# ULTRASONIC WAVE PROPAGATION STUDIES IN ANISOTROPIC

## PLATES WITH BUILT-IN MATERIAL DEGRADATION

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#### INTRODUCTION

Anomalies of several kind are often found unavoidable during the manufacturing process of fiber reinforced composite parts. Several ultrasonic wave propagation and feature based signal analysis techniques can be found for characterizing these defects. Such methods are currently established on a problem to problem basis. Hence, the NDE methods are unable to keep up with the rapidly progressing materials technology and there is a need for a quick turnaround generalized method for anomaly modeling and experimental simulation to physically study the anomaly influence process on the ultrasonic signal. This paper addresses issues regarding the emerging new methods of ultrasonics oblique incidence techniques for the non-destructive evaluation of anisotropic plates. In this paper, efforts on theoretical modeling of imperfect composite structures, with in-built anomalies, have been attempted. Some of the common types of anomalies which can be considered, includes micro and macro porosity, fiber fraction changes, fiber mis-orientation, improper lay-up, interfacial weakness, improper curing, etc..

The effects of these degradations on the overall elastic properties and consequently the ultrasonic wave generation, propagation, reflection and transmission characteristics are expected to yield algorithms capable of quantitatively discriminating the presence of one or many of the anomalies present at the same time. A model for studying the reflection/transmission characteristics of a generalized anisotropic plate model (with 21 independent elastic constants) immersed in water is used in the analysis. Both on-axis as well as off-axis cases were considered.

### THEORETICAL MODELING

If the material properties of the fiber and matrix are known, the overall elastic properties of a composite can be predicted through the use of micro-mechanics theory. One such theory (known as the method of cells) is "based on the analysis of a repeating cell in a unidirectional composite in which the fibers are distributed regularly in the

matrix, forming a doubly periodic array." Aboudi's [1] basic model assumes perfect bonding between the fiber and matrix. Gardner [2] has expanded Aboudi's two phase model to incorporate a third phase, the interphase layer between the fiber and matrix. The three phase unit cell is depicted in Figure 1. In other words, imperfect bonding exists between the fiber and matrix. The character of this bonding can have a measurable impact on the overall properties of the composite. Other parameters which can be studied include the fiber fraction and the interphase thickness. A brief overview of the theory behind the method of cells is given below.

According to the method of cells, the average subcell stresses are expressed through Hooke's law as

$$\sigma^{(\alpha\beta\gamma)} = C^{(\alpha\beta\gamma)} \epsilon^{(\alpha\beta\gamma)}$$
(1)

The average stresses in the composite are calculated from the average subcell stresses in the equation

$$\overline{\sigma}_{ij} = \frac{1}{V} \sum_{\alpha,\beta,\gamma} V^{(\alpha\beta\gamma)} S_{ij}^{(\alpha\beta\gamma)}$$
(2)

where V is the total volume of the unit cell. Next, by imposing the continuity of traction and displacements, the controlling equations for the unit cell can be obtained with the average composite strains given by

$$\overline{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial}{\partial x_i} w_j + \frac{\partial}{\partial x_j} w_i \right)$$
(3)

The effective Young's modulus  $(E_{11}^{*})$  and Poisson's ratio  $(\nu_{12}^{*})$  are then determined from

$$E_{11}^* = \overline{\sigma}_{11} / \overline{\epsilon}_{11} \tag{4}$$

$$\mathbf{v}_{12}^* = -\overline{\mathbf{e}}_{22}/\overline{\mathbf{e}}_{11} \tag{5}$$

where  $\overline{\sigma}_{11}$  is given by Equation 2. Then the effective shear moduli  $G_{12}^{*}$  is calculated from

$$G_{12}^* = \overline{\sigma}_{12}/2\overline{\epsilon}_{12} \tag{6}$$

where  $\overline{\sigma}_{12}$  is given from Equation 2. In an analogous process, equations are obtained for  $G_{13}^{\bullet}$ . For  $G_{23}^{\bullet}$ , symmetry conditions must also be used to obtain the proper system of equations. Full details for deriving these equations are given in Gardner[2]. At small void contents, the impact of porosity on the overall composite properties is due to its degradation of the matrix elastic properties[3]. The porosity was modeled by changing the matrix properties only. Details on this porosity model as well as building material properties using complex lay-up sequence, etc., is found in Balasubramaniam[4].



Figure 1. The three phase(fiber-interface-matrix) model representation.

The ultrasonic velocities are obtained for a generally anisotropic material using Christoffel's Equation [5] which is given below.

$$k^2 l_{ik} C_{kl} l_{j} u_j = \rho \omega^2 u_i \tag{7}$$

The phase velocities of the three wave modes: longitudinal, fast shear( $0^{\circ}$ ) and slow shear( $90^{\circ}$ ), as a function of angle of propagation, can then be derived from the eigenvalues of the determinant. The eigenvectors represent the polarization of the particle displacements.

The reflection/transmission factor model is based on the work by Nayfeh et. al.[6] for monoclinic plates and by Chedid-helou and Hemann[7] for a triclinic case. Here, a generally anisotropic plate is loaded on both sides by fluid half spaces and is subjected to an infinite harmonic longitudinal plane wave of arbitrary frequency originating in the upper fluid half-space and propagating at an arbitrary angle of incidence. The analytical expression for the reflection factor(R) and transmission factor(T) can be obtained by numerically solving the following equation.

$$\begin{bmatrix} E_{1} & E_{2} & E_{3} & E_{4} & E_{5} & E_{6} & -\frac{\lambda_{f}}{SIN(\Theta)} & 0 \\ E_{1}H_{1} & E_{2}H_{2} & E_{3}H_{3} & E_{4}H_{4} & E_{5}H_{5} & E_{6}H_{6} & 0 & I \\ W_{1} & W_{2} & W_{3} & W_{4} & W_{5} & W_{6} & -COS(\Theta) & 0 \\ W_{1}H_{1} & W_{2}H_{2} & W_{3}H_{3} & W_{4}H_{4} & W_{5}H_{5} & W_{6}H_{6} & 0 & J \\ F_{1} & F_{2} & F_{3} & F_{4} & F_{5} & F_{6} & 0 & 0 \\ G_{1} & G_{2} & G_{3} & G_{4} & G_{5} & G_{6} & 0 & 0 \\ G_{1}H_{1} & G_{2}H_{2} & G_{3}H_{3} & G_{4}H_{4} & G_{5}H_{5} & G_{6}H_{6} & 0 & 0 \\ \end{bmatrix}$$

$$\begin{bmatrix} B_{1}^{(1)} \\ B_{1}^{(2)} \\ B_{1}^{(3)} \\ B_{1}^{(3)} \\ B_{1}^{(6)} \\ B_{$$

Here, the  $E_i$ ,  $H_i$ ,  $W_i$ ,  $F_i$ , and  $G_i$  are functions of the material elastic constants of the anisotropic plate and corresponding wave numbers. The  $B^{(i)}_{1}$  represent the amplitudes of the 6 possible wave modes within the plate.

## **RESULTS AND DISCUSSION**

The model was used to obtain the relationship between the three wave modes which can practically be generated on a thin specimen. All of the modes propagated through the thickness of the specimen. The results of the theoretical prediction of longitudinal wave velocity is shown in Figure 2a. The shear with polarization perpendicular to the fiber direction ( $90^{\circ}$ ) are shown in Figures 2b and 2c respectively. As can be seen from the Figure 2, the sensitivity to the change in fiber fraction as well as porosity exist, but is small and a practical application of this method is not a reliable option. Hence, a new technique using the delta velocity (the difference between the two shear wave velocities) is studied using Figure 2d. It can be observed that the sensitivity to fiber volume fraction is significant, while there is not the same level of sensitivity to porosity changes. This method could thus be used to discriminate between the fiber volume faction and the porosity imperfection. Also, the delta velocity method is practically more reliable since it decreases errors due to couplant thickness, material thickness changes, instrument errors, etc..

The next set of studies was conducted to study the influence of the fiber-matrix interface properties. Here, the interface material properties were considered to be weak and the thickness of the interface was varied from 0% to 0.1% thereby artificially degrading the fiber-matrix interface properties. The dependence of shear elastic constant  $G_{23}$  for a unidirectional composite on the fiber volume fraction and porosity for different interface thickness is illustrated in Figure 3(a-d). It can be seen from Figure 3a that as the porosity increases or when the fiber fraction volume decreases, the value of  $G_{23}$  decreases, which can be anticipated. Now as the interface begins to degrade, change in this behavior is noticed. As can be seen from Figure 3(b,c,and d), if the interface thickness becomes sufficiently large, the interface properties contributes significantly to the shear properties of the laminate and the influence of the fiber volume fraction is significantly reduced. In fact, the 0.1% thickness at the interface causes the higher fiber fraction(70%) to have a lower  $G_{23}$  value than the 50% FF case.

The next set of studies focused on the reflection/transmission model and its response to anomalies within the composite laminate. The graphite-epoxy unidirectional laminated plate was considered with material properties of graphite and epoxy resin obtained from the literature[6,7]. Here, the anomalies which were introduced included, porosity in the matrix, fiber volume fraction and modification of the interface elastic property[E<sub>11</sub>] while keeping the interface thickness constant. The results are presented in Figure 4(a,b,c). Here, the reflection factors are plotted as a function of frequency\*plate thickness product. The various minima represent regions of resonance. At resonance, there is a strong possibility for plate waves to exist within the structure. It can be observed that at low values of f\*d, there is poor sensitivity to changes in material properties, while at higher values of f\*d the reflection factor function as well as the positions of the minima display significantly different behavior with changes in material degradation. The dependency of the interface material properties is much more than the porosity and the fibre fraction. These figures are for a reflection factor with an angle of incidence 30 degrees at an azimuthal angle of 90 degrees(perpendicular to the fibers).



Figure 2. The porosity and fiber volume fraction analysis on bulk wave velocities. a) Longitudinal, b)  $0^{\circ}$  Shear, c)  $90^{\circ}$  Shear, d) Delta ( $0^{\circ} - 90^{\circ}$ ) Shear.



Figure 3. The effect of interface thickness on the shear elastic constant( $G_{23}$ ) a)0%, b)0.01%, c)0.05%, and d) 0.1%.

These minima values could be quantified to predict relationships between the ultrasonic response and anomaly presence.

# CONCLUSION

A model capable of studying the effects of several types of imperfection has been developed. Initial results demonstrate the versatile capabilities of the model which could be effectively used to develop new NDE technique. Future efforts on obtaining quantitative anomaly discrimination algorithms using multiple measurements needs to be initiated. More experimental studies over a larger sample of specimens will have to be performed in order to further verify the results from this model.

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Figure 4. The reflection factor (R) as a function of material degradation a) porosity change, b) fiber volume fraction change, c) interface elastic constant( $E_{11}$ ).

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