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PROBABILISTIC PREDICTION OF THE JOMINY CURVE OF LOW ALLOY STEELS FROM COMPOSITION AND GRAIN SIZE.

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Probabilistic prediction of the Jominy curve of low alloy steels from composition and grain size

by

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NOMENCLATURE

C: Carbon content of alloy steel, percent

Mn: Manganese content of alloy steel, percent

Si: Silicon content of alloy steel, percent

Ni: Nickel content of alloy steel, percent

Cr: Chromium content of alloy steel, percent

Mo: Molybdenum content of alloy steel, percent

Cu: Copper content of alloy steel, percent

S: Sulphur content of alloy steel, percent

P: Phosphorus content of alloy steel, percent

D_{IC}: Carbon factor (ideal critical diameter) of alloy steel, in

D_I: Critical diameter of alloy steel, in

f_{Mn}: Multiplying factor for manganese in prediction of the hardenability of alloy steel

f_{Si}: Multiplying factor for silicon in prediction of the hardenability of alloy steel

f_{Ni}: Multiplying factor for nickel in prediction of the hardenability of alloy steel

f_{Cr}: Multiplying factor for chromiun in prediction of the hardenability of alloy steel

f_{Mo}: Multiplying factor for molybdenum in prediction of the hardenability of alloy steel

f_{Cu}: Multiplying factor for copper in the prediction of the hardenability of alloy steel

τ: Half temperature time or time constant, second

a: Thermal diffusivity of the steel, in^2/sec

G.S.: ASTM grain size of low alloy carbon steel

I.H.: Initial hardness, hardness at the extreme quenched end of the Jominy test piece, Rockwell C

D.H.: Distant hardness, hardness at some distance from the quenched end of the Jominy test piece, Rockwell C

Q_p: Mean of predicted Jominy hardness, Rockwell C

σ_{Q_p}: Standard deviation of predicted Jominy hardness, Rockwell C

Q_M: Mean of measured Jominy hardness, Rockwell C

σ_{Q_M}: Standard deviation of measured Jominy hardness, Rockwell C

μ_x: Mean of random variable x

σ_x: Standard deviation of random variable x

μ_y: Mean of random variable y

σ_y: Standard deviation of random variable y

$\sigma_{y/x}$: Standard deviation of y on x

R: Coefficient of correlation

N: Number of data points

INTRODUCTION

Adequate hardenability has long been recognized as one of the first requirements for producing certain desired mechanical properties in a heat-treated steel. Hardenability of a steel is the capacity, when it is quenched and tempered to obtain an essentially martensitic structure throughout the cross section The cooling rate during heat treatment and such factors as composition and grain size which control transformation at elevated temperature, affect the hardenability of a steel. In order to compare the hardenability of one steel with that of another steel, the cooling rate, composition, and grain size have to be considered in relation to the microstructure resulting from the transformation of austenite. "Critical diameter" has been widely accepted and used as an index for hardenability comparison of low carbon alloy steels.

The critical diameter of a low alloy steel is defined as the largest diameter of a right circular cylindrical bar that contains at its center a microstructure of 50% martensite after it is quenched in a medium with an infinite severity of quench. The method proposed by Grossmann (1) enables us to predict the critical diameter of a low carbon alloy steel from its grain size and chemical composition. A Jominy end-quench hardness curve can be predicted by using the empirical relationship between critical diameter and ratio of initial hardness, I.H. (which depends on carbon content of the steel alone), to distant hardness, D.H., as developed by Field (2). The critical diameter of a low carbon alloy steel can be correlated to the Jominy end-quench distance which has a microstructure of 50% martensite. Consequently, the hardenability of low carbon alloy steel can be characterized by its Jominy end-quench hardness curve.

The designer often makes a decision concerned with heat treatment before the steel to be used in the product is smelted. It is important that the designer be able to predict important properties from composition. Such predictions should include mean values as well as some measure of statistical dispersions.

Since a tempered hardness predictive procedure would incorporate a Jominy test (if the material is on hand) or a Jominy prediction, the first step toward a complete method of predicting quenched and tempered properties is to statistically estimate the Jominy signature of the material for those cases wherein the material is not on hand for test.

The methodology once developed can be automated using the computer to make routine Jominy signature predictions conveniently available to a designer.

It is the objective of this investigation to:

1. Identify a method for predicting the Jominy hardness profile of a material from its composition and

grain size.

2. Devise a method to predict the statistical dispersion in the Jominy hardness profile prediction.

3. Compare the results of the method with actual Jominy data from steel producers.

4. Incorporate the methodology if successful, into a computer program that is easy for the designer to use.

IDENTIFY A METHOD FOR PREDICTING THE JOMINY HARDNESS PROFILE OF A MATERIAL FROM ITS COMPOSITION AND GRAIN SIZE

Predict Hardenability from Chemical Composition and Grain Size

In 1942 Grossmann (1) proposed that the hardenability of a low carbon alloy steel may be calculated from its chemical composition and grain size. According to his thesis the steel is considered as having a base hardenability due to its carbon content without any other alloy element. The total hardenability of the steel is established by multiplying the base hardenability by a factor reflecting the contribution of each additional chemical element. The effect of the as-quenched grain size of the steel is incorporated in the base hardenability.

Critical diameter

Grossmann expressed the hardenability of an alloy carbon steel in terms of "critical diameter", namely the diameter of bar, in inches, that will just harden all the way through (absence of unhardened core) in an "ideal" quench. The hardenability may also be related to the Jominy hardness test through the 50% martensite hardness criterion of the steel.

Grossmann method for predicting the critical diameter of a low carbon alloy steel from chemical composition and grain size

In developing his thesis, Grossmann made the assumption that the effect of each alloy element on the hardenability of the steel is independent of every other and that the steel is heat-treated at an austenitizing temperature sufficiently high to dissolve all the carbides. Stable undissolved carbides are absent in the steel as-quenched. The amount of carbide that precipitates from austenite depends upon the quenching temperature and the time period during which the test piece is held at the quenching temperature prior to quenching. For simple alloy steels, the calculated hardenability correlated with the measured hardenability value (see Table 1 and Figure 1), but in case of complex alloy steel, especially in chromium steels, and chrome-molybdenum and chrome-vanadium steels, the calculation indicated only a maximum possible hardenability. The actual hardenability obtained may be much less. Grossmann explained this by reasoning that since chromium, molybdenum and vanadium have strong tendencies to form carbides, the smaller hardenability effect of complex steels is due to the decrease in alloy and carbon content of the austenite before quenching caused by the presence of undissolved carbides.

Steven (3), studying the effect on hardenability of chromium and molybdenum combinations, found that when both elements were present the hardenability was lower than that

Table 1. Calculated and actual hardenability (data from Table 18 of (1))

Average Grain Size	Composition of Steels (%)									Calculated	Actual
	C	Mn	P	S	Si	Ni	Cr	Cu	Mo	D_I in.	D_I in.
4-6	0.41	0.79	0.015	0.026	0.20	0.07	0.07	0.03		1.35	1.34
7	0.63	0.94	0.031	0.027	0.20	0.03	0.05	0.02		1.58	1.71
5	0.61	0.85	0.017	0.025	0.33	0.06	0.05	0.02		1.82	1.87
6.2	0.65	1.04	0.015	0.25	0.19	0.03	0.27	0.02		2.84	2.66
6.2	0.66	0.97	0.019	0.020	0.25	0.03	0.07	0.02	0.10	2.79	2.53
7	0.51	1.05	0.014	0.029	0.29		0.21	0.06		2.10	2.22
5	0.39	1.74	0.023	0.021	0.26	0.01	0.13	0.07		2.89	2.77
6	0.57	0.68	0.019	0.028	2.00		0.17	0.05		2.25	2.28
7	0.69	0.81	0.014	0.024	0.24	0.01	?	0.08	0.22	2.18	2.4
6	0.40	1.70	0.022	0.030	0.21	0.21	0.13	0.06	0.05	3.15	2.70
7	0.41	1.82	0.015	0.024	0.21	0.18	0.09	0.10	0.03	2.93	2.77
7	0.41	1.85	0.019	0.029	0.25	0.13	0.10	0.09	0.02	3.02	3.07
6	0.41	1.77	0.019	0.014	0.20	0.28	0.16	0.07	0.05	3.41	3.17
7	0.46	1.88	0.019	0.024	0.25	0.15	0.12	0.05	0.03	3.42	3.63
6	0.45	2.01	0.28	0.019	0.22	0.18	0.20	0.06	0.04	4.20	4.22

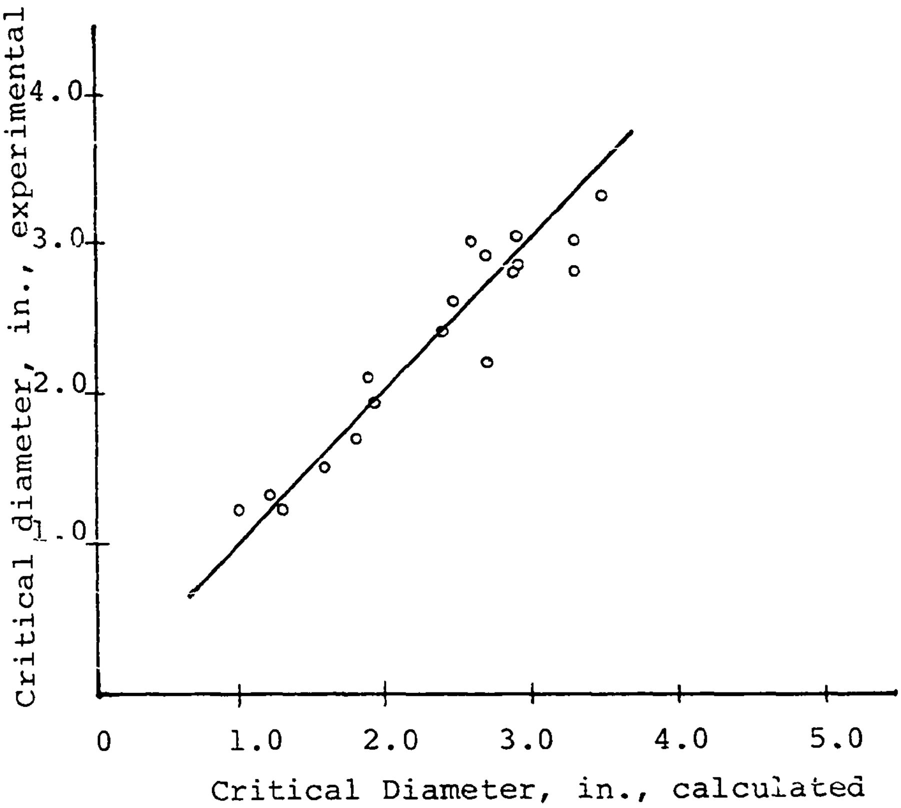

Figure 1. Relationship between calculated hardenability and that found by experiment (cited from Figure 1 of (1))

expected from a combination of the factors of the individual element (see Figure 2). It is possible that interactions may occur whenever two or more stable carbide-forming elements are present in the element alloy.

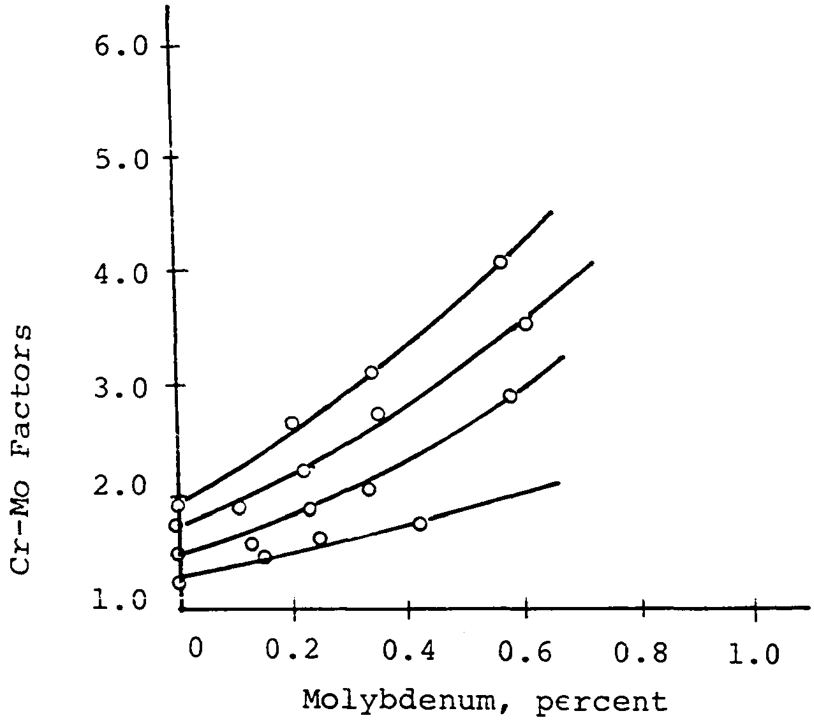

Figure 2. Multiplying factor chromium-molybdenum combination (cited from Figure 20 of (3))

Grossmann's multiplying factor of phosphorus, sulphur, silicon, manganese, nickel, chromium, and molybdenum

In order to predict the hardenability of a heat treated alloy steel from its chemical composition and grain size, Grossmann developed a set of graphs (multiplying factors) for alloy elements commonly present in commercial steels.

According to Grossmann's hypothesis the presence of a certain amount of chemical element multiplies the hardenability by a certain factor. The multiplying factor of phosphorus was originally determined by adding phosphorus to successive ingots of two steels, so that in each series the steels were substantially identical except for the phosphorus content. The composition of the two steels, which Grossmann used in his original experiment, was as shown in Table 2. The hardenabilities were measured in terms of critical diameter and results were plotted in Figure 3 as critical diameter (D_I) v.s. phosphorus content, percent. In the figure the two lines, one for each steel, were drawn to represent the increase in hardenability due to phosphorus. The slope of these lines are such that the increase in hardenability due to phosphorus is the same percentage in the high hardenability steel as in the low hardenability steel. Thus, the effect of phosphorus on hardenability can be derived by taking the ratio of the critical diameter of the steel in Table 2 to the intercept of the corresponding straight line in Figure 3, which is the critical diameter of the given steel with zero percentage of phosphorus.

Figure 4 shows the effect of phosphorus on hardenability in terms of multiplying factor. Similarly, the effect of sulphur silicon, nickel, chromium, manganese, molybdenum and copper (copper has approximately the same effect on hardenability as nickel) were determined and superposed as depicted in Figure 5. It was found that all the alloy elements mentioned above,

Table 2. Steels for study of effect of phosphorus (cited from Table 2 of (1))

Composition (%)								Average Grain Size	Ideal Diameter D_I, in.
C	Mn	P	S	Si	Ni	Cr	Cu		
0.62	0.98	0.020	0.018	0.22	0.05	0.05	0.02	4.5	1.95
0.62	0.98	0.038	0.018	0.22	0.05	0.05	0.02	4.5	1.99
0.62	0.98	0.056	0.018	0.22	0.05	0.05	0.02	4.5	2.08
0.62	0.98	0.077	0.018	0.22	0.05	0.05	0.02	4.5	2.22
0.62	0.98	0.097	0.018	0.22	0.05	0.05	0.02	4.5	2.38
0.63	0.94	0.031	0.027	0.20	0.03	0.05	0.02	7.0	1.71
0.63	0.94	0.047	0.027	0.20	0.03	0.05	0.02	7.0	1.81
0.63	0.94	0.067	0.027	0.20	0.03	0.05	0.02	7.0	1.86
0.63	0.94	0.086	0.027	0.20	0.03	0.05	0.02	7.0	1.91
0.63	0.94	0.104	0.027	0.20	0.03	0.05	0.02	7.0	1.97

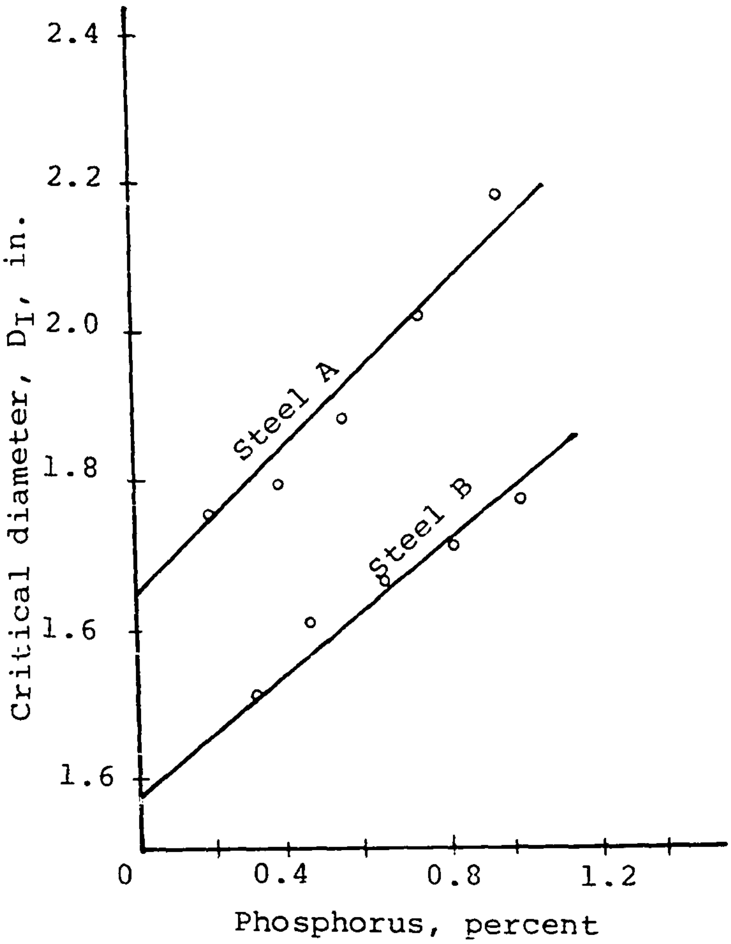

Figure 3. Effect of phosphorus content on hardenability determined experimentally (cited from Figure 3 of (1))

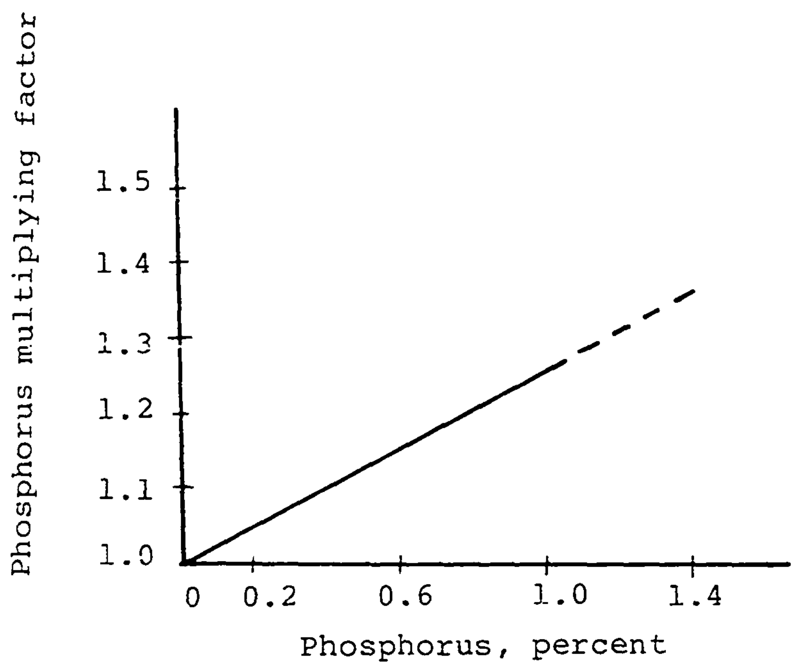

Figure 4. Multiplying factor for phosphorus (cited from Figure 4 of (1))

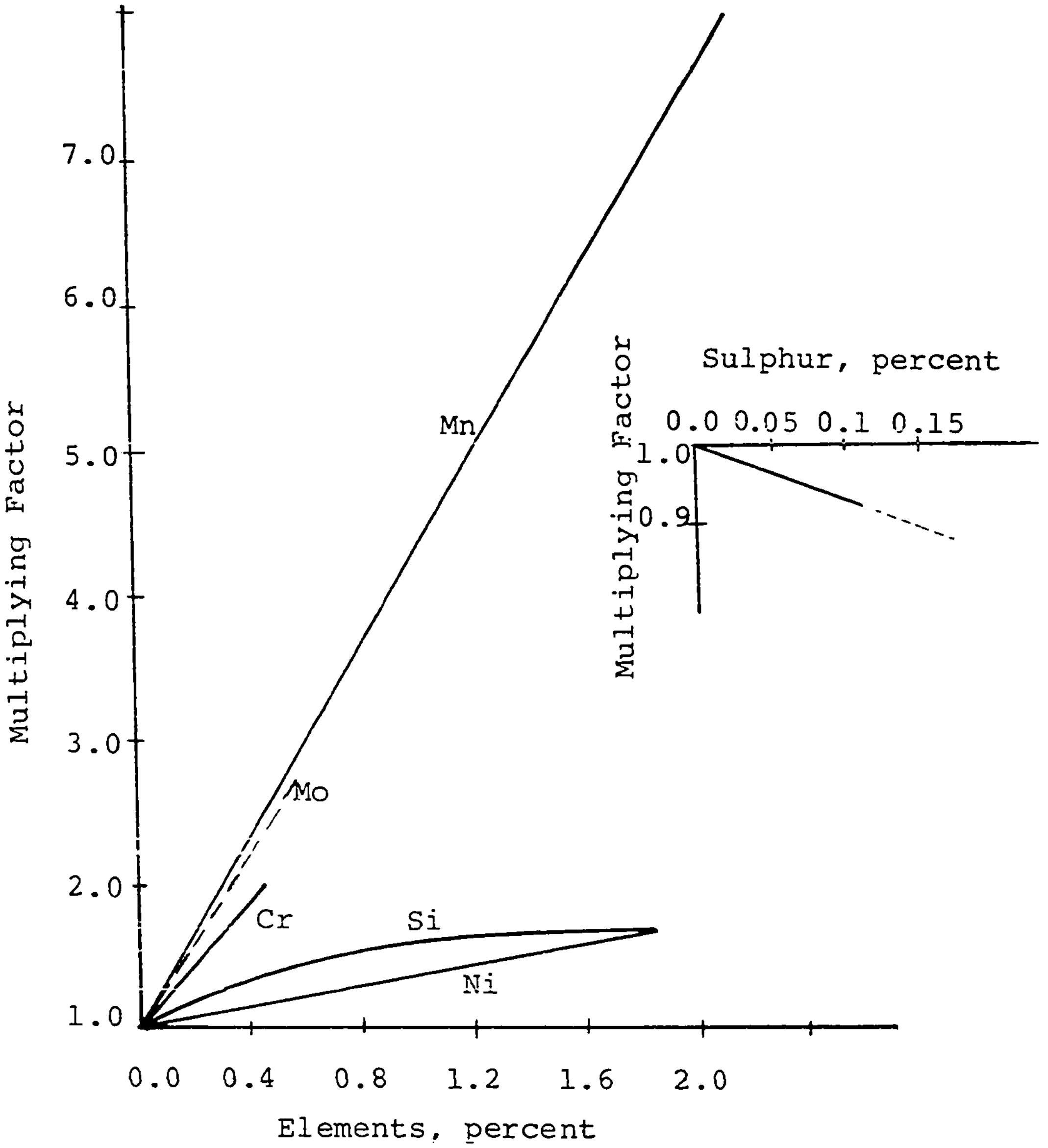

Figure 5. Multiplying factor of manganese, silicon, nickel, chromium, molybdenum, and sulphur (cited from Figure 27 in (1))

except sulphur, increase the hardenability of the steel.

Grossmann's carbon factor

Data on "pure steels" were not available in terms of "ideal critical diameter" as needed for determining the carbon factor (the base hardenability due to carbon content alone) of a steel. Grossmann first assumed that the cooling time (or cooling rate), in the range of 930° to 1100°F, that would just cause the steel to harden fully is a constant fraction of the "half temperature time"[1] (the time it takes for the metal to cool from the quenching temprature to a temperature halfway between the quenching temperature and temperature of the medium). Secondly, he assumed that the time for full hardening is a constant fraction of the time for half hardening. Digges (4) has shown that, there is a linear relationship between the carbon content and the cooling time from 1100° to 930°F, which will just provide full hardening of the steel. When the half temperature time is known, the ideal critical diameter (base hardenability due to carbon content alone) can be calculated by using the relation (5):

$$D_I = D_I^2 = K^*\tau$$

[1]Present day terminology would be "time constant".

where D_I is the "ideal critical diameter" K is a known constant, and τ is the half temperature time.

In Grossmann's original test, different amounts of carbon were added to successive ingots during an open-hearth heat. These successive ingots were substantially identical except for carbon content. The chemical analysis, as-quenched grain size and the measured ideal critical diameter D_{IC} corrected to ASTM No. 5 grain size, were as shown in Table 3. The ratio between the total hardenability of the steel and the product of the multiplying factors of all the other alloy elements, except carbon, will be the hardenability effect of the carbon alone. The results were tabulated as in Table 4.

If the linear relationship between carbon content and cooling time postulated by Digges (4) is true and the two assumptions mentioned are valid, then there should be a straight line relationship between carbon content and half temperature time τ is to be expected. Since D_I^2 is proportional to half temperature time $\tau(D_I^2=K*\tau)$, there should be a linear correlation between D_I^2 and carbon content. As is shown in Figure 6, the three available data points suggest a straight line containing the origin, suggesting zero hardenability at zero carbon content. (Coefficient of correlation between D_I^2 and carbon content is 0.996434, which is significant at five percent level). Grossmann presented this evidence of the "straight. line" relationship between carbon content and D_I^2. It is known that the pure

Table 3. Ingots with carbon additions (cited from Table 9 of (1))

Composition (%)								Average Grain Size	Ideal Diameter D_I, in. Corrected to No. 5 Grain Size
C	Mn	P	S	Si	Ni	Cr	Cu		
0.41	0.79	0.015	0.026	0.20	0.07	0.07	0.03	4.6	1.34
0.54	0.79	0.015	0.026	0.20	0.07	0.07	0.03	4.6	1.52
0.68	0.79	0.015	0.026	0.20	0.07	0.07	0.03	4.6	1.75

Table 4. Effect of carbon alone (cited from Table 11 of (1))

Carbon (%)	Ideal Diameter D_I at No. 5 Grain Size	Divided by Factor for other Elements	Ideal Diameter of Pure Iron-Carbon Alloy at No. 5 Grain Size D_I, in.	D_I^2
0.41	1.34	5.274	0.2540	0.0647
0.54	1.52	5.274	0.2882	0.0831
0.68	1.75	5.274	0.3316	0.1098

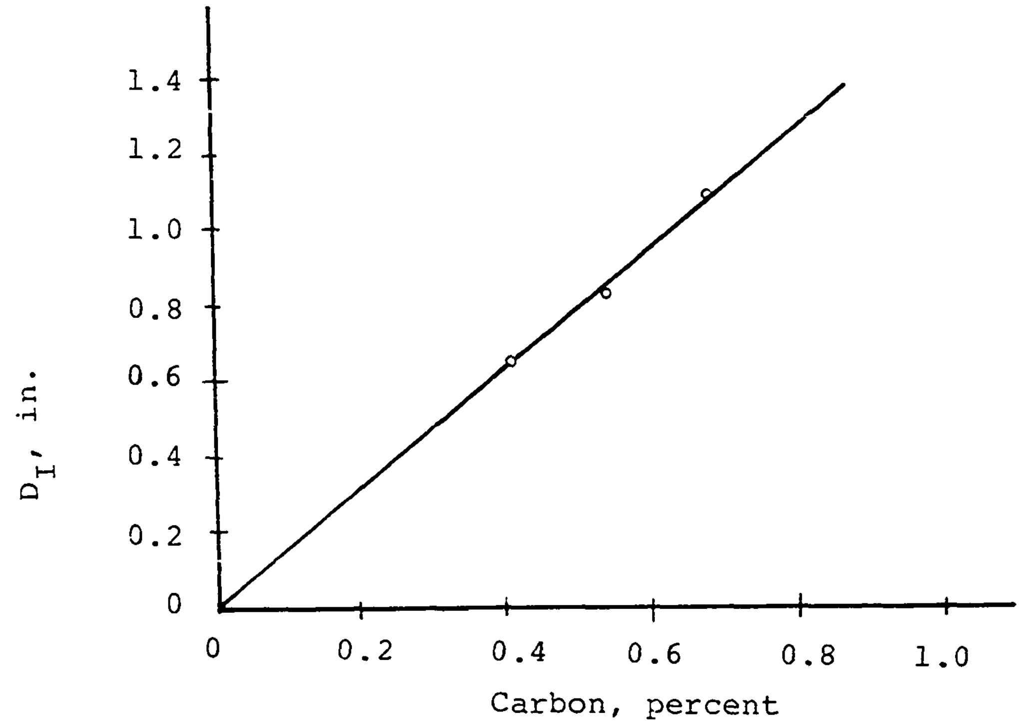

Figure 6. Relation of square of ideal diameter to carbon content (cited from Figure 17 of (1))

iron has practically negligible hardenability. From Figure 6 the carbon factor of grain size 5 was established and shown in Figure 7.

By the work of Grossmann and Stephenson (6) it had been shown that an increase of one grain size number caused a certain percentage increase in D_I^2, which means that the relationship between D_I^2 and grain size could be drawn as a series of parallel straight lines on semilogarithmic coordinate paper. If this was true, then it was clear that the

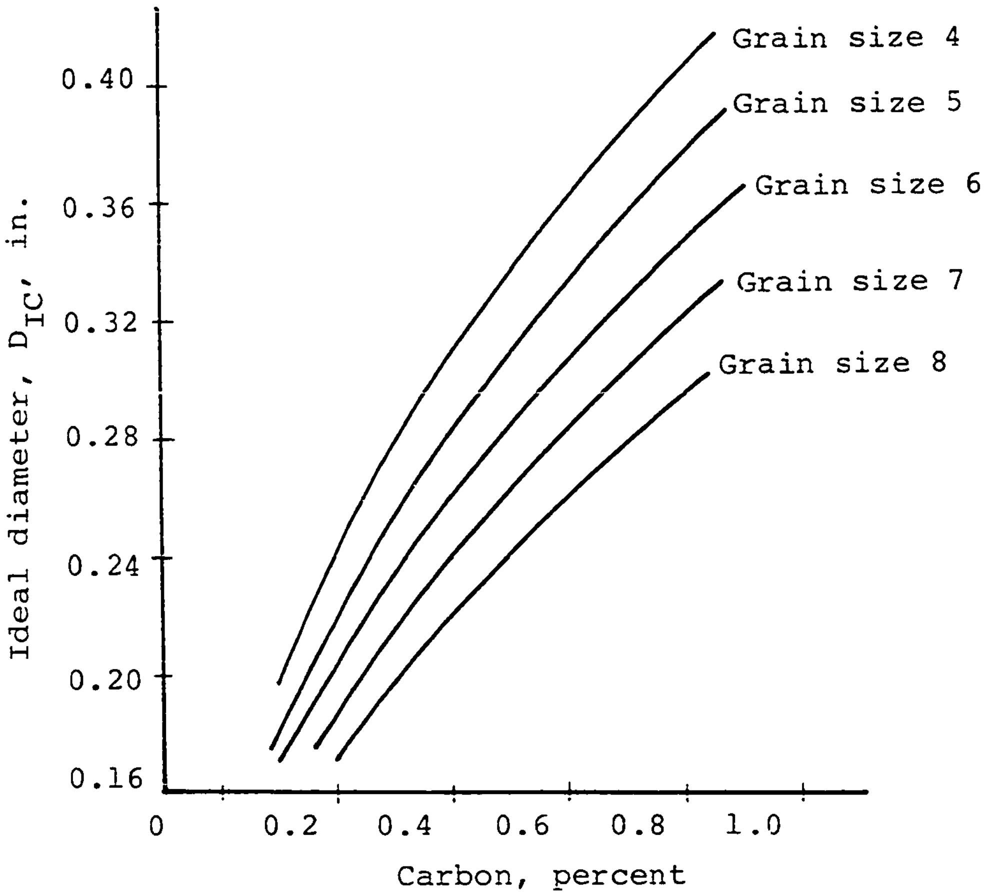

Figure 7. Hardenability of pure iron-carbon alloys, expressed as ideal diameter (cited from Figure 18 of (1))

relationship between grain size and D_I could likewise be drawn as a series of parallel straight lines on semilogarithmic coordinate paper as shown in Figure 8. The meaning of Figure 8 is merely that if the hardenability is known for a steel at one grain size, its hardenability at some other grain size can be read directly from Figure 8. Based on the carbon factor of grain size 5 and with the aid of Figure 8 the carbon factors for grain size 4, 6, 7 and 8 were established and are shown in Figure 7.

Calculation of critical diameter from chemical composition and grain size

So far the base hardenability due to carbon content alone and multiplying factors for different alloy elements have been determined. The general formula for calculating hardenability of a low carbon alloy steel from its chemical composition and grain size is as follows:

$$D_I = D_{IC}*f_{Mn}*f_{Si}*f_{Ni}, \ldots, \text{etc.}$$

where:

D_I - critical diameter, inches;

D_{IC}- ideal critical diameter for the carbon percentage and grain size from Figure 7; and

f - multiplying factor for alloy element from Figure 5.

Suppose we want to predict the hardenability of a steel which has the chemical composition and grain size as shown in the following table. Ideal critical diameter and multiplying

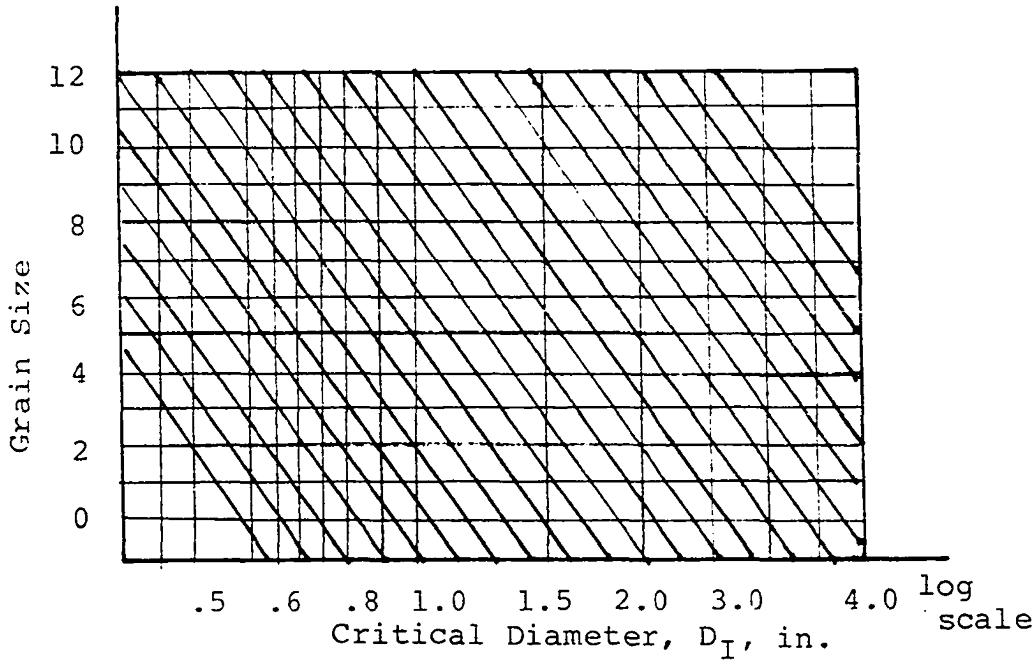

Figure 8. Effect of grain size on hardenability (cited from Figure 16 of (1))

factors for different alloy elements, shown in third column of the table were read from Figure 7 and Figure 5 for the given content of carbon and alloy elements. The product of the ideal critical diameter and multiplying factors is the ideal diameter of the given steel.

Example of hardenability calculation:

Element	Percentage in Steel	Multiplying Factor
Carbon	0.50	0.24
Manganese	0.90	4.00
Phosphorus	0.02	1.05
Sulphur	0.02	0.98
Silicon	0.10	1.10
Nickel	0.28	1.10
Chromium	0.30	1.70
Molybdenum	0.05	1.16
Copper	0.05	1.02
Product (critical diameter)		

Lamont's multiplying factor for manganese, silicon, nickel, chromium, molybdenum

Multiplying factors for alloy elements in steel established by Grossmann (1) were restricted to narrow range of alloy contents. Crafts and Lamont (7) extended the range of alloy contents of the multiplying factors and found that they agree fairly well with those published by Grossmann. In their experiments, Crafts assumed that Grossmann's factor for carbon, grain size, phosphorus, sulphur, and copper were correct. Using these factors and those determined experimentally, the ideal critical diameter was predicted on the assumption that the element under consideration accounted for all the change in the hardenability of the steel. The multiplying factor for a given element was determined as the ratio between the experimentally determined critical diameter and the critical diameter calculated by Grossmann's method from the other components of the composition. And the results are as shown in Figures 9 through 13. Crafts and Lamont also observed that the hardness at 50% martensite microstructure was influenced by alloying elements to a much greater degree than was indicated by Grossmann. The difference in hardness at 50% martensite microstructure appeared to be due to the character of the nonmartensitic part of the structure, the amount of which is determined largely by the amount and type of alloying elements in the steel.

Hardenability Concept

The hardenability of a steel is measured in terms of the severity of the cooling conditions necessary to avoid the pearlite and bainite transformation. The less rapid the

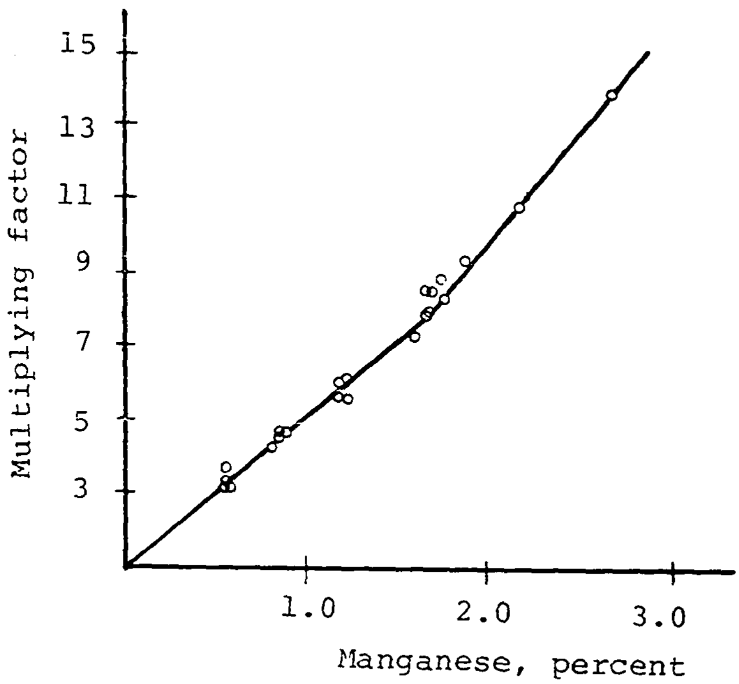

Figure 9. Multiplying factor for manganese (cited from Figure 1 of (7))

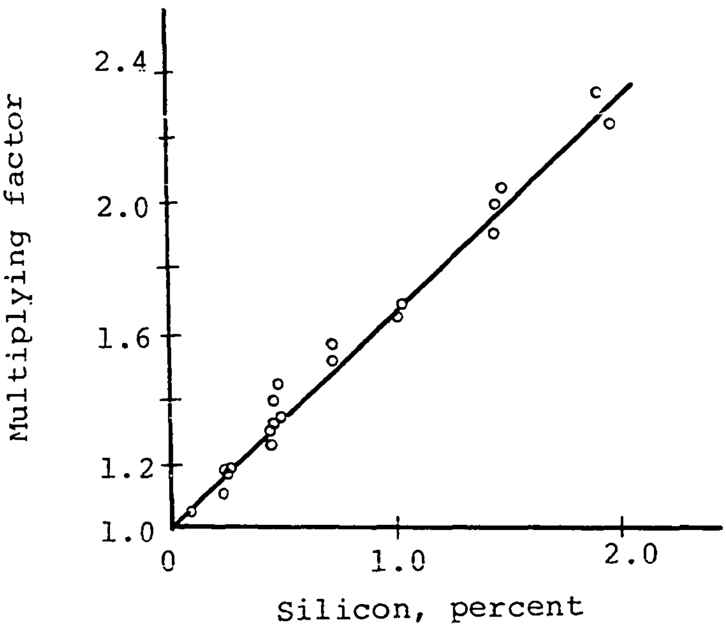

Figure 10. Multiplying factor of silicon (cited from Figure 2 of (7))

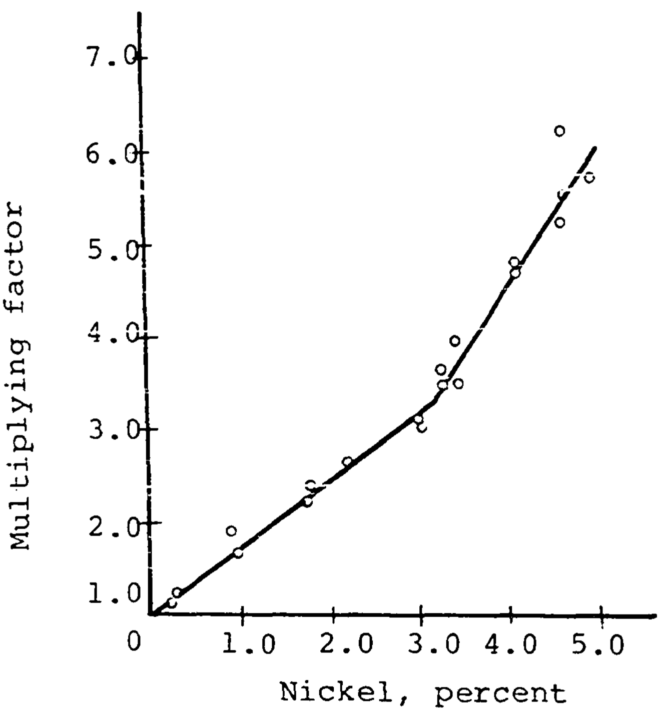

Figure 11. Multiplying factor for nickel (cited from Figure 4 of (7))

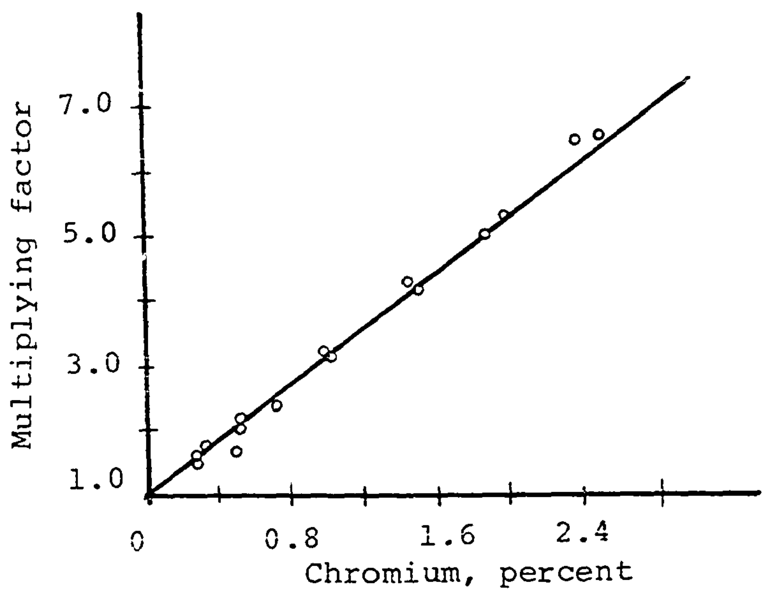

Figure 12. Multiplying factor for chromium (cited from Figure 5 of (7))

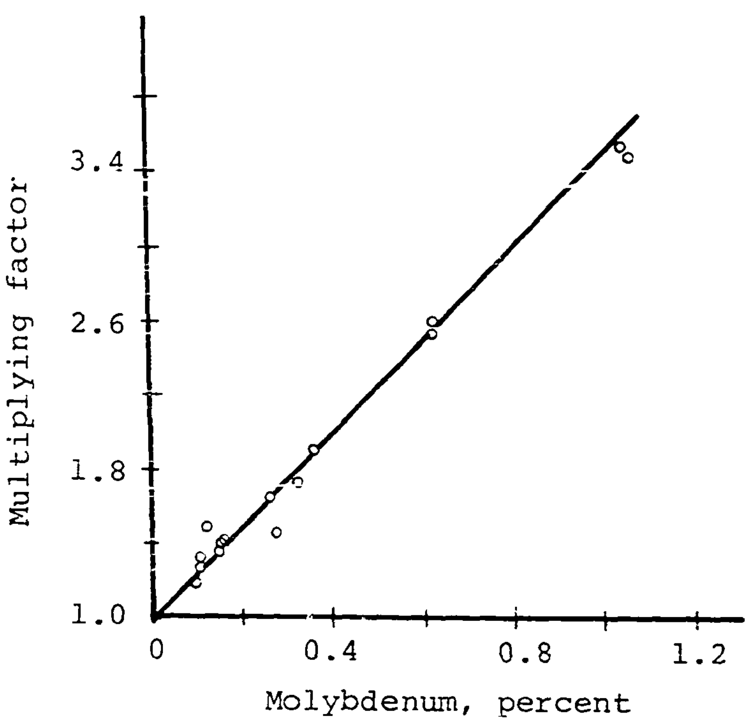

Figure 13. Multiplying factor for molybdenum (cited from Figure 6 of (7))

cooling necessary to prevent the formation of bainite and pearlite, the higher is the hardenability. Thus there are two hardenabilities, pearlitic and bainitic (8).

For plan carbon steel of moderate or small grain size which has been cooled at such a rate that it consists of 50% martensite and 50% nonmartensitic product, the nonmartensitic product has been reported by Grossmann, Asimow, and Urban (5) as pearlite. Therefore, if alloying elements that retard equally the pearlite and bainite transformations are added to plain carbon steels, the measured hardenability of the resulting steels, on the basis of 50% martensite, will be the pearlitic hardenability. If on the other hand, alloying elements are added that have a great retarding effect on the pearlite transformation than on the bainite, a composition will be reached in which bainite will restrict the formation of martensite, and the measurements of the further additions of the alloying elements will then apply to the bainitic hardenability.

The isothermal transformation data indicate that the elements that have the same or less of a carbide-forming tendency than iron retard the pearlite and the bainite transformations by approximately equal percentage. Thus the elements carbon, manganese, nickel, silicon, chromium (up to 0.5 percent) and molybdenum (up to 0.2 percent) do not appear to affect the nonmartensitic transformation selectively for small or moderate addition. The multiplying factors established by Grossmann can be assumed to be applied to the pearlite hardenability.

Hardenability Effects in Relation to the Percentage of Martensite

Hodge and Orehoski (9) established the relationship between hardenability and percentage of martensite in some low alloy steels. This enables us to predict the full martensitic hardenability from a calculation of the hardenability on a 50% martensite criterion and to determine from the hardness values of a hardenability test, such as an end-quench test, the point corresponding to a desired percentage of martensite. Orehoski expressed the hardness at 50%, 80%, 90%, 95%, and 99.9% martensitic microstructure as a function of carbon content of the steel. And the relationship between critical diameter values at the above percentage martensite criteria were also established. The results are as shown in Figure 14 through 22. It was found that the differences between the hardenability values based on 50% martensite and other percentage martensite criteria increased as the hardenability increased and that, in the steels of higher hardenability, these differences were fairly large.

Later Hodge and Orehoski (9) showed that the hardenability effects of the alloys may be represented by a single factor curve for 50, 95 and 99.9 percent martensite and they proposed

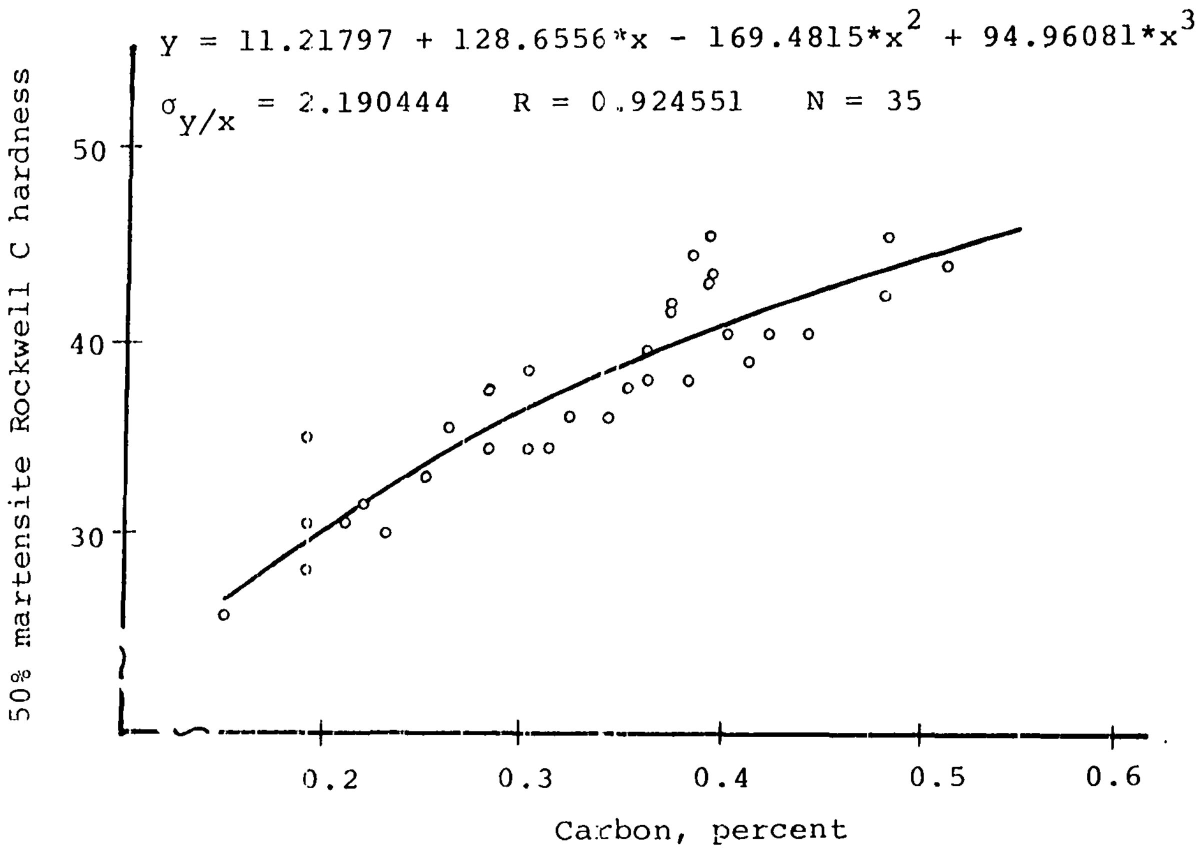

Figure 14. Relationship between carbon content and 50% martensite Rockwell C hardness

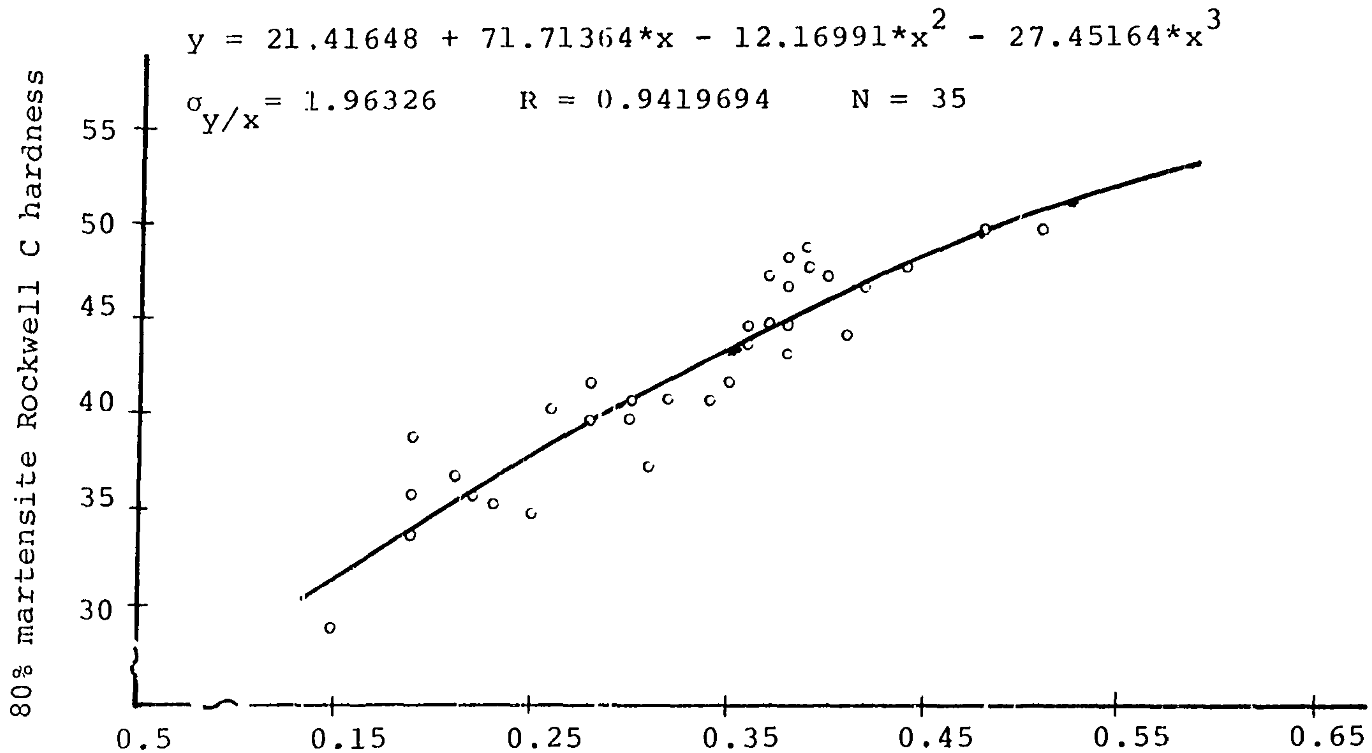

Figure 15. Relationship between carbon content and 80% martensite Rockwell C hardness (data cited from Table 4 of (9))

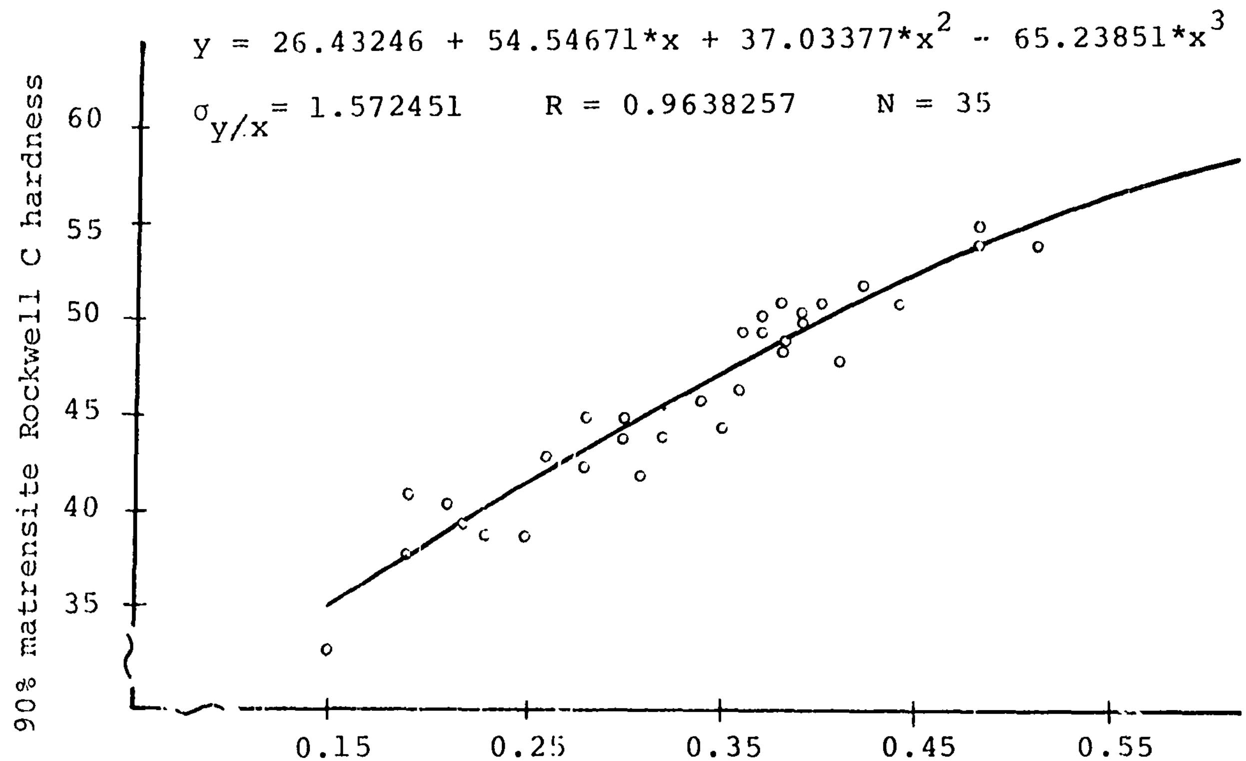

Figure 16. Relationship between carbon content and 90% martensite Rockwell C hardness (data cited from Table 4 of (9))

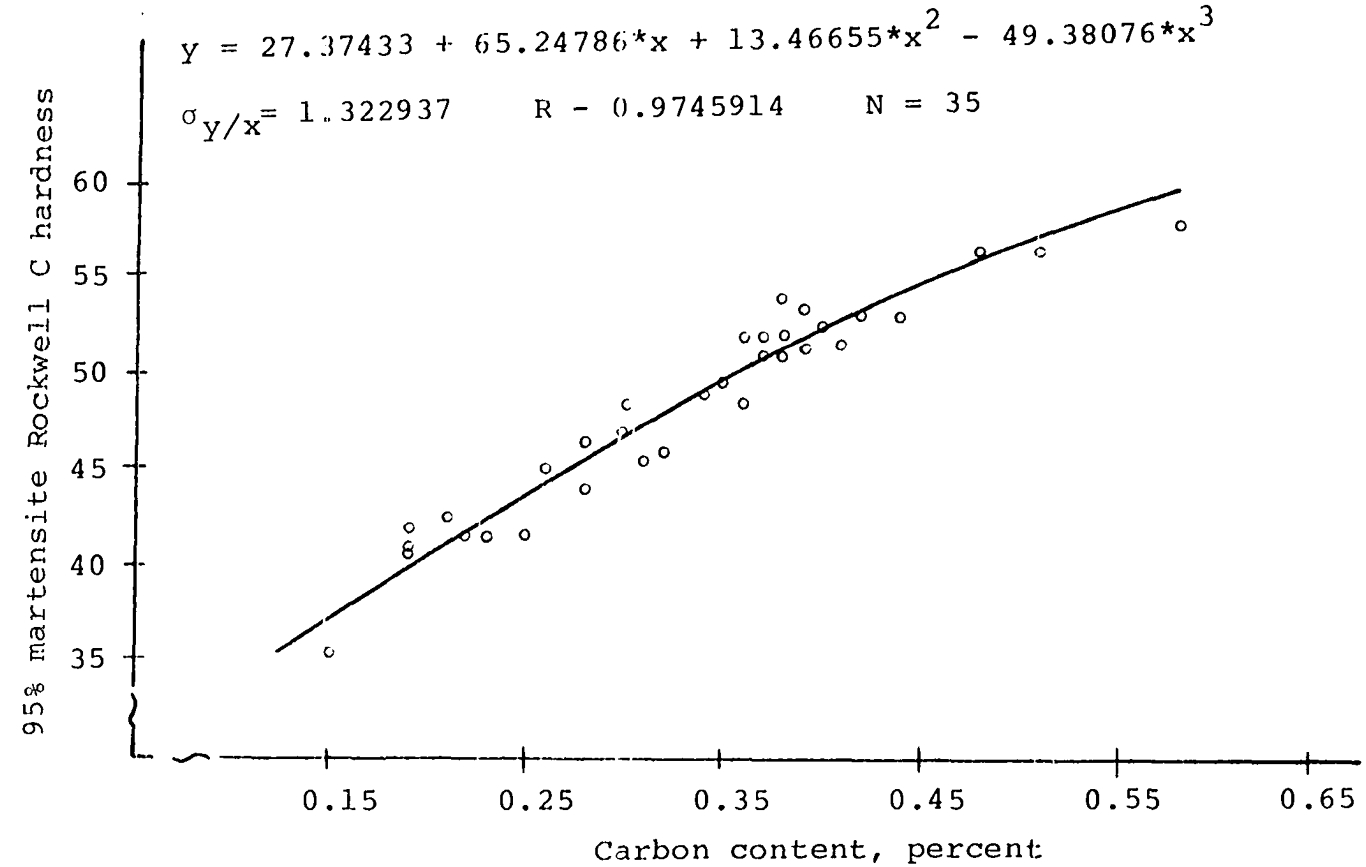

Figure 17. Relationship between carbon content and 95% (data cited from Table 4 of (9))

$$y = 28.61049 + 85.30015*x - 39.14815*x^2 - 14.49018*x^3$$

$\sigma_{yx} = 1.184942 \quad R = 0.9771282 \quad N = 35$

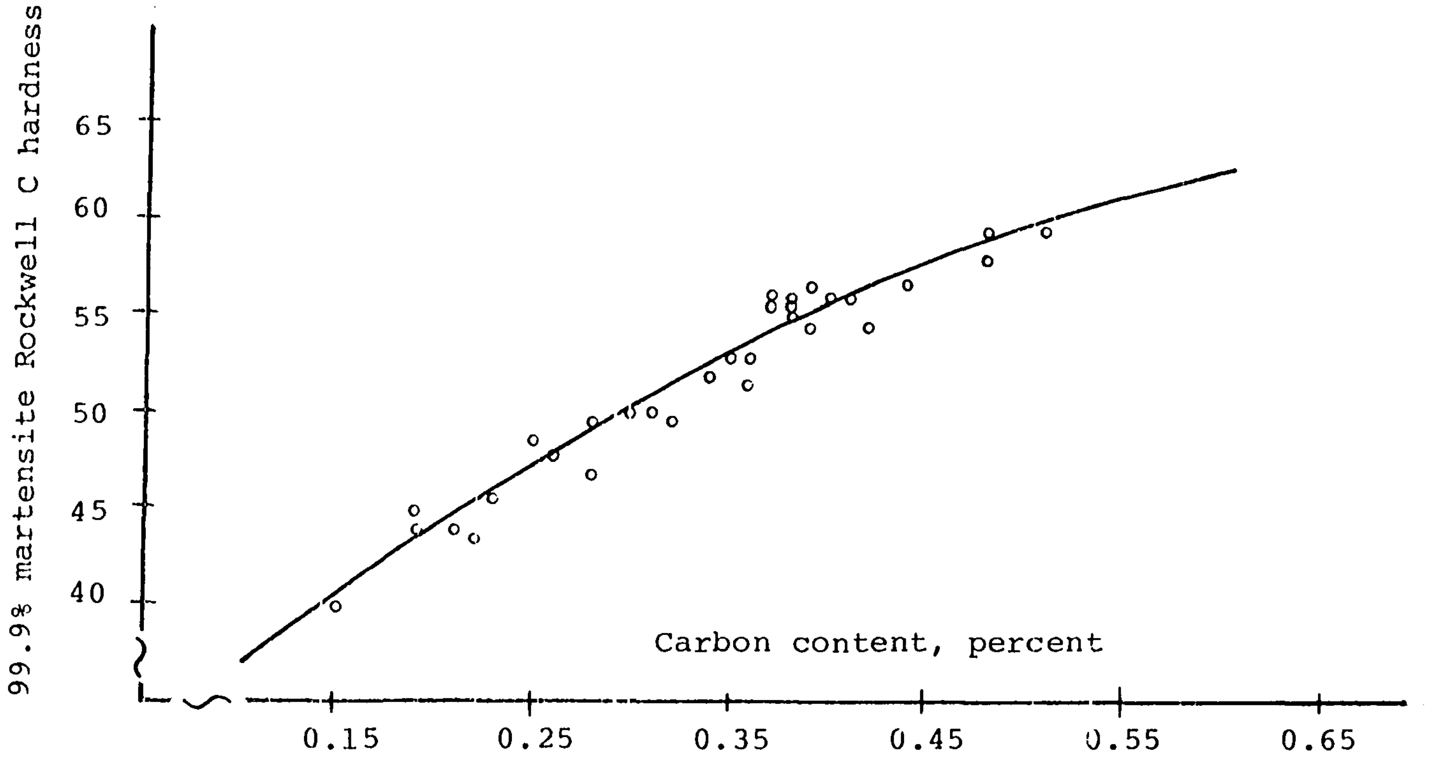

Figure 18. Relationship between carbon content and 99.9% martensite Rockwell C harness (data cited from Table 4 of (9))

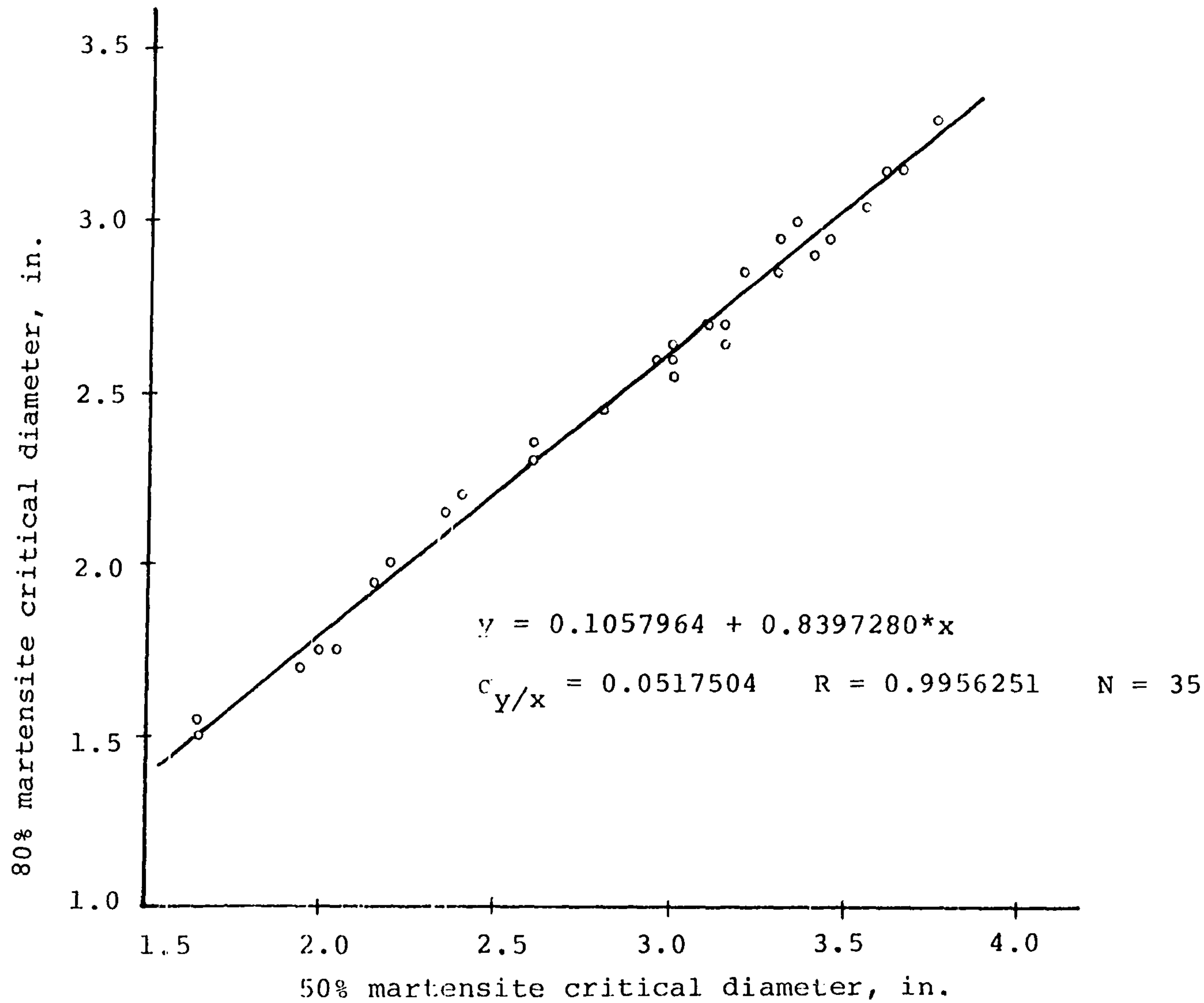

Figure 19. Relationship between hardenability based on 50% martensite and 80% (data cited from Table 2 of (9))

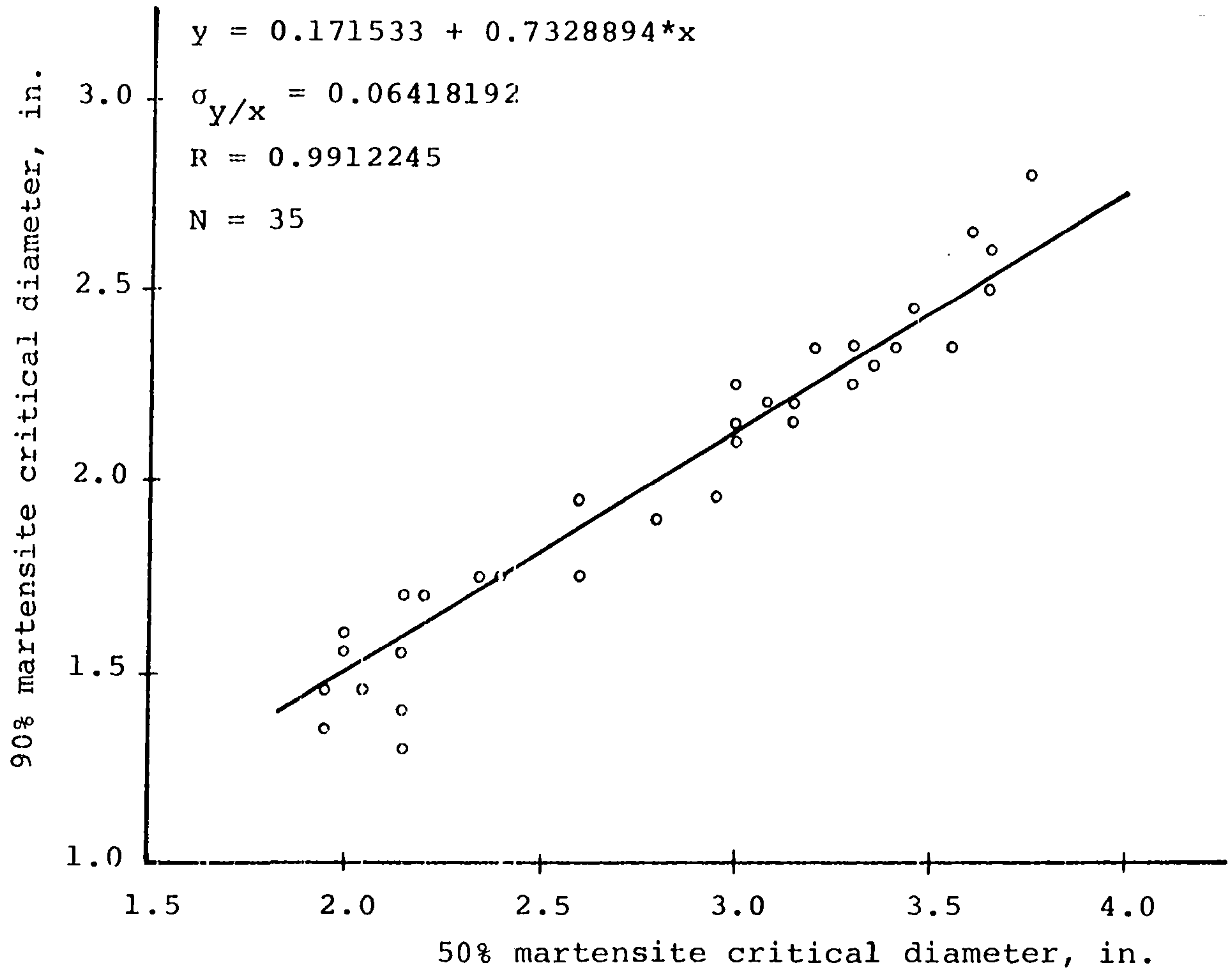

Figure 20. Relationship between hardenability based on 50% martensite and 90% martensite hardenability

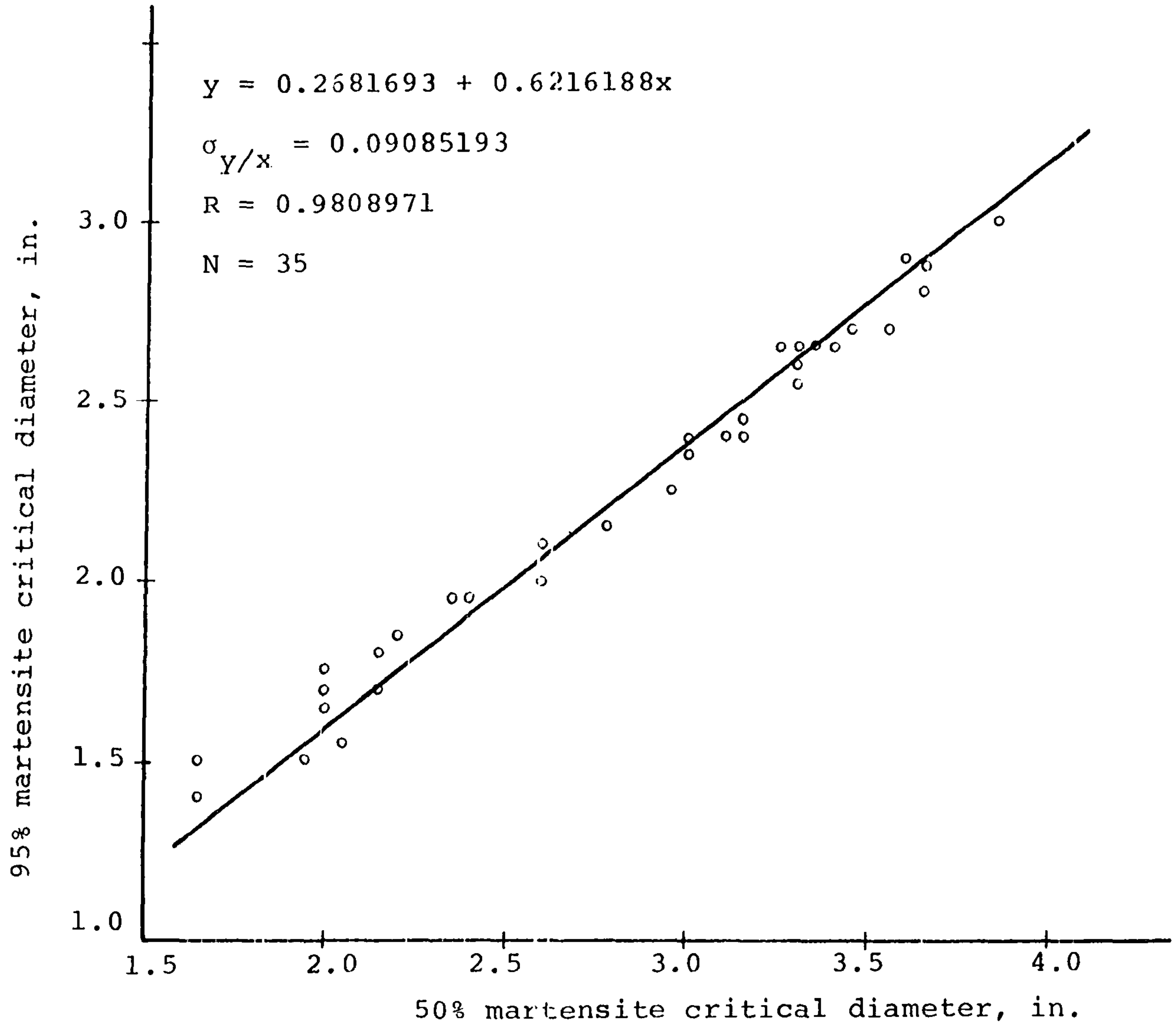

Figure 21. Relationship between hardenability absed on 50% martensite and 95% martenside hardenability (data cited from Table 2 of (9))

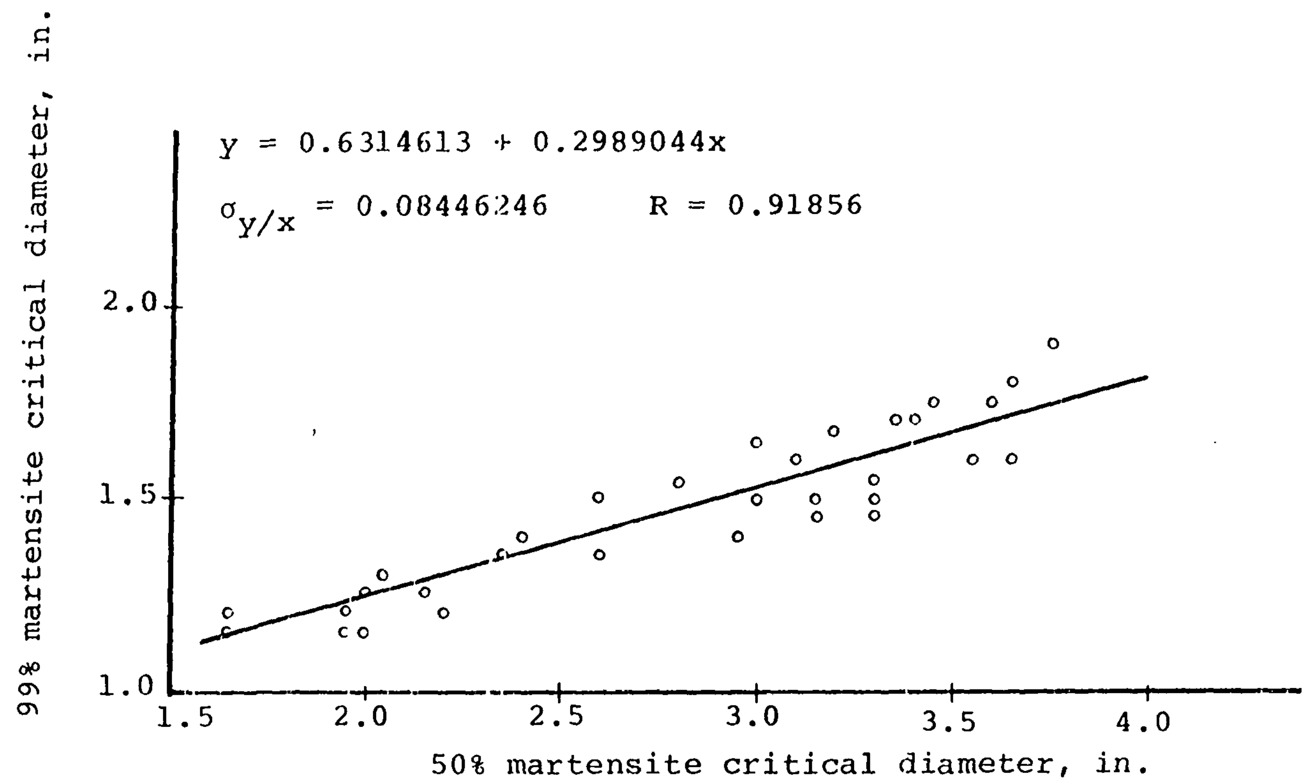

Figure 22. Relationship between hardenability based on 50% martensite and 99.9% martensite hardenability (data cited from Table 2 of (9))

that the relationship between full martensite hardenability and 50 percent martensite hardenability is a function of the base hardenability of the iron carbon alone.

Kramer's Multiplying Factor for Carbon, Manganese, Silicon, Nickel, Chromium, Molybdenum and Copper

The carbon factor curve obtained by Grossmann was dependent upon the factors for all the other alloying elements present in the steel. It has been found that the effect of manganese and silicon on hardenability was not a linear function of the alloy, which Grossmann had assumed in the determination of the multiplying factor curves. Kramer, Siegel and Brooks (10) reexamined the effect of carbon on hardenability and found that it is approximately three times higher than that reported by Grossmann.

To establish the level of the carbon curve, Kramer determined the critical diameter of the iron-carbon alloys very low in residual elements. The hardenability curves were then derived by a series of successive approximations in which the previously determined curves were initially used.

Kramer first used the manganese and silicon curves developed by Crafts and Lamont (7) to develop a carbon curve, this new carbon curve was then employed to develop new manganese and silicon curves, and the procedure was repeated until further cycling caused no changes. This process of cycling established the general shape of the carbon curve, whose level was determined from low-alloy steels. The balanced carbon, manganese, and silicon curves were then used to determine the effect of the other alloying elements on hardenability, and these factors curves were again cycled through a process of successive corrections until a completely balanced system was obtained. Kramer's multiplying factor curves showed that the effect of carbon (Figure 23 and Figure 24) on hardenability is approximately three times higher than that reported by Grossmann (1). The effect of grain size on hardenability for aluminum-killed steels is the same as predicted by Grossmann (1), but it is less on silicon-killed steels.

The factor curve for manganese (Figure 25) differs markedly as to shape and magnitude from those reported by Grossmann (1) and Crafts and Lamont (7). Manganese up to 1.00 per cent has relatively little effect on hardenability, but above this level it has a very large effect on hardenability. The factor curve for silicon (Figure 26) is lower than that reported by Grossmann (1) and Crafts and Lamont (7). In general, silicon has a relatively small effect on hardenability. The factor curve for nickel shown in Figure 27 shows good agreement with those determined by Crafts and Lamont up to approximately 3 percent nickel. The factor cuve for copper (Figure 28) is similar to that for nickel. Copper exerts a considerable influence on

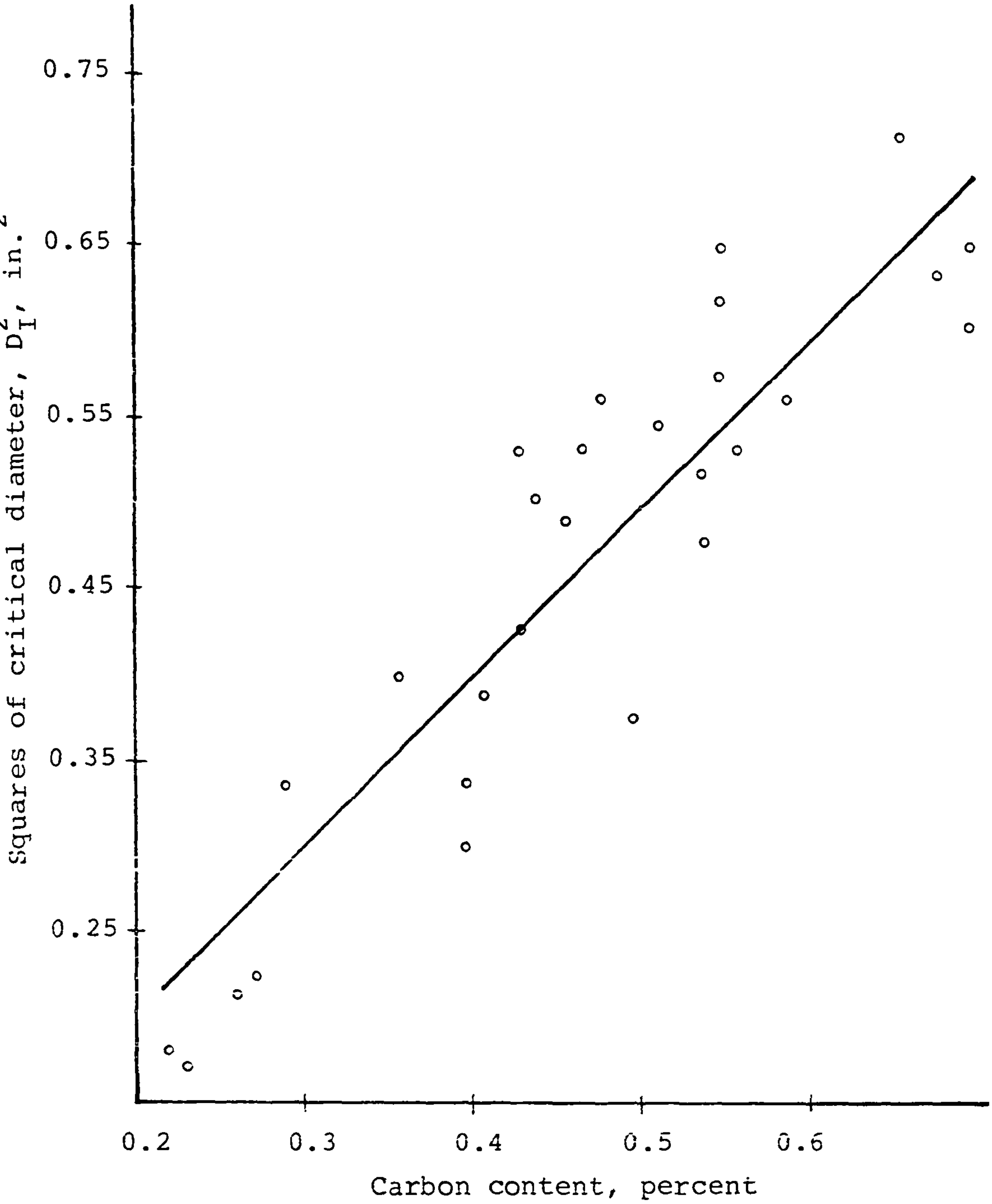

Figure 23. Relationship between carbon content and square of ideal critical diameter (data cited from Figure 8 of (10))

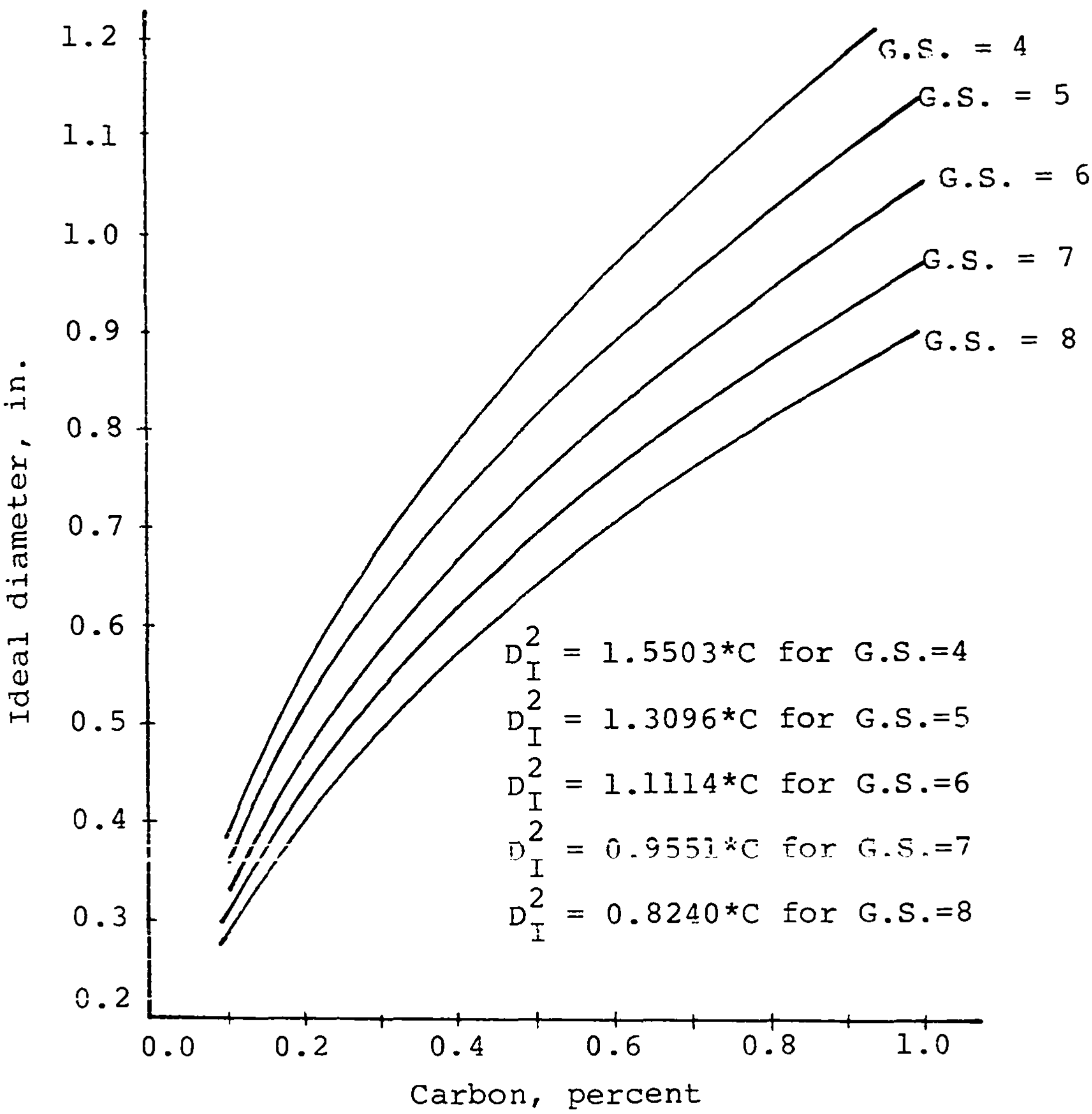

Figure 24. Relation between ideal diameter (D_I), carbon content, and grain size

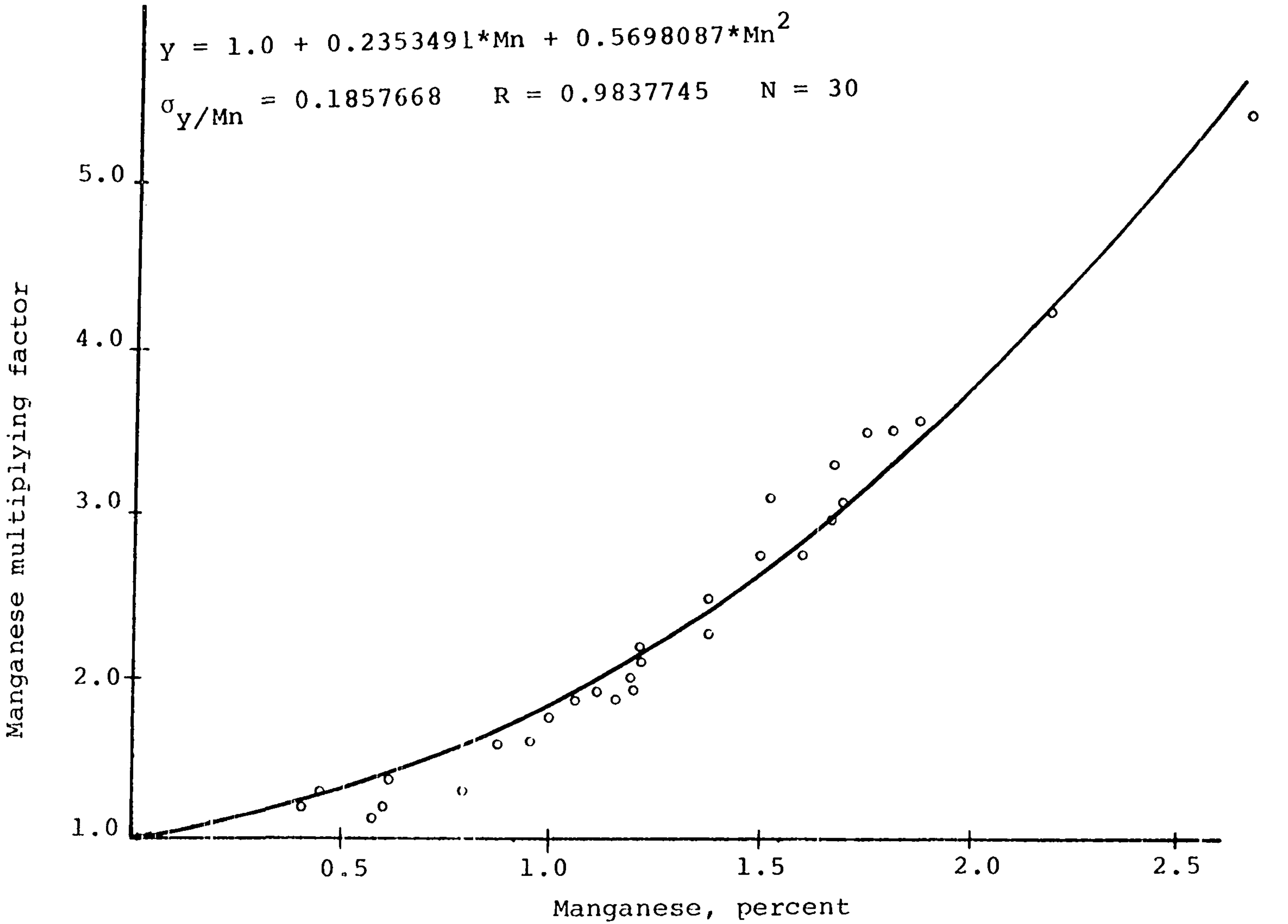

Figure 25. Multiplying factor for manganese (data cited from Figure 13 of (10))

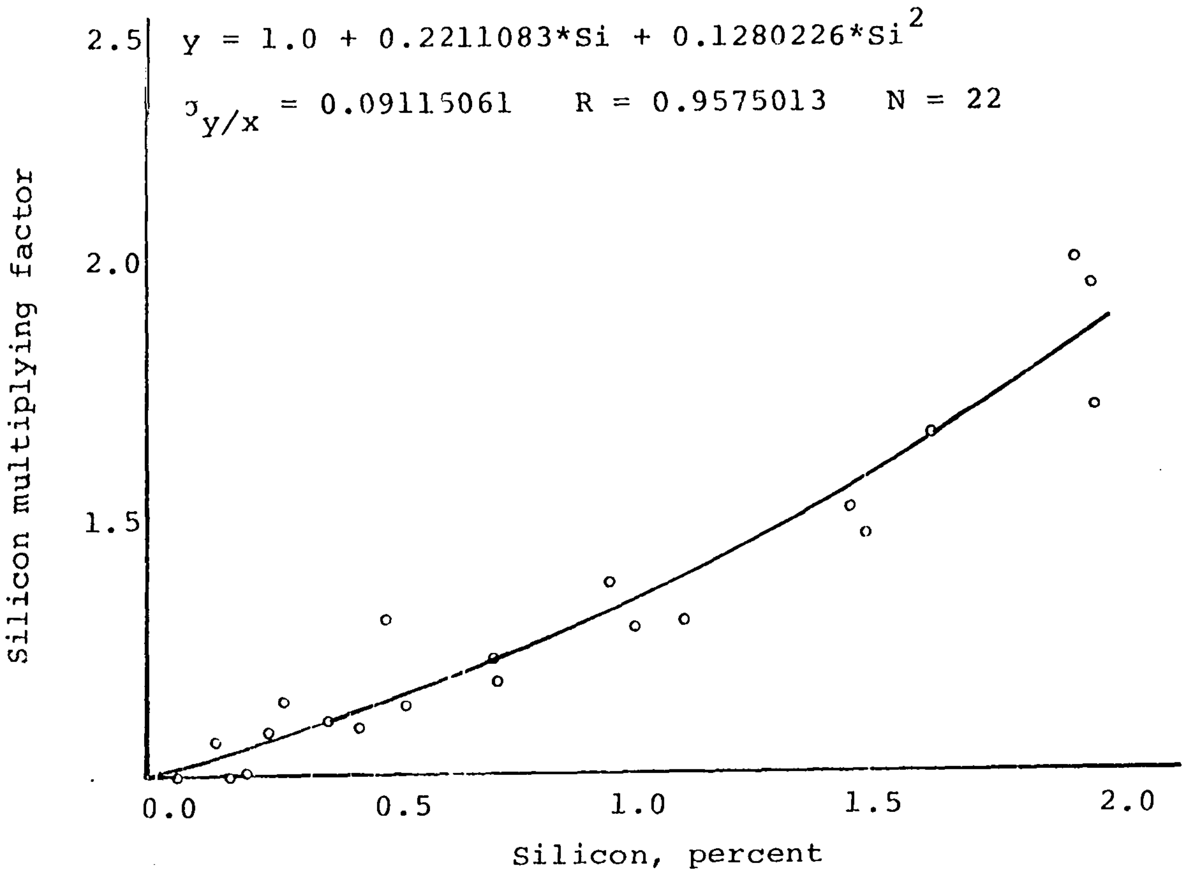

Figure 26. Multiplying factor for silicon (data cited from Figure 14 of (10))

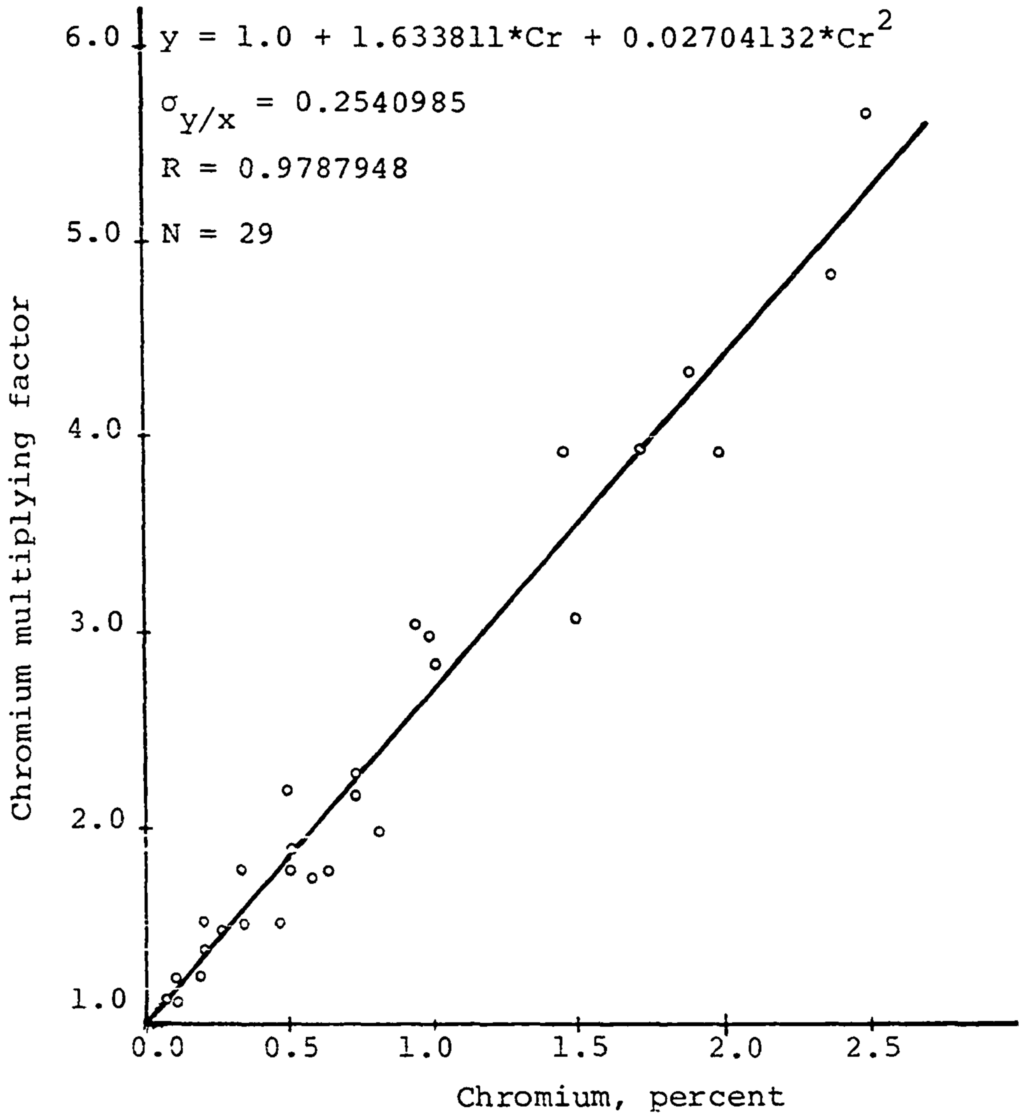

Figure 27. Multiplying factor for chromium (data cited from Figure 18 of (10))

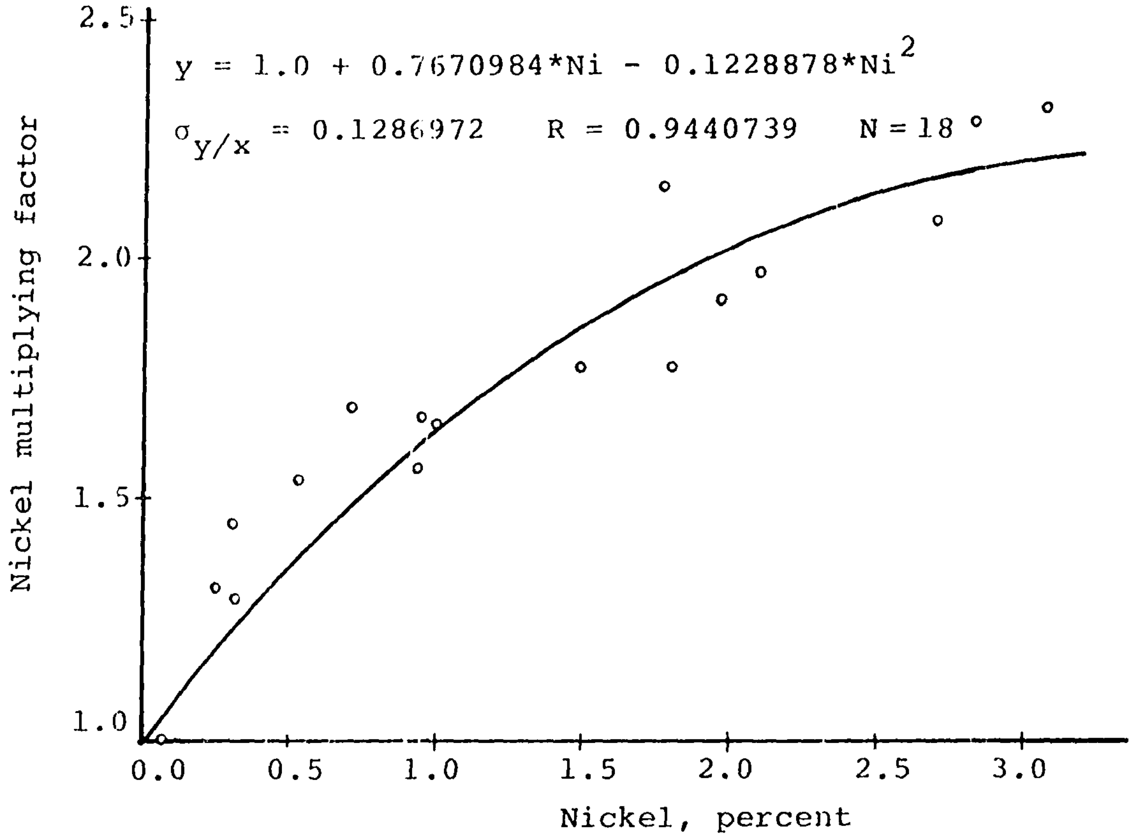

Figure 28. Multiplying factor of nickel (data cited from Figure 15 of (10))

the hardenability, as little as 0.10 percent copper raises the hardenability by almost 17 percent. The factor curve for chromium and molybdenum are shown in Figures 29 and 30 since chromium and molybdenum form stable carbides, their effect on hardenability is strongly dependent on heat treatment. This has resulted in large scattering of hardenability multiplying factors at high percentages of these alloying elements.

Calculation of the Jominy Curve by the Addition Method

Crafts and Lamont (11) proposed an addition method to predict the Jominy hardness curve, which characterizes the hardenability of a low and medium carbon alloy steel, by the addition of Rockwell C units proportional to the carbon and alloy content, grain size, and position in the Jominy test specimen. The calculation is started from a base that includes the effect of carbon content and position in the Jominy test specimen. Rockwell C units are added to the base in proportion to the alloy content and grain size. This sum represents the Rockwell C hardness up to the level at which a disproportionate increase of hardness is caused by the formation of martensite, and above this level an increment for martensite hardening is added. The effect of alloys are directly proportional to the amounts present and are

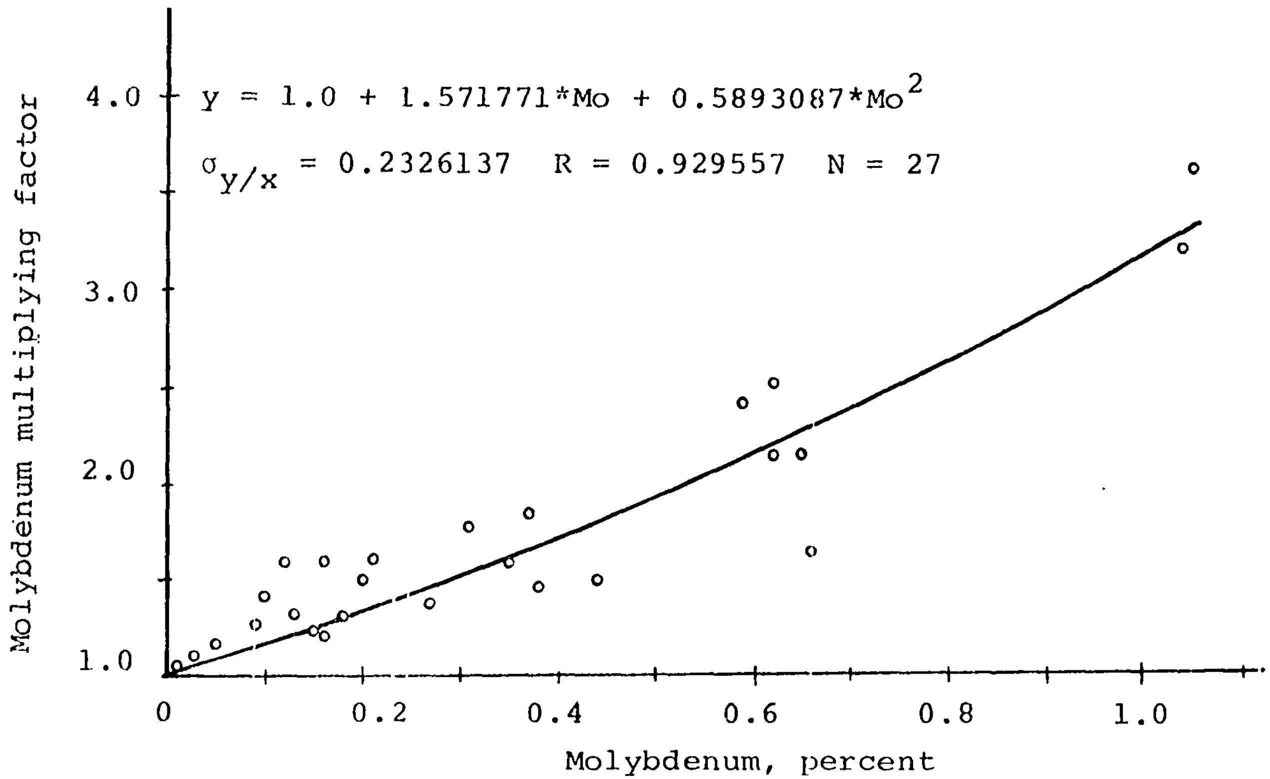

Figure 29. Multiplying factor for molybdenum (data cited from Figure 19 of (10))

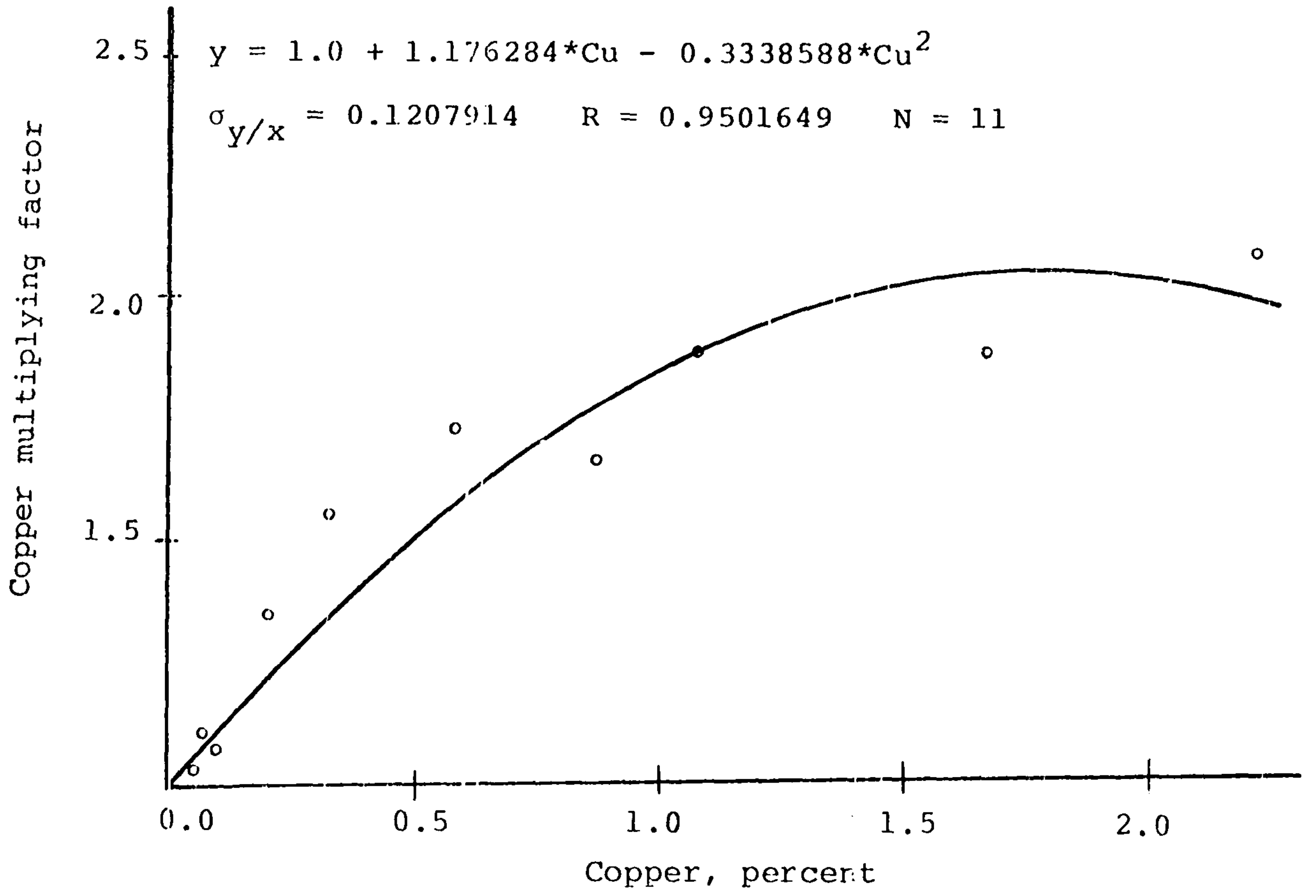

Figure 30. Multiplying factor for copper (data cited from Figure 16 of (10))

independent of each other, the carbon and the position in the specimen. Furthermore, the factors for determining the martensite increment are dependent only on carbon content and are independent of cooling rate and alloy content.

The addition method for calculating Rockwell C hardness of the Jominy hardenability test will be illustrated by the following example.

Assumed that it is desired to know the Jominy hardness curve for an 8645 steel of grain size 8 and of the following composition.

The alloy addition units (Figure 31) for each element and the actual grain size (Figure 32) are as shown in the third column of the table. If the sum of the total alloy and grain size addition units and the carbon base hardness at a specified Jominy station (Figures 33 and 34) for the given carbon content of the steel, is greater than the martensite-base hardness (Figure 33) for the given carbon content of the steel, there will be a martensite increment hardness. It is determined by subtracting the martensite base hardness from the sum of the carbon-base hardness and total alloy and grain size addition units and multiplying this difference by the martensite factor (Figure 35). The product is added to the martensite-base hardness to obtain the calculated Jominy hardness as indicated in the following table. If the hardness

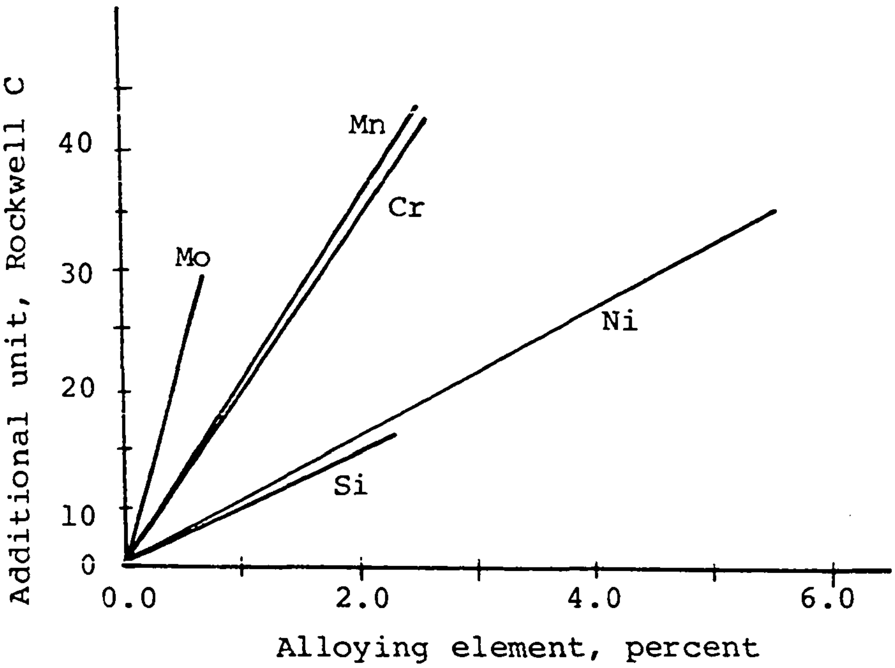

Figure 31. Addition units for alloying elements (cited from Figure 4 of (11))

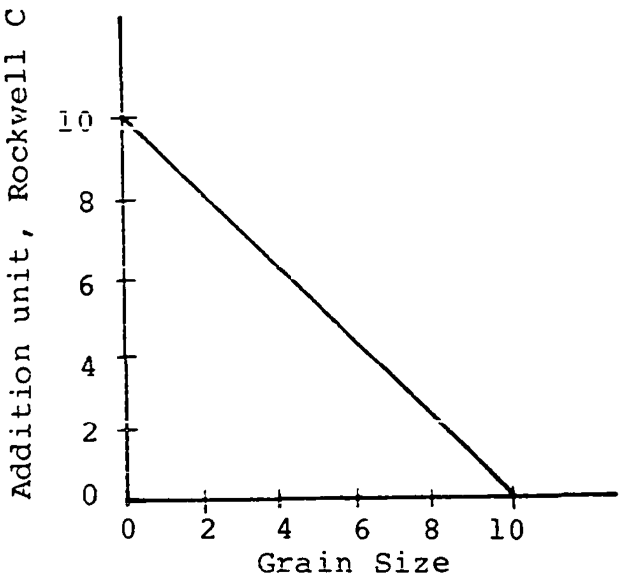

Figure 32. Addition unit for grain size (cited from Figure 4 of (11))

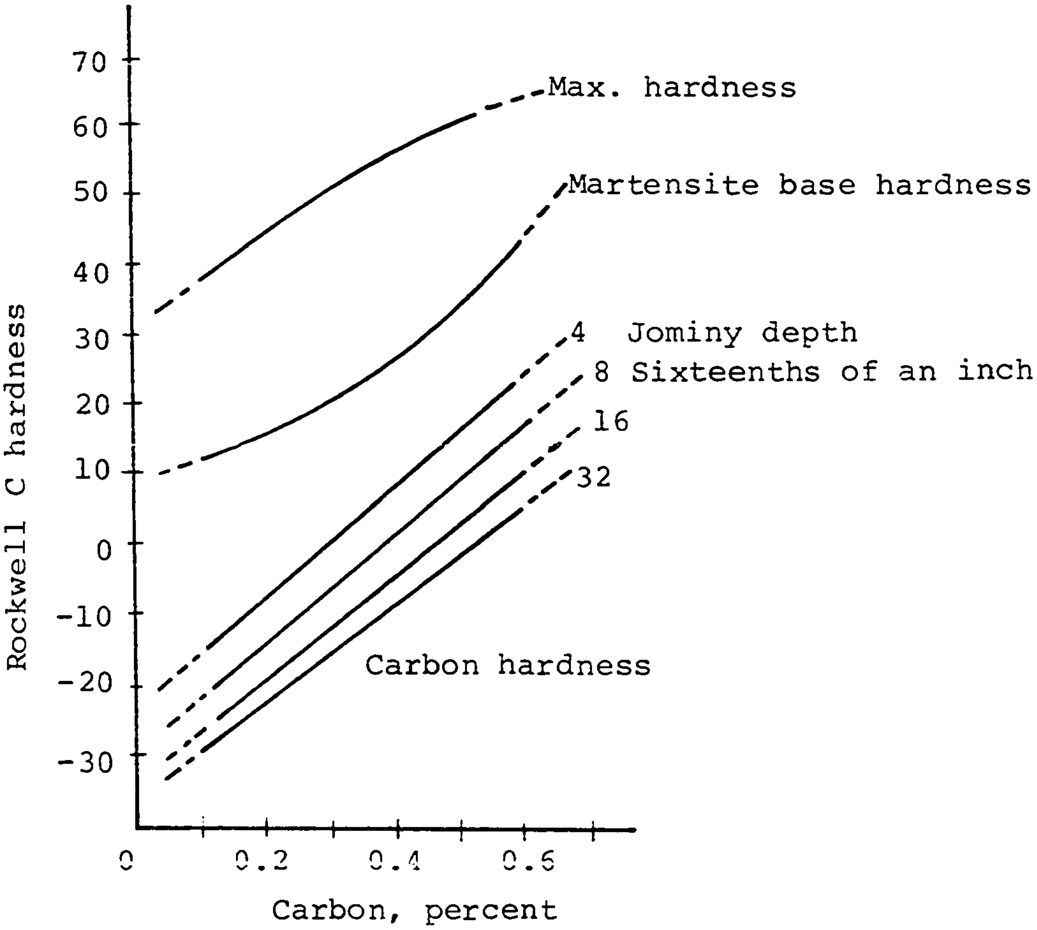

Figure 33. Carbon-base hardness, martensite-base hardness and maximum hardness with respect to carbon content (cited from Figure 2 of (11))

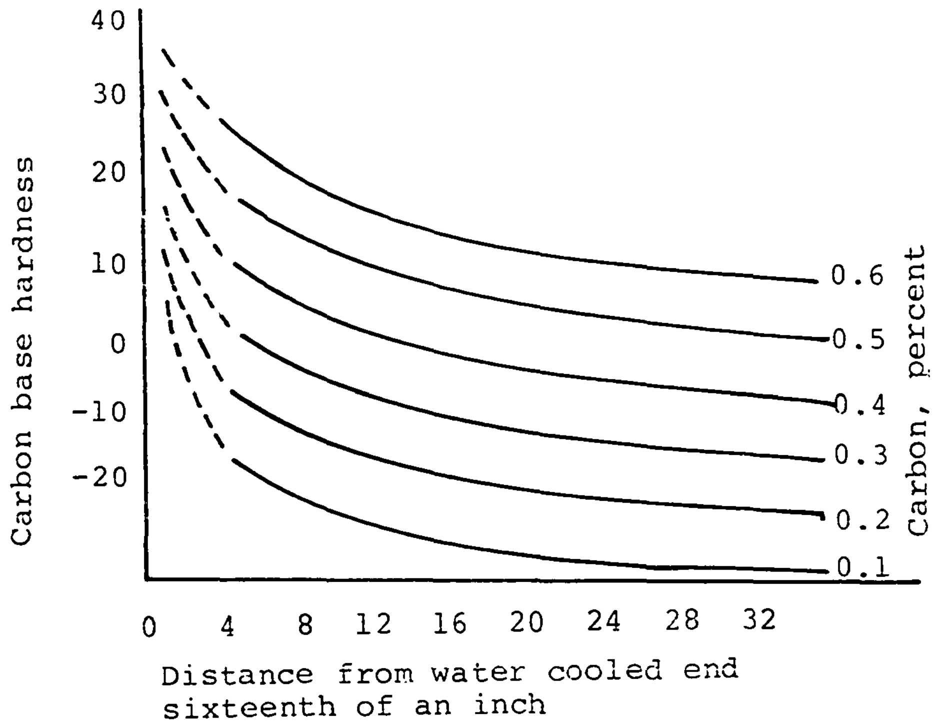

Figure 34. Carbon-base hardness with respect to position on the Jominy test specimen (cited from Figure 3 of (11))

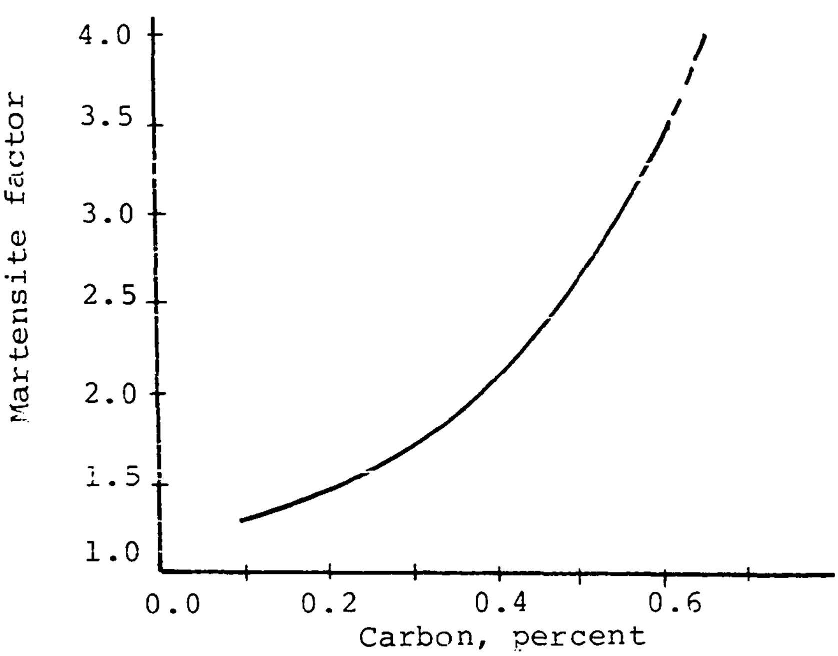

Figure 35. Factor of determining martensite increment (cited from Figure 5 of (11))

Jominy Rockwell C hardness calculation by addition method:

Step of Calculation	4/16	8/16	16/16	32/16
Carbon-base hardness (A)	12.1	5.4	-0.3	-5.0
(plus) Alloy-addition				
Units (B)	34.0	34.0	34.0	34.0
Sum	46.1	39.4	33.7	29.0
(minus) Martensite-Base				
Hardness (C), Figure 33	30.0	30.0	30.0	30.0
Difference	16.1	9.4	3.7	
(times) Martensite factor				
f, Figure 35				
Product	37.0	21.6	8.5	
(plus) Martensite-base				
Hardness, Figure 34	30.0	30.0	30.0	
Total (R_M)	67.0	51.6	38.5	
Calculated Hardness (R)	58.7	51.6	38.5	29.0

Composition, grain size of steel 8645:

Element	Percentage	Addition units from Figure 31
Carbon	0.45	
Manganese	0.85	13.2
Silicon	0.22	1.1
Nickel	0.50	2.7
Chromium	0.50	7.5
Molybdenum	0.20	7.2
ASTM No. 8 Grain size		2.0
Alloy and grain size addition units		34.0

from this calculation exceeds the maximum attainable hardness (Figure 33), which is 58.7 Rockwell C in a steel containing 0.45 percent carbon, the maximum hardness is considered to be the calculated hardness. If the sum of the carbon and the alloy units is less than the martensite-base hardness, this sum is the calculated hardness.

Correlation Between Jominy Test and Quenched Round Bars

For any particular steel, the extent to which it hardens when quenched varies with the cooling rate (cooling time) in the quench. Different cooling times occur along the length of a Jominy bar and various cooling times are also found at various positions in different size of quenched bars, quenched with various severity of quench. Asimow, Craig and Grossmann (12) established the correlation between Jominy test and quenched round bars by correlating the half-temperature time at various distance from water-cooled end of a Jominy specimen with that of different positions within a round bar when using a known severity of quenching. It was found that the extent of hardening for any particular steel correlated well with the "half-temperature time" in cooling. It is possible to predict from the results of a Jominy test what the hardness distribution will be on the cross section of a quenched round bar through the relation established by Asimow. The half-temperature time at various distances from water cooled end of a Jominy bar was measured by Jominy as shown in Table 5 and are plotted as shown in Figure 36. The calculated curve in Figure 36 was developed by Asimow and Craig by establishing that the water-quench end has an average severity of quench H = 2.33 and the severity of quench of the air cooling is 0.022. The cooling time at the center of bar subjected to

Table 5. Half temperature times at various distances from water-cooled end

Distance from water-cooled face, in.		Time-sec. to cool one-half temperature
1/16	0.0625	2.5
1/8	0.1250	10.5
3/16	0.1875	16
1/4	0.2500	22
3/8	0.3750	33
1/2	0.5000	52
5/8	0.6250	66
3/4	0.7500	81
9/8	1.1250	130
5/4	1.2500	150
2	2.00	224

[a]Cited from Table 1 of (11).

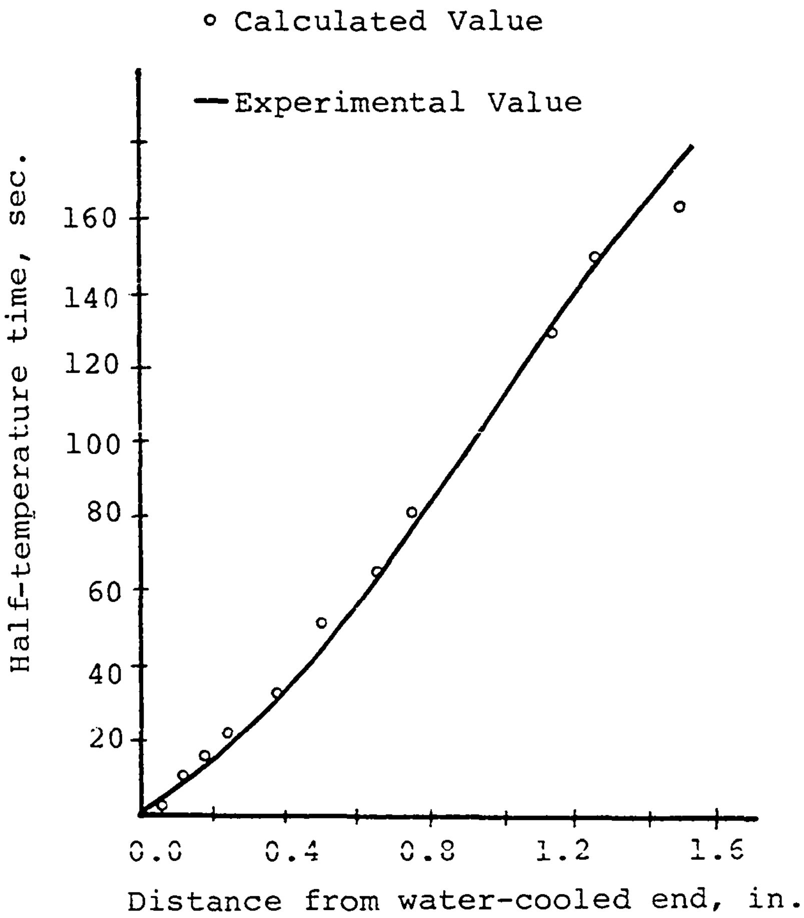

Figure 36. Half-temperature times - calculated and experimental values (cited from Figure (1) of (12))

ideal quench is related to the diameter of the bar according to the following relationship:

$$D = 2.0*(a*\tau)^{1/2}$$

where:

D is the diameter of the bar, inches.

a is the thermal diffusivity of the steel, L^2/T

τ is the half-temperature time.

Using a value of 0.009 in^2/sec. for a, the formula becomes,

$$D = 0.179*\tau$$

With the aid of the above expression and the relation between half-temperature time and distance from water quench end of a Jominy bar (Figure 36), it is possible to establish the relationship between the diameter of round in ideal quench with the distance from water-cooled end of the Jominy bar. The results are shown in Figure 37. Therefore, if we can determine the location from the water-cooled end of a Jominy bar, which has a 50% martensite microstructure or hardness, the critical diameter of the alloy steel can be determined from Figure 37.

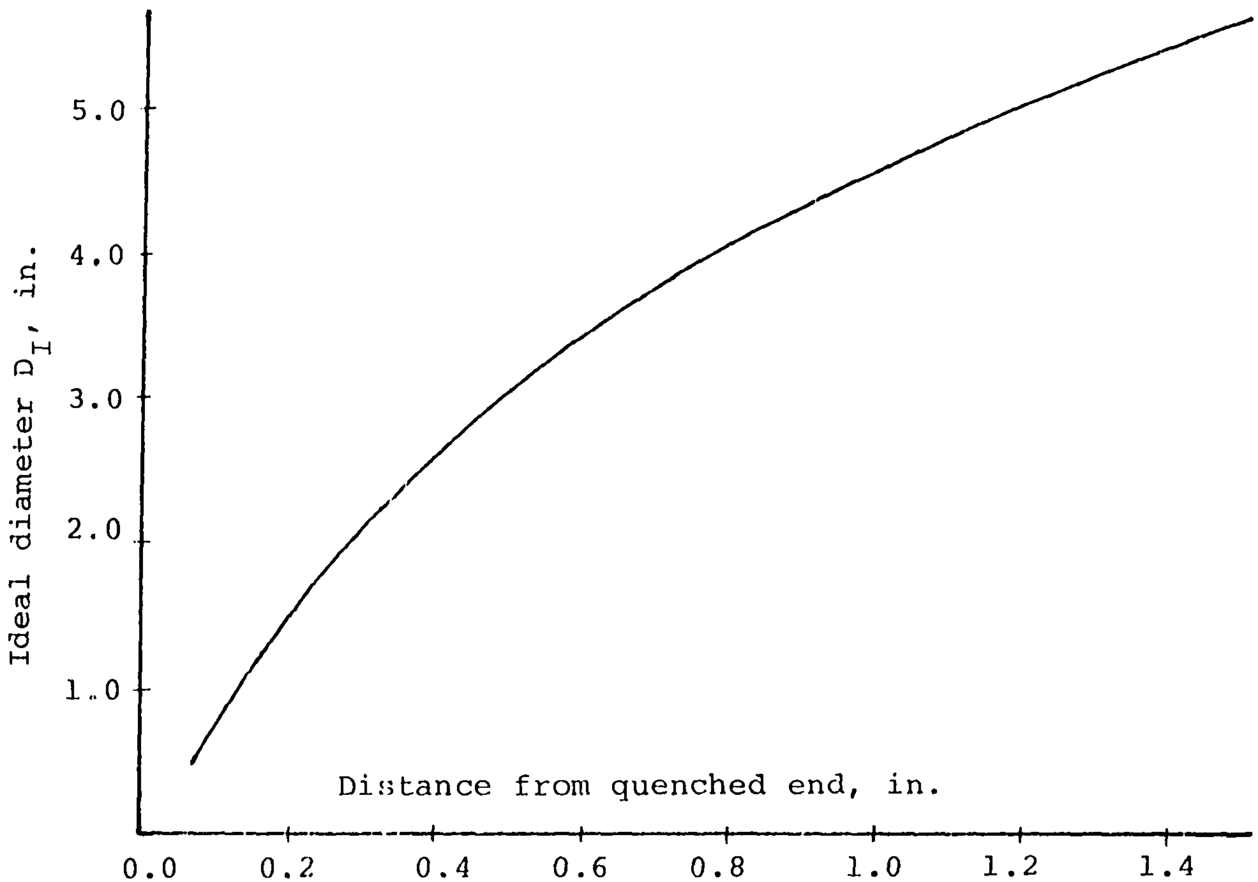

Figure 37. Relation between Jominy-Boegehold hardenability bar and diameter of round with ideal quench (cited from Figure 11 of (12))

Calculation of Jominy End-Quench Curve from Analysis

The Jominy hardness curve of a low and medium carbon alloy steel can be predicted from its critical diameter, which depends on the grain size and chemical composition of the steel, using the method proposed by Field (2). The method is based upon the following assumptions:

1. The hardness at the extreme quenched end of the Jominy test piece (initial hardness or I.H.) is a function only of the carbon content of the steel.
2. The hardness at any other distance from the end of the Jominy test piece (D.H.) is a function of carbon content, alloy content and grain size of the steel being tested.
3. The ratio of the initial hardness (I.H.) to the hardness at any other distance (D.H.) is a constant function of the critical diameter (D_I), which in turn is a function of carbon content, alloy content, and grain size of the steel being tested.

From the carbon content of the steel, the initial hardness (I.H.) of the quenched end (which is a function of carbon content only of the steel) can be determined from Figure 38. By Figure 39, the ratio of I.H./D.H. (ratio of initial hardness to distance hardness) at different Jominy

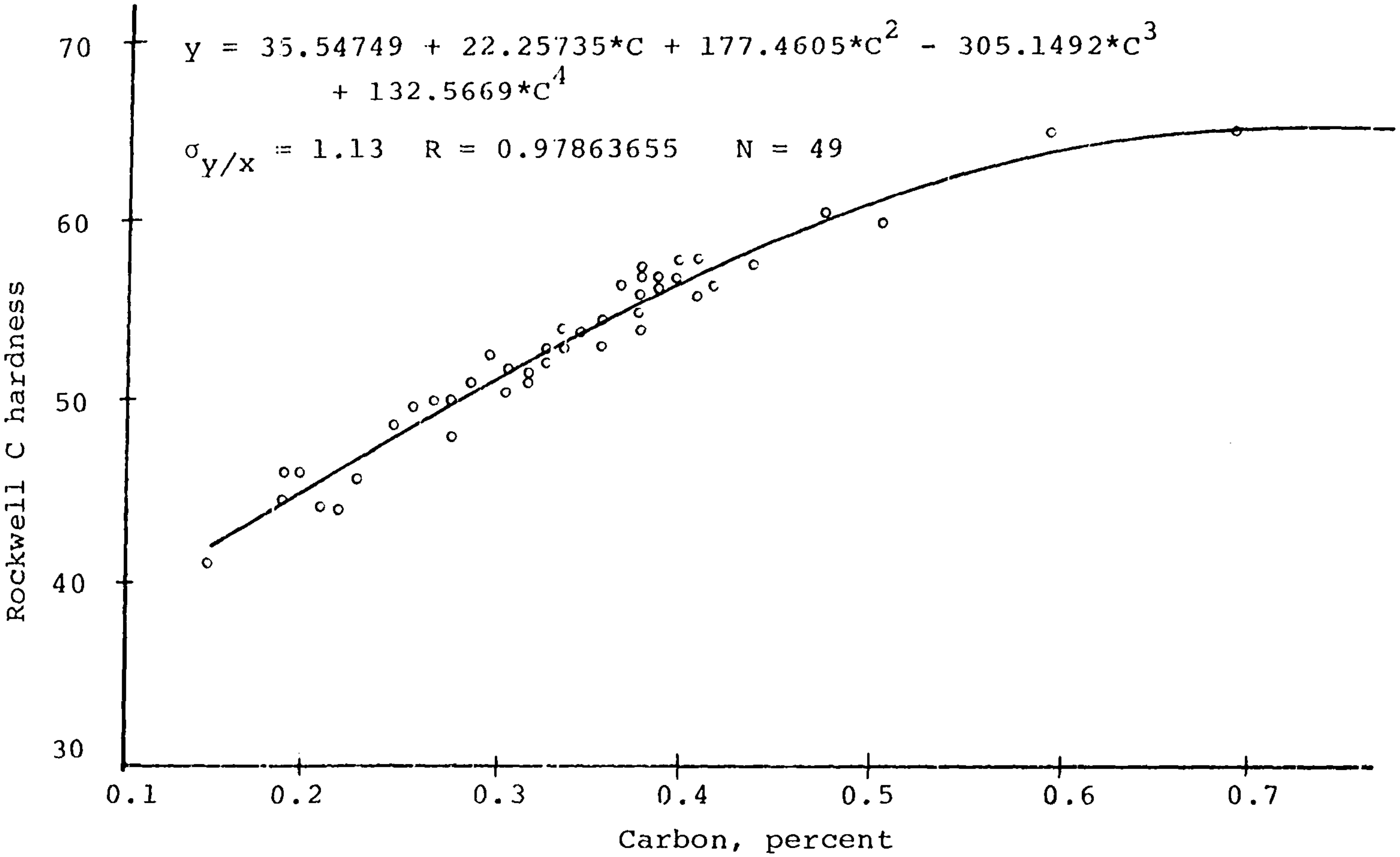

Figure 38. Relationship between (1/16)" quenched end Rockwell C hardness and carbon content (cited from Table 1 and 3 of (9), and Table I and II of (2))

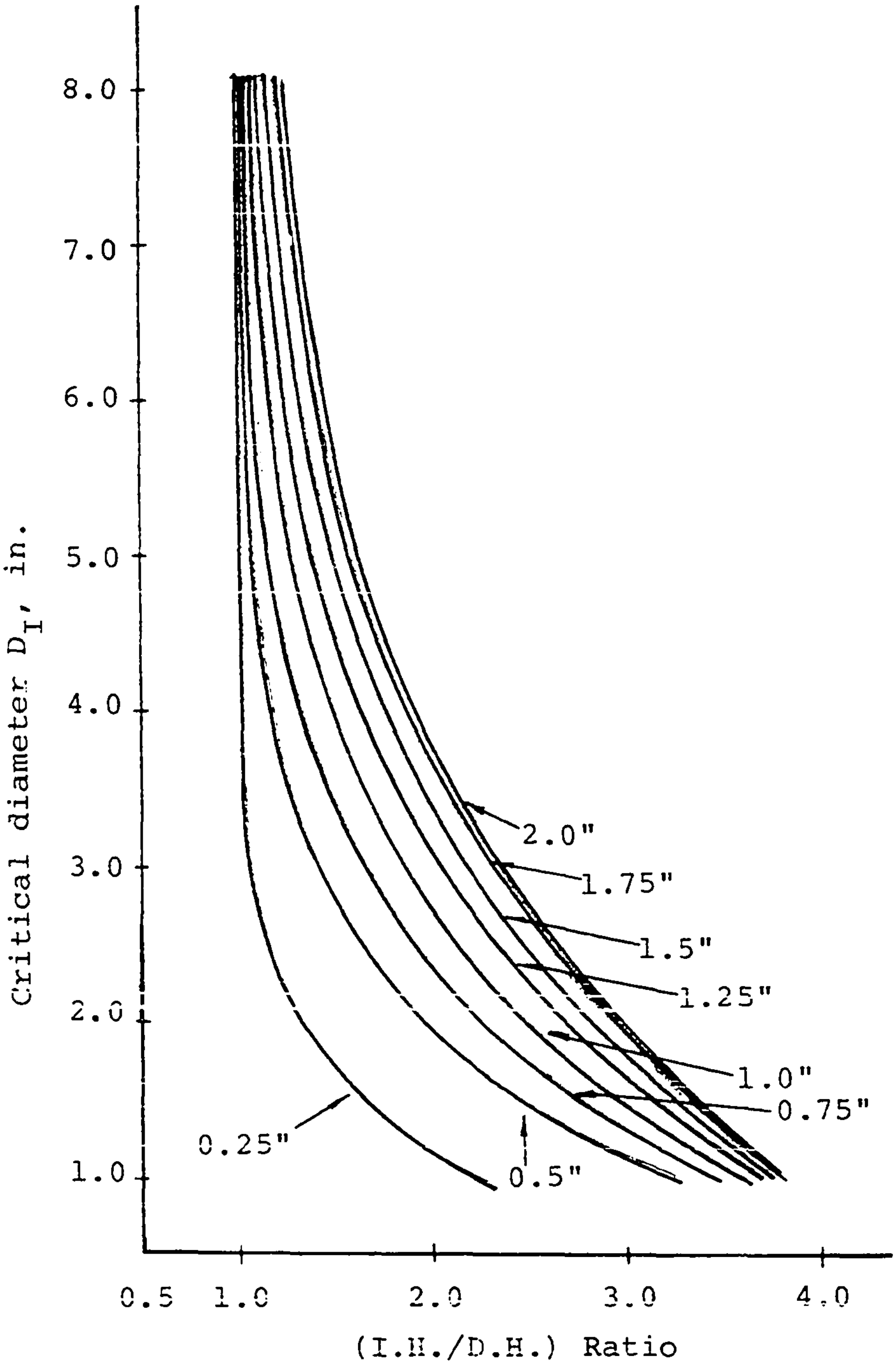

Figure 39. Relationship between ideal critical diameter and the ratio of initial hardness to distance hardness (I.H./D.H.)

stations can be determined from critical diameter, which can be measured experimentally, or predicted from grain size and composition of the steel. The Jominy hardness curve can be calculated by taking the ratios of the initial hardness I.H. and I.H./D.H. values.

Hardenability of High Carbon Alloy Steel

High carbon (hypereutectoid) alloy steels normally contain large quantities of undissolved carbides when hardened by commercial austenitizing procedures. The hardenability effect of a given quantity of alloy is influenced by the prior structure, prior carbide size and shape and distribution, and austenitizing time and temperature. In order to determine a single factor for the hardenability effect of an alloy element, the above conditions must be strictly controlled, so that the quantity of alloy and carbon in austenite solution would not be varied. Since the as-quenched grain size does not vary greatly from ASTM No. 6 to No. 8 when excess carbides are present, its effect on hardenability of high carbon steel is less important compared to the other factors.

Jatczak and Devine (13) developed hardenability factors for carbon, manganese, silicon, chromium, nickel and molybdenum in nominally 1.00 percent carbon steels by the end-quench test. The steels were austenized at 1475, 1525 and

1575°F and held for 20 to 40 minutes at quenching temperature from a normalized and spheroidize annealed prior structure. The criterion of hardenability (critical diameter) used in this investigation was the distance from the quenched end which has a hardness of 60 Rockwell C, and the first austenite decomposition product was pearlite.

Multiplying factors to be used for calculating hardenability of high carbon alloy steels that are hardened from 1475, 1525 or 1575°F and have a normalized prior structure, are shown in Figure 40, 41 and 42. Because these factors were calculated according to a base composition of 1.00C, 0.25 Mn, 0.25Si, 0.25Cr, and 0.25Ni, the factors originate at 0.25% content for the alloying elements and at 1.0% for carbon. When using these graphs the critical diameter values for the base composition should be considered and the contents of manganese, silicon, nickel, and chromium under 0.25% will have associated with them factors less than 1.00. It was found that if the nickel content of the composition exceeds 1.0%, the computed hardenability value was always lower than the measured value and the divergence increased with nickel content. The disagreement between measured and computed hardenability at the higher nickel levels lay in an interdependence of the effects of manganese and nickel on each other, in which the combined effect was far larger than consideration of their

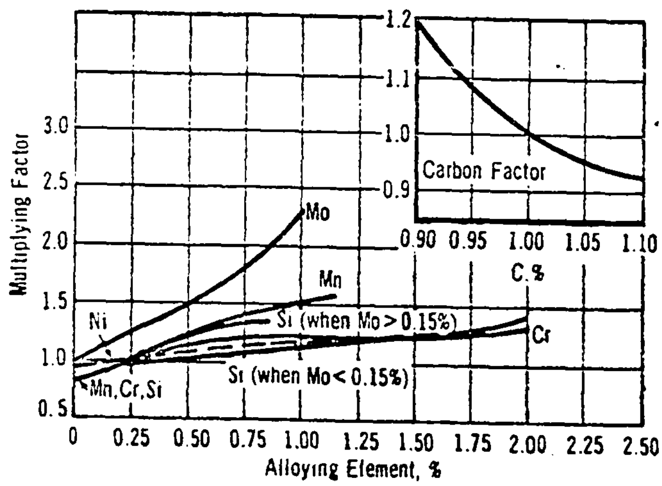

Figure 40. Multiplying factor for high-carbon steels austenitized at 1,475°F (cited from Figure 7 of (14))

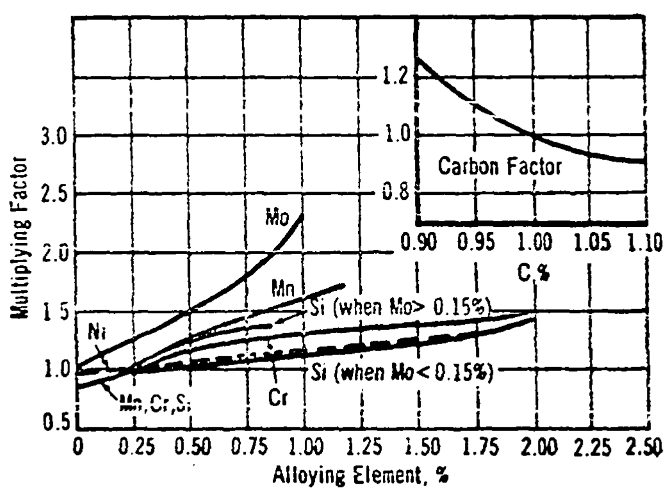

Figure 41. Multiplying factors for high-carbon steels austenitized at 1,525°F (cited from Figure 8 of (14))

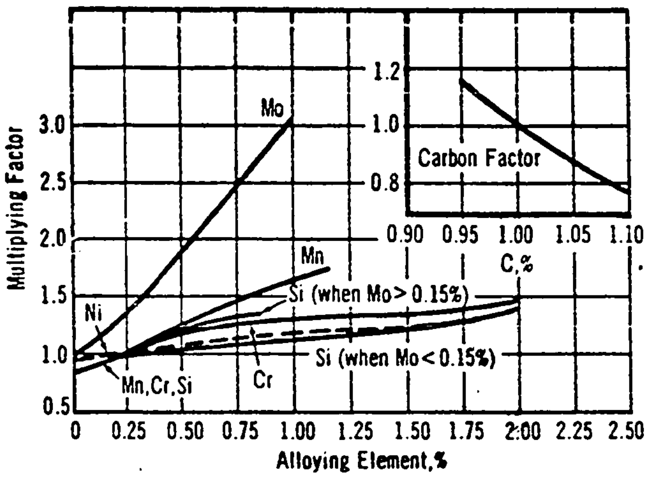

Figure 42. Multiplying factor for high carbon steels austenitized at 1,575°F (cited from Figure 9 of (14))

single effects indicated. The combined hardenability multiplying factors for nickel and manganese in normalized nickel-chromium-molybdenum 1.0% carbon steels at several hardening temperatures are shown in Figure 43a. In addition, it was found that while the molybdenum contribution to hardenability in the high nickel multi-alloy steels was apparently independent of other alloying elements, the specific effect was noticeably greater above 0.20% molybdenum than in molybdenum steels alone or in low nickel, chromium-nickel-molybdenum compositions. The chromium and silicon multiplying factors were found to be unchanged. Figure 43b gives the multiplying factors for these three elements appropriate for

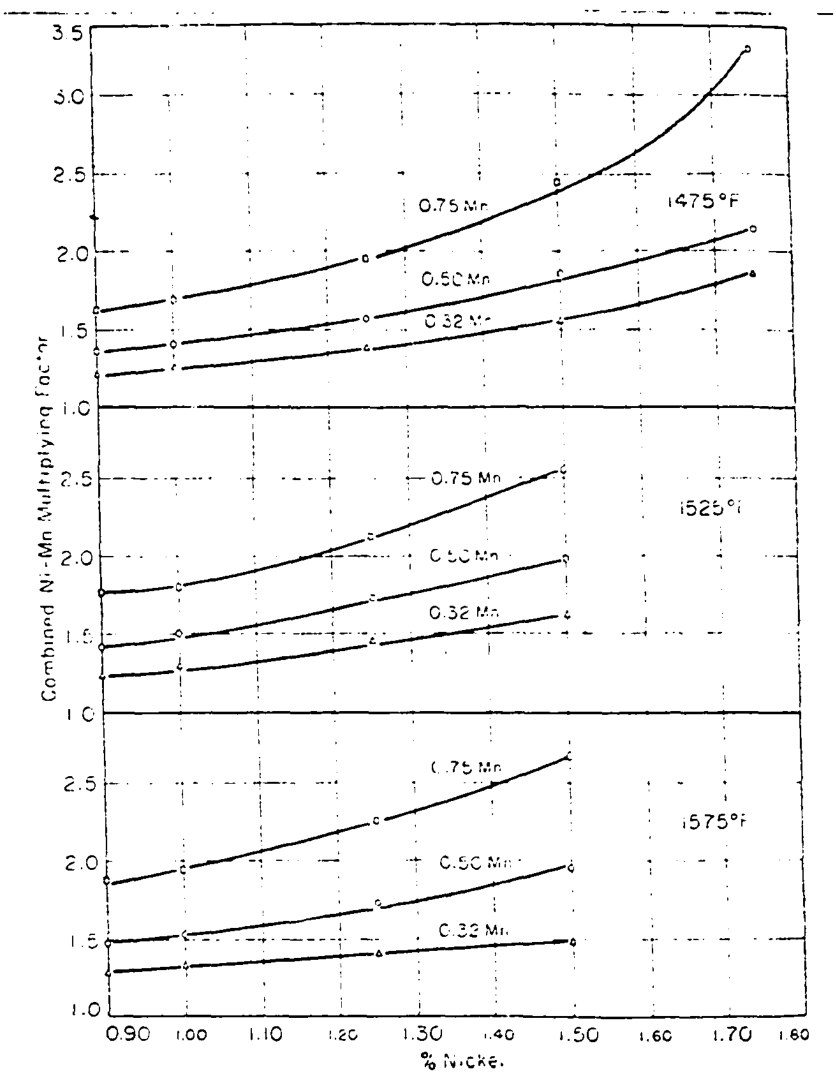

Figure 43a. Combined hardenability multiplying factors for nickel and manganese in normalized nickel-chromium-molybdenum 1.0% carbon steels at several hardenability temperatures (cited from Figure 7 of (13))

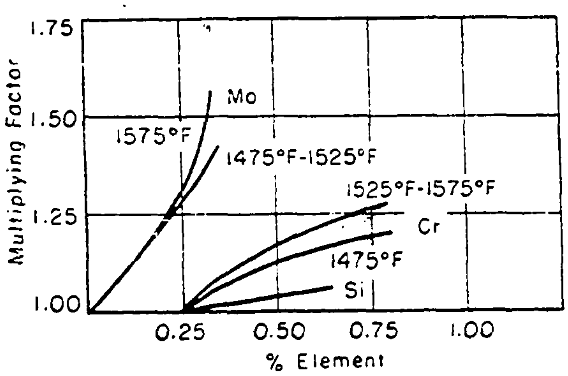

Figure 43b. Hardenability factors for molybdenum, chromium and silicon to be used with normalized multi-alloyed compositions containing more than 1.0% nickel (cited from Figure 8 of (13))

normalized chromium-nickel-molybdenum steels of greater than 1.0% nickel content.

Investigation of the existing data of effect of alloy elements on the hardenability of high carbon alloy steels end quenched from spheroidized annealed prior structure, showed almost complete dependence of each element upon the others for the value of its hardenability effect. Furthermore, some combinations such as nickel and molybdenum had a combined effect significantly greater than expected from their individual contributions. So it is concluded that the multiplying factor approach could not be used to predict the hardenability of annealed prior structure high carbon steels.

As shown in Figure 44, the hardenability, critical

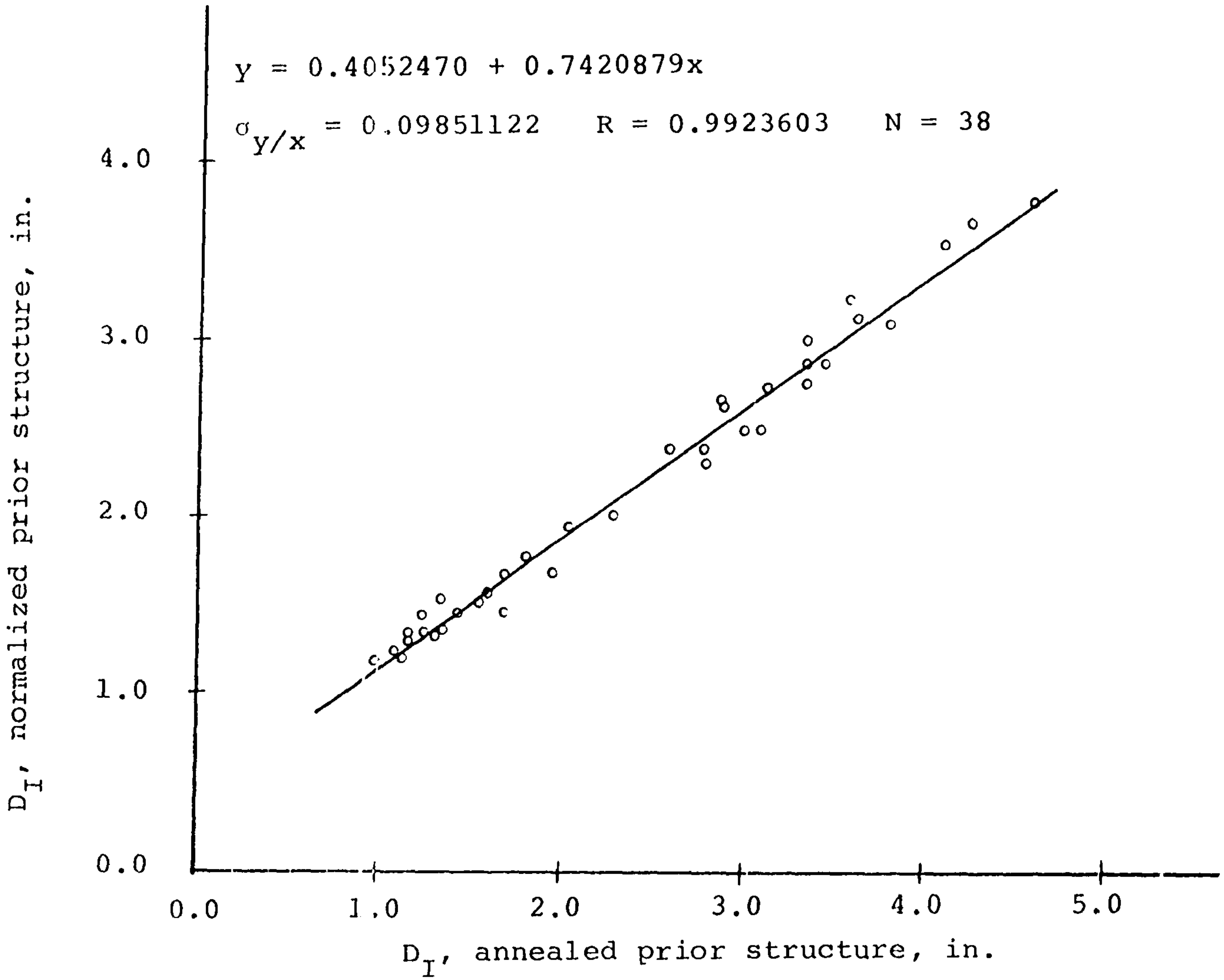

Figure 44. Correlation between hardenability based on normalized and annealed prior structure in alloyed 1.0% carbon steels (data cited from Table II of (13))

diameter, of annealed high carbon alloy steel can be predicted from that of normalized conditions through the simple linear relationship between them.

The figure also shows that, except for very low hardenability (critical diameter less than 1.50 inches) for the same steel annealed prior structure always has a higher hardenability than the normalized condition.

Jatczak and Girardi (15) used the distance as determined metallographically to 10% transformation to pearlite and/or bainite from the quenched end as the hardenability criterion. They determined the multiplying factor for the calculation of hardenability of hypereutectoid carbon steels hardened from 1700°F, with normalized and spheroidized prior structure and holding at austenitizing temperature for 35 to 40 minutes. The results are as shown in Figure 45 for normalized steel and Figure 46 for annealed steel. Good agreement has been found between the predicted and measured hardenability values provide that nickel content did not exceed 1.00% or silicon 0.50% when molybdenum content was in excess of 0.15%. But when the nickel or silicon in the presence of 0.15% or more molybdenum exceeded the above values, the steel hardenability proved somewhat higher than expected from the hardenability factors determined from the single alloy analysis. Metallographic examination disclosed that this anomalous behavior in hardenability was observed whenever the first product of transformation was bainite instead of pearlite. Since both nickel and silicon have a greater effect on bainitic hardenability than on pearlitic hardenability, new curves have been developed for silicon up to 0.75% and nickel up to 1.50%.

The factors shown in Figure 45 representing the normalized prior structure in 1.00% carbon steels were used in calculation of case hardenability of numerous single and multi-alloy carburizing steels quenched directly from the carburizer at 1700°F. In all instances except steels containing principally chromium, the agreement between calculated and measured case hardenability was good. Microscopic examination of the hardened chromium 1.00% carbon steels heat-treated from the normalized condition and case structures of the carburized chromium steels rehardened from 1700°F disclosed many excess carbide particles in the microstructure. However, very few were visible in the carburized chromium steels quenched directly from the carburizer. The greater effect of chromium in the direct quenched carburizing steels is obviously the result of the better solution of chromium and carbon. New factors for chromium, therefore, were developed for use in direct quenching of carburizing chromium steels and are shown by solid line on Figure 45.

Many commercial high-carbon alloy steels contain combinations of alloying elements which produce bainite as the first subcritical transformation product in normal hardening

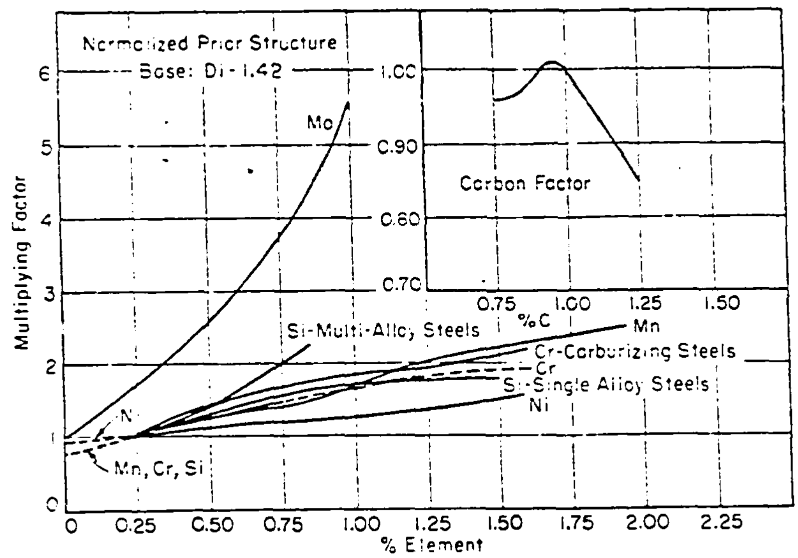

Figure 45. Multiplying factor for calculation of case hardenability in carburizing steels and of hardenability of prior normalized high carbon analyses (cited from Figure 4 of (15))

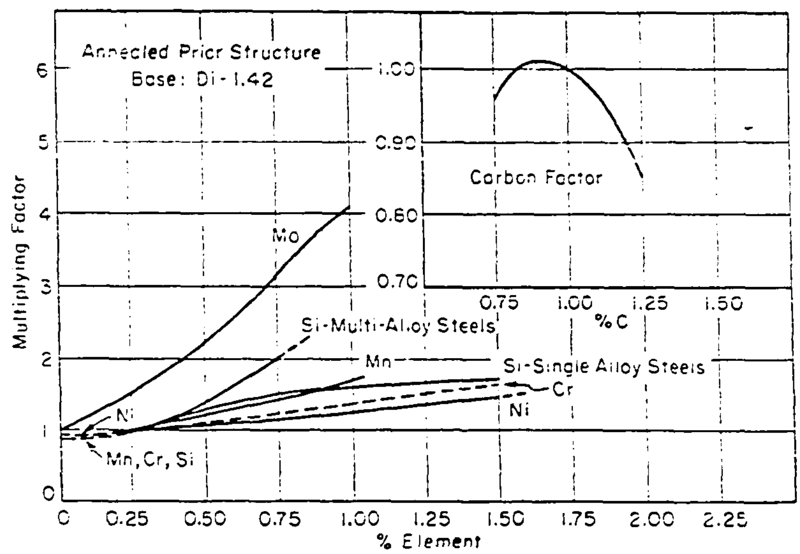

Figure 46. Multiplying factors for calculation of hardenability of prior spheroidized annealed high carbon analyses (cited from Figure 5 of (15))

operation and such steels are generally spheroidized for ease of machining prior to the hardening operation. Hardenability multiplying factor for carbon, manganese, silicon, chromium, nickel, and molybdenum have been developed, by Whittenberger, Burt and Carney (16), for hypereutectoid low-alloy steels in which bainite is the first subcritical transformation product. These factors permit the calculation of 95, 80 and 50 percent martensite hardenability from chemical composition when steels with spheroidized structure are quenched from 1475, 1525 and 1575°F. The 80% martensite multiplying factors for the various alloying elements in spheroidized hypereutectoid carbon steels austenized for 30 minutes at 1525°F are as shown in Figure 47.

It is observed that the carbon hardenability factor decreased as the carbon content increased from 0.75 to 1.25 percent. The relatively low hardenability of the higher-carbon steels is believed to reflect an effect of the large volume of undissolved carbides. Silicon has a higher effect on hardenability than that found in the work of Jatczak and Devine (13). They considered pearlite as the first transformation product. The difference between the two results was explained by the fact that silicon has a much greater effect on the retardation of the transformation of bainite than that on pearlite transformation.

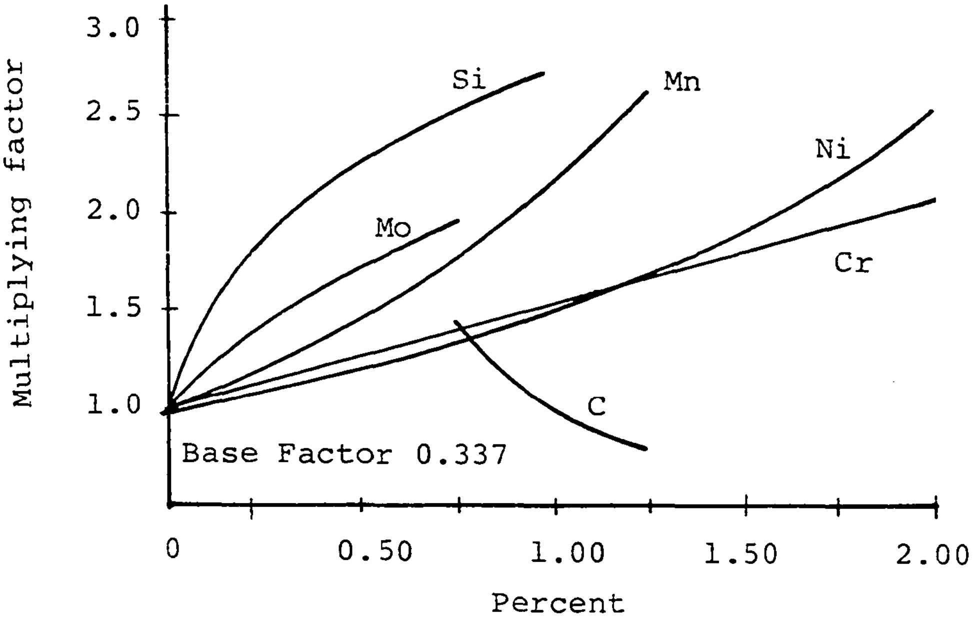

Figure 47. The 80% martensite multiplying factors for the various alloying elements in spheroidized hypereutectoid steels austenitized 30 min. at 1525°F (cited from Figure 8 of (16))

Summary

Hardenability of a steel is the capacity, when it is quenched and tempered, to obtain an essentially martensite structure throughout the cross section. For low carbon alloy steels, it is characterized by the "critical diameter".

Critical diameter of a low alloy steel is defined as the largest size of a cylindrical bar that contains at its center a microstructure of 50 percent martensite if it is quenched in a medium with an infinity serverity of quench.

When the alloy content of the steel is low, the effect of alloy elements on hardenability is independent of each other. For manganese, phosphorus, silicon, nickel, chromium molybdenum, and copper, these alloy elements increase the hardenability of steel.

Grossmann (1) predicted the hardenability of low carbon alloy steel from its chemical composition and grain size by associating a multiplying factor for each alloy element of the steel. The critical diameter of the steel will be the product of multiplying factors of each alloy elements present in the steel and the base hardenability due to carbon content and grain size of the steel.

Different sets of multiplying factors for alloy elements have been established empirically by Grossmann (1), Crafts and Lamont (7) and Kramer et al., (10). For low carbon alloy

steel, these multiplying factors predict the actual hardenability fairly well. Due to the carbide forming tendency and synergistic hardenability effects of alloy elements, these multiplying factors can only predict the maximum obtainable hardenability of complex and high alloy steels.

The relationship which correlated the quenched end distance of a Jominy hardenability test specimen, which has a microstructure of 50 percent martensite, and the critical diameter, enables us to predict hardenability of low carbon alloy steel from a Jominy hardenability test hardness profile. On the other hand if the hardenability, critical diameter, of a low carbon alloy steel are known, the Jominy hardness profile can be predicted using the relationship of ratio of initial hardness (I.H.), which depends on carbon content of the steel only, to distance hardness (D.H.) and critical diameter of Field (2).

DEVISE A METHOD TO PREDICT THE STATISTICAL DISPERSION IN THE JOMINY HARDNESS PROFILE PREDICTION

The critical diameter based on 50% martensite microstructure of low and medium carbon alloy steels, can be predicted, within ±15% accuracy (17) using the method of multiplying factors for alloy elements proposed by Grossmann (1). Different sets of multiplying factors for carbon, manganese, silicon, chromium, nickel, molybdenum and copper have been developed by Grossmann (1), Crafts and Lamont (7) and Kramer, Siegel and Brooks (10).

It has been recommended by several investigators (16, 17, 18) Kramer's factors should be used to calculate the hardenabilities (critical diameter) of low and medium carbon alloy steels which complete solution of carbon and alloy can be readily obtained. Calculations with Kramer's multiplying factors are accurate within ±15% at critical diameter values up to 4.5 inches (17). For steels with critical diameter greater than 4.5 inches, the Kramer's factors (and all others also) are not sufficiently accurate for practical use for the following reasons. First steels of high hardenability are primarily bainitic, and the hardenability effects of several elements (such as molybdenum) are very different when bainite is the first transformation product. Second, when some solution and carbide forming elements (such as nickel and molybdenum) are used together, they produce synergistic hardenability effect - that is the specific effect of each alloying element is larger than its effect when by itself in a composition. Third, steels of high hardenability usually contain large quantities of strong carbide forming elements which are often not completely dissolved.

Mean and Standard Deviation of Multiplying Factor for Carbon, Manganese, Silicon, Nickel, Chromium, Molybdenum and Copper

The mean and standard deviation of multiplying factor for carbon, manganese, silicon, nickel, chromium, molybdenum and copper will be developed from polynomials using the least-square regression method. The results can be incorporated into a digital computer program for the prediction of hardenabilities of low and medium carbon alloy steels.

Carbon factor

Grossmann (1) showed that for a given grain size, there is a linear relationship between carbon content of an alloy steel and the square of ideal critical diameter (carbon factor) which can be expressed by the following expression:

$$D_{IC}^2 = K*C$$

where

D_{IC} is the carbon factor or ideal critical diameter, inches

K is a constant

C is carbon content of the steel, percent

The variation of the data as a function of carbon content is such that a linear standard deviation regression model is appropriate. The standard deviation, σ, of D_{IC}^2 on C therefore is taken to be a linear function of C, i.e. $\sigma=\sigma_o*C$, where σ_o is a constant, which depends on the distribution and scattering of the data points.

Kramer's experimental carbon factors for low alloy steels corrected to grain size 7 with carbon content in the range of 0.2 to 0.7 percent, are tabulated in Table 6 and are as shown in Figure 23. With the aid of Iowa CADET (19), subroutine ME0238, the value of K and σ_o were determined.

$K = 0.9551$

$\sigma_o = 0.14022$

The coefficient of correlation between D_{IC}^2 and C is 0.9923 and for a sample size of 29 the fit is significant at 99% confidence level.

From the above result and using the relationship between ideal critical diameter and grain size (Figure 8) as developed by Grossmann (1), square of the ideal critical diameter at other grain sizes can be expressed as a function of carbon content of the steel (in the range of 0.2 to 0.7 percent) as follows:

Table 6. Carbon factor for low alloy steel[a]

C[b]	D_{IC}[c]	C[b]	D_{IC}[c]	C[b]	D_{IC}[c]
0.22	0.176	0.470	0.524	0.700	0.640
0.232	0.168	0.482	0.552		
0.261	0.209	0.500	0.371		
0.273	0.218	0.514	0.537		
0.291	0.332	0.541	0.471		
0.359	0.394	0.541	0.510		
0.400	0.295	0.550	0.566		
0.409	0.332	0.552	0.610		
0.411	0.383	0.552	0.640		
0.432	0.420	0.564	0.524		
0.432	0.524	0.591	0.552		
0.443	0.396	0.659	0.702		
0.459	0.483	0.682	0.624		
0.473	0.457	0.700	0.594		

[a]Data cited from Figure 8 of (10).

[b]C - carbon content of the steel, percent.

[c]D_{IC} - carbon factor, ideal critical diameter, inches.

<u>Grain size ASTM No. 4</u>:

$$D^2_{IC} = (1.5503)*C$$

<u>Grain size ASTM No. 5</u>:

$$D^2_{IC} = (1.3096)*C$$

<u>Grain size ASTM No. 6</u>:

$$D^2_{IC} = (1.1114)*C$$

<u>Grain size ASTM No. 7</u>:

$$D^2_{IC} = (0.9551)*C$$

<u>Grain size ASTM No. 8</u>:

$$D^2_{IC} = (0.8240)*C$$

and

$$\sigma = \sigma_o*C = (0.14022)*C$$

where σ is the standard deviation of D^2_{IC} on carbon content C of the steel, which was assumed to be the same for all grain size of the same carbon content.

If a random variable x has mean μ_x and standard deviation σ_x, the mean $\mu_{\sqrt{x}}$ and standard deviation $\sigma_{\sqrt{x}}$ of random variable $\sqrt{x}$ can be calculated by the following expression (20):

$$\mu_{\sqrt{x}} = \mu_x(1.0-0.125*(\sigma_x/\mu_x)^2)$$

and

$$\sigma_{\sqrt{x}} = \mu_x - \mu^2_{\sqrt{x}}$$

Therefore, the mean and standard deviation of the carbon factor D_{IC} can be calculated from that of D^2_{IC} by the above equations. The carbon factor D_{IC} is as shown in Figure 24.

If x is a random variable and

$$y_i = 1.0 + Ax_i + Bx_i^2 \quad i = 1,\ldots,n$$

where A and B are constants, under the assumption that the variance of x is small compared to that of y and thus can be neglected. The least square regression method gives us:

$$A = \frac{\sum_{i=1}^{n} x_i^4 \sum_{i=1}^{n} x_i(y_i-1) - \sum_{i=1}^{n} x_i^3(y_i-1)}{\sum_{i=1}^{n} x_i^4 \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i^3)^2}$$

$$B = \frac{\sum_{i=1}^{n} x_i^2 \sum_{i=1}^{n} x_i^2(y_i-1) - \sum_{i=1}^{n} x_i^3 \sum_{i=1}^{n} x_i(y_i-1)}{\sum_{i=1}^{n} x_i^4 \sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i^3)^2}$$

$$\sigma_y = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^{n} (y_i - \frac{\sum_{i=1}^{n} y_i}{n})^2}$$

$$\sigma_{y/x} = \sqrt{\frac{1}{(n-2)} \sum_{i=1}^{n} (y_i - 1.0 - A_x - B_x^2)^2}$$

$$R = \sqrt{1 - \left(\frac{\sigma_{y/x}}{\sigma_y}\right)^2}$$

where

σ_y - standard deviation of y.

$\sigma_{y/x}$ - standard deviation of y on x, which is independent of x and is a constant

R - coefficient of correlation between y and x $-1.0 \leq R \leq 1.0$

In practical engineering application, the chemical analyses of the alloying elements in the steel precised to $\pm$ 0.005 percent. Multiplying factors for the alloying elements were determined by the iteration method and their values are always greater than unity, it would be reasonable to assume that the variance of the alloying element content is so small compared to the variance of the multiplying factors of the alloy elements that use of Equation (1) through (5) to express alloy multiplying factor as a function of the alloying element content is justified.

Furthermore, Berkson (21) has showed that in the experiment in which one of the variates is a controlled observation (a controlled observation is one made when instead of wanting to know the value of some unknown quantity we wish to bring the quantity to a specified value) the line estimated by least square minimizing the sum of the squared residuals of the dependent uncontrolled variate, (as uncontrolled observation is made when, wishing to ascertain the value of some unknown quantity, we measure it with an instrument) is the same, where x and y is the controlled variate, that is, there is only one regression. The estimated line is not biased by the estimate of an error of observation in the independent controlled variate despite omission of account of it in the seast square fit. In determining the effect of alloy elements on the hardenability of a steel, a specified amount of a given alloy element is added to the steel, that is we try to bring the alloy content of the alloy to a specified value, so that the alloy content of the steel is a controlled observation. And the regression equations which express the multiplying factor of alloy elements as a function of alloy content will not be biased even though there is error of observation in the chemical analysis of the steel.

Multiplying factor for manganese

Based on Kramer's et al. (10) experimental data points of the hardenability effect of manganese (tabulated in Table 7 and shown in Figure 25), the multiplying factor of manganese, by Equations (1, 2, 3, 4 and 5), can be expressed as follows:

$$f_{Mn} = 1.0 + 0.23535*Mn + 0.56981*Mn^2$$

for Mn less than 3.0 percent

Table 7. Multiplying factor for manganese[a]

Mn[b]	f_{Mn}[c]	Mn[b]	f_{Mn}[c]	Mn[b]	f_{Mn}[c]
0.30	1.00	1.06	1.85	1.61	2.50
0.40	1.20	1.11	1.90	1.52	3.10
0.45	1.28	1.16	1.85	1.67	2.95
0.57	1.13	1.19	1.98	1.69	3.05
0.60	1.20	1.20	1.91	1.67	3.30
0.62	1.38	1.21	2.09	1.75	3.48
0.79	1.30	1.21	2.35	1.81	3.50
0.86	1.58	1.38	2.28	1.88	3.55
0.95	1.60	1.38	2.48	2.19	4.21
1.00	1.76	1.48	2.75	2.67	5.40

[a]Data cited from Table 13 of (10).

[b]Mn - manganese content of the steel, percent.

[c]f_{Mn} - multiplying factor for manganese.

$\sigma_{f_{Mn}} = 1.0354$

$\sigma_{f_{Mn}/Mn} = 0.18577$

$R = 0.98377$

with thirty data points, the curve fit is significant at one percent level.

The nomenclature is

f_{Mn} is the multiplying factor for manganese

Mn is the manganese content of the steel, percent

$\sigma_{f_{Mn}}$ is the standard deviation of multiplying factor for manganese

$\sigma_{f_{Mn}/Mn}$ is the standard deviation of multiplying factor for manganese on manganese content of the steel

R is the coefficient of correlation between multiplying factor for manganese and manganese content of the steel

Multiplying factor for silicon

In general, silicon has relatively small effect on hardenability. Based on Kramer et al. (10) experimental data points of the hardenability effect of silicon (tabulated in Table 8 and shown in Figure 26), the multiplying factor for silicon, by Equations (1, 2, 3, 4 and 5), can be expressed by the following expression:

$$f_{si} = 1.0 + 0.22111*Si + 0.12802*Si^2$$

for silicon content less than 2.0 percent

where

$$\sigma_{f_{Si}} = 0.31602$$

$$\sigma_{f_{Si}/Si} = 0.09115$$

$$R = 0.95750$$

with twenty-two data points, the curve fit is significant at

Table 8. Multiplying factor for silicon[a]

Si[b]	f_{Si}[c]	Si[b]	f_{Si}[c]
0.00	1.00	0.72	1.24
0.08	1.00	0.73	1.19
0.19	1.00	0.96	1.41
0.22	1.01	1.01	1.30
0.15	1.07	1.11	1.31
0.26	1.09	1.45	1.55
0.29	1.25	1.48	1.50
0.39	1.11	1.61	1.70
0.45	1.10	1.90	2.05
0.50	1.32	1.94	2.01
0.55	1.14	1.95	1.75

[a]Data cited from Figure 14 of (10).

[b]Si - silicon content of the steel, percent.

[c]f_{Si} - multiplying factor for silicon.

one percent level.

The nomenclature is

f_{Si} is the multiplying factor for silicon

Si is the silicon content of the steel, percent

$\sigma_{f_{Si}}$ is the standard deviation of multiplying factor for silicon

$\sigma_{f_{Si}/Si}$ is the standard deviation of multiplying factor for silicon on silicon content of the steel

R is the coefficient of correlation between multiplying factor for silicon and silicon content of the steel

Multiplying factor for nickel

For high nickel alloy steels, the 50% nonmartensite product was found to be bainite instead of pearlite. Results obtained by different investigators (1, 6, 9), showed good agreement with each other up to 3 percent nickel. Above 3 percent nickel, the hardenability effect of nickel showed large scattering in points.

Based on Kramer's et al. (10) experimental data points of the hardenability effect of nickel up to 3 percent nickel (tabulated in Table 9 and shown in Figure 27), the multiplying factor for nickel can be expressed by the following expression by using Equations (1, 2, 3, 4 and 5).

$$f_{Ni} = 1.0 + 0.76710*Ni - 0.12289*Ni^2$$

for nickel content less than 3.0 percent

Table 9. Multiplying factor for nickel[a]

Ni[b]	f_{Ni}[c]	Ni[b]	f_{Ni}[c]
0.00	1.00	1.00	1.65
0.07	1.00	1.49	1.77
0.25	1.309	1.79	1.77
0.32	1.29	1.97	1.91
0.31	1.44	1.77	2.15
0.53	1.53	2.09	1.97
0.77	1.68	2.71	2.08
0.93	1.56	2.82	2.28
0.94	1.66	3.07	2.31

[a]Data cited from Figure 15 of (10).

[b]Ni - nickel content of the steel, percent.

[c]f_{Ni} - multiplying factor for nickel.

$\sigma_{f_{Ni}} = 0.39031$

$\sigma_{f_{Ni}/Ni} = 0.12870$

$R = 0.94407$

with eighteen data points, the curve fit is significant at one percent level.

The nomenclature is

f_{Ni} is the multiplying factor for nickel

Ni is the nickel content of the steel, percent

$\sigma_{f_{Ni}}$ is the standard deviation of multiplying factor for nickel

$\sigma_{f_{Ni}/Ni}$ is the standard deviation of multiplying factor for nickel on the nickel content of the steel

R is the coefficient of correlation between the multiplying factor for nickel and nickel content of the steel

Multiplying factor for chromium

Chromium has a strong tendency to form stable carbide at high percentage content. The hardenability effect of chromium is strongly dependent on heat treatment. In general, the normalizing temperature had little effect on the hardenability factor, but as expected the austenitizing temperature and time had a marked influence.

Based on Kramer's et al. (10) experimental data points on the hardenability effect of chromium up to about 3 percent (tabulated in Table 10 and shown in Figure 28), the multiplying factor for chromium can be expressed by the following expression by using Equations (1, 2, 3, 4 and 5).

$$f_{Cr} = 1.0 + 1.6338*Cr + 0.02704*Cr^2$$

for chromium content less than 3.0 percent

$$\sigma_{f_{Cr}} = 1.2405$$

Table 10. Multiplying factor for chromium[a]

Cr[b]	f_{Cr}[c]	Cr[b]	f_{Cr}[c]	Cr[b]	f_{Cr}[c]
0.00	1.00	0.46	1.50	1.02	2.82
0.07	1.11	0.50	2.18	1.46	3.91
0.10	1.09	0.51	1.77	1.50	3.05
0.10	1.21	0.51	1.87	1.74	3.93
0.18	1.23	0.58	1.73	2.00	3.91
0.20	1.36	0.64	1.77	1.89	4.32
0.20	1.50	0.82	1.96	2.38	4.82
0.27	1.46	0.73	2.14	2.52	5.66
0.34	1.50	0.74	2.25		

[a]Data cited from Figure 18 of (10).

[b]Cr - chromium content of the steel, percent.

[c]f_{Cr} - multiplying factor for chromium.

$\sigma_{f_{Cr}/Cr} = 0.2541$

$R = 0.97879$

with twenty-nine data points, the curve fit is significant at one percent level.

The nomenclature is

f_{Cr} is multiplying factor for chromium

Cr is chromium content of the steel, percent

$\sigma_{f_{Cr}}$ is the standard deviation of multiplying factor for chromium

$\sigma_{f_{Cr}/Cr}$ is the standard deviation of multiplying factor for chromium on chromium content of steel

R is the coefficient of correlation of multiplying factor for chromium and chromium content of the steel

Multiplying factor for molybdenum

The hardenability effect of molybdenum is also strongly dependent on heat treatment. It has been found that, if both chromium and molybdenum were present, the hardenability was lower than that expected from a consideration of the factors of individual elements.

Based on Kramer's et al. (10) experimental data points of the hardenability effect of molybdenum up to about 1.1 percent molybdenum (tabulated in Table 11 and shown in Figure 29), the multiplying factor for molybdenum can be expressed by the following expression by using Equations (1, 2, 3, 4 and 5).

Table 11. Multiplying factor for molybdenum[a]

Mo[b]	f_{Mo}[c]	Mo[b]	f_{Mo}[c]	Mo[b]	f_{Mo}[c]
0.00	1.00	0.16	1.09	0.38	1.44
0.01	1.04	0.16	1.57	0.44	1.48
0.03	1.09	0.18	1.30	0.66	1.63
0.05	1.15	0.20	1.48	0.59	2.39
0.09	1.26	0.21	1.57	0.62	2.13
0.10	1.39	0.27	1.35	0.62	2.48
0.12	1.57	0.31	1.76	0.65	2.13
0.13	1.30	0.35	1.57	1.04	3.17
0.15	1.22	0.37	1.83	1.05	3.59

[a]Data cited from Figure 19 of (10).

[b]Mo - molybdenum content of the steel, percent.

[c]f_{Mo} - multiplying factor for molybdenum.

$$f_{Mo} = 1.0 + 1.5718*Mo + 0.58931*Mo^2$$

for molybdenum content less than 1.1 percent

$$\sigma_{f_{Mo}} = 0.63094$$

$$\sigma_{f_{Mo}/Mo} = 0.23261$$

$$R = 0.92956$$

with twenty-seven data points, the curve fit is significant at one percent level.

Where

f_{Mo} is the multiplying factor for molybdenum

Mo is the molybdenum content of the steel, percent

$\sigma_{f_{Mo}}$ is the standard deviation of multiplying factor for molybdenum

$\sigma_{f_{Mo}/Mo}$ is the standard deviation of multiplying factor for molybdenum to the molybdenum content of the steel

R is the coefficient of correlation between the multiplying factor for and molybdenum content of the steel

Multiplying factor for copper

Copper exerts a considerable influence on the hardenability and its effect is similar to that of nickel.

Based on the experimental data points of the hardenability effect of copper up to 2 percent copper (tabulated in Table 12 and shown in Figure 30), the multiplying factor for copper, by

Table 12. Multiplying factor for copper[a]

Cu[b]	f_{Cu}[c]
0.00	1.00
0.04	1.03
0.07	1.10
0.09	1.07
0.20	1.34
0.33	1.55
0.58	1.72
0.88	1.65
1.08	1.87
1.67	1.87
2.22	2.07

[a]Data cited from Figure 16 of (10).

[b]Cu - copper content of the steel, percent.

[c]f_{Cu} - multiplying factor for copper.

Equations (1, 2, 3, 4 and 5) can be expressed by the following expression.

$$f_{Cu} = 1.0 + 1.1763*Cu - 0.33386*Cu^2$$

for copper content less than 2.0 percent

$$\sigma_{f_{Cu}} = 0.38747$$

$$\sigma_{f_{Cu}/Cu} = 0.12079$$

$$R = 0.95016$$

with eleven data points, the curve fit is significant at one percent level.

Where

f_{Cu} is the multiplying factor for copper

Cu is the copper content of the steel, percent

$\sigma_{f_{Cu}}$ is the standard deviation of multiplying factor for copper

$\sigma_{f_{Cu}/Cu}$ is the standard deviation of multiplying factor for copper to the copper content of the steel

R is the coefficient of correlation between the multiplying factor for copper and copper content of steel

Jominy Hardenability Test

In order to obtain a reproducible and meaningful Jominy hardness curve for the comparison of hardenability of different low and medium alloy steels, the following items should be kept as nearly identical as possible in making duplicate tests:

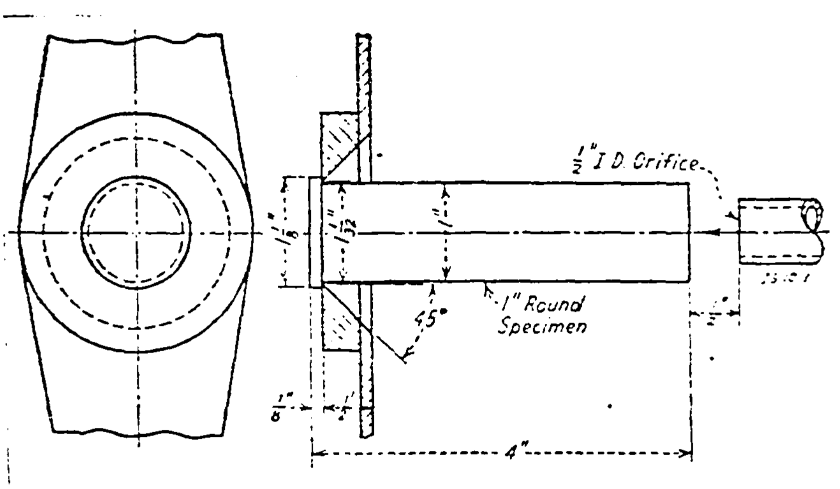

Figure 48. Hardenability test specimen in fixture for water quenching (cited from Figure 1 of (22))

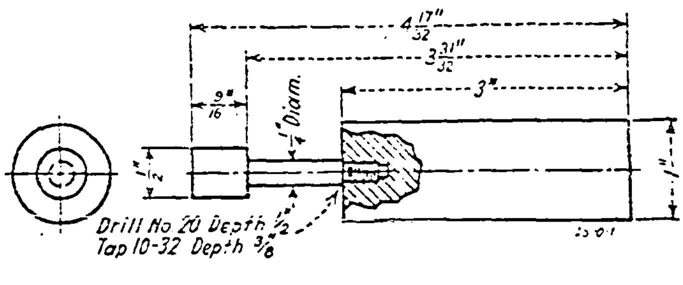

Figure 49. Optional specimen - a test (cited from Figure 3 of (22))

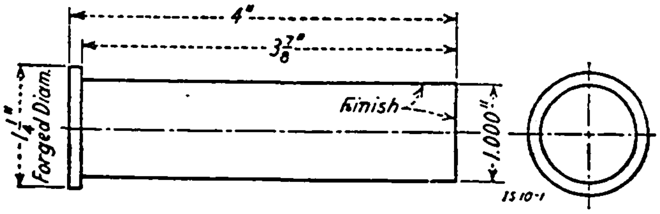

Figure 50. Preferred test specimen (cited from Figure 2 of (22))

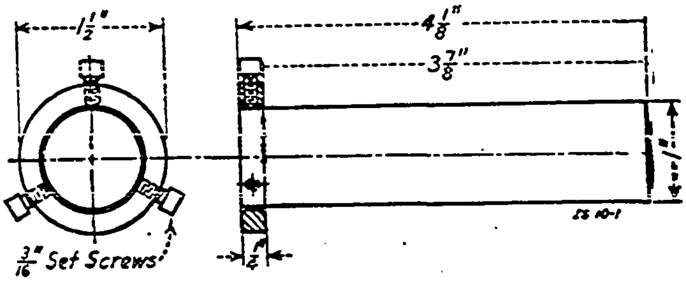

Figure 51. Optional specimen - B test (cited from Figure 4 of (22))

1. pretreatment of steel before the final quenching test is made;
2. surface finish of the bar to be hardened or of the face to be cooled;
3. rate of heating;
4. scale formation;
5. temperature to which piece is heated;
6. temperature and agitation of cooling medium, and
7. careful sectioning and grinding after hardening to avoid tempering in this operation.

Geometry and preparation of the test specimen; heat treatment temperature and time; and method for hardness measurement have been recommended and standardized, for the Jominy hardenability test, by the SAE (Society of Automotive Engineering) (22).

1. The test specimen: the test specimen is a 1 in. diameter cylinder 4 in. long with means for hanging it in a vertical position for end quenching. Figure 48 shows a test specimen in the fixture ready for quenching illustrating the preferred form of specimen. Figures 49, 50 and 51 give the details of the preferred specimen and two optional specimens.

The bar from which the specimen is machined shall be a forged or rolled $1\frac{1}{4}$ in. round representing the full cross section of the product. This point is of primary importance since any attempt to secure test specimens from any portion of the bloom, billet or bar other than the full cross section may introduce factors tending to affect the reproducibility of results. The condition of this hot formed bar shall be such that there is no decarburization on the one inch specimen machined from it.

2. Normalizing prior to heating for end-quenching: the forged or rolled round shall be normalized prior to machining the test specimen. This is of importance since the structure of material before quenching may materially affect the hardening characteristics. In order that variations in prior structure may be controlled as much as possible, the normalizing temperature listed in Table 13 should be used. The steel shall be held at such temperature for one hour and cooled in still air. If the normalized specimen is too hard it may be given a short time temper at about 100 degrees below the A_{c_1} temperature (temperature which steel transforms from pearlite and ferrite to ferrite and austenite) to improve machineability. The record of hardenability test results must always state the prior thermal history of the specimen tested.

3. Heating for end quenching: the specimen shall be heated to the austenitizing temperature shown in Table 13. The specimen shall be placed in a furnace which is at the specified temperature and shall be held at this temperature for 30 minutes. It is necessary to determine by means of a

Table 13. Standard Jominy end quench test specimen normalizing and austenitizing temperature

Maximum Carbon Content	Normalizing Temperature, Deg. F	Austenitizing Temperature, Deg. F
Steel series 1000, 1300, 3100, 3200, 4000, 4100, 4300, 4600, 5000, 5100, 6100,[a] 8600, 8700, 9400, 9700, 9300, 9900:		
Up to 0.25 incl.	1700	1700
0.26 to 0.36 incl.	1650	1600
0.37 and over	1600	1550
Steel series 2300, 2500, 3300, 4800, 9200:		
Up to 0.25 incl.	1700	1550
0.26 to 0.36 incl.	1650	1500
0.37 and over	1600	1475
0.50 and over	1650	1600

[a]Normalizing and austenitizing temperatures 50 deg. higher for 6100 series.

thermocouple the time required for a test specimen to come to the specified temperature to be sure that the above heating time and temperature requirements are met.

It is important that while heating the test specimen, care should be taken that its environment is such that practically no scaling or decarburization takes place on the end to be quenched. An adequately protective atmosphere in the furnace is suitable for meeting the above requirements. In the absence of such atmospheres, the specimen shall be inserted in a suitable container and placed with the bottom end of the specimen resting on a graphite or carbon disc.

4. Quenching: The test piece shall be placed on a fixture so that a column of water at a temperature of 40 to 85 degrees F and may be directed against the bottom face of the pice.

The column of water passing through an opening 1/2 in. in diameter shall rise to a free height of $2\frac{1}{2}$ in. above the opening. The fixture shall be dry at the beginning of each test.

In performing the test, the water supply shall be shut off with a quick opening valve and the hot specimen placed over the water pipe so that the bottom of the specimen is 1/2 in. from the opening of the water pipe and the water shall then be turned on. The time between removal of the specimen from the furnace and the beginning of the quench shall not be more than 5 seconds. The sample shall remain on the fixture for at least 10 minutes. A condition of still air shall be maintained around the piece during cooling.

5. Hardness measurement: Two flats 180 degrees apart shall be ground not less than 0.015 in. deep along the entire length of the bar and Rockwell C hardness measurements made along the length.

The preparation of the two flats must be carried out with considerable care. They should be mutually parallel and the grinding done in such a manner that no change in the quenched structure takes place.

Reading shall be taken in steps of 1/16 in. for the first 1 in. Distance between readings for the last 2 in. may be at the discretion of the tester.

Jominy Hardness curve prediction

If the initial hardness (I.H.) (hardness at 1/16 in. from quench end) of the steel is known and the ratio of initial hardness to distance hardness (I.H./D.H.) at different Jominy stations can be expressed as a function of critical diameter, which in turn can be determined from chemical composition and grain size of the steel, the Jominy hardness can be predicted.

Based on the experimental Jominy hardness curve of Field (2) and Hodge and Orehoski (9) (tabulated in Table 14

Table 14. Initial hardness (hardness at 1/16 in. from quench end)[a]

C[b]	I.H.[c]	C	I.H.	C	I.H.	C	I.H.
0.15	41.0	0.28	50.0	0.34	54.0	0.39	55.5
0.19	44.0	0.28	48.0	0.35	54.0	0.40	58.0
0.19	46.0	0.29	51.0	0.36	54.5	0.40	57.0
0.19	44.5	0.30	52.5	0.36	53.0	0.41	58.0
0.19	45.5	0.30	50.0	0.37	56.5	0.41	56.0
0.20	46.0	0.31	50.5	0.37	56.5	0.41	58.0
0.21	44.0	0.31	52.0	0.38	57.5	0.42	56.5
0.22	44.0	0.32	51.0	0.38	56.0	0.44	57.5
0.23	45.5	0.32	51.5	0.38	55.0	0.48	60.5
0.25	48.5	0.33	53.0	0.38	54.5	0.48	60.0
0.26	49.5	0.33	52.0	0.38	57.5	0.51	60.0
0.27	50.0	0.34	53.0	0.39	57.5	0.60	65.0
						0.79	65.0

[a]Data cited from Tables 1 and 3 of (10) and from Tables 1 and 2 of (2).

[b]C - carbon content of the steel, percent.

[c]I.H. - initial hardness, Rockwell C.

and shown in Figure 38), use the least square regression method and with the aid of Iowa CADET (19) library subroutine ME0225, the initial hardness of the steel can be expressed as a function of carbon content of the steel as follows:

$$I.H. = 35.54749 + 22.25735*C + 177.4605*C^2 - 305.1492*C^3 + 132.5669*C^4$$

for carbon content in the range of 0.2 to 0.66362 percent

and

$$I.H. = 65.0$$

for carbon content greater than 0.66362 percent

$$\sigma_{I.H.} = 5.4962$$

$$\sigma_{I.H./C} = 1.13$$

$$R = 0.97864$$

with thirty-seven data points, the curve fit is significant at one percent level.

The nomenclature is

I.H. is the initial hardness of the steel, Rockwell C

C is the carbon content of the steel, percent

$\sigma_{I.H.}$ is the standard deviation of the initial hardness, Rockwell C

$\sigma_{I.H./C}$ is the standard deviation of the initial hardness on carbon content of the steel, Rockwell C

Table 15a. Jominy Rockwell C hardness

No.	Jominy station (sixteenth of an inch)														Critical Diameter (in.)
	1	2	3	4	5	6	8	10	12	16	20	24	28	32	
1	64.0	64.0	64.0	64.0	64.0	64.0	63.0	62.0	61.0	57.0	50.0	45.0	42.0	41.0	5.30
2	65.0	65.0	65.0	65.0	65.0	65.0	64.0	63.0	63.0	62.0	61.0	57.0	52.0	48.0	6.33
3	59.0	58.5	58.8	58.0	58.0	58.0	57.5	56.5	55.3	52.0	49.5	45.8	44.8	42.5	6.37
4	60.8	60.3	59.0	59.0	58.8	58.8	58.0	57.8	57.8	55.0	52.5	49.8	46.0	42.3	6.38
5	61.3	60.0	59.8	59.0	59.5	59.3	58.3	58.3	56.8	54.3	50.5	47.5	44.8	41.5	6.37
6	60.5	60.0	59.0	58.3	58.0	58.0	57.5	56.8	55.5	53.5	49.8	47.3	43.5	42.5	6.32
7	59.0	58.3	57.5	57.3	57.3	56.8	56.0	54.5	53.3	49.3	47.5	44.8	41.0	42.0	6.22
8	60.5	59.8	59.0	58.8	59.0	58.8	58.0	57.0	56.8	55.0	48.5	47.5	43.3	41.8	6.19
9	60.3	60.0	59.3	59.3	58.8	58.3	58.8	57.5	56.8	54.3	51.0	48.3	44.0	41.0	6.15
10	60.5	59.8	60.0	59.3	59.5	59.5	58.0	57.0	55.5	51.0	48.5	44.8	44.3	44.8	6.05
11	61.8	60.8	60.8	60.8	60.3	58.8	58.5	57.8	57.3	54.0	49.8	46.5	44.0	41.0	5.96
12	59.8	60.0	59.0	58.0	58.0	57.0	56.0	56.0	55.8	53.0	48.8	46.5	43.5	41.8	5.95
13	60.8	60.3	60.0	59.0	59.0	58.8	57.5	56.8	56.0	51.8	48.0	44.5	42.0	39.3	5.82
14	61.0	60.3	59.5	59.8	59.8	59.3	58.3	57.8	56.5	53.0	49.3	45.3	43.0	41.0	5.74
15	61.0	61.0	60.5	60.0	59.0	59.0	58.3	57.8	56.3	53.3	47.8	45.0	42.3	39.3	5.66
16	58.3	57.3	56.5	56.0	56.0	55.8	54.5	53.8	52.3	47.0	42.8	40.8	39.3	37.5	5.53
17	59.5	59.0	58.5	57.5	57.5	57.0	56.3	55.3	53.0	49.5	46.0	42.3	39.5	39.8	5.52
18	60.8	59.8	59.3	58.8	58.5	58.5	58.3	57.0	54.5	50.0	45.3	42.3	40.5	40.0	5.48
19	59.8	59.3	59.0	58.8	58.3	58.0	57.0	55.8	53.8	51.0	47.8	40.8	41.0	40.3	5.46
20	60.3	59.3	58.5	58.8	58.5	58.3	57.0	55.5	54.5	49.8	45.0	42.5	40.5	39.3	5.43
21	59.0	58.5	58.0	57.0	57.0	57.0	56.8	56.0	53.8	49.0	44.3	42.0	39.5	38.0	5.40

Table 15a (Continued)

No.	Jominy station (sixteenth of an inch)														Critical Diameter (in.)
	1	2	3	4	5	6	8	10	12	16	20	24	28	32	
22	61.3	60.5	59.5	59.3	59.0	58.5	57.8	56.8	54.8	49.5	46.0	42.5	38.8	39.5	5.31
23	59.5	58.3	58.0	58.3	57.8	57.3	57.0	55.5	53.0	46.5	43.5	39.8	38.0	37.8	5.30
24	59.5	59.5	58.5	58.0	57.3	57.0	56.0	54.3	51.3	45.0	43.0	39.5	38.0	37.0	5.29
25	59.8	59.0	57.8	58.0	57.8	56.8	55.0	53.5	49.5	43.5	41.8	40.8	38.8	38.3	5.25
26	60.0	59.0	58.0	58.0	58.0	57.5	56.3	55.0	53.0	46.8	43.3	40.8	38.0	36.5	5.24
27	60.8	59.8	59.0	58.5	58.0	57.8	57.0	55.8	54.3	48.5	44.3	40.3	39.3	38.0	5.19
28	61.3	60.0	58.8	58.5	58.3	58.0	56.8	53.8	51.8	46.0	42.5	39.3	37.8	37.8	5.02
29	58.8	57.5	57.0	56.8	56.3	55.5	53.0	51.0	46.5	43.6	41.3	39.5	38.0	37.5	4.98
30	59.0	57.8	57.3	57.0	56.5	56.0	54.8	52.5	49.3	44.5	40.5	38.5	37.3	36.3	4.93
31	65.0	65.0	65.0	64.8	64.3	64.0	62.3	62.0	57.0	43.5	45.5	39.0	36.0	34.0	4.79
32	59.0	58.3	57.3	56.5	56.0	55.3	52.5	50.3	45.3	40.0	37.5	36.8	33.8	34.3	4.32
33	64.0	64.0	64.0	63.0	63.0	62.0	62.0	62.0	58.0	38.0	35.0	33.0	31.0	30.0	4.32
34	65.5	64.5	64.3	64.0	63.5	63.8	62.3	59.0	51.0	39.0	41.8	39.3	34.8	32.8	4.15
35	65.5	65.8	65.3	64.3	63.3	62.5	63.0	56.5	42.8	43.5	38.5	35.0	33.0	32.0	4.14
36	66.0	66.0	65.5	65.0	64.3	64.5	63.8	60.8	50.5	46.3	41.5	37.0	34.8	32.8	4.11
37	65.3	65.0	64.5	63.8	63.3	63.0	60.8	54.3	45.8	40.8	40.5	36.0	33.8	32.3	3.86
38	66.3	67.0	65.0	65.0	64.5	64.3	62.5	55.8	43.8	39.8	40.5	35.3	32.0	32.5	3.83
39	65.8	65.5	65.0	64.8	64.0	64.5	61.3	55.8	43.8	41.0	39.8	34.8	33.0	32.0	3.81
40	65.0	64.8	65.0	64.3	63.3	62.5	60.0	53.3	43.5	39.5	39.5	35.3	33.0	32.3	3.79
41	66.0	65.8	64.8	64.0	64.0	62.0	54.3	43.5	40.8	40.0	35.0	33.3	32.0	3.78	3.78
42	66.0	65.0	65.0	64.5	63.5	62.8	61.0	53.5	42.8	40.8	40.0	36.0	33.3	32.3	3.73

Table 15a (Continued)

No.	Jominy station (sixteenth of an inch)														Critical Diameter (in.)
	1	2	3	4	5	6	8	10	12	16	20	24	28	32	
43	65.0	65.0	65.0	64.0	63.3	63.3	61.5	53.8	42.0	39.3	40.5	36.0	32.8	31.8	3.66
44	58.0	57.0	56.5	56.0	54.0	52.5	46.8	43.0	41.0	38.5	36.5	35.0	33.0	32.5	3.65
45	65.3	65.0	65.0	63.8	63.3	62.8	60.3	51.5	40.8	39.5	38.8	34.3	32.0	31.0	3.59
46	62.0	61.5	61.0	60.3	59.3	58.3	54.0	46.5	39.8	35.3	37.5	31.5	27.0	26.8	3.57
47	65.0	64.8	64.3	63.8	63.0	62.3	58.0	49.3	40.8	40.5	38.8	34.5	32.5	31.8	3.54
48	65.8	65.3	64.8	64.5	64.3	63.5	62.0	55.8	44.3	41.0	40.3	36.3	33.8	32.3	3.53
49	65.3	65.8	65.0	64.0	63.5	63.0	59.8	48.5	41.8	41.8	37.0	34.0	32.3	30.8	3.53
50	64.5	64.0	63.3	62.8	62.0	60.5	57.5	47.8	40.5	39.3	37.5	33.5	31.5	30.0	3.53
51	63.3	62.5	61.3	60.3	59.0	57.5	52.0	45.0	38.8	35.0	38.5	32.5	28.8	26.5	3.52
52	66.5	66.0	65.0	64.0	63.0	61.8	56.3	45.3	38.8	42.0	38.0	32.3	31.5	30.5	3.51
53	65.0	65.3	65.0	64.0	63.0	62.3	59.0	49.0	40.0	39.8	38.3	33.8	32.0	31.3	3.51
54	65.0	65.3	64.3	64.0	63.0	62.5	58.0	46.5	38.8	41.3	37.3	33.3	30.8	30.5	3.51
55	64.5	64.3	62.8	62.5	61.8	60.3	55.8	45.0	38.5	39.0	36.8	33.0	30.8	29.3	3.49
56	54.8	53.5	52.8	52.3	51.8	50.5	45.0	40.0	37.8	33.5	31.8	31.0	29.0	29.8	3.49
57	63.0	62.8	62.0	61.0	60.0	59.0	54.3	45.8	39.5	38.3	35.3	30.8	28.3	27.3	3.48
58	62.8	62.5	61.8	61.3	60.5	58.3	52.8	42.5	37.5	36.8	35.3	31.0	28.5	27.0	3.39
59	65.5	66.0	64.8	64.0	64.0	53.5	55.3	41.8	38.8	40.5	35.0	33.0	31.3	30.5	3.34
60	58.8	57.8	56.5	56.0	54.5	52.5	45.8	39.3	35.3	31.8	30.0	29.3	28.5	28.5	3.32
61	65.5	64.8	64.0	63.3	62.8	60.0	51.0	40.8	37.5	41.5	35.3	32.3	30.8	29.3	3.17
62	44.0	44.0	42.0	39.0	36.0	34.0	30.0	28.0	26.0	24.0	23.0	22.0	22.0	21.0	3.16
63	64.8	64.3	63.3	62.0	61.0	58.0	47.8	40.0	35.0	40.5	35.3	31.5	29.5	29.3	3.00
64	54.8	53.5	52.5	50.0	46.5	42.5	37.5	33.8	31.8	30.0	23.5	20.0	17.0	17.0	2.92

Table 15a (Continued)

No.	Jominy station (sixteenth of an inch) 1	2	3	4	5	6	8	10	12	16	20	24	28	32	Critical Diameter (in.)
65	65.3	65.0	63.8	62.8	61.8	57.5	44.3	37.8	37.3	37.8	33.3	32.0	30.0	29.3	2.92
66	63.5	62.0	60.8	60.3	58.8	56.3	43.8	37.0	37.0	35.5	31.3	30.5	29.3	27.8	2.91
67	65.3	65.0	64.3	63.3	62.0	58.8	44.5	38.0	38.3	37.3	33.0	32.3	30.0	29.5	2.90
68	56.0	55.3	53.8	51.5	48.0	43.0	37.3	34.8	31.8	30.3	24.5	21.5	18.5	17.0	2.86
69	54.8	53.5	52.8	50.5	47.3	43.0	37.8	35.3	32.8	30.3	25.8	20.0	17.5	16.5	2.85
70	54.0	52.5	51.8	49.8	46.3	42.5	37.8	33.5	31.5	29.0	29.0	27.3	25.0	25.3	2.85
71	55.0	54.3	53.0	51.0	48.0	43.5	37.5	35.0	32.8	30.3	27.8	22.0	19.0	17.5	2.85
72	57.0	57.5	56.8	55.5	54.3	51.3	37.8	30.5	27.8	25.3	25.0	23.5	22.5	22.0	2.84
73	65.0	64.3	63.3	62.3	60.8	56.0	41.3	37.0	41.0	35.5	33.8	32.3	29.8	29.0	2.75
74	64.5	64.5	63.3	61.8	59.3	54.0	41.0	37.3	39.3	36.0	33.0	32.0	30.0	28.8	2.73
75	65.0	64.3	63.5	63.0	60.5	55.5	41.0	36.3	39.0	35.0	33.0	31.3	29.3	28.8	2.70
76	59.0	58.0	58.0	56.0	55.0	50.0	34.0	32.0	31.0	29.0	27.0	26.0	25.0	23.0	2.66
77	53.0	51.8	50.8	48.5	44.5	36.5	34.1	30.3	29.6	27.3	26.0	25.8	24.5	25.0	2.62
78	46.0	46.0	42.0	38.3	34.5	30.3	28.5	25.0	23.9	21.0	20.0	18.5	17.0	16.5	2.60
79	53.1	51.9	50.8	48.6	44.5	40.1	34.0	31.3	30.0	27.3	26.0	25.8	24.4	25.0	2.60
80	48.0	46.5	44.0	39.3	35.0	30.0	28.3	25.5	24.9	23.0	21.5	20.5	20.0	19.5	2.49
81	50.3	49.0	47.0	43.0	38.0	35.0	31.0	28.5	28.0	25.5	24.0	23.8	22.5	22.0	2.47
82	47.3	46.0	43.0	38.3	34.8	29.3	27.8	24.5	24.1	21.0	20.3	19.0	18.0	17.0	2.45
83	47.3	46.3	44.5	40.8	35.0	30.8	29.1	25.8	25.2	23.3	22.0	19.5	20.5	20.0	2.42
84	46.8	45.5	44.3	40.0	34.3	29.3	27.7	25.0	24.2	22.0	20.8	19.8	18.5	18.5	2.38
85	45.3	42.5	38.5	35.0	32.0	29.0	27.8	25.0	24.2	22.5	21.5	20.0	19.5	19.0	2.30

Table 15a (Continued)

No.	Jominy station (sixteenth of an inch) 1	2	3	4	5	6	8	10	12	16	20	24	28	32	Critical Diameter (in.)
86	51.0	50.0	49.0	46.0	39.0	33.0	27.0	24.0	23.0	20.0	18.0	17.0	17.0	15.0	2.30
87	48.5	47.5	46.5	44.0	37.0	28.0	26.0	23.5	23.1	20.0	18.0	17.5	17.5	17.0	2.26
88	59.0	58.0	55.0	52.5	46.0	38.0	28.0	28.0	26.5	24.5	23.5	23.0	16.5	21.0	2.26
89	47.0	46.3	41.8	36.5	32.8	28.5	27.1	24.8	24.1	22.3	21.0	20.3	19.5	19.5	2.25
90	60.3	59.0	57.5	53.0	45.8	39.0	31.5	29.5	28.3	26.3	24.5	24.0	23.5	22.0	2.25
91	47.3	46.0	41.3	35.5	32.0	28.0	26.8	24.0	23.3	21.0	19.5	19.0	17.5	16.5	2.24
92	51.0	49.5	47.5	43.0	35.5	27.0	25.5	23.0	22.7	21.0	19.5	18.5	17.0	17.0	2.16
93	46.8	45.8	43.0	37.3	32.0	27.3	26.0	24.3	23.6	22.0	20.0	19.0	18.0	18.0	2.15
94	46.5	46.0	42.0	36.5	32.0	27.5	26.3	23.5	23.2	20.5	19.5	18.5	17.5	17.0	2.14
95	50.0	49.0	47.5	43.0	35.5	27.0	25.4	23.0	22.1	19.5	18.0	16.0	14.5	16.0	2.13
96	45.5	42.0	36.5	34.0	30.8	27.5	26.4	24.3	23.6	21.5	20.5	20.0	19.0	18.2	2.13
97	58.3	56.8	54.5	49.3	41.8	35.8	29.3	27.5	25.5	24.3	23.0	22.5	21.5	20.8	2.13
98	45.3	41.0	36.3	32.5	30.5	27.3	26.2	23.0	22.7	20.5	19.5	18.0	17.0	16.5	2.13
99	47.0	46.3	42.5	36.5	32.0	27.0	25.9	23.5	23.3	20.8	19.5	17.5	16.0	15.0	2.13
100	47.0	46.0	43.0	37.0	33.0	28.0	26.6	24.3	23.5	21.0	20.0	20.0	18.0	17.5	2.11
101	58.5	56.8	54.8	49.5	41.5	35.3	29.5	27.0	25.5	24.3	23.5	22.3	21.5	21.0	2.11
102	50.5	49.5	48.0	43.5	35.5	26.5	24.7	22.0	21.9	20.5	19.0	18.0	17.0	17.0	2.10
103	50.0	48.5	47.0	41.0	34.0	26.5	25.1	22.5	22.2	20.0	18.5	17.5	17.0	17.5	2.04
104	49.5	48.0	46.5	40.0	32.5	26.0	24.2	22.0	21.5	19.5	18.0	17.0	13.0	15.5	2.04
105	62.3	61.0	60.0	58.0	40.0	33.0	32.3	32.0	31.0	28.8	27.0	24.8	21.8	19.5	2.04
106	46.3	45.0	39.8	34.0	30.3	26.5	25.1	23.0	22.5	19.5	18.5	17.0	15.5	15.5	2.04

Table 15a (Continued)

No.	Jominy station (sixteenth of an inch) 1	2	3	4	5	6	8	10	12	16	20	24	28	32	Critical Diameter (in.)
107	46.3	45.0	39.8	34.0	30.3	28.0	25.0	23.0	21.8	19.5	18.5	17.0	15.5	15.5	2.04
108	50.0	49.0	46.5	40.5	34.0	26.5	24.6	21.5	21.0	18.5	17.5	17.0	14.0	15.0	2.03
109	57.3	55.5	54.0	49.5	39.5	32.0	26.0	24.3	23.5	22.5	21.0	19.0	18.0	18.0	2.03
110	58.0	56.5	54.0	48.3	39.5	33.5	28.8	25.8	24.8	22.5	22.0	21.5	20.5	19.5	2.03
111	50.0	48.5	47.0	42.0	32.5	26.5	24.7	22.0	21.7	19.5	19.0	18.0	16.0	15.0	2.03
112	58.8	57.8	55.0	48.5	40.0	34.5	30.0	28.0	27.0	25.5	25.0	25.0	23.0	22.0	2.02
113	59.0	57.8	55.0	49.0	39.3	33.8	28.8	27.0	26.3	25.0	24.3	23.5	22.0	22.0	2.00
114	57.8	56.3	53.8	47.3	38.3	32.8	28.0	26.3	25.3	24.0	23.0	22.0	21.0	20.3	2.00
115	49.0	47.0	43.8	37.3	33.3	30.0	26.8	25.3	24.0	22.3	21.0	19.5	18.0	18.0	2.00
116	47.0	46.0	41.8	35.0	30.3	27.0	24.0	22.3	21.3	19.5	18.5	17.5	17.5	17.0	1.99
117	47.3	45.5	40.0	34.0	30.5	26.8	26.0	23.5	23.0	21.0	19.5	18.5	17.0	16.0	1.97
118	50.5	49.0	47.5	41.0	32.5	26.0	24.2	22.0	21.2	19.0	18.5	17.5	15.5	14.5	1.96
119	46.5	45.0	41.0	35.0	30.3	25.8	24.4	22.0	21.5	19.0	18.0	16.8	15.5	15.0	1.94
120	50.0	49.0	47.0	39.5	32.0	26.0	24.5	23.0	22.5	20.0	19.0	18.0	16.5	15.5	1.94
121	59.8	58.0	55.8	44.8	38.8	33.0	28.5	26.8	26.5	24.8	24.5	24.0	22.8	22.5	1.93
122	44.0	39.8	35.8	31.8	30.3	26.3	25.2	23.3	22.8	20.5	18.5	17.0	16.0	15.5	1.93
123	50.0	49.5	47.0	41.0	32.0	24.5	23.1	21.0	20.6	19.0	18.0	17.0	15.5	15.0	1.92
124	48.0	46.0	41.0	35.5	31.5	26.8	25.6	23.3	23.0	20.4	19.5	18.8	18.0	17.8	1.90
125	59.5	57.5	54.3	46.8	38.0	33.0	29.0	27.5	27.0	25.5	25.0	23.5	21.5	22.0	1.99
126	48.0	46.0	41.0	35.5	31.3	28.9	25.8	24.0	22.6	20.4	19.6	18.8	18.0	17.8	1.90
127	49.0	47.5	46.5	40.0	32.0	25.5	24.2	22.5	22.1	20.0	18.5	17.5	16.0	15.5	1.89
128	52.5	51.0	48.0	40.0	32.5	27.0	25.5	23.0	22.8	21.0	20.0	19.5	17.5	17.5	1.87

Table 15a (Continued)

No.	Jominy station (sixteenth of an inch) 1	2	3	4	5	6	8	10	12	16	20	24	28	32	Critical Diameter (in.)
129	52.3	51.0	48.8	40.0	31.0	28.0	24.8	23.5	22.3	20.5	18.0	18.0	16.5	15.5	1.85
130	57.5	55.5	51.8	42.5	34.5	30.0	26.8	24.5	23.5	21.8	21.0	20.3	19.0	19.3	1.83
131	45.0	41.0	35.0	31.5	29.5	27.8	26.9	24.5	24.0	22.0	20.5	19.5	18.5	17.0	1.83
132	58.5	56.5	52.5	44.0	37.3	33.3	29.5	28.1	26.9	25.9	24.9	23.9	22.5	20.8	1.82
133	57.5	55.5	51.8	42.5	33.8	29.5	26.3	24.3	23.5	22.3	21.3	19.8	19.5	19.5	1.80
134	46.5	42.0	37.3	33.3	31.0	27.5	26.2	24.0	23.7	21.5	19.8	18.0	17.0	16.0	1.80
135	56.8	54.8	51.3	41.5	32.8	28.8	25.0	24.0	22.8	22.0	20.8	20.0	19.0	18.5	1.80
136	51.0	50.0	47.5	38.0	30.0	25.0	24.0	22.0	21.6	19.5	18.0	17.0	15.5	14.5	1.79
137	52.5	50.5	48.5	39.0	30.0	25.5	24.0	21.0	21.0	20.0	19.0	17.5	15.0	15.0	1.78
138	59.3	57.3	53.3	43.5	34.8	30.3	27.5	26.0	25.5	23.5	22.8	21.8	20.8	21.0	1.78
139	58.5	56.5	52.8	42.0	34.0	30.0	27.3	26.0	24.8	23.3	22.3	21.3	20.0	19.5	1.77
140	59.0	56.3	52.5	42.3	33.5	29.0	26.5	25.5	25.0	23.0	22.8	21.3	20.3	20.0	1.75
141	57.0	54.5	49.5	39.3	32.3	28.6	24.5	24.0	22.5	21.0	19.8	19.5	18.0	17.0	1.74
142	53.8	51.8	48.0	39.0	31.5	27.0	22.3	21.3	20.0	18.3	17.5	16.3	14.8	14.8	1.71
143	58.0	55.5	48.5	37.8	31.3	28.6	26.0	25.0	24.0	23.0	22.0	20.8	19.5	19.3	1.62
144	56.8	54.0	47.5	35.5	28.5	25.5	22.5	22.0	21.3	18.5	17.5	17.0	15.0	15.5	1.57
145	54.0	50.0	38.5	30.8	27.3	25.5	23.8	22.5	21.6	20.0	18.5	16.8	15.8	15.5	1.35
146	50.8	48.8	45.0	38.8	33.5	30.0	26.0	23.8	22.0	19.0	17.0	16.0	15.0	14.0	2.01
147	50.0	49.0	47.5	41.0	33.5	26.5	26.2	23.5	22.8	20.5	19.5	18.0	17.0	12.5	1.99
148	52.0	49.0	48.0	46.0	42.0	38.3	33.3	30.3	28.5	24.3	19.0	17.0	14.0	13.0	2.56
149	53.8	52.3	51.0	47.3	42.3	38.3	33.8	31.0	29.3	25.8	19.5	17.5	14.5	14.0	2.52
150	51.8	50.5	48.3	44.3	39.3	35.3	32.0	29.0	27.0	24.0	19.0	15.5	13.0	11.5	2.36

Table 15a (Continued)

No.	Jominy station (sixteenth of an inch)														Critical Diameter (in.)
	1	2	3	4	5	6	8	10	12	16	20	24	28	32	
151	53.0	52.0	49.8	45.0	40.5	36.5	32.3	30.0	28.0	25.0	19.0	16.0	14.0	13.0	2.25
152	51.8	50.5	48.3	43.5	37.8	35.0	31.0	28.3	26.5	21.8	18.0	15.0	13.5	12.0	2.18
153	50.0	48.0	46.0	40.0	36.5	25.0	22.6	20.5	20.4	18.5	17.0	15.0	13.5	13.5	2.13
154	50.0	50.5	47.5	43.5	36.0	30.5	25.0	23.0	19.5	18.5	16.5	15.5	14.5	13.5	2.13
155	56.5	55.5	54.5	54.0	43.3	30.0	26.0	24.3	23.0	20.0	18.0	15.5	14.0	11.5	2.10
156	45.3	43.5	38.5	33.8	30.5	25.8	24.4	22.0	21.2	19.5	16.5	15.5	14.0	13.0	2.05
157	49.0	48.5	46.5	41.5	33.5	25.0	22.3	21.5	21.1	17.5	17.5	16.0	13.5	11.0	2.04
158	49.0	47.0	45.0	38.5	31.0	24.0	22.6	20.5	20.1	18.0	16.5	15.5	13.5	13.5	1.92
159	49.5	48.5	46.5	39.0	31.0	24.0	22.6	21.0	20.6	18.0	17.0	16.0	13.0	14.0	1.90
160	47.3	45.5	39.8	33.8	30.3	26.3	24.9	22.0	21.5	18.5	17.5	16.0	14.5	13.5	1.88

Table 15b. Composition of Jominy bar

No.	Percent times 100											
	C	Mn	Si	Ni	Cr	Mo	Cu	S	P	Al	V	B
1	60	83	32	53	48	21	4	1.5	1	4.3	0.0	0.0
2	66	61	27	122	61	22	7	0.9	1	3	0.0	0.0
3	45	96	29	3	99	24	2.1	1.8	1.7	4.8	0.6	0.0
4	46	94	26	3	92	18	4	1.8	1.6	4.3	0.6	0.0
5	44	96	26	1	92	20	3	1.6	1.3	5.7	0.5	0.0
6	43	96	30	3	97	17	3	2.2	0.9	5.9	0.5	0.0
7	43	95	30	3	95	23	2	1.7	1.2	6.5	0.5	0.0
8	45	91	29	2	92	19	3.1	1.8	1.7	9.6	0.6	0.0
9	45	92	28	3	92	18	3.2	1.8	1.2	5	0.6	0.0
10	48	91	29	2	92	21	2.6	1.9	1.2	0.5	0.6	0.0
11	48	88	28	1	92	18	2.1	2.5	0.8	5.7	0.3	0.0
12	47	95	32	1	93	23	2.0	1.3	1.4	5.4	0.5	0.0
13	44	94	26	8	90	22	1.6	1.0	1.2	4.9	1.2	0.0
14	48	96	26	1	95	20	2.6	1.7	2.0	4.4	0.6	0.0
15	47	88	25	3	93	20	3.0	1.8	1.3	2.7	0.5	0.0
16	38	99	30	1	96	21	2.0	3.0	0.8	7.6	0.4	0.0
17	43	87	27	4	92	20	3.4	1.4	1.0	4.4	0.4	0.0
18	43	86	22	2	86	20	2.0	1.5	0.9	2.1	0.7	0.0
19	44	95	27	2	93	21	3.1	1.8	1.0	5.2	0.6	0.0
20	44	97	28	3	92	20	3.0	1.8	1.0	4.2	0.5	0.0
21	45	95	27	3	95	24	2.6	1.7	1.5	5.8	0.5	0.0

Table 15b (Continued)

No.	C	Mn	Si	Ni	Cr	Mo	Cu	S	P	Al	V	B
	Percent times 100											
22	48	95	26	1	92	21	2.1	1.7	1.4	5.2	0.6	0.0
23	41	87	28	2	91	19	2.5	1.9	0.8	4.6	0.3	0.0
24	40	85	27	2	92	17	2.5	2.1	0.8	6.2	0.6	0.0
25	40	92	26	1	92	17	1.9	2.1	1.1	5.6	0.4	0.0
26	43	90	27	3	93	20	2.5	2.1	1.3	3.2	0.4	0.0
27	45	90	27	1	91	19	2.3	1.7	1.2	4.5	0.5	0.0
28	44	87	25	1	87	18	1.7	2.0	1.2	5.3	0.3	0.0
29	41	95	26	3	92	17	3.1	1.9	0.8	2.9	0.3	0.0
30	41	87	27	3	89	18	2.1	1.9	0.8	4.4	0.3	0.0
31	63	102	30	2	83	2	2.1	1.4	1	4.7	0.7	0.0
32	41	80	20	2	87	29	2.6	1.9	1.5	3.1	0.6	0.0
33	60	137	47	3	5	2	4.0	2.8	1.5	3.4	0.0	0.1
34	61	98	26	1	82	2	1	2.3	1.9	2.1	0.4	0.0
35	67	92	27	1	79	2	1.1	1.4	1.7	2.1	0.3	0.0
36	64	99	28	1	81	2	0.7	2.1	1.7	6.0	0.3	0.0
37	61	89	25	1	78	2	2.3	2.0	1.4	4.3	0.6	0.0
38	61	85	30	2	86	2	3.0	2.1	0.8	4.3	0.2	0.0
39	63	95	26	1	74	2	1.7	1.9	1.1	4.2	0.6	0.0
40	60	90	26	3	79	2	4.0	1.8	1.2	2.7	0.5	0.0
41	61	91	28	2	79	2	1.5	1.9	1.5	2.5	0.5	0.0
42	61	84	26	3	86	2	2.3	1.9	0.7	2.3	0.2	0.0

Table 15b (Continued)

No.	Percent times 100											
	C	Mn	Si	Ni	Cr	Mo	Cu	S	P	Al	V	B
43	62	93	27	1	79	2	2.0	2.9	1.3	3.0	0.4	0.0
44	42	88	24	3	82	19	2.6	2.7	1.3	6.1	0.7	0.0
45	63	95	26	1	74	2	1.7	1.9	1.1	4.2	0.6	0.0
46	50	83	28	1	90	2	1.5	1.9	0.6	3.5	0.2	0.0
47	59	91	26	1	79	2	1.5	1.8	0.9	3.5	0.6	0.0
48	61	93	27	1	78	2	0.8	1.6	1.6	3.8	0.2	0.0
49	61	92	27	1	77	2	4.0	1.7	1.5	4.3	0.5	0.0
50	56	85	27	1	77	2	1.0	2.4	0.8	2.0	0.2	0.0
51	50	78	27	4	90	2	5	1.9	0.7	1.6	0.2	0.0
52	60	76	29	3	82	2	5	2.3	0.9	1.6	0.2	0.0
53	60	91	25	2	75	2	2.5	2.0	1.8	1.5	0.3	0.0
54	58	73	30	5	79	2	1.0	2.1	0.5	2.8	0.2	0.0
55	59	88	29	2	78	2	1.5	2.3	0.8	2.0	0.2	0.0
56	38	66	27	2	92	20	2.8	1.9	1.9	4.7	0.0	0.0
57	52	81	34	4	91	2	0.7	2.4	0.7	4.4	0.2	0.0
58	53	81	20	1	89	2	1.0	2.0	0.9	5.4	0.2	0.0
59	61	86	28	1	75	2	1.0	2.1	0.7	2.1	0.2	0.0
60	41	83	25	53	47	18	2.3	1.8	1.0	3.3	0.0	0.0
61	59	87	22	3	80	2	1.1	1.8	0.6	2.1	0.2	0.0
62	18	80	25	103	42	35	3.0	1.5	1.5	5.5	0.0	0.0
63	58	79	25	1	75	2	1.4	2.4	0.5	0.7	0.2	0.0
64	32	82	29	1	93	2	0.6	2.3	1.5	4.2	0.2	0.0

Table 15b (Continued)

No.	Percent times 100											
	C	Mn	Si	Ni	Cr	Mo	Cu	S	P	Al	V	B
65	58	73	30	5	79	2	1.0	2.1	0.5	7.8	0.2	0.0
66	57	78	23	1	72	2	1.9	2.1	1.0	3.0	0.5	0.0
67	61	79	26	2	75	2	1.5	1.9	0.7	2.1	0.2	0.0
68	33	84	24	1	84	2	1.5	2.4	1.0	5.5	0.2	0.0
69	34	83	30	3	93	2	1.5	2.5	2.1	5.0	0.2	0.0
70	34	58	26	2	91	18	1.5	1.4	2.1	4.5	0.0	0.0
71	34	88	26	2	89	2	1.1	2.4	1	6	6	0.8
72	43	148	18	2	3	2	2.1	2.8	3.7	0.5	0.5	0.2
73	60	76	30	1	73	2	1.6	2.2	0.7	1.6	1.6	0.2
74	57	78	22	1	72	2	2.1	1.8	1.2	1.9	1.9	0.4
75	59	79	24	1	72	2	1.0	2.0	0.7	2.3	2.3	0.2
76	45	89	29	4	27	2	0.0	0.0	0.0	5.3	0.0	0.1
77	32	85	26	54	54	19	3.1	2.4	1.0	4.4	0.2	0.0
78	20	77	28	39	48	18	4.1	0.8	0.9	5.8	0.6	0.0
79	32	85	26	54	54	19	3.1	2.4	1.0	4.4	0.0	0.0
80	21	113	25	40	40	16	3.7	1.5	1.6	2.4	0.5	0.0
81	26	115	27	26	52	8	4.3	1.5	0.8	4.1	0.0	0.0
82	21	115	27	24	49	8	2.4	1.8	1.0	0.5	0.3	0.0
83	22	114	27	41	45	18	3.6	1.6	1.1	4.3	0.5	0.0
84	21	86	28	47	51	24	17	0.6	1.2	6.5	0.6	0.0
85	20	79	29	52	46	22	5.5	0.8	0.8	4.7	0.6	0.0

Table 15b (Continued)

No.	Percent times 100											
	C	Mn	Si	Ni	Cr	Mo	Cu	S	P	Al	V	B
86	27	84	25	3	6	20	2.3	4.6	0.8	4.5	0.4	0.0
87	25	84	25	3	6	20	2.3	4.6	0.8	4.5	0.4	0.0
88	41	160	20	1	7	2	3.4	2.1	0.8	3.2	0.1	0.0
89	21	117	31	43	39	15	3.5	1.5	1.1	3.1	0.3	0.0
90	43	157	28	3	7	2	4.6	1.9	1.1	3.4	0.2	0.0
91	20	124	28	25	48	8	2.7	2.3	0.9	2.2	0.2	0.0
92	25	90	24	1	13	23	2.9	2.8	0.7	3.7	0.4	0.0
93	21	87	26	44	51	22	2.6	1.6	1.0	5.3	0.5	0.0
94	21	78	27	48	46	22	6.0	0.9	0.8	4.8	0.7	0.0
95	28	91	25	2	13	24	2.0	1.4	1.1	3.7	0.4	0.0
96	20	78	33	44	50	18	7.4	0.8	0.9	6.1	1.0	0.0
97	39	158	25	1	7	2	2.0	1.6	1.2	2.7	0.2	0.0
98	19	84	27	40	50	18	6.0	0.8	0.7	5.2	0.3	0.0
99	21	83	27	55	50	21	2.2	1.3	1.7	4.9	0.4	0.0
100	23	87	27	62	46	17	3.5	1.7	0.9	3.8	0.2	0.0
101	39	149	24	5	8	2	5.5	2.3	1.5	3.3	0.2	0.0
102	27	84	25	3	6	20	2.3	4.6	0.8	4.5	0.4	0.0
103	27	83	25	3	11	26	4.0	3.7	1.5	8.0	0.5	0.0
104	25	93	24	2	13	24	3.4	2.9	0.9	3.6	0.4	0.0
105	51	73	22	1	9	2	2.5	1.7	0.5	4.4	0.2	0.0
106	20	81	26	69	47	20	3.2	1.6	1.1	3.1	0.3	0.0

Table 15b (Continued)

No.	C	Mn	Si	Ni	Cr	Mo	Cu	S	P	Al	V	B
	Percent times 100											
107	20	81	26	69	47	20	3.2	1.6	1.1	3.1	0.0	0.0
108	27	79	25	3	10	26	3.8	3.8	1.6	6.7	0.5	0.0
109	41	155	25	1	2	2	1.4	1.2	1.4	6.1	0.2	0.0
110	40	156	23	2	5	2	5.0	2.2	0.9	2.8	0.2	0.0
111	27	79	25	3	10	26	3.7	3.8	1.6	6.5	0.5	0.0
112	42	163	23	1	3	2	3.8	1.8	0.9	2.0	0.1	0.0
113	42	155	24	2	4	2	0.6	2.1	1.2	3.2	0.2	0.0
114	39	157	20	1	4	2	2.0	1.7	0.6	2.3	0.2	0.0
115	25	77	27	60	50	15	1.7	2.0	0.8	2.9	0.0	0.0
116	21	52	27	190	4	21	10	2.1	0.9	3.1	0.0	0.0
117	21	81	21	41	49	22	3.1	1.6	1.0	1.3	0.4	0.0
118	28	84	25	2	8	24	2.4	4.2	1.4	5.5	0.3	0.0
119	23	87	27	62	46	17	3.5	1.7	0.9	3.8	0.2	0.0
120	27	94	25	2	14	24	3.4	3.1	0.8	3.5	0.5	0.0
121	39	158	19	1	4	2	2.1	1.8	0.8	4.1	0.2	0.0
122	20	82	27	45	50	19	5.9	0.6	1.0	4.9	0.8	0.0
123	26	84	26	2	9	24	2.5	3.9	1.4	5.6	0.3	0.0
124	24	119	31	48	34	14	2.1	2.5	1.4	2.0	0.3	0.0
125	44	152	23	1	7	2	1.6	1.9	1.2	2.0	0.2	0.0
126	24	119	31	48	34	14	2.1	2.5	1.4	2.0	0.3	0.0
127	29	95	25	2	14	25	3.4	3.3	0.9	3.6	0.5	0.0

Table 15b (Continued)

No.	Percent times 100											
	C	Mn	Si	Ni	Cr	Mo	Cu	S	P	Al	V	B
128	30	88	25	1	3	25	2.5	2.1	1.0	7.6	0.3	0.0
129	30	87	28	1	4	26	2.0	2.3	0.9	8.5	0.4	0.0
130	37	149	24	3	4	2	3.6	2.0	0.7	3.1	0.2	0.0
131	20	82	25	38	47	21	5.5	0.7	1.1	4.5	0.6	0.0
132	42	104	24	1	29	2	3.8	1.6	0.6	1.0	0.0	0.0
133	38	157	20	1	3	2	1.5	1.4	0.8	4.1	0.2	0.0
134	23	80	28	52	52	20	1.3	1.9	1.2	5.3	0.3	0.0
135	36	154	19	2	3	2	1.5	1.9	0.8	2.3	0.2	0.0
136	29	80	26	1	7	19	1.5	4.5	0.9	5.3	0.4	0.0
137	30	87	28	1	4	26	2.0	2.3	0.9	8.5	0.4	0.0
138	42	147	22	4	5	2	4.5	2.1	0.8	2.5	0.2	0.0
139	39	140	24	1	2	2	1.1	1.4	0.7	2.8	0.2	0.0
140	41	149	24	2	2	2	1.2	1.8	0.5	3.0	0.2	0.0
141	36	147	23	1	7	2	2.0	1.8	0.7	2.6	0.2	0.0
142	36	138	22	1	1	1	1.4	1.7	1.6	4.9	0.0	0.0
143	40	139	21	4	5	2	4.3	2.1	0.6	1.3	0.2	0.0
144	39	138	23	1	6	2	4.6	2.2	0.8	3.7	0.2	0.0
145	34	83	22	1	5	20	2.6	1.6	0.8	1.5	0.2	0.0
146	26	162	24	1	19	2	2.4	1.5	1.2	3.6	0.2	0.0
147	26	76	26	4	12	25	3.1	4.0	0.8	5.1	0.5	0.0
148	30	79	27	1	91	2	1.3	2.0	1.0	2.6	0.2	0.0
149	31	80	27	1	90	2	1.3	1.5	1.2	4.6	0.2	0.0

Table 15b (Continued)

No.	Percent times 100											
	C	Mn	Si	Ni	Cr	Mo	Cu	S	P	Al	V	B
150	29	79	24	1	88	2	2.0	2.2	1.1	7.7	0.5	0.0
151	33	82	25	1	92	2	1.1	2.1	1.1	4.1	0.2	0.0
152	30	79	25	1	91	2	1.0	1.9	1.1	2.6	0.2	0.0
153	27	78	25	3	10	23	2.6	4.2	1.4	5.5	0.5	0.0
154	28	146	17	5	15	1	2.4	2.3	0.7	0.4	0.2	0.0
155	39	85	25	3	4	0	8.3	0.8	1.9	3.5	0.0	0.1
156	20	113	26	47	38	10	1.1	2.8	1.2	0.0	0.2	0.0
157	27	81	24	2	8	21	2.5	3.7	1.2	4.8	0.2	0.0
158	26	82	25	2	8	21	2.5	3.6	1.2	5.1	0.3	0.0
159	27	82	24	2	3	21	2.5	3.9	1.0	4.8	0.2	0.0
160	22	96	21	40	43	16	3.5	1.8	1.2	2.1	0.4	0.0

R is the coefficient of correlation between initial hardness and carbon content of the steel

Experimental Jominy hardness curve data were obtained from a steel company research laboratory, and are tabulated in Table 15a. The critical diameter of each alloy steel can be determined from its Jominy hardness curve through the relationship between the Jominy quench end distance of 50% martensite hardness and the critical diameter of the alloy steel (Figure 39). For example, an alloy steel which has a carbon content of 0.44 percent and the Jominy hardness curve were determined experimentally and are as shown in the following data:

Station No. 1/16 in.	1	2	3	4	5	6	10
Hardness Rockwell C	60.3	59.3	58.5	58.8	58.5	58.3	55.5

Station No. 1/16 in.	12	16	20	24	28	32
Hardness Rockwell C	54.5	49.8	45.0	42.5	40.5	39.3

For a carbon content of 0.44 percent, the 50% martensite hardness was found (from Figure 14) to be 43.1 Rockwell C. This hardness lies between the Jominy Hardness of station number 20, and station number 24. A polynomial of fourth degree will be determined by using least-square regression method based on the following six data points:

Station No. 1/16 in.	12	16	20	24	28	32
Hardness Rockwell C	54.5	49.8	45.0	42.5	40.5	39.3

and the polynomial was found as follows:

$$y = 45.87487 - 4.114609*x - 0.437111*x^2$$
$$+ 0.014989*x^3 - 0.0001709*x^4$$

where

y is the hardness in Rockwell C

x is the Jominy distance from the quench end in 1/16 of an inch

If we let y equal to the 50% martensite hardness corresponding to the carbon content of the steel, which in our example is 43.10 Rockwell C, the value of x can be determined by solving the above polynomial numerically. This was done by using Iowa CADET (19) library subroutine VARSEC, and the end quench distance x is found to be 1.431 inches.

From Figure 37, for an end quench distance of 1.431 inches, the critical diameter of the steel is 5.43 inches. Similarly the critical diameter of the steels in Table 15a were determined and are tabulated in the last column of Table 15a.

For Jominy station number 4, data in Table 15a showed that, the relationship between the ratio of initial hardness to distance hardness (I.H./D.H.) and critical diameter can be expressed by the following equations:

$$y = 1.0 - 0.008283*(x - 7.0)$$

$\sigma_{y/x} = 0.023$ for x in the range of 3.5 to 6.5

$R = 0.616354$

and the curve fit is significant at one percent level

$$y = 1.02899 - 0.0818586*(x-3.5) - 0.0923808*(x-3.5)^2 - 0.1002404*(x-3.5)^3$$

$\sigma_{y/x} = 0.063692$ for x in the range of 1.5 to 3.5

$R = 0.94286$

and the curve fit is significant at one percent level.

The nomenclature is

x is the critical diameter of the alloy steel, inches

y is the ratio of initial hardness to distance hardness I.H./D/H.

$\sigma_{y/x}$ is the standard deviation of I.H./D.H. on critical diameter

R is the coefficient of correlation between I.H./D.H. and critical diameter

For Jominy stations number 8, 12, 16, 20, 24, 28, and 32, the ratio of initial hardness to the distance hardness of a given Jominy station approaches unity as the value of the critical diameter becomes larger and larger. An exponential model has been fitted for the relationship between critical diameter and the ratio of initial hardness to distance hardness (I.H./D.H.).

$$y = 1.0 + A*e^{B*x} \tag{a}$$

or

$$y - 1.0 = A*e^{B*x}$$

$$\ln(y-1.0) = \ln A + B*x$$

$$u = K + B*x \tag{b}$$

where

y is the ratio of initial hardness to distance hardness at a given Jominy station

x is the critical diameter of the steel, inches

We can consider the critical diameters (x) and the ratio of initial hardness to distance hardness (I.H./D.H.), which can be calculated by taking the ratio of Jominy hardness at station 1 to that of Jominy station 8, 12, 16, 20, 24, 28, and 32 in Table 15a, as pairs of data points. The relationship between critical diameter and the ratio of initial hardness to distance hardness, that is the value of K and B in Equation (b), can be determined by using the least square regression method. This was done by using Iowa Cadet (19) library subroutine ME0226, and the results are summarized as follows:

<u>Jominy station number 8</u>:

$$u = 1.583563 - 0.85966*x$$

$\sigma_{u/x} = 0.39055$ for x in the range of 1.5 to 6.5

$R = -0.94885$

and the curve fit is significant at one percent level.

Jominy station number 12:

$u = 1.516167 - 0.65066*x$

$\sigma_{u/x} = 0.19035$ for x in the range of 1.5 to 6.5

$R = -0.97706$

and the curve fit is significant at one percent level.

Jominy station number 16:

$u = 1.471133 - 0.5444188*x$

$\sigma_{u/x} = 0.16144$ for x in the range of 1.5 to 6.5

$R = -0.97669$

and the curve fit is significant at one percent level.

Jominy station number 20:

$u = 1.46169 - 0.481393*x$

$\sigma_{u/x} = 0.159499$ for x in the range of 1.5 to 6.5

$R = -0.97166$

and the curve fit is significant at one percent level.

Jominy station number 24:

$u = 1.411127 - 0.41921*x$

$\sigma_{u/x} = 0.159686$ for x in the range of 1.5 to 6.5

$R = -0.964516$

and the curve fit is significant at one percent level.

Jominy station number 28:

$u = 1.392689 - 0.3797674*x$

$\sigma_{u/x} = 0.1529495$ for x in the range of 1.5 to 6.5

$R = -0.9622918$

and the curve fit is significant at one percent level.

Jominy station number 32:

$u = 1.372598 - 0.3577734*x$

$\sigma_{u/x} = 0.1528825$ for x in the range of 1.5 to 6.5

$R = -0.9580142$

and the curve fit is significant at one percent level.

The nomenclature is

u is equal to ln(y-1.0), and y is the ratio of initial hardness to distance hardness (I.H./D.H.)

x is the critical diameter of the alloy steel, inches

R is the coefficient of correlation between u and x

Two random variables x and y, if x = ln(y) is a normal distribution then y will be a log normal distribution. The mean and standard deviation of $x(\mu_x, \sigma_x)$ and $y(\mu_y, \sigma_y)$ are related to each other by the following expression (21):

$$\mu_y = \mathrm{Exp}(\mu_x + \frac{\sigma_x^2}{2.0})$$

$$\sigma_y^2 = (\mathrm{Exp}(\sigma_x^2)-1.0)\ (\mathrm{Exp}(2.0*\mu_x+\sigma_x^2))$$

In performing the least square regression to determine the value of K and B in Equation (b), it was assumed that $u = \ln(y-1.0)$ is normally distributed. Therefore, the mean μ_y and standard deviation σ_y (which is the mean and standard deviation of ratio of initial hardness to distance hardness I.H./D.H.) can be calculated easily from the mean and standard deviation of u by the above two relations and

$$\mu_y = \text{Exp}(\mu_u + \frac{\sigma^2_{u/x}}{2.0}) + 1.0$$

$$\sigma^2_{y/x} = \text{Exp}(\sigma^2_{u/x}) - 1.0) \quad (\text{Exp}(2.0 * \mu_u - \sigma^2_{u/x}))$$

COMPARE THE METHOD WITH THE JOMINY DATA FROM STEEL PRODUCERS

A computer program has been developed, which will predict the mean and standard deviation of Jominy hardness at stations 1, 4, 8, 12, 16, 20, 24, 28, and 32 from chemical composition and grain size of low and medium carbon alloy steel. This program is based on the expressions for the multiplying factors for carbon, manganese, silicon, nickel, chromium, molybdenum, copper, initial hardness, and the ratio of initial hardness to distance hardness at different Jominy stations which have been discussed earlier.

This computer program has been used to predict the Jominy hardness curve for a few alloy steels and the results showed good agreement with the experimental measured values (12).

The following symbols will be used in the tabulation of the results of the test of the program.

GS - ASTM grain size of the alloy steel

C - Carbon content of the alloy steel, percent

Mn - Manganese content of the steel, percent

Si - Silicon content of the alloy steel, percent

Ni - Nickel content of the alloy steel, percent

Cr - Chromium content of the alloy steel, percent

Mo - Molybdenum content of the alloy steel, percent

Cu - Copper content of the alloy steel, percent

f - Multiplying factor for alloy elements

S_f - Standard deviation of multiplying factors of alloy elements

D_I - Critical diameter of the alloy steel, inches

S_{D_I} - Standard deviation of critical diameter, inches

J - Jominy station number, sixteenth of an inch

I.H./D.H. - Ratio of initial hardness to distance hardness

$S_{I.H./D.H.}$ - Standard deviation of initial hardness to distance hardness

Q_p - Predicted hardness, Rockwell C

S_{Q_p} - Standard deviation of the predicted hardness, Rockwell C

Q_M - Experimental measured hardness, Rockwell C

Sensitivity Analysis of Content of Alloy Element on the Hardenability of Steel

The hardenability, critical diameter, of steel can be calculated by the following expression

$$D_I = D_{IC} f_{Mn} f_{Si} f_{Ni} f_{Cr} f_{Mo} f_{Cu}$$

The method of propagation of error gives us:

$$\delta_{D_I}^{\ 2} = \left(\frac{\partial D_I}{\partial D_{IC}}\right)^2 \delta_{D_{IC}}^2 + \left(\frac{\partial D_I}{\partial f_{Mn}}\right)^2 \delta_{f_{Mn}}^2 + \left(\frac{\partial D_I}{\partial f_{Si}}\right)^2 \delta_{Si}^{\ 2}$$
$$+ \left(\frac{\partial D_I}{\partial f_{Ni}}\right)^2 \delta_{Ni}^{\ 2} + \left(\frac{\partial D_I}{\partial f_{Cr}}\right)^2 \delta_{Cr}^{\ 2} + \left(\frac{\partial D_I}{\partial f_{Mo}}\right)^2 \delta_{Mo}^{\ 2}$$
$$+ \left(\frac{\partial D_I}{\partial f_{Cu}}\right)^2 \delta_{Cu}^{\ 2}$$

Where δ_x is the change in x. From the expressions of the carbon factor and multiplying factor for alloy elements.

$$\left(\frac{\partial D_I}{\partial D_{IC}}\right)^2 = \frac{0.38758}{C} \quad \text{for G.S. 4}$$

$$\left(\frac{\partial D_I}{\partial D_{IC}}\right)^2 = \frac{0.32740}{C} \quad \text{for G.S. 5}$$

$$\left(\frac{\partial D_I}{\partial D_{IC}}\right)^2 = \frac{0.27785}{C} \quad \text{for G.S. 6}$$

$$\left(\frac{\partial D_I}{\partial D_{IC}}\right)^2 = \frac{0.23878}{C} \quad \text{for G.S. 7}$$

$$\left(\frac{\partial D_I}{\partial D_{IC}}\right)^2 = \frac{0.20600}{C} \quad \text{for G.S. 8}$$

$$\left(\frac{\partial f_{Mn}}{\partial_{Mn}}\right)^2 = (0.23535 + 1.1396\ Mn)^2$$

$$\left(\frac{\partial f_{Si}}{\partial_{Si}}\right)^2 = (0.22111 + 0.25604\ Si)^2$$

$$\left(\frac{\partial f_{Ni}}{\partial_{Ni}}\right)^2 = (0.76710 - 0.24578\ Ni)^2$$

$$\left(\frac{\partial f_{Cr}}{\partial C_r}\right)^2 = (1.6338 + 0.05407\ Cr)^2$$

$$\left(\frac{\partial f_{Mo}}{\partial_{Mo}}\right)^2 = (1.5718 + 1.17862\ Mo)^2$$

$$\left(\frac{\partial f_{Cu}}{\partial_{Cu}}\right)^2 = (1.1763 - 0.66772\ Cu)^2$$

Consider a steel which has the following chemical composition and grain size

G.S. 6		f
C	0.33	0.604
Mn	0.88	1.648
Si	0.28	1.072
Ni	0.56	1.391
Cr	0.57	1.940
Mo	0.25	1.430
Cu	0.00	1.000

The mean and standard deviation of the critical diameter of the steel are $D_I = 4.121$ in and $\sigma_{DI} = 1.783$ in.

In practical engineering application the chemical analysis of the steel is precised to $\pm$ 0.005 percent.

For $\delta_C = \delta_{Mn} = \delta_{Si} = \delta_{Ni} = \delta_{Cr} = \delta_{Mo} = \delta_{Cu} = 0.05$ percent

$\delta_D{}^2 = 30.154 \times 10^{-4}\ \text{in}^2$

$\delta_D = 0.0549$ in

which is very small compared with the standard deviation of critical diameter.

Steel: NE9430[1] (Figure 52)

G.S.	7	f	S_f
C	0.32	0.551	0.041
Mn	1.34	2.339	0.186
Si	0.43	1.119	0.091
Ni	0.31	1.226	0.129
Cr	0.37	1.608	0.254
Mo	0.09	1.146	0.233
Cu	0.00	1.000	0.000

D_I = 3.26 inches, S_{D_I} = 1.43 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.00	52.2	1.13	51.0
4	1.045	0.064	50.2	3.28	48.5
8	1.319	0.129	40.0	4.13	41.5
12	1.556	0.107	33.7	2.46	35.0
16	1.748	0.122	30.0	2.22	30.5
20	1.909	0.146	27.5	2.22	28.0
24	2.060	0.170	25.5	2.22	27.0
28	2.181	0.182	24.1	2.12	26.0
32	2.244	0.191	23.5	2.10	25.5

[1] Steel composition, grain size and measured Jominy hardness were cited from Table I and II of (2).

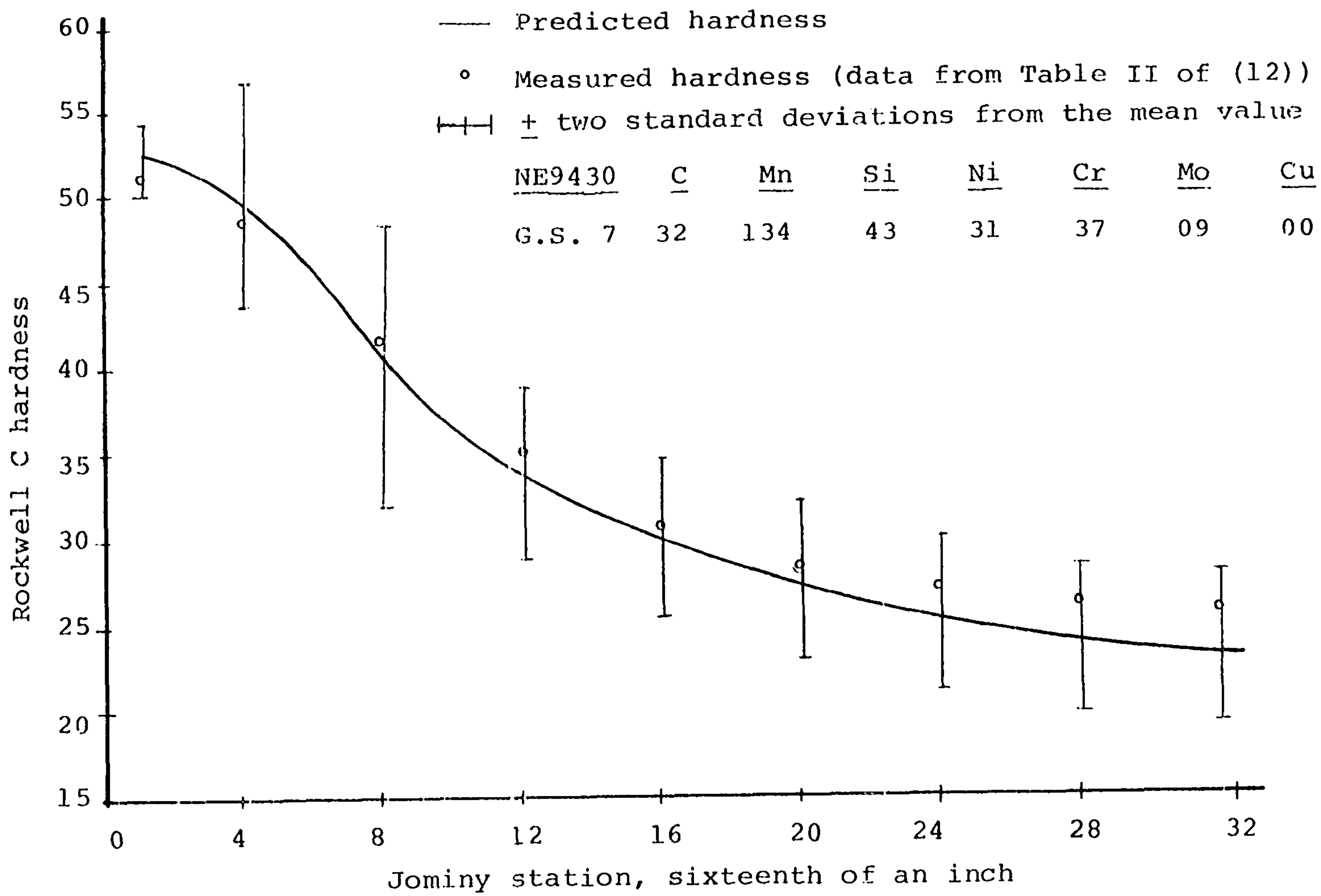

Figure 52. Jominy hardness curve of steel NE9430

Steel: NE9440[1] (Figure 53)

G.S.	7	f	S_f
C	0.41	0.624	0.046
Mn	0.90	1.673	0.186
Si	0.57	1.168	0.091
Ni	0.37	1.267	0.129
Cr	0.31	1.509	0.254
Mo	0.15	1.249	0.233
Cu	0.00	1.000	0.000

D_I = 2.912 inches, S_{D_I} = 1.27 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.000	57.2	1.13	56.0
4	1.066	0.064	53.9	3.43	53.0
8	1.430	0.175	40.6	5.23	44.5
12	1.700	0.134	33.9	2.81	38.0
16	1.904	0.147	30.2	2.45	33.0
20	2.075	0.173	27.8	2.42	30.0
24	2.225	0.200	25.9	2.40	29.0
28	2.348	0.207	24.6	2.27	28.0
32	2.408	0.217	24.0	2.25	27.0

[1] Steel composition and grain size, measured Jominy hardness were cited from Tables I and II of (2).

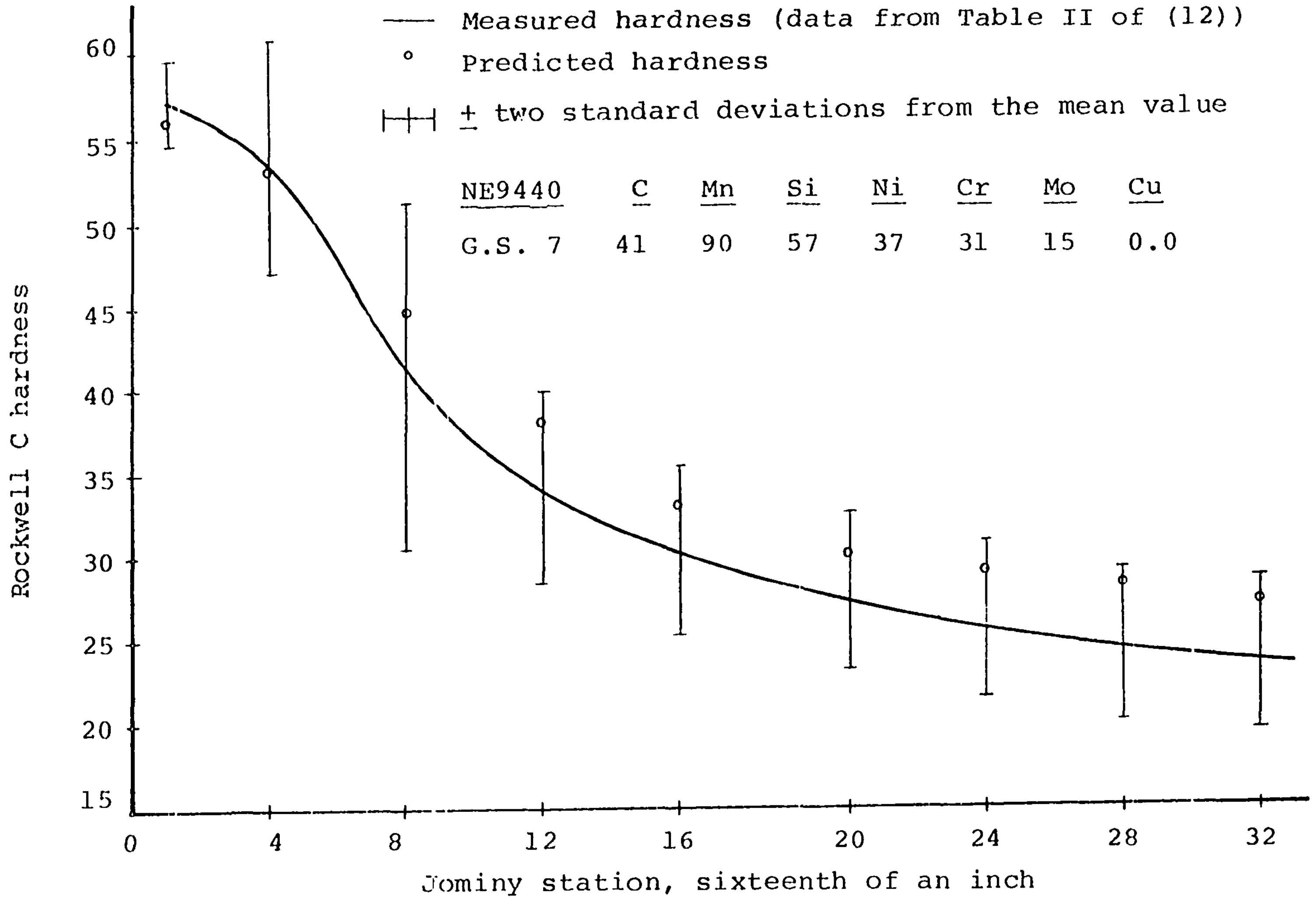

Figure 53. Jominy hardness curve of steel NE9440

Steel: A1340[1] (Figure 54)

G.S.	7	f	s_f
C	0.40	0.616	0.045
Mn	1.77	3.201	0.186
Si	0.19	1.047	0.091
Ni	0.10	1.075	0.129
Cr	0.08	1.131	0.254
Mo	0.02	1.032	0.233
Cu	0.00	1.000	0.000

D_I = 2.592 inches, s_{D_I} = 1.143 inches

J	I.H./D.H.	$s_{I.H./D.H.}$	Q_P	s_{Q_P}	Q_M
1	1.000	0.000	56.7	1.13	57.0
4	1.102	0.064	51.6	3.18	47.0
8	1.566	0.230	37.0	5.80	33.0
12	1.859	0.165	30.8	2.86	28.0
16	2.076	0.175	27.5	2.43	24.0
20	2.254	0.201	25.4	2.37	22.0
24	2.401	0.225	23.8	2.34	21.0
28	2.522	0.234	22.7	2.21	20.0
32	2.579	0.243	22.2	2.19	20.0

[1]Steel composition, grain size and measured Jominy hardness were cited from Table II of (2).

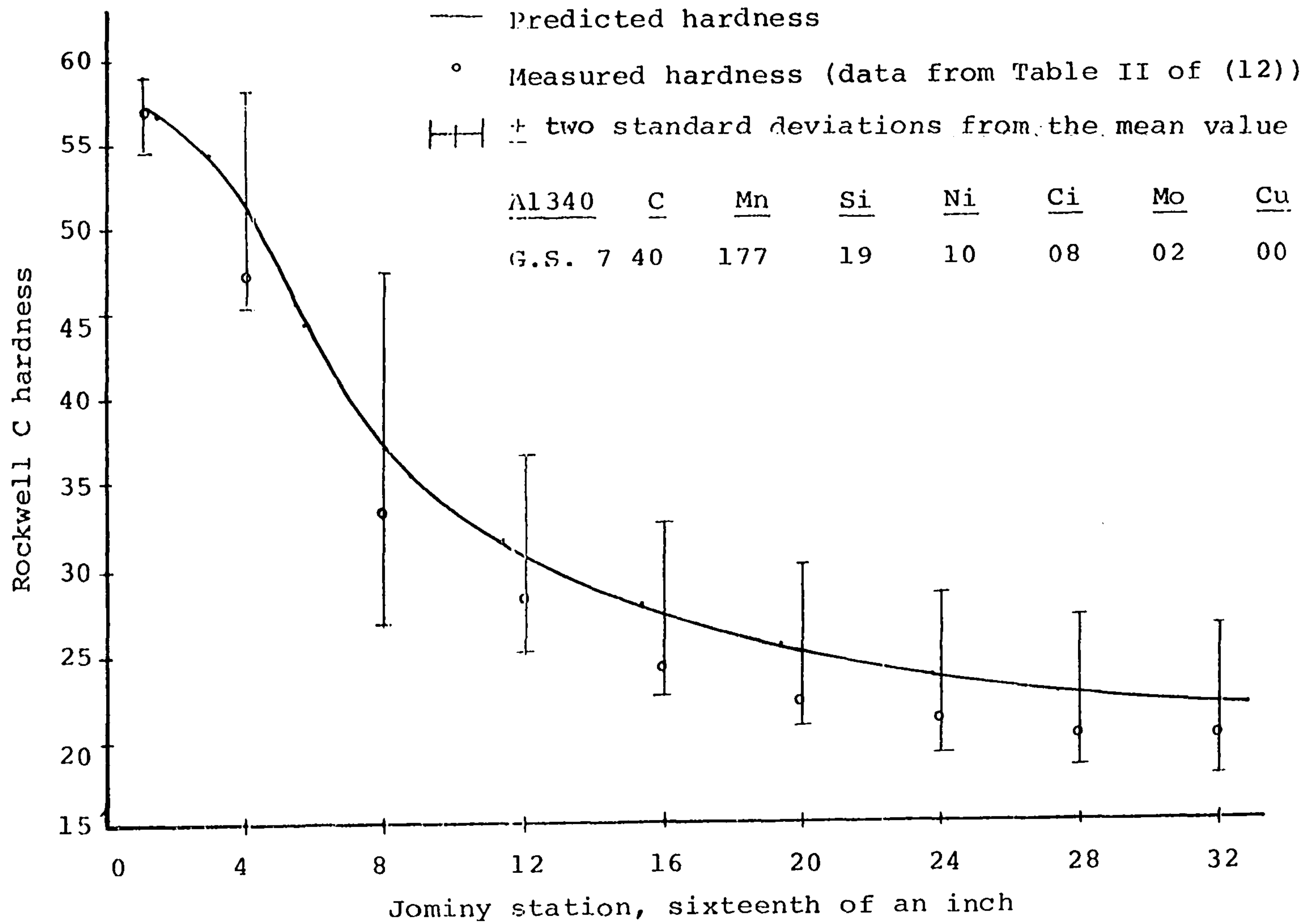

Figure 54. Jominy hardness curve of steel A1340

Steel: NE9630[1] (Figure 55)

G.S.	7	f	S_f
C	0.31	0.543	0.040
Mn	1.47	2.577	0.186
Si	0.50	1.143	0.091
Ni	0.003	1.023	0.129
Cr	0.52	1.857	0.254
Mo	0.00	1.000	0.000
Cu	0.00	1.000	0.000

D_I = 3.035 inches, S_{D_I} = 1.101 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.000	51.6	1.13	52.0
4	1.057	0.064	49.0	3.17	48.5
8	1.387	0.157	37.7	4.50	40.5
12	1.644	0.124	31.6	2.51	34.0
16	1.845	0.137	28.1	2.22	30.0
20	2.013	0.163	25.8	2.20	27.0
24	2.164	0.187	24.0	2.19	26.0
28	2.286	0.198	22.8	2.07	25.0
32	2.348	0.207	22.2	2.06	24.5

[1]Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of (2).

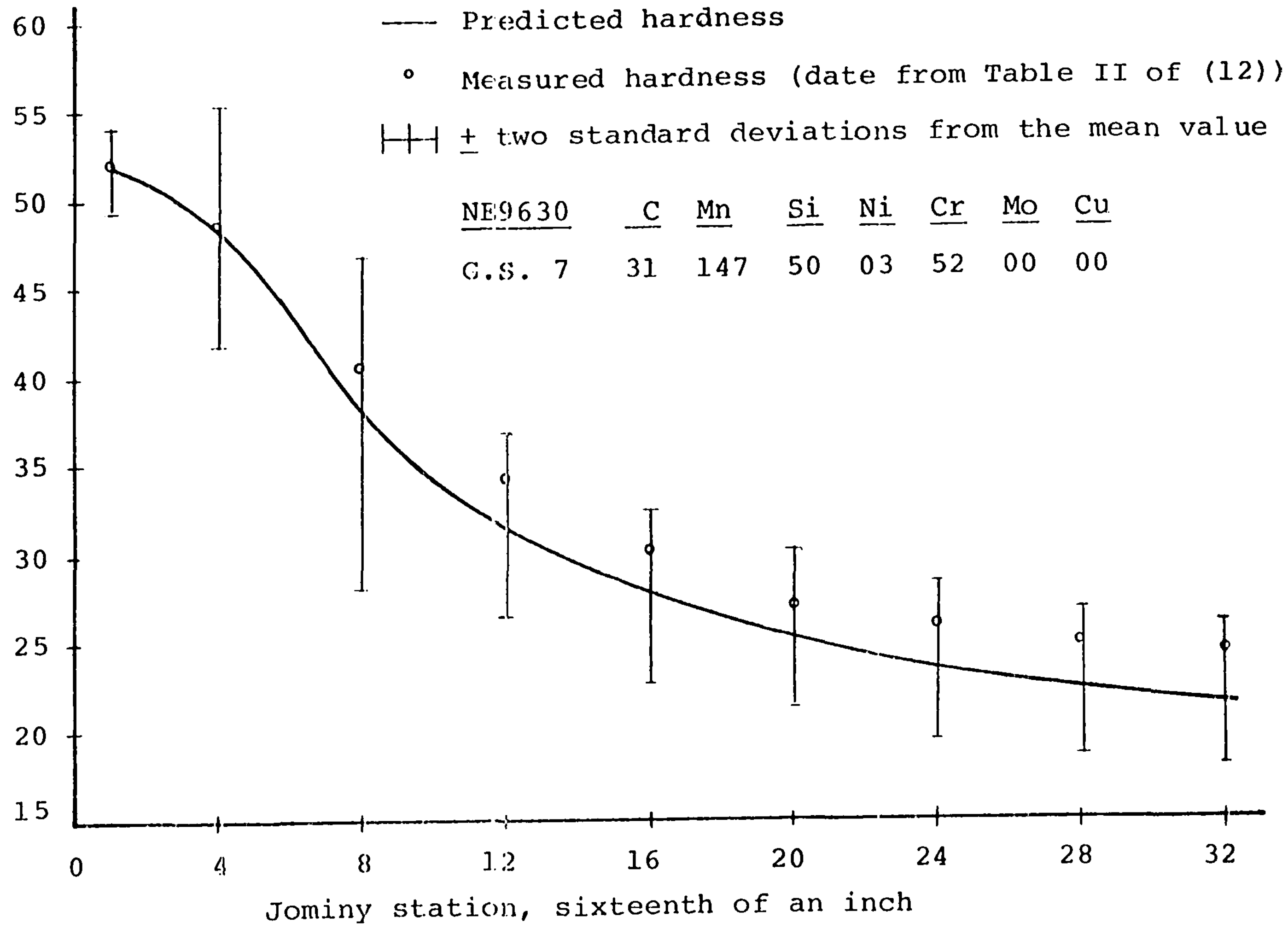

Figure 55. Jominy hardness curve of steel NE9630

Steel: NE9640[1] (Figure 56)

G.S.	8	f	S_f
C	0.38	0.558	0.048
Mn	1.48	2.596	0.186
Si	0.55	1.160	0.091
Ni	0.03	1.023	0.129
Cr	0.56	1.923	0.254
Mo	0.01	1.016	0.233
Cu	0.00	1.000	0.000

D_I = 3.357 inches, S_{D_I} = 1.481 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.000	55.7	1.13	56.0
4	1.039	0.064	53.8	3.51	53.0
8	1.293	0.119	43.4	4.19	46.0
12	1.522	0.100	36.7	2.56	39.0
16	1.709	0.115	32.7	2.33	33.5
20	1.800	0.139	30.0	2.35	31.0
24	2.017	0.163	27.8	2.36	30.0
28	2.138	0.175	26.2	2.25	29.0
32	2.201	0.185	25.5	2.24	28.0

[1]Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of (2).

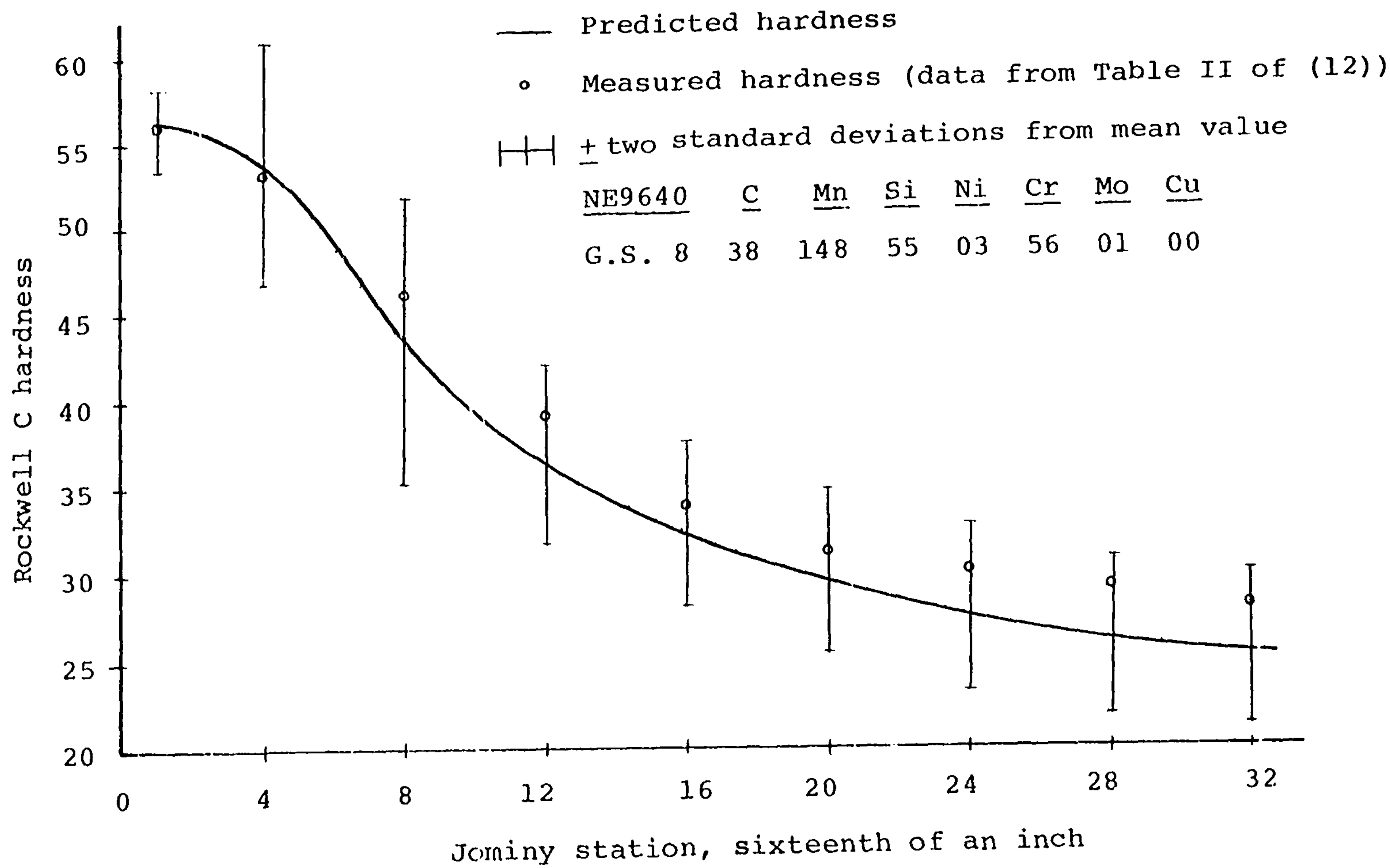

Figure 56. Jominy hardness curve of steel NE9640

Steel: NE8630[1] (Figure 57)

G.S.	8	f	S_f
C	0.34	0.527	0.045
Mn	0.88	1.648	0.186
Si	0.33	1.087	0.091
Ni	0.54	1.378	0.129
Cr	0.51	1.840	0.254
Mo	0.22	1.374	0.233
Cu	0.00	1.000	0.000

D_I = 3.294 inches S_{D_I} = 1.441 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.000	53.4	1.13	54.0
4	1.043	0.064	51.4	3.36	49.5
8	1.310	0.126	41.2	4.15	42.0
12	1.544	0.104	34.8	2.50	36.0
16	.734	0.119	30.9	2.26	31.0
20	1.895	0.144	28.4	2.27	28.5
24	2.044	0.168	26.3	2.27	27.0
28	2.166	0.179	24.8	2.16	26.5
32	2.229	0.189	24.1	2.15	26.0

[1] Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of (2).

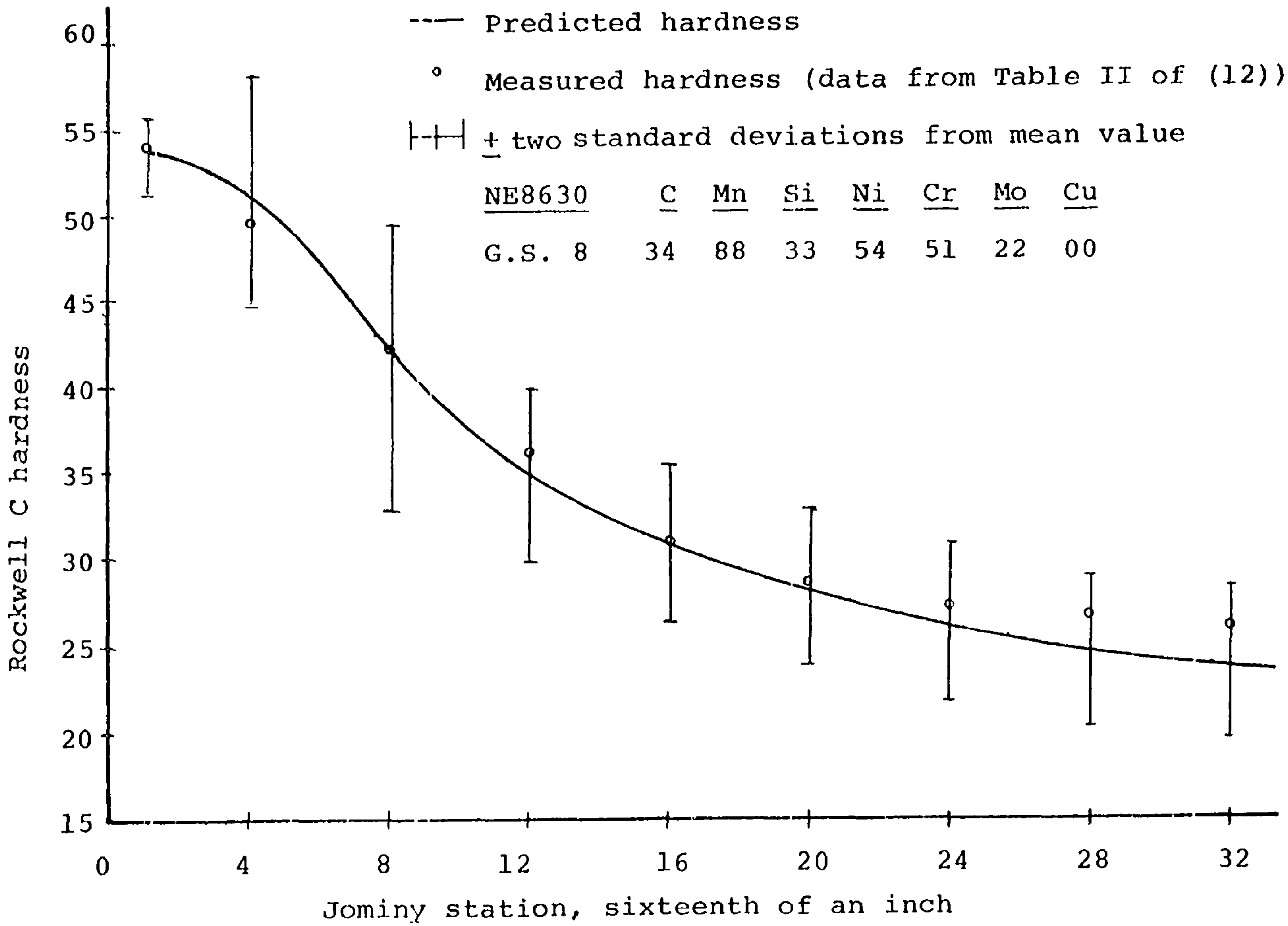

Figure 57. Jominy hardness curve of steel NE8630

Steel: NE8630[1] (Figure 58)

G.S.	7	f	S_f
C	0.33	0.560	0.041
Mn	0.87	1.636	0.186
Si	0.28	1.072	0.091
Ni	0.48	1.340	0.129
Cr	0.50	1.824	0.254
Mo	0.20	1.338	0.233
Cu	0.00	1.000	0.000

D_I = 3.210 inches, S_{D_I} = 1.398 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.000	2.8	1.13	52.0
4	1.047	0.064	50.6	3.30	49.0
8	1.333	0.135	40.0	4.27	41.5
12	1.574	0.110	33.7	2.50	35.0
16	1.768	0.125	30.0	2.25	30.0
20	1.932	0.150	27.5	2.25	27.5
24	2.081	0.174	25.6	2.25	26.5
28	2.204	0.185	24.1	2.13	26.0
32	2.266	0.195	23.5	2.12	25.0

[1]Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of (2).

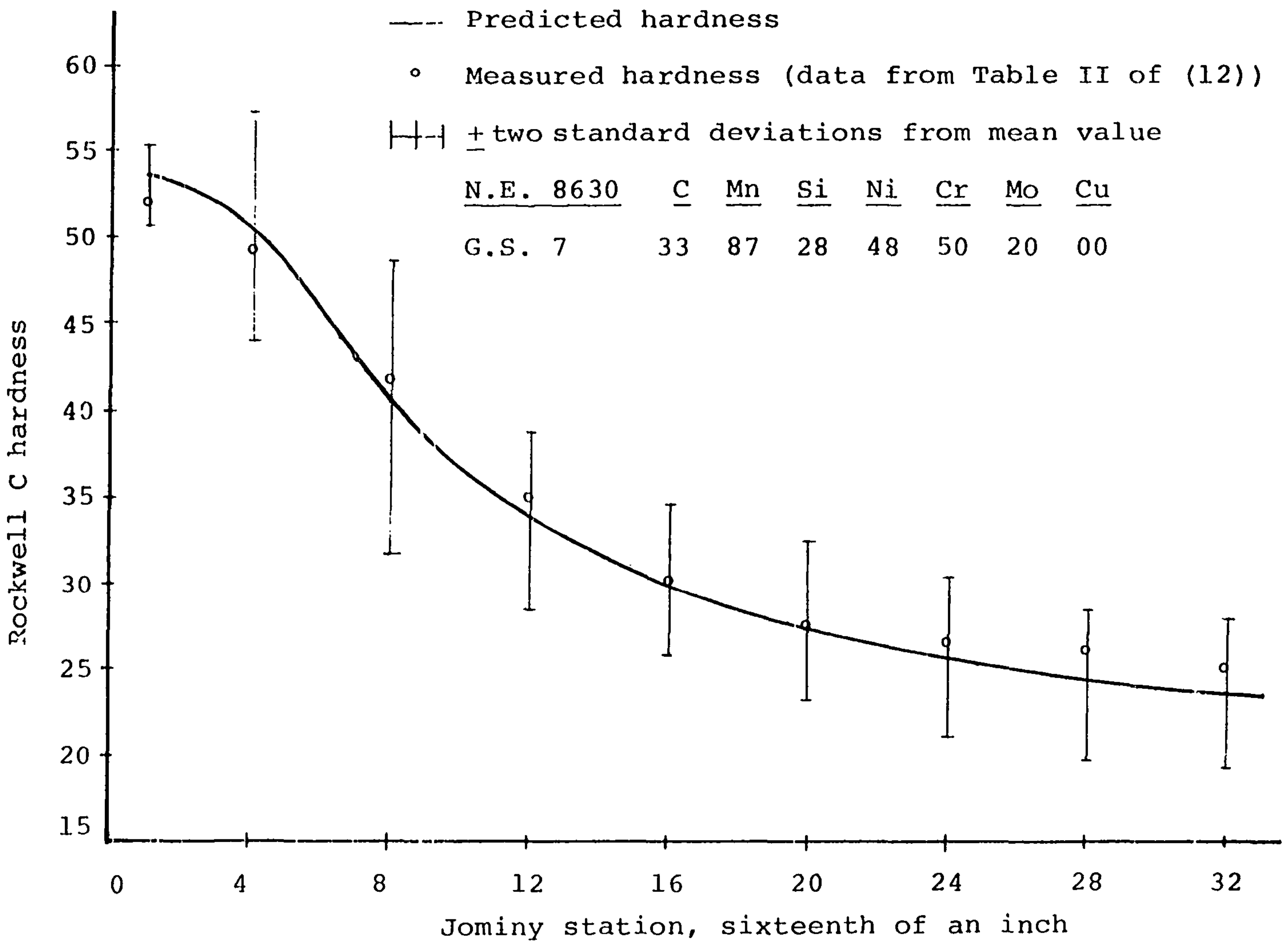

Figure 58. Jominy hardness curve of steel NE8630

Steel: NE8630[1] (Figure 59)

G.S.	6	f	S_f
C	0.33	0.604	0.038
Mn	0.88	1.648	0.168
Si	0.28	1.072	0.091
Ni	0.56	1.391	0.129
Cr	0.57	1.940	0.254
Mo	0.25	1.430	0.233
Cu	0.00	1.000	0.000

D_I = 4.121 inches, S_{D_I} = 1.783 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.000	52.8	1.13	53.0
4	1.024	0.023	51.6	1.60	52.5
8	1.152	0.062	46.0	2.67	47.5
12	1.318	0.061	40.2	2.06	42.0
16	1.468	0.076	36.1	2.04	37.5
20	1.601	0.096	33.1	2.14	35.0
24	1.738	0.118	30.5	2.21	33.0
28	1.852	0.131	28.7	2.15	32.0
32	1.914	0.141	27.8	2.15	31.0

[1] Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of (2).

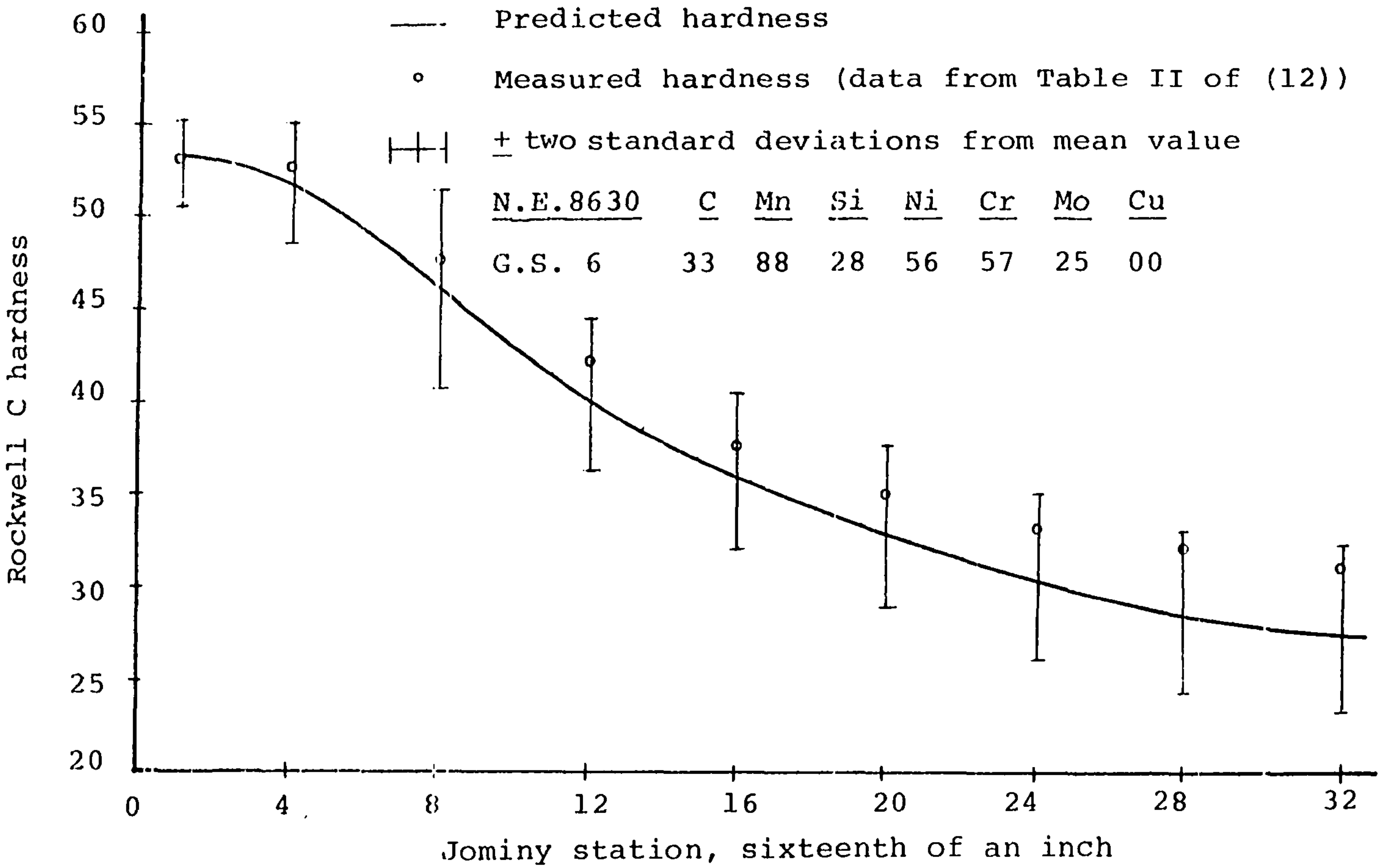

Figure 59. Jominy hardness curve of steel NE8630

Steel: NE8630[1] (Figure 60)

G.S.	7	f	S_f
C	0.29	0.525	0.039
Mn	0.82	1.576	0.186
Si	0.28	1.072	0.091
Ni	0.50	1.353	0.129
Cr	0.52	1.859	0.254
Mo	0.21	1.356	0.233
Cu	0.00	1.000	0.000

D_I = 3.021 inches, S_{D_I} = 1.315 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.000	50.4	1.13	51.0
4	1.058	0.064	47.8	3.10	47.0
8	1.392	0.159	36.7	4.43	39.0
12	1.650	0.125	30.7	2.46	32.0
16	1.852	0.138	27.4	2.17	28.0
20	2.020	0.164	25.1	2.15	25.5
24	2.171	0.188	23.4	2.14	24.5
28	2.293	0.199	22.2	2.03	24.0
32	2.355	0.208	21.6	2.01	23.0

[1] Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of [(2)].

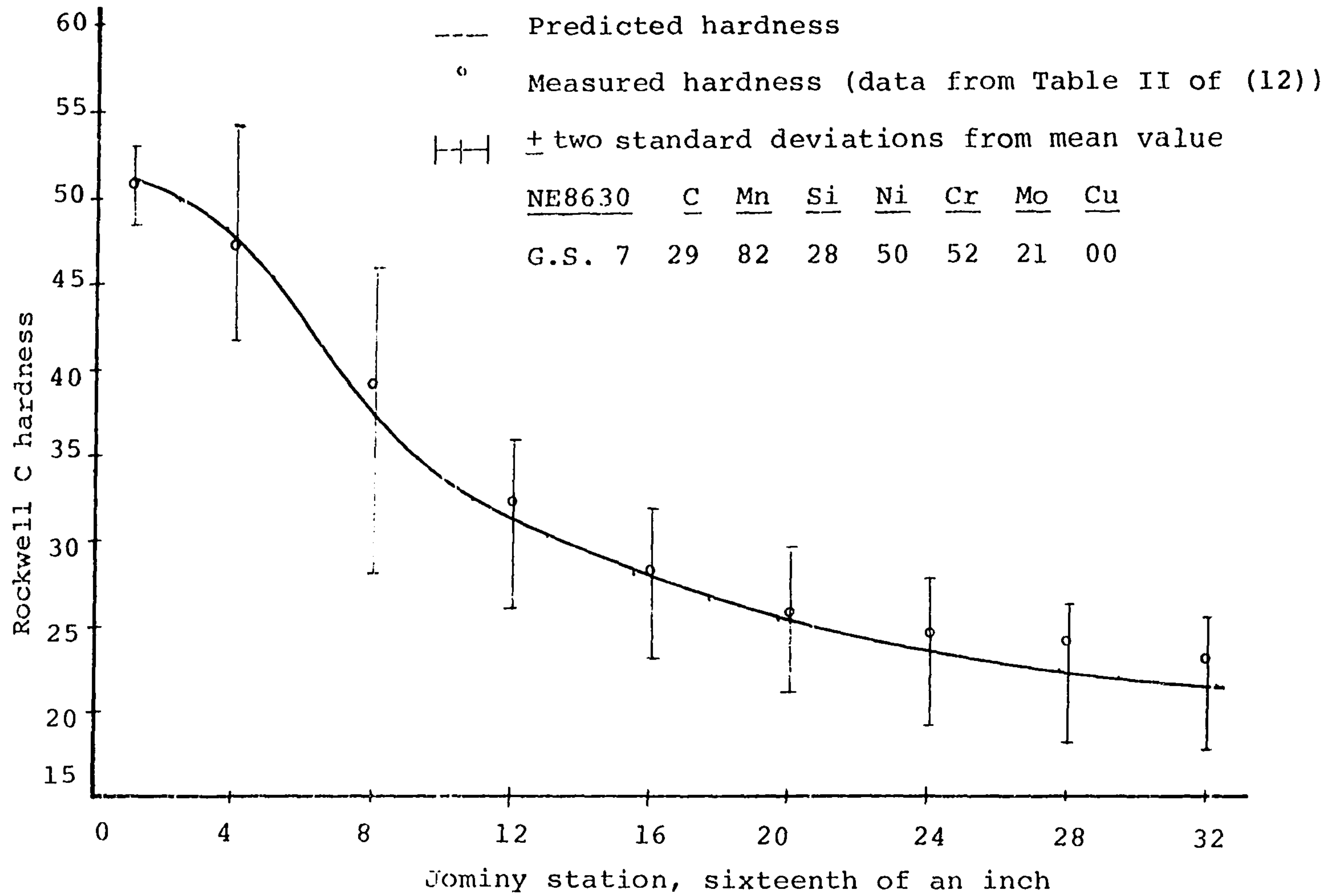

Figure 60. Jominy hardness curve NE8630

Steel: NE8630[1] (Figure 61)

G.S.	8	f	S_f
C	0.27	0.470	0.040
Mn	0.78	1.530	0.186
Si	0.28	1.072	0.091
Ni	0.40	1.287	0.129
Cr	0.47	1.774	0.254
Mo	0.18	1.302	0.233
Cu	0.00	1.000	0.000

D_I = 2.292 inches, S_{D_I} = 1.004 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.000	0.000	49.2	1.13	50.0
4	1.170	0.0637	42.2	2.51	43.0
8	1.733	0.298	29.3	5.46	32.0
12	2.044	0.201	24.3	2.51	26.0
16	2.267	0.206	21.9	2.10	23.0
20	2.449	0.233	20.3	2.03	21.0
24	2.589	0.255	19.2	2.00	20.0
28	2.705	0.262	18.4	1.88	19.0
32	2.758	0.270	18.0	1.86	19.0

[1] Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of (2).

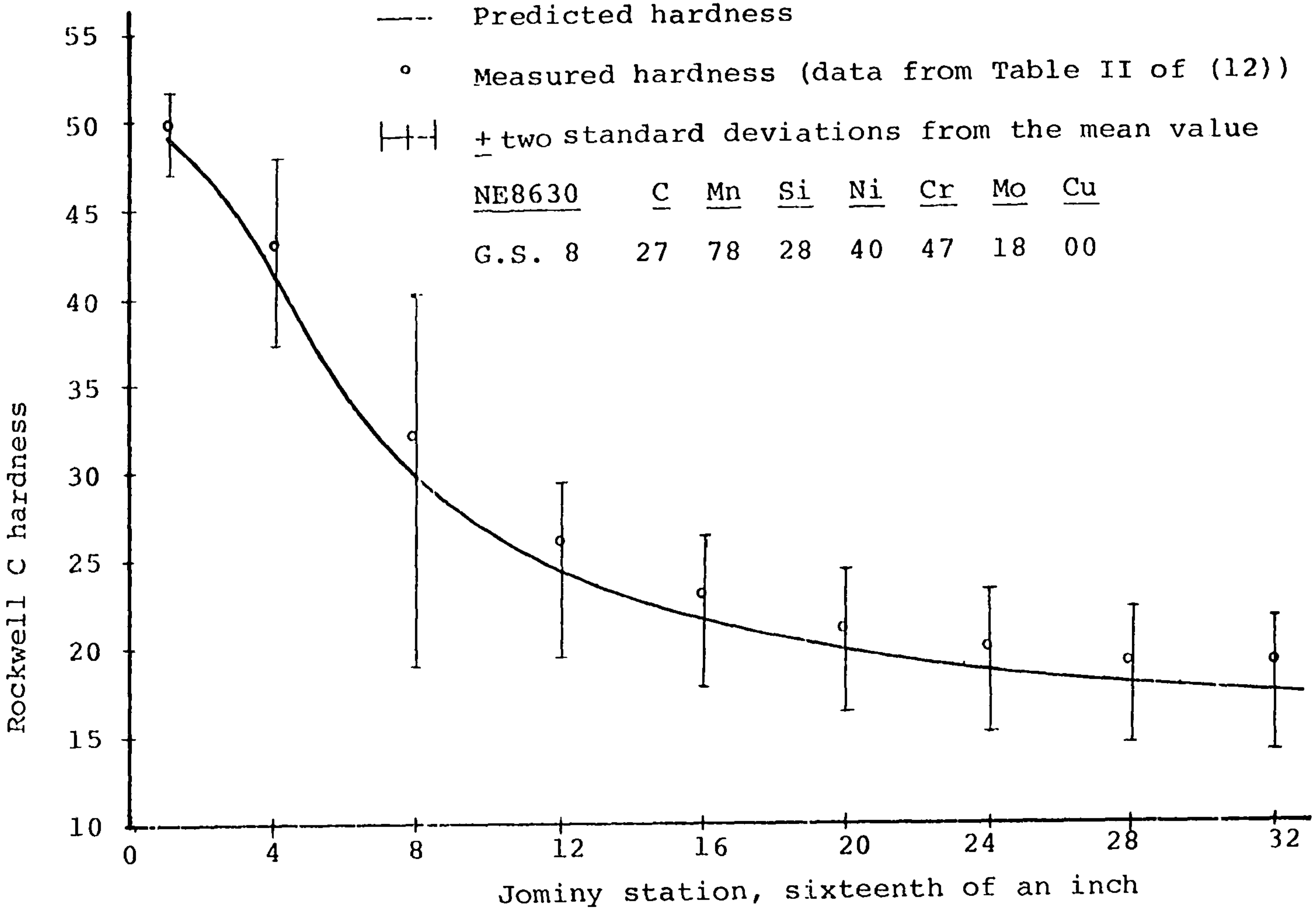

Figure 61. Jominy hardness curve of steel NE8630

Steel: NE8720[1] (Figure 62)

G.S.	7	f	s_f
C	0.20	0.436	0.032
Mn	0.75	1.497	0.186
Si	0.24	1.060	0.091
Ni	0.58	1.404	0.129
Cr	0.42	1.691	0.254
Mo	0.27	1.467	0.233
Cu	0.00	1.000	0.000

D_I = 2.410 inches, s_{D_I} = 1.048 inches

J	I.H./D.H.	$s_{I.H./D.H.}$	Q_P	s_{Q_P}	Q_M
1	1.000	0.000	44.9	1.13	46.0
4	1.138	0.064	39.5	2.45	40.5
8	1.662	0.269	27.8	4.85	31.5
12	1.967	0.186	23.0	2.30	26.0
16	2.188	0.193	20.7	1.94	22.0
20	2.369	0.220	19.1	1.88	21.0
24	2.512	0.243	18.0	1.85	19.5
28	2.631	0.251	17.2	1.74	19.0
32	2.686	0.259	16.9	1.73	18.5

[1]Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of (2).

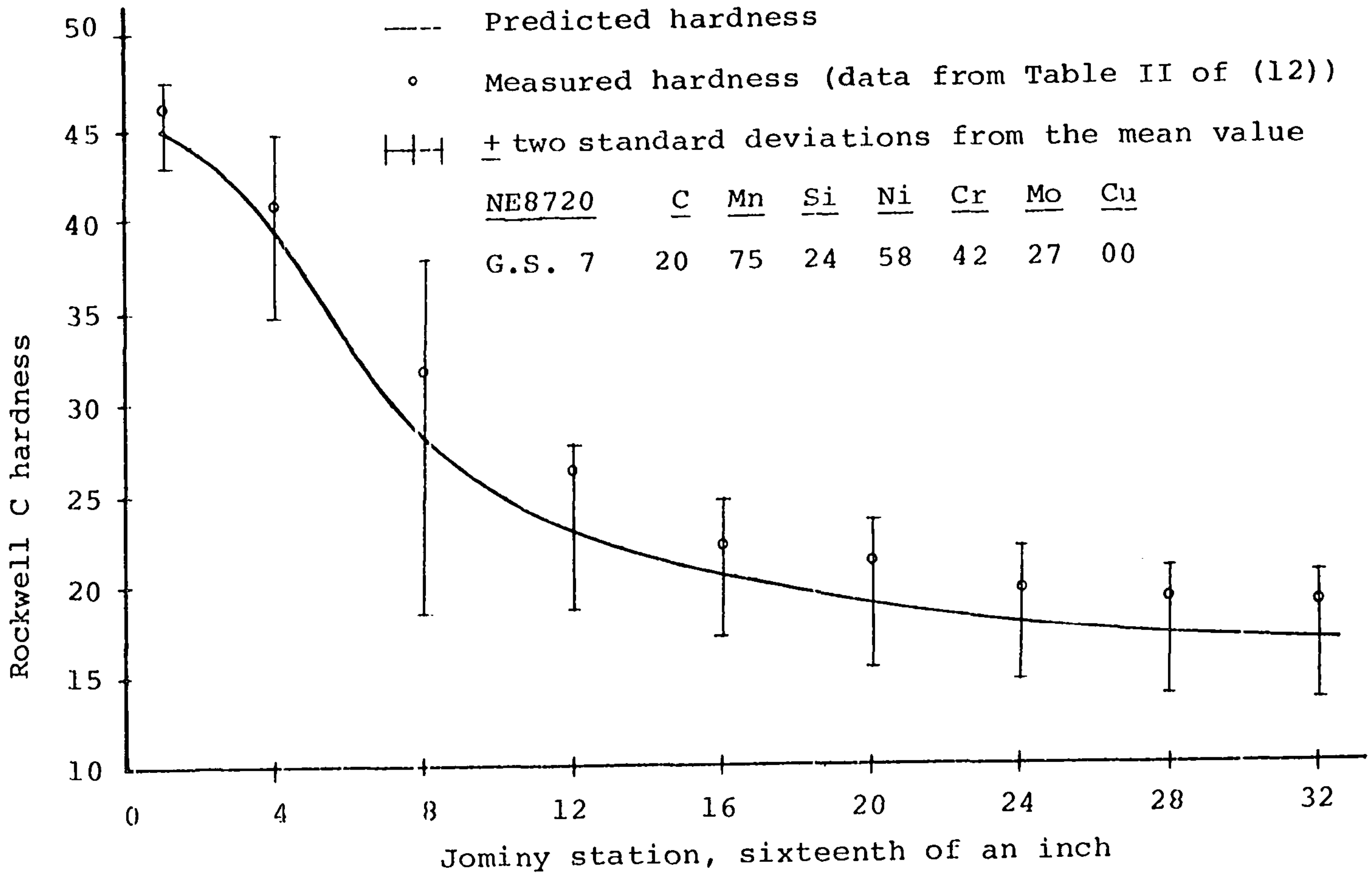

Figure 62. Jominy hardness curve of steel NE8720

Steel: NE8270[1] (Figure 63)

G.S.	8	f	S_f
C	0.19	0.394	0.034
Mn	0.73	1.475	0.186
Si	0.25	1.063	0.091
Ni	0.6	1.416	0.129
Cr	0.45	1.741	0.254
Mo	0.32	1.563	0.233
Cu	0.0	1.000	0.000

D_I = 2.383 inches, S_{D_I} = 1.040 inches

J	I.H./D.H.	$S_{I.H./D.H.}$	Q_P	S_{Q_P}	Q_M
1	1.0000	0.000	44.3	1.13	45.5
4	1.145	0.064	38.8	2.39	38.5
8	1.678	0.275	27.1	4.82	28.5
12	1.984	0.189	22.5	2.28	24
16	2.205	0.196	20.2	1.91	20
20	2.387	0.223	18.7	1.85	19
24	2.529	0.246	17.7	1.82	18
28	2.648	0.254	16.9	1.71	17
32	2.702	0.262	16.5	1.70	17

[1]Steel composition, grain size and measured Jominy hardness were cited from Tables I and II of (2).

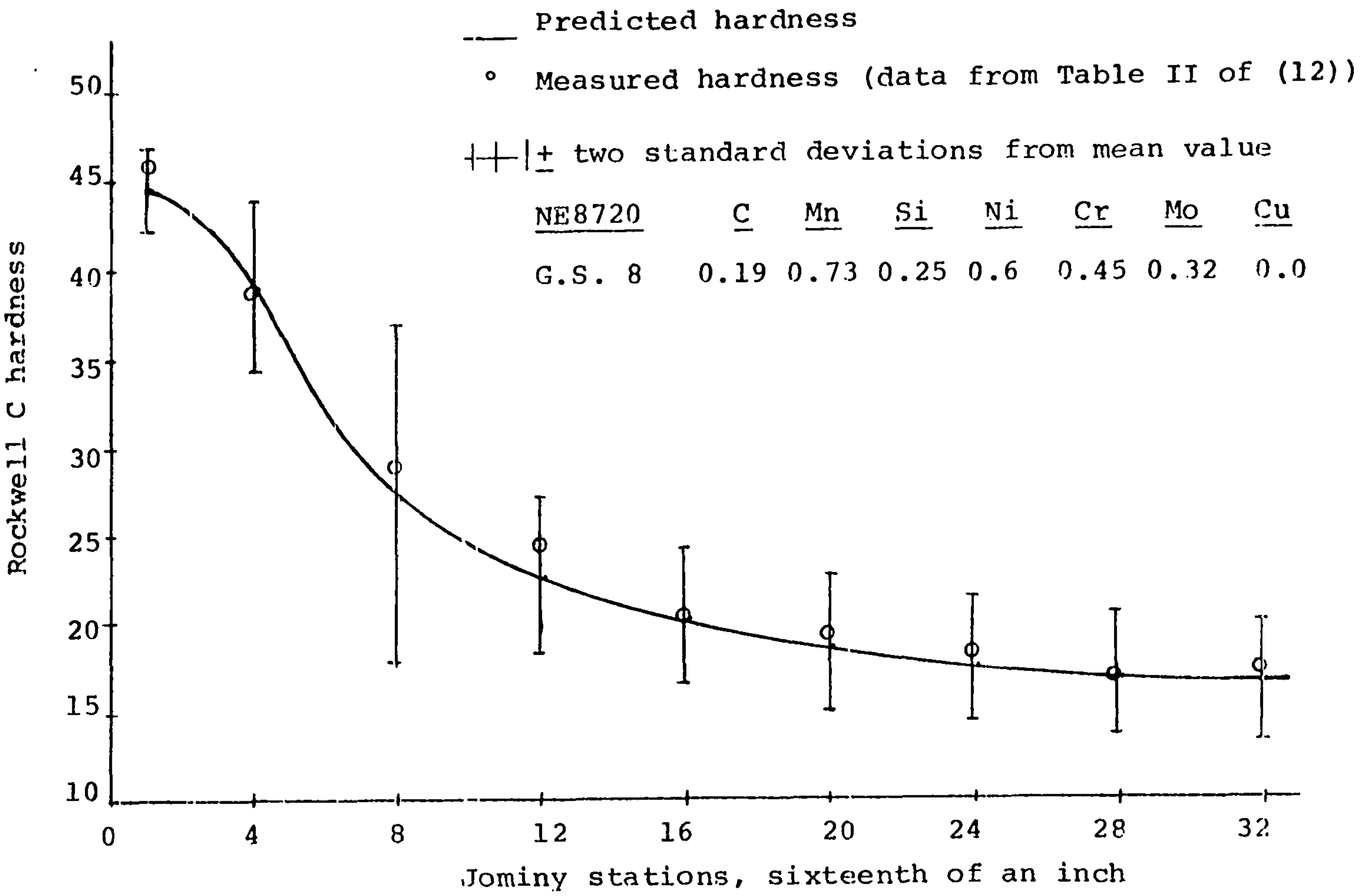

Figure 63. Jominy hardness curve of steel NE8720

The Iron and Steel Committee of the War Engineering Board and the Iron and Steel Division, General Standards Committee, of the Society of Automotive Engineering, Inc., and the Technical Committee on Alloy Steel of American Iron and Steel Institute have devised hardenability bands for commercial alloy steels (23).

Maximum and minimum hardness at different Jominy stations of several alloy commercial steels have been predicted and compared the results with the devised hardenability band values. The maximum and minimum predicted hardness are based respectively on the upper bound and lower bound chemical compositions of the alloy steel. Commercial alloy steels usually have fine grain size, an ASTM No. 8 grain size have been assumed in the prediction.

Predicted and devised hardenability band of steel 8650H: (Figure 64)

Jominy Station	Hardenability Band[1]		Predicted Hardness			
			Max.		Min.	
	Max.	Min.	Q_P	S_{Q_P}	Q_P	S_{Q_P}
1	65.0	58.0	62.5	1.13	59.6	1.13
4	65.0	55.0	62.1	1.81	51.1	2.97
8	62.0	48.0	60.8	1.29	35.5	6.60
12	58.0	41.0	57.5	1.37	29.4	3.02
16	56.0	38.0	53.9	1.56	26.5	2.52
20	51.0	32.5	50.7	1.80	24.5	2.44
24	49.0	31.0	47.3	2.06	23.2	2.40
28	47.0	30.0	44.6	2.15	22.2	2.25
32	6.0	29.5	43.1	2.22	21.8	2.24

Steel 8650H[1]	C	Mn	Si	Ni	Cr	Mo	Cu
ASTM G.S. 8	46/54	70/105	20/35	35/75	35/65	15/25	0.0

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[1] Steel composition and devised hardenability band were cited from Chart 24 of (23).

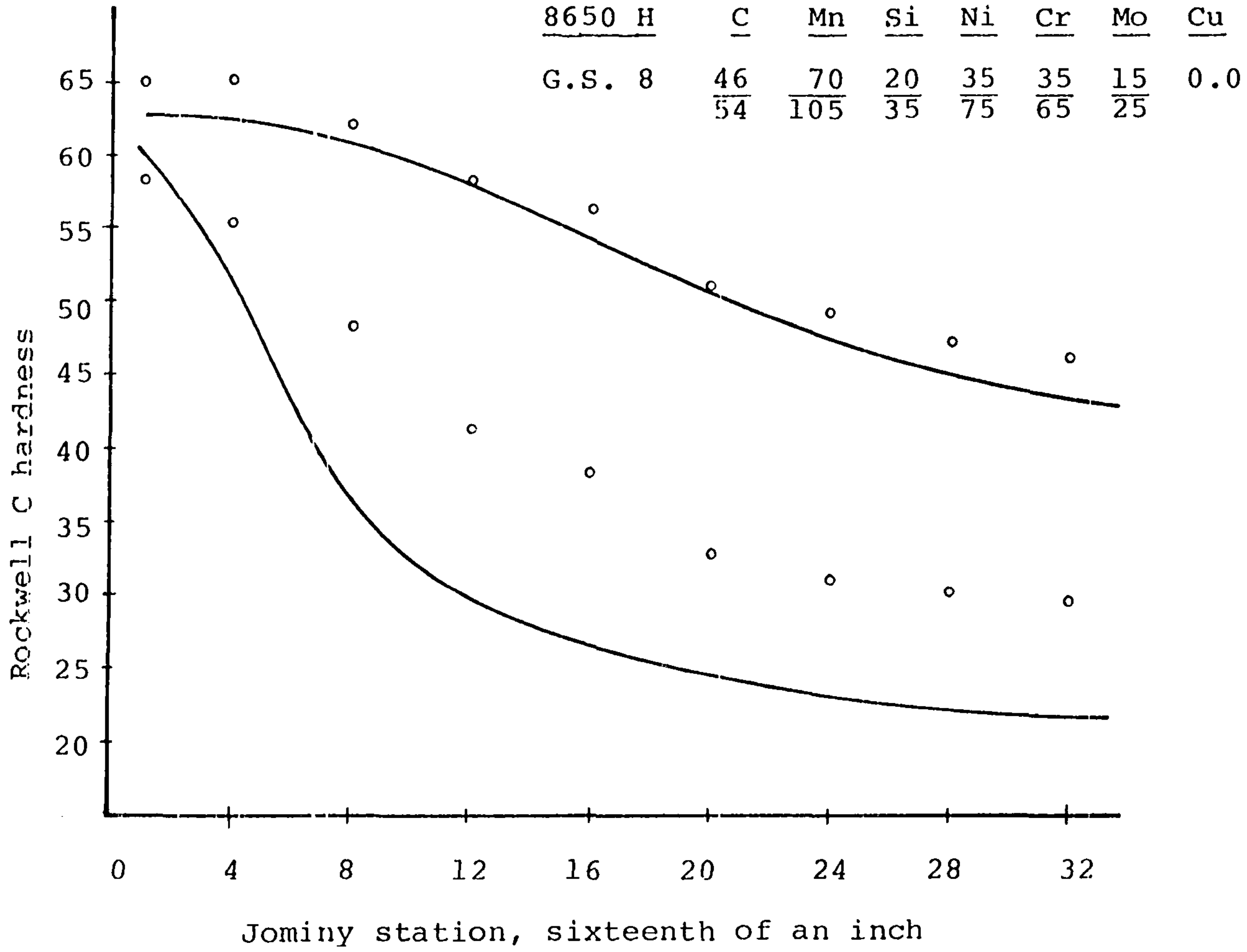

Figure 64. Hardenability band of steel 8650H

Predicted and devised hardenability band of steel 434021: (Figure 65)

Jominy Station	Hardenability Band[1]		Predicted Hardness			
			Max.		Min.	
	Max.	Min.	Q_P	S_{Q_P}	Q_P	S_{Q_P}
1	60.0	52.5	59.1	1.13	55.1	1.13
4	60.0	52.5	59.1	1.13	53.8	1.64
8	60.0	52.0	59.0	1.13	47.5	2.90
12	60.0	51.0	58.5	1.12	41.4	2.18
16	60.0	50.0	57.5	1.13	37.1	2.14
20	60.0	48.0	56.3	1.16	34.1	2.24
24	60.0	46.0	54.5	1.25	31.4	2.31
28	60.0	44.5	52.7	1.34	29.5	2.24
32	60.0	43.0	51.6	1.41	28.6	2.24

Steel 4340H[1]	C	Mn	Si	Ni	Cr	Mo	Cu
ASTM G.S. 8	37/45	60/95	20/35	150/200	65/95	20/30	0.00

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[1]Steel composition and devised hardenability band were cited from Chart 1 of (23).

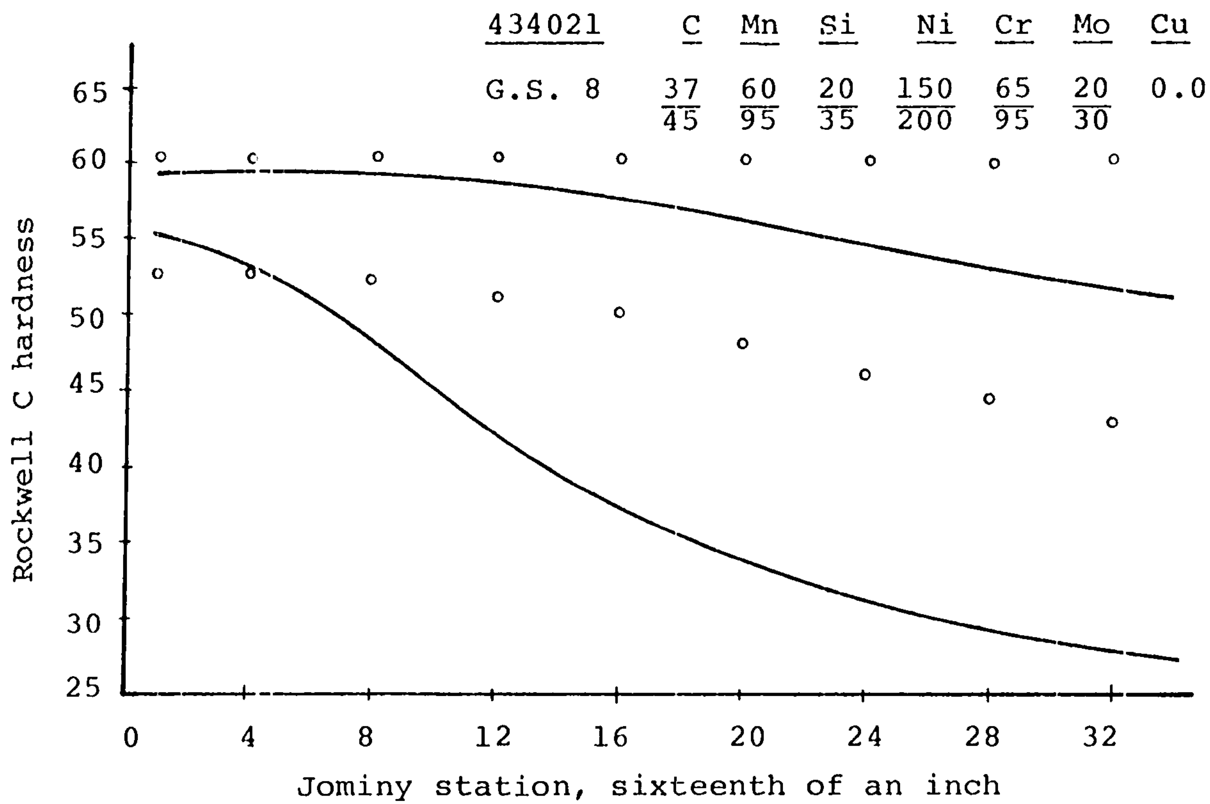

Figure 65. Hardenability band of steel 434021

Predicted and devised hardenability band of steel 8750H: (Figure 66)

Jominy Station	Hardenability Band[1] Max.	Min.	Predicted Hardness Max. Q_p	S_{Q_p}	Min. Q_p	S_{Q_p}
1	65.0	58.0	62.5	1.13	9.6	1.13
4	65.0	56.5	62.3	1.82	53.0	3.18
8	64.0	51.0	61.3	1.21	37.4	6.34
12	62.0	43.5	58.6	1.28	31.0	3.01
16	60.0	38.5	55.4	1.44	27.8	2.53
20	57.5	35.0	52.5	1.66	25.7	2.47
24	55.5	33.5	49.2	1.92	24.2	2.43
28	54.5	32.5	46.5	2.03	23.1	2.29
32	53.5	32.5	45.0	2.12	22.6	2.27

Steel 8750H[1]	C	Mn	Si	Ni	Cr	Mo	Cu
ASTM G.S. 8	46/54	70/105	20/35	35/75	35/65	20/30	0.00

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[1] Steel composition and devised hardenability band (cited from Chart 37 of (23)).

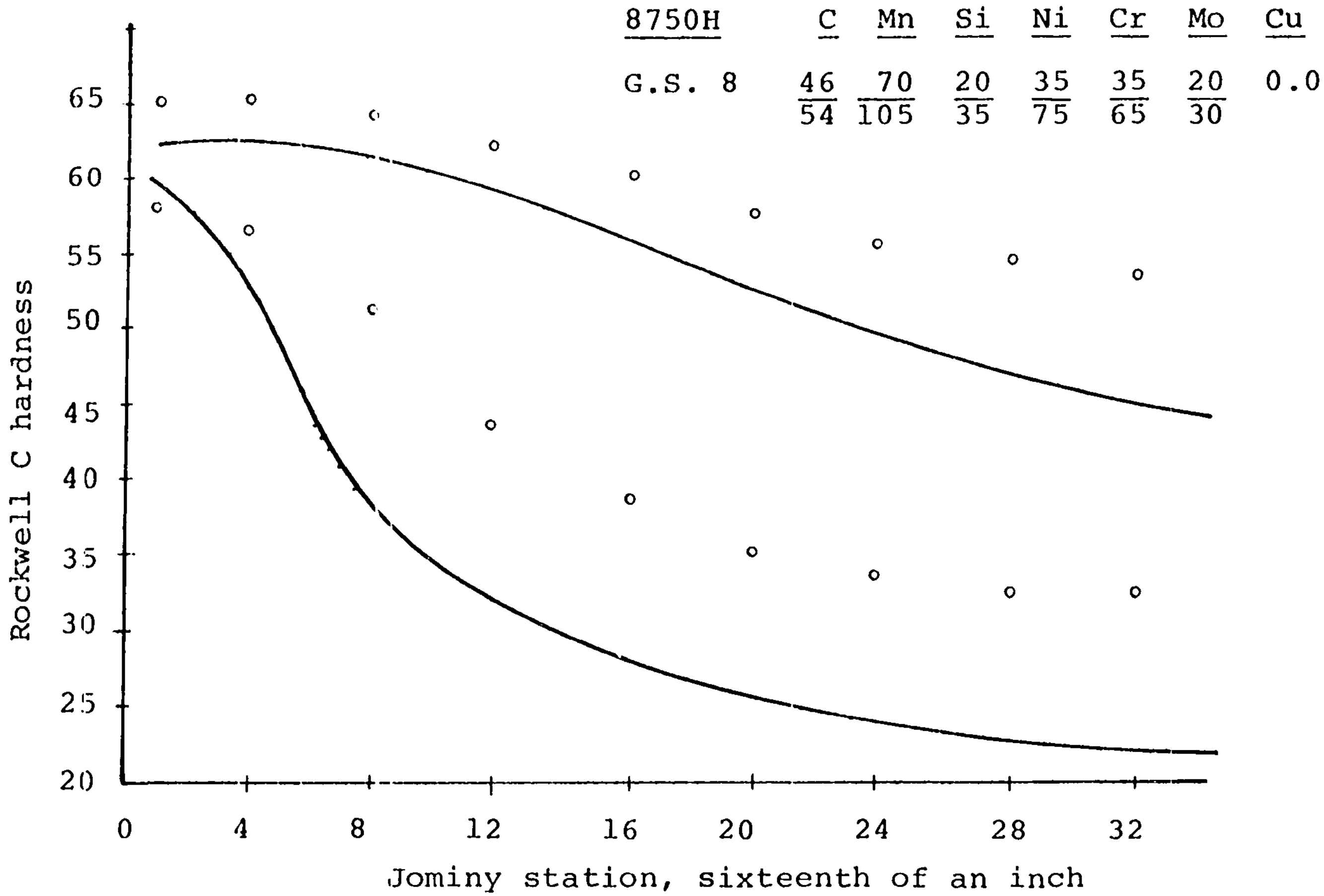

Figure 66. Hardenability band of steel 8750H

Predicted and devised hardenability band of steel 8622H: (Figure 67

Jominy Station	Hardenability Band[1]		Predicted Hardness			
			Max.		Min.	
	Max.	Min.	Q_P	S_{Q_P}	Q_P	S_{Q_P}
1	49.5	42.0	49.2	1.13	44.9	1.13
4	43.5	27.5	48.0	1.54	26.2	1.18
8	34.0	21.0	42.2	2.70	18.8	5.31
12	29.0	-	36.6	2.01	16.0	2.12
16	27.0	-	32.8	1.95	14.9	1.72
20	25.5	-	30.1	2.03	14.1	1.66
24	24.5	-	27.8	2.09	13.8	1.64
28	24.0	-	26.1	2.02	13.4	1.54
32	24.0	-	25.3	2.02	13.3	1.53

Steel 8622H[1]	C	Mn	Si	Ni	Cr	Mo	Cu
ASTM G.S. 8	20/27	60/95	20/35	35/75	35/65	15/25	0.00

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[1] Steel composition and devised hardenability band (cited from Chart 13 of (23)).

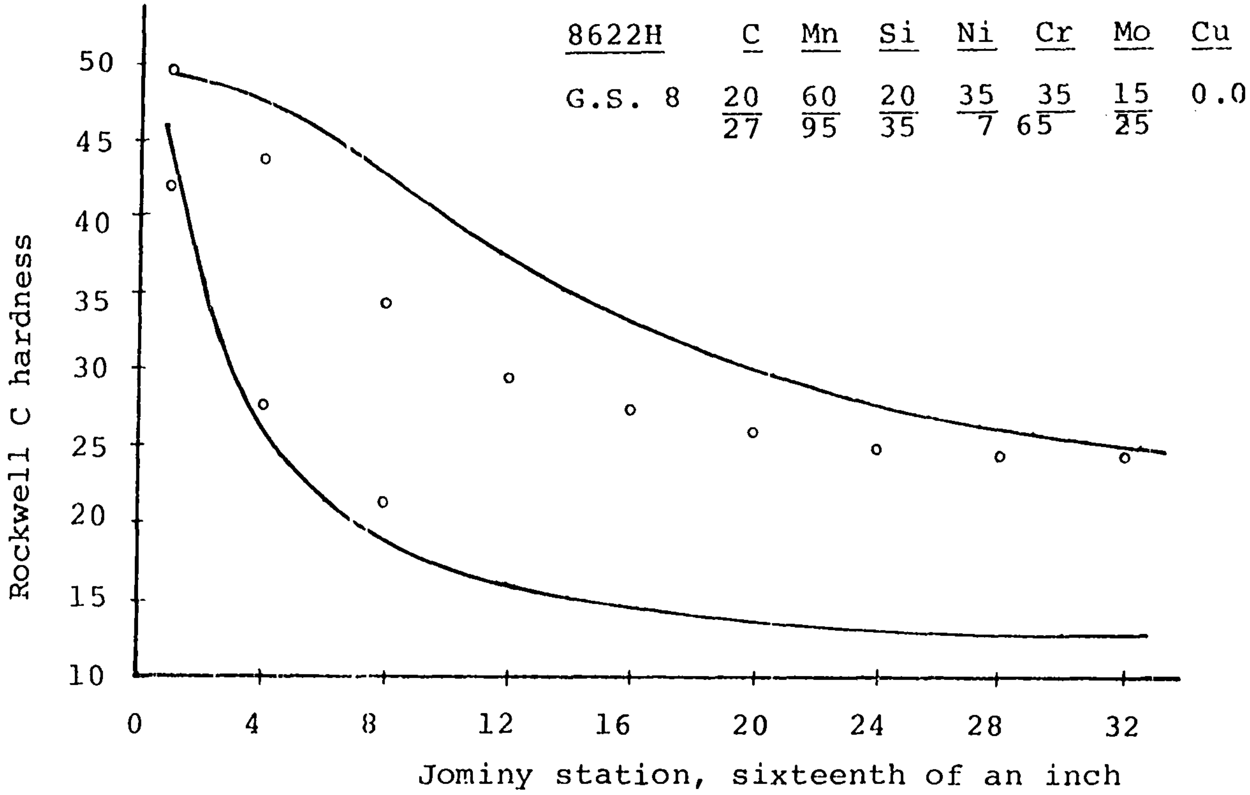

Figure 67. Hardenability band of steel 8622H

Predicted and devised hardenability band of steel 8722H:
(Figure 68)

Jominy Station	Hardenability Band[1]		Predicted Hardness			
			Max.		Min.	
	Max.	Min.	Q_P	S_{Q_P}	Q_P	S_{Q_P}
1	49.5	42.0	49.2	1.13	44.9	1.13
4	44.0	28.5	48.1	1.55	27.8	1.30
8	35.5	22.5	43.4	2.35	19.7	5.35
12	31.0	-	38.1	1.88	16.6	2.16
16	28.5	-	34.3	1.88	15.4	1.75
20	27.0	-	31.5	1.98	14.6	1.68
24	26.5	-	29.1	2.06	14.2	1.66
28	26.0	-	27.3	2.01	13.8	1.56
32	26.0	-	26.4	2.01	13.7	1.56

Steel 8722H[1]	C	Mn	Si	Ni	Cr	Mo	Cu
ASTM G.S. 8	20/27	60/95	20/35	35/75	35/65	20/30	0.00

---

[1] Steel composition and devised hardenability band (cited from Chart 26 of (23)).

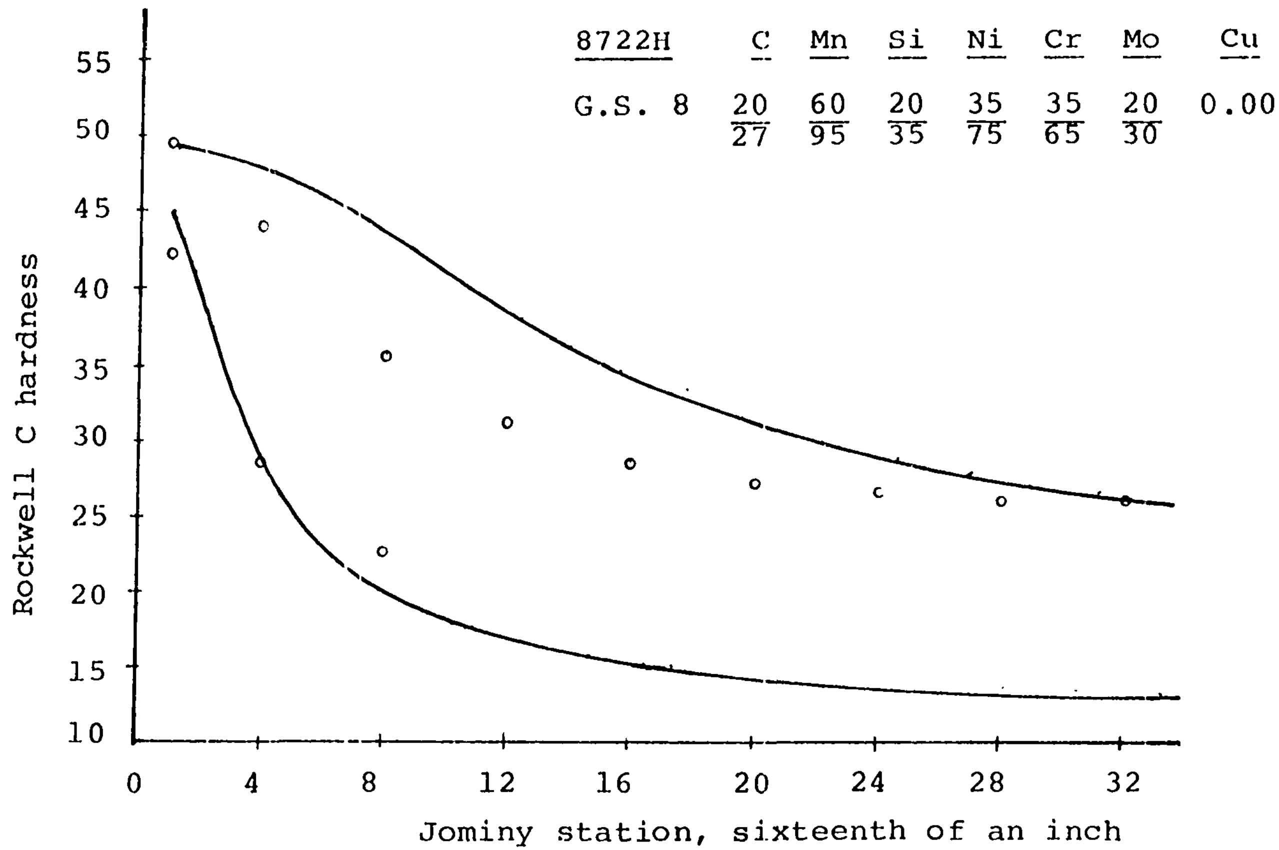

Figure 68. Hardenability band of steel 8722H

Predicted and devised hardenability band of steel 4150H: (Figure 69)

Jominy Station	Hardenability Band[1] Max.	Hardenability Band[1] Min.	Predicted Hardness Max. Q_p	Predicted Hardness Max. S_{Q_p}	Predicted Hardness Min. Q_p	Predicted Hardness Min. S_{Q_p}
1	65.0	58.0	62.5	1.13	59.6	1.13
4	65.0	57.0	62.4	1.83	57.0	3.67
8	64.0	53.0	61.5	1.18	45.0	4.82
12	63.0	47.0	59.1	1.24	37.9	2.81
16	62.0	42.0	56.2	1.38	33.7	2.51
20	61.0	39.0	53.4	1.59	30.9	2.51
24	60.0	37.0	50.2	1.85	28.7	2.52
28	59.0	36.0	47.5	1.97	27.1	2.39
32	58.0	35.5	46.0	2.06	26.4	2.38

Steel 4150H[1]	C	Mn	Si	Ni	Cr	Mo	Cu
ASTM G.S. 8	46/54	70/105	20/35	Mx. 25	80/115	15/25	0.00

---

[1]Steel composition and hardenability band (cited from Chart 9 of (23)).

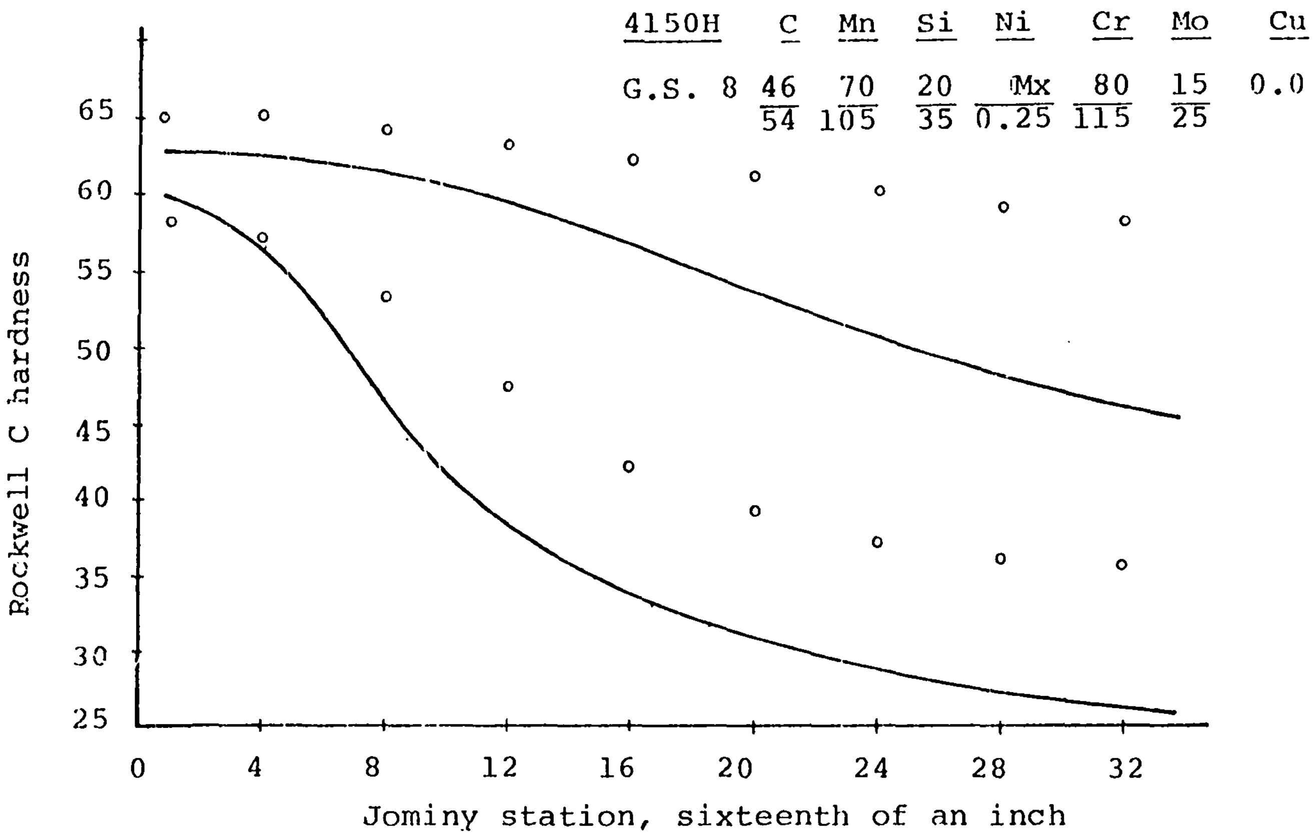

Figure 69. Hardenability band of steel 4150H

Predicted and devised hardenability band of steel 4130H: (Figure 70)

Jominy Station	Hardenability Band[1]		Predicted Hardness			
			Max.		Min.	
	Max.	Min.	Q_P	S_{Q_P}	Q_P	S_{Q_P}
1	54.0	46.5	53.4	1.13	49.2	1.13
4	52.0	39.0	52.1	1.61	37.8	2.05
8	43.5	30.0	45.7	2.94	26.0	5.77
12	37.5	26.0	39.6	2.16	21.6	2.48
16	33.5	23.0	35.5	2.10	19.7	2.04
20	31.5	21.0	32.5	2.19	18.4	1.96
24	30.5	20.0	30.0	2.25	17.6	1.93
28	30.0	-	28.2	2.18	16.9	1.81
32	29.5	-	27.3	2.18	16.7	1.80

Steel 4130H[1]	C	Mn	Si	Ni	Cr	Mo	Cu
ASTM G.S. 8	27/34	35/65	20/35	Mx.25	80/115	15/25	0.00

---

[1] Steel composition and hardenability band (cited from Chart 1 of (23)).

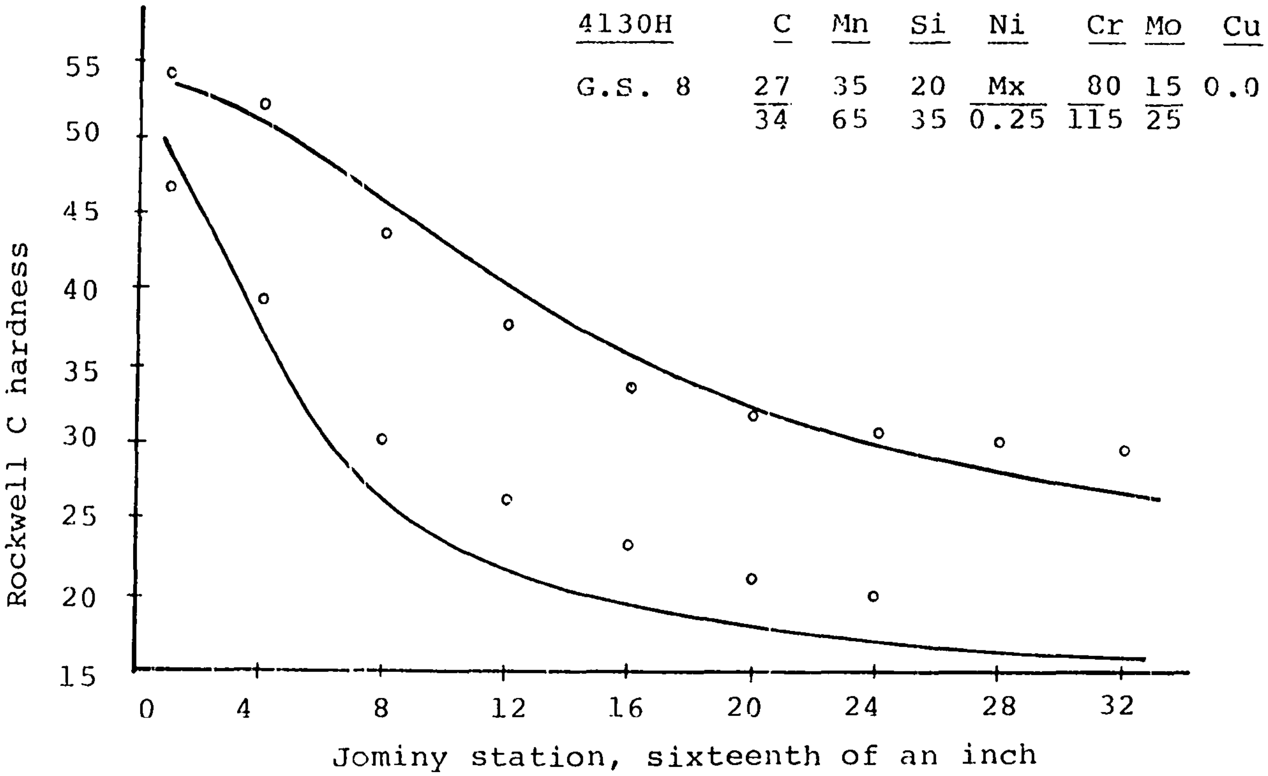

Figure 70. Hardenability band of steel 4130H

INCORPORATE THE METHODOLOGY INTO A COMPUTER PROGRAM THAT IS EASY FOR THE DESIGNER TO USE

A computer program has been developed to predict the mean and standard deviation of the Jominy hardness profile from chemical composition and grain size of the low alloy carbon steel. The computer program consists of four subroutines. Subroutine JOMINY, Subroutine RATIOP, Subroutine DIDEAL and Subroutine RANDOM, and have been listed and documented in the Appendix.

SUMMARY AND CONCLUSIONS

Based on Kramer's multiplying factors for alloy elements, the prediction of the Jominy hardness curve from chemical composition and grain size of low alloy carbon steels, agreed fairly well with the measured Jominy hardness data only for simple steels and steels with low alloy content. For complex steels and steel with high alloy content, the effect of each alloy element on hardenability of steel will not be independent of each other, especially when some carbide forming elements (such as nickel and molybdenum) are used together, they produce synergistic hardenability effects - that is the specific effect of each alloying element is larger than its effect when by itself in a composition. Steels of high hardenability usually contain large quantities of strong carbide forming elements, which are often not completely dissolved. The correlation between alloying elements on the hardenability of steels and the effect of heat treatment history and time on the carbide forming tendency of the alloying elements are necessary to investigate in order to have a good prediction of the Jominy hardness profile of complex and high alloy content steels.

The criterion used to determine the hardenability of low alloy carbon steels was the 50 percent martensite transformation point, and the other 50 percent transformation the bias may be different.

Recommendations

1. With the computer program available, a concerted effort should be made to compare as many predictions as possible for the classes of steels of interest to the user. A correlation can be made between predictions and tests at every principle station, and any bias of the mean for such classes can be assessed and the mean prediction adjusted.

2. With the computer program available, it is recommended that the addition method of Crafts and Lamont (11) for predicting tempered hardness be investigated and the data upon which it is based examined, in order that any bias in the prediction and the dispersion in the prediction of tempered hardness can be quantitatively assessed.

3. In estimating critical diameter, the point of interest is the size of bar in which the structure is 50 percent martensite at the center. In order to determine the critical diameter of low alloy steel by Jominy test, it would be better to locate the distance from the quench end of the Jominy bar, which has a 50 percent martensite microstructure and measure the corresponding hardness, instead of predicting the 50 percent martensite hardness from carbon content of the steel alone.

4. For complex steels and steel with high alloy

content, the effect of each alloy element on hardenability of steel will not be independent of others, especially when some carbide-forming elements (such as nickel and molybdenum) are used together. They produce synergistic hardenability effects - that is the specific effect of each alloying element is larger than its effect when by itself in a composition. Steels of high hardenability usually contain large quantities of strong carbide forming elements, which are often not completely dissolved. The correlation between alloying elements on the hardenability of steels and the effect of heat treatment history and time on the carbide forming tendency of the alloying elements must be investigated in order to have a good prediction of the Jominy hardness profile of complex and high alloy content steels.

5. The criterion used to determine the hardenability of low alloy carbon steel was the 50 percent martensite transformation point, and the other 50 percent transformation product may be bainite or pearlite. The effect of alloying elements on the nonmartensite product transformation of the steel should be investigated for more precise hardenability predictions.

6. There are numerous alloy combinations which exhibit specific hardenability of a low alloy carbon steel. The opportunity is now present to estimate the best combination

to meet a specified Jominy hardness profile in the light of the alloy market and inventory conditions. By linear programming and optimization techniques, the cost of the steel can be reduced to a minimum.

REFERENCES

1. M. A. Grossmann. "Hardenability Calculated from Chemical Composition." Trans. Am. Inst. Min. Met. Eng. 150 (1942):227-259.

2. J. Field. "Calculation of Jominy End-Quench Curve from Analysis." Metal Progress 43 (1943):402-405.

3. W. Steven. "The Effect on the Hardenability of Small Addition of Chromium and Molybdenum to a Grain-size-Controlled 0.9% Nickel-Steel." Int. Iron and Steel Inst. 149 (1944):239-252.

4. T. G. Digges. "Effect of Carbon on the Hardenability of High Purity Iron-Carbon Alloys." Trans. Amer. Soc. Metals 26 (1938):408-421.

5. M. A. Grossmann, M. Asimow and S. F. Urban. "Hardenability, its Relation to Quenching, and Some Quantitative Data," pp. 124-196. In Hardenability of Alloy Steels. Cleveland, Ohio: American Society of Metals, 1939.

6. M. A. Grossmann and R. L. Stephenson. "The Effect of Grain Size on Hardenability." Trans. Amer. Soc. Metals 29 (1941):1-13.

7. W. Crafts, and J. L. Lamont. "Effect of Some Elements on Hardenability." Trans. Am. Inst. Min. Met. Eng. 158 (1944):157-167.

8. J. H. Hollsman and L. D. Jazze. "The Hardenability Concept." Trans. Am. Inst. Min. Met. Eng. 167 (1946):601-616.

9. J. M. Hodge and M. A. Orehoski. "Relationship between Hardenability and Percentage of Martensite in some Low-Alloy Steels." Trans. Am. Inst. Min. Met. Eng. 167 (1946):627-642.

10. I. R. Kramer, S. Siegel and J. G. Brooks. "Factor for the Calculation of Hardenability." Trans. Am. Inst. Min. Met. Eng. 167 (1946):670-679.

11. W. Crafts and J. L. Lamont. "Addition Method for Calculating Rockwell C Hardness of the Jominy Hardenability Test." Trans. Am. Inst. Min. Met. Eng. 167 (1946):698-718.

12. A. Asimow, W. F. Craig and M. A. Grossmann. "Correlation Between Jominy Test and Quenched Round Bars." SAE Trans. 49 (1941):283-292.

13. C. F. Jatczak and R. W. Devine Jr. "Calculation of Hardenability in High Carbon Alloy Steels." Trans. Amer. Soc. for Metals 47 (1955):748-765.

14. C. F. Jatczak. "Determining Hardenability from Composition." Metal Progress 100 (September 1971):60-65.

15. C. F. Jatczak and D. J. Girardi. "Multiplying Factors for the Hardenability of Hypereutectoid Steels Hardened from 1700°F." ASM Trans. 51 (1959):335-352.

16. E. J. Whittenberger, R. R. Burt and D. J. Carney. "Hardenability Factors for Hypereutectoid Low Alloy Steels." Trans. Amer. Inst. of Mining and Metallurgical Engineering 206 (August 1956): 1008-1016.

17. Anthony, F. de Retane and Douglas V. Doane. "Predicting Hardenability of Carburizing Steels." Metal Progress 100 (September 1971):65-69.

18. Cobbert H. Robert. "How Steel Plant Applies Calculated Hardenabilities." Metal Progress 111 (May 1977):35-37.

19. Iowa CADET, Computer Aided Design Subroutine Library, Department of Mechanical Engineering, Iowa State University, Ames, Iowa, 1974.

20. C. Mischke. "A Rationale for Mechanical Design to a Reliability Specification." First Design Technology Transfer Conference of the A.S.M.E., October 1974, Proceedings, p. 221.

21. J. Berkson. "Are There Two Regressions?" Journal of American Association 45 (1950):164-180.

22. Society of Automotive Engineers. "SAE Handbook." New York: The Society 1948.

23. Tentative Hardenability Bands. "Contributions to the Metallurgy of Steel, No. 11." New York: American Iron and Steel Institute, 1946.

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APPENDIX

Subroutine JOMINY (A1, A2, A3, A4, A5, A6, A7, A8, B1, B2, B3, B4, B5, B6, B7, B8):

This subroutine determines the mean and standard deviation of the Jominy hardness curve of a low alloy carbon steel from its composition and grain size.

The critical diameter D_I of a low carbon alloy steel can be predicted by the following formula:

$$D_I = f_C * f_{Mn} * f_{Si} * f_{Ni} * f_{Cr} * F_{Mo} * f_{Cu}$$

where f is the multiplying factor associate with each alloy element in the steel. The Jominy hardness curve will be predicted through the relationship between critical diameter and ratio of initial hardness (I.H. which is function of carbon content of the steel only) and the distant hardness (D.H.).

For detailed information see "Probabilistic Prediction of the Jominy Hardness Curve of Low Allcy Steel from Composition and Grain Size", by T. K. Ho, Ph.D. thesis, Iowa State University, Ames, Iowa, 1978.

Calling program requirements:

Provide the equivalent of the following statement.

Dimension B3(7), B4(7), B5(9), B6(9), B7(9), B8(9)

Subroutine RANDOM, RATIOP, DIDEAL must be available for the call of this subroutine:

A1: ASTM grain size of the low alloy carbon steel

A2: Carbon content of the low alloy carbon steel, percentage

A3: Manganese content of the low alloy carbon steel, percent

A4: Silicon content of the low alloy carbon steel, percent

A5: Nickel content of the low alloy carbon steel, percent

A6: Chromium content of the low alloy carbon steel, percent

A7: Molybdenum content of the low alloy carbon steel, percent

A8: Copper content of the low alloy carbon steel, percent

B1: Mean critical diameter of the low alloy carbon steel, inch

B2: Standard deviation of critical diameter of the low alloy carbon steel, inch

B3: Column vector of mean multiplying factor for carbon, manganese, silicon, nickel, chromium, molybdenum, copper

B4: Column vector of standard deviation of multiplying factor for carbon, manganese, silicon, nickel, chromium, molybdenum, and copper

B5: Mean ratio of initial hardness (I.H.) to distant hardness (D.H.) at Jominy stations 1, 4, 8, 12, 16, 20, 24, 28, 32

B6: Standard deviation of ratio of initial hardness (I.H.) to distant hardness (D.H.) at Jominy stations 1, 4, 8, 12, 16, 20, 24, 28, 32

B7: Mean Jominy hardness at Jominy stations 1, 4, 8, 12, 16, 20, 24, 28, 32, Rockwell C

B8: Standard deviation of Jominy hardness at Jominy stations 1, 4, 8, 12, 16, 20, 24, 28, 32, Rockwell C

Preempted names

Subroutine RANDOM

Subroutine RATIOP

Subroutine DIDEAL

Size: 13220 Bytes WATFIV compiler

Subroutine RATIOP (I1, A1, B1, B2):

This subroutine calculates the mean and standard deviation of ratio of initial hardness (I.H.) to distant hardness (D.H.) at different Jominy stations from ideal critical diameter of the steel.

If x is the mean critical diameter of the steel, y is the ratio of initial hardness to distant hardness at a given Jominy station, and u = ln(y-1). Then x and y can be related by the following expressions:

Jominy station 4:

$\mu_y = 1.0-0.008283*(x-7.0)$

$\sigma_{y/x} = 0.023005$ for $3.5 \leq x \leq 7.0$

and

$\mu_y = 1.02899-0.0818586*(x-3.5)$
$-0.0923808*(x-3.5)^2-0.1002404*(x-3.5)^2$

$\sigma_{y/x} = 0.063692$

for

$1.5 \leq x \leq 3.5$

Jominy station 8:

$\mu_u = 1.583563-0.858657*x$

$\sigma_{u/x} = 0.390554$

for

$1.5 \leq x \leq 7.0$

Jominy station 12:

$$\mu_u = 1.516167-0.650661*x$$

$$\sigma_{u/x} = 0.190353$$

for

$$1.5 \leq x \leq 7.0$$

Jominy station 16:

$$\mu_u = 1.471133-0.544419*x$$

$$\sigma_{u/x} = 0.161444$$

for

$$1.5 \leq x \leq 7.0$$

Jominy station 20:

$$\mu_u = 1.461690-0.481383*x$$

$$\sigma_{u/x} = 0.159499$$

for

$$1.5 \leq x \leq 7.0$$

Jominy station 24:

$$\mu_u = 1.411127-0.419207*x$$

$$\sigma_{u/x} = 0.15986$$

for

$$1.5 \leq x \leq 7.0$$

Jominy station 28:

$$\mu_u = 1.392689-0.379767*x$$

$$\sigma_{u/x} = 0.152950$$

for

$$1.5\leq x\leq 7.0$$

Jominy station 32:

$$\mu_u = 1.372598-0.357773*x$$

$$\sigma_{u/x} = 0.152883$$

for

$$1.5\leq x\leq 7.0$$

With the assumption that u has a normal distribution, the mean and standard deviation μ_y and σ_y of y can be related to the mean and standard deviation μ_u and $\sigma_{u/x}$ of u by the following equations:

$$\mu_y = \mathrm{Exp}\{\mu_u + \frac{\sigma^2_{u/x}}{2}\}$$

and

$$\sigma^2_y = \{\mathrm{Exp}(\sigma^2_{u/x})-1\}\{\mathrm{Exp}(2\mu_u+\sigma^2_{u/x})\}$$

For detailed information see "Probabilistic Prediction of the Jominy Hardness Curve of Low Alloy Steels from Composition and Grain Size", by T. K. Ho, Ph.D. thesis, Iowa State University, Ames, Iowa, 1978.

Calling program requirements:

None

Call list arguments:

I1: Index for Jominy station

= 1 Jominy station No. 4

= 2 Jominy station No. 8

= 3 Jominy station No. 12

= 4 Jominy station No. 16

= 5 Jominy station No. 20

= 6 Jominy station No. 24

= 7 Jominy station No. 28

= 8 Jominy station No. 32

A2: Critical diameter of the low carbon alloy steel, inch

B1: Mean of ratio of initial hardness (I.H.) to distant hardness (D.H.) at a given Jominy station

B2: Standard deviation of ratio of initial hardness (I.H.) to distant hardness (D.H.) at a given Jominy station

Preempted names:

None

Size:

1576 Bytes WATFIV compiler

Subroutine DIDEAL (I1, A2, B1, B2):

This subroutine determines the mean and standard deviation of carbon factor (ideal critical diameter) of an alloy low carbon steel form its carbon content and grain size. The carbon factor at different grain sizes were expressed as a function of carbon content by the following equations:

Grain size ASTM No. 4

$$\mu_{D^2_{IC}} = (1.550281)*C$$

Grain size ASTM No. 5

$$\mu_{D^2_{IC}} = (1.309604)*C$$

Grain size ASTM No. 6

$$\mu_{D^2_{IC}} = (1.111438)*C$$

Grain size ASTM No. 7

$$\mu_{D^2_{IC}} = (0.955099)*C$$

Grain size ASTM No. 8

$$\mu_{D^2_{IC}} = (0.823952)*C$$

and

$$\sigma^2_{D^2_{I/C}} = (0.140223)*C \text{ for grain sizes 4 to 8}$$

where

$\mu_{D_I^2}$ is the mean of square of carbon factor

$\sigma_{D_{I/C}^2}$ is the standard deviation of square of carbon factor on carbon content of the steel

The mean μ_{D_I} and standard deviation $\sigma_{D_{I/C}}$ of the carbon factor can be related to $\mu_{D_I^2}$ and $\sigma_{D_{I/C}^2}$ by the following expressions:

$$\mu_{D_I} = \mu_{D_I^2}\{1.0-0.125(\frac{\sigma_{D_{I/C}^2}}{\mu_{D_I^2}})^2\}$$

$$\sigma^2_{D_I} = \mu_{D_I^2} - \mu^2_{D_I}$$

For detailed information see "Probabilistic Prediction of the Jominy Hardness Curve of Low Alloy Steels from Composition and Grain Size", by T. K. Ho, Ph.D. thesis, Iowa State University, Ames, Iowa, 1978. Calling program requirements:

None

Call List Arguments:

I1: Index for ASTM grain size

= 1 ASTM grain size 4

= 2 ASTM grain size 5

= 3 ASTM grain size 6

= 4 ASTM grain size 7

= 5 ASTM grain size 8

A2: Carbon content of the alloy steel, percent

B1: Mean carbon factor (ideal critical diameter), in.

B2: Standard deviation of carbon factor, in.

Preempted names:

None

Size:

880 bytes WATFIV compiler

Subroutine RANDOM:

The subroutine RANDOM has no call list. It is called in order to load a battery of Function subprograms which aid the user in executing the algebra of random variables.

Programming is based upon the following equations given in "A Rationale for Mechanical Design to a Reliability Specification," C. Mischke, First Design Technology Transfer Conference of the A.S.M.E., October 1974, Proceedings, p. 221.

When the exponent n is fractional, the mean of x must be positive.

Real Function	Algebraic Operation	Value of Function
MEAN1 $(\mu_x, \sigma_x, \mu_y, \sigma_y, \rho)$	$z = x + y$	μ_z
SIGMA1 $(\mu_x, \sigma_x, \mu_y, \sigma_y, \rho)$		σ_z
MEAN2 $(\mu_x, \sigma_x, \mu_y, \sigma_y, \rho)$	$z = x - y$	μ_z
SIGMA2 $(\mu_x, \sigma_x, \mu_y, \sigma_y, \rho)$		σ_z
MEAN3 $(\mu_x, \sigma_x, \mu_y, \sigma_y, \rho)$	$z = xy$	μ_z
SIGMA3 $(\mu_x, \sigma_x, \mu_y, \sigma_y, \rho)$		σ_z
MEAN4 $(\mu_x, \sigma_x, \mu_y, \sigma_y, \rho)$	$z = x/y$	μ_z
SIGMA4 $(\mu_x, \sigma_x, \mu_y, \sigma_y, \rho)$		σ_z
MEAN5 $(\mu_x, \sigma_x, \mu_y, \sigma_y)$	$z = x^y$	μ_z
SIGMA5 $(\mu_x, \sigma_x, \mu_y, \sigma_y)$		σ_z

CALLING PROGRAM REQUIREMENTS:

Declaration: REAL MEAN1, MEAN2, MEAN3, MEAN4, MEAN5

FUNCTION CALL LIST ARGUMENTS:

μ_x = mean of the variate x, real FORTRAN variable

σ_x = standard deviation of the variate x, real FORTRAN variable

μ_y = mean of the variate y, real FORTRAN variable

σ_y = standard deviation of the variate y, real FORTRAN variable

ρ = correlation coefficient, real FORTRAN variable

PREEMPTED NAMES:

None.

SIZE: 11732 bytes WATFIV compiler

Table A1. Subroutine JOMINY

```
      SUBROUTINE JOMINY(GS,C,MN,SI,NI,CR,MO,CU,DI,SIGDI,FACTOR,SIGFAC,
     1   RATIO,SIGRAT,QJ,SIGQJ)
C
C
      REAL MN,NI,MO,MEAN3,MEAN4
C
      DIMENSION XT(5),YT(5),FACTOR(7),SIGFAC(7),RATIO(9),SIGRAT(9),QJ(9)
     1     ,SIGQJ(9),SIGYT(5)
C
      ND=6
      GO TO 10
    5 CONTINUE
C
C ***********************************************************************
C *                                                                     *
C *       *****CRITICAL DIAMETER OF THE MATERIAL*****                   *
C *                                                                     *
C ***********************************************************************
C
C     CARBON MULTIPLYING FACTOR.
C
      DO 1000 I=1,5
      XT(I)=3.0+FLOAT(I)
      CALL DIDEAL(I,C,Y,SIGY)
      IF(GS.EQ.XT(I))GO TO 101
      YT(I)=Y
      SIGYT(I)=SIGY
 1000 CONTINUE
C
C     INTERPOLATION OF MEAN AND STANDARD DEVIATION OF CARBON
C     MULTIPLICATION FACTOR (IDEAL CRITICAL DIAMETER).
C
      CALL INTER(5,XT,YT,GS,Y,IJK1)
      CALL INTER(5,XT,SIGYT,GS,SIGY,IJK1)
```

```
  101 FACTOR(1)=Y
      SIGFAC(1)=SIGY
C
C     MANGANESE MULTIPLYING FACTOR.
C
  200 FACTOR(2)=1.0
      SIGFAC(2)=0.0
      IF(MN.EQ.0.0)GO TO 210
      SIGFAC(2)=0.1857668
      FACTOR(2)=1.0+0.235349*MN+0.569809*MN*MN
C
C     SILICON MULTIPLYING FACTOR.
C
  210 FACTOR(3)=1.0
      SIGFAC(3)=0.0
      IF(SI.EQ.0.0)GO TO 220
      SIGFAC(3)=0.0911506
      FACTOR(3)=1.0+0.221108*SI+0.128023*SI*SI
C
C     NICKEL MULTIPLYING FACTOR.
C
  220 SIGFAC(4)=0.0
      FACTOR(4)=1.0
      IF(NI.EQ.0.0)GO TO 230
      SIGFAC(4)=0.1286972
      FACTOR(4)=1.0+0.767098*NI-0.122888*NI*NI
C
C     CHROMIUM MULTIPLYING FACTOR.
C
  230 FACTOR(5)=1.0
      SIGFAC(5)=0.0
      IF(CR.EQ.0.0)GO TO 240
      SIGFAC(5)=0.254099
      FACTOR(5)=1.0+1.633811*CR+0.027041*CR*CR
C
```

Table A1 (Continued)

```
C      MOLYBDENUM MULTIPLYING FACTOR.
C
  240 FACTOR(6)=1.0
      SIGFAC(6)=0.0
      IF(MO.EQ.0.0)GO TO 250
      SIGFAC(6)=0.2326137
      FACTOR(6)=1.0+1.571771*MO+0.589309*MO*MO
C
C      COPPER MULTIPLYING FACTOR.
C
  250 SIGFAC(7)=0.0
      FACTOR(7)=1.0
      IF(CU.EQ.0.0)GO TO 260
      SIGFAC(7)=0.120791
      FACTOR(7)=1.0+1.176284*CU-0.333859*CU*CU
C
C      CRITICAL DIAMETER.
C
  260 DI=FACTOR(1)
      SIGDI=SIGFAC(1)
C
      CALL RANDCM
C
      DO 270 I=2,7
C
C      MEAN OF CRITICAL DIAMETER OF THE MATERIAL.
C
      DI=MEAN3(DI,SIGDI,FACTOR(I),SIGFAC(I),0.0)
C
C      STANDARD DEVIATION OF THE CRITICAL DIAMETER OF THE MATERIAL.
C
      SIGDI=SIGMA3(DI,SIGDI,FACTOR(I),SIGFAC(I),0.0)
  270 CONTINUE
C
C *****************************************************************
```

Table A1 (Continued)

```
C *                                                                      *
C *        *****ROCKELL HARDNESS AT JOMINY STATION 1*****                *
C *                                                                      *
C ************************************************************************
C
      QJ(1)=35.54749+22.25735*C+177.4605*C*C-305.1492*C*C*C
     1         +132.5669*C*C*C*C
      SIGQJ(1)=1.13
      IF(C.GE.0.66362)QJ(1)=65.0
      RATIO(1)=1.0
      SIGRAT(1)=0.0
C
C ************************************************************************
C *                                                                      *
C *        *****JOMINY HARDNESS CURVE*****                               *
C *                                                                      *
C ************************************************************************
C
  370 DO 380 I=2,9
C
C ************************************************************************
C *                                                                      *
C *        *****RATIO OF IH/DH*****                                      *
C *                                                                      *
C ************************************************************************
C
C
      II=I-1
      CALL RATIOP(II,DI,RATIO(I),SIGRAT(I))
C
C
C     MEAN OF JOMINY STATION HARDNESS.
C
      QJ(I)=MEAN4(QJ(1),SIGQJ(1),RATIO(I),SIGRAT(I),0.0)
C
```

Table A1 (Continued)

```
C     STANDARD DEVIATION OF JOMINY STATION HARDNESS.
C
      SIGQJ(I)=SIGMA4(QJ(1),SIGQJ(1),RATIO(I),SIGRAT(I),0.0)
  380 CONTINUE
      RETURN
C
C ****************************************************************************
C *                                                                          *
C *        *****PROTECTIONS*****                                             *
C *                                                                          *
C ****************************************************************************
C
   10 IFERR=0
      IF(GS.LT.4.0.OR.GS.GT.8.0)GO TO 20
   25 IF(C.LT.0.2.OR.C.GT.0.8)GO TO 30
   35 IF(MN.LT.0.0.OR.MN.GT.3.0)GO TO 40
   45 IF(SI.LT.0.0.OR.SI.GT.2.0)GO TO 50
   55 IF(NI.LT.0.0.OR.NI.GT.3.1)GO TO 60
   65 IF(CR.LT.0.0.OR.CR.GT.2.5)GO TO 70
   75 IF(MO.LT.0.0.OR.MO.GT.1.05)GO TO 80
   85 IF(CU.LT.0.0.OR.CU.GT.2.2)GO TO 90
C
C
      IF(IFERR)5,5,100
C
   20 IFERR=1
      WRITE(NO,15)
   15 FORMAT(1X,'*****ERROR MESSAGE SUBROUTINE JOMINY*****')
      WRITE(NO,21)GS
   21 FORMAT(1X,'A1 = ',G15.7,1X,'GRAIN SIZE OF THE MATERIAL MUST BE ',
     1   /,' IN THE RANGE OF 4.0 AND 8.0')
      GO TO 25
C
   30 IFERR=1
      WRITE(NO,15)
```

Table A1 (Continued)

```
      WRITE(NO,31)C
   31 FORMAT(1X,'A2 = ',G15.7,1X,'CARBON CONTENT OF THE MATERIAL MUST ',
     1   /,' BE IN THE RANGE OF 0.2 AND 0.8 PERCENT.')
      GO TO 35
C
   40 IFERR=1
      WRITE(NO,15)
      WRITE(NO,41)MN
   41 FORMAT(1X,'A3 = ',G15.7,1X,'MANGANESE CONTENT OF THE MATERIAL MUST
     1 ',/,' BE IN THE RANGE OF 0.0 AND 3.0 PERCENT.')
      GO TO 45
C
   50 IFERR=1
      WRITE(NO,15)
      WRITE(NO,51)SI
   51 FORMAT(1X,'A4 = ',G15.7,1X,' SILCON CONTENT OF THE MATERIAL MUST '
     1   ,/,' BE IN THE RANGE OF 0.0 AND 2.0 PERCENT.')
      GO TO 55
C
   60 IFERR=1
      WRITE(NO,15)
      WRITE(NO,61)NI
   61 FORMAT(1X,'A5 = ',G15.7,1X,'NICKEL CONTENT OF THE MATERIAL MUST ',
     1   /,' BE IN THE RANGE OF 0.0 AND 3.1 PERCENT.')
      GO TO 65
C
   70 IFERR=1
      WRITE(NO,15)
      WRITE(NO,71)CR
   71 FORMAT(1X,'A6 = ',G15.7,1X,'CHROMIUM CONTENT OF THE MATERIAL MUST'
     1     ,/,'  BE IN THE RANGE OF 0.0 AND 2.5 PERCENT.')
      GO TO 75
C
   80 IFERR=1
      WRITE(NO,15)
```

Table A1 (Continued)

```
      WRITE(NO,81)MO
   81 FORMAT(1X,'A7 = ',G15.7,1X,'MOLYBDENUM CONTENT OF THE MATERIAL ',
     1   /,' MUST BE IN THE RANGE OF 0.0 AND 1.05 PERCENT.')
      GO TO 85
C
   90 WRITE(NO,15)
      WRITE(NO,91)CU
   91 FORMAT(1X,'A8 = ',G15.7,1X,'COPPER CONTENT OF THE MATERIAL MUST ',
     1   /,' BE IN THE RANGE OF 0.0 AND 2.2 PERCENT.')
C
  100 RETURN
      END
```

Table A2. Subroutine RATIOP

```
      SUBROUTINE RATIOP(I,X,R,SIGR)
C
C     THIS SUBROUTINE CALCULATES THE RATIO OF IH/DH AT DIFFERENT JOMINY
C     STATIONS FROM CRITICAL DIAMETER OF THE STEEL.
C
      GO TO (10,20,30,40,50,60,70,80),I
C
C
C     JOMINY STATION 4.
C
   10 IF(X.LT.3.5)GO TO 91
      R=1.0-0.008283*(X-7.0)
      IF(R.LT.1.00)R=1.00
      SIGR=0.023005
      RETURN
   91 XX=X-3.5
      R=1.02899-0.0818586*XX-0.0923808*XX*XX-0.1002404*XX*XX*XX
      SIGR=0.063692
      RETURN
C
C     JOMINY STATION 8.
C
   20 Y=1.583563-0.859657*X
      SIGMA=0.390554
      GO TO 200
C
C     JOMINY STATION 12.
C
   30 Y=1.516167-0.650661*X
      SIGMA=0.190353
      GO TO 200
C
C     JOMINY STATION 16.
```

Table A2 (Continued)

```
C
   40 Y=1.471133-0.544419*X
      SIGMA=0.161444
      GO TO 200
C
C     JOMINY STATION 20.
C
   50 Y=1.461690-0.481393*X
      SIGMA=0.159499
      GO TO 200
C
C     JOMINY STATION 24.
C
   60 Y=1.411127-0.419207*X
      SIGMA=0.159686
      GO TO 200
C
C     JOMINY STATION 28.
C
   70 Y=1.392689-0.379767*X
      SIGMA=0.152950
      GO TO 200
C
C     JOMINY STATION 32.
C
   80 Y=1.372598-0.357773*X
      SIGMA=0.152883
C
C     MEAN OF RATIO IH/DH.
C
  200 T=SIGMA*SIGMA
      T1=Y+T/2.0
      R=EXP(T1)+1.0
C
C     STANDARD DEVIATION OF RATIO IH/DH.
```

Table A2 (Continued)

```
C
      T1=EXP(T)-1.0
      T2=2.0*Y+T
      T2=EXP(T2)
      SIGR=T1*T2
      SIGR=SQRT(SIGR)
      RETURN
      END
```

Table A3. Subroutine DIDEAL

```
      SUBROUTINE DIDEAL(I,C,DI,SIGDI)
C
C     THIS SUBROUTINE DETERMINES THE IDEAL CRITICAL DIAMETER OF A STEEL
C     FROM ITS CARBON CONTENT AND GRAIN SIZE.
C
C
      SIGMA=0.140223*C
      GO TO (10,20,30,40,50),I
C
C     *****GRAIN SIZE 4*****
C
   10 Y=1.550281*C
      GO TO 60
C
C     *****GRAIN SIZE 5*****
C
   20 Y=1.309604*C
      GO TO 60
C
C     *****GRAIN SIZE 6*****
C
   30 Y=1.111438*C
      GO TO 60
C
C     *****GRAIN SIZE 7*****
C
   40 Y=0.955099*C
      GO TO 60
C
C     *****GRAIN SIZE 8*****
C
   50 Y=0.823952*C
C
C     MEAN OF IDEAL CRITICAL DIAMETER.
```

Table A3 (Continued)

```
   60 DI=SQRT(Y)*(1.0-0.125*SIGMA*SIGMA/(Y*Y))
C
C     STANDARD DEVIATION OF IDEAL CRITICAL DIAMETER.
C
      SIGDI=Y-DI*DI
      SIGDI=SQRT(SIGDI)
      RETURN
      END
```

Table A4. Subroutine RANDOM

```
      SUBROUTINE RANDOM
      RETURN
      END
C
      REAL FUNCTION MEAN1(XBAR,SIGMAX,YBAR,SIGMAY,P)
      GO TO 10
    5 MEAN1=XBAR+YBAR
      RETURN
C
C     ..... PROTECTION .....
C
   10 IFERR=0
      IF(SIGMAX)100,11,11
   11 IF(SIGMAY)100,12,12
   12 IF(P.LT.-1..OR.P.GT.1.)GO TO 110
      IF(IFERR)5,5,14
  100 IFERR=1
      WRITE(6,101)SIGMAX,SIGMAY
  101 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN1*****',/,6X,'A2=',G15.7,
     16X,'A4=',G15.7,/,6X,'THESE VARIABLES MUST BE POSITIVE.')
      GO TO 12
  110 IFERR=1
      WRITE(6,111)P
  111 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN1*****',/,6X,'A5=',G15.7,
     1/,6X,'THIS VARIABLE MUST NOT BE LESS THAN -1. OR GREATER THAN 1.')
   14 RETURN
      END
      FUNCTION SIGMA1(XBAR,SIGMAX,YBAR,SIGMAY,P)
      GO TO 10
    5 SIGMA1=SQRT(SIGMAX*SIGMAX+SIGMAY*SIGMAY+2.*P*SIGMAX*SIGMAY)
      RETURN
C
C     ..... PROTECTION .....
C
   10 IFERR=0
```

Table A4 (Continued)

```
      IF(SIGMAX)100,11,11
   11 IF(SIGMAY)100,12,12
   12 IF(P.LT.-1..OR.P.GT.1.)GO TO 110
      IF(IFERR)5,5,14
  100 IFERR=1
      WRITE(6,101)SIGMAX,SIGMAY
  101 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA1*****',/,6X,'A2=',G15.7
     1,6X,'A4=',G15.7,/,6X,'THESE VARIABLES MUST BE POSITIVE.')
      GO TO 12
  110 IFERR=1
      WRITE(6,111)P
  111 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA1*****',/,6X,'A5=',G15.7
     1,/,6X,'THIS VARIABLE MUST NOT BE LESS THAN -1. OR GREATER THAN 1.'
     2)
   14 RETURN
      END
      REAL FUNCTION MEAN2(XBAR,SIGMAX,YBAR,SIGMAY,P)
      GO TO 10
    5 MEAN2=XBAR-YBAR
      RETURN
C
C     ..... PROTECTION .....
C
   10 IFERR=0
      IF(SIGMAX)100,11,11
   11 IF(SIGMAY)100,12,12
   12 IF(P.LT.-1..OR.P.GT.1.)GO TO 110
      IF(IFERR)5,5,14
  100 IFERR=1
      WRITE(6,101)SIGMAX,SIGMAY
  101 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN2*****',/,6X,'A2=',G15.7,
     16X,'A4=',G15.7,/,6X,'THESE VARIABLES MUST BE POSITIVE.')
      GO TO 12
  110 IFERR=1
      WRITE(6,111)P
```

Table A4 (Continued)

```
  111 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN2*****',/,6X,'A5=',G15.7,
     1/,6X,'THIS VARIABLE MUST NOT BE LESS THAN -1. OR GREATER THAN 1.')
   14 RETURN
      END
      FUNCTION SIGMA2(XBAR,SIGMAX,YBAR,SIGMAY,P)
      GO TO 10
    5 SIGMA2=SQRT(SIGMAX*SIGMAX+SIGMAY*SIGMAY-2.*P*SIGMAX*SIGMAY)
      RETURN
C
C     ..... PROTECTION .....
C
   10 IFERR=0
      IF(SIGMAX)100,11,11
   11 IF(SIGMAY)100,12,12
   12 IF(P.LT.-1..OR.P.GT.1.)GO TO 110
      IF(IFERR)5,5,14
  100 IFERR=1
      WRITE(6,101)SIGMAX,SIGMAY
  101 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA2*****',/,6X,'A2=',G15.7
     1,6X,'A4=',G15.7,/,6X,'THESE VARIABLES MUST BE POSITIVE.')
      GO TO 12
  110 IFERR=1
      WRITE(6,111)P
  111 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA2*****',/,6X,'A5=',G15.7
     1,/,6X,'THIS VARIABLE MUST NOT BE LESS THAN -1. OR GREATER THAN 1.'
     2)
   14 RETURN
      END
      REAL FUNCTION MEAN3(XBAR,SIGMAX,YBAR,SIGMAY,P)
      GO TO 10
    5 MEAN3=XBAR*YBAR+P*SIGMAX*SIGMAY
      RETURN
C
C     ..... PROTECTION .....
C
```

Table A4 (Continued)

```
   10 IFERR=0
      IF(SIGMAX)100,11,11
   11 IF(SIGMAY)100,12,12
   12 IF(P.LT.-1..OR.P.GT.1.)GO TO 110
      IF(IFERR)5,5,14
  100 IFERR=1
      WRITE(6,101)SIGMAX,SIGMAY
  101 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN3*****',/,6X,'A2=',G15.7,
     16X,'A4=',G15.7,/,6X,'THESE VARIABLES MUST BE POSITIVE.')
      GO TO 12
  110 IFERR=1
      WRITE(6,111)P
  111 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN3*****',/,6X,'A5=',G15.7,
     1/,6X,'THIS VARIABLE MUST NOT BE LESS THAN -1. OR GREATER THAN 1.')
   14 RETURN
      END
      FUNCTION SIGMA3(XBAR,SIGMAX,YBAR,SIGMAY,P)
      GO TO 10
    5 T1=XBAR*YBAR
      T2=SIGMAX/XBAR
      T3=SIGMAY/YBAR
      T4=T2*T3
      T5=T2*T2+T3*T3+T4*T4+2.*P*T4+P*P*T4*T4
      SIGMA3=XBAR*YBAR*(1.+P*T4)*SQRT(T5)
      RETURN
C
C     ..... PROTECTION .....
C
   10 IFERR=0
      IF(SIGMAX)100,11,11
   11 IF(SIGMAY)100,12,12
   12 IF(P.LT.-1..OR.P.GT.1.)GO TO 110
   14 IF(XBAR)15,120,15
   15 IF(YBAR)16,120,16
   16 IF(IFERR)5,5,17
```

Table A4 (Continued)

```
  100 IFERR=1
      WRITE(6,101)SIGMAX,SIGMAY
  101 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA3*****',/,6X,'A2=',G15.7
     1,6X,'A4=',G15.7,/,6X,'THESE VARIABLES MUST BE POSITIVE.')
      GO TO 12
  110 IFERR=1
      WRITE(6,111)P
  111 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA3*****',/,6X,'A5=',G15.7
     1,/,6X,'THIS VARIABLE MUST NOT BE LESS THAN -1. OR GREATER THAN 1.'
     2)
      GO TO 14
  120 WRITE(6,121)XBAR,YBAR
  121 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA3*****',/,6X,'A5=',G15.7
     1,6X,'A3=',G15.7,/,6X,'THESE VARIABLES MUST NOT BE ZERO.')
   17 RETURN
      END
      REAL FUNCTION MEAN4(XBAR,SIGMAX,YBAR,SIGMAY,P)
      GO TO 10
    5 T1=SIGMAX/XBAR
      T2=SIGMAY/YBAR
      MEAN4=(XBAR/YBAR)*(1.+T2*(T2-P*T1)*(1.+3.*(T2*T2)))
      RETURN
C
C     ..... PROTECTION .....
C
   10 IFERR=0
      IF(SIGMAX)100,11,11
   11 IF(SIGMAY)100,12,12
   12 IF(P.LT.-1..OR.P.GT.1.)GO TO 110
   14 IF(XBAR)15,120,15
   15 IF(YBAR)16,120,16
   16 IF(IFERR)5,5,17
  100 IFERR=1
      WRITE(6,101)SIGMAX,SIGMAY
  101 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN4*****',/,6X,'A2=',G15.7,
```

Table A4 (Continued)

```
    16X,'A4=',G15.7,/,6X,'THESE VARIABLES MUST BE POSITIVE.')
      GO TO 12
  110 IFERR=1
      WRITE(6,111)P
  111 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN4*****',/,6X,'A5=',G15.7,
     1/,6X,'THIS VARIABLE MUST NOT BE LESS THAN -1. OR GREATER THAN 1.')
      GO TO 14
  120 WRITE(6,121)XBAR,YBAR
  121 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN4*****',/,6X,'A1=',G15.7,
     16X,'A3=',G15.7,/,6X,'THESE VARIABLES MUST NOT BE ZERO.')
   17 RETURN
      END
      FUNCTION SIGMA4(XBAR,SIGMAX,YBAR,SIGMAY,P)
      GO TO 10
    5 T1=SIGMAX/XBAR
      T2=SIGMAY/YBAR
      T3=T2*T2+T1*T1-2.*P*T1*T2+8.*T2*T2*T2*T2-16.*P*T2*T2*T2*T1+3.*T1*T
     11*T2*T2+5.*P*P*T1*T1*T2*T2
      SIGMA4=(XBAR/YBAR)*SQRT(ABS(T3))
      RETURN
C
C     ..... PROTECTION .....
C
   10 IFERR=0
      IF(SIGMAX)100,11,11
   11 IF(SIGMAY)100,12,12
   12 IF(P.LT.-1..OR.P.GT.1.)GO TO 110
   14 IF(XBAR)15,120,15
   15 IF(YBAR)16,120,16
   16 IF(IFERR)5,5,17
  100 IFERR=1
      WRITE(6,101)SIGMAX,SIGMAY
  101 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA4*****',/,6X,'A2=',G15.7
     1,6X,'A4=',G15.7,/,6X,'THESE VARIABLES MUST BE POSITIVE.')
      GO TO 12
```

Table A4 (Continued)

```
 110 IFERR=1
     WRITE(6,111)P
 111 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA4*****',/,6X,'A5=',G15.7
    1,/,6X,'THIS VARIABLE MUST NOT BE LESS THAN -1. OR GREATER THAN 1.'
    2)
     GO TO 14
 120 WRITE(6,121)XBAR,YBAR
 121 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA4*****',/,6X,'A1=',G15.7
    1,6X,'A3=',G15.7,/,6X,'THESE VARIABLES MUST NOT BE ZERO.')
  17 RETURN
     END
     REAL FUNCTION MEAN5(XBAR,SIGMAX,N)
     REAL N
     GO TO 10
   5 IF(N .EQ. 0.) GO TO 1
     IF(N .EQ. 1.) GO TO 2
     MEAN5=(1.+0.5*N*(N-1.)*(SIGMAX/XBAR)*(SIGMAX/XBAR))*XBAR**N
     RETURN
   1 MEAN5=1.
     RETURN
   2 MEAN5=XBAR
     RETURN
C
C    ..... PROTECTION .....
C
  10 IFERR=0
     IF(SIGMAX)110,12,12
  12 IF(XBAR)120,120,14
  14 IF(IFERR)5,5,15
 110 IFERR=1
     WRITE(6,111)SIGMAX
 111 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN5*****',/,6X,'A2=',G15.7,
    1/,6X,'THIS VARIABLE MUST BE POSITIVE.')
     GO TO 12
 120 WRITE(6,121)XBAR
```

Table A4 (Continued)

```
121 FORMAT(' *****ERROR MESSAGE FUNCTION MEAN5*****',/,6X,'A1=',G15.7,
   1/,6X,'THIS VARIABLE MUST BE GREATER THAN ZERO.')
 15 RETURN
    END
    FUNCTION SIGMA5(XBAR,SIGMAX,N)
    REAL N
    GO TO 10
  5 IF(N .EQ. 0.) GO TO 1
    IF(N .EQ. 1.) GO TO 2
    SIGMA5=ABS(N)*SIGMAX*(1.+0.25*(N-1.)*(N-1.)*(SIGMAX/XBAR)*(SIGMAX/
   1XBAR))*XBAR**(N-1.)
    RETURN
  1 SIGMA5=0.
    RETURN
  2 SIGMA5=SIGMAX
    RETURN
C
C     ..... PROTECTION .....
C
 10 IFERR=0
    IF(SIGMAX)110,12,12
 12 IF(XBAR)120,120,14
 14 IF(IFERR)5,5,15
110 IFERR=1
    WRITE(6,111)SIGMAX
111 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA5*****',/,6X,'A2=',G15.7
   1,/,6X,'THIS VARIABLE MUST BE POSITIVE.')
    GO TO 12
120 WRITE(6,121)XBAR
121 FORMAT(' *****ERROR MESSAGE FUNCTION SIGMA5*****',/,6X,'A1=',G15.7
   1,/,6X,'THIS VARIABLE MUST BE GREATER THAN ZERO.')
 15 RETURN
    END
```