Groundwater management by a dual-pipe subirrigation system

DON KIRKHAM & ROBERT HORTON
*Agronomy Department, Iowa State University, Ames, Iowa 50011,
USA*

Abstract An analytical solution of Laplace's equation is
presented for appropriate boundary conditions associated with
the problem of dual-pipe subirrigation and drainage. The
solution describes stream function and hydraulic head within
the groundwater flow region and likewise the associated
water table shape. The solution is general and flexible.
Response of the groundwater system can be studied relative
to thickness of saturated zone, position of subirrigation and
drainage pipes, heads in the subirrigation and drainage
pipes, crop evapotranspiration, fraction of inflowing
subirrigation water that exits the drains, and the aquifer
hydraulic conductivity. Calculations are presented showing how
pipe spacing affects shape of water table. The general
solution can be used also to predict salt flow through the
system as well as water flow.

Gestion de la nappe phréatique par un système d'irrigation souterraine à deux tuyaux

Résumé Une solution analytique de l'équation de Laplace est
présentée pour les conditions aux limites qui sont associées aux
problèmes d'irrigation et de drainage souterrains à deux tuyaux.
La solution décrit la fonction courante et la charge
hydraulique dans la région d'écoulement souterrain et la forme
de la surface de la nappe. La solution est générale et souple.
On peut étudier la réponse du système des eaux souterraines
par rapport à l'épaisseur de la zone saturée, aux profondeurs
des tuyaux d'irrigation et drainage, aux charges des tuyaux
d'irrigation et drainage, à l'évapotranspiration des cultures,
à la proportion d'eaux entrantes qui ressort par les drains, et
à la conductivité hydraulique du milieu poreux. Les calculs
presentés montrent comment change la forme de la surface du
niveau d'eau quand l'intervalle entre les tuyaux est changé. On
peut utiliser la solution générale pour prévoir le mouvement
des sels vers le système aussi bien que l'écoulement d'eau.

INTRODUCTION

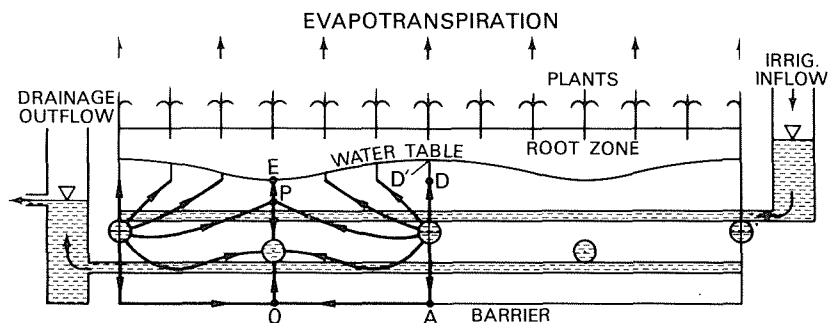Figures 1 and 2, after Kirkham & Horton (1986), illustrate the problem. It is

Fig. 1   *The dual-pipe subirrigation-drainage system.*

to determine, with the aid of the stream function, heads, velocities, and other hydrological quantities for a dual-pipe subirrigation-drainage system patented by Thornton (1985). Some analysis of the problem has already been done whereby, instead of dual cylindrical pipes, rectangular (slit) tubes were used (Kirkham & Horton, 1986). The work is now extended to cylindrical (circle) tubes.

Literature on subirrigation is scarce. One may see Ernst (1962), Brandyk & Wesseling (1987), Willardson (1987), Van Bakel (1988), Cooper & Fouss (1988) and Arthur (1988). For practice, this paper identifies system parameters and shows how they enter into groundwater management for clean water and crop growth; a main aim is to find formulas for the water table arch z with maximum height H as they depend on the tube spacing, evapotranspiration and the other parameters of Fig. 2.
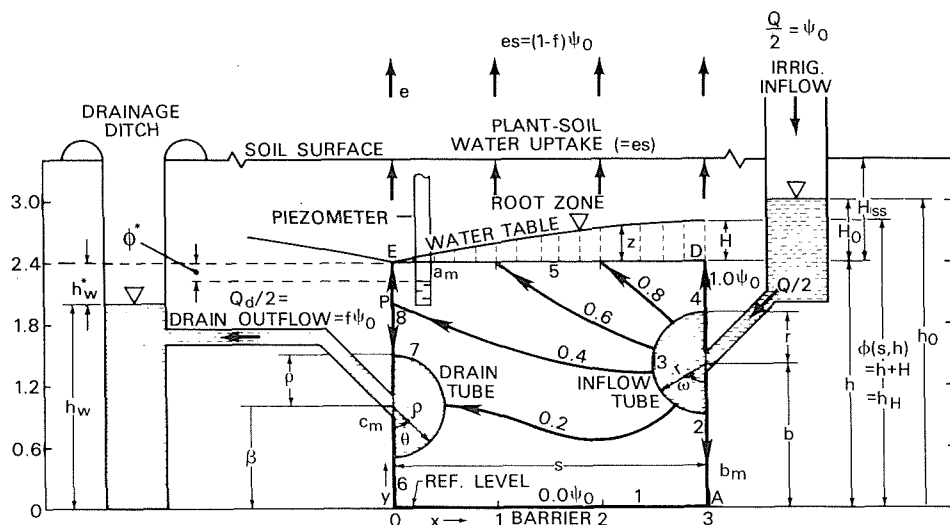

Fig. 2   *Geometry, hydraulics, and most symbols for a typical flow semisection of the dual-pipe subirrigation-drainage system.*

ANALYSIS

Streamlines serve as a graphical representation of the stream function and are indicated in Figs 1 and 2. Some of the streamlines go to drainage outflow (a sink) and some to evapotranspiration (another sink). All streamlines originate at the irrigation tube (the source). Figure 1 has symmetry and because of it we may consider flow only in the space *OAD'EPO*, less the space occupied by tubes and by a water table arch *DD'E*. In this work it is to be understood that the flow region (problem domain) extends a distance of 1 m, perpendicular to the plane of the figure.

Denoting the stream function by ψ [m^3 m^{-1} day^{-1}], streamlines are indicated graphically in Fig. 2 as a set of arrowed curves $0.0\psi_0$, $0.2\psi_0$, $0.4\psi_0$, ..., $1.0\psi_0$. Here ψ_0 is given by the relation:

$$\psi_0 = Q/2 \tag{1}$$

where $Q/2$ is the total flow (per 1 m length of inflow tube) moving to the left of the flow region.

In Fig. 2 the streamline $0.4\psi_0$ is particularly important because the quantity $0.4\psi_0$ is selected to be the amount of water ($0.4\psi_0 - 0.0\psi_0 = 0.4\psi_0$) that enters the drain tube sink. In general, let f denote the fraction of flow going to the drain tube. Therefore an amount $(1 - f)\psi_0$ goes into evapotranspiration. The $0.4\psi_0$ streamline in Fig. 2 is branched at point P. One branch, $0.4\psi_0$, moves up and another branch, also $0.4\psi_0$, moves down. The $0.4\psi_0$ and other streamlines in the figure are approximate. Let e [m^3 m^{-2} day^{-1}] represent the evapotranspiration coefficient and take s to be the distance between an adjacent pair of tube centres. The extremely important continuity equation is:

$$es = (1 - f)\psi_0 \tag{2}$$

where e is taken as a constant.

Laplace's equation is needed to solve the problem. It will be solved first for fictitious narrow rectangular vertical (slit) "tubes" assumed to replace the circle tubes in Fig. 2. These slits will then be shrunk to points which in turn become centres of drain tube circles, a procedure used in Kirkham (1958), and again in Kirkham & Horton (1986).

In Fig. 2, but now with the slits replacing the circle tubes, take the bottom of the left slit at (O,α), its centre at (O,β) and its top at (O,γ). The corresponding values for the right slits are (s,a), (s,b), and (s,c). The slits are considered to be so thin that flow passing into and out of the top and bottom of a slit may be neglected. These slits will be used to get a stream function solution of Laplace's equation. From ψ the velocity potentials and heads will then be derived.

The stream function $\psi = \psi(x,y)$ [m^3 m^{-2} day^{-1}] satisfies Laplace's equation as is proved by Kirkham & Powers (1984, p. 108). The equation for Cartesian x,y coordinates is:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \tag{3}$$

The equation is like that for Laplace's equation when $\Phi = k\phi$ or just ϕ is the potential function. A previous groundwater example is in Kirkham (1957). In order to solve this current problem, boundary condition equations have to be stated. To do this first choose an origin O of x,y coordinates. Choose O at the lower left-hand corner of the flow region similar to the "standard" as in Walker (1933, pp. 9 and 10). Boundaries in Fig. 2 are numbered 1, 2, ..., 8, and the boundary conditions may now be expressed similar to those in Kirkham & Horton (1986) and Kirkham & Powers (1984, pp. 99-101).

With Fig. 2 in view, consider the circle tubes shown to be replaced by fictitious slit tubes and now write down the boundary conditions for bottom, right, top, and left boundaries for ψ, for the slit tube system, as follows:

Bottom:

1. $\psi(x,O) = 0$ $0 \leqslant x \leqslant s$

Right b_m

2. $\psi(s,y) = 0$ $0 \leqslant y \leqslant a$

3. $\psi(s,y) = \dfrac{y - a}{c - a}\, \psi_0$ $a \leqslant y \leqslant c$

4. $\psi(s,y) = \psi_0$ $c \leqslant y < h$

Top a_m

5. $\psi(x,h) = f\psi_0 + (1 - f)\dfrac{x}{s}\psi_0$ $0 \leqslant x < s$

Left c_m

6. $\psi(O,y) = 0$ $0 \leqslant y \leqslant \alpha$

7. $\psi(O,y) = \dfrac{y - \alpha}{\gamma - \alpha}\, f\psi_0$ $\alpha \leqslant y \leqslant \gamma$

8. $\psi(O,y) = f\psi_0$ $\gamma \leqslant y \leqslant h$ (4)

To satisfy the boundary conditions, except possibly for points (O,h) and (s,h), which may, for now, be considered as removed from the flow medium, select (Kirkham & Powers, 1972, pp. 91–101) the function ψ as:

$$\frac{\psi}{\psi_0} = \Sigma\, a_m \sin(m\pi x/s)\frac{\sinh(m\pi y/s)}{\sinh(m\pi h/s)} + \Sigma\, b_m \sin(m\pi y/h)\frac{\sinh(m\pi x/h)}{\sinh(m\pi s/h)}$$

$$+ \Sigma\, c_m \sin(m\pi y/h)\frac{\sinh[m\pi(s - x)/h]}{\sinh(m\pi s / h)} \tag{5}$$

where the summing is for $m = 1, 2, ..., \infty$ (which is to be understood in the following for the infinite sums, unless otherwise stated).

The assembled a_m, b_m and c_m for circle tubes are found to be:

$$a_m = \frac{2}{\pi} \left[\frac{f}{m} - \frac{\cos m\pi}{m} \right] \tag{6}$$

$$b_m = \frac{2}{\pi} \left[\frac{1}{m} \cos \frac{m\pi b}{h} - \frac{1}{m} \cos m\pi \right] \tag{7}$$

$$c_m = \frac{2}{\pi} f \left[\frac{1}{m} \cos \frac{m\pi\beta}{h} - \frac{1}{m} \cos m\pi \right] \tag{8}$$

The constants a_m, b_m and c_m of equations (6)–(8) when substituted in equation (5) satisfy the boundary conditions of Fig. 3 for line sources and sinks and hence for the circle tubes for the normalized stream function ψ/ψ_0. To get needed heads and velocities for the flow region convert ψ of equation (5) to a velocity potential Φ defined by:

$$\Phi = k\phi \quad [\Phi(\mathrm{m}^2 \ \mathrm{day}^{-1}), \ k(\mathrm{m} \ \mathrm{day}^{-1}), \ \phi(\mathrm{m})] \tag{9}$$

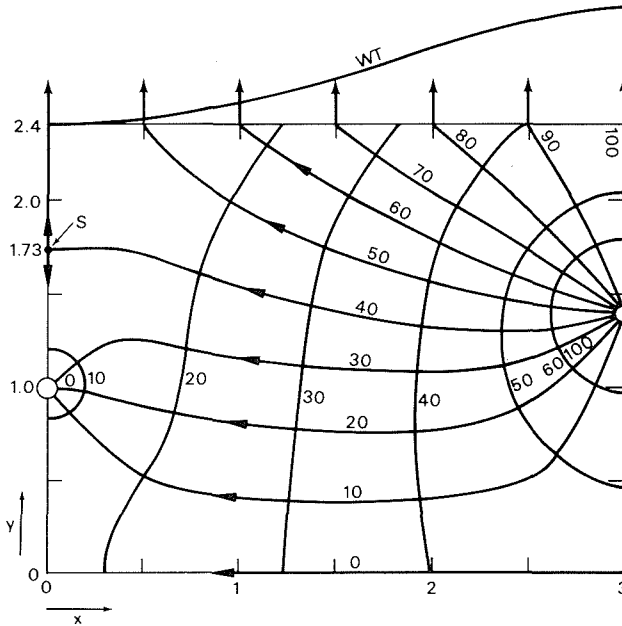*Fig. 3   A computed flow net with water table WT curve: for f = 0.4, s = 3 m, h = 2.4, β = 1.0, b = 1.4, ρ = 0.05, r = 0.0375, and e/k = 0.2; maximum height of the water table is 0.6286.*

where k is the hydraulic conductivity, assumed a constant of the flow medium, and ϕ is the hydraulic head. Choose the reference level of ϕ to be at $y = 0$.

To convert ψ to Φ Cauchy-Riemann (CR) relations will be used which are

$$\partial\Phi/\partial x = \partial\psi/\partial y; \qquad \partial\Phi/\partial y = -\partial\Psi/\partial x \tag{10}$$

These CR differential equations are derived and calculated in some integrated forms in Kirkham & Powers (1984, pp. 105, 106, 491–494).

To get Φ of equation (9) from ψ of equation (5) with the a_m, b_m, and c_m as in equations (6)–(8) and Φ subject to equation (10) proceed as follows.

Define a variable $V(x,y)$ the form of which we write down by selecting Φ elements in Kirkham & Powers (1984, Table 3-1) from ψ elements of equation (5). The form of $V(x,y)$ found for Φ (independently of whether we use the a_m, b_m and c_m for slits or for circles — because points for centres of circles are special cases of the slits) given by:

$$V(x,y) = \Sigma \, a_m(-1) \cos(m\pi x/s) \frac{\cosh(m\pi y/s)}{\sinh(m\pi h/s)} + \Sigma \, b_m \cos(m\pi y/h) \frac{\cosh(m\pi x/h)}{\sinh(m\pi s/h)}$$

$$+ \Sigma \, c_m(-1) \cos(m\pi y/h) \frac{\cosh[m\pi(s-x)/h]}{\sinh(m\pi s/h)} \tag{11}$$

and one can write a general integrated form of the Φ of equation (10) as:

$$\Phi(x,y) = \psi_0 \, V(x,y) + G \tag{12}$$

where G is a constant (Kirkham & Powers, 1984, Table 3-1).

To get G write equation (13) via equation (9) as:

$$k\phi(x,y) = \psi_0 \, V(x,y) + G \tag{13}$$

In Fig. 2 the water table by definition is at atmospheric pressure and this is true in particular for the point (O,h), where h is the height above the barrier which is also the reference level. Therefore $\phi(O,h) = h$ (see Kirkham & Powers, 1972, pp. 98–99) and equation (13) becomes:

$$kh = \psi_0 \, V(O,h) + G \tag{13a}$$

which gives:

$$G = kh - \psi_0 \, V(O,h) \tag{13b}$$

Put G as given by equation (13b) in (13), divide the result by k and find after rearranging:

$$\phi(x,y) - h = (\psi_0/k)[V(x,y) - V(O,h)] \tag{14}$$

Equation (14) is an important equation but does not involve the evapotranspiration coefficient e. Returning to equation (2) and dividing both sides by the hydraulic conductivity k equation (2) becomes:

$$\psi_0/k = (e/k)[s/(1 - f)] \tag{14a}$$

Putting the right-hand side of equation (14a) in equation (14) yields:

$$\phi(x,y) - h = \frac{e}{k} \frac{s}{1 - f} [V(x,y) - V(O,h)] = \phi^*(x,y) \tag{14b}$$

which is a key equation involving e. In equation (14b) a number of parameters do not appear. A more complete form of $V(x,y)$ is $V(x,y; f, s, h, \beta, b)$ (as is seen from equations (11) and (6)–(8)).

From equation (14b) the water table arch heights z and H can be found as they are shown in Fig. 2. Putting $y = h$ into equation (14b) z must be given by:

$$z = \phi(x,h) - h \tag{14b'}$$

Changing y to h in equation (14b) and then using equation (14b') gives:

$$z = \frac{e}{k} \frac{s}{1 - f} [V(x,h) - V(O,h)] \tag{14c}$$

For H, substitute $x = s$ in equation (14c) and change, in view of Fig. 2, z in equation (14c) to H and find H as:

$$H = \frac{e}{k} \frac{s}{1 - f} [V(s,h) - V(O,h)] \tag{14d}$$

Some more important heads h_w and h_0 seen in Fig. 2 can be obtained from equation (14b). The head h_w is for drainage ditch or sink (well head), and the head h_0 is for irrigation inflow or (originating) head. Therefore, for h_w, put $x = \rho$ and $y = \beta$ in equation (14b) and get:

$$h_w - h = \frac{e}{k} \frac{s}{1 - f} [V(\rho,\beta) - V(O,h)] = h_w^* \tag{14e}$$

and similarly put $x = s - r$ and $y = h$ in equation (14b) to yield:

$$h_0 - h = \frac{e}{k} \frac{s}{1 - f} \; [V(s - r, b) - V(O,h)] = h_0^* \tag{14f}$$

and the three heads ϕ^*, h_w^* and h_0^* are nothing more (see Fig. 2) than heads for the flow region points (x,y), (ρ,β) and $(s - r, b)$, each with respect to the level $y = h$.

The heights z and H of the water table arch (see Fig. 2) can be obtained in terms of starred values as:

$$\phi^*(x,h) = z \tag{14g}$$

which for the maximum value H of z becomes:

$$\phi^*(s,h) = H \tag{14h}$$

In all the preceding equations the value of $V(x,y)$ in terms of the original parameters is not available. In order to resolve this, $V(x,y)$ will be found in terms of the parameters f, s, h, β, b and of the variables x, y.

Putting a_m, b_m and c_m of equations (6)–(8) in equation (11) gives:

$$V(x,y) = \Sigma \frac{2}{\pi} \left[\frac{f}{m} - \frac{\cos m\pi}{m} \right] (-1) \cos(m\pi x/s) \frac{\cosh(m\pi y/s)}{\sinh(m\pi h/s)}$$

$$+ \Sigma \frac{2}{\pi} \left[\frac{1}{m} \cos(m\pi b/h) - \frac{\cos m\pi}{m} \right] \cos(m\pi y/h) \frac{\cosh(m\pi x/h)}{\sinh(m\pi s/h)}$$

$$+ \Sigma \frac{2}{\pi} f \left[\frac{1}{m} \cos(m\pi\beta/h) - \frac{\cos m\pi}{m} \right] (-1) \cos(m\pi y/h) \frac{\cosh[m\pi(s - x)/h]}{\sinh(m\pi s/h)}$$

$$\tag{15}$$

Following Kirkham (1958) to overcome convergence difficulties in equation (15):

$$V(x,y) = V_a + V_b + V_c + V_d + T_2 + T_4 + T_6 \tag{16}$$

where

$$V_a(x,y) = (1 - f) \left[\frac{h - y}{s} - \frac{\ln 2}{\pi} \right] \tag{16a}$$

$$V_b(x,y) = \frac{1}{\pi} f \ln \frac{\cosh[\pi(h - y)/s] - \cos(\pi x/s)}{\cosh(\pi x/h) + \cos(\pi y/h)} \qquad (x,y) \neq (O,h) \tag{16b}$$

$$V_c(x,y) = \frac{1}{\pi} \ln \frac{\cosh[\pi(s - x)/h] - \cos(\pi y/h)}{\cosh[\pi(h - y)/s + \cos(\pi x/s)} \qquad (x,y) \neq (s,h) \tag{16c}$$

$$V_d(x,y) = \frac{1}{2\pi} \ln \Big[\{[\cosh(\pi x/h) - \cos(\pi(\beta + y)/h)][\cosh(\pi x/h) - \cos(\pi(\beta - y)/h)]\}^f$$
$$\times \{[\cosh(\pi(s - x)/h) - \cos(\pi(b + y)/h)][\cosh(\pi(s - x)/h) - \cos(\pi(b - y)/h)]\}^{-1} \Big]$$

(16d)

$$T_2(x,y) = \Sigma \frac{2}{\pi} \frac{1}{m} (f - \cos m\pi)(-1) \cos(m\pi x/s) \exp(-m\pi h/s) \ .$$

$$\times \frac{\cosh[m\pi(h - y)/s]}{\sinh(m\pi h / s)}$$

(16e)

$$T_4(x,y) = \Sigma \frac{2}{\pi} \frac{1}{m} (\cos(m\pi b/h) - \cos m\pi) \cos(m\pi y/h) \exp(-m\pi s/h) \ .$$

$$\times \frac{\cosh[m\pi(s - x)/h]}{\sinh(m\pi s / h)}$$

(16f)

$$T_6(x,y) = \Sigma \frac{2}{\pi} f \frac{1}{m} (\cos(m\pi\beta/h) - \cos m\pi)(-1) \cos(m\pi y/h) \exp(-m\pi s/h) \ .$$

$$\times \frac{\cosh(m\pi x/h)}{\sinh(m\pi s/h)}$$

(16g)

Further simplifications (not included here) of equations (16)–(16g) have been made for special values of $V(x,y)$, namely, $(V(O,h)$, $V(s,h)$, $V(\rho, \beta)$, $V(s - r,b)$, and $V(s, h) - V(O, h)$ for $H)$.   In equation (16b) use $(O + \epsilon, y - \delta)$ for (x,y) and let ϵ and δ approach zero to find $V(O,h) = (f(/\pi)$ $\ln(h^2/s^2)$. Equation (16c) gives $V(s,h) = (1/\pi) \ln(s^2/h^2)$.

RESULTS AND DISCUSSION

To present results one needs to have "knowns" and "unkowns" (independent and dependent variables) in mind. They are (see Fig. 2 and its legend):

knowns: $x, y, e, k, f, h, \beta, b, \rho, r$

unknowns: $Q/2$, $\phi(x,y)$, $\phi^*(x,y)$, z, H, $h_w{}^*$, $h_0{}^*$; also a flow net; a set of graphs of z vs. s with k as parameter, and graphs of $h_w{}^*$, $h_0{}^*$ and H all vs. s.

(17)

Considering the "unknowns" in order:
From equations (1) and (2) the formula for $Q/2$ is:

$$Q/2 = (e/s)/(1 - f)$$

(18)

Note that $Q/2$ depends on e, s and f only (not on k, h, β, b, ρ or r).
From equation (14b) (with $V(x,y)$ as in equations (16)–(16g)):

$$\phi = h + [(e/k)s/(1 - f)][V(x,y) - V(O,h)] \tag{19}$$

By definition:

$$\phi^*(x,y) = \phi - h \tag{19a}$$

The height z and H of the water table arch can be obtained from equations (14c) and (14d), respectively.

There are two points about H of equation (14d):

(a) If in Fig. 2, the thickness of the land overburden is less than H there will be a "blowout" or seep above the inflow tube.

(b) Drain radii ρ and r of the "knowns" in equation (17) do not appear explicitly in H of equation (14d) (nor in equations (18), (19), (14c)). Nevertheless the problem tacitly assumes that the tube radii are non-zero; otherwise no water would flow through the flow region and there would be no problem. The larger the radii of the tubes the less head difference will be required to get a certain amount $Q/2$ [= $es/(1 - f)$ of equation (18)] of flow through to the flow region. The reason is that large tubes cut down convergence loss of head in the flow region.

Because H of equation (14d) is so important, equation (14d) will be expanded. With some simplification one finds, from equation (14d), that H, the water table arch height, is given by:

$$H = \frac{e}{k}\frac{s}{1 - f}(A + B + C + D + E) \qquad \begin{array}{l} 0 \leqslant f < 1 \\ 0 < \rho < h - \beta \\ 0 < r < h - b \end{array} \tag{20}$$

where

$$A = \frac{2}{\pi}(1 + f)\ln\frac{s/h}{\sinh(\pi s/2h)}$$

$$B = \frac{1}{\pi}\ln\left[\frac{\cosh(\pi s/h) + \cos(\pi\beta/h)}{2\cos^2(\pi\beta/2h)}\right]^f \frac{\cosh(\pi s/h) + \cos(\pi b/h)}{2\cos^2(\pi b/2h)}$$

$$C = \Sigma_{m=1,3,5,\ldots}\frac{4}{\pi}\frac{1}{m}(1 + f)[\coth(m\pi h/s) - 1]$$

$$D = \Sigma_{m=1,3,5,\ldots}\frac{4}{\pi}\frac{1}{m}[\cos^2(m\pi b/2h) + f\cos^2(m\pi\beta/2h)]\exp(-m\pi s/h)\ .$$
$$\times\ \tanh(m\pi s/2h)$$

$$E = \Sigma_{m=2,4,6,\ldots}\frac{4}{\pi}\frac{1}{m}[\sin^2(m\pi b/2h) + f\sin^2(m\pi\beta/2h)]\exp(-m\pi s/h)\ .$$
$$\times\ \tanh(m\pi s/2h)$$

In equation (20), f may not be 1 because of the factor $1/(1 - f)$; and neither β nor b may be equal to h because of $2\cos^2(\pi\beta/2h)$ and $2\cos^2(\pi b/2h)$, in B.

Coming now to h_w^* and h_0^* of the "unknowns". (Note that the stars mean heads with respect to level $y = h$ in Fig. 2.) The formula h_w^* is found from equation (14e), and the formula for h_0^* is found from equation (14f).

A flow net (Fig. 3) is the next "unknown". It consists of normalized equipotentials and streamlines (and a water table (WT) curve z *vs.* x of equation (14c)) derived from the ψ and Φ functions for the parameters:

$$e/k = 0.20 \qquad f = 0.40 \qquad s = 3 \text{ m} \qquad h = 2.4 \text{ m}$$

$$\beta = 1.0 \text{ m} \qquad b = 1.4 \text{ m} \qquad \rho = 0.05 \text{ m} \qquad r = 0.0375 \text{ m} \qquad (21)$$

In the net, streamlines and equipotentials intersect at 90°, as they should. The 40% ($f = 0.4$) streamline is important. It separates flow that goes into the drain tube from flow that goes into evapotranspiration. The 40 line is branched. It branches at a stagnation point labelled S at (0, 1.73). One branch goes to the top left corner of the flow region, and the other branch goes down the left side of the flow region to the drain tube. The 40% streamline approaches the left side of the flow region at the point S at 90°. In Fig. 3, the net is valid for $e/k = 0.20$, but it is also valid for other values of e/k because the net is normalized and the ratio e/k cancels in the normalization process. At the top of Fig. 3 a water table curve z *vs.* x of equation (14c) is shown for the parameters of equation (21). This Fig. 3 water table curve is flat at $x = 0$ and again at $x = s$ which must be as is seen from the symmetry of Fig. 1. The flatness property is useful in using a spline to connect plotted points, especially if only the two head points $\phi^*(O,h)$ and $\phi^*(s,h)$ of a water table are known.

The next "unkown" of equation (17) is a set of graphs of z *vs.* x with k as a parameter; Fig. 4 gives such a set for $k = 0.025, 0.050, 0.100$ and 1.00 m
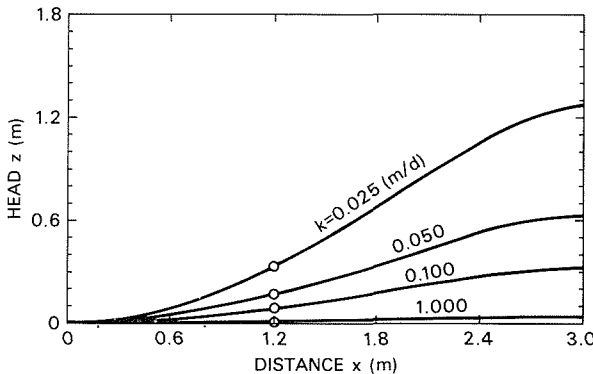

Fig. 4   Water table height z vs. x for several values of k when $e = 0.01$ m day^{-1}, $f = 0.40$, $s = 3$ m, $h = 2.4$, $\beta = 1.0$, $b = 1.4$, $\rho = 0.05$, $r = 0.375$ m; example values of z (at $x = 1.2$ m, circled points) are $z = 0.0084, 0.0840, 0.1680$ and 0.3360 m.

Table 1 *Points for z vs. x of Fig. 4, where $\phi^*(x,h)$ = z of equation (14c) and parameters are e = 0.01 m day^{-1}, f = 0.4, s = 3.0 m, h = 2.4 m, β = 1.0 m, b = 1.4 m and ρ and r are any values that are sufficiently small to approximate radii of circles; x and z are in metres and k in metres per day*

		x					
		0	0.6	1.2	1.8	2.4	3.0
k	(0.05/k)	z	z	z	z	z	z
0.025	2	0.0	0.0926	0.3360	0.6802	1.0578	1.2572
0.05	1	0.0	0.0463	0.1680	0.3401	0.5289	0.6286
0.1	0.50	0.0	0.0232	0.0840	0.1701	0.2645	0.3143
1.0	0.05	0.0	0.0023	0.0084	0.0170	0.0264	0.0314

day^{-1} where the other parameters (the same for each curve) are e = 0.01 m day^{-1}, f = 0.4, s = 3.0 m, h = 2.4 m, β = 1.0 m, b = 1.4 m (and ρ and r for streamlines can be any values, but for equipotentials one must specify ρ and r and choose them as ρ = 0.05 and r = 0.0375).

The curve for k = 0.05 of Fig. 4, because it is for e = 0.01 (and k = 0.05) is also for e/k = 0.01/0.05 = 0.2 and hence is the same as at the top of Fig. 3.

In Fig. 4 there are four values of k, viz. 0.025, 0.50, 0.10 and 1 m day^{-1}. Three of the curves were obtained from the curve k = 0.05 m day^{-1} which is the curve at the top of Fig. 3 as just explained. Values are presented in Table 1. If (e/k) is not 0.2 but is say (e_1/k_1), then multiply each z-value by $[(e_1/k_1)/0.2]$ to get a new z-set. As a sample calculation:

$$[(0.01/0.025)/(0.2)](0.1680) = 0.3360$$

which is the top singled out z in Fig. 4. In this calculation the number 0.1680 is taken from Table 1.

Coming next to the "graphs of h_w^*, h_0^*, and H all *vs.* s", of the "unkowns", for these graphs values of h_w^*, h_0^* and H are needed for a number of values of s. Choosing s = 2, 2.5, 3, 5, 10, 20 and 40 m and using as before f = 0.4, h = 2.4, β = 1.0, b = 1.4 and (h_w^* and h_0^*) ρ = 0.05 and r = 0.0375 (H does not need radii values, except that they be small as noted in the heading of Table 1). Figure 5 (including the case of f = 0) can be used to calculate values of h_w^*, h_0^*, and H for other values of e/k. Thus, it is a practical and useful representation of the mathematical solution of subirrigation and drainage.

FUTURE WORK

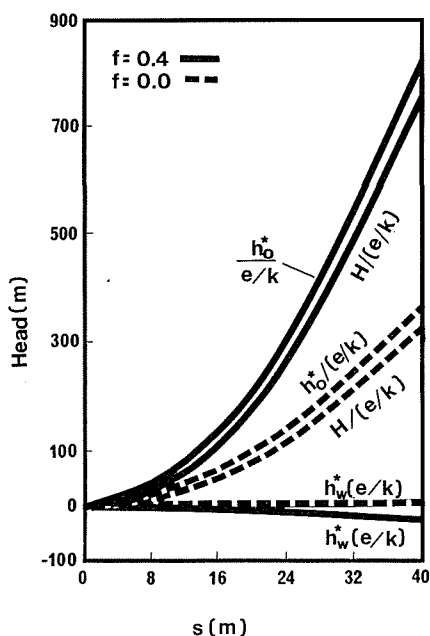This water flow solution is being investigated further for additional practical

Fig. 5   Normalized heads $h_w^*/(e/k)$, $H/(e/k)$. and $h_0^*/(e/k)$ vs. s for $f = 0.4$, and $f = 0$ and for $h = 2.4$, $\beta = 1$, $b = 1.4$, $\rho = 0.05$, $r = 0.0375$ m; multiply ordinates by the numerical value of e/k to get the heads in metres; for example to get the head for the real value of the normalized head $h_0^*/(e/k) = 800$ m when $s = 40$, $e = 0.01$ m day^{-1} and $k = 100$ m day^{-1} (coarse sand), get $h_0^* = 800 (0.01/100) = 0.08$ m, and for $f = 0$, $h_0^* = 340(0.01/100) = 0.034$ m.

relationships useful to groundwater management. The effects of size of irrigation and drainage pipes on the inflow heads and outflow heads will be analysed. The positioning of irrigation and drainage pipes, and the use of "negative" drainage (use of drain pipe for additional subirrigation) will be analysed for effects on water table shape, inflow and outflow heads, and water travel times within the flow region.

REFERENCES

Arthur, T. (1988)   Storing water for dry spells. *Agric. Consultant* **16**, fall.
Brandyk, T. & Wesseling, J. G. (1987)   Soil moisture flow in drainage-subirrigation systems. *J. Irrig. Drain. Div. ASCE* **113** (1), 86-97.
Cooper, J. L. & Fouss, J. L. (1988)   Rainfall probability system for water table control.

Agric. Wat. Manage. 15, 47-59.

Ernst, L. F. (1962) Grondwaterstromingen in de verzadigde zone en hun berekening bij aanwesigheid van horizontale evenwijdige open leidingen (in Dutch). *Verslag. Landbouwk. Onderz.* 67 (15). PUDOC, Wageningen.

Kirkham, D. (1957) Theory of land drainage. In: *Drainage of Agricultural Lands* (ed by J. N. Luthin), p.162. Agron. Ser. vol. 7, American Society of Agronomy, Madison, Wisconsin.

Kirkham, D. (1958) Seepage of steady rainfall through soil into drains. *Trans. AGU* 39 (5), 802-908.

Kirkham, D. & Powers, W. L. (1972) *Advanced Soil Physics* (first edn). Wiley-Interscience, New York.

Kirkham, D. & Powers, W. L. (1984) *Advanced Soil Physics* (revised edn). Robert E. Krieger Publishing Co., Malabar, Florida 32950.

Kirkham, D. & Horton, R. (1986) Subirrigation with drainage for drought prone areas. In: *Proc. Int. Conf. on Water Resources and Needs in Drought Prone Areas,* Part 2, 957-980. Sudan Engrs Soc., Khartoum, UNESCO.

Thornton, K. O. (1985) *"Irrigation System".* US Patent no. 4 538 377, US Dept of Commerce Patent and Trademark Office, Washington, DC 20231.

Van Bakel, P. J. T. (1988) Using drainage systems for supplementary irrigation. *Irrigation and drainage Systems* 2, 125-137.

Walker, M. (1933) *Conjugate Functions for Engineers,* 9-10. Oxford University Press, London.

Willardson, L. S. (1987) Methods of irrigation. In: *Reports Prepared for a Conference on Agricultural and Water Resources Development in the Arab World,* 34-41. Iowa State University, Ames, Iowa.