RECONSTRUCTION OF THE GEOMETRY OF A SURFACE-BREAKING CRACK

D. M. Tow

Idaho National Engineering Laboratory
EG&G Idaho, Inc., P. O. Box 1625
Idaho Falls, ID 83415

INTRODUCTION

The crack reconstruction method presented here employs a ray-tracing field code described in another session of this conference [1]. This field code calculates the transducer response to a crack of known geometry by modeling it as a collection of point scattering elements. The model is linear in that the transducer response to a collection of point scattering elements is equal to the sum of the transducer responses of the individual elements. The field code is currently only capable of modeling two-dimensional geometries. In two dimensions a crack is modeled as a linear array of scattering elements. A transducer is similarly modeled as a linear array of point sources. All results discussed in this paper are two-dimensional simulated results; Figure 1 depicts a typical 2-D inspection geometry.

The reconstruction algorithm determines the collection of point scattering elements constituting the "best" estimate of the crack geometry. The estimate is "best" in the sense that it is an optimal Bayes estimate. In mathematical terms the reconstruction task can be expressed as follows:

Let $f(x,y)$ $\equiv$ crack geometry

$g_i(x,y)$ $\equiv$ point scattering function, and

$r(\cdot)$ $\equiv$ transducer response of a scattering function.

From measurements $\mathbf{z}$

$$\mathbf{z} = r(f) + \mathbf{v} \tag{1}$$

where $\mathbf{v}$ is noise, we desire an estimate of the form

$$\hat{f}(x,y) = \Sigma w_i g_i(x,y) \tag{2}$$

that is optimal from the point of view of bayesian estimation theory.

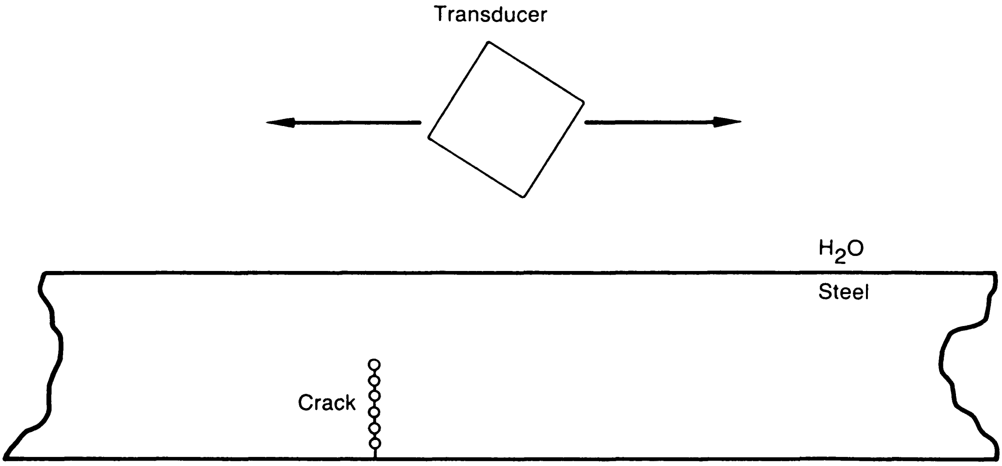

Fig. 1. Typical 2-D inspection geometry.

The steps involved in sizing a crack by this method are illustrated in Figure 2. The first step involves detecting the crack and determining the region that contains it. The second step is reconstructing the crack on a rectangular grid of scattering elements as depicted in Figure 2b. If the reconstruction is successful, grid points far from the crack will be assigned values of 0; grid points near the crack will be assigned values near 1.

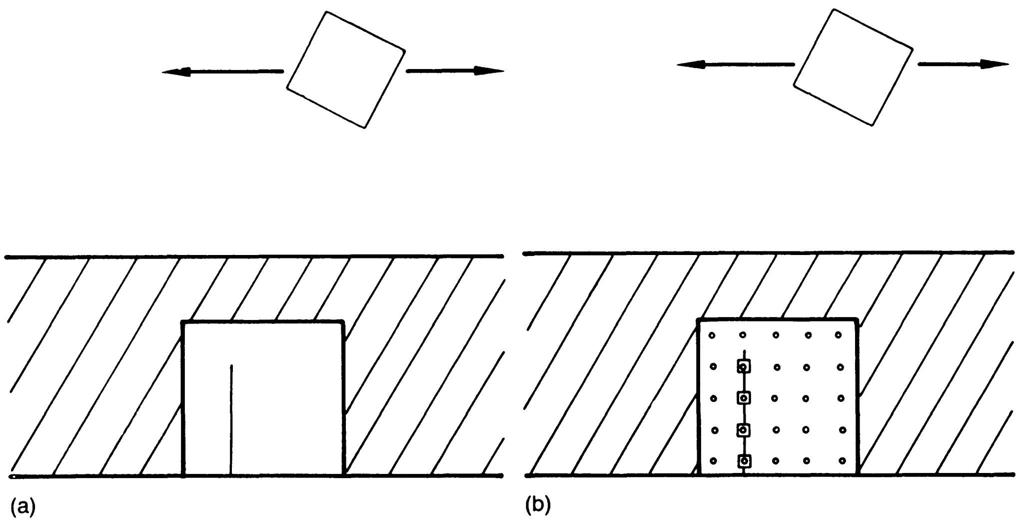

Fig. 2. Steps involved in reconstructing a crack. (a) Detect and determine roughly the region that contains the crack. (b) Reconstruct the crack on a grid filling the region.

TRANSDUCER RESPONSE

The geometry of a crack is reconstructed using the transducer response at one or many transducer positions. For this study, it was assumed that the vicinity of a surface breaking crack is examined by an ultrasonic transducer scanning at a fixed elevation above a flat plate containing the crack. Each pulse-echo is transformed into the frequency domain. The total transducer response is thus a two-dimensional complex-valued array that is a function of frequency and transducer position, as shown in Figure 3. The data array is sampled and the sampled values are arranged into a transducer response vector **z**. Although every element in the two-dimensional data array could be used, usually the data are thinned by sampling to reduce the dimension of **z** to 2 to 3 times the number of points on the reconstruction grid. Samples are selected from a band of frequencies centered about the center frequency of the transducer. The empirically-chosen sampling scheme used here had a center frequency of 5 MHz and sampled band width of about 5 MHz.

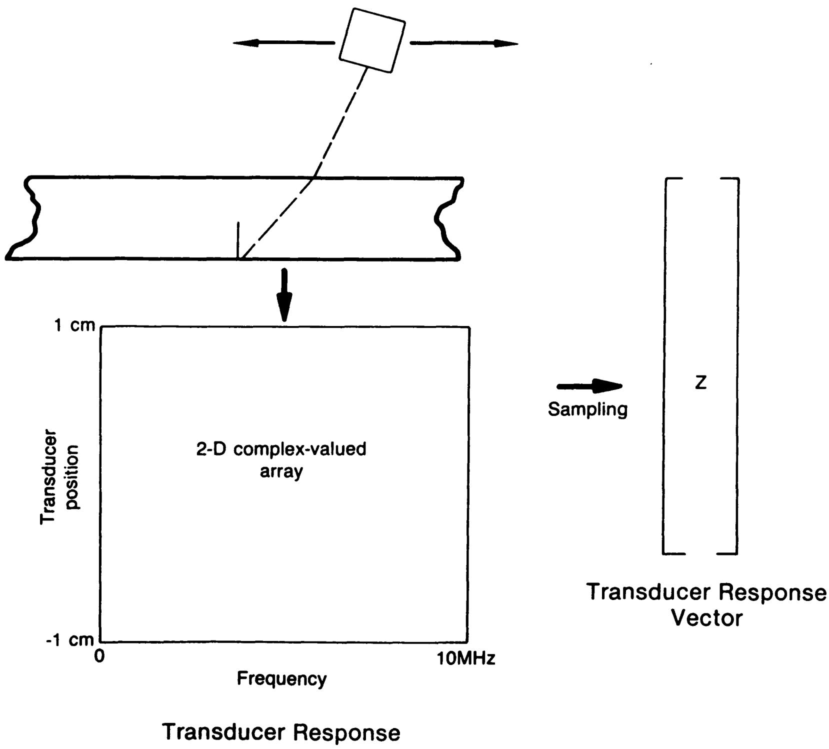

Fig. 3. Transducer response and the transducer response vector.

NOISE-FREE CASE

In the absence of any noise, including round-off noise, the reconstruction task is a simple matrix inversion problem:

$$\mathbf{z} = Q\,\mathbf{w} \tag{3}$$

where **z** is the transducer response vector of length N of the unknown crack, and Q is the N x M matrix whose column vectors are the transducer response vectors of the M scattering elements in the reconstruction grid. The field code is used to calculate Q. **w** is the vector of length M whose elements are the weights to be associated with points on the reconstruction grid. Assuming Q is full rank and $N \geq M$, a unique generalized inverse exists,

$$Q^{-} = (Q^{*}Q)^{-1} Q^{*} \tag{4}$$

and the reconstruction task is accomplished by simply multiplying the transducer response vector by the generalized inverse:

$$\mathbf{w} = Q^{-} \mathbf{z} \tag{5}$$

Experience has shown that it is not difficult to construct a Q that is full rank. However it is difficult to construct a well-conditioned **Q.** An ill-conditioned Q means that the column vectors are almost linearly dependent. The calculation of Q^{-} and the reconstruction of **w** become very sensitive to noise when Q is ill-conditioned. For this reason the simple matrix inversion approach does not work.

ESTIMATION IN NOISE

Reconstruction must be viewed as an estimation problem. A solution is sought to the following equation:

$$\mathbf{z} = Q\mathbf{w} + \mathbf{v} \tag{6}$$

where **v** is a noise term. For this work we have elected to use a maximum a posteriori (MAP) estimator [2] to estimate **w**. The form of the estimator is

$$\hat{\mathbf{w}}_{MAP} = \left(\frac{\sigma_{\mathbf{v}}}{\sigma_{\mathbf{w}}} \; I + Q^{*}Q \right)^{-1} Q^{*}\mathbf{z} \tag{7}$$

where we have assumed that **w** and **v** are independent and distributed in a gaussian fashion so that

$$E\{\mathbf{w}\} = 0 \quad \text{var} \; \{\mathbf{w}\} = \sigma_{\mathbf{w}} I$$

$$E\{\mathbf{v}\} = 0 \quad \text{var} \; \{\mathbf{v}\} = \sigma_{\mathbf{v}} I$$

Notice that when $\sigma_{\mathbf{v}} = 0$ the MAP estimator is just the generalized inverse of **Q.**

EXPERIMENTAL RESULTS

The field code was used to simulate the transducer responses of 2-D cracks. The transducer responses of the scattering points on the rectangular reconstruction grid were also computed using the field code. The matrix Q was constructed from the transducer responses of the reconstruction grid points. Because the transducer responses of the cracks and the matrix Q were computed using the same model, one would expect the reconstruction algorithm to work very well. Indeed, when the points modeling the crack are coincident with the grid points there is essentially no noise and perfect reconstructions may be obtained using the generalized inverse Q^- for the estimator. When the crack points are not coincident with the grid points, modeling noise is introduced and the MAP estimator must be used with a nonzero value for σ_v.

The reconstructions were performed on a grid with 5 columns and 24 rows. The grid points were spaced 0.25 mm apart. Figure 4 depicts the reconstruction grid and a 3 mm crack. In Figure 4a the crack points are coincident with the grid points, whereas Figure 4b depicts the situation with the greatest modeling noise: the crack lies between grid columns and the crack point spacing is different than the separation between grid points. The modeling noise in the situation depicted in Figure 4b was empirically determined to be approximately equivalent to additive white noise with a signal-to-noise ratio (SNR) of 100. This SNR value was used in the filter design for several of the reconstructions appearing here.

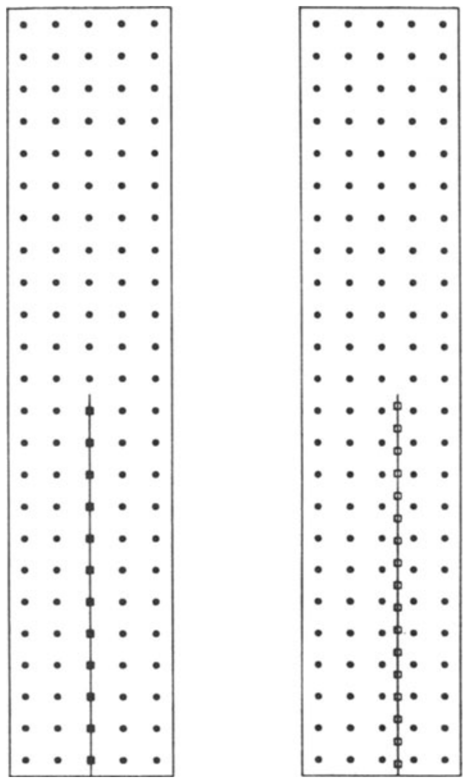

Fig. 4. Reconstruction grid and 3 mm crack. (a) Noise-free situation; (b) Modeling noise introduced because crack points are not coincident with grid points.

The results of the reconstructions are displayed by plotting the 5 columns of the reconstruction grid. Figure 5a shows the reconstruction of a 3 mm crack when no modeling noise is present and the generalized inverse Q^- is used for the estimator. In this case the crack points are coincident with the grid points as in Figure 4a. If the MAP estimator is designed for SNR = 100, the reconstruction shown in Figure 5b is obtained from the noise-free data. Figures 5c-f show reconstructions of cracks which are 1.5, 2.0, 3.0 and 4.5 mm in length, respectively. The cracks are modeled, as illustrated in Figure 4b, so

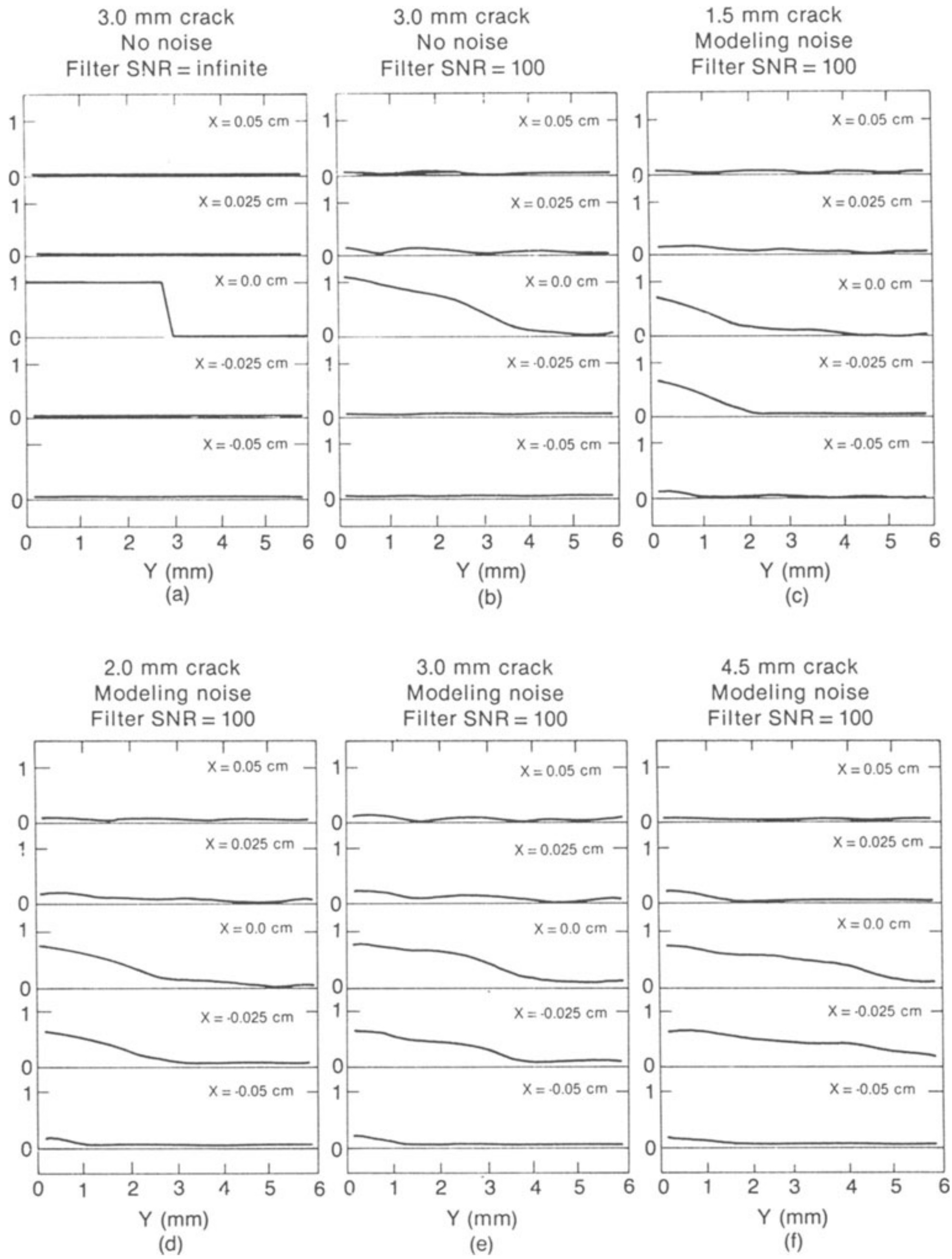

Fig. 5. Crack reconstructions.

that modeling noise will be maximized. The MAP estimator with SNR = 100 was used for the reconstructions. Notice that the crack signals are distributed between the two grid columns adjacent to the simulated cracks.

CONCLUSIONS

A distinct shoulder indicating the tip of the crack does not appear in Figures 5b-f. Because of this it would be difficult to size the cracks with much accuracy by viewing each reconstruction independently. However, it is possible to order the cracks by size from examination of the group of reconstructions. This suggests that an unknown crack might be accurately sized by comparing its reconstruction with a library of reconstructions of known cracks.

Better reconstructions might be obtained by reducing the modeling noise. One way of doing this would be to use a finer reconstruction grid. However, this tactic has the negative effect of causing the matrix **Q** to become even more poorly conditioned. This in turn causes the estimator to be more sensitive to noise. So reducing the modeling noise by reducing the grid point spacing may not accomplish the desired objective. In the examples presented here, the condition of Q was such that computer round-off noise would have become a problem had the grid point spacing been reduced. A finer reconstruction grid was therefore not tried.

More accurate reconstructions could be obtained by improving the condition of **Q**. Q is ill-conditioned because two or more linear combinations of grid points result in almost identical transducer responses. This situation might be avoided by using different scanning geometries. For example, a crack could be scanned from two or more transducer elevations, or two or more transducer pointing angles could be used. It may even prove worthwhile to use two or more different transducers. A search light beam scanning mode may also be well suited to the problem. The reconstruction algorithm is easily adapted to use any combination of these techniques, and others, to arrive at an optimal estimate of the crack geometry. Such flexibility makes this an attractive imaging method.

ACKNOWLEDGMENT

This work was supported by the U.S. Department of Energy, Office of Energy Research, Office of Basic Energy Sciences under DOE Contract No. DE-AC07-76ID01570.

REFERENCES

1. D. M. Tow and J. A. Johnson, Review of Progress in Quantitative Nondestructive Evaluation 5, (Plenum Press, New York).

2. A. P. Sage and J. L. Melsa, Estimation Theory with Applications to Communications and Control, (Robert E. Krieger Publishing, Huntington, NY, 1979), pp. 175-194.