Allocation and Inventory Policies for Reels in Printed Circuit Board Assemblies

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Abstract

In Printed Circuit Board (PCB) assemblies, various types of reels loaded with different components are extensively utilized. We examine the inventory and allocation policies across assembly lines when the number and size of the reels substantially affect the assembly efficiency (e.g., when the number of available slots in a chip shooter is relatively limited). Critical features of the policies are illustrated via numerical examples.

Keywords

Printed Circuit Board Assemblies (PCBA), Inventory, Scheduling, Reel Operations.

1. Introduction

In this paper, we first propose and examine reel allocation policies and their impact on idle time of assembly lines. We then proceed to formulate a reel inventory policy integrating the allocation and idle time aspects as a major model component.

Specifically, we consider a PCB production facility consisting of a multiple number of assembly lines. For each line, the primary bottleneck is assumed to be a single high-speed chip shooter that places various chips on PCB's. Each type of chips for the placement is from a specific reel that is set up on either side of the chip shooter. Since the number of types of chips and the number of types of boards are generally large, the coordination of the inventory and allocation policies is all the more difficult.

As a first step to study this complex problem, in this paper, we will study the coordination of such policies focusing on a single type of chips that is critical for the assembly of numerous types of PCB's. For this study, we will first examine the allocation policy across the assembly lines. We will then examine the inventory policy given our analyses of the allocation policy based on an extension of a (Q, R) inventory model. Finally, we will provide a numerical example followed by concluding remarks.

2. Reel Allocation Policies

In PCB manufacturing, the interaction between process planning, production planning, and scheduling may depend on three important decisions. Namely, Grouping, Allocation, and Sequencing (Arrangement) [1]. Grouping is the selection of machine groups for part families; allocation is sending the right components to the right machines; and sequencing is arranging these components on the feeder. This section focuses on the issue of right allocation with the objective of machine idle time reduction and setup reduction [2].

Allocating the right reel to the right line may reduce a sizeable portion of the machine idle time. This problem is similar to that of tool allocation. There are two primary issues here, one is that of selecting the right tool and the other is that of selecting the right tool request [3]. For example, a tool with the highest life may be selected as the right tool while a tool request with the shortest processing time (SPT) may be selected as the right tool request. We note that the literature on reel allocation itself is quite limited (e.g., [4] does discuss how to maximize the length of an uninterrupted machine production run. However, the idle time due to reel changes is often inevitable when the availability of slots is constrained).

In the case of PCBA, a reel with the largest number of chips remaining may be selected as the right reel and an assembly line with the least amount of chips demanded may be selected as the right reel request. We will expand upon this observation with the following three reel allocation policies. For simplicity, let us

suppose that we have L lines and L reels at the beginning of the planning horizon of X days, and the daily demand of the chips for each line is known.

2.1 Standard Policy – The first approach is, at the beginning of each day, to send the reel with the maximum number of components remaining to the line with the maximum daily demand [3]. With this approach, there may be a more uniform rate of consumption of the components. Hence, we are likely to operate with the L reels for a longer time (as compared to the alternative policy shown next). From the data we obtained from an anonymous company, this policy appears to work well for demands which do not exceed the reel size (i.e., the original number of the chips put in a new reel) and when the number of reels either exceeds or equals to the number of lines.

2.2 Alternative Policy - The second approach is, at the beginning of each day, to send the reel with the maximum number of components remaining to the line with the minimum daily demand. This policy is a straightforward modification of the standard policy with interesting implications. Since the reel with the maximum number of components is being used up at a slower rate (as it goes to the line with least demand), the other reels are being used up more rapidly. Eventually, one or more reels may be completely exhausted. From the same data this policy appears to work well for demands which exceed the reel size and when the number of reels is strictly less than the number of lines.

We now present examples for both policies. Here we define the idle time as the sum of the time spent waiting for a reel and the setup time for a new reel when the old reel is exhausted. In this example, the reel setup time is assumed to be 10 minutes. The placement rate of each line is assumed to be 1000 chips per hour.

The first example illustrates the standard policy. There are 3 reels of 49612 chips each, and at no point does the demand on any line exceed the reel size.

	L1	L2	L3
W1	19659	9484	2154
W2	20634	9663	20634
W3	17956	9721	2098
W4	24527	10561	1745

Table 1: Relatively Even Demand Data across Assembly Lines

The standard policy results in the maximum idle time of a day on a line to be 20 minutes while the alternative policy results in the maximum idle time of a day on a line to be 1010 minutes.

On the other hand, with the relatively uneven demand data below, the alternative policy is shown to be superior to the standard policy. Specifically, there are three reels of 8048 chips each. We note that the demand for the June 4 on Line 1 is 8165, which is more than the reel size.

	L1	L2	L3
3-Jun	1092	1335	2052
4-Jun	8165	1029	272
5-Jun	3441	4380	2378

Table 2: Relatively Uneven Demand Data across Assembly Lines

The alternative policy results in the maximum idle time of a day on a line to be 163 minutes while the standard policy results in the maximum idle time of a day on a line to be 226 minutes.

We observe that which policy is superior depends on the pattern of the daily demand. Based on this observation, we note that various hybrid policies are possible by mixing the standard and alternative policies depending on the number of remaining reels, the number of remaining chips on each reel, and the pattern of the remaining daily demands.

Thus far, we have examined the allocation policy of the reels. Let us now proceed to investigate the inventory as well as allocation policies of the reels by formulating an approximate mathematical model. Here, as a first step, we will look into an elementary chip inventory model based on a (Q, R) model (see e.g., [5] and/or [6]) in Section 3. This implies that the daily demand will now be stochastic (in practice, the demand in the immediate future will be relatively deterministic while the demand in the distant future will be relatively stochastic). This will be followed by an integrated model linking the inventory policy to the allocation policy in Section 4.

3. (Q, R) Model

The basic mechanics of the (Q, R) model is as follows: Demand occurs randomly. When the inventory position reaches the reorder point R, a replenishment order for quantity Q is placed. Thus the inventory position is at $R+Q$. After the lead time, the order is received. The problem is how to determine the appropriate number of chips and the number of reels given the following assumptions and definitions.

3.1 Assumptions
1. The inventory will be reviewed continuously. 2. Demands occur one at a time (no overshoot). 3. Unfilled demand is backordered. 4. Replenishment lead times are fixed and known.

3.2 Notations and the model
- A: Ordering cost
- c: Unit cost for regular order
- h: Inventory holding cost
- Q: Order quantity
- R: Re-order point
- D: Annual demand
- $\overline{D}$: Average Annual Demand
- t: Lead time
- D_t: Demand during lead time
- Π: Backorder cost
- $\overline{b}$ (R): The expected number of stock outs in a cycle
- F(R): The probability of demand during lead time greater than re-order point
- TCPC: Total cost per cycle
- TCPUT: Total cost per unit time
- Q*: Optimal order quantity
- R*: Optimal reorder point

TCPUT can be obtained by dividing TCPC by the cycle time, resulting in

$$TCPUT = \frac{A\overline{D}}{Q} + c\overline{D} + h\left(\frac{Q}{2} + R - \overline{D}\,t\right) + \Pi\frac{\overline{D}}{Q}\left(\overline{b}(R)\right) \tag{1}$$

From the first order necessary conditions for minimizing TCPUT, we obtain

$$Q^* = \sqrt{\frac{2\overline{D}[A + \Pi\overline{b}(R)]}{h}}$$

$$1 - F(R^*) = \frac{hQ}{\Pi \overline{D}}$$

These two equations can be solved iteratively for the optimal order quantity and the optimal reorder point.

Based on this (Q, R) model as well as the allocation policies developed previously, we propose the following integrated inventory and allocation model.

4. Integrated Model for Inventory and Allocation

Let us first define the following notations for the integrated inventory and allocation model.

4.1 Notations and the model

- m = Number of reels ordered in a given order of Q chips
- n = Number of chips per reel
- M = Number of storage slots originally assigned for the reels of a critical chip.
- I^+ = Cost per reel for storage when the number of reels exceeds M
- I^- = Cost per reel for storage when the number of reels is less than or equal to M
- C_r = Cost of a blank reel in \$ per reel
- C_c = Cost of a chip in \$ per chip
- P_I = Penalty for idle time in \$ per hour.
- T_I = Total idle time in hours in a year.

The integrated model of inventory and allocation considers the optimal number of reels (m^*) as well as the optimal number of chips per reel (n^*) in an order quantity of Q chips. At the same time, the idle time in hours in a year (T_I) in the assembly facility is incorporated in this model. The idle time is defined as the sum of the time spent waiting for a reel and the setup time for a new reel when the old reel is exhausted. The total idle time is defined to be the sum of all idle times of all assembly lines of all days in a year. We note that the idle time aspect can be represented by different measurements. For example, the maximum daily idle time of a line. Hence, different versions of $P_I T_I$ are possible.

The objective of the integrated model is to minimize the Expected Modified Cost Per Unit Time (EMCPUT) where EMCPUT = Setup cost + Purchase cost + Holding cost + Storage cost + Shortage cost + Penalty due to idle time. Mathematically,

$$EMCPUT = A\frac{\overline{D}}{Q} + (C_r m + C_c n)\frac{\overline{D}}{Q} + h\left(\frac{Q}{2} + R - \overline{D}_t\right) + \max\left[(m-M)I^+, (M-m)I^-\right] + \Pi \overline{b}(R)\frac{\overline{D}}{Q} + P_I T_I$$

Since $Q = mn$

$$EMCPUT = A\frac{\overline{D}}{mn} + (C_r m + C_c n)\frac{\overline{D}}{mn} + h\left(\frac{mn}{2} + R - \overline{D}_t\right) + \max\left[(m-M)I^+, (M-m)I^-\right] + \Pi \overline{b}(R)\frac{\overline{D}}{mn} + P_I T_I$$

(2)

The decision variables are m, n, and R. We now elaborate on the relationship of the inventory and allocation policies.

4.2 On Optimal m^*, n^* and R^*

For some initial values of m, n and R, the corresponding idle time can be obtained based on an allocation policy. Given this idle time, Equation (2) can be used to compute the minimum EMCPUT. In the neighborhood of this initial point, it is possible to find a steepest descent direction and perform a line search. This will result in a new set of m, n, and R values. Given the new values of m, n and R, we can find the corresponding idle time, and this iterative process can continue until we have converging m, n and R values. Such values will be the approximate optimal values of m^*, n^* and R^*.

We now present a simplified numerical example that illustrates the relationship between the allocation and inventory policies.

4.3 Numerical Example:
For the numerical example, the following assumptions are employed.
- The blank reel cost is relatively small, hence only the chip cost will be considered.
- Initially the number of reels is equal to the number of lines (i.e., no extra reel is available.).
- Reorder point is when all the chips get exhausted (R=0).
- There is no lead time and order is delivered immediately.

In addition, we will assume that the idle time is represented by the total idle time of the facility in a year. Finally, let us assume the following hypothetical data.

The average annual demand $\bar{D}$ = 1592604 (chips/year) is obtained by multiplying the demand for the month of June (from the anonymous company) by 12. Also, let Cc = $1/unit, C_r = $0/unit, A= $50, h = i* Cc = $0.25/unit/year, M = 2, I^+ = $500/year/reel, I^- = $100/year/reel, P_I = $100/hr. Since R = 0 and lead time = 0 days, there is no shortage, hence $\bar{b}$ (R) = 0. For Q = one month demand = 132717, we now can evaluate EMCPUT for different values of m under different allocation policies.

4.3.1 Standard Policy
If m = 3 reels, then $n = Q/m = \dfrac{132717}{3} = 44239$ chips/reel. The corresponding total idle time per year under the standard allocation policy can be obtained to be T_I =12 hrs. In obtaining this number, we assume that the demand date for the month of June will repeat every month, and the set up time required for a reel is 30 minutes.

Based on these values of parameters and computation, the EMCPUT is given by,
$EMCPUT$ = $ 549757.625

Similarly, for the given Q value, using m = 2 and m = 1, we can obtain the corresponding $EMCPUT$ values. The overall results are summarized in the table along with the corresponding n and T_I values

m (no. of reels)	n (no. of chips/reel)	T_I (hour/year)	EMCPUT ($)
1	132717	2894.4	1899333.625
2	66359	487.8	862271.625
3	44239	12	549757.625

Table 3: Numerical Results under Standard Policy

4.3.2 Alternative Policy
The numerical results under the alternative policy for the same Q value and the corresponding m, n, and T_I values are summarized as below.

m (no. of reels)	n (no. of chips/reel)	T_I (hour/year)	EMCPUT ($)
1	132717	959.6	1705853.625
2	66359	219	835391.625
3	44239	112.4	559797.625

Table 4: Numerical Results under Alternative Policy

As stated earlier, in the neighborhood of these initial points, it is possible to find a steepest descent direction and perform a line search. This will result in a new set of *m, n* and *R* values. This iterative process can continue until we have converging *m, n* and *R* values. Also, based on our computational experience on

the data from the anonymous company thus far, a hybrid allocation policy (to start with the standard policy and then switch to the alternative policy when one of the reels gets exhausted) appeared to be superior to both the pure standard and alternative policies. Finally, in our opinion, the allocation policy that is optimal within the assembly operation only may not be optimal when both assembly and inventory operations are simultaneously considered.

5. Conclusion

In this paper, we have explored the allocation and inventory policies for the reels used in a PCB production facility consisting of a multiple number of assembly lines. As a first step to study this complex problem, our focus has been on a single type of chips that is critical for the assembly of numerous types of PCB's. Specifically, we have developed a couple of allocation policies and demonstrated how they can be executed via numerical examples. We observed that various hybrid policies are possible by mixing the standard and alternative policies depending on the number of remaining reels, the number of remaining chips on each reel, and the pattern of the remaining daily demands.

Also, we have integrated these policies into the traditional (Q, R) inventory model and showed how EMCPUT can be computed given a set of m, n, and R values. In addition, we have suggested how to achieve a set of optimal m, n, and R values using line search along the direction of steepest descent. Finally, we noted that the allocation policy that is optimal within the assembly operation only may not be optimal when both assembly and inventory operations are simultaneously considered.

As for future studies, we plan to work on an integrated simulation model. The demand will be randomly generated, and the corresponding EMCPUT will be computed based on a pre-determined rules on the allocation and inventory policies. A set of optimal m, n, and R values may be obtained via stochastic optimization using some type of experimental design (e.g., Central Composite Design [7]). It will be of interest to compare and contrast the approximate approach in this paper and the suggested simulation approach.

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