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SOLVENT EXTRACTION STUDY OF THE
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NITRATES - NITRIC ACID - WATER - TRI-
BUTYL PHOSPHATE.

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SOLVENT EXTRACTION STUDY OF THE SYSTEM MONAZITE RARE EARTH
NITRATES - NITRIC ACID - WATER - TRIBUTYL PHOSPHATE

by

Brooks Martin Sharp

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NOMENCLATURE

The following three paragraphs are explanatory notes on the nomenclature used in this thesis.

(1) All molalities are expressed in terms of the nitrate group associated with the solute in question. For example, a solution containing one kilogram of water and one molecular weight (324.9 gms.) of $\text{La}(\text{NO}_3)_3$ would be three molal $\text{La}(\text{NO}_3)_3$.

(2) For a doubly subscripted quantity, the first subscript is the solute subscript and the second subscript is a "location" subscript, referring to the phase, stream, or stage with which the quantity is associated.

(3) If used as a subscript S, R, and F denote that the quantity subscripted refers to a stream in an extraction cascade. If used as a variable S, R, and F denote the solvent flow rate in a stream in an extraction cascade. In the text, a reference to stream R_n or S_n denotes a reference to the stream as a whole.

- M - molality, moles of a solute per kilogram of solvent
- K - distribution coefficient, ratio of molality of a solute in an organic phase to molality of the solute in the equilibrium aqueous phase
- β - separation factor, ratio of two distribution coefficients
- X - ratio of molality of a solute in aqueous solution to total molality of the solution

- x - ratio of molality of a rare earth solute in aqueous solution to total rare earth molality of the solution
- Y - ratio of molality of a solute in organic solution to total molality of the solution
- y - ratio of molality of a rare earth solute in organic solution to total rare earth molality of the solution
- S - flow rate of solvent in an organic stream, kgm. TBP/unit time
- R - flow rate of solvent in an aqueous stream, kgm. H₂O/unit time
- F - flow rate of solvent in a feed stream, kgm. H₂O/unit time
- T - total number of solutes
- α - flow rate ratio, S/R

Subscripts

- i,j - arbitrary solute
- t - total solutes present, expressed as equivalents of nitrate
- RE - total rare earth nitrate solutes present
- La - lanthanum nitrate
- Pr - praseodymium nitrate
- Nd - neodymium nitrate
- Sm - samarium nitrate
- HNO₃ - nitric acid
- n - arbitrary stage in extraction cascade
- org. - organic phase
- aq. - aqueous phase

SUMMARY

Using single stage equilibrium data for the systems shown below, plus the assumption of mutual immiscibility of water and tributyl phosphate (TBP), a calculation method was developed to give the stagewise conditions in a cascade of equilibrium stages with all species present in the flow streams.

- I $\text{La}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$
- II $\text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$
- III $\text{Nd}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$
- IV $\text{Sm}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$
- V $\text{La}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$
- VI $\text{Nd}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$
- VII $\text{Sm}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$

The calculation method was checked via a series of simulated column experiments. The agreement between predicted and experimental stagewise conditions was considered reasonable, and the calculation is considered useful for engineering work.

As the calculation is tedious and time consuming, digital computer programs were written for the IBM 7074 to calculate the stagewise conditions in cascades of interest. It is hoped these programs will be useful in design and optimization studies.

INTRODUCTION

The elements of atomic number 57 to 71 are commonly known as the rare earths or lanthanides. They occur in nature as rather complex mixtures, and due to their chemical similarity are quite difficult to separate. The term rare earths is a misnomer, and these elements represent a potentially rich source of metals. The abundance, industrial potential, physical properties, and chemistry of the rare earths have been discussed extensively in the recent literature (1,2,3,4, 5,6,7) and will not be elaborated on here.

Spedding and Powell (8) have developed ion exchange techniques which have been used at the Ames Laboratory of the Atomic Energy Commission and by several chemical companies to prepare the pure lanthanides in industrial quantities. The work discussed in this thesis is a continuation of a long term project sponsored by the Chemical Engineering Division of Ames Laboratory to investigate the application of solvent extraction to the separation of the rare earths. It is thought that either a solvent extraction process or a combination of solvent extraction and ion exchange might possess certain advantages over the present process, such as more continuous operation and larger throughputs per dollar of capital investment.

Specifically, the purpose of this research was to propose and verify a calculation method to give the stagewise conditions

in a multistage cascade operating with the system $\text{La}(\text{NO}_3)_3$ - $\text{Pr}(\text{NO}_3)_3$ - $\text{Nd}(\text{NO}_3)_3$ - $\text{Sm}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O . If considered desirable, as was the case, the method was to be programmed for digital computer calculation.

The basic problem in this project, as in many projects of this type, was to develop a method of calculating the conditions in one equilibrium phase with the contacting equilibrium phase completely specified. Because of the large number of solutes present it is practically impossible to systematically investigate the equilibrium of the system of interest over the range of composition and concentration encountered in a solvent extraction cascade. Therefore the basic approach was to use experimentally determined equilibrium data for contributing systems to predict the equilibrium of the more complex system of interest.

The function of the nitric acid is threefold. The solvent, TBP, has a relatively high viscosity and a specific gravity approximately equal to that of water (about 0.98 at 20°C.). These characteristics have led to slow phase separation and emulsion formation in the past. The inclusion of nitric acid promotes rapid phase separation and inhibits emulsion formation. Also, the separation factors between the rare earth nitrates have been found to increase with increasing nitrate concentration of the system. The inclusion of nitric acid allows high nitrate concentrations without the

phase separation and emulsion problems mentioned above. On the debit side of the discussion, nitric acid is also extracted by TBP and complicates the system as an extra solute which must be considered.

PREVIOUS WORK

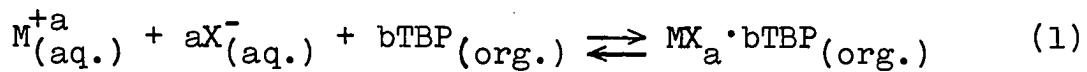
The first study of solvent extraction separation of the rare earths was made in 1937 by Fischer et al. (9). Since that time a large number of systems have been investigated, primarily by single stage "shakeups", and often using radioactive rare earth tracers. Typical studies are (10,11,12,13, 14,15). The field has also been reviewed extensively (16,17, 18,19).

In recent years most workers have followed one of two distinct approaches. The more fundamental approach has been based on the thermodynamic equilibrium constants for postulated complexing reactions between the solutes in the aqueous phase and the organic extractant. In contrast, several workers have approached the problem from a pragmatic point of view, obtaining equilibrium data for simple systems and then making approximations to extrapolate the available data to systems of interest. These two approaches are considered in more detail in the following sections.

Thermodynamic Equilibrium Constant Approach

Many thermodynamic studies have been reported in the literature, typical studies being (20,21,22,23,24,25). In general, the conclusions were that the extraction of many

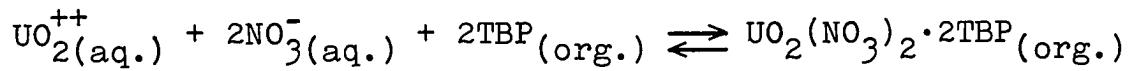
solutes such as nitric acid and rare earth nitrates from aqueous solution by TBP may be considered the result of a reaction of the type



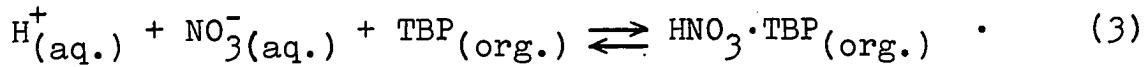
where b is the solvation number, that is, the number of TBP molecules complexed with one molecule of the extracted compound. For the rare earth nitrates b has been established to be three, and for nitric acid b is usually taken as one, although for nitric acid solubility considerations show such a simple picture to be impossible. That is, the solubility of nitric acid in TBP grossly exceeds the theoretical maximum as given by complete formation of $HNO_3 \cdot TBP$ in the organic phase. The present trend (26,27,28) is to consider the nitric acid extraction as an initial formation of $HNO_3 \cdot TBP$ followed by solubility of excess acid in the complex.

Rozen and Khorkhorina (29) have approached the equilibrium of the system $UO_2(NO_3)_2 - HNO_3 - TBP$ plus diluent - H_2O by a consideration of the proposed complexing reactions

(2)



and



The equilibrium constant values for these reactions plus

material balances make possible a calculation of the equilibrium organic concentrations if the aqueous concentrations are known. Several questionable assumptions are, however, necessary. No activity data are available for the reactants in the organic phase and so the activity coefficients of all organic constituents (the free TBP and the solvated complexes) are assumed to be unity. Activity measurements of uranyl nitrate in aqueous solution and of nitric acid in aqueous solution are available (30), but no justification is given by the authors for their use of these activity data for mixtures of uranyl nitrate and nitric acid. Also, as pointed out previously, the proposed reaction involving nitric acid is inadequate over the allowable nitric acid concentration range.

Jury and Whatley (31) have written a digital computer subroutine based on the equations of Rozen and Khorkhorina to calculate equilibrium points in the system discussed above. They do not compare their results with distribution data. Referring to their calculated distribution curves the authors conclude "Beyond the insipid observation that these curves seem to follow the behavior of what is known about the system, there is no quantitative criteria available for evaluating their accuracy". Although many of their calculated distributions appear reasonable, it is to be noted that under some conditions such as low values of the TBP concentration and high

values of the uranyl nitrate concentration the equations predict negative values of the solvent phase nitric acid concentration, which is, of course, impossible.

Olander (32) has written a digital computer program for multistage, multicomponent cascade design making use of equations of the form of Equation 1 alternating with material balance calculations. He assumed:

- (1) unit activity coefficients for all species,
- (2) complete dissociation of the solutes in the aqueous phase, and
- (3) no competing side reactions.

The first two assumptions are certainly in error to some undetermined extent, and the third is open to question. His program is for a center fed cascade and involves a multiple trial and error, the extract and raffinate conditions first being assumed and then working toward the center stage by stage from both ends with a match at the feed point the criteria of correct choice of the end streams. He states that it is impossible at present to estimate activity coefficients of mixed electrolytes containing more than two solutes and also to estimate the activity coefficients of the complexes and the unbound TBP in the organic phase. Consequently his calculation must be regarded solely as an order of magnitude approximation to the true number of theoretical stages.

Empirical Approach

Weaver et al. (33) produced better than a kilogram of 95% pure gadolinium oxide starting with a mixture of rare earth oxides containing about 30% gadolinium oxide. They used a 60:40 mixture of TBP in a hydrocarbon diluent and extracted a highly acidified solution of the rare earth nitrates many times in both batch extractions and counter-current extractions in a York Scheibel column. Their approach was primarily trial and error, empirically adjusting the operating conditions from extraction to extraction.

Peppard and Mason (34) report several simulated column extractions of milligram quantities of mixed rare earths. The aqueous phases were either acidified chloride or nitrate solutions and the organic extractant was TBP either undiluted or mixed with a light hydrocarbon diluent. No design procedure was given. In each case they obtained two enriched fractions, the light rare earths preferring the aqueous and the heavies preferentially extracting into the organic.

A series of reports by Michigan Chemical Corporation (35, 36, 37, 38, 39, 40, 41, 42, 43) describe the bench and pilot plant scale separation of yttrium from the rare earths and the division of the rare earths into a light enriched and a heavy enriched fraction. Some work was also done in attempting to strip lanthanum from a rare earth and yttrium mixture. The

solvent used in the latter study was monododecyl phosphoric acid plus hydrocarbon diluent and the aqueous phase acidified chloride or acetate solutions of the rare earths. In some cases a chelating agent was added to the aqueous strip. Yttrium of 98% purity was obtained.

At Ames Laboratory of the Atomic Energy Commission Bochinski (44) demonstrated the partial separation of the rare earths. He distributed the acid free nitrates between undiluted, water equilibrated TBP and water. He developed a design procedure based on two assumptions concerning the equilibrium distribution of the system. They are:

- (1) an average equilibrium curve may be used for all mixtures of the rare earths and
- (2) the separation factors between the rare earths are independent of the composition of the rare earth solutes and a function only of the total concentration of solutes present in an equilibrium stage.

Bochinski then "pinched" the total solute concentration, thus by assumption 2 holding the separation factors constant. He based his calculations on an oxides per liter basis.

Using Bochinski's method Schoenherr (45) set up a pilot plant separation run. Because of the aforementioned phase separation and emulsion formation problems he acidified the system with nitric acid. The stages were preloaded with a fixed nitric acid concentration, the feed and strip were also

at this acid concentration, and the organic was pre-equilibrated with an aqueous of the specified acidity. Under these conditions it was hoped that the acid concentration in the system would remain constant and Bochinski's method could be used to predict the stagewise rare earth compositions and concentrations. This condition was not realized and it was found that the acid, if included, must be considered as a solute. Schoenherr (46), Sharp (47), and Dinnin (48) then collected distribution data for various acidified systems, the object being to predict the equilibrium of the complex acidified systems of interest.

The project described in this thesis is essentially a continuation of this work carried out at the Ames Laboratory. After a consideration of the state of development of the field of thermodynamic chemistry applied to concentrated, multicomponent ionic solutions, it was the opinion of the author that an empirical approach of this type was more likely to yield a realistic design procedure than the thermodynamic equilibrium constant approach.

METHOD OF CALCULATION

In this section the proposed calculation method will be discussed in detail. The data used in the calculation method are single stage "shake-up" distributions for the systems

- I $\text{La}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$,
- II $\text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$,
- III $\text{Nd}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$,
- IV $\text{Sm}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$,
- V $\text{La}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$,
- VI $\text{Nd}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$,
- VII $\text{Sm}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$.

The data for systems I, II, V, VI, and VII were collected by the author by the method described in detail by Sharp (47). The bulk of the data for systems III and IV were taken from the work of Schoenherr (46), supplemented to a small degree by work of the author. A complete compilation of the equilibrium data used is given in Appendix B.

Equilibrium Phase Calculation

The method to be discussed in this section is the calculation of all concentrations of one equilibrium phase with all concentrations of the contacting phase specified. The general procedure was:

- (1) to obtain values of K_t and K_{HNO_3} by interpolation using data obtained from the contributing two solute systems and then use the defining equations

$$K_t = (M_t)_{org.} / (M_t)_{aq.}, \quad (4)$$

$$K_{HNO_3} = (M_{HNO_3})_{org.} / (M_{HNO_3})_{aq.}, \quad (5)$$

$$M_{RE} = M_t - M_{HNO_3}, \quad (6)$$

to calculate the total molality, HNO_3 molality, and the rare earth molality of the unspecified phase, followed by

- (2) the use of separation factors between the rare earths obtained from data for the contributing three solute systems in one of the equations

$$(M_i)_{org.} = \frac{(M_{RE})_{org.} \times \beta_{i-Pr} \times (M_i)_{aq.}}{\sum_{i=1}^T (\beta_{i-Pr} \times (M_i)_{aq.})}, \quad (7)$$

$$(M_i)_{aq.} = \frac{(M_{RE})_{aq.} \times (M_i)_{org.}}{\beta_{i-Pr} \times \sum_{i=1}^T ((M_i)_{org.} / \beta_{i-Pr})}, \quad (8)$$

to calculate the rare earth molalities of the unspecified phase. Equations 7 and 8 follow directly from the definition of separation factor and are applicable, of course, only to the rare earths present.

Considering this calculation in more detail, the data from systems I, II, III, and IV were processed to give a series of plots of K_t and K_{HNO_3} versus the composition of an equilibrium phase with the total molality of the same phase as parameter. Typical plots are shown in Figures 1 through 8. A complete compilation of the values from these plots is given in Appendix D.

The separation factors from systems V, VI, and VII were plotted versus the total molality of the organic phase as shown in Figure 9. In each case the rare earth separation factors are seen to be, to a good approximation, a function of the total nitrate molality of the equilibrium organic phase and relatively independent of the phase composition. This observation parallels Bochinski's (44) observation of the acid free system. The curves of Figure 9 were fitted to straight lines by the least squares method to give

$$\beta_{La-Pr} = 0.8187 - 0.1106(M_t)_{org.}, \quad (9)$$

$$\beta_{Nd-Pr} = 1.0448 + 0.09874(M_t)_{org.}, \quad (10)$$

$$\beta_{Sm-Pr} = -0.3795 + 0.9214(M_t)_{org.}. \quad (11)$$

These correlations were assumed to hold with $(M_t)_{org.}$ greater than 1.75.

At this point a gross assumption was made. It was assumed that Equations 9, 10, and 11 would hold also in the five

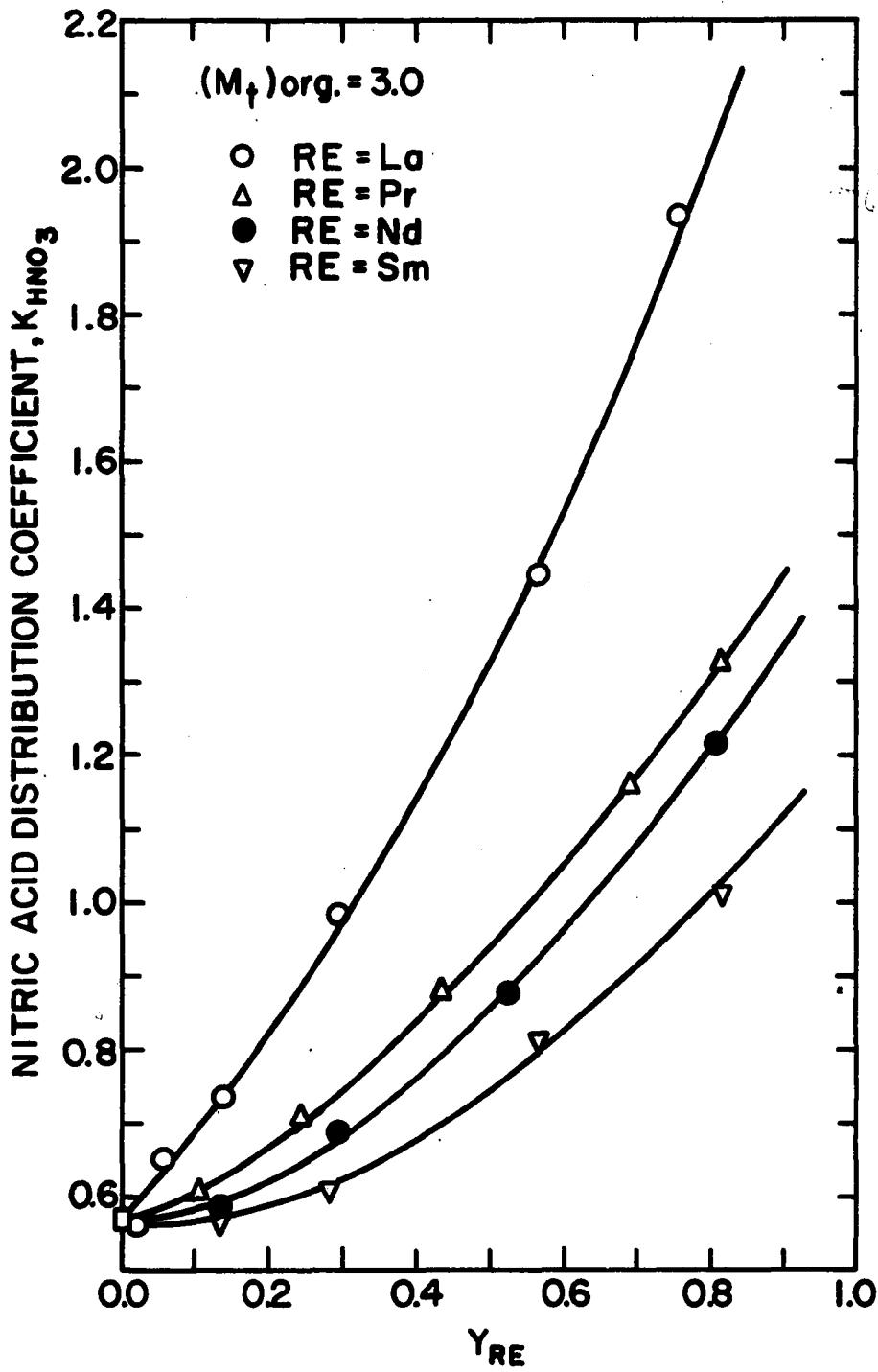


Figure 1. Nitric acid distribution coefficient for $RE(NO_3)_3$ - HNO_3 - TBP - H_2O systems as a function of the composition of the organic phase at $(M_t)_{org.}$ equal to 3.0

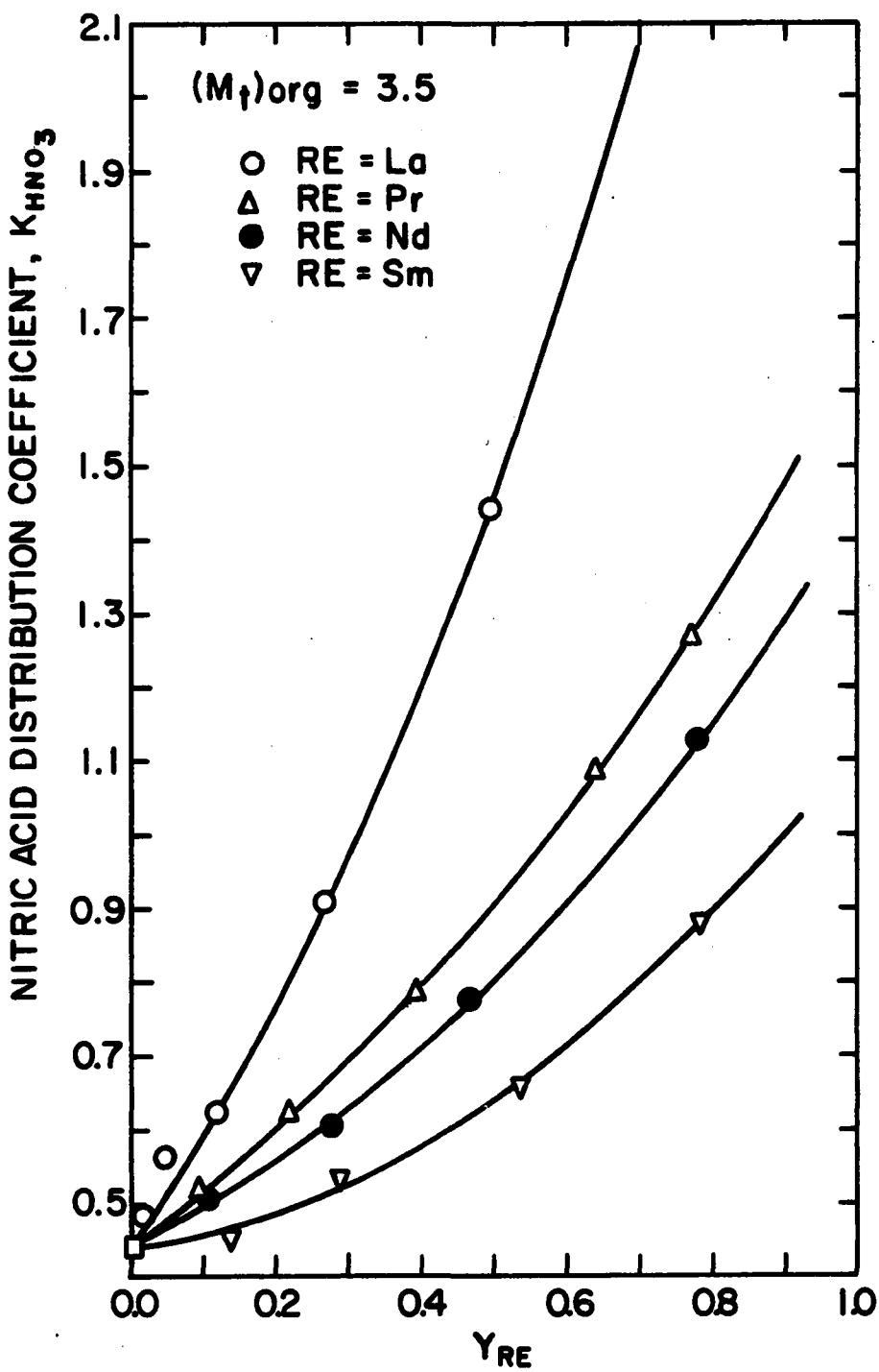
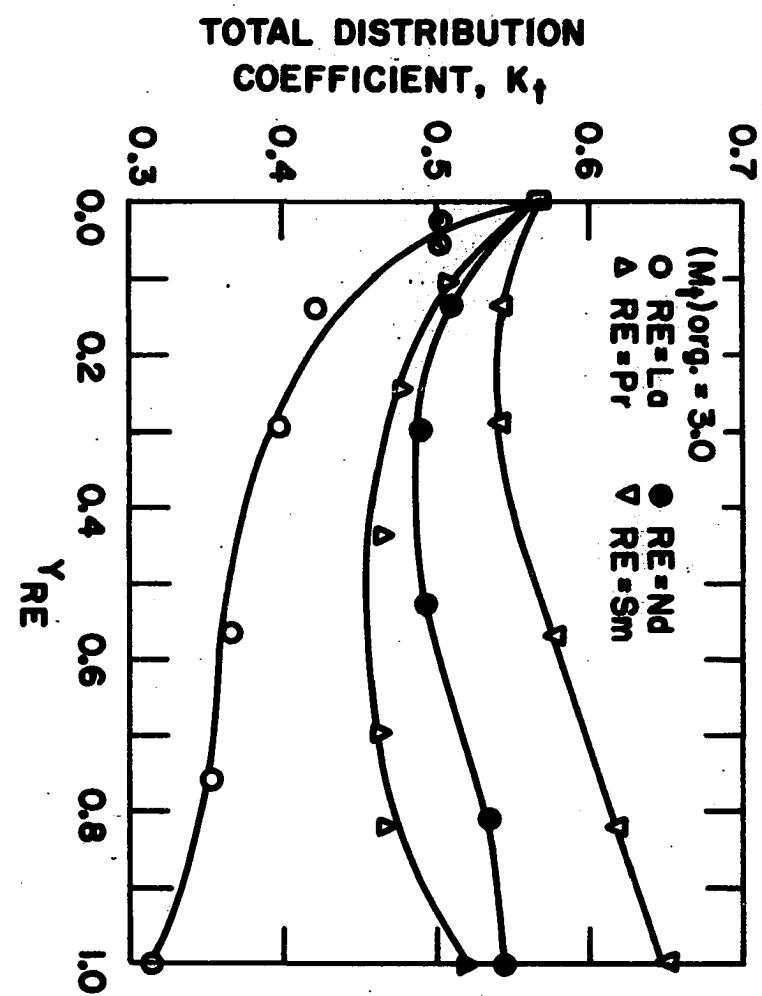
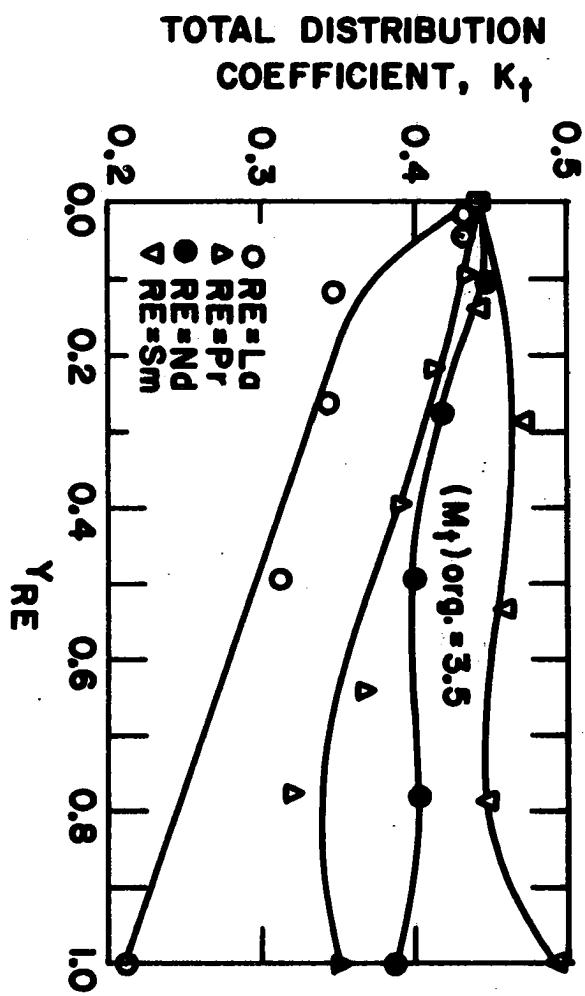


Figure 2. Nitric acid distribution coefficient for $RE(NO_3)_3$ - HNO_3 - TBP - H_2O systems as a function of the composition of the organic phase at $(M_t)_{org}$ equal to 3.5

Figure 3. Total distribution coefficient for $\text{RE}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O systems as a function of the composition of the organic phase at $(M_t)_{\text{org}}$ equal to 3.0

Figure 4. Total distribution coefficient for $\text{RE}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O systems as a function of the composition of the organic phase at $(M_t)_{\text{org}}$ equal to 3.5



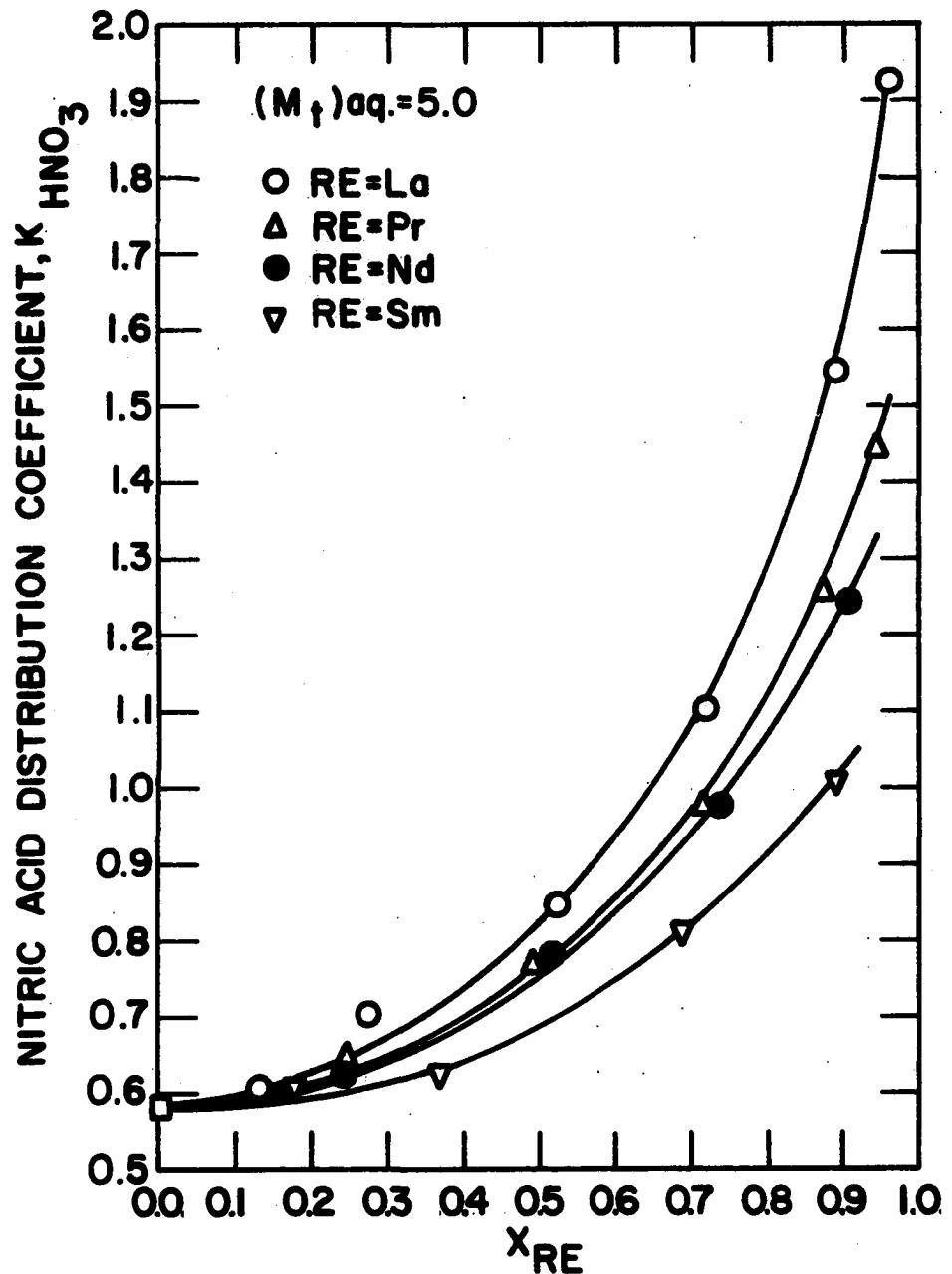


Figure 5. Nitric acid distribution coefficient for $RE(NO_3)_3$ - HNO_3 - TBP - H_2O systems as a function of the composition of the aqueous phase at $(M_t)_{aq.}$ equal to 5.0

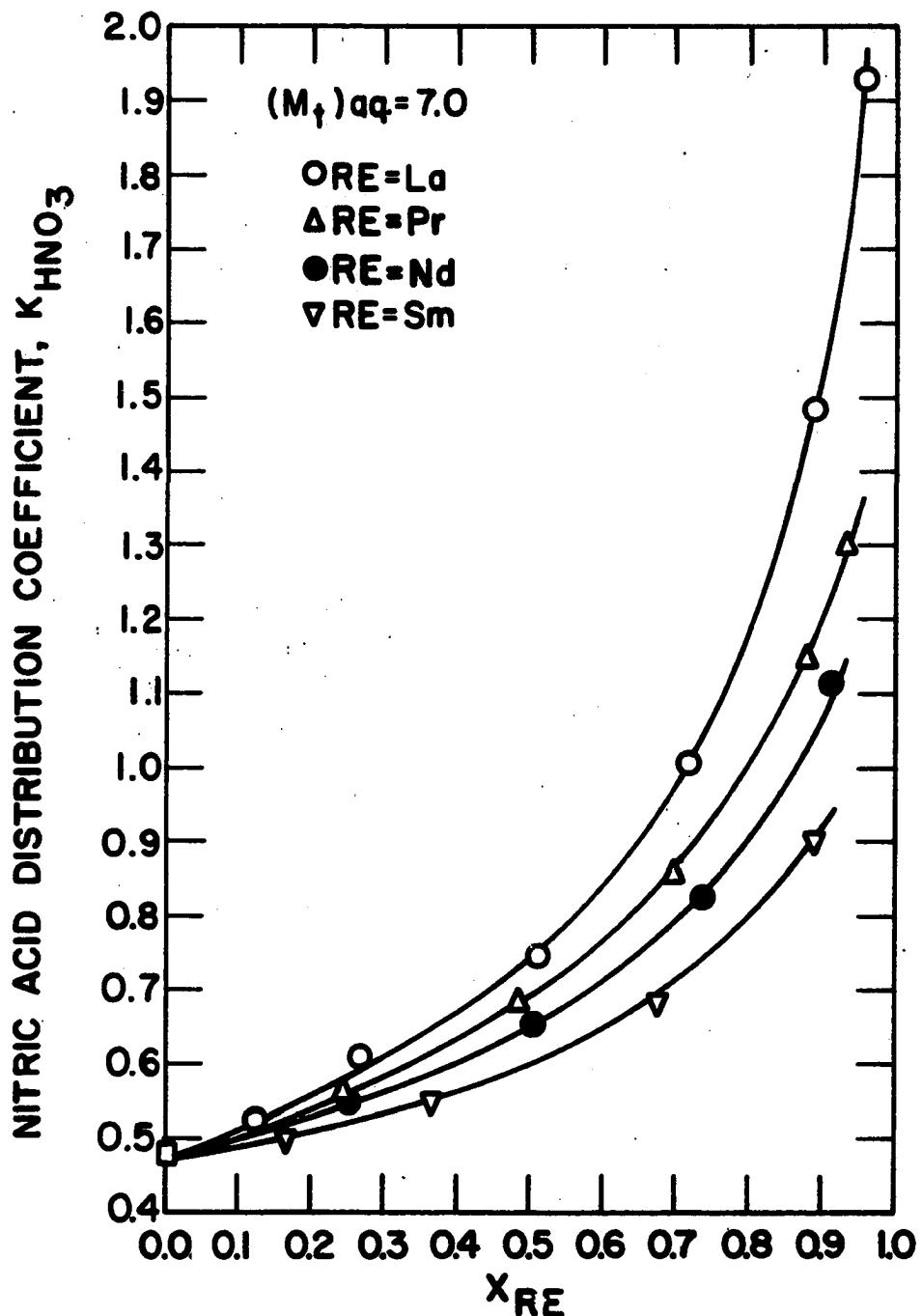
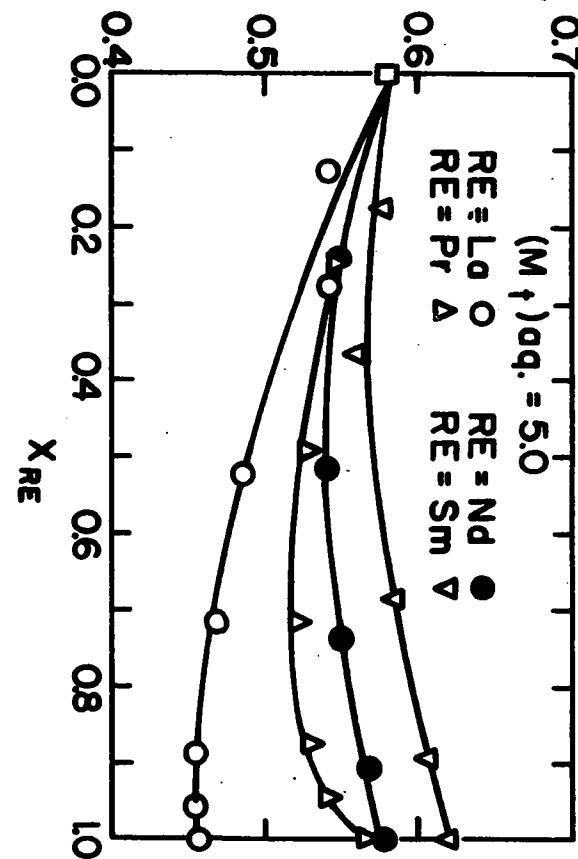
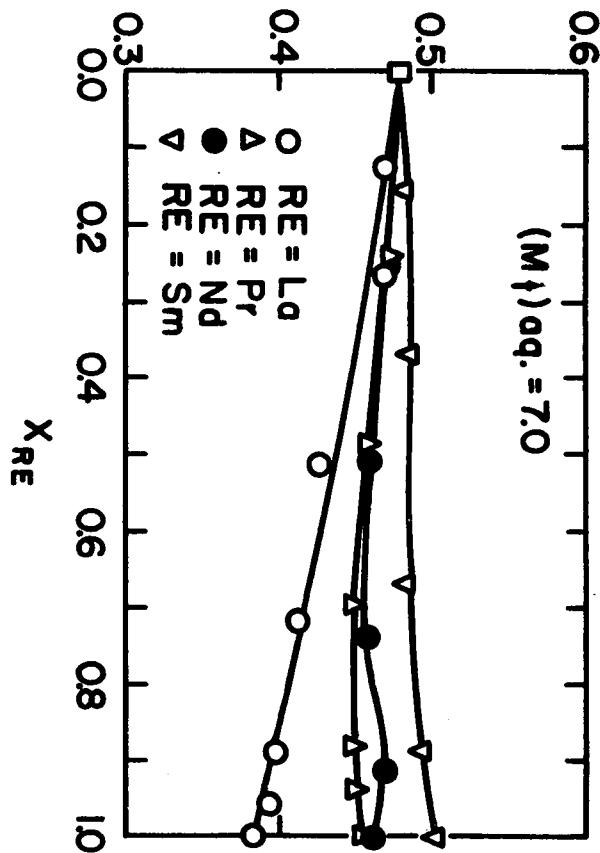


Figure 6. Nitric acid distribution coefficient for $\text{RE}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O systems as a function of the composition of the aqueous phase at $(M_t)_{aq}$ equal to 7.0

Figure 7. Total distribution coefficient for $\text{RE}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O systems as a function of the composition of the aqueous phase at $(M_t)_{\text{aq}}$. equal to 5.0

Figure 8. Total distribution coefficient for $\text{RE}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O systems as a function of the composition of the aqueous phase at $(M_t)_{\text{aq}}$. equal to 7.0

TOTAL DISTRIBUTION COEFFICIENT, K_t



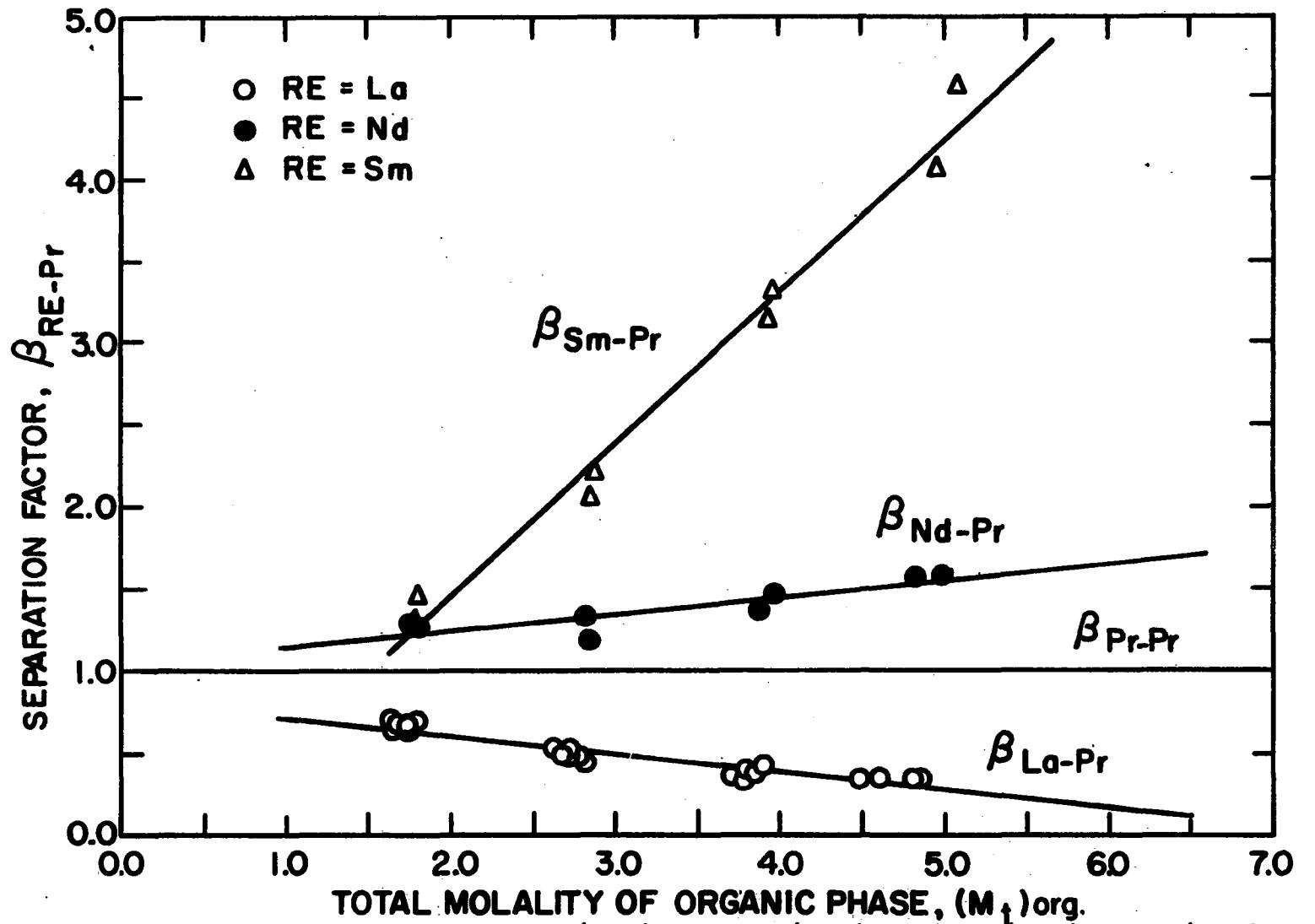


Figure 9. Separation factor between $RE(NO_3)_3$ and $Pr(NO_3)_3$ for $RE(NO_3)_3 - Pr(NO_3)_3 - HNO_3 - TBP - H_2O$ systems as a function of concentration of the organic phase

solute system $\text{La}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{Nd}(\text{NO}_3)_3 - \text{Sm}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$. The limitations of such a sweeping assumption may only be ascertained empirically, therefore the usefulness of the approximation must be judged by the comparison with experimental results in the Verification of Model section.

Sample calculation

A sample calculation is now given to illustrate this method. It was assumed that an equilibrium organic phase has the following concentrations:

$$(M_t)_{\text{org.}} = 3.0261 ,$$

$$(M_{\text{HNO}_3})_{\text{org.}} = 2.3590 ,$$

$$(M_{\text{La}})_{\text{org.}} = 0.0397 ,$$

$$(M_{\text{Pr}})_{\text{org.}} = 0.0460 ,$$

$$(M_{\text{Nd}})_{\text{org.}} = 0.0706 ,$$

$$(M_{\text{Sm}})_{\text{org.}} = 0.5108 ,$$

$$(M_{\text{RE}})_{\text{org.}} = 0.6671 .$$

From these data were calculated

$$Y_{\text{HNO}_3} = (M_{\text{HNO}_3})_{\text{org.}} / (M_t)_{\text{org.}} = 2.3590 / 3.0261 = 0.7796 ,$$

$$Y_{\text{RE}} = (M_{\text{RE}})_{\text{org.}} / (M_t)_{\text{org.}} = 0.6671 / 3.0261 = 0.2204 ,$$

$$y_{\text{La}} = (M_{\text{La}})_{\text{org.}} / (M_{\text{RE}})_{\text{org.}} = 0.0397 / 0.6671 = 0.0595 ,$$

$$y_{\text{Pr}} = \frac{(M_{\text{Pr}})_{\text{org.}}}{(M_{\text{RE}})_{\text{org.}}} = 0.0460/0.6671 = 0.0690 ,$$

$$y_{\text{Nd}} = \frac{(M_{\text{Nd}})_{\text{org.}}}{(M_{\text{RE}})_{\text{org.}}} = 0.0706/0.6671 = 0.1058 ,$$

$$y_{\text{Sm}} = \frac{(M_{\text{Sm}})_{\text{org.}}}{(M_{\text{RE}})_{\text{org.}}} = 0.5108/0.6671 = 0.7657 .$$

To obtain the value of K_t it was first assumed that all the rare earth nitrates present were $\text{La}(\text{NO}_3)_3$. That is, $y_{\text{RE}} = y_{\text{La}} = 0.2204$. Then, by straight line interpolation between the total molality parameters of Figures 3 and 4, a value of K_t was obtained that would hold for the case of $(M_t)_{\text{org.}} = 3.0261$ and $y_{\text{RE}} = y_{\text{La}} = 0.2204$. This was $K_t = 0.4031$. Exactly the same procedure was then followed to get values for K_t assuming, in succession, that all the rare earths present were $\text{Pr}(\text{NO}_3)_3$, $\text{Nd}(\text{NO}_3)_3$, and $\text{Sm}(\text{NO}_3)_3$. To then get the K_t used in the calculation each K_t derived from the assumption of only one rare earth solute was multiplied by the fraction of the rare earths present as that particular solute. In other words, the contribution from the $\text{La}(\text{NO}_3)_3$ system to the final value of K_t was assumed to be equal to the value of K_t for the $\text{La}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ system multiplied by y_{La} . Or arithmetically, the contribution was $(0.4031) \times (0.0595) = 0.0240$. The sum of the four contributions was then the value of K_t used.

Exactly the same procedure, using Figures 1 and 2 was then used to get a value of K_{HNO_3} for the case in question.

The values so obtained were

$$K_t = 0.5200 ,$$

$$K_{HNO_3} = 0.6114 .$$

The values of $(M_t)_{aq.}$, $(M_{HNO_3})_{aq.}$, and $(M_{RE})_{aq.}$ were then calculated from Equations 4, 5 and 6 to be

$$(M_t)_{aq.} = (M_t)_{org.}/K_t = 3.0261/0.5200 = 5.8194 ,$$

$$(M_{HNO_3})_{aq.} = (M_{HNO_3})_{org.}/K_{HNO_3} = 2.3590/0.6114 = 3.8584 ,$$

$$(M_{RE})_{aq.} = (M_t)_{aq.} - (M_{HNO_3})_{aq.} = 5.8194 - 3.8584 = 1.9610 .$$

Using $(M_t)_{org.} = 3.0261$ the separation factors were calculated as

$$\beta_{La-Pr} = 0.4840 ,$$

$$\beta_{Nd-Pr} = 1.3436 ,$$

$$\beta_{Sm-Pr} = 2.4087 .$$

Note that, by definition, β_{Pr-Pr} is equal to 1.0.

The rare earth molalities of the aqueous phase were then calculated by Equation 8 as follows:

$$\sum_{i=1}^T ((M_i)_{org.}/\beta_{i-Pr}) = (M_{La})_{org.}/\beta_{La-Pr} + (M_{Pr})_{org.}/\beta_{Pr-Pr} + (M_{Nd})_{org.}/\beta_{Nd-Pr} + (M_{Sm})_{org.}/\beta_{Sm-Pr} ,$$

$$\sum_{i=1}^T ((M_i)_{org.}/\beta_{i-Pr}) = 0.0397/0.4840 + 0.0460/1.0000 + 0.0706/1.3436 + 0.5108/2.4087 ,$$

$$\sum_{i=1}^T ((M_i)_{org.}/\beta_{i-Pr}) = 0.3926$$

$$(M_{La})_{aq.} = \frac{(M_{RE})_{aq.} \times (M_{La})_{org.}}{\beta_{La-Pr} \times \sum_{i=1}^T ((M_i)_{org.}/\beta_{i-Pr})}$$

$$= \frac{(1.9610)(0.0397)}{(0.4840)(0.3926)} = 0.4097$$

and in exactly the same manner

$$(M_{Pr})_{aq.} = 0.2298 ,$$

$$(M_{Nd})_{aq.} = 0.2625 ,$$

$$(M_{Sm})_{aq.} = 1.0592 .$$

If the reverse calculation had been desired, the same procedure would have been followed, first using plots of the type portrayed in Figures 5, 6, 7, and 8 to get K_t and K_{HNO_3} , calculating then $(M_t)_{org.}$, $(M_{HNO_3})_{org.}$, and $(M_{RE})_{org.}$, and then, after calculating the separation factors, using Equation 7 to calculate the rare earth molalities.

Application to Multistage Calculations

It was desired to apply the model to the calculation of the stagewise conditions in an ideal multistage cascade. Referring to Figures 10 and 11 it is seen that if any point on

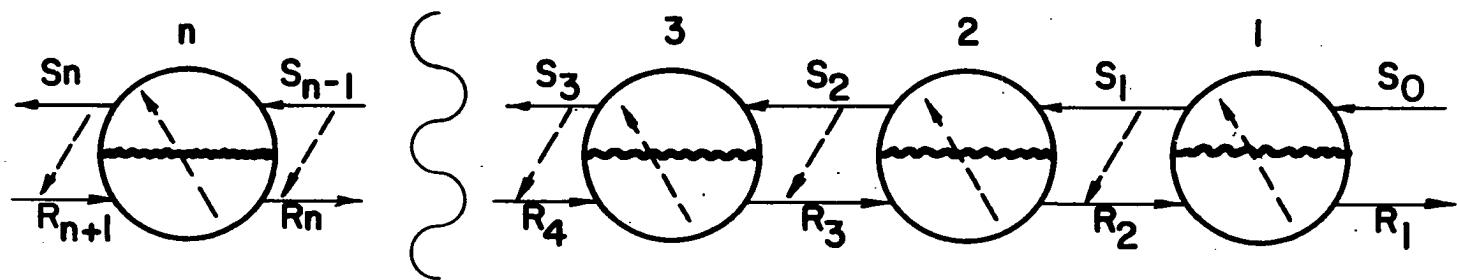


Figure 10. Cascade I

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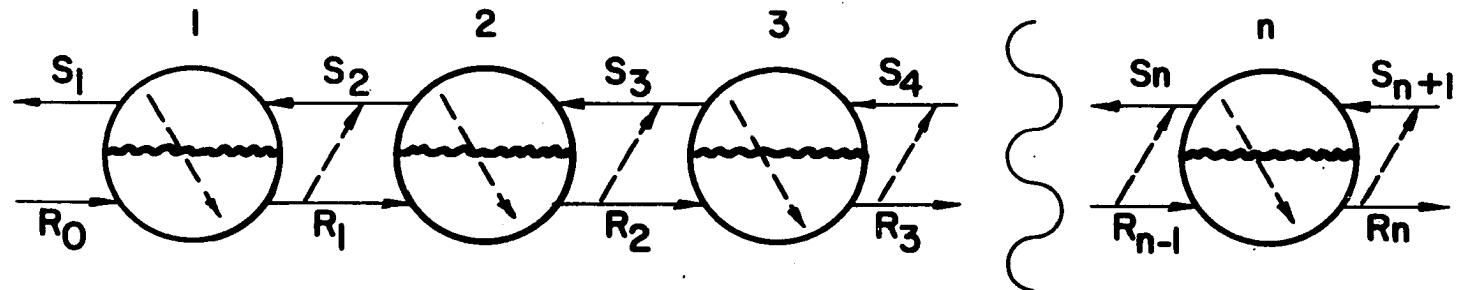


Figure 11. Cascade II

the operating line is known, that is, if two streams passing each other are completely defined, the conditions for any number of stages may be calculated "backwards" through the column if the operating line equation is known. The direction such a calculation would follow is shown by the dashed arrows in Figures 10 and 11. Obviously, but to ensure clarity, the calculation would be composed of alternating applications of the equilibrium model and the operating line.

In the present work, the assumption of mutual immiscibility of water and TBP was made. Writing a general operating line for the cascade pictured in Figure 10 gives

$$\left[(M_i)_{S_0} \times S_0 \right] + \left[(M_i)_{R_n} \times R_n \right] = \left[(M_i)_{S_{n-1}} \times S_{n-1} \right] + \left[(M_i)_{R_1} \times R_1 \right]. \quad (12)$$

By the assumption of mutual immiscibility $S_0 = S_{n-1}$ and $R_n = R_1$ and so the subscripts may be deleted, and after rearrangement

$$(M_i)_{R_n} = \alpha(M_i)_{S_{n-1}} - \alpha(M_i)_{S_0} + (M_i)_{R_1}. \quad (13)$$

The assumption of mutual immiscibility of water and TBP was felt to be justified. The solubility of TBP in water is very low, on the order of tenths of a gram per liter, and the solubility of water in TBP does not exceed approximately six percent by weight. Because the solvent is normally equi-

librated with water before use, the assumption of constant water content does not introduce a large error.

Using the proposed equilibrium model and operating lines of the form of Equation 13 several calculations of the stage-wise conditions in various cascades were carried out. The procedure was to arbitrarily choose the end conditions, either R_1 and S_0 for the cascade in Figure 10, or R_0 and S_1 for the cascade in Figure 11, and then to proceed as explained previously. The purposes of these calculations were to obtain a "feel" for the possible separations that could be made with the system and to select several predictions for experimental verification.

At this point an unexpected problem arose. In several of the calculations the total molality was "pinched" at a high value. This was done to impress high separation factors on the system, since, with the equilibrium model proposed the separation factors increase with increased total concentration. In each case in which this was done the individual concentrations also quickly "pinched". When the end conditions and flow ratio were chosen so that the total concentration changed from stage to stage no such "pinch" of the individual solutes was calculated.

The study of this "pinch" based on the limit of a sequence that can be written describing the molality of a general component in successive streams of a cascade is an inter-

esting problem the author has not been able to solve completely. A discussion of this approach is given in Appendix C.

As the stagewise calculation is quite tedious and time consuming, four digital computer programs were written for various cascades of interest. These programs were compiled and debugged on the IBM 7074. Although some calculations were made with each program they are intended primarily for future use in design and optimization studies, and as guides for further programming. A complete discussion of these programs is given in Appendix D.

VERIFICATION OF MODEL

A specification of the source and method of preparation of the materials used in the experimental phase of this project follows:

Tributyl phosphate Commercial grade TBP, obtained from Commercial Solvents Corporation, was pre-equilibrated with distilled water before use as the solvent. The TBP was stored in contact with a water phase to ensure equilibrium.

Nitric acid Reagent grade nitric acid was used in all laboratory work.

Rare earth nitrate solutions The oxides of La, Pr, Nd, and Sm, 99.9 percent pure, were obtained from the ion exchange group of the Ames Laboratory of the Atomic Energy Commission. Stock nitrate solutions of each rare earth were prepared by dissolving the oxides in boiling, concentrated nitric acid. After complete reaction, the excess nitric acid was boiled off and distilled water added. All feed solutions for the simulated column experiments were prepared by mixing these stock solutions, reagent grade concentrated nitric acid and distilled water.

As mentioned previously, the simulated column technique was used to obtain experimental stagewise conditions for comparison with the predicted values. Briefly, a simulated column experiment is a batchwise approximation to a continuous counter-

current cascade. The error in the approximation may be made arbitrarily small by increasing the number of cycles the experiment is run. The mechanics and mathematical demonstrations involved will not be gone into here as this technique has been discussed extensively by Scheibel (49,50,51,52).

Four of the prediction calculations discussed in the Method of Calculation section were chosen for simulation. The experiments were carried out in laboratory glassware at room temperature, using 1/100 horsepower laboratory stirrers for the mixing steps. A minimum of two minutes mixing and three minutes settling time was maintained for each contact.

Along with the phases from the final cycle, the leaving aqueous phase from each cycle was retained and analyzed as a check on the approach to steady state. In all cases the number of cycles was felt to be sufficient.

All organic and aqueous phases from the final cycle were analyzed for total oxides, nitric acid content, and rare earth composition. The analytical methods used are discussed in Appendix A.

A complete compilation of the experimental conditions and the final experimental-predicted comparisons from these four simulations is given on pages 32 to 52. In each case, the first figure gives all pertinent experimental conditions, including the simulation pattern and the continuous cascade being simu-

lated, and is followed by both tabular and graphic experimental-predicted concentration and composition comparisons.

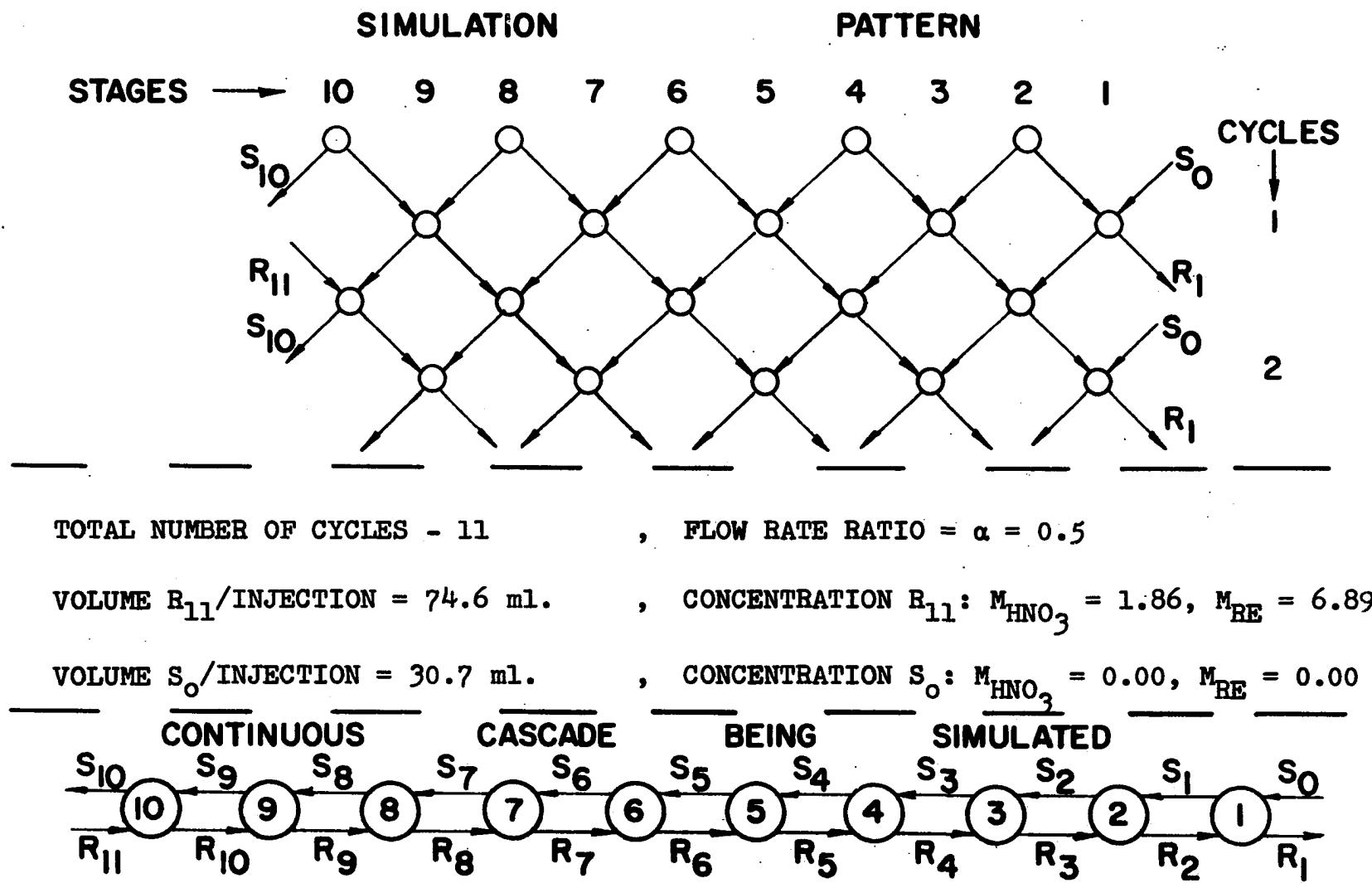


Figure 12. General information pertaining to Simulated Column Experiment I

Table 1. Comparison of experimental and predicted compositions and concentrations for Simulated Column Experiment I

	Weight % La_2O_3 in total oxides		Weight % $\text{Pr}_{6\text{O}}_{11}$ in total oxides		Weight % Nd_2O_3 in total oxides		Weight % Sm_2O_3 in total oxides		Grams total oxides per gram of so- lution		Grams HNO_3 per gram of solution	
	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.
R ₁	32.3	32.3	25.2	25.3	25.6	25.0	17.0	17.3	0.191	0.195	0.039	0.037
R ₂	30.0	29.5	26.8	24.8	25.0	25.6	18.2	20.1	0.208	0.211	0.055	0.053
R ₃	28.0	29.2	25.4	24.5	25.9	25.4	20.7	21.0	0.206	0.208	0.060	0.060
R ₄	28.8	29.3	24.7	24.4	25.7	25.3	20.8	21.1	0.204	0.207	0.062	0.062
R ₅	28.4	29.3	26.4	24.4	25.4	25.2	19.8	21.0	0.204	0.207	0.062	0.062
R ₆	29.9	29.3	25.2	24.4	25.0	25.2	19.9	21.0	0.203	0.207	0.062	0.063
R ₇	29.7	29.3	24.8	24.4	25.4	25.2	20.2	21.0	0.202	0.207	0.062	0.063
R ₈	29.1	29.3	24.4	24.4	28.8	25.2	17.8	21.0	0.202	0.207	0.062	0.063
R ₉	29.2	29.3	24.8	24.4	26.5	25.2	19.5	21.0	0.202	0.207	0.062	0.063
R ₁₀	30.1	29.3	24.9	24.4	26.0	25.2	18.9	21.0	0.205	0.207	0.062	0.063
R ₁₁	29.0	29.3	24.9	24.4	26.5	25.2	19.6	21.0	0.204	0.207	0.062	0.063
S ₁	12.1	13.1	22.4	21.6	28.4	28.8	37.1	36.6	0.088	0.088	0.054	0.054

Table 1 (Continued)

	Weight % La_2O_3 in total oxides		Weight % Pr_6O_{11} in total oxides		Weight % Nd_2O_3 in total oxides		Weight % Sm_2O_3 in total oxides		Grams total oxides per gram of so- lution		Grams HNO_3 per gram of solution
	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.
S ₂	8.0	10.0	19.9	19.1	28.8	27.4	43.3	43.5	0.085	0.083	0.071 0.074
S ₃	9.4	9.6	18.8	18.6	28.1	26.7	43.8	45.2	0.082	0.080	0.076 0.080
S ₄	9.3	9.5	19.1	18.5	27.5	26.6	44.1	45.4	0.081	0.078	0.078 0.082
S ₅	9.4	9.6	19.0	18.5	27.1	26.6	44.5	45.4	0.080	0.078	0.078 0.083
S ₆	10.0	9.6	18.7	18.5	27.0	26.6	44.4	45.3	0.081	0.078	0.078 0.083
S ₇	10.1	9.6	19.0	18.5	27.2	26.6	43.6	45.3	0.080	0.078	0.078 0.083
S ₈	7.5	9.6	20.5	18.5	28.3	26.6	43.7	45.3	0.081	0.078	0.078 0.083
S ₉	10.2	9.6	19.0	18.5	26.0	26.6	44.8	45.3	0.081	0.078	0.077 0.083
S ₁₀	8.2	9.6	19.8	18.5	27.5	26.6	44.5	45.3	0.081	0.078	0.077 0.083

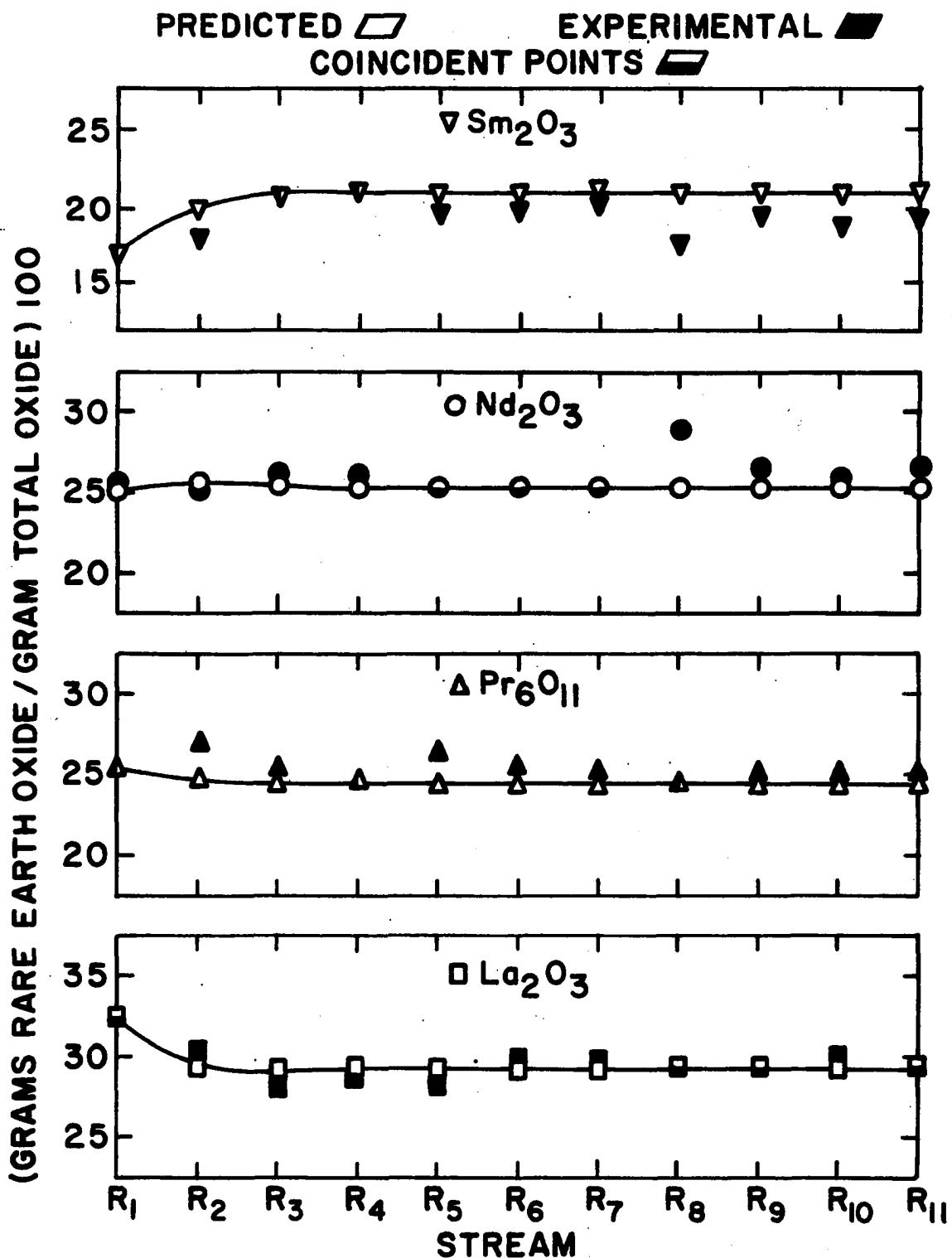


Figure 13. Stagewise aqueous rare earth oxide composition for Simulated Column Experiment I

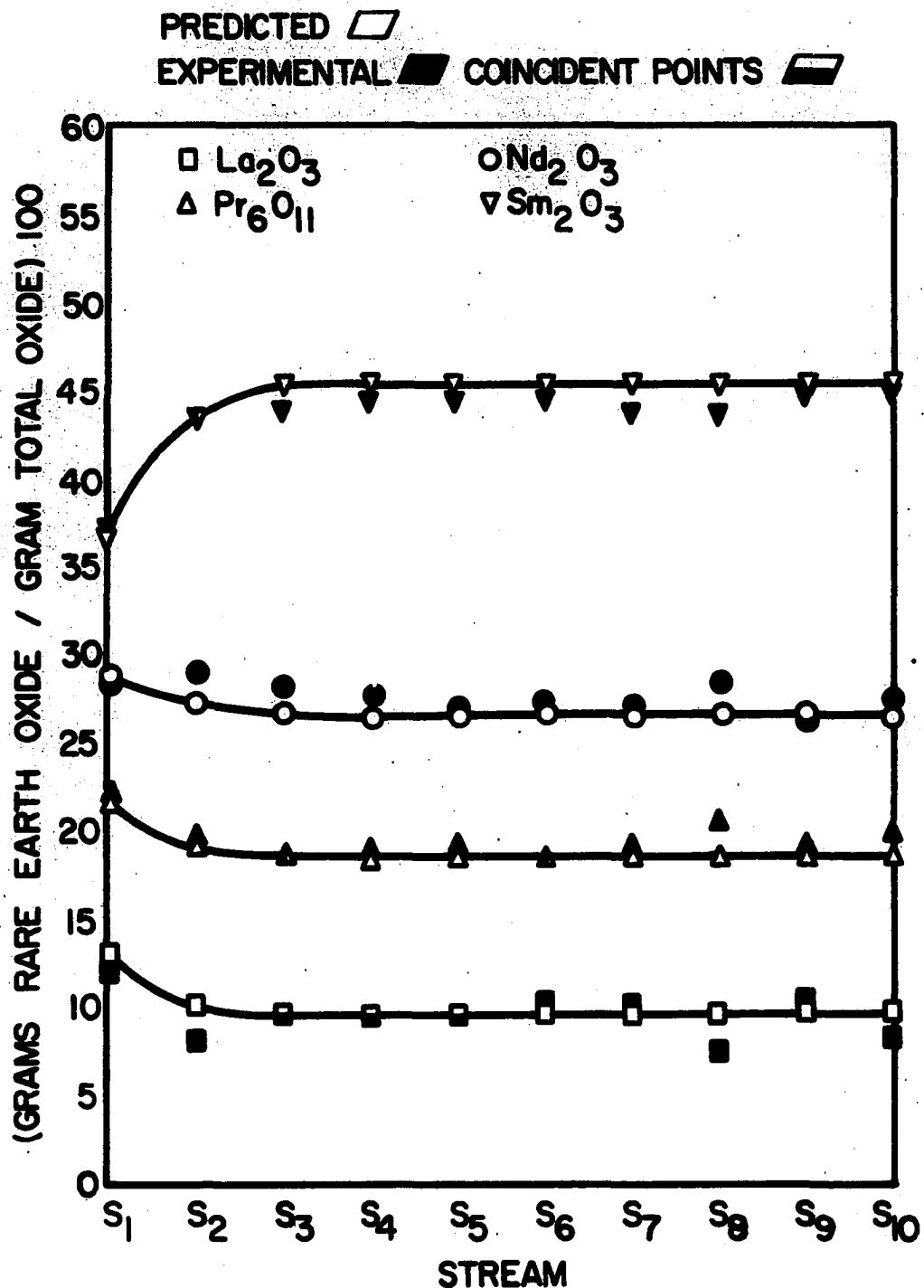


Figure 14. Stagewise organic rare earth oxide composition for Simulated Column Experiment I

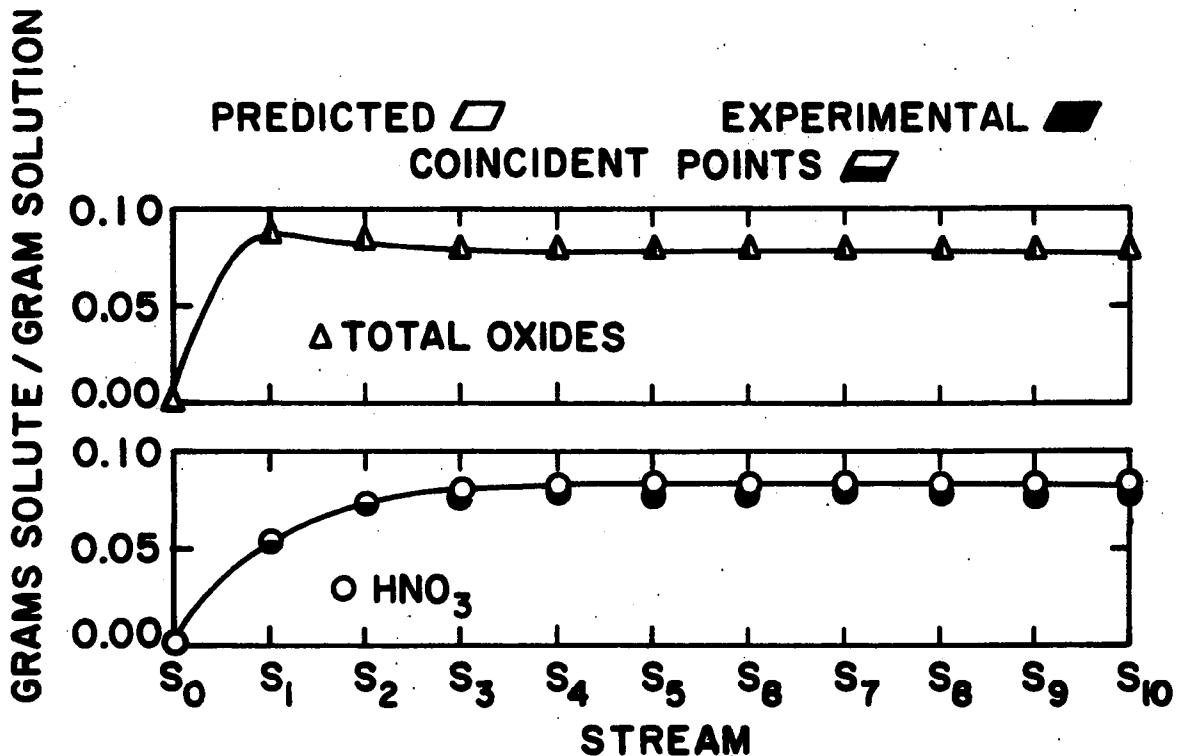


Figure 15. Stagewise organic rare earth oxide and nitric acid concentrations for Simulated Column Experiment I

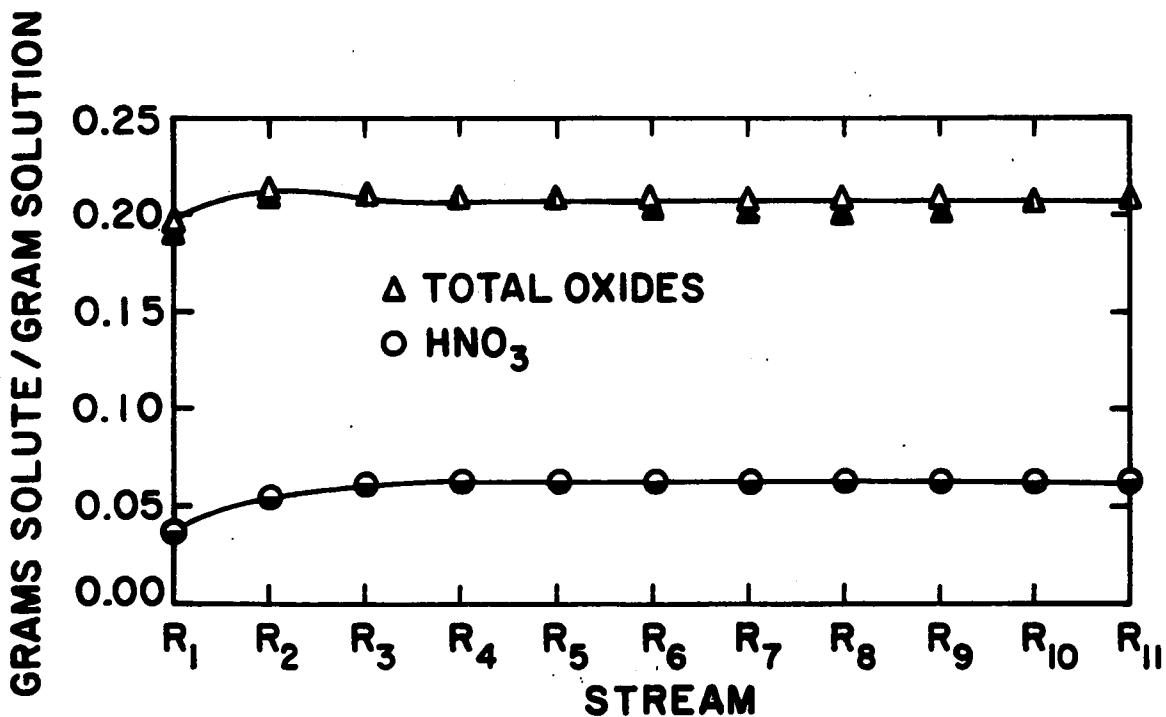


Figure 16. Stagewise aqueous rare earth oxide and nitric acid concentrations for Simulated Column Experiment I

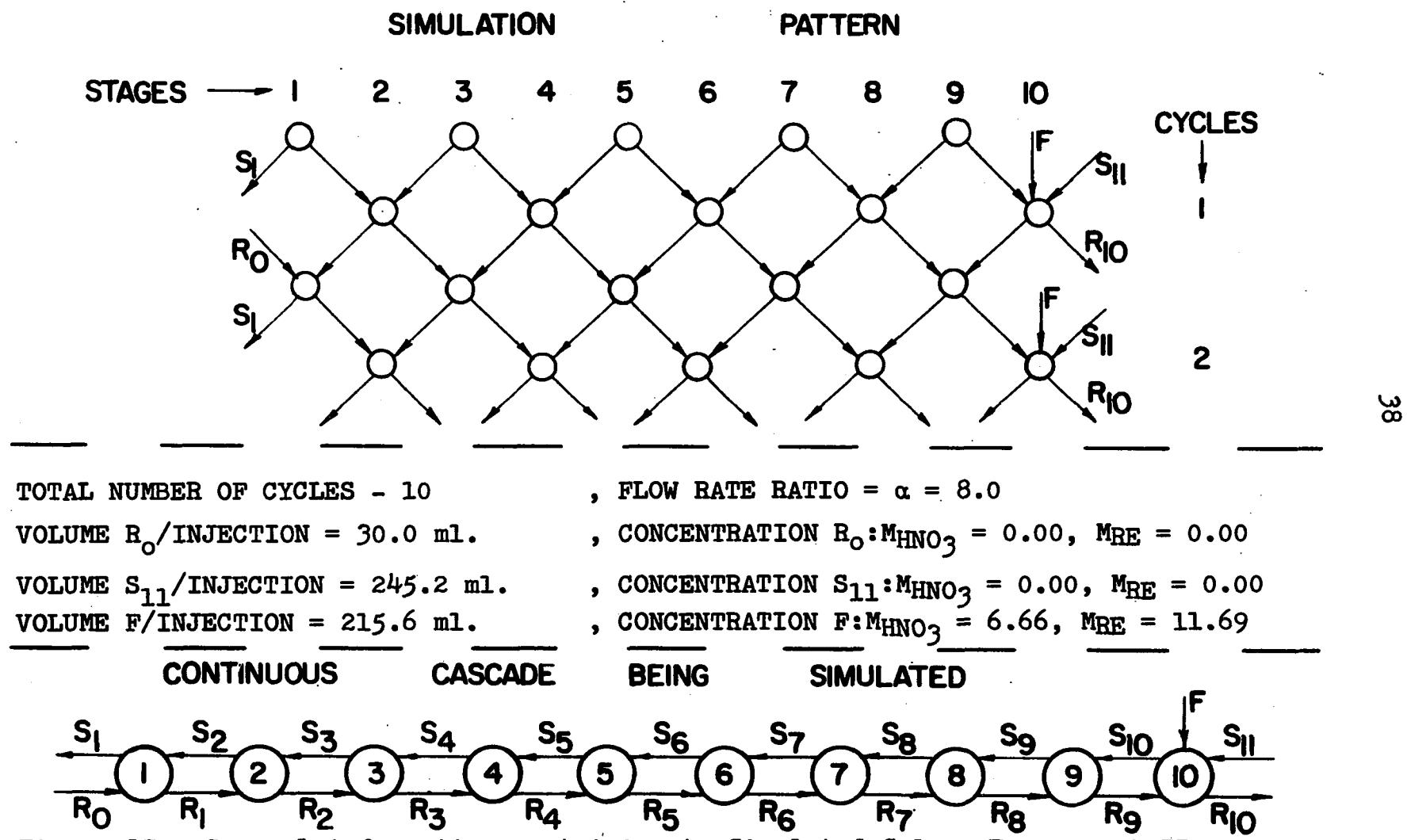


Figure 17. General information pertaining to Simulated Column Experiment II

Table 2. Comparison of experimental and predicted compositions and concentrations for Simulated Column Experiment II

	Weight % La_2O_3 in total oxides	Weight % $\text{Pr}_{6\text{--}11}$ in total oxides	Weight % Nd_2O_3 in total oxides	Weight % Sm_2O_3 in total oxides	Grams total oxides per gram of so- lution	Grams HNO_3 per gram of solution
	exp. pred.	exp. pred.	exp. pred.	exp. pred.	exp. pred.	exp. pred.
R ₁	7.3 11.2	10.4 9.5	11.4 12.1	70.9 67.3	0.052 0.058	0.139 0.136
R ₂	14.6 19.9	12.9 11.7	11.9 13.2	60.6 55.2	0.072 0.076	0.168 0.167
R ₃	26.4 31.0	15.0 12.3	10.5 12.6	48.2 44.1	0.087 0.092	0.172 0.171
R ₄	37.4 43.4	14.7 11.5	8.6 10.9	39.3 34.2	0.103 0.108	0.165 0.164
R ₅	50.6 55.4	12.0 9.8	7.0 8.7	30.4 26.0	0.122 0.126	0.152 0.151
R ₆	62.8 65.8	9.8 7.9	4.2 6.7	23.2 19.6	0.140 0.145	0.136 0.136
R ₇	72.9 74.0	7.5 6.1	2.6 5.1	17.0 14.9	0.163 0.166	0.117 0.119
R ₈	82.3 80.0	3.4 4.6	1.2 3.9	13.2 11.4	0.184 0.186	0.102 0.104
R ₉	81.3 84.4	6.1 3.6	2.5 3.1	10.1 8.9	0.204 0.207	0.088 0.090
R ₁₀	86.4 87.8	5.8 2.8	1.9 2.4	5.8 7.0	0.220 0.228	0.076 0.077
F	83.0 81.2	2.9 3.0	0.8 3.0	13.2 12.8	0.235 0.238	0.153 0.156
S ₁	0.0 3.8	5.7 5.9	10.8 9.7	83.5 80.6	0.023 0.024	0.108 0.107

Table 2 (Continued)

	Weight % La_2O_3 in total oxides		Weight % $\text{Pr}_{6\text{--}11}$ in total oxides		Weight % Nd_2O_3 in total oxides		Weight % Sm_2O_3 in total oxides		Grams total oxides per gram of so- lution		Grams HNO_3 per gram of solution	
	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.
S ₂	3.4	5.6	7.1	6.8	11.9	10.3	77.6	77.3	0.029	0.031	0.124	0.122
S ₃	6.5	9.0	8.2	7.8	12.5	10.9	72.8	72.3	0.032	0.035	0.127	0.126
S ₄	10.5	14.1	8.9	8.4	12.4	10.8	68.3	66.7	0.034	0.037	0.129	0.127
S ₅	18.6	20.7	9.2	8.3	11.2	10.2	61.0	60.7	0.037	0.040	0.128	0.126
S ₆	25.5	28.3	9.3	7.8	10.5	9.3	54.6	54.6	0.040	0.044	0.126	0.125
S ₇	36.3	36.1	7.4	6.9	9.9	8.2	46.5	48.8	0.044	0.047	0.124	0.122
S ₈	45.4	43.4	5.7	6.0	8.4	7.1	40.5	43.5	0.047	0.052	0.121	0.119
S ₉	51.2	49.8	5.4	5.1	7.5	6.2	35.9	38.8	0.052	0.056	0.117	0.116
S ₁₀	50.7	55.5	5.0	4.4	5.7	5.5	38.6	34.6	0.057	0.062	0.115	0.114

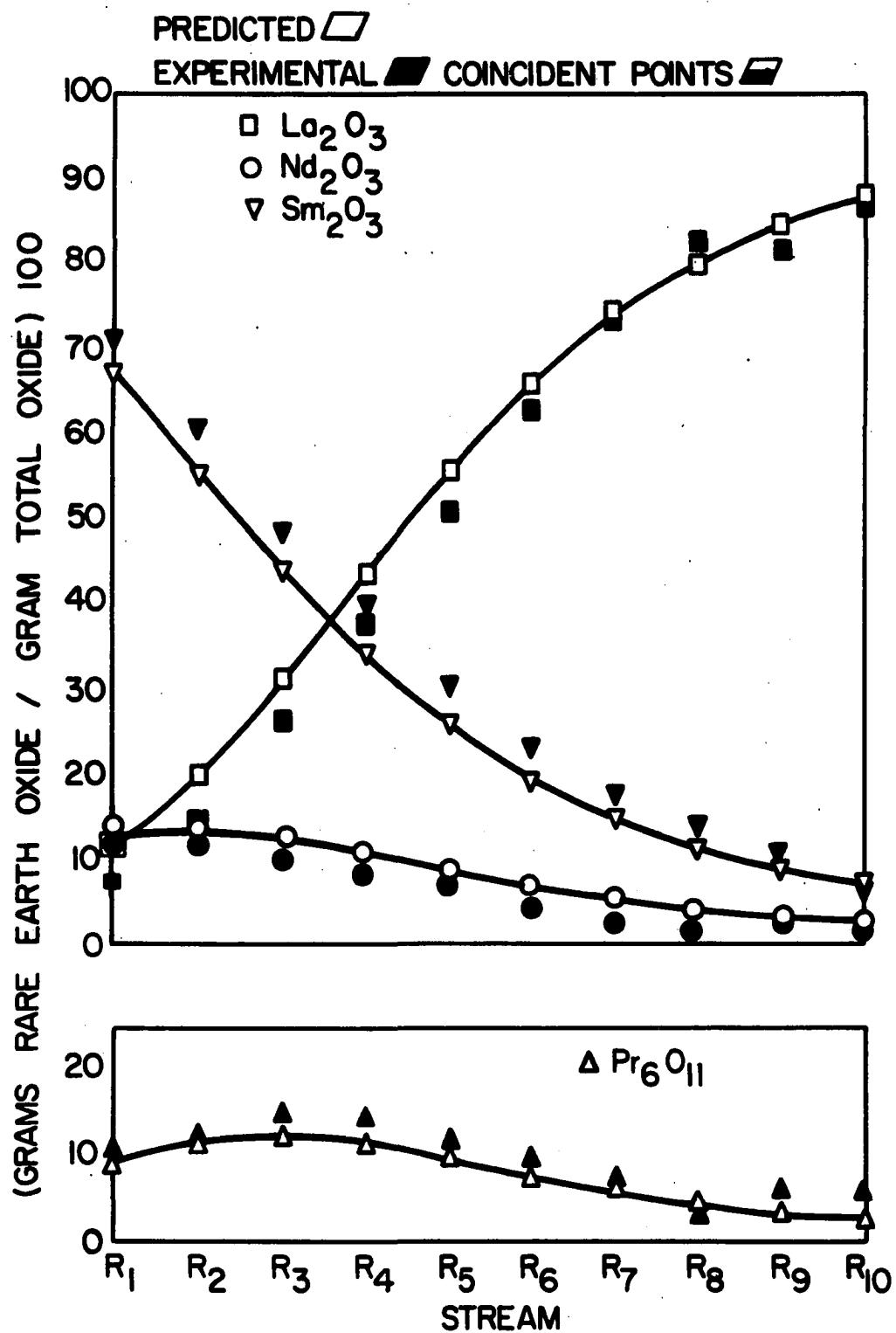


Figure 18. Stagewise aqueous rare earth oxide composition for Simulated Column Experiment II

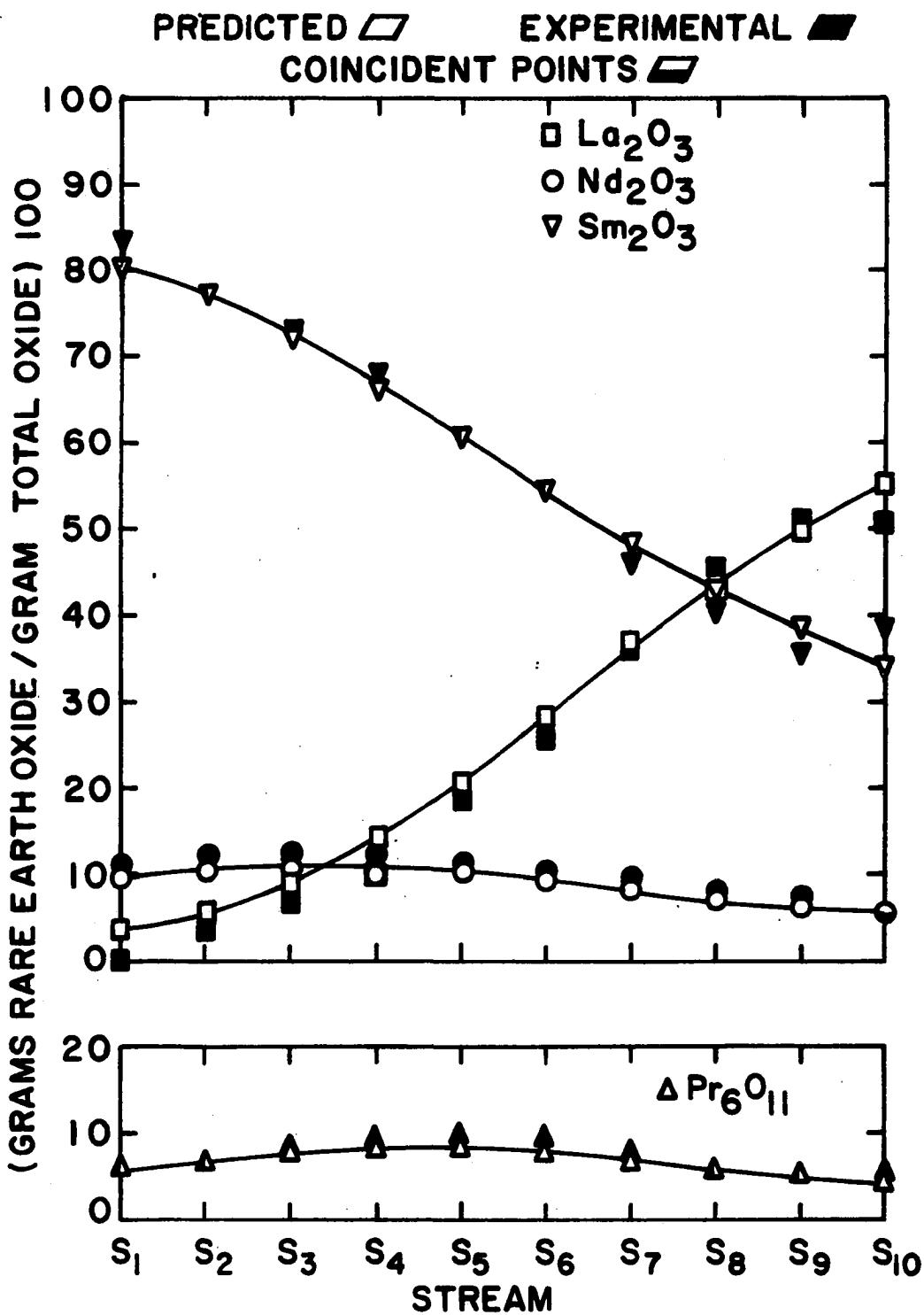


Figure 19. Stagewise organic rare earth oxide composition for Simulated Column Experiment II

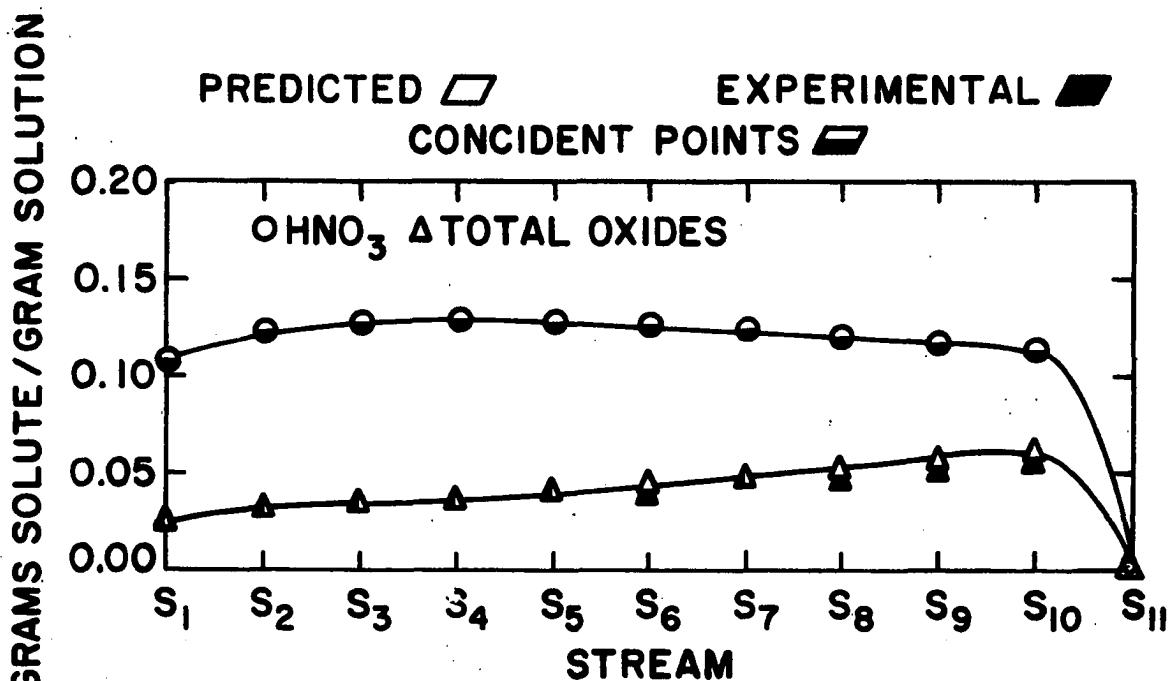


Figure 20. Stagewise organic rare earth oxide and nitric acid concentrations for Simulated Column Experiment II

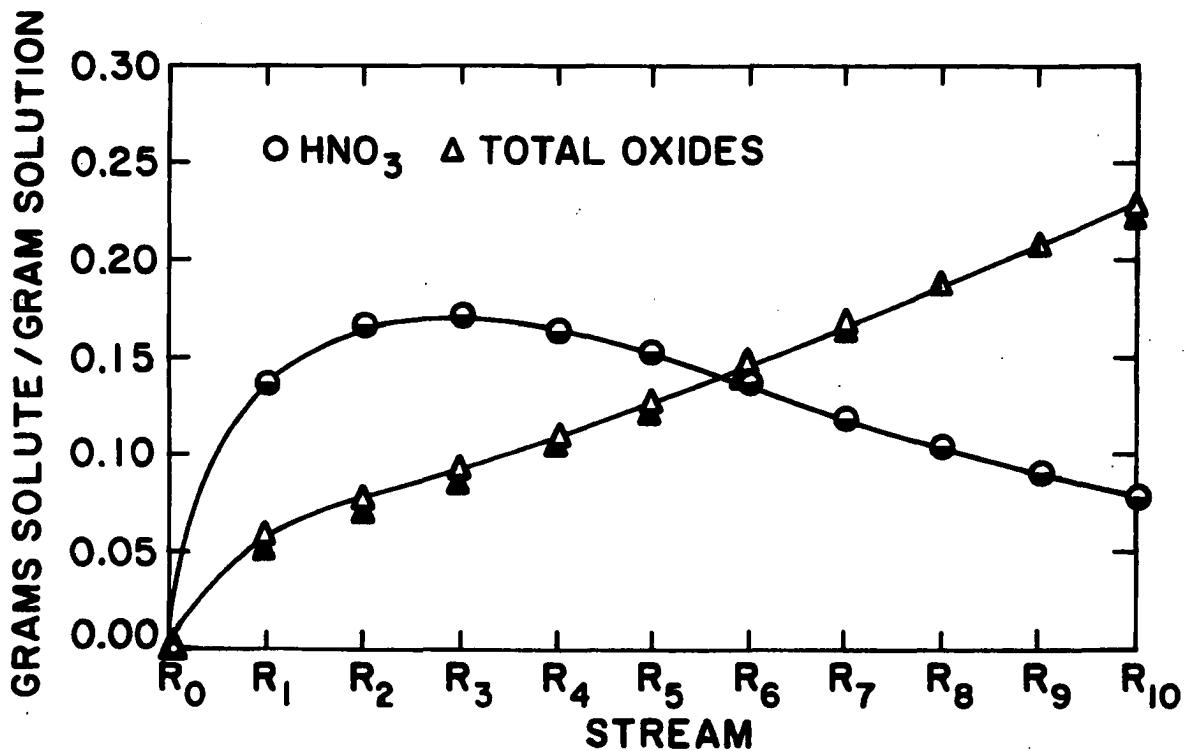


Figure 21. Stagewise aqueous rare earth oxide and nitric acid concentrations for Simulated Column Experiment II

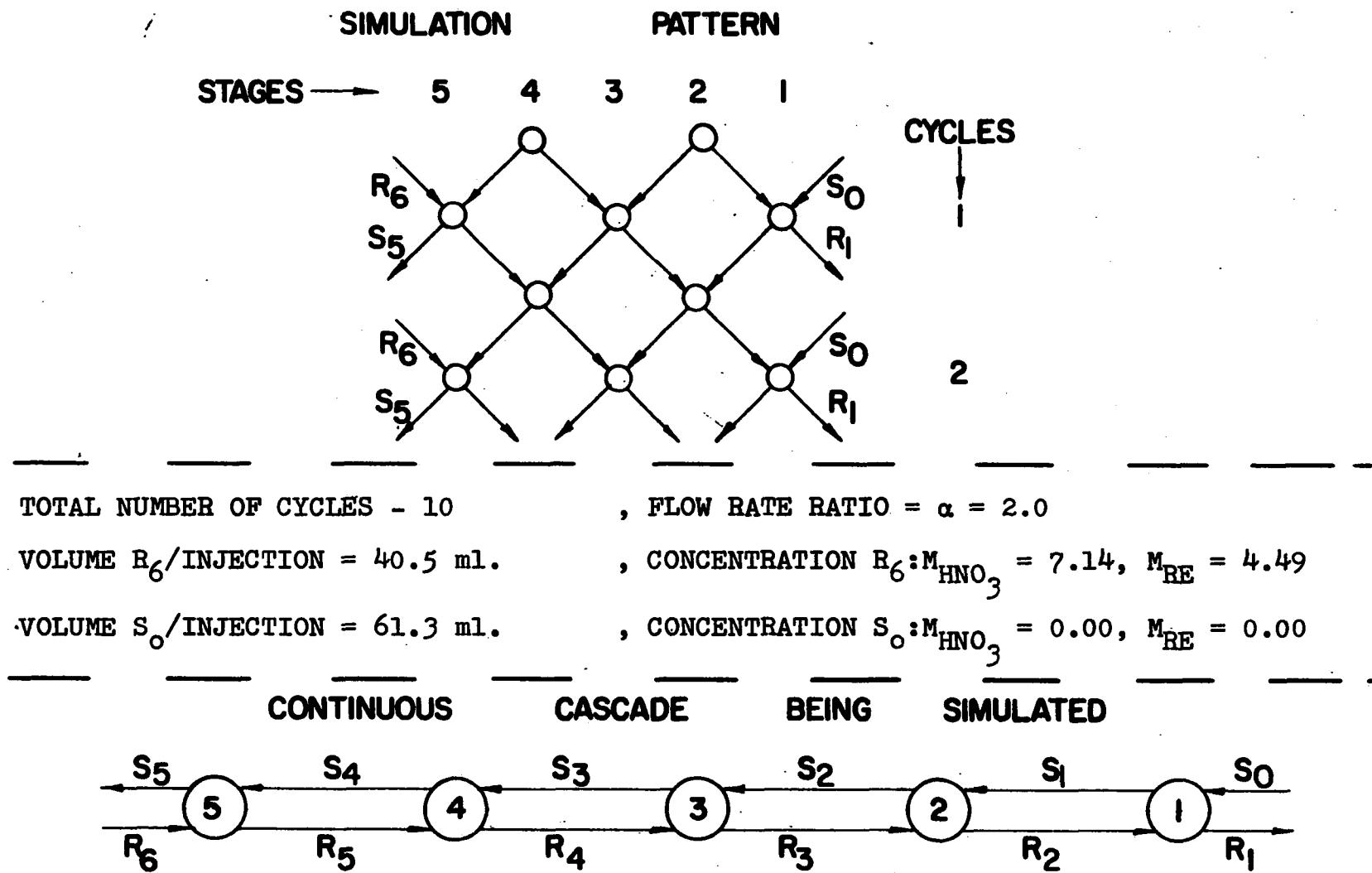


Figure 22. General information pertaining to Simulated Column Experiment III

Table 3. Comparison of experimental and predicted compositions and concentrations for Simulated Column Experiment III

	Weight % La_2O_3 in total oxides		Weight % Pr_6O_{11} in total oxides		Weight % Nd_2O_3 in total oxides		Weight % Sm_2O_3 in total oxides		Grams total oxides per gram of so- lution		Grams HNO_3 per gram of solution	
	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.
S ₁	65.8	66.8	10.5	11.4	11.5	12.1	12.3	9.7	0.049	0.060	0.044	0.031
S ₂	43.7	47.4	12.8	12.8	16.7	16.7	26.8	23.1	0.050	0.061	0.098	0.083
S ₃	29.1	28.5	10.7	11.1	17.0	17.4	43.3	43.1	0.039	0.046	0.137	0.128
S ₄	26.9	19.6	7.1	8.2	13.5	14.1	52.5	58.1	0.032	0.035	0.156	0.154
S ₅	17.0	17.8	6.7	6.7	10.2	11.3	66.0	64.3	0.030	0.030	0.163	0.166
R ₁	79.6	79.3	8.2	8.3	7.4	7.2	4.9	5.3	0.118	0.127	0.025	0.016
R ₂	77.8	73.7	9.3	9.7	5.2	9.4	7.7	7.3	0.163	0.183	0.081	0.055
R ₃	64.5	64.3	10.2	10.4	11.3	11.6	13.9	13.6	0.158	0.174	0.148	0.122
R ₄	61.8	58.7	8.7	9.4	9.9	11.3	19.5	20.6	0.139	0.151	0.199	0.182
R ₅	62.4	58.8	7.8	8.3	8.6	9.5	21.2	23.4	0.130	0.136	0.223	0.216
R ₆	63.9	60.4	7.1	7.8	7.5	8.4	21.4	23.4	0.126	0.128	0.232	0.232

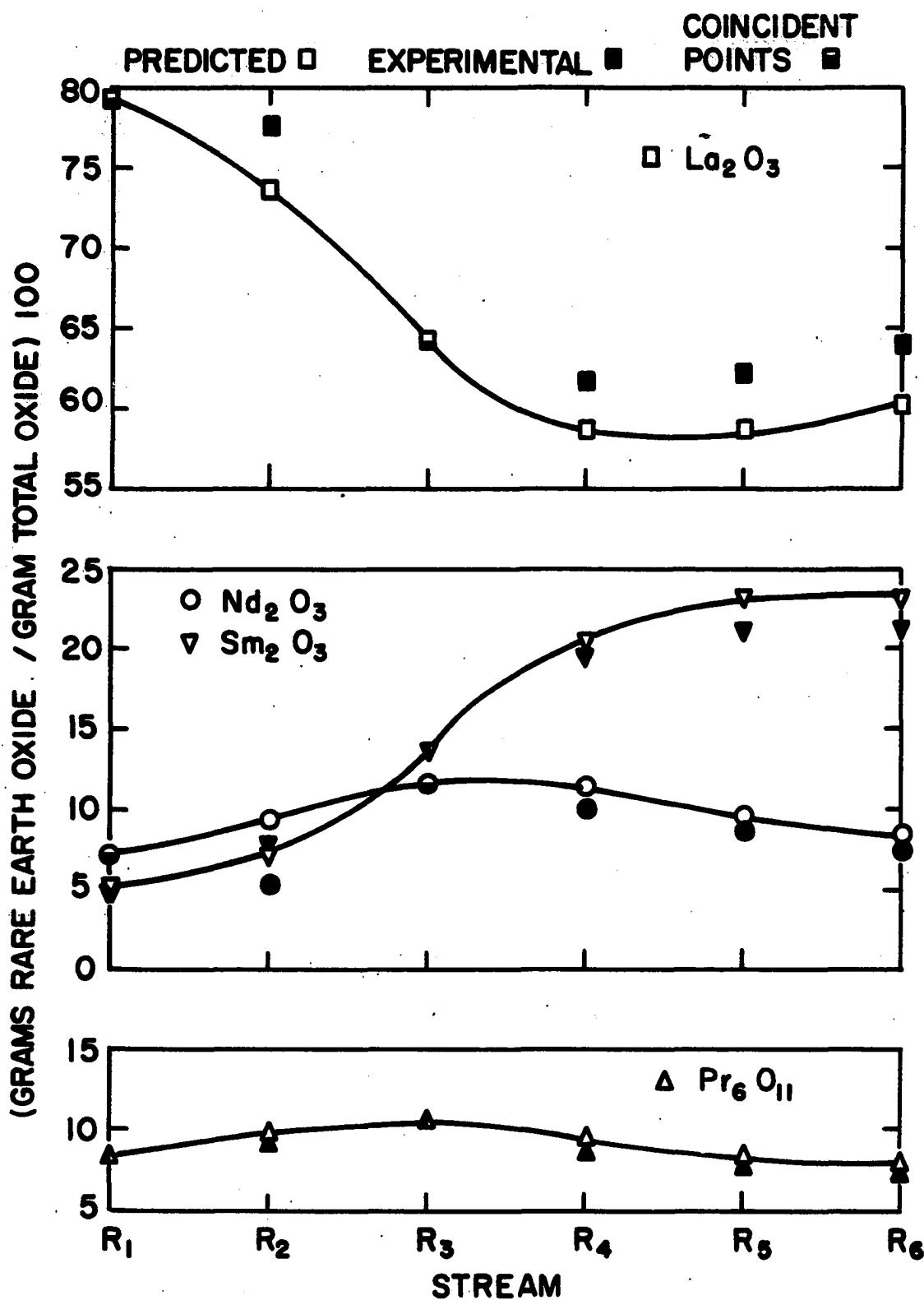


Figure 23. Stagewise aqueous rare earth oxide composition for Simulated Column Experiment III

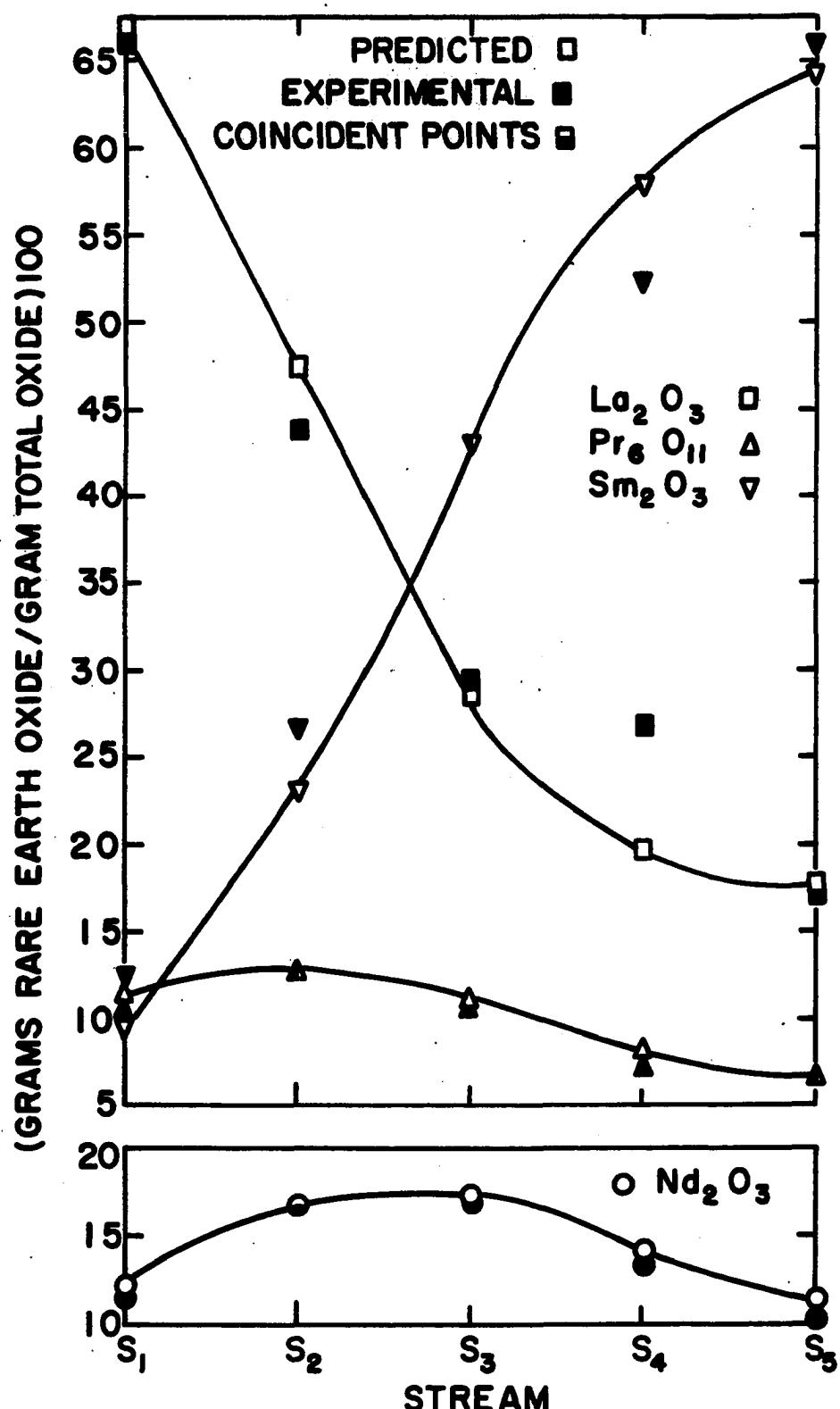


Figure 24. Stagewise organic rare earth oxide composition for Simulated Column Experiment III

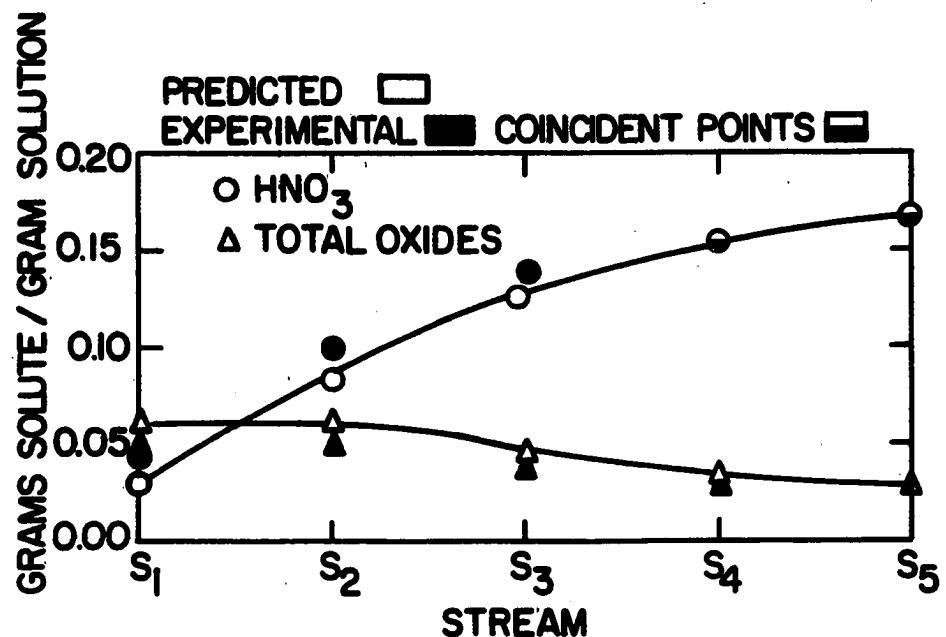


Figure 25. Stagewise organic rare earth oxide and nitric acid concentrations for Simulated Column Experiment III

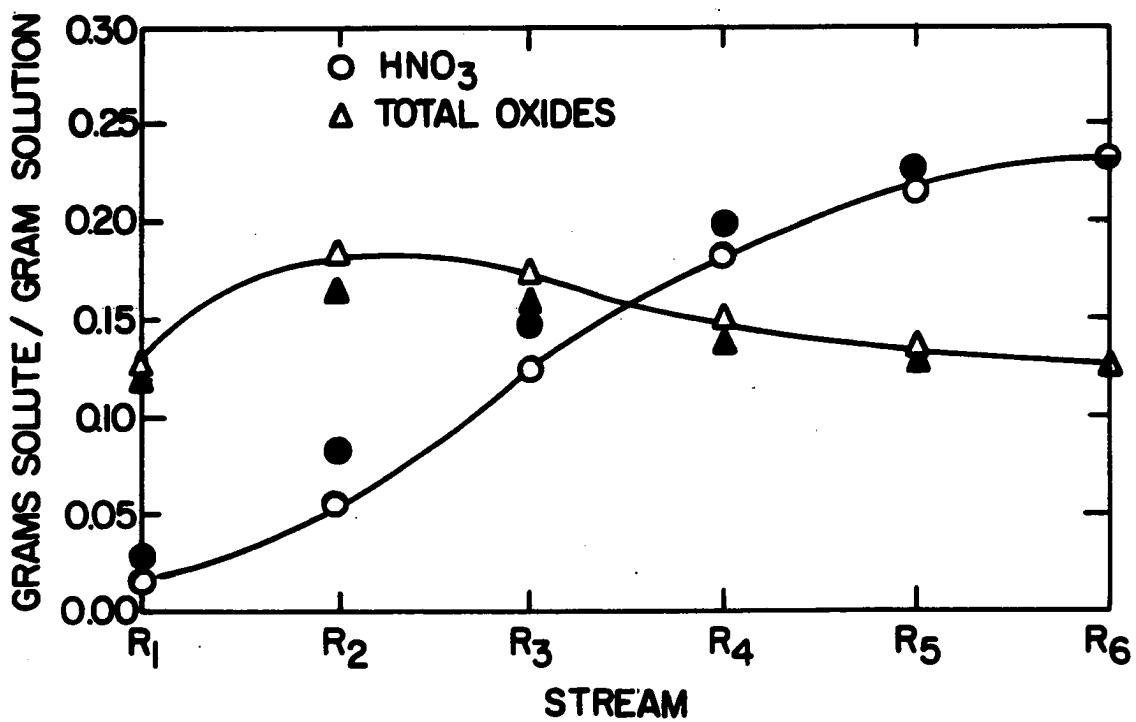


Figure 26. Stagewise aqueous rare earth oxide and nitric acid concentrations for Simulated Column Experiment III

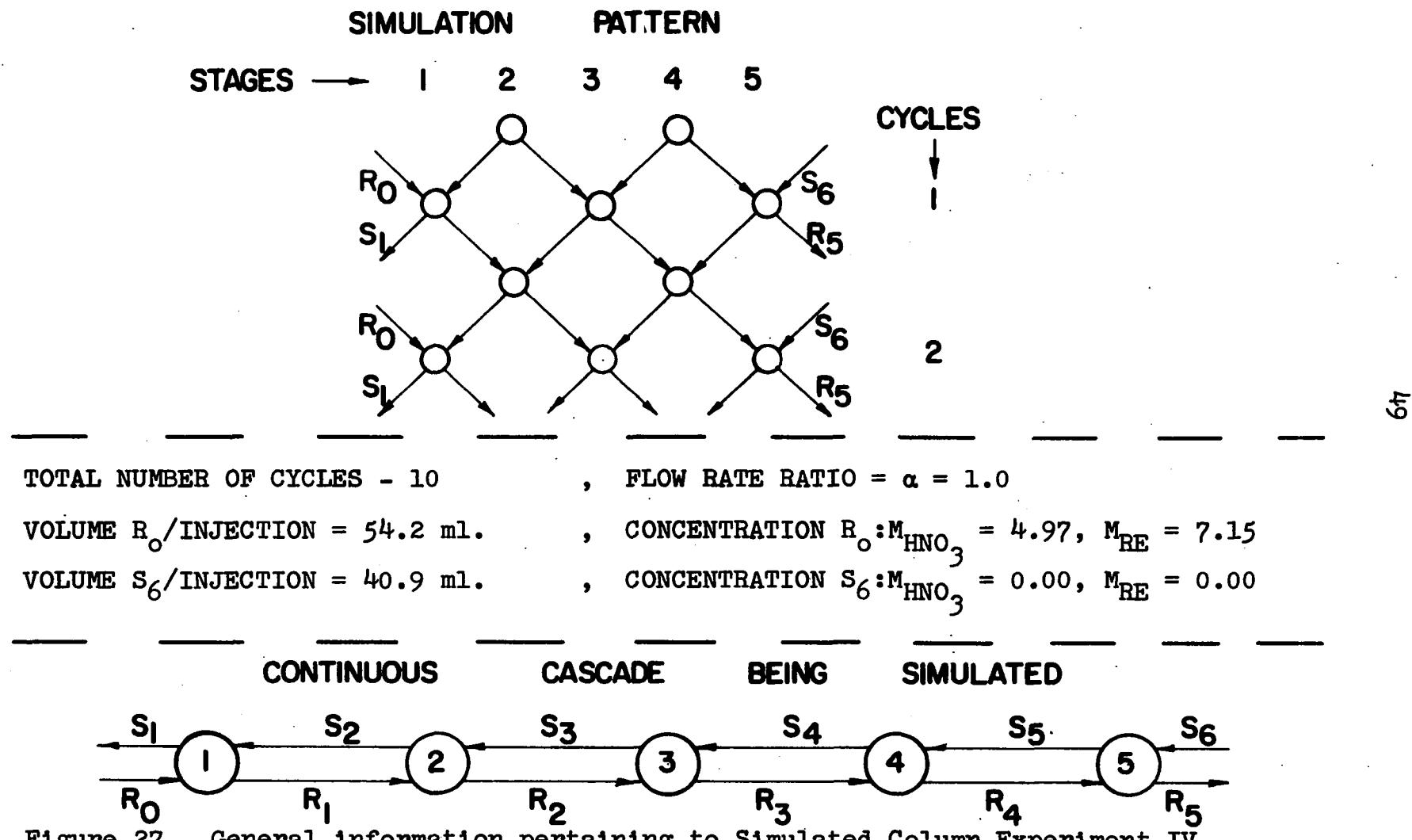


Figure 27. General information pertaining to Simulated Column Experiment IV

Table 4. Comparison of experimental and predicted compositions and concentrations for Simulated Column Experiment IV

	Weight % La_2O_3 in total oxides		Weight % $\text{Pr}_{6}\text{O}_{11}$ in total oxides		Weight % Nd_2O_3 in total oxides		Weight % Sm_2O_3 in total oxides		Grams total oxides per gram of so- lution		Grams HNO_3 per gram of solution	
	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.	exp.	pred.
S ₁	7.0	9.1	17.9	19.0	21.8	21.8	53.4	50.0	0.052	0.050	0.136	0.142
S ₂	6.9	9.1	17.9	19.0	22.3	21.9	52.9	50.1	0.053	0.050	0.135	0.142
S ₃	7.1	9.1	18.2	19.2	22.7	22.2	51.9	49.5	0.054	0.052	0.132	0.138
S ₄	7.7	10.1	19.7	20.7	25.1	23.6	47.5	45.7	0.059	0.058	0.122	0.127
S ₅	14.3	16.0	24.5	24.7	27.6	25.2	33.6	34.1	0.067	0.067	0.088	0.090
R ₀		34.0		25.8		20.4		19.8		0.191		0.149
R ₁	33.4	34.0	23.9	25.8	20.7	20.4	22.0	19.9	0.193	0.191	0.147	0.149
R ₂	33.2	33.9	24.2	25.8	21.7	20.5	21.0	19.9	0.194	0.192	0.145	0.147
R ₃	33.5	33.6	25.9	25.9	21.9	20.7	18.8	19.7	0.197	0.196	0.138	0.140
R ₄	34.8	34.3	25.8	26.6	22.0	21.1	17.4	18.0	0.204	0.204	0.118	0.117
R ₅	36.4	38.9	24.6	27.1	21.8	20.1	17.2	13.9	0.185	0.188	0.072	0.071

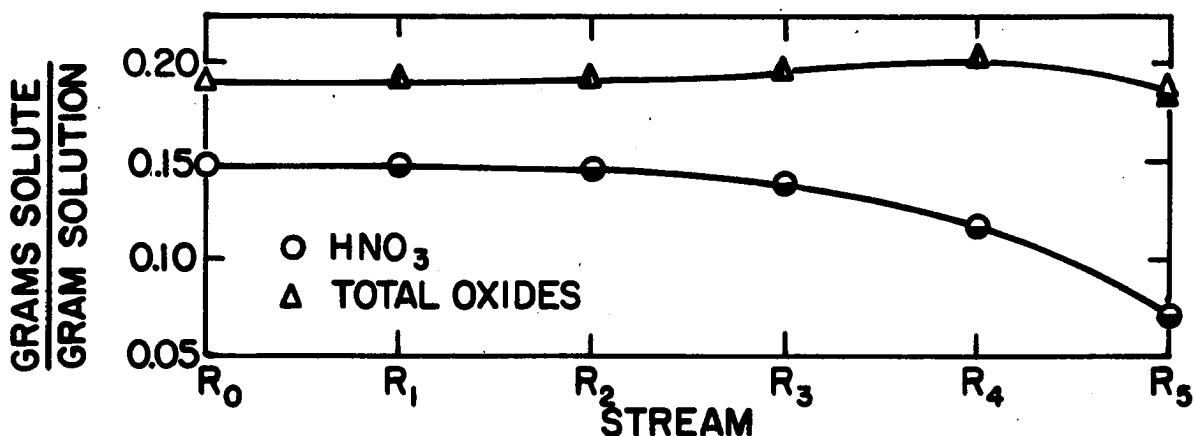


Figure 28. Stagewise aqueous rare earth oxide and nitric acid concentrations for Simulated Column Experiment IV

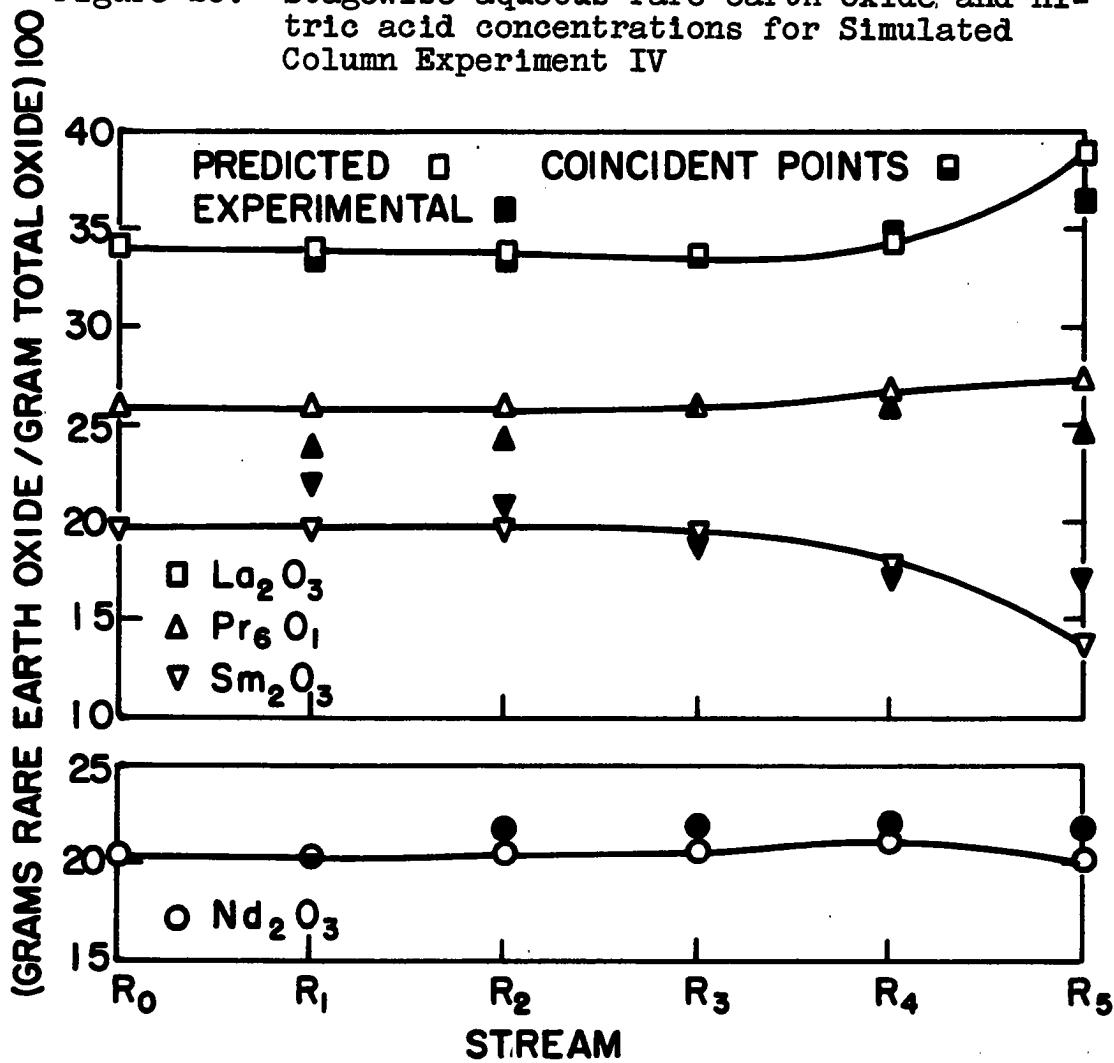


Figure 29. Stagewise aqueous rare earth oxide composition for Simulated Column Experiment IV

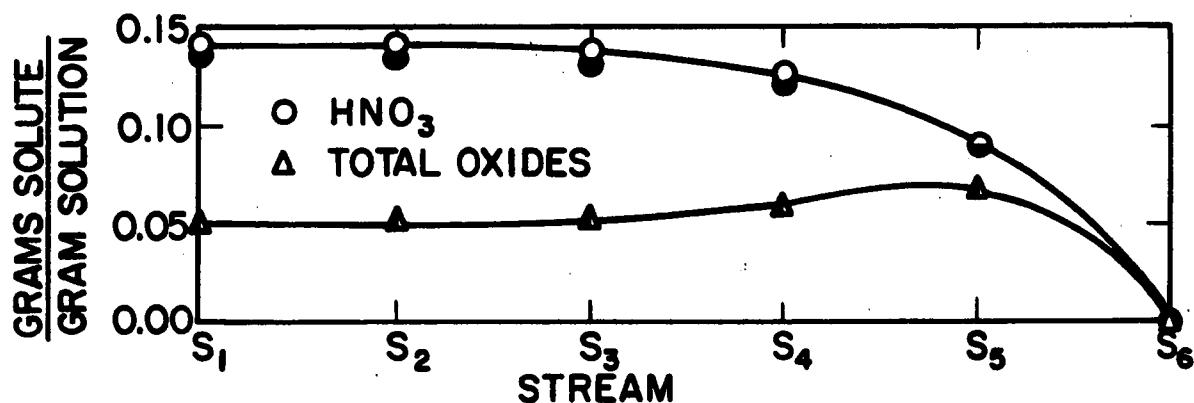


Figure 30. Stagewise organic rare earth oxide and nitric acid concentrations for Simulated Column Experiment IV

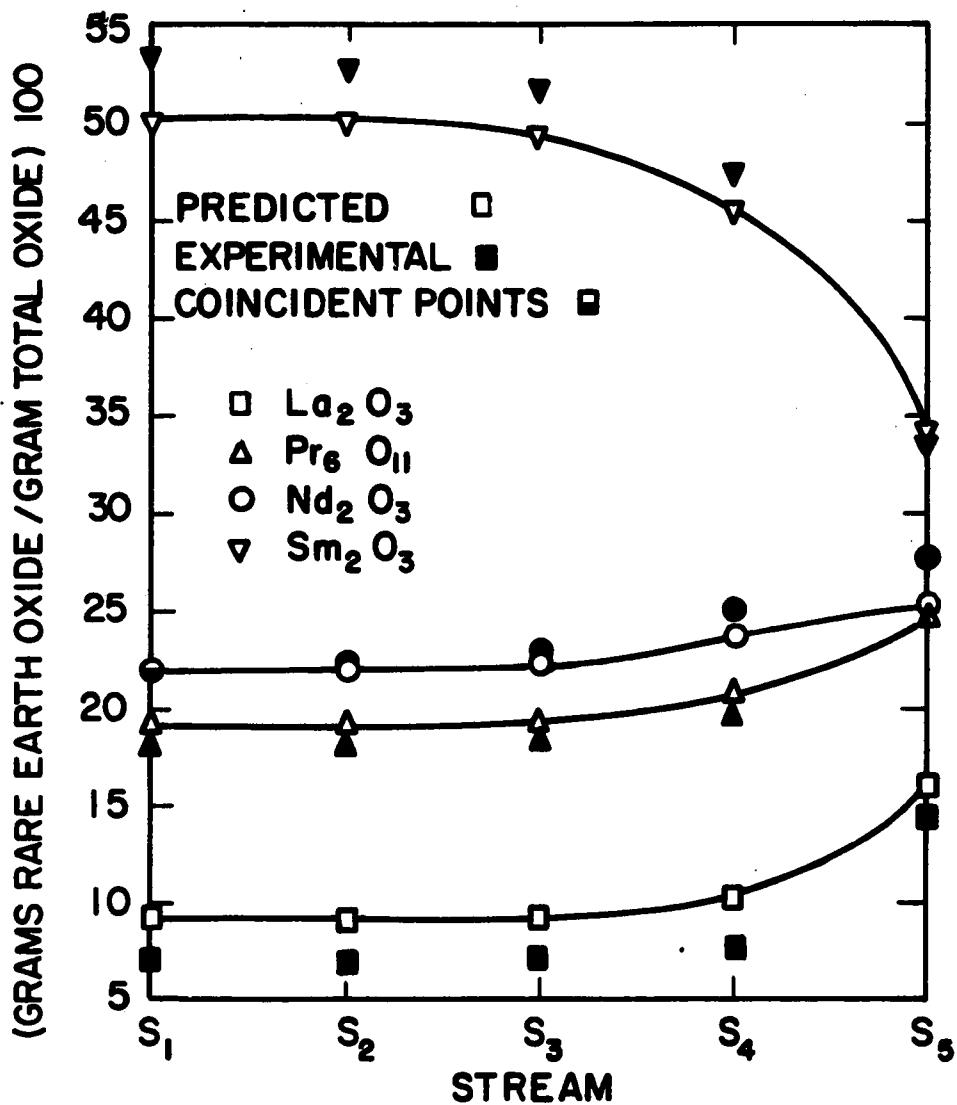


Figure 31. Stagewise organic rare earth oxide composition for Simulated Column Experiment IV

CONCLUSIONS

1. The calculation method developed for the multistage multisolute system gave good agreement with experimental data from four simulated column experiments. The conditions for each of the four simulated column experiments were quite different, and the agreement between predicted values and experimental data was believed to be a valid test of the method.

2. The assumption made that the separation factor between two rare earth nitrate solutes was a function only of the total molality of the organic phase, when $(M_t)_{\text{org.}}$ was greater than 1.75, was found to give surprisingly reliable results. The reason for the validity of this assumption is not known, but it appears to be a good approximation.

3. The digital computer technique for deriving distribution coefficients from a series of arrays was accurate and convenient. The technique makes it possible to use experimentally determined equilibrium data directly in the computer solution rather than introduce the data in terms of a mathematical correlation.

4. The digital computer programs for stage by stage calculations should make it possible to carry out various studies on the effects of process variables, such as the effect of acidity of an internal feed stage on recovery in an

end stream, etc. Such studies without the use of machine calculations would be prohibitively time consuming.

5. The flame spectroscopic analytical technique developed by the analytical chemistry group headed by Dr. V. A. Fassel of the Ames Laboratory of the Atomic Energy Commission has been found to give reliable results for each of the rare earths used in this study. The fact that lanthanum can now be determined directly, rather than by difference, is a significant scientific advance in studies of this type.

RECOMMENDATIONS FOR FUTURE WORK

There are several logical extensions of the present work, the most straightforward being an incorporation into the calculation method of systems with a greater number of rare earth nitrate solutes.

Assuming that it was desired to incorporate a rare earth nitrate solute denoted $\text{RE}(\text{NO}_3)_3$ into the method it would be necessary to acquire single stage equilibrium data for the systems $\text{RE}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ and $\text{RE}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ by the methods described by Sharp (47). These data could then be processed in the manner described in the Method of Calculation section, and the calculation for the six solute system $\text{La}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{Nd}(\text{NO}_3)_3 - \text{Sm}(\text{NO}_3)_3 - \text{RE}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ would follow exactly the steps also given in that section. Also, the modification of the computer programs for the six solute system would be completely straightforward, as no changes would be made in the logic flow as described in Appendix D.

Such incorporation might be a worthwhile program if it was desired to apply this work to the design of ore processing equipment.

An interesting computer extension would be the writing of a program for a center fed cascade, the calculation to

start from both ends from an initial assumption of end conditions, and, continuously modifying the end concentrations, repeat the calculation a large number of times, endeavoring to match all concentrations at the feed stage.

There are also possibilities of using the existing computer programs, plus others that could be written, in various optimization studies. Because of the large number of variables such a project would probably have to have a statistical basis, and would probably be primarily of academic interest.

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APPENDIX A. ANALYTICAL PROCEDURES

Weighed samples of each equilibrium phase were analyzed. The solutes in the organic samples were transferred to aqueous solution prior to analysis. This was accomplished by contacting the organic samples a minimum of three times with distilled water. The aqueous phases were combined and analyzed in the same manner as the aqueous phase samples.

For the determination of total rare earths the aqueous sample was diluted to approximately 200 ml. with distilled water, brought to a boil on a hot plate, and the rare earths completely precipitated with a saturated solution of oxalic acid. The oxalates were filtered and converted to the oxides at approximately 870° C. in a muffle furnace. The weight of the oxides was determined on an analytical balance.

The composition of the final rare earth oxide sample was determined by a flame photometric method developed by members of the analytical chemistry group headed by Dr. V. A. Fassel of the Ames Laboratory of the Atomic Energy Commission.

To determine the nitric acid content, the samples were diluted to approximately 200 ml. and the rare earths present precipitated by addition of an excess of saturated potassium ferrocyanide. The nitric acid content was then determined by titration with standardized sodium hydroxide. An automatic titrator was used and the end point was taken at a pH of 8.7.

It was found that the nitric acid content of aqueous solutions of nitric acid and potassium ferrocyanide diminishes with time. This was thought to be due to oxidation by the acid. To obtain satisfactory analyses for the nitric acid it was necessary to titrate the aqueous solutions of nitric acid, potassium ferrocyanide, and precipitated rare earth ferrocyanides within about 15 minutes after addition of the potassium ferrocyanide solution.

APPENDIX B. EQUILIBRIUM DATA

Table 5. Equilibrium data for the system $\text{La}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$

	Aqueous phase			Organic phase		
	Grams of $\text{La}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution	Grams of $\text{La}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution
1	0.127	0.065	0.808	0.026	0.075	0.899
2	0.170	0.087	0.743	0.035	0.095	0.871
3	0.204	0.112	0.684	0.037	0.113	0.849
4	0.258	0.147	0.595	0.037	0.143	0.820
5	0.309	0.173	0.519	0.035	0.161	0.804
6	0.169	0.036	0.796	0.051	0.054	0.896
7	0.214	0.050	0.736	0.061	0.070	0.869
8	0.278	0.064	0.658	0.074	0.087	0.839
9	0.351	0.082	0.567	0.078	0.112	0.810
10	0.419	0.093	0.488	0.078	0.134	0.788
11	0.070	0.099	0.831	0.011	0.090	0.899
12	0.089	0.131	0.781	0.014	0.115	0.871
13	0.111	0.173	0.716	0.015	0.140	0.845
14	0.142	0.229	0.629	0.013	0.170	0.817
15	0.162	0.283	0.555	0.012	0.191	0.798
16	0.032	0.126	0.842	0.004	0.097	0.898
17	0.046	0.166	0.789	0.006	0.121	0.873
18	0.055	0.232	0.713	0.005	0.154	0.841
19	0.072	0.288	0.640	0.006	0.177	0.817
20	0.089	0.343	0.568	0.005	0.194	0.801
21	0.211	0.013	0.776	0.094	0.027	0.880
22	0.272	0.020	0.709	0.114	0.037	0.849
23	0.342	0.025	0.633	0.131	0.049	0.819
24	0.432	0.031	0.537	0.145	0.066	0.789
25	0.226	0.006	0.768	0.116	0.014	0.869

Table 5 (Continued)

	Aqueous phase			Organic phase		
	Grams of La(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution	Grams of La(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution
26	0.287	0.009	0.704	0.140	0.021	0.839
27	0.360	0.010	0.631	0.169	0.024	0.807
28	0.457	0.012	0.531	0.191	0.034	0.776
29	0.091	0.399	0.511	0.006	0.195	0.799
30	0.102	0.444	0.454	0.008	0.213	0.779
31	0.180	0.332	0.488	0.016	0.209	0.775
32	0.195	0.363	0.443	0.020	0.223	0.757
33	0.110	0.507	0.383	0.016	0.267	0.717
34	0.338	0.210	0.453	0.038	0.187	0.775
35	0.511	0.036	0.453	0.143	0.089	0.768
36	0.531	0.019	0.450	0.177	0.061	0.763
37	0.457	0.111	0.432	0.071	0.156	0.773
38	0.529	0.051	0.420	0.120	0.115	0.765
39	0.358	0.229	0.413	0.042	0.198	0.760
40	0.461	0.138	0.402	0.065	0.170	0.765
41	0.208	0.432	0.360	0.035	0.253	0.712

Table 6. Equilibrium data for the system $\text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$

	Aqueous phase			Organic phase		
	Grams of $\text{Pr}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution	Grams of $\text{Pr}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution
1	0.079	0.134	0.787	0.022	0.108	0.870
2	0.099	0.179	0.721	0.027	0.133	0.841
3	0.130	0.237	0.634	0.028	0.160	0.812
4	0.158	0.287	0.554	0.030	0.179	0.792
5	0.173	0.332	0.495	0.034	0.195	0.771
6	0.146	0.087	0.766	0.050	0.087	0.864
7	0.187	0.113	0.700	0.061	0.103	0.836
8	0.245	0.150	0.605	0.067	0.130	0.803
9	0.294	0.184	0.522	0.065	0.153	0.782
10	0.328	0.211	0.461	0.069	0.168	0.763
11	0.230	0.041	0.729	0.102	0.055	0.842
12	0.268	0.065	0.667	0.108	0.075	0.817
13	0.336	0.085	0.579	0.116	0.095	0.789
14	0.409	0.101	0.489	0.116	0.117	0.768
15	0.455	0.115	0.430	0.116	0.133	0.752
16	0.467	0.143	0.390	0.113	0.151	0.763
17	0.258	0.022	0.720	0.137	0.036	0.828
18	0.328	0.027	0.646	0.167	0.040	0.793
19	0.416	0.032	0.552	0.185	0.050	0.765
20	0.491	0.040	0.470	0.183	0.067	0.750
21	0.536	0.051	0.412	0.170	0.088	0.742
22	0.269	0.011	0.720	0.164	0.020	0.816
23	0.343	0.012	0.645	0.198	0.021	0.781
24	0.437	0.019	0.544	0.211	0.032	0.757
25	0.522	0.018	0.459	0.219	0.038	0.743
26	0.184	0.372	0.444	0.043	0.209	0.749
27	0.349	0.241	0.410	0.081	0.180	0.739
28	0.195	0.401	0.404	0.055	0.220	0.725
29	0.352	0.270	0.378	0.091	0.192	0.717
30	0.064	0.103	0.834	0.017	0.094	0.889

Table 6 (Continued)

	Aqueous phase			Organic phase		
	Grams of Pr(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution	Grams of Pr(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution
31	0.124	0.066	0.809	0.040	0.072	0.888
32	0.172	0.039	0.789	0.069	0.054	0.877
33	0.208	0.014	0.778	0.113	0.026	0.860
34	0.224	0.007	0.770	0.139	0.014	0.847
35	0.201	0.424	0.375	0.068	0.226	0.706

Table 7. Equilibrium data for the system Nd(NO₃)₃ - HNO₃ - TBP - H₂O

	Aqueous phase			Organic phase		
	Grams of Nd(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution	Grams of Nd(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution
1	0.301	0.017	0.681	0.185	0.025	0.790
2	0.352	0.021	0.627	0.201	0.029	0.770
3	0.377	0.023	0.600	0.207	0.032	0.762
4	0.355	0.019	0.626	0.205	0.028	0.767
5	0.458	0.023	0.519	0.222	0.037	0.742
6	0.483	0.026	0.491	0.218	0.043	0.739
7	0.454	0.093	0.454	0.142	0.110	0.748
8	0.427	0.076	0.498	0.151	0.091	0.757
9	0.397	0.081	0.522	0.144	0.091	0.764
10	0.370	0.074	0.556	0.146	0.081	0.773

Table 7 (Continued)

	Aqueous phase			Organic phase		
	Grams of Nd(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution	Grams of Nd(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution
11	0.348	0.071	0.581	0.145	0.078	0.778
12	0.325	0.068	0.607	0.141	0.073	0.786
13	0.295	0.060	0.645	0.135	0.068	0.797
14	0.287	0.060	0.653	0.134	0.068	0.798
15	0.350	0.207	0.444	0.093	0.163	0.744
16	0.346	0.178	0.476	0.093	0.150	0.757
17	0.312	0.178	0.510	0.085	0.147	0.768
18	0.295	0.164	0.591	0.087	0.138	0.776
19	0.284	0.159	0.557	0.085	0.135	0.780
20	0.262	0.152	0.586	0.083	0.124	0.793
21	0.238	0.134	0.628	0.084	0.113	0.804
22	0.225	0.130	0.644	0.082	0.109	0.809
23	0.204	0.384	0.411	0.073	0.206	0.722
24	0.200	0.341	0.459	0.059	0.198	0.744
25	0.183	0.324	0.493	0.049	0.191	0.761
26	0.172	0.290	0.538	0.043	0.179	0.778
27	0.169	0.286	0.545	0.042	0.177	0.781
28	0.154	0.264	0.582	0.040	0.167	0.794
29	0.142	0.236	0.622	0.040	0.156	0.804
30	0.133	0.227	0.640	0.038	0.152	0.809
31	0.151	0.446	0.404	0.064	0.227	0.709
32	0.151	0.398	0.451	0.050	0.211	0.740
33	0.140	0.376	0.485	0.040	0.202	0.758
34	0.134	0.343	0.523	0.034	0.192	0.774
35	0.129	0.331	0.541	0.032	0.187	0.781
36	0.119	0.306	0.576	0.029	0.179	0.792
37	0.111	0.277	0.612	0.028	0.168	0.804
38	0.100	0.254	0.645	0.028	0.162	0.811
39	0.144	0.353	0.503	0.036	0.195	0.768
40	0.270	0.236	0.494	0.066	0.167	0.767

Table 7 (Continued)

	Aqueous phase			Organic phase		
	Grams of $\text{Nd}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution	Grams of $\text{Nd}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution
41	0.330	0.191	0.479	0.084	0.152	0.764
42	0.463	0.057	0.480	0.173	0.078	0.749
43	0.453	0.055	0.492	0.176	0.077	0.747
44	0.367	0.125	0.508	0.112	0.124	0.764
45	0.265	0.217	0.518	0.068	0.160	0.772
46	0.143	0.331	0.527	0.036	0.189	0.775
47	0.535	0.062	0.403	0.179	0.096	0.726
48	0.468	0.138	0.394	0.133	0.144	0.724
49	0.404	0.183	0.413	0.120	0.155	0.725
50	0.211	0.423	0.366	0.091	0.226	0.683
51	0.238	0.475	0.287	0.132	0.250	0.618
52	0.596	0.005	0.399	0.273	0.007	0.720
53	0.512	0.055	0.434	0.182	0.085	0.733
54	0.436	0.147	0.417	0.121	0.146	0.733
55	0.273	0.339	0.388	0.091	0.205	0.704
56	0.231	0.441	0.328	0.112	0.235	0.653
57	0.335	0.257	0.408	0.096	0.182	0.723
58	0.473	0.115	0.412	0.140	0.132	0.728
59	0.529	0.047	0.424	0.194	0.079	0.727
60	0.223	0.478	0.299	0.125	0.242	0.633
61	0.201	0.009	0.790	0.124	0.018	0.858
62	0.175	0.030	0.795	0.082	0.043	0.875
63	0.126	0.059	0.815	0.046	0.066	0.888
64	0.061	0.097	0.843	0.018	0.088	0.894
65	0.265	0.014	0.721	0.166	0.023	0.811
66	0.225	0.043	0.732	0.112	0.055	0.834
67	0.163	0.085	0.752	0.064	0.083	0.853
68	0.079	0.137	0.784	0.026	0.107	0.867
69	0.202	0.111	0.687	0.073	0.102	0.825
70	0.100	0.181	0.719	0.035	0.128	0.837

Table 8. Equilibrium data for the system $\text{Sm}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$

	Aqueous phase			Organic phase		
	Grams of $\text{Sm}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution	Grams of $\text{Sm}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution
1	0.487	0.029	0.484	0.226	0.047	0.727
2	0.447	0.030	0.523	0.228	0.037	0.735
3	0.386	0.027	0.587	0.225	0.031	0.744
4	0.341	0.023	0.637	0.217	0.027	0.756
5	0.307	0.020	0.672	0.207	0.026	0.767
6	0.313	0.022	0.665	0.208	0.026	0.766
7	0.277	0.018	0.705	0.194	0.023	0.783
8	0.248	0.016	0.737	0.177	0.018	0.805
9	0.400	0.108	0.492	0.160	0.098	0.742
10	0.376	0.096	0.528	0.163	0.086	0.751
11	0.324	0.087	0.589	0.159	0.076	0.765
12	0.282	0.072	0.646	0.154	0.066	0.781
13	0.261	0.063	0.676	0.148	0.063	0.789
14	0.259	0.071	0.670	0.146	0.063	0.791
15	0.234	0.059	0.707	0.136	0.057	0.808
16	0.178	0.043	0.779	0.102	0.049	0.850
17	0.269	0.283	0.448	0.113	0.161	0.726
18	0.261	0.260	0.480	0.104	0.156	0.740
19	0.230	0.228	0.542	0.092	0.142	0.766
20	0.205	0.196	0.599	0.088	0.129	0.783
21	0.188	0.179	0.633	0.088	0.123	0.790
22	0.176	0.174	0.650	0.083	0.120	0.798
23	0.162	0.159	0.680	0.081	0.113	0.807
24	0.142	0.140	0.718	0.072	0.102	0.826
25	0.129	0.432	0.439	0.090	0.197	0.657
26	0.123	0.402	0.475	0.074	0.189	0.684
27	0.119	0.344	0.537	0.056	0.176	0.768
28	0.103	0.294	0.604	0.047	0.164	0.789
29	0.099	0.283	0.618	0.046	0.160	0.794
30	0.094	0.271	0.636	0.044	0.153	0.803

Table 8 (Continued)

	Aqueous phase			Organic phase		
	Grams of Sm(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution	Grams of Sm(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution
31	0.085	0.243	0.672	0.041	0.148	0.811
32	0.055	0.142	0.804	0.026	0.108	0.869
33	0.222	0.291	0.487	0.097	0.165	0.739
34	0.217	0.280	0.503	0.093	0.160	0.747
35	0.213	0.269	0.518	0.089	0.156	0.754
36	0.202	0.245	0.553	0.085	0.147	0.769
37	0.222	0.292	0.486	0.095	0.166	0.739
38	0.219	0.285	0.497	0.092	0.162	0.746
39	0.214	0.270	0.516	0.088	0.158	0.754
40	0.204	0.244	0.553	0.082	0.151	0.767
41	0.129	0.526	0.345	0.126	0.222	0.653
42	0.200	0.383	0.418	0.114	0.188	0.698
43	0.278	0.268	0.454	0.116	0.161	0.723
44	0.422	0.130	0.448	0.156	0.116	0.728
45	0.484	0.064	0.452	0.202	0.073	0.725
46	0.541	0.043	0.416	0.222	0.065	0.713
47	0.579	0.007	0.416	0.277	0.009	0.714
48	0.101	0.597	0.302	0.138	0.248	0.614
49	0.075	0.088	0.837	0.032	0.080	0.888
50	0.113	0.066	0.821	0.052	0.067	0.882
51	0.163	0.034	0.803	0.092	0.047	0.862
52	0.194	0.014	0.792	0.132	0.022	0.847
53	0.144	0.093	0.763	0.075	0.081	0.844

Table 9. Equilibrium data for the system $\text{La}(\text{NO}_3)_3$ - $\text{Pr}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O

	Aqueous phase				Organic phase			
	Grams of $\text{La}(\text{NO}_3)_3$ per gram of solution	Grams of $\text{Pr}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution	Grams of $\text{La}(\text{NO}_3)_3$ per gram of solution	Grams of $\text{Pr}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution
1	0.075	0.130	0.019	0.776	0.025	0.069	0.036	0.870
2	0.142	0.065	0.19	0.774	0.054	0.035	0.037	0.873
3	0.057	0.096	0.051	0.796	0.014	0.036	0.066	0.885
4	0.106	0.049	0.049	0.797	0.027	0.018	0.067	0.887
5	0.029	0.055	0.093	0.824	0.005	0.015	0.087	0.893
6	0.057	0.027	0.089	0.827	0.010	0.008	0.087	0.895
7	0.119	0.205	0.036	0.639	0.031	0.108	0.056	0.805
8	0.233	0.098	0.033	0.636	0.072	0.059	0.058	0.811
9	0.086	0.152	0.089	0.673	0.017	0.058	0.098	0.828
10	0.168	0.074	0.087	0.671	0.034	0.031	0.102	0.833
11	0.052	0.080	0.161	0.707	0.007	0.024	0.131	0.838
12	0.092	0.040	0.160	0.707	0.013	0.013	0.131	0.843
13	0.178	0.300	0.054	0.468	0.026	0.127	0.091	0.757

Table 9 (Continued)

	Aqueous phase				Organic phase			
	Grams of La(NO ₃) ₃ per gram of solution	Grams of Pr(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution	Grams of La(NO ₃) ₃ per gram of solution	Grams of Pr(NO ₃) ₃ per gram of solution	Grams of HNO ₃ per gram of solution	Grams of solvent per gram of solution
14	0.334	0.145	0.050	0.417	0.064	0.078	0.096	0.763
15	0.126	0.233	0.142	0.500	0.013	0.065	0.143	0.779
16	0.247	0.112	0.138	0.503	0.031	0.036	0.148	0.785
17	0.069	0.132	0.256	0.544	0.006	0.026	0.176	0.792
18	0.144	0.273	0.176	0.407	0.013	0.072	0.167	0.748
19	0.289	0.130	0.171	0.409	0.031	0.041	0.171	0.757
20	0.086	0.154	0.328	0.433	0.007	0.039	0.203	0.752
21	0.167	0.076	0.325	0.433	0.015	0.021	0.207	0.756

Table 10. Equilibrium data for the system $\text{Nd}(\text{NO}_3)_3$ - $\text{Pr}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O

	Aqueous phase				Organic phase			
	Grams of $\text{Pr}(\text{NO}_3)_3$ per gram of solution	Grams of $\text{Nd}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution	Grams of $\text{Pr}(\text{NO}_3)_3$ per gram of solution	Grams of $\text{Nd}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution
1	0.074	0.126	0.021	0.780	0.032	0.071	0.035	0.861
2	0.064	0.033	0.087	0.816	0.018	0.012	0.083	0.888
3	0.211	0.105	0.039	0.645	0.092	0.060	0.053	0.795
4	0.048	0.102	0.151	0.699	0.014	0.036	0.119	0.830
5	0.146	0.328	0.059	0.467	0.042	0.130	0.083	0.745
6	0.148	0.079	0.236	0.538	0.028	0.022	0.167	0.783
7	0.266	0.137	0.195	0.402	0.055	0.045	0.166	0.734
8	0.088	0.182	0.301	0.429	0.017	0.056	0.190	0.737

Table 11. Equilibrium data for the system $\text{Sm}(\text{NO}_3)_3 - \text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$

	Aqueous phase				Organic phase			
	Grams of $\text{Pr}(\text{NO}_3)_3$ per gram of solution	Grams of $\text{Sm}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution	Grams of $\text{Pr}(\text{NO}_3)_3$ per gram of solution	Grams of $\text{Sm}(\text{NO}_3)_3$ per gram of solution	Grams of HNO_3 per gram of solution	Grams of solvent per gram of solution
1	0.073	0.118	0.022	0.787	0.033	0.079	0.033	0.855
2	0.064	0.025	0.088	0.824	0.020	0.010	0.082	0.888
3	0.221	0.087	0.040	0.652	0.090	0.072	0.049	0.789
4	0.052	0.083	0.154	0.711	0.013	0.047	0.116	0.825
5	0.176	0.288	0.062	0.474	0.031	0.159	0.075	0.735
6	0.158	0.062	0.236	0.545	0.027	0.035	0.159	0.779
7	0.265	0.108	0.220	0.408	0.043	0.071	0.162	0.724
8	0.102	0.154	0.311	0.433	0.013	0.090	0.173	0.723

APPENDIX C. MATHEMATICAL ANALYSIS OF PINCH

In this section an attempt to determine mathematically the generality of the "pinch" properties of the multicomponent, multistage calculation discussed in the body of this report is presented. The treatment is quite incomplete, however, the results indicate that the "pinch" of solute concentrations may be general over a wide range of system parameters. The cascade portrayed in Figure 10 was considered.

The approach used was to combine the defining equation of separation factor with a material balance and write an equation for the molality of solute j in organic stream S_n . Remember that the calculation to be considered is applied to the end of the cascade, first defining the solvent (S_0), the raffinate (R_1), and the flow ratio (α), and then working back through the cascade toward the feed.

By definition

$$(\beta_{j-Pr})_n = \frac{(M_j)_{S_n} / (M_j)_{R_n}}{(K_{Pr})_n} \quad (14)$$

or

$$(K_{Pr})_n (\beta_{j-Pr})_n (M_j)_{R_n} = (M_j)_{S_n} \quad . \quad (15)$$

Summing each side of Equation 15 over the solutes in the system get

$$(K_{Pr})_n \sum_{i=1}^T (\beta_{i-Pr})_n (M_i)_{R_n} = \sum_{i=1}^T (M_i)_{S_n} = (M_t)_{S_n} \quad . \quad (16)$$

Substituting Equation 16 into 15 get

$$(M_j)_{S_n} = \frac{(M_t)_{S_n} (\beta_{j-Pr})_n (M_j)_{R_n}}{\sum_{i=1}^T (\beta_{i-Pr})_n (M_i)_{R_n}} . \quad (17)$$

Writing a material balance over the right hand end of the cascade gives

$$\begin{aligned} & [(M_j)_{R_1} \times R_1] + [(M_j)_{S_{n-1}} \times S_{n-1}] = \\ & [(M_j)_{S_0} \times S_0] + [(M_j)_{R_n} \times R_n] , \end{aligned} \quad (18)$$

and assuming immiscible solvents

$$(M_j)_{R_n} = \alpha(M_j)_{S_{n-1}} + (M_j)_{R_1} - \alpha(M_j)_{S_0} . \quad (19)$$

Substituting into Equation 17 get

$$(M_j)_{S_n} = \frac{(M_t)_{S_n} (\beta_{j-Pr})_n [\alpha(M_j)_{S_{n-1}} + (M_j)_{R_1} - \alpha(M_j)_{S_0}]}{\sum_{i=1}^T (\beta_{i-Pr})_n [\alpha(M_i)_{S_{n-1}} + (M_i)_{R_1} - \alpha(M_i)_{S_0}]} . \quad (20)$$

The assumptions of "pinched" total molality and constant separation factors were now applied. The general situation in which the total molality is "pinched" at a high value is shown on a McCabe Thiele diagram in Figure 32. The equations to be considered below apply to the circled section on the drawing, that is to the "pinched" stages. The fact that the following equations assume a "pinch" in stage 1 causes no loss in generality, as in a case in which the total molality of

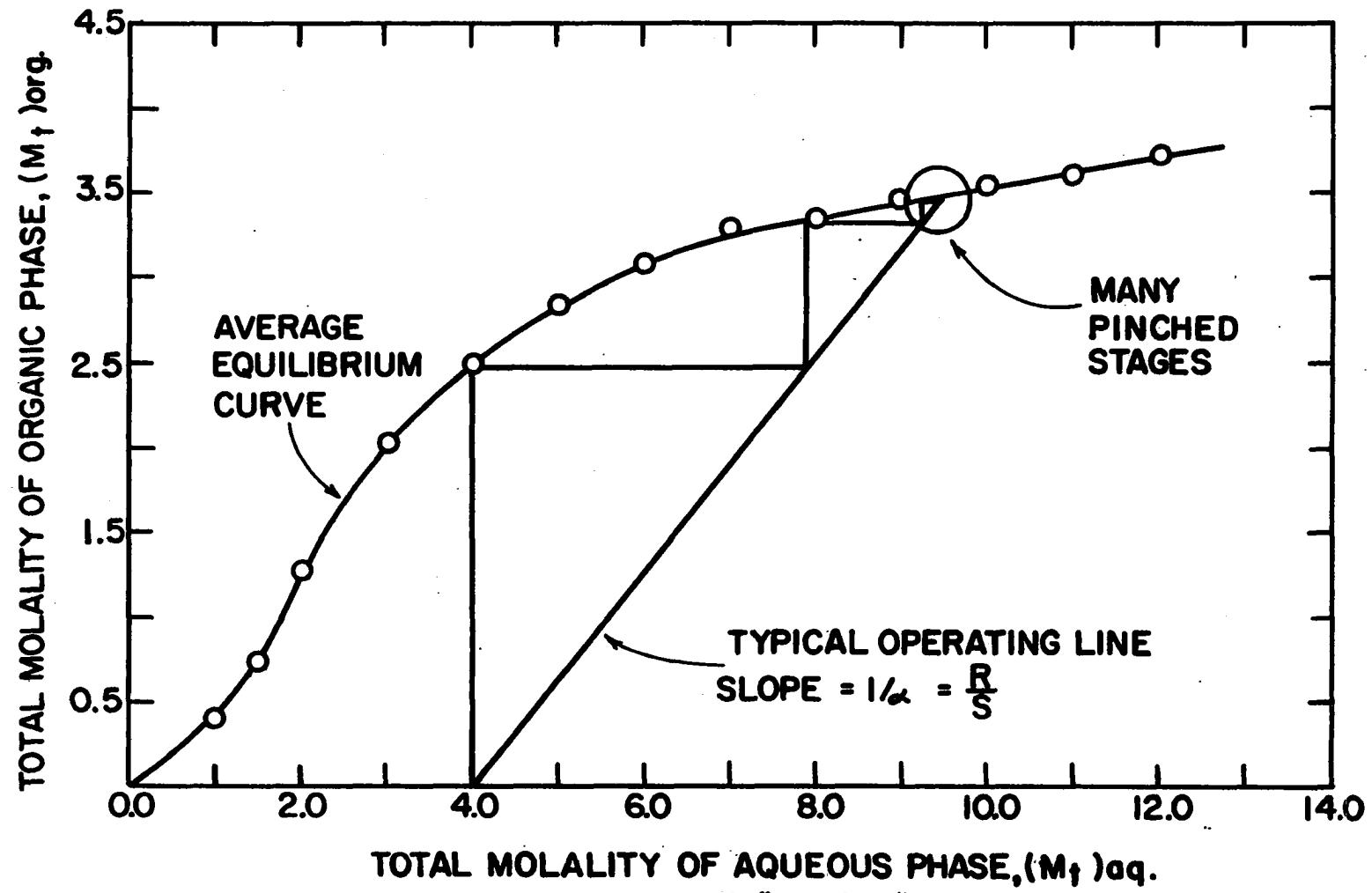


Figure 32. McCabe-Thiele representation of "pinched" total molality

several of the right hand stages is not constant stage 1 may be chosen as the first stage in which the organic molality attains its "pinch" value.

For this equilibrium model the stream and stage designations of the total molalities and separation factors may be deleted, and also, for convenience $(M_i)_n$ will be inserted for $(M_i)_{S_n}$ and so

$$(M_j)_n = \frac{\alpha M_t \beta_{j-Pr} (M_j)_{n-1} + M_t \beta_{j-Pr} [(M_j)_{R_1} - \alpha (M_j)_{S_0}]}{\alpha \sum_{i=1}^T \beta_{i-Pr} (M_i)_{n-1} + \sum_{i=1}^T \beta_{i-Pr} [(M_i)_{R_1} - \alpha (M_i)_{S_0}]} . \quad (21)$$

Note that for specified end conditions and a specified "pinch" value of the total molality Equation 21 gives the molality of solute j in stream S_n as a function of the molalities of all solutes in stream S_{n-1} . By letting $n = 1, 2, 3, \dots$, a sequence was generated which gives the molality of solute j in successive organic streams in terms of the system parameters. If the sequence could be shown to converge to a non-zero value the "pinch" would be proven general. The author has written a large number of terms of the sequence but has not been able to write the general term.

For the special case of $T = 2$, that is two solutes, an analytical treatment of the general equation was developed and the limit taken. This case is now discussed. For convenience it was assumed that stream S_0 is pure solvent. This is not

necessary but simplifies the following equations algebraically. For this case, denoting quantities that pertain to the two solutes by the subscripts 1 and 2

$$(M_1)_n = \frac{M_t \beta_{1-Pr}^{(M_1)_{n-1}} + (M_t/\alpha) \beta_{1-Pr}^{(M_1)_{R_1}}}{\left[\beta_{1-Pr}^{(M_1)_{n-1}} + \beta_{2-Pr}^{(M_2)_{n-1}} \right] + (1/\alpha) \sum_{i=1}^2 \beta_{i-Pr}^{(M_i)_{R_1}}} , \quad (22)$$

and

$$(M_2)_n = \frac{M_t \beta_{2-Pr}^{(M_2)_{n-1}} + (M_t/\alpha) \beta_{2-Pr}^{(M_2)_{R_1}}}{\left[\beta_{1-Pr}^{(M_1)_{n-1}} + \beta_{2-Pr}^{(M_2)_{n-1}} \right] + (1/\alpha) \sum_{i=1}^2 \beta_{i-Pr}^{(M_i)_{R_1}}} . \quad (23)$$

These equations are of exactly the same form. By use of finite difference techniques they were solved for the molalities of the solutes as a function of the system parameters and stage number, and the limit taken as n becomes large. This technique is now illustrated by solving Equation 22 and then a numerical example given.

For the two solute case with "pinched" total molality

$$(M_1)_{n-1} + (M_2)_{n-1} = M_t , \quad (24)$$

and so substituting into Equation 22 get

$$(M_1)_n = \frac{M_t \beta_{1-Pr}^{(M_1)_{n-1}} + (M_t/\alpha) \beta_{1-Pr}^{(M_1)_{R_1}}}{(\beta_{1-Pr} - \beta_{2-Pr})(M_1)_{n-1} + \beta_{2-Pr} M_t + (1/\alpha) \sum_{i=1}^2 \beta_{i-Pr}^{(M_i)_{R_1}}} . \quad (25)$$

Now let

$$a_1 = M_t \beta_{1-Pr} ,$$

$$b_1 = (M_t / \alpha) \beta_{1-Pr} (M_1)_{R_1} ,$$

$$c_1 = \beta_{1-Pr} - \beta_{2-Pr} ,$$

$$d_1 = \beta_{2-Pr} M_t + (1/\alpha) \sum_{i=1}^2 \beta_{i-Pr} (M_i)_{R_1} .$$

Substitution into Equation 25 and simplification gives

$$(M_1)_n (M_1)_{n-1} + a_1 (M_1)_n + \theta_1 (M_1)_{n-1} + \gamma_1 = 0 , \quad (26)$$

with

$$a_1 = d_1 / c_1 ,$$

$$\theta_1 = -a_1 / c_1 ,$$

$$\gamma_1 = -b_1 / c_1 .$$

At this point it is to be noted that the limit of $(M_1)_n$ as n becomes large can be determined without solving Equation 26. This can be done by recognizing that if a "pinch" takes place $(M_1)_n = (M_1)_{n-1}$. If this substitution is made in Equation 26 and the resulting quadratic solved for $(M_1)_n$ two roots are obtained, one of which must be discarded on physical grounds.

Alternatively, Equation 26 is a Riccati difference equation and may be reduced to a linear difference equation with constant coefficients by the substitution

$$(M_1)_{n-1} = (V_1)_n (V_1)_{n-1} - a_1 . \quad (27)$$

For the methods of solution the reader is referred to Boole (53) and Brand (54).

After substitution and simplification get

$$(v_1)_{n+1} + E_1(v_1)_n + F_1(v_1)_{n-1} = 0 \quad (28)$$

with

$$E_1 = \theta_1 - \alpha_1 ,$$

$$F_1 = \gamma_1 - \alpha_1 \theta_1 .$$

A trial solution of the form $(v_1)_n = Z^n$ gives the characteristic equation

$$Z^2 + E_1 Z + F_1 = 0 , \quad (29)$$

and so

$$Z = \frac{-E_1 \pm (E_1^2 - 4F_1)^{0.5}}{2} . \quad (30)$$

To complete the solution it was necessary to know the nature of the roots of the characteristic quadratic. For all calculations considered the roots were real and distinct. The author has attempted to prove that the discriminant of the quadratic is positive by expanding it in terms of the original system parameters. A proof has not been possible and so it was assumed that this is the case and the roots are real and distinct.

Let

$$G_{1+} = \frac{-E_1 + (E_1^2 - 4F_1)^{0.5}}{2}$$

and

$$G_{1-} = \frac{-E_1 - (E_1^2 - 4F_1)^{0.5}}{2}$$

Then the solution of Equation 28 is

$$(V_1)_n = C_1 G_{1+}^n + C_2 G_{1-}^n \quad (31)$$

with C_1 and C_2 constants, and the solution of Equation 26 is

$$(M_1)_n = \frac{C_1 G_{1+}^{n+1} + C_2 G_{1-}^{n+1}}{C_1 G_{1+}^n + C_2 G_{1-}^n} - \alpha_1 \quad (32)$$

The limit of Equation 32 was taken by noting that the roots are real and distinct. Then necessarily either $|G_{1+}| > |G_{1-}|$ or $|G_{1+}| < |G_{1-}|$. Assuming that $|G_{1+}| > |G_{1-}|$ the numerator and denominator of the fraction on the right hand side of Equation 32 was divided by G_{1+}^n giving

$$(M_1)_n = \frac{C_1 G_{1+} + C_2 G_{1-} (G_{1-}/G_{1+})^n}{C_1 + C_2 (G_{1-}/G_{1+})^n} - \alpha_1 \quad , \quad (33)$$

and so

$$\lim_{n \rightarrow \infty} (M_1)_n = G_{1+} - \alpha_1 \quad . \quad (34)$$

It should be noted at this point that the algebraic combinations of system parameters a_1 , b_1 , c_1 , d_1 , α_1 , θ_1 , γ_1 , E_1 , F_1 , G_{1+} , G_{1-} all refer to solute 1. Corresponding expressions for solute 2 may be obtained by interchanging the solute subscripts in the defining equations. Exactly the same treatment was applied to Equation 23. Alternatively, the limit of the differences of the molalities in successive

stages, that is $[(M_1)_{n+1} - (M_1)_n]$ and $[(M_2)_{n+1} - (M_2)_n]$ may be shown to approach zero as n becomes large.

As a numerical example the following values were chosen:

$$\alpha = 1.0 ,$$

$$M_t = 3.50 ,$$

$$\beta_{1-Pr} = 0.43 ,$$

$$\beta_{2-Pr} = 1.00 ,$$

$$(M_t)_{R_1} = 8.00 ,$$

$$(M_1)_{R_1} = 4.00 ,$$

$$(M_2)_{R_2} = 4.00 .$$

Using these values

$$G_{1+} = -3.472 ,$$

$$G_{1-} = -15.344 ,$$

$$\alpha_1 = -16.175 ,$$

$$G_{2+} = 15.344 ,$$

$$G_{2-} = 3.472 ,$$

$$\alpha_2 = 12.675 ,$$

were calculated. The limits of $(M_1)_n$ and $(M_2)_n$ were then calculated to be

$$\lim_{n \rightarrow \infty} (M_1)_n = -15.344 + 16.175 = 0.831 ,$$

$$\lim_{n \rightarrow \infty} (M_2)_n = 15.344 - 12.675 = 2.669$$

Using alternate material balances and the chosen equilibrium model the stagewise values shown in Table 12 were calculated for this numerical example.

Table 12. Molalities of solutes 1 and 2 in organic streams of ideal cascade

	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7
M_1	0.0000	1.0520	0.8820	0.8429	0.8340	0.8320	0.8315	0.8314
M_2	0.0000	2.4480	2.6180	2.6571	2.6660	2.6680	2.6685	2.6686

The stagewise concentrations "pinch" rather rapidly to the limiting values. Note however that this is not an independent check of the validity of the process, as the equation which was used to solve for and take the limit of the molalities is simply a generalization of the stage by stage calculation used to determine the stagewise concentrations in the preceding table. This result indicates only that the solution of Equations 22 and 23 by finite difference techniques and the limiting process were carried out correctly.

Two comments should be made concerning the preceding treatment. First the form of the solution is dependent on the

assumption of real and distinct roots of the characteristic quadratic. As pointed out previously, this has been true in all specific cases studied but no proof has been given. Secondly, it may be that the derived result, that is "pinched" concentrations of the individual solutes under the assumptions made and the situations considered, is or should be intuitively obvious from simple material balance considerations. No definite conclusion has been reached concerning this second statement.

APPENDIX D. DIGITAL COMPUTER PROGRAMS

The calculation which is discussed in the body of this thesis to give the stagewise conditions in a cascade of equilibrium stages was programmed in the Full Fortran Language for the IBM 7074 digital computer for four cascades of interest. These cascades are shown in Figures 33, 34, 35, and 36 with the direction of calculation denoted by dashed arrows.

In the computer operation the values of K_t and K_{HNO_3} were derived by straight line interpolation in two dimensional arrays prepared from the parametric plots of K_t and K_{HNO_3} versus equilibrium phase composition discussed in the Method of Calculation section. These two dimensional arrays are shown in Tables 13 through 28. All non-zero values in these arrays were taken from plots of the type depicted in Figures 1 through 8. The zero values have no physical meaning but were inserted for convenience in making "branches" in the programs.

The principal variable names used in the programs are given in Figure 37. A compilation of the four programs is given in Figures 38 through 45. In each case the logic flow is diagrammed and is followed by the Fortran program. The program designation corresponds to the cascade designation in Figures 33, 34, 35, and 36.

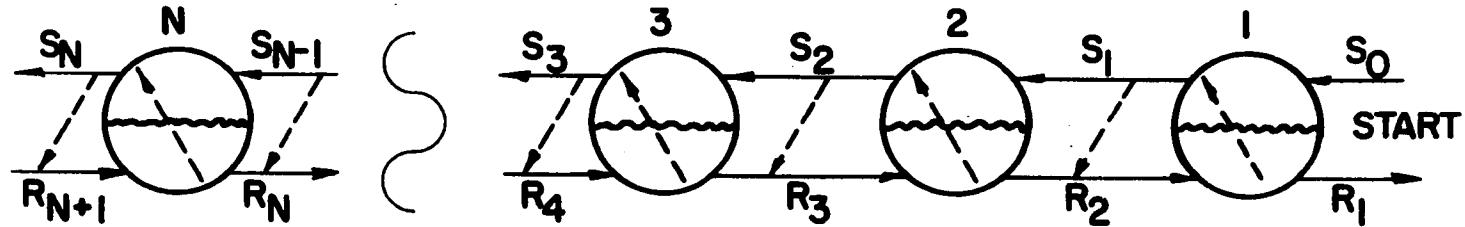


Figure 33. Cascade I

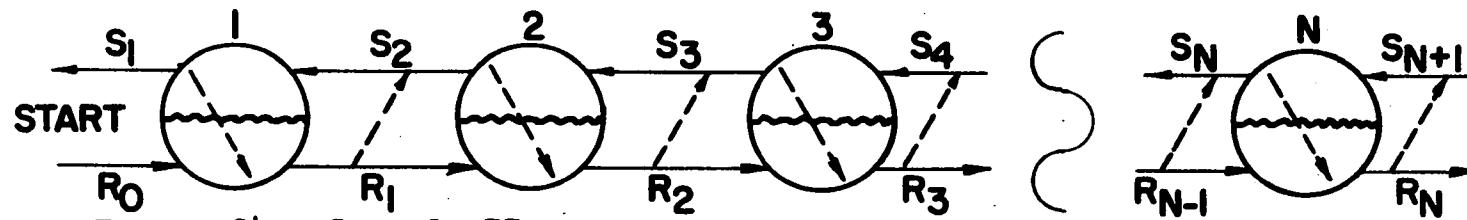


Figure 34. Cascade II

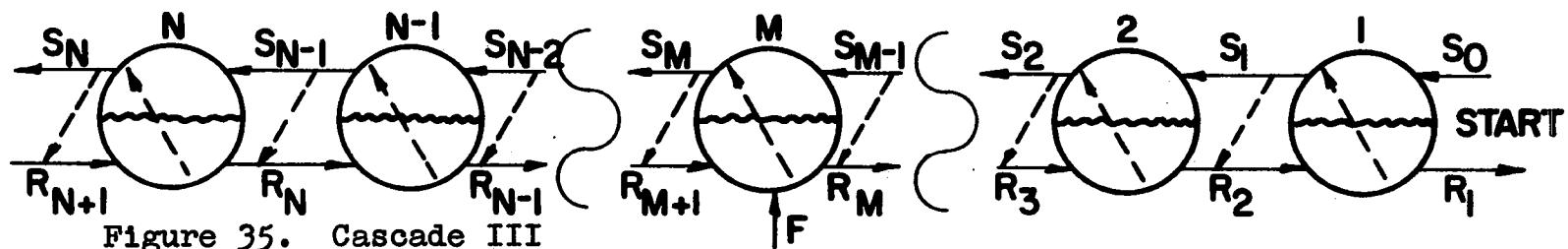


Figure 35. Cascade III

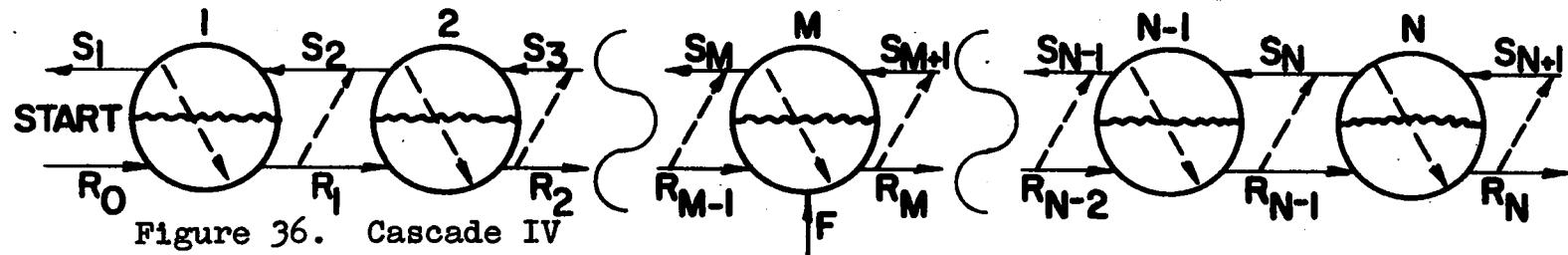


Figure 36. Cascade IV

Table 13. Array of values of K_t for the system $\text{La}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{La}

x_{La}	$(M_t)_{\text{aq.}}$						
	3.0	4.0	5.0	7.0	10.0	13.0	16.0
0.00	0.713	0.640	0.580	0.478	0.382	0.330	0.296
0.05	0.695	0.626	0.569	0.475	0.382	0.329	0.298
0.10	0.676	0.612	0.560	0.471	0.382	0.328	0.299
0.15	0.659	0.598	0.550	0.468	0.382	0.326	0.299
0.20	0.643	0.586	0.541	0.464	0.382	0.325	0.299
0.25	0.628	0.575	0.533	0.460	0.381	0.324	0.298
0.30	0.614	0.564	0.524	0.456	0.380	0.322	0.297
0.35	0.601	0.554	0.516	0.452	0.378	0.320	0.295
0.40	0.589	0.545	0.508	0.448	0.376	0.318	0.293
0.45	0.578	0.537	0.501	0.443	0.374	0.316	0.289
0.50	0.568	0.529	0.494	0.438	0.371	0.314	0.285
0.55	0.559	0.523	0.487	0.433	0.369	0.311	0.281
0.60	0.549	0.516	0.481	0.428	0.366	0.308	0.276
0.65	0.541	0.511	0.474	0.423	0.363	0.306	0.271
0.70	0.533	0.506	0.469	0.417	0.359	0.303	0.266
0.75	0.526	0.500	0.464	0.412	0.355	0.301	0.259
0.80	0.521	0.495	0.460	0.407	0.349	0.296	0.252
0.85	0.518	0.491	0.456	0.401	0.336	0.288	0.244
0.90	0.519	0.489	0.455	0.396	0.331	0.278	0.236
0.95	0.524	0.491	0.454	0.389	0.319	0.266	0.226
1.00	0.534	0.497	0.456	0.384	0.303	0.248	0.217

Table 14. Array of values of K_t for the system $\text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{Pr}

x_{Pr}	$(M_t)_{\text{aq.}}$						
	3.0	4.0	5.0	7.0	10.0	13.0	16.0
0.00	0.713	0.640	0.580	0.478	0.382	0.330	0.296
0.05	0.702	0.628	0.571	0.476	0.388	0.331	0.298
0.10	0.690	0.617	0.564	0.474	0.383	0.332	0.299
0.15	0.679	0.608	0.556	0.470	0.383	0.332	0.299
0.20	0.668	0.601	0.550	0.468	0.385	0.331	0.299
0.25	0.659	0.595	0.545	0.465	0.384	0.330	0.298
0.30	0.650	0.588	0.540	0.462	0.383	0.330	0.297
0.35	0.642	0.583	0.535	0.460	0.382	0.328	0.296
0.40	0.634	0.578	0.532	0.458	0.381	0.327	0.295
0.45	0.627	0.574	0.529	0.455	0.380	0.325	0.294
0.50	0.622	0.570	0.526	0.453	0.378	0.324	0.292
0.55	0.616	0.566	0.525	0.451	0.376	0.322	0.290
0.60	0.612	0.565	0.523	0.450	0.374	0.320	0.288
0.65	0.608	0.563	0.523	0.450	0.373	0.317	0.285
0.70	0.605	0.562	0.523	0.449	0.370	0.315	0.282
0.75	0.600	0.562	0.523	0.449	0.368	0.312	0.000
0.80	0.601	0.564	0.524	0.448	0.365	0.309	0.000
0.85	0.605	0.570	0.527	0.449	0.361	0.306	0.000
0.90	0.615	0.582	0.535	0.450	0.358	0.302	0.000
0.95	0.631	0.599	0.547	0.451	0.354	0.298	0.000
1.00	0.657	0.619	0.567	0.454	0.350	0.294	0.000

Table 15. Array of values of K_t for the system $\text{Nd}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{Nd}

x_{Nd}	$(M_t)_{\text{aq.}}$						
	3.0	4.0	5.0	7.0	10.0	13.0	16.0
0.00	0.713	0.640	0.580	0.478	0.382	0.330	0.296
0.05	0.702	0.626	0.570	0.476	0.383	0.332	0.301
0.10	0.693	0.615	0.562	0.474	0.385	0.335	0.303
0.15	0.683	0.605	0.555	0.471	0.386	0.337	0.305
0.20	0.673	0.597	0.550	0.469	0.387	0.338	0.305
0.25	0.664	0.593	0.547	0.467	0.387	0.338	0.306
0.30	0.656	0.587	0.544	0.465	0.388	0.338	0.306
0.35	0.649	0.585	0.542	0.463	0.388	0.338	0.306
0.40	0.643	0.583	0.541	0.461	0.388	0.337	0.305
0.45	0.637	0.582	0.540	0.460	0.387	0.336	0.305
0.50	0.631	0.582	0.540	0.459	0.386	0.335	0.303
0.55	0.626	0.582	0.540	0.458	0.385	0.335	0.302
0.60	0.622	0.583	0.542	0.457	0.382	0.333	0.300
0.65	0.620	0.585	0.544	0.457	0.380	0.330	0.298
0.70	0.620	0.587	0.546	0.457	0.337	0.329	0.295
0.75	0.620	0.591	0.548	0.457	0.374	0.326	0.000
0.80	0.623	0.597	0.552	0.460	0.370	0.324	0.000
0.85	0.628	0.605	0.556	0.463	0.366	0.320	0.000
0.90	0.640	0.615	0.562	0.466	0.362	0.314	0.000
0.95	0.660	0.627	0.568	0.464	0.358	0.307	0.000
1.00	0.697	0.643	0.577	0.457	0.352	0.000	0.000

Table 16. Array of values of K_t for the system $\text{Sm}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{Sm}

x_{Sm}	$(M_t)_{\text{aq.}}$						
	3.0	4.0	5.0	7.0	10.0	13.0	16.0
0.00	0.713	0.640	0.580	0.478	0.382	0.330	0.296
0.05	0.706	0.635	0.576	0.480	0.388	0.335	0.309
0.10	0.699	0.630	0.573	0.479	0.391	0.339	0.316
0.15	0.693	0.628	0.570	0.479	0.394	0.342	0.323
0.20	0.687	0.625	0.569	0.479	0.396	0.345	0.327
0.25	0.681	0.622	0.567	0.479	0.398	0.348	0.332
0.30	0.675	0.620	0.566	0.480	0.399	0.350	0.335
0.35	0.672	0.619	0.565	0.480	0.399	0.350	0.338
0.40	0.668	0.618	0.566	0.480	0.399	0.351	0.341
0.45	0.666	0.619	0.566	0.481	0.398	0.351	0.343
0.50	0.665	0.619	0.568	0.483	0.396	0.351	0.344
0.55	0.665	0.621	0.570	0.480	0.395	0.350	0.000
0.60	0.667	0.623	0.573	0.486	0.394	0.348	0.000
0.65	0.670	0.625	0.577	0.488	0.393	0.346	0.000
0.70	0.674	0.630	0.581	0.490	0.390	0.343	0.000
0.75	0.681	0.637	0.586	0.492	0.389	0.340	0.000
0.80	0.688	0.648	0.591	0.494	0.387	0.335	0.000
0.85	0.697	0.666	0.598	0.496	0.384	0.328	0.000
0.90	0.708	0.682	0.604	0.498	0.380	0.319	0.000
0.95	0.731	0.693	0.612	0.501	0.375	0.307	0.000
1.00	0.770	0.700	0.620	0.504	0.370	0.290	0.000

Table 17. Array of values of K_{HNO_3} for the system $\text{La}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{La}

Table 18. Array of values of K_{HNO_3} for the system $\text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{Pr}

Table 19. Array of values of K_{HNO_3} for the system $\text{Nd}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{Nd}

Table 20. Array of values of K_{HNO_3} for the system $\text{Sm}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{Sm}

Table 21. Array of values of K_t for the system $\text{La}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{org.}}$ and Y_{La}

Y_{La}	$(M_t)_{\text{org.}}$					
	2.0	2.5	3.0	3.5	4.0	4.5
0.00	0.769	0.667	0.566	0.441	0.358	0.314
0.05	0.610	0.547	0.484	0.394	0.344	0.304
0.10	0.568	0.505	0.452	0.370	0.329	0.295
0.15	0.545	0.482	0.430	0.354	0.314	0.283
0.20	0.529	0.466	0.413	0.345	0.300	0.270
0.25	0.516	0.457	0.399	0.337	0.284	0.254
0.30	0.505	0.450	0.389	0.329	0.269	0.000
0.35	0.498	0.445	0.380	0.320	0.254	0.000
0.40	0.492	0.442	0.374	0.312	0.239	0.000
0.45	0.486	0.438	0.370	0.303	0.000	0.000
0.50	0.484	0.435	0.367	0.295	0.000	0.000
0.55	0.482	0.432	0.365	0.286	0.000	0.000
0.60	0.480	0.430	0.364	0.278	0.000	0.000
0.65	0.480	0.427	0.361	0.270	0.000	0.000
0.70	0.481	0.425	0.357	0.262	0.000	0.000
0.75	0.483	0.424	0.352	0.253	0.000	0.000
0.80	0.485	0.422	0.346	0.245	0.000	0.000
0.85	0.487	0.420	0.339	0.237	0.000	0.000
0.90	0.490	0.419	0.331	0.228	0.000	0.000
0.95	0.494	0.417	0.322	0.220	0.000	0.000
1.00	0.498	0.415	0.313	0.212	0.000	0.000

Table 22. Array of values of K_t for the system $\text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{org.}}$ and Y_{Pr}

Y_{Pr}	$(M_t)_{\text{org.}}$					
	2.0	2.5	3.0	3.5	4.0	4.5
0.00	0.769	0.667	0.566	0.441	0.358	0.314
0.05	0.693	0.603	0.531	0.437	0.358	0.315
0.10	0.662	0.575	0.509	0.433	0.355	0.314
0.15	0.639	0.559	0.495	0.426	0.351	0.310
0.20	0.620	0.549	0.484	0.420	0.346	0.305
0.25	0.605	0.544	0.475	0.413	0.340	0.298
0.30	0.595	0.540	0.469	0.406	0.331	0.290
0.35	0.588	0.537	0.464	0.398	0.323	0.280
0.40	0.584	0.535	0.461	0.389	0.314	0.269
0.45	0.582	0.533	0.459	0.379	0.304	0.000
0.50	0.583	0.533	0.456	0.368	0.000	0.000
0.55	0.584	0.534	0.455	0.359	0.000	0.000
0.60	0.588	0.535	0.455	0.352	0.000	0.000
0.65	0.593	0.539	0.455	0.349	0.000	0.000
0.70	0.599	0.544	0.458	0.346	0.000	0.000
0.75	0.607	0.551	0.461	0.345	0.000	0.000
0.80	0.616	0.561	0.467	0.345	0.000	0.000
0.85	0.625	0.574	0.475	0.345	0.000	0.000
0.90	0.636	0.588	0.485	0.346	0.000	0.000
0.95	0.648	0.602	0.499	0.349	0.000	0.000
1.00	0.660	0.619	0.519	0.351	0.000	0.000

Table 23. Array of values of K_t for the system $\text{Nd}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{org.}}$ and Y_{Nd}

Y_{Nd}	$(M_t)_{\text{org.}}$					
	2.0	2.5	3.0	3.5	4.0	4.5
0.00	0.769	0.667	0.566	0.441	0.358	0.314
0.05	0.700	0.611	0.540	0.445	0.368	0.321
0.10	0.665	0.584	0.522	0.445	0.374	0.326
0.15	0.643	0.569	0.508	0.442	0.378	0.329
0.20	0.627	0.564	0.499	0.435	0.377	0.329
0.25	0.618	0.562	0.492	0.424	0.375	0.326
0.30	0.612	0.564	0.487	0.415	0.368	0.321
0.35	0.611	0.565	0.485	0.408	0.357	0.315
0.40	0.610	0.566	0.485	0.403	0.345	0.306
0.45	0.611	0.569	0.488	0.400	0.327	0.000
0.50	0.614	0.572	0.490	0.400	0.307	0.000
0.55	0.616	0.575	0.494	0.401	0.000	0.000
0.60	0.620	0.580	0.500	0.404	0.000	0.000
0.65	0.625	0.584	0.508	0.405	0.000	0.000
0.70	0.630	0.590	0.519	0.405	0.000	0.000
0.75	0.636	0.596	0.527	0.403	0.000	0.000
0.80	0.645	0.603	0.533	0.400	0.000	0.000
0.85	0.655	0.613	0.537	0.398	0.000	0.000
0.90	0.669	0.624	0.540	0.395	0.000	0.000
0.95	0.684	0.637	0.542	0.390	0.000	0.000
1.00	0.702	0.653	0.543	0.386	0.000	0.000

Table 24. Array of values of K_t for the system $\text{Sm}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{org.}}$ and y_{Sm}

y_{Sm}	$(M_t)_{\text{org.}}$					
	2.0	2.5	3.0	3.5	4.0	4.5
0.00	0.769	0.667	0.566	0.441	0.358	0.314
0.05	0.724	0.641	0.555	0.449	0.370	0.328
0.10	0.700	0.626	0.548	0.454	0.377	0.339
0.15	0.684	0.615	0.543	0.458	0.383	0.345
0.20	0.672	0.608	0.541	0.460	0.388	0.349
0.25	0.665	0.605	0.541	0.461	0.390	0.350
0.30	0.660	0.603	0.544	0.462	0.390	0.349
0.35	0.659	0.604	0.548	0.461	0.388	0.346
0.40	0.659	0.606	0.553	0.461	0.386	0.341
0.45	0.660	0.610	0.560	0.460	0.382	0.000
0.50	0.664	0.616	0.567	0.459	0.377	0.000
0.55	0.669	0.623	0.575	0.475	0.372	0.000
0.60	0.676	0.630	0.582	0.455	0.365	0.000
0.65	0.685	0.640	0.590	0.450	0.359	0.000
0.70	0.695	0.652	0.598	0.450	0.351	0.000
0.75	0.707	0.665	0.605	0.449	0.343	0.000
0.80	0.720	0.679	0.614	0.450	0.000	0.000
0.85	0.733	0.694	0.622	0.455	0.000	0.000
0.90	0.748	0.710	0.630	0.463	0.000	0.000
0.95	0.763	0.727	0.638	0.475	0.000	0.000
1.00	0.780	0.746	0.646	0.492	0.000	0.000

Table 25. Array of values of K_{HNO_3} for the system $\text{La}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{org.}}$ and Y_{La}

Table 26. Array of values of K_{HNO_3} for the system $\text{Pr}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{org.}}$ and Y_{Pr}

Table 27. Array of values of K_{HNO_3} for the system $\text{Nd}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O as a function of $(M_t)_{\text{org.}}$ and Y_{Nd}

Table 28. Array of values of K_{HNO_3} for the system $\text{Sm}(\text{NO}_3)_3$ - HNO_3 - TBP - H_2O as a function of $(M_t)_{\text{org.}}$ and Y_{Sm}

N	- total number of stages in an extraction cascade
M	- feed stage in an extraction cascade with internal feed
S	- flow rate of solvent in organic streams, kgm. TBP/unit time
F	- flow rate of solvent in a feed stream, kgm. H ₂ O/unit time
SOVR	- flow rate ratio, S/R
ROVS	- flow rate ratio, R/S
REX	- flow rate of solvent in aqueous streams on extract side of internally fed cascade, kgm. H ₂ O/unit time
RSC	- flow rate of solvent in aqueous streams on scrub side of internally fed cascade
TMAQ	- (M _t) _{aq.}
AMAQ	- (M _{HNO₃}) _{aq.}
XLAMAQ	- (M _{La}) _{aq.}
PRMAQ	- (M _{Pr}) _{aq.}
XNDMAQ	- (M _{Nd}) _{aq.}
SMMAQ	- (M _{Sm}) _{aq.}
REMAQ	- (M _{RE}) _{aq.}
XHNO ₃	- X _{HNO₃}
XLA	- X _{La}
XPR	- X _{Pr}
XND	- X _{Nd}
XSM	- X _{Sm}

Figure 37. Principal variable names

XRE	- x_{RE}
XLASF	- x_{La}
XPRSF	- x_{Pr}
XNDSF	- x_{Nd}
XSMSF	- x_{Sm}
TMOR	- $(M_t)_{org.}$
AMOR	- $(M_{HNO_3})_{org.}$
XLAMOR	- $(M_{La})_{org.}$
PRMOR	- $(M_{Pr})_{org.}$
XNDMOR	- $(M_{Nd})_{org.}$
SMMOR	- $(M_{Sm})_{org.}$
REMOR	- $(M_{RE})_{org.}$
YHNO ₃	- y_{HNO_3}
YLA	- y_{La}
YPR	- y_{Pr}
YND	- y_{Nd}
YSM	- y_{Sm}
YRE	- y_{RE}
YLASF	- y_{La}
YPRSF	- y_{Pr}
YNDSF	- y_{Nd}
YSMSF	- y_{Sm}

Figure 37. (Continued)

XKTRE	- K_t for the system $\text{RE}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{RE} ; RE = La, Pr, Nd, Sm
XXKARE	- K_{HNO_3} for the system $\text{RE}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{aq.}}$ and x_{RE} ; RE = La, Pr, Nd, Sm
YKTRE	- K_t for the system $\text{RE}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{org.}}$ and y_{RE} ; RE = La, Pr, Nd, Sm
YKARE	- K_{HNO_3} for the system $\text{RE}(\text{NO}_3)_3 - \text{HNO}_3 - \text{TBP} - \text{H}_2\text{O}$ as a function of $(M_t)_{\text{org.}}$ and y_{RE} ; RE = La, Pr, Nd, Sm
BLAPR	- $\beta_{\text{La-Pr}}$
BPRPR	- $\beta_{\text{Pr-Pr}}$
BNDPR	- $\beta_{\text{Nd-Pr}}$
BSMPR	- $\beta_{\text{Sm-Pr}}$
TOTK	- K_t
HNO3K	- K_{HNO_3}
FTM	- $(M_t)_F$
FAM	- $(M_{\text{HNO}_3})_F$
FLAM	- $(M_{\text{La}})_F$
FPRM	- $(M_{\text{Pr}})_F$
FNDM	- $(M_{\text{Nd}})_F$
FSMM	- $(M_{\text{Sm}})_F$
FREM	- $(M_{\text{RE}})_F$
XAF	- $(x_{\text{HNO}_3})_F$

Figure 37. (Continued)

XLAFF - $(x_{La})_F$

XPRF - $(x_{Pr})_F$

XNDF - $(x_{Nd})_F$

XSMF - $(x_{Sm})_F$

XREF - $(x_{RE})_F$

XLASFF - $(x_{La})_F$

XPRSFF - $(x_{Pr})_F$

XNDSFF - $(x_{Nd})_F$

XSMSFF - $(x_{Sm})_F$

For Programs 1 and 3:

R1TM - $(M_t)_{R_1}$

R1AM - $(M_{HNO_3})_{R_1}$

R1LAM - $(M_{La})_{R_1}$

R1PRM - $(M_{Pr})_{R_1}$

R1NDM - $(M_{Nd})_{R_1}$

R1SMM - $(M_{Sm})_{R_1}$

SOTM - $(M_t)_{S_0}$

SOAM - $(M_{HNO_3})_{S_0}$

SOLAM - $(M_{La})_{S_0}$

SOPRM - $(M_{Pr})_{S_0}$

Figure 37. (Continued)

SONDM - $(M_{Nd})_{S_0}$

SOSMM - $(M_{Sm})_{S_0}$

For Programs 2 and 4:

ROTM - $(M_t)_{R_0}$

ROAM - $(M_{HNO_3})_{R_0}$

ROLAM - $(M_{La})_{R_0}$

ROPRM - $(M_{Pr})_{R_0}$

RONDM - $(M_{Nd})_{R_0}$

ROSMM - $(M_{Sm})_{R_0}$

S1TM - $(M_t)_{S_1}$

S1AM - $(M_{HNO_3})_{S_1}$

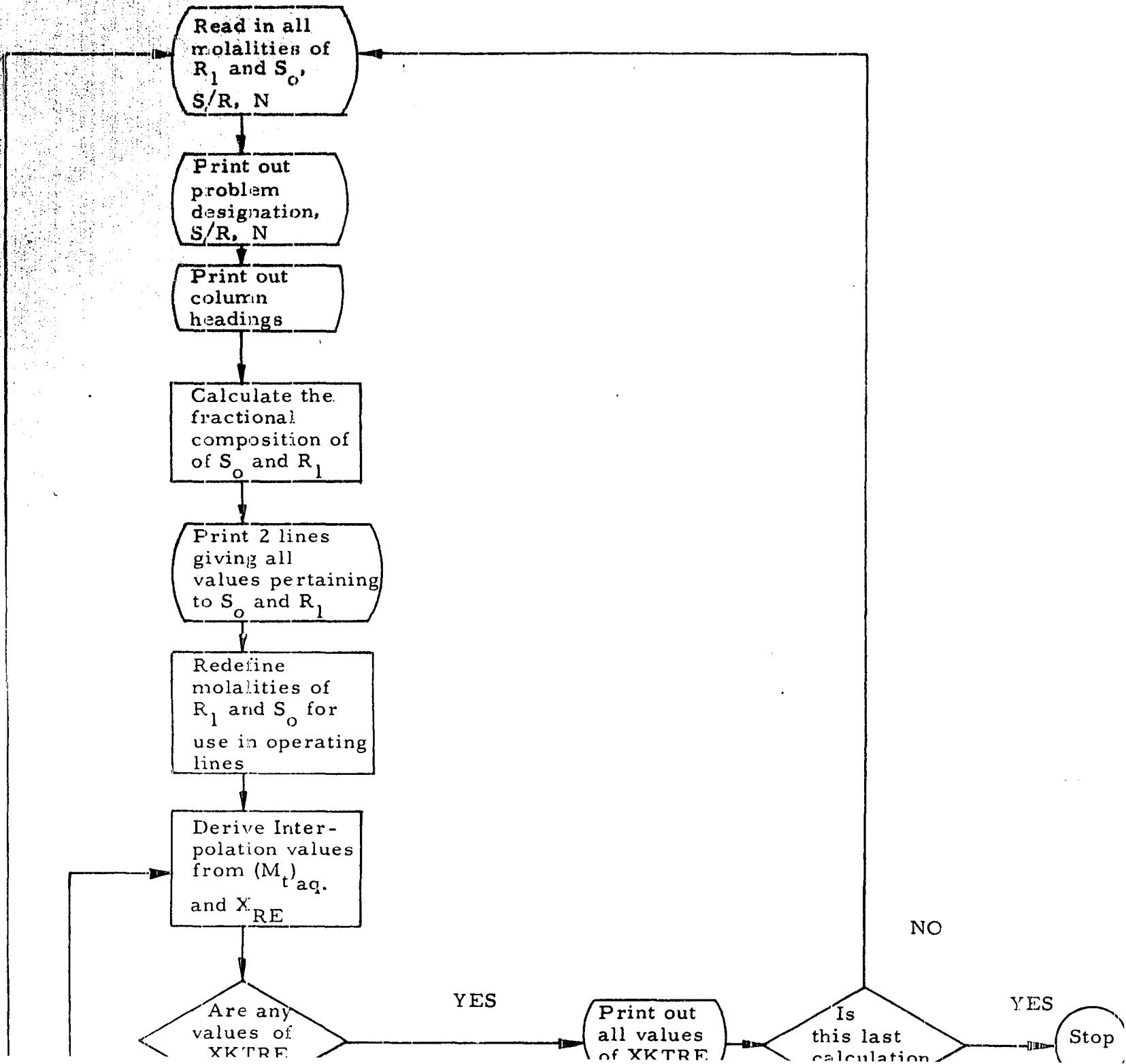
S1LAM - $(M_{La})_{S_1}$

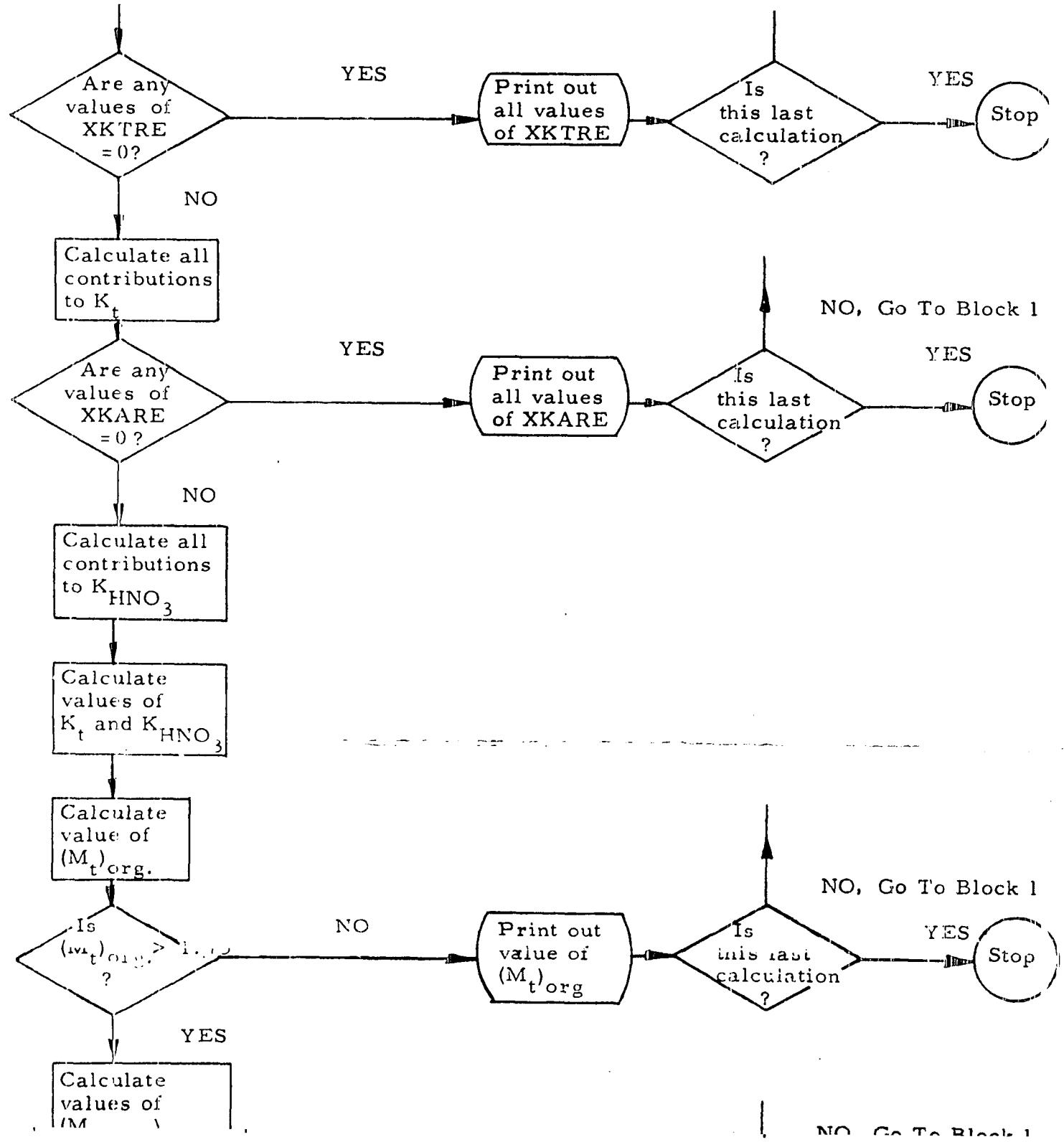
S1PRM - $(M_{Pr})_{S_1}$

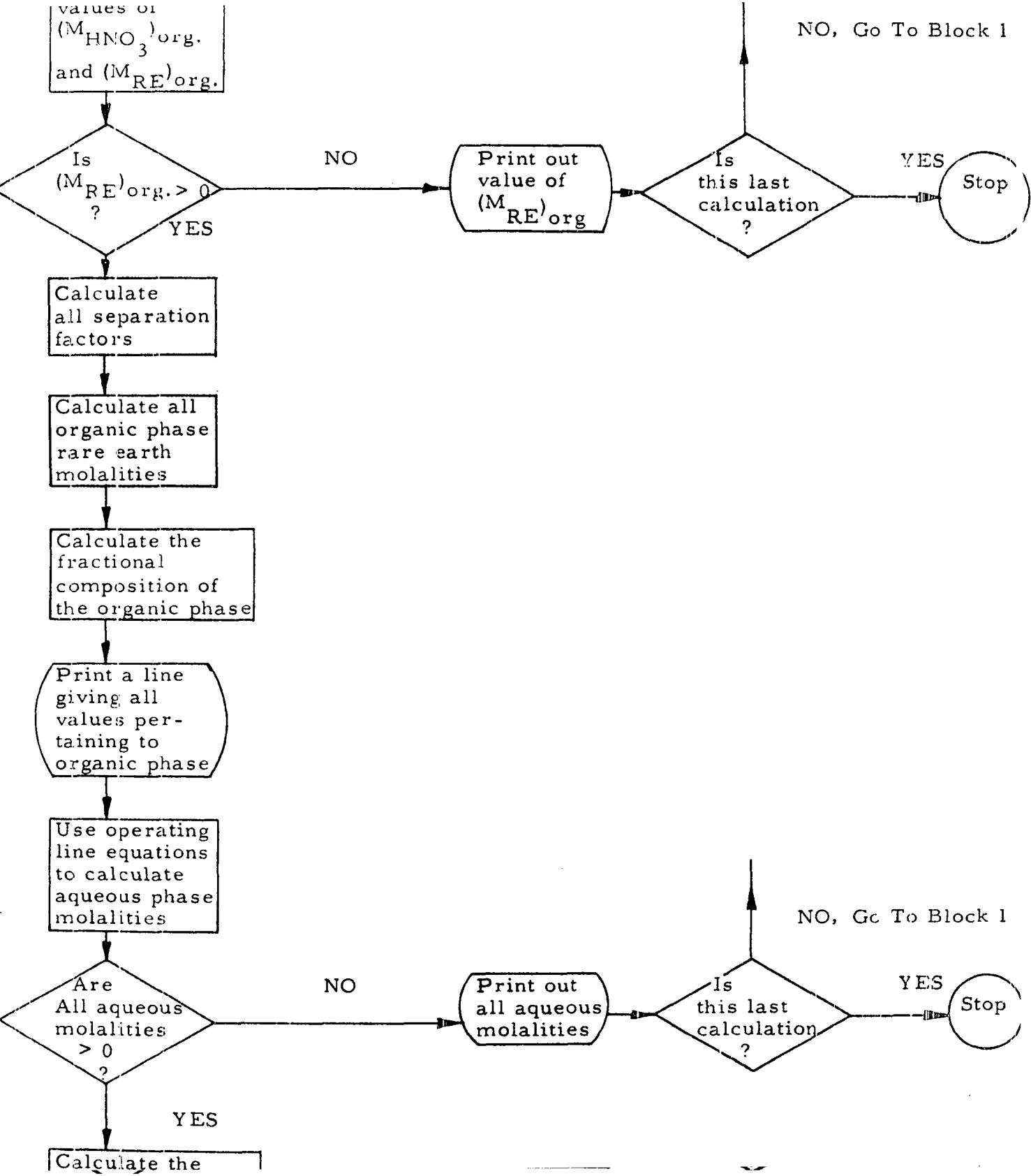
S1NDM - $(M_{Nd})_{S_1}$

S1SMM - $(M_{Sm})_{S_1}$

Figure 37. (Continued)

Block 1





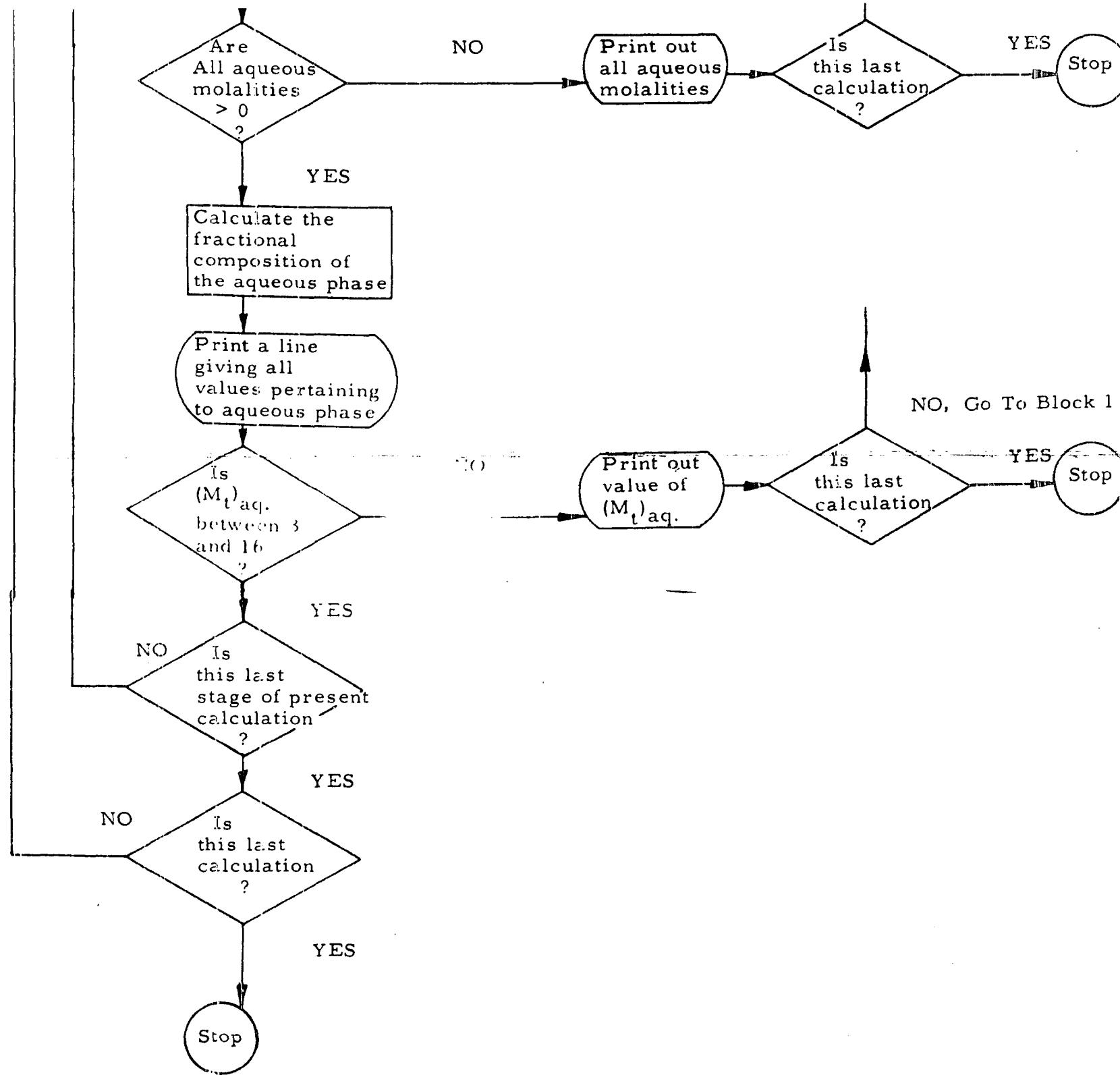


Figure 38. Flow diagram, Program I

```

      DIMENSION XKTLA(21,7),XKTPR(21,7),XKTND(21,7),XKTSH(21,7),XKALA(21BMS1 001
1,7),XKAPR(21,7),XKAND(21,7),XKASM(21,7)                                BMS1 002
      DO 5 I = 1,21                                                       BMS1 C1
5       READ INPUT TAPE 5, 500, (XKTLA(I,J),J = 1,7)                         BMS1 C2
      DO 10 I = 1,21                                                       BMS1 C3
10      READ INPUT TAPE 5, 500, (XKTPR(I,J),J = 1,7)                         BMS1 C4
      DO 15 I = 1,21                                                       BMS1 C5
15      READ INPUT TAPE 5, 500, (XKTND(I,J),J = 1,7)                         BMS1 C6
      DO 20 I = 1,21                                                       BMS1 C7
20      READ INPUT TAPE 5, 500, (XKTSH(I,J),J = 1,7)                         BMS1 C8
      DO 25 I = 1,21                                                       BMS1 C9
25      READ INPUT TAPE 5, 500, (XKALA(I,J),J = 1,7)                         BMS1 C10
      DO 30 I = 1,21                                                       BMS1 C11
30      READ INPUT TAPE 5, 500, (XKAPR(I,J),J = 1,7)                         BMS1 C12
      DO 35 I = 1,21                                                       BMS1 C13
35      READ INPUT TAPE 5, 500, (XKAND(I,J),J = 1,7)                         BMS1 C14
      DO 40 I = 1,21                                                       BMS1 C15
40      READ INPUT TAPE 5, 500, (XKASM(I,J),J = 1,7)                         BMS1 C16
45      READ INPUT TAPE 5, 501, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMS1 035
      READ INPUT TAPE 5, 502, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,BMS1 036
1SOVR,N,K
      PRINT 503, SOVR,N
      PRINT 504
      XHN03 = AMAQ/TMAQ
      XLA = XLAMAQ/TMAQ
      XPR = PRMAQ/TMAQ
      XND = XNDMAQ/TMAQ
      XSM = SMMAQ/TMAQ
      XRE = REMAQ/TMAQ
      SXA = XHN03 + XLA + XPR + XND + XSM
      XLASF = XLAMAQ/REMAQ
      XPRSF = PRMAQ/REMAQ
      XNDSF = XNDMAQ/REMAQ
      XSMSF = SMMAQ/REMAQ
      SXREA = XLASF + XPRSF + XNDSF + XSMSF
      YHN03 = AMOR/TMOR
      YLA = XLAMOR/TMOR
      YPR = PRMOR/TMOR
      YND = XNDMOR/TMOR
      YSM = SMMOR/TMOR
      YRE = REMOR/TMOR
      SYO = YHN03 + YLA + YPR + YND + YSM
      YLASF = XLAMOR/REMR
      YPRSF = PRMOR/REMR
      YNDSF = XNDMOR/REMR
      XSMSF = SMMOR/REMR
      SYREO = YLASF + YPRSF + YNDSF + XSMSF
      PRINT 505, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,YHN03,YLA,YPRBMS1 064
1,YND,YSM,YRE,YLASF,YPRSF,YNDSF,YSMSF,SYO,SYREO
      PRINT 506, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHN03,XLA,XPRBMS1 066
1,XND,XSM,XRE,XLASF,XPRSF,XNDSF,XSMSF,SXA,SXREA
      R1TH = TMAQ
      R1AM = AMAQ
      R1LM = XLAMAQ
      R1PR = PRMAQ
      R1ND = XNDMAQ
      R1SM = SMMAQ
      SOTH = TMOR
      SOAM = AMOR
      SOLAM = XLAMOR
      SOPRM = PRMOR
      SONDM = XNDMOR
      BMS1 067
      BMS1 068
      BMS1 069
      BMS1 070
      BMS1 071
      BMS1 072
      BMS1 073
      BMS1 074
      BMS1 075
      BMS1 076
      BMS1 077
      BMS1 078

```

Figure 39. Fortran statements, Program I

```

SOSMM = SMMOR
50 DO 150 L = 1,N                                BMSI 079
XREI = (XRE+100.0 + 5.0)/5.0                  BMSI 080
IX = XREI                                         BMSI 081
XI = IX                                           BMSI 082
FRX = XREI - XI                                 BMSI 083
55 IF(5.0 - TMAQ) 60,70,65                         BMSI 084
60 IF(7.0 - TMAQ) 75,75,70                         BMSI 085
65 Z = TMAQ - 2.0                                  BMSI 086
GO TO 80                                         BMSI 087
70 Z * (TMAQ + 1.0)/2.0                           BMSI 088
GO TO 80                                         BMSI 089
75 Z = (TMAQ + 5.0)/3.0                           BMSI 090
80 KZ = Z                                         BMSI 091
ZK = KZ                                         BMSI 092
FRZ = Z - ZK                                     BMSI 093
SKT = MIN1F(XKTLA(IX,KZ),XKTLA(IX+1,KZ),XKTLA(IX,KZ+1),XKTLA(IX+1,BMSI 095
1KZ+1),XKTPR(IX,KZ),XKTPR(IX+1,KZ),XKTPR(IX,KZ+1),XKTPR(IX+1,KZ+1),BMSI 096
2XKTN(IX,KZ),XKTN(IX+1,KZ),XKTN(IX,KZ+1),XKTN(IX+1,KZ+1),XKTS(BMSI 097
3IX,KZ),XKTS(IX+1,KZ),XKTS(IX,KZ+1),XKTS(IX+1,KZ+1))   BMSI 098
IF(SKT) 85,85,90                                 BMSI 099
85 PRINT 507, XKTLA(IX,KZ),XKTLA(IX+1,KZ),XKTLA(IX,KZ+1),XKTLA(IX+1,BMSI 100
1Z+1),XKTPR(IX,KZ),XKTPR(IX+1,KZ),XKTPR(IX,KZ+1),XKTPR(IX+1,KZ+1),XBMSI 101
2KTN(IX,KZ),XKTN(IX+1,KZ),XKTN(IX,KZ+1),XKTN(IX+1,KZ+1),XKTS(BMSI 102
3X,KZ),XKTS(IX+1,KZ),XKTS(IX,KZ+1),XKTS(IX+1,KZ+1),TMAQ,XRE   BMSI 103
IF(K) 495,495,45                               BMSI 104
90 XKTLA1 = XKTLA(IX,KZ)*(1.0 - FRX) + XKTLA(IX+1,KZ)*FRX  BMSI 105
XKTLA2 = XKTLA(IX,KZ+1)*(1.0 - FRX) + XKTLA(IX+1,KZ+1)*FRX BMSI 106
XKTLA3 = XKTLA1*(1.0 - FRZ) + XKTLA2*FRZ             BMSI 107
CKTLA = XKTLA3 * XLASF                          BMSI 108
XKTPR1 = XKTPR(IX,KZ)*(1.0 - FRX) + XKTPR(IX+1,KZ)*FRX  BMSI 109
XKTPR2 = XKTPR(IX,KZ+1)*(1.0 - FRX) + XKTPR(IX+1,KZ+1)*FRX BMSI 110
XKTPR3 = XKTPR1*(1.0 - FRZ) + XKTPR2*FRZ             BMSI 111
CKTPR = XKTPR3 * XPRS(F)                        BMSI 112
XKTN1 = XKTN(IX,KZ)*(1.0 - FRX) + XKTN(IX+1,KZ)*FRX  BMSI 113
XKTN2 = XKTN(IX,KZ+1)*(1.0 - FRX) + XKTN(IX+1,KZ+1)*FRX BMSI 114
XKTN3 = XKTN1*(1.0 - FRZ) + XKTN2*FRZ              BMSI 115
CKTN = XKTN3 * XNDS(F)                          BMSI 116
XKTS1 = XKTS(IX,KZ)*(1.0 - FRX) + XKTS(IX+1,KZ)*FRX  BMSI 117
XKTS2 = XKTS(IX,KZ+1)*(1.0 - FRX) + XKTS(IX+1,KZ+1)*FRX BMSI 118
XKTS3 = XKTS1*(1.0 - FRZ) + XKTS2*FRZ              BMSI 119
CKTS = XKTS3 * XSMS(F)                          BMSI 120
SKA = MIN1F(XKALA(IX,KZ),XKALA(IX+1,KZ),XKALA(IX,KZ+1),XKALA(IX+1,BMSI 121
1KZ+1),XKAPR(IX,KZ),XKAPR(IX+1,KZ),XKAPR(IX,KZ+1),XKAPR(IX+1,KZ+1),BMSI 122
2XKAND(IX,KZ),XKAND(IX+1,KZ),XKAND(IX,KZ+1),XKAND(IX+1,KZ+1),XKASM(BMSI 123
3IX,KZ),XKASM(IX+1,KZ),XKASM(IX,KZ+1),XKASM(IX+1,KZ+1))   BMSI 124
IF(SKA) 95,95,100                               BMSI 125
95 PRINT 508, XKALA(IX,KZ),XKALA(IX+1,KZ),XKALA(IX,KZ+1),XKALA(IX+1,BMSI 126
1Z+1),XKAPR(IX,KZ),XKAPR(IX+1,KZ),XKAPR(IX,KZ+1),XKAPR(IX+1,KZ+1),XBMSI 127
2KAND(IX,KZ),XKAND(IX+1,KZ),XKAND(IX,KZ+1),XKAND(IX+1,KZ+1),XKASM(BMSI 128
3X,KZ),XKASM(IX+1,KZ),XKASM(IX,KZ+1),XKASM(IX+1,KZ+1),TMAQ,XRE   BMSI 129
IF(K) 495,495,45                               BMSI 130
100 XKALA1 = XKALA(IX,KZ)*(1.0 - FRX) + XKALA(IX+1,KZ)*FRX  BMSI 131
XKALA2 = XKALA(IX,KZ+1)*(1.0 - FRX) + XKALA(IX+1,KZ+1)*FRX BMSI 132
XKALA3 = XKALA1*(1.0 - FRZ) + XKALA2*FRZ             BMSI 133
CKALA = XKALA3 * XLASF                          BMSI 134
XKAPR1 = XKAPR(IX,KZ)*(1.0 - FRX) + XKAPR(IX+1,KZ)*FRX  BMSI 135
XKAPR2 = XKAPR(IX,KZ+1)*(1.0 - FRX) + XKAPR(IX+1,KZ+1)*FRX BMSI 136
XKAPR3 = XKAPR1*(1.0 - FRZ) + XKAPR2*FRZ             BMSI 137
CKAPR = XKAPR3 * XPRS(F)                        BMSI 138
XKAND1 = XKAND(IX,KZ)*(1.0 - FRX) + XKAND(IX+1,KZ)*FRX  BMSI 139
XKAND2 = XKAND(IX,KZ+1)*(1.0 - FRX) + XKAND(IX+1,KZ+1)*FRX BMSI 140

```

Figure 39. (Continued)

```

XKAND3 = XKAND1*(1.0 - FRZ) + XKAND2*FRZ          BMSI 141
CKAND = XKAND3 * XNDSF                            BMSI 142
XKASM1 = XKASM(IX,KZ)*(1.0 - FRX) + XKASM(IX+1,KZ)*FRX   BMSI 143
XKASM2 = XKASM(IX,KZ+1)*(1.0 - FRX) + XKASM(IX+1,KZ+1)*FRX   BMSI 144
XKASM3 = XKASM1*(1.0 - FRZ) + XKASM2*FRZ          BMSI 145
CKASM = XKASM3 * XSMSF                            BMSI 146
TOTK = CKTLA + CKTPR + CKTND + CKTSM            BMSI 147
HNO3K = CKALA + CKAPR + CKAND + CKASH           BMSI 148
TMOR = THAQ*TOTK                                BMSI 149
IF(TMOR - 1.75) 105,110,110
105 PRINT 509, TMOR, TMAQ, TOTK                  BMSI 150
IF(K) 495,495,45
110 AMOR = AMAQ*HNO3K                            BMSI 151
REMOR = TMOR - AMOR                            BMSI 152
IF(REMOR) 115,115,120                          BMSI 153
115 PRINT 510, TMOR, AMOR, REMOR, HNO3K         BMSI 154
IF(K) 495,495,45
120 BLAPR = 0.8187 - 0.1106*TMOR              BMSI 155
BPRPR = 1.0
BNDPR = 1.0448 + 0.09874*TMOR                BMSI 156
BSMPR = -0.3795 + 0.9214*TMOR                BMSI 157
DEMS = XLAMAQ*BLAPR + PRMAQ*BPRPR + XNDMAQ*BNDPR + SMMAQ*BSMPR BMSI 158
XLAMOR = REMOR*BLAPR*XLAMAQ/DEMS            BMSI 159
PRMOR = REMOR*BPRPR*PRMAQ/DEMS            BMSI 160
XNDMOR = REMOR*BNDPR*XNDMAQ/DEMS            BMSI 161
SMMOR = REMOR*BSMPR*SMMAQ/DEMS            BMSI 162
YHNO3 = AMOR/TMOR                            BMSI 163
YLA = XLAMOR/TMOR                           BMSI 164
YPR = PRMOR/TMOR                           BMSI 165
YND = XNDMOR/TMOR                           BMSI 166
YSM = SMMOR/TMOR                           BMSI 167
YRE = REMOR/TMOR                           BMSI 168
SYO = YHNO3 + YLA + YPR + YND + YSM          BMSI 169
YLASF = XLAMOR/REMOR                         BMSI 170
YPRSF = PRMOR/REMOR                         BMSI 171
YNDSF = XNDMOR/REMOR                         BMSI 172
YSMSF = SMMOR/REMOR                         BMSI 173
SYREQ = YLASF + YPRSF + YNDSF + YSMSF        BMSI 174
PRINT 511,L,TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,YHNO3,YLA,YPBMSI 175
1R,YND,YSM,YRE,YLASF,YPRSF,YNDSF,YSMSF,SYO,SYREQ      BMSI 176
TMAQ = TMOR*SOVR - SOTM*SOVR + R1TM          BMSI 177
AMAQ = AMOR*SOVR - SOAM*SOVR + R1AM          BMSI 178
XLAMAQ = XLAMOR*SOVR - SOLAM*SOVR + R1LAM        BMSI 179
PRMAQ = PRMOR*SOVR - SOPRM*SOVR + R1PRM        BMSI 180
XNDMAQ = XNDMOR*SOVR - SONDM*SOVR + R1NDM        BMSI 181
SMMAQ = SMMOR*SOVR - SOSMM*SOVR + R1SMM        BMSI 182
REMAQ = TMAQ - AMAQ                           BMSI 183
SAM = MIN1(TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ) BMSI 184
IF(SAM) 125,125,130                          BMSI 185
125 PRINT 512,TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMSI 186
IF(K) 495,495,45
130 XHN03 = AMAQ/TMAQ                           BMSI 187
XLA = XLAMAQ/TMAQ                           BMSI 188
XPR = PRMAQ/TMAQ                           BMSI 189
XND = XNDMAQ/TMAQ                           BMSI 190
XSM = SMMAQ/TMAQ                           BMSI 191
XRE = REMAQ/TMAQ                           BMSI 192
XKA = XHN03 + XLA + XPR + XND + XSM          BMSI 193
XLASF = XLAMAQ/REMAQ                         BMSI 194
XPRSF = PRMAQ/REMAQ                         BMSI 195
XNDSF = XNDMAQ/REMAQ                         BMSI 196
XSMSF = SMMAQ/REMAQ                         BMSI 197

```

Figure 39. (Continued)

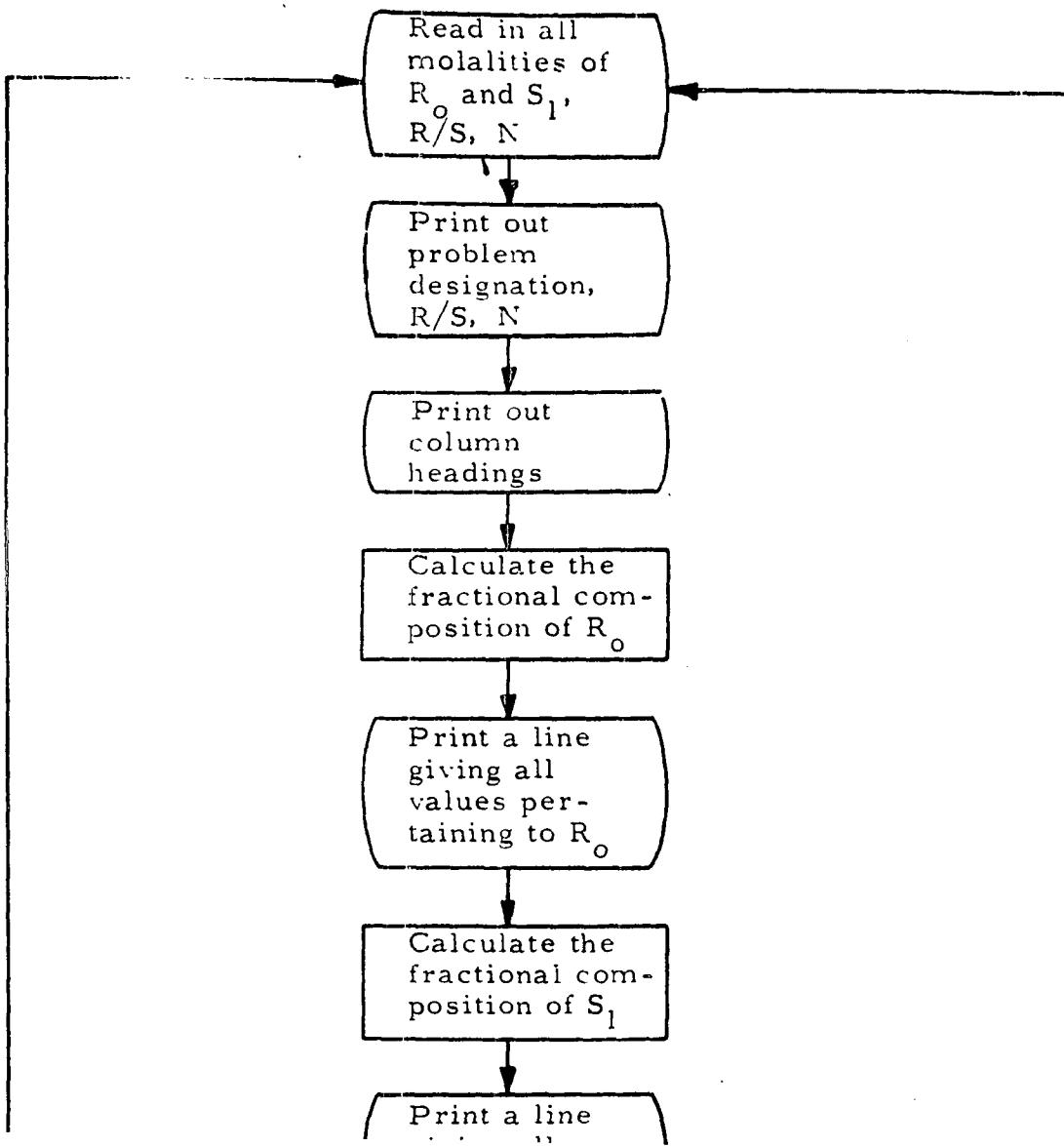
```

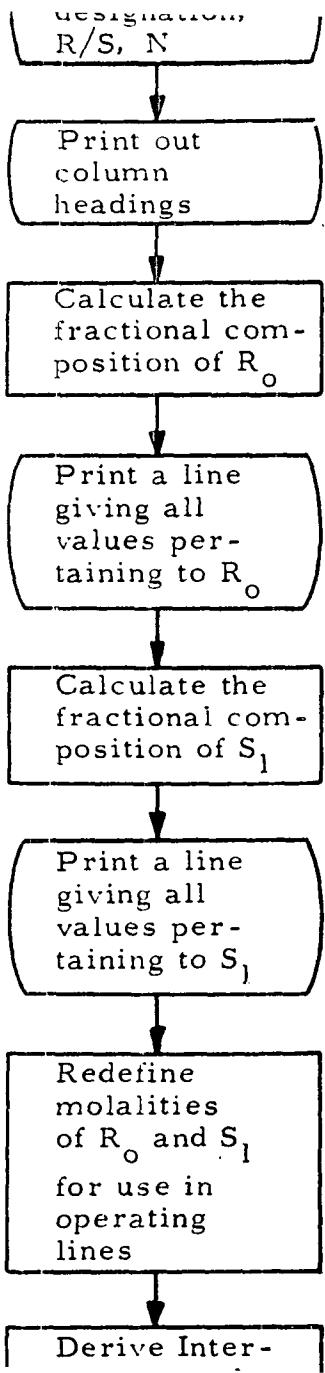
SXREA = XLASF + XPRSF + XNDSF + XSMSF          BMSI 203
LL = L+1                                         BMSI 204
PRINT 513,LL,TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHNO3,XLA,XBMSI 205
1PR,XND,XSM,XRE,XLASF,XPRSFXNDSF,XSMSFSXA,SXREA      BMSI 206
!IF(TMAQ - 3.0) 135,140,140                      BMSI 207
135 PRINT 514, TMAQ                                BMSI 208
IF(K) 495,495,45                                  BMSI 209
140 IF(TMAQ - 16.0) 150,145,145                  BMSI 210
145 PRINT 514,TMAQ                                BMSI 211
IF(K) 495,495,45                                  BMSI 212
150 CONTINUE                                     BMSI 213
IF(K)495,495,45                                  BMSI 214
500 FORMAT (7F10.3)                                BMSI 217
501 FORMAT (7F9.4)                                 BMSI 218
502 FORMAT (7F7.4,F8.4,I3,I3)                   BMSI 219
503 FORMAT (31H1EXTRACT SIDE CALCULATION SOVR= F8.4,4H N= I3) BMSI 220
504 FORMAT (120HJ TOTALMOLHNO3 MOL LA MOL PR MOL ND MOL SM MOL BMSI 221
IRE MOL ACFR LAFR PRFR NDFR SMFR REFLA/REPR/REND/RESM/RE STFRSREFRBMSI 222
2)
505 FORMAT (4HJS 0 7F8.4,12F5.3)                 BMSI 223
506 FORMAT (4H R 1 7F8.4,12F5.3)                 BMSI 224
507 FORMAT (8HJXKTR=0 16F6.3,F8.4,F6.3)        BMSI 225
508 FORMAT (8HJXKAR=0 16F6.3,F8.4,F6.3)        BMSI 226
509 FORMAT (6HJTMR= F10.4,6H TMAQ= F8.4,6H TOTK= F8.4) BMSI 227
510 FORMAT (6HJTMR= F10.4,6H AMOR= F10.8,7H REMOR= F10.8,7H HNO3K= F18BMSI 228
10.8)                                         BMSI 229
511 FORMAT (2HJS I2,7F8.4,12F5.3)                BMSI 230
512 FORMAT (6HJTMAQ= F10.4,6H AMAQ= F10.4,8H XLAMAQ= F10.4,7H PRMAQ= FBMSI 231
110.4,8H XNDMAQ= F10.4,7H SMMAQ= F10.4,7H REMAQ= F10.4) BMSI 232
513 FORMAT (2H R I2,7F8.4,12F5.3)                BMSI 233
514 FORMAT (6HJTMAQ= F10.4)                      BMSI 234
495 STOP 89                                      BMSI 235
END                                            BMSI 215
                                                 BMSI 216

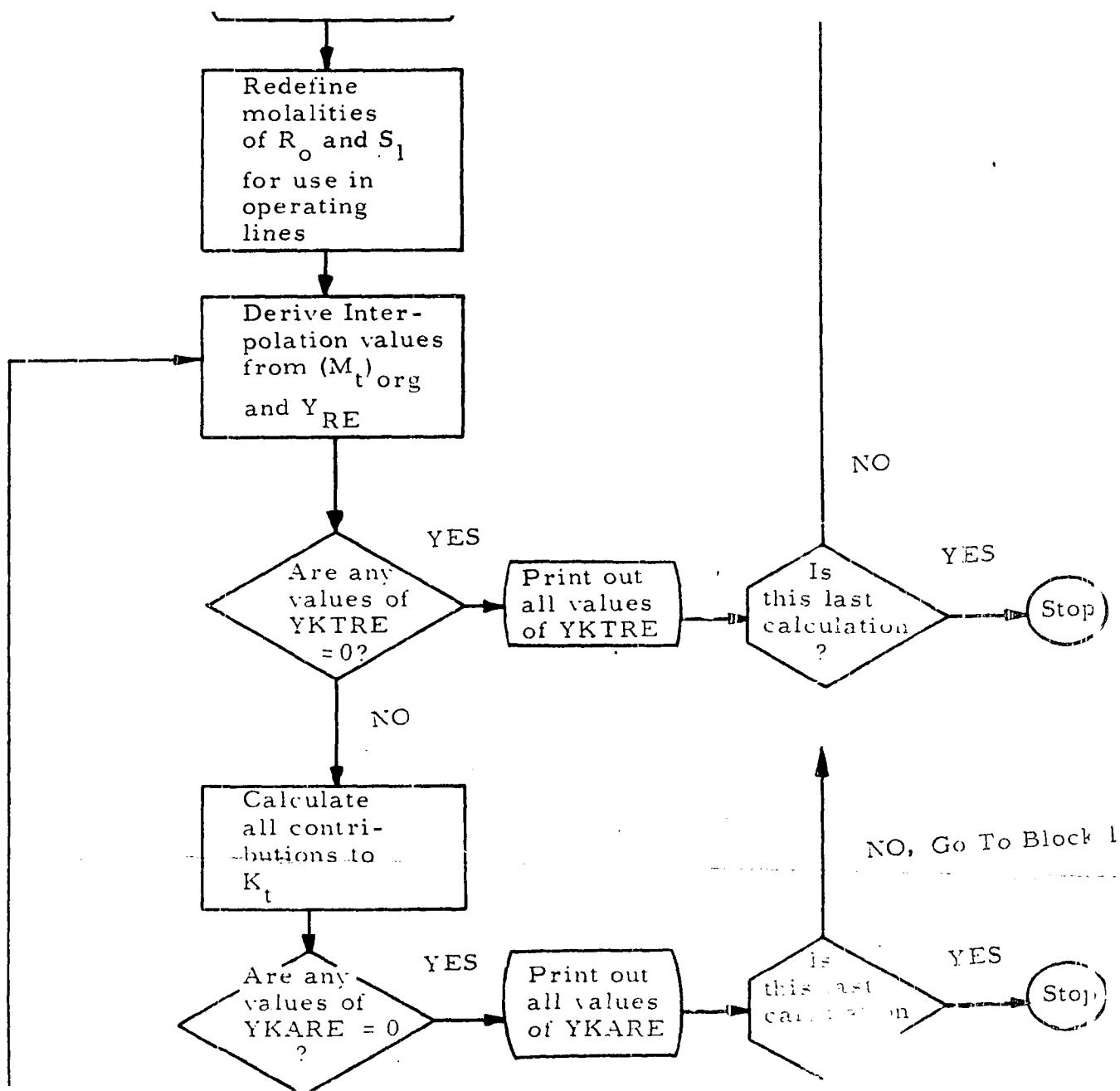
```

Figure 39. (Continued)

Block 1







Calculate
all contri-
butions to
 K_t

Are any
values of
 $YKARE = 0$
?

YES

Print out
all values
of YKARE

NO, Go To Block 1

Is this last
calculation
?

YES

Stop

Calculate
all contri-
butions to
 K_{HNO_3}

Calculate
values of
 K_t and
 K_{HNO_3}

Calculate
values of
 $(M_t)_{aq.}$,
 $(M_{HNO_3})_{aq.}$,
 $(M_{RE})_{aq.}$

Is
 M_t
NO

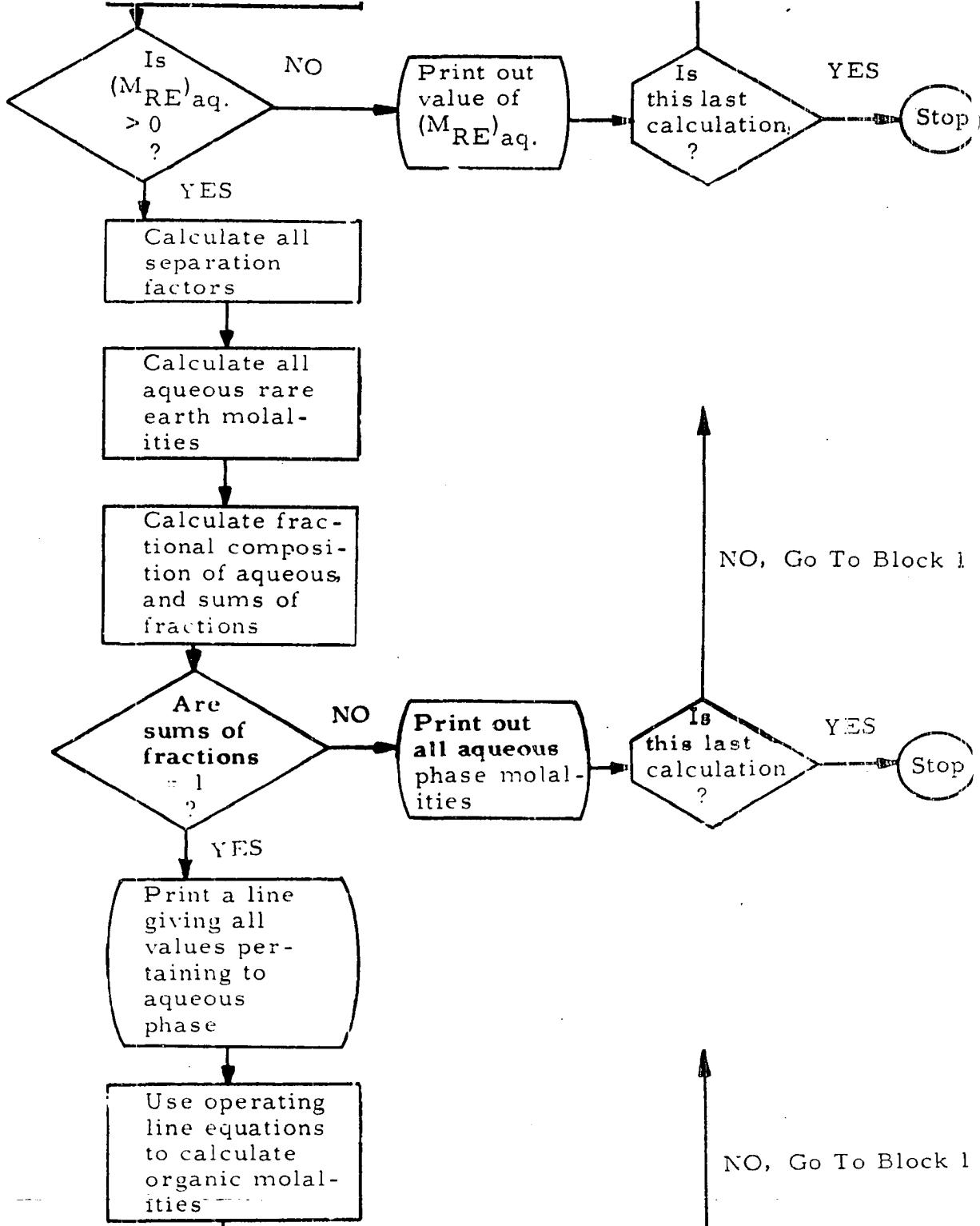
NO

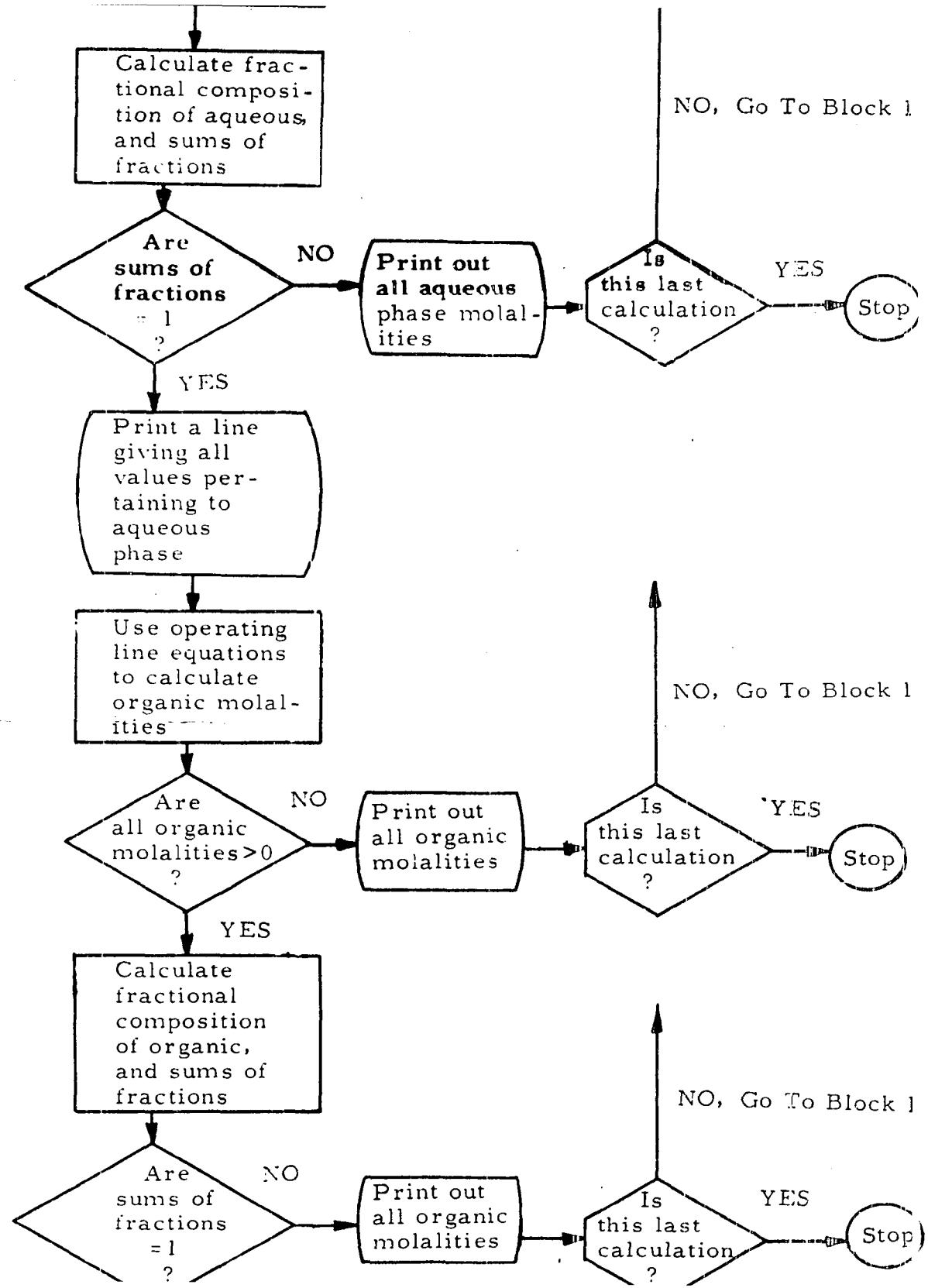
Print out

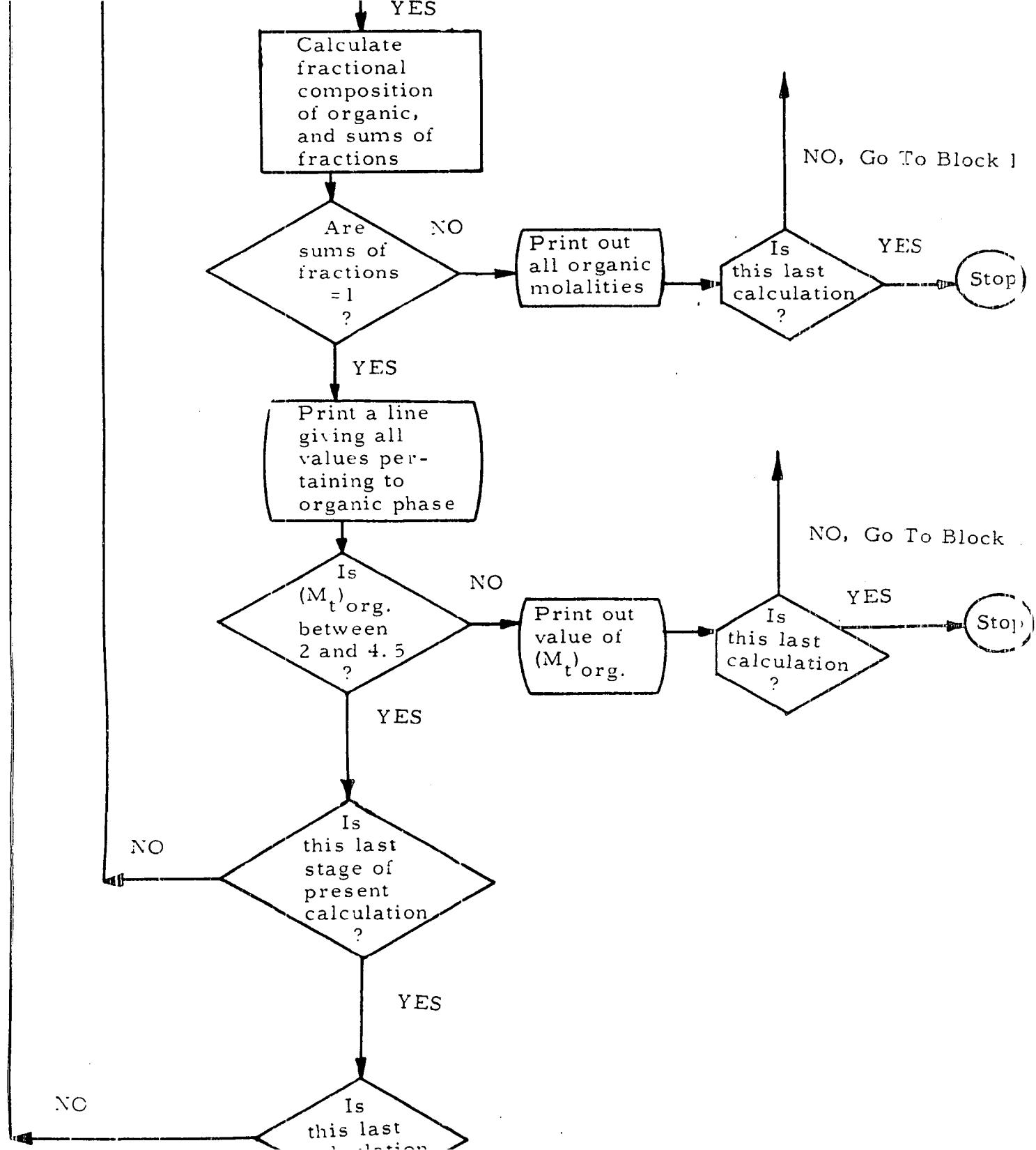
NO, Go To Block 1

Is
 M_t
YES

YES







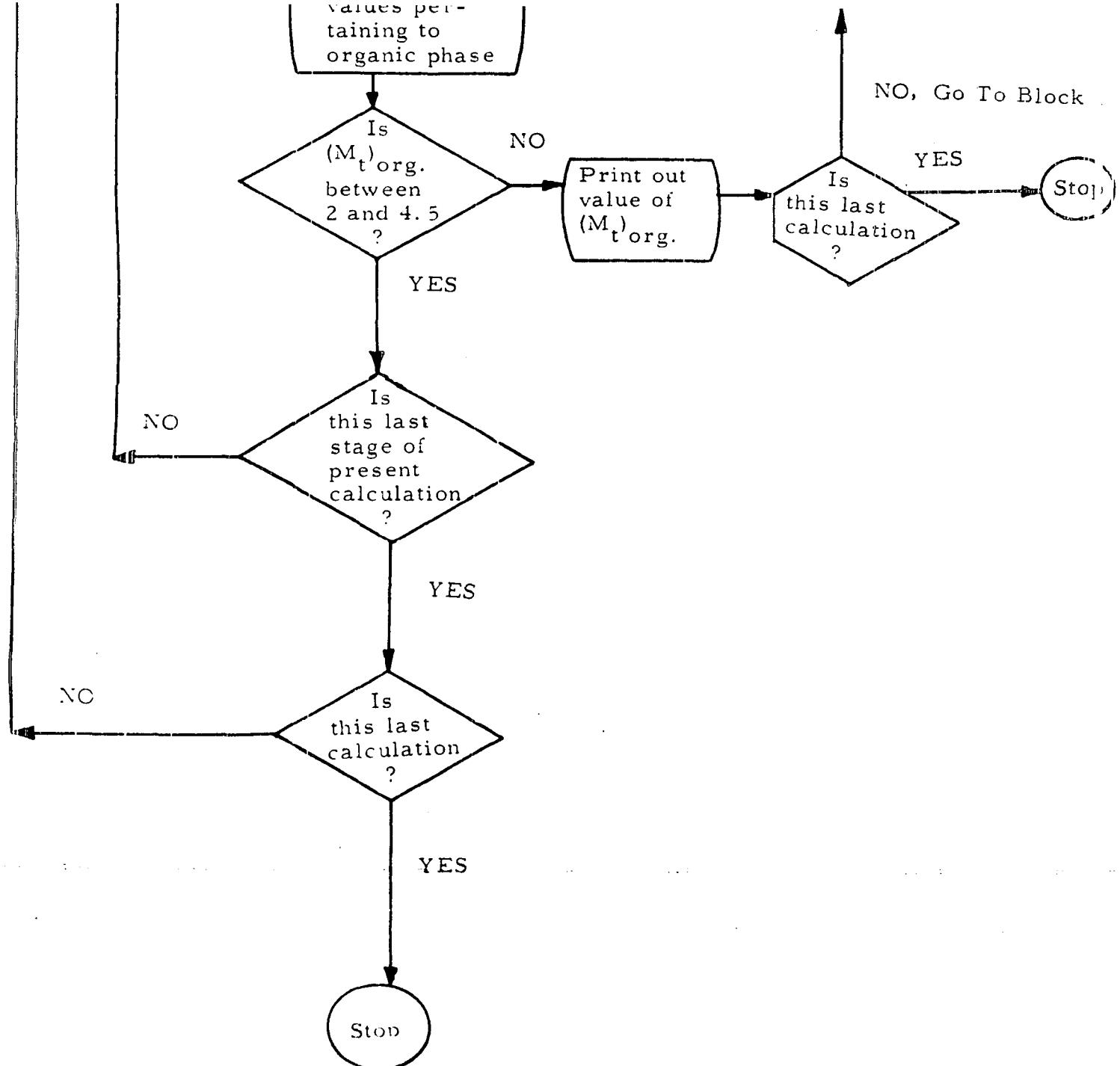


Figure 40. Flow diagram, Program II

```

DIMENSION YKTLA(21,6),YKTPR(21,6),YKTND(21,6),YKTSM(21,6),YKALA(21BMS2 001
1,6),YKAPR(21,6),YKAND(21,6),YKASM(21,6)                                BMS2 002
DO 5 I = 1,21                                                               BMS2 003
5   READ INPUT TAPE 5, 500, (YKTLA(I,J),J = 1,6)                           BMS2 004
DO 10 I = 1,21                                                              BMS2 005
10  READ INPUT TAPE 5, 500, (YKTPR(I,J),J = 1,6)                           BMS2 006
DO 15 I = 1,21                                                               BMS2 007
15  READ INPUT TAPE 5, 500, (YKTND(I,J),J = 1,6)                           BMS2 008
DO 20 I = 1,21                                                               BMS2 009
20  READ INPUT TAPE 5, 500, (YKTSM(I,J),J = 1,6)                           BMS2 010
DO 25 I = 1,21                                                               BMS2 011
25  READ INPUT TAPE 5, 500, (YKALA(I,J),J = 1,6)                           BMS2 012
DO 30 I = 1,21                                                               BMS2 013
30  READ INPUT TAPE 5, 500, (YKAPR(I,J),J = 1,6)                           BMS2 014
DO 35 I = 1,21                                                               BMS2 015
35  READ INPUT TAPE 5, 500, (YKAND(I,J),J = 1,6)                           BMS2 016
DO 40 I = 1,21                                                               BMS2 017
40  READ INPUT TAPE 5, 500, (YKASM(I,J),J = 1,6)                           BMS2 018
45  READ INPUT TAPE 5, 501, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ      BMS2 019
    READ INPUT TAPE 5, 502, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,      BMS2 020
    IRUVS,N,K                                                               BMS2 021
    PRINT 503, ROVS,N                                         BMS2 022
    PRINT 504                                                               BMS2 023
    IF(TMAQ) 600,65,50                                         BMS2 024
50   XHNO3 = AMAQ/TMAQ                                         BMS2 025
    XLA = XLAMAQ/TMAQ                                         BMS2 026
    XPR = PRMAQ/TMAQ                                         BMS2 027
    XND = XNDMAQ/TMAQ                                         BMS2 028
    XSM = SMMAQ/TMAQ                                         BMS2 029
    XRE = REMAQ/TMAQ                                         BMS2 030
    IF(REMAQ) 600,60,55                                         BMS2 031
55   XLASF = XLAMAQ/REMAQ                                         BMS2 032
    XPRSF = PRMAQ/REMAQ                                         BMS2 033
    XNDSF = XNDMAQ/REMAQ                                         BMS2 034
    XSMSF = SMMAQ/REMAQ                                         BMS2 035
    PRINT 505, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHNO3,XLA,XPRBMS2 036
1,XND,XSM,XRE,XLASF,XPRSF,XNDSF,XSMSF                         BMS2 037
    GO TO 70                                                 BMS2 038
60   PRINT 506, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHNO3,XLA,XPRBMS2 039
1,XND,XSM,XRE
    GO TO 70                                                 BMS2 040
65   PRINT 507, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ      BMS2 041
70   YHNO3 = AMOR/TMOR                                         BMS2 042
    YLA = XLAMOR/TMOR                                         BMS2 043
    YPR = PRMOR/TMOR                                         BMS2 044
    YND = XNDMOR/TMOR                                         BMS2 045
    YSM = SMMOR/TMOR                                         BMS2 046
    YRE = REMOR/TMOR                                         BMS2 047
    YLASF = XLAMOR/REMR                                         BMS2 048
    YPRSF = PRMOR/REMR                                         BMS2 049
    YNDSF = XNDMOR/REMR                                         BMS2 050
    YSMSF = SMMOR/REMR                                         BMS2 051
    PRINT 508, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,YHNO3,YLA,YPRBMS2 053
1,YND,YSM,YRE,YLASF,YPRSF,YNDSF,YSMSF                         BMS2 054
    ROTM = TMAQ                                         BMS2 055
    ROAM = AMAQ                                         BMS2 056
    RCLAM = XLAMAQ                                         BMS2 057
    ROPRM = PRMAQ                                         BMS2 058
    ROND = XNDMAQ                                         BMS2 059
    POSMM = SMMAQ                                         BMS2 060
    SITM = TMOR                                         BMS2 061
    SIAM = AMOR                                         BMS2 062

```

Figure 41. Fortran statements, Program II

```

S1LAM = XLAMOR                                BMS2 063
S1PRM = PRMOR                                 BMS2 064
S1NDM = XNDMOR                                BMS2 065
S1SMM = SMMOR                                 BMS2 066
DO 185 L = 1,N                                BMS2 067
YREI = (YRE*100.0 + 5.01/5.0                  BMS2 068
IY = YREI                                     BMS2 069
IY = IY                                       BMS2 070
FRY = YREI - YI                               BMS2 071
Z = (TMOR*10.0 - 15.0)/5.0                   BMS2 072
KZ = Z                                         BMS2 073
ZK = KZ                                       BMS2 074
FRZ = Z - ZK                                  BMS2 075
SKT = MIN1F(YKTLA(IY,KZ),YKTLA(IY+1,KZ),YKTLA(IY,KZ+1),YKTLA(IY+1,BMS2 076
1KZ+1),YKTPR(IY,KZ),YKTPR(IY+1,KZ),YKTPR(IY,KZ+1),YKTPR(IY+1,KZ+1),BMS2 077
2YKTND(IY,KZ),YKTND(IY+1,KZ),YKTND(IY,KZ+1),YKTND(IY+1,KZ+1),YKTSM(BMS2 078
3IY,KZ),YKTSM(IY+1,KZ),YKTSM(IY,KZ+1),YKTSM(IY+1,KZ+1))                BMS2 079
IF (SKT) 75,75,80                            BMS2 080
75 PRINT 509, YKTLA(IY,KZ),YKTLA(IY+1,KZ),YKTLA(IY,KZ+1),YKTLA(IY+1,KBMS2 081
1Z+1),YKTPR(IY,KZ),YKTPR(IY+1,KZ),YKTPR(IY,KZ+1),YKTPR(IY+1,KZ+1),YBMS2 082
2KTNDO(IY,KZ),YKTND(IY+1,KZ),YKTND(IY,KZ+1),YKTND(IY+1,KZ+1),YKTSM(BMS2 083
3IY,KZ),YKTSM(IY+1,KZ),YKTSM(IY,KZ+1),YKTSM(IY+1,KZ+1),TMOR,YRE        BMS2 084
IF (K) 600,600,45                            BMS2 085
80 YKTLA1 = YKTLA(IY,KZ)*(1.0 - FRY) + YKTLA(IY+1,KZ)*FRY                  BMS2 086
YKTLA2 = YKTLA(IY,KZ+1)*(1.0 - FRY) + YKTLA(IY+1,KZ+1)*FRY                BMS2 087
YKTLA3 = YKTLA1*(1.0 - FRZ) + YKTLA2*FRZ                                BMS2 088
CKTLA = YKTLA3 * YLASF                           BMS2 089
YKTPR1 = YKTPR(IY,KZ)*(1.0 - FRY) + YKTPR(IY+1,KZ)*FRY                  BMS2 090
YKTPR2 = YKTPR(IY,KZ+1)*(1.0 - FRY) + YKTPR(IY+1,KZ+1)*FRY                BMS2 091
YKTPR3 = YKTPR1*(1.0 - FRZ) + YKTPR2*FRZ                                BMS2 092
CKTPR = YKTPR3 * YPRSF                           BMS2 093
YKTND1 = YKTND(IY,KZ)*(1.0 - FRY) + YKTND(IY+1,KZ)*FRY                  BMS2 094
YKTND2 = YKTND(IY,KZ+1)*(1.0 - FRY) + YKTND(IY+1,KZ+1)*FRY                BMS2 095
YKTND3 = YKTND1*(1.0 - FRZ) + YKTND2*FRZ                                BMS2 096
CKTND = YKTND3 * YNDSF                           BMS2 097
YKTSM1 = YKTSM(IY,KZ)*(1.0 - FRY) + YKTSM(IY+1,KZ)*FRY                  BMS2 098
YKTSM2 = YKTSM(IY,KZ+1)*(1.0 - FRY) + YKTSM(IY+1,KZ+1)*FRY                BMS2 099
YKTSM3 = YKTSM1*(1.0 - FRZ) + YKTSM2*FRZ                                BMS2 100
CKTSM = YKTSM3 * YSMSF                           BMS2 101
SKA = MIN1F(YKALA(IY,KZ),YKALA(IY+1,KZ),YKALA(IY,KZ+1),YKALA(IY+1,BMS2 102
1KZ+1),YKAPR(IY,KZ),YKAPR(IY+1,KZ),YKAPR(IY,KZ+1),YKAPR(IY+1,KZ+1),BMS2 103
2YKAND(IY,KZ),YKAND(IY+1,KZ),YKAND(IY,KZ+1),YKAND(IY+1,KZ+1),YKASM(BMS2 104
3IY,KZ),YKASM(IY+1,KZ),YKASM(IY,KZ+1),YKASM(IY+1,KZ+1))                BMS2 105
IF (SKA) 85,85,90                            BMS2 106
85 PRINT 510, YKALA(IY,KZ),YKALA(IY+1,KZ),YKALA(IY,KZ+1),YKALA(IY+1,KBMS2 107
1Z+1),YKAPR(IY,KZ),YKAPR(IY+1,KZ),YKAPR(IY,KZ+1),YKAPR(IY+1,KZ+1),YBMS2 108
2KAND(IY,KZ),YKAND(IY+1,KZ),YKAND(IY,KZ+1),YKAND(IY+1,KZ+1),YKASM(BMS2 109
3IY,KZ),YKASM(IY+1,KZ),YKASM(IY,KZ+1),YKASM(IY+1,KZ+1),TMOR,YRE        BMS2 110
IF (K) 600,600,45                            BMS2 111
90 YKALA1 = YKALA(IY,KZ)*(1.0 - FRY) + YKALA(IY+1,KZ)*FRY                  BMS2 112
YKALA2 = YKALA(IY,KZ+1)*(1.0 - FRY) + YKALA(IY+1,KZ+1)*FRY                BMS2 113
YKALA3 = YKALA1*(1.0 - FRZ) + YKALA2*FRZ                                BMS2 114
CKALA = YKALA3 * YLASF                           BMS2 115
YKAPR1 = YKAPR(IY,KZ)*(1.0 - FRY) + YKAPR(IY+1,KZ)*FRY                  BMS2 116
YKAPR2 = YKAPR(IY,KZ+1)*(1.0 - FRY) + YKAPR(IY+1,KZ+1)*FRY                BMS2 117
YKAPR3 = YKAPR1*(1.0 - FRZ) + YKAPR2*FRZ                                BMS2 118
CKAPR = YKAPR3 * YPRSF                           BMS2 119
YKAND1 = YKAND(IY,KZ)*(1.0 - FRY) + YKAND(IY+1,KZ)*FRY                  BMS2 120
YKAND2 = YKAND(IY,KZ+1)*(1.0 - FRY) + YKAND(IY+1,KZ+1)*FRY                BMS2 121
YKAND3 = YKAND1*(1.0 - FRZ) + YKAND2*FRZ                                BMS2 122
CKAND = YKAND3 * YNDSF                           BMS2 123
YKASM1 = YKASM(IY,KZ)*(1.0 - FRY) + YKASM(IY+1,KZ)*FRY                  BMS2 124

```

Figure 41. (Continued)

```

YKASM2 = YKASM(IY,KZ+1)*(1.0 - FRY) + YKASM(IY+1,KZ+1)*FRY      BMS2 125
YKASM3 = YKASM1*(1.0 - FRZ) + YKASM2*FRZ                          BMS2 126
CKASM = YKASM3 * YSMSF                                         BMS2 127
TOTK = CKTLA + CKTPR + CKTND + CKTSW                           BMS2 128
HNO3K = CKALA + CKAPR + CKAND + CKASM                           BMS2 129
TMAQ = TMOR/TOTK                                         BMS2 130
AMAQ = AMOR/HNO3K                                         BMS2 131
REMAQ = TMAQ - AMAQ                                         BMS2 132
IF(REMAQ) 95,95,100                                         BMS2 133
95 PRINT 511, TMAQ,AMAQ,REMAQ,HNO3K                           BMS2 134
IF (K) 600,600,45                                         BMS2 135
100 BLAPR = 0.8187 - 0.1106*TMOR                           BMS2 136
BPRPR = 1.0                                         BMS2 137
BNDPR = 1.0448 + 0.09874*TMOR                           BMS2 138
BSMPR = -0.3795 + 0.9214*TMOR                           BMS2 139
DEMS = XLAMOR/BLAPR + PRMOR/BPRPR + XNDMOR/BNDPR + SMMOR/BSMPR BMS2 140
XLAMAQ = XLAMOR*REMAQ/(BLAPR*DEMS)                      BMS2 141
PRMAQ = PRMOR*REMAQ/(BPRPR*DEMS)                      BMS2 142
XNDMAQ = XNDMOR*REMAQ/(BNDPR*DEMS)                      BMS2 143
SMMAQ = SMMOR*REMAQ/(BSMPR*DEMS)                      BMS2 144
XHNO3 = AMAQ/TMAQ                                         BMS2 145
XLA = XLAMAQ/TMAQ                                         BMS2 146
XPR = PRMAQ/TMAQ                                         BMS2 147
XND = XNDMAQ/TMAQ                                         BMS2 148
XSM = SMMAQ/TMAQ                                         BMS2 149
XRE = REMAQ/TMAQ                                         BMS2 150
XLASF = XLAMAQ/REMAQ                                         BMS2 151
XPRSF = PRMAQ/REMAQ                                         BMS2 152
XNDSF = XNDMAQ/REMAQ                                         BMS2 153
XSMSF = SMMAQ/REMAQ                                         BMS2 154
SXA = XHNO3 + XLA + XPR + XND + XSM                         BMS2 155
IF (0.9999 - SXA) 105,115,110                           BMS2 156
105 IF (1.0001 - SXA) 110,115,115                           BMS2 157
110 PRINT 512, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMS2 158
IF (K) 600,600,45                                         BMS2 159
115 SXREA = XLASF + XPRSF + XNDSF + XSMSF               BMS2 160
IF (0.9999 - SXREA) 120,130,125                           BMS2 161
120 IF (1.0001 - SXREA) 125,130,130                           BMS2 162
125 PRINT 513, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMS2 163
IF (K) 600,600,45                                         BMS2 164
130 PRINT 514,L,TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHNO3,XLA,XPRBMS2 165
1,XND,XSM,XRE,XLASF,XPRSF,XNDSF,XSMSF                  BMS2 166
TMOR = TMAQ * ROVS + S1TM - ROTM*ROVS                 BMS2 167
AMOR = AMAQ*ROVS + S1AM - ROAM*ROVS                 BMS2 168
XLAMOR = XLAMAQ*ROVS + S1LAM - ROLAM*ROVS             BMS2 169
PRMOR = PRMAQ*ROVS + S1PRM - ROPRM*ROVS              BMS2 170
XNDMOR = XNDMAQ*ROVS + S1NDM - ROND M*ROVS            BMS2 171
SMMOR = SMMAQ*ROVS + S1SMM - ROSMM*ROVS              BMS2 172
REMOR = TMOR - AMOR                                     BMS2 173
SOM = MIN1(TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR) BMS2 174
IF (SOM) 135,135,140                                     BMS2 175
135 PRINT 515, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR BMS2 176
IF (K) 600,600,45                                         BMS2 177
140 YHNO3 = AMOR/TMOR                                         BMS2 178
YLA = XLAMOR/TMOR                                         BMS2 179
YPR = PRMOR/TMOR                                         BMS2 180
YND = XNDMOR/TMOR                                         BMS2 181
YSM = SMMOR/TMOR                                         BMS2 182
YRE = REMOR/TMOR                                         BMS2 183
YLASF = XLAMOR/REMOR                                         BMS2 184
YPRSF = PRMOR/REMOR                                         BMS2 185
YNDSF = XNDMOR/REMOR                                         BMS2 186

```

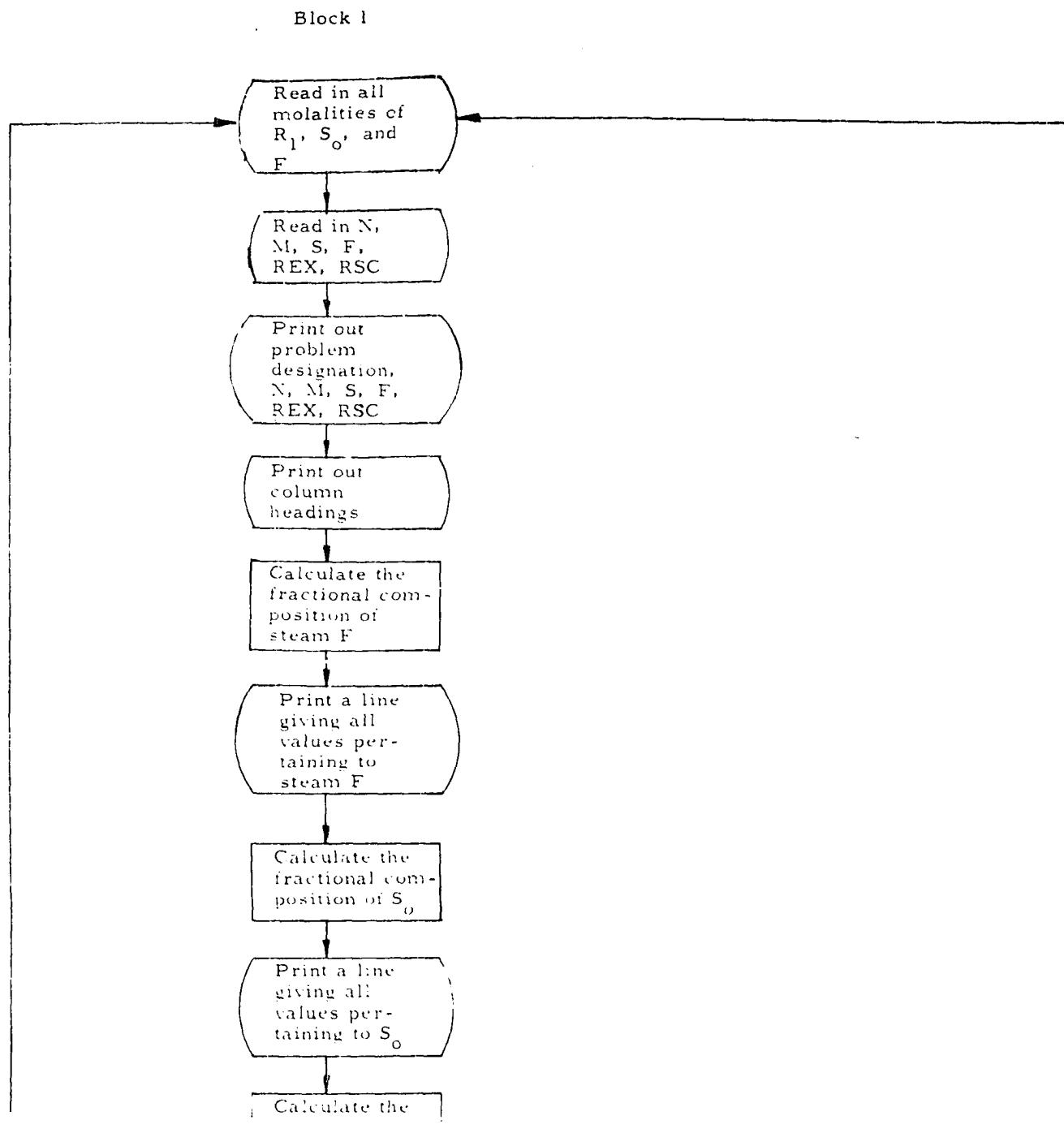
Figure 41. (Continued)

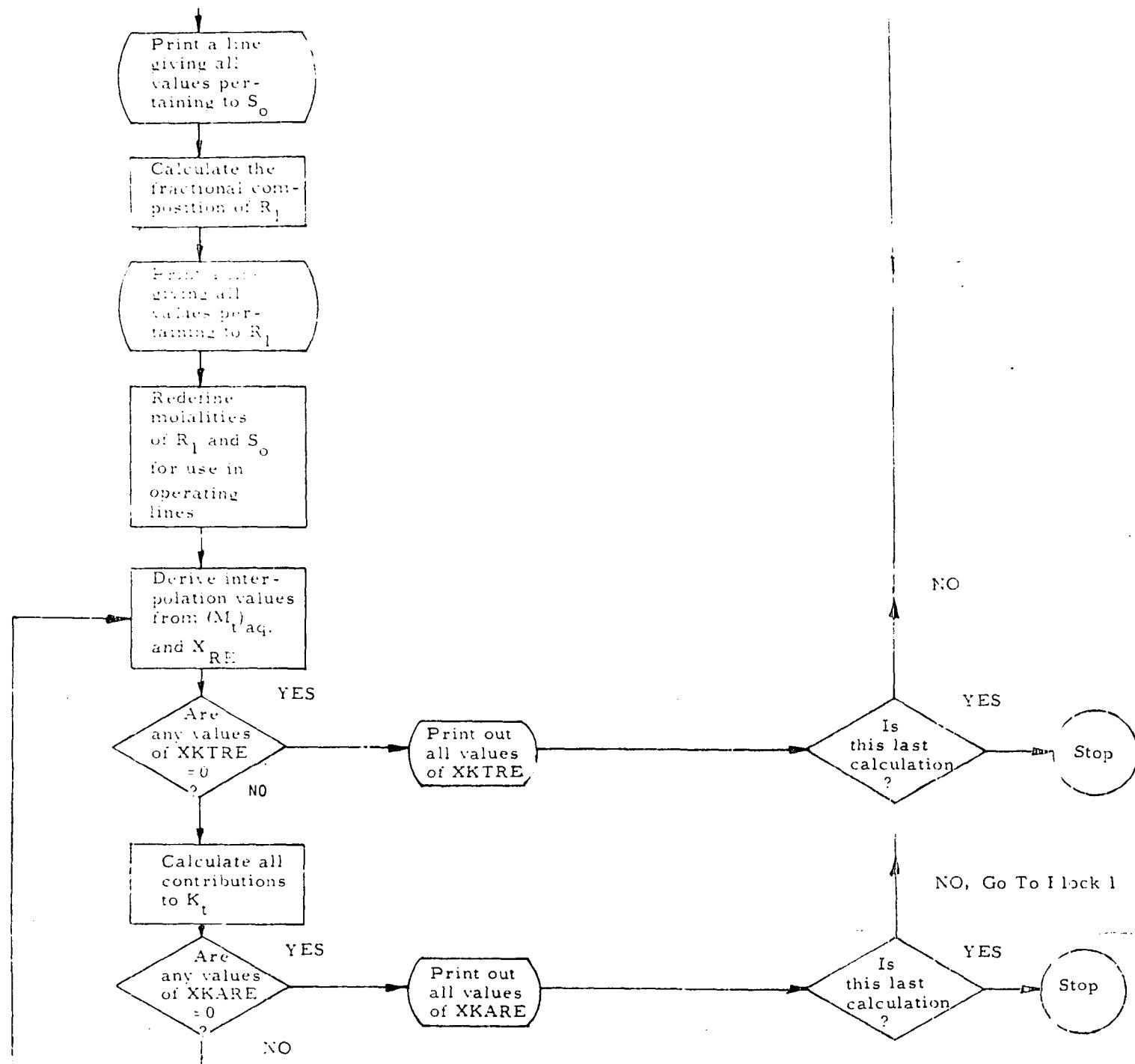
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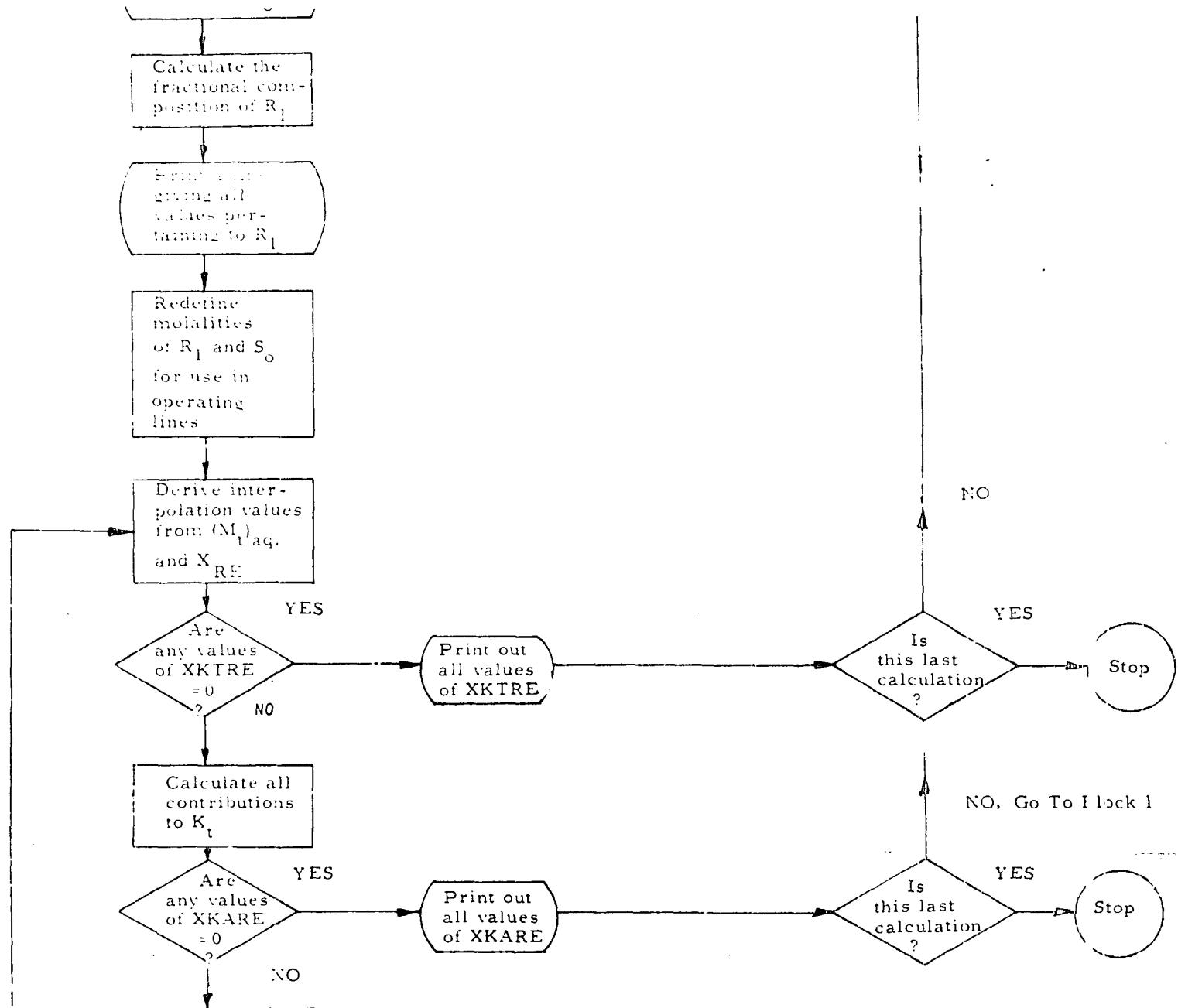
YSMSF = SMMOR/REMOR                                BMS2 187
SYO = YHN03 + YLA + YPR + YND + YSM              BMS2 188
IF (0.9999 - SYO) 145,155,150                      BMS2 189
145 IF (1.0001 - SYO) 150,155,155                  BMS2 190
150 PRINT 516, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR BMS2 191
      IF (K) 600,600,45                               BMS2 192
155 SYREO = YLASF + YPRSF + YNDSF + YSMSF          BMS2 193
      IF (0.9999 - SYREO) 160,170,165                BMS2 194
160 IF (1.0001 - SYREO) 165,170,170                BMS2 195
165 PRINT 517, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR BMS2 196
      IF (K) 600,600,45                               BMS2 197
170 LL = L + 1                                     BMS2 198
      PRINT 518, LL,TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,YHN03,YLA,BMS2 199
      1YPR,YND,YSM,YRE,YLASF,YPRSF,YNDSF,YSMSF        BMS2 200
      IF (TMOR - 2.0) 175,180,180                  BMS2 201
175 PRINT 519, TMOR                                BMS2 202
      IF (K) 600,600,45                               BMS2 203
180 IF (TMOR - 4.5) 185,175,175                  BMS2 204
185 CONTINUE                                         BMS2 205
      IF (K) 600,600,45                               BMS2 206
500 FORMAT (6F10.3)                                 BMS2 207
501 FORMAT (7F9.4)                                  BMS2 208
502 FORMAT (7F7.4,F8.4,I3,I3)                      BMS2 209
503 FORMAT (29H1SCRUB SIDE CALCULATION ROVS=F8.4,4H N=I3) BMS2 210
504 FORMAT (120HJ TOTALMOLHN03 MOL LA MOL PR MOL ND MOL SM MOL BMS2 211
      1RE MOL ACFR LAFR PRFR NDFR SMFR REFR LA/RE PR/RE ND/RE SM/REBMS2 212
      2)
      BMS2 213
505 FORMAT (4HJR 07F8.4,10F6.3)                   BMS2 214
506 FURMAT (4HJR 07F8.4,6F6.3)                   BMS2 215
507 FORMAT (4HJR 07F8.4)                           BMS2 216
508 FORMAT (4H S 17F8.4,10F6.3)                   BMS2 217
509 FORMAT (8HJYKTRE=016F6.3,F8.4,F6.3)          BMS2 218
510 FORMAT (8HJYKARE=016F6.3,F8.4,F6.3)          BMS2 219
511 FORMAT (18HJREMAQ LESS THAN 04F10.4)          BMS2 220
512 FORMAT (16HJSXA NOT EQUAL 17F10.4)            BMS2 221
513 FORMAT (18HJSXREA NOT EQUAL 17F10.4)           BMS2 222
514 FORMAT (2HJR!2,7F8.4,10F6.3)                  BMS2 223
515 FORMAT (6HJTMOR=F10.4,6H AMOR=F10.4,8H XLAMOR=F10.4,7H PRMOR=F10.4BMS2 224
      1,8H XNDMOR=F10.4,7H SMMOR=F10.4,7H REMOR=F10.4) BMS2 225
516 FORMAT (16HJSYO NOT EQUAL 17F10.4)            BMS2 226
517 FORMAT (18HJSYREO NOT EQUAL 17F10.4)           BMS2 227
518 FORMAT (2H S!2,7F8.4,10F6.3)                  BMS2 228
519 FORMAT (6HJTMOR=F10.4)                         BMS2 229
600 STOP 89                                         BMS2 230
      END                                            BMS2 231

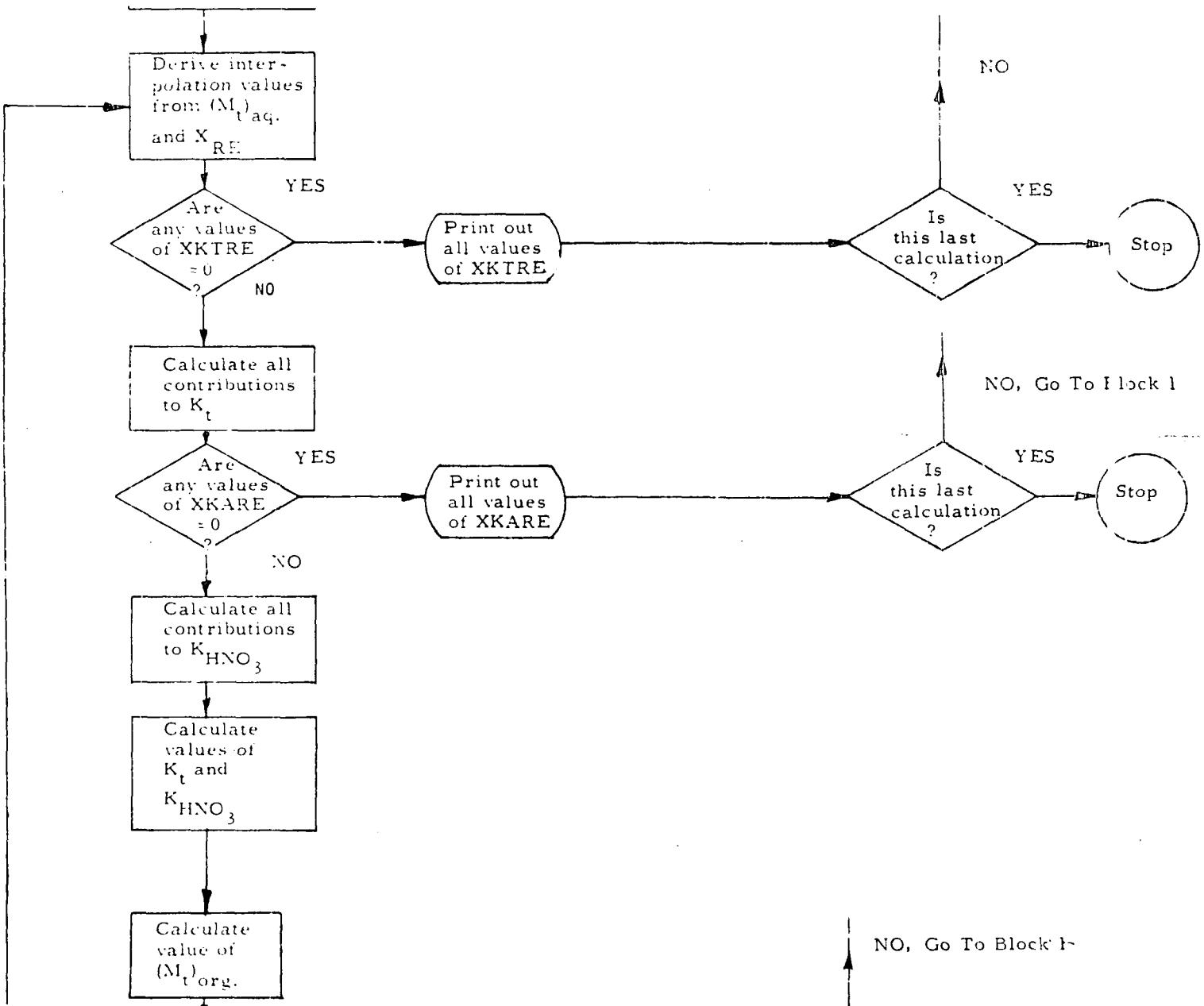
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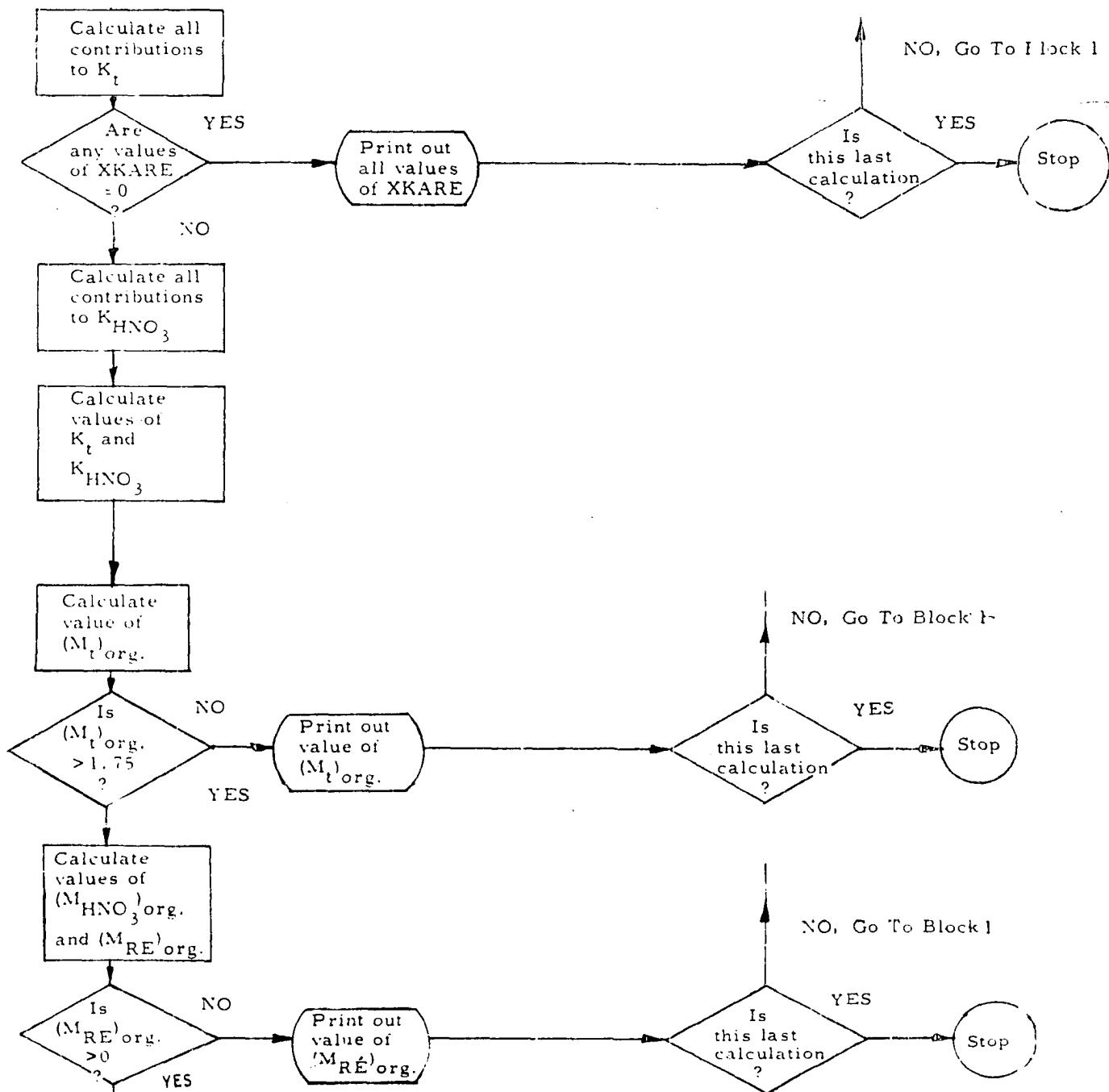
Figure 41. (Continued)

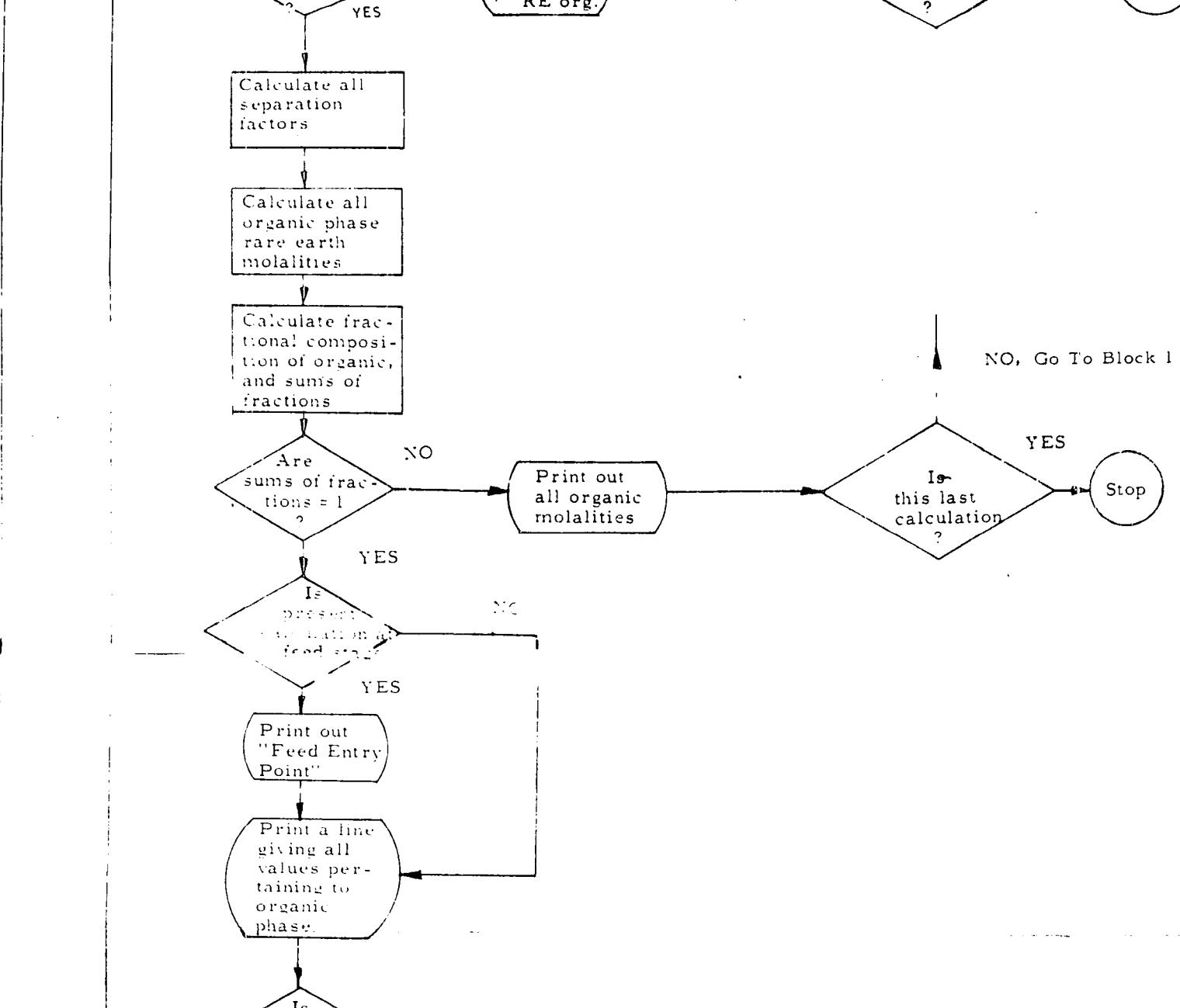


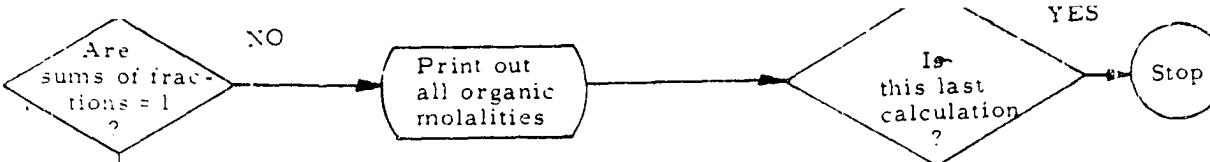












Is present calculation at feed stage?

YES → Print out "Feed Entry Point"

Print a line giving all values pertaining to organic phase.

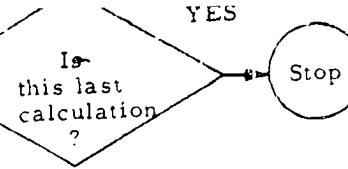
Is present stage an extract side stage?

YES → Use extract side operating lines to calculate aqueous molalities

Use scrub side operating lines to calculate aqueous molalities

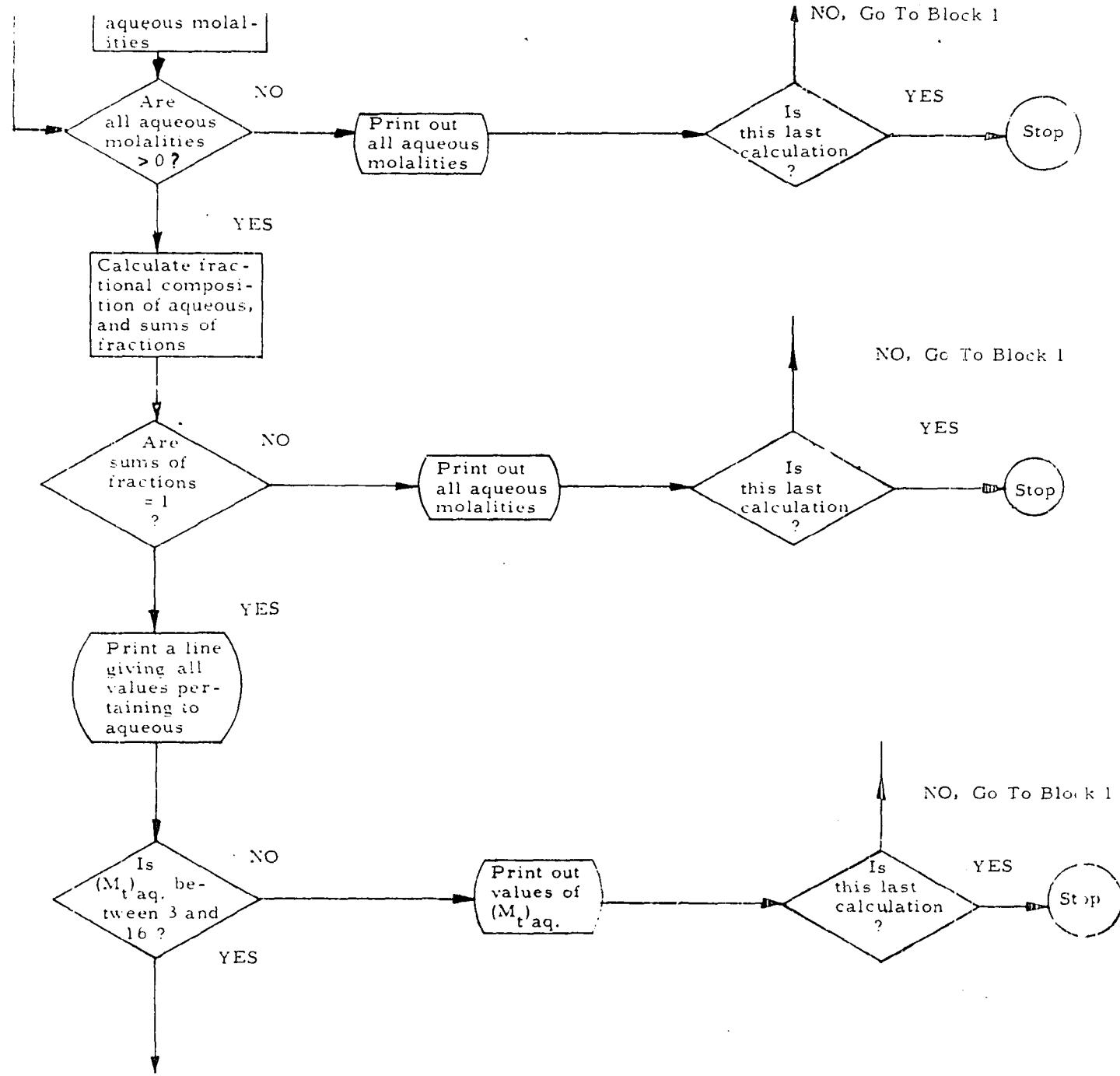
Are

NO



NO, Go To Block 1

VFC



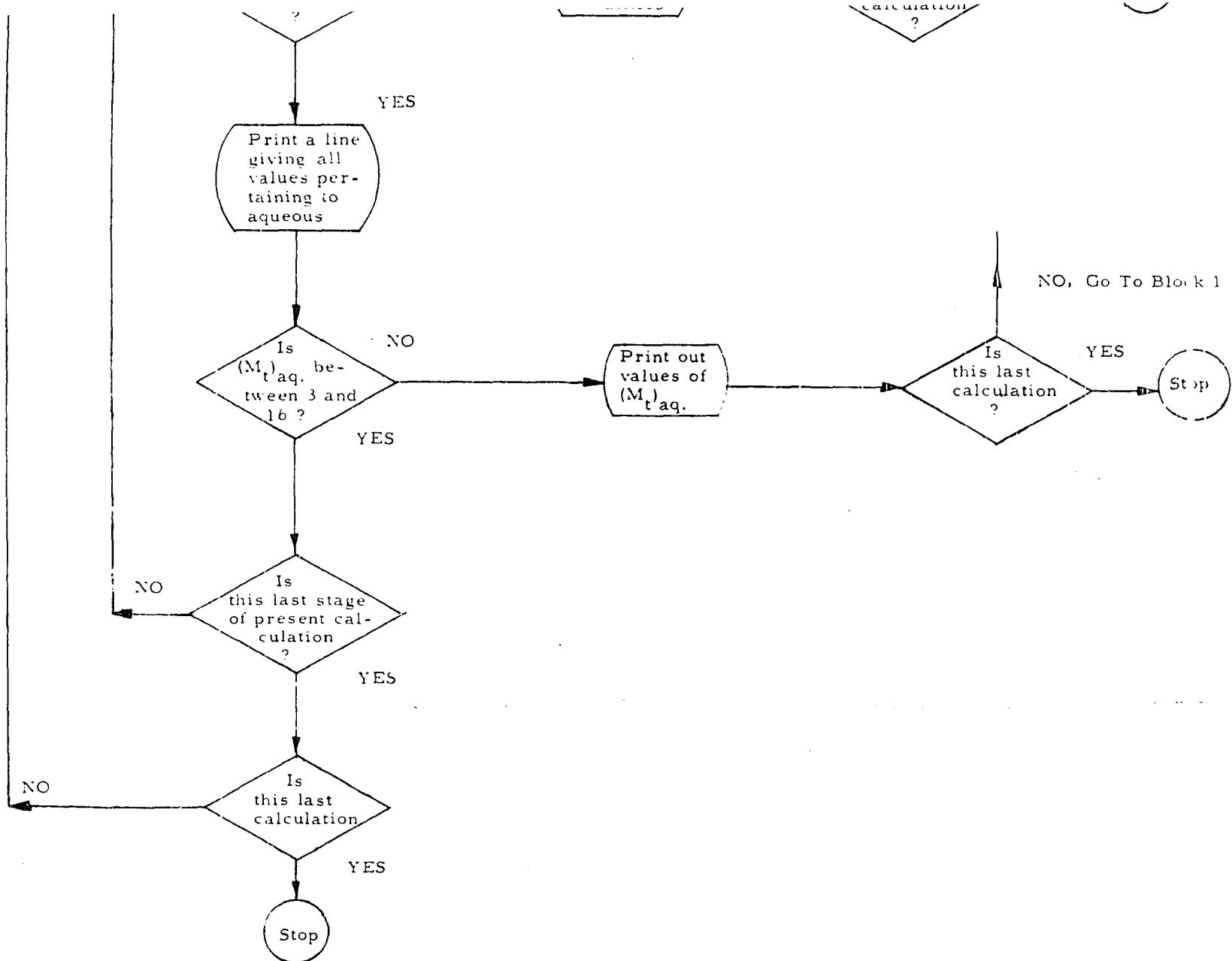


Figure 42. Flow diagram, Program III

```

DIMENSION XKTLA(21,7),XKTPR(21,7),XKTND(21,7),XKTS(21,7),XKALA(21BMS3 001
1,7),XKAPR(21,7),XKAND(21,7),XKASM(21,7)                                BMS3 002
DO 5 I = 1,21                                                               BMS3 003
5   READ INPUT TAPE 5, 500, (XKTLA(I,J),J = 1,7)                           BMS3 004
DO 10 I = 1,21                                                               BMS3 005
10  READ INPUT TAPE 5, 500, (XKTPR(I,J),J = 1,7)                           BMS3 006
DO 15 I = 1,21                                                               BMS3 007
15  READ INPUT TAPE 5, 500, (XKTND(I,J),J = 1,7)                           BMS3 008
DO 20 I = 1,21                                                               BMS3 009
20  READ INPUT TAPE 5, 500, (XKTS(I,J),J = 1,7)                            BMS3 010
DO 25 I = 1,21                                                               BMS3 011
25  READ INPUT TAPE 5, 500, (XKALA(I,J),J = 1,7)                           BMS3 012
DO 30 I = 1,21                                                               BMS3 013
30  READ INPUT TAPE 5, 500, (XKAPR(I,J),J = 1,7)                           BMS3 014
DO 35 I = 1,21                                                               BMS3 015
35  READ INPUT TAPE 5, 500, (XKAND(I,J),J = 1,7)                           BMS3 016
DO 40 I = 1,21                                                               BMS3 017
40  READ INPUT TAPE 5, 500, (XKASM(I,J),J = 1,7)                           BMS3 018
45  READ INPUT TAPE 5, 501, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMS3 019
READ INPUT TAPE 5, 501, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR BMS3 020
READ INPUT TAPE 5, 502, FTM,FAM,FLAM,FPRM,FNDM,FSMM,FREM,S,F,REX,RBMS3 021
1SC,N,M,K
PRINT 503, S,F,REX,RSC,N,M
PRINT 504
IF (FTM) 600,65,50
50  XAF = FAM/FTM
    XLAF = FLAM/FTM
    XPRF = FPRM/FTM
    XNDF = FNDM/FTM
    XSMP = FSMM/FTM
    XREF = FREM/FTM
    IF (FREM) 600,60,55
55  XLASFF = FLAM/FREM
    XPRSFF = FPRM/FREM
    XNDSFF = FNDM/FREM
    XSMSFF = FSMM/FREM
    PRINT 505, FTM,FAM,FLAM,FPRM,FNDM,FSMM,FREM,XAF,XLAF,XPRF,XNDF,XSMBMS3 037
1F,XREF,XLASFF,XPRSFF,XNDSFF,XSMSFF
      GO TO 70
      BMS3 038
      BMS3 039
60  PRINT 506, FTM,FAM,FLAM,FPRM,FNDM,FSMM,FREM,XAF,XLAF,XPRF,XNDF,XSMBMS3 040
1F,XREF
      GO TO 70
      BMS3 041
      BMS3 042
65  PRINT 507, FTM,FAM,FLAM,FPRM,FNDM,FSMM,FREM
      IF (TMOR) 600,90,75
70  YHNO3 = AMOR/TMOR
      YLA = XLAMOR/TMOR
      YPR = PRMOR/TMOR
      YND = XNDMOR/TMOR
      YSM = SMMOR/TMOR
      YRE = REMOR/TMOR
      IF (REMOR) 600,85,80
80  YLASF = XLAMOR/REMOR
      YPRSF = PRMOR/REMOR
      YNDSF = XNDMOR/REMOR
      YSMSF = SMMOR/REMOR
      PRINT 508, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,YHNO3,YLA,YPRBMS3 056
1,YND,YSM,YRE,YLASF,YPRSF,YNDSF,YSMSF
      GO TO 95
      BMS3 057
      BMS3 058
85  PRINT 509, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,YHNO3,YLA,YPRBMS3 059
1,YND,YSM,YRE
      GO TO 95
      BMS3 060
      BMS3 061
90  PRINT 510, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR
      BMS3 062

```

Figure 43. Fortran statements, Program III

```

95    XHN03 = AMAQ /TMAQ                                BMS3 063
      XLA = XLAMAQ/TMAQ                               BMS3 064
      XPR = PRMAQ/TMAQ                               BMS3 065
      XND = XNDMAQ/TMAQ                               BMS3 066
      XSM = SMMAQ/TMAQ                               BMS3 067
      XRE = REMAQ/TMAQ                               BMS3 068
      XLASF = XLAMAQ/REMAQ                               BMS3 069
      XPRSF = PRMAQ/REMAQ                               BMS3 070
      XNDSF = XNDMAQ/REMAQ                               BMS3 071
      XSMSF = SMMAQ/REMAQ                               BMS3 072
      PRINT 511, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHN03,XLA,XPRBMS3 073
1,XND,XSM,XRE,XLASF,XPRSF,XNDSF,XSMSF                               BMS3 074
      R1TM = TMAQ                               BMS3 075
      R1AM = AMAQ                               BMS3 076
      R1LAM= XLAMAQ                               BMS3 077
      R1PRM = PRMAQ                               BMS3 078
      R1NDM = XNDMAQ                               BMS3 079
      R1SMM = SMMAQ                               BMS3 080
      SOTM = TMOR                               BMS3 081
      SOAM = AMOR                               BMS3 082
      SOLAM = XLAMOR                               BMS3 083
      SOPRM = PRMOR                               BMS3 084
      SONDM = XNDMOR                               BMS3 085
      SOSMM = SMMOR                               BMS3 086
      DD 270 L = 1,N                               BMS3 087
      XREI = (XRE*100.0 + 5.0)/5.0                BMS3 088
      IX = XREI                               BMS3 089
      XI = IX                               BMS3 090
      FRX = XREI - XI                               BMS3 091
      IF (5.0 - TMAQ) 100,110,105                BMS3 092
100     IF (7.0 - TMAQ) 115,115,110                BMS3 093
105     Z = TMAQ - 2.0                               BMS3 094
      GO TO 120                               BMS3 095
110     Z = (TMAQ + 1.0)/2.0                BMS3 096
      GO TO 120                               BMS3 097
115     Z = (TMAQ + 5.0)/3.0                BMS3 098
120     KZ = Z                               BMS3 099
      ZK = KZ                               BMS3 100
      FRZ = Z - ZK                               BMS3 101
      SKT = MIN1F(XKTLA(IX,KZ),XKTLA(IX+1,KZ),XKTLA(IX,KZ+1),XKTLA(IX+1,BMS3 102
1KZ+1),XKTPR(IX,KZ),XKTPR(IX+1,KZ),XKTPR(IX,KZ+1),XKTPR(IX+1,KZ+1),BMS3 103
2XKTND(IX,KZ),XKTND(IX+1,KZ),XKTND(IX,KZ+1),XKTND(IX+1,KZ+1),XKTS(BMS3 104
3IX,KZ),XKTS(IX+1,KZ),XKTS(IX,KZ+1),XKTS(IX+1,KZ+1))                BMS3 105
      IF (SKT) 125,125,130                BMS3 106
125     PRINT 512, XKTLA(IX,KZ),XKTLA(IX+1,KZ),XKTLA(IX,KZ+1),XKTLA(IX+1,KBMS3 107
1Z+1),XKTPR(IX,KZ),XKTPR(IX+1,KZ),XKTPR(IX,KZ+1),XKTPR(IX+1,KZ+1),XBMS3 108
2KTND(IX,KZ),XKTND(IX+1,KZ),XKTND(IX,KZ+1),XKTND(IX+1,KZ+1),XKTS(BMS3 109
3X,KZ),XKTS(IX+1,KZ),XKTS(IX,KZ+1),XKTS(IX+1,KZ+1),TMAQ,XRE                BMS3 110
      IF (K) 500,600,45                BMS3 111
130     XKTLA1 = XKTLA(IX,KZ)*(1.0 - FRX) + XKTLA(IX+1,KZ)*FRX                BMS3 112
      XKTLA2 = XKTLA(IX,KZ+1)*(1.0 - FRX) + XKTLA(IX+1,KZ+1)*FRX                BMS3 113
      XKTLA3 = XKTLA1*(1.0-FRZ) + XKTLA2*FRZ                BMS3 114
      CKTLA = XKTLA3*XLASF                BMS3 115
      XKTPR1 = XKTPR(IX,KZ)*(1.0-FRX) + XKTPR(IX+1,KZ)*FRX                BMS3 116
      XKTPR2 = XKTPR(IX,KZ+1)*(1.0-FRX) + XKTPR(IX+1,KZ+1)*FRX                BMS3 117
      XKTPR3 = XKTPR1*(1.0-FRZ) + XKTPR2 *FRZ                BMS3 118
      CKTPR = XKTPR3* XPRSF                BMS3 119
      XKTN1 = XKTND(IX,KZ)*(1.0-FRX) + XKTND(IX+1,KZ)*FRX                BMS3 120
      XKTN2 = XKTND(IX,KZ+1)*(1.0 - FRX) + XKTND(IX+1,KZ+1)*FRX                BMS3 121
      XKTN3 = XKTND1* (1.0- FRZ) + XKTND2 * FRZ                BMS3 122
      CKTND = XKTND3 * XNDSF                BMS3 123
      XKTS1 = XKTS(IX,KZ)*(1.0-FRX) + XKTS(IX+1,KZ)*FRX                BMS3 124

```

Figure 43. (Continued)

```

XKTSMD = XKTSM(IX,KZ+1)*(1.0 -FRX) + XKISM(IX+1,KZ+1)*FRX      BMS3 125
XKTSM3 = XKTSM1*(1.0 -FRZ) + XKTSMD * FRZ                         BMS3 126
CKTSM = XKTSM3 * XSMSF                                              BMS3 127
SKA = MINF(XKALA(IX,KZ),XKALA(IX+1,KZ),XKALA(IX,KZ+1),XKALA(IX+1,BMS3 128
1KZ+1),XKAPR(IX,KZ),XKAPR(IX+1,KZ),XKAPR(IX,KZ+1),XKAPR(IX+1,KZ+1),BMS3 129
2XKAND(IX,KZ),XKAND(IX+1,KZ),XKAND(IX,KZ+1),XKAND(IX+1,KZ+1),XKASM(BMS3 130
3IX,KZ),XKASM(IX+1,KZ),XKASM(IX,KZ+1),XKASM(IX+1,KZ+1))          BMS3 131
IF (SKA) 135,135,140                                               BMS3 132
135 PRINT 513, XKALA(IX,KZ),XKALA(IX+1,KZ),XKALA(IX,KZ+1),XKALA(IX+1,BMS3 133
1Z+1),XKAPR(IX,KZ),XKAPR(IX+1,KZ),XKAPR(IX,KZ+1),XKAPR(IX+1,KZ+1),XBMS3 134
2KAND(IX,KZ),XKAND(IX+1,KZ),XKAND(IX,KZ+1),XKAND(IX+1,KZ+1),XKASM(BMS3 135
3X,KZ),XKASM(IX+1,KZ),XKASM(IX,KZ+1),XKASM(IX+1,KZ+1),TMAQ,XRE     BMS3 136
IF (K) 600,600,45                                                 BMS3 137
140 XKALA1 = XKALA(IX,KZ)*(1.0 - FRX) + XKALA(IX+1,KZ)*FRX      BMS3 138
XKALA2 = XKALA(IX,KZ+1)*(1.0-FRX) + XKALA(IX+1,KZ+1)*FRX       BMS3 139
XKALA3 = XKALA1*(1.0-FRZ) + XKALA2*FRZ                          BMS3 140
CKALA = XKALA3 * XLASF                                           BMS3 141
XKAPR1 = XKAPR(IX,KZ)*(1.0 - FRX) + XKAPR(IX+1,KZ)* FRX        BMS3 142
XKAPR2 = XKAPR(IX,KZ+1)*(1.0 -FRX) + XKAPR(IX+1,KZ+1)*FRX       BMS3 143
XKAPR3 = XKAPR1*(1.0 -FRZ) + XKAPR2 * FRZ                      BMS3 144
CKAPR = XKAPR3 * XPRSF                                         BMS3 145
XKAND1 = XKAND(IX,KZ)*(1.0 -FRX) + XKAND(IX+1,KZ)*FRX           BMS3 146
XKAND2 = XKAND(IX,KZ+1)*(1.0 -FRX) + XKAND(IX+1,KZ+1)*FRX       BMS3 147
XKAND3 = XKAND1*(1.0 -FRZ) + XKAND2 * FRZ                      BMS3 148
CKAND = XKAND3 * XNDSF                                         BMS3 149
XKASM1 = XKASM(IX,KZ)*(1.0 - FRX) + XKASM(IX+1,KZ)*FRX         BMS3 150
XKASM2 = XKASM(IX,KZ+1)*(1.0 -FRX) + XKASM(IX+1,KZ+1)*FRX       BMS3 151
XKASM3 = XKASM1*(1.0 -FRZ) + XKASM2 *FRZ                      BMS3 152
CKASM = XKASM3 * XSMSF                                         BMS3 153
TOTK = CKTLA + CKTPR + CKTND + CKTSM                           BMS3 154
HNO3K = CKALA + CKAPR + CKAND + CKASM                         BMS3 155
TMOR = TMAQ * TOTK                                            BMS3 156
IF(TMOR - 1.75) 145,150,150                                     BMS3 157
145 PRINT 514, TMOR,TMAQ,TOTK                                    BMS3 158
IF (K) 600,600,45                                              BMS3 159
150 AMOR = AMAQ * HNO3K                                         BMS3 160
REMOR = TMOR - AMOR                                           BMS3 161
IF(REMOR) 155,155,160                                         BMS3 162
155 PRINT 515, TMOR,AMOR,REMOR,HNO3K                           BMS3 163
IF (K) 600,600,45                                              BMS3 164
160 BLAPR = 0.8187 - 0.1106*TMOR                                BMS3 165
BPRPR = 1.0                                                       BMS3 166
BNDPR = 1.0448 + 0.09874*TMOR                                 BMS3 167
BSMPR = -0.3795 + 0.9214*TMOR                                BMS3 168
DEMS = XLAMAQ*BLAPR + PRMAQ* BPRPR + XNDMAQ*BNDPR + SMMAQ*BSMPR BMS3 169
XLAMOR = REMOR*BLAPR*XLAMAQ/DEMS                            BMS3 170
PRMOR = REMOR*BPRPR*PRMAQ/DEMS                             BMS3 171
XNDMOR= REMOR*BNDPR*XNDMAQ/DEMS                            BMS3 172
SMMOR = REMOR*BSMPR*SMMAQ/DEMS                            BMS3 173
YHN03 = AMOR/TMOR                                            BMS3 174
YLA = XLAMOR/TMOR                                           BMS3 175
YPR = PRMOR/TMOR                                           BMS3 176
YND = XNDMOR/TMOR                                           BMS3 177
YSM = SMMOR/TMOR                                           BMS3 178
YRE = REMOR/TMOR                                           BMS3 179
SYO = YHN03 + YLA + YPR + YND + YSM                         BMS3 180
IF (0.9999 - SYO) 165,175,170                               BMS3 181
165 IF (1.0001 - SYO) 170,175,175                           BMS3 182
170 PRINT 516, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR   BMS3 183
IF (K) 600,600,45                                              BMS3 184
175 YLASF = XLAMOR/REMOR                                      BMS3 185
YPRSF = PRMOR/REMOR                                         BMS3 186

```

Figure 43. (Continued)

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YNDSF = XNDMOR/REMOR          BMS3 187
YSMSF = SMMOR/REMOR          BMS3 188
SYREO = YLASF + YPRSF + YNDSF + YSMSF BMS3 189
IF (0.9999 - SYREO) 180,190,185      BMS3 190
180 IF (1.0001 - SYREO) 185,190,190      BMS3 191
185 PRINT 517, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR BMS3 192
    IF (K) 600,600,45                  BMS3 193
190 IF (L - M) 200,195,200          BMS3 194
195 PRINT 518                      BMS3 195
200 PRINT 519,L,TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,YHNO3,YLA,YPBMS3 196
    LR,YND,YSM,YRE,YLASF,YPRSF,YNDSF,YSMSF      BMS3 197
    IF (L-M) 205,210,210          BMS3 198
205 TMAQ = TMOR*S/REX - SOTM*S/REX + R1TM      BMS3 199
    AMAQ = AMOR*S/REX - SOAM*S/REX + R1AM      BMS3 200
    XLAMAQ = XLAMOR*S/REX - SOLAM*S/REX + R1LAM BMS3 201
    PRMAQ = PRMOR*S/REX - SOPRM*S/REX + R1PRM BMS3 202
    XNDMAQ = XNDMOR*S/REX - SONDM*S/REX + R1NDM BMS3 203
    SHMAQ = SMMOR*S/REX - SOSMM*S/REX + R1SMM BMS3 204
    GO TO 215                      BMS3 205
210 TMAQ = TMOR*S/RSC - SOTM*S/RSC - FTM*F/RSC + R1TM*REX/RSC BMS3 206
    AMAQ = AMOR*S/RSC - SOAM*S/RSC - FAM*F/RSC + R1AM*REX/RSC BMS3 207
    XLAMAQ = XLAMOR*S/RSC - SOLAM*S/RSC - FLAM*F/RSC + R1LAM*REX/RSC BMS3 208
    PRMAQ = PRMOR*S/RSC - SOPRM*S/RSC - FPRM*F/RSC + R1PRM*REX/RSC BMS3 209
    XNDMAQ = XNDMOR*S/RSC - SONDM*S/RSC - FNDM*F/RSC + R1NDM*REX/RSC BMS3 210
    SMMAQ = SMMOR*S/RSC - SOSMM*S/RSC - FSMM*F/RSC + R1SMM*REX/RSC BMS3 211
215 REMAQ = TMAQ - AMAQ          BMS3 212
    SAM = MINIF(TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ) BMS3 213
    IF(SAM) 220,220,225          BMS3 214
220 PRINT 520, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMS3 215
    IF(K) 600,600,45          BMS3 216
225 XHNO3 = AMAQ/TMAQ          BMS3 217
    XLA = XLAMAQ/TMAQ          BMS3 218
    XPR = PRMAQ/TMAQ          BMS3 219
    XND = XNDMAQ/TMAQ          BMS3 220
    XSM = SMMAQ/TMAQ          BMS3 221
    XRE = REMAQ/TMAQ          BMS3 222
    SXA = XHNO3 + XLA + XPR + XND + XSM          BMS3 223
    IF (0.9999 - SXA) 230,240,235      BMS3 224
230 IF (1.0001 - SXA) 235,240,240      BMS3 225
235 PRINT 521, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMS3 226
    IF (K) 600,600,45          BMS3 227
240 XLASF = XLAMAQ/REMAQ          BMS3 228
    XPRSF = PRMAQ/REMAQ          BMS3 229
    XNDSF = XNDMAQ/REMAQ          BMS3 230
    XSMSF = SMMAQ/REMAQ          BMS3 231
    SXREA = XLASF + XPRSF + XNDSF + XSMSF      BMS3 232
    IF(0.9999 - SXREA) 245,255,250      BMS3 233
245 IF(1.0001 - SXREA) 250,255,255      BMS3 234
250 PRINT 522, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMS3 235
    IF(K) 600,600,45          BMS3 236
255 LL = L+1                      BMS3 237
    PRINT 523,LL,TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHNO3,XLA,XBMS3 238
    1PR,XND,XSM,XRE,XLASF,XPRSF,XNDSF,XSMSF      BMS3 239
    IF (TMAQ - 3.0) 260,265,265      BMS3 240
260 PRINT 524, TMAQ          BMS3 241
    IF (K) 600,600,45          BMS3 242
265 IF (TMAQ - 16.0) 270,260,260      BMS3 243
270 CONTINUE                      BMS3 244
    IF (K) 600,600,45          BMS3 245
500 FORMAT (7F10.3)          BMS3 246
501 FORMAT (7F10.4)          BMS3 247
502 FORMAT (7F7.4,4F5.2,3I3)      BMS3 248

```

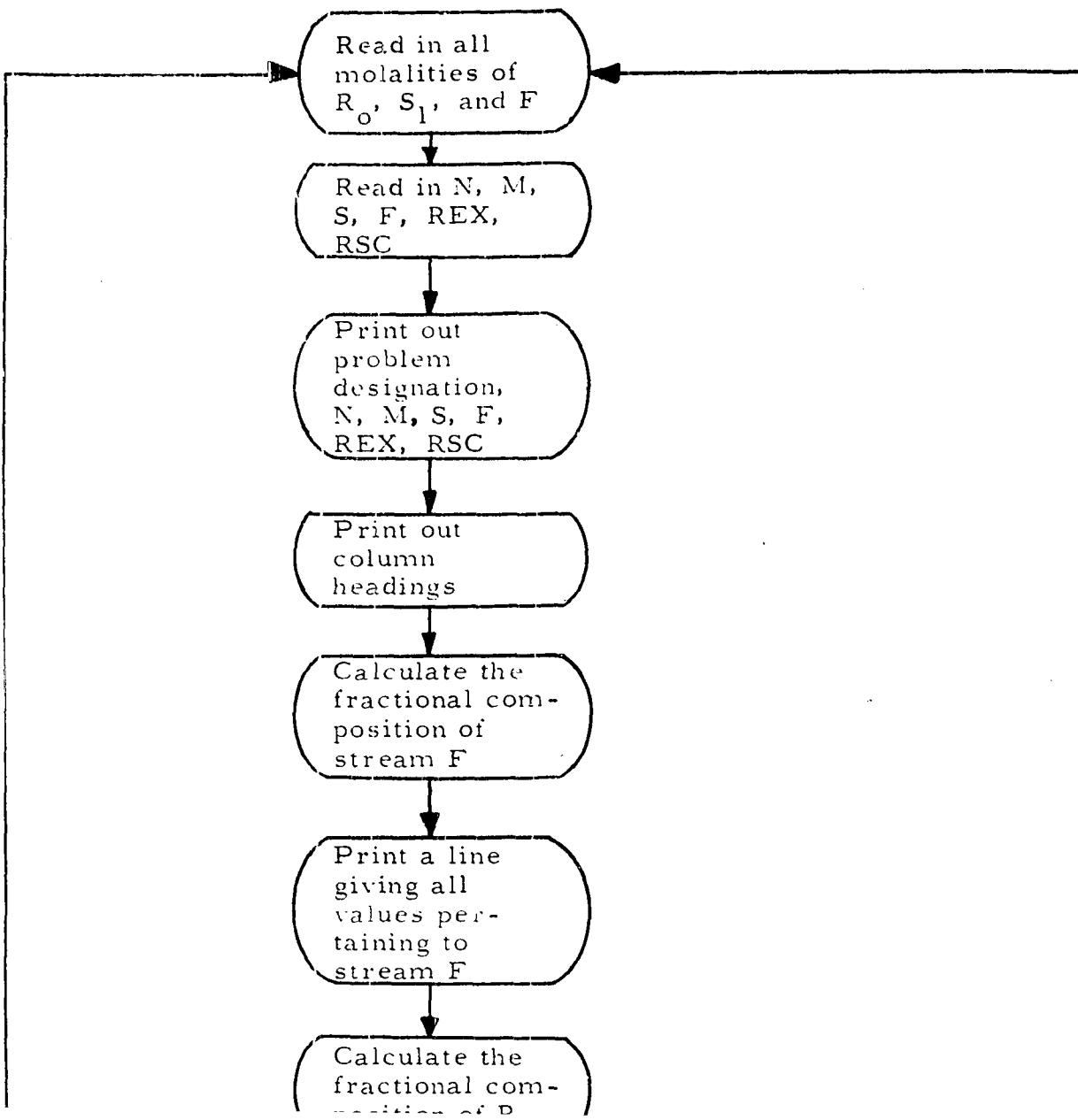
Figure 43. (Continued)

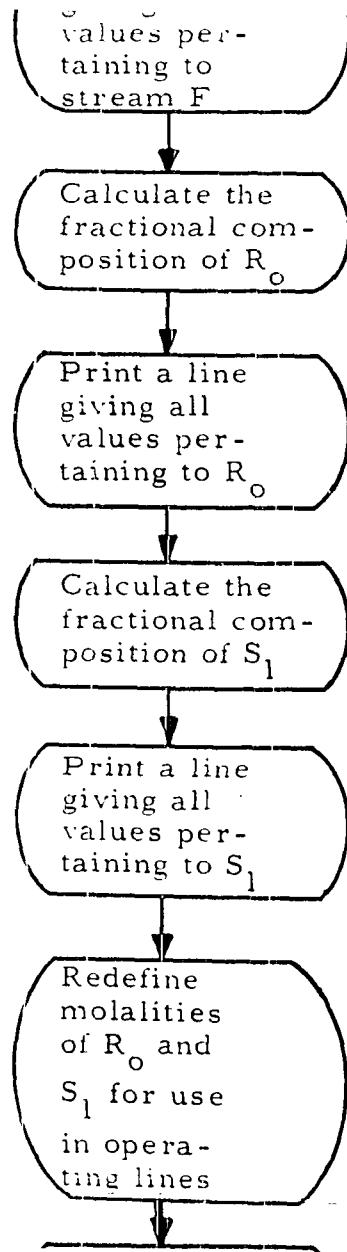
```

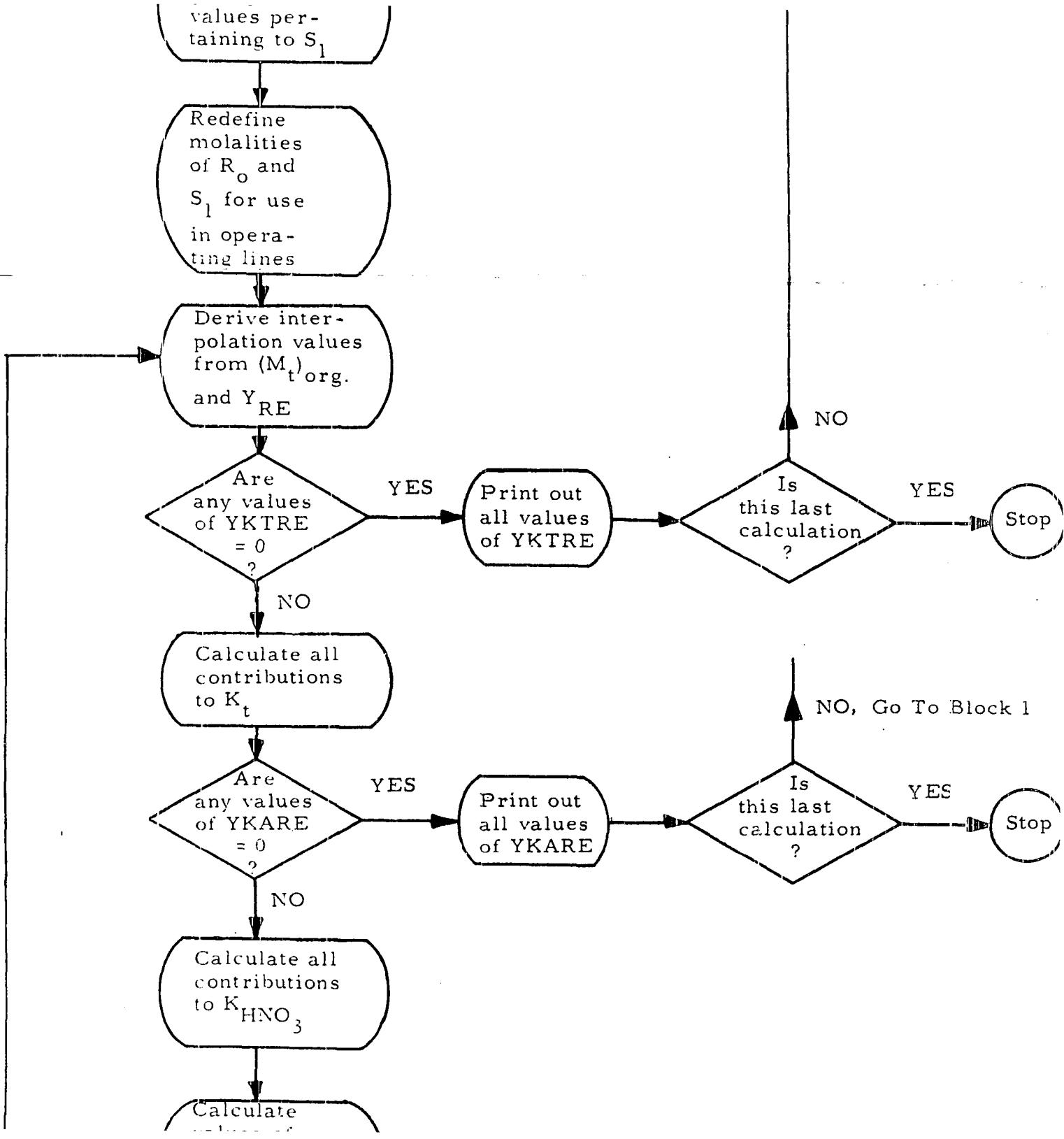
503 FORMAT (36HI INTERNAL FEED,EXTRACT SIDE START S=F5.2,3H F=F5.2,5H RBMS3 249
1EX=F5.2,5H RSC=F5.2,3H N=I3,3H M=I3) BMS3 250
504 FORMAT (120HJ TOTALMOLHNO3 MOL LA MOL PR MOL ND MOL SM MOL BMS3 251
1RE MOL ACFR LAFR PRFR NDFR SMFR REFR LA/RE PR/RE ND/RE SM/REBMS3 252
?) BMS3 253
505 FORMAT (4HJ F7F8.4,10F6.3) BMS3 254
506 FORMAT (4HJ F7F8.4,6F6.3) BMS3 255
507 FORMAT (4HJ F7F8.4) BMS3 256
508 FORMAT (4HJS 07F8.4,10F6.3) BMS3 257
509 FORMAT (4HJS 07F8.4,6F6.3) BMS3 258
510 FORMAT (4HJS 07F8.4) BMS3 259
511 FORMAT (4H R 17F8.4,10F6.3) BMS3 260
512 FORMAT (8HJXKTR=E016F6.3,F8.4,F6.3) BMS3 261
513 FORMAT (8HJXKARE=016F6.3,F8.4,F6.3) BMS3 262
514 FORMAT (6HJTMR=F8.4,6H TMAQ=F8.4,6H TOTK=F8.4) BMS3 263
515 FORMAT (18HJREMOR LESS THAN 04F10.4) BMS3 264
516 FORMAT (16HJSYD NOT EQUAL 17F10.4) BMS3 265
517 FORMAT (18HJSYREQ NOT EQUAL 17F10.4) BMS3 266
518 FORMAT (17HJFEED ENTRY POINT) BMS3 267
519 FORMAT (2HJSI2,7F8.4,10F6.3) BMS3 268
520 FORMAT (6HJTMAQ=F10.4,6H AMAQ=F10.4,8H XLAMAQ=F10.4,7H PRMAQ=F10.4BMS3 269
1,8H XNDMAQ=F10.4,7H SMMAQ=F10.4,7H REMAQ=F10.4) BMS3 270
521 FORMAT (16HJSXA NOT EQUAL 17F10.4) BMS3 271
522 FORMAT (18HJSXREA NOT EQUAL 17F10.4) BMS3 272
523 FORMAT (2H RI2,7F8.4,10F6.3) BMS3 273
524 FORMAT (6HJTMAQ=F10.4) BMS3 274
600 STOP 89 BMS3 275
END BMS3 276

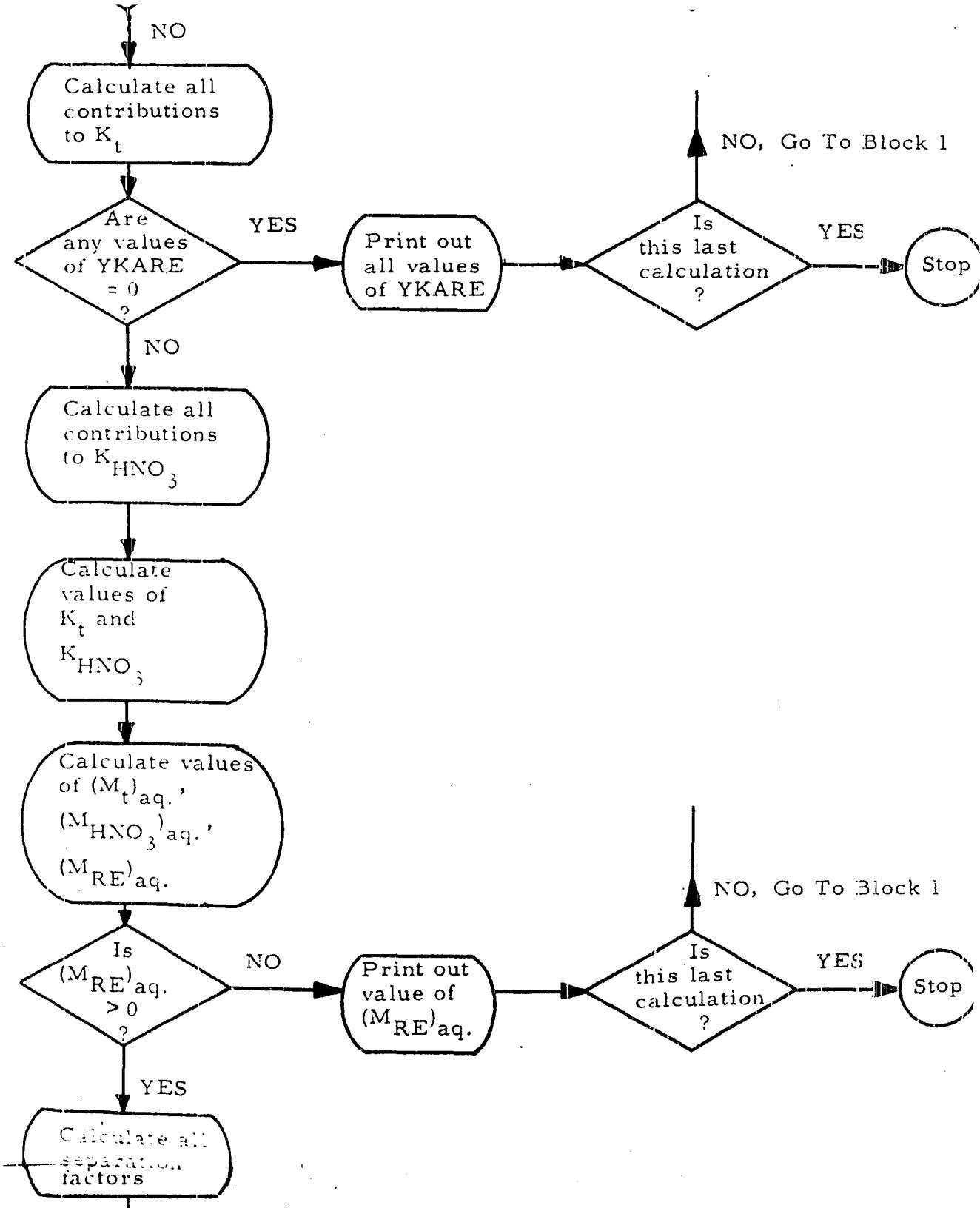
```

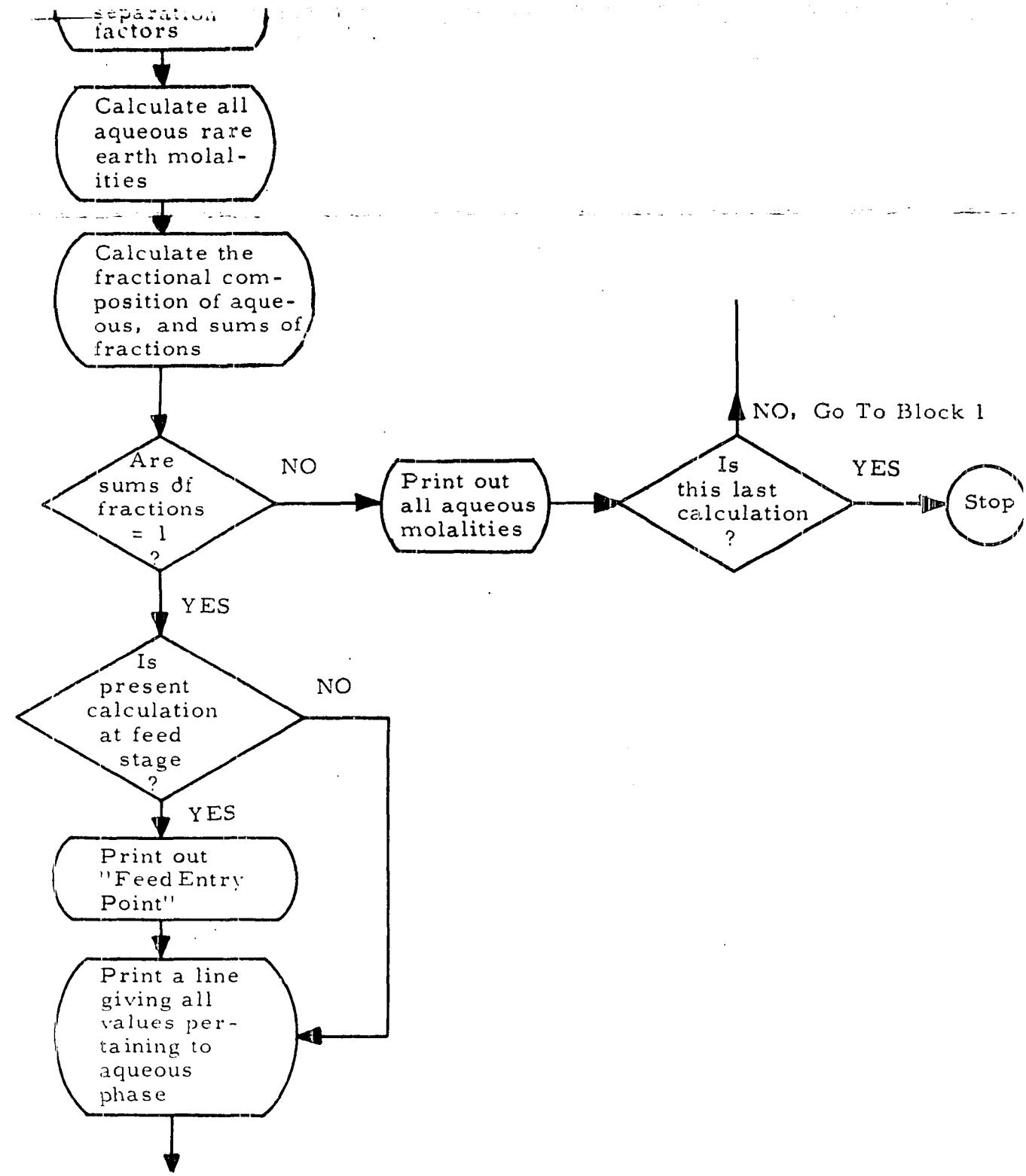
Figure 43. (Continued)

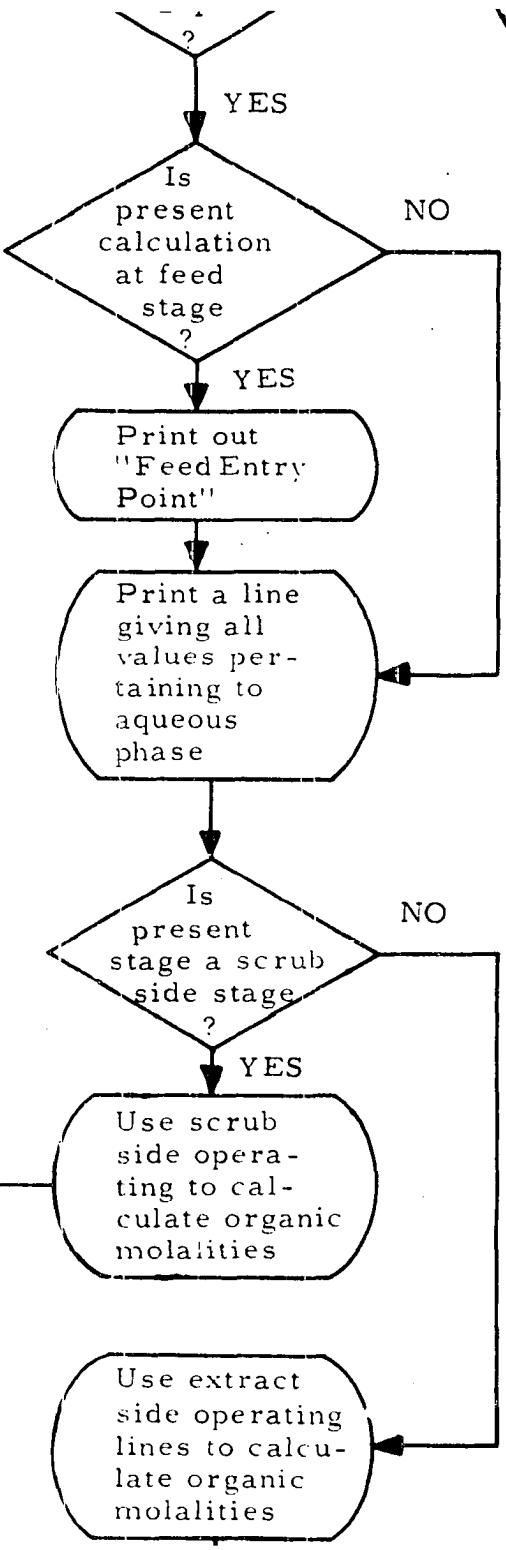
Block 1

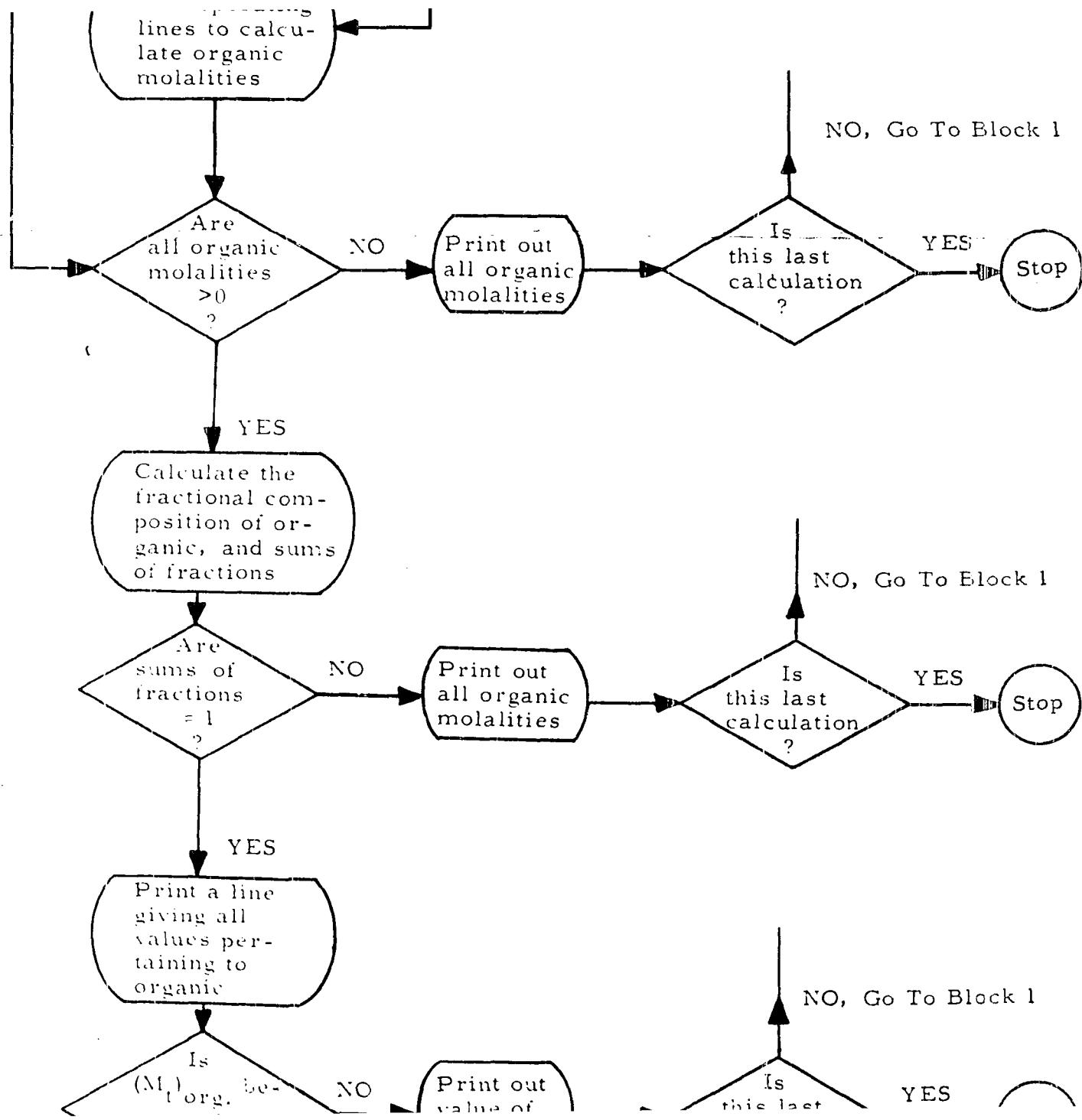












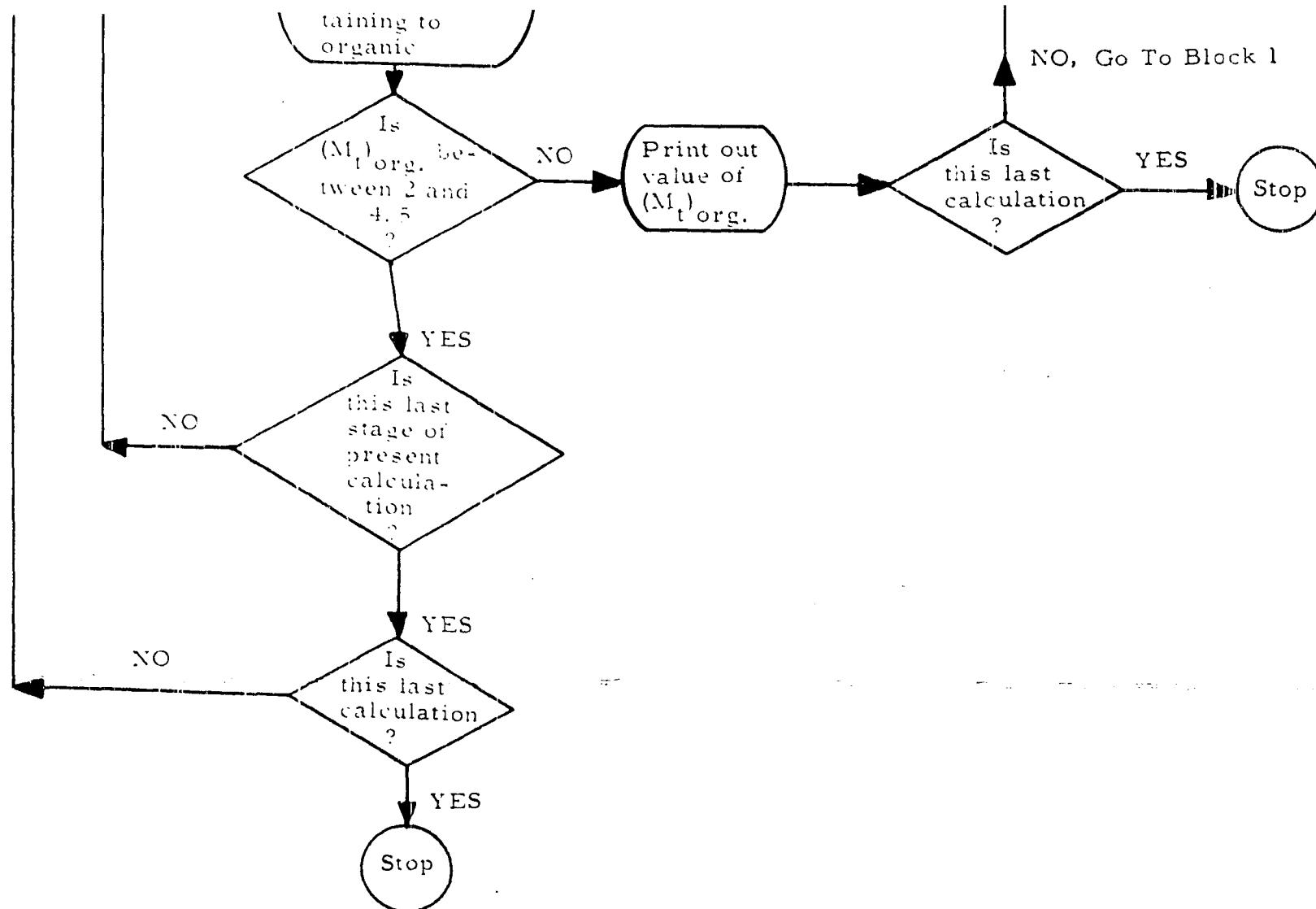


Figure 44. Flow diagram, Program IV

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DIMENSION YKTLA(21,6),YKTPR(21,6),YKTND(21,6),YKTSM(21,6),YKALA(21BMS4 001
1,6),YKAPR(21,6),YKAND(21,6),YKASM(21,6)                                BMS4 002
DO 5 I = 1,21                                                               BMS4 003
5   READ INPUT TAPE 5, 500, (YKTLA(I,J),J = 1,6)                           BMS4 004
DC 10 I = 1,21                                                               BMS4 005
10  READ INPUT TAPE 5, 500, (YKTPR(I,J),J = 1,6)                           BMS4 006
DO 15 I = 1,21                                                               BMS4 007
15  READ INPUT TAPE 5, 500, (YKTND(I,J),J = 1,6)                           BMS4 008
DO 20 I = 1,21                                                               BMS4 009
20  READ INPUT TAPE 5, 500, (YKTSM(I,J),J = 1,6)                           BMS4 010
DO 25 I = 1,21                                                               BMS4 011
25  READ INPUT TAPE 5, 500, (YKALA(I,J),J = 1,6)                           BMS4 012
DO 30 I = 1,21                                                               BMS4 013
30  READ INPUT TAPE 5, 500, (YKAPR(I,J),J = 1,6)                           BMS4 014
DO 35 I = 1,21                                                               BMS4 015
35  READ INPUT TAPE 5, 500, (YKAND(I,J),J = 1,6)                           BMS4 016
DO 40 I = 1,21                                                               BMS4 017
40  READ INPUT TAPE 5, 500, (YKASM(I,J),J = 1,6)                           BMS4 018
45  READ INPUT TAPE 5, 501, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ BMS4 019
READ INPUT TAPE 5, 501, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMUR,REMOR BMS4 020
READ INPUT TAPE 5, 502, FTM,FAM,FLAM,FPRM,FNDM,FSMM,FREM,S,F,REX,RBM4 021
ISC,N,M,K
PRINT 503, S,F,REX,RSC,N,M                                              BMS4 022
PRINT 504                                                               BMS4 023
IF (FTM) 600,65,50
50  XAF = FAM/FTM                                              BMS4 025
    XLAF = FLAM/FTM                                              BMS4 026
    XPRF = FPRM/FTM                                              BMS4 027
    XNDF = FNDM/FTM                                              BMS4 028
    XSMF = FSMM/FTM                                              BMS4 029
    XREF = FREM/FTM                                              BMS4 030
    IF (FREM) 600,60,55
55  XLASFF = FLAM/FREM                                              BMS4 031
    XPRSFF = FPRM/FREM                                              BMS4 032
    XNDSFF = FNDM/FREM                                              BMS4 033
    XSMSFF = FSMM/FREM                                              BMS4 034
    PRINT 505, FTM,FAM,FLAM,FPRM,FNDM,FSMM,FREM,XAF,XLAF,XPRF,XNDF,XSMBMS4 035
1F,XREF,XLASFF,XPRSFF,XNDSFF,XSMSFF
1F,XREF
GU TO 70
60  PRINT 506, FTM,FAM,FLAM,FPRM,FNDM,FSMM,FREM,XAF,XLAF,XPRF,XNDF,XSMBMS4 040
1F,XREF
GO TO 70
65  PRINT 507, FTM,FAM,FLAM,FPRM,FNDM,FSMM,FREM
70  IF (TMAQ) 600,90,75
75  XHNO3 = AMAQ/TMAQ
    XLA = XLAMAQ/TMAQ
    XPR = PRMAQ/TMAQ
    XND = XNDMAQ/TMAQ
    XSM = SMMAQ/TMAQ
    XRE = REMAQ/TMAQ
    IF (REMAQ) 600,95,80
80  XLASF = XLAMAQ/REMAQ
    XPRS = PRMAQ/REMAQ
    XNDSF = XNDMAQ/REMAQ
    XSMSF = SMMAQ/REMAQ
    PRINT 508, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHNO3,XLA,XPRBMS4 056
1,XND,XSM,XRE,XLASF,XPRS,XNDSF,XSMSF
    GO TO 95
85  PRINT 509, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHNO3,XLA,XPRBMS4 059
1,XND,XSM,XRE
    GO TO 95
90  PRINT 510, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ

```

Figure 45. Fortran statements, Program IV

```

95      YHNO3 = AMOR/TMOR          BMS4 063
      YLA = XLAMOR/TMOR          BMS4 064
      YPR = PRMOR/TMOR          BMS4 065
      YND = XNDMOR/TMOR          BMS4 066
      YSM = SMMOR/TMOR          BMS4 067
      YRE = REMOR/TMOR          BMS4 068
      YLASF = XLAMOR/REMO           BMS4 069
      YPRSF = PRMOR/REMO           BMS4 070
      YNDSF = XNDMOR/REMO           BMS4 071
      YSMSF = SMMOR/REMO           BMS4 072
      PRINT 511, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMO,YHNO3,YLA,YPRBMS4 073
1,YNDSF,YSMF
      ROTM = TMAQ          BMS4 074
      ROAM = AMAQ          BMS4 075
      ROLAM = XLAMAQ          BMS4 076
      ROPRM = PRMAQ          BMS4 077
      RCNDM = XNDMAQ          BMS4 078
      ROSMM = SMMAQ          BMS4 079
      SITM = TMOR          BMS4 080
      SIAM = AMOR          BMS4 081
      SILAM = XLAMOR          BMS4 082
      SIPRM = PRMOR          BMS4 083
      SINDM = XNDMOR          BMS4 084
      SISMM = SMMOR          BMS4 085
      DU 235 L = 1,N          BMS4 086
      YREI = (YRE*100.0 + 5.0)/5.0          BMS4 087
      IY = YREI          BMS4 088
      YI = IY          BMS4 089
      FRY = YREI - YI          BMS4 090
      Z = (TMOR*10.0 - 15.0)/5.0          BMS4 091
      KZ = Z          BMS4 092
      ZK = KZ          BMS4 093
      FRZ = Z - ZK          BMS4 094
      SKT = MIN1F(YKTLA(IY,KZ),YKTLA(IY+1,KZ),YKTLA(IY,KZ+1),YKTLA(IY+1,BMS4 096
      1KZ+1),YKTPR(IY,KZ),YKTPR(IY+1,KZ),YKTPR(IY,KZ+1),YKTPR(IY+1,KZ+1),BMS4 097
      2YKTND(IY,KZ),YKTND(IY+1,KZ),YKTND(IY,KZ+1),YKTND(IY+1,KZ+1),YKTSM(BMS4 098
      3IY,KZ),YKTSM(IY+1,KZ),YKTSM(IY,KZ+1),YKTSM(IY+1,.Z+1))          BMS4 099
      IF (SKT) 100,100,105          BMS4 100
100      PRINT 512, YKTLA(IY,KZ),YKTLA(IY+1,KZ),YKTLA(IY,KZ+1),YKTLA(IY+1,KBMS4 101
      1Z+1),YKTPR(IY,KZ),YKTPR(IY+1,KZ),YKTPR(IY+1,KZ),YKTPR(IY+1,KZ+1),YBMS4 102
      2KTND(IY,KZ),YKTND(IY+1,KZ),YKTND(IY,KZ+1),YKTND(IY+1,KZ+1),YKTSM(BMS4 103
      3Y,KZ),YKTSM(IY+1,KZ),YKTSM(IY,KZ+1),YKTSM(IY+1,KZ+1),TMOR,YRE          BMS4 104
      IF (K) 500,600,45          BMS4 105
105      YKTLA1 = YKTLA(IY,KZ)*(1.0 - FRY) + YKTLA(IY+1,KZ)*FRY          BMS4 106
      YKTLA2 = YKTLA(IY,KZ+1)*(1.0 - FRY) + YKTLA(IY+1,KZ+1)*FRY          BMS4 107
      YKTLA3 = YKTLA1*(1.0 - FRZ) + YKTLA2*FRZ          BMS4 108
      CKTLA = YKTLA3 * YLASF          BMS4 109
      YKTPR1 = YKTPR(IY,KZ)*(1.0 - FRY) + YKTPR(IY+1,KZ)*FRY          BMS4 110
      YKTPR2 = YKTPR(IY,KZ+1)*(1.0 - FRY) + YKTPR(IY+1,KZ+1)*FRY          BMS4 111
      YKTPR3 = YKTPR1*(1.0 - FRZ) + YKTPR2*FRZ          BMS4 112
      CKTPR = YKTPR3 * YPRSF          BMS4 113
      YKTND1 = YKTND(IY,KZ)*(1.0 - FRY) + YKTND(IY+1,KZ)*FRY          BMS4 114
      YKTND2 = YKTND(IY,KZ+1)*(1.0 - FRY) + YKTND(IY+1,KZ+1)*FRY          BMS4 115
      YKTND3 = YKTND1*(1.0 - FRZ) + YKTND2*FRZ          BMS4 116
      CKTND = YKTND3 * YNDSF          BMS4 117
      YKTSM1 = YKTSM(IY,KZ)*(1.0 - FRY) + YKTSM(IY+1,KZ)*FRY          BMS4 118
      YKTSM2 = YKTSM(IY,KZ+1)*(1.0 - FRY) + YKTSM(IY+1,KZ+1)*FRY          BMS4 119
      YKTSM3 = YKTSM1*(1.0 - FRZ) + YKTSM2*FRZ          BMS4 120
      CKTSM = YKTSM3 * YSMSF          BMS4 121
      SKA = MIN1F(YKALA(IY,KZ),YKALA(IY+1,KZ),YKALA(IY,KZ+1),YKALA(IY+1,BMS4 122
      1KZ+1),YKAPR(IY,KZ),YKAPR(IY+1,KZ),YKAPR(IY,KZ+1),YKAPR(IY+1,KZ+1),BMS4 123
      2YKAND(IY,KZ),YKAND(IY+1,KZ),YKAND(IY,KZ+1),YKAND(IY+1,KZ+1),YKASM(BMS4 124

```

Figure 45. (Continued)

```

3IY,KZ),YKASM(IY+1,KZ),YKASM(IY,KZ+1),YKASM(IY+1,KZ+1))          BMS4 125
110   IF (SKA) 110,110,115                                              BMS4 126
      PRINT 513, YKALA(IY,KZ),YKALA(IY+1,KZ),YKALA(IY,KZ+1),YKALA(IY+1,KBMS4 127
      1Z+1),YKAPR(IY,KZ),YKAPR(IY+1,KZ),YKAPR(IY,KZ+1),YKAPR(IY+1,KZ+1),YBMS4 128
      2KAND(IY,KZ),YKAND(IY+1,KZ),YKAND(IY,KZ+1),YKAND(IY+1,KZ+1),YKASM(1BMS4 129
      3Y,KZ),YKASM(IY+1,KZ),YKASM(IY,KZ+1),YKASM(IY+1,KZ+1),TMOR,YRE     BMS4 130
      IF (K) 600,600,45                                              BMS4 131
115   YKALAL1 = YKALA(IY,KZ)*(1.0 - FRY) + YKALA(IY+1,KZ)*FRY          BMS4 132
      YKALAL2 = YKALA(IY,KZ+1)*(1.0 - FRY) + YKALA(IY+1,KZ+1)*FRY        BMS4 133
      YKALAL3 = YKALAL1*(1.0 - FRZ) + YKALAL2*FRZ                      BMS4 134
      CKALA = YKALAL3 * YLASF                                         BMS4 135
      YKAPR1 = YKAPR(IY,KZ)*(1.0 - FRY) + YKAPR(IY+1,KZ)*FRY          BMS4 136
      YKAPR2 = YKAPR(IY,KZ+1)*(1.0 - FRY) + YKAPR(IY+1,KZ+1)*FRY        BMS4 137
      YKAPR3 = YKAPR1*(1.0 - FRZ) + YKAPR2*FRZ                      BMS4 138
      CKAPR = YKAPR3 * YPRSF                                         BMS4 139
      YKAND1 = YKAND(IY,KZ)*(1.0 - FRY) + YKAND(IY+1,KZ)*FRY          BMS4 140
      YKAND2 = YKAND(IY,KZ+1)*(1.0 - FRY) + YKAND(IY+1,KZ+1)*FRY        BMS4 141
      YKAND3 = YKAND1*(1.0 - FRZ) + YKAND2*FRZ                      BMS4 142
      CKAND = YKAND3 * YNDSF                                         BMS4 143
      YKASM1 = YKASM(IY,KZ)*(1.0 - FRY) + YKASM(IY+1,KZ)*FRY          BMS4 144
      YKASM2 = YKASM(IY,KZ+1)*(1.0 - FRY) + YKASM(IY+1,KZ+1)*FRY        BMS4 145
      YKASM3 = YKASM1*(1.0 - FRZ) + YKASM2*FRZ                      BMS4 146
      CKASM = YKASM3 * YSMSF                                         BMS4 147
      TOTK = CKTLA + CKTPR + CKTND + CKTSM                         BMS4 148
      HNO3K = CKALA + CKAPR + CKAND + CKASM                         BMS4 149
      TMAQ = TMOR/TOTK                                         BMS4 150
      AMAQ = AMOR/HNO3K                                         BMS4 151
      REMAQ = TMAQ - AMAQ                                         BMS4 152
      IF (REMAQ) 120,120,125                                         BMS4 153
120   PRINT 514, TMAQ,AMAQ,REMAQ,HNO3K                           BMS4 154
      IF (K) 600,600,45                                              BMS4 155
125   BLAPR = 0.8187 - 0.1106*TMOR                               BMS4 156
      BPRPR = 1.0                                              BMS4 157
      BNDPR = 1.0448 + 0.09874*TMOR                               BMS4 158
      BSMPR = -0.3795 + 0.9214*TMOR                               BMS4 159
      DEMS = XLAMOR/BLAPR + PRMOR/BPRPR + XNDMOR/BNDPR + SMMOR/BSMPR    BMS4 160
      XLAMAQ = XLAMOR*REMAQ/(BLAPR*DEMS)                         BMS4 161
      PRMAQ = PRMOR*REMAQ/(BPRPR*DEMS)                         BMS4 162
      XNDMAQ = XNDMOR*REMAQ/(BNDPR*DEMS)                         BMS4 163
      SMMAQ = SMMOR*REMAQ/(BSMPR*DEMS)                         BMS4 164
      XHNO3 = AMAQ/TMAQ                                         BMS4 165
      XLA = XLAMAQ/TMAQ                                         BMS4 166
      XPR = PRMAQ/TMAQ                                         BMS4 167
      XND = XNDMAQ/TMAQ                                         BMS4 168
      XSM = SMMAQ/TMAQ                                         BMS4 169
      XRE = REMAQ/TMAQ                                         BMS4 170
      XLASF = XLAMAQ/REMAQ                                         BMS4 171
      XPRSF = PRMAQ/REMAQ                                         BMS4 172
      XNDSF = XNDMAQ/REMAQ                                         BMS4 173
      XSMSF = SMMAQ/REMAQ                                         BMS4 174
      SXA = XHNO3 + XLA + XPR + XND + XSM                         BMS4 175
      IF (0.9999 - SXA) 130,140,135                               BMS4 176
130   IF (1.0001 - SXA) 135,140,140                               BMS4 177
135   PRINT 515, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ       BMS4 178
      IF (K) 600,600,45                                              BMS4 179
140   SXREA = XLASF + XPRSF + XNDSF + XSMSF                     BMS4 180
      IF (0.9999 - SXREA) 145,155,150                               BMS4 181
145   IF (1.0001 - SXREA) 150,155,155                               BMS4 182
150   PRINT 516, TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ       BMS4 183
      IF (K) 600,600,45                                              BMS4 184
155   IF (L - M) 165,160,165                                         BMS4 185
160   PRINT 517                                              BMS4 186

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Figure 45. (Continued)

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165 PRINT 518,L,TMAQ,AMAQ,XLAMAQ,PRMAQ,XNDMAQ,SMMAQ,REMAQ,XHNO3,XLA,XPBMS4 187
1R,XND,XSM,XRE,XLASF,XPRSF,XNDSF,XSMSF
170 IF (L - M) 170,175,175
    TMOR = ((TMAQ - ROTM)*RSC/S) + S1TM
    AMOR = ((AMAQ - ROAM)*RSC/S) + S1AM
    XLAMOR = ((XLAMAQ - ROLAM)*RSC/S) + S1LAM
    PRMOR = ((PRMAQ - ROPRM)*RSC/S) + S1PRM
    XNDMOR = ((XNDMAQ - ROND)*RSC/S) + S1NDM
    SMMOR = ((SMMAQ - ROSMM)*RSC/S) + S1SMM
    GO TO 180
175 TMOR = TMAQ*REX/S - FTM*F/S - ROTM*RSC/S + S1TM
    AMOR = AMAQ*REX/S - FAM*F/S - ROAM*RSC/S + S1AM
    XLAMOR = XLAMAQ*REX/S - FLAM*F/S - ROLAM*RSC/S + S1LAM
    PRMOR = PRMAQ*REX/S - FPRM*F/S - ROPRM*RSC/S + S1PRM
    XNDMOR = XNDMAQ*REX/S - FNDM*F/S - ROND*RSC/S + S1NDM
    SMMOR = SMMAQ*REX/S - FSMM*F/S - ROSMM*RSC/S + S1SMM
    REMOR = TMOR - AMOR
    SOM = MINIF(TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR)
    IF (SOM) 185,185,190
180 PRINT 519, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR
    IF (K) 600,600,45
185 YHNO3 = AMOR/TMOR
    YLA = XLAMOR/TMOR
    YPR = PRMOR/TMOR
    YND = XNDMOR/TMOR
    YSM = SMMOR/TMOR
    YRE = REMOR/TMOR
    YLASF = XLAMOR/REMOR
    YPRSF = PRMOR/REMOR
    YNDSF = XNDMOR/REMOR
    YSMSF = SMMOR/REMOR
    SYO = YHNO3 + YLA + YPR + YND + YSM
    IF (0.9999 - SYO) 195,205,200
190 IF (1.0001 - SYO) 200,205,205
200 PRINT 520, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR
    IF (K) 600,600,45
205 SYREO = YLASF + YPRSF + YNDSF + YSMSF
    IF (0.9999 - SYREO) 210,220,215
210 IF (1.0001 - SYREO) 215,220,220
215 PRINT 521, TMOR,AMOR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR
    IF (K) 600,600,45
220 LL = L + 1
    PRINT 522, LL,TMOR,AMUR,XLAMOR,PRMOR,XNDMOR,SMMOR,REMOR,YHNO3,YLA,BMS4 229
    1YPR,YND,YSM,YRE,YLASF,YPRSF,YNDSF,YSMSF
    IF (TMOR - 2.0) 225,230,230
225 PRINT 523, TMOR
    IF (K) 600,600,45
230 IF (TMOR - 4.5) 235,225,225
235 CONTINUE
    IF (K) 600,600,45
500 FORMAT (1F10.3)
501 FORMAT (7F10.4)
502 FORMAT (7F7.4,4F5.2,3I3)
503 FORMAT (14H1INTERNAL FEED,SCRUB SIDE START S=F5.2,3H F=F5.2,5H REXBMS4 240
1=F5.2,5H RSC=F5.2,3H N=I3,3H M=I3)
504 FORMAT (120HJ .TOTALMULHN03 MOL LA MOL PR MOL ND MOL SM MUL BMS4 242
1RE MOL ACFR LAFR PRFR NDFR SMFR REFR LA/RE PR/RE ND/RE SM/REBMS4 243
2)
505 FORMAT (4HJ F7F8.4,10F6.3) BMS4 244
506 FORMAT (4HJ F7F8.4,6F6.3) BMS4 245
507 FORMAT (4HJ F7F8.4) BMS4 246
508 FORMAT (4HJR 07F8.4,10F6.3) BMS4 247
      BMS4 248

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Figure 45. (Continued)

509	FORMAT (4HJR 07F8.4,6F6.3)	BMS4 249
510	FORMAT (4HJR 07F8.4)	BMS4 250
511	FORMAT (4H S 17F8.4,10F6.3)	BMS4 251
512	FORMAT (8HJYKTR=016F6.3,F8.4,F6.3)	BMS4 252
513	FORMAT (8HJYKARE=016F6.3,F8.4,F6.3)	BMS4 253
514	FORMAT (18HJREMAQ LESS THAN 04F10.4)	BMS4 254
515	FORMAT (16HJSXA NOT EQUAL 17F10.4)	BMS4 255
516	FORMAT (18HJSXREA NOT EQUAL 17F10.4)	BMS4 256
517	FORMAT (17HJFEED ENTRY POINT)	BMS4 257
518	FORMAT (2HJRI2,7F8.4,10F6.3)	BMS4 258
519	FORMAT (6HJTMOR=F10.4,6H AMOR=F10.4,8H XLAMOR=F10.4,7H PRMOR=F10.4BMS4 259 1,8H XNDMDR=F10.4,7H SMMOR=F10.4,7H REMOR=F10.4)	BMS4 260
520	FORMAT (16HJSYO NOT EQUAL 17F10.4)	BMS4 261
521	FORMAT (18HJSYREO NOT EQUAL 17F10.4)	BMS4 262
522	FORMAT (2H SI2,7F8.4,10F6.3)	BMS4 263
523	FORMAT (6HJTMOR=F10.4)	BMS4 264
600	STOP 89	BMS4 265
	END	BMS4 266

Figure 45. (Continued)