

INVERSE RAY TRACING IN ANISOTROPIC ELASTIC SOLIDS

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ABSTRACT

The problem of inverse ray tracing in a homogeneous anisotropic elastic solid is considered, with specific application to crack sizing. The data is assumed to be in the form of travel times of diffracted ultrasonic signals between transducers positioned on an exterior surface of the body. Both pulse-echo and pitch-catch data are considered. First, it is assumed that the wave speeds are unknown and must be obtained as part of the inversion procedure. The specific problem of locating a crack tip in a two-dimensional geometry is investigated. It is found that travel time data on the exterior surface suffices to locate the crack tip only if the material is isotropic. If the material is anisotropic, we must be able to move the source and/or receiver in the direction normal to the surface. The same problem is considered with the source and receiver positioned in a surrounding isotropic material, e.g., a water bath. It is shown that the ray inversion is now possible only if the solid is isotropic, the problem being underdetermined for an anisotropic solid. Numerical results are presented for a synthetic experiment in which a finite crack is present in some anisotropic elastic solids. Next, the problem is considered when the speeds are known a-priori. It is shown that a crack edge can be mapped by a local approximation procedure.

INTRODUCTION

Metals are usually neither homogeneous nor isotropic. In most applications the inhomogeneity can be ignored in the frequency range of interest. However, anisotropy is an inherent property of

many metals, which should not be ignored in an inversion algorithm. In this paper we focus on the problem of locating and sizing a crack in a homogeneous anisotropic body.

The inversion methods discussed are based on ray theory and the geometrical theory of diffraction [1]. These theories predict that diffracted wavefronts appear to emanate from flash points on the crack edge. In the direct problem the flash points are readily calculated using Fermat's principle of stationary time. We formulate the inverse problem as that of finding the flash points. If a large number of flash points are known, the crack edge can be inferred by "joining the dots". The input data are the travel times of diffracted signals. These can be measured directly from first arrival times or indirectly from high frequency spectra [2].

This paper is divided into two parts. In the first part the anisotropy of the material is not known a-priori, but must be learned as part of the inversion procedure. A simple two dimensional problem is considered and the necessary and sufficient data required are discussed. The same problem is addressed in Section 3 with the source and receiver located in a surrounding water bath. The second part of the paper discusses the crack mapping problem when the material anisotropy is known. The methods presented here are an extension of those in Ref. 3 to include anisotropy.

2. A Homogeneous Solid of Unknown Anisotropy

The necessary and sufficient data required for locating a flash point in a homogeneous anisotropic solid are now described. The data are in the form of elapsed travel times along diffracted rays. The diffracted signals are assumed to be generated and received by transducers located on an exterior surface of the specimen. In the pulse-echo arrangement, a single transducer acts as source and receiver; in the pitch-catch arrangement, the source and receiver are separate transducers. We shall consider the two arrangements separately. For simplicity, we assume a two dimensional configuration. This would correspond for example, to all rays propagating in a symmetry plane of a material of cubic symmetry, as is considered in the numerical example of Section 2.3.

2.1 Pulse-Echo Data

Consider a transducer S positioned on the exterior surface of a specimen which contains an interior crack tip F, see Fig. 1. A signal is emitted from S, diffracted at F and subsequently received at S after a time delay of $2T$. The speeds of propagation to and from the flash point are identical. Thus

$$T = R/c, \tag{1}$$

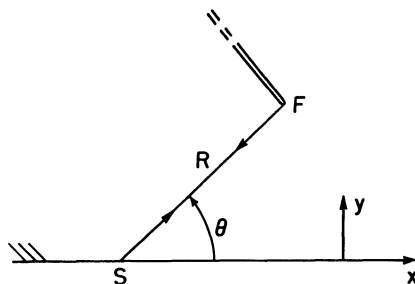


Fig. 1. Two dimensional pulse-echo configuration. The flash point is the crack tip F.

where $c = c(\theta)$. Explicit differentiation gives

$$T_x = - (1/c) (\cos\theta + \gamma \sin\theta) , \quad (2)$$

$$T_y = - (1/c) (\sin\theta - \gamma \cos\theta) , \quad (3)$$

where

$$\gamma(\theta) \equiv \frac{1}{c} \frac{dc(\theta)}{d\theta} ,$$

and T_x, T_y are the derivatives of T at S . Further differentiation of eqs.(2) and (3) gives

$$T_{xx} = (1/Rc) \sin^2\theta (1 + \gamma^2 - \gamma') \quad (4)$$

$$T_{yy} = (1/Rc) \cos^2\theta (1 + \gamma^2 - \gamma') \quad (5)$$

$$T_{xy} = -(1/Rc) \cos\theta \sin\theta (1 + \gamma^2 - \gamma') \quad (6)$$

where $\gamma' = d\gamma/d\theta$. The derivatives T_x and T_{xx} can be computed by shifting the transducer tangentially on the surface at S , to some new point S_1 . The travel time of the pulse-echo signal is measured at S_1 , say T_1 . Then a finite difference approximation to T_x can

be formed. Similarly, a finite difference procedure can be used to approximate T_{xx} if two shifts are performed. The derivatives in the y direction are more difficult to handle experimentally. An approximate evaluation of T_y , T_{xy} or T_{yy} by any finite difference scheme requires shifting the transducer in the direction normal to the solid surface. One way of achieving this is by introducing a thin slab of the same material at S such that the principal directions of the slab and the specimen are exactly aligned.

We now discuss the necessary and sufficient data required to find the flash point F. Suppose first that the medium is either isotropic or anisotropic and that the speed $c(\theta)$ is known as a function of angle. Then a knowledge of T and T_x provides enough information to determine the two unknowns R and θ by eqs.(1) and (2).

Next, suppose that the medium is isotropic but the speed c (a constant) is unknown. Now there are three unknowns R, θ and c , and we require three pieces of information. In eqs.(2)-(6) we have $\gamma = \gamma' = 0$. Therefore, we may use eqs.(1),(2) and either of eqs. (3) and (4). However, use of eq.(3) means computing the y-derivative of T , which is much more cumbersome experimentally. Using T , T_x and T_{xx} we have $R = cT$ and

$$\theta = \tan^{-1}[-(T T_{xx})^{1/2}/T_x] , \quad (7)$$

$$c = [2/(T^2)_{xx}]^{1/2}. \quad (8)$$

If the medium is anisotropic with unknown speed $c(\theta)$, there is no way to avoid using y derivatives of T . By inspection of eqs.(1)-(6), it is apparent that since γ and γ' are non-zero, we must solve for the five unknowns R, c, θ, γ and γ' . Thus, we require knowledge of five of the six quantities on the left hand sides of eqs.(1)-(6). We choose as our quantities T, T_x, T_y, T_{xy} , since these do not involve second derivatives with respect to y. We find that

$$c = (T_{xx}^2 + T_{xy}^2)^{1/2} / |T_{xx}T_y + T_{xy}T_x| , \quad (9)$$

$$\theta = \tan^{-1}(-T_{xx}/T_{xy}) \quad (10)$$

and the other quantities follow simply.

Finally, before we leave the discussion of the pulse-echo problem, we remark on the problem when mode conversion occurs.

In this case the signal has different speeds c_1 and c_2 as it propagates to and from the flash point. The equations (1)-(6) are replaced by similar equations with the term $\frac{1}{2}(1/c_1 + 1/c_2)$ substituted for $1/c$. Therefore, any inversion of these equations cannot produce c_1 and c_2 separately, but only in the combination $(1/c_1 + 1/c_2)$.

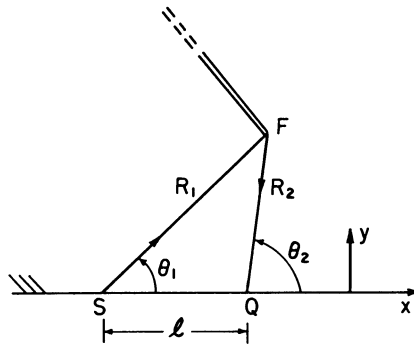


Fig. 2. Two dimensional pitch-catch configuration.

2.2 Pitch-Catch Data

Now suppose the source S and receiver Q are distinct, see Fig. 2. Then the travel time of a pitch-catch signal is T, where

$$T = R_1/c_1 + R_2/c_2 \quad (11)$$

and $c_j(\theta_j)$ $j = 1, 2$ are the two wave speeds on the rays. The ray inversion is complicated by the presence of the two different wave speeds, which introduces more unknowns into the problem. However, we can also measure more data than in the pulse-echo case. For instance, by shifting the source transducer slightly

in the x-direction, we can compute T_{1x} , the x-derivative of T with respect to source position. Similarly, shifting the receiver determines T_{2x} , the x-derivative with respect of receiver position. In the same manner we define the quantities T_{jy} , T_{jxx} , T_{jxy} and T_{jyy} , $j = 1, 2$ where the subscripts 1 and 2 refer to source and receiver respectively. We now summarize the necessary and sufficient conditions for the ray inversion. It is assumed that the wave speeds are not known a-priori, otherwise the flash point may be found quite simply for example, from T and T_{1x} .

First, suppose that the material is isotropic and the speed on both rays is the same, say c . Then three pieces of data are required. The three simplest to obtain are T , T_{1x} and T_{2x} , involving no normal or second derivatives. The five unknowns R_1 , R_2 , θ_1 , θ_2 and c can be solved explicitly. However, for brevity we refer the reader to Ref. 4 for further details.

Next, suppose the material is again isotropic with speeds unknown, but the diffracted ray is mode converted. For example, the incident ray might be one of longitudinal motion, and the diffracted ray of transverse motion. We need one additional piece of information compared with the previous case, since there are now six unknowns: R_j , θ_j and c_j , $j = 1, 2$. In order to avoid using normal derivatives, we choose T_{1xx} as the extra datum. The resulting expressions for the unknowns can be found in Ref. 4.

The problem of finding the flash point becomes more complicated if the material is anisotropic with unknown speed dependence upon angle. There are nine unknowns, R_j , θ_j , c_j , γ_j , $j = 1, 2$ and γ'_1 , requiring nine equations to solve them. The minimum required data for inversion is T , T_{jx} , T_{jy} , $j = 1, 2$, T_{1xx} and T_{1xy} . We note that we could use T_{1yy} instead of T_{1xy} , but the latter requires only a first derivative normal to the surface. In either case, it is obvious that normal derivatives are necessary to obtain a closed system of equations. Again, the unknowns can be obtained explicitly in terms of the measured quantities [4].

We have discussed the inversion of pulse-echo and pitch-catch data through a homogeneous solid. Sometimes it is possible to obtain both types of data simultaneously. For example, consider a two-dimensional configuration in which there are separate source and receiver transducers, but the source transducer also acts as a receiver. Then the two travel times $T_1 = R_1/c_1$ and $T_2 = R_2/c_2$

can be determined directly. It turns out that this single piece of extra datum simplifies the ray inversion considerably. For example, in the anisotropic case, the required data is T_j , T_{jx} and T_{jy} , $j = 1, 2$ which does not include either of the two second derivatives necessary in pure pitch-catch. However, we still need to compute two normal derivatives. This requirement of normal derivatives seems to be unavoidable, as we shall see when we consider a two-media problem below. But first we describe a numerical example involving the inversion of synthetic travel time data.

2.3 Inversion of Synthetic Data for a Solid of Cubic Symmetry

Many metals possess cubic symmetry due to their underlying crystal structure. As a result, the wave modes are not purely longitudinal or transverse, but depend upon the orientation of the ray direction to the symmetry ones. For most materials, the fastest wave speed is that of the quasi-longitudinal mode. Consider a quasi-longitudinal ray propagating in a plane formed by two symmetry axes, and let the ray be at the angle θ to one axis. The anisotropic wave speed is approximately

$$c(\theta) \approx c(0)[1 + \epsilon \sin^2 2\theta] \quad (12)$$

where

$$\epsilon = (2c_{44} + c_{12} - c_{11})(c_{11} + c_{12}) / [8 c_{11}(c_{11} - c_{44})] \quad (13)$$

and c_{11}, c_{12}, c_{44} are the elastic moduli [5]. The approximation in eq.(12) is based on the assumption that $|\epsilon|$ is small compared with unity. This is generally the case, as can be seen from Table 1. Therefore, in the following numerical examples we have used eq.(12).

Table 1. The anisotropy parameter for some cubic metals, from [5].

Material	ϵ
Aluminum	.024
Silicon	.114
Silver	.174
Iron	.236
Copper	.242
Nickel	.249

In order to compute synthetic travel times, we have considered a two-dimensional configuration with the solid occupying $y > 0$ and the symmetry axes in the x and y directions. Time and distance are made non-dimensional by taking the speed in the x -direction to be unity. A slit of length one half and describing a 45 degree angle to the positive x -axis is located such that the tip nearest the free surface $y = 0$ is at the coordinates $x = 0$, $y = 5$. Two transducers are situated on the free surface $y = 0$ at $x = -2$ and $x = 4$.

First, we shall locate the flash point at $(0,5)$ using pitch-catch and pulse-echo data simultaneously. Let T_1 be the travel time from the "source" transducer at $(-2,0)$ to the flash point, and T_2 be the travel time from the flash point to the "receiver" at $(4,0)$. In the synthetic experiment, the values of T_1 and T_2 are computed for a given ϵ in eq.(12). Then the partial derivatives T_{1x} , T_{1y} , T_{2x} and T_{2y} are computed by a finite difference procedure. For example, the "transducer" at $(-2,0)$ is shifted a distance h in the x direction and the new travel time T'_1 is computed. The quantity $(T'_1 - T_1)/h$ is then used as the "experimental" value of T_{1x} . Thus, exact travel time data is assumed, and the only error enters in the evaluation of the partial derivatives. We are interested in estimating the error in the flash point location as a function of the shift distance h .

Three methods of inversion are considered. The first, method A, assumes anisotropy and requires as data all four partial derivatives, T_{jx} , T_{jy} , $j = 1,2$. The second method B, assumes isotropy but the wave speeds on the two rays are different. For this we need only the two x -derivatives. In method C, isotropy is assumed with only one wave speed. This requires only one x -derivative in addition to the travel times T_1 and T_2 . Of course, as $h \rightarrow 0$, only method A will give exact agreement, but we consider methods B and C since they both require only x -derivatives of the travel time. In addition, B and C will be exact for isotropy, as $h \rightarrow 0$.

In Fig. 3 we have plotted the error, defined as the distance between the estimated crack tip and the actual tip $(0,5)$, as a function of shift h . Two anisotropic materials are considered, and the isotropic case is shown for comparison. In each case method A shows a linear error growth. Methods B and C are both approximations for the case of anisotropy. Similar plots for nickel, which is highly anisotropic, gave completely unacceptable results with methods B and C.

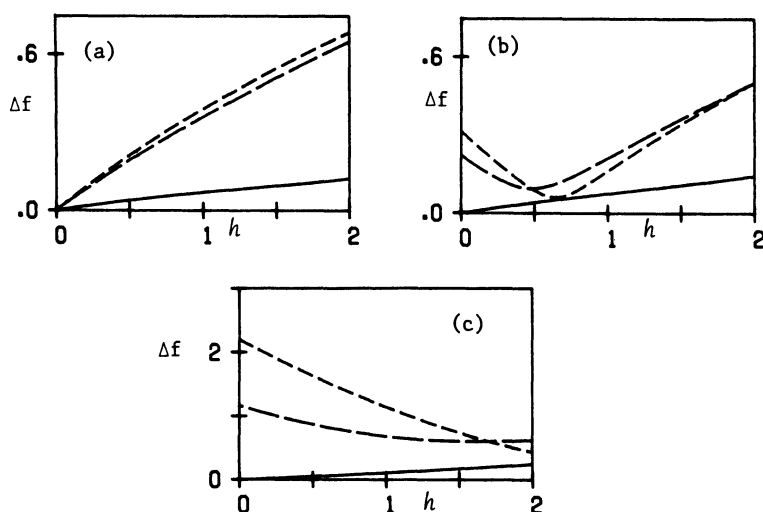


Fig. 3. Error (Δf) in the estimated flash point position versus the shift distance h for various anisotropic solids: (a) Isotropic, (b) Aluminum, and (c) Silicon. Method (A), —; method (B) — —; method (C) — · —.

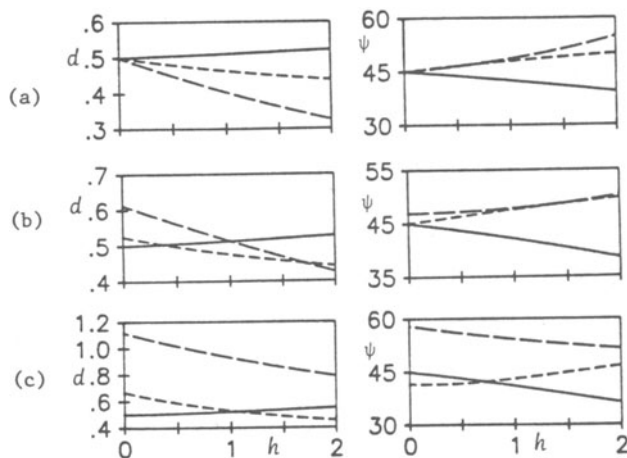


Fig. 4. Estimated crack length d and orientation ψ versus shift distance h . (a) Isotropic, (b) Aluminum, (c) Silicon. Method (A) —; method (B) — —; method (C) — · —.

In Fig. 4 we have plotted the estimated crack length and orientation, the exact values being .5 and 45 degrees, respectively. The curves in Fig. 4 were calculated by estimating the second flash point in a similar manner to that used for the near flash point. The conclusion to be made from these curves is that methods B and C can only be used for small anisotropy; otherwise method A must be used, requiring knowledge of the normal derivatives of the travel time.

3. Inverse Ray Tracing through an Interface

A simple example of a two-media problem is that in which the source and receiver are located in a water bath surrounding the solid specimen. For simplicity, the scattering is assumed to be pulse-echo and the interface is taken as flat. The geometry is again two dimensional. The source S is a distance b from the interface, which is the exterior surface of the material, see Fig. 5. The pulse-echo diffracted ray makes angles θ_f and θ with the interface in the fluid and in the solid respectively. Let $2T$ be the time taken for the signal to propagate from S to F and back again. Referring to Fig. 5, we have the relationships

$$b = R_f \sin \theta_f, \quad (14)$$

$$2T = R_f/c_f + R/c, \quad (15)$$

where subscript f indicates a quantity in the fluid. The fluid is assumed to be homogeneous and isotropic, so that c_f is constant.

The wave speed in the solid, may, because of anisotropy, depend upon θ . Snell's law at the interface is

$$\cos \theta_f / c_f = \cos \theta (1 + \gamma \tan \theta) / c \quad (16)$$

where $\gamma(\theta)$ is as before. Equation (16) may be shown to be a consequence of Fermat's Principle, or alternatively, it can be interpreted as the matching of the phase velocity components on the interface.

The spatial gradient of T at the source-receiver S is

$$\nabla T = -\mathbf{p}/c_f, \quad (17)$$

where $\mathbf{p} = (\cos \theta_f, \sin \theta_f)$. The angle θ_f is determined by any single component of ∇T , and the distance R_f is got from eq.(14). Thus, a knowledge of T and a single first order derivative of T

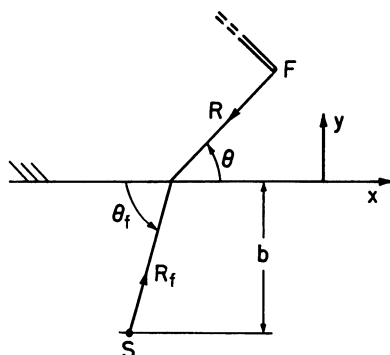


Fig. 5 Pulse-echo arrangement with the source-receiver in water.

determines the quantities R/c and $(\cos\theta + \gamma\sin\theta)/c$ by eqs.(15) and (16). This is analogous to ray tracing in a solid only, in which we know the left hand sides of eqs.(1) and (2).

Now consider second order derivatives of T with respect to the position S . If \underline{n} is any direction vector, we have that

$$(\underline{n} \cdot \nabla)^2 T = |\underline{n} \wedge \underline{p}|^2 / c_f a \quad (18)$$

where a is the radius of curvature of the wavefront at S ;

$$a = R_f + R \frac{\sin\theta_f \tan\theta_f}{\sin\theta \tan\theta} \frac{1 + \gamma \tan\theta}{1 + \gamma^2 - \gamma'} \quad (19)$$

and $\gamma'(\theta)$ is as before. Assume a to be known by measurement of some second order derivative of T . Also, the quantity $z \equiv (\cos\theta + \gamma\sin\theta)/c$ is known from first derivatives. We note from eq.(19) that the quantity $z \sin\theta_f \tan\theta_f / (a - R_f)$ is precisely the right hand side of eq.(4). Thus, we have reduced the ray inversion problem in the fluid-solid to a similar problem in the solid only, where the quantities T, T_x and T_{xx} of eqs.(1),(2) and

(4) are known. But, as discussed above, this information is sufficient for finding the flash point only if the solid is isotropic, see eqs.(7) and (8). If the material is anisotropic, further information is required, specifically the left hand sides of eqs.(2),(4),(5) and (6). These quantities all involve normal derivatives of the travel time at the interface. However, according to eq.(16), on transversing the interface the ray transmits only information concerning the tangential derivative. All information about the normal derivatives in one medium is lost as the ray enters a different medium. We conclude that the general problem of ray inversion in unknown anisotropic media is under-determined if the measurements are made only within some other medium.

4. Inverse Ray Tracing when the Wave Speeds are Known

We now turn to the simpler problem of finding the flash points when the constitutive nature of the material is known a-priori. The inversion reduces to finding the intersection or congruence of a set of curves in the solid, each of which corresponds to a given travel time. For example, in a two dimensional isotropic configuration where the fastest wave speed is c (a constant), let $2T_1$ and $2T_2$ be the first arriving pulse-echo signal times from two different transducers. Then the nearest flash point is found as the intersection of the circles of radii cT_1 and cT_2 , centered at the transducers. Now suppose the material is anisotropic, the circles become curves of equal travel time defined by $r(\theta) = T_1/c(\theta)$ etc. where $c(\theta)$ is the anisotropic speed. Finding the intersection of these curves can require a lot of numerical work, since in general $c(\theta)$ is a far more complicated function than in eq.(12).

Briefly, the algorithm for calculating $c(\theta)$ is as follows: First the phase velocity $v(\phi)$ must be known as a function of the angle ϕ ; in linear elasticity $v(\phi)$ is generally an algebraic function of $\sin\phi$. The angle $\delta = \delta(\phi)$ is defined by

$$\tan\delta = (1/v) \partial v(\phi) / \partial \phi \quad (20)$$

For a given angle θ , the wave speed $c(\theta)$ is

$$c(\theta) = v(\phi) \sec\delta, \quad (21)$$

where $\phi = \phi(\theta)$ is such that

$$\phi + \delta(\theta) = \theta. \quad (22)$$

A similar algorithm can be defined for three dimensional anisotropy. In three dimensions, the flash points must be found from the intersections of a large set of equi-travel time surfaces. Since each point on a surface requires computing the relevant wave speed, the operation of mapping a crack edge could use an enormous amount of computer time. We now describe an alternative, approximate intersection method which converges to the exact solution, and is numerically much more efficient.

4.1 Approximation by Planes

Consider a pulse echo arrangement in an anisotropic solid with $2T$ as the measured travel time of a diffracted signal from an unknown flash point at the position vector \underline{x} . Let \underline{x}_0 be some arbitrary point in the solid, and c_0, T_0 be the speed and travel time of a ray from the transducer to this point. Since the travel time T is strictly a function of the flash point position \underline{x} , we can write the difference $(T-T_0)$ as a Taylor's series,

$$T-T_0 = (\underline{x}-\underline{x}_0) \cdot (\nabla T)|_0 + \frac{1}{2}(\underline{x}-\underline{x}_0)(\underline{x}-\underline{x}_0)(\nabla \nabla T)|_0 + \dots \quad (23)$$

where the derivatives are evaluated at \underline{x}_0 . The first term on the right of eq.(23) can be simplified as follows: let $c_0 \underline{p}_0$ be the wave velocity vector at \underline{x}_0 , i.e. the unit vector \underline{p}_0 represents the ray direction from the source to the point \underline{x} . Let the corresponding phase velocity vector be $\underline{v}_0 \underline{n}_0$, which is related to the wave velocity through eqs.(20)-(22). In Ref. 3 it is shown that

$$\nabla T|_0 = (1/v_0) \underline{n}_0. \quad (24)$$

The phase and wave velocities are identical for isotropy, in which case eq.(24) becomes self-evident.

Now suppose that the distance $d \equiv |\underline{x}-\underline{x}_0|$ is small as compared with the ray length r_0 from the transducer to the point \underline{x}_0 . Then the second term on the right of eq.(23) is of order $(d^2/c_0 r_0)$ in magnitude. This may be seen by noting that $||\nabla \nabla T|| \leq |\nabla^2 T|$; and the Laplacian of T is proportional to $1/c_0$ times the curvature of the wavefront at \underline{x}_0 , which is approximately $1/r_0$. Observing that $(d^2/c_0 r_0)$ is of order (d/r_0) times smaller than $(T-T_0)$, we may neglect second and higher derivatives in eq.(23), and rewrite it in the approximate form

$$(\underline{x} - \underline{x}_0) \cdot \underline{v}_0^{-1} \underline{n}_0 - (T - T_0) = 0. \quad (25)$$

We can interpret this equation as follows: the wavefront near the point \underline{x}_0 at time T_0 is locally planar. At times close to T_0 , the local plane approximation to the wavefront is parallel to the plane at time T_0 , but shifted slightly in the normal direction. Because of anisotropy, the normal direction is not \underline{p}_0 the ray direction, but \underline{n}_0 the phase propagation direction. Finally, the plane approximation holds over lengths small in comparison with the radius of curvature, $d/r_0 \ll 1$.

4.2 Finding the Flash Points: The Local Edge Mapping

Suppose we have two pulse-echo travel times for one flash point in a two dimensional geometry. A base point \underline{x}_0 is chosen, which is assumed to be near the flash point. Some a-priori knowledge may be used in the choice of the base point location. For example, a cursory application of one of the procedures described in Section 2 would suggest the neighborhood of the flash point. Next, two lines (the 2-D analogy of planes) are formed relative to \underline{x}_0 using eq.(23) with T replaced by the two pulse-echo times. In each line, the travel time T_0 is the same, having been computed for the chosen base point. Also, the phase speed c_0 and direction \underline{n}_0 , which depend upon \underline{x}_0 , are known. The only unknown is the point \underline{x} , which is found from the intersection of the lines. This point is a first approximation to the flash point, hence we call this procedure a local mapping technique, in contrast to the global methods of Section 2.

The completion of the algorithm, and it's generalization to three-dimensional cracks are the same as those for the isotropic local edge mapping method, which is fully described in the paper by Norris and Achenbach [3]. A comparison of analytical and experimental results can be found in Ref.[6].

4.3 Small Anisotropy

The travel time T_0 to the base point is equal to $(r_0 c_0)$, where r_0 is the ray path length. In calculating T_0 for eq.(23), the wave speed c_0 must be computed quite accurately, otherwise the relatively large distance r_0 could magnify the error. However, we may approximate the vector $\underline{v}_0^{-1} \underline{n}_0$, since the distance multi-

plying it is by assumption small compared with r_o . We say the anisotropy is small if the angle δ of eq.(20) satisfies $|\delta| \ll 1$. If the anisotropy is small, we have by eqs.(20) and (21) that

$$v_o \approx c_o \quad (26)$$

and

$$\delta \approx \dot{c}_o / c_o \quad (27)$$

where $\dot{c} = \partial c(\theta) / \partial \theta$. The direction vector \underline{n}_o follows from eqs.(22) and (27). Let $\underline{p}_o = (\cos\theta, \sin\theta)$ be the ray direction. Then we have

$$\underline{n}_o^{-1} \approx c_o^{-1} (\cos[\theta - \dot{c}_o / c_o], \sin[\theta - \dot{c}_o / c_o]) \quad (28)$$

A similar approximation can be determined for three dimensional anisotropy.

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DISCUSSION

R.B. Thompson (Ames Laboratory): In describing the experimental configuration with a transducer and fluid, and trying to obtain a normal derivative, I believe you said that if you move the transducer back and forth, you'd only get something relating to spatial derivatives?

A.N. Norris (Northwestern University): Yes.

R.B. Thompson: My question is: Do you assume that the angle is fixed and, if so, couldn't you get normal derivatives by slightly varying the angle of the transducer as well as its position?

A.N. Norris: No. Whatever way you move it, both in the ray direction or transverse to that direction, it only gives you horizontal information.

R.B. Thompson: Intuitively, I would think that what you're trying to do is vary the direction of the ray in the material.

A.N. Norris: Yes.