type operation, minimum program demands for all products are satisfied. In no case is the maximum demand exceeded. The maximum total tonnage of 309,623 tons does not exceed the 310,000 ton-limitation imposed by the board of directors of TVA. The full capacity of the liquid mixed

Table 19. Optimum product mix TVA plant fiscal year 1967

Product	Sales limits	Optimum product mix	Product mix requirements Tons P_4	Tons H_3PO_4	Tons NH_3	Tons HNO_3
P4D	4,500- 8,000	4,500	4,500	-	-	-
P4A	3,000- 4,000	3,000	3,000	-	-	-
HPO	7,000- 10,000	7,000	2,240	7,000	-	-
CSP	15,000- 20,000	15,000	2,760	8,640	-	-
HAP	4,000- 10,000	4,000	748	2,332	-	-
LFA	50,000-120,000	110,000	18,040	56,540	15,070	-
LFB	10,000- 20,000	10,000	1,780	5,550	1,780	-
NH3	20,000- 40,000	20,473	-	-	20,473	-
HNA	15,000- 20,000	15,000	-	-	4,500	15,000
HND	2,000- 5,000	2,000	-	-	600	2,000
ANZ	500- 1,000	1,000	-	-	460	800
ANS	15,000- 25,000	20,479	-	-	8,683	13,926
APN	30,000- 45,000	45,000	2,448	7,650	18,045	29,025
PNA	15,000- 25,000	15,000	1,890	5,925	5,055	6,750
APP	3,000- 10,000	3,000	804	2,511	588	-
NPA	6,000- 15,000	6,000	307	960	1,566	2,364
DAP	15,000- 25,000	20,171	5,043	15,733	5,547	-
CUB	4,000- 5,000	4,000	440	1,360	1,120	2,400
NSL	1,000- 3,000	3,000	-	-	1,443	2,505
UNS	1,000- 3,000	1,000	-	-	70	230
Total	221,000-394,000	309,623	44,000	114,201	85,000	75,000

fertilizer facility is used. The full capacities of the facilities to produce phosphorus, ammonia, and nitric acid are used.

The "dual" solution to the problem shows that additional amounts of each intermediate could be used to improve the level of marginal income if there were any way to increase their production without increasing variable costs per ton of product.

The dual solution is also quite valuable in guiding management decisions on plant operations. The dual solution assigns dollar values to each of the limiting restrictions, whether they be supply (production) restrictions or demand restrictions. A thorough analysis of the dual solution to this program for plant operation may be as important as, or even more important than, that of the primary solution. The primary solution gives the optimum product mix and indicates general direction of activity. The dual solution indicates the value of an additional ton of scarce intermediate product; at the same time, it indicates the cost to the total program of "forcing" into the product mix some products having smaller marginal incomes than those which could alternatively be produced.

Table 20 summarizes results of the dual solution to the problem. Thirteen products are listed under the heading "$/Ton to reduce limit." From a purely economic approach, it would be profitable to reduce these products below their present minimum production levels if program requirements could be met with other fertilizer materials. Under the column headed "$/Ton to increase limit" the dollar values to be attached to additional amounts of certain materials and certain production facilities are given.

For example, an additional ton of phosphorus should be valued in terms of marginal income at $193.14 per ton rather than its marginal value of

Table 20. Summary of the dual solution to example of TVA plant operations

Restriction	Limit	$/Ton to reduce limit	$/Ton to increase limit
P4D	Lower	$22.99	$ -
P4A	Lower	52.99	-
HPO	Lower	4.84	-
CSP	Lower	4.17	-
HAP	Lower	12.61	-
LFB	Lower	5.36	-
HNA	Lower	6.12	-
HND	Lower	8.62	-
ANZ	Upper	-	2.81
APN	Upper	-	3.00
PNA	Lower	6.96	-
APP	Lower	11.12	-
NPA	Lower	1.72	-
CUB	Lower	5.97	-
NSL	Upper	-	1.64
UNS	Lower	22.21	-
Liq. fert. prod. facility	Upper	-	2.16
P_4 production	Upper	-	193.14
NH_3 production	Upper	-	24.78
HNO_3 production	Upper	-	18.03

$170.15 if sold for defense purposes or $140.15 if used in agricultural programs. The higher value, i.e., $193.14, results from the fact that if additional phosphorus were available a new product mix would be possible. More DAP could be produced and slightly less ammonia sold with the resulting net change in total marginal income of $193.14 per ton of additional phosphorus.

Thus, both the optimum solution to the linear programming problem and its dual solution can be of great assistance to management in deciding proper direction of effort, proper program emphasis on products, and general direction of total product mix. This application has limitations as outlined; but when these limitations are realized and proper interpretation is made of the results, the results can be useful in guiding management decisions on production scheduling.

THE APPLICATION OF A SPATIAL MODEL TO TURKEY

Background

One of the major problems facing developing nations in their quest for additional food to prevent famine in the coming decades is that of an efficient fertilizer production, transportation, and distribution system. Supplies of fertilizers and fertilizer raw materials appear ample for the world as a whole. Many of these supplies, however, are not located where they are most urgently needed. Many developed nations have temporary surpluses of production facilities. Many of the developing nations have deficits of finished fertilizer products. The obvious question is how to satisfy most economically the fertilizer requirements of the deficit nations.

Almost without exception, the developing nations have severe shortages of foreign exchange. On the other hand, they have surpluses of labor and varying other resources which can be used to produce finished fertilizer products. Thus, there are many alternative ways for them to import raw materials such as naphtha, raw rock phosphate, and muriate of potash and produce the required amounts of finished fertilizer products for use by their farmers in producing additional food.

These raw materials can usually be imported to one or more locations within the nation. Thus, the potential exists to produce finished fertilizers at more than one location. Production costs may differ at each location. Types and grades of fertilizer can differ. Transportation and distribution costs from the production points to the end-use area will vary both by type of material and with different production points.

It is the objective of this chapter to develop a "spatial" or "transportation" linear programming model into which all of these different costs can be inserted. This spatial program will encompass the area requirements for fertilizer--both total requirements and requirements of specific types of materials. At the same time, it will include provision for various production possibilities.

The objective function of the model is to minimize the cost of the total fertilizer industry required by the nation.

Generally, such transportation or spatial problems are established with the supply exactly equal to the demand. In the particular case chosen, however, it is necessary to allow for the possibility of oversupply, since demand is growing at a rapid rate. Establishing the programs in such a way has the added advantage of allowing for an analysis of future direction of decisions on production facilities.

The problem country and its fertilizer requirements

The nation selected for this problem is Turkey.[a] Fertilizer use in the country is a relatively new development. In the period following World War II and extending through the 1950's, additional food was made available by rapid expansion of cropland acreage. By 1960, however, the further expansion of cropland acreage was no longer feasible. Thus, emphasis began to be placed upon increased use of fertilizer coupled with better strains of crops, better cropping practices, and increasing amounts of irrigation.

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[a]Most of the input-output data and requirements resulted from the author's assignment to Turkey in 1966 as head of a team to assist that nation develop a second 5-year fertilizer plan. All the data used in this problem are empirical data--thus, the method can be applied to other countries.

Table 21 summarizes the past decade's use of plant nutrients in Turkey. During the past five years the rate of increase in use of nitrogen and phosphate has been exceptionally high, while use of potash has been erratic. Potash content of soils is very high. For this reason, only nitrogen and phosphatic fertilizers are considered in this model.

Table 21. Actual plant nutrient consumption in Turkey 1955-65

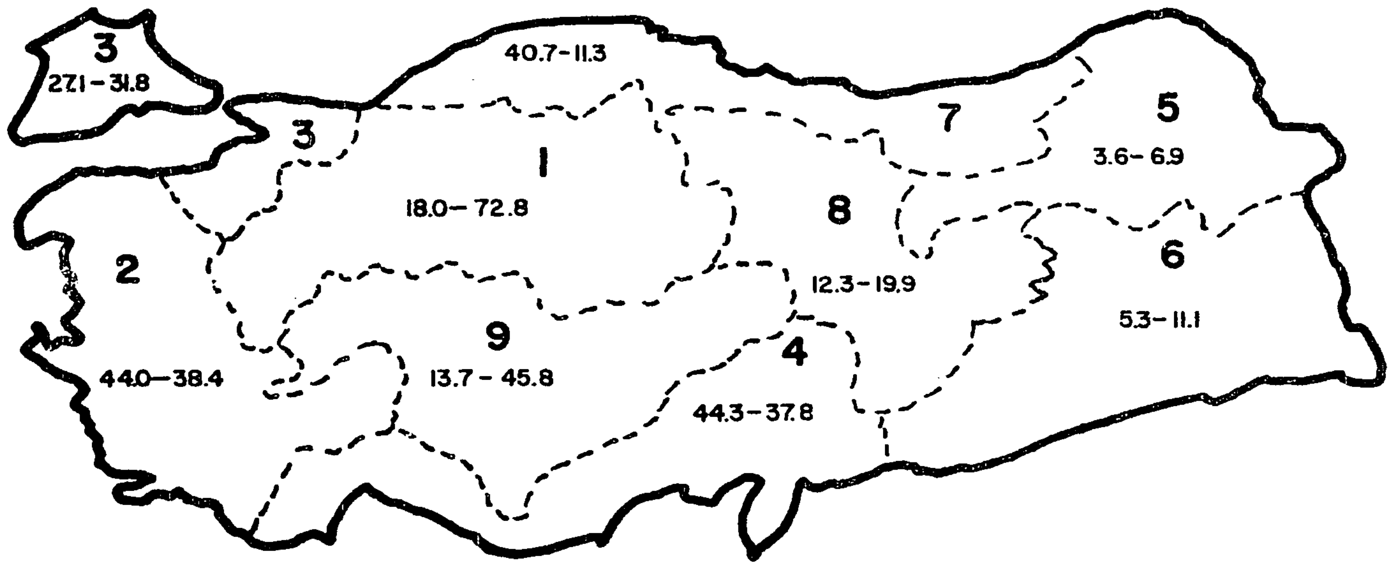
Year	N	P_2O_5	K_2O
		(Thousand metric tons)	
1955	11	13	0.6
1956	9	4	6.1
1957	10	5	-
1958	10	4	-
1959	25	9	-
1960	10	9	0.7
1961	29	12	-
1962	38	17	5.2
1963	39	35	10.5
1964	54	42	4.5
1965	71	71	3.2

All interested governmental agencies in Turkey within the past five years have made it a policy to encourage greater use of fertilizers. Through an agricultural bank, large amounts of credit are being extended to farmers for fertilizer purchases. Additional funds are being made available to the extension service and experiment stations. Import taxes on fertilizer raw materials and finished products have been removed. Support prices for food grains and other crops have encouraged greater use of fertilizers. Money has been made available for training agricultural technicians both inside Turkey and abroad. Money, technicians, and other resources have been made available for detailed analyses of past, present, and future fertilizer requirements of the total nation and of its various areas. In short, governmental policy has been such as to encourage a rapid expansion of fertilizer use.

The nine major geographical areas of Turkey are shown on Map 3. The map also lists the projected total requirements of nitrogen and phosphate fertilizers in terms of thousands of metric tons of N and P_2O_5. These estimates are the result of a 2-year detailed study by a Turkish governmental work group. This study analyzed statistics of each crop in each geographic area, its potential, and how much of this potential can reasonably be expected to be reached within the next six years.

Turkey currently has over 1,700 retail sales locations for the distribution of fertilizers. Costs of distribution have been carefully studied by the State Planning Office for use in planning purposes. Costs of transportation, for example, are a function of ton-mileage using highway mileages from point to point. Other distribution costs vary both with tonnage and with value of the material distributed, since a portion of these distribution

Map 3. Projected nitrogen and phosphate fertilizer use in Turkey 1972



LEGEND: FIRST FIGURE = NITROGEN USE 1972
SECOND FIGURE = P_2O_5 USE 1972

costs are for insurance, losses, and inventory costs.

Studies by the various planning offices of the nation's agricultural agencies have determined the centers of fertilizer use of each area. These studies are useful both for general economic planning and for determination of possible locations of large wholesale distribution points where storage warehouses must be constructed to serve the rapidly expanding demand.

Other studies have been made of the current and expected use of specific types of fertilizer materials by crop and by geographic area. It is realized that not all fertilizers are equally suitable for all crops. It is further recognized that once farmers in an area have begun using a specific type of fertilizer they will change to new and improved types only over a long period of time.

In short, the fertilizer demand situation has been carefully studied by agricultural technicians of Turkey. Information from these analyses is used in designing the spatial program to minimize total economic cost of fertilizer production and distribution at the end of the second 5-year plan.

Fertilizer supply situation in Turkey

Turkey has only three fertilizer production facilities. A plant at Kutahya produces 150,000 tons annually of ammonium sulfate (20-0-0). An expansion is under way which will increase capacity to 590,000 tons equivalent of ammonium sulfate. Production costs of this material are high, but are expected to decline when the new expansion is completed in 1968.

Two normal superphosphate production facilities are located at Izmit and at Iskenderun. Each has an annual capacity of 122,200 tons of normal superphosphate (NSP, 18-0-0). Production costs are similar at each location since they are of identical construction and operation and their raw material costs are similar.

A new normal superphosphate plant is to be constructed at Ergani. This plant is to be almost twice the size of those at Izmit and Iskenderun. It is being constructed to use a surplus of sulfuric acid at that location for which there is no alternative use and for which transportation facilities are not available. It is sized so as to make full use of the surplus sulfuric acid. The production costs of its product will be low in relation to the production costs of the other two plants, since the sulfuric acid production costs will be low. The quantity of sulfuric acid available is sufficient for economical operation of a normal superphosphate (NSP) plant, but not enough to produce triple superphosphate (TSP) at an efficient level of operation.

By 1968 there will be a surplus of sulfuric acid at Samsun. A fertilizer production facility is planned to utilize this material. In this case, the quantity of sulfuric acid is sufficient to warrant production of triple superphosphate (TSP). The facility will produce a maximum of 222,200 tons of material at a relatively low cost per unit of P_2O_5. Since there is no refinery located nearby and no natural gas or other inexpensive source of hydrogen for use in producing ammonia, it will produce only a straight phosphate fertilizer.

In addition to the five plants described above, there is a requirement for a large complex plant to produce both a nitrogen and a phosphatic fertilizer. To produce the nitrogen, large amounts of naphtha or natural gas are required. Turkey has insufficient natural gas reserves for such a plant. Naphtha can be made available at either of two locations, Mersin or

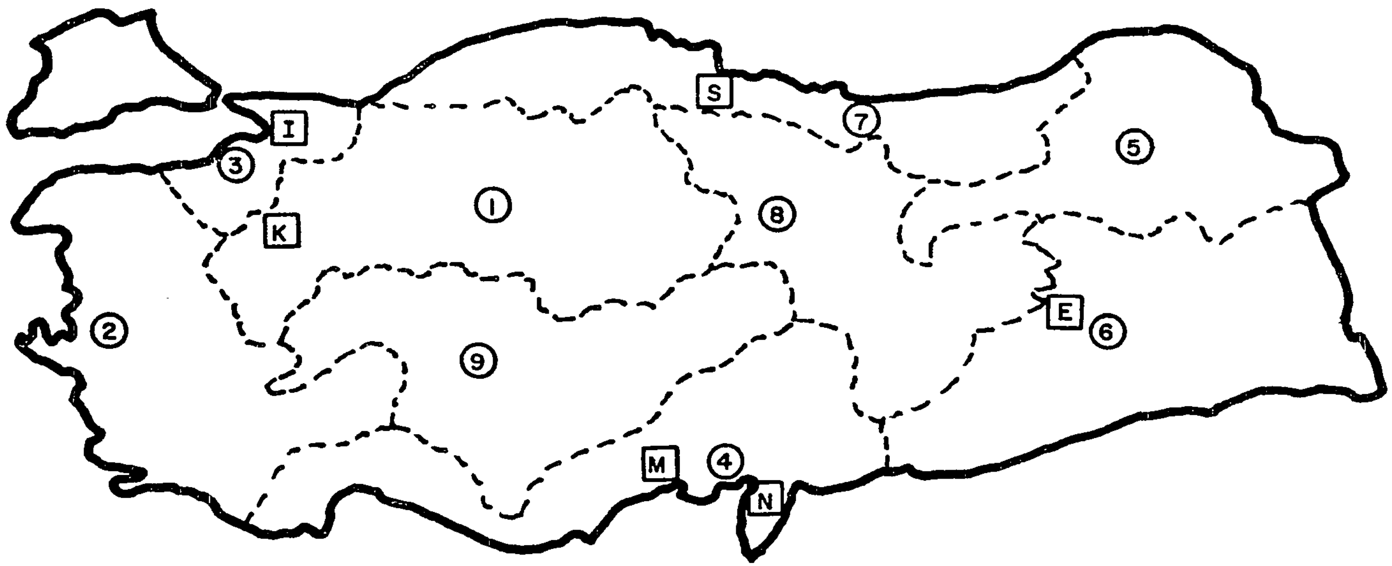
Izmit. Both locations have refineries. At Mersin the refinery has excess capacity, while at Izmit the total capacity of the refinery is already being used. Thus, naphtha for fertilizer use at Izmit would be slightly more costly than at Mersin.

Sulfuric acid for either location can be produced from flotation pyrites produced as a byproduct of a copper smelter at Ergani. In each case the flotation pyrites would be transported as such from Ergani to the fertilizer production unit where they would be converted to sulfuric acid, thence to the production of phosphoric acid, and thence into final fertilizer product. Transportation facilities are available for transporting the flotation pyrites, but are not available for transporting sulfuric acid or phosphoric acid. The transportation costs of the flotation pyrites are estimated at $5 per ton more to Izmit than to Mersin.

On the other hand, Izmit may be closer to the area of final use of the fertilizer product to be produced in the complex plant facility. If so, transportation costs from production facility to the end user will be less than if the complex is located at Mersin, and the economies of transportation of the final product may overweigh the diseconomies of production costs brought about by higher delivered costs of some of the raw materials.

Map 4 shows the six locations of the present and proposed plant production facilities, along with the area "gravity centers" for use of fertilizer. Minimization of the production, transportation, and other distribution costs of the required amounts and types of fertilizers to each of the nine areas is essentially the spatial problem to be solved.

Map 4. Location of fertilizer plants and proposed sites together with geographic centers of fertilizer use Turkey 1972



LEGEND: □ – PRODUCTION POINT

○ – AREA CENTER OF GRAVITY FOR FERTILIZER USE.

▬▬ – NATION BOUNDARY

- - - - – AREA BOUNDARY

The mathematical model

Mathematically, the cost minimization model used is stated as:

$$\text{Minimize } f(c) = \sum_{i=1}^{72} c(j)\, X_i$$

in which

$c(j)$ = the average costs per ton of production, distribution, and marketing of each type of fertilizer considered feasible for the Turkish fertilizer industry.

X_i = amount of fertilizer to be produced at each plant and distributed to each of the nine major use regions.

With this highly simplified minimization equation, in essence there are established 72 real activities for the spatial model. Each real activity consists of the possibility of producing one type of fertilizer material at one production point and distributing this material to one of the nine using regions. The model includes provision for production at six locations with one of the six plants producing three grades of fertilizers. Thus, mathematically there are eight "production units" with the possibility of selling fertilizers to nine using regions--a total of 72 real activities.

The minimization equation is solved subject to basic sets of demand and supply restrictions and equations which simulate mathematically the Turkish fertilizer industry in the time period of 1972. A third set of restrictive inequalities is imposed to simulate mathematically various refinements relative to regional demand for different types of fertilizers.

Equations 71 through 88 simulate mathematically the regional demand for nitrogen and phosphate fertilizers (in terms of N and P_2O_5 content) for the year 1972. These are expressed as equalities. They are based upon an extensive study completed by the State Planning Organization of Turkey in cooperation with the Turkish Department of Agriculture.

Equations 89 through 98 simulate mathematically the supply potential for nitrogen and phosphatic fertilizers to be produced within Turkey. The first four of these are stated in terms of equalities to simulate an express governmental policy that existing plants will continue to operate at full capacity. The remaining equations are expressed as inequalities, and represent mathematically the upper limits of production capacity either for a total plant operation or for a specific technical "bottleneck" or restriction within the proposed complex fertilizer facility.

Finally, the third general set of equalities and/or inequalities are entered in Equations 99 through 134. In the model used, these equations are all entered as inequalities and all relate to refinements in the regional demand pattern for types and grades of fertilizers in 1972. They represent mathematically some of the judgments of agronomists and other market researchers as to the most appropriate types of fertilizer materials.

The model used is, in fact, a simplified presentation of a highly complex industrial production, transportation, and distribution system, as first suggested by Heady and Candler (18, p. 373-374). The c(j) statistic as used is actually the summation of three different statistics: (1) the production cost of the i-th ton of material, plus (2) the average transportation cost of the i-th ton of material from production point to the region indicated by the real activity, plus (3) the normal marketing costs per i-th ton of material. By using this c(j) statistic in the simplified system, however, it has been possible to minimize net total cost of the fertilizer production and distribution system of Turkey in one solution.

The matrix

The real activities in the matrix of the spatial problem consist of the possibilities of producing and shipping each type of fertilizer from each particular production facility to each use area (1 through 9). Thus, the Kutahya production unit generates nine real activities: ammonium sulfate produced at Kutahya and used in areas 1 through 9. Each fertilizer produced at each possible plant location then generates nine possible real activities.

The c(j) of each activity is composed of the total cost of production per ton of specific type of fertilizer at its plant location plus the accumulated costs of transportation and distribution.

The input-output coefficients are the respective amounts of N, P_2O_5, straight N, etc., contained in each specific type of fertilizer which could be used.

The problem or program then has the objective function of minimizing total costs of producing and distributing the fertilizer from the five plants, i.e., Kutahya, Izmit, Iskenderun, Ergani, and Samsun, plus the plant complex at Mersin or the plant complex if it were relocated at Izmit. It does not solve for Mersin or Izmit location of the complex plant facility at the same time and on the same computer run.

Activity costs

As indicated above, the activity costs c(j) are composed of the production costs of each material at each plant location plus respective transportation and distribution costs to each of the nine use areas.

The estimated production and marketing costs of each type of fertilizer material at each location are indicated in Table 22. In this table the term "marketing costs" does not include costs of transportation but refers only to sales and storage costs.

Table 22. Estimated production and marketing costs of various fertilizers

Production location	Material	Grade	Estimated cost per ton	
			Production	Marketing
Kutahya	Am. Sulfate	20-0-0	$95.00	$ 9.55
Izmit	NSP	0-18-0	32.95	6.67
Iskenderun	NSP	0-18-0	32.95	6.67
Ergani	NSP	0-18-0	26.49	6.67
Samsun	TSP	0-45-0	51.45	10.80
Mersin	TSP	0-45-0	56.49	10.80
Mersin	DP	18-46-0	72.79	13.50
Mersin	Urea	46-0-0	40.74	8.75
Izmit	TSP	0-45-0	60.23	10.80
Izmit	DP	18-46-0	79.07	13.50
Izmit	Urea	46-0-0	42.72	8.75

These production and marketing costs were all taken from unpublished records of the fertilizer study group--except for the production costs of TSP, DP, and urea at Izmit. These three production costs were estimated by using the costs at Mersin as a base starting point and increasing them by the amount of increased costs of raw materials delivered to Izmit.

Thus, the estimated basic production cost of urea at Mersin was \$40.74 per ton. Naphtha for the Izmit plant was estimated to cost \$4 per ton more than at Mersin. Naphtha requirements per ton of ammonia will be 0.862 ton; therefore, ammonia cost at Izmit will increase over that at Mersin by \$3.45 per ton. Urea requires 0.575 ton of ammonia per ton of product output, thus leading to an increased cost of urea at Izmit of 0.575 x \$3.45, or \$1.98 per ton. Adding the \$1.98 to the basic cost of \$40.74 (production cost at Mersin) results in a production cost of urea at Izmit of \$42.72 per ton.

Using the same method, flotation pyrites delivered to Izmit cost \$5 per ton more than at Mersin. Pyrite requirements per ton of sulfuric acid are 0.876 ton, thus cost of sulfuric acid is increased by \$4.38 per ton. This results in an increased cost of P_2O_5 in phosphoric acid of \$11.69 (2.67 tons sulfuric acid per ton of P_2O_5 in phosphoric acid). This, in turn, results in an increased cost of \$3.74 per ton of TSP at Izmit (\$11.69 per ton P_2O_5 in phosphoric acid with 0.32 ton P_2O_5 from phosphoric acid used per ton of TSP produced). Adding the \$3.74 per ton of TSP to the basic cost of TSP at Mersin (\$56.49), results in a production cost of TSP at Ismit of \$60.23 per ton of material.

The same type analysis leads to a production cost of DP at Izmit of (1) \$72.79 (the basic cost at Mersin) plus (2) \$0.79 (increased cost of ammonia requirement per ton of product) plus (3) \$5.49 (increased cost of P_2O_5 from phosphoric acid per ton of product)--a total of \$79.07 per ton.

To arrive at the total real activity costs, however, the cost of transportation from each producing point to each using area must be added to the respective production and distribution costs. Table 23 is a detailed

Table 23. Summary of average transportation costs per ton of material from production points to the using areas

To	Kutahya	Izmit	Isken-derun	Ergani	Samsun	Mersin
Area 1	$ 6.98	$ 7.82	$14.27	$18.80	$ 9.44	$10.62
Area 2	8.13	10.42	25.82	33.36	24.56	22.16
Area 3	4.22	2.58	22.22	27.11	17.27	18.56
Area 4	15.13	18.64	3.44	13.20	17.09	1.53
Area 5	29.29	28.04	17.67	8.60	15.36	20.67
Area 6	27.42	28.82	10.82	1.00	18.71	13.82
Area 7	21.18	19.36	21.89	15.49	4.76	20.11
Area 8	16.93	17.78	13.36	8.87	7.69	11.58
Area 9	7.38	12.58	11.20	20.04	15.22	7.53

summary of the applicable transportation costs from the six possible production points to each of the nine using areas. These have been computed based upon road mileage and a fixed price per ton-mile, since it is the custom in Turkey to transport fertilizer by truck and a fixed price per ton-mile is normally charged. This cost is equal to or lower than the cost of the alternative forms of transportation.

When the costs of production, transportation, and distribution through the local retail outlets are summarized, the total activity costs, c(j), are complete. Table 24 summarizes these costs. Included in this table are the costs of each type material from each production point to each using

Table 24. Summarization of real activity costs for the Turkish fertilizer industry 1972

Type material and production point	Cost per ton delivered to areas 1	2	3	4	5	6	7	8	9
AS - Kutahya	$111.53	$112.68	$108.77	$119.68	$133.84	$131.97	$125.73	$121.48	$111.93
NSP - Izmit	47.44	50.04	42.20	58.26	67.66	68.44	58.98	57.40	52.20
NSP - Iskenderun	53.89	65.44	61.84	43.06	57.29	50.44	61.51	52.98	50.82
NSP - Ergani	51.96	66.52	60.27	46.36	41.76	34.16	48.63	42.03	53.20
TSP - Samsun	71.69	86.81	79.52	79.34	77.61	80.96	67.01	69.94	77.47
If complex located at Mersin									
TSP - Mersin	77.91	89.45	85.85	68.82	87.96	81.11	87.40	78.87	74.82
DP - Mersin	96.91	108.45	104.85	87.82	106.96	100.11	106.40	97.87	93.82
Urea- Mersin	60.11	71.65	68.05	51.02	70.16	63.31	69.60	61.07	57.02
If complex located at Izmit									
TSP - Izmit	78.85	81.45	73.61	89.67	99.07	99.85	90.39	88.81	83.61
DP - Izmit	100.39	102.99	95.15	111.21	120.61	121.39	111.93	110.33	105.15
Urea- Izmit	59.29	61.89	54.05	70.11	79.51	80.29	70.83	69.25	64.05

area. Included are the costs from the complex fertilizer plants at both of the possible locations.

With this background of statistical data, then, it is possible to begin constructing the matrix of the spatial problem of linear programming. This matrix will be, in effect, a mathematical simulation of the supply and demand situation for fertilizers in Turkey in 1972.

<u>Nitrogen and phosphate fertilizer requirements, Turkey, 1972</u>

For each of the nine areas of Turkey, a specific requirement has been established for 1972 consumption of both nitrogen and phosphate. The first set of 18 equations in the spatial matrix will simulate these 18 requirements, i.e., the use of nitrogen and phosphate in each of nine areas.

Three types of materials can be used to supply the nitrogen requirements. Each ton of ammonium sulfate from Kutahya will supply 0.20 ton of nitrogen. Each ton of diammonium phosphate from Mersin will supply 0.18 ton of nitrogen. Urea from Mersin contains 0.46 ton of nitrogen. Thus, the nine equations simulating the requirements of nitrogen for use in each area become:

	For	Total N requirements		Supplied by
(71)	Area 1	18,000	=	$.20X_1 + .18X_{55} + .46X_{64}$
(72)	Area 2	44,000	=	$.20X_2 + .18X_{56} + .46X_{65}$
(73)	Area 3	27,100	=	$.20X_3 + .18X_{57} + .46X_{66}$
(74)	Area 4	44,300	=	$.20X_4 + .18X_{58} + .46X_{67}$
(75)	Area 5	3,600	=	$.20X_5 + .18X_{59} + .46X_{68}$
(76)	Area 6	5,300	=	$.20X_6 + .18X_{60} + .46X_{69}$
(77)	Area 7	40,700	=	$.20X_7 + .18X_{61} + .46X_{70}$

(78)	Area 8	12,300	$= .20X_8 + .18X_{62} + .46X_{71}$
(79)	Area 9	13,700	$= .20X_9 + .18X_{63} + .46X_{72}$

There are six possible sources of supply for the required amounts of phosphate for use in each of the nine areas. Three production facilities will have available normal superphosphate (0-18-0) and each ton used will supply 0.18 ton of the P_2O_5 required. Two production facilities may produce triple superphosphate (0-45-0) and one may supply diammonium phosphate (18-46-0) which supplies 0.46 ton of P_2O_5 per ton of material. Thus, the nine equations for phosphate requirements for each area become:

	For	Total P_2O_5	Supplied by
(80)	Area 1	72,800	$= .18(X_{10} + X_{19} + X_{28}) + .45(X_{37} + X_{46}) + .46X_{55}$
(81)	Area 2	38,400	$= .18(X_{11} + X_{20} + X_{29}) + .45(X_{38} + X_{47}) + .46X_{56}$
(82)	Area 3	31,800	$= .18(X_{12} + X_{21} + X_{30}) + .45(X_{39} + X_{48}) + .46X_{57}$
(83)	Area 4	37,800	$= .18(X_{13} + X_{22} + X_{31}) + .45(X_{40} + X_{49}) + .46X_{58}$
(84)	Area 5	6,900	$= .18(X_{14} + X_{23} + X_{32}) + .45(X_{41} + X_{50}) + .46X_{59}$
(85)	Area 6	11,000	$= .18(X_{15} + X_{24} + X_{33}) + .45(X_{42} + X_{51}) + .46X_{60}$
(86)	Area 7	11,300	$= .18(X_{16} + X_{25} + X_{34}) + .45(X_{43} + X_{52}) + .46X_{61}$
(87)	Area 8	19,900	$= .18(X_{17} + X_{26} + X_{35}) + .45(X_{44} + X_{53}) + .46X_{62}$
(88)	Area 9	45,800	$= .18(X_{18} + X_{27} + X_{36}) + .45(X_{45} + X_{54}) + .46X_{63}$

With these 18 equations, Equations 71 through 88, the area requirements for total N and P_2O_5 have been programmed or simulated. There are other limiting restrictions on the demand side, such as requirements for specific types of materials for each area, but first the equations expressing the supplies of fertilizers will be discussed.

Supplies of fertilizers, Turkey, 1972

The first four equations simulating the supply portion of the spatial matrix are expressed in equalities. Generally this would not be the case in such a program with a known oversupply of both nitrogen and phosphate. In this particular case, however, the equalities are inserted in order to simulate a stated governmental policy that these four particular production facilities will operate at full capacity before other facilities are allowed to supply the market. These four equations are:

	Production point	Production level (Tons)	
(89)	Kutahya	590,000	$= \sum_{i=1}^{9} X_i$
(90)	Izmit	122,200	$= \sum_{i=10}^{18} X_i$
(91)	Iskenderun	122,200	$= \sum_{i=19}^{27} X_i$
(92)	Ergani	222,200	$= \sum_{i=28}^{36} X_i$

The next four equations express the supply possibilities of the plants at Samsun and Mersin. These are inserted as inequalities with an upper limit expressing the maximum production possibilities of each material at each location. Each, of course, will require the appropriate slack vector to convert it to an equality for machine operation and solution. In the program used, the slack or surplus vectors are automatically inserted; thus in the equations below they are omitted.

	Production point	Production level	
(93)	TSP production Samsun	222,200	$\geq \sum_{i=37}^{45} X_i$
(94)	TSP production Mersin	281,250	$\geq \sum_{i=46}^{54} X_i$
(95)	DP production Mersin	191,490	$\geq \sum_{i=55}^{63} X_i$
(96)	Urea production Mersin	264,000	$\geq \sum_{i=64}^{72} X_i$

Two additional supply restrictions must be inserted. The Mersin production facility is a highly flexible complex fertilizer plant. It has an oversized granulator which can produce either 281,250 tons of TSP or 191,490 tons of DP or some combination of the two. Its total production of these two materials is limited by the size of the phosphoric acid unit which in turn is limited by the supply of sulfuric acid which in turn is limited by the availability of flotation pyrites. Phosphoric acid is required for both TSP and DP production. There will be available a maximum of 90,000 tons of P_2O_5 in the form of phosphoric acid at this location. For each ton of TSP produced and shipped, 0.32 ton of P_2O_5 in the form of phosphoric acid will be required. For each ton of DP produced, 0.47 ton of P_2O_5 in the form of phosphoric acid will be required. Thus, the equation simulating this restriction on availability of phosphoric acid becomes:

	Production capacity of phosphoric acid		Use of phosphoric acid
(97)	90,000	$\geq$	$.32 \sum_{i=46}^{54} X_i + .47 \sum_{i=55}^{63} X_i$

The Mersin production facility also has a restriction in the amount of anhydrous ammonia which can be produced. The ammonia can all be used to produce urea at a level of 260,000 tons per year. It can also be divided and a portion used to produce DP with a portion or all of the remainder being used in the production of urea.

In production of urea, 0.575 ton of ammonia will be required per ton of product. In production of DP, 0.23 ton of ammonia will be required per ton of product. Thus the next restrictive equation becomes:

	Productive capacity of ammonia		Use of ammonia
(98)	148,500	$\geq$	$.23 \sum_{i=55}^{63} X_i + .575 \sum_{x=64}^{72} X_i$

The above ten equations simulate the supply portion of the linear programming matrix. As outlined above, the matrix simulates the governmental policy decision that the first four plants will operation at full capacity and that the Mersin location will be chosen for the highly flexible complex facility.

Additional demand restrictions

There are 36 additional demand restrictions which must be imposed. These are based upon a combination of agronomic and sociological factors which are considered pertinent to the fertilizer situation in Turkey. In essence, they simulate refinements to the fertilizer demand situation.

Turkish farmers are accustomed to using ammonium sulfate. Although the governmental agronomists do not proclaim a mandatory agronomic requirement for this specific type of fertilizer, governmental sociologists and market research personnel insist that customs and habits can only slowly be changed. Thus, those farmers who now use ammonium sulfate will tend to continue using ammonium sulfate. It is estimated that by 1972 farmers in each area will demand at least 80 percent as much ammonium sulfate as is now being used. Rather than attempting to force a change at an unduly high rate, the requirement is stipulated that in 1972 at least 80 percent as much ammonium sulfate as is now used will be made available to each geographic area. The following restrictive equations result:

	For	1972 level of ammonium sulfate used		Amount of ammonium sulfate shipped
(99)	Area 1	16,000	$\leq$	X_1
(100)	Area 2	75,200	$\leq$	X_2
(101)	Area 3	49,600	$\leq$	X_3
(102)	Area 4	48,000	$\leq$	X_4
(103)	Area 5	4,000	$\leq$	X_5
(104)	Area 6	800	$\leq$	X_6
(105)	Area 7	77,600	$\leq$	X_7
(106)	Area 8	20,400	$\leq$	X_8
(107)	Area 9	16,000	$\leq$	X_9

Following the same line of reasoning, a set of restrictions is entered into the matrix for use of NSP (0-18-0). These are expressed in terms of P_2O_5 although they could just as easily be expressed in terms of tons of materials. The equations are:

	For	1972 level of NSP		Amount of NSP shipped
(108)	Area 1	22,600	$\leq$	$.18 \sum X_{10}, X_{19}, X_{28}$
(109)	Area 2	5,400	$\leq$	$.18 \sum X_{11}, X_{20}, X_{29}$
(110)	Area 3	11,500	$\leq$	$.18 \sum X_{12}, X_{21}, X_{30}$

(111)	Area 4	10,600	$\leq$	$.18\sum X_{13}, X_{22}, X_{31}$
(112)	Area 5	700	$\leq$	$.18\sum X_{14}, X_{23}, X_{32}$
(113)	Area 6	100	$\leq$	$.18\sum X_{15}, X_{24}, X_{33}$
(114)	Area 7	1,900	$\leq$	$.18\sum X_{16}, X_{25}, X_{34}$
(115)	Area 8	3,400	$\leq$	$.18\sum X_{17}, X_{26}, X_{35}$
(116)	Area 9	7,200	$\leq$	$.18\sum X_{18}, X_{27}, X_{36}$

Much of Turkey's cropland is devoted to dryland wheat farming. Past experiments on large scales have shown that the only fertilizer which is necessary for this dryland wheat acreage is some form of straight phosphatic material. No nitrogen is considered useful for this purpose. Thus, there must be available for each area a minimum amount of straight phosphate fertilizer material. NSP and TSP are equally suitable, but DP is eliminated since it also contains nitrogen. This requirement establishes nine other equations with the limiting level being the minimum requirement level which the agronomists have established. They are expressed below in terms of P_2O_5.

	For	Straight P_2O_5 required		P_2O_5 supplied by
(117)	Area 1	56,300	$\leq$	$(.18\sum X_{10}, X_{19}, X_{28}) + (.45\sum X_{37}, X_{46})$
(118)	Area 2	4,600	$\leq$	$(.18\sum X_{11}, X_{20}, X_{29}) + (.45\sum X_{38}, X_{47})$
(119)	Area 3	17,200	$\leq$	$(.18\sum X_{12}, X_{21}, X_{30}) + (.45\sum X_{39}, X_{48})$
(120)	Area 4	12,200	$\leq$	$(.18\sum X_{13}, X_{22}, X_{31}) + (.45\sum X_{40}, X_{49})$
(121)	Area 5	6,100	$\leq$	$(.18\sum X_{14}, X_{23}, X_{32}) + (.45\sum X_{41}, X_{50})$
(122)	Area 6	3,500	$\leq$	$(.18\sum X_{15}, X_{24}, X_{33}) + (.45\sum X_{42}, X_{51})$
(123)	Area 7	8,100	$\leq$	$(.18\sum X_{16}, X_{25}, X_{34}) + (.45\sum X_{43}, X_{52})$
(124)	Area 8	9,300	$\leq$	$(.18\sum X_{17}, X_{26}, X_{35}) + (.45\sum X_{44}, X_{53})$

(125) Area 9 31,600 (.18 X_{18}, X_{27}, X_{36}) + (.45 X_{45}, X_{54})

The agronomists also have stipulated a minimum amount of straight nitrogen materials for each area. These materials are for use as side dressing and on certain vegetable and horticultural crops. The following equations simulate the straight nitrogen requirements:

	For	Straight N required		Straight N supplied
(126)	Area 1	7,400	$\leq$	$\sum .20X_1, .46X_{64}$
(127)	Area 2	19,000	$\leq$	$\sum .20X_2, .46X_{65}$
(128)	Area 3	18,200	$\leq$	$\sum .20X_3, .46X_{66}$
(129)	Area 4	21,100	$\leq$	$\sum .20X_4, .46X_{67}$
(130)	Area 5	2,600	$\leq$	$\sum .20X_5, .46X_{68}$
(131)	Area 6	1,100	$\leq$	$\sum .20X_6, .46X_{69}$
(132)	Area 7	37,400	$\leq$	$\sum .20X_7, .46X_{70}$
(133)	Area 8	4,800	$\leq$	$\sum .20X_8, .46X_{71}$
(134)	Area 9	6,700	$\leq$	$\sum .20X_9, .46X_{72}$

Solution and interpretation

This completes the restrictive equations as proposed for the spatial matrix. Thus, there are 64 restrictive equations (Equations 71 through 134 above) and 72 real activities. The program used for this problem requires no slack or surplus vectors (artificial activities) for equalities, but one is required for each inequality. In the restrictive equations all but the first 22 are inequalities; thus, there will be 42 artificial activities. The total matrix then becomes 64 x 114, subject to the minimization function.

The minimization function is:

$$(135) \sum_{i=1}^{72} c(j)\, X_i = \text{minimum}$$

In the solution to this problem, one feasible but not necessarily optimal solution is found. Table 25 is a summary of such a solution. It satisfies all the restrictive equations. The total cost of the fertilizer production and distribution system to Turkey, using this solution, would be $131,755,588. As it turns out, the first feasible solution very nearly simulates the optimal solution.

Table 26 is a summary of the optimal solution (with the complex located at Mersin). The minimum cost under this solution is reduced to $131,741,893, a reduction of only $13,695 from the first feasible solution as it appeared in the machine printout.

A detailed analysis of the optimal solution to this spatial program can be very helpful in establishing guidelines for policy decisions in Turkey in coming years. Help is available in determining in which geographic areas to place emphasis on an introductory program for high-analysis fertilizers. Where should emphasis be placed on urea? Where should the emphasis remain on ammonium sulfate? Where should programs for introduction and use of TSP and DP be emphasized? Guideline answers to these and many other questions are found from an analysis of the printout solution.

In the original set of restrictions the Kutahya plant to produce ammonium sulfate was "forced" into full capacity operation without regard to economic costs (a governmental decision based upon factors other than economics). The output of the plant under the optimal solution should be

Table 25. A feasible solution to the spatial problem (plant complex located at Mersin)

Product and location produced	Fertilizers used in area 1	2	3	4	5	6	7	8	9	Total
Amm. sulf. - Kutahya	90,000	162,459	135,500	48,000	4,000	800	77,600	20,400	51,241	590,000
NSP - Izmit	28,311	30,000	63,889	-	-	-	-	-	-	122,200
NSP - Iskenderun	4,489	-	-	77,711	-	-	-	-	40,000	122,200
NSP - Ergani	92,756	-	-	-	38,333	61,667	10,555	18,889	-	222,200
Subtotal NSP	(125,556)	(30,000)	(63,889)	(77,711)	(38,333)	(61,667)	(10,555)	(18,889)	(40,000)	(466,600)
P_2O_5 in NSP	(22,600)	(5,400)	(11,500)	(14,000)	(6,900)	(11,100)	(1,900)	(3,400)	(7,200)	(84,000)
TSP - Samsun	111,556	7,978	45,111	-	-	-	20,889	36,666	-	222,200
TSP - Mersin	-	-	-	-	-	-	-	-	66,175	66,175
DAP - Mersin	-	63,935	-	51,765	-	-	-	-	19,176	134,876
Urea- Mersin	-	-	-	55,179	6,087	11,174	54,739	17,870	-	145,049
Total N	18,000	44,000	27,100	44,300	3,600	5,300	40,700	12,300	13,700	209,000
Total P_2O_5	72,800	38,400	31,800	37,800	6,900	11,100	11,300	19,900	45,800	275,800
Straight N	18,000	32,491	27,100	34,982	3,600	5,300	40,700	12,300	10,248	184,721
Total NH_3 - Mersin	-	14,705	-	43,635	3,500	6,425	31,475	10,275	4,410	114,425
Total H_3PO_4 - Mersin	-	30,049	-	24,330	-	-	-	-	30,189	84,568

Table 26. Optimal solution to spatial problem (complex located at Mersin)

Product and location produced	Fertilizers used in area 1	2	3	4	5	6	7	8	9	Total
Amm. sulf. - Kutahya	90,000	169,087	135,500	48,000	4,000	800	77,600	20,400	44,613	590,000
NSP - Izmit	28,311	30,000	63,889	-	-	-	-	-	-	122,200
NSP - Iskenderun	23,311	-	-	58,889	-	-	-	-	40,000	122,200
NSP - Ergani	73,933	-	-	-	38,333	61,667	10,556	37,711	-	222,200
Subtotal NSP	(125,555)	(30,000)	(63,889)	(58,889)	(38,333)	(61,667)	(10,556)	(37,711)	(40,000)	(466,600)
P_2O_5 in NSP	(22,600)	(5,400)	(11,500)	(10,600)	(6,900)	(11,100)	(1,900)	(6,800)	(7,200)	(84,000)
TSP - Samsun	111,556	15,507	45,111	-	-	-	20,889	29,137	-	222,200
TSP - Mersin	-	-	-	3,556	-	-	-	-	58,646	62,202
DAP - Mersin	-	56,570	-	55,652	-	-	-	-	26,542	138,764
Urea- Mersin	-	-	-	53,658	6,087	11,174	54,739	17,870	-	143,528
Total N	18,000	44,000	27,100	44,300	3,600	5,300	40,700	12,300	13,700	209,000
Total P_2O_5	72,800	38,400	31,800	37,800	6,900	11,100	11,300	19,900	45,800	275,800
Straight N	18,000	33,817	27,100	34,283	3,600	5,300	40,700	12,300	8,923	184,023
Str. P_2O_5	72,800	12,390	31,800	12,200	6,900	11,100	11,300	19,900	33,590	211,980
Total NH_3 - Mersin	-	13,011	-	43,653	3,500	6,425	31,475	10,275	6,105	114,444
Total H_3PO_4 - Mersin	-	26,588	-	27,294	-	-	-	-	31,242	85,124

concentrated in Areas 1, 2, 3, and 9. In Areas 4 through 8 only the minimum amounts as required because of custom and habit should be used.

The dual solution to the program provides some idea of the economic cost of these customs and habits (Table 27). An analysis of the dual solution for Area 4 indicates that substitution of urea or some other nitrogen material for ammonium sulfate would produce a saving equal to $4.83 per ton of ammonium sulfate.

However, these dual solutions should be viewed as only guidelines. They represent the value of one more or one less unit of a particular restriction with all other factors remaining constant.

Thus, many answers can be found from the solution to the spatial problem. Some of the answers may be definitive; others only give directions of emphasis needed. All are based upon an integrated fertilizer industry. So far in the solution, no answer has been attempted for determining the proper plant location.

Plant location

It was assumed that the proposed plant would be a highly flexible complex and that it would be the same general construction and have the same production capabilities regardless of which location is chosen. Thus, the problem is narrowed down to a very simple one. Should such a complex be located at Mersin or at Izmit?

Under these assumptions, the total matrix for this second model is composed of the restrictive equations and real activities as they were with the plant complex located at Mersin. Equations 71 through 134 are valid for the Izmit location. The only change is in the cost of the real activities

Table 27. Optimal dual solution to spatial problem (complex plant location at Mersin)

Restrictive real activity or restriction	Limit	$/Ton value of Increased limit	$/Ton value of Decreased limit
Production AS - Kutahya	590,000		92.67
Production NSP - Izmit	122,200		8.87
Production NSP - Iskenderun	122,200		15.32
Production NSP - Ergani	222,200		13.39
Production TSP - Samsun	222,200	1.66	
AS use in area 4	48,000		4.83
AS use in area 5	4,000		10.67
AS use in area 6	800		11.77
AS use in area 7	77,600		2.80
AS use in area 8	20,400		2.26
NSP use in area 1	22,600		51.28
NSP use in area 2	5,400		32.12
NSP use in area 3	11,500		4.77
NSP use in area 4	10,600		1.18
NSP use in area 7	1,900		43.18
NSP use in area 9	7,200		30.96
Straight P_2O_5 use in area 4	12,200		5.42

in the objective function, i.e., the c(j), of the activities X_{46} through X_{72} (since these are the real activities resulting from the complex facility).

The solution of this second model, with the complex plant located at Izmit, results in several small changes in product mix. See Table 28. The complex at Izmit would produce more DP and less TSP and urea than that at Mersin.

Distribution of its product would also be changed, with less going to Areas 4 and 9 and more going to Areas 2 and 3. The optimal production, transportation, and distribution costs involved would be $134,274,362.

Thus, the cost of the total fertilizer program with the complex located at Izmit instead of at Mersin would be increased by more than $2.5 million. It was recommended that the location of the complex fertilizer facility be at Mersin.

Effect of governmental decisions

The foregoing spatial models have each been constructed in such a manner as to simulate the governmental policy that the production capacity of plants such as that at Kutahya will be used without regard to the economic logic. Equations 89 through 92 have been inserted into the matrix as equalities, thus ensuring that total capacity of these four plants is used prior to use of materials from the proposed plants at Samsun and at Mersin or Izmit, as the case may be. This was done as a result of a governmental decision--one taking into account other possible gains as well as the economics of producing fertilizer.

Once the spatial model has been constructed and solved, it becomes a simple task to analyze the economic costs of such a decision based upon

Table 28. Optimal solution to spatial problem (complex located at Izmit)

Product and location produced	Fertilizers used in areas									
	1	2	3	4	5	6	7	8	9	Total
Amm. sulf. - Kutahya	90,000	155,435	135,500	65,548	4,000	800	77,600	20,400	40,717	590,000
NSP - Izmit	28,311	30,000	63,889	-	-	-	-	-	-	122,200
NSP - Iskenderun	4,489	-	-	77,711	-	-	-	-	40,000	122,200
NSP - Ergani	92,756	-	-	-	38,333	61,667	10,555	18,889	-	222,200
Subtotal NSP	(125,556)	(30,000)	(63,889)	(77,711)	(38,333)	(61,667)	(10,555)	(18,889)	(40,000)	(446,600)
Subtotal P_2O_5 in NSP	(22,600)	(5,400)	(11,500)	(14,000)	(6,900)	(11,100)	(1,900)	(3,400)	(7,200)	(84,000)
TSP - Samsun	111,555	-	-	-	-	-	20,889	36,667	53,089	222,200
TSP - Izmit	-	-	45,111	-	-	-	-	-	1,133	46,244
DAP - Izmit	-	71,739	-	51,765	-	-	-	-	30,870	154,374
Urea- Izmit	-	-	-	47,549	6,087	11,174	54,739	17,870	-	137,419
Total N	18,000	44,000	27,100	44,300	3,600	5,300	40,700	12,300	13,700	209,000
Total P_2O_5	72,800	38,400	31,800	37,800	6,900	11,100	11,300	19,900	45,800	275,800
Straight N	18,000	31,087	27,100	34,982	3,600	5,300	40,700	12,300	8,143	181,212
Straight P_2O_5	72,800	5,400	31,800	14,000	6,900	11,100	11,300	19,900	31,600	204,800
Total NH_3 - Izmit	-	16,500	-	39,247	3,500	6,425	31,475	10,275	7,100	114,522
Total H_3PO_4 - Izmit	-	33,717	14,436	24,330	-	-	362	-	14,509	87,354

factors other than simple economic efficiency.

The only change necessary to the spatial model in this case, in order to analyze the added cost of full capacity operation of the four specific plants, is to change Equations 89 through 92 to inequalities, i.e., delete Equations 89 through 92 from the matrix and insert the following:

$$(136) \quad 590{,}000 \geq \sum_{i=1}^{9} X_i$$

$$(137) \quad 122{,}200 \geq \sum_{i=10}^{18} X_i$$

$$(138) \quad 122{,}200 \geq \sum_{i=19}^{27} X_i$$

$$(139) \quad 222{,}200 \geq \sum_{i=28}^{36} X_i$$

The optimal solution to the spatial model, when these changes are made and Mersin is chosen as a site for the complex facility, indicates a total cost of producing and distributing the required amounts of fertilizer of only $119,393,198. Thus, the added costs of the governmental decision to run these four production facilities at full capacity will exceed $12 million--almost 10 percent of the total cost of the Turkish fertilizer industry.

When Equations 136 through 139 are substituted into the matrix simulating the Izmit location, the optimal solution is reached at a cost of $122,152,652. This total cost is also approximately $12 million below the original solution in which the restrictive governmental decision was included, but it is about $3 million higher than the optimal cost with the complex fertilizer facility located at Mersin.

Thus, it appears that regardless of whether the complex facility is located at Mersin or at Izmit, the economic cost of the restrictive governmental policy decision approximates $12 million per year. Whether such a policy is warranted becomes a decision to be based upon factors other than economics.

The optimal solution, using this second set of equations, i.e., those which are not restrictive as to the productive output of specified plants, indicates that changes would be made in the geographic distribution of many of the total fertilizer products. Urea would be introduced into all areas other than that immediately adjacent to the Kutahya plant. One of the high-analysis phosphatic materials, either TSP or DP, would be introduced to every area of the nation. The total productive capacity of the complex at Mersin would be utilized.

Practical limitations of model

There are three major limitations to use of this model under practical situations. These limitations in some cases prevent the model from being a true presentation of the fertilizer industry as it will exist. The limitations also preclude any probability that the exact level of total costs will be attained. At the same time, the answers obtained as to direction of planning efforts can be useful to those who are charged with the planning responsibilities.

The model is static, not dynamic. The industry which it simulates is dynamic. Thus, any one application of the model simulates only the conditions which are expected to exist at one point of time in the future. However, once the basic matrix has been developed (i.e., the matrix representing the already existing plant production facilities), different levels of regional demand and different types of possible new plant configurations and locations may be easily simulated mathematically. Thus, the model may be used to simulate a dynamic situation, but only by successive changes and solutions to the matrix.

The model assumes that regional demand levels are known and fixed at an absolute level for a particular point of time. Although much research effort has been directed to improving predictions of fertilizer demand, the task of making regional predictions in a developing country imposes in many cases a severe limitation on practical application of the model. Few developing countries have available the detailed statistics on cropping patterns and fertilizer use which are considered necessary for such long-range projections.

The third major limitation of the model with respect to practical applications is that the inherent assumption is made that production costs of each fertilizer material remain constant regardless of level of plant output. This assumption of constant cost per unit of output with varying levels of operation is not a valid assumption. In the fertilizer industry, however, this assumption is a close approximation to reality over a relatively wide range of the possible operating levels. Within the range of 80 to 100 percent of capacity operation in a phosphatic fertilizer complex, the average unit cost curve is relatively horizontal. This results from the high ratio of variable costs to fixed costs within this range. Thus, although the assumption of constant average unit costs is not a truly valid one, it can be used within the model as an approximation to reality.

One additional minor limitation is imposed by using the assumption of constant average unit costs. This assumption approximates reality over only a limited range of the average cost curve. Thus, the planner, in constructing his mathematical simulation of the supply of fertilizers, should insist that the simulations of plant size do not appreciably exceed total demand.

Versatility of spatial models

Once the basic spatial model of the industry of one of the developing nations has been constructed, it becomes possible to investigate the relative costs of any proposed new fertilizer venture. With one computer run a new type plant can be simulated and inserted into the total production and distribution matrix. Thus, a total "systems analysis" is possible.

If the relative prices of input factors change--as frequently happens--there are relatively few changes to be made in the basic matrix before recomputing the optimal solution. With changes in prices of input factors and no technological changes, only the $c(j)$'s--the real activity costs--are changed.

With changing technology, only that portion of the matrix in which technology is changed is affected. The remainder of the matrix remains constant. Thus, the spatial model of the fertilizer industry can be helpful to administrators charged with the responsibility of aiding the economic planners of developing nations.

The model which has been presented herein has been used in this manner. The optimum solution, as presented in table 26, was accepted as the official 5-year goal for Turkey in mid-1966.

By early 1967, three different foreign firms had approached the Undersecretary and Director of the State Planning Organization of Turkey, Mr. Turgot Ozal, with proposals of slightly modified plant configurations.

It has been possible to simulate each of these proposals mathematically and to substitute the new proposals for the portion of the optimal plan for which they would act as substitutes.

None of the proposed alternatives has as yet led to a more economical total industry, although one has closely approached the optimum.

These plans are considered of a highly proprietary nature and thus cannot be presented. They would not add more to the thesis except in mere bulk, since each follows the pattern used herein in substituting the Izmit location and costs for the Mersin optimum configuration.

SUMMARY

Background

The general objective of this thesis was to investigate possible applications of linear programming techniques to the developing fertilizer industry of the United States and of the world.

The world population is expanding rapidly. These increased numbers of people must be fed if widespread famine is to be averted. Increased use of fertilizer along with other inputs of agricultural production, such as, insecticides, improved varieties of seed, and improved farming practices, offers one means of increasing food production.

The specific objectives of this study were (1) to analyze the potential capacities of the fertilizer industry to produce nitrogen, phosphate, and potash fertilizer with relation to expected future needs, (2) to analyze earlier applications of linear programming for determining least-cost mixes and to investigate the practical applications which might be made thereof, (3) to prepare a practical linear programming model of a complex fertilizer facility of the type which is being developed by current technology and to analyze the practical applicability of such a mathematical simulation of an integrated production facility, and (4) to simulate the entire fertilizer production and distribution facilities of an emerging nation--to develop a modified spatial equilibrium model which can be used to reduce the cost of production and distribution of the required amounts of fertilizers.

Present and future capacity of the world fertilizer industry

This study indicates that there now exists within the world sufficient fertilizer productive capacity to supply fertilizer for use at rates much higher than has been the case in the past and higher than those expected for the immediate future. Much of this productive capacity, however, is located in the developed areas of the world. Little of the productive capacity is located in the developing areas where the greatest increases in use are needed.

Changes in technology, however, have made it possible to locate production facilities within the emerging nations. With new technology of nitrogen production, this nutrient can be produced almost anywhere in the world. Phosphate and potash deposits are limited to a very few locations, but production facilities to process these raw materials can be located almost anywhere.

In summary, then, the raw material input factors to produce finished fertilizer products are amply available. Technology for producing finished products is available. A major problem remaining is to determine the most economical total system to produce and distribute this fertilizer to farmers.

Linear programming applications to least-cost mixes

The applications of simple linear programming techniques to determining least-cost mixes of dry blended fertilizer materials have been studied in detail. It has been found that this application is one which can be highly useful for determining the minimum-cost combination of a large group of multinutrient fertilizer materials when a specified end product such as a 17-17-17 grade fertilizer is required.

As a byproduct of this linear programming operation, it has been found that the shadow pricing concept can be highly useful in pricing of new and unique types of multinutrient fertilizer materials.

Linear programming application for a complex fertilizer facility

With the introduction of complex technology and complex fertilizer materials to the fertilizer industry, it becomes imperative that a total systems analysis be made available to management of these complex facilities to aid in the necessary decision-making process. The linear programming approach is one such system which may be useful.

The production operations of the highly complex fertilizer facility of the Tennessee Valley Authority has been simulated mathematically in one linear programming matrix. Into this one matrix have been incorporated the upper and lower demand requirements for each of the end products. Also, the upper and lower limits of production capacities (supply limitations) have been incorporated in all cases wherein these production capacity limitations present foreseeable bottlenecks. Restrictions in the capacities to produce both end products and intermediate products have been considered and mathematically simulated.

An analysis of the solution to the total plant mathematical matrix proves helpful to management decisions with respect to both the amounts and the types of products to be produced. Analysis of the results also indicate the most profitable direction of efforts in various introductory programs.

An analysis of the dual solution to the programming problem can be quite helpful in management considerations of plant facility improvement. The analysis of the dual solution attaches specific economic values to the removal of process and product restrictions in the form of upper limits of production capabilities. Thus, a specific economic return can be computed from any specific proposed addition to the total production facilities.

A spatial model for Turkey

A modified spatial equilibrium model has been prepared for the entire fertilizer production and distribution facilities for one emerging nation--Turkey. The input-output coefficients for this model are those which were collected while the second 5-year fertilizer plan for Turkey was being prepared during early 1966.

Analysis of the results of the optimal solution to this problem is useful in determining the most economical total production and distribution system for that nation at a future time. The basic matrix, once it is completed, can be used as a starting point in analyzing additional facilities or types of fertilizer materials for future development of the nation's industry. Alternative plant locations are analyzed to determine relative economic costs. The economic effect of governmental policies on bases other than economic have been analyzed.

It is suggested that use of this spatial model of a nation's fertilizer industry may be of considerable aid to policymakers in their attempts to assist developing nations. The basic model with minor variations and the necessary changes resulting from different geographic locations can be readily adapted to other nations.

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