Design of packet-based soft-modem for a band-limited radio channel

by

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Abstract

The performance of early modems is mainly limited by the hardware; the signal processing of modulation and demodulation are carried out by analog circuitry, which imposed severe restriction on complexity of the algorithms. With the advent of Digital Signal Processors, most of the modem functions are now performed in software. The ability to program the DSP with any communication software makes the modem adapt to more than one communication network. Such a modem is defined as a soft-modem. The communications software today implements most powerful algorithms by utilizing the high processing speeds of the DSPs.

This work aims at designing an inexpensive soft-modem between two computers separated by a distance of 100-300 meters. An ideal software radio has all the communication blocks of Modulation, Synchronization, Equalization and Channel coding performed by signal processing software running on a DSP rather than a dedicated analog and digital circuitry. The soft-modem, on the other hand, captures most of the features of a software radio but it replaces the expensive DSPs with the processing capabilities of a PC. The IF up/down converters are replaced by FM transceivers. The DAC and ADC on the soundcards synthesize and capture the analog waveforms. The channel is modeled as linear gaussian channel with ISI. A ML joint synchronization and equalization is derived and implemented for a QAM constellation size of 256 points. Various coding methods to achieve high coding gains on this soft-modem are discussed.

1 Definitions of software-defined radio and the soft-modem

1.1 Introduction

Traditionally, the analog signals such as audio and video were the only signals used for communication, but with the invention of digital representation of analog signals (sampling theorem by Fourier) and the important discovery of the fundamental unit of data communication (BIT by Shannon) the world of digital communication came into existence. Since the channels are still analog, orthonormal basis functions are used to map the bits to channel waveforms. Traditionally, analog filters with the help of mixers and antennas used for mapping the digital signals into analog waveforms and vice versa (modulation and demodulation). Microprocessors were interfaced to the communication devices through digital logic circuitry that writes and reads the transmitted and received data bits from and into the memory. Over the years with the advances in Digital Signal Processing algorithms as well as the architectures, the computing power of the DSPs has reached to an extent that the most of the signal processing blocks in a communication device are performed on a microprocessor in digital domain or in other words the software running on the microprocessor defines the functionality of the communication device. In the world of wire-line communications (in a cable modem, for example), the software running on the DSPs controls all the blocks of transmission and reception over the cable but in the wireless world, few blocks of up-conversion and down-conversion still remain analog. This chapter begins with a discussion on ideal software-defined radio, the state of art of present day's digital radio architecture and finally describes the architecture of soft-modem.

1.2 Software radio definition

An ideal software defined radio has all the communications signal processing blocks implemented on digital signal processors; The only analog blocks (mixed-signal) are the ADCs and DACs, which are used to synthesize the RF waveforms from the digital samples. A rigorous and exact definition of the (SW) software radio concept does not yet exist, but the SW radio has the following broad objectives:

- Flexible TX/RX architecture, controlled and programmable by SW.

- Signal processing able to replace as much as possible, radio functionalities
- Air interface downloadability: radio equipment dynamically reconfigurable by downloadable SW at every level of the protocol stack
- SW realization of terminals "multiple mode/standard"
- Transceiver where the following can be defined by SW:
    1. Frequency band and radio channel bandwidth
    2. Modulation and coding scheme
    3. Radio resource and mobility management protocols
    4. User applications

1.3 The software radio transceiver

The transmitters and receivers employed in radio mobile systems are based on the traditional super-heterodyne scheme (Figure 1.1), where RF and IF stages are totally analog, while digital component is only present in the base-band state usually built in ASIC technology. On the other hand, the ideal scheme of a SW radio transceiver has a very reduced analog stage. The only analog components are the antenna, the band-pass filter, and the low noise amplifier (LNA) (Figure 1.2). A/D conversion is done immediately at RF in order to digitally elaborate the signal on a completely re-programmable board.

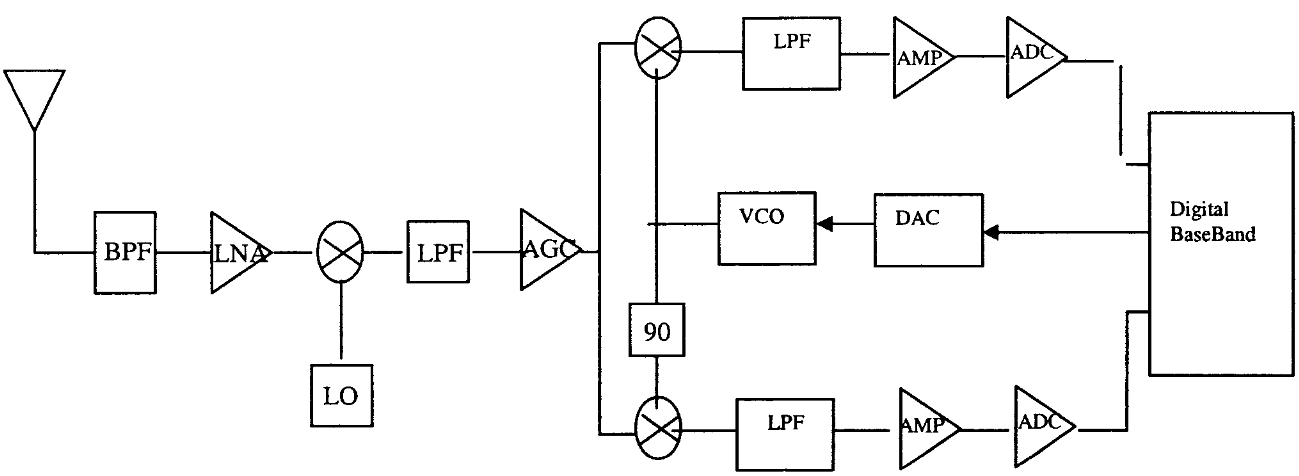

Figure 1.1 A traditional super-heterodyne receiver

The SW radio receiver shown in Figure 1.2 has been defined as ideal because there are several matters that make it, at the moment, far from realizable. Firstly, it is not reasonable to use a single RF stage for a multi-band system due to the impossibility of building antennas and LNAs on a bandwidth ranging from hundreds of MHz to units or tens of GHz. The only way to guarantee the multi-band feature will be to have more RF stages, depending on the radio band used for the SW radio system (900MHz for 2G mobile systems such as GSM and 2 GHz for 3G systems). Also, jitter effects make A/D conversion directly at RF very difficult.

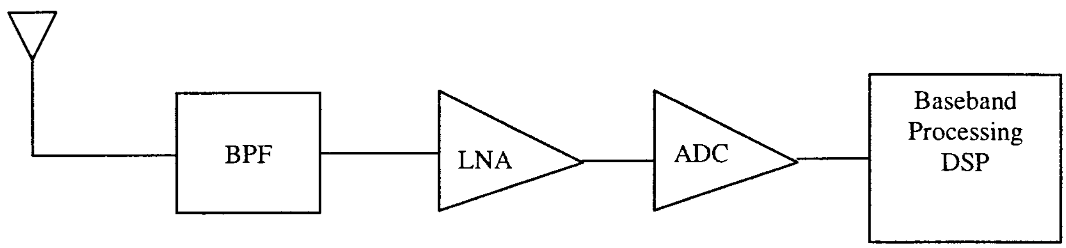

Figure 1.2 An ideal SW radio receiver

The most promising solution, at the moment, is known as the *digital radio transceiver,* whose receiver section is shown in Figure 1.3. Its structure is very similar to the wideband transceiver; with RF stage completely analog and digital extending toward IF. The A/D converter samples the overall spectrum allocated to the system, while the programmable downconverter provides the following operations:

1. *Downconversion:* Digital conversion from IF to BB, using a lookup table containing the samples of a sinusoidal carrier. The lookup table replaces the local oscillator used in an analog downconverter.
2. *Channelization:* Selection of carrier and channel to be elaborated by digital filtering. This operation in analog filter, with very stringent requirements, before BB conversion.
3. *Sample rate Adaptation:* Undersampling of the signal output of the channelization filter to match the sample rate to the selected channel bandwidth, which is a narrowband signal, compared to a wider-spectrum A/D input signal.

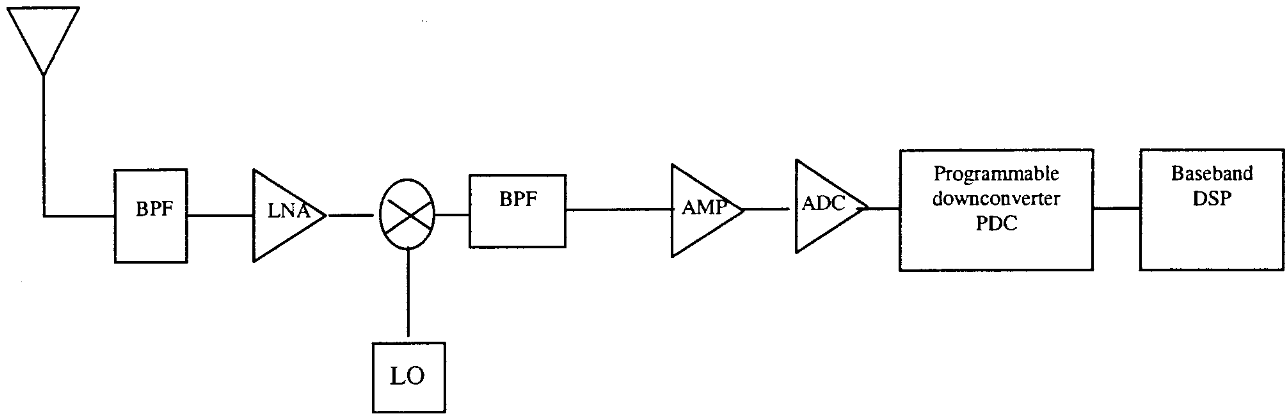

Figure 1.3 A digital radio receiver

The development of a digital radio transceiver presents a few difficulties at IF and BB stages. At IF stage the ADC and DAC performance is the bottleneck. There is a tradeoff between sampling rate and the resolution; higher the sampling rate, the lower is the resolution.

Today's technology allows reaching a 1 GHz sampling rate with 6-8 resolution bits, decreasing to 100MHz with 10 bits and 150KHz with 16 bits. The bit number can be insufficient, taking into account that the signals to be sampled can have high dynamic range. The GSM signal, for example, has a dynamic range of around 100dB from -104dbM for the minimum received signal to 13dBm for the maximum. The limited bandwidth of the converters, frequency jitter, and inter-modulation products are the problems afflicting the sampled signal, which remain to be solved. It has been shown that effective representation of GSM and UMTS waveforms with their dynamic range requires 17-20 bits of resolution.

The BB also has a few difficulties to be overcome due to processing power and power consumption of the DSP engines. The processing power has to be enough to allow real-time execution of SW-implemented radio interfaces. This could require the use of several DSPs in parallel, depending on the complexity of the radio interface to be implemented. Since a SW radio system should adapt to different standards, it is necessary to dimension processing power to the worst-case situation. The solutions adopting multi-user detection or beam-forming algorithms cause exponential growth of needed processing power. (10s of GIPS).

1.4 The soft-modem

The soft-modem is constructed using inexpensive radios utilizing the ADC and DAC capabilities of the soundcard. As mentioned in the previous section, the digital radio transceiver (Figure 1.3) is currently, the closest we can reach to the ideal software defined radio (Figure 1.2). But the high speed ADCs and the programmable down converter (PDC) of the IF stage are still very expensive, instead, the RF and the digital IF stages of the soft-modem are replaced by analog FM transceivers. The base-band of the soft-modem is fully digital, unlike the base-band of super-heterodyne receiver, where in analog oscillators convert the IF signal to its quadrature components.

The soft-modem consists of three components as shown in Figure 1.4:

1. A digital signal processor (PC) and the ADC, DAC (sound card) that facilitate waveform synthesis and capture
2. An FM transceiver (handheld radio) that replaces the RF and the IF stages of the digital radio transceiver
3. An interfacing circuit to switch the radios between Transmit and Receive modes

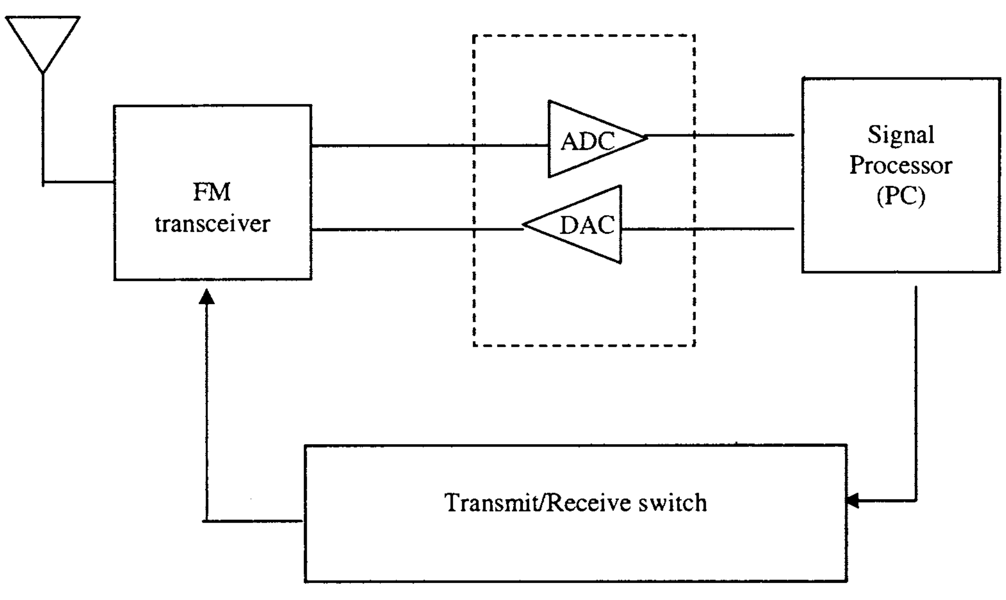

Figure 1.4 Components of soft-modem

All base-band processing is carried on the PC (It has a 1GHz Pentium III processor with a memory of 256MB). These specifications of the PC are not important because the modem is designed and tested using Matlab in non-real time design. But they play a key role when the soft-modem is implemented in real time on a linux OS. The waveforms are synthesized using the DAC present on the sound card. It offers sampling rates up to 48KHz at 16 bits per sample and a bandwidth of 22.05KHz. The hand-held radios have a bandwidth of only 3KHz, hence the effective bandwidth available to the soft-modem is only 3KHz. The Line-In present on the sound card provides a convenient interface with the ADC present on the PCI board. The ADC also can sample up to a sampling rate of 48KHz at 16 bits per sample and has an anti-aliasing filter of 22.05KHz. Chapter 2 discusses the channel characteristics of the sound card and the handheld radios in detail.

The hand-held radios are designed to provide a half-duplex FM link. The modulating (audio) signal has a bandwidth of 3KHz and a frequency deviation Δf of 12.5KHz. The FM transceiver is in receive-mode by default and listens on of the 14 carriers in the licensed band of 462.5625MHz to 467.7125MHz. The PTT (Press To Talk) switches the transceiver to transmit-mode. The PTT is driven by the computer's COM port through its RTS pin.

The real-time architecture of the soft-modem is shown in the Figure 1.5. RT (real-time) linux would provide various clock signals used for Transmitter and Receiver operation. The transmitter and the receiver modules would be a part of sound card driver running in kernel space. The Ethernet driver (kernel space) would communicate with the sound driver providing a transparent TCP/IP interface between the linux OS and the soft-modem.

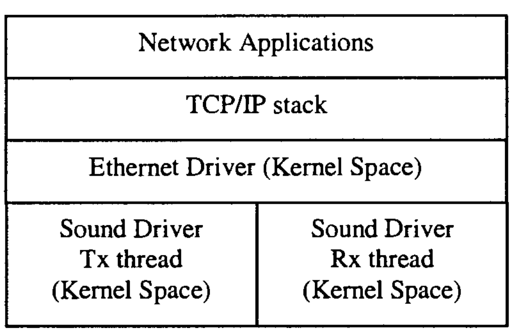

Figure 1.5 The real-time architecture of the soft-modem

The second chapter describes the equivalent channel model and modulation scheme of the soft-modem. Synchronization and Equalization are described in the third chapter. Lastly, the fourth chapter discusses the efficient coding schemes for this bandwidth limited channel.

2 Channel model and modulation

2.1 Introduction

This chapter describes the mathematical model of the channel. It is shown the combined FM transceiver system and the radio link can be reduced to an equivalent ISI channel corrupted by colored noise. The modulation and signal design used in the soft-modem are described. The later part of the chapter describes the conversion of linear continuous channel to an equivalent discrete-time channel.

2.2 Channel model

Let us now derive the channel model using the description of the soft-modem system in the first chapter. Figure 2.1 shows the system equivalent of the soft modem. The dashed line encloses the handheld radios and the radio link together of which constitute the channel for the soft-modem. Since the radios are used to transmit 3KHz speech signals, the input to the Frequency Modulator is filtered with a LPF $a(t)$. Also, the output of the discriminator is filtered with a LPF $b(t)$.

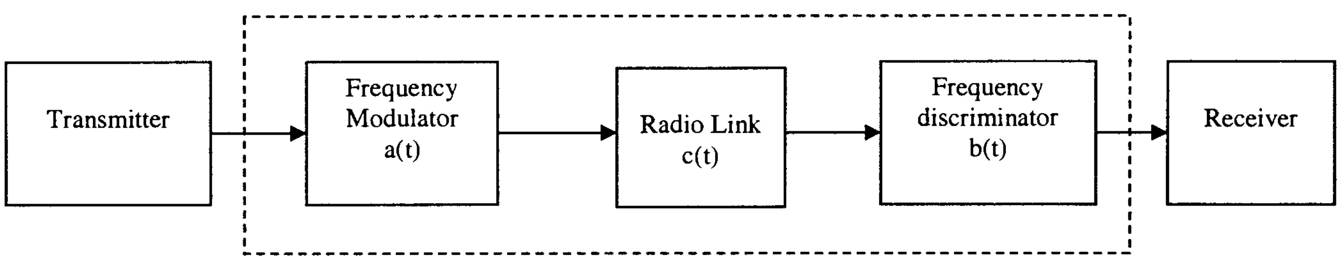

Figure 2.1 The system equivalent of the soft modem

The impulse response of the radio channel (at 465MHz), $c(t)$ is almost flat since the multi-path components are negligible at the low symbol rates of 3Ksym/s. If we assume the Frequency discriminator perfectly tracks the frequency and acts as an ideal low-pass filter so that the thermal noise $w(t)$ at the RF end of the handheld radios remains white at the output of discriminator, the mathematical model of the channel simplifies to AWGN channel with ISI shown in Figure 2.2.

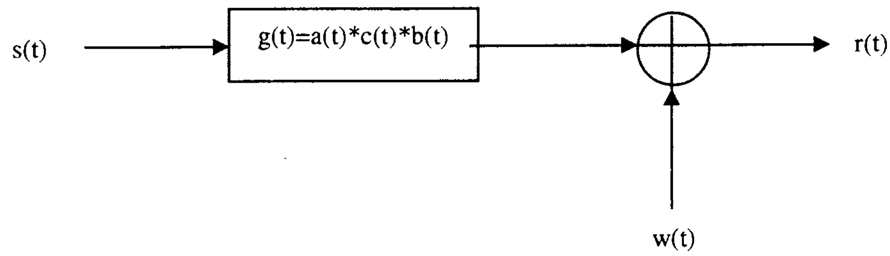

Figure 2.2 Simplified channel model, white noise

In reality the frequency discriminator is equivalent to a differentiator in cascade with envelope detector (an ideal low pass filter). Thus, the thermal noise when passes thorough the Frequency discriminator becomes colored. But since the signal is Frequency Modulated, it remains intact and the discriminator acts as LPF on the signal. The model can be reduced to an ISI channel with equivalent impulse response $g(t)$ but corrupted by Additive Colored Gaussian Noise $n(t)$ as shown in Figure 2.4

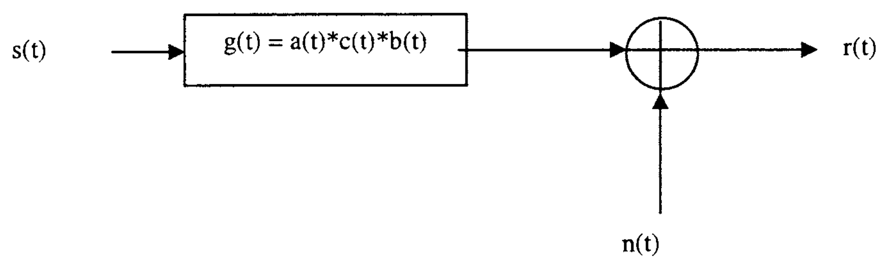

Figure 2.3 A more realistic channel model, colored noise

An approximate channel response $|G(f)|^2$ is shown in the Figure 2.4. A sinusoidal signal with linearly varying frequency (chirp signal) is transmitted through the first radio. The measured spectrum of received signal at the second radio, thus gives the approximate response. Also the phase response is observed to be linear over the transmission band. The 3 dB bandwidth of the system is about 3KHz. As the channel is not flat over the 3KHz bandwidth used for transmission, this is an ISI channel.

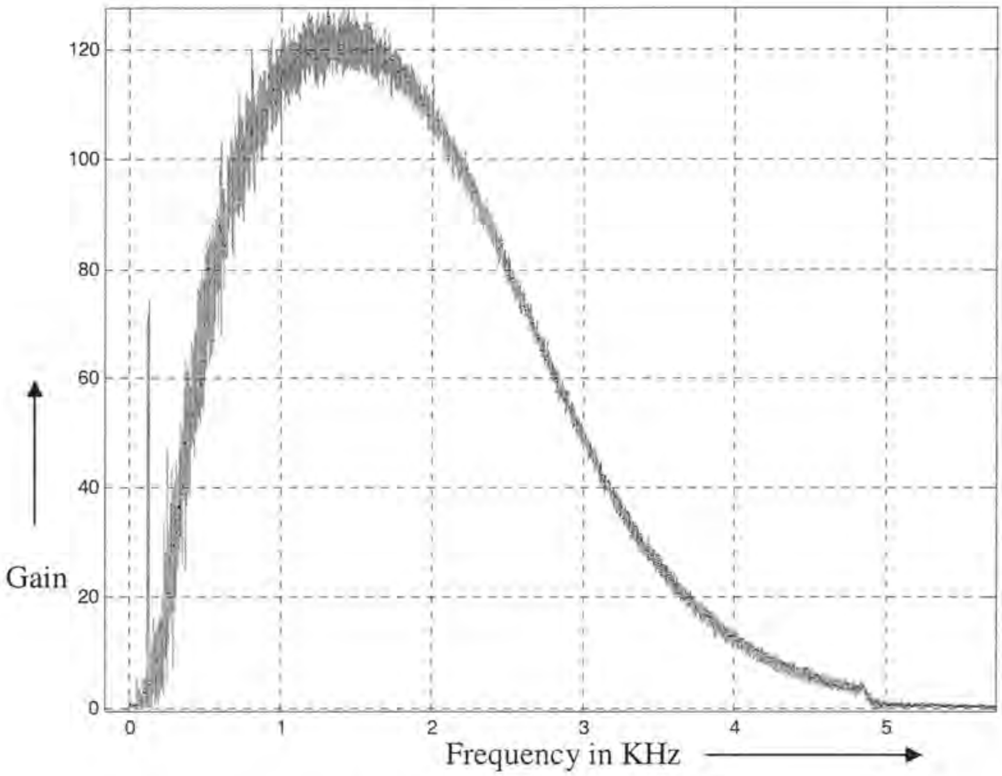

Figure 2.4 An approximate channel response

2.3 Modulation and signal design

Like all standard modems the soft-modem uses QAM on this high SNR channel with two orthogonal components sine and cosine centered at 1.5 KHz. The modulated waveform $s(t)$ is given by

$$s(t) = \mathrm{Re}\left(\sum_k a(k) g_T(t - kT) e^{-2\pi f_c t} \right)$$

and its equivalent complex envelope is

$$s_l(t) = \sum_k a(k) g_T(t - kT)$$

The center frequency f_c is 1.5KHz. The bit stream $b(k)$ is gray coded into M -ary QAM constellation with $a_i(k)$ and $a_q(k)$ as in-phase and quadrature phase components. These are compactly represented by one complex variable,

$$a(k) = a_i(k) + a_q(k)$$

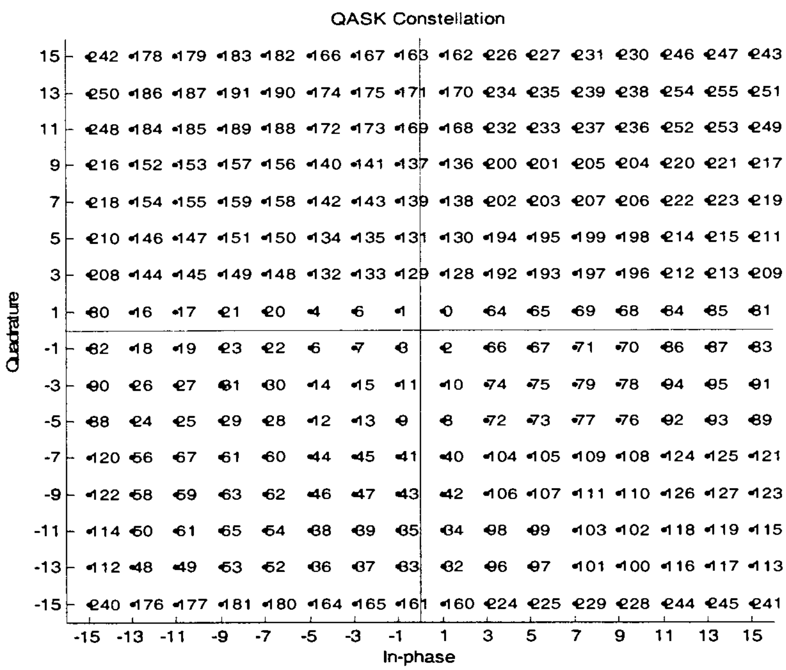

Figure 2.5 256 gray code constellation

A 256 gray code constellation is shown in the Figure 2.5. $s(t)$ is synthesized at a sampling rate, f_s of 48KHz and is reconstructed by DAC present on the soundcard. The bit period T is chosen to be $1/f_c$ so that one period of carrier represents one-symbol -duration. The resultant uncoded bit rate is, therefore, 12Kbps (=1.5*8Kbps). It will be shown later, since the waveforms are synthetically generated, carrier phase and symbol timing are dependent and only one of them needs to be recovered by the receiver. Since f_s/f_c =32 is an integer, the carrier could be easily generated using a lookup table of sinusoidal samples.

Figure 2.6 shows the mathematical model of the entire system including the transmitter and receiver filters. The channel $g_l(t)$, the transmitter filter $g_T(t)$ and the receiver filter $g_R(t)$ all represent low pass complex envelope equivalents. It is well known that the water-pouring solution gives the optimum transmitter power spectral density. Referring back to equivalent representation of the channel shown in the Figure 2.7, the optimal transmitted $psd, P_s^o(f)$ is given by

$$P_s^o(f) = \begin{cases} K - 1/SNR_c(f) & f \in B \\ 0 & f \notin B \end{cases}$$

where $B = \{f : P_s^o(f) > 0\}$ is called the capacity-achieving band, and K is the constant chosen such that

$$\int_{f>0} P_s(f)df = P$$

where P is the total power available for transmission. The channel SNR function is defined by $|G(f)|^2/N(f)$.

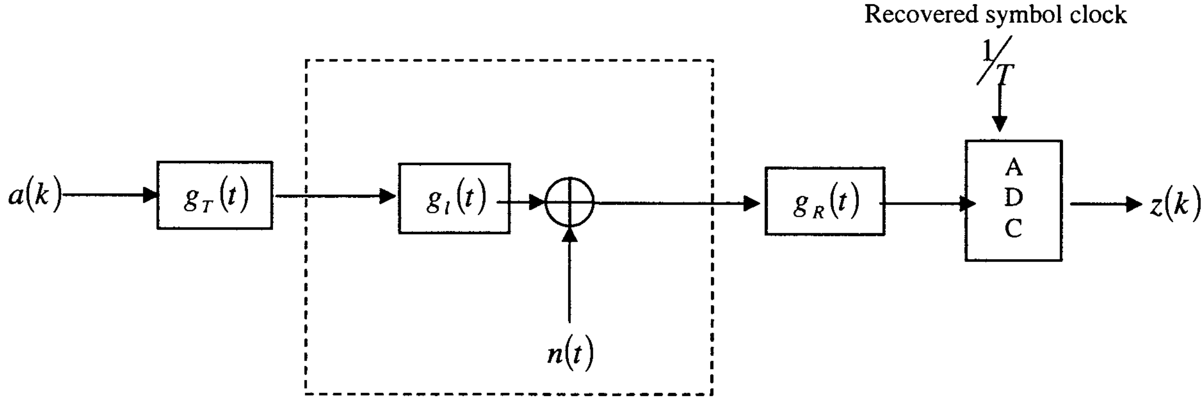

Figure 2.6 Mathematical model of the entire system

Intuitively, the preferred transmission band is where $SNR_c(f)$ is largest. The optimum $P_s(f)$ maximizes the mutual information between channel input and output subject to the power constraint. Graphically, the water pouring solution is shown in the Figure 2.7.

To build a transmitter filter $g_T(t)$ that achieves capacity, we thus need to estimate the channel SNR function. In other words, we need to estimate the channel $g_l(t)$ as well as the power spectrum (or autocorrelation) of the noise $n(t)$. However, the soft-modem uses square-root raised cosine filter shown in Figure 2.8 and avoids estimation of the noise spectral density.

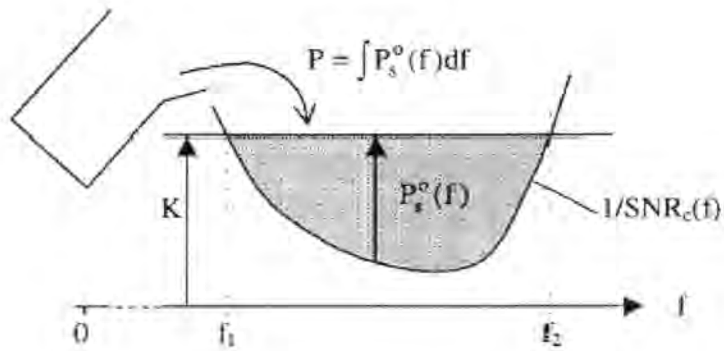

Figure 2.7 Water pouring solution

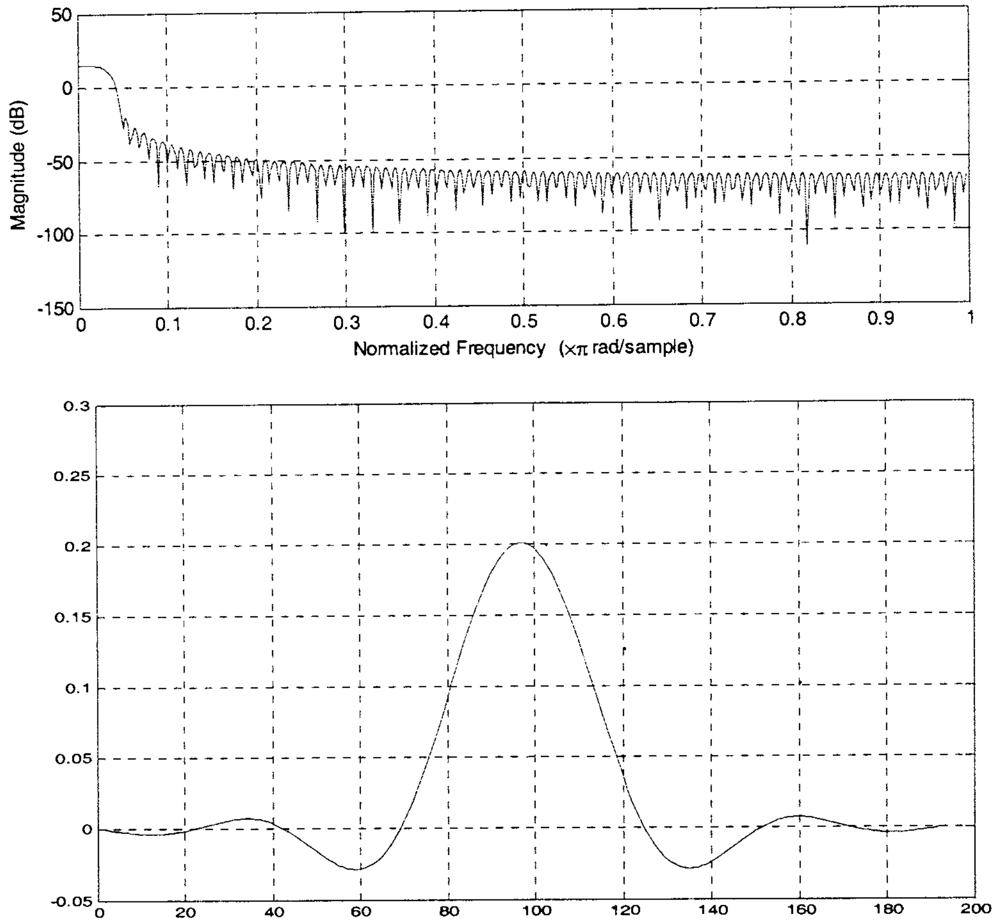

Figure 2.8 Raised cosine frequency and time domain plots

The receiver filter $g_R(t)$ could be designed such that the waveform channel with combined response of the transmitter filter $g_T(t)$, the channel $g_l(t)$ and the receiver filter $g_R(t)$ along with the colored noise $n(t)$ is transformed to a discrete-time channel with ISI and i.i.d Gaussian noise. To do this the received signal $r(t)$is first passed through the noise whitening filter $g_w(t)$, the transfer function of which is chosen such that

$$|G_w(f)|^2 = 1/N(f)$$

over the band B. The resulting complex signal is then

$$r'(t) = \sum_k a(k)v(t - kT) + w(t)$$

where $v(t)$ is a symbol response with Fourier transform $V(f) = G_T(f)G_l(f)G_w(f)$ and $w(t)$ is normalized AWGN with p.s.d 1 over the signal band B.

Since the set of responses $\{v(t-kT)\}$ represents a basis for the signal space, by principles of optimum detection theory the set $\{z'(k)\}$ of T-sampled matched-filter outputs $z'(k) = \int r'(t)v^*(t-kT)dt$ of a matched filter (MF) with response $v^*(-t)$ is a set of sufficient statistics of the detection of the symbol sequence $\{a(k)\}$. Thus no loss of mutual information or optimality occurs in the course of reducing $r(t)$ to the sequence $\{z'(k)\}$. The composite receive filter consisting of the noise-whitening filter and the MF has the transfer function $G_R^{MF}(f) = G_w(f)V^*(f)$ and the Fourier transform of the end-to-end symbol response is given by $Q(f) = G_T(f)G_l(f)G_R^{MF}(f)$

The sampled output sequence of the MF is given by

$$z'(k) = \sum q(l)a(k-l) + n(k)$$

where $q(k)$ are the sample values $q(kT)$, of the end-to-end symbol response $q(t)$.

So far, we have obtained without loss of optimality an equivalent discrete-time channel model, which can be written in D-transform notation as

$$z'(D) = a(D)q(D) + n(D)$$

By Spectral Factorization theorem the above equation can be written as

$$z'(D) = a(D)A^2h(D)h^*(D^{-1}) + w'(D)Ah^*(D^{-1})$$

in which $w'(D)$ is an i.i.d Gaussian noise sequence with symbol variance 1. Filtering $z'(D)$ by $\frac{1}{A^2}h^*(D^{-1})$ yields the channel model of the equivalent discrete time Gaussian channel

$$z(D) = a(D)h(D) + w'(D)$$

Thus the waveform channel in the Figure 2.3 reduces to a discrete time channel shown in the figure 2.9, and the receiver filter $G_R(f)$ is given by the *Whitened Matched filter*,

$$G_R^{WMF}(f) = \frac{G_R^{MF}(f)}{A^2H^*(f)}$$

Now, Viterbi algorithm could be used to for the Maximum Likelihood detection of the sequence $a(n)$ in the presence of finite ISI due to $h(n)$ and the i.i.d gaussian noise $w(n)$. A combination of Decision Feed-back Equalizer with the viterbi algorithm eliminates the ISI with a reduced complexity.

The soft-modem, however, uses the Square root-raised cosine filter with roll of factor of 0.5 as the receiver filter $g_R(t)$. The Whitening Matched filter is estimated as a part of equalizer. Chapter 4 describes Equalization in detail.

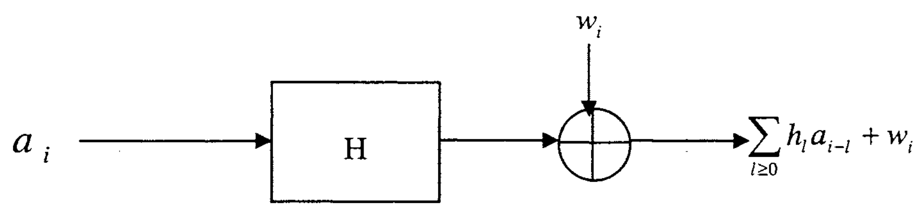

Figure 2.9 Equivalent discrete time channel model

3 Synchronization and channel estimation

3.1 Introduction

It is important to recover the carrier phase and the symbol timing accurately at the receiver to demodulate the data symbols. Traditionally, joint symbol timing and carrier tracking loops are used to trace changes in the phase and estimate the start and end of the symbol. But in a packet-based system such as soft-modem, the variations in phase and symbol timing are negligible over one packet duration and can be accurately estimated using a training sequence at the start of the packet. The non-ideal channel of the soft-modem introduces a considerable amount of ISI which when left unequalized leads to large BER. A Maximum Likelihood (ML) joint synchronization and Channel estimation is derived in this chapter. The problem could be compactly, posed as estimating the noise whitening filter and the effective channel filter which can be solved using well-known Weiner solution. Finally the physical layer packet structure of the modem is described and a few results on the linear equalization are presented.

3.2 Carrier and symbol synchronization

Since the propagation delay from the transmitter to the receiver is generally unknown to the receiver, symbol timing must be estimated from the received signal in order to synchronously sample the output of the demodulator. This is referred to as symbol synchronization. Also, in the general case, the phase of the carrier is independent of the symbol timing and hence has to be estimated independently. This is referred to as carrier synchronization.

3.3 Signal parameter estimation

Let a signal $s(t)$, be transmitted through an AWGN channel. Assuming that the channel has infinite bandwidth (we will consider the case of finite-bandwidth soft-modem later), if τ is the propagation delay in the channel and $n(t)$ is the AWGN, the received signal $r(t)$ can be expressed as

$$r(t) = s(t-\tau) + n(t)$$

where

$$s(t) = \mathrm{Re}\left[s_l(t)e^{j2\pi f_c t}\right]$$

$s_l(t)$ is the low-pass equivalent of $s(t)$. The received signal can be expressed as

$$r(t) = \mathrm{Re}\left\{\left[s_l(t-\tau)e^{j\phi} + z(t)\right]e^{2\pi f_c t}\right\}$$

where the carrier phase ϕ, due to propagation delay τ, is $\phi = -2\pi f_c \tau$. Now, from the formulation, it may appear that there is only one signal parameter to be estimated, either the propagation delay τ or the carrier phase ϕ, since if one is known the other can be derived knowing the value of the carrier frequency f_c. But the oscillator that generates the carrier signal for demodulation at the receiver is generally not synchronous in phase with that at the transmitter. Further, the two oscillators may be drifting slowly with time. Thus the received carrier phase is not only dependent on the time delay τ. Also the precision to which one must synchronize in time is for the purpose of demodulating the received signal depends on the symbol interval T. Usually, the estimation error in estimating τ must be a relatively small fraction of T. However, this level of precision is generally inadequate for estimating the carrier phase, even if ϕ depends only on τ. This is due to the fact f_c is generally large and hence, a small estimation error in τ causes a large phase error. In effect, we must estimate both parameters τ and ϕ to demodulate and coherently detect the received signal.

But if both the transmitter as well as the receiver is implemented on a signal processor as in the case of the soft-modem, there is hardly any drift in the oscillators since they are numerically controlled. This of course, assumes the ADC and the DAC used for digitizing the waveforms have negligible time-jitter, which is usually a good assumption. Also, if we could estimate first ϕ to a desired degree of accuracy, the symbol timing τ, to a very good degree of precision, is given by

$$\tau = \frac{\phi}{2\pi f_c}$$

Thus, in the case of soft-modem we need only to estimate the carrier phase ϕ.

There are basically two criteria that are widely applied to signal parameter estimation: maximum likelihood (ML) criterion and the maximum a posteriori probability (MAP) criterion. In the MAP criterion, the signal parameter (carrier phase ϕ, in our case) is modeled as random and characterized by an a priori probability density function $p(\phi)$. In the maximum-likelihood criterion, the signal parameter vector ϕ is treated as deterministic but unknown.

By performing an orthonormal expansion of $r(t)$ using N orthonormal functions $\{f_k(t)\}$, we may represent $r(t)$ by the vector of coefficients $\lfloor r_1\ r_{2 \ldots} r_N \rfloor \equiv \mathbf{r}$. The joint PDF of the random variables $\lfloor r_1\ r_{2 \ldots} r_N \rfloor$ in the expression can be expressed as $p(\mathbf{r} \mid \phi)$. Then, the ML estimate of ϕ is the value that maximizes $p(\mathbf{r} \mid \phi)$. On the other hand, the MAP estimate is the value of ϕ that maximizes the a posteriori probability density function

$$p(\phi \mid \mathbf{r}) = \frac{p(\mathbf{r} \mid \phi)p(\phi)}{p(\mathbf{r})}$$

Since we have no prior knowledge of the carrier phase, we may assume $p(\phi)$ is uniform over 2π. Thus the value that maximizes $p(\mathbf{r} \mid \phi)$ also maximizes $p(\phi \mid \mathbf{r})$. Therefore, the MAP and ML estimates are identical.

3.4 The likelihood function

Let us consider the ML phase estimation for the class of linear modulation techniques for which the received equivalent low-pass signal may be expressed as

$$r_l(t) = s_l(t-\tau)e^{-j2\pi f_c \tau} + z(t)$$

Now, since $\tau = \dfrac{\phi}{2\pi f_c}$, we have

$$r_l(t) = s_l(t,\phi)e^{-j\phi} + z(t)$$

If we assume that the orthogonal functions $\{f_k(t)\}$ in the section 3.2, are time shifted *sinc* functions $\left\{\sin c\left(\dfrac{t-kT_s}{T_s}\right)\right\}$, then the coefficients of vector $\mathbf{r}$ are just the time samples of $r_l(t)$ sampled at Nyquist rate $1/T_s$, after low-pass filtering using a filter of bandwidth $1/2T_s$. The filter bandwidth is chosen to be greater than or equal to bandwidth of the signal, $s_l(t,\phi)$. Thus in discrete form the equation reduces to

$$r_l(k) = s_l(k,\phi)e^{-j\phi} + z(k)$$

Note that the samples of noise at Nyquist rate are uncorrelated and hence $z(k)$ are i.i.d Gaussian variables. We have assumed, so far that the signal $s(t)$, passes through the channel undistorted and the noise is white. In other words, we have assumed the channel is ideal over the bandwidth of transmission. But in chapter 2 we have noticed that the channel is a non-ideal and is corrupted by colored noise. Hence the samples of the received signal $r_l(k)$ is given by

$$r_l(k) = g_l(k) \otimes s_l(k,\phi)e^{-j\phi} + n(k)$$

Now, the joint PDF $p(\mathbf{r} \mid \phi)$ could be expressed in a simple form provided, if the noise samples $n(k)$ are uncorrelated. To do that, we should first whiten the noise component, $n(k)$. Following the argument in chapter 2, passing $r(k)$ through the whitening filter, we have

$$r'_l(k) = g_{lw}(k) \otimes s_l(k,\phi)e^{-j\phi} + w(k)$$

where $w(k)$ is i.i.d complex Gaussian noise and g_{wl} is the result of convolving the channel $g_l(t)$and the noise whitening filter $g_w(t)$.

$$g_{lw}(t) = g_l(t) \otimes g_w(t)$$

The joint PDF can now be expressed as,

$$p(\mathbf{r}' \mid \phi) = \left(\frac{1}{\sqrt{2\pi\sigma}}\right)^N \exp\left\{-\sum_{k=1}^{N} \frac{\left|r'_l(k) - g_{lw}(k) \otimes s(k,\phi)e^{-j\phi}\right|^2}{2\sigma^2}\right\}$$

The ML estimate of ϕ is obtained by maximizing $p(\mathbf{r}' \mid \phi)$, which is equivalent to minimizing the function,

$$\Lambda(\phi) = \sum_{k=1}^{N} \left|g_w(k) \otimes r(k) - g_{lw}(k) \otimes s(k,\phi)e^{-j\phi}\right|^2$$

3.5 Joint phase and channel estimation

The carrier phase ϕ, can be estimated by maximizing the likelihood function $\Lambda(\phi)$. Two situations arise. In the first case, the transmitter sends the training sequence for the receiver to synchronize. In this case, if the training sequence $\{I_k\}$ is known to the receiver. Then the transmitted signal $s(t)$ is given by,

$$s(t) = \sum_k I_k g_T(t - kT)$$

Thus, the estimation of ϕ boils down to maximizing $\Lambda(\phi)$, in the presence of unknown filters, $g_w(t)$ and $g_{lw}(t)$. In the second case, the receiver does not have the prior knowledge of the transmitted sequence. Thus, in addition to unknown filters $g_w(t)$ and $g_{lw}(t)$, the transmitted signal $s(t)$ is an unknown. In this case, we also need to find the sequence $\{I_k\}$ for which the likelihood attains maximum. Since the complexity of this search grows exponentially with N, it is difficult to implement this algorithm even for a moderate length of the synchronization window.

Now, if the training sequence $\{I_k\}$ is known as in the first case, or in other words, the transmitted signal $s(t)$ is given, it is interesting to note that the ϕ dependency of the maximum likelihood function $\Lambda(\phi)$, could be absorbed into the filter $g_{lw}(t)$. To do this let us express the delayed signal $g_{lw}(t) \otimes s(t-\tau)e^{-j\phi}$ as,

$$g_{lw}(t) \otimes s(t-\tau)e^{-j\phi} = g_{lw}(t)s(t) \otimes \delta(t-\tau)e^{-j\phi} = g^*_{lw}(t) \otimes s(t)$$

where the new filter $g^*_{lw}(t)$ is given by

$$g^*_{lw}(t) = g_{lw}(t-\tau)e^{-j\phi}$$

This means the likelihood function, $\Lambda(\phi)$ could now be written as

$$\Lambda(g_w, g^*_{lw}) = \sum_{k=0}^{N-1} \left| g_w(k) \otimes r(k) - g^*_{lw}(k) \otimes s(k) \right|^2$$

Note that g^*_{lw} now becomes non-causal. Thus instead of estimating the phase and the channel separately, we estimate the whitening filter g_w and the effective channel filter g^*_{lw}. To do this, let us reformulate the estimation problem in terms of least squares estimate. Minimizing the likelihood function Λ is equivalent to minimizing the mean square error between the vector $\mathbf{x}$, given by

$$x(k)=\sum_{n} r(k-n)g_{w}(n)-s(k-n+1)g_{lw}^{*}(n-1)$$

and the desired signal vector $\mathbf{d}$, given by

$$d(k)=s(k)$$

where the variable $k \in \{0,1,2,\ldots N-1\}$. These can be seen as Wiener-Hopf equations,

$$\mathbf{Xg}=\mathbf{d}$$

where it is assumed $g_{lw}^{*}(0)=1$ and $\mathbf{X}$ is Toeplitz given by,

$$\mathbf{X}=[\mathbf{R} \quad \mathbf{S}]$$

$$\mathbf{R}=\begin{bmatrix} r(M)\,r(M+1) & \ldots & r(M+L-1) \\ r(M+1)\,r(M+2) & \ldots & r(M+L) \\ \ldots & \ldots\ \ldots & \\ & & \\ r(N)\,r(N+1)\ldots & & r(N+L-1) \end{bmatrix}$$

$$\mathbf{S}=\begin{bmatrix} s(M-1)\,s(M-2) & \ldots & s(0) \\ s(M)\,s(M-1) & \ldots & s(1) \\ \ldots & \ldots & .. \\ & & \\ s(N-1)\,s(N-2) & \ldots & s(N-M) \end{bmatrix}$$

The vector $\mathbf{g}$ is given by,

$$\mathbf{g}=\begin{bmatrix} g_{w}(-L+1) \\ g_{w}(-L+2) \\ \ldots \\ g_{w}(0) \\ g_{lw}^{*}(1) \\ g_{lw}^{*}(2) \\ \ldots \\ g_{lw}^{*}(M-1) \end{bmatrix}$$

Having expressed in the well-known form, the least squares solution to the equation is given by

$$\mathbf{g} = \left(\mathbf{X}^H \mathbf{X}\right)^{-1} \mathbf{X}^H \mathbf{d}$$

Figures 3.1 and 3.2 show the estimated whitened matched filter and the effective channel filter.

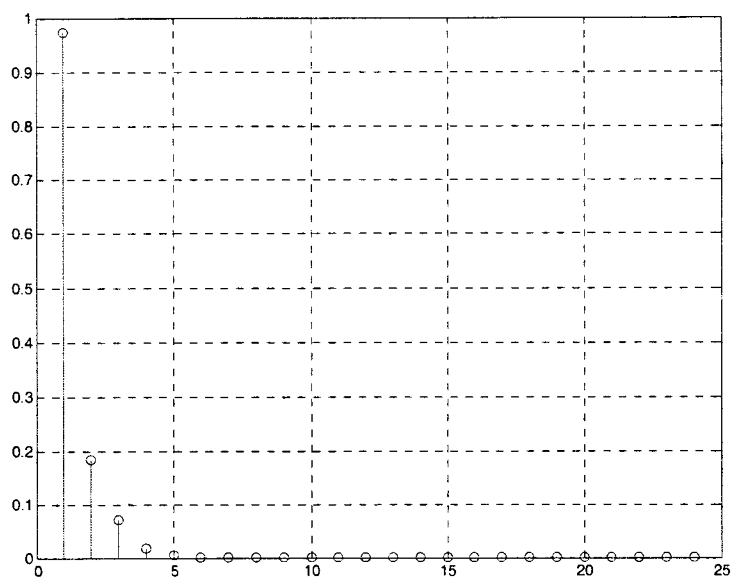

Figure 3.1 Estimated whitened matched filter

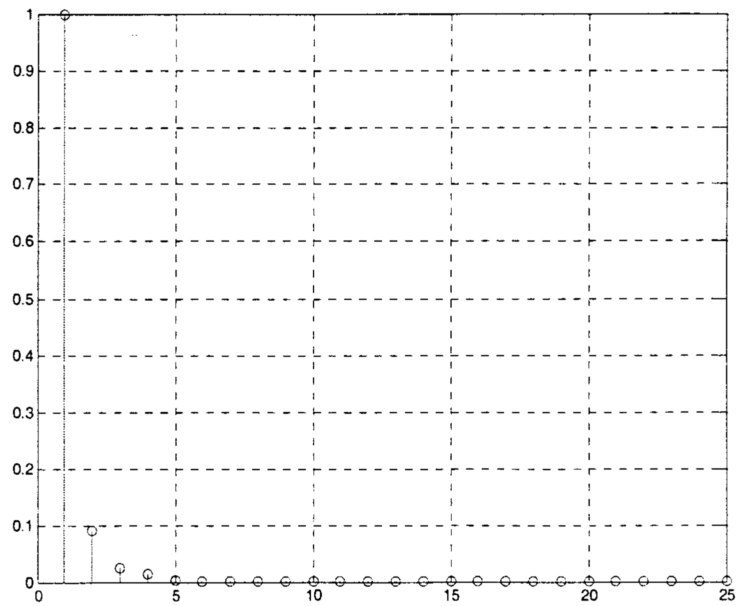

Figure 3.2 Estimated effective channel

3.6 Packet structure and synchronization in soft-modem

Although the link between the radios is one to one (unlike GSM where the traffic on the downlink and the uplinks is Time Division Mutliplexed among different users) the communication between the soft-modems is packet based (as in GSM). The packet structure of soft-modem also resembles the structure of the GSM's Broad Cast Carrier (BCC). The BCC of GSM comprises of Frequency Correction Channel (FCCH) for coarse synchronization and a Synchronization Channel (SCH) for fine synchronization. The mobile detects the FCCH, a sequence of 142 zeros (unmodulated carrier). If detected, the mobile then looks for the SCH in the adjacent Time Slot (TS). Similarly, the soft-modem, at the start of the packet, transmits a sequence of 48 symbols that correspond to the point (15,15) on the QAM constellation (see Figure 2.5 in chapter 2). This results in a carrier offset by a phase of $+45^{o}$. A 1024 symbol training sequence follows the unmodulated carrier, which is used for synchronization and equalization. The rest of the packet is filled with encoded data (2048 symbols). Figure 3.3 below shows the physical layer packet structure of the modem.

FCCH 48 symbols	SCH 1024 symbols	Encoded data 2048 symbols

Figure 3.3 Physical layer packet structure

The receiver of soft-modem correlates the received signal $r(k)$ with unmodulated carrier. When the correlation exceeds certain threshold τ, it marks the start of he packet. Mathematically,

$$R(n) = \sum_k r(k)c^*(k-n) > \tau$$

is the criterion for presence of carrier, which in turn marks the start of the packet. Once the packet is detected, the receiver collects the training sequence from SCH. This is used to jointly estimate the carrier phase and the channel $\mathbf{g}$, using the ML estimate discussed in the section 3.5. A typical packet transmitted by the modem is shown in the Figure 3.4

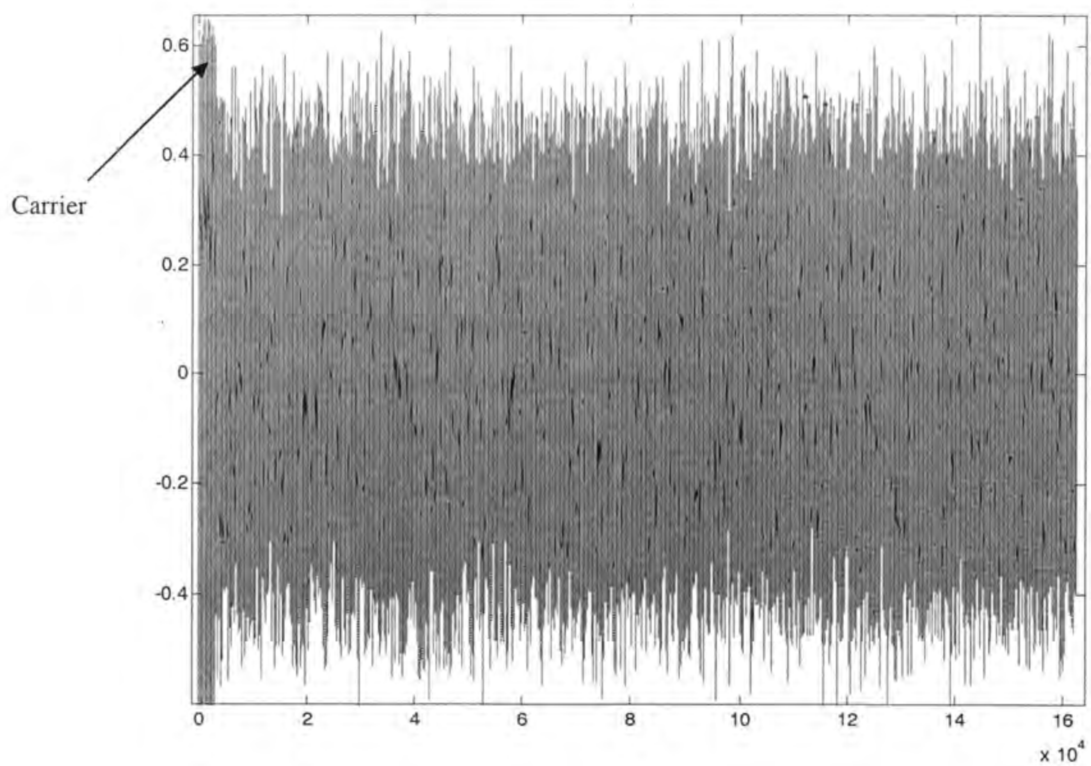

Figure 3.4 Transmitted packet

3.7 Linear equalization

In the section 3.5 we estimated the channel in presence of colored noise. Knowing the channel estimate, in order to detect the symbols in presence of ISI optimally a complex MLSE should be implemented. Moreover, the computational complexity of the MLSE grows exponentially with the length of the channel dispersion. If the size of the symbol alphabet is M and the number of interfering symbols contributing to ISI is L, the Viterbi algorithm computes M^{L+1} metrics for each new row received symbol. In most channels of practical interest, such a large computational complexity is prohibitively expensive to implement.

A sub-optimum approach eliminates the channel dependency on the received symbol as follows. If $r_l(k)$ denote the unequalized symbols, then a linear equalizer passes the symbols to be equalized through a linear filter $c(k)$of order $2K$. The filter coefficients are determined by minimizing the mean square error (MSE) $e(k)$, between the output symbol $r^{'}(k)$ and the training sequence $s(k)$. Figure 3.5 depicts the system model.

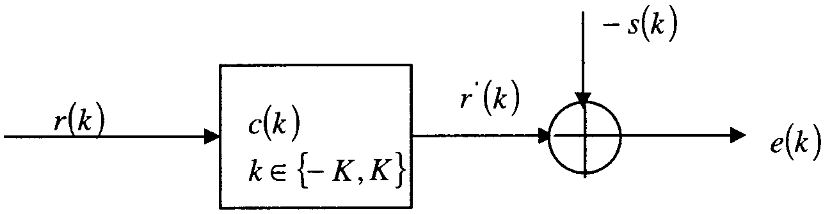

Figure 3.5 Linear equalization

The equalizer coefficients $c(k)$ can be seen to be the solution of well-known weiner filter. Thus,

$$\mathbf{c} = \mathbf{R}_{rr}^{-1}\mathbf{R}_{sr}$$

where $\mathbf{R}_{rr}$ is the autocorrelation matrix of the unequalized symbols $r(k)$ and $\mathbf{R}_{rs}$ is the cross-correlation matrix of $r(k)$ and $s(k)$.

The results of linear equalization on the symbols transmitted through the soft-modem are shown below. Figure 3.6 shows the unequalized 100-point constellation. Although the SNR is high, due to ISI the symbol boundaries are not clearly defined. Figure 3.7 shows the equalized symbol constellation $r'(k)$. The magnitude of the equalizer taps $\{c(k)\}$ is plotted in Figure 3.8.

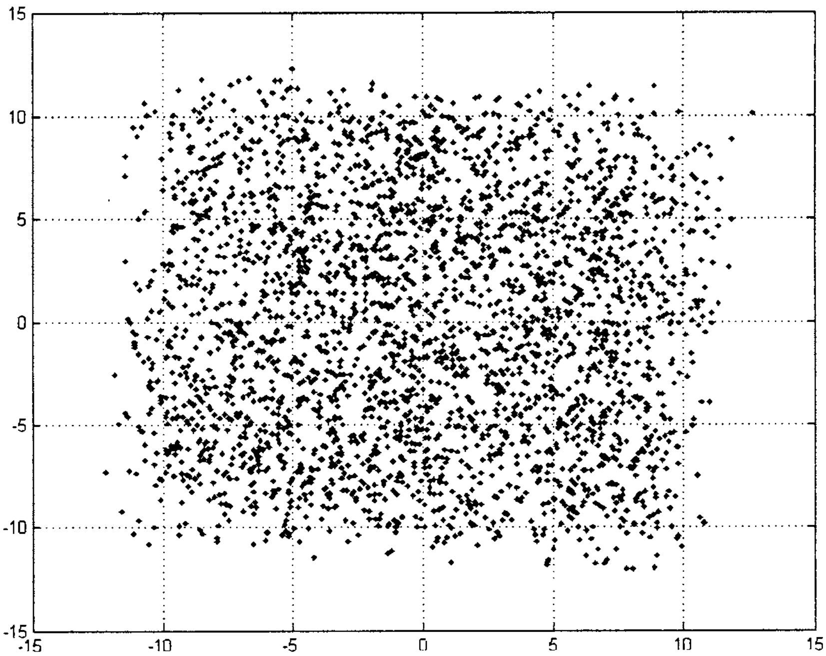

Figure 3.6 Unequalized 100 point QAM constellation

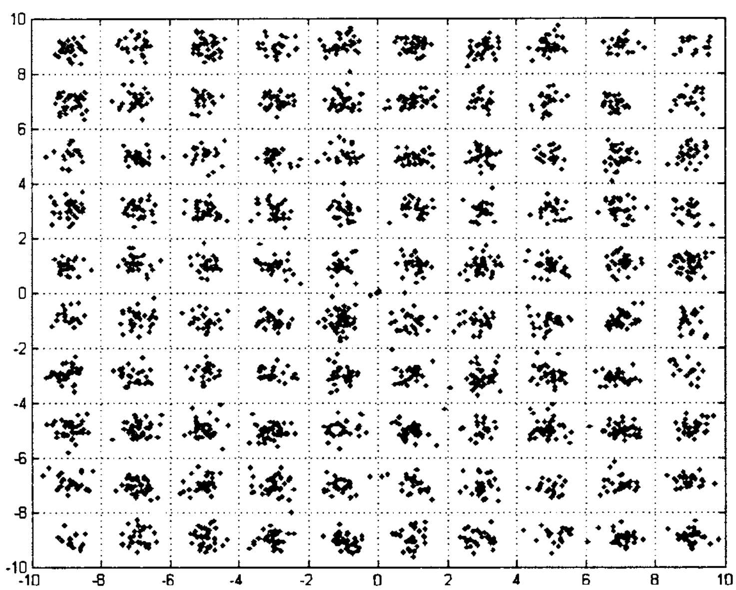

Figure 3.7 Linearly equalized 100 point QAM constellation

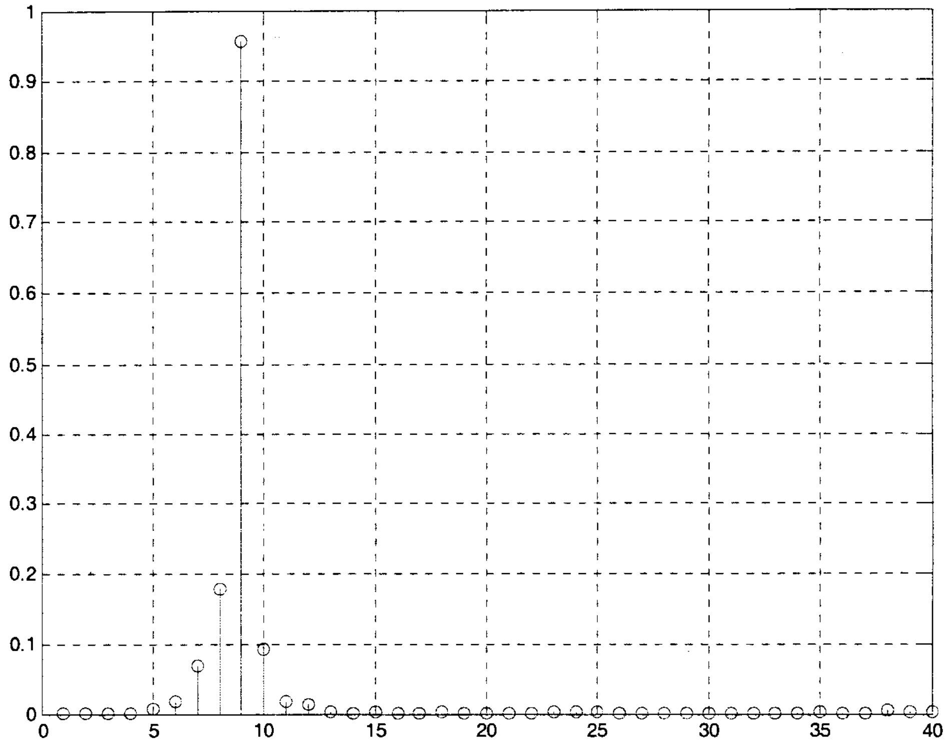

Figure 3.8 Magnitude of equalizer taps

4 Equalization and channel coding

4.1 Introduction

In the last section of the previous chapter, we have designed a linear equalizer that cancels the ISI introduced by the channel. The linear equalizer essentially combats ISI by filtering the complex base-band signal with the inverse channel filter. This not only enhances the noise variance in trying to flatten the response of the channel but also makes it colored. One way to achieve perfect equalization is to use non-linear equalization such as Zero-Forcing (ZF) Decision Feed back Equalizer (DFE). A ZF-DFE not only cancels the ISI effectively, but also keeps the SNR intact, achieving considerable gain over linear equalization. It also preserves the whiteness of the noise making ML Viterbi detection possible.

Although the channel coding is not fully implemented on the soft-modem, it is a very essential to close in the gap between the theoretical capacity and the best possible data rates that could be achieved on the modem. The maximum coding gain is 9 dB on a high SNR band-limited channel. Good trellis codes with coding gains up to 6 dB and shaping-gains of 1.5 dB have been discovered over the last decade (1.5 dB away from Shannon limit). Since reliable decisions are not available from the Viterbi decoder without considerable delay the ZF-DFE is not an attractive solution to cancel ISI. Precoding, a transmitter side equalization technique, allows combining channel coding with equalization elegantly. In fact v.34 telephone modem uses combined precoding and coded modulation to achieve large coding gains. It also provides an efficient way of incorporating shaping (trellis precoding), thus making it the most popular equalization-coding method used in modern modems.

In this chapter we discuss the DFE-ZF followed by precoding after pointing out the disadvantages of DFE when used with coded modulation. Combined precoding and coded modulation is discussed followed by a brief summary of Trellis precoding: a joint precoding, coded modulation and shaping. The chapter concludes with a discussion of differences in coding objectives and coding gains of high SNR and low SNR regimes.

4.2 Equalization

The channel response as seen by the soft-modem is shown in Figure 4.1. Notice it goes to zero at DC and has a considerable attenuation towards the edge of the transmission band. The transmission band is from 0.375 to 2.625KHz. The QAM symbols are pulse-shaped using raised cosine pulse shaping with a roll of factor of 0.5 is as shown in the figure 4.2. The pulse shaping helps in regulating ISI, by reducing the number of equalizer tapes required or equivalently reducing the length of effective channel response $h(D)$. Equalization (or precoding) is thus, inevitable to achieve optimum rate on this channel.

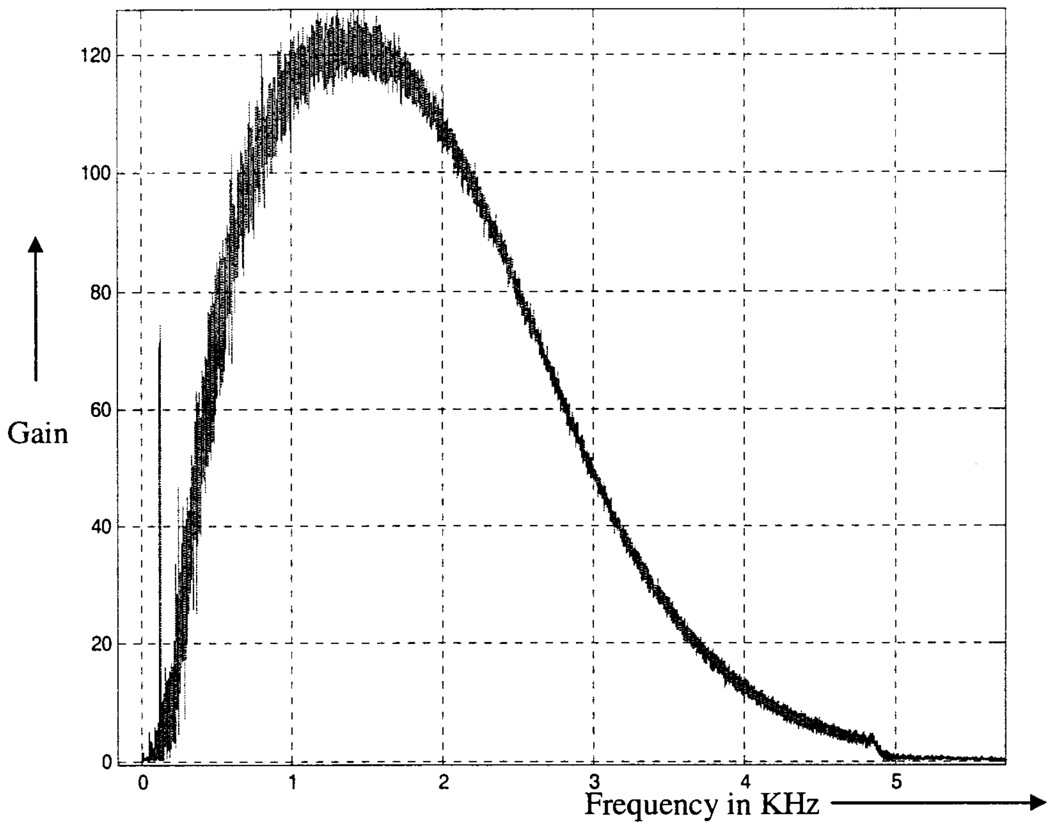

Figure 4.1 Approximate channel response

Let us now discuss the classical equalization methods. Without loss of optimality, subject to the technical conditions, the front of a receiver in a QAM modem may be taken to consist of a whitened matched filter (WMF) and a symbol-rate sampler. Thus, the equivalent discrete-time channel shown in figure 4.3 is obtained. The received sequence $\{r(k)\}$ is given by

$$r(k) = \sum_{j \geq 0} h(j) x(k-j) + n(k)$$

where $x(k)$ is the transmitted sequence $h(k)$ is the sequence of coefficients of an equivalent discrete-time channel response, and $n(k)$ is a discrete-time white Gaussian noise sequence. The term $y(k) = \sum_{j \geq 0} h(j) x(k-j)$ is the noiseless output (In a QAM system, all of these quantities are complex numbers). In D-transform notation

$$r(D) = x(D)h(D) + n(D) = y(D) + n(D)$$

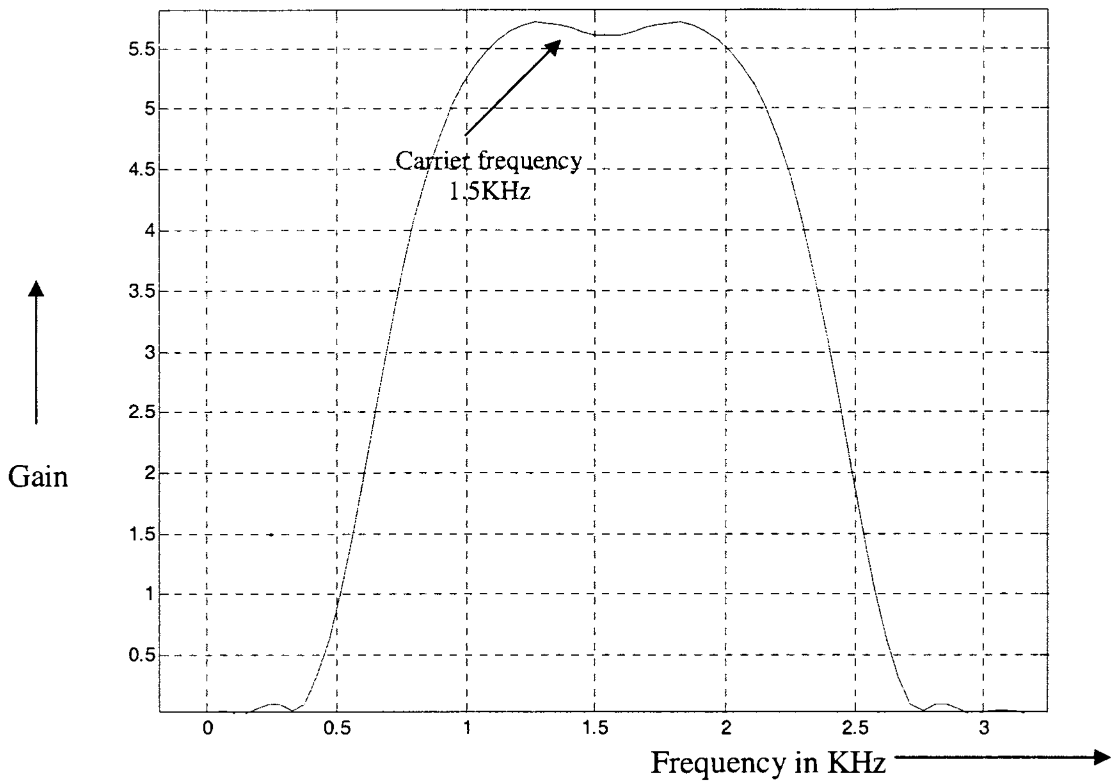

Figure 4.2 Raised cosine pulse shaping

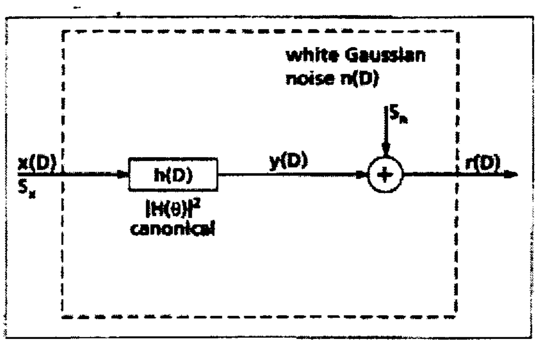

Figure 4.3 Equivalent discrete-time channel

The equivalent discrete-time channel response $h(D)$ is canonical, i.e, causal ($h(j)=0, if\ j<0$), monic ($h(0)=1$), and minimum phase (all poles are outside the unit circle, and all zeroes are on or outside the unit circle). Its frequency response is $H(\theta), -\pi<\theta\leq\pi$. The energy of the response is

$$\|\mathbf{h}\|^2=\sum_j |h_j|^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}|H(\theta)|^2\,d\theta$$

$\|\mathbf{h}\|^2$ is finite, and $\|h\|^2\geq 1$, with equality if and only if the channel is ideal, $h(D)=1$.

Both $x(D)$and $n(D)$are white (iid) sequences, with average energies S_x and S_n per symbol, respectively.

Zero-Forcing linear equalization (ZE-LE discussed in 3.7) may be used if the channel response $h(D)$ has a well-defined $1/h(D)$ (results in 3.7 indicate the channel in fact, has a well defined inverse). With ZF-LE, the received sequence $r(D)$ is simply filtered by $1/h(D)$ to produce an equalized response

$$r'(D)=r(D)/h(D)=x(D)+n(D)/h(D)=x(D)+n'(D)$$

as shown in Figure 4.4. Symbol-by-symbol decisions can then be made on the transmitted sequence $x(D)$. ISI is eliminated, but the average noise energy is enhanced by the energy $\|1/h\|^2$ of the inverse filter response $1/h(D)$:

$$S_{n'} = S_n \left\| \frac{1}{\mathbf{h}} \right\|^2 = S_n \left(\tfrac{1}{2\pi} \right) \int_{-\pi}^{\pi} \tfrac{1}{|H(\theta)|^2} \, d\theta$$

The signal-to-noise ratio at the decision point is thus equal to

$$SNR_{ZF-LE} = \tfrac{S_x}{S_n} \Big/ \left[\left(\tfrac{1}{2\pi} \right) \int_{-\pi}^{\pi} \left(\tfrac{1}{|H(\theta)|^2} \right) d\theta \right]$$

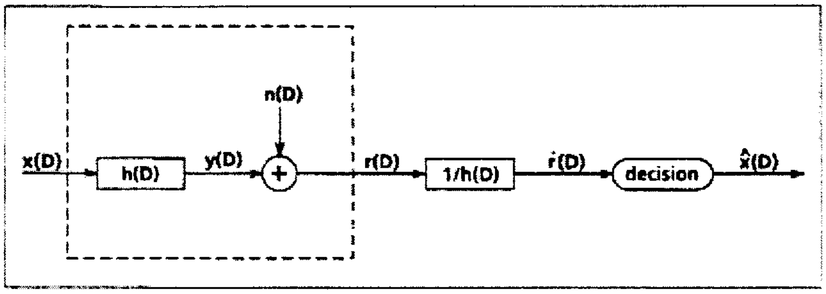

Figure 4.4 Linear equalizer

The noise enhancement factor $\left\| \tfrac{1}{h} \right\|^2$ is greater than or equal to 1, with equality if and only if the channel is ideal, $h(D)=1$. If the response $|H(\theta)|^2$ is reasonably constant over the Nyquist band $\{-\pi < \theta < \pi\}$, then the noise enhancement of ZF-LE is not very serious. However, if $|H(\theta)|^2$ has a near-null, then noise enhancement can become large. Notice that $|H(\theta)|^2$ for the soft modem is nearly constant, so ZF-LE should perform well at high SNRs. But if the power is increased to accommodate larger bandwidth, there will considerable attenuation towards the edge of the (optimum) transmission band, thus use of ZF-LE severely degrades the SNR. The most popular non-linear equalizer structure is decision-feedback equalization (DFE). Zero-forcing DFE is illustrated in Figure 4.5. The basic idea, when estimating x_k, is to assume that all previous estimates are correct: $\hat{x}(k-j) = x(k-j),\ j \geq 1$ (the ideal DFE assumption). The the tail $\sum_{j\geq 1} h(k)x(k-j)$ can be removed by subtraction, i.e., the equalized sample is

$$r'(k) = r(k) - \sum_{j \geq 1} h(j)\hat{x}(k-j) = x(k) + n(k)$$

where the second equality depends on the ideal DFE assumption and the face that $h_0 = 1$. In D-transform notation,

$$r'(D) = r(D) - \hat{x}(D)[h(D) - 1] = x(D) + n(D)$$

Thus, intersymbol interference is completely removed, and the noise is white. The SNR is simply,

$$SNR_{ZF-DFE} = \frac{S_x}{S_n}$$

which improves on SNR_{ZF-LE} by the noise enhancement factor $\left\|\frac{1}{h}\right\|^2$. Thus, $SNR_{ZF-DFE} \geq SNR_{ZF-LE}$, with equality if and only if $h(D) = 1$.

The optimum equalization structure for a discrete transmitted sequence $x(D)$ in the presence of ISI generally is considered to be maximum likelihood sequence estimation (MLSE). If the transmitted symbols $x(k)$ are drawn from an M-point signal set, and the channel response $h(D)$ has length ν with M-state memory elements, i.e., as an M^{ν}-state machine. An M^{ν}-state Viterbi algorithm (VA) may be used to implement MLSE for such a finite-state system. If M or ν is even moderately large, however, an M^{ν}-state VA generally is considered to be too complex to implement.

On many channels without severe ISI an MLSE detector achieves effective SNR of the matched filter bound,

$$SNR_{MFB} = \frac{S_x \|h\|^2}{S_n}$$

The matched filter bound often is taken as a bound on the best possible SNR achievable on a channel with response $h(D)$. When ISI is severe, however, even MLSE can fail to achieve this SNR.

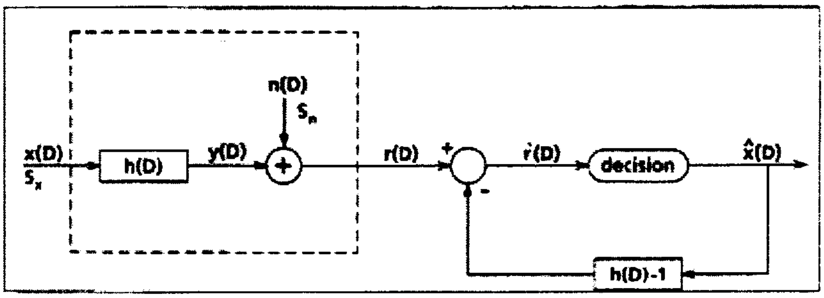

Figure 4.5 Decision feedback equalization

4.3 Tomlinson-harashima precoding

Unlike those techniques in which all equalization takes place in the receiver, precoding is a transmitter technique. It is more closely tied to specific signal set than the previous techniques, although it can be generalized to all cases of interest. Originally it was proposed for use with M-point PAM, where the signal set A consists of M equally spaced levels,

$$A = \{\pm 1, \pm 3, ..., \pm (M-1)\}$$

the set of all odd integers in the interval $[-M, M]$.

Tomlinson-Harashima precoding works as follows, as illustrated in Figure 4.6. The equivalent discrete time channel response $h(D)$ is assumed to be known at the transmitter. The transmitter generates a data sequence $d(D)$ whose symbols $d(k)$ are in the PAM signal set A. The transmitted signal $x(k)$ is formed by first subtracting the tail $\sum_{j\geq 1} h(j)x(k-j)$ due to previously transmitted signals from $d(k)$ (decision feedback in the transmitter), and then reducing $d(k)-\sum_{j\geq 1} h(j)x(k-j)$ modulo 2M to the half-open interval (-M, M].

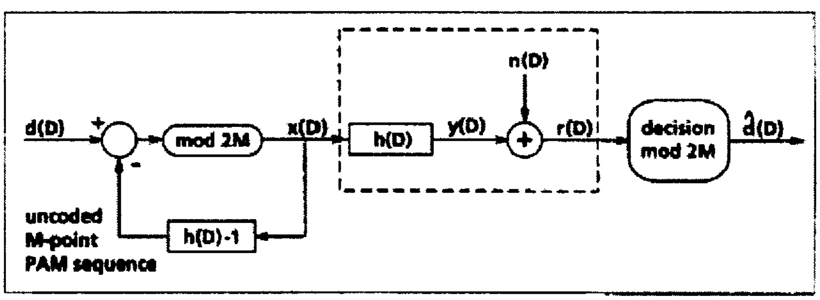

Figure 4.6 Tomlinson-Harashima precoding

The modulo 2M operation can be characterized in various ways. By definition, the transmitted symbol $x(k)$ is the unique number that satisfies the two constraints,

$$x(k) = d(k) - \sum\nolimits_{j\geq 1} h(j)x(k-j) \quad \text{mod}\mathit{ulo} \quad 2M$$
$$x(k) \in (-M, \quad M]$$

In other words, the transmitter finds the unique integer $z(k)$ such that

$$x(k) = d(k) + 2Mz(k) - \sum\nolimits_{j\geq 1} h(j)x(k-j)$$

is in the interval $(-M, M]$. In D-transform notation,

$$x(D) = d(D) + 2Mz(D) - x(D)[h(D) - 1]$$

This reduces to

$$x(D) = {[d(D) + 2Mz(D)]}\big/{h(D)} = {y(D)}\big/{h(D)}$$

where $y(D)$ is a sequence of odd-integer modified data symbols $y(k) = d(k) + 2Mz(k)$. Consequently, the received signal is

$$r(D) = x(D)h(D) + n(D) = y(D) + n(D)$$

Thus, in the absence of noise the received sequence is the modified data sequence $y(D) = d(D) + 2Mz(D)$, which may be detected on a symbol-by-symbol basis to give an estimated sequence $\hat{y}(D)$. There is no ISI, and the error probability is the same as if the original sequence $d(D)$ were sent on the same channel and detected with a ZF-DFE, or if $d(D)$ were sent on an ideal channel with signal-to-noise ratio $SNR_{ZF-DFE} = \frac{S_x}{S_n}$. An estimate $\hat{d}(k)$ of the original data symbol $d(k)$ can then be retrieved by reducing $\hat{y}(k)$ to the interval $(-M, M]$ with a modulo 2M operation. Note that, precoding woks even if $h(D)$ is not invertible.

4.4 Combined precoding and coded modulation

Now suppose that it is desired to use non-linear equalization in combination with trellis-coded modulation. With DFE, a major problem then arises: symbol-by –symbol decisions are unreliable, but reliable decoded decisions are not available until after a decoding delay.

One approach is to use an interleaver to shuffle the symbols in such a way to provide reliable decisions for feedback most of the time. Such an interleaver, however, must have a large span in order for most feedback decisions to be reliable, and thus may introduce considerable delay.

Another approach that is optimum in principle is to consider the combination of the trellis encoder and the channel as one large finite-state machine, whose state space is the product of the encoder and channel state spaces, and then to perform MLSE via Viterbi algorithm in the entire system. But this is very complex computationally.

Often when the transmitter and receiver can cooperate, precoding is a more attractive approach. Precoding can be combined with coded modulation essentially with no glue as follows. Practically all known good codes work by constraining the one or two least significant bits in a binary representation of the data symbols $d(k)$ according to the possible code sequences in a binary convolutional code. In other words, a PAM data sequence $d(D)$ is a legitimate word in a given trellis code C if, and only if, the least significant bits of the components $d(k)$ satisfy certain constraints. The key point is that if $d(D)$ is a coded PAM sequence in such a trellis code C, if $z(D)$ is any integer sequence and if M is a multiple of 4, then the modified data sequence

$$y(D) = d(D) + 2Mz(D)$$

is also a sequence in C.

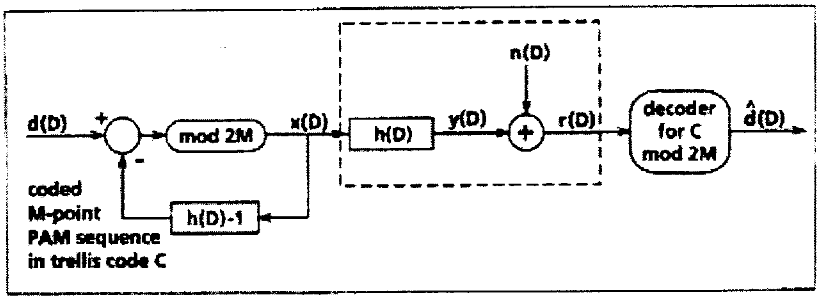

Figure 4.7 Combined coding and precoding

A combined coder-precoder can therefore operate as shown in Figure 4.7. The transmitter first generates a coded PAM sequence $d(D)$ in a given trellis code C. It then precodes it as in the previous section to generate a transmitted sequence $x(D)=\frac{y(D)}{h(D)}$, where $y(D)=d(D)+2Mz(D)$. If M is a multiple of 4, then $y(D)$ also is in C.

The received sequence is then $r(D)=y(D)+n(D)$, i.e., a code sequence in C plus white Gaussian noise. An ordinary Viterbi algorithm can operate as usual on $r(D)$ to find the closest code sequence $\hat{y}(D)$ to $r(D)$, which can then be reduced modulo 2M to $\hat{y}(D)$. For large M, the performance complexity of such a decoder will be essentially the same as if coded sequences $d(D)$ were sent through an ideal Gaussian channel with signal-to-noise ratio $SNR_{ZF-DFE}={}^{S_x}\!/_{S_n}$ and detected with a VA decoder for C.

In summary, precoding can be combined with coding in a very natural way. The combined system obtains the equalization gain of ideal DFE in combination with the coding gain of the given trellis code C.

4.5 Trellis precoding: combined precoding, coding and shaping

In the combined coding-precoding system of Figure 4.7, as in Tomlinson-Harashima precoding, the transmitted symbols are essentially uniformly distributed within the interval (-M, M]. This makes it impossible to achieve any shaping gain.

Briefly, shaping gain is a reduction in average signal energy that can be obtained by shaping N-dimensional signal sets more like an N-sphere than an N-cube, or equivalently by causing the probability distribution of one or two-dimensional transmitted signals to be more like Gaussian distribution. The maximum possible shaping gain is essentially independent of coding gain, and the two problems can be addressed separately.

While shaping gain is limited to 1.53 dB, and therefore is, much less than typical coding gains, it nonetheless is worth pursuing. On ideal channels, simple techniques have been developed that achieve of the order 1 dB of shaping gain with modest complexity. Given a trellis code with 3 dB or 4 dB coding gain, it is much easier to obtain the next 1 dB with such shaping than by more complex coding.

4.6 Low-SNR and high SNR regimes

We see from Figure 4.8 that in the low-SNR regime an equiprobable binary alphabet is nearly optimal. For SNR<1 the reduction in capacity is negligible. In the high SNR regime, the capacity of equi-probable M-PAM constellations asymptotically approaches a straight line parallel to the capacity of the AWGN channel. The asymptotic loss of $\pi e/6$ (1.53dB) is due to using a uniform rather than a Gaussian distribution over the signal set. To achieve capacity, the use of powerful coding with equiprobable M-PAM signals is not enough. To obtain the remaining 1.53dB, constellation-shaping techniques that produce a Gaussian-like distribution over an M-PAM constellation are required. Same holds for QAM.

Thus coding techniques for the low-SNR and high-SNR regimes are quite different. In the low-SNR regime, binary codes are nearly optimal and no constellation shaping is required. On the other hand, in the high-SNR regime, non-binary signal constellations must be used. To approach capacity, coding techniques must be supplemented with constellation-shaping techniques. Moreover, on bandwidth-limited channels as in the case of soft-modem, ISI is dominant impairment and practical techniques for combined coding, shaping and equalization are required to approach capacity.

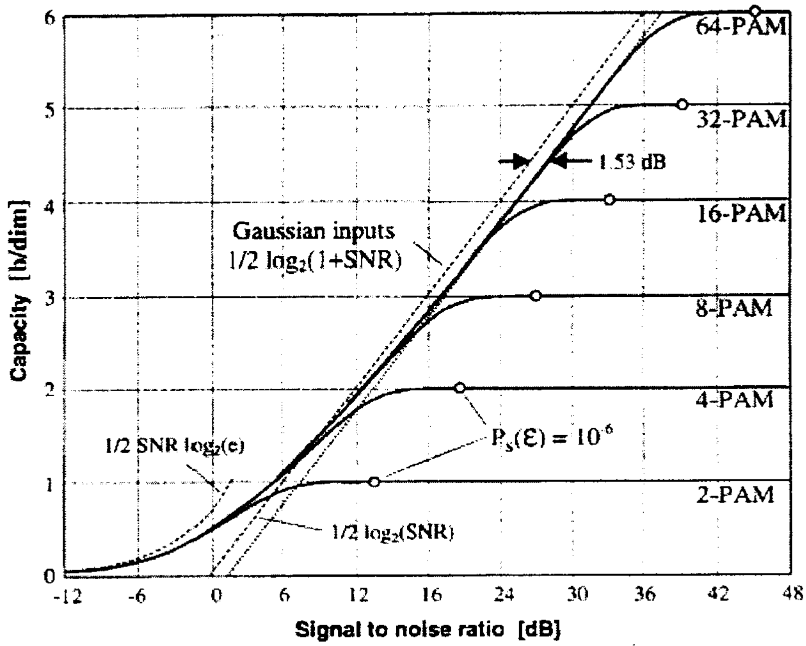

Figure 4.8 Capacity of AWGN channel, gaussian and equiprobable PAM inputs

4.7 Baseline performance of uncoded M-PAM and normalized SNR

In an uncoded M-PAM system, $R = \log_2 M$ information bits are independently encoded into each M-PAM symbol transmitted. In the receiver, the optimum decoding rule is then to make independent symbol-by-symbol decisions. The probability that a Gaussian noise variable w_i exceeds half of the distance d_0 between adjacent M-PAM symbols is $Q\left({d_0}/{2\sigma_w}\right)$. The error probability per symbol is given by $P_s(\varepsilon \mid outer) = Q\left({d_0}/{2\sigma_w}\right)$ for the two outer points, and by $P_s(\varepsilon \mid inner) = 2Q\left({d_0}/{2\sigma_w}\right)$, so the average error probability per symbol is

$$P_s(\varepsilon) = \frac{2(M-1)}{M} Q\left(\frac{d_0}{2\sigma_w}\right) = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{3SNR}{M^2-1}}\right)$$

Thus $P_s(\varepsilon)$ is a function only of M and SNR. The circles in figure 4.8 indicate the values of SNR for which $P_s(\varepsilon)=10^{-6}$ is achieved. The capacity formula can be rewritten as $SNR/(2^{2C}-1)=1$. This suggests defining normalized SNR

$$SNR_{norm}=SNR/(2^{2R}-1)$$

where R is the actual data rate of a given modulation and coding scheme. For a capacity-achieving scheme, R equals the channel capacity C and $SNR_{norm}=1\,(0\ dB)$. If $R<C$, as will be always the case in practice, then $SNR_{norm}>1$. The value of SNR_{norm} thus signifies how far a system is operating from Shannon limit. (the gap to capacity). For uncoded M-PAM, since, we have

$$SNR/M^2-1=SNR/2^{2R}-1=SNR_{norm}$$

Therefore, the average error probability per symbol of uncoded M-PAM can be written as

$$P_s(\varepsilon)=\frac{2(M-1)}{M}Q\left(\sqrt{3SNR_{norm}}\right)\approx 2Q\left(\sqrt{3SNR_{norm}}\right)(M\ \mathrm{Large})$$

Note that the baseline M-PAM performance curve of $P_s(\varepsilon)$ versus SNR_{norm} is nearly independent of M, if M is large. This shows that SNR_{norm} is appropriately normalized for rate in the high-SNR regime.

The effective coding gain of a coded modulation scheme is measured by the reduction in required E_b/N_0 or SNR_{norm} to achieve a certain target error probability relative to a baseline uncoded scheme. In low-SNR regime, the baseline will be taken as 2-PAM; in the high-SNR regime, the baseline will be taken as M-PAM (M large).

Figure 4.9 gives probability of BER $P_b(\varepsilon)$ for uncoded 2-PAM as a function of both SNR_{norm} and E_b/N_0. At $P_b(\varepsilon)=10^{-6}$, the baseline uncoded binary modulation scheme operates about 12.5 dB away from the Shannon limit. Therefore, a coding gain of up to 12.56 dB in E_b/N_0 is in principle possible at this $P_b(\varepsilon)$, provided that bandwidth can be expanded sufficiently to permit the use of powerful very-low rate binary codes $R << 1$. If bandwidth can be expanded by a factor of only 2, then with binary codes of rate $R = \frac{1}{2}$ a coding gain of up to about 10.8 dB can in principle be achieved.

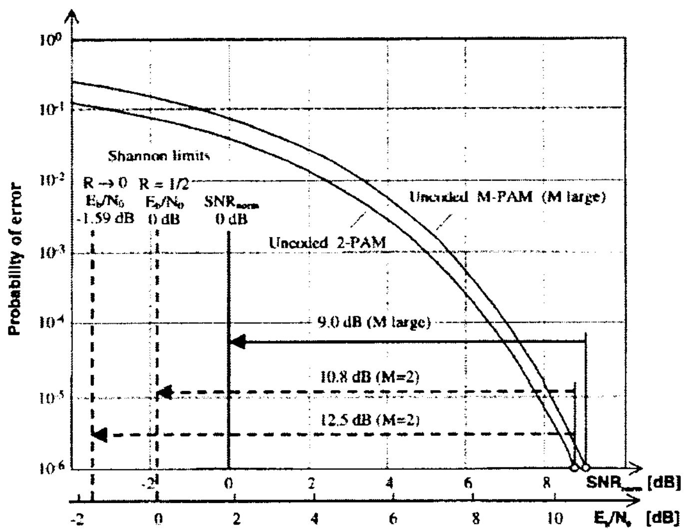

Figure 4.9 Coding gains for low-SNR and high-SNR cases

Figure 4.9 also depicts the BER for uncoded M-PAM as a function of SNR_{norm} for large M. At $P_s(\varepsilon) = 10^{-6}$, a baseline uncoded M-PAM modulation scheme operates about 9dB away from the Shannon limit in the bandwidth-limited regime. Thus if bandwidth is a fixed, nonexpendable resource, a coding gain up to about 9 dB in SNR_{norm} is in principle possible at $P_s(\varepsilon) = 10^{-6}$.

5 Conclusions and future work

An inexpensive modem has been designed in this thesis. The proposed architecture uses the processing capabilities of PC to carry out the digital signal processing. FM transceivers provide the radio channel. The DAC and ADC present on the sound cards provide the analog interface between the processor and the FM transceivers. The communication between the modems is packet based and is assumed that the carrier phase and the symbol timing do not change considerably in one packet duration. The preamble of packet consists of an unmodulated carrier and a synchronization sequence. The carrier is used to detect the start of the packet and the synchronization sequence is used for channel estimation. Coded data symbols make up the rest of the packet structure. Although channel coding is not implemented, various capacity approaching techniques have been discussed. Having estimated the channel, it is proposed that precoding combined with Trellis coding modulation and shaping would yield good coding gains over this band-limited radio channel. A multi-dimensional trellis-precoding, similar to the coding used in V.34 needs to be incorporated and its performance needs to be evaluated.

The modem is designed and tested in non-real time using the Matlab's communication and signal processing toolboxes. GlodWave, a sound editing software, is used to playback and record the waveforms, which are ported to the Matlab for offline processing. A real-time implementation of the modem requires the Matlab functions to be rewritten in C. Linux operating system is most suitable for real-time implementation when used with rt-linux kernel (rt stands for real-time). In addition, the kernel drivers are modularized and can be easily stacked up to provide the network interface to the modem drivers.

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